第3章b:向量与向量组的线性组合

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● n 维行向量

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- 零向量 O = (0, 0, ···, 0)

•
$$\mathfrak{P} \alpha = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}, \ \beta = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}, \ k \in \mathbb{R}, \ \mathbb{M}$$

$$\alpha + \beta =$$
 , $\alpha - \beta =$, $k\alpha =$

• 设
$$\alpha = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$
, $\beta = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$, $k \in \mathbb{R}$, 则

$$\alpha + \beta = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{pmatrix}, \quad \alpha - \beta = \qquad , \quad k\alpha =$$

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• 行向量类似

• 给定向量组

$$\alpha_1 = \begin{pmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{m1} \end{pmatrix}, \ \alpha_2 = \begin{pmatrix} \alpha_{12} \\ \alpha_{22} \\ \vdots \\ \alpha_{m2} \end{pmatrix}, \dots, \ \alpha_n = \begin{pmatrix} \alpha_{1n} \\ \alpha_{2n} \\ \vdots \\ \alpha_{mn} \end{pmatrix}$$

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设 k_1 , k_2 , · · · , k_n 为任意数,则称

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_n\alpha_n$$

为向量组 α_1 , α_2 , ..., α_n 的 **线性组合** 。

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• 问题 给定向量 $\beta = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b \end{pmatrix}$,问 β 能否由 $\alpha_1, \alpha_2, \cdots, \alpha_n$ 线性表示?

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即:是否存在数 k_1, k_2, \ldots, k_n 使得:

$$\beta = k_1 \alpha_1 + k_2 \alpha_2 + \dots + k_n \alpha_n?$$

线性组合

例 判断 β 能否由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式 $\beta = k_1\alpha_1 + k_2\alpha_2 + k_3\alpha_3$ 。

$$\begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} \qquad \begin{pmatrix} \alpha_1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \alpha_2 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \alpha_3 \\ 0 \\ 0 \\ 2 \end{pmatrix}$$

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$$\begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} = -\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + --\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + -\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

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$$\begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} = \mathbf{2} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \mathbf{2} \begin{pmatrix} \alpha_1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \mathbf{3} \begin{pmatrix} \alpha_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

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$$\begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} = \frac{2}{2} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \frac{-7}{2} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \frac{5}{2} \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

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$$\overrightarrow{P}$$

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所以
$$\beta = 2\alpha_1 - 7\alpha_2 + \frac{5}{2}\alpha_3$$
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所以 $\beta = \frac{2\alpha_1 - 7\alpha_2 + \frac{5}{2}\alpha_3}{3}$; β 能由 α_1 , α_2 , α_3 线性表示

(2)
$$\stackrel{\beta}{\bowtie}$$

$$\begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} \qquad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

线性组合

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1 \\
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0
\end{pmatrix} + \frac{-7}{2} \begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix} + \frac{5}{2} \begin{pmatrix}
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\end{pmatrix}$$

所以 $\beta = 2\alpha_1 - 7\alpha_2 + \frac{5}{2}\alpha_3$; β 能由 α_1 , α_2 , α_3 线性表示

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 β α_1 α_2 α_3 α_4 α_5 α_5

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所以 $\beta = \frac{2\alpha_1 - 7\alpha_2}{3} + \frac{5}{2}\alpha_3$; β 能由 α_1 , α_2 , α_3 线性表示

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$$\begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

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(2)
$$\triangleright$$

$$\begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} \times \begin{pmatrix} \alpha_1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \alpha_2 \\ 2 \\ 3 \\ 0 \end{pmatrix} + \begin{pmatrix} \alpha_3 \\ 0 \\ 2 \\ 0 \end{pmatrix}$$

所以 β 不能由 α_1 , α_2 , α_3 线性表示!

例 问
$$\begin{pmatrix} \beta & \alpha_1 & \alpha_2 & \alpha_3 \\ 2 & 3 & 0 \\ 5 & 0 & 1 \\ 2 & 0 & 1 \end{pmatrix}$$
 + $-\begin{pmatrix} 2 & \alpha_3 & \alpha_4 & \alpha_5 \\ -1 & 1 & 1 \\ 1 & -2 & 1 \end{pmatrix}$ + $-\begin{pmatrix} 3 & 2 & \alpha_5 & \alpha_5 & \alpha_5 \\ 2 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$

即: β 能否由 α_1 , α_2 , α_3 线性表示? 如果能,线性表达式是什么?

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例 问
$$\begin{bmatrix} \beta & \alpha_1 & \alpha_2 & \alpha_3 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} + - \begin{bmatrix} 2 & \alpha_3 & \alpha_3 \\ -1 & 1 \\ 1 & -2 \end{bmatrix} + - \begin{bmatrix} 3 & 2 & \alpha_3 & \alpha_3 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

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问题

• 一般地,如何判断 β 能否由 $\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性表示?

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$$\begin{bmatrix} \beta & \alpha_1 & \alpha_2 & \alpha_3 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} + \dots \begin{bmatrix} \alpha_2 & \alpha_3 & \alpha_3 \\ 2 & -1 & 1 \\ 1 & -2 \end{bmatrix} + \dots \begin{bmatrix} \alpha_3 & \alpha_3 & \alpha_3 & \alpha_3 & \alpha_3 \\ 2 & 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

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问题

- 一般地,如何判断 β 能否由 $\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性表示?
- 如果能线性表出,如何求出 k₁, k₂,..., kn 使

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_n\alpha_n = \beta$$
?

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$$\begin{bmatrix} \beta & \alpha_1 & \alpha_2 & \alpha_3 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} + - \begin{pmatrix} 2 & \alpha_3 & \alpha_3 \\ -1 & 1 & 1 \\ 1 & -2 \end{pmatrix} + - \begin{pmatrix} 3 & 2 & \alpha_3 & \alpha_3 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

即: β 能否由 α_1 , α_2 , α_3 线性表示? 如果能,线性表达式是什么?

问题

- 一般地,如何判断 β 能否由 $\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性表示?
- 如果能线性表出,如何求出 *k*₁, *k*₂, . . . , *k*_n 使

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_n\alpha_n = \beta$$
?

不难看出, k_1, \dots, k_n 的求解可归结为线性方程组的求解。

 $\begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} \qquad \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} \qquad \cdots \qquad \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} \qquad \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{mn} \end{pmatrix}$

 β 可由 $\alpha_1, \alpha_2, \cdots, \alpha_n$ 线性表示

$lpha_1$	α_2	α_n	β
$\begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix}$	$\begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix}$	$\begin{pmatrix} a_{1n} \\ a_{2n} \end{pmatrix}$	$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$
:	:		
(a_{m1})	$\langle a_{m2} \rangle$	$\langle a_{mn} \rangle$	$\setminus b_m$

$$\Leftrightarrow k_1 \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_2 \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \dots + k_n \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

$$\Leftrightarrow k_{1} \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_{2} \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \cdots + k_{n} \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\begin{pmatrix} a_{1} & a_{2} & \alpha_{n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} k_{1} \\ k_{2} \\ \vdots \\ k_{n} \end{pmatrix}$$

$$\Leftrightarrow k_{1} \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_{2} \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \cdots + k_{n} \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} k_{1} \\ k_{2} \\ \vdots \\ k_{n} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

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eta可由 $lpha_1$, $lpha_2$, \cdots , $lpha_n$ 线性表示

$$\Leftrightarrow k_{1} \begin{pmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{m1} \end{pmatrix} + k_{2} \begin{pmatrix} \alpha_{12} \\ \alpha_{22} \\ \vdots \\ \alpha_{m2} \end{pmatrix} + \cdots + k_{n} \begin{pmatrix} \alpha_{1n} \\ \alpha_{2n} \\ \vdots \\ \alpha_{mn} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} \end{pmatrix} \begin{pmatrix} k_{1} \\ k_{2} \\ \vdots \\ k_{n} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

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$$\Leftrightarrow k_{1} \begin{pmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{m1} \end{pmatrix} + k_{2} \begin{pmatrix} \alpha_{12} \\ \alpha_{22} \\ \vdots \\ \alpha_{m2} \end{pmatrix} + \cdots + k_{n} \begin{pmatrix} \alpha_{1n} \\ \alpha_{2n} \\ \vdots \\ \alpha_{mn} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} \alpha_{1n} & \alpha_{2} & \alpha_{n} \\ \alpha_{1n} & \alpha_{2} & \alpha_{n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} \end{pmatrix} \begin{pmatrix} k_{1} \\ k_{2} \\ \vdots \\ k_{n} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

 \Leftrightarrow $Ax = \beta$ 有解

$$\beta$$
可由 α_1 , α_2 , \cdots , α_n 线性表示

$$\Leftrightarrow k_{1} \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_{2} \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \cdots + k_{n} \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} k_{1} \\ k_{2} \\ \vdots \\ k_{n} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow$$
 $Ax = \beta$ 有解 $(k_1, \dots, k_n$ 是方程的解)

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$$\beta$$
可由 $\alpha_1, \alpha_2, \cdots, \alpha_n$ 线性表示

$$\Leftrightarrow k_1 \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_2 \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \dots + k_n \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

$$\Leftrightarrow Ax = \beta f \text{m} \qquad (k_1, \dots, k_n) E f \text{meth}$$

