

§3.2 向量与向量组的线性组合

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向量

- n 维行向量

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- 零向量 $O = (0, 0, \dots, 0)$

向量的线性运算

• 设 $\alpha = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$, $\beta = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$, $k \in \mathbb{R}$, 则

$$\alpha + \beta = \quad , \quad \alpha - \beta = \quad , \quad k\alpha =$$

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- 行向量类似

线性组合问题

- 给定向量组

$$\alpha_1 = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}, \alpha_2 = \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix}, \dots, \alpha_n = \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix}$$

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设 k_1, k_2, \dots, k_n 为任意数, 则称

$$k_1\alpha_1 + k_2\alpha_2 + \dots + k_n\alpha_n$$

为向量组 $\alpha_1, \alpha_2, \dots, \alpha_n$ 的线性组合。

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即: 是否存在数 k_1, k_2, \dots, k_n 使得:

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例 判断 β 能否由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示, 若能, 写出线性表示等式

$$\beta = k_1\alpha_1 + k_2\alpha_2 + k_3\alpha_3.$$

• (1) 问

$$\beta \quad \begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix}$$

$$\alpha_1 \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\alpha_2 \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

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$$\text{所以 } \beta = 2\alpha_1 - 7\alpha_2 + \frac{5}{2}\alpha_3;$$

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所以 $\beta = 2\alpha_1 - 7\alpha_2 + \frac{5}{2}\alpha_3$; β 能由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示

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$$\begin{matrix} \beta & \alpha_1 & \alpha_2 & \alpha_3 \\ \begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} & \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \end{matrix}$$

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所以 β 不能由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示!

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$$\text{例 问 } \overset{\beta}{\begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}} = - \overset{\alpha_1}{\begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}} + - \overset{\alpha_2}{\begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}} + - \overset{\alpha_3}{\begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}}$$

即： β 能否由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示？如果能，线性表达式是什么？

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- 如果能线性表出，如何求出 k_1, k_2, \dots, k_n 使

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$$k_1\alpha_1 + k_2\alpha_2 + \dots + k_n\alpha_n = \beta$$

不难看出， k_1, \dots, k_n 的求解可归结为线性方程组的求解。

$$\begin{array}{cccc}
 \alpha_1 & \alpha_2 & & \alpha_n & \beta \\
 \left(\begin{array}{c} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{array} \right) & \left(\begin{array}{c} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{array} \right) & \cdots & \left(\begin{array}{c} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{array} \right) & \left(\begin{array}{c} b_1 \\ b_2 \\ \vdots \\ b_m \end{array} \right)
 \end{array}$$

β 可由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示

$$\begin{matrix} \alpha_1 & \alpha_2 & & \alpha_n & \beta \\ \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} & \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} & \cdots & \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} & \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix} \end{matrix}$$

β 可由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示

$$\Leftrightarrow k_1 \overset{\alpha_1}{\begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}} + k_2 \overset{\alpha_2}{\begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix}} + \dots + k_n \overset{\alpha_n}{\begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix}} = \overset{\beta}{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}$$

β 可由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示

$$\Leftrightarrow k_1 \overset{\alpha_1}{\begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}} + k_2 \overset{\alpha_2}{\begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix}} + \dots + k_n \overset{\alpha_n}{\begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix}} = \overset{\beta}{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}$$
$$\begin{pmatrix} \overset{\alpha_1}{a_{11}} & \overset{\alpha_2}{a_{12}} & \cdots & \overset{\alpha_n}{a_{1n}} \\ \overset{\alpha_1}{a_{21}} & \overset{\alpha_2}{a_{22}} & \cdots & \overset{\alpha_n}{a_{2n}} \\ \vdots & \vdots & & \vdots \\ \overset{\alpha_1}{a_{m1}} & \overset{\alpha_2}{a_{m2}} & \cdots & \overset{\alpha_n}{a_{mn}} \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix}$$

β 可由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示

$$\Leftrightarrow k_1 \overset{\alpha_1}{\begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}} + k_2 \overset{\alpha_2}{\begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix}} + \dots + k_n \overset{\alpha_n}{\begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix}} = \overset{\beta}{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}$$

$$\Leftrightarrow \begin{matrix} \alpha_1 & \alpha_2 & & \alpha_n \\ \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \end{matrix} \begin{matrix} \\ \\ \\ \beta \\ \end{matrix} \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

