#### 第3章b: 洛必达法则

数学系 梁卓滨

2019-2020 学年 I

#### **Outline**



考虑极限

$$\lim \frac{f(x)}{g(x)}$$

对以下情况,极限的商公式  $\lim \frac{f(x)}{g(x)} = \frac{\lim f(x)}{\lim g(x)}$  不再成立:

•  $\lim f(x) = 0$  和  $\lim g(x) = 0$  时,

•  $\lim f(x) = \infty$  和  $\lim g(x) = \infty$  时,

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一般地,这样的  $\lim \frac{f(x)}{g(x)}$  可能不存在,可能存在,存在的话,极限的值与具体问题有关,因此称为未定式。

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**定理** 假设  $\lim \frac{f(x)}{g(x)}$  是  $\frac{0}{0}$  型或  $\frac{\infty}{\infty}$  型未定式,则(在一定条件下)成立:

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**例1** 计算  $\lim_{x\to 1} \frac{x^3-3x+2}{x^3-x^2-x+1}$ 



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$$\lim_{x\to 0} \frac{\sin ax}{\sin bx}$$
,  $(b \neq 0)$ ; **(2)**  $\lim_{x\to 0} \frac{x-\sin x}{x^3}$ 



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$$\lim_{x \to 0} \frac{x - \sin x}{x^3} = \lim_{x \to 0} \frac{(x - \sin x)'}{(x^3)'} = \lim_{x \to 0} \frac{1 - \cos x}{3x^2}$$

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(利用洛必达法则: 
$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{(\sin x)'}{x'}$$
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$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{(\sin x)'}{x'} = \lim_{x\to 0} \frac{\cos x}{1} = 1$$
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注**1** 事实上 
$$\lim_{n \to \infty} \frac{x^n}{e^x} = 0$$
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$$\lim_{x \to +\infty} \frac{\ln x}{x}$$
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 $\mathbf{m}$  (1) 为  $\stackrel{\text{o}}{\sim}$  型未定式,利用洛必达法则:

$$\lim_{x \to +\infty} \frac{\ln x}{x} = \lim_{x \to +\infty} \frac{(\ln x)'}{x'} = \lim_{x \to +\infty} \frac{\frac{1}{x}}{1} = 0.$$

(2) 为  $\frac{8}{8}$  型未定式,利用洛必达法则:

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**注1** 事实上 
$$\lim_{r\to +\infty} \frac{x^r}{e^x} = 0$$
.

**注 2** 尽管  $x^n$ , $\ln x$ , $e^x$  都是无穷大(当  $x \to +\infty$ ),但趋于  $+\infty$  的速度不一样: $e^x$  最快, $x^n$  次之, $\ln x$  最慢.

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● ∞ – ∞ 型不定式,

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● 0.∞ 型未定式,例如

$$\lim_{x \to +\infty} (\frac{\pi}{2} - \arctan x)x, \quad \lim_{x \to 0^+} x \ln x$$

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$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x), \quad \lim_{x \to 0} (\frac{1}{\sin x} - \frac{1}{x})$$

0<sup>0</sup> 型不定式,例如

$$\lim_{x\to 0^+} x^x$$

∞<sup>0</sup> 型不定式,例如

$$\lim_{x\to +\infty} x^{\frac{1}{2}}$$

0⋅∞型未定式,例如

$$\lim_{x \to +\infty} (\frac{\pi}{2} - \arctan x)x, \quad \lim_{x \to 0^+} x \ln x$$

∞ – ∞ 型不定式,例如

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x), \quad \lim_{x \to 0} (\frac{1}{\sin x} - \frac{1}{x})$$

0<sup>0</sup> 型不定式,例如

$$\lim_{x\to 0^+} x^x$$

∞<sup>0</sup> 型不定式,例如

$$\lim_{x \to \infty} x^{\frac{1}{x}}$$

1∞ 等等



$$\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x$$



$$\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x = \lim_{x \to +\infty} \frac{\frac{\pi}{2} - \arctan x}{\frac{1}{x}}$$



例 4 计算极限 (1) 
$$\lim_{x\to +\infty} (\frac{\pi}{2} - \arctan x)x$$
; (2)  $\lim_{x\to 0^+} x \ln x$ 

$$\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x = \lim_{x \to +\infty} \frac{\frac{\pi}{2} - \arctan x}{\frac{1}{2}} \qquad \frac{0}{0}$$
型未定式



$$\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x = \lim_{x \to +\infty} \frac{\frac{\pi}{2} - \arctan x}{\frac{1}{x}} \qquad \frac{0}{0}$$
型未定式
$$= \lim_{x \to +\infty} \frac{-\frac{1}{1+x^2}}{-\frac{1}{2}}$$

