Light Curve Simulation and Shape Inversion for Human-Made Space Objects

Liam Robinson

Purdue Space Information Dynamics Group

11/13/2023

Outline

Motivation

- Space is becoming more crowded
- We have an imperfect understanding of the space environment
- Understanding the shape of debris objects is important for orbit propagation and active removal

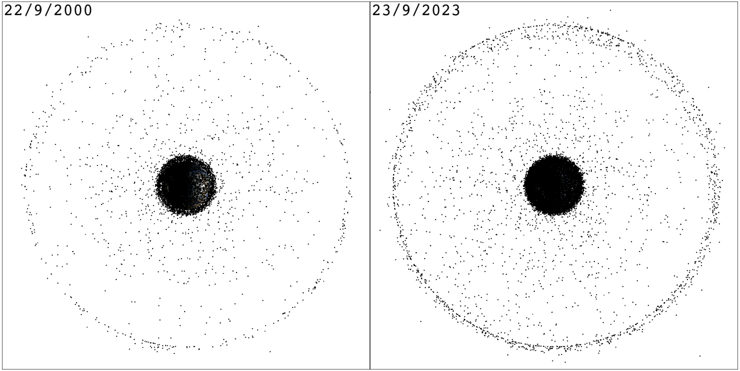


Figure: Publicly tracked objects in 2000 and 2023

Research Objectives

- Developing a high-fidelity light curve simulation framework to simulate the physics of real measurements
- Adapting direct shape inversion techniques for human-made space objects
- Designing a new method for inverting shape with noisy measurements and estimating nonconvex geometry

State of The Art: Simulation

- Light curve simulation literature often assumes:
  - Uniform material properties
  - Self-shadowing is negligible
  - Object geometry is simple [1]
  - Measurement noise is negligible or Gaussian [2]

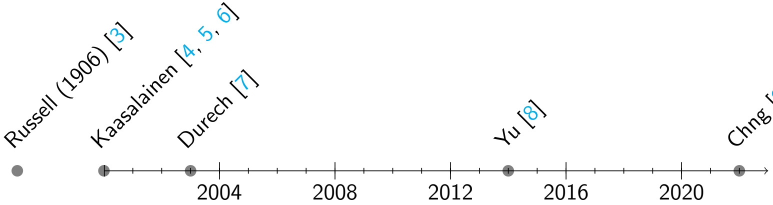
State of The Art: Shape Inversion

- Direct shape inversion
  - Require *a priori* knowledge of material properties and attitude
  - Highest fidelity shape estimates
- Filter-based methods
  - Attempt to estimate shape and attitude simultaneously
  - Limited to simple geometries
- Deep learning
  - Trains models to classify objects by their light curves
  - Unpredictable behavior outside the training set

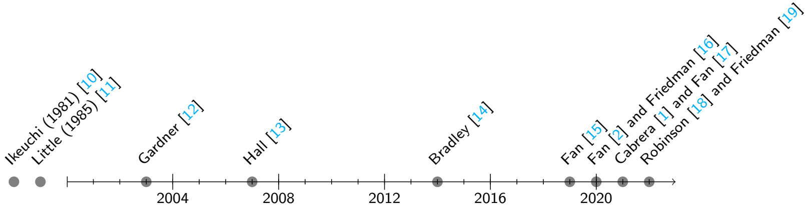
This work presents improvements to direct shape inversion

Direct Inversion Literature Timeline

Asteroid Inversion



Human-made Object Inversion



Outline

Attitude Motion

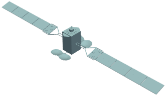
- Environmental torques do exist on orbit, but can be neglected on the scale of minutes to hours
- Torque-free attitude motion for rigid bodies is described by Euler's equations of motion for an inertia tensor J and body frame angular velocity ω:

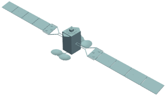
$$\dot{\omega} = J^{-1}\left[(J\omega) \times \omega\right]$$

- These EOMs are integrated with a quaterion $\mathbf{q} = [q_1, q_2, q_3, q_4]^T$ to track the orientation of the body in inertial space:

$$\begin{bmatrix} \dot{\epsilon}_1 \\ \dot{\epsilon}_2 \\ \dot{\epsilon}_3 \\ \dot{\epsilon}_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \epsilon_4 & -\epsilon_3 & \epsilon_2 & \epsilon_1 \\ \epsilon_3 & \epsilon_4 & -\epsilon_1 & \epsilon_2 \\ -\epsilon_2 & \epsilon_1 & \epsilon_4 & \epsilon_3 \\ -\epsilon_1 & -\epsilon_2 & -\epsilon_3 & \epsilon_4 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ 0 \end{bmatrix}.$$

Types of Torque-Free Motion





Bidirectional Reflectance Distribution Functions (BRDFs)

