

# **BAPC 2023**

Solutions presentation

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The BAPC 2023 jury

October 28, 2023

# A: APT Upgrade

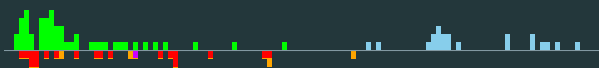
Problem Author: Ragnar Groot Koerkamp



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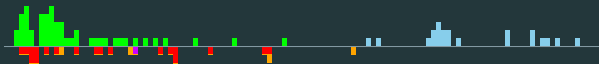


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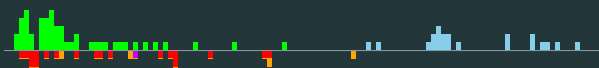
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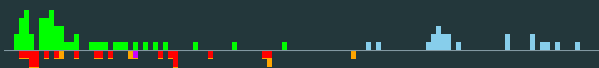
**Observation 2:** The packages underway need to be the next largest, and at 99.9%.

**Solution:** Sort the list, sum the largest  $m + k$  packages, divide by the total sum, multiply by 100:

$$\frac{\sum_{i=1}^{m+k} s_i}{\sum_{i=1}^n s_i} \cdot 100 \quad (\text{assuming } s_i \text{ are sorted from large to small})$$

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Statistics: 95 submissions, 50 accepted, 21 unknown

# B: Battle Bots

Problem Author: Mees de Vries



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**Naive solution:** Just try all possible combinations:  $S, C, SS, SC, CS, CC, SSS, SSC, \dots$ , until you find one that works.

If  $m$  is the answer, this runs in  $\mathcal{O}(m2^m)$ . Since  $m \approx \log_2(n)$ , this is  $\mathcal{O}(n \log(n))$ . Too slow!

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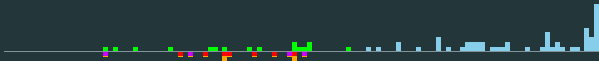
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Statistics: 127 submissions, 50 accepted, 12 unknown



# C: Compressing Commands

Problem Author: Ragnar Groot Koerkamp



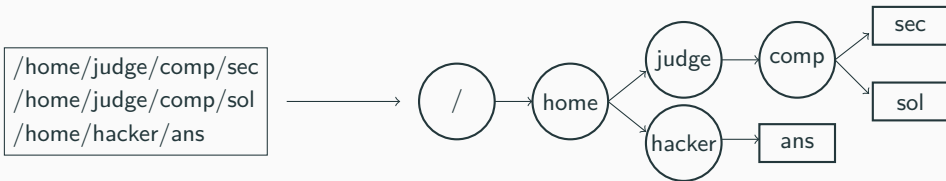
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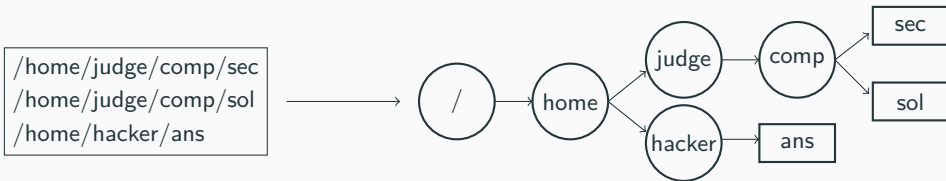


