

# Supplemental file to “Testing for heterogeneous rates of discrete character evolution on phylogenies” – hidden-rate analysis of *Anolis* caudal vertebrae number

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```
## load packages
library(phytools)

## Loading required package: ape
## Loading required package: maps
library(foreach)
library(doParallel)

## Loading required package: iterators
## Loading required package: parallel

## load data & results from non-HRM analysis
load("../morph-analysis/morph-analysis.Rdata")

## build hidden-rates model
nn<-c(colnames(anolis_vert),paste(colnames(anolis_vert),"*",sep=""))
MODEL<-matrix(0,length(nn),length(nn),dimnames=list(nn,nn))

for(i in 1:(ncol(anolis_vert)-1)){
  MODEL[i,i+1]<-1
  MODEL[i+1,i]<-2
  MODEL[i+ncol(anolis_vert),i+1+ncol(anolis_vert)]<-3
  MODEL[i+1+ncol(anolis_vert),i+ncol(anolis_vert)]<-4
}
for(i in 1:ncol(anolis_vert)){
  MODEL[i,i+ncol(anolis_vert)]<-5
  MODEL[i+ncol(anolis_vert),i]<-6
}
anolis_vert.hidden<-cbind(anolis_vert,anolis_vert)
colnames(anolis_vert.hidden)<-nn

## fit hidden-rates model
## set number of optimization iterations per model
niter<-20
## open cluster for parallelized optimization
ncores<-min(niter,parallel::detectCores()-2)
mc<-makeCluster(ncores,type="PSOCK")
registerDoParallel(cl=mc)
fits<-foreach(i=1:niter)%dopar%{
```

```

phytools::fitMk(pruned.anolis_tree,
  anolis_vert.hidden,model=MODEL,
  rand_start=TRUE)
}
stopCluster(cl=mc)
logL<-sapply(fits,logLik)
print(logL)

```

```

## [1] -302.7425 -297.4101 -297.4689 -296.3706 -302.7425 -296.3706 -296.3706
## [8] -296.5782 -296.3706 -296.5664 -296.3706 -296.3706 -305.0230 -296.3706
## [15] -302.7425 -300.8120 -296.3706 -296.3706 -296.3706 -296.3706

```

