§6.4 微积分基本定理

2015-2016 **学年** II



教学要求









Outline of §6.4

1. 变上限的定积分

2. 微积分基本定理: 牛顿一莱布尼茨公式

We are here now...

1. 变上限的定积分

2. 微积分基本定理: 牛顿一莱布尼茨公式

$$\int_{a}^{x} f(t)dt, \quad \forall x \in [a, b]$$

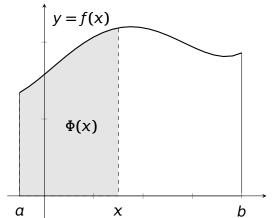
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为变上限的定积分。

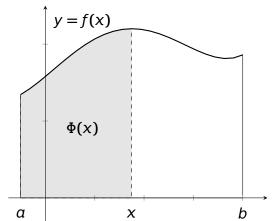
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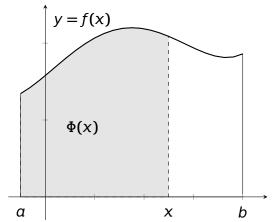
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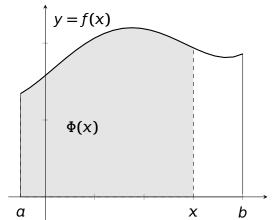
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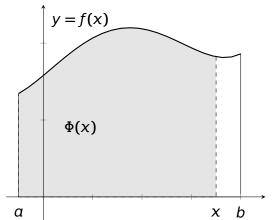
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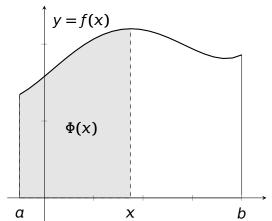
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为变上限的定积分。



$$\Phi(x) = \int_{a}^{x} f(t)dt \qquad \forall x \in [a, b]$$



$$\Phi'(x) = \left[\int_a^x f(t)dt\right]' = ? \quad \forall x \in [a, b]$$



$$\Phi'(x) = \left[\int_a^x f(t)dt\right]' = f(x) \quad \forall x \in [a, b]$$



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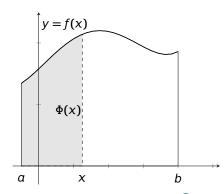
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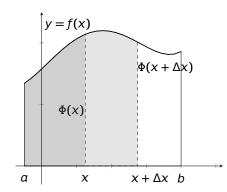
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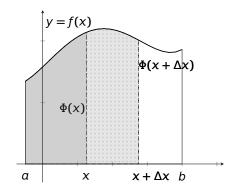
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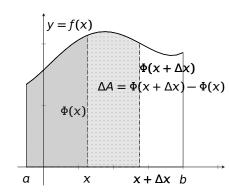
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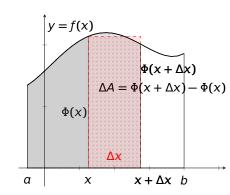
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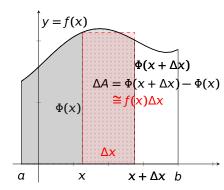
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$$\frac{\Phi(x + \Delta x)}{\Delta x} - \Phi(x)$$

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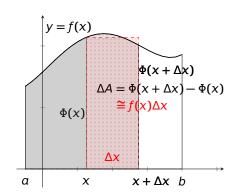
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$$\Phi'(x) = \lim_{\Delta x \to 0} \frac{\Phi(x + \Delta x) - \Phi(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left[\int_{a}^{x + \Delta x} f(t) dt - \int_{a}^{x} f(t) dt \right]$$



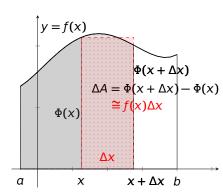
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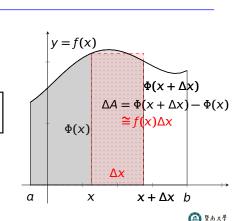
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$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} (f(\xi) \Delta x)$$



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证明

$$\Phi'(x) = \lim_{\Delta x \to 0} \frac{\Phi(x + \Delta x) - \Phi(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left[\int_{a}^{x + \Delta x} f(t) dt - \int_{a}^{x} f(t) dt \right]$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \int_{x}^{x + \Delta x} f(t) dt$$

 $= \lim_{\Delta x \to 0} \frac{1}{\Delta x} (f(\xi) \Delta x) = \lim_{\Delta x \to 0} f(\xi)$

y = f(x) $\Phi(x + \Delta x)$ $\Delta A = \Phi(x + \Delta x) - \Phi(x)$ $\cong f(x)\Delta x$ а $x + \Delta x b$ х

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§6.4 微积分基本定理

微积分基本定理 $\Phi'(x) = \left[\int_a^x f(t)dt\right]' = f(x), \quad \forall x \in [a, b]$

例
$$\left[\int_2^x e^{-t} \sin(t^2) dt\right]' = \underline{e^{-x} \sin(x^2)}.$$

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例
$$\left[\int_{x}^{0} e^{-t} \sin(t^{2}) dt\right]' = \underline{\qquad}$$

