§8.4 偏导数与全微分

2017-2018 学年 II

Outline of §8.4

1. 二元函数偏导数定义

3. 全微分的定义与计算

We are here now...

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3. 全微分的定义与计算

偏导数引入

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$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

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两种方式各有优点，要灵活运用

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

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解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y +$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} =$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

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$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} =$$

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所以

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$$\frac{\partial z}{\partial x}=(xy+\frac{x}{y})'_x=(xy)'_x+(\frac{x}{y})'_x=y+\frac{1}{y}$$

$$\frac{\partial z}{\partial y}=(xy+\frac{x}{y})'_y=$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}}=(y+\frac{1}{y})\Big|_{\substack{x=2\\y=1}}=1+\frac{1}{1}=2$$

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所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

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$$\frac{\partial z}{\partial y}=(xy+\frac{x}{y})'_y=(xy)'_y+(\frac{x}{y})'_y=x-\frac{x}{y^2}$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}}=(y+\frac{1}{y})\Big|_{\substack{x=2\\y=1}}=1+\frac{1}{1}=2$$

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所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = \left.(y + \frac{1}{y})\right|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

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$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\bigg|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\bigg|_{y=y_0}$$

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二阶偏导数

$$
\begin{tikzcd}
 & \dfrac{\partial z}{\partial x} \\
z = f(x, y) \arrow[ur] \arrow[dr] & \\
 & \dfrac{\partial z}{\partial y}
\end{tikzcd}
$$

二阶偏导数

$$
\begin{array}{ccccc}
 & & & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right) \\
 & & & \nearrow & \\
 & & \dfrac{\partial z}{\partial x} & & \\
 & \nearrow & & \searrow & \\
z = f(x, y) & & & & \\
 & \searrow & & & \\
 & & \dfrac{\partial z}{\partial y} & &
\end{array}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
 z=f(x,y) \arrow[uur] \arrow[dr] & & \\
 & \frac{\partial z}{\partial y} &
\end{tikzcd}
$$

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\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & {}
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)
\end{tikzcd}
$$

二阶偏导数

$$\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial y}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}$$

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$$
\begin{tikzcd}
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x\partial x}, \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
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 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y\partial x}, \\
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 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial x}, \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial y}, \\
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 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y \partial x}, \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y \partial y},
\end{tikzcd}
$$

二阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial y},$$

$$\frac{\partial z}{\partial y}$$

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$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \\
z = f(x, y) \arrow[uuur, shift left=0, to=2-2] \arrow[dddr, to=6-2] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial y},
\end{tikzcd}
$$

二阶偏导数

$$
\begin{array}{lllll}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x \partial x}, & z_{xx}, \\
 & \nearrow\frac{\partial z}{\partial x} & & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x \partial y}, & z_{xy}, \\
z = f(x, y) & & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y \partial x}, & z_{yx}, \\
 & \searrow\frac{\partial z}{\partial y} & & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y \partial y}, &
\end{array}
$$

二阶偏导数

$$\begin{tikzcd}[row sep=small]
& & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x\partial x}, \quad z_{xx}, \\
& \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
& & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x\partial y}, \quad z_{xy}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & \\
& & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y\partial x}, \quad z_{yx}, \\
& \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
& & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y\partial y}, \quad z_{yy},
\end{tikzcd}$$

二阶偏导数

$$
\begin{tikzcd}[column sep=large, row sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial x}, \quad z_{xx}, \quad f_{xx} \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial y}, \quad z_{xy}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y\partial x}, \quad z_{yx}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y\partial y}, \quad z_{yy},
\end{tikzcd}
$$

二阶偏导数

$$z=f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x\partial x}, \quad z_{xx}, \quad f_{xx}$$

$$z=f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x\partial y}, \quad z_{xy}, \quad f_{xy}$$

$$z=f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y\partial x}, \quad z_{yx},$$

$$z=f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y\partial y}, \quad z_{yy},$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x\partial x},\quad z_{xx},\quad f_{xx} \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x\partial y},\quad z_{xy},\quad f_{xy} \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y\partial x},\quad z_{yx},\quad f_{yx} \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y\partial y},\quad z_{yy},
\end{tikzcd}
$$

二阶偏导数

$$\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right):
\end{tikzcd}$$

$$\frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \quad f_{xx}$$

$$\frac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \quad f_{xy}$$

$$\frac{\partial^2 z}{\partial y \partial x}, \quad z_{yx}, \quad f_{yx}$$

$$\frac{\partial^2 z}{\partial y \partial y}, \quad z_{yy}, \quad f_{yy}$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x =$$

$$z_y =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} +$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} +$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

暨南大學 JINAN UNIVERSITY

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=$$

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例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

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$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

