§3.1 线性方程组的消元解法

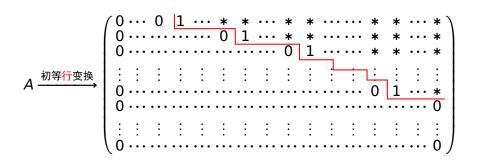
数学系 梁卓滨

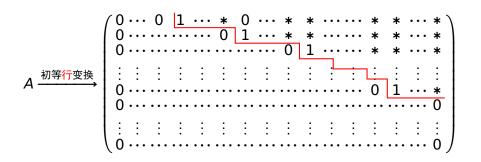
2018 - 2019 学年上学期

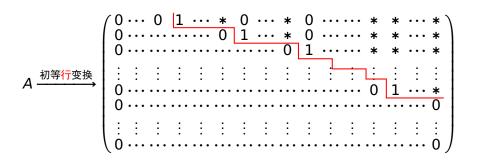


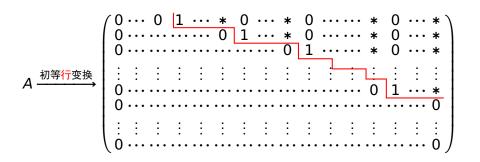


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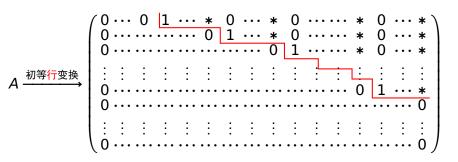












后者称为简化的阶梯型矩阵。



记号

考虑 n 个未知量 m 个方程的线性方程组:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

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可以,等价地,改写成矩阵形式

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

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整个方程组的信息包含在:

含性:
$$(A : b) = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{pmatrix}$$



$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

可以,等价地,改写成矩阵形式

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整个方程组的信息包含在:

增广矩阵
$$(A:b) = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{pmatrix}$$



$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 4x_1 + 7x_2 - x_3 = -1 \\ 3x_1 + 4x_2 - 2x_3 = 3 \end{cases}$$

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例 解方程组

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$$\begin{cases} x_1 & -2x_3 = 5 \\ x_2 + & x_3 = -3 \\ & 0 = 0 \\ & 0 = 0 \end{cases} \Rightarrow$$



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$$(A:b) = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 0 & -1 \\ 4 & 7 & -1 & -1 \\ 3 & 4 & 2 & 3 \end{pmatrix}$$

$$\begin{cases} x_1 & -2x_3 = 5 \\ x_2 + & x_3 = -3 \\ & 0 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 5 + 2x_3 \\ x_2 = -3 - x_3 \end{cases}$$

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$$(3x_1 + 4x_2 - 2x_3 = 3) \xrightarrow{(4)^{-3} \times (1)} (x_2 + x_3 = -3) \xrightarrow{(4)^{-(2)}} (0 = 0)$$

$$(A:b) = \begin{pmatrix} \boxed{1} & 1 & & & & & \\ 1 & & & & & \\ \hline{1} & & & & \\ \hline{1} & & & & & \\ \hline{1} & & & & \\$$

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$$\begin{cases} x_1 & -2x_3 = 5 \\ x_2 + & x_3 = -3 \\ & 0 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 5 + 2x_3 \\ x_2 = -3 - x_3 \end{cases}$$



例 解方程组

$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 4x_1 + 7x_2 - x_3 = -1 \\ 3x_1 + 4x_2 - 2x_3 = 3 \end{cases} \xrightarrow{(2)-(1)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \\ 3x_2 + 3x_3 = -9 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(1)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \end{cases} \xrightarrow{(4)-(2)} \begin{cases} x_1 + x$$

$$(A:b) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 0 & -1 \\ 4 & 7 & -1 & -1 \\ 3 & 4 & 2 & 3 \end{pmatrix} \xrightarrow[r_4 - 3r_1]{r_3 - 4r_1} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & 1 & -3 \\ 0 & 3 & 3 & -9 \\ 0 & 1 & 1 & -3 \end{pmatrix} \xrightarrow[r_4 - r_2]{r_1 - r_2} \begin{pmatrix} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & -3 \\ 0 & 3 & 3 & -9 \\ 0 & 1 & 1 & -3 \end{pmatrix}$$

$$\begin{cases} x_1 & -2x_3 = 5 \\ x_2 + & x_3 = -3 \\ & 0 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 5 + 2x_3 \\ x_2 = -3 - x_3 \end{cases}$$



例 解方程组

$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 4x_1 + 7x_2 - x_3 = -1 \\ 3x_1 + 4x_2 - 2x_3 = 3 \end{cases} \xrightarrow{(3)-4\times(1)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \\ 3x_2 + 3x_3 = -9 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \\ x_2 + x_3 = -3 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x_1 + x_2 - x_3 = 2 \end{cases} \xrightarrow{(3)-3\times(2)} \begin{cases} x$$

$$(A:b) = \begin{pmatrix} \boxed{1} & \boxed{1} & \boxed{1} & \boxed{1} & \boxed{1} & \boxed{1} \\ \boxed{0} & \boxed{1} & \boxed{1} & \boxed{3} \\ \boxed{0} & \boxed{3} & \boxed{3} & \boxed{9} \\ \boxed{0} & \boxed{1} & \boxed{1} & \boxed{3} \\ \boxed{0} & \boxed{1} & \boxed{1} & \boxed{3} \\ \boxed{0} & \boxed{1} & \boxed{1} & \boxed{3} \\ \boxed{0} & \boxed{0} & \boxed{0} & \boxed{0} \\ \boxed{0}$$

所以

$$\begin{cases} x_1 & -2x_3 = 5 \\ x_2 + & x_3 = -3 \\ & 0 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 5 + 2x_3 \\ x_2 = -3 - x_3 \end{cases}$$

自由变量

$$\begin{cases} x_1 - 2x_3 = 5 \\ x_2 + x_3 = -3 \\ 0 = 0 \end{cases}$$

$$\begin{array}{c}
0 = 0 \\
0 = 0
\end{array}$$

步骤:

- 1. $Ax = b \implies (A : b) \xrightarrow{\eta + \eta + \eta}$ 简化的阶梯型矩阵
- 2. 确定主元、自由变量

步骤:

- 1. $Ax = b \implies (A \cdot b) \xrightarrow{\text{formal parts}}$ 简化的阶梯型矩阵
- 2. 确定主元、自由变量

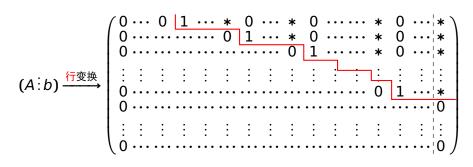
例如



步骤:

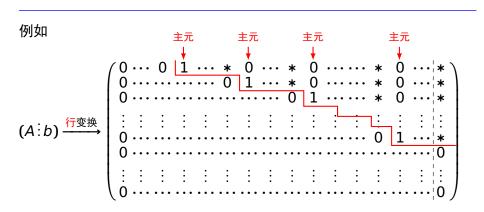
- 1. $Ax = b \implies (A \cdot b) \xrightarrow{\text{formal parts}}$ 简化的阶梯型矩阵
- 2. 确定主元、自由变量

例如



步骤:

- 1. $Ax = b \implies (A \cdot b) \xrightarrow{\text{Most Top}}$ 简化的阶梯型矩阵
- 2. 确定主元、自由变量

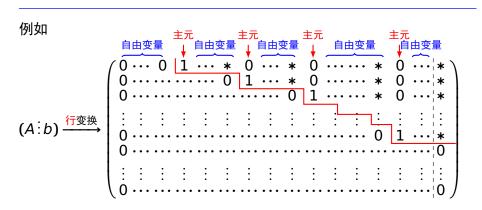


初等行变换求解线性方程组

步骤:

1.
$$Ax = b \implies (A \cdot b) \xrightarrow{\text{formal formal for$$

2. 确定主元、自由变量



例 1 解方程组: $\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$

例 1 解方程组: $\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$

例 1 解方程组: $\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$ $\begin{pmatrix} 1 & 1 - 1 & 2 \\ 1 & 2 & 0 & -1 \\ 2 & 5 & 1 & -5 \\ -2 & -3 & 1 & -1 \end{pmatrix}$

§3.1

线性方程组的消元解法

例 1 解方程组:
$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 0 & -1 \\ 2 & 5 & 1 & -5 \\ -2 & -3 & 1 & -1 \end{pmatrix} \xrightarrow[r_3 - 2r_1]{r_3 - 2r_1}{r_4 + 2r_1}$$

例 1 解方程组:
$$\begin{cases} x_1 + x_2 - x_3 = 0 \\ x_1 + 2x_2 = 0 \\ 2x_1 + 5x_2 + x_3 = 0 \end{cases}$$

例 1 解方程组:
$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 0 & -1 \\ 2 & 5 & 1 & -5 \\ -2 & -3 & 1 & -1 \end{pmatrix} \xrightarrow[r_3 - 2r_1]{r_3 - 2r_1}{r_4 + 2r_1} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 1 & 1 & -1 & 2 \\ 1 & 1 & 1 & -1 & 2 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

例 1 解方程组:
$$\begin{cases} x_1 + x_2 - x_3 = 0 \\ x_1 + 2x_2 = 0 \\ 2x_1 + 5x_2 + x_3 = 0 \end{cases}$$

例 1 解方程组:
$$\begin{cases} 2x_1 + 5x_2 + x_3 = -1 \\ -2x_1 - 3x_2 + x_3 = -1 \\ 1 & 1 - 1 \\ 2 & 1 - 1 \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & 1 - 1 & 2 \\ 1 & 2 & 1 - 1 \\ 1 & 2 & 1 - 1 \end{pmatrix} \frac{r_2 - r_1}{r_2 - r_1}$$

例 1 解方程组:
$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 0 & -1 \\ 2 & 5 & 1 & -5 \\ -2 & -3 & 1 & -1 \end{pmatrix} \xrightarrow[r_3 - 2r_1]{r_3 - 2r_1} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & 1 & -3 \end{pmatrix}$$

例 1 解方程组:
$$\begin{cases} x_1 + x_2 - x_3 = 0 \\ x_1 + 2x_2 = 0 \\ 2x_1 + 5x_2 + x_3 = 0 \end{cases}$$

例 1 解方程组:
$$\begin{cases} x_1 + 2x_2 &= -x_1 + 2x_2 &= -x_2 + x_3 = -x_3 + x_3 =$$

例 1 解方程组:
$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 0 & -1 \\ 2 & 5 & 1 & -5 \\ -2 & -3 & 1 & -1 \end{pmatrix} \xrightarrow[r_4+2r_1]{r_3-2r_1} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & 1 & -1 & 2 \\ 0 & 3 & 3 & -9 \end{pmatrix}$$



1 解方程组:
$$\begin{cases} 2x_1 + 5x_2 + x_3 = -4 \\ -2x_1 - 3x_2 + x_3 = -4 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 2 \\ 1 & 1$$

例 1 解方程组:
$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 0 & -1 \\ 2 & 5 & 1 & -5 \\ -2 & -3 & 1 & -1 \end{pmatrix} \xrightarrow[r_3 - 2r_1]{r_3 - 2r_1} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & -1 & -1 & 3 \end{pmatrix}$$

例 1 解方程组:
$$\begin{cases} x_1 + x_2 - x_3 = x_1 \\ x_1 + 2x_2 = x_2 \\ 2x_1 + 5x_2 + x_3 = x_3 \end{cases}$$

例 1 解方程组:
$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 0 & -1 \\ 2 & 5 & 1 & -5 \\ -2 & -3 & 1 & -1 \end{pmatrix} \xrightarrow[r_4 + 2r_1]{r_3 - 2r_1} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & 3 & 3 & -9 \\ 0 & -1 & -1 & 3 \end{pmatrix}$$



例 1 解方程组: $\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$ $(A:b) = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 0 & -1 \\ 2 & 5 & 1 & -5 \\ -2 & -3 & 1 & -1 \end{pmatrix} \xrightarrow[r_4+2r_1]{r_2-r_1} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 3 & 3 & -9 \\ 0 & -1 & -1 & 3 \end{pmatrix}$

$$\xrightarrow{r_1-r_2}$$

$$\xrightarrow{r_3-3r_2}$$



例 1 解方程组:
$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 0 & -1 \\ 2 & 5 & 1 & -5 \\ -2 & -3 & 1 & -1 \end{pmatrix} \xrightarrow[r_3 - 2r_1]{r_3 - 2r_1} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 3 & 3 & -9 \\ 0 & -1 & -1 & 3 \end{pmatrix}$$

$$\xrightarrow[r_3-3r_2]{r_1-r_2} \left(\begin{array}{ccc|c} 0 & 1 & 1 & -3 \end{array}\right)$$



例 1 解方程组:
$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 0 & -1 \\ 2 & 5 & 1 & -5 \\ -2 & -3 & 1 & -1 \end{pmatrix} \xrightarrow[r_3 - 2r_1]{r_3 - 2r_1} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 3 & 3 & -9 \\ 0 & -1 & -1 & 3 \end{pmatrix}$$