β 可由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示

$$\Leftrightarrow k_1 \overset{\alpha_1}{\begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}} + k_2 \overset{\alpha_2}{\begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix}} + \dots + k_n \overset{\alpha_n}{\begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix}} = \overset{\beta}{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}$$

$$\Leftrightarrow \underbrace{\begin{pmatrix} \overset{\alpha_1}{a_{11}} & \overset{\alpha_2}{a_{12}} & \cdots & \overset{\alpha_n}{a_{1n}} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_A \underbrace{\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix}}_x = \overset{\beta}{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}$$

β 可由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示

$$\Leftrightarrow k_1 \overset{\alpha_1}{\begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}} + k_2 \overset{\alpha_2}{\begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix}} + \dots + k_n \overset{\alpha_n}{\begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix}} = \overset{\beta}{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}$$

$$\Leftrightarrow \underbrace{\begin{pmatrix} \overset{\alpha_1}{a_{11}} & \overset{\alpha_2}{a_{12}} & \cdots & \overset{\alpha_n}{a_{1n}} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_A \underbrace{\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix}}_x = \overset{\beta}{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}$$

$$\Leftrightarrow Ax = \beta \text{有解}$$

β 可由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示

$$\Leftrightarrow k_1 \overset{\alpha_1}{\begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}} + k_2 \overset{\alpha_2}{\begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix}} + \dots + k_n \overset{\alpha_n}{\begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix}} = \overset{\beta}{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}$$

$$\Leftrightarrow \underbrace{\begin{pmatrix} \overset{\alpha_1}{a_{11}} & \overset{\alpha_2}{a_{12}} & \cdots & \overset{\alpha_n}{a_{1n}} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_A \underbrace{\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix}}_x = \overset{\beta}{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}$$

$$\Leftrightarrow Ax = \beta \text{有解} \quad (k_1, \dots, k_n \text{是方程的解})$$

β 可由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示

$$\Leftrightarrow k_1 \overset{\alpha_1}{\begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}} + k_2 \overset{\alpha_2}{\begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix}} + \dots + k_n \overset{\alpha_n}{\begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix}} = \overset{\beta}{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}$$

$$\Leftrightarrow \underbrace{\begin{pmatrix} \overset{\alpha_1}{a_{11}} & \overset{\alpha_2}{a_{12}} & \cdots & \overset{\alpha_n}{a_{1n}} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_A \underbrace{\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix}}_x = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix} \overset{\beta}$$

$$\Leftrightarrow Ax = \beta \text{有解} \quad (k_1, \dots, k_n \text{是方程的解})$$

$$\Leftrightarrow r(A) = r(A|\beta)$$

β 可由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示

$$\Leftrightarrow k_1 \begin{matrix} \alpha_1 \\ \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} \end{matrix} + k_2 \begin{matrix} \alpha_2 \\ \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} \end{matrix} + \dots + k_n \begin{matrix} \alpha_n \\ \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} \end{matrix} = \begin{matrix} \beta \\ \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix} \end{matrix}$$

$$\Leftrightarrow \underbrace{\begin{pmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_n \\ \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \end{pmatrix}}_A \underbrace{\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix}}_x = \begin{pmatrix} \beta \\ \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix} \end{pmatrix}$$

$$\Leftrightarrow Ax = \beta \text{有解} \quad (k_1, \dots, k_n \text{是方程的解})$$

$$\Leftrightarrow r(A) = r(A|\beta) \quad \Leftrightarrow (\alpha_1 \alpha_2 \dots \alpha_n) \quad (\alpha_1 \alpha_2 \dots \alpha_n \beta)$$

β 可由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示

$$\Leftrightarrow k_1 \begin{matrix} \alpha_1 \\ \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} \end{matrix} + k_2 \begin{matrix} \alpha_2 \\ \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} \end{matrix} + \dots + k_n \begin{matrix} \alpha_n \\ \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} \end{matrix} = \begin{matrix} \beta \\ \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix} \end{matrix}$$

$$\Leftrightarrow \underbrace{\begin{pmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_n \\ \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \end{pmatrix}}_A \underbrace{\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix}}_x = \begin{pmatrix} \beta \\ \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix} \end{pmatrix}$$

$$\Leftrightarrow Ax = \beta \text{有解} \quad (k_1, \dots, k_n \text{是方程的解})$$

$$\Leftrightarrow r(A) = r(A:\beta) \quad \Leftrightarrow r(\alpha_1 \alpha_2 \dots \alpha_n) = r(\alpha_1 \alpha_2 \dots \alpha_n \beta)$$