例
$$\left[\int_2^x e^{-t} \sin(t^2) dt\right]' = \underline{e^{-x} \sin(x^2)}.$$

$$\left[\int_{x}^{0} e^{-t} \sin(t^{2}) dt\right]' = \left[-\int_{0}^{x} e^{-t} \sin(t^{2}) dt\right]'$$

例
$$\left[\int_2^x e^{-t} \sin(t^2) dt\right]' = \underline{e^{-x}} \sin(x^2).$$

例
$$\left[\int_{x}^{0} e^{-t} \sin(t^{2}) dt\right]' = \underline{\hspace{1cm}}$$

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例
$$\left[\int_{x}^{-2} e^{\sin t} dt\right]' = \underline{\qquad}$$

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例
$$\left[\int_{x}^{-2} e^{\sin t} dt\right]' = \underline{-e^{\sin x}}$$

$$\therefore \left[\int_{x}^{-2} e^{\sin t} dt \right]' = \left[-\int_{-2}^{x} e^{\sin t} dt \right]'$$

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$$\left[\int_2^x e^{-t} \sin(t^2) dt\right]' = \underline{e^{-x} \sin(x^2)}.$$

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$$\left[\int_{x}^{-2} e^{\sin t} dt\right]' = \underline{-e^{\sin x}}$$

$$\therefore \left[\int_{x}^{-2} e^{\sin t} dt \right]' = \left[- \int_{2}^{x} e^{\sin t} dt \right]' = - e^{\sin x}.$$

$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' =$$

$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)].$$

$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

$$\left[\int_{1}^{x^{2}} \cos t dt\right]' =$$
_____; $\left[\int_{2x}^{-1} \sqrt{1 + t^{2}} dt\right]' =$ _____.



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

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$$\left[\int_{1}^{x^{2}} \cos t dt\right]' = \cos(x^{2}).$$



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

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_____; $\left[\int_{2x}^{-1} \sqrt{1+t^{2}} dt\right]' =$ _____

$$\left[\int_{1}^{x^{2}} \cos t dt\right]' = \cos(x^{2}) \cdot (x^{2})'$$



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

$$\left[\int_{1}^{x^{2}} \cos t dt\right]' = \underline{\qquad}; \left[\int_{2x}^{-1} \sqrt{1+t^{2}} dt\right]' = \underline{\qquad}$$

$$\left[\int_{1}^{x^2} \cos t dt\right]' = \cos(x^2) \cdot (x^2)' = 2x \cos(x^2)$$



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

$$\left[\int_{1}^{x^{2}} \cos t dt\right]' =$$
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$$\left[\int_{1}^{x^2} \cos t dt\right]' = \cos(x^2) \cdot (x^2)' = 2x \cos(x^2)$$

$$\left[\int_{2t}^{-1} \sqrt{1+t^2} dt \right]' =$$

$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

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$$\left[\int_{1}^{x^2} \cos t dt\right]' = \cos(x^2) \cdot (x^2)' = 2x \cos(x^2)$$

$$\left[\int_{2x}^{-1} \sqrt{1+t^2} dt \right]' = - \left[\int_{-1}^{2x} \sqrt{1+t^2} dt \right]' =$$

$$\left[\int_{0}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

$$\left[\int_{1}^{x^{2}} \cos t dt\right]' =$$
______; $\left[\int_{2x}^{-1} \sqrt{1 + t^{2}} dt\right]' =$ ______

$$\left[\int_{1}^{x^2} \cos t dt\right]' = \cos(x^2) \cdot (x^2)' = 2x \cos(x^2)$$

$$\left[\int_{2x}^{-1} \sqrt{1 + t^2} dt \right]' = - \left[\int_{-1}^{2x} \sqrt{1 + t^2} dt \right]' = \sqrt{1 + 4x^2}.$$



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

$$\left[\int_{1}^{x^{2}} \cos t dt\right]' = \underline{\qquad}; \left[\int_{2x}^{-1} \sqrt{1+t^{2}} dt\right]' = \underline{\qquad}$$

$$\left[\int_{1}^{x^2} \cos t dt\right]' = \cos(x^2) \cdot (x^2)' = 2x \cos(x^2)$$

$$\left[\int_{2x}^{-1} \sqrt{1+t^2} dt\right]' = -\left[\int_{-1}^{2x} \sqrt{1+t^2} dt\right]' = \sqrt{1+4x^2} \cdot (2x)'$$



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

$$\left[\int_{1}^{x^{2}} \cos t dt\right]' = \underline{\qquad} ; \left[\int_{2x}^{-1} \sqrt{1+t^{2}} dt\right]' = \underline{\qquad}$$

$$\left[\int_{1}^{x^2} \cos t dt\right]' = \cos(x^2) \cdot (x^2)' = 2x \cos(x^2)$$

$$\left[\int_{2x}^{-1} \sqrt{1+t^2} dt\right]' = -\left[\int_{-1}^{2x} \sqrt{1+t^2} dt\right]' = -\sqrt{1+4x^2} \cdot (2x)'$$



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$$= -2\sqrt{1+4x^2}$$



注

$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

例
$$\left[\int_{x^3}^{x^2} \ln(1+t)dt\right]' = \underline{\qquad \qquad }$$

$$\left[\int_{x^3}^{x^2} \ln(1+t)dt\right]'$$



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

例
$$\left[\int_{x^3}^{x^2} \ln(1+t)dt\right]' = \underline{\qquad}$$

$$\left[\int_{x^3}^{x^2} \ln(1+t)dt \right]' = \left[\int_{x^3}^0 \ln(1+t)dt + \int_0^{x^2} \ln(1+t)dt \right]'$$