$$\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x = \lim_{x \to +\infty} \frac{\frac{\pi}{2} - \arctan x}{\frac{1}{x}} \qquad \frac{0}{0}$$
型未定式
$$= \lim_{x \to +\infty} \frac{-\frac{1}{1+x^2}}{-\frac{1}{2}} = \lim_{x \to +\infty} \frac{x^2}{1+x^2}$$

$$\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x = \lim_{x \to +\infty} \frac{\frac{\pi}{2} - \arctan x}{\frac{1}{x}} \qquad \frac{0}{0}$$
 型未定式
$$= \lim_{x \to +\infty} \frac{-\frac{1}{1+x^2}}{-\frac{1}{x^2}} = \lim_{x \to +\infty} \frac{x^2}{1+x^2} = \lim_{x \to +\infty} \frac{1}{1+\frac{1}{x^2}}$$

$$\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x = \lim_{x \to +\infty} \frac{\frac{\pi}{2} - \arctan x}{\frac{1}{x}} \qquad \frac{0}{0}$$
型未定式
$$= \lim_{x \to +\infty} \frac{-\frac{1}{1+x^2}}{\frac{1}{x}} = \lim_{x \to +\infty} \frac{x^2}{0} = \lim_{x \to +\infty} \frac{1}{1+x^2} = \lim_{x \to +\infty} \frac{x^2}{0} = \lim_{x \to +\infty} \frac{1}{1+x^2} = \lim_{x \to +\infty} \frac{x^2}{0} = \lim_{x \to +\infty} \frac{1}{1+x^2} = \lim_{x \to +\infty} \frac{x^2}{0} = \lim_{x \to +\infty} \frac{1}{1+x^2} = \lim_{x \to +\infty} \frac{x^2}{0} = \lim_{x \to +\infty} \frac{1}{1+x^2} = \lim_{x \to +\infty} \frac{x^2}{0} = \lim_{x \to +\infty} \frac{1}{1+x^2} = \lim_{x \to +\infty} \frac{x^2}{0} = \lim_{x \to +\infty} \frac{1}{1+x^2} = \lim_{x \to +\infty} \frac{x^2}{0} = \lim_{x \to +\infty} \frac{1}{1+x^2} = \lim_{x \to +\infty} \frac{x^2}{0} = \lim_{x \to +\infty} \frac{$$

$$= \lim_{x \to +\infty} \frac{1}{2} - \operatorname{dictal}(x) = \lim_{x \to +\infty} \frac{1}{\frac{1}{x}} = \lim_{x \to +\infty} \frac{x^2}{1 + x^2} = \lim_{x \to +\infty} \frac{1}{1 + \frac{1}{x^2}} = 1.$$



$$\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x = \lim_{x \to +\infty} \frac{\frac{\pi}{2} - \arctan x}{\frac{1}{x}} \qquad \frac{0}{0}$$
  $= \lim_{x \to +\infty} \frac{-\frac{1}{1+x^2}}{\frac{1}{x}} = \lim_{x \to +\infty} \frac{x^2}{0} = \lim_{x \to +\infty} \frac{x^2}{0}$ 

$$x \to +\infty \begin{pmatrix} 2 & x \to +\infty & \frac{1}{x} & 0 \\ = \lim_{x \to +\infty} \frac{-\frac{1}{1+x^2}}{-\frac{1}{x^2}} = \lim_{x \to +\infty} \frac{x^2}{1+x^2} = \lim_{x \to +\infty} \frac{1}{1+\frac{1}{x^2}} = 1.$$

(2) 为 0⋅∞ 型未定式:



**解(1)**为0·∞型未定式:

$$\lim_{x \to +\infty} \left(\frac{\pi}{2} - \arctan x\right) x = \lim_{x \to +\infty} \frac{\frac{\pi}{2} - \arctan x}{\frac{1}{x}} \qquad \frac{0}{0}$$
型未定式
$$= \lim_{x \to +\infty} \frac{-\frac{1}{1+x^2}}{-\frac{1}{x^2}} = \lim_{x \to +\infty} \frac{x^2}{1+x^2} = \lim_{x \to +\infty} \frac{1}{1+\frac{1}{x^2}} = 1.$$

$$\lim_{x\to 0^+} x \ln x$$



$$\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x = \lim_{x \to +\infty} \frac{\frac{\pi}{2} - \arctan x}{\frac{1}{x}} \qquad \frac{0}{0}$$
型未定式
$$= \lim_{x \to +\infty} \frac{-\frac{1}{1+x^2}}{-\frac{1}{x^2}} = \lim_{x \to +\infty} \frac{x^2}{1+x^2} = \lim_{x \to +\infty} \frac{1}{1+\frac{1}{x^2}} = 1.$$