 \Leftrightarrow $r(A) = r(A:\beta)$

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$$\beta$$
可由 $\alpha_1, \alpha_2, \cdots, \alpha_n$ 线性表示

$$\Leftrightarrow$$
 $Ax = β$ 有解 $(k_1, \dots, k_n$ 是方程的解)

$$\Leftrightarrow r(A) = r(A : \beta) \iff (\alpha_1 \alpha_2 \cdots \alpha_n) \quad (\alpha_1 \alpha_2 \cdots \alpha_n \beta)$$

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$$\beta$$
可由 $\alpha_1, \alpha_2, \cdots, \alpha_n$ 线性表示

$$\Leftrightarrow k_{1} \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_{2} \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \cdots + k_{n} \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} k_{1} \\ k_{2} \\ \vdots \\ k_{n} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow A_{X} = a_{1} + a_{22} + \cdots + a_{mn} + a_{$$

$$\Leftrightarrow$$
 $Ax = β$ 有解 $(k_1, \dots, k_n$ 是方程的解)

$$\Leftrightarrow$$
 $r(A) = r(A:\beta)$ \Leftrightarrow $r(\alpha_1 \alpha_2 \cdots \alpha_n) = r(\alpha_1 \alpha_2 \cdots \alpha_n \beta)$

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$$eta$$
可由 $lpha_1$, $lpha_2$, \cdots , $lpha_n$ 线性表示

$$\Leftrightarrow k_1 \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_2 \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \cdots + k_n \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

$$\Leftrightarrow Ax = \beta f g g \qquad (k_1, \cdots, k_n) \xi f \xi h g h g$$

$$\Leftrightarrow r(A) = r(A : \beta) \Leftrightarrow r(\alpha_1 \alpha_2 \cdots \alpha_n) = r(\alpha_1 \alpha_2 \cdots \alpha_n \beta)$$

定理 β 可由 $\alpha_1, \alpha_2, \cdots, \alpha_n$ 线性表示 $\Leftrightarrow r(\alpha_1 \alpha_2 \cdots \alpha_n) = r(\alpha_1 \alpha_2 \cdots \alpha_n \beta)$

$$\Leftrightarrow k_{1}\begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_{2}\begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \cdots + k_{n}\begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} k_{1} \\ k_{2} \\ \vdots \\ k_{n} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow Ax = \beta f \beta \beta \qquad (k_{1}, \cdots, k_{n}) \xi f \xi b \beta \beta$$

定理 β 可由 $\alpha_1, \alpha_2, \cdots, \alpha_n$ 线性表示 $\Leftrightarrow r(\alpha_1 \alpha_2 \cdots \alpha_n) = r(\alpha_1 \alpha_2 \cdots \alpha_n \beta)$

 \Leftrightarrow $r(A) = r(A:\beta) \Leftrightarrow r(\alpha_1 \alpha_2 \cdots \alpha_n) = r(\alpha_1 \alpha_2 \cdots \alpha_n \beta)$

 $\mathbf{k}_1, \dots, \mathbf{k}_n$ 的求解不需要特意解方程 $\mathbf{k}_1, \dots, \mathbf{k}_n$ 的求解不需要特意解方程 $\mathbf{k}_2 = \mathbf{k}_3$,方法见下例

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (1)

$$(\alpha_1 \ \alpha_2 \ \alpha_3 | \beta) = \begin{pmatrix} 1 & 2 & 3 | 2 \\ 0 & -1 & 2 | 3 \\ 1 & 1 & 0 | 0 \\ 2 & -2 & 1 | 5 \end{pmatrix}$$

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$$r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta) = 3,$$

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (1)

$$(\alpha_1 \ \alpha_2 \ \alpha_3 \ | \) = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 1 & 1 & 0 \\ 2 & -2 & 1 \end{pmatrix} \xrightarrow{\text{初等行变换}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

•
$$r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta) = 3$$

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• 所以 $r(\alpha_1\alpha_2\alpha_3) = 3$, $r(\alpha_1\alpha_2\alpha_3\beta) = 3$,

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (1)

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• 所以
$$r(\alpha_1\alpha_2\alpha_3) = 3$$
, $r(\alpha_1\alpha_2\alpha_3\beta) = 3$,成立
$$r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta)$$

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例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (1)

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• 所以
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$$r(\alpha_1\alpha_2\alpha_3)=r(\alpha_1\alpha_2\alpha_3\beta)$$

 β 可由 α_1 , α_2 , α_3 线性表示。

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例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。

(1)
$$(\alpha_1 \ \alpha_2 \ \alpha_3 \ | \ \beta \) = \begin{pmatrix} 1 & 2 & 3 \ | & 2 \ 0 & -1 & 2 \ | & 3 \ 1 & 1 & 0 \ | & 0 \ 2 & -2 & 1 \ | & 5 \end{pmatrix} \xrightarrow{\overline{\text{初等行变换}}} \begin{pmatrix} \alpha_1' & \alpha_2' & \alpha_3' & \beta' \\ 0 & 1 & 0 & | & 1 \ 0 & 0 & 1 \ | & 1 \ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

• 所以 $r(\alpha_1\alpha_2\alpha_3) = 3$, $r(\alpha_1\alpha_2\alpha_3\beta) = 3$, 成立

$$r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta)$$

 β 可由 α_1 , α_2 , α_3 线性表示。

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• 所以
$$r(\alpha_1\alpha_2\alpha_3) = 3$$
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 β 可由 α_1 , α_2 , α_3 线性表示。

• 显然 $\beta' = \alpha'_1 - \alpha'_2 + \alpha'_3$,

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。

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• 所以
$$r(\alpha_1\alpha_2\alpha_3) = 3$$
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$$r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta)$$

 β 可由 α_1 , α_2 , α_3 线性表示。

• 显然 $\beta' = \alpha'_1 - \alpha'_2 + \alpha'_3$,是否也有 $\beta = \alpha_1 - \alpha_2 + \alpha_3$?

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。

(1)
$$(\alpha_1 \ \alpha_2 \ \alpha_3 \ | \ \beta \) = \begin{pmatrix} 1 & 2 & 3 \ | & 2 \ 0 & -1 & 2 \ | & 3 \ 1 & 1 & 0 \ | & 0 \ 2 & -2 & 1 \ | & 5 \end{pmatrix} \xrightarrow{\overline{\text{初等行变换}}} \begin{pmatrix} \alpha_1' & \alpha_2' & \alpha_3' & \beta' \\ 1 & 0 & 0 \ | & 1 \ 0 \\ 0 & 0 & 1 \ | & 1 \ 0 \\ 0 & 0 & 0 \ | & 0 \end{pmatrix}$$

• 所以
$$r(\alpha_1\alpha_2\alpha_3) = 3$$
, $r(\alpha_1\alpha_2\alpha_3\beta) = 3$,成立

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是的

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。

(1)
$$(\alpha_1 \ \alpha_2 \ \alpha_3 \ | \ \beta \) = \begin{pmatrix} 1 & 2 & 3 \ | & 2 \ 0 & -1 & 2 \ | & 3 \ 1 & 1 & 0 \ | & 0 \ 2 & -2 & 1 \ | & 5 \end{pmatrix} \xrightarrow{\overline{ay}} \begin{pmatrix} \alpha_1' \ \alpha_2' \ \alpha_3' \ \beta' \ 0 & 1 & 0 \ | & 1 \ 0 & 0 & 1 \ | & 1 \ 0 & 0 & 0 \ | & 0 \end{pmatrix}$$

• 所以
$$r(\alpha_1\alpha_2\alpha_3) = 3$$
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$$r(\alpha_1\alpha_2\alpha_3)=r(\alpha_1\alpha_2\alpha_3\beta)$$

 β 可由 α_1 , α_2 , α_3 线性表示。

• 显然
$$\beta' = \alpha'_1 - \alpha'_2 + \alpha'_3$$
,是否也有 $\beta = \alpha_1 - \alpha_2 + \alpha_3$?