- BRDFs describe how light is reflected from a surface
  - $f_r(L, O)$ describes the fraction of incident light L reflected in the direction of an observer O
- Many formulations exist, but to be relevant to this work, they must [20]:
  - Conserve energy for all L:

$$\int_{O \in \mathbb{S}^2} f_r(L, O) \, d\mathbb{S}^2 \leq 1$$

  - Be nonnegative: $f_r(L, O) \geq 0$ for all L and O
  - Be reciprocal: $f_r(L, O) = f_r(O, L)$ for all L and O
- Popular relevant BRDFs include:
  - Lambertian [20]
  - Phong [21]
  - Cook-Torrance [22]

BRDFs in Action

Figure: BRDFs implemented for this work

Outline

Outline

Outline

The Earth

center

EGI Optimization

Equation

[1] David Vallverdú Cabrera, Jens Utzmann, and Roger Förstner.
Inversion of the shape of space debris from non-resolved optical measurements within spook.
In *Proceedings of the 22nd Advanced Maui Optical and Space Surveillance Technologies Conference*, pages 1–18, 2021.

[2] Siwei Fan.
The Light Curve Simulation and Its Inversion Problem for Human-Made Space Objects.
PhD thesis, Purdue University, August 2020.

[3] Henry Norris Russell.
On the light-variations of asteroids and satellites.
The Astrophysical Journal, 24(1):1–18, 1906.

[4] M. Kaasalainen and J. Torppa.
Optimization methods for asteroid lightcurve inversion: I. shape determination.
Icarus, 153(1):24–36, 2001.

[5] Mikko Kaasalainen, Johanna Torppa, and Karri Muinonen.
Optimization methods for asteroid lightcurve inversion: Ii. the complete inverse problem.
Icarus, 153(1):37–51, 2001.

[6] M Kaasalainen, J Torppa, and J Piironen.
Models of twenty asteroids from photometric data.
Icarus, 159(2):369–395, 2002.

[7] J Durech and M Kaasalainen.
Photometric signatures of highly nonconvex and binary asteroids.
Astronomy & Astrophysics, 404(2):709–714, 06 2003.

[8] LiangLiang Yu, Jianghui Ji, and Su Wang.
Shape, thermal and surface properties determination of a candidate spacecraft target asteroid (175706) 1996 fg3.
Monthly Notices of the Royal Astronomical Society, 439(4):3357–3370, 2014.

[9] Chee-Kheng Chng, Michele Sasdelli, and Tat-Jun Chin.
Globally optimal shape and spin pole determination with light-curve inversion.
Monthly Notices of the Royal Astronomical Society, 513(1):311–332, 01 2022.

[10] Katsushi Ikeuchi.
Recognition of 3-d objects using the extended gaussian image.
In *IJCAI*, pages 595–600, 1981.

[11] James J. Little.
Extended gaussian images, mixed volumes, shape reconstruction.
In *Proceedings of the First Annual Symposium on Computational Geometry*, SCG '85, pages 15–23, New York, NY, USA, 1985. Association for Computing Machinery.

[12] Richard Gardner and Peyman Milanfar.
Reconstruction of convex bodies from brightness functions.
Discrete and Computational Geometry, 29:279–303, 07 2003.

[13] Doyle Hall, Brandoch Calef, Keith Knox, Mark Bolden, and Paul Kervin.
Separating attitude and shape effects for non-resolved objects.
In *The 2007 AMOS Technical Conference Proceedings*, pages 464–475. Maui Economic Development Board, Inc. Kihei, Maui, HI, 2007.

[14] Ben K. Bradley and Penina Axelrad.
Lightcurve inversion for shape estimation of geo objects from space-based sensors.
In *Proceedings of the International Symposium on Space Flight Dynamics*, pages 1–20, 2014.

[15] Siwei Fan and Carolin Frueh.
A direct light curve inversion scheme in the presence of measurement noise.
The Journal of the Astronautical Sciences, 67, 08 2019.

[16] Alex M Friedman.
Observability Analysis for Space Situational Awareness.
PhD thesis, Purdue University, Apr 2020.

[17] S Fan and C Frueh.
Multi-hypothesis light curve inversion scheme for convex objects with minimal observations.
In *Proceedings of the 8th European Conference on Space Debris*, pages 1–7. ESA Space Debris Office, 2021.

[18] Liam Robinson and Carolin Frueh.

Light curve inversion for reliable shape reconstruction of human-made space objects.
In *Proceedings of the 32nd AIAA/AAS Astrodynamics Specialist Conference*, pages 1–19, 09 2022.

[19] Alex M. Friedman and Carolin Frueh.
Observability of light curve inversion for shape and feature determination exemplified by a case analysis.
Journal of the Astronautical Sciences, 69(2):537–569, April 2022.

[20] Rosana Montes Soldado and Carlos Ureña Almagro.
An overview of brdf models.
2012.

[21] Bui Tuong Phong.
Illumination for Computer Generated Pictures, pages 95–101.
Association for Computing Machinery, New York, NY, USA, 1998.

[22] R. L. Cook and K. E. Torrance.
A reflectance model for computer graphics.
ACM Trans. Graph., 1(1):7–24, jan 1982.