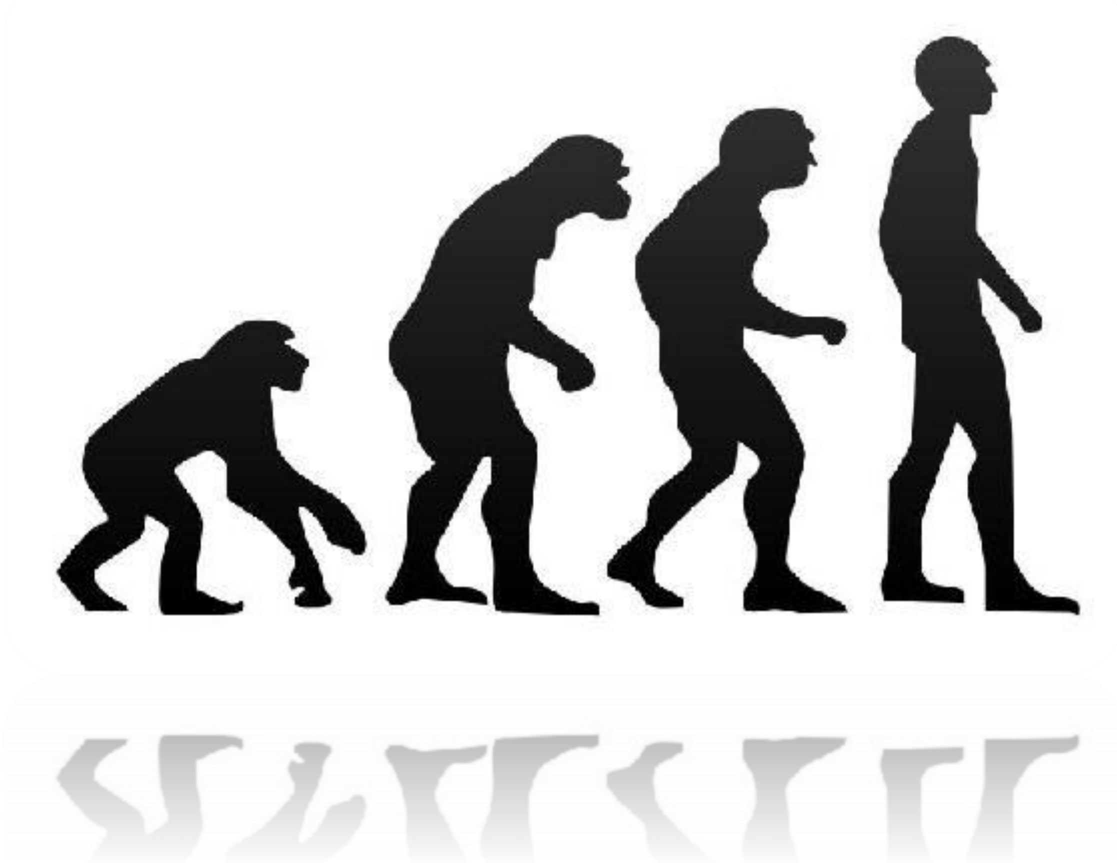
JUNE 2, 2014



EVOLUTIONARY ALGORITHMS

PROJECT REPORT

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Abstract

This paper presents the results of the application of Evolutionary algorithms on various problems. The problems are different, but the modelling remane the same. The first section presents a generic class EA implementing the base mechanism of genetic algorithm. For each problem, we must find a suitable representation for the individuals (tuple, matrix ...), then we must implement a fitness function and a mutation/crossover operator.

I chose **Python** as a programming langage.

