FORMULA SHEET

Trigonometric Functions

$$\tan\theta = \frac{sin\theta}{cos\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$sin^2\theta + cos^2\theta = 1$$

$$1 + tan^2\theta = sec^2\theta$$

$$1 + cot^2\theta = csc^2\theta$$

Angle Sum and Difference Identities

$$sin(\alpha \pm \beta) = sin\,\alpha\, cos\,\beta \pm cos\,\alpha\, sin\,\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

Double-Angle Identities

$$\sin(2x) = 2\sin x\cos x$$

$$cos(2x) = cos^2 x - sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\,sin^2 x$$

$$tan(2x) = \frac{2\,tan\,x}{1 - tan^2 x}$$

Half-Angle Identities

$$sin^2\theta = \frac{1 - cos\,\theta}{2}$$

$$sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - cos\,\theta}{2}}$$

$$= \pm\sqrt{\frac{1 + cos\,\theta}{2}}$$

$$cos^2\theta = \frac{1 + cos\,2\theta}{2}$$

$$tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - cos\,\theta}{1 + cos\,\theta}}$$

Distance Formula

$$d = \sqrt{x^2 + y^2}$$

Product to Sum Formulas

$$sin(x)\,sin(y) = \frac{1}{2}(cos(y - x) - cos(y + x))$$

$$sin^2(x) = \frac{1}{2}(1 - cos(2x))$$

$$cos(x)\,cos(y) = \frac{1}{2}(cos(y + x) + cos(y - x))$$

$$cos^2(x) = \frac{1}{2}(cos(2x) + 1)$$

$$\sin(x)\cos(y) = \frac{1}{2}(\sin(y + x) - \sin(y - x))$$

$$\cos x\sin x = \frac{1}{2}\sin(2x)$$

Definition of a Derivative

$$f'(x) = \lim_{h\to 0}\frac{f(x + h) - f(x)}{h}$$

Standard Differentials

$$\frac{d}{dx}sin\,x = cos\,x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$$

$$\frac{d}{dx}sec\,x = tan\,x\,sec\,x$$

$$\frac{d}{dx}cosec\,x = -cosec\,x\,cot\,x$$

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}tan^{-1}(ax) = \frac{a}{1 + (ax)^2}$$

$$\frac{d}{dx}\cot^{-1}(x) = \frac{-1}{1 + x^2}$$

$$\frac{d}{dx}\sec^{-1}(x) - \frac{1}{|x|\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}\operatorname{cosec}^{-1}(x) = \frac{-1}{|x|\sqrt{x^2 - 1}}$$

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Product Rule

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$

Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Chain Rule

$$\left(f\big(g(x)\big)\right)' = f'(g(x))g'(x)$$

Definition of an Integral

$$\int_a^b f(x) = \lim_{\Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

Standard Integrals

$$\int \frac{1}{\sqrt{a^2 - u^2}}\,du = sin^{-1}\left(\frac{u}{a}\right) + c$$

$$\int \frac{1}{a^2 + u^2}\,du = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + c$$

$$\int \frac{1}{u\sqrt{u^2 - a^2}}\,du = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + c$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax + c$$