```

fit.hrm<-fits[[which.max(logL)[1]]]
fit.hrm

```

```

## Object of class "fitMk".
##

```

```

## Fitted (or set) value of Q:

```

	34	35	36	37	38	39	40
## 34	-0.067088	0.050700	0.000000	0.000000	0.000000	0.000000	0.000000
## 35	0.010347	-0.077435	0.050700	0.000000	0.000000	0.000000	0.000000
## 36	0.000000	0.010347	-0.077435	0.050700	0.000000	0.000000	0.000000
## 37	0.000000	0.000000	0.010347	-0.077435	0.050700	0.000000	0.000000
## 38	0.000000	0.000000	0.000000	0.010347	-0.077435	0.050700	0.000000
## 39	0.000000	0.000000	0.000000	0.000000	0.010347	-0.077435	0.050700
## 40	0.000000	0.000000	0.000000	0.000000	0.000000	0.010347	-0.077435
## 41	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.010347
## 42	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 43	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 44	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 45	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 46	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 47	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 48	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 49	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 50	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 51	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 52	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 53	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 54	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 55	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 34*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 35*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 36*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 37*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 38*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 39*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 40*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 41*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 42*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 43*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 44*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 45*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 46*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 47*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

[illegible]

[illegible]

[illegible]

[illegible]

## 39*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 40*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 41*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 42*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 43*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 44*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 45*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 46*	0.466505	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
## 47*	-1.040769	0.466505	0.000000	0.000000	0.000000	0.000000	0.000000
## 48*	0.574264	-1.040769	0.466505	0.000000	0.000000	0.000000	0.000000
## 49*	0.000000	0.574264	-1.040769	0.466505	0.000000	0.000000	0.000000
## 50*	0.000000	0.000000	0.574264	-1.040769	0.466505	0.000000	0.000000
## 51*	0.000000	0.000000	0.000000	0.574264	-1.040769	0.466505	0.000000
## 52*	0.000000	0.000000	0.000000	0.000000	0.574264	-1.040769	0.466505
## 53*	0.000000	0.000000	0.000000	0.000000	0.000000	0.574264	-1.040769
## 54*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.574264
## 55*	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
##	54*	55*					
## 34	0.000000	0.000000					
## 35	0.000000	0.000000					
## 36	0.000000	0.000000					
## 37	0.000000	0.000000					
## 38	0.000000	0.000000					
## 39	0.000000	0.000000					
## 40	0.000000	0.000000					
## 41	0.000000	0.000000					
## 42	0.000000	0.000000					
## 43	0.000000	0.000000					
## 44	0.000000	0.000000					
## 45	0.000000	0.000000					
## 46	0.000000	0.000000					
## 47	0.000000	0.000000					
## 48	0.000000	0.000000					
## 49	0.000000	0.000000					
## 50	0.000000	0.000000					
## 51	0.000000	0.000000					
## 52	0.000000	0.000000					
## 53	0.000000	0.000000					
## 54	0.016388	0.000000					
## 55	0.000000	0.016388					
## 34*	0.000000	0.000000					
## 35*	0.000000	0.000000					
## 36*	0.000000	0.000000					
## 37*	0.000000	0.000000					
## 38*	0.000000	0.000000					
## 39*	0.000000	0.000000					
## 40*	0.000000	0.000000					
## 41*	0.000000	0.000000					
## 42*	0.000000	0.000000					
## 43*	0.000000	0.000000					
## 44*	0.000000	0.000000					
## 45*	0.000000	0.000000					
## 46*	0.000000	0.000000					
## 47*	0.000000	0.000000					

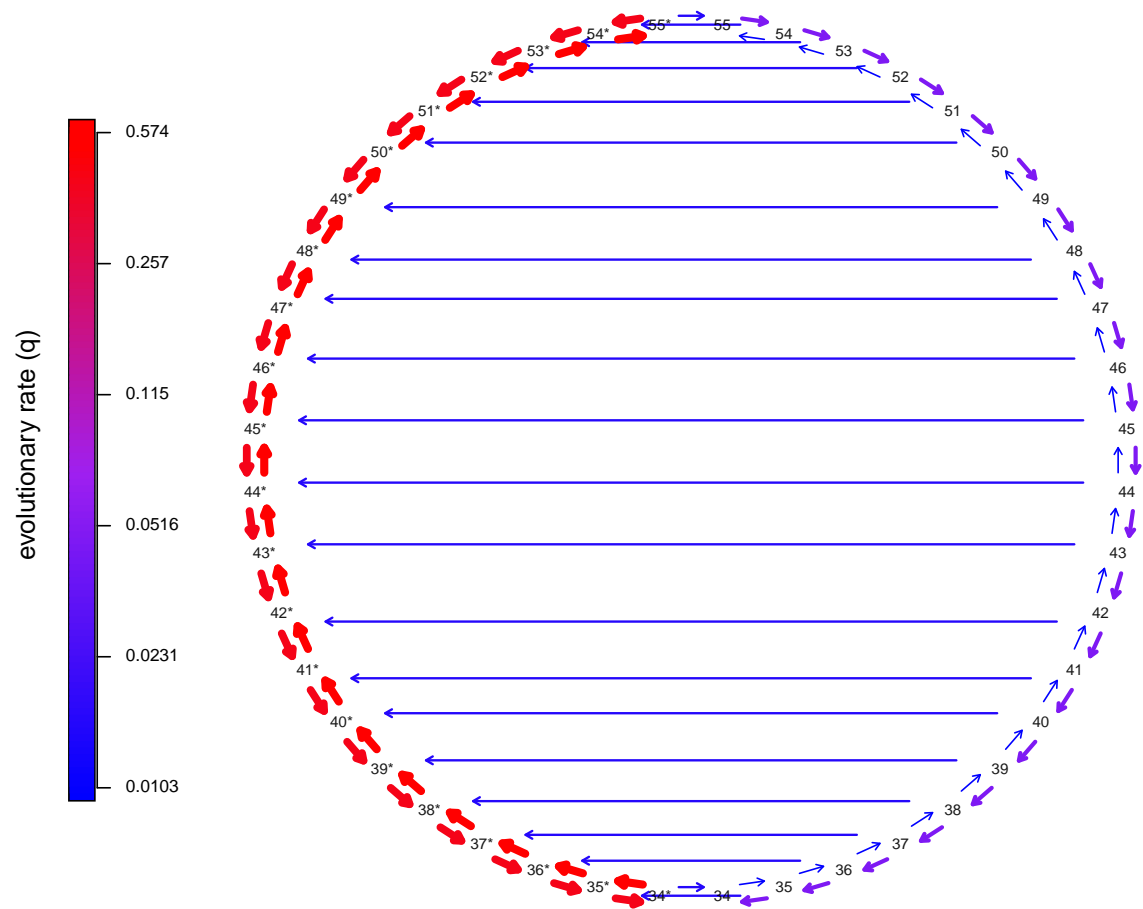
```

## 48* 0.000000 0.000000
## 49* 0.000000 0.000000
## 50* 0.000000 0.000000
## 51* 0.000000 0.000000
## 52* 0.000000 0.000000
## 53* 0.466505 0.000000
## 54* -1.040769 0.466505
## 55* 0.574264 -0.574264
##
## Fitted (or set) value of pi:
##      34      35      36      37      38      39      40      41
## 0.022727 0.022727 0.022727 0.022727 0.022727 0.022727 0.022727 0.022727
##      42      43      44      45      46      47      48      49
## 0.022727 0.022727 0.022727 0.022727 0.022727 0.022727 0.022727 0.022727
##      50      51      52      53      54      55      34*      35*
## 0.022727 0.022727 0.022727 0.022727 0.022727 0.022727 0.022727 0.022727
##      36*      37*      38*      39*      40*      41*      42*      43*
## 0.022727 0.022727 0.022727 0.022727 0.022727 0.022727 0.022727 0.022727
##      44*      45*      46*      47*      48*      49*      50*      51*
## 0.022727 0.022727 0.022727 0.022727 0.022727 0.022727 0.022727 0.022727
##      52*      53*      54*      55*
## 0.022727 0.022727 0.022727 0.022727
## due to treating the root prior as (a) flat.
##
## Log-likelihood: -296.37057
##
## Optimization method used was "nlminb"
##
## R thinks it has found the ML solution.
Q<-as.Qmatrix(fit.hrm)
nc<-ncol(Q)
ind<-c(1:(nc/2),(nc/2)+1:nc)
Q<-Q[ind,ind]
Q<-as.Qmatrix(Q)