$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{1/x}$$



例 4 计算极限 (1) 
$$\lim_{x\to+\infty} (\frac{\pi}{2} - \arctan x)x$$
; (2)  $\lim_{x\to 0^+} x \ln x$ 

$$\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x = \lim_{x \to +\infty} \frac{\frac{\pi}{2} - \arctan x}{\frac{1}{x}} \qquad \frac{0}{0}$$
 型未定式
$$= \lim_{x \to +\infty} \frac{-\frac{1}{1+x^2}}{-\frac{1}{x^2}} = \lim_{x \to +\infty} \frac{x^2}{1+x^2} = \lim_{x \to +\infty} \frac{1}{1+\frac{1}{x^2}} = 1.$$

$$\lim_{x \to 0^{+}} x \ln x = \lim_{x \to 0^{+}} \frac{\ln x}{1/x}$$

$$\lim_{x \to 0^{+}} x \ln x = \lim_{x \to 0^{+}} \frac{x}{\frac{1}{\ln x}}$$

例 4 计算极限 (1) 
$$\lim_{x \to +\infty} (\frac{\pi}{2} - \arctan x)x$$
; (2)  $\lim_{x \to 0^+} x \ln x$ 

解 (1) 为 0·∞ 型未定式:

$$\lim_{n \to \infty} \left( \frac{\pi}{n} - \arctan x \right) x =$$

$$\lim_{n \to \infty} \left( \frac{\pi}{2} - \arctan x \right) x = 1$$

 $\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x = \lim_{x \to +\infty} \frac{\frac{\pi}{2} - \arctan x}{1} \qquad \frac{0}{0}$ 型未定式

$$\lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) x = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan} x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \operatorname{arctan}$$

$$\int_{x\to+\infty}^{x\to+\infty} \frac{1}{2} = \lim_{x\to+\infty} \frac{1}{2}$$

(2) 为 0 · ∞ 型未定式: (黨型未定式)

 $\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{1/x}$ 

 $\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{x}{\frac{1}{1-x}}$ 

$$\lim_{x \to \infty} \left( \frac{n}{2} - \arctan x \right) x = \lim_{x \to \infty} \frac{1}{n}$$

$$\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x = \lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) = \lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right$$

$$\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x = \lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) = \lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right$$

λ→+ω	$\frac{-}{x}$	U	
- Lina	$-\frac{1}{1+x^2}$ — $\lim$	<i>x</i> <sup>2</sup>	lina

$$\frac{\overline{1+x^2}}{-\frac{1}{x^2}} = \lim_{x \to +\infty} \frac{x^2}{1+x^2} = \lim_{x \to +\infty} \frac{1}{1+x^2}$$

$$= \lim_{x \to +\infty} \frac{-\frac{1}{1+x^2}}{-\frac{1}{x^2}} = \lim_{x \to +\infty} \frac{x^2}{1+x^2} = \lim_{x \to +\infty} \frac{1}{1+\frac{1}{x^2}} = 1.$$

$$\lim_{x \to +\infty} \frac{\frac{1}{1+x^2}}{-\frac{1}{x^2}} = \lim_{x \to +\infty} \frac{x}{1+x^2} = \lim_{x \to +\infty} \frac{1}{1+x^2}$$

3b 洛必达法则

例 4 计算极限 (1) 
$$\lim_{x \to +\infty} (\frac{\pi}{2} - \arctan x)x$$
; (2)  $\lim_{x \to 0^+} x \ln x$ 

$$\lim_{n \to \infty} \left( \frac{\pi}{n} - \arctan x \right) x =$$

$$\lim_{n \to \infty} \left( \frac{\pi}{--\arctan x} \right) x = 1$$

$$\lim_{x \to \infty} \left( \frac{\pi}{2} - \arctan x \right) x = 1$$

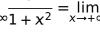
$$\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x = \lim_{x \to +\infty} \frac{\frac{\pi}{2} - \arctan x}{\frac{1}{x}} \qquad \frac{0}{0}$$
 型未定式

$$\lim_{x \to +\infty} \left( \frac{n}{2} - \arctan x \right) x = \lim_{x \to \infty} \frac{1}{n}$$

(2) 为 0 · ∞ 型未定式: (黨型未定式)

 $\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{x}{\frac{1}{\ln x}}$ 

 $\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{1/x} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x}}$ 



 $= \lim_{x \to +\infty} \frac{-\frac{1}{1+x^2}}{-\frac{1}{2}} = \lim_{x \to +\infty} \frac{x^2}{1+x^2} = \lim_{x \to +\infty} \frac{1}{1+\frac{1}{x^2}} = 1.$ 