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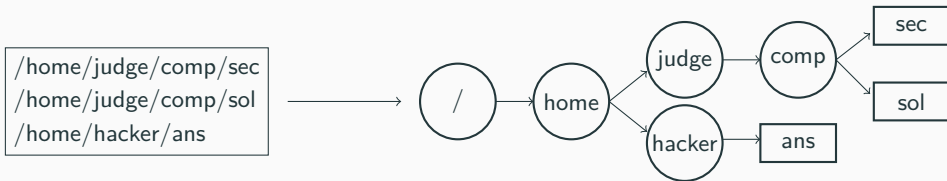
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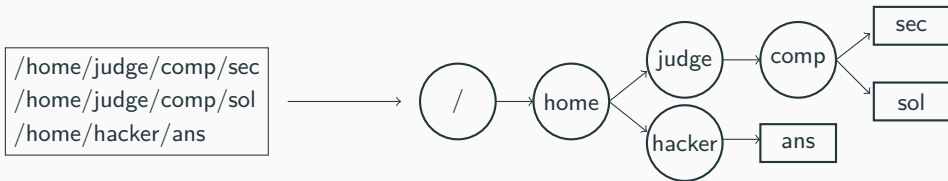
1.  $cost("/") = \#total\_path\_components$ .
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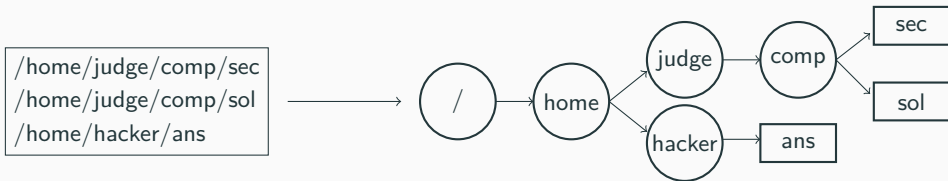
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# D: Democratic Naming

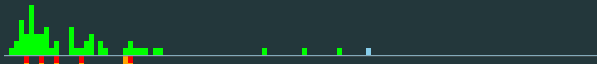
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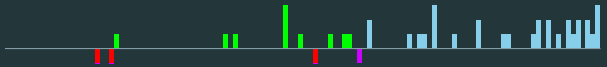
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(spoiler: they solved it! 🎈)

# E: Exam Study Planning

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**Problem:** Given an exam schedule, determine how many exams you can pass with optimal scheduling.

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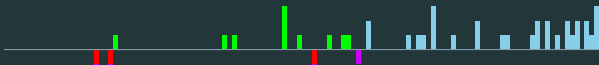
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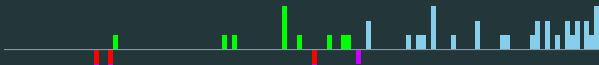
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Use dynamic programming:

$$\text{DP}(i, j) = \begin{cases} x, & \text{max extra study time at } e_i \text{ with } j \text{ exams passed,} \\ -\infty & \text{if it's impossible to pass } j \text{ exams at } e_i. \end{cases}$$

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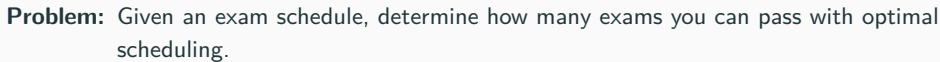
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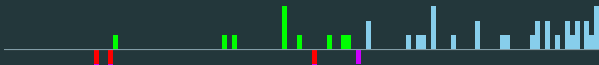
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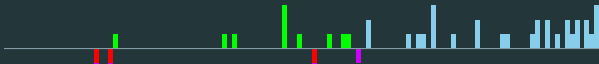
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Take the maximum of these options!

Note: you can only pass exam  $i$  if you have time to prep:

$$DP(i-1, j-1) + s_i - e_{i-1} \geq a_i.$$

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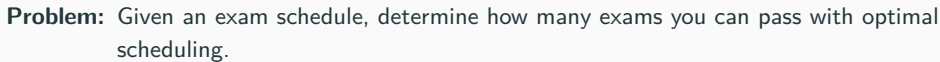
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# F: Funicular Frenzy

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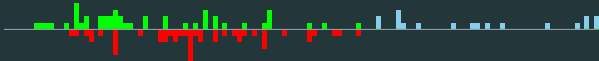


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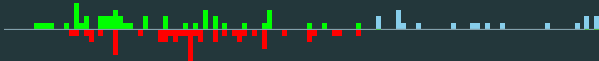
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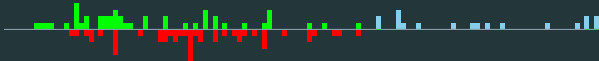
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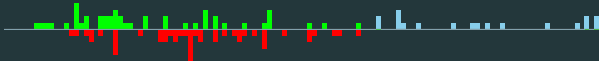
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**Run time:**  $\mathcal{O}(n)$ .

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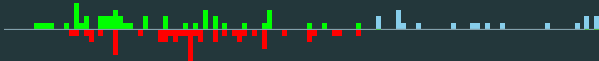
- Start the queue with 0 passengers.
- For every minute  $i$ , add  $a_i$ , save the current queue length, and subtract  $c$ .
- The queue length can not go negative.
- Find the minute for which the queue length was the shortest.

