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Forecasting CPI: A Comparative Study on U.S. inflation rate with AR and ADL Models

Abstract
This essay forecasts the Consumer Price Index (CPI) over 12 months using two models: a CPI component model and an ADL(2) model with corporate bond spreads. Both models yield similar point forecasts, but the ADL(2) model offers narrower intervals, reflecting higher confidence.

Introduction

The Consumer Price Index (CPI) has been widely used as an economic indicator that measures the average change in prices over time for a fixed basket of goods and services typically consumed by households. The “basket” includes categories like food, housing, clothing, transportation, healthcare, education and recreation, which is based on household surveys to reflect typical consumption patterns. For a long time, CPI indicates the purchasing power of money and so the changes of living cost overtime. Government and central banks use CPI to guide monetary policy and adjust economic strategies, while markets often use it as a benchmark to adjust wages, pensions and other contracts to maintain purchasing power.

In this project, I attempted to forecast CPI for the upcoming 12 months. Federal Reserve Bank releases CPI monthly, and the baseline value of CPI is 100 corresponding to the period 1982–1984 (Figure 1). A couple of models have been considered to forecast future CPI. And selection criteria included CPI’s inner time-series property and predictive power of other indicators or indexes. Among these models, I determined a component model and second order autoregressive distributed lag model with corporate (denoted as corporate ADL(2)).

In addition to forecasting future CPI, this essay tried to reveal the correlation of unemployment rate, Gross Domestic Product (GDP), Treasury bills and bonds, and corporate bond yields with CPI. Traditional economic models suggest that lower unemployment, which is associated with rising wages, leads to increased inflationary pressures (Seydl & Gao, 2024). Similar to the CPI, the GDP tracks changes in prices for consumer goods and services but also includes price changes for goods and services bought by businesses, governments, and foreign buyers (Church, 2016). changes in corporate bond corresponds to movements in economic conditions, which are closely linked to inflation trends and, consequently, to CPI movements (Linder, 2020). Treasury yield curve has empirical association with U.S recessions, which directly related to CPI (Benzoni et al., 2018).

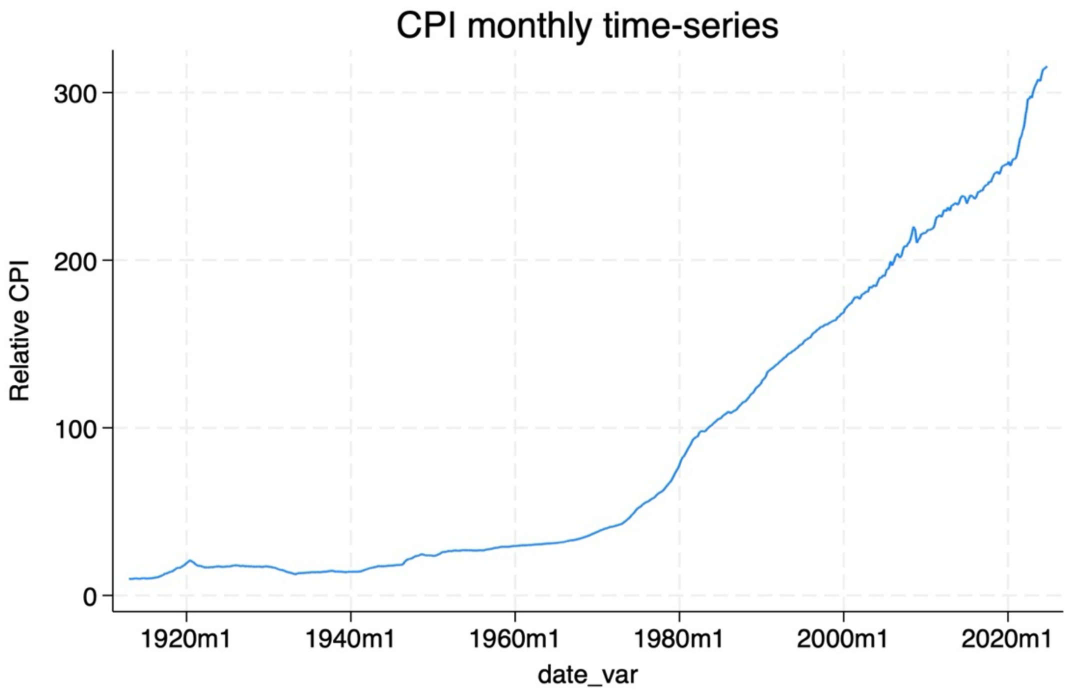

Figure 1. A monthly time series plot of CPI from 1913. The base value in this data set is 100, corresponding to the period 1982–1984.