Libraries used:

- numpy/scipy: provides powerful tools to manipulate arrays and generate random numbers
- pydot: allows to easily draw graphs from Python
- pygame: for image rendering
- pyalgotrade: provides financial tools, used only in the last program

Contents

1 Implementation of the GA algorithm (file "ga.py")

The EA class implmenets the (self adapt) $(1 + (\lambda, \lambda))$ GA in a generic form. The constructor is:

```
def __init__(self, fitness, mutation=default_mutation, crossover=default_crossover,
                l_distribution=np.random.binomial)
```

It requires:

- a fitness function to evaluate individuals,
- a mutation operator, by default, this operator is used:

```
def default_mutation(l, x):
        bits_to_change = random.sample(range(len(x)), l)
        x_mut = list(x)
        for b in bits_to_change:
                x_mut[b] = 1 - x_mut[b]
        return x_mut
```

- a crossover operator

```
def default_crossover(c, x, xx):
        parent = [x, xx]
        parent_choice = bernoulli.rvs(c, size=len(x))
        return [ parent[p][i] for i, p in enumerate(parent_choice) ]
```

- a function that generates l from some distribution. By default, l is generated from a binomial distribution.

```
def default_rule(offspring_size, better_solution_found):
        return offspring_size
```

Then, the *run* function does all the work. It generate the children, select the best one, does the crossover and mutation phase, adapt λ if needed, and then repeat the process *n generations* times.

```
def run(self, n, x_init, offspring_size=5, n_generations=10,
                                self_adapt_rule=default_rule, max_fitness=None )
```

- n is the size of individuals
- x_init is the initial individual
- $offspring\_size$ is λ
- $n\_generations$ is the maximum number of generations before the function ends
- $self\_adapt\_rule$ is function that updates λ in each loop. The default rule is the rule that does nothing:

  ```
  def default_rule(offspring_size, better_solution_found):
          return offspring_size
  ```

  The $1/5^{th}$ rule would be:

  ```
  def one_fifth_rule(offspring_size, better_solution_found):
          F = 1.5
          if better_solution_found:
                          return max(1, offspring_size / F)
          else:
                          return offspring_size * (F**(0.25))
  ```

- $max\_fitness$: when the fitness of x is greater than $max_fitness$, the function returns immediately

2 The One-Max problem (file "onemax.py")

The implmentation of the one-max problem is straight forward. We start with a random vector in $\{0,1\}^n$. Default mutation and crossover operator work fine. For the fitness function, we sum all the bits in x (there is a builtin function *sum* provided with *Python*). The code is as follow:

```
ea_algo = ga.EA(fitness=sum)
x_init         = np.random.random_integers(0, 1, size=n)
best_x         = ea_algo.run(n, x_init, offspring_size=5, n_generations=100)
```

2.1 Stats

All numbers reported are derived from 100 independent runs of the respective algorithms. In all experiments reported below, we set $p = \frac{\lambda}{n}$ and $c = \frac{1}{\lambda}$.

For raw data, see "'onemaxstats.zip"'. Stat files contains data in the following format:
n λ
N
a_1 a_2 ... a_N
Where a_i is the result obtained for the i^{th} realisation.

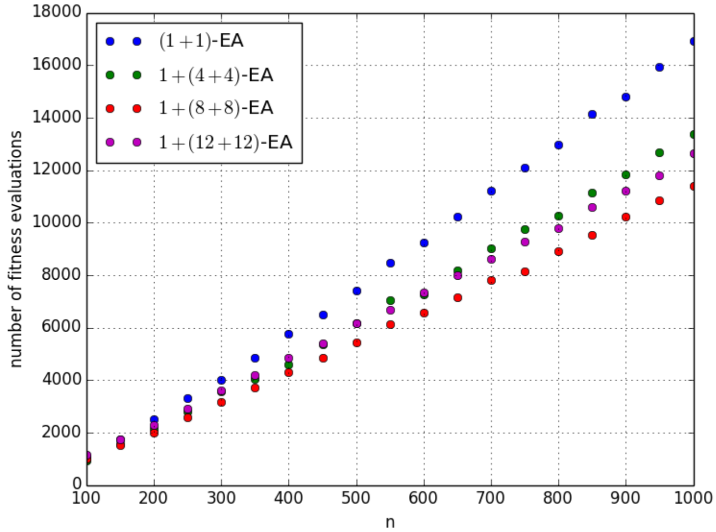


Figure 1: Number of fitness evaluations for $\lambda \in \{1, 4, 8, 12\}$

The $1 + (8 + 8)$-EA gives the best results, better than the classic (1+1)-EA.
I tried the self-adaptive rule, that is:

$$\lambda = \begin{cases} \lambda/F \text{ after a successful iteration} \\ \lambda * F^{1/a} \text{ otherwise} \end{cases}$$

with $a \in \{3, 4, 6, 9\}$

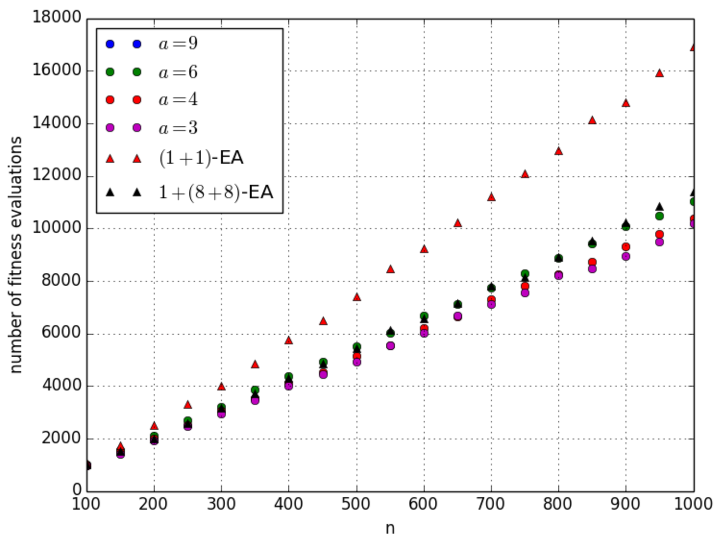


Figure 2: Number of fitness evaluations for $a \in \{3, 4, 6, 9\}$

Whatever the initial value for λ, the self-adaptive version of the algorithm behave like the best version of the $1+(\lambda+\lambda)$-EA, that is $\lambda = 8$.

3 The Maximum Matching (file "matching.py")

We represent an undirected graph by it's adjacency list. For example:

```
vertices = range(n)
edges = [(1, 2), (3, 4)]
```

The function responsible for calculating the degree of a vertex v in ine the subgraph consisting of the edges of M:

```
def deg(M):
    deg_m = np.zeros(n)
    for i, (e, f) in enumerate(edges):
        if M[i]:
            deg_m[e] += 1
            deg_m[f] += 1
    return sum([max(0, d-1) for d in deg_m])
```

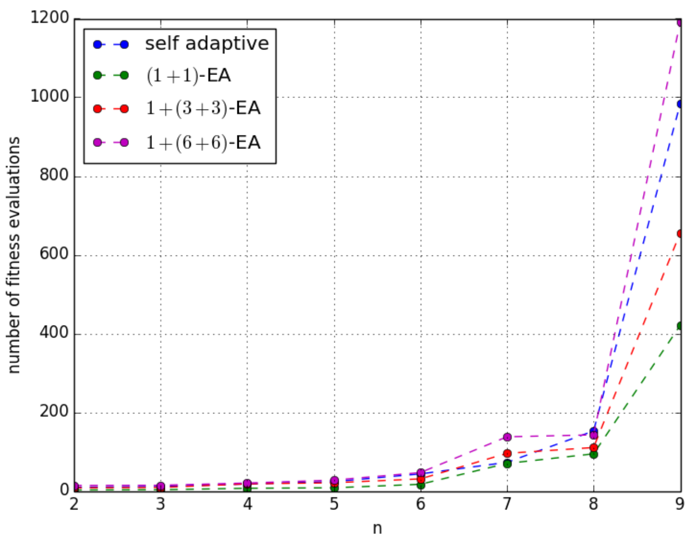
And the fitness function:

```
def fitness(M):
    return sum(M) - m * deg(M)
```

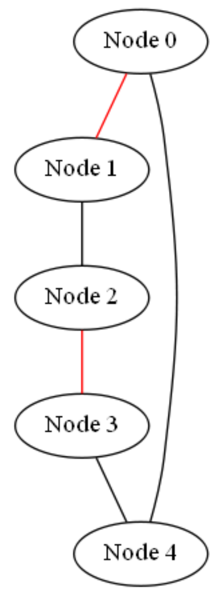
3.1 Stats

For raw data see "'**matchingstats.zip**"'.