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

$$\left[\int_{x^3}^{x^2} \ln(1+t)dt \right]' = \left[\int_{x^3}^0 \ln(1+t)dt + \int_0^{x^2} \ln(1+t)dt \right]'$$
$$= \left[-\int_0^{x^3} \ln(1+t)dt + \int_0^{x^2} \ln(1+t)dt \right]'$$



$$= \left[\int_{a}^{\varphi(x)} f(t) dt \right]' = f[\varphi(x)] \cdot \varphi'(x).$$

 $= \left| -\int_0^{x^3} \ln(1+t)dt + \int_0^{x^2} \ln(1+t)dt \right|^{x^2}$

 $= -\left| \int_0^{x^3} \ln(1+t)dt \right|^2 + \left| \int_0^{x^2} \ln(1+t)dt \right|^2$

 $\left| \int_{x^3}^{x^2} \ln(1+t)dt \right| = \left| \int_{x^3}^0 \ln(1+t)dt + \int_0^{x^2} \ln(1+t)dt \right|$

例 $\left[\int_{x^3}^{x^2} \ln(1+t) dt \right]' =$

注:
$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

 $= \left| -\int_0^{x^3} \ln(1+t)dt + \int_0^{x^2} \ln(1+t)dt \right|$

 $= - \left| \int_0^{x^3} \ln(1+t)dt \right| + \left| \int_0^{x^2} \ln(1+t)dt \right|'$





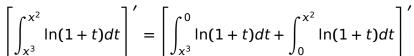
例 $\left[\int_{x^3}^{x^2} \ln(1+t) dt \right]' =$

- $\left| \int_{x^3}^{x^2} \ln(1+t)dt \right| = \left| \int_{x^3}^0 \ln(1+t)dt + \int_0^{x^2} \ln(1+t)dt \right|$

$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

 $= \left| -\int_0^{x^3} \ln(1+t)dt + \int_0^{x^2} \ln(1+t)dt \right|$

 $= - \left[\int_0^{x^3} \ln(1+t)dt \right]' + \left[\int_0^{x^2} \ln(1+t)dt \right]'$



解:

例 $\left[\int_{x^3}^{x^2} \ln(1+t) dt \right]' =$





注:
$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$
例
$$\left[\int_{x^{3}}^{x^{2}} \ln(1+t)dt\right]' = ;$$

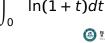
$$\int_{0}^{\infty} \int_{0}^{\infty} \ln(1+$$

$$\left| \int_{x^3}^{x^2} \ln(1+t)dt \right| = \left| \int_{x^3}^0 \ln(1+t)dt + \int_0^{x^2} \ln(1+t)dt \right|$$

$$= \left[-\int_0^{x^3} \ln(1+t)dt + \int_0^{x^2} \ln(1+t)dt \right]'$$

$$= \left[-\int_{0} \ln(1+t) \right]$$

$$= - \left[\int_0^{x^3} \ln(1+t)dt \right]' + \left[\int_0^{x^2} \ln(1+t)dt \right]'$$



例
$$\left[\int_{x^3}^{x^2} \ln(1+t)dt\right]' =$$

$$\int x^2 dx$$

$$\left[\int_{x^3}^{x^2} \ln(1+t)dt \right]' = \left[\int_{x^3}^{0} \ln(1+t)dt + \int_{0}^{x^2} \ln(1+t)dt \right]'$$

 $\left[\int_{-\infty}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$

 $= \left| -\int_0^{x^3} \ln(1+t)dt + \int_0^{x^2} \ln(1+t)dt \right|$

 $= -\left[\int_0^{x^3} \ln(1+t)dt \right]' + \left[\int_0^{x^2} \ln(1+t)dt \right]'$

例
$$\left[\int_{x^3}^{x^2} \ln(1+t)dt\right]' =$$

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 $\left[\int_{-\infty}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$

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 $= -\left[\int_0^{x^3} \ln(1+t)dt \right]' + \left[\int_0^{x^2} \ln(1+t)dt \right]'$

We are here now...

1. 变上限的定积分

2. 微积分基本定理: 牛顿一莱布尼茨公式

牛顿一莱布尼茨公式

$$\int_a^b f(x)dx =$$

$$\int_{a}^{b} f(x)dx = F(b) - F(a)$$

$$\int_{a}^{b} f(x)dx = F(b) - F(a) = F(x)\Big|_{a}^{b}.$$

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

$$\int_{a}^{b} f(x)dx = F(b) - F(a) = F(x)\Big|_{a}^{b}.$$

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

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$$\Phi(x) = \int_{a}^{x} f(t)dt \mathcal{L}f(x)$$
 的一个原函数