是的

注 可证明:作初等行变换不改变列与列之间的"线性关系"。

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (2)

$$(\alpha_1 \ \alpha_2 \ \alpha_3 | \beta) = \begin{pmatrix} 1 & 2 & -1 & | & 4 \\ 2 & -1 & 3 & | & 3 \\ -1 & 1 & -2 & | & 0 \\ 5 & 1 & 4 & | & 11 \end{pmatrix}$$

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (2)

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•
$$r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta) = 3$$

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (2)

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•
$$r(\alpha_1\alpha_2\alpha_3) = 2$$
, $r(\alpha_1\alpha_2\alpha_3\beta) = 3$,

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (2)

$$(\alpha_1 \ \alpha_2 \ \alpha_3 \ | \ \beta \) = \begin{pmatrix} 1 & 2 & -1 & | & 4 \\ 2 & -1 & 3 & | & 3 \\ -1 & 1 & -2 & | & 0 \\ 5 & 1 & 4 & | & 11 \end{pmatrix} \xrightarrow{\text{初等行变换}} \begin{pmatrix} 1 & 2 & -1 & | & 4 \\ 0 & 1 & -1 & | & 1 \\ 0 & 0 & 0 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

• 所以
$$r(\alpha_1\alpha_2\alpha_3) = 2$$
, $r(\alpha_1\alpha_2\alpha_3\beta) = 3$,成立
$$r(\alpha_1\alpha_2\alpha_3) \neq r(\alpha_1\alpha_2\alpha_3\beta)$$

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例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (2)

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$$r(\alpha_1\alpha_2\alpha_3) = 2$$
, $r(\alpha_1\alpha_2\alpha_3\beta) = 3$,成立
$$r(\alpha_1\alpha_2\alpha_3) \neq r(\alpha_1\alpha_2\alpha_3\beta)$$

 β 不能由 α_1 , α_2 , α_3 线性表示。

<mark>问题</mark> β 能否由 $α_1$, $α_2$, ..., $α_n$ 线性表示? 若能,写出线性表示等式。

步骤

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步骤

$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n | \beta)$$

问题 β 能否由 $\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性表示? 若能,写出线性表示等式。 步骤

$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n | \beta) \xrightarrow{\overline{\text{NS}_{7}^{\oplus}}}$$

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$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n | eta) \xrightarrow{ ext{ instance}} (\alpha_1' \ \alpha_2' \ \cdots \ \alpha_n' | eta')$$
 (简化)

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$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n | eta) \xrightarrow{ ext{初等行变换}} (\alpha_1' \ \alpha_2' \ \cdots \ \alpha_n' | eta')$$
 (简化)

1. β 由 α_1 , α_2 , ..., α_n 线性表示 $\Leftrightarrow r(\alpha_1 \cdots \alpha_n) = r(\alpha_1 \cdots \alpha_n \beta)$.

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$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n | eta) \xrightarrow{\overline{\eta}$$
 $\overline{\eta}$ $\overline{\eta$

1. β 由 α_1 , α_2 , ..., α_n 线性表示 \Leftrightarrow $r(\alpha_1 \cdots \alpha_n) = r(\alpha_1 \cdots \alpha_n \beta)$. 也就是,在阶梯型矩阵中

$$(\alpha'_1\alpha'_2\cdots\alpha'_n)$$
 非零行数 = $(\alpha'_1\alpha'_2\cdots\alpha'_n|\beta)$ 非零行数

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$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n | \beta) \xrightarrow{\eta \in \uparrow \circ \circ} (\alpha_1' \ \alpha_2' \ \cdots \ \alpha_n' | \beta')^{(简化)}_{N R 型矩阵}$$

1. β 由 α_1 , α_2 , ..., α_n 线性表示 $\Leftrightarrow r(\alpha_1 \cdots \alpha_n) = r(\alpha_1 \cdots \alpha_n \beta)$. 也就是,在阶梯型矩阵中

$$(\alpha_1'\alpha_2'\cdots\alpha_n')$$
 非零行数 = $(\alpha_1'\alpha_2'\cdots\alpha_n'|\beta)$ 非零行数

2. 行变换前后列与列的线性关系不变,即:

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$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n | \beta) \xrightarrow{\overline{\eta + \alpha_0^*}} (\alpha_1' \ \alpha_2' \ \cdots \ \alpha_n' | \beta')^{(简化)}_{\overline{n}$$
 (简化)

1. β 由 α_1 , α_2 , ..., α_n 线性表示 \Leftrightarrow $r(\alpha_1 \cdots \alpha_n) = r(\alpha_1 \cdots \alpha_n \beta)$. 也就是,在阶梯型矩阵中

$$(\alpha_1'\alpha_2'\cdots\alpha_n')$$
 非零行数 = $(\alpha_1'\alpha_2'\cdots\alpha_n'|\beta)$ 非零行数

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$$\beta' = k_1 \alpha_1' + \dots + k_n \alpha_n' \Rightarrow$$

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$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n \,|\, eta\) \stackrel{{\overline{\eta}}$$
 $\xrightarrow{\overline{\eta}}$ $(\alpha_1' \ \alpha_2' \ \cdots \ \alpha_n' \,|\, eta'\)$ (简化)

1. β 由 α_1 , α_2 , ..., α_n 线性表示 \Leftrightarrow $r(\alpha_1 \cdots \alpha_n) = r(\alpha_1 \cdots \alpha_n \beta)$. 也就是,在阶梯型矩阵中

$$(\alpha_1'\alpha_2'\cdots\alpha_n')$$
 非零行数 = $(\alpha_1'\alpha_2'\cdots\alpha_n'|\beta)$ 非零行数

2. 行变换前后列与列的线性关系不变,即:

$$\beta' = k_1 \alpha_1' + \dots + k_n \alpha_n' \Rightarrow \beta = k_1 \alpha_1 + \dots + k_n \alpha_n$$

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例1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

例1
$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\mathbf{H}$$
 α_1 α_2 α_3 β

$$\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & -1 & 2 & 3 \\
1 & 1 & 0 & 0 \\
2 & -2 & 1 & 5
\end{pmatrix}$$

例 1
$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\mathbf{H}$$
 α_1 α_2 α_3 β

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \xrightarrow{r_4 - 2r_1}$$

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$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\mathbf{H}$$
 α_1 α_2 α_3 β

$$\begin{pmatrix}
1 & 2 & 3 & | & 2 \\
0 & -1 & 2 & | & 3 \\
1 & 1 & 0 & | & 0 \\
2 & -2 & 1 & | & 5
\end{pmatrix}
\xrightarrow[r_4-2r_1]{r_3-r_1}
\begin{pmatrix}
1 & 2 & 3 & | & 2 \\
0 & -1 & 2 & | & 3 \\
0 & -1 & 2 & | & 3
\end{pmatrix}$$

例 1
$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\mathbf{H}$$
 α_1 α_2 α_3 β

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \end{pmatrix}$$

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 α_1 α_2 α_3 β

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

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$$\mathbf{H}$$
 α_1 α_2 α_3 β

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2}$$

例 1
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能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\mathbf{H}$$
 α_1 α_2 α_3 λ

$$\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & -1 & 2 & 3 \\
1 & 1 & 0 & 0 \\
2 & -2 & 1 & 5
\end{pmatrix}
\xrightarrow{r_3 - r_1}
\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & -1 & 2 & 3 \\
0 & -1 & -3 & -2 \\
0 & -6 & -5 & 1
\end{pmatrix}
\xrightarrow{(-1) \times r_2}
\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & 1 & -2 & -3 \\
0 & -1 & -3 & -2 \\
0 & -6 & -5 & 1
\end{pmatrix}$$

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$$\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & -1 & 2 & 3 \\
1 & 1 & 0 & 0 \\
2 & -2 & 1 & 5
\end{pmatrix}
\xrightarrow[r_4-2r_1]{r_3-r_1}
\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & -1 & 2 & 3 \\
0 & -1 & -3 & -2 \\
0 & -6 & -5 & 1
\end{pmatrix}
\xrightarrow[r_4-2r_1]{(-1)\times r_2}
\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & 1 & -2 & -3 \\
0 & -1 & -3 & -2 \\
0 & -6 & -5 & 1
\end{pmatrix}$$

$$r_3+r_2$$

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$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
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$$\mathbf{H}$$
 α_1 α_2 α_3 β

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{r_4+6r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \end{pmatrix}$$

例 1
$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\mathbf{H}$$
 α_1 α_2 α_3 β

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\begin{array}{c|cccc}
r_{3}+r_{2} \\
\hline
r_{4}+6r_{2}
\end{array}
\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & 1 & -2 & -3 \\
0 & 0 & -5 & -5
\end{pmatrix}$$

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$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\begin{array}{c|cccc}
r_3 + r_2 \\
\hline
r_4 + 6r_2
\end{array}
\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & 1 & -2 & -3 \\
0 & 0 & -5 & -5 \\
0 & 0 & -17 & -17
\end{pmatrix}$$

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$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\begin{array}{c}1 & 2 & 3 & 2\\0 & 1 & -2 & -3\\0 & 0 & -5 & -5\\0 & 0 & -17 & -17\end{array}} \longrightarrow \begin{pmatrix}1 & 2 & 3 & 2\\0 & 1 & -2 & -3\\0 & 0 & 1 & 1\\0 & 0 & 1 & 1\end{pmatrix}$$

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$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\begin{array}{c|cccc} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \\ \end{array}} \rightarrow \left(\begin{array}{c|cccc} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ \end{array}\right) \xrightarrow{r_4-r_3}$$

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$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\begin{array}{c|cccc}1&2&3&2\\0&1&-2&-3\\0&0&-5&-5\\0&0&-17\end{array}} \longrightarrow \begin{pmatrix}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&1&1\end{pmatrix} \xrightarrow[r_4-r_3]{\begin{array}{c|cccc}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&0&0\end{array}}$$

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$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\begin{array}{c|cccc}1&2&3&2\\0&1&-2&-3\\0&0&-5&-5\\0&0&-17&-17\end{array}} \longrightarrow \begin{pmatrix}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&1&1\end{pmatrix}\xrightarrow[r_4-r_3]{\begin{pmatrix}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&0&0\end{array}}$$

$$\frac{r_2-2r_3}{r_1-3r_3}$$

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$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\begin{array}{c}1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17\end{array}} \longrightarrow \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1\end{pmatrix} \xrightarrow[r_4-r_3]{\begin{array}{c}1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0\end{array}}$$

$$\frac{r_2 - 2r_3}{r_1 - 3r_3} \left(\begin{array}{c} & & \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