例 1 解方程组:
$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & 1 & -1 & 2 \\ 1 & 2 & 0 & -1 \\ 2 & 5 & 1 & -5 \\ -2 & -3 & 1 & -1 \end{pmatrix} \xrightarrow[r_3 - 2r_1]{r_3 - 2r_1} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 3 & 3 & -9 \\ 0 & -1 & -1 & 3 \end{pmatrix}$$

$$\begin{array}{c|ccccc}
 & -2 & -3 & 1 & -1 & 7 & \frac{1}{r_4} \\
\hline
 & \frac{r_1 - r_2}{r_3 - 3r_2} & \begin{pmatrix} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\
 & \frac{r_1 + r_2}{r_4} & \frac{1}{r_4} & \frac{1}{r_4} & \frac{1}{r_4} & \frac{1}{r_4} \\
\end{array}$$

例 1 解方程组:
$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$$

$$\begin{pmatrix} 1 & 1 & -1 & 2 \\ 2 & 5 & 1 & -5 \\ -2 & -3 & 1 & -1 \end{pmatrix} \xrightarrow[r_3 - 2r_1]{r_3 - 2r_1} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 3 & 3 & -9 \\ 0 & -1 & -1 & 3 \end{pmatrix}$$

$$\begin{array}{c|ccccc}
 & r_{1} - r_{2} \\
\hline
 & r_{1} - r_{2} \\
\hline
 & r_{3} - 3r_{2} \\
 & r_{4} + r_{2}
\end{array}
\begin{pmatrix}
1 & 0 & -2 & 5 \\
0 & 1 & 1 & -3 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}$$



例 1 解方程组:
$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$$

$$\begin{pmatrix} 1 & 1 & -1 & 2 \\ 2 & 5 & 1 & -5 \\ -2 & -3 & 1 & -1 \end{pmatrix} \xrightarrow[r_3 - 2r_1]{r_3 - 2r_1} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 3 & 3 & -9 \\ 0 & -1 & -1 & 3 \end{pmatrix}$$

例 1 解方程组:
$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$$

$$\begin{pmatrix} 1 & 1 & -1 & 2 \\ 2 & 5 & 1 & -5 \\ -2 & -3 & 1 & -1 \end{pmatrix} \xrightarrow[r_3 - 2r_1]{r_3 - 2r_1} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 3 & 3 & -9 \\ 0 & -1 & -1 & 3 \end{pmatrix}$$

$$\xrightarrow[r_4+r_2]{r_1-r_2} \begin{pmatrix} \boxed{1} & 0 & -2 & | & 5 \\ \boxed{0} & \boxed{1} & 1 & | & -3 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

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-2x_1 - 3x_2 & x_3 & - \\
1 & 1 & -1 & 2 \\
1 & 2 & 0 & -1 \\
2 & 5 & 1 & -5 \\
-2 & -3 & 1 & -1
\end{pmatrix} \xrightarrow[r_4+2r_1]{r_2-r_1} \begin{pmatrix}
1 & 1 & -1 & 2 \\
0 & 3 & 3 & -9 \\
0 & -1 & -1 & 3
\end{pmatrix}$ $\xrightarrow[r_3-3r_2]{r_1-r_2} \left(\begin{array}{c|c} 1 & 0 & -2 & -5 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array} \right)$

 x_1, x_2 为主元, x_3 为自由变量。

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$$\begin{cases} x_1 + & -2x_3 = 5 \\ x_2 + & x_3 = -3 \end{cases}$$



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$$\begin{cases} x_1 + & -2x_3 = 5 \\ x_2 + & x_3 = -3 \end{cases} \iff \begin{cases} x_1 + & = 5 + 2x_3 \\ x_2 = -3 - x_3 \end{cases}$$

● 整角大型 ANAN UMIYERS 例 1 解方程组: $\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 = -1 \\ 2x_1 + 5x_2 + x_3 = -5 \\ -2x_1 - 3x_2 + x_3 = -1 \end{cases}$ $\mathbf{A}:b) = \begin{pmatrix} \mathbf{1} & 1 & -1 & 2 \\ 1 & 2 & 0 & -1 \\ 2 & 5 & 1 & -5 \\ -2 & -3 & 1 & -1 \end{pmatrix} \xrightarrow[r_1+2r_1]{r_2-r_1} \begin{pmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & 1 & -2 \\ 0 & 3 & 3 & -9 \\ 0 & -1 & -1 & 3 \end{pmatrix}$ $\xrightarrow[r_3-3r_2]{r_1-r_2} \left(\begin{array}{c|c} 1 & 0 & -2 & -5 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array} \right)$ x_1, x_2 为主元, x_3 为自由变量。所以原方程组等价于 $\begin{cases} x_{1} + -2x_{3} = 5 \\ x_{2} + x_{3} = -3 \end{cases} \Leftrightarrow \begin{cases} x_{1} + = 5 + 2x_{3} \\ x_{2} = -3 - x_{3} \end{cases}$ 所以通解是: $(c_1$ 为任意常数)

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例 2 解方程组:
$$\begin{cases} x_1 + 2x_2 + 4x_3 = 28 \\ -2x_1 - 3x_2 - 9x_3 = -53 \\ 3x_1 + 6x_2 + 13x_3 = 88 \\ 5x_1 + 9x_2 + 22x_3 = 141 \end{cases}$$

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$$\mathbf{(A:b)} = \begin{pmatrix}
-\frac{1}{2} & -\frac{2}{3} & -\frac{4}{9} & -\frac{28}{53} \\
-\frac{2}{3} & 6 & 13 & 88 \\
5 & 9 & 22 & 141
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$$\mathbf{(A:b)} = \begin{pmatrix}
\frac{1}{2} & \frac{2}{3} & \frac{4}{9} & \frac{28}{-53} \\
\frac{3}{3} & \frac{6}{6} & \frac{13}{13} & \frac{88}{141}
\end{pmatrix} \xrightarrow[r_4-5r_1]{r_2+2r_1} \begin{pmatrix}
1 & 2 & 4 & 28 \\
0 & 1 & -1 & 3 \\
0 & 0 & 1 & 4 \\
0 & -1 & 2 & 1
\end{pmatrix}$$



例 2 解方程组:
$$\begin{cases} -2x_1 - 3x_1 + 3x_1 + 3x_1 \end{cases}$$

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$$r_1 - 2r_2$$



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$$\begin{cases} 5x_1 + 9x_2 + 22x_3 = 141 \\ \begin{pmatrix} 1 & 2 & 4 \\ -2 & -3 & -9 \\ -53 & \end{pmatrix} t_{2} + 2t_1 & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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$$\xrightarrow[r_4+r_2]{r_1-2r_2} \left(\begin{array}{cc|c} 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 4 \end{array} \right)$$



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1 & 2 & 4 & 28 \\
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\end{pmatrix}$$

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\boxed{1} & 2 & 4 & 28 \\
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0 & 0 & 1 & 4 \\
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\end{pmatrix}$$

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$$\frac{r_{1}-2r_{2}}{r_{4}+r_{2}} \left(\begin{array}{ccc} 1 & 0 & 6 & 22 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 4 \end{array} \right) \xrightarrow[r_{2}-r_{3}]{r_{1}-6r_{3}} \left(\begin{array}{ccc} 0 & 0 & 1 & 4 \end{array} \right)$$

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$$\xrightarrow[r_4+r_2]{\begin{array}{c} 1 & 0 & 6 & 22 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 4 \\ \end{array}} \xrightarrow[r_4+r_2]{\begin{array}{c} 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 4 \\ \end{array}} \xrightarrow[r_6+r_3]{\begin{array}{c} 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 4 \\ \end{array}}$$

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$$\xrightarrow[r_4+r_2]{r_4+r_2} \begin{pmatrix} 1 & 0 & 6 & 22 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 4 \end{pmatrix} \xrightarrow[r_4-r_2]{r_1-6r_3} \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

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$$\frac{r_{1}-2r_{2}}{r_{4}+r_{2}} \left(\begin{array}{ccc|c} 1 & 0 & 6 & 22 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 4 \end{array} \right) \xrightarrow[r_{4}-r_{3}]{r_{1}-6r_{3}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

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\end{pmatrix} \xrightarrow{r_2 + 2r_1} \begin{pmatrix}
1 & 2 & 4 & 28 \\
0 & 1 & -1 & 3 \\
0 & 0 & 1 & 4
\end{pmatrix}$$

$$\xrightarrow{r_1 - 2r_2} \begin{pmatrix}
1 & 0 & 6 & 22 \\
0 & 1 & -1 & 3 \\
0 & 0 & 1 & 4
\end{pmatrix} \xrightarrow{r_2 + r_3} \begin{pmatrix}
1 & 0 & 0 & -2 \\
0 & 1 & 0 & 7 \\
0 & 0 & 1 & 4
\end{pmatrix}$$

 x_1, x_2, x_3 为主元,没有自由变量。

例 2 解方程组:
$$\begin{cases} x_1 + 2x_2 + 4x_3 = 28 \\ -2x_1 - 3x_2 - 9x_3 = -53 \\ 3x_1 + 6x_2 + 13x_3 = 88 \\ 5x_1 + 9x_2 + 22x_3 = 141 \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & 2 & 4 & 28 \\ -2 & -3 & -9 & -53 \\ 3 & 6 & 13 & 88 \\ 5 & 9 & 22 & 141 \end{pmatrix} \xrightarrow{r_2 + 2r_1} \begin{pmatrix} 1 & 2 & 4 & 28 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & -1 & 2 & 1 \end{pmatrix}$$

$$\frac{r_1 - 2r_2}{r_4 + r_2} \begin{pmatrix}
1 & 0 & 6 & 22 \\
0 & 1 & -1 & 4 \\
0 & 0 & 1 & 4
\end{pmatrix} \xrightarrow[r_2 + r_3]{r_2 + r_3} \begin{pmatrix}
1 & 0 & 0 & -2 \\
0 & 1 & 0 & 7 \\
0 & 0 & 1 & 4 \\
0 & 0 & 0 & 0
\end{pmatrix}$$

 x_1, x_2, x_3 为主元,没有自由变量。所以原方程组等价于

$$\begin{cases} x_1 & = -2 \\ x_2 & = 7 \\ x_3 & = 4 \end{cases}$$

例 3 解方程组:
$$\begin{cases} 4x_1 + 2x_2 - 7x_3 = -3\\ 2x_1 + x_2 - 4x_3 = -1\\ 5x_1 + 3x_2 - 11x_3 = 2\\ x_1 + x_2 - 4x_3 = 2 \end{cases}$$

$$(A \vdots b) = \begin{pmatrix} 4 & 2 & -7 & -3 \\ 2 & 1 & -4 & -1 \\ 5 & 3 & -11 & 2 \\ 1 & 1 & -4 & 2 \end{pmatrix}$$

$$(A : b) = \begin{pmatrix} 4 & 2 & -7 & | & -3 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 1 & 1 & -4 & | & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4}$$

$$(A : b) = \begin{pmatrix} 4 & 2 & -7 & -3 \\ 2 & 1 & -4 & -1 \\ 5 & 3 & -11 & 2 \\ 1 & 1 & -4 & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 1 & 1 & -4 & 2 \\ 2 & 1 & -4 & -1 \\ 5 & 3 & -11 & 2 \\ 4 & 2 & -7 & -3 \end{pmatrix}$$

$$(A : b) = \begin{pmatrix} 4 & 2 & -7 & -3 \\ 2 & 1 & -4 & -1 \\ 5 & 3 & -11 & 2 \\ 1 & 1 & -4 & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 1 & 1 & -4 & 2 \\ 2 & 1 & -4 & -1 \\ 5 & 3 & -11 & 2 \\ 4 & 2 & -7 & -3 \end{pmatrix}$$

$$(A:b) = \begin{pmatrix} 4 & 2 & -7 & | & -3 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 1 & 1 & -4 & | & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \xrightarrow{r_3 - 5r_1} \xrightarrow{r_4 - 4r_1}$$



$$\mathbf{(A:b)} = \begin{pmatrix} 42 & -7 & -3 \\ 21 & -4 & -1 \\ 53 & -11 & 2 \\ 11 & -4 & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 11 & -4 & 2 \\ 21 & -4 & -1 \\ 53 & -11 & 2 \\ 42 & -7 & -3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \xrightarrow{r_3 - 5r_1} \xrightarrow{r_4 - 4r_1}$$

$$\begin{pmatrix} 1 & 1-4 & 2 \\ & & \end{pmatrix}$$

$$(A:b) = \begin{pmatrix} 4 & 2 & -7 & | & -3 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -1 & 1 & | & 2 \\ 1 & 1 & -4 & | & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} \boxed{1} & 1 & -4 & | & 2 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -1 & 1 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \frac{r_2 - 2r_1}{r_3 - 5r_1}$$
$$\begin{pmatrix} \boxed{1} & \boxed{1} & -4 & | & 2 \\ 7 & 1 & 4 & | & -5 \end{pmatrix}$$

$$(A:b) = \begin{pmatrix} 4 & 2 & -7 & | & -3 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 1 & 1 & -4 & | & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & -1 & | & -4 & | & -2 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & -1 & | & -4 & | & -2 \\ 0 & -1 & 4 & | & -5 \end{pmatrix}$$

$$\begin{pmatrix}
1 & -1 & -4 \\
0 & -1 & 4 \\
0 & -2 & 9
\end{pmatrix}
\begin{pmatrix}
2 \\
-8
\end{pmatrix}$$

$$(A:b) = \begin{pmatrix} 4 & 2 & -7 & | & -3 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 1 & 1 & -4 & | & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 0 & -1 & 4 & | & -5 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 & | & -8 \\ 0 & -2 & 9 &$$

$$\begin{pmatrix}
1 & 1 - 4 & 2 \\
0 - 1 & 4 & -5 \\
0 - 2 & 9 & -8 \\
0 - 2 & 9 & -11
\end{pmatrix}$$

$$(A:b) = \begin{pmatrix} 4 & 2 & -7 & | & -3 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 1 & 1 & -4 & | & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \xrightarrow{r_3 - 5r_1} \xrightarrow{r_4 - 4r_1} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 0 & -1 & 4 & | & -5 \\ 0 & -2 & 9 & | & -11 \end{pmatrix}$$

$$(A:b) = \begin{pmatrix} 4 & 2 & -7 & | & -3 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -1 & 1 & | & 2 \\ 1 & 1 & -4 & | & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} \boxed{1} & 1 & -4 & | & 2 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -1 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} \boxed{1} & 1 & -4 & | & 2 \\ 5 & 3 & -1 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} \boxed{1} & 1 & -4 & | & 2 \\ 7 & 3 & -1 & | & 2 \\ 7 & 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \xrightarrow{r_4 - 4r_1} \begin{pmatrix} \boxed{1} & 1 & -4 & | & 2 \\ 7 & 3 & -1 & | & 2 \\ 7 & 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \xrightarrow{r_4 - 4r_1} \begin{pmatrix} \boxed{1} & 1 & -4 & | & 2 \\ 7 & 3 & -1 & | & 2 \\ 7 & 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \xrightarrow{r_4 - 4r_1} \begin{pmatrix} \boxed{1} & 1 & -4 & | & 2 \\ 7 & 3 & -1 & | & 2 \\ 7 & 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_4 - 4r_1} \xrightarrow{r_5 - 2r_4} \xrightarrow{r_5 - 2r_5} \xrightarrow{r_5 - 2$$