3.1.1 First example: rings of size n



Number of fitness evaluations

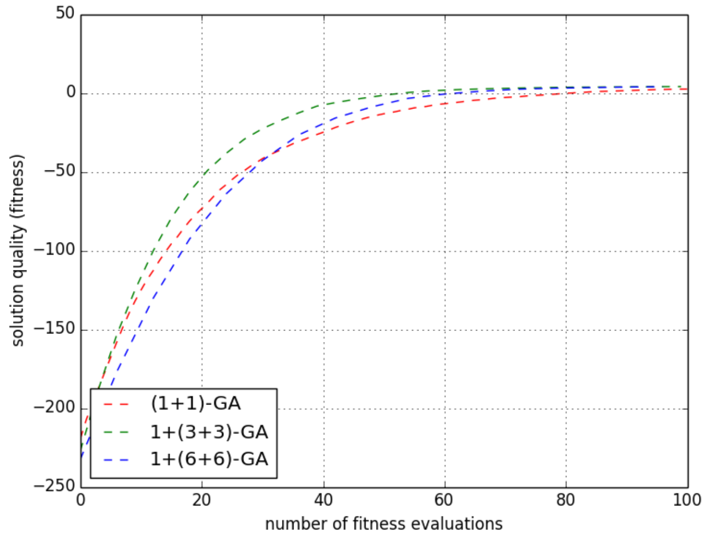


Instance

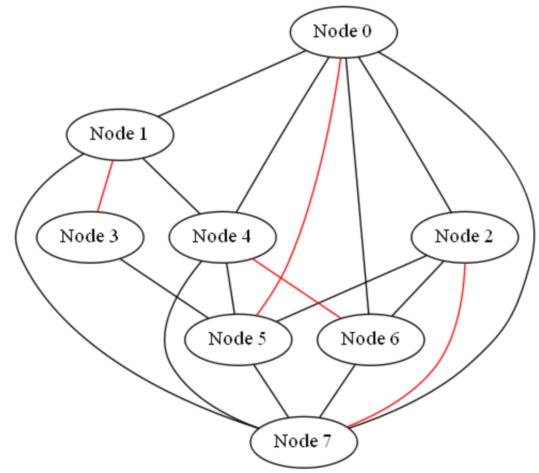
Unlike the onemax problem, the traditional $(1+1)-EA$ gives the best results.

3.1.2 Second example: random graphs

Protocol: We took $n = 10$ fixed and $\lambda \in \{1, 3, 6\}$. I generated $N = 1000$ independant random graph with n vertices, Each of them chooses uniformly at random 3 friends. The following graph shows how the fitness evolves over time in average.



Solution quality (fitness) evolution over time



Random instance

4 Euler cycles (file "eulerian cycle.py")

We represent graphes by their adjacency matrix.

Individuals are represented by a list of matchings, ie $x = \{M_v \mid v \in V\}$ where each M_v is a dictionnary.