**Run time:**  $\mathcal{O}(n)$ .

**Edge case:** The answer is “impossible” when the current queue length in minute  $i$  is never smaller than  $c \cdot (n - i)$ .

# F: Funicular Frenzy

Problem Author: Ragnar Groot Koerkamp



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**Solution:** Simulation.

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- For every minute  $i$ , add  $a_i$ , save the current queue length, and subtract  $c$ .
- The queue length can not go negative.
- Find the minute for which the queue length was the shortest.

**Run time:**  $\mathcal{O}(n)$ .

**Edge case:** The answer is “impossible” when the current queue length in minute  $i$  is never smaller than  $c \cdot (n - i)$ .

Statistics: 111 submissions, 45 accepted, 18 unknown

# G: Geometry Game

Problem Author: Jorke de Vlas



**Problem:** Determine the *most restrictive* type of quadrilateral from four points.



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**Possible solution:** There are multiple ways of determining the shapes, this is one of them:

- If all four sides have equal length, output “square” if the two diagonals have equal length, else “rhombus”.
- If two pairs of opposite sides each have equal length, output “rectangle” if the two diagonals have equal length, else “parallelogram”.
- If two pairs of adjacent sides each have equal length, output “kite”.
- If two pairs of opposite sides are parallel, output “trapezium”, else “none”.

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**Parallel test:** Check if out-product of two vectors equals zero:

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**Float note:** Calculating the length of an edge ( $\sqrt{x^2 + y^2}$ ) requires 18 digits (59 bits) of precision. double only has 53!

I.e. 64-bit integers (without  $\sqrt{\phantom{x}}$ ) or C++ long double with an epsilon of  $10^{-19}$  works.



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Statistics: 122 submissions, 32 accepted, 48 unknown

# H: Hidden Art

Problem Author: Reinier Schmiermann



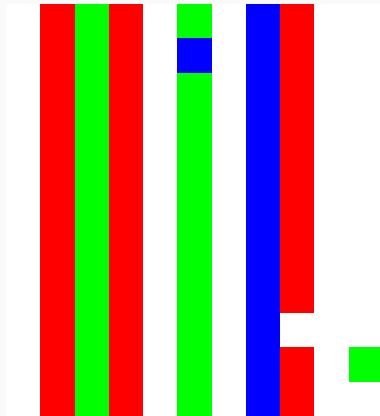
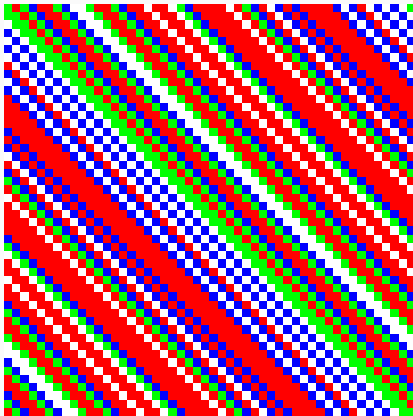
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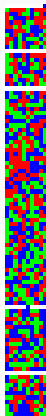


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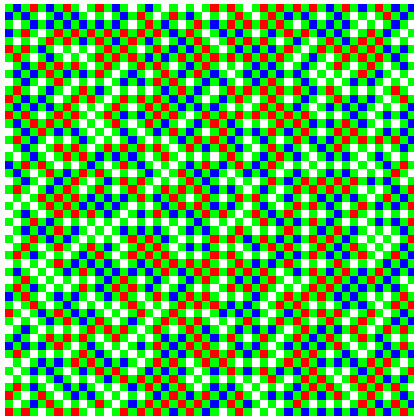


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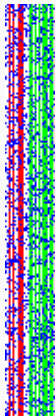
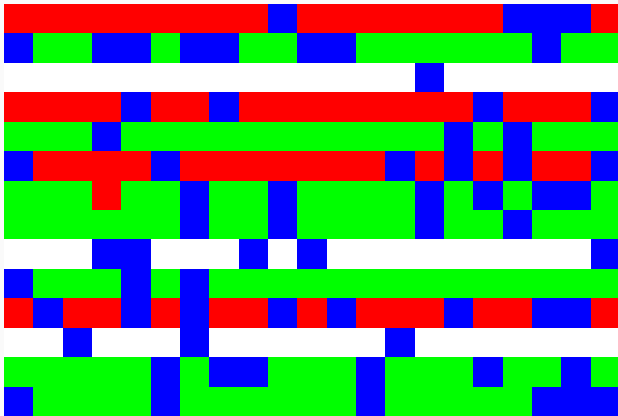