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$$(A:b) = \begin{pmatrix} 4 & 2 & -7 & | & -3 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 1 & 1 & -4 & | & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 1 & 1 & -4 & 2 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 5 & 3 & -11 & | & 2 \\ 7 & 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 5 & 3 & -11 & | & 2 \\ 7 & 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 7 & 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 7 & 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 7 & 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 7 & 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 2 & 1 & -4 & | & -1 \\ 2 & 1 & 2 & | & -7 & | & -1 \end{pmatrix}$$



$$(A:b) = \begin{pmatrix} 4 & 2 & -7 & | & -3 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 1 & 1 & -4 & | & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 1 & 1 & -4 & 2 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & -4 & | & 2 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 + 2r_2} \begin{pmatrix} 1 & 0 & 0 & | & -3 \\ 0 & -1 & 4 & | & -5 \\ 0 & -2 & 9 & | & -11 \end{pmatrix} \xrightarrow{r_3 + 2r_2} \begin{pmatrix} 1 & 0 & 0 & | & -3 \\ 0 & -1 & 4 & | & -5 \\ 0 & -2 & 9 & | & -11 \end{pmatrix}$$



$$(A:b) = \begin{pmatrix} 4 & 2 & -7 & | & -3 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -1 & 1 & | & 2 \\ 1 & 1 & -4 & | & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 1 & 1 & -4 & 2 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -1 & 1 & | & 2 \\ 5 & 3 & -1 & 1 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & -4 & | & 2 \\ 5 & 3 & -1 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 2r_2} \begin{pmatrix} 1 & 0 & 0 & | & -3 \\ 0 & -1 & 4 & | & -5 \\ 0 & 0 & 2 & 9 & | & -11 \end{pmatrix} \xrightarrow{r_3 + 2r_2} \begin{pmatrix} 1 & 0 & 0 & | & -3 \\ 0 & -1 & 4 & | & -5 \\ 0 & 0 & 1 & | & 2 \end{pmatrix}$$



$$(A:b) = \begin{pmatrix} 4 & 2 & -7 & | & -3 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 1 & 1 & -4 & | & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 1 & 1 & -4 & 2 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \xrightarrow{r_3 - 5r_1} \begin{pmatrix} 1 & -4 & | & 2 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \xrightarrow{r_3 - 5r_1} \xrightarrow{r_4 - 4r_1} \begin{pmatrix} 1 & -4 & | & 2 \\ 0 & -1 & | & 2 \\ 0 & -2 & 9 & | & -11 \end{pmatrix} \xrightarrow{r_1 + r_2} \begin{pmatrix} 1 & 0 & 0 & | & -3 \\ 0 & 0 & 1 & | & -5 \\ 0 & 0 & 1 & | & -1 \end{pmatrix}$$



$$(A:b) = \begin{pmatrix} 4 & 2 & -7 & | & -3 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -1 & 1 & | & 2 \\ 1 & 1 & -4 & | & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -1 & 1 & | & 2 \\ 5 & 3 & -1 & 1 & | & 2 \\ 4 & 2 & -7 & | & 3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \xrightarrow{r_3 - 5r_1} \xrightarrow{r_4 - 4r_1} \begin{pmatrix} 1 & 1 & -4 & | & 2 \\ 1 & 1 & -4 & | & 2 \\ 5 & 3 & -1 & | & 2 \\ 4 & 2 & -7 & | & 3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \xrightarrow{r_3 - 5r_1} \xrightarrow{r_4 - 4r_1} \begin{pmatrix} 1 & 1 & 0 & 0 & | & -3 \\ 0 & -1 & 4 & | & -5 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$



$$(A:b) = \begin{pmatrix} 4 & 2 & -7 & | & -3 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 1 & 1 & -4 & | & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} \boxed{1} & 1 & -4 & | & 2 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & 3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \xrightarrow{r_3 - 5r_1}$$

$$(A:b) = \begin{pmatrix} 2 & 1 & -4 & -1 \\ 5 & 3 & -11 & 2 \\ 1 & 1 & -4 & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 2 & 1 & -4 & -1 \\ 5 & 3 & -11 & 2 \\ 4 & 2 & -7 & -3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{bmatrix} r_2 - 2r_1 & r_3 - 5r_1 & r_4 - 4r_1 \\ r_4 - 4r_1 & r_4 - 4r_1 & r_4 - 4r_1 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 1 - 4 & 2 & 1 \\ 0 & -1 & 4 & -5 \\ 0 & -2 & 9 & -11 \end{pmatrix} \xrightarrow{r_1 + r_2} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \xrightarrow{r_1 + r_2} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$$(A:b) = \begin{pmatrix} 4 & 2 & -7 & | & -3 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 1 & 1 & -4 & | & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} \boxed{1} & 1 & -4 & | & 2 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -11 & | & 2 \\ 4 & 2 & -7 & | & -3 \end{pmatrix} \frac{r_2 - 2r_1}{r_3 - 5r_1}$$

$$(A : b) = \begin{pmatrix} \frac{7}{2} & \frac{7}{4} & -\frac{7}{4} \\ \frac{7}{5} & \frac{7}{3} & -\frac{11}{2} \\ \frac{7}{1} & \frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} \frac{7}{2} & \frac{7}{4} & -\frac{7}{4} \\ \frac{7}{5} & \frac{7}{3} & -\frac{11}{2} \\ \frac{7}{5} & \frac{7}{3} & -\frac{7}{1} \\ \frac{7}{4} & \frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_2 \to r_1} \begin{pmatrix} \frac{1}{5} & \frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & \frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 + r_2} \begin{pmatrix} \frac{1}{5} & \frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 + r_2} \begin{pmatrix} \frac{1}{5} & \frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_2 \to r_1} \begin{pmatrix} \frac{1}{5} & \frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_2 \to r_1} \begin{pmatrix} \frac{1}{5} & \frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 \to r_2} \begin{pmatrix} \frac{1}{5} & \frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_2 \to r_1} \begin{pmatrix} \frac{1}{5} & \frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 \to r_2} \begin{pmatrix} \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 \to r_2} \begin{pmatrix} \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 \to r_2} \begin{pmatrix} \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 \to r_2} \begin{pmatrix} \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 \to r_2} \begin{pmatrix} \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 \to r_2} \begin{pmatrix} \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 \to r_2} \begin{pmatrix} \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 \to r_2} \begin{pmatrix} \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 \to r_2} \begin{pmatrix} \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 \to r_2} \begin{pmatrix} \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 \to r_2} \begin{pmatrix} \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 \to r_2} \begin{pmatrix} \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 \to r_2} \begin{pmatrix} \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \\ \frac{7}{4} & -\frac{7}{4} & -\frac{7}{4} \end{pmatrix} \xrightarrow{r_1 \to r_2} \begin{pmatrix} \frac{7}{4} & -\frac{7}{4}$$



$$(A:b) = \begin{pmatrix} 3x_1 + 3x_2 - 11x_3 - 2 \\ x_1 + x_2 - 4x_3 = 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 1 & 1 & -4 & 2 \\ 2 & 1 & -4 & -1 \\ 5 & 3 - 11 & 2 \\ 1 & 1 & -4 & 2 \end{pmatrix} \xrightarrow{r_2 \to r_4} \begin{pmatrix} 1 & 1 & -4 & 2 \\ 2 & 1 & -4 & -1 \\ 5 & 3 - 11 & 2 \\ 4 & 2 & -7 & -3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \xrightarrow{r_3 - 5r_1} \xrightarrow{r_3 - 4r_3}$$

 $\begin{pmatrix} 1 & 1 & -4 \\ 0 & -1 & 4 \\ 0 & -2 & 9 \\ 0 & -2 & 9 \\ -11 \end{pmatrix} \xrightarrow[r_3+2r_2]{} \begin{pmatrix} 1 & 0 & 0 & | & -3 \\ 0 & -1 & 4 & | & -5 \\ 0 & 0 & 1 & | & 2 \\ 0 & 0 & 1 & | & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & | & -3 \\ 0 & 1 & 0 & | & 13 \\ 0 & 0 & 1 & | & 2 \\ 0 & 0 & 0 & | & -3 \end{pmatrix}$ 所以原方程组等价于 $\begin{cases} x_1 & = -3 \\ x_2 & = 13 \\ x_3 & = 2 \\ 0 & = -3 \end{cases}$

$$\begin{cases} 5x_1 + 3x_2 - 11x_3 = 2x_1 \\ x_1 + x_2 - 4x_3 = 2x_2 \end{cases}$$

$$\vdots b) = \begin{pmatrix} 4 & 2 & -7 & -3 \\ 2 & 1 & -4 & -1 \\ 5 & 3 & -11 & 2 \\ 1 & 1 & -4 & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & -1 \\ 5 & 3 & -1 \\ 4 & 2 & -1 \end{pmatrix}$$

 $(A:b) = \begin{pmatrix} 4 & 2 & -7 & | & -3 \\ 2 & 1 & -4 & | & -1 \\ 5 & 3 & -1 & 1 & | & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 1 & 1 & 1 & -4 & | & 2 \\ 2 & 1 & -4 & | & -1 & | & 2 \\ 5 & 3 & -1 & 1 & | & 2 & | & r_{3} - 5r_{1} \\ 4 & 2 & -7 & | & -3 & | & r_{3} - 5r_{1} \\ 7 & 3 & 7 & 7 & 7 & | & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7 & 7 \\ 7 & 3 & 7$

 $\begin{pmatrix} 1 & 1 & -4 \\ 0 & -1 & 4 \\ 0 & -2 & 9 \\ 0 & -2 & 9 \\ -11 \end{pmatrix} \xrightarrow[r_3+2r_2]{} \begin{pmatrix} 1 & 0 & 0 & | & -3 \\ 0 & -1 & 4 & | & -5 \\ 0 & 0 & 1 & | & 2 \\ 0 & 0 & 1 & | & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & | & -3 \\ 0 & 1 & 0 & | & 13 \\ 0 & 0 & 1 & | & 2 \\ 0 & 0 & 0 & | & -3 \end{pmatrix}$

所以原方程组等价于 $\begin{cases} x_1 & = -3 \\ x_2 & = 13 \\ x_3 & = 2 \\ 0 & = -3 \end{cases}$ \Rightarrow 无解!