Model selection for component model

The CPI component model was composed of trend, seasonal component, cyclical component and mean shift as seen in (1). Trend component was represented by a function of time t, where t was a monthly time series, as seen in (2). Seasonal component included season dummies as seen in (3). Cyclical component was represented by an Autoregressive model of CPI, as seen in (4). Besides these components, CPI component model included a dummy variable d_t which indicated the break date of the time series. The break date set as 1983, since monetary policy changed in the 1980s.

$$CPI_t = T_t + S_t + C_t + \beta d_t + \varepsilon_t \tag{1}$$

$$T_t = \mu_1 + \mu_2 t \tag{2}$$

$$S_t = \sum_{n=1}^{11} \gamma_n D_{n,t} \tag{3}$$

$$C_t = \sum_{n=1}^{p} \beta_n CPI_{t-n} \tag{4}$$

To determine the autocorrelation of cyclical component, I employed an autocorrelation analysis. According to Frisch-Waugh-Lovell (FWL) theorem, I first removed the trend, subtracted mean, and adjusted seasonal effects. After de-trend, de-mean, and seasonal adjusted, the autocorrelation analysis purely represented the cyclical autocorrelation. According to the Figure 2, CPI is highly persistent. Therefore, I decided to forecast the growth rate of CPI (dCPI), which is always less persistent. Repeated the same autocorrelation analysis on growth rate, the growth rate was shown with a less persistent autocorrelation (Figure 3).

Autocorrelation figure of dCPI showed non-zero autocorrelation of 19 lags. This suggested autoregressive models (AR) outweighed the moving-average models (MA). As a rule of thumb, I tested only one year worth of lags, meaning only up to 12 lags were considered. To determine the most appropriate lags included in AR models, AIC and BIC were introduced. Both AIC and BIC balanced the least sum of square error (SSR) with the penalty of number of parameters in the models. Besides these two in-sample model selection processes, each model's predictive least squares (PLS) were also tested. In Table 1, AR(4) had the smallest AIC, BIC, and PLS, indicating it was the best fitted model.

Table 1. AIC, BIC, and PLS Test results of 12 AR models.

Model	AIC	BIC	PLS
AR1	-10,082.77	-10072.37	0.0000347
AR2	-10,118.97	-10103.37	0.0000343
AR3	-10,127.82	-10107.02	0.0000344
AR4	**-10,166.09**	**-10140.1**	**0.0000333**
AR5	-10,161.79	-10130.61	0.0000337
AR6	-10,153.75	-10117.37	0.0000343
AR7	-10,148.39	-10106.82	0.0000356
AR8	-10,145.73	-10098.97	0.0000366
AR9	-10,137.36	-10085.42	0.0000459
AR10	-10,130.24	-10073.11	0.0000425
AR11	-10,146.24	-10083.93	0.0000392
AR12	-10,138.04	-10070.54	0.000041