例 1
$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\begin{array}{c|cccc}1&2&3&2\\0&1&-2&-3\\0&0&-5&-5\\0&0&-17&-17\end{array}} \longrightarrow \left(\begin{array}{ccccc}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&1&1\end{array}\right) \xrightarrow[r_4-r_3]{\left(\begin{array}{cccccc}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&0&0\end{array}\right)}$$

$$\begin{array}{c|c} r_2 - 2r_3 \\ \hline r_1 - 3r_3 \end{array} \left(\begin{array}{c|c} 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

例 1
$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\begin{array}{c|cccc}1&2&3&2\\0&1&-2&-3\\0&0&-5&-5\\0&0&-17&-17\end{array}} \longrightarrow \left(\begin{array}{ccccc}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&1&1\end{array}\right) \xrightarrow[r_4-r_3]{\left(\begin{array}{cccccc}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&0&0\end{array}\right)}$$

$$\frac{r_2 - 2r_3}{r_1 - 3r_3} \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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例 1
$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\begin{array}{c|cccc}1&2&3&2\\0&1&-2&-3\\0&0&-5&-5\\0&0&-17&-17\end{array}} \longrightarrow \left(\begin{array}{ccccc}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&1&1\end{array}\right) \xrightarrow[r_4-r_3]{\left(\begin{array}{cccccc}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&0&0\end{array}\right)}$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{r_1-2r_2}$$

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例 1
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能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\left(\begin{array}{ccc|c}1&2&3&2\\0&1&-2&-3\\0&0&-5&-5\\0&0&-17&-17\end{array}\right)} \rightarrow \left(\begin{array}{ccc|c}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&1&1\end{array}\right)\xrightarrow[r_4-r_3]{\left(\begin{array}{ccc|c}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&0&0\end{array}\right)}$$

$$\xrightarrow[r_1-3r_3]{\begin{array}{c}1&2&0&|-1\\0&1&0&|-1\\0&0&1&1\\0&0&0&0\end{array}}\xrightarrow[r_1-2r_2]{\begin{array}{c}1&0&0&1\\0&1&0&|-1\\0&0&1&1\\0&0&0&0\end{array}}$$

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例 1
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$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\left(\begin{array}{ccc|c}1&2&3&2\\0&1&-2&-3\\0&0&-5&-5\\0&0&-17&-17\end{array}\right)} \rightarrow \left(\begin{array}{ccc|c}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&1&1\end{array}\right)\xrightarrow[r_4-r_3]{\left(\begin{array}{ccc|c}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&0&0\end{array}\right)}$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \begin{pmatrix} 1 & 2 & 0 & | & -1 \\ 0 & 1 & 0 & | & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow[r_1-2r_2]{r_1-2r_2} \begin{pmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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例 1
$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

 \mathbf{f} \mathbf{g} \mathbf{g} \mathbf{g} \mathbf{g}

$$\begin{pmatrix}
1 & 2 & 3 & | & 2 \\
0 & -1 & 2 & | & 3 \\
1 & 1 & 0 & | & 0 \\
2 & -2 & 1 & | & 5
\end{pmatrix}
\xrightarrow{r_3 - r_1}
\begin{pmatrix}
1 & 2 & 3 & | & 2 \\
0 & -1 & 2 & | & 3 \\
0 & -1 & -3 & | & -2 \\
0 & -6 & -5 & | & 1
\end{pmatrix}
\xrightarrow{(-1) \times r_2}
\begin{pmatrix}
1 & 2 & 3 & | & 2 \\
0 & 1 & -2 & | & -3 \\
0 & -1 & -3 & | & -2 \\
0 & -6 & -5 & | & 1
\end{pmatrix}$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow[r_1-2r_2]{r_1-2r_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

所以 $r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta)$,

例 1
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$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\begin{array}{c|ccccc}
r_{3}+r_{2} \\
\hline
r_{4}+6r_{2}
\end{array}
\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & 1 & -2 & -3 \\
0 & 0 & -5 & -5 \\
0 & 0 & -17 & -17
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & 1 & -2 & -3 \\
0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1
\end{pmatrix}
\xrightarrow{r_{4}-r_{3}}
\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & 1 & -2 & -3 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0
\end{pmatrix}$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow[r_1-2r_2]{r_1-2r_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

所以 $r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta)$,能线性表示

例 1
$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

 \mathbf{f} α_1 α_2 α_3

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow[r_1-2r_2]{r_1-2r_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

所以 $r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta)$,能线性表示,且 $\beta = \alpha_1 - \alpha_2 + \alpha_3$

例 2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 2 & -1 & 4 & | & 1 \\ -1 & 3 & 3 & | & 2 \\ 1 & -2 & 0 & | & -1 \\ 1 & 4 & 11 & | & 6 \end{pmatrix}$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3}$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & -2 & 0 & | -1 \\ -1 & 3 & 3 & | & 2 \\ 2 & -1 & 4 & | & 1 \\ 1 & 4 & 11 & | & 6 \end{pmatrix}$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & -2 & 0 & | -1 \\ -1 & 3 & 3 & | & 2 \\ 2 & -1 & 4 & | & 1 \\ 1 & 4 & 11 & | & 6 \end{pmatrix}$$

$$r_2+r_1$$
 r_3-2r_1
 r_4-r_1

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} \alpha_{1} & \alpha_{2} & \alpha_{3} & \beta \\ 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{pmatrix} \xrightarrow{r_{1} \leftrightarrow r_{3}} \begin{pmatrix} 1 & -2 & 0 & | -1 \\ -1 & 3 & 3 & | & 2 \\ 2 & -1 & 4 & | & 1 \\ 1 & 4 & 11 & | & 6 \end{pmatrix}$$

$$\xrightarrow[r_{4}]{r_{2}+r_{1}} \left(\begin{array}{ccc} 1-2 & 0 & | -1 \\ & & & \\ & & & \\ \end{array} \right)$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} \alpha_{1} & \alpha_{2} & \alpha_{3} & \beta \\ 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{pmatrix} \xrightarrow{r_{1} \leftrightarrow r_{3}} \begin{pmatrix} 1 & -2 & 0 & | -1 \\ -1 & 3 & 3 & | & 2 \\ 2 & -1 & 4 & | & 1 \\ 1 & 4 & 11 & | & 6 \end{pmatrix}$$

$$r_{2} + r_{1} \quad \begin{pmatrix} 1 - 2 & 0 & | -1 \\ 0 & 1 & 3 & | & 1 \end{pmatrix}$$

$$\xrightarrow[r_4-r_1]{r_2-r_1} \begin{pmatrix} 1-2 & 0 & | -1 \\ 0 & 1 & 3 & | 1 \\ & & & & | \end{pmatrix}$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} \alpha_{1} & \alpha_{2} & \alpha_{3} & \beta \\ 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{pmatrix} \xrightarrow{r_{1} \leftrightarrow r_{3}} \begin{pmatrix} 1 & -2 & 0 & | -1 \\ -1 & 3 & 3 & | & 2 \\ 2 & -1 & 4 & | & 1 \\ 1 & 4 & 11 & | & 6 \end{pmatrix}$$

$$\xrightarrow[r_{4}-r_{1}]{r_{2}-2r_{1}} \begin{pmatrix} 1-2 & 0 & | & -1 \\ 0 & 1 & 3 & | & 1 \\ 0 & 3 & 4 & | & 3 \end{pmatrix}$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & -2 & 0 & | -1 \\ -1 & 3 & 3 & | & 2 \\ 2 & -1 & 4 & | & 1 \\ 1 & 4 & 11 & | & 6 \end{pmatrix}$$

$$\xrightarrow[r_4-r_1]{r_2-r_1} \begin{pmatrix}
1-2 & 0 & | -1 \\
0 & 1 & 3 & | 1 \\
0 & 3 & 4 & | 3 \\
0 & 6 & 11 & | 7
\end{pmatrix}$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & -2 & 0 & | -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{pmatrix}$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix}
\alpha_{1} & \alpha_{2} & \alpha_{3} & \beta \\
2 & -1 & 4 & 1 \\
-1 & 3 & 3 & 2 \\
1 & -2 & 0 & -1 \\
1 & 4 & 11 & 6
\end{pmatrix}
\xrightarrow{r_{1} \leftrightarrow r_{3}}
\begin{pmatrix}
1 & -2 & 0 & | -1 \\
-1 & 3 & 3 & 2 \\
2 & -1 & 4 & 1 \\
1 & 4 & 11 & 6
\end{pmatrix}$$

$$\xrightarrow{r_{2} + r_{1}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 3 & 4 & 3
\end{pmatrix}
\xrightarrow{r_{3} - 3r_{2}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & 1
\end{pmatrix}$$

$$\xrightarrow[r_{4}-r_{1}]{r_{3}-2r_{1}} \begin{pmatrix}
1-2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 3 & 4 & 3 \\
0 & 6 & 11 & 7
\end{pmatrix}
\xrightarrow[r_{4}-6r_{2}]{r_{3}-3r_{2}} \begin{pmatrix}
1-2 & 0 & | -1 \\
0 & 1 & 3 & | 1 \\
0 & 1 & 3 & | 1
\end{pmatrix}$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix}
\alpha_{1} & \alpha_{2} & \alpha_{3} & \beta \\
2 & -1 & 4 & 1 \\
-1 & 3 & 3 & 2 \\
1 & -2 & 0 & -1 \\
1 & 4 & 11 & 6
\end{pmatrix}
\xrightarrow{r_{1} \leftrightarrow r_{3}}
\begin{pmatrix}
1 & -2 & 0 & | -1 \\
-1 & 3 & 3 & 2 \\
2 & -1 & 4 & 1 \\
1 & 4 & 11 & 6
\end{pmatrix}$$

$$\xrightarrow{r_{2} + r_{1}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 3 & 4 & 3 \\
0 & 6 & 11 & 7
\end{pmatrix}
\xrightarrow{r_{3} - 3r_{2}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 0 & -5 & 0
\end{pmatrix}$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