3b 洛必达法则

8/11 ⊲ ⊳ ∆ ⊽

例 4 计算极限 (1) 
$$\lim_{x \to +\infty} (\frac{\pi}{2} - \arctan x)x$$
; (2)  $\lim_{x \to 0^+} x \ln x$ 

$$\lim_{n \to \infty} \left( \frac{\pi}{n} - \arctan x \right) x = 0$$

$$\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x = \lim_{x \to +\infty} \frac{\frac{\pi}{2} - \arctan x}{\frac{1}{x}} \qquad \frac{0}{0}$$
 型未定式

$$\lim_{x \to +\infty} \left( \frac{1}{2} - \arctan x \right) x = \lim_{x \to +\infty} \left( \frac{1}{2} - \arctan x \right) = \lim_{x \to +\infty} \left( \frac{1}{2} - \arctan x \right$$

$$= \lim_{x \to +\infty} \frac{-\frac{1}{1+x^2}}{-\frac{1}{2}} = \lim_{x \to +\infty} \frac{x^2}{1+x^2} = \lim_{x \to +\infty} \frac{1}{1+\frac{1}{2}} = 1.$$

$$x \to +\infty(2$$
 )  $x \to +\infty(2)$ 

(2) 为 
$$0 \cdot \infty$$
 型未定式:  $(\frac{\infty}{\infty}$ 型未定式)
$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{1/x} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \to 0^+} -x$$

 $\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{x}{\frac{1}{\ln x}}$ 

3b 洛必达法则

例 4 计算极限 (1) 
$$\lim_{x \to +\infty} (\frac{\pi}{2} - \arctan x)x$$
; (2)  $\lim_{x \to 0^+} x \ln x$ 

$$\lim_{n \to \infty} \left( \frac{\pi}{n} - \arctan x \right) x =$$

$$\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x = \lim_{x \to +\infty} \frac{\frac{\pi}{2} - \arctan x}{\frac{1}{x}} \qquad \frac{0}{0}$$
 型未定式

$$\xrightarrow{\rightarrow +\infty} (2 \qquad x \xrightarrow{\rightarrow +\infty}$$

$$-\lim_{x \to \infty} \frac{-\frac{1}{1+x^2}}{-\lim_{x \to \infty} \frac{x^2}{-\lim_{x \to \infty} \frac{x^2}{-$$

$$= \lim_{x \to +\infty} \frac{-\frac{1}{1+x^2}}{-\frac{1}{2}} = \lim_{x \to +\infty} \frac{x^2}{1+x^2} = \lim_{x \to +\infty} \frac{1}{1+\frac{1}{2}} = 1.$$

$$\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{1/x} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \to 0^+} -x = 0.$$

$$\lim_{x \to 0^{+}} x \ln x = \lim_{x \to 0^{+}} \frac{x}{\ln x}$$

例 4 计算极限 (1) 
$$\lim_{x\to +\infty} (\frac{\pi}{2} - \arctan x)x$$
; (2)  $\lim_{x\to 0^+} x \ln x$ 

$$\lim_{n \to \infty} \left( \frac{\pi}{n} - \arctan x \right) x = \lim_{n \to \infty} \frac{\pi}{n}$$

$$= \lim_{x \to +\infty} \frac{-\frac{1}{1+x^2}}{-\frac{1}{2}} = \lim_{x \to +\infty} \frac{x^2}{1+x^2} = \lim_{x \to +\infty} \frac{1}{1+\frac{1}{2}} = 1.$$

$$\lim_{x \to 0^{+}} x \ln x = \lim_{x \to 0^{+}} \frac{\ln x}{1/x} = \lim_{x \to 0^{+}} \frac{\frac{1}{x}}{-\frac{1}{x^{2}}} = \lim_{x \to 0^{+}} -x = 0.$$

$$\lim_{x \to 0^{+}} x \ln x = \lim_{x \to 0^{+}} \frac{x}{\frac{1}{\ln x}} = \lim_{x \to 0^{+}} \frac{1}{-\frac{1}{\ln^{2} x} \cdot \frac{1}{x}}$$



解 (1) 为 0·∞ 型未定式:

$$\lim_{n \to \infty} \left( \frac{\pi}{-} - \arctan x \right) x =$$

 $\lim_{x \to +\infty} \left( \frac{\pi}{2} - \arctan x \right) x = \lim_{x \to +\infty} \frac{\frac{\pi}{2} - \arctan x}{1} \qquad \frac{0}{0}$ 型未定式

$$= \lim_{x \to +\infty} \frac{\frac{1}{x}}{x} = \lim_{x \to +\infty} \frac{x^2}{1+x^2} = \lim_{x \to +\infty} \frac{x^2}$$

$$= \lim_{x \to +\infty} \frac{-\frac{1}{1+x^2}}{-\frac{1}{x^2}} = \lim_{x \to +\infty} \frac{x^2}{1+x^2} = \lim_{x \to +\infty} \frac{1}{1+\frac{1}{x^2}} = 1.$$

$$= \lim_{x \to +\infty} \frac{1+x^2}{-\frac{1}{x^2}} = \lim_{x \to +\infty} \frac{1+x^2}{1+x^2} = \lim_{x \to +\infty} \frac{1+x^2}{1+x^2}$$