β 可由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示

$$\Leftrightarrow k_1 \begin{matrix} \alpha_1 \\ \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} \end{matrix} + k_2 \begin{matrix} \alpha_2 \\ \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} \end{matrix} + \dots + k_n \begin{matrix} \alpha_n \\ \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} \end{matrix} = \begin{matrix} \beta \\ \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix} \end{matrix}$$

$$\Leftrightarrow \underbrace{\begin{pmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \end{pmatrix}}_A \underbrace{\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix}}_x = \begin{pmatrix} \beta \\ \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix} \end{pmatrix}$$

$$\Leftrightarrow Ax = \beta \text{有解} \quad (k_1, \dots, k_n \text{是方程的解})$$

$$\Leftrightarrow r(A) = r(A|\beta) \quad \Leftrightarrow r(\alpha_1 \alpha_2 \cdots \alpha_n) = r(\alpha_1 \alpha_2 \cdots \alpha_n \beta)$$

定理 β 可由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示 $\Leftrightarrow r(\alpha_1 \alpha_2 \cdots \alpha_n) = r(\alpha_1 \alpha_2 \cdots \alpha_n \beta)$

β 可由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示

$$\Leftrightarrow k_1 \begin{matrix} \alpha_1 \\ \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} \end{matrix} + k_2 \begin{matrix} \alpha_2 \\ \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} \end{matrix} + \dots + k_n \begin{matrix} \alpha_n \\ \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} \end{matrix} = \begin{matrix} \beta \\ \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix} \end{matrix}$$

$$\Leftrightarrow \underbrace{\begin{pmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \end{pmatrix}}_A \underbrace{\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix}}_x = \begin{pmatrix} \beta \\ \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix} \end{pmatrix}$$

$$\Leftrightarrow Ax = \beta \text{有解} \quad (k_1, \dots, k_n \text{是方程的解})$$

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定理 β 可由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示 $\Leftrightarrow r(\alpha_1 \alpha_2 \cdots \alpha_n) = r(\alpha_1 \alpha_2 \cdots \alpha_n \beta)$

注 实际中, k_1, \dots, k_n 的求解不需要特意解方程 $Ax = \beta$, 方法见下例

初等行变换求线性表示问题——例 1

例 判断 β 是否能由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示, 若能, 写出线性表示等式。

(1)

$$(\alpha_1 \ \alpha_2 \ \alpha_3 | \beta) = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right)$$

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• $r(\alpha_1 \alpha_2 \alpha_3) = \quad , \quad r(\alpha_1 \alpha_2 \alpha_3 \beta) = \quad ,$

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• $r(\alpha_1 \alpha_2 \alpha_3) =$, $r(\alpha_1 \alpha_2 \alpha_3 \beta) =$,

初等行变换求线性表示问题——例 1

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• $r(\alpha_1 \alpha_2 \alpha_3) =$, $r(\alpha_1 \alpha_2 \alpha_3 \beta) = 3,$

初等行变换求线性表示问题——例 1

例 判断 β 是否能由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示, 若能, 写出线性表示等式。

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$$(\alpha_1 \ \alpha_2 \ \alpha_3 \mid) = \left(\begin{array}{ccc|c} 1 & 2 & 3 & \\ 0 & -1 & 2 & \\ 1 & 1 & 0 & \\ 2 & -2 & 1 & \end{array} \right) \xrightarrow{\text{初等行变换}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & \end{array} \right)$$

• $r(\alpha_1 \alpha_2 \alpha_3) =$, $r(\alpha_1 \alpha_2 \alpha_3 \beta) = 3,$

初等行变换求线性表示问题——例 1

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- 所以 $r(\alpha_1 \alpha_2 \alpha_3) = 3$, $r(\alpha_1 \alpha_2 \alpha_3 \beta) = 3$,

初等行变换求线性表示问题——例 1

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• 所以 $r(\alpha_1 \alpha_2 \alpha_3) = 3$, $r(\alpha_1 \alpha_2 \alpha_3 \beta) = 3$, 成立

$$r(\alpha_1 \alpha_2 \alpha_3) = r(\alpha_1 \alpha_2 \alpha_3 \beta)$$

初等行变换求线性表示问题——例 1

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$$r(\alpha_1 \alpha_2 \alpha_3) = r(\alpha_1 \alpha_2 \alpha_3 \beta)$$