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例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$-\frac{1}{5} \times r_3$$

线性组合

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

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例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

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例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

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例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix}
\alpha_{1} & \alpha_{2} & \alpha_{3} & \beta \\
-1 & 3 & 3 & 2 \\
1 & -2 & 0 & -1 \\
1 & 4 & 11 & 6
\end{pmatrix}
\xrightarrow{r_{1} \leftrightarrow r_{3}}
\begin{pmatrix}
1 & -2 & 0 & | -1 \\
-1 & 3 & 3 & 2 \\
2 & -1 & 4 & 1 \\
1 & 4 & 11 & 6
\end{pmatrix}$$

$$\xrightarrow{r_{2} + r_{1}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 3 & 4 & 3 \\
0 & 6 & 11 & 7
\end{pmatrix}
\xrightarrow{r_{3} - 3r_{2}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 0 & -5 & 0 \\
0 & 0 & -7 & 1
\end{pmatrix}$$

$$\xrightarrow{-\frac{1}{5} \times r_{3}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 0 & 1 & 3 \\
0 & 0 & 7 & 1
\end{pmatrix}
\xrightarrow{r_{4} + 7r_{3}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 0 & 1 & 3 \\
0 & 0 & 1 & 0
\end{pmatrix}$$

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例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\xrightarrow{-\frac{1}{5} \times r_3} \begin{pmatrix} 1 - 2 & 0 & | -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -7 & 1 \end{pmatrix} \xrightarrow{r_4 + 7r_3} \begin{pmatrix} 1 - 2 & 0 & | -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

可见 $r(\alpha_1\alpha_2\alpha_3\beta) = 4 > 3 = r(\alpha_1\alpha_2\alpha_3)$,

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix}
\alpha_{1} & \alpha_{2} & \alpha_{3} & \beta \\
-1 & 3 & 3 & 2 \\
1 & -2 & 0 & -1 \\
1 & 4 & 11 & 6
\end{pmatrix}
\xrightarrow{r_{1} \leftrightarrow r_{3}}
\begin{pmatrix}
1 & -2 & 0 & | -1 \\
-1 & 3 & 3 & 2 \\
2 & -1 & 4 & 1 \\
1 & 4 & 11 & 6
\end{pmatrix}$$

$$\xrightarrow{r_{2} + r_{1}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 3 & 4 & 3 \\
0 & 6 & 11 & 7
\end{pmatrix}
\xrightarrow{r_{3} - 3r_{2}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 0 & -5 & 0 \\
0 & 0 & -7 & 1
\end{pmatrix}$$

$$\xrightarrow{-\frac{1}{5} \times r_{3}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 7 & 1
\end{pmatrix}
\xrightarrow{r_{4} + 7r_{3}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}$$

可见 $r(\alpha_1\alpha_2\alpha_3\beta) = 4 > 3 = r(\alpha_1\alpha_2\alpha_3)$,所以不能线性表示。

定义 设有两个向量组

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

定义 设有两个向量组

 $(A): \alpha_1, \alpha_2, \ldots, \alpha_s$

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如果中(A)中每一向量均可由(B)线性表示,则称向量组(A)可由向量组(B)线性表示。

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例

向量组 α_1 , α_2 可由向量组 β_1 , β_2 , β_3 线性表示

定义 设有两个向量组

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例

向量组 α_1 , α_2 可由向量组 β_1 , β_2 , β_3 线性表示

$$\Rightarrow \left\{ \begin{array}{ll} \alpha_1 = & \beta_1 + & \beta_2 + & \beta_3 \\ \alpha_2 = & \beta_1 + & \beta_2 + & \beta_3 \end{array} \right.$$

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定义 设有两个向量组

$$(A): \alpha_1, \alpha_2, \ldots, \alpha_s$$

(B):
$$\beta_1, \beta_2, \ldots, \beta_t$$

如果中(A)中每一向量均可由(B)线性表示,则称向量组(A)可由向量组(B)线性表示。

例

向量组 α_1 , α_2 可由向量组 β_1 , β_2 , β_3 线性表示

$$\Rightarrow \left\{ \begin{array}{l} \alpha_1 = \alpha_{11}\beta_1 + \alpha_{21}\beta_2 + \alpha_{31}\beta_3 \\ \alpha_2 = \beta_1 + \beta_2 + \beta_3 \end{array} \right.$$

线性组合

定义 设有两个向量组

$$(A): \alpha_1, \alpha_2, \ldots, \alpha_s$$

(B):
$$\beta_1, \beta_2, \ldots, \beta_t$$

如果中(A)中每一向量均可由(B)线性表示,则称向量组(A)可由向量组(B)线性表示。

例

向量组 α_1 , α_2 可由向量组 β_1 , β_2 , β_3 线性表示

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$$\Rightarrow \left\{ \begin{array}{l} \alpha_1 = \alpha_{11}\beta_1 + \alpha_{21}\beta_2 + \alpha_{31}\beta_3 \\ \alpha_2 = \alpha_{12}\beta_1 + \alpha_{22}\beta_2 + \alpha_{32}\beta_3 \end{array} \right.$$

线性组合

A = BP

$$A = BP$$

$$\Leftrightarrow \underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1s} \\ a_{21} & a_{22} & \cdots & a_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{ms} \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1t} \\ b_{21} & b_{22} & \cdots & b_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mt} \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{t1} & p_{t2} & \cdots & p_{tn} \end{pmatrix}}_{P}$$

$$A = BP$$

$$\alpha_{1} \quad \alpha_{2} \quad \alpha_{5}$$

$$(a_{11} \quad a_{12} \cdots a_{15}) \quad a_{21} \quad a_{22} \cdots a_{25}) \quad a_{21} \quad a_{22} \cdots a_{ms}) = \underbrace{\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1t} \\ b_{21} & b_{22} & \cdots & b_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mt} \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{t1} & p_{t2} & \cdots & p_{tn} \end{pmatrix}}_{P}$$

$$A = BP \atop \alpha_{1} \quad \alpha_{2} \quad \alpha_{s} \\ \Leftrightarrow \underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1s} \\ a_{21} & a_{22} & \cdots & a_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{ms} \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1t} \\ b_{21} & b_{22} & \cdots & b_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mt} \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{t1} & p_{t2} & \cdots & p_{tn} \end{pmatrix}}_{P}$$

$$A = BP$$

$$\alpha_{1} \quad \alpha_{2} \quad \alpha_{s}$$

$$\alpha_{11} \quad \alpha_{12} \cdots \alpha_{1s}$$

$$\alpha_{21} \quad \alpha_{22} \cdots \alpha_{2s}$$

$$\vdots \quad \vdots \quad \ddots \quad \vdots$$

$$\alpha_{m1} \quad \alpha_{m2} \cdots \alpha_{ms}$$

$$A = BP$$

$$\beta_{1} \quad \beta_{2} \quad \beta_{t}$$

$$b_{11} \quad b_{12} \cdots b_{1t}$$

$$b_{21} \quad b_{22} \cdots b_{2t}$$

$$\vdots \quad \vdots \quad \ddots \quad \vdots$$

$$b_{m1} \quad b_{m2} \cdots b_{mt}$$

$$B = P$$

$$\beta_{11} \quad \beta_{22} \cdots \beta_{1n}$$

$$\beta_{21} \quad \beta_{22} \cdots \beta_{2n}$$

$$\vdots \quad \vdots \quad \ddots \quad \vdots$$

$$\beta_{t1} \quad \beta_{t2} \cdots \beta_{tn}$$

$$\Leftrightarrow (\alpha_1 \ \alpha_2 \cdots \alpha_s) = (\beta_1 \ \beta_2 \cdots \beta_t) \begin{pmatrix} p_{11} \ p_{12} \cdots p_{1n} \\ p_{21} \ p_{22} \cdots p_{2n} \\ \vdots \ \vdots \ \ddots \ \vdots \\ p_{t1} \ p_{t2} \cdots p_{tn} \end{pmatrix}$$