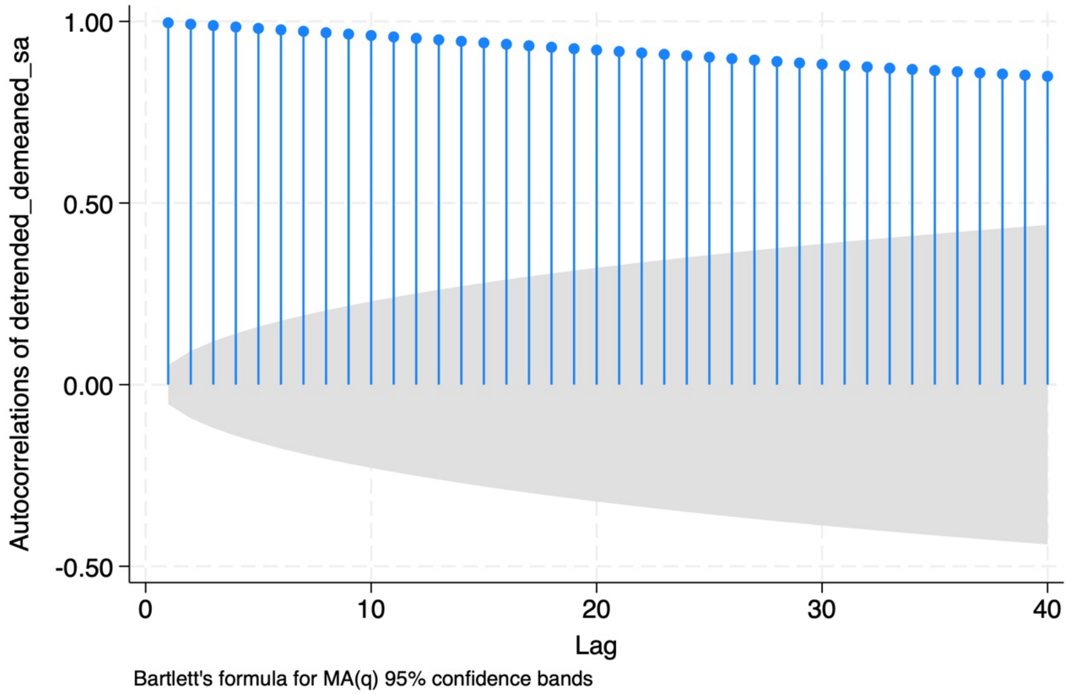

Figure 2. Autocorrelation graph of detrended, demended, and seasonal adjusted CPI.

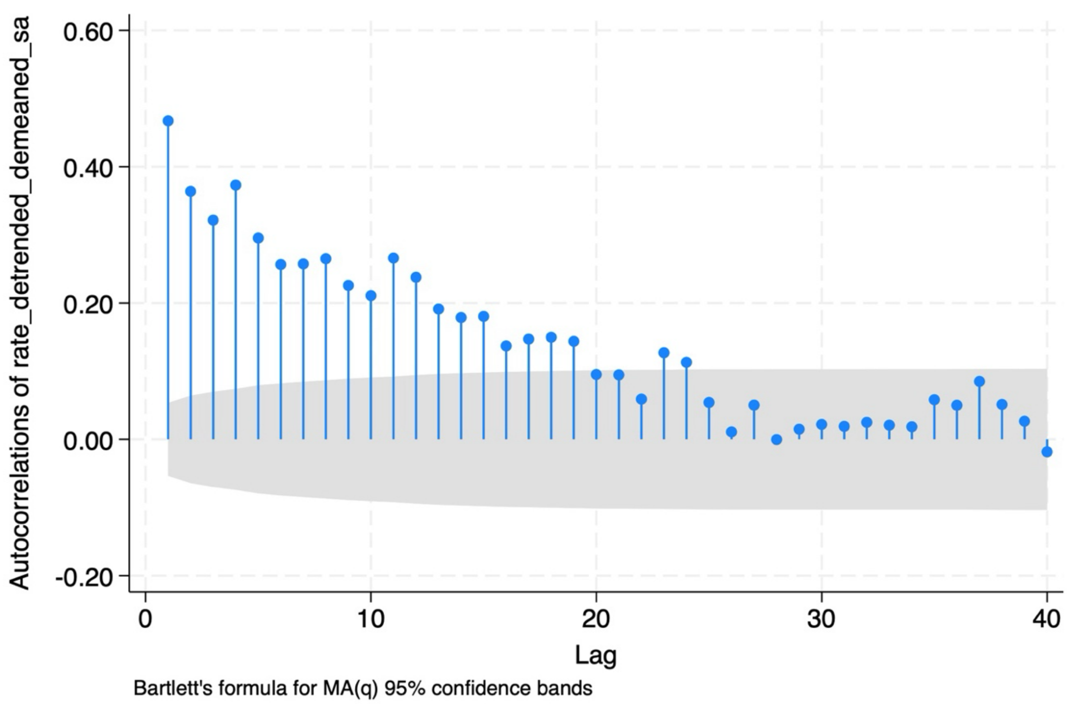

Figure 3. Autocorrelation graph of detrended, demended, and seasonal adjusted CPI growth rate.

Model selection for ADL model

I considered Real GDP, unemployment rate, 10-Year Treasury Yield (GS10), 3-Month Treasury Bill Yield (TB3MS), 1-Year Treasury Yield (GS1), Moody's BAA Corporate Bond Yield (BAA), and Moody's AAA Corporate Bond Yield (AAA). Among these index or yield rate, several combinations were considered as potential explanatory variables as shown in Table 2. Spread 2 represented the 10-Year vs. 3-Month Treasury Spread; Dt12 represented difference in the 1-Year Treasury Yield; dt3 indicated change in the 3-Month Treasury Yield; spread 1 represented 1-Year vs. 3-Month Treasury Spread; And corporate determined spread between BAA and AAA corporate bond yields.

