第 9 章 c：多元复合函数的求导法则

数学系　梁卓滨

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二元复合函数求导

设有二元函数 $z = f(u, v)$

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- 设 $u = \varphi(t)$，$v = \psi(t)$，则 $z = f(\varphi(t), \psi(t))$

问 $\frac{dz}{dt} = ?$

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& u \arrow[r] \arrow[dr] & x \\
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\end{tikzcd}$$

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二元复合函数求导公式——中间变量是一元函数

公式 设 $z=f(u,v)$，$u=\varphi(t)$，$v=\psi(t)$，则 $z=f(\varphi(t),\psi(t))$ 的全导数

$$\frac{dz}{dt}=$$

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 & u \arrow[r] & t \\
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$$\frac{dz}{dt} =$$

$$\begin{tikzcd}[column sep=large]
 & u \arrow[r, "\frac{du}{dt}"] & t \\
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$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt}$$

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$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt} \qquad \frac{\partial z}{\partial v} \cdot \frac{dv}{dt}$$

$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
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$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt} + \frac{\partial z}{\partial v} \cdot \frac{dv}{dt}$$

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例 设 $z=uv$，而 $u=e^{-t}$，$v=\sin t$，求全导数 $\frac{dz}{dt}$

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解法一

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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$

$$=(uv)'_u\cdot$$

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$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt} + \frac{\partial z}{\partial v} \cdot \frac{dv}{dt}$$

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&= v\cdot
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&= v \cdot (-e^{-t}) + u \cdot \cos t \\
&= \sin t \cdot (-e^{-t}) + e^{-t} \cdot \cos t \\
&= e^{-t}(\cos t - \sin t)
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解法二

$$\because \quad z=uv=$$

例 设 $z=uv$，而 $u=e^{-t}$，$v=\sin t$，求全导数 $\frac{dz}{dt}$

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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$
$$=(uv)'_u\cdot(e^{-t})'_t+(uv)'_v\cdot(\sin t)'_t$$
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$$\because\quad z=uv=e^{-t}\cdot\sin t$$

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解法二

$$\because\quad z=uv=e^{-t}\cdot\sin t$$

$$\therefore\quad \frac{dz}{dt}=\frac{d}{dt}(e^{-t}\sin t)=(e^{-t})'_t\cdot\sin t+$$

例 设 $z = uv$，而 $u = e^{-t}$，$v = \sin t$，求全导数 $\frac{dz}{dt}$

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$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt} + \frac{\partial z}{\partial v} \cdot \frac{dv}{dt}$$
$$= (uv)'_u \cdot (e^{-t})'_t + (uv)'_v \cdot (\sin t)'_t$$
$$= v \cdot (-e^{-t}) + u \cdot \cos t$$
$$= \sin t \cdot (-e^{-t}) + e^{-t} \cdot \cos t$$
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解法二

$$\because \quad z = uv = e^{-t} \cdot \sin t$$

$$\therefore \quad \frac{dz}{dt} = \frac{d}{dt}(e^{-t} \sin t) = (e^{-t})'_t \cdot \sin t + e^{-t} \cdot (\sin t)'_t$$

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$$\because \quad z=uv=e^{-t}\cdot\sin t$$

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\therefore \quad \frac{dz}{dt} &= \frac{d}{dt}(e^{-t}\sin t)=(e^{-t})'_t\cdot\sin t+e^{-t}\cdot(\sin t)'_t \\
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例 设 $z = \frac{y}{x}$，而 $x = e^t$，$y = 1 - e^{2t}$，求全导数 $\frac{dz}{dt}$

解

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$$\begin{tikzcd} & x \arrow[r, "\frac{dx}{dt}"] & t \\ z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[dr, "\frac{\partial z}{\partial y}"'] & & \\ & y \arrow[r, "\frac{dy}{dt}"'] & t \end{tikzcd}$$

例 设 $z=\frac{y}{x}$，而 $x=e^t$，$y=1-e^{2t}$，求全导数 $\frac{dz}{dt}$

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$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=$$