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$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} +$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

$$z_{xy}=(ye^{xy}+2y^2)'_y=(ye^{xy})'_y+(2y^2)'_y=e^{xy}+xye^{xy}+4y$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

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$$z_{yy} =$$

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解

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例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

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例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

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$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

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$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

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$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

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$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2e^{xy} +$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

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$$z_{yx}=(xe^{xy}+4xy)'_x=(xe^{xy})'_x+(4xy)'_x=e^{xy}+xye^{xy}+4y$$

$$z_{yy}=(xe^{xy}+4xy)'_y=(xe^{xy})'_y+(4xy)'_y=x^2e^{xy}+4x$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2e^{xy} + 4x$$

注 此例成立 $z_{xy} = z_{yx}$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) & \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z = f(x, y) \arrow[uuur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right)
\end{tikzcd}
$$

三阶偏导数

$$z=f(x,y)\to\frac{\partial z}{\partial x}\to\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\qquad\frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$\frac{\partial z}{\partial x}\to\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$z=f(x,y)\to\frac{\partial z}{\partial y}\to\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial z}{\partial y}\to\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$z = f(x, y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$z = f(x, y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial x}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): &
\end{tikzcd}
$$

三阶偏导数

$$z=f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$z=f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial x}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial y}, \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial x\partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): &
\end{tikzcd}
$$

三阶偏导数

$$z=f(x,y) \to \frac{\partial z}{\partial x},\quad \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial z}{\partial x} \qquad \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$z = f(x, y) \qquad \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial z}{\partial y} \qquad \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x}, \; z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y}, \; z_{xxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z = f(x, y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$z = f(x, y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z=f(x,y)\to\frac{\partial z}{\partial x}\to\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial z}{\partial x}\to\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$z=f(x,y)\to\frac{\partial z}{\partial y}\to\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial z}{\partial y}\to\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z=f(x,y)\to\frac{\partial z}{\partial x}\to\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$z=f(x,y)\to\frac{\partial z}{\partial y}\to\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x}, \ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y}, \ z_{xxy},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x}, \ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y}, \ z_{xyy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x}, \ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y}, \ z_{yxy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x}, \ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z=f(x,y) \arrow[uuur] \arrow[dddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & \frac{\partial z}{\partial y} \arrow[uur] \arrow[r] & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right)
\end{tikzcd}
$$

$$
\begin{aligned}
\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) &: \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx}, \\
\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) &: \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy}, \\
\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) &: \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx}, \\
\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) &: \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy}, \\
\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) &: \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx}, \\
\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) &: \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy}, \\
\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) &: \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx}, \\
\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) &: \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},
\end{aligned}
$$

三阶偏导数

$$z = f(x, y)$$

$$z = f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right), \quad \frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$z = f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right), \quad \frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial x \partial x}, \ z_{xxx}, \ f_{xxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial x \partial y}, \ z_{xxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial y \partial x}, \ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial y \partial y}, \ z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial x \partial x}, \ z_{yxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial x \partial y}, \ z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial y \partial x}, \ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial y \partial y}, \ z_{yyy},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \qquad \frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \qquad \frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \qquad \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z = f(x, y)$$

$$z = f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},\ f_{xyy}$$

$$z = f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},\ z_{yxy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$z=f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \qquad \frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \qquad \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},\ f_{yyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)$$

$$z=f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$z=f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},\ f_{yyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},\ f_{yyy}$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x =$$

$$z_y =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解 $z_x =$

$z_y =$

$z_{xx} =$

$z_{xy} =$

$z_{yx} =$

$z_{yy} =$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解 $z_x =$

$z_y =$

$z_{xx} =$

$z_{xy} =$

$z_{yx} =$

$z_{yy} =$

$z_{xxx} =$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解 $$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2$$

$$z_y=$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

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例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$
$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$
$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$
$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y =$$
$$z_{yx} =$$
$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$
$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$
$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$
$$z_{yx}=$$
$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$

$$z_{yx}=(2x^3y-9xy^2-x)'_x=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$

$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$
$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$
$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$
$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2$$
$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$

$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2-1$$

$$z_{yy}=(2x^3y-9xy^2-x)'_y=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$

$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2-1$$

$$z_{yy}=(2x^3y-9xy^2-x)'_y=2x^3-18xy$$

$$z_{xxx}=(6xy^2)'_x=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$
$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$
$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$
$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$
$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} = (6xy^2)'_x = 6y^2$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$
$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$
$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$
$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2-1$$
$$z_{yy}=(2x^3y-9xy^2-x)'_y=2x^3-18xy$$

$$z_{xxx}=(6xy^2)'_x=6y^2$$

注 此例成立 $z_{xy}=z_{yx}$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x =$$
$$z_y =$$
$$z_{xx} =$$
$$z_{xy} =$$
$$z_{yx} =$$
$$z_{yy} =$$
$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$
$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$
$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$
$$z_{yx} = (3x\cos(3y))'_x =$$
$$z_{yy} =$$
$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$
$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$
$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$
$$z_{yy} = (3x\cos(3y))'_y =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$

$$z_{yy} = (3x\cos(3y))'_y = -9x\sin(3y)$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$

$$z_{yy} = (3x\cos(3y))'_y = -9x\sin(3y)$$

$$z_{xyy} = (3\cos(3y))'_y =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$