The start date of the time series has changed after considered these variables, since some of them had no record until 1986 which was after the break date set previously. I tested the autocorrelation of CPI again by AIC and BIC, and it showed the best fitted model was AR(2). Therefore, Granger-cause test and ADL models preliminary included models with two lags.

The Granger-cause test results showed that under 0.05 significance level ($\alpha = 0.05$), corporate, dt12, dt3, real GDP, and GDP growth rates had Granger-cause on dCPI. This means these explanatory variables had predictive power for predicting dCPI.

After determined these Granger-cause variables, I tested AIC, BIC and PLS on their ADL(1) and ADL(2) models. The results are shown in Table 3. The ADL(2) of corporate had smallest AIC and BIC. Thus, I decided to use ADL(2) of corporate to forecast dCPI.

Table 2. Different combinations of bond yield as potential explanatory variables

spread2 = GS10 - TB3MS
dt12 = GS1 - L.GS1
dt3 = TB3MS - L.TB3MS
spread1 = GS1 - TB3MS
corporate = BAA - AAA

Table 3. AIC, BIC, and PLS Test results of ADL models

Model	AIC	BIC	PLS
corporateADL1	-4106.642	-4094.222	0.0000159
corporateADL2	**-4123.479**	**-4102.79**	0.0000159
dt12ADL1	-4110.356	-4097.936	0.0000154
dt12ADL2	-4120.716	-4100.027	0.0000151
dt3ADL1	-4111.884	-4099.464	0.0000153
dt3ADL2	-4121.113	-4100.424	0.0000151
gdpADL1	-4087.165	-4074.759	0.0000159
gdpADL2	-4099.438	-4078.771	0.0000149
dgdpADL1	-4093.604	-4081.197	0.0000149
dgdpADL2	-4099.765	-4079.098	**0.0000146**

After determining the lags number, we now have the complete version of the dCPI component model, as shown in (5) and the corporate ADL(2) model, as shown in (6).

$$dCPI_t = \mu_1 + \mu_2 t + \sum_{n=1}^{11} \gamma_n D_{n,t} + \sum_{n=1}^{4} \beta_n dCPI_{t-n} + \phi d_t + \varepsilon_t \tag{5}$$

$$dCPI_t = \lambda_1 + \alpha_1 dCPI_{t-1} + \alpha_2 dCPI_{t-2} + \theta_1 cor_{t-1} + \theta_2 cor_{t-2} + \varepsilon_t \tag{6}$$

Results

The coefficients of both models were estimated by STATA. Estimation of dCPI component model used CPI data from 1918, while estimation of ADL model used CPI and corporate data from 1986. Table 4 lists estimated value and standard error (SE) of each coefficient in ADL(2) model. Since I used direct-method to forecast ADL model, only the one-step-ahead regression value is included in Table 4. Table 5 lists estimated value and SE of each coefficient in dCPI component model.

Table 4. Corporate ADL(2) model's estimated values and standard error of coefficients.

Variables	**Coef.**	**value**	**Std. err.**	**Variables**	**Coef.**	**value**	**Std. err.**
L1.	θ_1	0.5324	0.0481	**Cor L1.**	α_1	0.0046	0.0014
L2.	θ_2	0.1814	0.0460	**Cor L2.**	α_2	0.0043	0.0014
intercept	λ_1	0.0017	0.4276				

Table 5. dCPI component model's estimated values and standard error of coefficients.

Variables	Coef.	value	Std. err.	Variables	Coef.	value	Std. err.
L1.	β_1	0.3291	0.0269	**m3**	γ_3	0.0030	0.0007
L2.	β_2	0.1088	0.0284	**m4**	γ_4	0.0036	0.0007
L3.	β_3	0.0554	0.0284	**m5**	γ_5	0.0020	0.0007
L4.	β_4	0.1997	0.0269	**m6**	γ_6	0.0030	0.0007
breakdate	ϕ	-0.0007	0.0006	**m7**	γ_7	0.0022	0.0007
time	μ_2	8.48E-07	6.99E-07	**m8**	γ_8	0.0008	0.0007
intercept	μ_1	-0.0008	0.0005	**m9**	γ_9	0.0027	0.0007
m1	γ_1	0.0019	0.0007	**m10**	γ_10	0.0010	0.0007
m2	γ_2	0.0013	0.0007	**m11**	γ_11	0.0005	0.0007