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

$$\int_{a}^{b} f(x)dx = F(b) - F(a) = F(x)\Big|_{a}^{b}.$$

$$\therefore F(x) = \Phi(x) + C$$

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

$$\int_{a}^{b} f(x)dx = F(b) - F(a) = F(x)\Big|_{a}^{b}.$$

$$\Phi(x) = \int_{a}^{x} f(t)dt \mathcal{L}f(x)$$
 的一个原函数

$$\therefore F(x) = \Phi(x) + C$$

$$\therefore F(b) - F(a) = () - ()$$

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

$$\int_{a}^{b} f(x)dx = F(b) - F(a) = F(x)\Big|_{a}^{b}.$$

$$: \Phi(x) = \int_{a}^{x} f(t)dt \mathcal{L}f(x)$$
 的一个原函数

$$\therefore F(x) = \Phi(x) + C$$

$$\therefore F(b) - F(a) = (\Phi(b) + C) - ($$

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

$$\int_{a}^{b} f(x)dx = F(b) - F(a) = F(x)\Big|_{a}^{b}.$$

$$: \Phi(x) = \int_{a}^{x} f(t)dt \mathcal{L}f(x)$$
 的一个原函数

$$F(x) = \Phi(x) + C$$

$$\therefore F(b) - F(a) = (\Phi(b) + C) - (\Phi(a) + C)$$

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

$$\int_{a}^{b} f(x)dx = F(b) - F(a) = F(x)\Big|_{a}^{b}.$$

$$\Phi(x) = \int_{a}^{x} f(t)dt \mathcal{L}f(x)$$
 的一个原函数

$$\therefore F(x) = \Phi(x) + C$$

$$\therefore F(b) - F(a) = (\Phi(b) + C) - (\Phi(a) + C)$$
$$= \Phi(b) - \Phi(a)$$

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

$$\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b.$$

$$F(x) = \Phi(x) + C$$

$$\therefore F(b) - F(a) = (\Phi(b) + C) - (\Phi(a) + C)$$
$$= \Phi(b) - \Phi(a)$$
$$= \int_{a}^{b} f(t)dt - \int_{a}^{a} f(t)dt$$

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

$$\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b.$$

$$F(x) = \Phi(x) + C$$

$$\therefore F(b) - F(a) = (\Phi(b) + C) - (\Phi(a) + C)$$
$$= \Phi(b) - \Phi(a)$$

$$= \int_{a}^{b} f(t)dt - \int_{a}^{a} f(t)dt = \int_{a}^{b} f(t)dt$$

牛顿一莱布尼茨公式
$$\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b$$
.

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

牛顿一莱布尼茨公式
$$\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b$$
.

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

$$\iint_0^1 x^2 dx =$$



牛顿一莱布尼茨公式
$$\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b$$
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$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

$$\iint_0^1 x^2 dx = \frac{1}{3}x^3$$



牛顿一莱布尼茨公式
$$\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b$$
.

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1}$$

牛顿一莱布尼茨公式
$$\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b$$
.

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0$$

牛顿一莱布尼茨公式
$$\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b$$
.

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_0^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

$$\int_0^{\pi/2} \sin x dx =$$

$$\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2} =$$

$$\int_{2}^{-1} \frac{dx}{x} =$$

例 计算定积分

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_0^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

$$\int_{0}^{\pi/2} \sin x dx = -\cos x$$

 $\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2} =$

$$\int_{-2}^{-1} \frac{dx}{x} =$$



$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{\pi/2} \sin x dx; \quad \int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}}; \quad \int_{2}^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

$$\iint_{0}^{\pi/2} \sin x dx = -\cos x \Big|_{0}^{\pi/2}$$

$$\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2} = \int_{2}^{-1} \frac{dx}{x} = \int_{2}^{-1} \frac{d$$

$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{\pi/2} \sin x dx; \quad \int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

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$$\int_{1}^{\sqrt{3}} \frac{dx}{1 + x^{2}} =$$

$$\int_{-2}^{-1} \frac{dx}{x} =$$



例 计算定积分

$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{\pi/2} \sin x dx; \quad \int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}}; \quad \int_{2}^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

$$\iint_{0}^{\pi/2} \sin x dx = -\cos x \Big|_{0}^{\pi/2} = -(\cos \frac{\pi}{2} - \cos 0) = 1$$

$$\iint_{1}^{\sqrt{3}} \frac{dx}{1 + x^{2}} = \frac{1}{3} - \cos x = \frac{1}{3} - \cos x = \frac{1}{3}$$



 $\int_{-2}^{-1} \frac{dx}{x} =$

$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{\pi/2} \sin x dx; \quad \int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

$$\iint_0^1 x^2 dx = \frac{1}{3} x^3 \Big|_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$$

$$\int_{0}^{\pi/2} \sin x dx = -\cos x \Big|_{0}^{\pi/2} = -(\cos \frac{\pi}{2} - \cos 0) = 1$$

$$\int_{1}^{\sqrt{3}} \frac{dx}{1 + x^{2}} = \arctan x$$

$$\int_{-2}^{-1} \frac{dx}{x} =$$



$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{\pi/2} \sin x dx; \quad \int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

$$\int_0^{\pi/2} \sin x dx = -\cos x \Big|_0^{\pi/2} = -(\cos \frac{\pi}{2} - \cos 0) = 1$$

$$\int_1^{\sqrt{3}} \frac{dx}{1 + x^2} = \arctan x \Big|_1^{\sqrt{3}}$$

$$\int_{-2}^{-1} \frac{dx}{x} =$$



例 计算定积分

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

$$\iint_0^1 x^2 dx = \frac{1}{3} x^3 \Big|_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$$

