Light Curve Simulation and Shape Inversion for Human-Made Space Objects

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Outline

Motivation

- Space is becoming more crowded
- We have an imperfect understanding of the space environment
- Understanding the shape of debris objects is important for orbit propagation and active removal

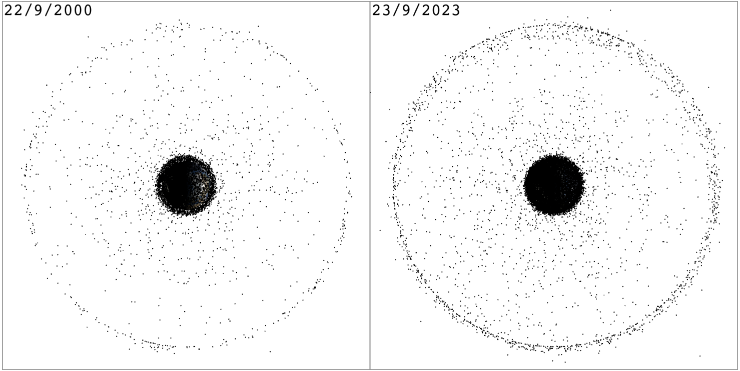


Figure: Publicly tracked objects in 2000 and 2023

Research Objectives

- Developing a high-fidelity light curve simulation framework to simulate the physics of real measurements
- Adapting direct shape inversion techniques for human-made space objects
- Designing a new method for inverting shape with noisy measurements and estimating nonconvex geometry

State of The Art: Simulation

- Light curve simulation literature often assumes:
  - Uniform material properties
  - Self-shadowing is negligible
  - Object geometry is simple [1]
  - Measurement noise is negligible or Gaussian [2]

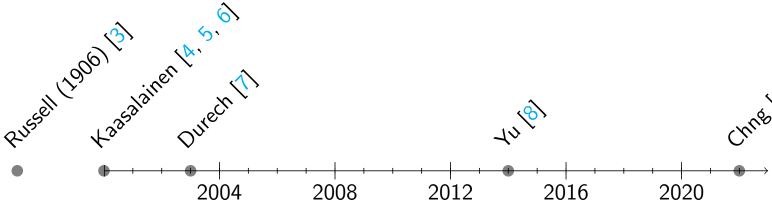
State of The Art: Shape Inversion

- Direct shape inversion
  - Require *a priori* knowledge of material properties and attitude
  - Highest fidelity shape estimates
- Filter-based methods
  - Attempt to estimate shape and attitude simultaneously
  - Limited to simple geometries
- Deep learning
  - Trains models to classify objects by their light curves
  - Unpredictable behavior outside the training set

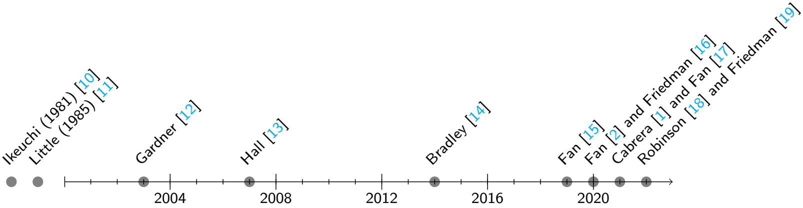
This work presents improvements to direct shape inversion

Direct Inversion Literature Timeline

Asteroid Inversion



Human-made Object Inversion



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Attitude Motion

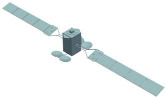
- Environmental torques do exist on orbit, but can be neglected on the scale of minutes to hours
- Torque-free attitude motion for rigid bodies is described by Euler's equations of motion for an inertia tensor J and body frame angular velocity ω:

$$\dot{\omega} = J^{-1}\left[(J\omega) \times \omega\right]$$

- These EOMs are integrated with a quaterion $\mathbf{q} = [q_1, q_2, q_3, q_4]^T$ to track the orientation of the body in inertial space:

$$\begin{bmatrix} \dot{\epsilon}_1 \\ \dot{\epsilon}_2 \\ \dot{\epsilon}_3 \\ \dot{\epsilon}_4 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \epsilon_4 & -\epsilon_3 & \epsilon_2 & \epsilon_1 \\ \epsilon_3 & \epsilon_4 & -\epsilon_1 & \epsilon_2 \\ -\epsilon_2 & \epsilon_1 & \epsilon_4 & \epsilon_3 \\ -\epsilon_1 & -\epsilon_2 & -\epsilon_3 & \epsilon_4 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ 0 \end{bmatrix}.$$

Types of Torque-Free Motion



Bidirectional Reflectance Distribution Functions (BRDFs)

- BRDFs describe how light is reflected from a surface
  - $f_r(L, O)$ describes the fraction of incident light L reflected in the direction of an observer O
- Many formulations exist, but to be relevant to this work, they must [20]:
  - Conserve energy for all L:

$$\int_{O \in \mathbb{S}^2} f_r(L, O)\, d\mathbb{S}^2 \leq 1$$

  - Be nonnegative: $f_r(L, O) \geq 0$ for all L and O
  - Be reciprocal: $f_r(L, O) = f_r(O, L)$ for all L and O
- Popular relevant BRDFs include:
  - Lambertian [20]
  - Phong [21]
  - Cook-Torrance [22]

BRDFs in Action

Figure: BRDFs implemented for this work

Outline

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The Earth

center

EGI Optimization

Equation

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