β 可由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示。

初等行变换求线性表示问题——例 1

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• 所以 $r(\alpha_1 \alpha_2 \alpha_3) = 3$, $r(\alpha_1 \alpha_2 \alpha_3 \beta) = 3$, 成立

$$r(\alpha_1 \alpha_2 \alpha_3) = r(\alpha_1 \alpha_2 \alpha_3 \beta)$$

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初等行变换求线性表示问题——例 1

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$$r(\alpha_1 \alpha_2 \alpha_3) = r(\alpha_1 \alpha_2 \alpha_3 \beta)$$

β 可由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示。

- 显然 $\beta' = \alpha'_1 - \alpha'_2 + \alpha'_3$,

初等行变换求线性表示问题——例 1

例 判断 β 是否能由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示, 若能, 写出线性表示等式。

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$$r(\alpha_1 \alpha_2 \alpha_3) = r(\alpha_1 \alpha_2 \alpha_3 \beta)$$

β 可由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示。

- 显然 $\beta' = \alpha'_1 - \alpha'_2 + \alpha'_3$, 是否也有 $\beta = \alpha_1 - \alpha_2 + \alpha_3$?

初等行变换求线性表示问题——例 1

例 判断 β 是否能由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示, 若能, 写出线性表示等式。

(1)

$$(\alpha_1 \ \alpha_2 \ \alpha_3 | \beta) = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow{\text{初等行变换}} \left(\begin{array}{ccc|c} \alpha'_1 & \alpha'_2 & \alpha'_3 & \beta' \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

- 所以 $r(\alpha_1 \alpha_2 \alpha_3) = 3$, $r(\alpha_1 \alpha_2 \alpha_3 \beta) = 3$, 成立

$$r(\alpha_1 \alpha_2 \alpha_3) = r(\alpha_1 \alpha_2 \alpha_3 \beta)$$

β 可由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示。

- 显然 $\beta' = \alpha'_1 - \alpha'_2 + \alpha'_3$, 是否也有 $\beta = \alpha_1 - \alpha_2 + \alpha_3$?

是的

初等行变换求线性表示问题——例 1

例 判断 β 是否能由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示, 若能, 写出线性表示等式。

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是的

注 可证明: 作初等行变换不改变列与列之间的“线性关系”。

初等行变换求线性表示问题——例 2

例 判断 β 是否能由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示, 若能, 写出线性表示等式。

(2)

$$(\alpha_1 \ \alpha_2 \ \alpha_3 | \beta) = \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 2 & -1 & 3 & 3 \\ -1 & 1 & -2 & 0 \\ 5 & 1 & 4 & 11 \end{array} \right)$$

初等行变换求线性表示问题——例 2

例 判断 β 是否能由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示, 若能, 写出线性表示等式。

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$$(\alpha_1 \ \alpha_2 \ \alpha_3 | \beta) = \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 2 & -1 & 3 & 3 \\ -1 & 1 & -2 & 0 \\ 5 & 1 & 4 & 11 \end{array} \right)$$

• $r(\alpha_1 \alpha_2 \alpha_3) = \quad , \quad r(\alpha_1 \alpha_2 \alpha_3 \beta) = \quad ,$

初等行变换求线性表示问题——例 2

例 判断 β 是否能由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示, 若能, 写出线性表示等式。

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$$(\alpha_1 \ \alpha_2 \ \alpha_3 | \beta) = \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 2 & -1 & 3 & 3 \\ -1 & 1 & -2 & 0 \\ 5 & 1 & 4 & 11 \end{array} \right) \xrightarrow{\text{初等行变换}} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

• $r(\alpha_1 \alpha_2 \alpha_3) =$, $r(\alpha_1 \alpha_2 \alpha_3 \beta) =$,

初等行变换求线性表示问题——例 2

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$$(\alpha_1 \ \alpha_2 \ \alpha_3 | \beta) = \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 2 & -1 & 3 & 3 \\ -1 & 1 & -2 & 0 \\ 5 & 1 & 4 & 11 \end{array} \right) \xrightarrow{\text{初等行变换}} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

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初等行变换求线性表示问题——例 2

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• $r(\alpha_1 \alpha_2 \alpha_3) = \quad , \quad r(\alpha_1 \alpha_2 \alpha_3 \beta) = 3,$

初等行变换求线性表示问题——例 2

例 判断 β 是否能由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示, 若能, 写出线性表示等式。

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- $r(\alpha_1 \alpha_2 \alpha_3) = 2, r(\alpha_1 \alpha_2 \alpha_3 \beta) = 3,$

初等行变换求线性表示问题——例 2

例 判断 β 是否能由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示, 若能, 写出线性表示等式。

(2)

$$(\alpha_1 \ \alpha_2 \ \alpha_3 | \beta) = \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 2 & -1 & 3 & 3 \\ -1 & 1 & -2 & 0 \\ 5 & 1 & 4 & 11 \end{array} \right) \xrightarrow{\text{初等行变换}} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