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$$\int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}} = \arctan x \Big|_{1}^{\sqrt{3}} = \arctan \sqrt{3} - \arctan 1$$

$$\int_{2}^{-1} \frac{dx}{x} =$$





例 计算定积分

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

 $\int_0^{\pi/2} \sin x dx = -\cos x \Big|_0^{\pi/2} = -(\cos \frac{\pi}{2} - \cos 0) = 1$

$$\int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}} = \arctan x \Big|_{1}^{\sqrt{3}} = \arctan \sqrt{3} - \arctan 1 = \frac{\pi}{3} - \frac{\pi}{4}$$



例 计算定积分

$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{\pi/2} \sin x dx; \quad \int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}}; \quad \int_{2}^{-1} \frac{dx}{x}$$

$$\iint_0^1 x^2 dx = \frac{1}{3} x^3 \Big|_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$$

 $\int_{0}^{\pi/2} \sin x dx = -\cos x \Big|_{0}^{\pi/2} = -(\cos \frac{\pi}{2} - \cos 0) = 1$

$$\int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}} = \arctan x \Big|_{1}^{\sqrt{3}} = \arctan \sqrt{3} - \arctan 1 = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

$$\int_{-3}^{-1} \frac{dx}{x} = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$



例 计算定积分

$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{\pi/2} \sin x dx; \quad \int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}}; \quad \int_{2}^{-1} \frac{dx}{x}$$

$$\int_0^\infty x^2 dx; \quad \int_0^\infty \sin x dx; \quad \int_1^\infty \frac{dx}{1+x^2}; \quad \int_{-2}^\infty \frac{dx}{x}$$

 $\iint_{1}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$

$$\int_0^{\pi/2} \sin x dx = -\cos x \Big|_0^{\pi/2} = -(\cos \frac{\pi}{2} - \cos 0) = 1$$

 $\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2} = \arctan x \Big|_{1}^{\sqrt{3}} = \arctan \sqrt{3} - \arctan 1 = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$





例 计算定积分

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_0^{-1} \frac{dx}{x}$$

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 $\int_{0}^{\pi/2} \sin x dx = -\cos x \Big|_{0}^{\pi/2} = -(\cos \frac{\pi}{2} - \cos 0) = 1$

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例 计算定积分

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_0^{-1} \frac{dx}{x}$$

$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{1} \sin x dx; \quad \int_{1}^{1} \frac{1}{1 + x^{2}}; \quad \int_{-2}^{1} \frac{1}{x} dx$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

 $\int_{0}^{\pi/2} \sin x \, dx = -\cos x \Big|_{0}^{\pi/2} = -(\cos \frac{\pi}{2} - \cos 0) = 1$

 $\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2} = \arctan x \Big|_{1}^{\sqrt{3}} = \arctan \sqrt{3} - \arctan 1 = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$

 $\int_{-2}^{-1} \frac{dx}{x} = \ln|x|\Big|_{-2}^{-1} = \ln|-1| - \ln|-2|$





例 计算定积分

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_0^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

 $\int_0^{\pi/2} \sin x \, dx = -\cos x \Big|_0^{\pi/2} = -(\cos \frac{\pi}{2} - \cos 0) = 1$

 $\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2} = \arctan x \Big|_{1}^{\sqrt{3}} = \arctan \sqrt{3} - \arctan 1 = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$



 $\int_{2}^{-1} \frac{dx}{x} = \ln|x|\Big|_{-2}^{-1} = \ln|-1| - \ln|-2| = -\ln 2$

$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
$$\int (2x-5)dx$$
, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
$$\int (2x-5)dx$$
, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

解

$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
$$\int (2x-5)dx$$
, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

解

$$\int_0^2 (2x-5)dx =$$

$$\int_{4}^{9} \frac{1}{\sqrt{x}} dx =$$

$$\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
$$\int (2x-5)dx$$
, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^2 (2x - 5) dx = (x^2 - 5x)$$

$$\int_{4}^{9} \frac{1}{\sqrt{x}} dx =$$

$$\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

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$$\int_0^2 (2x - 5) dx = (x^2 - 5x) \Big|_0^2$$

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$$\int_0^9 \frac{1}{\sqrt{x}} dx = \int_0^9 x^{-1/2} dx = 0$$

$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

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$$\int_0^9 \frac{1}{\sqrt{x}} dx = \int_0^9 x^{-1/2} dx = 2\sqrt{x}$$

$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

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$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$

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$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$



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提示: 先求出
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$$\int_0^9 \frac{1}{\sqrt{x}} dx = \int_0^9 x^{-1/2} dx = 2\sqrt{x} \Big|_0^9 = 2(3 - 2) = 2$$

$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

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$$\int_0^2 (2x - 5) dx = (x^2 - 5x) \Big|_0^2 = -6 - 0 = -6$$

