$$
\begin{aligned}
g_s^1 &= z(pg_{s+1}^1 + qg_{s-1}^{-1}) \\
g_{s+1}^1 &= \frac{1}{zp}g_s^1 - \frac{q}{p}g_{s-1}^{-1}
\end{aligned}
$$

$$
\begin{aligned}
g_s^{-1} &= z(qg_{s+1}^1 + pg_{s-1}^{-1}) \\
g_{s+1}^1 &= \frac{1}{zq}g_s^{-1} - \frac{p}{q}g_{s-1}^{-1}
\end{aligned}
$$

$$
\begin{aligned}
g_s^{-1} &= z(qg_{s+1}^1 + pg_{s-1}^{-1}) \\
g_{s+1}^{-1} &= z(qg_{s+2}^1 + pg_s^{-1}) \\
g_{s+1}^{-1} &= z(\frac{q}{zp}g_{s+1}^1 - \frac{q^2}{p}g_s^{-1} + pg_s^{-1}) \\
g_{s+1}^{-1} &= z(\frac{q}{zp}g_{s+1}^1 + (p - \frac{q^2}{p})g_s^{-1}) \\
g_{s+1}^{-1} &= \frac{q}{p}g_{s+1}^1 + z(p - \frac{q^2}{p})g_s^{-1} \\
g_{s+1}^{-1} &= \frac{q}{p}(\frac{1}{zq}g_s^{-1} - \frac{p}{q}g_{s-1}^{-1}) + z(p - \frac{q^2}{p})g_s^{-1} \\
g_{s+1}^{-1} &= (\frac{1}{zp} + z(p - \frac{q^2}{p}))g_s^{-1} - g_{s-1}^{-1}g_{s+1}^{-1} \qquad = (\frac{1}{zp} + z(2 - \frac{1}{p}))g_s^{-1} - g_{s-1}^{-1}
\end{aligned}
$$

$$
\begin{aligned}
G_{s+1} = \begin{pmatrix} g_{s+1}^1 \\ g_{s+1}^{-1} \\ g_s^1 \\ g_s^{-1} \end{pmatrix} &= \begin{pmatrix} \frac{1}{zp} & 0 & 0 & -\frac{q}{p} \\ 0 & \frac{1}{zp} + z(2 - \frac{1}{p}) & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} g_s^1 \\ g_s^{-1} \\ g_{s-1}^1 \\ g_{s-1}^{-1} \end{pmatrix} \\
&= A(z).G_s
\end{aligned}
$$