• 所以 $r(\alpha_1 \alpha_2 \alpha_3) = 2$, $r(\alpha_1 \alpha_2 \alpha_3 \beta) = 3$, 成立

$$r(\alpha_1 \alpha_2 \alpha_3) \neq r(\alpha_1 \alpha_2 \alpha_3 \beta)$$

初等行变换求线性表示问题——例 2

例 判断 β 是否能由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示, 若能, 写出线性表示等式。

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$$(\alpha_1 \ \alpha_2 \ \alpha_3 | \beta) = \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 2 & -1 & 3 & 3 \\ -1 & 1 & -2 & 0 \\ 5 & 1 & 4 & 11 \end{array} \right) \xrightarrow{\text{初等行变换}} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

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$$r(\alpha_1 \alpha_2 \alpha_3) \neq r(\alpha_1 \alpha_2 \alpha_3 \beta)$$

β 不能由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示。

初等行变换求线性表示问题——总结

问题 β 能否由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示？若能，写出线性表示等式。

步骤

初等行变换求线性表示问题——总结

问题 β 能否由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示？若能，写出线性表示等式。

步骤 作初等 **行** 变换：

$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n \mid \beta)$$

初等行变换求线性表示问题——总结

问题 β 能否由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示？若能，写出线性表示等式。

步骤 作初等 **行** 变换：

$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n \mid \beta) \xrightarrow{\text{初等行变换}}$$

初等行变换求线性表示问题——总结

问题 β 能否由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示？若能，写出线性表示等式。

步骤 作初等 **行** 变换：

$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n | \beta) \xrightarrow{\text{初等行变换}} (\alpha'_1 \ \alpha'_2 \ \cdots \ \alpha'_n | \beta') \begin{matrix} \text{(简化)} \\ \text{阶梯型矩阵} \end{matrix}$$

初等行变换求线性表示问题——总结

问题 β 能否由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示? 若能, 写出线性表示等式。

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1.

$$\beta \text{ 由 } \alpha_1, \alpha_2, \dots, \alpha_n \text{ 线性表示} \iff r(\alpha_1 \cdots \alpha_n) = r(\alpha_1 \cdots \alpha_n \ \beta)$$

初等行变换求线性表示问题——总结

问题 β 能否由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示? 若能, 写出线性表示等式。

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$$\Updownarrow$$

$$r(\alpha'_1 \cdots \alpha'_n) = r(\alpha'_1 \cdots \alpha'_n \ \beta')$$

初等行变换求线性表示问题——总结

问题 β 能否由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示？若能，写出线性表示等式。

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2. 行变换前后列与列的线性关系不变，即：

初等行变换求线性表示问题——总结

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步骤 作初等 **行** 变换:

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2. 行变换前后列与列的线性关系不变, 即:

$$\beta' = k_1 \alpha'_1 + \cdots + k_n \alpha'_n \Rightarrow$$

初等行变换求线性表示问题——总结

问题 β 能否由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示? 若能, 写出线性表示等式。

步骤 作初等 **行** 变换:

$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n | \beta) \xrightarrow{\text{初等行变换}} (\alpha'_1 \ \alpha'_2 \ \cdots \ \alpha'_n | \beta') \begin{matrix} \text{(简化)} \\ \text{阶梯型矩阵} \end{matrix}$$

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$$\beta \text{ 由 } \alpha_1, \alpha_2, \dots, \alpha_n \text{ 线性表示} \iff r(\alpha_1 \cdots \alpha_n) = r(\alpha_1 \cdots \alpha_n \ \beta)$$

$$\Updownarrow$$

$$r(\alpha'_1 \cdots \alpha'_n) = r(\alpha'_1 \cdots \alpha'_n \ \beta')$$

2. 行变换前后列与列的线性关系不变, 即:

$$\beta' = k_1 \alpha'_1 + \cdots + k_n \alpha'_n \Rightarrow \beta = k_1 \alpha_1 + \cdots + k_n \alpha_n$$

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right)$$

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$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4 - 2r_1]{r_3 - r_1}$$

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解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4 - 2r_1]{r_3 - r_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2}$$

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

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$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4 - 2r_1]{r_3 - r_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

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$$\xrightarrow[r_4 + 6r_2]{r_3 + r_2}$$

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 1 & 1 & 0 & | & 0 \\ 2 & -2 & 1 & | & 5 \end{pmatrix} \xrightarrow[r_4 - 2r_1]{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix}$$
$$\xrightarrow[r_4 + 6r_2]{r_3 + r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & -1 & | & -5 \\ 0 & 0 & 1 & | & -17 \end{pmatrix}$$