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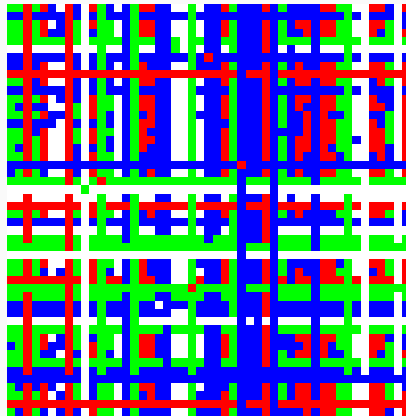
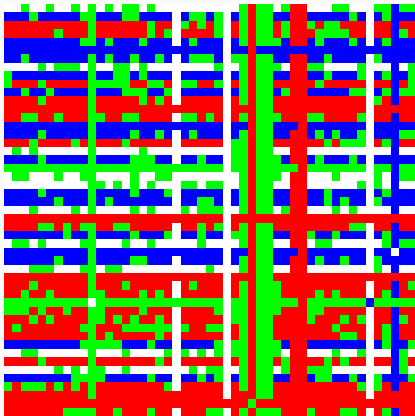


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**Observation:** If you have a solution in the infinite grid, then it forms a rectangle in the original grid.

w	r	w	r	w	r
w	g	w	g	w	g
b	g	b	g	b	g
w	r	w	r	w	r
w	g	w	g	w	g
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**Observation:** However, not all rectangles in the original grid make squares.

g	w	b	w	g	w	b	w
w	w	r	w	w	w	r	w
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w	w	r	w	w	w	r	w

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**Observation:** For an  $x \times y$  rectangle in a  $w \times h$  grid, we can obtain all rectangles  $(x + kh) \times (y + \ell w)$ .

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In the example above: it doesn't work, because  $x - y = 2 - 1 = 1$  while  $\gcd(w, h) = \gcd(4, 2) = 2$ .

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**Naive solution:** For every rectangle in the grid, check if its corners have all four colors, and if the difference between height and width is divisible by  $g = \gcd(w, h)$ .

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**Observation:** There are not that many combinations of colors possible.

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r	b	w	g	r	r
r	w	b	b	g	r
g	r	b	b	w	w
b	b	b	w	g	r
g	r	g	g	r	b
w	w	w	b	w	g
r	w	b	w	g	w
w	r	g	w	b	w
w	r	b	g	w	b

Row 0 (mod 3): { }

Row 1 (mod 3): { }

Row 2 (mod 3): { }

**Solution:** Fix two columns. Then check all colour combinations in those two columns, and store them by their row (mod  $g$ ).

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r	w	b	b	g	r
g	r	b	b	w	w
b	b	b	w	g	r
g	r	g	g	r	b
w	w	w	b	w	g
r	w	b	w	g	w
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b	b	b	w	g	r
g	r	g	g	r	b
w	w	w	b	w	g
r	w	b	w	g	w
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r	w	b	b	g	r
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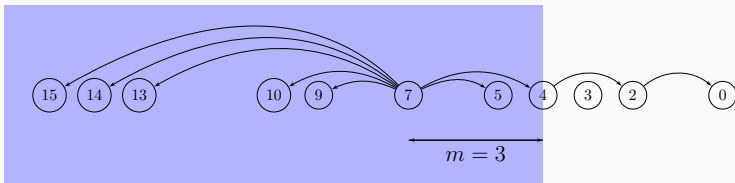
**Run time:**  $\mathcal{O}(hw^2)$  – fast enough, but program efficiently, especially in Python!

Statistics: 63 submissions, 1 accepted, 41 unknown

# I: International Irregularities

Problem Author: Ragnar Groot Koerkamp

**Problem:** Given are  $n \leq 10^5$  countries with ascending infection rates  $r_i$ , and quarantine times  $t_i$ .



- *Hop*: if  $r_j \geq r_i - m$ , go without quarantine (1 day).
- *Jump*: go with quarantine ( $1 + t_j$  days).