Helper functions:

```
# Matches e and f
def match(Mv, e, f):
        Mv[e] = f
        Mv[f] = e

def find_match_and_remove(Mv, e):
        # Look for an edge f matched to e
        f = Mv.pop(e, None)
        # Remove the matching (if possible)
        if f is not None:
                        del Mv[f]
        return f

def count_walks(x):
        # k is the number of independant paths induced by the matching x
        k = 0
        xx = deepcopy(x)
        # Each vertex v, we follow and remove the path induced by the matching
        for v in vertices:
                while len(xx[v]) > 0:
                        e, f = xx[v].popitem()
                        del xx[v][f]
                        is_cycle = False
                        # follow the path in both directions
                        for i, j in [ (e, f), (f, e)]:
                                # if we have found a cycle, there is no need
                                # to follow the path in the opposite direction
                                if is_cycle: break

                                # s is the current vertex, i the precedent, and j the next
                                # i -> s -> j
                                i, s = v, j
```

```
                            while j != None:
                                    j = find_match_and_remove(xx[s], i)
                                    i, s = s, j
                            # we have found a cycle if the last vertex visited == v
                            is_cycle = (i == v)
                            k += 1
        return k
```

We can use the default **crossover** operator.
We implement the **mutation** operator as suggested:

```
def mutation(l, x):
        x_mut = deepcopy(x)
        for _ in range(l):
                # Choose a random vertex v
                v = random.randint(0, n-1)
                Mv = x_mut[v]
                deg_v = sum([edges[v][i] for i in vertices if i != v])
                if deg_v < 2:
                        raise Exception('The graph is not eulerian because
                        vertex %d has deg = %d' % (v, deg_v))
                # if deg(v) == 2, a trivial perfect matching is [ (i, j) ]
                elif deg_v == 2:
                        i, j = [w for w in vertices if edges[v][w] and v != w]
                        match(Mv, i, j)
                else:
                        # Choose a random edge e incident to v,
                        e = random.choice([i for i in vertices if i != v and edges[v][i]])
                        # See if it is matched. If it's the case, remove the match
                        f = find_match_and_remove(Mv, e)

                        # Same thing for (ee, ff)
                        ee = random.choice([i for i in vertices if (not i in[ v, e, f]) and edges[v][i]])
                        ff = find_match_and_remove(Mv, ee)

                        # match (e, ee) and (f, ff) if possible
                        match(Mv, e, ee)
                        if f and ff:
                                match(Mv, f, ff)

        return x_mut
```

The fitness function:

```
def fitness(x):
        k = count_walks(x)
        cycles_penality = nb_edges - sum(map(lambda Mv: len(Mv), x))/2
        return  - cycles_penality  - (k-1)
```

We run the program as follow:

```
def poisson_distribution(n, p):
    return 1 + np.random.poisson(1)

# Start with empty matchings
x_init = [ {} for _ in range(n)]
best_x = ea_algo.run(n, x_init=x_init, offspring_size=4, n_generations=1000,
                                max_fitness=0, l_distribution=poisson_distribution)
```

After the algorithm has finished, we can reconstruct the tour from the maximum matching:

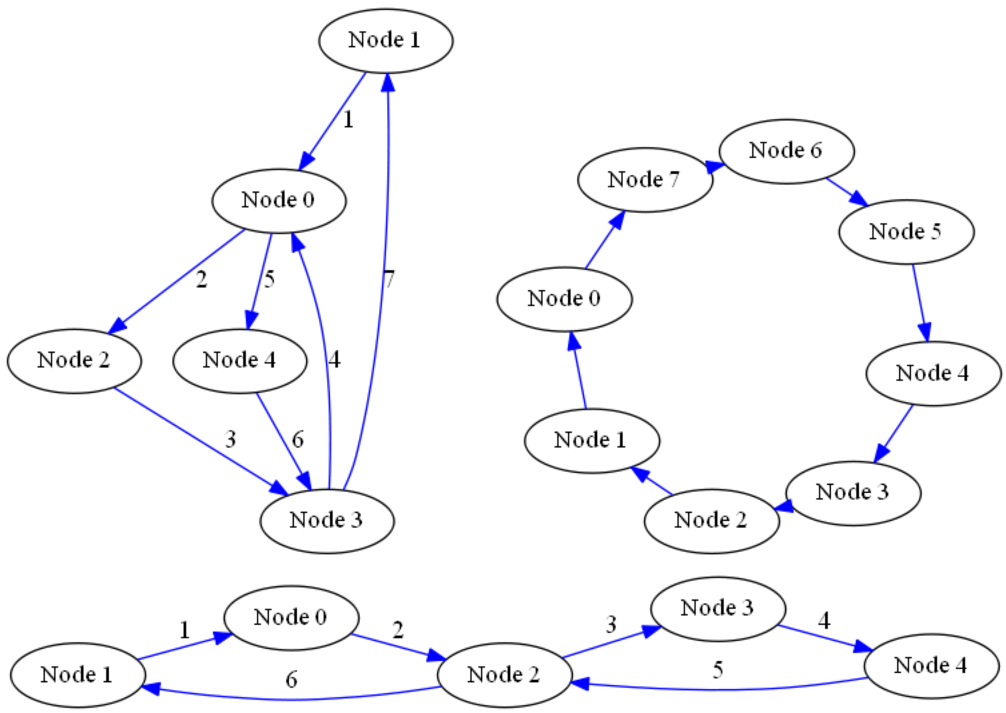
```
# Reconstruct the tour induced by x
def reconstruct_tour(x, start=0):
```

```
xx = deepcopy(x)
if len(xx[start]) == 0: return []

# v is the current vertex, i the precedent, and j the next
# i -> v -> j
i, j = xx[start].popitem()
path = [i, start, j]
i, v = start, j
while j is not None:
    j = find_match_and_remove(xx[v], i)
    path.append(j)
    i, v = v, j
return path[:-1]
```

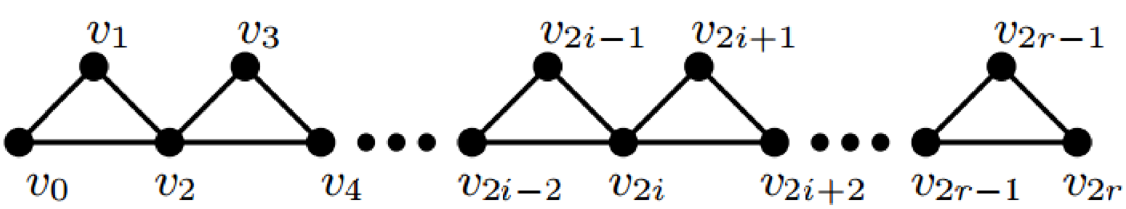
4.1 Some Tests

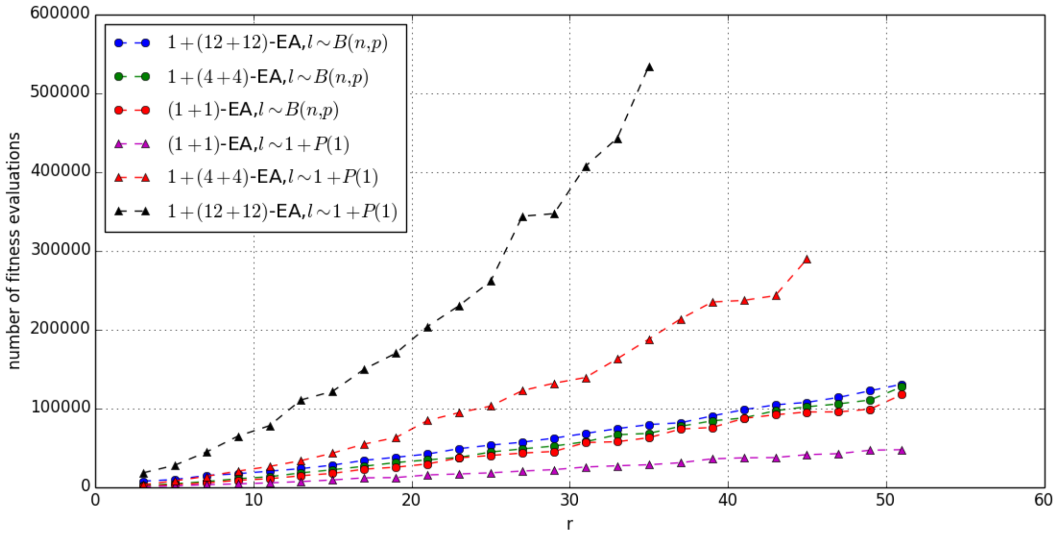
See the file "graph.py" for graph creation and rendering.