Future 12 months from November 2024 were forecasted by both dCPI component model (denoted as AR(4) model) and corporate ADL(2) model. For AR(4) model, I used simulation method to forecast. The simulation method was that first regressed AR(4) model on existed data to get and store estimated values of coefficients, and then use estimated regression to generate future 12 months dCPI. The future dCPI generation was repeated by one thousand times and the point estimated and standard deviation of simulated values were recorded. The estimated dCPI and its forecast interval is shown in Figure 4. Forecast CPIs were transformed given the value of dCPIs and shown in Figure 5. The specific CPI values of each forecast months are shown in Table 4. Specifically, The CPI in November 2024 is predicted to be 315.8, and the forecast interval is [312.6, 319.1].

In addition to AR(4) model, I used direct method to forecast dCPI with the ADL(2) model. The direct method estimated the h-step-ahead forecast directly from the data, rather than sequentially forecasting intermediate steps (such as generate future corporate first). The Figure 6 shows the forecast value and interval of dCPI by ADL(2) model. Similarly, Forecast CPIs were transformed by the value of dCPIs and they are depicted in Figure 7. In Table 5, the specific CPI values of each forecast months are shown. ADL(2) model predicts the CPI in November 2024 as 316.3 and the forecast interval is [314.6, 318.1].

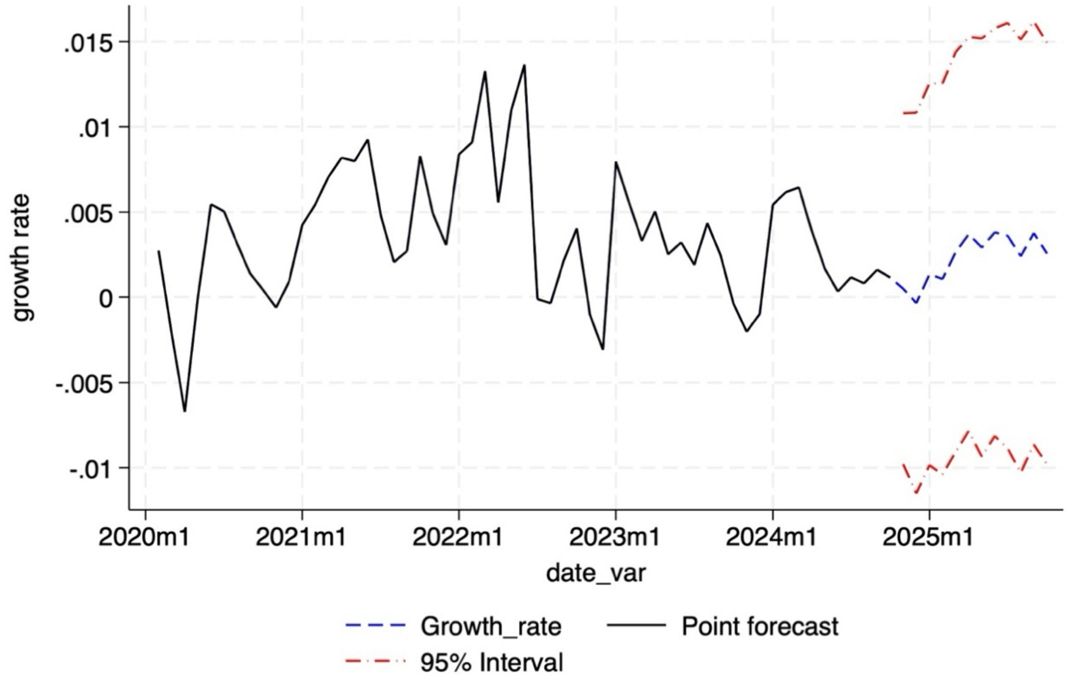

Figure 4. dCPI point forecast and 95% forecast interval. The forecast was made by dCPI component model.