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 1 & 1 & 0 & | & 0 \\ 2 & -2 & 1 & | & 5 \end{pmatrix} \xrightarrow[r_4 - 2r_1]{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix}$$

$$\xrightarrow[r_4 + 6r_2]{r_3 + r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & -5 & | & -5 \\ 0 & 0 & -5 & | & -5 \end{pmatrix}$$

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 1 & 1 & 0 & | & 0 \\ 2 & -2 & 1 & | & 5 \end{pmatrix} \xrightarrow[r_4 - 2r_1]{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix}$$
$$\xrightarrow[r_4 + 6r_2]{r_3 + r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & -5 & | & -5 \\ 0 & 0 & -17 & | & -17 \end{pmatrix}$$

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 1 & 1 & 0 & | & 0 \\ 2 & -2 & 1 & | & 5 \end{pmatrix} \xrightarrow[r_4 - 2r_1]{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix}$$

$$\xrightarrow[r_4 + 6r_2]{r_3 + r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & -5 & | & -5 \\ 0 & 0 & -17 & | & -17 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & 1 & | & 1 \\ 0 & 0 & 1 & | & 1 \end{pmatrix}$$

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 1 & 1 & 0 & | & 0 \\ 2 & -2 & 1 & | & 5 \end{pmatrix} \xrightarrow[r_4 - 2r_1]{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix}$$

$$\xrightarrow[r_4 + 6r_2]{r_3 + r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & -5 & | & -5 \\ 0 & 0 & -17 & | & -17 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & 1 & | & 1 \\ 0 & 0 & 1 & | & 1 \end{pmatrix} \xrightarrow{r_4 - r_3}$$

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 1 & 1 & 0 & | & 0 \\ 2 & -2 & 1 & | & 5 \end{pmatrix} \xrightarrow[r_4 - 2r_1]{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix}$$

$$\xrightarrow[r_4 + 6r_2]{r_3 + r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & -5 & | & -5 \\ 0 & 0 & -17 & | & -17 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & 1 & | & 1 \\ 0 & 0 & 1 & | & 1 \end{pmatrix} \xrightarrow{r_4 - r_3} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & 1 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3}$$

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left(\begin{array}{ccc|c} & & & \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left(\begin{array}{ccc|c} 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{r_1-2r_2}$$

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{r_1-2r_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{r_1-2r_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{r_1-2r_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

所以 $r(\alpha_1 \alpha_2 \alpha_3) = r(\alpha_1 \alpha_2 \alpha_3 \beta)$,

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{r_1-2r_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

所以 $r(\alpha_1 \alpha_2 \alpha_3) = r(\alpha_1 \alpha_2 \alpha_3 \beta)$, 能线性表示

例 1 $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

解 $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{r_1-2r_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

所以 $r(\alpha_1 \alpha_2 \alpha_3) = r(\alpha_1 \alpha_2 \alpha_3 \beta)$, 能线性表示, 且 $\beta = \alpha_1 - \alpha_2 + \alpha_3$

例 2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

例 2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \end{array}$$

例 2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) & \xrightarrow{r_1 \leftrightarrow r_3} & & \end{array}$$

例 2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) & \xrightarrow{r_1 \leftrightarrow r_3} & \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \end{array}$$

例 2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) & \xrightarrow{r_1 \leftrightarrow r_3} & \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \end{array}$$

$$\begin{array}{l} \xrightarrow{r_2 + r_1} \\ r_3 - 2r_1 \\ r_4 - r_1 \end{array}$$

例 2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{cccc}
 \alpha_1 & \alpha_2 & \alpha_3 & \beta \\
 \left(\begin{array}{ccc|c}
 2 & -1 & 4 & 1 \\
 -1 & 3 & 3 & 2 \\
 1 & -2 & 0 & -1 \\
 1 & 4 & 11 & 6
 \end{array} \right) & \xrightarrow{r_1 \leftrightarrow r_3} & \left(\begin{array}{ccc|c}
 1 & -2 & 0 & -1 \\
 -1 & 3 & 3 & 2 \\
 2 & -1 & 4 & 1 \\
 1 & 4 & 11 & 6
 \end{array} \right) \\
 & \xrightarrow{\substack{r_2+r_1 \\ r_3-2r_1 \\ r_4-r_1}} & \left(\begin{array}{ccc|c}
 1 & -2 & 0 & -1 \\
 & & & \\
 & & & \\
 & & &
 \end{array} \right)
 \end{array}$$