$$\int \cos ax\,dx = \frac{1}{a}\sin x + c$$

$$\int \tan ax\,dx$$

$$= -\frac{1}{a}\ln|\cos ax| + c$$

$$= \frac{1}{a}\ln|\sec ax| + c$$

$$\int \cot(x) = \ln(\sin(x)) + c$$

$$\int \sec^2(x)\,dx = \tan(x) + C$$

$$\int cosec^2(x)\,dx = -\cot(x) + C$$

$$\int \sec(x)\tan(x)\;\,dx = \sec(x) + C$$

$$\int cosec(x)\cot(x)\;\,dx = -cosec(x) + C$$

$$\int \sec(x)\,dx = \ln|sec(x) + tan(x)| + C$$

Integral Properties

$$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

$$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$$

$$\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$$

$$If\ f(x) \geq 0\ for\ a \leq x \leq b\ then\ \int_a^b f(x)\,dx \geq 0$$

$$If\ f(x) \geq g(x) for\ a \leq x \leq b$$

$$then\ \int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$$

$$If\ m \leq f(x) \leq M\ for\ a \leq x \leq b$$

$$then\ m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$

$$If\ f(x)\ is\ even\ then\ \int_{-x}^{x} f(t)\,dt = 2\int_0^x f(t)\,dt$$

$$If\ f(x)\ is\ odd\ then\ \int_{-x}^{x} f(t)\,dt = 0$$

Integration by Substitution

$$\int f\big(g(x)\big)g'(x)\,dx = \int f(u)\,du\ \ if\ u = g(x)$$

Integration by Parts

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

The Area Between Curves

$$A = \int_a^b [f(x) - g(x)]\,dx$$

The Volume of a Solid

$$V = \int_a^b A(x)\,dx$$

Arc Length

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

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Work

$$W = \int_a^b f(x)\,dx$$

Differential Equations

Logistic Growth

$$\frac{dy}{dt} = ky(M-y)$$

Separable equations

$$\frac{dy}{dx} = g(x)f(y)$$

Surface Area of Revolution

$$S = 2\pi \int_a^b f(x)\sqrt{1+[f'(x)]^2}\,dx$$

$$S = \int 2\pi y\,ds \ \ about\ the\ x-axis$$

$$S = \int 2\pi x\,ds \ \ about\ the\ y-axis$$

The Improper Integral

$$\int_a^\infty f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx$$

$$\int_{-\infty}^b f(x)\,dx = \lim_{t\to-\infty}\int_t^b f(x)\,dx$$

$$\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^a f(x)\,dx + \int_a^\infty f(x)\,dx$$

$$\int_a^b f(x)\,dx = \lim_{t\to b^-}\int_a^t f(x)\,dx$$

$If\ f\ is\ continuous\ on\ [a,b)\ and\ is\ discontinuous\ at\ b$

$$\int_a^b f(x)\,dx = \lim_{t\to a^+}\int_t^b f(x)\,dx$$

$If\ f\ is\ continuous\ on\ (a,b]\ and\ is\ discontinuous\ at\ a$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Limits

Properties of Direct Substitution

$$\lim_{x\to a} f(x) = f(a)\ provided\ f(x)\ is\ continuous$$

Standard Limits

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x} = 0$$

Limit Laws

Standard limits page 35

$given\ c\ is\ an\ arbitrary\ constant\ and$

$the\ limits\ \lim_{x\to a} f(x)\ and\ \lim_{x\to a} g(x)\ \ exist$

$$\lim_{x\to a}[f(x)\pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$$

$$\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$$

$$\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x)\lim_{x\to a} g(x)$$

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}\ iff \lim_{x\to a} g(x) \neq 0$$

$$\lim_{x\to a}[f(x)^n] = \left[\lim_{x\to a} f(x)\right]^n \ \ \forall n > 0$$

L'Hopital's Rule

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

$provided\ the\ right\ hand\ side\ limit\ exists$

Logarithms

Definition of the logarithm

$$\ln(x) = \int_1^x \frac{1}{t}\,dt \ \ x > 0$$

Change of base formula

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

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Exponentials

Definition of an exponent

$$\frac{d}{dx}e^x = \int e^x\,dt - C = e^x$$

Properties of an exponent

$$a^x = e^{xln(a)}$$

$$a^{x+y} = a^x a^y$$

$$a^{x-y} = \frac{a^x}{a^y}$$

$$(a^x)^y = a^{xy}$$

$$(ab)^x = a^x b^x$$

$$\log_a x = \frac{\log(x)}{\log(a)}$$

$$\frac{d}{dx}(\log_a x = \frac{1}{xln(a)}$$

$$\frac{d}{dx}(a^x) = a^x \ln(a)$$

$$\int a^x\,dx = \frac{a^x}{\ln(a)} + C$$

Series

Geometric series

$$a_n = ar^{n-1}\, for\, n\ \in \mathbb{Z}^+ > 0$$

$$s_n = \frac{a(1-r^n)}{1-r}$$

$$s_\infty = \frac{a}{1-r}\ |r| < 1$$

Series properties

$$\sum_{n=1}^{\infty} ca_n = c\sum_{n=1}^{\infty} a_n$$

$$\sum_{n=1}^{\infty}(a_n \pm b_n) = \sum_{n=1}^{\infty} a_n \pm \sum_{n=1}^{\infty} b_n$$

Power Series

$$f(x) = \sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$
$$= f(a) + f'(a)(x-a)$$
$$+ f''(a)(x-a)^2 + \cdots$$

The Remainder

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1}\ x < z < a$$

Standard power series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n\ \ R = 1$$

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}\ \ R = \infty$$

$$\sin(x) = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}\ R = \infty$$

$$\cos(x) = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}\ \ R = \infty$$

$$\arctan(x) = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}\ R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}\ R = 1$$

$$(x+a)^n = \sum_{n=0}^{\infty}\binom{k}{n}x^n a^{k-n}\ \ R = 1$$