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$$A = BP \atop \alpha_{1} \quad \alpha_{2} \quad \alpha_{s} \\ \Leftrightarrow \underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1s} \\ a_{21} & a_{22} & \cdots & a_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{ms} \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1t} \\ b_{21} & b_{22} & \cdots & b_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mt} \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{t1} & p_{t2} & \cdots & p_{tn} \end{pmatrix}}_{P}$$

$$\Leftrightarrow (\alpha_1 \ \alpha_2 \cdots \alpha_s) = (\beta_1 \ \beta_2 \cdots \beta_t) \begin{pmatrix} p_{11} \ p_{12} \cdots p_{1n} \\ p_{21} \ p_{22} \cdots p_{2n} \\ \vdots \ \vdots \ \ddots \ \vdots \\ p_{t1} \ p_{t2} \cdots p_{tn} \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} \alpha_1 = p_{11}\beta_1 + p_{21}\beta_2 + \dots + p_{t1}\beta_t \\ \end{cases}$$

$$\Leftrightarrow (\alpha_1 \ \alpha_2 \cdots \alpha_s) = (\beta_1 \ \beta_2 \cdots \beta_t) \begin{pmatrix} p_{11} \ p_{12} \cdots p_{1n} \\ p_{21} \ p_{22} \cdots p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{t1} \ p_{t2} \cdots p_{tn} \end{pmatrix}$$

$$\iff \begin{cases} \alpha_{1} = p_{11}\beta_{1} + p_{21}\beta_{2} + \dots + p_{t1}\beta_{t} \\ \alpha_{2} = p_{12}\beta_{1} + p_{22}\beta_{2} + \dots + p_{t2}\beta_{t} \end{cases}$$

$$A = BP \atop \alpha_{1} \quad \alpha_{2} \quad \alpha_{s} \\ \Leftrightarrow \underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1s} \\ a_{21} & a_{22} & \cdots & a_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{ms} \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1t} \\ b_{21} & b_{22} & \cdots & b_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mt} \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{t1} & p_{t2} & \cdots & p_{tn} \end{pmatrix}}_{P}$$

$$\Leftrightarrow (\alpha_1 \ \alpha_2 \cdots \alpha_s) = (\beta_1 \ \beta_2 \cdots \beta_t) \begin{pmatrix} \beta_1 1 \ \beta_1 2 \cdots \beta_{1n} \\ \beta_{21} \ \beta_{22} \cdots \beta_{2n} \\ \vdots \ \vdots \ \ddots \ \vdots \\ \beta_{t1} \ \beta_{t2} \cdots \beta_{tn} \end{pmatrix}$$

$$\iff \begin{cases} \alpha_{1} = p_{11}\beta_{1} + p_{21}\beta_{2} + \dots + p_{t1}\beta_{t} \\ \alpha_{2} = p_{12}\beta_{1} + p_{22}\beta_{2} + \dots + p_{t2}\beta_{t} \\ \vdots \\ \alpha_{s} = p_{1s}\beta_{1} + p_{2s}\beta_{2} + \dots + p_{ts}\beta_{t} \end{cases}$$

$$\Leftrightarrow (\alpha_1 \ \alpha_2 \cdots \alpha_s) = (\beta_1 \ \beta_2 \cdots \beta_t) \begin{pmatrix} p_{11} \ p_{12} \cdots p_{1n} \\ p_{21} \ p_{22} \cdots p_{2n} \\ \vdots \ \vdots \ \ddots \ \vdots \\ p_{t1} \ p_{t2} \cdots p_{tn} \end{pmatrix}$$

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 $\Leftrightarrow \{\alpha_1, \alpha_2, \cdots \alpha_s\}$ 由 $\{\beta_1, \beta_2 \cdots \beta_t\}$ 线性表示

$$A = BP \atop \alpha_{1} \quad \alpha_{2} \quad \alpha_{s} \\ \Leftrightarrow \underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1s} \\ a_{21} & a_{22} & \cdots & a_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{ms} \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1t} \\ b_{21} & b_{22} & \cdots & b_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mt} \end{pmatrix}}_{B} \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{t1} & p_{t2} & \cdots & p_{tn} \end{pmatrix}}_{P}$$

$$\Leftrightarrow (\alpha_1 \ \alpha_2 \cdots \alpha_s) = (\beta_1 \ \beta_2 \cdots \beta_t) \begin{pmatrix} p_{11} \ p_{12} \cdots p_{1n} \\ p_{21} \ p_{22} \cdots p_{2n} \\ \vdots \ \vdots \ \ddots \ \vdots \\ p_{t1} \ p_{t2} \cdots p_{tn} \end{pmatrix}$$
$$(\alpha_1 = p_{11}\beta_1 + p_{21}\beta_2 + \cdots + p_{t1}\beta_t$$

$$\Leftrightarrow \begin{cases} \alpha_{1} = p_{11}\beta_{1} + p_{21}\beta_{2} + \dots + p_{t1}\beta_{t} \\ \alpha_{2} = p_{12}\beta_{1} + p_{22}\beta_{2} + \dots + p_{t2}\beta_{t} \\ \vdots \\ \alpha_{s} = p_{1s}\beta_{1} + p_{2s}\beta_{2} + \dots + p_{ts}\beta_{t} \end{cases}$$

 $\leftrightarrow \{\alpha_1, \alpha_2, \cdots \alpha_s\}$ 由 $\{\beta_1, \beta_2 \cdots \beta_t\}$ 线性表示,P的列是线性表示系数

定理(向量组线性表示的传递性) 假设向量组 (A), (B), (C) 满足: (A)

可由 (B) 线性表示,(B) 可由 (C) 线性表示,则 (A) 可由 (C) 线性表示。

定理(向量组线性表示的传递性) 假设向量组 (A), (B), (C) 满足: (A) 可由 (B) 线性表示,(B) 可由 (C) 线性表示,则 (A) 可由 (C) 线性表示。

证明设向量均为列向量。

设向量组 α_1 , α_2 , ..., α_s 可由向量组 β_1 , β_2 , ..., β_t 线性表示:

向量组 β_1 , β_2 , ..., β_t 可由向量组 γ_1 , γ_2 , ..., γ_k 线性表示:

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定理(向量组线性表示的传递性) 假设向量组 (*A*), (*B*), (*C*) 满足: (*A*) 可由 (*B*) 线性表示,(*B*) 可由 (*C*) 线性表示,则 (*A*) 可由 (*C*) 线性表示。

证明 设向量均为列向量。

设向量组 α_1 , α_2 , ..., α_s 可由向量组 β_1 , β_2 , ..., β_t 线性表示:

$$(\alpha_1, \alpha_2, \ldots, \alpha_s) = (\beta_1, \beta_2, \ldots, \beta_t) A_{t \times s}.$$

向量组 β_1 , β_2 , ..., β_t 可由向量组 γ_1 , γ_2 , ..., γ_k 线性表示:

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向量组 β_1 , β_2 , ..., β_t 可由向量组 γ_1 , γ_2 , ..., γ_k 线性表示:

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线性组合

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将第2式代入第1式,可得

线性组合

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将第2式代入第1式,可得

$$(\alpha_1, \alpha_2, \ldots, \alpha_s) = (\gamma_1, \gamma_2, \ldots, \gamma_k) B_{k \times t} A_{t \times s}$$

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$$(\beta_1, \beta_2, \ldots, \beta_t) = (\gamma_1, \gamma_2, \ldots, \gamma_k) B_{k \times t}.$$

将第2式代入第1式,可得

$$(\alpha_1, \alpha_2, \ldots, \alpha_s) = (\gamma_1, \gamma_2, \ldots, \gamma_k) \underbrace{B_{k \times t} A_{t \times s}}_{C_{k \times s}}$$

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证明 设向量均为列向量。

设向量组 α_1 , α_2 , ..., α_s 可由向量组 β_1 , β_2 , ..., β_t 线性表示:

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向量组 β_1 , β_2 , ..., β_t 可由向量组 γ_1 , γ_2 , ..., γ_k 线性表示:

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将第2式代入第1式,可得

$$(\alpha_1, \alpha_2, \ldots, \alpha_s) = (\gamma_1, \gamma_2, \ldots, \gamma_k) \underbrace{B_{k \times t} A_{t \times s}}_{C} = (\gamma_1, \gamma_2, \ldots, \gamma_k) C.$$

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所以向量组 α_1 , α_2 , ..., α_s 可由向量组 γ_1 , γ_2 , ..., γ_k 线性表示。

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证明设向量均为列向量。

设向量组 α_1 , α_2 , ..., α_s 可由向量组 β_1 , β_2 , ..., β_t 线性表示:

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向量组 β_1 , β_2 , ..., β_t 可由向量组 γ_1 , γ_2 , ..., γ_k 线性表示:

$$(\beta_1, \beta_2, \ldots, \beta_t) = (\gamma_1, \gamma_2, \ldots, \gamma_k)B_{k \times t}.$$

将第2式代入第1式,可得

$$(\alpha_1, \alpha_2, \ldots, \alpha_s) = (\gamma_1, \gamma_2, \ldots, \gamma_k) \underbrace{B_{k \times t} A_{t \times s}}_{C_{k \times s}} = (\gamma_1, \gamma_2, \ldots, \gamma_k) C.$$

所以向量组 α_1 , α_2 , ..., α_s 可由向量组 γ_1 , γ_2 , ..., γ_k 线性表示。(并且,线性组合的系数就是矩阵 C 的列。)

 $\left. egin{array}{ll} & \alpha_1, \ \alpha_2 ext{由} eta_1, \ eta_2 ext{线性表示} \\ & \beta_1, \ eta_2 ext{由} \gamma_1, \ \gamma_2, \ \gamma_3 ext{线性表示} \end{array}
ight.
ight.$