例2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) & \xrightarrow{r_1 \leftrightarrow r_3} & \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\ & \xrightarrow{\substack{r_2+r_1 \\ r_3-2r_1 \\ r_4-r_1}} & \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ & & & \\ & & & \end{array} \right) \end{array}$$

例 2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{cccc}
 \alpha_1 & \alpha_2 & \alpha_3 & \beta \\
 \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) & \xrightarrow{r_1 \leftrightarrow r_3} & \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_3 - 2r_1]{r_2 + r_1, r_4 - r_1} & \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \end{array} \right)
 \end{array}$$

例 2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right)
 \end{array}$$

例 2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2}
 \end{array}$$

例2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -5 & -5 \end{array} \right)
 \end{array}$$

例 2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -5 & 0 \end{array} \right)
 \end{array}$$

例2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right)
 \end{array}$$

例2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \\
 \xrightarrow{-\frac{1}{5} \times r_3} \rightarrow
 \end{array}$$

例2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \\
 \xrightarrow{-\frac{1}{5} \times r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right)
 \end{array}$$

例2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \\
 \xrightarrow{-\frac{1}{5} \times r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \xrightarrow{r_4 + 7r_3}
 \end{array}$$

例2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \\
 \xrightarrow{-\frac{1}{5} \times r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \xrightarrow{r_4 + 7r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right)
 \end{array}$$

例2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow{\substack{r_2+r_1 \\ r_3-2r_1 \\ r_4-r_1}} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow{\substack{r_3-3r_2 \\ r_4-6r_2}} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \\
 \xrightarrow{-\frac{1}{5} \times r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \xrightarrow{r_4+7r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right)
 \end{array}$$

例2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \\
 \xrightarrow{-\frac{1}{5} \times r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \xrightarrow{r_4 + 7r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right)
 \end{array}$$

可见 $r(\alpha_1 \alpha_2 \alpha_3 \beta) = 4 > 3 = r(\alpha_1 \alpha_2 \alpha_3)$,

例2 $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$ 能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left(\begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \\
 \xrightarrow{-\frac{1}{5} \times r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \xrightarrow{r_4 + 7r_3} \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right)
 \end{array}$$

可见 $r(\alpha_1 \alpha_2 \alpha_3 \beta) = 4 > 3 = r(\alpha_1 \alpha_2 \alpha_3)$, 所以不能线性表示。

向量组的线性组合

定义 设有两个向量组

$$(A): \alpha_1, \alpha_2, \dots, \alpha_s$$

$$(B): \beta_1, \beta_2, \dots, \beta_t$$

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$$\Longrightarrow \begin{cases} \alpha_1 = & \beta_1 + & \beta_2 + & \beta_3 \\ \alpha_2 = & \beta_1 + & \beta_2 + & \beta_3 \end{cases}$$

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其中的系数就是 A 的第 j 列 $\begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{sj} \end{pmatrix}$ 。

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$$= (\beta_1, \beta_2, \dots, \beta_t) \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1s} \\ a_{21} & a_{22} & \cdots & a_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ a_{t1} & a_{t2} & \cdots & a_{ts} \end{pmatrix}$$

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注 3 反之, 若 $P = QA$, 则说明 P 的列向量组可由的 Q 列向量组线性表示

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定理（向量组线性表示的传递性） 假设向量组 (A) , (B) , (C) 满足： (A) 可由 (B) 线性表示， (B) 可由 (C) 线性表示，则 (A) 可由 (C) 线性表示。

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$$(\alpha_1, \alpha_2, \dots, \alpha_s) = (\beta_1, \beta_2, \dots, \beta_t)A_{t \times s}.$$

向量组 $\beta_1, \beta_2, \dots, \beta_t$ 可由向量组 $\gamma_1, \gamma_2, \dots, \gamma_k$ 线性表示：

$$(\beta_1, \beta_2, \dots, \beta_t) = (\gamma_1, \gamma_2, \dots, \gamma_k)B_{k \times t}.$$

将第 2 式代入第 1 式，可得

$$(\alpha_1, \alpha_2, \dots, \alpha_s) = (\gamma_1, \gamma_2, \dots, \gamma_k) \underbrace{B_{k \times t} A_{t \times s}}_{C_{k \times s}} = (\gamma_1, \gamma_2, \dots, \gamma_k)C.$$