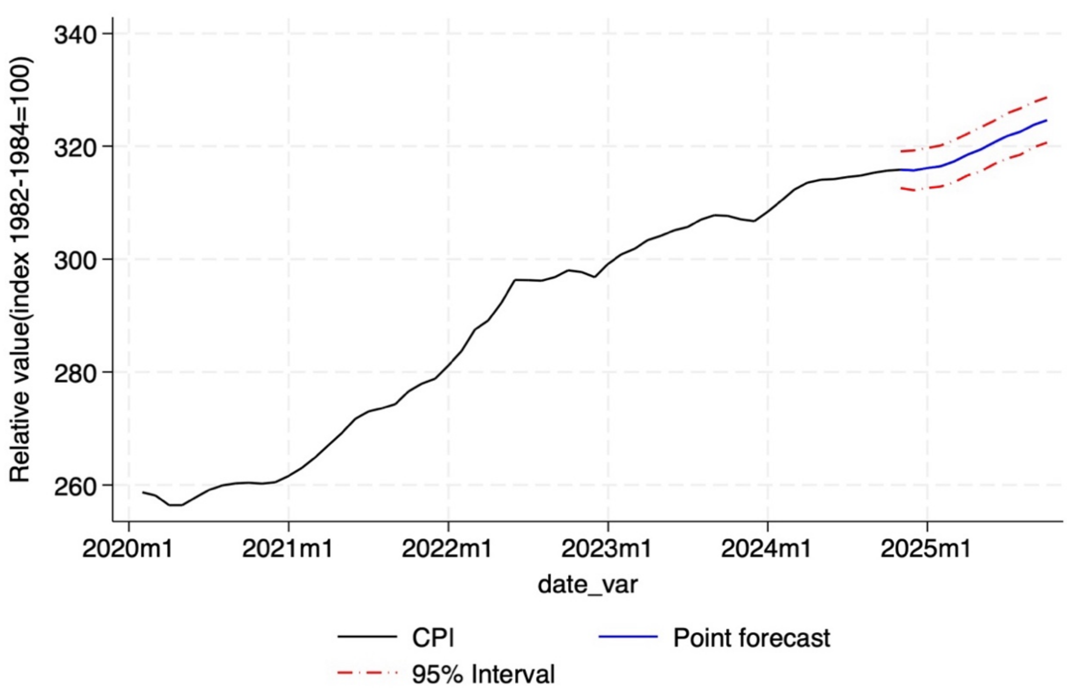

Figure 5. CPI point forecast and 95% forecast interval. This CPI point forecast was transformed from dCPI forecast value by dCPI component model.

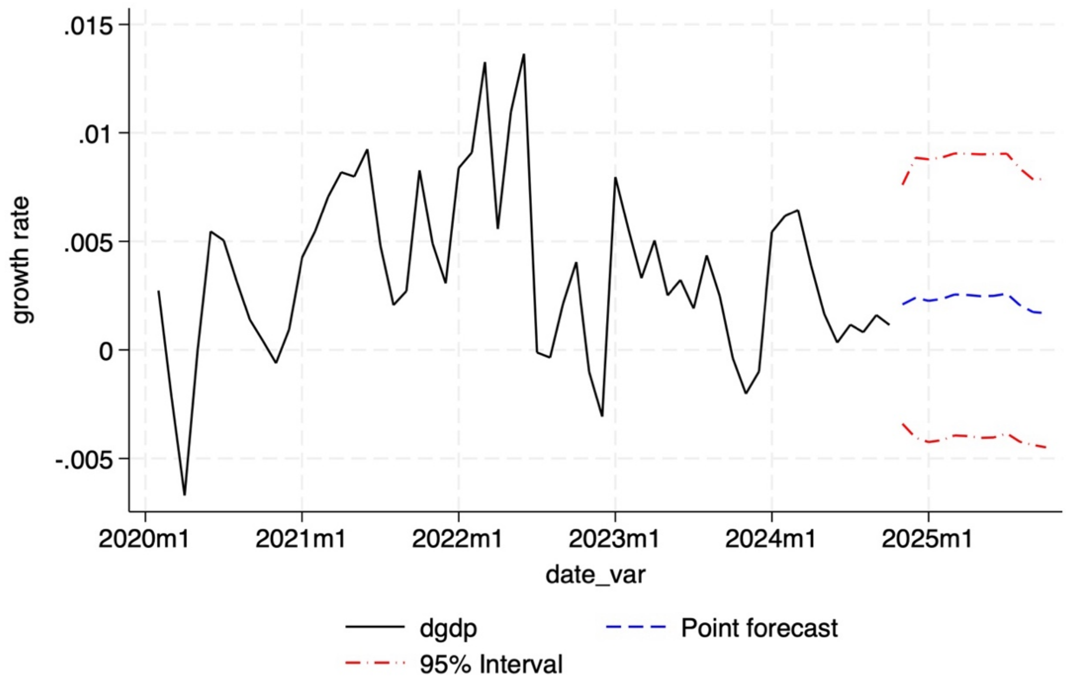

Figure 6. dCPI point forecast and 95% forecast interval. This forecast was made by corporate ADL(2) model.