Answer  $10^5$  queries: What is the fastest route from  $x$  to  $y$ .

# I: International Irregularities

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**Solution** If  $r_x < r_y$ : We can hop directly, so print 1.

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# I: International Irregularities

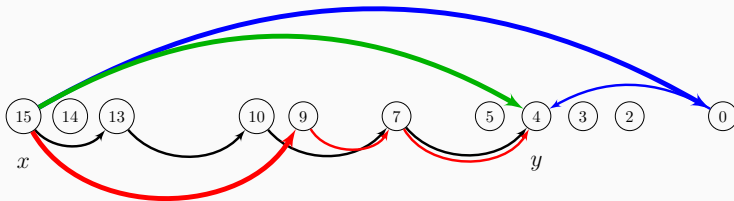
Problem Author: Ragnar Groot Koerkamp

**Solution** If  $r_x < r_y$ : We can hop directly, so print 1.

**Observation** Jump at most once, and only in the very beginning.

If  $r_x > r_y$ , four options:

1. *Hop* to the right up to  $m$  at a time.
2. *Jump* directly to  $y$ .
3. *Jump* right of  $y$ , then hop left once.
4. *Jump* left of  $y$ , then hop right some times.



# I: International Irregularities

Problem Author: Ragnar Groot Koerkamp



**Case 1:** Hop to the right up to  $m$  at a time.

Define  $H_k(i)$  as the rightmost country reachable within  $2^k$  hops.

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To compute hops from  $x$  to  $y$ :

Try to go right  $2^k$  steps without overshooting  $y$ , for decreasing  $k$ .



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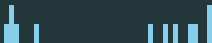
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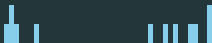
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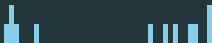
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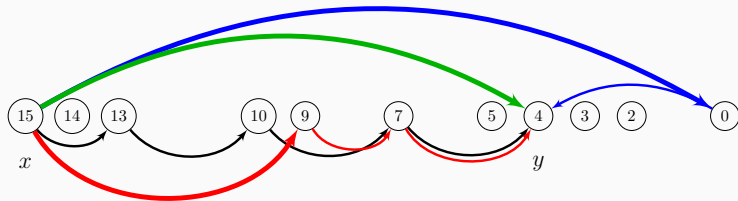
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Add one for the hop.

# I: International Irregularities

Problem Author: Ragnar Groot Koerkamp

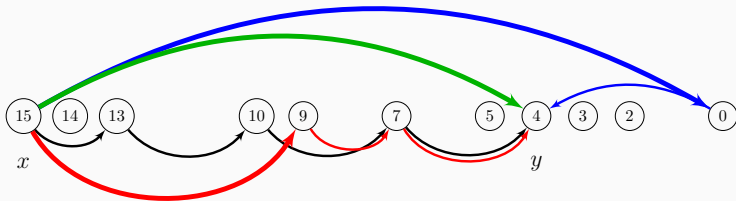
**Case 4:** Hop to the left of  $y$ , then hop right some times.



Iterate through the countries from left to right, keeping track of the best country to jump to first.



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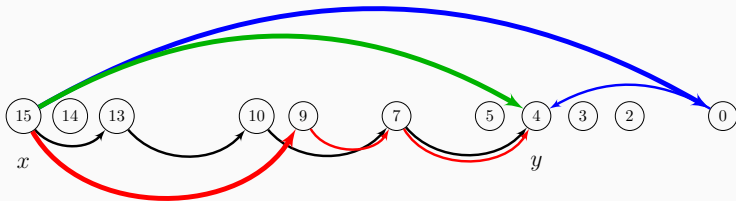
For each country, either:

- jump to the stored best and hop from there, or
- jump directly and update the stored best.

# I: International Irregularities

Problem Author: Ragnar Groot Koerkamp

**Case 4:** Hop to the left of  $y$ , then hop right some times.



Iterate through the countries from left to right, keeping track of the best country to jump to first.