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\left. egin{array}{ll} & \alpha_1, \ \alpha_2 ext{由} eta_1, \ eta_2 ext{线性表示} \\ & \beta_1, \ eta_2 ext{由} \gamma_1, \ \gamma_2, \ \gamma_3 ext{线性表示} \end{array}
ight.
ight.$$

$$\begin{cases} \alpha_1 = \alpha_{11}\beta_1 + \alpha_{21}\beta_2 \\ \alpha_2 = \alpha_{12}\beta_1 + \alpha_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\left. egin{array}{ll} & \alpha_1, \ \alpha_2 ext{由} eta_1, \ eta_2 ext{线性表示} \\ & \beta_1, \ eta_2 ext{由} \gamma_1, \ \gamma_2, \ \gamma_3 ext{线性表示} \end{array}
ight.
ight.$$

$$\begin{cases} \alpha_1 = \alpha_{11}\beta_1 + \alpha_{21}\beta_2 \\ \alpha_2 = \alpha_{12}\beta_1 + \alpha_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\alpha_1 =$$

$$\alpha_2 =$$

例
$$\alpha_1, \alpha_2 \oplus \beta_1, \beta_2$$
线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示
具体地,设
$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$
 则
$$\alpha_1 = a_{11} () + a_{21} ($$

 $\alpha_2 =$

例
$$\alpha_1, \alpha_2$$
由 β_1, β_2 线性表示 β_1, β_2 由 $\gamma_1, \gamma_2, \gamma_3$ 线性表示 β_1, β_2 由 $\gamma_1, \gamma_2, \gamma_3$ 线性表示
具体地,设
$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\alpha_1 = a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}($$

$$\alpha_2 =$$

$$\left. egin{array}{ccc} & \alpha_1, \ \alpha_2 ext{由} eta_1, \ eta_2 ext{线性表示} \\ & \beta_1, \ eta_2 ext{ e} \gamma_1, \ \gamma_2, \ \gamma_3 ext{线性表示} \end{array}
ight.
ight.$$

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\alpha_1 = a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

$$\alpha_2 =$$

例
$$\alpha_1, \alpha_2$$
由 β_1, β_2 线性表示 β_1, β_2 由 $\gamma_1, \gamma_2, \gamma_3$ 线性表示 β_1, β_2 由 $\gamma_1, \gamma_2, \gamma_3$ 线性表示 β_1, β_2 由 β_2 中 β_2, β_3

$$\alpha_1 = \alpha_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + \alpha_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

$$= ()\gamma_1 + ()\gamma_2 + ()\gamma_3$$

$$\alpha_2 =$$

例
$$\alpha_1$$
, α_2 由 β_1 , β_2 线性表示 β_1 , β_2 由 γ_1 , γ_2 , γ_3 线性表示 β_1 , β_2 由 β_1 , β_2 ط β_1 , β_2 由 β_1 , β_2 ط β_1 , β_2

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\alpha_1 = a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

= $(a_{11}b_{11} + a_{21}b_{12})\gamma_1 + ($ $)\gamma_2 + ($ $)\gamma_3$

$$\alpha_2 =$$

$$\left\{egin{align*} & \alpha_1, \ \alpha_2 ext{db} eta_1, \ eta_2 ext{db} \ eta_1, \ eta_2 ext{db} eta_1, \ eta_2 ext{db} \ eta_2, \ eta_3 ext{db} \ eta_3, \ eta_4 ext{db} \ eta_4, \ eta_5 ext{db} \ eta_5 \ eta_5 \ eta_5, \ eta_5 ext{db} \ eta_5, \$$

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + ()\gamma_3 \end{aligned}$$

$$\alpha_2 =$$

$$\left\{egin{align*} & \alpha_1, \ \alpha_2 ext{db} eta_1, \ eta_2 ext{db} eta_1, \ eta_2 ext{db} \gamma_1, \ \gamma_2, \ \gamma_3 ext{db} ext{db} ext{db} eta_3, \ eta_2 ext{db} \gamma_1, \ \gamma_2, \ \gamma_3 ext{db} ext{db} ext{db} \ \end{array}
ight\} \Rightarrow lpha_1, \ lpha_2 ext{db} \gamma_1, \ \gamma_2, \ \gamma_3 ext{db} ext{db} ext{db} ext{db} ext{db} \ .$$

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \end{aligned}$$

$$\alpha_2 =$$

例
$$\alpha_1, \alpha_2 \oplus \beta_1, \beta_2$$
线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 $\beta_2, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示
具体地,设
$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$
 则
$$\alpha_1 = a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

$$= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3$$

$$\alpha_2 = a_{12}() + a_{22}()$$

例
$$\alpha_1, \alpha_2$$
由 β_1, β_2 线性表示 β_1, β_2 的性表示 β_1, β_2 的 β_1, β_2 的大 β_1 的大 β_2 的大 β_1 的大 β_2 的大 β_2 的大 β_2 的大 β_2 的大 β_2 的大 β_2 的大 β_3 的大 β_4 的大 β_2 的大 β_3 的大 β_4 的大 β

$$\left. \begin{array}{l} \alpha_1, \ \alpha_2 \oplus \beta_1, \ \beta_2 \ \text{线性表示} \\ \beta_1, \ \beta_2 \oplus \gamma_1, \ \gamma_2, \ \gamma_3 \ \text{线性表示} \end{array} \right\} \Rightarrow \alpha_1, \ \alpha_2 \oplus \gamma_1, \ \gamma_2, \ \gamma_3 \ \text{线性表示}$$

$$\begin{cases} \alpha_1 = \alpha_{11}\beta_1 + \alpha_{21}\beta_2 \\ \alpha_2 = \alpha_{12}\beta_1 + \alpha_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \end{aligned}$$

$$\alpha_2 = \alpha_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + \alpha_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

例
$$\alpha_1, \alpha_2$$
由 β_1, β_2 线性表示 β_1, β_2 3 (力) β_1, β_2 (力) β_2 (力) β_1 (力) β_2 (力) β_2 (力) β_3 (力) β_4 (力)

 $\alpha_2 = a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$

 $)\gamma_2 + ($

 $)\gamma_3$

 $)\gamma_{1} + ($

= (

例
$$\alpha_1, \alpha_2 \oplus \beta_1, \beta_2$$
线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示
具体地,设
$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$
 则
$$\alpha_1 = a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

$$= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3$$

 $\alpha_2 = a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$

 $)\gamma_2 + ($

 $)\gamma_3$

 $=(a_{12}b_{11}+a_{22}b_{12})\gamma_1+($

线性组合

$$\left. \begin{array}{l} \alpha_1, \ \alpha_2 \oplus \beta_1, \ \beta_2$$
线性表示 $\beta_1, \ \beta_2 \oplus \gamma_1, \ \gamma_2, \ \gamma_3$ 线性表示 $\beta_1, \ \beta_2 \oplus \gamma_1, \ \gamma_2, \ \gamma_3$ 线性表示

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\alpha_1 = a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

= $(a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3$

$$\alpha_2 = a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

= $(a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + ($

 $)\gamma_3$

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\alpha_1 = a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

= $(a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3$

$$\alpha_2 = a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

= $(a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3$

例
$$\alpha_1$$
, α_2 由 β_1 , β_2 线性表示 β_1 , β_2 由 γ_1 , γ_2 , γ_3 线性表示 β_1 , β_2 由 γ_1 , γ_2 , γ_3 线性表示 具体地,设
$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\begin{split} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + \\ \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{split}$$

例
$$\alpha_1, \alpha_2$$
由 β_1, β_2 线性表示 β_1, β_2 由 $\gamma_1, \gamma_2, \gamma_3$ 线性表示 β_1, β_2 由 $\gamma_1, \gamma_2, \gamma_3$ 线性表示 具体地,设
$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\begin{split} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + \\ \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{split}$$

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \\ \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{aligned}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\begin{split} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \\ \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + \end{split}$$

例
$$\alpha_1$$
, α_2 由 β_1 , β_2 线性表示 β_1 , β_2 由 γ_1 , γ_2 , γ_3 线性表示 β_1 , β_2 由 γ_1 , γ_2 , γ_3 线性表示 具体地,设
$$\begin{cases} \alpha_1 = \alpha_{11}\beta_1 + \alpha_{21}\beta_2 \\ \alpha_2 = \alpha_{12}\beta_1 + \alpha_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\begin{split} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \\ \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + \end{split}$$

例
$$\alpha_1, \alpha_2$$
由 β_1, β_2 线性表示 β_1, β_2 由 $\gamma_1, \gamma_2, \gamma_3$ 线性表示 β_1, β_2 由 $\gamma_1, \gamma_2, \gamma_3$ 线性表示 β_1, β_2 由 β_2 的 β_2 中 β_1, β_2 中 β_2 中 β_2 中 β_3 中 β_2 中 β_2 中 β_3 中 β_3 中 β_3 中 β_1 中 β_2 中 β_3 中 β

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \\ \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$

线性组合

例
$$\alpha_1, \alpha_2 = \beta_1, \beta_2$$
线性表示 $\beta_1, \beta_2 = \beta_1, \beta_2$ 线性表示 $\beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_2$ 线性表示 $\beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1$