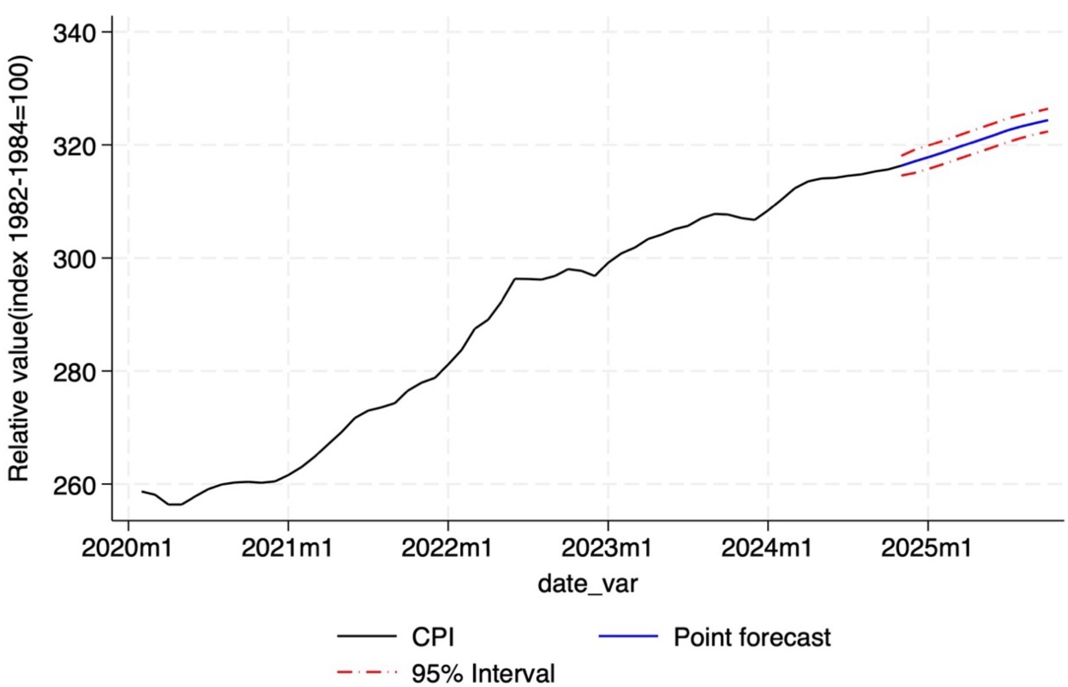

Figure 7. CPI point forecast and 95% forecast interval. This CPI point forecast was transformed from dCPI forecast value by corporate ADL(2) model.

Table 6. CPI point forecast and forecast interval values of dCPI component model (AR(4))

Month	**AR forecast**	Lower bound	Upper bound
2024m11	**315.8**	312.6	319.1
2024m12	**315.7**	312.2	319.3
2025m1	**316.1**	312.6	319.7
2025m2	**316.5**	312.9	320.1
2025m3	**317.3**	313.6	321.1
2025m4	**318.5**	314.8	322.2
2025m5	**319.4**	315.5	323.4
2025m6	**320.6**	316.8	324.5
2025m7	**321.8**	317.8	325.8
2025m8	**322.6**	318.5	326.7
2025m9	**323.8**	319.8	327.8
2025m10	**324.6**	320.6	328.7

Table 7. CPI Point forecast and forecast interval values of corporate ADL(2) model

Month	**ADL forecast**	Lower bound	Upper bound
2024m11	**316.3**	314.6	318.1
2024m12	**317.1**	315.1	319.1
2025m1	**317.8**	315.7	319.9
2025m2	**318.6**	316.5	320.6
2025m3	**319.4**	317.3	321.5
2025m4	**320.2**	318.1	322.3
2025m5	**321.0**	318.9	323.1
2025m6	**321.8**	319.7	323.9
2025m7	**322.6**	320.5	324.7
2025m8	**323.3**	321.3	325.3
2025m9	**323.8**	321.9	325.8
2025m10	**324.4**	322.4	326.4

Discussion

Compared to AR(4) model, ADL(2) model has the similar point forecast but a narrower forecast interval. The similar point forecast indicates these two models predicted an almost same result, while two models were built in a totally different logic. AR(4) model attempted to figure out the inner patterns of CPI while ADL(2) model employed corporate as an explanatory power. The narrower forecast interval of ADL(2) infers it has a higher confidence, which is reasonable given the smaller PLS value of ADL(2). One possible explanation of a broader forecast interval of AR(4) model is more variables included in AR(4). In Figure 8 of forecast comparison, we can see AR(4) model has a subtle seasonal component making the prediction curved, while ADL(2) model is almost straight.