Example of tours found by the algorithm

For the following simulations, I compare the average optimization time of the $(1+(\lambda,\lambda))$ GA for l following the $B(n,p)$ distribution and $P(1)$ distribution. As graph instance, I took triangular graphs:





when l follows a binomial distribution, the performance is independant from the value of λ. When l follows a poisson distribution, it's better to choose $\lambda = 1$.

For raw data, see "eulerstats.zip".

5 Own Idea

5.1 The N-Queen's Problem (file "chess.py")

In chess, a queen can move horizontally, vertically, or diagonally. Given a chess board with N rows and N columns, N-Queen's problem asks how to place N queens on the chess board so that none of them can hit any other in one move.

Since the number of queens is equal to the number of rows/columns, in each row there is exactly one queen, and two different queens are placed in two different columns. Therefore a configuration (position of the queens on the chess board) is represented with a permutation of $\{0, ..., N-1\}$. An individual is a N-tuple $x = (x_1, ..., x_N)$, where $0 \leq x_i < N$ is the column of the queen placed on the row i. We start with the N-tuple $(0, ..., N-1)$:

```
x_init = range(n)
```

With this representation, there will be no queens on the same row/column. We only have to check for possible collisions diagonally. The **fitness** function counts the number of those collisions:

```
def fitness(t):
        x = np.zeros((n , n))
        for i, j in enumerate(t):
                x[i][j] = 1

        s = 0
        # diags
        for k in range(-n + 1, n):
                # diag (i, i+k) (n-i-1, i+k)
                q_diag = sum([ x[i][i + k] for i in range(max(0, -k), min(n, n - k))])
                s += max(0, q_diag - 1)
                q_diag = sum([ x[n - i - 1][i + k] for i in range(max(0, -k), min(n, n - k))])
                s += max(0, q_diag - 1)
        return -s
```

The **mutation** operator select l random elements, and permutates them randomly:

```
def mutation(l, x):
        # selection of the bits to mutate
        bits_to_change = random.sample(range(len(x)), l)

        # apply a random permutation to bits_to_change
```

```python
        rand_permutation = list(bits_to_change)
        random.shuffle(rand_permutation)

        x_mut = list(x)

        for i in range(l):
                x_mut[bits_to_change[i]] = x[rand_permutation[i]]

        return x_mut
```

The **crossover** operator select a parent p: x with probabilty c and xx with probability $1 - c$. Then it find an element that is not already present in the child y, add it to y, and then repeat n times:

```python
def crossover(c, x, xx):
        parent = [x, xx]
        parent_choice = bernoulli.rvs(c, size=len(x))
        y = []
        for p in parent_choice:
                # Avoids repeating the same number in the permutation y
                for b in parent[p]:
                        if not (b in y):
                                y.append(b)
                                break
        return y
```

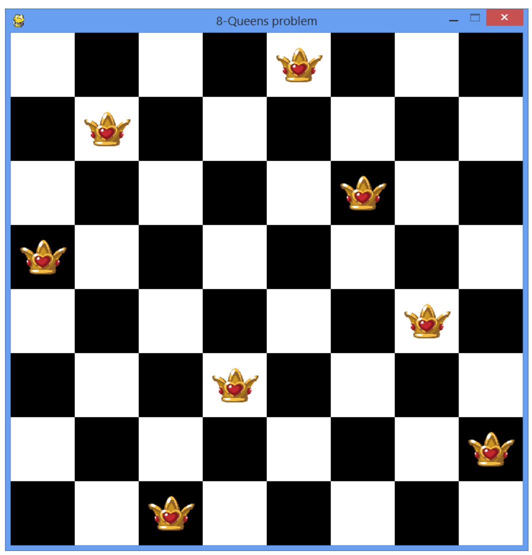


Figure 3: A Valid configuration

(see the file "draw.py" for drawing function)