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解

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=(\frac{y}{x})'_x\cdot$$

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$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} = (\frac{y}{x})'_x \cdot (e^t)'_t + (\frac{y}{x})'_y \cdot$$

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$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=(\frac{y}{x})'_x\cdot(e^t)'_t+(\frac{y}{x})'_y\cdot(1-e^{2t})'_t$$

$$=$$

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$$=-\frac{y}{x^2}\cdot$$

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$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=$$

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解

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=(\frac{y}{x})'_x\cdot(e^t)'_t+(\frac{y}{x})'_y\cdot(1-e^{2t})'_t$$

$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=-\frac{1-e^{2t}}{e^{2t}}\cdot e^t+$$

$$\begin{tikzcd}
 & x \arrow[r, "\frac{dx}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[dr, "\frac{\partial z}{\partial y}"'] & & \\
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$$=$$

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$$
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\frac{dz}{dt} &= \frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=\left(\frac{y}{x}\right)'_x\cdot(e^t)'_t+\left(\frac{y}{x}\right)'_y\cdot(1-e^{2t})'_t \\
&= -\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t}) = -\frac{1-e^{2t}}{e^{2t}}\cdot e^t+\frac{1}{e^t}\cdot(-2e^{2t}) \\
&= -e^{-t}-e^t
\end{aligned}
$$

$$
\begin{tikzcd}
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三元复合函数求导公式——中间变量是一元函数

公式 设 $z=f(u,v,w)$，$u=\varphi(t)$，$v=\psi(t)$，$w=\omega(t)$，则 $z=f(\varphi(t),\psi(t),\omega(t))$ 的全导数

$$\frac{dz}{dt}=$$

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$$\begin{tikzcd}
 & u \arrow[r] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"] & v \arrow[r] & t \\
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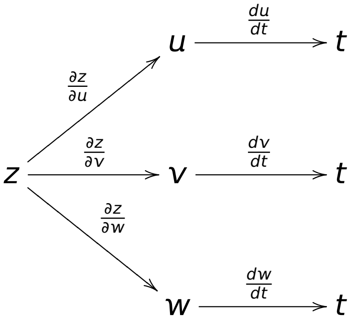

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的偏导数是：

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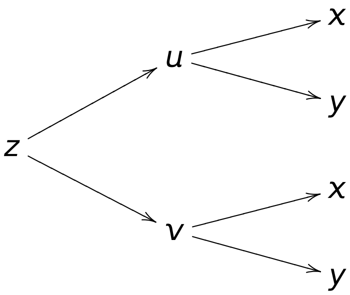
图示



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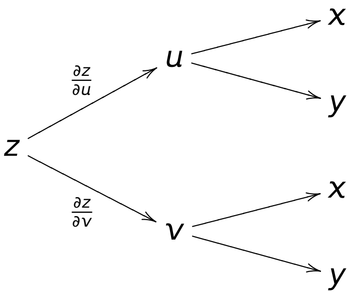

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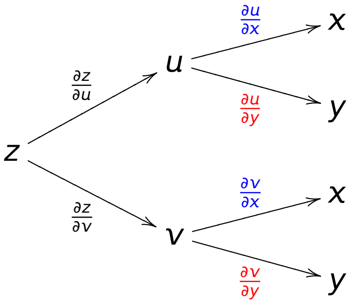

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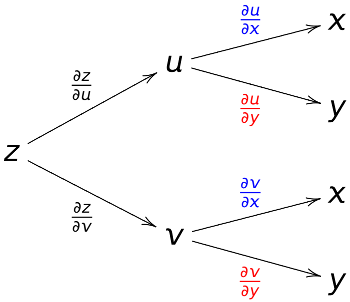

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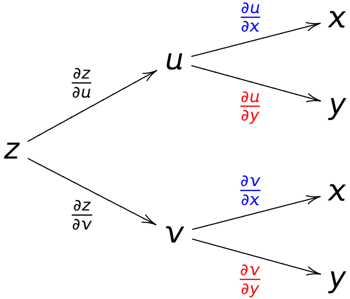
图示