(2) 为 0 · ∞ 型未定式: (黨型未定式)

 $\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{\ln x}{1/x} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \to 0^+} -x = 0.$ 

 $\lim_{x \to 0^+} x \ln x = \lim_{x \to 0^+} \frac{x}{\frac{1}{\ln x}} = \lim_{x \to 0^+} \frac{1}{-\frac{1}{\ln^2 x} \cdot \frac{1}{x}} = \cdots (\text{行不通})$ 

▲ 暨南大學

例 5 计算极限 (1)  $\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$ ; (2)  $\lim_{x \to 0} (\frac{1}{\sin x} - \frac{1}{x})$ 



例 5 计算极限 (1) 
$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$$
; (2)  $\lim_{x \to 0} (\frac{1}{\sin x} - \frac{1}{x})$ 

#### 解 (1)

$$\lim_{x\to \frac{\pi}{2}}(\sec x - \tan x)$$



例 5 计算极限 (1)  $\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$ ; (2)  $\lim_{x \to 0} (\frac{1}{\sin x} - \frac{1}{x})$ 

解 (1) 为 ∞ – ∞ 型未定式:

 $\lim_{x\to \frac{\pi}{2}}(\sec x - \tan x)$ 

例 5 计算极限 (1) 
$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$$
; (2)  $\lim_{x \to 0} (\frac{1}{\sin x} - \frac{1}{x})$ 

解 (1) 为 ∞ – ∞ 型未定式:

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x}$$

$$\mathbf{M}$$
 (1) 为  $\infty - \infty$  型未定式: ( $\frac{0}{0}$ 型未定式)

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x}$$

$$\mathbf{K}$$
 (1) 为  $\infty - \infty$  型未定式:  $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ 型未定式)

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} = \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x}$$

例 5 计算极限 (1) 
$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$$
; (2)  $\lim_{x \to 0} (\frac{1}{\sin x} - \frac{1}{x})$ 

$$\mathbf{K}$$
 (1) 为  $\infty - \infty$  型未定式:  $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ 型未定式)

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} = \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0.$$

例 5 计算极限 (1) 
$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$$
; (2)  $\lim_{x \to 0} (\frac{1}{\sin x} - \frac{1}{x})$ 

$$\mathbf{K}$$
 (1) 为  $\infty - \infty$  型未定式: ( $\frac{0}{0}$ 型未定式)

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} = \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0.$$

(2)

$$\lim_{x\to 0}(\frac{1}{\sin x}-\frac{1}{x})$$



解 (1) 为 
$$\infty$$
 –  $\infty$  型未定式: ( $\frac{0}{0}$ 型未定式)

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} = \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0.$$

(2) 为 
$$\infty$$
 –  $\infty$  型未定式:

$$\lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$$



$$\mathbf{K}$$
 (1) 为  $\infty - \infty$  型未定式:  $(\frac{0}{0}$ 型未定式)

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} = \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0.$$

(2) 为 ∞ - ∞ 型未定式:

$$\lim_{x \to 0} \left( \frac{1}{\sin x} - \frac{1}{x} \right) = \lim_{x \to 0} \frac{x - \sin x}{x \sin x}$$

$$\mathbf{K}$$
 (1) 为  $\infty - \infty$  型未定式: ( $\frac{0}{0}$ 型未定式)

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} = \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0.$$

(2) 为  $\infty - \infty$  型未定式:

$$\lim_{x \to 0} \left( \frac{1}{\sin x} - \frac{1}{x} \right) = \lim_{x \to 0} \frac{x - \sin x}{x \sin x} = \lim_{x \to 0} \frac{x - \sin x}{x^2}$$

$$\mathbf{H}$$
 (1) 为  $\infty - \infty$  型未定式: ( $\frac{0}{0}$ 型未定式)

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} = \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0.$$

(2) 为 
$$\infty$$
 –  $\infty$  型未定式:

$$\lim_{x \to 0} \left( \frac{1}{\sin x} - \frac{1}{x} \right) = \lim_{x \to 0} \frac{x - \sin x}{x \sin x} = \lim_{x \to 0} \frac{x - \sin x}{x^2}$$

$$\mathbf{K}$$
 (1) 为  $\infty - \infty$  型未定式: ( $\frac{0}{0}$ 型未定式)

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} = \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0.$$

(2) 为 
$$\infty$$
 –  $\infty$  型未定式:

$$\lim_{x \to 0} \left( \frac{1}{\sin x} - \frac{1}{x} \right) = \lim_{x \to 0} \frac{x - \sin x}{x \sin x} = \lim_{x \to 0} \frac{x - \sin x}{x^2}$$
$$= \lim_{x \to 0} \frac{1 - \cos x}{2x}$$

$$\mathbf{H}$$
 (1) 为  $\infty - \infty$  型未定式: ( $\frac{0}{0}$ 型未定式)