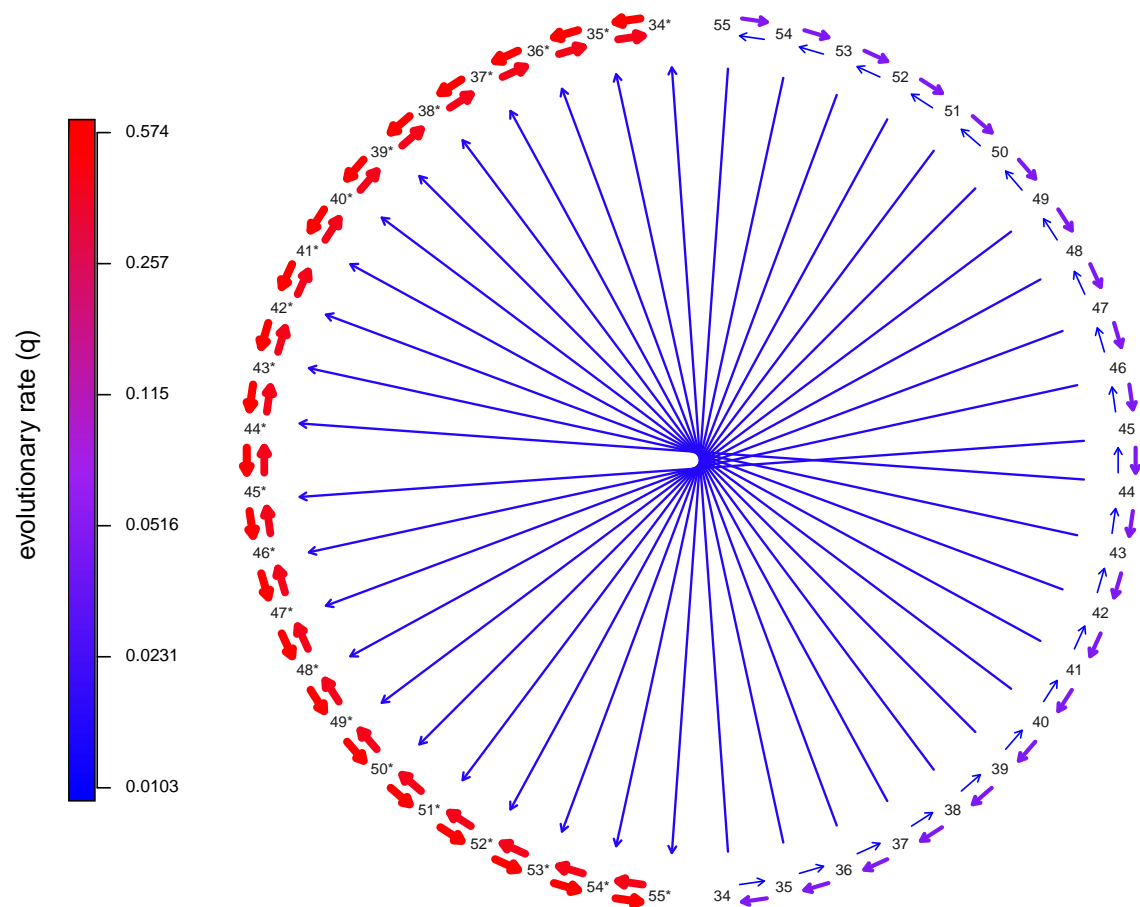
plot(Q,text=FALSE,cex.traits=0.6,width=TRUE,
     max.lwd=5,color=TRUE,xlim=c(-1.5,1),ylim=c(-1,1),
     show.zeros=FALSE)

```





```
## pdf
## 2
plot(fit.hrm,text=FALSE,cex.traits=0.6,width=TRUE,
     max.lwd=5,color=TRUE,xlim=c(-1.5,1),ylim=c(-1,1),
     show.zeros=FALSE)
```



```
## pdf
## 2

best_so_far<-fit.hrm
## compare models
anova(fit.single,fit.multi,fit.hrm)

##          log(L) d.f.      AIC      weight
## fit.single -305.0839   2 614.1679 0.008872567
## fit.multi  -304.3583   4 616.7166 0.002480877
## fit.hrm    -296.3706   6 604.7411 0.988646556

## save results
save(fit.hrm,file="morph-hrm-analysis.rda")
```