$$\begin{cases} x_{1} + & x_{2} - & x_{3} = 2 \\ x_{1} + & 2x_{2} & = -1 \\ 2x_{1} + & 5x_{2} + & x_{3} = -5 \\ -2x_{1} - & 3x_{2} + & x_{3} = -1 \end{cases} \qquad \begin{cases} x_{1} + & 2x_{2} + & 4x_{3} = & 28 \\ -2x_{1} - & 3x_{2} - & 9x_{3} = & -53 \\ 3x_{1} + & 6x_{2} + & 13x_{3} = & 88 \\ 5x_{1} + & 9x_{2} + & 22x_{3} = & 141 \end{cases} \qquad \begin{cases} 4x_{1} + & 2x_{2} - & 7x_{3} = & -3 \\ 2x_{1} + & x_{2} - & 4x_{3} = & -1 \\ 5x_{1} + & 3x_{2} - & 11x_{3} = & 2 \\ x_{1} + & x_{2} - & 4x_{3} = & 2 \end{cases}$$

$$\begin{cases} x_{1} + & x_{2} - & x_{3} = 2 \\ x_{1} + & 2x_{2} & = -1 \\ 2x_{1} + & 5x_{2} + & x_{3} = -5 \\ -2x_{1} - & 3x_{2} + & x_{3} = -1 \end{cases} \qquad \begin{cases} x_{1} + & 2x_{2} + & 4x_{3} = & 28 \\ -2x_{1} - & 3x_{2} - & 9x_{3} = & -53 \\ 3x_{1} + & 6x_{2} + & 13x_{3} = & 88 \\ 5x_{1} + & 9x_{2} + & 22x_{3} = & 141 \end{cases} \qquad \begin{cases} 4x_{1} + & 2x_{2} - & 7x_{3} = & -3 \\ 2x_{1} + & x_{2} - & 4x_{3} = & -1 \\ 5x_{1} + & 3x_{2} - & 11x_{3} = & 2 \\ x_{1} + & x_{2} - & 4x_{3} = & 2 \end{cases}$$

$$\downarrow \qquad \qquad \downarrow \qquad$$









主元数 = 独立方程数



主元数 = 独立方程数 < n



主元数 = 独立方程数 < nr(A) = r(A : b) < n

$$\begin{cases} x_{1} + x_{2} - x_{3} &= 2 \\ x_{1} + 2x_{2} &= -1 \\ 2x_{1} + 5x_{2} + x_{3} &= -5 \\ -2x_{1} - 3x_{2} + x_{3} &= -1 \end{cases}$$

$$\begin{cases} x_{1} + 2x_{2} + 4x_{3} &= 28 \\ -2x_{1} - 3x_{2} - 9x_{3} &= -53 \\ 3x_{1} + 6x_{2} + 13x_{3} &= 88 \\ 5x_{1} + 9x_{2} + 22x_{3} &= 141 \end{cases}$$

$$\begin{cases} 4x_{1} + 2x_{2} - 7x_{3} &= -3 \\ 2x_{1} + x_{2} - 4x_{3} &= -1 \end{cases}$$

$$\begin{cases} 2x_{1} + x_{2} - 4x_{3} &= -1 \\ 5x_{1} + 3x_{2} - 11x_{3} &= 2 \\ x_{1} + x_{2} - 4x_{3} &= 2 \end{cases}$$

$$\begin{cases} A : b \end{cases}$$

$$(A : b)$$

$$($$



r(A) = r(A : b) < n

主元数 = 独立方程数 < n 主元数 = 独立方程数

主元数 = 独立方程数 < n 主元数 = 独立方程数 = n



r(A) = r(A : b) < n

主元数 = 独立方程数 = n

r(A) = r(A : b) < n r(A) = r(A : b) = n



主元数 = 独立方程数 < n

$$\begin{pmatrix} 1 & 0 & -2 & 5 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

无穷多解

主元数 = 独立方程数
$$< n$$
 $r(A) = r(A : b) < n$

$$\left\{ \begin{array}{lllll} x_1+&2x_2+&4x_3=&28\\ -2x_1-&3x_2-&9x_3=&-53\\ 3x_1+&6x_2+&13x_3=&88\\ 5x_1+&9x_2+&22x_3=&141 \end{array} \right. \left\{ \begin{array}{llllllll} 4x_1+&2x_2-&7x_3=&-3\\ 2x_1+&x_2-&4x_3=&-1\\ 5x_1+&3x_2-&11x_3=&2\\ x_1+&x_2-&4x_3=&2 \end{array} \right.$$

$$x_{11} + 2x_{21} + 4x_{3} = 26$$

 $-2x_{1} - 3x_{2} - 9x_{3} = -53$
 $3x_{1} + 6x_{2} + 13x_{3} = 88$
 $5x_{1} + 9x_{2} + 22x_{3} = 141$

初等↓行变换

$$\begin{pmatrix} 1 & 0 & 0 & | & -2 \\ 0 & 1 & 0 & | & 7 \\ 0 & 0 & 1 & | & 4 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

唯一解

主元数 = 独立方程数=
$$n$$

 $r(A) = r(A : b) = n$

(A:b)

初等 ↓ 行变换

$$\begin{pmatrix} 1 & 0 & 0 & | & -3 \\ 0 & 1 & 0 & | & 13 \\ 0 & 0 & 1 & | & 2 \\ 0 & 0 & 0 & | & -3 \end{pmatrix}$$

无解

主元数 < 独立方程数



$$\begin{pmatrix} 1 & 0 & -2 & | & 5 \\ 0 & 1 & 1 & | & -3 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

无穷多解

主元数 = 独立方程数
$$< n$$

 $r(A) = r(A : b) < n$

初等↓行变换

$$\begin{pmatrix}
1 & 0 & 0 & -2 \\
0 & 1 & 0 & 7 \\
0 & 0 & 1 & 4 \\
0 & 0 & 0 & 0
\end{pmatrix}$$

唯一解

主元数 = 独立方程数=
$$n$$

 $r(A) = r(A : b) = n$

$$\begin{cases} 4x_{1} + 2x_{2} - 7x_{3} = -3\\ 2x_{1} + x_{2} - 4x_{3} = -1\\ 5x_{1} + 3x_{2} - 11x_{3} = 2\\ x_{1} + x_{2} - 4x_{3} = 2 \end{cases}$$

(A:b)

初等 ↓ 行变换

$$\begin{pmatrix} 1 & 0 & 0 & | & -3 \\ 0 & 1 & 0 & | & 13 \\ 0 & 0 & 1 & | & 2 \\ 0 & 0 & 0 & | & -3 \end{pmatrix}$$

无解

主元数 < 独立方程数 r(A) < r(A : b)



总结 定理 方程组 $\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$

$$⇔ Ax = b$$
 的



总结
定理 方程组
$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

$$⇔ Ax = b$$
 的

$$1. r(A : b) = r(A)$$

2.
$$r(A) \neq r(A : b)$$

总结
定理 方程组
$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

$$1. r(A : b) = r(A)$$

$$r(A) \neq r(A : b) \iff r(A) < r(A : b)$$

解有如下情形:

1.
$$r(A : b) = r(A)$$
$$r(A) = r(A : b) < n$$
$$r(A) = r(A : b) = n$$

 $r(A) \neq r(A : b) \Leftrightarrow r(A) < r(A : b)$

2.

1.
$$r(A : b) = r(A)$$

$$r(A) = r(A \dot{:} b) < n$$

• 只有唯一解
$$\Leftrightarrow$$
 $r(A) = r(A : b) = n$

2.
$$r(A) \neq r(A : b) \Leftrightarrow r(A) < r(A : b)$$

 $\Leftrightarrow Ax = b$ 的

$$1. r(A : b) = r(A)$$

- 有无穷多解 \Leftrightarrow r(A) = r(A : b) < n
- 只有唯一解 \Leftrightarrow r(A) = r(A : b) = n
- $r(A) \neq r(A : b) \Leftrightarrow r(A) < r(A : b)$ 2.

$$⇔ Ax = b$$
 的

- 1. 有解 \Leftrightarrow r(A:b) = r(A)
 - 有无穷多解 \Leftrightarrow r(A) = r(A : b) < n
 - 只有唯一解 \Leftrightarrow r(A) = r(A : b) = n
- $r(A) \neq r(A : b) \Leftrightarrow r(A) < r(A : b)$ 2.

$$⇔ Ax = b$$
 的

- 1. 有解 \Leftrightarrow r(A:b) = r(A)
 - 有无穷多解 \Leftrightarrow r(A) = r(A : b) < n
 - 只有唯一解 \Leftrightarrow r(A) = r(A : b) = n
- 2. 无解 \Leftrightarrow $r(A) \neq r(A : b) \Leftrightarrow r(A) < r(A : b)$

解有如下情形:

- 1. 有解 \Leftrightarrow r(A:b) = r(A)
 - 有无穷多解 \Leftrightarrow r(A) = r(A : b) < n
 - 只有唯一解 \Leftrightarrow r(A) = r(A : b) = n
- 2. 无解 \Leftrightarrow $r(A) \neq r(A : b) \Leftrightarrow r(A) < r(A : b)$

注

• r(A:b) = r(A) 的值,相当于方程组中"独立"方程个数,此时



 $\Leftrightarrow Ax = b$ 的

解有如下情形:

- 1. 有解 \Leftrightarrow r(A:b) = r(A)
 - 有无穷多解 ⇔ r(A) = r(A:b) < n
 - 只有唯一解 \Leftrightarrow r(A) = r(A : b) = n
- \mathcal{E} 无解 \Leftrightarrow $r(A) \neq r(A : b) \Leftrightarrow r(A) < r(A : b)$

注

- r(A:b) = r(A) 的值,相当于方程组中"独立"方程个数,此时
 - n − r(A) 为自由变量的个数



练习 1 求解 $\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1\\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3\\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7\\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$

练习 1 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1\\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3\\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7\\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$
 解

$$(A:b) = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 4 & 3 & 1 & 1 & 3 \\ -1 & -2 & 1 & 3 & -3 & 7 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix}$$

练习 1 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1\\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3\\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7\\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$
解

$$(A:b) = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 4 & 3 & 1 & 1 & 3 \\ -1 & -2 & 1 & 3 & -3 & 7 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix} \xrightarrow{r_2 - 2r_1} r_3 + r_1$$



练习 1 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1 \\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3 \\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7 \\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$

$$(A : b) = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 4 & 3 & 1 & 1 & 3 \\ -1 & -2 & 1 & 3 & -3 & 7 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 2 & 4 & -2 & 8 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix}$$



练习1求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1 \\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3 \\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7 \\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$

$$(A \vdots b) = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 4 & 3 & 1 & 1 & 3 \\ -1 & -2 & 1 & 3 & -3 & 7 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 2 & 4 & -2 & 8 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix}$$

$$\frac{r_3-2r_2}{r_3-2r_3}$$



练习 1 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1 \\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3 \\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7 \\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$

解

$$(A : b) = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 4 & 3 & 1 & 1 & 3 \\ -1 & -2 & 1 & 3 & -3 & 7 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 2 & 4 & -2 & 8 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix}$$

$$\frac{r_3 - 2r_2}{r_4 - 2r_2} \left(\begin{array}{ccccc}
1 & 2 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & -1 & -1 & 1 \\
0 & 0 & 0 & 6 & 0 & 6 \\
0 & 0 & 0 & 7 & 0 & 7
\end{array} \right)$$



练习 1 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1 \\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3 \\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7 \\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$

胖

$$(A:b) = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 4 & 3 & 1 & 1 & 3 \\ -1 & -2 & 1 & 3 & -3 & 7 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 2 & 4 & -2 & 8 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix}$$

$$\xrightarrow[r_4-2r_2]{ \begin{array}{c} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 6 & 0 & 6 \\ 0 & 0 & 0 & 7 & 0 & 7 \end{array} } \xrightarrow[\bar{\tau}\times r_4]{\frac{1}{6}\times r_3}$$



练习 1 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1\\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3\\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7\\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$

脌

$$(A:b) = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 4 & 3 & 1 & 1 & 3 \\ -1 & -2 & 1 & 3 & -3 & 7 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 2 & 4 & -2 & 8 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix}$$

$$\frac{r_{3-2r_{2}}}{r_{4-2r_{2}}} \left(\begin{array}{cccccc}
1 & 2 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & -1 & -1 & 1 \\
0 & 0 & 0 & 6 & 0 & 6 \\
0 & 0 & 0 & 7 & 0 & 7
\end{array} \right) \xrightarrow{\frac{1}{6} \times r_{3}} \left(\begin{array}{cccccc}
1 & 2 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & -1 & -1 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 1
\end{array} \right)$$



练习1求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1\\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3\\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7\\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$

解

$$(A:b) = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 4 & 3 & 1 & 1 & 3 \\ -1 & -2 & 1 & 3 & -3 & 7 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 2 & 4 & -2 & 8 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix}$$

$$\xrightarrow{r_3 - 2r_2} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 6 & 0 & 6 \\ 0 & 0 & 0 & 7 & 0 & 7 \end{pmatrix} \xrightarrow{\frac{1}{6} \times r_3} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$r_4-r_3$$



练习 1 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1 \\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3 \\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7 \\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$

解

$$(A:b) = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 4 & 3 & 1 & 1 & 3 \\ -1 & -2 & 1 & 3 & -3 & 7 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 2 & 4 & -2 & 8 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix}$$

$$\frac{r_{3}-2r_{2}}{r_{4}-2r_{2}} \left(\begin{array}{cccccc} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 6 & 0 & 6 \\ 0 & 0 & 0 & 7 & 0 & 7 \end{array} \right) \xrightarrow{\frac{1}{6}\times r_{3}} \left(\begin{array}{ccccccc} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{r_{4}-r_{3}} \left(\begin{array}{ccccccc} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$



练习 1 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1 \\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3 \\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7 \\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$

脌

$$(A:b) = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 4 & 3 & 1 & 1 & 3 \\ -1 & -2 & 1 & 3 & -3 & 7 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 2 & 4 & -2 & 8 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix}$$

$$\frac{r_{3}-2r_{2}}{r_{4}-2r_{2}} \left(\begin{array}{cccccc} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 6 & 0 & 6 \\ 0 & 0 & 0 & 7 & 0 & 7 \end{array} \right) \xrightarrow{\frac{1}{6}\times r_{3}} \left(\begin{array}{ccccccc} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{array} \right) \xrightarrow{\frac{1}{6}\times r_{3}} \left(\begin{array}{cccccccc} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{\frac{1}{7}\times r_{4}} \left(\begin{array}{ccccccc} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$



练习 1 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1\\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3\\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7\\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$

解

$$(A:b) = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 4 & 3 & 1 & 1 & 3 \\ -1 & -2 & 1 & 3 & -3 & 7 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 2 & 4 & -2 & 8 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix}$$