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性质 设有二元函数 $z = f(x, y)$。若 $\frac{\partial^2 z}{\partial y \partial x}$ 和 $\frac{\partial^2 z}{\partial x \partial y}$ 均连续，则
$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$$

We are here now...

1. 二元函数偏导数定义

3. 全微分的定义与计算

回顾一元函数的微分

- 函数 $y = f(x)$ 的增量

$$\Delta y = f(x + \Delta x) - f(x)$$

回顾一元函数的微分

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此时可用 $f'(x)\Delta x$ 近似代替 Δy，称为函数 $y=f(x)$ 的微分，记为：

$$dy=f'(x)dx \quad 或 \quad df=f'(x)dx$$

多元函数的全微分

- 二元函数 $z = f(x, y)$

$$f(x+\Delta x, y+\Delta y) - f(x, y)$$

多元函数的全微分

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$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y)\\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \approx A\Delta x + B\Delta y\end{aligned}$$

多元函数的全微分

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例 设 $z = f(x, y) = x^2 + y^2$,

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例 设 $z = f(x, y) = x^2 + y^2$，则

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多元函数的全微分

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例 设 $z = f(x, y) = x^2 + y^2$，则

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

$$= \left[(x + \Delta x)^2 + (y + \Delta y)^2\right] - \left[x^2 + y^2\right]$$

多元函数的全微分

- 二元函数 $z=f(x,y)$的全增量

$$\Delta z=f(x+\Delta x,y+\Delta y)-f(x,y)$$

- 称 $z=f(x,y)$可微是指：

$$\begin{aligned}\Delta z&=f(x+\Delta x,y+\Delta y)-f(x,y)\\&=A\Delta x+B\Delta y+o\left(\sqrt{(\Delta x)^2+(\Delta y)^2}\right)\approx A\Delta x+B\Delta y\end{aligned}$$

例 设 $z=f(x,y)=x^2+y^2$，则

$$\begin{aligned}\Delta z&=f(x+\Delta x,y+\Delta y)-f(x,y)\\&=\left[(x+\Delta x)^2+(y+\Delta y)^2\right]-\left[x^2+y^2\right]\\&=2x\Delta x+2y\Delta y+\left[(\Delta x)^2+(\Delta y)^2\right]\end{aligned}$$

多元函数的全微分

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所以 $z=x^2+y^2$ 可微。

多元函数的全微分 (Cont.)

- 若 $z = f(x, y)$可微，则连续，且存在偏导数 z_x, z_y，还有

$$\Delta z = f(x + \Delta x) - f(x)$$

多元函数的全微分 (Cont.)

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$$\Delta z = f(x + \Delta x) - f(x)$$

$$= z_x(x, y)\Delta x + z_y(x, y)\Delta y +$$

多元函数的全微分 (Cont.)

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$$= z_x(x, y)\Delta x + z_y(x, y)\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right)$$

$$\approx z_x(x, y)\Delta x + z_y(x, y)\Delta y$$

多元函数的全微分 (Cont.)

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$$\begin{aligned}\Delta z &= f(x + \Delta x) - f(x) \\ &= z_x(x, y)\Delta x + z_y(x, y)\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \\ &\approx z_x(x, y)\Delta x + z_y(x, y)\Delta y\end{aligned}$$

$z = f(x, y)$ 的全微分：$dz = z_x(x, y)dx + z_y(x, y)dy$

多元函数的全微分 (Cont.)

- 若 $z=f(x,y)$可微，则连续，且存在偏导数 z_x, z_y，还有

$$\begin{aligned}\Delta z &= f(x+\Delta x)-f(x)\\ &= z_x(x,y)\Delta x + z_y(x,y)\Delta y + o\left(\sqrt{(\Delta x)^2+(\Delta y)^2}\right)\\ &\approx z_x(x,y)\Delta x + z_y(x,y)\Delta y\end{aligned}$$

$z=f(x,y)$ 的全微分：$dz = z_x(x,y)dx + z_y(x,y)dy$

- 若 $z=f(x,y)$ 可微，则 $\Delta z \approx dz$

多元函数的全微分 (Cont.)

- 对三元函数 $u = \varphi(x, y, z)$，其全微分

$$du = u_x dx + u_y dy + u_z dz$$

多元函数的全微分 (Cont.)