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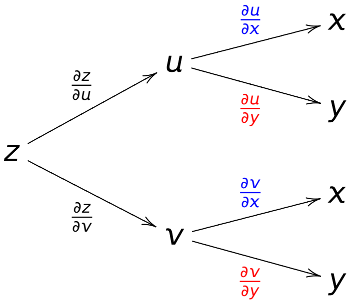

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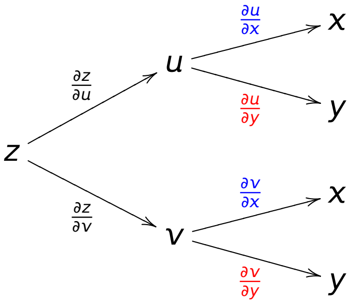
图示



例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

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解
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$$
$$=$$

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$$= (e^{2u} \sin v)'_u \cdot$$

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$$= 2e^{2u}\sin v \cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
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解
$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_x + (e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\ &= 2e^{2u}\sin v\cdot 3x^2y + e^{2u}\cos v\cdot 2x\\ &=\end{aligned}$$

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$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot (x^2 + y^2)'_x$$

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$$= 2e^{2x^3 y}\sin(x^2 + y^2) \cdot$$

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解

$$\begin{aligned}
\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x} \\
&= (e^{2u}\sin v)'_u\cdot(x^3y)'_x + (e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x \\
&= 2e^{2u}\sin v\cdot 3x^2y + e^{2u}\cos v\cdot 2x \\
&= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y + e^{2x^3y}\cos(x^2+y^2)\cdot 2x
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$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x$$
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例 设 $z = e^{2u}\sin v$，$u = x^3y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$\begin{aligned}\frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_y+\end{aligned}$$

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解
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x$$
$$=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$
\begin{aligned}
\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x} \\
&= (e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x \\
&= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x \\
&= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y} \\
&= (e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y \\
&=
\end{aligned}
$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解 $$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

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$$=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$

$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y$$

$$=2e^{2u}\sin v\cdot$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot (x^2 + y^2)'_x$$

$$= 2e^{2u}\sin v \cdot 3x^2 y + e^{2u}\cos v \cdot 2x$$

$$= 2e^{2x^3 y}\sin(x^2 + y^2) \cdot 3x^2 y + e^{2x^3 y}\cos(x^2 + y^2) \cdot 2x$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y}$$

$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_y + (e^{2u}\sin v)'_v \cdot (x^2 + y^2)'_y$$

$$= 2e^{2u}\sin v \cdot x^3 +$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$

$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x$$

$$=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$

$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y$$

$$=2e^{2u}\sin v\cdot x^3+e^{2u}\cos v\cdot$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$$
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$$= 2e^{2u}\sin v \cdot 3x^2 y + e^{2u}\cos v \cdot 2x$$
$$= 2e^{2x^3 y}\sin(x^2 + y^2) \cdot 3x^2 y + e^{2x^3 y}\cos(x^2 + y^2) \cdot 2x$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y}$$
$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_y + (e^{2u}\sin v)'_v \cdot (x^2 + y^2)'_y$$
$$= 2e^{2u}\sin v \cdot x^3 + e^{2u}\cos v \cdot 2y$$
$$=$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
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例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\begin{aligned}\frac{\partial z}{\partial x}&=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\&=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\&=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\&=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial y}&=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\\&=(e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y\\&=2e^{2u}\sin v\cdot x^3+e^{2u}\cos v\cdot 2y\\&=2e^{2x^3y}\sin(x^2+y^2)\cdot x^3+\end{aligned}$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$
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\end{aligned}
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\end{aligned}
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$$= 2e^{2u}\sin v\cdot 3x^2y + e^{2u}\cos v\cdot 2x$$
$$= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y + e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$
$$= (e^{2u}\sin v)'_u\cdot(x^3y)'_y + (e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y$$
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三元复合函数求导公式：举例

公式 设 $z = f(x, y, u)$，$u = u(x, y)$，

三元复合函数求导公式：举例

公式 设 $z = f(x, y, u)$，$u = u(x, y)$，则复合函数

$$z = f(x, y, u(x, y))$$

的偏导数是：

$$\frac{\partial z}{\partial x} = \qquad\qquad\qquad , \quad \frac{\partial z}{\partial y} =$$