练习 1 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1\\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3\\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7\\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$

脌

$$(A:b) = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 4 & 3 & 1 & 1 & 3 \\ -1 & -2 & 1 & 3 & -3 & 7 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 2 & 4 & -2 & 8 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix}$$

$$\frac{r_{3}-2r_{2}}{r_{4}-2r_{2}} \left(\begin{array}{cccccc} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 6 & 0 & 6 \\ 0 & 0 & 0 & 7 & 0 & 7 \end{array} \right) \frac{1}{6} \times r_{3} \left(\begin{array}{ccccccc} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{array} \right) \frac{1}{7} \times r_{4} \left(\begin{array}{cccccc} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \frac{1}{7} \times r_{4} \left(\begin{array}{cccccc} 1 & 2 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$r_1-r_2$$



练习 1 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1\\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3\\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7\\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$
解

$$(A : b) = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 4 & 3 & 1 & 1 & 3 \\ -1 & -2 & 1 & 3 & -3 & 7 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 2 & 4 & -2 & 8 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix}$$

$$\xrightarrow{r_3 - 2r_2} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 6 & 0 & 6 \\ 0 & 0 & 0 & 7 & 0 & 7 \end{pmatrix} \xrightarrow{\frac{1}{6} \times r_3} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{r_4 - r_3} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \xrightarrow{r_2 + r_3} \begin{pmatrix} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



练习 1 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1\\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3\\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7\\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$
解

$$(A:b) = \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 2 & 4 & 3 & 1 & 1 & 3 \\ -1 & -2 & 1 & 3 & -3 & 7 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 2 & 4 & -2 & 8 \\ 0 & 0 & 2 & 5 & -2 & 9 \end{pmatrix}$$

$$\xrightarrow{r_3 - 2r_2} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 6 & 0 & 6 \\ 0 & 0 & 0 & 7 & 0 & 7 \end{pmatrix} \xrightarrow{\frac{1}{6} \times r_3} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{r_4 - r_3} \begin{pmatrix} 1 & 2 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{r_2 + r_3} \begin{pmatrix} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



练习 2 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1 \\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3 \\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7 \\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$
解

$$(A:b) \longrightarrow \begin{pmatrix} 1 & 2 & 0 & 0 & 2 & -2 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



练习 2 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1 \\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3 \\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7 \\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$

解

$$(A:b) \longrightarrow \begin{pmatrix} 1 & 2 & 0 & 0 & 2 & -2 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

• 可见
$$r(A) = r(A : b) = 3 < 5$$
,有无穷多的解,

练习 2 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1 \\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3 \\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7 \\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$

解

$$(A:b) \longrightarrow \begin{pmatrix} 1 & 2 & 0 & 0 & 2 & -2 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

• 可见 r(A) = r(A : b) = 3 < 5,有无穷多的解,含 5 - 3 = 2 个自由变量



练习 2 求解
$$\begin{cases} x_1 + & 2x_2 + & x_3 + & x_4 + & x_5 = 1 \\ 2x_1 + & 4x_2 + & 3x_3 + & x_4 + & x_5 = 3 \\ -x_1 - & 2x_2 + & x_3 + & 3x_4 - & 3x_5 = 7 \\ & & & 2x_3 + & 5x_4 - & 2x_5 = 9 \end{cases}$$

$$(A:b) \longrightarrow \begin{pmatrix} 1 & 2 & 0 & 0 & 2 & -2 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- 可见 r(A) = r(A : b) = 3 < 5,有无穷多的解,含 5 3 = 2 个自由变量
- 由既约阶梯形矩阵可知,原方程组等价于

$$\begin{cases} x_1 + 2x_2 & + 2x_5 = -2 \\ x_3 & - x_5 = 2 \\ x_4 & = 1 \end{cases}$$

练习 2 求解
$$\begin{cases} x_1 + & 2x_2 + & x_3 + & x_4 + & x_5 = 1 \\ 2x_1 + & 4x_2 + & 3x_3 + & x_4 + & x_5 = 3 \\ -x_1 - & 2x_2 + & x_3 + & 3x_4 - & 3x_5 = 7 \\ & & & 2x_3 + & 5x_4 - & 2x_5 = 9 \end{cases}$$

$$(A:b) \longrightarrow \begin{pmatrix} 1 & 2 & 0 & 0 & 2 & -2 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- 可见 r(A) = r(A : b) = 3 < 5,有无穷多的解,含 5 3 = 2 个自由变量
- 由既约阶梯形矩阵可知,原方程组等价于

$$\begin{cases} x_1 + 2x_2 & + 2x_5 = -2 \\ x_3 & - x_5 = 2 \\ x_4 & = 1 \end{cases} \Rightarrow \begin{cases} x_1 = -2 - 2x_2 - 2x_5 \\ x_3 = 2 + x_5 \\ x_4 = 1 \end{cases}$$

练习 2 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1 \\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3 \\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7 \\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$
解

$$(A:b) \longrightarrow \begin{pmatrix} 2 & 2 & 0 & 0 & 2 & 2 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- 可见 r(A) = r(A : b) = 3 < 5,有无穷多的解,含 5 3 = 2 个自由变量
- 由既约阶梯形矩阵可知。原方程组等价于

$$\begin{cases} x_1 + 2x_2 & + 2x_5 = -2 \\ x_3 & - x_5 = 2 \\ x_4 & = 1 \end{cases} \Rightarrow \begin{cases} x_1 = -2 - 2x_2 - 2x_5 \\ x_3 = 2 + x_5 \\ x_4 = 1 \end{cases}$$

$$\begin{cases} x_1 = \\ x_2 = \\ x_3 = \\ x_4 = \\ x_7 = \end{cases}$$

练习 2 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1 \\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3 \\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7 \\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$
解

$$(A:b) \longrightarrow \begin{pmatrix} 2 & 2 & 0 & 0 & 2 & 2 \\ \hline 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- 可见 r(A) = r(A : b) = 3 < 5,有无穷多的解,含 5 3 = 2 个自由变量
- 由既约阶梯形矩阵可知。原方程组等价于

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练习 2 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1 \\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3 \\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7 \\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$
 解

$$(A:b) \longrightarrow \begin{pmatrix} 2 & 2 & 0 & 0 & 2 & 2 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- 可见 r(A) = r(A : b) = 3 < 5,有无穷多的解,含 5 3 = 2 个自由变量
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$$\begin{cases} x_1 + 2x_2 & + 2x_5 = -2 \\ x_3 & - x_5 = 2 \\ x_4 & = 1 \end{cases} \Rightarrow \begin{cases} x_1 = -2 - 2x_2 - 2x_5 \\ x_3 = 2 + x_5 \\ x_4 = 1 \end{cases}$$

所以連肼是
$$\begin{cases} x_1 = -2 - 2c_1 - 2c_2 \\ x_2 = c_1 \\ x_3 = \\ x_4 = \\ x_5 = c_2 \end{cases} (c_1, c_2 为任意常数)$$

练习 2 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1 \\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3 \\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7 \\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$
 解

鹏

$$(A:b) \longrightarrow \begin{pmatrix} 2 & 2 & 0 & 0 & 2 & 2 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- 可见 r(A) = r(A : b) = 3 < 5,有无穷多的解,含 5 3 = 2 个自由变量
- 由既约阶梯形矩阵可知,原方程组等价于

$$\begin{cases} x_1 + 2x_2 & + 2x_5 = -2 \\ x_3 & - x_5 = 2 \\ x_4 & = 1 \end{cases} \Rightarrow \begin{cases} x_1 = -2 - 2x_2 - 2x_5 \\ x_3 = 2 + x_5 \\ x_4 = 1 \end{cases}$$

是
$$\begin{cases} x_1 = -2 - 2c_1 - 2c_2 \\ x_2 = c_1 \\ x_3 = 2 + c_2 \\ x_4 = \\ x_5 = c_2 \end{cases} \qquad \textbf{(c_1, c_2为任意常数)}$$

练习 2 求解
$$\begin{cases} x_1 + 2x_2 + x_3 + x_4 + x_5 = 1 \\ 2x_1 + 4x_2 + 3x_3 + x_4 + x_5 = 3 \\ -x_1 - 2x_2 + x_3 + 3x_4 - 3x_5 = 7 \\ 2x_3 + 5x_4 - 2x_5 = 9 \end{cases}$$
 解

$$(A:b) \longrightarrow \begin{pmatrix} 1 & 2 & 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- 可见 r(A) = r(A : b) = 3 < 5,有无穷多的解,含 5 3 = 2 个自由变量
- 由既约阶梯形矩阵可知。原方程组等价于

$$\begin{cases} x_1 + 2x_2 & + 2x_5 = -2 \\ x_3 & - x_5 = 2 \\ x_4 & = 1 \end{cases} \Rightarrow \begin{cases} x_1 = -2 - 2x_2 - 2x_5 \\ x_3 = 2 + x_5 \\ x_4 = 1 \end{cases}$$

所以通解是
$$\begin{cases} x_1 = -2 - 2c_1 - 2c_2 \\ x_2 = c_1 \\ x_3 = 2 + c_2 \\ x_4 = 1 \\ x_5 = c_2 \end{cases} \qquad \textbf{(c_1, c_2为任意常数)}$$



例 1 讨论 a, b 取何值时,方程组

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ x_2 + 2x_3 + 2x_4 = 1 \\ -x_2 + (a-3)x_3 - 2x_4 = b \\ 3x_1 + 2x_2 + x_3 + ax_4 = -1 \end{cases}$$
 有无穷解、唯一解,及无解?

 Θ 1 讨论 α , b 取何值时,方程组

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ x_2 + 2x_3 + 2x_4 = 1 \\ -x_2 + (\alpha - 3)x_3 - 2x_4 = b \\ 3x_1 + 2x_2 + x_3 + \alpha x_4 = -1 \end{cases}$$
有无穷解、唯一解,及无解?

$$(A \vdots b) = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & a - 2 & 2 & 1 \\ 0 & -1 & a - 3 & -2 & b \\ 3 & 2 & 1 & a - 1 \end{pmatrix}$$

例 1 讨论 a, b 取何值时, 方程组

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ x_2 + 2x_3 + 2x_4 = 1 \\ -x_2 + (a-3)x_3 - 2x_4 = b \\ 3x_1 + 2x_2 + x_3 + ax_4 = -1 \end{cases}$$
 有无穷解、唯一解,及无解?

$$(A \vdots b) = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & a - 3 & -2 & b \\ 3 & 2 & 1 & a - 1 \end{pmatrix} \xrightarrow{r_4 - 3r_1}$$

例 1 讨论 a, b 取何值时, 方程组

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ x_2 + 2x_3 + 2x_4 = 1 \\ -x_2 + (a-3)x_3 - 2x_4 = b \\ 3x_1 + 2x_2 + x_3 + ax_4 = -1 \end{cases}$$
 有无穷解、唯一解,及无解?

$$(A \vdots b) = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & a - \frac{2}{3} & -\frac{2}{3} & | & 1 \\ 0 & -1 & a & | & a & | & 1 \end{pmatrix} \xrightarrow{r_4 - 3r_1} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 2 & 2 & 1 \\ 0 & -1 & a - \frac{3}{3} & -\frac{2}{3} & | & b \\ 0 & -1 & -2 & a - 3 & | & -1 \end{pmatrix}$$



 M_1 讨论 a, b 取何值时,方程组

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ x_2 + 2x_3 + 2x_4 = 1 \\ -x_2 + (a-3)x_3 - 2x_4 = b \\ 3x_1 + 2x_2 + x_3 + ax_4 = -1 \end{cases}$$
 有无穷解、唯一解,及无解?

$$(A \vdots b) = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & a - \frac{2}{3} & -\frac{2}{3} & | & 1 \\ 0 & -1 & a & | & -1 \end{pmatrix} \xrightarrow{r_4 - 3r_1} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 & | & 1 \\ 0 & -1 & a - \frac{3}{3} & -\frac{2}{3} & | & 1 \\ 0 & -1 & -2 & a - 3 & | & -1 \end{pmatrix}$$

$$r_3+r_2$$



例 1 讨论 a, b 取何值时,方程组

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ x_2 + 2x_3 + 2x_4 = 1 \\ -x_2 + (\alpha - 3)x_3 - 2x_4 = b \\ 3x_1 + 2x_2 + x_3 + ax_4 = -1 \end{cases}$$
 有无穷解、唯一解,及无解?

$$(A \vdots b) = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 2 & 1 \\ 0 & -1 & a - 3 & -2 & b \\ 3 & 2 & a - 1 & a - 1 \end{pmatrix} \xrightarrow{r_4 - 3r_1} \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 2 & 1 \\ 0 & -1 & a - 3 & -2 & b \\ 0 & -1 & a - 3 & -2 & a - 3 & -1 \end{pmatrix}$$