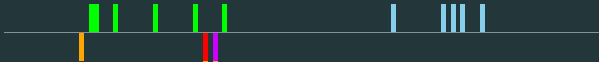
For each country, either:

- jump to the stored best and hop from there, or
- jump directly and update the stored best.

Statistics: 14 submissions, 0 accepted, 12 unknown

# J: Jungle Job

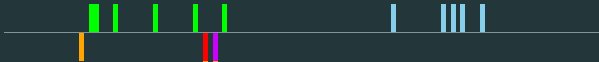
Problem Author: Jorke de Vlas and Mike de Vries



**Problem:** Given a tree, can you find the number of connected subtrees of each size? (modulo  $10^9 + 7$  because the answer is huge).

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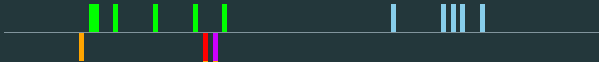


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**Observation:** Let's define  $F(v, c)$  - the number of connected subtrees, that have node  $v$  as the root and have exactly  $c$  nodes.

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$$\sum_{i \in V} F(i, c).$$

**Solution:** Use dynamic programming

**Problem Author:** Jorke de Vlas and Mike de Vries



**Base case:** If  $v$  is a leaf:

$$F(v, c) = 1 \text{ if } c \text{ is } 0 \text{ or } 1.$$
$$F(v, c) = 0 \text{ if } c \geq 2.$$

# J: Jungle Job

Problem Author: Jorke de Vlas and Mike de Vries

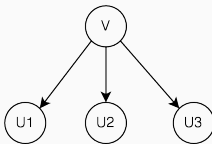


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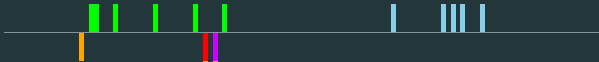
**DP idea:** Consider the following subtree:





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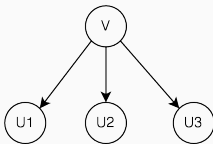


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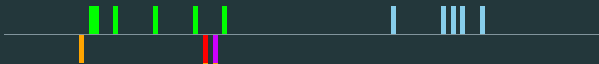


To calculate  $F(v, c)$  we need to consider every way to distribute  $c - 1$  remaining nodes among three child subtrees of  $v$ :

$$F(v, c) = \sum_{c_1=0}^{c-1} \sum_{c_2=0}^{c-1-c_1} F(u_1, c_1) F(u_2, c_2) F(u_3, c - c_1 - c_2)$$

# J: Jungle Job

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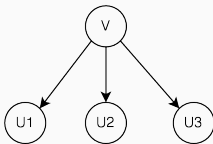


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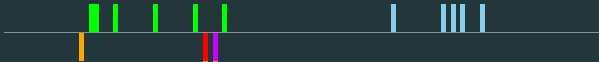
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**Problem:** For a node with many children  $m$ , this will hit the time limit:

$$F(v, c) = \sum_{c_1=0}^{c-1} \sum_{c_2=0}^{c-1-c_1} \dots \sum_{c_{m-1}=0}^{c-1-\dots} \prod_{i=1}^m F(u_i, c_i)$$

# J: Jungle Job

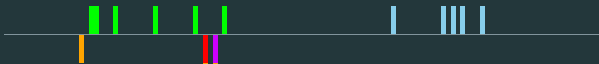
Problem Author: Jorke de Vlas and Mike de Vries



**Fix:** Introduce  $F'(v, i, c)$  - the number of connected subtrees, that have node  $v$  as the root, have exactly  $c$  nodes and only include first  $i$  children of node  $v$ .

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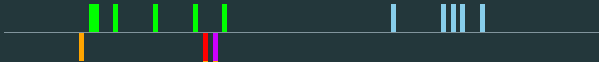
Base cases for node  $v$  that has  $m$  children:

$$F(v, c) = F'(v, m, c),$$

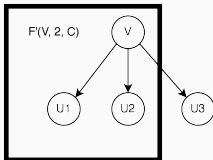
$$F'(v, 1, c) = F(u_1, c - 1),$$

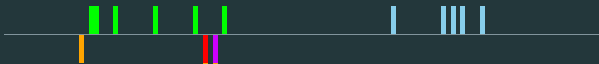
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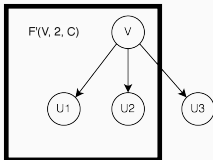