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{pmatrix}$$

例
$$\alpha_1, \alpha_2 \oplus \beta_1, \beta_2$$
线性表示 β_1, β_2 分 β_1, β_2 分 β_1, β_2 β_2 β_1, β_2 β_2 β_1, β_2 β_2 β_1, β_2 β_2 β_3 β_2 β_3 β_2 β_3 β_3 β_4 β_4 β_5 β_5 β_5 β_6 β

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

例
$$\alpha_1, \alpha_2 = \beta_1, \beta_2$$
线性表示 $\beta_1, \beta_2 = \beta_1, \beta_2$ 经性表示 $\beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_2 = \beta_1, \beta_1 = \beta_1, \beta_2 = \beta_1, \beta_2$

$$\alpha_{1} = a_{11}(b_{11}\gamma_{1} + b_{21}\gamma_{2} + b_{31}\gamma_{3}) + a_{21}(b_{12}\gamma_{1} + b_{22}\gamma_{2} + b_{32}\gamma_{3})$$

$$= (a_{11}b_{11} + a_{21}b_{12})\gamma_{1} + (a_{11}b_{21} + a_{21}b_{22})\gamma_{2} + (a_{11}b_{31} + a_{21}b_{32})\gamma_{3}$$

$$= c_{11}\gamma_{1} + c_{21}\gamma_{2} + c_{31}\gamma_{3}$$

$$\alpha_{2} = a_{12}(b_{11}\gamma_{1} + b_{21}\gamma_{2} + b_{31}\gamma_{3}) + a_{22}(b_{12}\gamma_{1} + b_{22}\gamma_{2} + b_{32}\gamma_{3})$$

$$= (a_{12}b_{11} + a_{22}b_{12})\gamma_{1} + (a_{12}b_{21} + a_{22}b_{22})\gamma_{2} + (a_{12}b_{31} + a_{22}b_{32})\gamma_{3}$$

$$= c_{12}\gamma_{1} + c_{22}\gamma_{2} + c_{32}\gamma_{3}$$

$$(c_{ij}) = \left(\begin{array}{ccc} a_{11}b_{11} + a_{12}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{array}\right) = \left(\begin{array}{ccc} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{array}\right) \left(\begin{array}{ccc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array}\right)$$

$$\alpha_1, \alpha_2 \oplus \beta_1, \beta_2$$
线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示

$$\left\{ \begin{array}{l} \alpha_1 = \alpha_{11}\beta_1 + \alpha_{21}\beta_2 \\ \alpha_2 = \alpha_{12}\beta_1 + \alpha_{22}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) = (\beta_1, \; \beta_2) \left(\begin{array}{ll} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{array} \right)$$

$$\left\{ \begin{array}{l} \beta_1 = b_{11} \gamma_1 + b_{21} \gamma_2 + b_{31} \gamma_3 \\ \beta_2 = b_{12} \gamma_1 + b_{22} \gamma_2 + b_{32} \gamma_3 \end{array} \right. \Rightarrow \left(\beta_1, \, \beta_2\right) = \left(\gamma_1, \, \gamma_2, \, \gamma_3\right) \left(\begin{array}{ll} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{array} \right)$$

则

$$\begin{split} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \\ \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{split}$$

其中

 $= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3$

$$(c_{ij}) = \left(\begin{array}{ccc} a_{11}b_{11} + a_{12}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{array}\right) = \left(\begin{array}{ccc} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{array}\right) \left(\begin{array}{ccc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array}\right)$$

$$\left\{egin{align*} & \alpha_1, \ \alpha_2 \pm eta_1, \ eta_2 \ & \beta_1, \ eta_2 \pm \gamma_1, \ \gamma_2, \ \gamma_3 \ & \gamma_3 \ & \gamma_3, \ & \gamma$$

$$\left\{ \begin{array}{l} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) = (\beta_1, \; \beta_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) = (\beta_1, \; \beta_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) = (\beta_1, \; \beta_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) = (\beta_1, \; \beta_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) = (\beta_1, \; \beta_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) = (\beta_1, \; \beta_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) = (\beta_1, \; \beta_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) = (\beta_1, \; \beta_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) = (\beta_1, \; \beta_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{21}\beta_1 + a_{22}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{21}\beta_2 \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{21}\beta_1 + a_{22}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{21}\beta_2 \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{21}\beta_1 + a_{22}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{12}\beta_2 \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{21}\beta_1 + a_{22}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{12}\beta_2 \right)}_{\left. \begin{array}{ll} \alpha_1 = a_{11}\beta_1 + a_{22}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) \underbrace{\left(\begin{array}{ll} a_{11} & a_{12}\beta_2 \right)}_{\left.$$

$$\left\{ \begin{array}{l} \beta_1 = b_{11} \gamma_1 + b_{21} \gamma_2 + b_{31} \gamma_3 \\ \beta_2 = b_{12} \gamma_1 + b_{22} \gamma_2 + b_{32} \gamma_3 \end{array} \right. \Rightarrow \left(\beta_1, \, \beta_2\right) = \left(\gamma_1, \, \gamma_2, \, \gamma_3\right) \left(\begin{array}{ll} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{array} \right)$$

则

$$\alpha_{1} = a_{11}(b_{11}\gamma_{1} + b_{21}\gamma_{2} + b_{31}\gamma_{3}) + a_{21}(b_{12}\gamma_{1} + b_{22}\gamma_{2} + b_{32}\gamma_{3})$$

$$= (a_{11}b_{11} + a_{21}b_{12})\gamma_{1} + (a_{11}b_{21} + a_{21}b_{22})\gamma_{2} + (a_{11}b_{31} + a_{21}b_{32})\gamma_{3}$$

$$= c_{11}\gamma_{1} + c_{21}\gamma_{2} + c_{31}\gamma_{3}$$

$$\alpha_{2} = a_{12}(b_{11}\gamma_{1} + b_{21}\gamma_{2} + b_{31}\gamma_{3}) + a_{22}(b_{12}\gamma_{1} + b_{22}\gamma_{2} + b_{32}\gamma_{3})$$

$$= (a_{12}b_{11} + a_{22}b_{12})\gamma_{1} + (a_{12}b_{21} + a_{22}b_{22})\gamma_{2} + (a_{12}b_{31} + a_{22}b_{32})\gamma_{3}$$

$$= c_{12}\gamma_{1} + c_{22}\gamma_{2} + c_{32}\gamma_{3}$$

$$(c_{ij}) = \left(\begin{array}{ccc} a_{11}b_{11} + a_{12}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{array} \right) = \left(\begin{array}{ccc} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{array} \right) \left(\begin{array}{ccc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)$$

$$\left\{ \begin{array}{ll} \alpha_{1}, \ \alpha_{2} \oplus \beta_{1}, \ \beta_{2} \ \text{线性表示} \\ \beta_{1}, \ \beta_{2} \oplus \gamma_{1}, \ \gamma_{2}, \ \gamma_{3} \ \text{线性表示} \end{array} \right\} \Rightarrow \alpha_{1}, \ \alpha_{2} \oplus \gamma_{1}, \ \gamma_{2}, \ \gamma_{3} \ \text{线性表示}$$

$$\left\{\begin{array}{ll} \alpha_1=\alpha_{11}\beta_1+\alpha_{21}\beta_2\\ \alpha_2=\alpha_{12}\beta_1+\alpha_{22}\beta_2 \end{array}\right.\Rightarrow (\alpha_1,\;\alpha_2)=(\beta_1,\;\beta_2)\underbrace{\left(\begin{array}{cc} \alpha_{11} & \alpha_{12}\\ \alpha_{21} & \alpha_{22} \end{array}\right)}_{A}$$

$$\left\{ \begin{array}{l} \beta_1 = b_{11} \gamma_1 + b_{21} \gamma_2 + b_{31} \gamma_3 \\ \beta_2 = b_{12} \gamma_1 + b_{22} \gamma_2 + b_{32} \gamma_3 \end{array} \right. \Rightarrow \left(\beta_1, \, \beta_2\right) = \left(\gamma_1, \, \gamma_2, \, \gamma_3\right) \underbrace{\left(\begin{array}{cc} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{array} \right)}_{B}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \\ \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{aligned}$$

其中

 $= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3$

$$(c_{ij}) = \left(\begin{array}{ccc} a_{11}b_{11} + a_{12}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{array}\right) = \left(\begin{array}{ccc} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{array}\right) \left(\begin{array}{ccc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array}\right)$$

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases} \Rightarrow (\alpha_1, \alpha_2) = (\beta_1, \beta_2) \underbrace{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}}_{A}$$

$$\left\{ \begin{array}{l} \beta_1 = b_{11} \gamma_1 + b_{21} \gamma_2 + b_{31} \gamma_3 \\ \beta_2 = b_{12} \gamma_1 + b_{22} \gamma_2 + b_{32} \gamma_3 \end{array} \right. \Rightarrow \left(\beta_1, \, \beta_2\right) = \left(\gamma_1, \, \gamma_2, \, \gamma_3\right) \underbrace{\left(\begin{array}{cc} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{array} \right)}_{B}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \\ \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{aligned}$$

其中

 $= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3$

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = BA$$

定义 设有两个向量组

(A):
$$\alpha_1, \alpha_2, \ldots, \alpha_s$$

(B):
$$\beta_1, \beta_2, \ldots, \beta_t$$

如果 (A) 与 (B) 可相互线性表示,则称向量组 (A) 与 (B) 等价。

线性组合 16/16 ▷ ▷