例 2 讨论 α , b 取何值时, 方程组

$$\begin{cases} x_{1}+ & x_{2}+ & x_{3}+ & x_{4}=0\\ & x_{2}+ & 2x_{3}+ & 2x_{4}=1\\ & -x_{2}+ & (\alpha-3)x_{3}- & 2x_{4}=b\\ 3x_{1}+ & 2x_{2}+ & x_{3}+ & \alpha x_{4}=-1 \end{cases}$$
 有无穷解、唯一解,及无

$$-x_2 + (a-3)x_3 - 2x_4 = 0$$

$$3x_1 + 2x_2 + x_3 + ax_4 = -1$$

例 2 讨论 α , b 取何值时, 方程组

$$\begin{cases} x_1 + & x_2 + & x_3 + x_4 = 0 \\ & x_2 + & 2x_3 + 2x_4 = 1 \\ & -x_2 + (\alpha - 3)x_3 - 2x_4 = b \\ 3x_1 + & 2x_2 + & x_3 + ax_4 = -1 \end{cases}$$
 有无穷解、唯一解,及无

- 当 a ≠ 1 时
- 当 a = 1 时

例 2 讨论 α , b 取何值时, 方程组

$$\begin{cases} x_{1}+&x_{2}+&x_{3}+&x_{4}=&0\\ &x_{2}+&2x_{3}+&2x_{4}=&1\\ &-x_{2}+&(\alpha-3)x_{3}-&2x_{4}=&b\\ 3x_{1}+&2x_{2}+&x_{3}+&\alpha x_{4}=&-1 \end{cases}$$

$$\neq A$$

$$(A:b) \longrightarrow \begin{pmatrix} \begin{vmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} b + \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- 当 a ≠ 1 时
- 当 a = 1 时



例 2 讨论 a, b 取何值时,方程组

$$\begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ x_2 + 2x_3 + 2x_4 = 1 \\ -x_2 + (a-3)x_3 - 2x_4 = b \end{cases}$$
 有无穷解、唯一解,及无 $3x_1 + 2x_2 + x_3 + ax_4 = -1$

$$(A:b) \longrightarrow \begin{pmatrix} \begin{vmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

- 当 α ≠ 1 时 (b 为任意数), r(A) = r(A : b) = 4,
- 当 a = 1 时

例 2 讨论 a, b 取何值时,方程组

$$\begin{cases} x_{1}+&x_{2}+&x_{3}+&x_{4}=&0\\ &x_{2}+&2x_{3}+&2x_{4}=&1\\ &-x_{2}+&(\alpha-3)x_{3}-&2x_{4}=&b\\ 3x_{1}+&2x_{2}+&x_{3}+&ax_{4}=&-1 \end{cases}$$
 from all α from α fro

$$(A:b) \longrightarrow \begin{pmatrix} \begin{vmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 2 & 2 & 1 \\ 0 & 0 & 2a - 1 & 0 & b + 1 \\ 0 & 0 & 0 & 2a - 1 & 0 \end{pmatrix}$$

- 当 α ≠ 1 时(b 为任意数), r(A) = r(A · b) = 4, 有唯一解;
- 当 a = 1 时



$$\begin{cases} x_1 + & x_2 + & x_3 + & x_4 = & 0 \\ & x_2 + & 2x_3 + & 2x_4 = & 1 \\ & -x_2 + & (\alpha - 3)x_3 - & 2x_4 = & b \\ 3x_1 + & 2x_2 + & x_3 + & \alpha x_4 = & -1 \end{cases}$$
有无穷解、唯一解,及无

解?

- 当 $\alpha \neq 1$ 时(b 为任意数),r(A) = r(A : b) = 4,有唯一解;
- 当 a = 1 时

例 2 讨论 a, b 取何值时,方程组

$$\begin{cases} x_1 + & x_2 + & x_3 + & x_4 = 0 \\ & x_2 + & 2x_3 + & 2x_4 = 1 \\ & -x_2 + & (\alpha - 3)x_3 - & 2x_4 = b \\ 3x_1 + & 2x_2 + & x_3 + & ax_4 = -1 \end{cases}$$
 有无穷解、唯一解,及无

解?

- 当 α ≠ 1 时(b 为任意数), r(A) = r(A · b) = 4, 有唯一解;
- 当 a = 1 时

$$(A : b) \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} b + \frac{1}{0}$$



例 2 讨论 a, b 取何值时,方程组 $\begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ x_2 + 2x_3 + 2x_4 = 1 \\ -x_2 + (\alpha - 3)x_3 - 2x_4 = b \end{cases}$ 有无穷解、唯一解,及无 $3x_1 + 2x_2 + x_3 + ax_4 = -1$

$$\begin{cases}
-x_2 + (a-3)x_3 - 2x_4 = b \\
3x_1 + 2x_2 + x_3 + ax_4 = -1
\end{cases}$$

$$\text{#?}$$

$$(A:b) \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & a-1 & 0 & b+1 \\ 0 & 0 & a-1 & 0 & a-1 \end{pmatrix}$$

- 当 $\alpha \neq 1$ 时(b为任意数), $r(A) = r(A \cdot b) = 4$, 有唯一解;
- 当 a = 1 时

$$(A:b) \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & b+1 \\ 0 & 0 & 0 & 0 & b \end{pmatrix}$$

- a = 1, b = -1 时
- $a = 1, b \neq -1$ 时



例 2 讨论 a, b 取何值时,方程组 $\begin{cases} x_1 + x_2 + x_3 + x_4 = 0 \\ x_2 + 2x_3 + 2x_4 = 1 \\ -x_2 + (\alpha - 3)x_3 - 2x_4 = b \end{cases}$ 有无穷解、唯一解,及无 $3x_1 + 2x_2 + x_3 + ax_4 = -1$

$$\begin{cases}
-x_2 + (a-3)x_3 - 2x_4 = b \\
3x_1 + 2x_2 + x_3 + ax_4 = -1
\end{cases}$$

$$\text{#?}$$

$$(A:b) \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & a-1 & 0 & b+1 \\ 0 & 0 & a-1 & 0 & a-1 \end{pmatrix}$$

- 当 $\alpha \neq 1$ 时(b为任意数), $r(A) = r(A \cdot b) = 4$, 有唯一解;
- 当 a = 1 时

$$(A:b) \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & b+1 \\ 0 & 0 & 0 & 0 & b \end{pmatrix}$$

- a = 1, b = -1 时
- $a = 1, b \neq -1$ 时



 Θ 2 讨论 α , b 取何值时,方程组 $\begin{cases} x_1 + & x_2 + & x_3 + & x_4 = & 0 \\ & x_2 + & 2x_3 + & 2x_4 = & 1 \\ & -x_2 + & (\alpha - 3)x_3 - & 2x_4 = & b \\ 3x_1 + & 2x_2 + & x_3 + & ax_4 = & -1 \end{cases}$ 有无穷解、唯一解,及无

解?
$$(A:b) \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 & 2 & 2 \\ 0 & 0 & a - 1 & 0 & b + 1 \\ 0 & 0 & 0 & a - 1 & 0 \end{pmatrix}$$

- 当 α ≠ 1 时(b 为任意数), r(A) = r(A · b) = 4, 有唯一解;
- 当 a = 1 时

$$(A:b) \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & b+1 \\ 0 & 0 & 0 & 0 & b \end{pmatrix}$$

-
$$a = 1$$
, $b = -1$ 时, $r(A) = r(A : b) = 2 < 4$,

-
$$a = 1, b \neq -1$$
 时



例 2 讨论 a, b 取何值时,方程组 $\begin{cases} x_1 + & x_2 + & x_3 + & x_4 = & 0 \\ & x_2 + & 2x_3 + & 2x_4 = & 1 \\ & -x_2 + & (\alpha - 3)x_3 - & 2x_4 = & b \\ 3x_1 + & 2x_2 + & x_3 + & ax_4 = & -1 \end{cases}$ 有无穷解、唯一解,及无

- 当 α ≠ 1 时(b 为任意数), r(A) = r(A · b) = 4, 有唯一解;
- 当 a = 1 时

•
$$\exists a = 1 \text{ FT}$$

$$(A : b) \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & b+1 \\ 0 & 0 & 0 & 0 & b+1 \end{pmatrix}$$

- $-\alpha = 1, b = -1$ 时, r(A) = r(A : b) = 2 < 4, 有无穷多解
- $a = 1, b \neq -1$ 时



例 2 讨论 a, b 取何值时,方程组 $\begin{cases} x_1 + & x_2 + & x_3 + & x_4 = & 0 \\ & x_2 + & 2x_3 + & 2x_4 = & 1 \\ & -x_2 + & (\alpha - 3)x_3 - & 2x_4 = & b \\ 3x_1 + & 2x_2 + & x_3 + & ax_4 = & -1 \end{cases}$ 有无穷解、唯一解,及无

- 当 α ≠ 1 时(b 为任意数), r(A) = r(A · b) = 4, 有唯一解;

•
$$\exists a = 1 \exists f$$

$$(A : b) \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & b+1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- $-\alpha = 1, b = -1$ 时, r(A) = r(A : b) = 2 < 4, 有无穷多解
- $a = 1, b \neq -1$ 时



例 2 讨论 a, b 取何值时,方程组 $\begin{cases} x_1 + & x_2 + & x_3 + & x_4 = & 0 \\ & x_2 + & 2x_3 + & 2x_4 = & 1 \\ & -x_2 + & (\alpha - 3)x_3 - & 2x_4 = & b \\ 3x_1 + & 2x_2 + & x_3 + & ax_4 = & -1 \end{cases}$ 有无穷解、唯一解,及无

- 当 α ≠ 1 时(b 为任意数), r(A) = r(A · b) = 4, 有唯一解;

- $-\alpha = 1, b = -1$ 时, r(A) = r(A : b) = 2 < 4, 有无穷多解
- a = 1, $b \neq -1$ 时, r(A) = 2 < 3 = r(A : b),



例 2 讨论 a, b 取何值时,方程组 $\begin{cases} x_1 + & x_2 + & x_3 + & x_4 = 0 \\ & x_2 + & 2x_3 + & 2x_4 = 1 \\ & -x_2 + & (\alpha - 3)x_3 - & 2x_4 = b \\ 3x_1 + & 2x_2 + & x_3 + & ax_4 = -1 \end{cases}$ 有无穷解、唯一解,及无

- 当 α ≠ 1 时(b 为任意数), r(A) = r(A · b) = 4, 有唯一解;

•
$$\[\exists \ a = 1 \ \] \]$$

$$(A : b) \longrightarrow \begin{pmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 2 & 1 \\ 0 & 0 & 0 & 0 & b+1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- $-\alpha = 1, b = -1$ 时, r(A) = r(A : b) = 2 < 4, 有无穷多解
- $\alpha = 1, b \neq -1$ 时, r(A) = 2 < 3 = r(A : b), 无解



$$(A:b) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & a \end{pmatrix} - \frac{1}{b}$$



$$(A:b) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & a \end{pmatrix} - \frac{1}{b} \frac{r_2 - 2r_1}{r_3 - 3r_1}$$

$$(A:b) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & \alpha \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ b \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & 2 & 3 \\ & & 1 \end{pmatrix}$$



$$(A:b) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & a \end{pmatrix} - \begin{pmatrix} 1 \\ b \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 \\ 0 & -1 \end{pmatrix} - \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$



$$(A:b) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & a \end{pmatrix} - \begin{pmatrix} 1 \\ b \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 0 & -2 & a - 9 \end{pmatrix} \begin{pmatrix} 1 \\ b - 3 \end{pmatrix}$$



$$(A:b) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & a \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ b \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 \\ 0 \\ -1 \\ -2 \end{pmatrix} \xrightarrow{r_3 - 3r_1} \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \xrightarrow{r_3 - 2r_2}$$



例 3 讨论
$$a$$
, b 取何值时,方程组
$$\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 + 3x_2 + 5x_3 = -1 \\ 3x_1 + 4x_2 + ax_3 = b \end{cases}$$

$$(A : b) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & a \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ b \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 \\ 0 & -1 \\ 0 & -2 & a - 9 \end{pmatrix} \begin{pmatrix} -1 \\ b - 3 \end{pmatrix}$$

$$\xrightarrow{r_3 - 2r_2} \begin{pmatrix} 1 \\ 0 & -1 \\ 0 & a - 7 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ b + 3 \end{pmatrix}$$



例 3 讨论
$$a$$
, b 取何值时,方程组
$$\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 + 3x_2 + 5x_3 = -1 \\ 3x_1 + 4x_2 + ax_3 = b \end{cases}$$

$$(A:b) = \begin{pmatrix} \frac{1}{2} & \frac{2}{3} & \frac{3}{5} & -\frac{1}{b} \\ \frac{2}{3} & \frac{3}{4} & \frac{5}{a} & -\frac{1}{b} \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} \frac{1}{0} & -\frac{2}{1} & -\frac{3}{1} \\ 0 & -\frac{2}{0} & a - \frac{9}{1} & -\frac{3}{3} \end{pmatrix}$$

$$\xrightarrow{r_3 - 2r_2} \begin{pmatrix} \frac{1}{0} & -\frac{2}{1} & -\frac{3}{1} \\ 0 & 0 & a - \frac{7}{1} & b + \frac{3}{3} \end{pmatrix}$$

- 当 a ≠ 7 时
- 当 a = 7 时

例 3 讨论
$$a$$
, b 取何值时,方程组
$$\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 + 3x_2 + 5x_3 = -1 \\ 3x_1 + 4x_2 + ax_3 = b \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & 2 & 3 & 5 \\ 2 & 3 & 5 & -1 \\ 3 & 4 & a & -1 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & -2 & -3 \\ 0 & -2 & a - 9 \\ -2 & a - 9 & b - 3 \end{pmatrix}$$

$$\xrightarrow{r_3 - 2r_2} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 0 & 0 & a - 7 \\ 0 & 0 & a - 7 \\ 0 & 0 & b + 3 \end{pmatrix}$$