**DP** Let's calculate  $F'(v, 2, c)$  for this graph:

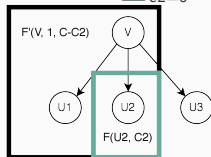




**DP** Let's calculate  $F'(v, 2, c)$  for this graph:



For that we just need to decide how many nodes will be in the subtree of the second child and then we can recurse:  $F'(v, 2, c) = \sum_{c_2=0}^{c-1} F'(v, 1, c - c_2)F(u_2, c_2)$



**Problem Author:** Jorke de Vlas and Mike de Vries

**Runtime:** Computing  $F'(v, i, c)$  for all  $c$  takes  $O(|u_i| \cdot \sum_j |u_j|)$  time, where  $|u_i|$  denotes the size of the subtree at  $u_i$ .

**Problem Author:** Jorke de Vlas and Mike de Vries

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Total time spent at  $|v|$  is  $O(\sum_i \sum_j |u_i| \cdot |u_j|)$ .



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**Observation:**  $\sum_i \sum_j |u_i| \cdot |u_j|$  is the number of pairs of nodes with lowest common ancestor  $v$ .

# J: Jungle Job

Problem Author: Jorke de Vlas and Mike de Vries



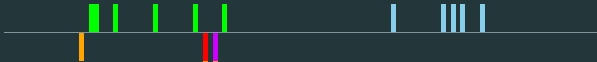
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Since every pair of nodes has one LCA, the total runtime is  $O(n^2)$ .

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Statistics: 14 submissions, 6 accepted, 5 unknown

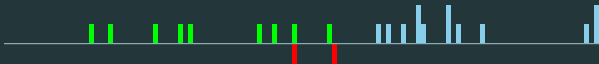
## D – Kindergarten Excursion

- If a 1 is to the left of a 0, these two have to be swapped at some point. The same is true for 2/0 and 2/1.
- Process the sequence from left to right. Keep track of the number of 1's and 2's to the left of current number and calculate the result.
- Watch out for overflow.
- Linear time solution.

Statistics: 60 submissions, 12 correct, first at 1:13:57.

# L: Locking Doors

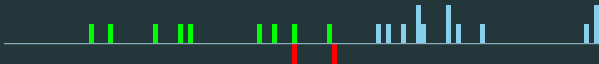
Problem Author: Jorke de Vlas and Mike de Vries



**Problem:** Given the layout of a building, with doors that lock from only one side, how many exits on the outside do we need to close all doors?

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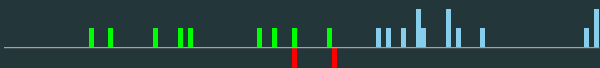


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**Observation:** If a room  $a$  has an exit, then which doors can we close using that exit?

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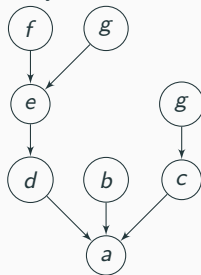
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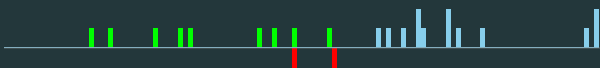
Write  $b \rightarrow a$  to mean there is a door you can close from side  $a$ . Then consider:



If there is an exit at  $a$ , you can close all these doors: just start at any leaf, close that door, and repeat.

# L: Locking Doors

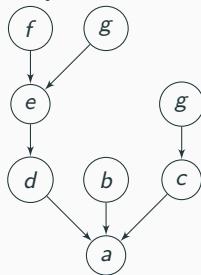
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If there is an exit at  $a$ , you can close all these doors: just start at any leaf, close that door, and repeat.

Maybe you can close *more* doors, but definitely these ones.

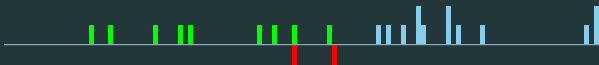


**Problem Author:** Jorke de Vlas and Mike de Vries

**Strongly connected:** For  $a, b$  nodes, if you can walk from  $a$  to  $b$  via arrows, and also  $b$  to  $a$ , we call  $a$  and  $b$  *strongly connected*.

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**SCC:** We can collect strongly connected nodes into groups, called *strongly connected components*.