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$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^{2}}} = \arcsin x \Big|_{0}^{1/2}$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
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$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}} = \arcsin x \Big|_{0}^{1/2} = \arcsin \frac{1}{2} - \arcsin 0$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出 $\int (2x-5)dx$, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^2 (2x - 5) dx = (x^2 - 5x) \Big|_0^2 = -6 - 0 = -6$$

$$\int_{4}^{9} \frac{1}{\sqrt{x}} dx = \int_{4}^{9} x^{-1/2} dx = 2\sqrt{x} \Big|_{4}^{9} = 2(3-2) = 2$$

$$\int_{4}^{1/2} \frac{dx}{\sqrt{x}} dx = \int_{4}^{1/2} x^{1/2} dx = 2\sqrt{x}|_{4} = 2(3-2) = 2$$

$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^{2}}} = \arcsin x|_{0}^{1/2} = \arcsin \frac{1}{2} - \arcsin 0 = \frac{\pi}{6} - 0$$

$$\int_{0}^{2} (2x-5)dx; \quad \int_{4}^{9} \frac{1}{\sqrt{x}} dx; \quad \int_{0}^{1/2} \frac{dx}{\sqrt{1-x^{2}}}$$
提示: 先求出 $\int_{0}^{2} (2x-5)dx$, $\int_{1}^{2} \frac{1}{\sqrt{x}} dx$, $\int_{1}^{2} \frac{dx}{\sqrt{1-x^{2}}}$

解

$$\int_0^2 (2x - 5) dx = (x^2 - 5x) \Big|_0^2 = -6 - 0 = -6$$

$$\int_{4}^{9} \frac{1}{\sqrt{x}} dx = \int_{4}^{9} x^{-1/2} dx = 2\sqrt{x} \Big|_{4}^{9} = 2(3-2) = 2$$

$$\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}} = \arcsin x \Big|_0^{1/2} = \arcsin \frac{1}{2} - \arcsin 0 = \frac{\pi}{6} - 0 = \frac{\pi}{6}$$

§6.4 微积分基本定理

例 计算定积分
$$\int_0^2 |1-x| dx.$$

$$\text{解} \int_0^2 |1-x| dx$$

$$\mathbf{m} \int_0^2 |1 - x| dx$$

$$= \int_0^1 |1 - x| dx + \int_1^2 |1 - x| dx$$

$$\mathbf{m} \int_0^2 |1 - x| dx$$

$$= \int_0^1 |1-x| dx + \int_1^2 |1-x| dx = \int_0^1 (1-x) dx + \int_0^2 |1-x| dx = \int_0^2 (1-x) dx + \int_0^2 (1-x) dx = \int_0^2 (1-x) dx + \int_0^2 (1-x) dx = \int_0^2 (1-x) dx + \int_0^2 (1-x) dx = \int_$$



$$\iiint_0^2 |1-x| dx$$

$$= \int_0^1 |1-x| dx + \int_1^2 |1-x| dx = \int_0^1 (1-x) dx + \int_1^2 (x-1) dx$$



$$\lim_{x \to \infty} \int_0^2 |1-x| dx$$

$$\int_{0}^{1} |1 - x| dx + \int_{1}^{2} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{2} (x - 1) dx$$
$$= (x - \frac{1}{2}x^{2})$$

$$\lim_{x \to \infty} \int_0^2 |1-x| dx$$

$$\int_{0}^{1} |1 - x| dx + \int_{1}^{2} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{2} (x - 1) dx
= (x - \frac{1}{2}x^{2}) + (\frac{1}{2}x^{2} - x)$$

$$\lim_{x \to \infty} \int_0^2 |1-x| dx$$

$$= \int_{0}^{1} |1 - x| dx + \int_{1}^{2} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{2} (x - 1) dx$$
$$= (x - \frac{1}{2}x^{2}) \Big|_{0}^{1} + (\frac{1}{2}x^{2} - x)$$

$$\lim_{x \to 0} \int_0^2 |1-x| dx$$

$$\int_{0}^{1} |1 - x| dx + \int_{1}^{2} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{2} (x - 1) dx
= (x - \frac{1}{2}x^{2}) \Big|_{0}^{1} + (\frac{1}{2}x^{2} - x) \Big|_{1}^{2}$$

$$\lim_{x \to \infty} \int_{0}^{2} |1-x| dx$$

$$\int_{0}^{1} |1 - x| dx$$

$$= \int_{0}^{1} |1 - x| dx + \int_{1}^{2} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{2} (x - 1) dx$$

$$= (x - \frac{1}{2}x^{2}) \Big|_{0}^{1} + (\frac{1}{2}x^{2} - x) \Big|_{1}^{2} = [\frac{1}{2} - 0] +$$

$$\lim_{x \to \infty} \int_{0}^{2} |1-x| dx$$

$$= \int_{0}^{1} |1 - x| dx + \int_{1}^{2} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{2} (x - 1) dx$$
$$= (x - \frac{1}{2}x^{2}) \Big|_{0}^{1} + (\frac{1}{2}x^{2} - x) \Big|_{1}^{2} = [\frac{1}{2} - 0] + [0 - (-\frac{1}{2})]$$

$$\lim_{x \to \infty} \int_{0}^{2} |1-x| dx$$

$$= \int_{0}^{1} |1 - x| dx + \int_{1}^{2} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{2} (x - 1) dx$$
$$= (x - \frac{1}{2}x^{2}) \Big|_{0}^{1} + (\frac{1}{2}x^{2} - x) \Big|_{1}^{2} = [\frac{1}{2} - 0] + [0 - (-\frac{1}{2})] = 1$$