- 当 a ≠ 7 时
- 当 α = 7 时

例 3 讨论
$$a$$
, b 取何值时,方程组
$$\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 + 3x_2 + 5x_3 = -1 \\ 3x_1 + 4x_2 + ax_3 = b \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & 2 & 3 & 5 \\ 2 & 3 & 5 & -1 \\ 3 & 4 & a & -1 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & -2 & -3 \\ 0 & -2 & a - 9 \\ -3 & -2 & -3 \end{pmatrix} \xrightarrow{r_3 - 2r_2} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -3 \\ 0 & 0 & a - 7 \\ 0 & 0 & a - 7 \end{pmatrix} \xrightarrow{r_3 - 2r_2} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -3 \\ 0 & 0 & a - 7 \\ 0 & 0 & a - 7 \end{pmatrix}$$

- 当 α ≠ 7 时(b 为任意数), r(A · b) = r(A) = 3,
- 当 a = 7 时

例 3 讨论
$$a$$
, b 取何值时,方程组
$$\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 + 3x_2 + 5x_3 = -1 \\ 3x_1 + 4x_2 + ax_3 = b \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & 2 & 3 & 5 \\ 2 & 3 & 5 & -1 \\ 3 & 4 & a & -1 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 & -2 & -3 \\ 0 & -2 & a - 9 & b - 3 \end{pmatrix}$$

$$\xrightarrow{r_3 - 2r_2} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 0 & 0 & a - 7 & b + 3 \end{pmatrix}$$

- 当 $\alpha \neq 7$ 时 (b 为任意数), r(A : b) = r(A) = 3, 有唯一解;
- 当α=7时

例 3 讨论
$$a$$
, b 取何值时,方程组
$$\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 + 3x_2 + 5x_3 = -1 \\ 3x_1 + 4x_2 + ax_3 = b \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & a \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ b \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 \\ 0 & -1 \\ 0 & -2 & a - 9 \end{pmatrix} \begin{vmatrix} -1 \\ b - 3 \end{pmatrix}$$

$$\xrightarrow{r_3 - 2r_2} \begin{pmatrix} 1 \\ 0 & -1 \\ 0 & a - 7 \end{vmatrix} \begin{vmatrix} -3 \\ b + 3 \end{vmatrix}$$

- 当 α ≠ 7 时(b 为任意数), r(A : b) = r(A) = 3, 有唯一解;
- 当 a = 7 时

$$(A:b) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & a \end{pmatrix} - \begin{pmatrix} 1 \\ b \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \xrightarrow{r_3 - 3r_1} \begin{pmatrix} 1 \\ 0 \\ -2 \\ a - 9 \end{pmatrix} \begin{pmatrix} -1 \\ b - 3 \end{pmatrix}$$

$$\xrightarrow{r_3 - 2r_2} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ a - 7 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ b + 3 \end{pmatrix}$$

- 当 $\alpha \neq 7$ 时(b为任意数), r(A : b) = r(A) = 3, 有唯一解;
- 当 a = 7 时 $(A:b) \longrightarrow \begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 0 & b+3 \end{pmatrix}$



例 3 讨论 a, b 取何值时,方程组 $\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 + 3x_2 + 5x_3 = -1 \end{cases}$ 有无 $3x_1 + 4x_2 + ax_3 = b$ 穷解、唯一解.及无解?

 $(A:b) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & a \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ b \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 \\ 0 & -1 \\ 0 & -2 & a-9 \end{pmatrix} \begin{pmatrix} 1 \\ b-3 \\ 0 & -3 \end{pmatrix}$

$$\frac{r_3-2r_2}{0} \begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -3 & -3 \\ 0 & a-7 & b+3 \end{pmatrix}$$
• 当 $a \neq 7$ 时 $(b$ 为任意数), $r(A:b) = r(A) = 3$, 有唯一解;
• 当 $a = 7$ 时 $(A:b) \longrightarrow \begin{pmatrix} 1 & 2 & 3 & -1 \\ 0 & -1 & -1 & b+3 \end{pmatrix}$

- a = 7. $b \neq -3$ 时

-a = 7, b = -3 时

 $(A:b) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 \\ 0 \\ -1 \\ -2 \\ -2 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ -3 \end{pmatrix}$

$$\xrightarrow{r_3-2r_2} \begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -1 & -3 \\ 0 & a-7 & b+3 \end{pmatrix}$$
• 当 $a \neq 7$ 时(b 为任意数), $r(A:b) = r(A) = 3$,有唯一解;

- 当 a = 7 时
- $(A:b) \longrightarrow \begin{pmatrix} 1 & 2 & 3 & -1 \\ 0 & -1 & -1 & -3 \end{pmatrix}$
 - -a=7. b=-3 时
 - a = 7, $b \neq -3$ 时



$$(A:b) = \begin{pmatrix} \frac{1}{2} & \frac{2}{3} & \frac{3}{5} & | -\frac{1}{1} \\ \frac{2}{3} & \frac{3}{4} & \frac{5}{a} & | -\frac{1}{1} \\ \frac{7}{3} - 3r_1 & | \frac{1}{3} & | -\frac{2}{3} & | -\frac{3}{1} \\ \frac{r_3 - 2r_2}{3} & | \frac{1}{3} & | -\frac{2}{3} & | -\frac{3}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{1}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{1}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{1}{3} & | -\frac{1}{$$

- 当 $a \neq 7$ 时 (b 为任意数), r(A : b) = r(A) = 3, 有唯一解;
- - a = 7, b = -3 时, r(A : b) = r(A) = 2 < 3,
 - $a = 7, b \neq -3$ 时



$$(A:b) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & a \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ b \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 \\ 0 \\ -1 \\ -2 \\ a - 9 \end{pmatrix} \begin{pmatrix} 3 \\ b - 3 \end{pmatrix}$$

$$\xrightarrow{r_3 - 2r_2} \begin{pmatrix} 1 \\ 0 \\ -1 \\ a - 7 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ b + 3 \end{pmatrix}$$

- 当 $\alpha \neq 7$ 时 (b 为任意数), r(A : b) = r(A) = 3, 有唯一解;
- - -a = 7, b = -3 时, r(A : b) = r(A) = 2 < 3, 有无穷多解
 - $a = 7, b \neq -3$ 时



例 3 讨论 a, b 取何值时,方程组 $\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 + 3x_2 + 5x_3 = -1 \end{cases}$ 有无 $3x_1 + 4x_2 + ax_3 = b$ 穷解、唯一解,及无解?

$$\mathbf{(A:b)} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & a \end{pmatrix} - \begin{pmatrix} 1 \\ b \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \xrightarrow{r_3 - 3r_1} \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \begin{pmatrix} 1 \\ a - 9 \\ b - 3 \end{pmatrix}$$

$$\xrightarrow{r_3 - 2r_2} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \xrightarrow{r_3 - 3r_1} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$

- 当 $\alpha \neq 7$ 时 (b 为任意数), r(A : b) = r(A) = 3, 有唯一解;
- -a = 7, b = -3 时,r(A : b) = r(A) = 2 < 3,有无穷多解
 - a = 7, b ≠ -3 时

- 当 α ≠ 7 时(b 为任意数), r(A · b) = r(A) = 3, 有唯一解;
- $\stackrel{\text{def}}{=} a = 7 \text{ pt}$ $(A \stackrel{\text{def}}{=} b) \longrightarrow \begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 0 & b+3 \end{pmatrix}$
 - -a = 7, b = -3 时, r(A : b) = r(A) = 2 < 3, 有无穷多解
 - $a = 7, b \neq -3$ 时



例 3 讨论 a, b 取何值时,方程组 $\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 + 3x_2 + 5x_3 = -1 \end{cases}$ 有无 $3x_1 + 4x_2 + ax_3 = b$

$$(A:b) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & a \end{pmatrix} - \begin{pmatrix} 1 \\ b \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \xrightarrow{r_3 - 3r_1} \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \xrightarrow{a - 9} \begin{pmatrix} 1 \\ b - 3 \end{pmatrix}$$

$$\frac{r_{3}-2r_{2}}{0} \begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -1 & -3 \\ 0 & a-7 & b+3 \end{pmatrix}$$
• 当 $a \neq 7$ 时(b 为任意数), $r(A : b) = r(A) = 3$,有唯一解;

- 当α=7时
- $(A:b) \longrightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$
 - $-\alpha = 7, b = -3$ 时,r(A : b) = r(A) = 2 < 3,有无穷多解
 - a = 7, $b \neq -3$ 时, $r(A : b) = 3 \neq 2 = r(A)$,



$$(A:b) = \begin{pmatrix} \frac{1}{2} & \frac{2}{3} & \frac{3}{5} & | -\frac{1}{1} \\ \frac{2}{3} & \frac{3}{4} & \frac{5}{a} & | -\frac{1}{1} \\ \frac{7}{3} - 3r_1 & | \frac{1}{3} & | -\frac{2}{3} & | -\frac{3}{1} \\ \frac{r_3 - 2r_2}{3} & | \frac{1}{3} & | -\frac{2}{3} & | -\frac{3}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} & | -\frac{3}{3} & | -\frac{1}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{1}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{1}{3} \\ \frac{1}{3} & | \frac{1}{3} & | -\frac{1}{3} & | -\frac{1}{$$

- 当 $\alpha \neq 7$ 时(b为任意数), r(A:b) = r(A) = 3, 有唯一解;
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 - -a = 7, b = -3 时, r(A : b) = r(A) = 2 < 3, 有无穷多解
 - a = 7, $b \neq -3$ 时, $r(A : b) = 3 \neq 2 = r(A)$, 无解



• 一般线性方程组 $A_{m \times n} x = b$ (m 个方程, n 个未知量)

Ax = b	有无穷解	有唯一解	无解
	r(A) = r(A : b) < n	r(A) = r(A : b) = n	r(A) < r(A : b)

• 一般线性方程组 $A_{m \times n} x = b$ (m 个方程, n 个未知量)

Ax = b	有无穷解	有唯一解	无解
	r(A) = r(A : b) < n	r(A) = r(A : b) = n	r(A) < r(A : b)

• 齐次线性方程组 $A_{m \times n} x = 0$,一定有解(至少有零解), $r(A) = r(A \stackrel{!}{\cdot} 0)$



• 一般线性方程组 $A_{m \times n} x = b$ (m 个方程, n 个未知量)

Ax = b	有无穷解	有唯一解	无解
	r(A) = r(A : b) < n	r(A) = r(A : b) = n	r(A) < r(A : b)

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Ax = 0	有无穷解	有唯一解(零解)

• 一般线性方程组 $A_{m \times n} x = b$ (m 个方程, n 个未知量)

Ax = b	有无穷解	有唯一解	无解
	r(A) = r(A : b) < n	r(A) = r(A : b) = n	r(A) < r(A : b)

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Ax = 0	有无穷解	有唯一解(零解)
	r(A) < n	

• 一般线性方程组 $A_{m \times n} x = b$ (m 个方程, n 个未知量)

Ax = b	有无穷解	有唯一解	无解
	r(A) = r(A : b) < n	r(A) = r(A : b) = n	r(A) < r(A : b)

• 齐次线性方程组 $A_{m\times n}x = 0$,一定有解(至少有零解), $r(A) = r(A \stackrel{!}{\cdot} 0)$

Ax = 0	有无穷解	有唯一解(零解)
	r(A) < n	r(A) = n

例解齐次线性方程组
$$\begin{cases} x_1-&x_2+&5x_3-&x_4&=0\\ x_1+&x_2-&2x_3+&3x_4&=0\\ 3x_1-&x_2+&8x_3+&x_4&=0\\ x_1+&3x_2-&9x_3+&7x_4&=0 \end{cases}$$

例解齐次线性方程组 $\begin{cases} x_1- & x_2+ 5x_3- & x_4=0\\ x_1+ & x_2- 2x_3+ 3x_4=0\\ 3x_1- & x_2+ 8x_3+ x_4=0\\ x_1+ 3x_2- 9x_3+ 7x_4=0 \end{cases}$

$$(A \vdots b) = \begin{pmatrix} 1 & -1 & 5 & -1 & 0 \\ 1 & 1 & -2 & 3 & 0 \\ 3 & -1 & 8 & 1 & 0 \\ 1 & 3 & -9 & 7 & 0 \end{pmatrix}$$



例解齐次线性方程组 $\begin{cases} x_1- & x_2+ 5x_3- & x_4=0\\ x_1+ & x_2- 2x_3+ 3x_4=0\\ 3x_1- & x_2+ 8x_3+ x_4=0\\ x_1+ 3x_2- 9x_3+ 7x_4=0 \end{cases}$

$$(A:b) = \begin{pmatrix} 1 & -1 & 3 & -1 & 0 \\ 1 & 1 & -2 & 3 & 0 \\ 3 & -1 & 8 & 1 & 0 \\ 1 & 3 & -9 & 7 & 0 \end{pmatrix} \xrightarrow[r_3 - 3r_1]{r_3 - 3r_1} \xrightarrow[r_4 - r_1]{r_4 - r_1}$$