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} = \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0.$$

(2) 为 
$$\infty$$
 –  $\infty$  型未定式:

$$\lim_{x \to 0} \left( \frac{1}{\sin x} - \frac{1}{x} \right) = \lim_{x \to 0} \frac{x - \sin x}{x \sin x} = \lim_{x \to 0} \frac{x - \sin x}{x^2}$$
$$= \lim_{x \to 0} \frac{1 - \cos x}{2x} = \lim_{x \to 0} \frac{\sin x}{2}$$

$$\mathbf{K}$$
 (1) 为  $\infty - \infty$  型未定式: ( $\frac{0}{0}$ 型未定式)

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} = \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0.$$

(2) 为 
$$\infty$$
 –  $\infty$  型未定式:

$$\lim_{x \to 0} \left( \frac{1}{\sin x} - \frac{1}{x} \right) = \lim_{x \to 0} \frac{x - \sin x}{x \sin x} = \lim_{x \to 0} \frac{x - \sin x}{x^2}$$
$$= \lim_{x \to 0} \frac{1 - \cos x}{2x} = \lim_{x \to 0} \frac{\sin x}{2} = 0.$$



例 5 计算极限 (1) 
$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$$
; (2)  $\lim_{x \to 0} (\frac{1}{\sin x} - \frac{1}{x})$ 

$$\mathbf{F}$$
 (1) 为  $\infty - \infty$  型未定式: ( $\frac{0}{0}$ 型未定式)

$$\lim_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos x} = \lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0.$$

(<sup>0</sup>型未定式)

$$\lim_{x \to 0} \left( \frac{1}{\sin x} - \frac{1}{x} \right) = \lim_{x \to 0} \frac{x - \sin x}{x \sin x} = \lim_{x \to 0} \frac{x - \sin x}{x^2}$$
$$= \lim_{x \to 0} \frac{1 - \cos x}{2x} = \lim_{x \to 0} \frac{\sin x}{2} = 0.$$

**例 6** 计算极限 **(1)**  $\lim_{x\to 0^+} x^x$ ; **(2)**  $\lim_{x\to +\infty} x^{\frac{1}{x}}$ 



## 解 (1)

$$\lim_{x\to 0^+} x^x =$$



$$\lim_{x\to 0^+} x^x =$$

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x}$$

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x}$$

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x} = e^{\lim_{x \to 0^+} x \ln x}$$

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x} = e^{\lim_{x \to 0^+} x \ln x} = e^0 = 1$$

**例 6** 计算极限 (1) 
$$\lim_{x\to 0^+} x^x$$
; (2)  $\lim_{x\to +\infty} x^{\frac{1}{x}}$ 

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x} = e^{\lim_{x \to 0^+} x \ln x} = e^0 = 1$$

**(2)** 



**例 6** 计算极限 (1) 
$$\lim_{x\to 0^+} x^x$$
; (2)  $\lim_{x\to +\infty} x^{\frac{1}{x}}$ 

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x} = e^{\lim_{x \to 0^+} x \ln x} = e^0 = 1$$

(2) 为 ∞ <sup>0</sup> 型未定式:

$$\lim_{x \to \infty} x^{\frac{1}{x}}$$

**例 6** 计算极限 (1) 
$$\lim_{x\to 0^+} x^x$$
; (2)  $\lim_{x\to +\infty} x^{\frac{1}{x}}$ 

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x} = e^{\lim_{x \to 0^+} x \ln x} = e^0 = 1$$

(2) 为 ∞0 型未定式:

$$\lim_{x \to +\infty} x^{\frac{1}{x}} = \lim_{x \to +\infty} e^{\ln x^{\frac{1}{x}}}$$

**例 6** 计算极限 (1) 
$$\lim_{x\to 0^+} x^x$$
; (2)  $\lim_{x\to +\infty} x^{\frac{1}{x}}$ 

解(1)为00型未定式:

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x} = e^{\lim_{x \to 0^+} x \ln x} = e^0 = 1$$

(2) 为 ∞0 型未定式:

$$\lim_{x \to +\infty} x^{\frac{1}{x}} = \lim_{x \to +\infty} e^{\ln x^{\frac{1}{x}}} = \lim_{x \to +\infty} e^{\frac{1}{x} \ln x}$$

**例 6** 计算极限 **(1)** 
$$\lim_{x\to 0^+} x^x$$
; **(2)**  $\lim_{x\to +\infty} x^{\frac{1}{x}}$ 

解(1)为00型未定式:

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x} = e^{\lim_{x \to 0^+} x \ln x} = e^0 = 1$$

