1 Astrodynamics and Space Applications Qualifying Exam Notes

gps

orb

quals

sad

stm

2 Satellite Navigation Past Problems

3 Orbit Mechanics Past Problems

3.1 Problem 1

3.1.1 Problem statement

Consider a hyperbolic flyby of a planet

1. Determine the values of the periapsis flyby radius r_p and hyperbolic excess speed v_∞ that yield the *maximum possible* magnitude of the equivalent Δv_{eq} for the spacecraft due to the flyby. Express your answer for r_p in terms of the planet radius r_s; include the constraint that $r_p \geq r_s$.
2. Determine this maximum Δv_{eq} in terms of v_s, the circular speed at the surface of the planet. Also determine the numerical values for the corresponding turn angle δ and the hyperbolic eccentricity e.

3.1.2 Solution

We know that the angle between the incoming and outgoing hyperbolic asymptotes is given by:

$$\begin{aligned}\delta &= 2\sin^{-1}\left(\frac{1}{e}\right)\\ &= 2\sin^{-1}\left(\frac{\Delta v_{eq}}{2v_\infty}\right)\end{aligned}$$

We'll use these two expressions for δ to solve for the conditions that maximize Δv. First, we have to find a way to introduce r_p into the equation. We know that the distance from the attracting focus to the center of the hyperbola is given by:

$$\begin{aligned}ae &= r_p + a\\ e &= \frac{r_p}{a} + 1\end{aligned}$$

We also know that by conservation of energy at $r = \infty$, we can express the semi-major axis a in terms of the hyperbolic excess speed v_∞:

$$\begin{aligned}\frac{v_\infty^2}{2} &= \frac{\mu}{2a}\\ a &= \frac{\mu}{v_\infty^2}\end{aligned}$$

Substituting this into the expression for e:

$$\begin{aligned}e &= \frac{r_p}{\mu/v_\infty^2} + 1\\ &= \frac{r_p v_\infty^2}{\mu} + 1\end{aligned}$$

Such that we can equate the two expressions for δ:

$$\begin{aligned}2\sin^{-1}\left(\frac{\Delta v_{eq}}{2v_\infty}\right) &= 2\sin^{-1}\left(\frac{1}{\frac{r_p v_\infty^2}{\mu} + 1}\right)\\ \frac{\Delta v_{eq}}{2v_\infty} &= \frac{1}{\frac{r_p v_\infty^2}{\mu} + 1}\\ \Delta v_{eq} &= \frac{2v_\infty}{\frac{r_p v_\infty^2}{\mu} + 1}\end{aligned}$$

This tells us that for any given v_∞, minimizing r_p will maximize Δv_{eq}. The minimum value of r_p is r_s, the radius of the planet. Solving for the v_∞ that corresponds to this minimum r_p requires taking the derivative of the Δv_{eq} expression with respect to v_∞ and looking for critical points:

$$
\begin{aligned}
\frac{\partial \Delta v_{eq}}{\partial v_\infty} &= \frac{2}{\frac{r_p v_\infty^2}{\mu}+1} - \frac{2v_\infty}{\left(\frac{r_p v_\infty^2}{\mu}+1\right)^2}\frac{2r_p v_\infty}{\mu} \\
&= \frac{\frac{2r_p v_\infty^2}{\mu}+2-\frac{4r_p v_\infty^2}{\mu}}{\left(\frac{r_p v_\infty^2}{\mu}+1\right)^2} \\
&= \frac{2-\frac{2r_p v_\infty^2}{\mu}}{\left(\frac{r_p v_\infty^2}{\mu}+1\right)^2}
\end{aligned}
$$

We notice that the denominator is always positive, so we can simply set the numerator to zero:

$$
\begin{aligned}
2-\frac{2r_p v_\infty^2}{\mu} &= 0 \\
\frac{2r_p v_\infty^2}{\mu} &= 2 \\
v_\infty^2 &= \frac{\mu}{r_p} \\
v_\infty &= \sqrt{\frac{\mu}{r_p}}
\end{aligned}
$$

This is an interesting result! We have found that the hyperbolic excess velocity for maximum Δv_{eq} is equal to the circular velocity at the surface of the planet. Solving for the corresponding value of Δv_{eq}:

$$
\begin{aligned}
\Delta v_{eq} &= \frac{2v_\infty}{\frac{r_p v_\infty^2}{\mu}+1} \\
&= \frac{2\sqrt{\frac{\mu}{r_p}}}{\frac{r_p\left(\sqrt{\frac{\mu}{r_p}}\right)^2}{\mu}+1} \\
&= \frac{2\sqrt{\frac{\mu}{r_p}}}{\frac{\mu}{\mu}+1} \\
&= \frac{2\sqrt{\frac{\mu}{r_p}}}{2} \\
&= \sqrt{\frac{\mu}{r_p}}
\end{aligned}
$$

We can also solve for the corresponding values of δ:

$$
\begin{aligned}
\delta &= 2\sin^{-1}\left(\frac{1}{e}\right) \\
&= 2\sin^{-1}\left(\frac{\Delta v_{eq}}{2v_\infty}\right) \\
&= 2\sin^{-1}\left(\frac{\sqrt{\frac{\mu}{r_p}}}{2\sqrt{\frac{\mu}{r_p}}}\right) \\
&= 2\sin^{-1}\left(\frac{1}{2}\right) \\
&= 60^\circ
\end{aligned}
$$

And e:

$$
\begin{aligned}
e &= \frac{r_p}{a} + 1 \\
&= \frac{r_p}{\frac{\mu}{v_\infty^2}} + 1 \\
&= \frac{r_p v_\infty^2}{\mu} + 1 \\
&= \frac{\mu}{\mu} + 1 \\
&= 2
\end{aligned}
$$

4 Attitude Dynamics Past Problems

5 Orbit Determination Past Problem