例解齐次线性方程组
$$\begin{cases} x_1- & x_2+ 5x_3- & x_4=0\\ x_1+ & x_2- 2x_3+ 3x_4=0\\ 3x_1- & x_2+ 8x_3+ & x_4=0\\ x_1+ 3x_2- 9x_3+ 7x_4=0 \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & -1 & 5 & -1 & 0 \\ 1 & 1 & -2 & 3 & 0 \\ 3 & -1 & 8 & 1 & 0 \\ 1 & 3 & -9 & 7 & 0 \end{pmatrix} \xrightarrow[r_4-r_1]{r_2-r_1} \begin{pmatrix} 1 & -1 & 5 & -1 & 0 \\ r_3-3r_1 & r_4-r_1 & r_4-r_1 & r_4-r_1 & r_4-r_1 \end{pmatrix}$$



例解齐次线性方程组
$$\begin{cases} x_1- & x_2+ 5x_3- & x_4=0\\ x_1+ & x_2- 2x_3+ 3x_4=0\\ 3x_1- & x_2+ 8x_3+ x_4=0\\ x_1+ 3x_2- 9x_3+ 7x_4=0 \end{cases}$$



例解齐次线性方程组
$$\begin{cases} x_1-&x_2+&5x_3-&x_4&=0\\ x_1+&x_2-&2x_3+&3x_4&=0\\ 3x_1-&x_2+&8x_3+&x_4&=0\\ x_1+&3x_2-&9x_3+&7x_4&=0 \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & -1 & 5 & -1 & 0 \\ 1 & 1 & -2 & 3 & 0 \\ 3 & -1 & 8 & 1 & 0 \\ 1 & 3 & -9 & 7 & 0 \end{pmatrix} \xrightarrow[r_4-r_1]{r_2-r_1} \begin{pmatrix} 1 & -1 & 5 & -1 & 0 \\ 0 & 2 & -7 & 4 & 0 \\ 0 & 2 & -7 & 4 & 0 \end{pmatrix}$$



例解齐次线性方程组
$$\begin{cases} x_1- & x_2+ 5x_3- x_4=0\\ x_1+ & x_2- 2x_3+ 3x_4=0\\ 3x_1- & x_2+ 8x_3+ x_4=0\\ x_1+ 3x_2- 9x_3+ 7x_4=0 \end{cases}$$

$$(A:b) = \begin{pmatrix} 1 & -1 & 5 & -1 & 0 \\ 1 & 1 & -2 & 3 & 0 \\ 3 & -1 & 8 & 1 & 0 \\ 1 & 3 & -9 & 7 & 0 \end{pmatrix} \xrightarrow[r_3 - 3r_1]{r_3 - 3r_1} \begin{pmatrix} 1 & -1 & 5 & -1 & 0 \\ 0 & 2 & -7 & 4 & 0 \\ 0 & 2 & -7 & 4 & 0 \\ 0 & 4 & -14 & 8 & 0 \end{pmatrix}$$



例解齐次线性方程组
$$\begin{cases} x_1-&x_2+&5x_3-&x_4&=0\\ x_1+&x_2-&2x_3+&3x_4&=0\\ 3x_1-&x_2+&8x_3+&x_4&=0\\ x_1+&3x_2-&9x_3+&7x_4&=0 \end{cases}$$

$$(A : b) = \begin{pmatrix} 1 & -1 & 5 & -1 & 0 \\ 1 & 1 & -2 & 3 & 0 \\ 3 & -1 & 8 & 1 & 0 \\ 1 & 3 & -9 & 7 & 0 \end{pmatrix} \xrightarrow[r_4 - r_1]{r_2 - r_1} \begin{pmatrix} 1 & -1 & 5 & -1 & 0 \\ 0 & 2 & -7 & 4 & 0 \\ 0 & 2 & -7 & 4 & 0 \\ 0 & 4 & -14 & 8 & 0 \end{pmatrix}$$

$$\frac{r_3 - r_2}{r_4 - 2r_2}$$



例解齐次线性方程组
$$\begin{cases} x_1- & x_2+ 5x_3- & x_4=0\\ x_1+ & x_2- 2x_3+ 3x_4=0\\ 3x_1- & x_2+ 8x_3+ & x_4=0\\ x_1+ 3x_2- 9x_3+ 7x_4=0 \end{cases}$$



例解齐次线性方程组
$$\begin{cases} x_1- & x_2+ 5x_3- & x_4=0\\ x_1+ & x_2- 2x_3+ 3x_4=0\\ 3x_1- & x_2+ 8x_3+ x_4=0\\ x_1+ 3x_2- 9x_3+ 7x_4=0 \end{cases}$$

$$\frac{1}{2} \times r_2$$



例解齐次线性方程组 $\begin{cases} x_1- & x_2+ 5x_3- & x_4=0\\ x_1+ & x_2- 2x_3+ 3x_4=0\\ 3x_1- & x_2+ 8x_3+ x_4=0\\ x_1+ 3x_2- 9x_3+ 7x_4=0 \end{cases}$

$$(A:b) = \begin{pmatrix} 1 & -1 & 5 & -1 & 0 \\ 1 & 1 & -2 & 3 & 0 \\ 3 & -1 & 8 & 1 & 0 \\ 1 & 3 & -9 & 7 & 0 \end{pmatrix} \xrightarrow[r_{4}-r_{1}]{r_{2}-r_{1}} \begin{pmatrix} 1 & -1 & 5 & -1 & 0 \\ 0 & 2 & -7 & 4 & 0 \\ 0 & 2 & -7 & 4 & 0 \\ 0 & 4 & -14 & 8 & 0 \end{pmatrix}$$

$$\begin{array}{c}
r_{3}-r_{2} \\
\hline
r_{4}-2r_{2}
\end{array}
\begin{pmatrix}
1 & -1 & 5 & -1 & 0 \\
0 & 2 & -7 & 4 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{array}{c}
\frac{1}{2} \times r_{2} \\
\hline
0 & 1 & -7/2 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}$$



例解齐次线性方程组 $\begin{cases} x_1-&x_2+&5x_3-&x_4&=0\\ x_1+&x_2-&2x_3+&3x_4&=0\\ 3x_1-&x_2+&8x_3+&x_4&=0\\ x_1+&3x_2-&9x_3+&7x_4&=0 \end{cases}$

$$(A:b) = \begin{pmatrix} 1 & -1 & 5 & -1 & 0 \\ 1 & 1 & -2 & 3 & 0 \\ 3 & -1 & 8 & 1 & 0 \\ 1 & 3 & -9 & 7 & 0 \end{pmatrix} \xrightarrow[r_4-r_1]{r_2-r_1} \begin{pmatrix} 1 & -1 & 5 & -1 & 0 \\ 0 & 2 & -7 & 4 & 0 \\ 0 & 2 & -7 & 4 & 0 \\ 0 & 4 & -14 & 8 & 0 \end{pmatrix}$$

$$\frac{r_{3}-r_{2}}{r_{4}-2r_{2}} \begin{pmatrix}
1 & -1 & 5 & -1 & 0 \\
0 & 2 & -7 & 4 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

$$\frac{\frac{1}{2}\times r_{2}}{1} \begin{pmatrix}
1 & -1 & 5 & -1 & 0 \\
0 & 1 & -7/2 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
\xrightarrow{r_{1}+r_{2}}$$



例解齐次线性方程组 $\begin{cases} x_1- & x_2+ 5x_3- & x_4=0\\ x_1+ & x_2- 2x_3+ 3x_4=0\\ 3x_1- & x_2+ 8x_3+ & x_4=0\\ x_1+ & 3x_2- 9x_3+ 7x_4=0 \end{cases}$

$$(A:b) = \begin{pmatrix} 1 & -1 & 5 & -1 & 0 \\ 1 & 1 & -2 & 3 & 0 \\ 3 & -1 & 8 & 1 & 0 \\ 1 & 3 & -9 & 7 & 0 \end{pmatrix} \xrightarrow[r_4-r_1]{r_2-r_1} \begin{pmatrix} 1 & -1 & 5 & -1 & 0 \\ 0 & 2 & -7 & 4 & 0 \\ 0 & 2 & -7 & 4 & 0 \\ 0 & 4 & -14 & 8 & 0 \end{pmatrix}$$

例解齐次线性方程组 $\begin{cases} x_1-&x_2+5x_3-x_4=0\\ x_1+&x_2-2x_3+3x_4=0\\ 3x_1-&x_2+8x_3+x_4=0\\ x_1+3x_2-9x_3+7x_4=0 \end{cases}$



例解齐次线性方程组 $\begin{cases} x_1- & x_2+ 5x_3- & x_4=0\\ x_1+ & x_2- 2x_3+ 3x_4=0\\ 3x_1- & x_2+ 8x_3+ & x_4=0\\ x_1+ & 3x_2- 9x_3+ 7x_4=0 \end{cases}$

解

所以原方程组等价于



例解齐次线性方程组 $\begin{cases} x_1-&x_2+&5x_3-&x_4&=0\\ x_1+&x_2-&2x_3+&3x_4&=0\\ 3x_1-&x_2+&8x_3+&x_4&=0\\ x_1+&3x_2-&9x_3+&7x_4&=0 \end{cases}$

解

所以原方程组等价于

$$\begin{cases} x_1 + & \frac{3}{2}x_3 + x_4 = 0 \\ & x_2 - \frac{7}{2}x_3 + 2x_4 = 0 \end{cases}$$

例解齐次线性方程组 $\begin{cases} x_1-&x_2+5x_3-x_4=0\\ x_1+&x_2-2x_3+3x_4=0\\ 3x_1-&x_2+8x_3+x_4=0\\ x_1+3x_2-9x_3+7x_4=0 \end{cases}$

解

所以原方程组等价于

$$\begin{cases} x_1 + & \frac{3}{2}x_3 + x_4 = 0 \\ x_2 - \frac{7}{2}x_3 + 2x_4 = 0 \end{cases} \iff \begin{cases} x_1 + & = -\frac{3}{2}x_3 - x_4 \\ x_2 = \frac{7}{2}x_3 - 2x_4 \end{cases}$$

ション 登布大学 JINAN UMVERSITY 例解齐次线性方程组 $\begin{cases} x_1-&x_2+5x_3-x_4=0\\ x_1+&x_2-2x_3+3x_4=0\\ 3x_1-&x_2+8x_3+x_4=0\\ x_1+3x_2-9x_3+7x_4=0 \end{cases}$

所以原方程组等价于
$$\begin{cases} x_1 + \frac{3}{2}x_3 + x_4 = 0 \\ x_2 - \frac{7}{2}x_3 + 2x_4 = 0 \end{cases} \iff \begin{cases} x_1 + \frac{3}{2}x_3 - x_4 \\ x_2 = \frac{7}{2}x_3 - 2x_4 \end{cases}$$

所以 $\begin{cases} x_3 = c_1 \\ x_4 = c_2 \end{cases}$



例解齐次线性方程组 $\begin{cases} x_1-&x_2+&5x_3-&x_4&=0\\ x_1+&x_2-&2x_3+&3x_4&=0\\ 3x_1-&x_2+&8x_3+&x_4&=0\\ x_1+&3x_2-&9x_3+&7x_4&=0 \end{cases}$ 解

所以原方程组等价于
$$\begin{cases} x_1 + \frac{3}{2}x_3 + x_4 = 0 \\ x_2 - \frac{7}{2}x_3 + 2x_4 = 0 \end{cases} \iff \begin{cases} x_1 + \frac{3}{2}x_3 - x_4 \\ x_2 = \frac{7}{2}x_3 - 2x_4 \end{cases}$$

所以 $\begin{cases} x_1 = -\frac{3}{2}c_1 - c_2 \\ x_3 = c_1 \\ x_4 = c_2 \end{cases}$



例解齐次线性方程组 $\begin{cases} x_1-&x_2+&5x_3-&x_4&=0\\ x_1+&x_2-&2x_3+&3x_4&=0\\ 3x_1-&x_2+&8x_3+&x_4&=0\\ x_1+&3x_2-&9x_3+&7x_4&=0 \end{cases}$ 解

所以原方程组等价于
$$\begin{cases} x_1 + \frac{3}{2}x_3 + x_4 = 0 \\ x_2 - \frac{7}{2}x_3 + 2x_4 = 0 \end{cases} \iff \begin{cases} x_1 + \frac{3}{2}x_3 - x_4 \\ x_2 = \frac{7}{2}x_3 - 2x_4 \end{cases}$$

所以 $\begin{cases} x_1 = -\frac{3}{2}c_1 - c_2 \\ x_2 = \frac{7}{2}c_1 - 2c_2 \\ x_3 = c_1 \\ x_4 = c_2 \end{cases}$

例解齐次线性方程组 $\begin{cases} x_1-&x_2+&5x_3-&x_4&=0\\ x_1+&x_2-&2x_3+&3x_4&=0\\ 3x_1-&x_2+&8x_3+&x_4&=0\\ x_1+&3x_2-&9x_3+&7x_4&=0 \end{cases}$ 解

所以 $\begin{cases} x_1 = -\frac{3}{2}c_1 - c_2 \\ x_2 = \frac{7}{2}c_1 - 2c_2 \\ x_3 = c_1 \\ x_4 = c_2 \end{cases}$

所以原方程组等价于 $\begin{cases} x_1 + \frac{3}{2}x_3 + x_4 = 0 \\ x_2 - \frac{7}{2}x_3 + 2x_4 = 0 \end{cases} \iff \begin{cases} x_1 + \frac{3}{2}x_3 - x_4 \\ x_2 = \frac{7}{2}x_3 - 2x_4 \end{cases}$

(注自由变量个数 = 2 = 4 - r(A))