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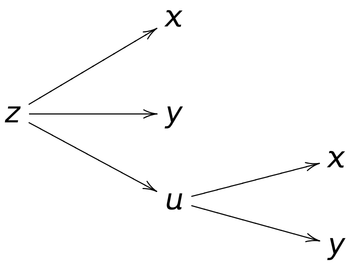
图示



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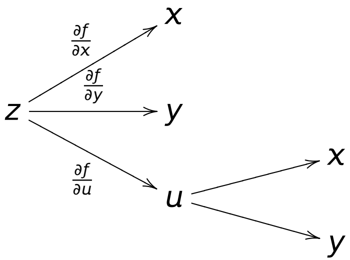

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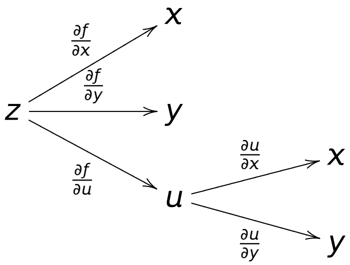
图示



三元复合函数求导公式：举例

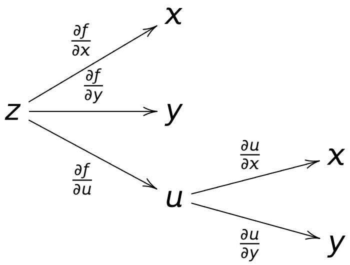
公式 设 $z = f(x, y, u)$，$u = u(x, y)$，则复合函数

$$z = f(x, y, u(x, y))$$

的偏导数是：

$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} + \qquad\qquad , \quad \frac{\partial z}{\partial y} =$$

图示



三元复合函数求导公式：举例

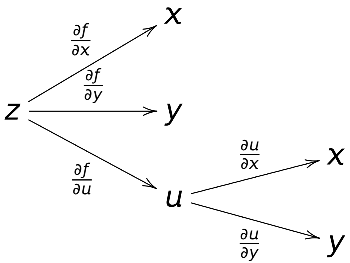
公式 设 $z = f(x, y, u)$，$u = u(x, y)$，则复合函数

$$z = f(x, y, u(x, y))$$

的偏导数是：

$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x}, \qquad \frac{\partial z}{\partial y} =$$

图示



三元复合函数求导公式：举例

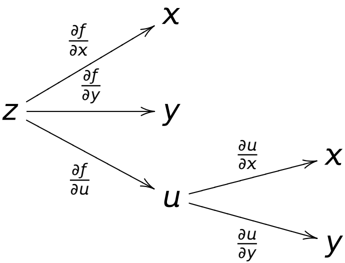
公式 设 $z=f(x,y,u)$，$u=u(x,y)$，则复合函数

$$z=f(x,y,u(x,y))$$

的偏导数是：

$$\frac{\partial z}{\partial x}=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x},\qquad \frac{\partial z}{\partial y}=\frac{\partial f}{\partial y}+$$

图示



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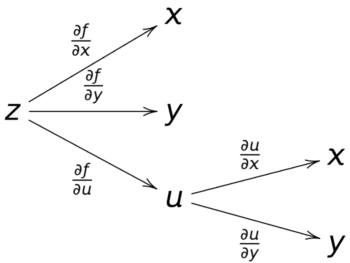
公式 设 $z = f(x, y, u)$，$u = u(x, y)$，则复合函数

$$z = f(x, y, u(x, y))$$

的偏导数是：

$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x}, \qquad \frac{\partial z}{\partial y} = \frac{\partial f}{\partial y} + \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial y}$$