(2) 为 ∞ 型未定式:

$$\lim_{x \to +\infty} x^{\frac{1}{x}} = \lim_{x \to +\infty} e^{\ln x^{\frac{1}{x}}} = \lim_{x \to +\infty} e^{\frac{1}{x} \ln x} = e^{\lim_{x \to +\infty} \frac{1}{x} \ln x}$$

**例 6** 计算极限 **(1)** 
$$\lim_{x\to 0^+} x^x$$
; **(2)**  $\lim_{x\to +\infty} x^{\frac{1}{x}}$ 

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x} = e^{\lim_{x \to 0^+} x \ln x} = e^0 = 1$$

(2) 为 ∞ <sup>0</sup> 型未定式:

$$\lim_{x \to +\infty} x^{\frac{1}{x}} = \lim_{x \to +\infty} e^{\ln x^{\frac{1}{x}}} = \lim_{x \to +\infty} e^{\frac{1}{x} \ln x} = e^{\lim_{x \to +\infty} \frac{1}{x} \ln x} = e^{0} = 1$$



例 6 计算极限 (1) 
$$\lim_{x\to 0^+} x^x$$
; (2)  $\lim_{x\to +\infty} x^{\frac{1}{x}}$ 

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x} = e^{\lim_{x \to 0^+} x \ln x} = e^0 = 1$$

(2) 为 ∞ <sup>0</sup> 型未定式:

$$\lim_{x \to +\infty} x^{\frac{1}{x}} = \lim_{x \to +\infty} e^{\ln x^{\frac{1}{x}}} = \lim_{x \to +\infty} e^{\frac{1}{x} \ln x} = e^{\lim_{x \to +\infty} \frac{1}{x} \ln x} = e^{0} = 1$$

例 7 
$$\lim_{x \to \infty} (1 + \frac{1}{x})^x = e$$

**例 6** 计算极限 **(1)** 
$$\lim_{x\to 0^+} x^x$$
; **(2)**  $\lim_{x\to +\infty} x^{\frac{1}{x}}$ 

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x} = e^{\lim_{x \to 0^+} x \ln x} = e^0 = 1$$

(2) 为 ∞ 型未定式:

$$\lim_{x \to +\infty} x^{\frac{1}{x}} = \lim_{x \to +\infty} e^{\ln x^{\frac{1}{x}}} = \lim_{x \to +\infty} e^{\frac{1}{x} \ln x} = e^{\lim_{x \to +\infty} \frac{1}{x} \ln x} = e^{0} = 1$$

例7 
$$\lim_{x\to\infty}(1+\frac{1}{x})^x=e$$

验证 
$$\lim_{x \to \infty} (1 + \frac{1}{x})^x = \lim_{x \to \infty} e^{\ln(1 + \frac{1}{x})^x}$$

**例 6** 计算极限 (1) 
$$\lim_{x\to 0^+} x^x$$
; (2)  $\lim_{x\to +\infty} x^{\frac{1}{x}}$ 

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x} = e^{\lim_{x \to 0^+} x \ln x} = e^0 = 1$$

(2) 为 ∞ 型未定式:

$$\lim_{x \to +\infty} x^{\frac{1}{x}} = \lim_{x \to +\infty} e^{\ln x^{\frac{1}{x}}} = \lim_{x \to +\infty} e^{\frac{1}{x} \ln x} = e^{\lim_{x \to +\infty} \frac{1}{x} \ln x} = e^{0} = 1$$

例7 
$$\lim_{x\to\infty}(1+\frac{1}{x})^x=e$$

验证 
$$\lim_{x \to \infty} (1 + \frac{1}{x})^x = \lim_{x \to \infty} e^{\ln(1 + \frac{1}{x})^x} = \lim_{x \to \infty} e^{x \ln(1 + \frac{1}{x})}$$

**例 6** 计算极限 (1) 
$$\lim_{x\to 0^+} x^x$$
; (2)  $\lim_{x\to +\infty} x^{\frac{1}{x}}$ 

解(1)为00型未定式:

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x} = e^{\lim_{x \to 0^+} x \ln x} = e^0 = 1$$

(2) 为 ∞0 型未定式:

$$\lim_{x \to +\infty} x^{\frac{1}{x}} = \lim_{x \to +\infty} e^{\ln x^{\frac{1}{x}}} = \lim_{x \to +\infty} e^{\frac{1}{x} \ln x} = e^{\lim_{x \to +\infty} \frac{1}{x} \ln x} = e^{0} = 1$$

例7 
$$\lim (1+\frac{1}{2})^x = 0$$

例7 
$$\lim_{x \to \infty} (1 + \frac{1}{x})^x = e$$
  
验证  $\lim_{x \to \infty} (1 + \frac{1}{x})^x = \lim_{x \to \infty} e^{\ln(1 + \frac{1}{x})^x} = \lim_{x \to \infty} e^{x \ln(1 + \frac{1}{x})} = e^{\lim_{x \to \infty} x \ln(1 + \frac{1}{x})}$ 