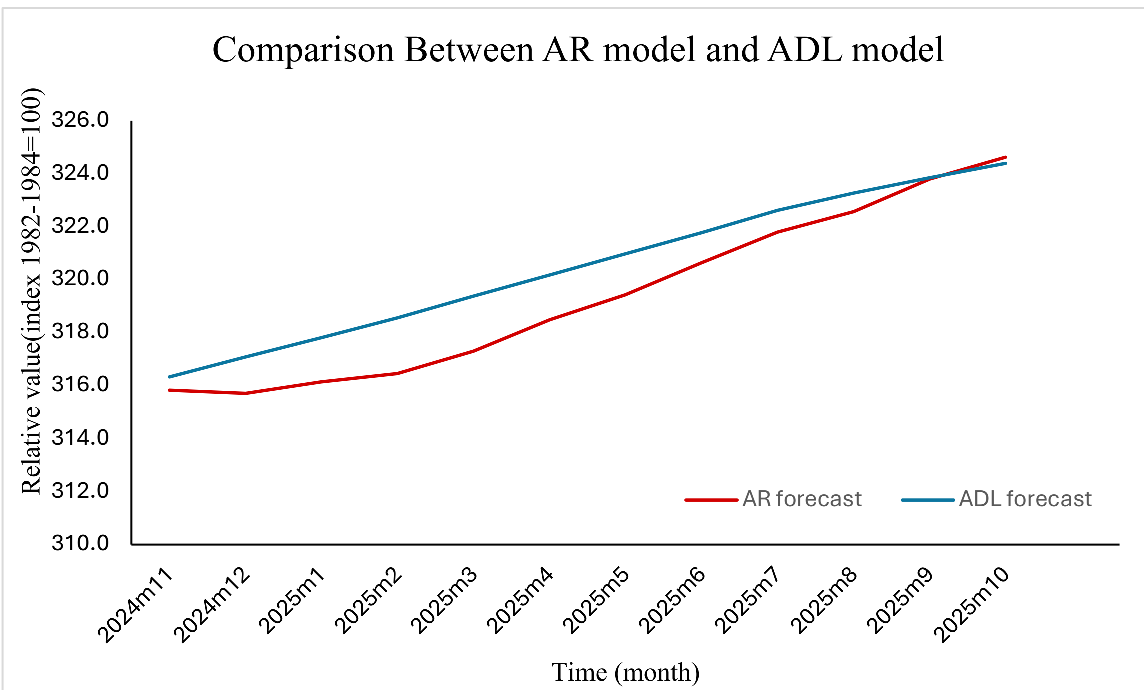

Figure 8. Comparison of point forecast values between AR forecast and ADL forecast

Reference

1. Seydl, J., & Gao, Z. (2024, February 26). *Predicting inflation: What have we learned?* J.P. Morgan Private Bank U.S. https://privatebank.jpmorgan.com/nam/en/insights/markets-and-investing/predicting-inflation-what-have-we-learned?utm_source=chatgpt.com
2. Church, J. (2016, March). *Comparing the Consumer Price Index with the gross domestic product price index and gross domestic product implicit price deflator*. U.S. Bureau of Labor Statistics. https://www.bls.gov/opub/mlr/2016/article/comparing-the-cpi-with-the-gdp-price-index-and-gdp-implicit-price-deflator.htm
3. Linder, M. H. (2020). *Using interest rates to predict economic growth: Are corporate bonds useful?* International Journal of Forecasting. https://doi.org/10.1002/ijfe.2867
4. Benzoni, L., Chyruk, O., & Kelley, D. (2018). *Why Does the Yield-Curve Slope Predict Recessions?* Federal Reserve Bank of Chicago. https://www.chicagofed.org/publications/chicago-fed-letter/2018/404?utm_source=chatgpt.com