- 对三元函数 $u = \varphi(x, y, z)$，其全微分

$$du = u_x dx + u_y dy + u_z dz$$

此时

$$\Delta u = \varphi(x + \Delta x, y + \Delta y, z + \Delta z) - \varphi(x, y, z)$$

多元函数的全微分 (Cont.)

- 对三元函数 $u = \varphi(x, y, z)$，其全微分

$$du = u_x dx + u_y dy + u_z dz$$

此时

$$\Delta u = \varphi(x + \Delta x, y + \Delta y, z + \Delta z) - \varphi(x, y, z) \approx du$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

解

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解

$$z_x =$$

$$z_y =$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x =$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x =$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数 $z=\frac{y}{x}$ 的全微分

解

$$z_x=\left(\frac{y}{x}\right)'_x=-\frac{y}{x^2}$$

$$z_y=\left(\frac{y}{x}\right)'_y=$$

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例 计算函数 $z = \frac{y}{x}$ 的全微分

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$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy =$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

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$$z_x = \left(\frac{y}{x}\right)_x' = -\frac{y}{x^2}$$

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例 计算函数 $z = x^2y + y^2$ 的全微分

例 计算函数 $z = \frac{y}{x}$ 的全微分

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例 计算函数 $z = x^2y + y^2$ 的全微分

解

$$z_x = (x^2y + y^2)'_x =$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数 $z=\frac{y}{x}$ 的全微分

解

$$z_x=\left(\frac{y}{x}\right)'_x=-\frac{y}{x^2}$$

$$z_y=\left(\frac{y}{x}\right)'_y=\frac{1}{x}$$

$$dz=z_x dx+z_y dy=-\frac{y}{x^2}dx+\frac{1}{x}dy$$

例 计算函数 $z=x^2y+y^2$ 的全微分

解

$$z_x=(x^2y+y^2)'_x=(x^2y)'_x+(y^2)'_x=$$

$$z_y=$$

$$dz=z_x dx+z_y dy=$$

例 计算函数 $z=\frac{y}{x}$ 的全微分

解

$$z_x=\left(\frac{y}{x}\right)'_x=-\frac{y}{x^2}$$

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例 计算函数 $z=x^2y+y^2$ 的全微分

解

$$z_x=(x^2y+y^2)'_x=(x^2y)'_x+(y^2)'_x=2xy$$

$$z_y=$$

$$dz=z_x dx+z_y dy=$$

例 计算函数 $z=\frac{y}{x}$ 的全微分

解

$$z_x=\left(\frac{y}{x}\right)'_x=-\frac{y}{x^2}$$

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$$z_x=(x^2y+y^2)'_x=(x^2y)'_x+(y^2)'_x=2xy$$

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$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

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例 计算函数 $z = x^2y + y^2$ 的全微分

解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x = 2xy$$

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例 计算函数 $z = \frac{y}{x}$ 的全微分

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$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

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解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x = 2xy$$

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$$dz = z_x dx + z_y dy =$$

例 计算函数 $z=\frac{y}{x}$ 的全微分

解

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$$z_y=\left(\frac{y}{x}\right)'_y=\frac{1}{x}$$

$$dz=z_xdx+z_ydy=-\frac{y}{x^2}dx+\frac{1}{x}dy$$

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$$z_x=(x^2y+y^2)'_x=(x^2y)'_x+(y^2)'_x=2xy$$

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$$dz=z_xdx+z_ydy=$$

例 计算函数 $z=\frac{y}{x}$ 的全微分

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$$z_x=\left(\frac{y}{x}\right)_x'=-\frac{y}{x^2}$$

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解

$$z_x=(x^2y+y^2)_x'=(x^2y)_x'+(y^2)_x'=2xy$$

$$z_y=(x^2y+y^2)_y'=(x^2y)_y'+(y^2)_y'=x^2+2y$$

$$dz=z_xdx+z_ydy=2xydx+(x^2+2y)dy$$

例 求 $z = xy$ 在点 $(2, 3)$ 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解

例 求 $z = xy$ 在点 $(2, 3)$ 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解 $z_x =$ ， $z_y =$

例 求 $z=xy$ 在点 $(2,3)$ 处，关于 $\Delta x=0.1$，$\Delta y=0.2$ 的全增量 Δz 及全微分 dz。

解

$$z_x = \qquad\qquad , \qquad z_y =$$

$$dz = z_x dx + z_y dy =$$

例 求 $z = xy$ 在点 $(2, 3)$ 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解

$$z_x = (xy)'_x = \quad , \qquad z_y =$$

$$dz = z_x dx + z_y dy =$$

例 求 $z = xy$ 在点 $(2, 3)$ 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解
$$z_x = (xy)'_x = y\,, \qquad z_y =$$
$$dz = z_x dx + z_y dy =$$

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