图示



复合函数的高阶导数

公式 设 $z=f(u,v)$，$u=u(x,y)$，$v=v(x,y)$，则复合函数

$$z=f(u(x,y),v(x,y))$$

的偏导数是：

$$z_x=z_u\cdot u_x+z_v\cdot v_x,$$

$$z_y=z_u\cdot u_y+z_v\cdot v_y,$$

复合函数的高阶导数

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的偏导数是：

$z_x=z_u\cdot u_x+z_v\cdot v_x,$

$z_y=z_u\cdot u_y+z_v\cdot v_y,$

$z_{xx}=$

$z_{xy}=$

$z_{yy}=$

复合函数的高阶导数

公式 设 $z = f(u, v)$，$u = u(x, y)$，$v = v(x, y)$，则复合函数

$$z = f(u(x, y), v(x, y))$$

的偏导数是：

$$z_x = z_u \cdot u_x + z_v \cdot v_x,$$

$$z_y = z_u \cdot u_y + z_v \cdot v_y,$$

$$z_{xx} = (z_x)'_x$$

$$z_{xy} =$$

$$z_{yy} =$$

复合函数的高阶导数

公式 设 $z = f(u, v)$，$u = u(x, y)$，$v = v(x, y)$，则复合函数

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复合函数的高阶导数

公式 设 $z=f(u,v)$，$u=u(x,y)$，$v=v(x,y)$，则复合函数

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$$z_x=z_u\cdot u_x+z_v\cdot v_x,$$

$$z_y=z_u\cdot u_y+z_v\cdot v_y,$$

$$\begin{aligned} z_{xx}&=(z_x)'_x=(z_u\cdot u_x+z_v\cdot v_x)'_x \\ &=(z_u)'_x\cdot u_x+z_u\cdot u_{xx}+(z_v)'_x\cdot v_x+z_v\cdot v_{xx} \end{aligned}$$

$$z_{xy}=$$

$$z_{yy}=$$

复合函数的高阶导数

公式 设 $z = f(u, v)$，$u = u(x, y)$，$v = v(x, y)$，则复合函数

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$$\begin{aligned} z_{xx} &= (z_x)'_x = (z_u \cdot u_x + z_v \cdot v_x)'_x \\ &= (z_u)'_x \cdot u_x + z_u \cdot u_{xx} + (z_v)'_x \cdot v_x + z_v \cdot v_{xx} \\ &= (\qquad\qquad\qquad) \cdot u_x + z_u \cdot u_{xx} + (\qquad\qquad\qquad) \cdot v_x + z_v \cdot v_{xx} \end{aligned}$$

$$z_{xy} =$$

$$z_{yy} =$$

复合函数的高阶导数

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$$z_y=z_u\cdot u_y+z_v\cdot v_y,$$

$$\begin{aligned}z_{xx}&=(z_x)'_x=(z_u\cdot u_x+z_v\cdot v_x)'_x\\&=(z_u)'_x\cdot u_x+z_u\cdot u_{xx}+(z_v)'_x\cdot v_x+z_v\cdot v_{xx}\\&=(z_{uu}\cdot u_x+z_{uv}\cdot v_x)\cdot u_x+z_u\cdot u_{xx}+(\qquad\qquad\qquad\qquad)\cdot v_x+z_v\cdot v_{xx}\end{aligned}$$

$$z_{xy}=$$

$$z_{yy}=$$

复合函数的高阶导数

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$$z_y = z_u \cdot u_y + z_v \cdot v_y,$$

$$\begin{aligned}
z_{xx} &= (z_x)'_x = (z_u \cdot u_x + z_v \cdot v_x)'_x \\
&= (z_u)'_x \cdot u_x + z_u \cdot u_{xx} + (z_v)'_x \cdot v_x + z_v \cdot v_{xx} \\
&= (z_{uu} \cdot u_x + z_{uv} \cdot v_x) \cdot u_x + z_u \cdot u_{xx} + (z_{vu} \cdot u_x + z_{vv} \cdot v_x) \cdot v_x + z_v \cdot v_{xx}
\end{aligned}$$

