Quantum Teleportation

APM - IPSS - LJT

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The state of qubit 0 will be sent by utilising entanglement between qubit 1 and qubit 2. Alice (the sender) has qubit 0 and qubit 1, whilst Bob (the reciever) has qubit 2.
The state to teleport is: $|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle$.

1. Firstly, produce the β_{00} state using qubit 1 and qubit 2;

$$|\psi\rangle|00\rangle = (\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle)|00\rangle \rightarrow (\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle)(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle)$$

$$= \frac{1}{\sqrt{2}}\Big[\frac{\sqrt{3}}{2}|0\rangle(|00\rangle + |11\rangle) + \frac{1}{2}|1\rangle(|00\rangle + |11\rangle)\Big]$$

2. Next, qubit 0 and qubit 1 are entangled, with qubit 0 the control and qubit 1 as the target;

$$\frac{1}{\sqrt{2}}\Big[\frac{\sqrt{3}}{2}|0\rangle(|00\rangle + |11\rangle) + \frac{1}{2}|1\rangle(|00\rangle + |11\rangle)\Big] \rightarrow \frac{1}{\sqrt{2}}\Big[\frac{\sqrt{3}}{2}|0\rangle(|00\rangle + |11\rangle) + \frac{1}{2}|1\rangle(|10\rangle + |01\rangle)\Big]$$

3. Qubit 0 is then put through the gate, that would map $|1\rangle \rightarrow |-\rangle$;

$$\frac{1}{\sqrt{2}}\Big[\frac{\sqrt{3}}{2}|0\rangle(|00\rangle + |11\rangle) + \frac{1}{2}|1\rangle(|10\rangle + |01\rangle)\Big] \rightarrow \frac{1}{\sqrt{2}}\Big[\frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \frac{1}{2}\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)\Big]$$

$$= \frac{1}{2}\Big[\frac{\sqrt{3}}{2}(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) + \frac{1}{2}(|0\rangle - |1\rangle)(|10\rangle + |01\rangle)\Big]$$

$$= \frac{1}{2}\Big[\frac{\sqrt{3}}{2}(|000\rangle + |011\rangle + |100\rangle + |111\rangle) + \frac{1}{2}(|001\rangle + |010\rangle - |110\rangle - |101\rangle)\Big]$$

$$= \frac{1}{2}\Big[|00\rangle(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle) + |01\rangle(\frac{\sqrt{3}}{2}|1\rangle + \frac{1}{2}|0\rangle) + |10\rangle(\frac{\sqrt{3}}{2}|0\rangle - \frac{1}{2}|1\rangle) + |11\rangle(\frac{\sqrt{3}}{2}|1\rangle - \frac{1}{2}|0\rangle)\Big]$$

After Correction;

$$\Rightarrow \frac{1}{2}\Big[|00\rangle(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle) + |01\rangle\ \boxed{\text{X}}\ (\frac{\sqrt{3}}{2}|1\rangle + \frac{1}{2}|0\rangle) + |10\rangle\ \boxed{\text{Z}}\ (\frac{\sqrt{3}}{2}|0\rangle - \frac{1}{2}|1\rangle) + |11\rangle\ \boxed{\text{X}}\boxed{\text{Z}}\ (\frac{\sqrt{3}}{2}|1\rangle - \frac{1}{2}|0\rangle)\Big]$$

$$= \frac{1}{2}\Big[|00\rangle(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle) + |01\rangle(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle) + |10\rangle(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle) + |11\rangle(\frac{\sqrt{3}}{2}|0\rangle + \frac{1}{2}|1\rangle)\Big]$$

Alice (the sender) must measure her two qubits (qubit 0 and qubit 1), then send the results classically to Bob (the receiver). Alice's measurements allow the following deductions about Bob's state;

Alice's Measurements	Bob's State	Correction Required
$\lvert 00\rangle$	$\frac{\sqrt{3}}{2}\lvert 0\rangle + \frac{1}{2}\lvert 1\rangle$	-
$\lvert 01\rangle$	$\frac{\sqrt{3}}{2}\lvert 1\rangle + \frac{1}{2}\lvert 0\rangle$	$\boxed{\text{X}}$
$\lvert 10\rangle$	$\frac{\sqrt{3}}{2}\lvert 0\rangle - \frac{1}{2}\lvert 1\rangle$	$\boxed{\text{Z}}$
$\lvert 11\rangle$	$\frac{\sqrt{3}}{2}\lvert 1\rangle - \frac{1}{2}\lvert 0\rangle$	$\boxed{\text{X}}\ \boxed{\text{Z}}$