$$\iiint_0^2 |1-x| dx$$

$$= \int_0^1 |1 - x| dx + \int_1^2 |1 - x| dx = \int_0^1 (1 - x) dx + \int_1^2 (x - 1) dx$$
$$= (x - \frac{1}{2}x^2) \Big|_0^1 + (\frac{1}{2}x^2 - x) \Big|_1^2 = [\frac{1}{2} - 0] + [0 - (-\frac{1}{2})] = 1$$



$$\iiint_0^2 |1-x| dx$$

$$\int_{0}^{1} |1-x| dx$$

$$= \int_{0}^{1} |1-x| dx + \int_{1}^{2} |1-x| dx = \int_{0}^{1} (1-x) dx + \int_{1}^{2} (x-1) dx$$

$$= (x - \frac{1}{2}x^{2}) \Big|_{0}^{1} + (\frac{1}{2}x^{2} - x) \Big|_{1}^{2} = [\frac{1}{2} - 0] + [0 - (-\frac{1}{2})] = 1$$

练习 计算定积分
$$\int_0^3 |2-x| dx$$

$$\Re$$
 $\int_{0}^{3} |2-x| dx$

$$\lim_{x \to \infty} \int_{0}^{2} |1-x| dx$$

$$\int_{0}^{1} |1-x| dx$$

$$= \int_{0}^{1} |1-x| dx + \int_{1}^{2} |1-x| dx = \int_{0}^{1} (1-x) dx + \int_{1}^{2} (x-1) dx$$

$$= (x - \frac{1}{2}x^{2}) \Big|_{0}^{1} + (\frac{1}{2}x^{2} - x) \Big|_{1}^{2} = [\frac{1}{2} - 0] + [0 - (-\frac{1}{2})] = 1$$

$$\mathbf{H} \int_0^3 |2-x| dx$$

$$= \int_{0}^{2} |2-x| dx + \int_{2}^{3} |2-x| dx$$



$$\lim_{x \to 0} \int_0^2 |1-x| dx$$

$$\int_{0}^{1} |1-x| dx$$

$$= \int_{0}^{1} |1-x| dx + \int_{1}^{2} |1-x| dx = \int_{0}^{1} (1-x) dx + \int_{1}^{2} (x-1) dx$$

$$= (x - \frac{1}{2}x^{2}) \Big|_{0}^{1} + (\frac{1}{2}x^{2} - x) \Big|_{1}^{2} = [\frac{1}{2} - 0] + [0 - (-\frac{1}{2})] = 1$$

$$\lim_{x \to 0} \int_0^3 |2-x| dx$$

$$= \int_{0}^{2} |2-x| dx + \int_{2}^{3} |2-x| dx = \int_{0}^{2} (2-x) dx + \int_{0}^{2} (2-x) dx + \int_{0}^{2} (2-x) dx = \int_{0}^{2} (2-x) dx = \int_{0}^{2} (2-x) dx = \int_{0}^{2} (2-x) dx + \int_{0}^{2} (2-x) dx = \int_{0}$$

$$\iint_{0}^{2} |1-x| dx$$

$$\begin{aligned}
&= \int_{0}^{1} |1 - x| dx \\
&= \int_{0}^{1} |1 - x| dx + \int_{1}^{2} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{2} (x - 1) dx \\
&= (x - \frac{1}{2}x^{2}) \Big|_{0}^{1} + (\frac{1}{2}x^{2} - x) \Big|_{1}^{2} = [\frac{1}{2} - 0] + [0 - (-\frac{1}{2})] = 1
\end{aligned}$$

$$\lim_{x \to 0} \int_0^3 |2-x| dx$$

$$= \int_{0}^{2} |2-x| dx + \int_{2}^{3} |2-x| dx = \int_{0}^{2} (2-x) dx + \int_{2}^{3} (x-2) dx$$



例 计算定积分
$$\int_0^2 |1-x| dx$$
.

 $\mathbf{H} \int_0^2 |1-x| dx$

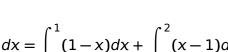
$$\mathbf{H} \int_0^2 |1 - x| dx$$

练习 计算定积分 $\int_0^3 |2-x| dx$

 $\lim_{x \to \infty} \int_{0}^{3} |2-x| dx$

 $=(2x-\frac{1}{2}x^2)$

$$|1-x|dx$$



$$= \int_0^1 |1-x| dx + \int_1^2 |1-x| dx = \int_0^1 (1-x) dx + \int_1^2 (x-1) dx$$

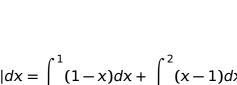
$$-x|dx + \int_{0}^{2} |1-x|dx = \int_{0}^{1} (1-x)^{2} dx$$

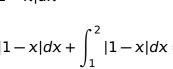
 $= \int_{0}^{2} |2-x| dx + \int_{0}^{3} |2-x| dx = \int_{0}^{2} (2-x) dx + \int_{0}^{3} (x-2) dx$