$$z_{xy} =$$

$$z_{yy} =$$

复合函数的高阶导数

公式 设 $z=f(u,v)$，$u=u(x,y)$，$v=v(x,y)$，则复合函数

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的偏导数是：

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$$\begin{aligned} z_{xx} &= (z_x)'_x = (z_u \cdot u_x + z_v \cdot v_x)'_x \\ &= (z_u)'_x \cdot u_x + z_u \cdot u_{xx} + (z_v)'_x \cdot v_x + z_v \cdot v_{xx} \\ &= (z_{uu}\cdot u_x + z_{uv}\cdot v_x)\cdot u_x + z_u\cdot u_{xx} + (z_{vu}\cdot u_x + z_{vv}\cdot v_x)\cdot v_x + z_v\cdot v_{xx} \\ &= z_{uu}u_x^2 + 2z_{uv}u_x v_x + z_{vv}v_x^2 + z_u u_{xx} + z_v v_{xx} \end{aligned}$$

$$z_{xy} =$$

$$z_{yy} =$$

复合函数的高阶导数

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$$\begin{aligned} z_{xx} = (z_x)'_x &= (z_u \cdot u_x + z_v \cdot v_x)'_x \\ &= (z_u)'_x \cdot u_x + z_u \cdot u_{xx} + (z_v)'_x \cdot v_x + z_v \cdot v_{xx} \\ &= (z_{uu} \cdot u_x + z_{uv} \cdot v_x) \cdot u_x + z_u \cdot u_{xx} + (z_{vu} \cdot u_x + z_{vv} \cdot v_x) \cdot v_x + z_v \cdot v_{xx} \\ &= z_{uu} u_x^2 + 2 z_{uv} u_x v_x + z_{vv} v_x^2 + z_u u_{xx} + z_v v_{xx} \end{aligned}$$

$$z_{xy} = ?$$

$$z_{yy} = ?$$

复合函数的高阶导数

公式 设 $z=f(u,v)$，$u=u(x,y)$，$v=v(x,y)$，则复合函数

$$z=f(u(x,y),v(x,y))$$

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$$\begin{aligned}z_{yy}&=(z_y)'_y=(z_u\cdot u_y+z_v\cdot v_y)'_y\\&=(z_u)'_y\cdot u_y+z_u\cdot u_{yy}+(z_v)'_y\cdot v_y+z_v\cdot v_{yy}\end{aligned}$$

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$$\begin{aligned}
z_{yy}&=(z_y)'_y=(z_u\cdot u_y+z_v\cdot v_y)'_y\\
&=(z_u)'_y\cdot u_y+z_u\cdot u_{yy}+(z_v)'_y\cdot v_y+z_v\cdot v_{yy}\\
&=(z_{uu}\cdot u_y+z_{uv}\cdot v_y)\cdot u_y+z_u\cdot u_{yy}+(z_{vu}\cdot u_y+z_{vv}\cdot v_y)\cdot v_y+z_v\cdot v_{yy}
\end{aligned}$$

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$$\begin{aligned} z_{yy} &= (z_y)'_y = (z_u \cdot u_y + z_v \cdot v_y)'_y \\ &= (z_u)'_y \cdot u_y + z_u \cdot u_{yy} + (z_v)'_y \cdot v_y + z_v \cdot v_{yy} \\ &= (z_{uu} \cdot u_y + z_{uv} \cdot v_y) \cdot u_y + z_u \cdot u_{yy} + (z_{vu} \cdot u_y + z_{vv} \cdot v_y) \cdot v_y + z_v \cdot v_{yy} \\ &= z_{uu}u_y^2 + 2z_{uv}u_y v_y + z_{vv}v_y^2 + z_u u_{yy} + z_v v_{yy} \end{aligned}$$

例 设 $z = f(xy^2, x^2y)$，求 $\frac{\partial^2 z}{\partial x \partial y}$

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$$=2yf_u+y^2\cdot(f_{uu}\cdot u_y+f_{uv}\cdot v_y)+2xf_v+2xy\cdot(f_{vu}\cdot u_y+f_{vv}\cdot v_y)$$

$$=2yf_u+y^2\cdot(2xyf_{uu}+x^2f_{uv})+2xf_v+2xy\cdot(\qquad\qquad)$$

例 设 $z=f(xy^2, x^2y)$，求 $\frac{\partial^2 z}{\partial x\partial y}$

解 设 $z=f(u, v)$，$u=xy^2$，$v=x^2y$，则

$$\frac{\partial z}{\partial x}=f_u\cdot u_x+f_v\cdot v_x=f_u\cdot(xy^2)'_x+f_v\cdot(x^2y)'_x=y^2f_u+2xyf_v$$