5.2 2D Image reconstruction (file "2d reconstruct.py")

This program tries to rebuild a 2D grey scale image using only disks. Individuals are strings representing a list of 256 circles $\{(x, y, radius, color, alpha)\}$ in binary format where:

- x and y are the position of the center of the circle
- *color* is an integer between 0 and 255 giving the tone of grey the circle must be filled with
- *alpha* is the transparency

The fitness function is a per-pixel RGB comparison:

```python
def fitness(x):
        draw_spheres(x)
        pixels = array2d(srf)
        return -np.linalg.norm((reference_pixels - pixels) / M)
```

Since we use a string based representation, we can use the default crossover and mutation operators as in the first algorithm.

Here is the result:

Figure 4: Source image

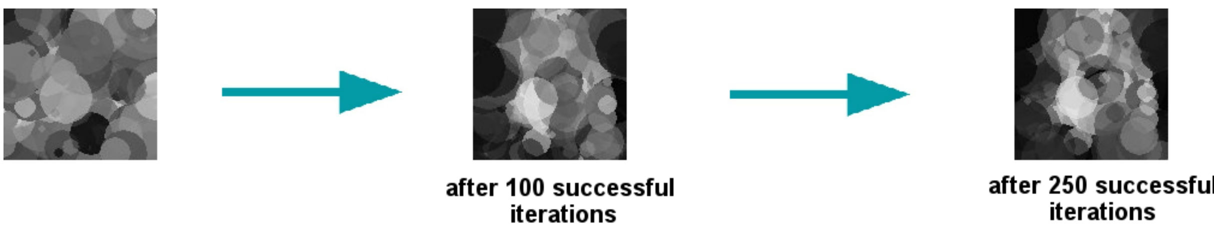


Figure 5: Evolution

5.3 Finance (file "finance.py")

Genetic algorithm can be used in trading. Trading strategies (individuals) can be represented by boolean trees, for instance:

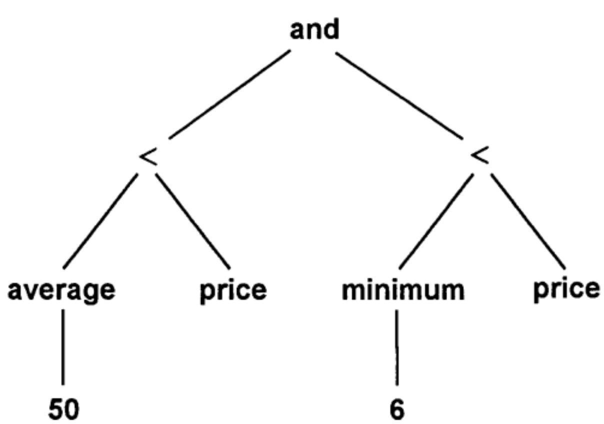


Figure 6: A tree representing a trading strategy

It can be interpreted as a boolean value like this: If:

- the average of the 50 last stock values is less the actual price and the min of the 6 last stock values is less the actual price. then answer True, else False

If the answer is True, then send a BUY signal, and SELL signal otherwise.

See the file **"'rules.py"'** for the tree representaion.

- The fitness is the benefit we gain if we apply this strategy daily during a year to a given share (ORACLE for example). I use the library **pyalgotrade** to calculate the fitness.
- The mutation operator changes some random nodes of the tree.
- The crossover operator takes a sub tree of x and replace it by a sub tree of xx.

The output of the program is a tree like this:

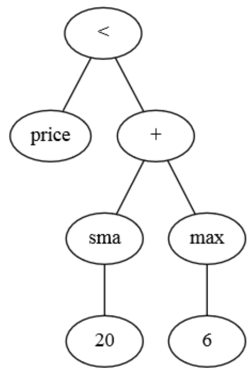


Figure 7: Best trading strategy

This strategy has a fitness of 76$.