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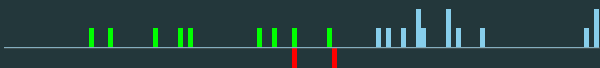


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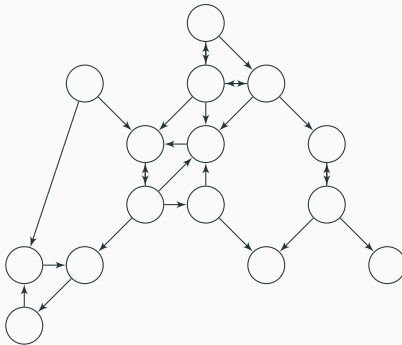
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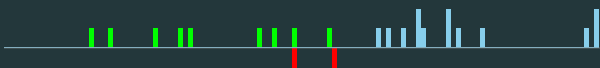
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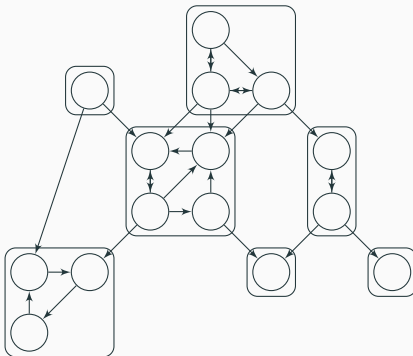
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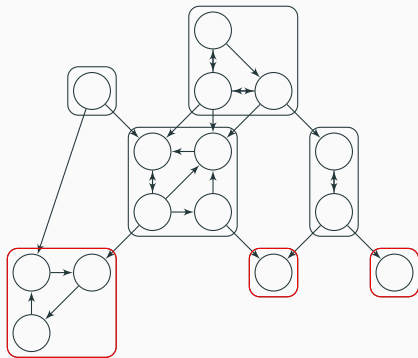
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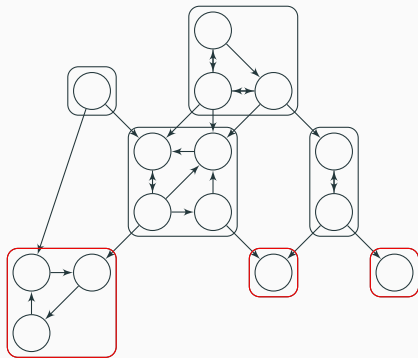
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**Necessary** How many exits does this graph need? We need *at least one* in the (red) components without outgoing edges. Otherwise you can never leave it once you close the last incoming door.

# L: Locking Doors

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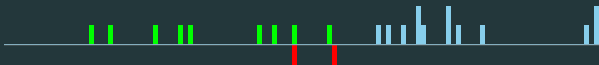


**Necessary** How many exits does this graph need? We need *at least one* in the (red) components without outgoing edges. Otherwise you can never leave it once you close the last incoming door.

**Sufficient** That is also *enough exits*: from any node you can follow the arrows to one of those components, which we saw is enough to close all doors.

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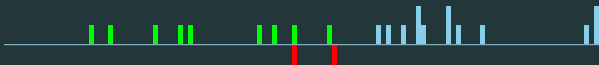


**Solution** Find the strongly connected components, e.g. with Tarjan's algorithm. Output the number of SCCs without outgoing edges.



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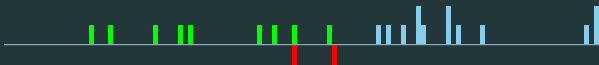


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Since we only have to count root-SCCs, simpler algorithms are also possible.

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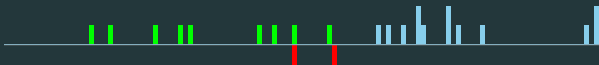
**Solution** Find the strongly connected components, e.g. with Tarjan's algorithm. Output the number of SCCs without outgoing edges.

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**Note** Be careful with recursion on python. Use a stack instead.

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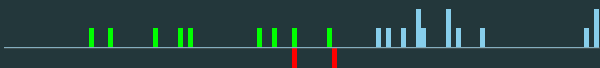
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**Runtime** Runs in  $\mathcal{O}(m)$ .

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Statistics: 24 submissions, 9 accepted, 13 unknown