**例 6** 计算极限 **(1)** 
$$\lim_{x\to 0^+} x^x$$
; **(2)**  $\lim_{x\to +\infty} x^{\frac{1}{x}}$ 

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x} = e^{\lim_{x \to 0^+} x \ln x} = e^0 = 1$$

## (2) 为 ∞0 型未定式:

$$\lim_{x \to +\infty} x^{\frac{1}{x}} = \lim_{x \to +\infty} e^{\ln x^{\frac{1}{x}}} = \lim_{x \to +\infty} e^{\frac{1}{x} \ln x} = e^{\lim_{x \to +\infty} \frac{1}{x} \ln x} = e^{0} = 1$$

$$\sqrt{5}$$
 7  $\lim_{x \to 0} (1 + \frac{1}{x})^x = 0$ 

例7 
$$\lim_{x \to \infty} (1 + \frac{1}{x})^x = e$$
  
验证  $\lim_{x \to \infty} (1 + \frac{1}{x})^x = \lim_{x \to \infty} e^{\ln(1 + \frac{1}{x})^x} = \lim_{x \to \infty} e^{x \ln(1 + \frac{1}{x})} = e^{\lim_{x \to \infty} x \ln(1 + \frac{1}{x})}$ 

$$= e^{\lim_{t \to 0} \frac{\ln(1+t)}{t}}$$



**例** 6 计算极限 (1) 
$$\lim_{x\to 0^+} x^x$$
; (2)  $\lim_{x\to +\infty} x^{\frac{1}{x}}$ 

解(1)为00型未定式:

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x} = e^{\lim_{x \to 0^+} x \ln x} = e^0 = 1$$

(2) 为 ∞0 型未定式:

$$\lim_{x \to +\infty} x^{\frac{1}{x}} = \lim_{x \to +\infty} e^{\ln x^{\frac{1}{x}}} = \lim_{x \to +\infty} e^{\frac{1}{x} \ln x} = e^{\lim_{x \to +\infty} \frac{1}{x} \ln x} = e^{0} = 1$$

例7 
$$\lim (1+\frac{1}{2})^x = \epsilon$$

例 7 
$$\lim_{x\to\infty} (1+\frac{1}{x})^x = e$$

验证  $\lim_{x \to \infty} (1 + \frac{1}{x})^x = \lim_{x \to \infty} e^{\ln(1 + \frac{1}{x})^x} = \lim_{x \to \infty} e^{x \ln(1 + \frac{1}{x})} = e^{\lim_{x \to \infty} x \ln(1 + \frac{1}{x})}$  $= \underset{t \to 0}{\lim} \frac{\ln(1+t)}{t} = \underset{t \to 0}{\lim} \frac{\frac{1}{1+t}}{1}$ 



**例** 6 计算极限 (1) 
$$\lim_{x\to 0^+} x^x$$
; (2)  $\lim_{x\to +\infty} x^{\frac{1}{x}}$ 

解(1)为00型未定式:

$$\lim_{x \to 0^+} x^x = \lim_{x \to 0^+} e^{\ln x^x} = \lim_{x \to 0^+} e^{x \ln x} = e^{\lim_{x \to 0^+} x \ln x} = e^0 = 1$$

(2) 为 ∞0 型未定式:

$$\lim_{x \to +\infty} x^{\frac{1}{x}} = \lim_{x \to +\infty} e^{\ln x^{\frac{1}{x}}} = \lim_{x \to +\infty} e^{\frac{1}{x} \ln x} = e^{\lim_{x \to +\infty} \frac{1}{x} \ln x} = e^{0} = 1$$

例7 
$$\lim_{x \to \infty} (1 + \frac{1}{x})^x = e$$
  
验证  $\lim_{x \to \infty} (1 + \frac{1}{x})^x = \lim_{x \to \infty} e^{\ln(1 + \frac{1}{x})^x} = \lim_{x \to \infty} e^{x \ln(1 + \frac{1}{x})} = e^{\lim_{x \to \infty} x \ln(1 + \frac{1}{x})}$ 

 $= e^{\lim_{t \to 0} \frac{\ln(1+t)}{t}} = e^{\lim_{t \to 0} \frac{1}{1+t}} - e^{\lim_{t \to 0} \frac{1}{1+t}}$ 





例 8 分析下面的做法正确与否?



例 8 分析下面的做法正确与否?

$$\lim_{x \to \infty} \frac{1}{x} \xrightarrow{\underline{\text{ABVS}} \pm \underline{\text{Mim}}} \lim_{x \to \infty} \frac{(1)'}{(x)'} = \lim_{x \to \infty} \frac{0}{1} = 0$$

这是  $\frac{1}{2}$  的. 原因是  $\frac{1}{2}$  不是未定式,所以不能用洛必达法则.