$$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial}{\partial y}\left(y^2f_u+2xyf_v\right)$$

$$=(y^2)'_y\cdot f_u+y^2\cdot(f_u)'_y+(2xy)'_y\cdot f_v+2xy\cdot(f_v)'_y$$

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$$\begin{aligned}\frac{\partial^2 z}{\partial x\partial y}&=\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial}{\partial y}\left(y^2f_u+2xyf_v\right)\\&=(y^2)'_y\cdot f_u+y^2\cdot(f_u)'_y+(2xy)'_y\cdot f_v+2xy\cdot(f_v)'_y\\&=2yf_u+y^2\cdot(f_{uu}\cdot u_y+f_{uv}\cdot v_y)+2xf_v+2xy\cdot(f_{vu}\cdot u_y+f_{vv}\cdot v_y)\\&=2yf_u+y^2\cdot(2xyf_{uu}+x^2f_{uv})+2xf_v+2xy\cdot(2xyf_{vu}+x^2f_{vv})\\&=2yf_u+2xf_v+2xy^3f_{uu}+x^2y^2f_{uv}+4x^2y^2f_{vu}+2x^3yf_{vv}\end{aligned}$$

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$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}\left(y^2 f_u + 2xyf_v\right)$$

$$= (y^2)'_y \cdot f_u + y^2 \cdot (f_u)'_y + (2xy)'_y \cdot f_v + 2xy \cdot (f_v)'_y$$

$$= 2yf_u + y^2 \cdot (f_{uu} \cdot u_y + f_{uv} \cdot v_y) + 2xf_v + 2xy \cdot (f_{vu} \cdot u_y + f_{vv} \cdot v_y)$$

$$= 2yf_u + y^2 \cdot (2xyf_{uu} + x^2 f_{uv}) + 2xf_v + 2xy \cdot (2xyf_{vu} + x^2 f_{vv})$$

$$= 2yf_u + 2xf_v + 2xy^3 f_{uu} + x^2y^2 f_{uv} + 4x^2y^2 f_{vu} + 2x^3yf_{vv}$$

$$= 2yf_u + 2xf_v + 2xy^3 f_{uu} + 5x^2y^2 f_{uv} + 2x^3yf_{vv}$$

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例 设 $z=f(\sin x, \cos y, e^{x+y})$，求 $\frac{\partial^2 z}{\partial x \partial y}$

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$$=\cos x \cdot f_u+e^{x+y} f_w$$

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$$=\cos x \cdot f_u+e^{x+y} f_w$$

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$$= \cos x \cdot f_u + e^{x+y} f_w$$

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$$= \cos x \cdot (f_u)'_y + (e^{x+y})'_y \cdot f_w + e^{x+y} \cdot (f_w)'_y$$

$$= \cos x \cdot (\qquad\qquad\qquad\qquad)$$

$$+ e^{x+y} f_w + e^{x+y} \cdot (\qquad\qquad\qquad\qquad)$$

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$$= \cos x \cdot (f_u)'_y + (e^{x+y})'_y \cdot f_w + e^{x+y} \cdot (f_w)'_y$$

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$$+e^{x+y}f_w+e^{x+y}\cdot(f_{wu}\cdot u_y+f_{wv}\cdot v_y+f_{ww}\cdot w_y)$$

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$$+e^{x+y} f_w+e^{x+y} \cdot (f_{wu} \cdot u_y+f_{wv} \cdot v_y+f_{ww} \cdot w_y)$$

$$=\cos x \cdot (-\sin y \cdot f_{uv}+e^{x+y} f_{uw})$$

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$$=e^{x+y} f_w-\cos x \sin y \cdot f_{uv}+\cos x e^{x+y} f_{uw}-\sin y e^{x+y} f_{wv}+e^{2x+2y} f_{ww}$$