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将第 2 式代入第 1 式，可得

$$(\alpha_1, \alpha_2, \dots, \alpha_s) = (\gamma_1, \gamma_2, \dots, \gamma_k) \underbrace{B_{k \times t} A_{t \times s}}_{C_{k \times s}} = (\gamma_1, \gamma_2, \dots, \gamma_k)C.$$

所以向量组 $\alpha_1, \alpha_2, \dots, \alpha_s$ 可由向量组 $\gamma_1, \gamma_2, \dots, \gamma_k$ 线性表示。

定理（向量组线性表示的传递性） 假设向量组 (A) , (B) , (C) 满足： (A) 可由 (B) 线性表示， (B) 可由 (C) 线性表示，则 (A) 可由 (C) 线性表示。

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将第 2 式代入第 1 式，可得

$$(\alpha_1, \alpha_2, \dots, \alpha_s) = (\gamma_1, \gamma_2, \dots, \gamma_k) \underbrace{B_{k \times t} A_{t \times s}}_{C_{k \times s}} = (\gamma_1, \gamma_2, \dots, \gamma_k)C.$$

所以向量组 $\alpha_1, \alpha_2, \dots, \alpha_s$ 可由向量组 $\gamma_1, \gamma_2, \dots, \gamma_k$ 线性表示。

（并且，线性组合的系数就是矩阵 C 的列。）

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

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具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

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则

$$\alpha_1 =$$

$$\alpha_2 =$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

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则

$$\alpha_1 = a_{11}(\quad) + a_{21}(\quad)$$

$$\alpha_2 =$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

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则

$$\alpha_1 = a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(\quad)$$

$$\alpha_2 =$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

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则

$$\alpha_1 = a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

$$\alpha_2 =$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

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$$\alpha_2 =$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

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$$\alpha_2 = a_{12}(\quad) + a_{22}(\quad)$$

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则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{aligned}$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{aligned}$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{aligned}$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{aligned}$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + \end{aligned}$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + \end{aligned}$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$

其中

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{pmatrix}$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$

其中

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{11}b_{21} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{12} & a_{21}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{12}b_{32} & a_{21}b_{31} + a_{22}b_{32} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases} \Rightarrow (\alpha_1, \alpha_2) = (\beta_1, \beta_2) \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$

其中

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{11}b_{21} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{12} & a_{21}b_{21} + a_{22}b_{22} \\ a_{31}b_{11} + a_{32}b_{12} & a_{31}b_{21} + a_{32}b_{22} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases} \Rightarrow (\alpha_1, \alpha_2) = (\beta_1, \beta_2) \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases} \Rightarrow (\beta_1, \beta_2) = (\gamma_1, \gamma_2, \gamma_3) \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$

其中

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{11}b_{21} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{12} & a_{21}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{12}b_{32} & a_{21}b_{31} + a_{22}b_{32} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases} \Rightarrow (\alpha_1, \alpha_2) = (\beta_1, \beta_2) \underbrace{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}}_A$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases} \Rightarrow (\beta_1, \beta_2) = (\gamma_1, \gamma_2, \gamma_3) \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$

其中

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{11}b_{21} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{12} & a_{21}b_{21} + a_{22}b_{22} \\ a_{31}b_{11} + a_{32}b_{12} & a_{31}b_{21} + a_{32}b_{22} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases} \Rightarrow (\alpha_1, \alpha_2) = (\beta_1, \beta_2) \underbrace{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}}_A$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases} \Rightarrow (\beta_1, \beta_2) = (\gamma_1, \gamma_2, \gamma_3) \underbrace{\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}}_B$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$

其中

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{11}b_{21} + a_{12}b_{22} \\ a_{11}b_{31} + a_{12}b_{32} & a_{21}b_{11} + a_{22}b_{12} \\ a_{21}b_{21} + a_{22}b_{22} & a_{21}b_{31} + a_{22}b_{32} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

例 $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases} \Rightarrow (\alpha_1, \alpha_2) = (\beta_1, \beta_2) \underbrace{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}}_A$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases} \Rightarrow (\beta_1, \beta_2) = (\gamma_1, \gamma_2, \gamma_3) \underbrace{\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}}_B$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$

其中

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{11}b_{21} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{12} & a_{21}b_{21} + a_{22}b_{22} \\ a_{31}b_{11} + a_{32}b_{12} & a_{31}b_{21} + a_{32}b_{22} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = BA$$

定义 设有两个向量组

$$(A): \alpha_1, \alpha_2, \dots, \alpha_s$$

$$(B): \beta_1, \beta_2, \dots, \beta_t$$

如果 (A) 与 (B) 可相互线性表示, 则称向量组 (A) 与 (B) 等价。