- $= \left(x \frac{1}{2}x^2\right)\Big|_0^1 + \left(\frac{1}{2}x^2 x\right)\Big|_1^2 = \left[\frac{1}{2} 0\right] + \left[0 \left(-\frac{1}{2}\right)\right] = 1$

$$\mathbf{H} \int_0^2 |1 - x| dx$$

$$\int_0^2 |1-x| dx$$





 $\lim_{x \to \infty} \int_{0}^{3} |2-x| dx$

 $= \int_{0}^{1} |1-x| dx + \int_{1}^{2} |1-x| dx = \int_{0}^{1} (1-x) dx + \int_{1}^{2} (x-1) dx$

 $= \left(x - \frac{1}{2}x^2\right)\Big|_0^1 + \left(\frac{1}{2}x^2 - x\right)\Big|_1^2 = \left[\frac{1}{2} - 0\right] + \left[0 - \left(-\frac{1}{2}\right)\right] = 1$

练习 计算定积分 $\int_0^3 |2-x| dx$ $= \int_{0}^{2} |2-x| dx + \int_{0}^{3} |2-x| dx = \int_{0}^{2} (2-x) dx + \int_{0}^{3} (x-2) dx$

 $=(2x-\frac{1}{2}x^2)+(\frac{1}{2}x^2-2x)$

例 计算定积分
$$\int_0^2 |1-x| dx.$$

$$\mathbf{H} \int_0^2 |1-x| dx$$

$$\mathbf{H} \int_{0}^{2} |1 - x| dx$$

$$= \int_{0}^{1} |1 - x| dx + \int_{1}^{2} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{2} (x - 1) dx$$

练习 计算定积分 $\int_0^3 |2-x| dx$

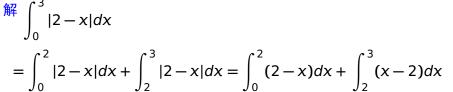
$$|1-x|dx$$

$$|1-x|dx$$

$$-x|dx$$







$$\int_{0}^{1} \int_{0}^{1} \int_{0$$

 $= \left(x - \frac{1}{2}x^2\right)\Big|_0^1 + \left(\frac{1}{2}x^2 - x\right)\Big|_1^2 = \left[\frac{1}{2} - 0\right] + \left[0 - \left(-\frac{1}{2}\right)\right] = 1$

$$\mathbf{H} \int_0^2 |1-x| dx$$

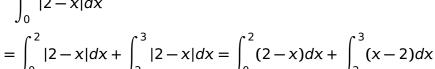
$$\mathbf{H} \int_0^2 |1 - x| dx$$

 $= \int_{0}^{1} |1-x| dx + \int_{1}^{2} |1-x| dx = \int_{0}^{1} (1-x) dx + \int_{1}^{2} (x-1) dx$

 $= \left(x - \frac{1}{2}x^2\right)\Big|_0^1 + \left(\frac{1}{2}x^2 - x\right)\Big|_1^2 = \left[\frac{1}{2} - 0\right] + \left[0 - \left(-\frac{1}{2}\right)\right] = 1$







$$= (2x - \frac{1}{2}x^2)|_0^2 + (\frac{1}{2}x^2 - 2x)|_2^3 = [2 - 0] +$$

 $\lim_{x \to \infty} \int_{0}^{3} |2-x| dx$

例 计算定积分
$$\int_0^2 |1-x| dx.$$

$$\mathbf{H} \int_0^2 |1-x| dx$$

$$\iint_{0}^{2} |1 - x| dx$$

$$= \int_{0}^{1} |1 - x| dx + \int_{1}^{2} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{2} (x - 1) dx$$

 $\lim_{x \to \infty} \int_{0}^{3} |2-x| dx$ $= \int_{0}^{2} |2-x| dx + \int_{3}^{3} |2-x| dx = \int_{0}^{2} (2-x) dx + \int_{3}^{3} (x-2) dx$

 $= \left(x - \frac{1}{2}x^2\right)\Big|_0^1 + \left(\frac{1}{2}x^2 - x\right)\Big|_1^2 = \left[\frac{1}{2} - 0\right] + \left[0 - \left(-\frac{1}{2}\right)\right] = 1$

练习 计算定积分 $\int_0^3 |2-x| dx$

 $= (2x - \frac{1}{2}x^2)\Big|_0^2 + (\frac{1}{2}x^2 - 2x)\Big|_2^3 = [2 - 0] + [-\frac{3}{2} - (-2)]$

$$\mathbf{H} \int_0^2 |1 - x| dx$$

 $= \int_{0}^{1} |1-x| dx + \int_{1}^{2} |1-x| dx = \int_{0}^{1} (1-x) dx + \int_{1}^{2} (x-1) dx$

 $\lim_{x \to \infty} \int_{0}^{3} |2-x| dx$

练习 计算定积分 $\int_0^3 |2-x| dx$

 $= \left(x - \frac{1}{2}x^2\right)\Big|_0^1 + \left(\frac{1}{2}x^2 - x\right)\Big|_1^2 = \left[\frac{1}{2} - 0\right] + \left[0 - \left(-\frac{1}{2}\right)\right] = 1$

 $= \int_{0}^{2} |2-x| dx + \int_{3}^{3} |2-x| dx = \int_{0}^{2} (2-x) dx + \int_{3}^{3} (x-2) dx$

 $= (2x - \frac{1}{2}x^2)\Big|_0^2 + (\frac{1}{2}x^2 - 2x)\Big|_2^3 = [2 - 0] + [-\frac{3}{2} - (-2)] = \frac{5}{3}$