NCPC 2023 Presentation of solutions

2023-10-07

Problems prepared by

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Test solver

Björn Martinsson

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October 5, 2024

Problems prepared by

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Problem

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Solution

ullet A good idea is to write out the string for K=3 or 4 on paper, and look for patterns.

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- Guess: the string is generated by first letting s = 0, and then applying s = s * k + 1 k times, and then append infinitely many ones.
- ② There are now some recursive ways of counting the number of ones in $\mathcal{O}(\log(N))$.

Problem

Given bounds a,b how many partitions ρ of n satisfy $|\rho| \leq b$, $\max \rho \leq a$ and $\rho_i + i$ is constant across all indices i such that $\rho_i \neq \rho_{i+1}$.

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- First consider the case without the bounds a, b.
- ② We define f(n, d) as the number of partitions of n with $\rho_i + i = d + 1$ for the aforementioned indices.
- The problem can now be solved using dynamic programming.

Solution

• Consider the maximum index k such that $\rho_1=\rho_k$ and $\rho_k\neq\rho_{k+1}$. Then $\rho_k+k=d+1$. Then by cutting off these front values we remove k values and a sum of k(d+1-k). This gives us the recurrence

$$f(n,d) = \sum_{k=1}^{d} f(n-k(d+1-k), d-k)$$

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② This is $\mathcal{O}(n^3)$ which is not quite good enough. But f(n-k(d+1-k),d-k) has the first argument < 0 for all but the first and last \sqrt{n} terms. So we can skip those and get a time complexity of $\mathcal{O}(n^{2.5})$.

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- **3** Then we simply subtract all partitions that have $|\rho| > a$ or $\max \rho > b$, then add back all the ones that violate both to get the right answer by inclusion-exclusion.

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- Then we simply subtract all partitions that have $|\rho| > a$ or $\max \rho > b$, then add back all the ones that violate both to get the right answer by inclusion-exclusion.

Statistics at 4-hour mark: 14 submissions, 0 accepted, first after ??:??

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- For each digit in a number, check if it is smaller than anything to the right. In that case, subtract it.
- **1** Make sure this runs faster than $\mathcal{O}(L^2)!$
- This can be done by looping from the end of the string, and keeping track of the largest digit seen so far.

Problem

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Statistics at 4-hour mark: 214 submissions, 23 accepted, first after 00:10

Problem

There are N types of components, each with f_i copies and placement time t_i . In one move you can place two components of different types i and j in $\max(t_i, t_j)$ nanoseconds. Find the minimum total time to place all components.

Problem

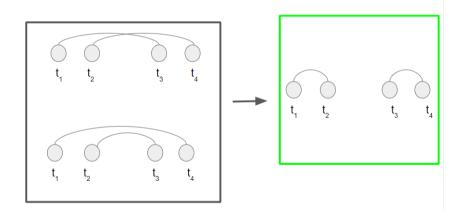
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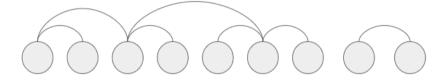
- Greedily place two component types with maximum t_i ? Doesn't work on second sample.
- ② If t_i values are similar, then it is more important to match all components.

Solution

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- ② DP(i, u) =answer of first i components, if there are u unmatched components that we already "paid" for.
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- Write out the DP transitions carefully. They can be reduced to RMQ:s on a sliding window.
- **3** Use segment tree, RMQ, or priority queue in the transition: $\mathcal{O}(NF \log(F))$.
- **o** Bonus: can you solve it in $\mathcal{O}(NF)$ (without fancy RMQ)?

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- ① Detect the outer face (the leftmost vertex (break even on y coordinate) and its neighbor with angle highest $\leq 90^{\circ}$)
- Use the shoelace algorithm for computing the area of each face/polygon.

H — Heroes of Velmar

Problem

Given the description of a card game and a final state of the game, implement the rules for resolving which player won the match.

- Process each player and location pair individually to apply abilities and add all the power levels of the cards together.
 - $oldsymbol{0}$ If the card is Seraphina and k is the number of friendly cards at the location, add k-1 power to the location.
 - If the card is Thunderheart and the location has 4 friendly cards, add 6 power to the location.
 - If the card is Zenith, and the location is the center location, add 5 power to the location.
- Count the number of locations where each player wins.
- 3 If one of the players wins more locations than the other, that player wins.

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- We now solve the characters and numbers separately.
- **3** For the letters we use a lazy propagation segment tree that allows for a range update where the first element is incremented by b, the next by a+b, the third by 2a+b and so on.

• The segment tree will keep track of the number of occurrences of each character in the input string as we iterate over the suffix array. Let S be the i-th element of the suffix array and L be the (i-1)-st element of the longest common prefix array.

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- Then n-S-L new prefixes start at position S. The first L possible substrings starting at S are duplicates. So we increment the segment tree by n-S-L at positions S through S+L-1. Then we increment the segment tree by n-S-L, n-S-L-1, n-S-L-2, and so on at positions S+L through n-1. At the end of this all we can read off the number of occurrences of each character from the segment tree.

• Finally we consider the digits. For this we use a lazy propagation segment tree that allows incrementing values on a range and then a collect operation that both counts the digits in the values on a range before then zeroing them out.

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- ② The other is to do things more by hand. Let b(x) be the number of binary digits in x. Then to check if he goes broke before his first payment, check whether b(m) < d. To check whether he never goes broke, check whether $d \le b(s)$. Then if b(d) > 12 just print b(m). Finally simulate the salary periods one by one and print when he runs out of money.

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Statistics at 4-hour mark: 575 submissions, 81 accepted, first after 00:12

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Partition N items among M scouts so that no one has more than two items and the maximum total size is minimised.

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- ② If $N \leq M$, the answer is the size of largest item.
- If N > M, pair up the M+1:th largest item with the M:th largest, the M+2:th largest item with the M-1:th largest, and so on until we have M groups of pairs and single elements. Report the maximum size.

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- If N > M, pair up the M+1:th largest item with the M:th largest, the M+2:th largest item with the M-1:th largest, and so on until we have M groups of pairs and single elements. Report the maximum size.
- This greedy strategy works since for any four sizes $a \le b \le c \le d$, the best way to divide them in two pairs minimising the maximum is a + d and b + c.

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Statistics at 4-hour mark: 687 submissions, 152 accepted, first after 00:04