§8.4 二元函数偏导数与全微分

2016-2017 学年 II

教学要求

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Outline of §8.4

1. 二元函数偏导数定义

3. 全微分的定义与计算

We are here now...

1. 二元函数偏导数定义

3. 全微分的定义与计算

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偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 x 的偏增量：

$$f(x_0 + \Delta x, y_0) - f(x_0, y_0)$$

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例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

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解法二 利用

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解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \quad [f(x, y_0)] \quad ,$$

例 设 $z=xy+\frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2,1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0},$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

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$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

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$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \qquad [f(x_0, y)]$$

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解法二 利用

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所以

$$f(x, 1)$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以

$$f(x, 1) = 2x$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

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$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \qquad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以

$$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] =$$

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所以

$$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$$

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所以

$$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$$

$$\Rightarrow \frac{\partial z}{\partial x}\Big|_{\substack{x=2\\y=1}} = \frac{d}{dx}[f(x, 1)]\Big|_{x=2} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

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所以 $f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$

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$$f(x,1)=2x \quad\Rightarrow\quad \frac{d}{dx}[f(x,1)]=2$$

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$$f(2,y)=2y+\frac{2}{y} \quad\Rightarrow\quad \frac{d}{dy}[f(2,y)]=2-\frac{2}{y^2}$$

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$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0},\qquad \frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\Big|_{y=y_0}$$

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$$\Rightarrow\quad \frac{\partial z}{\partial y}\Big|_{\substack{x=2\\y=1}}=\frac{d}{dy}[f(2,y)]\Big|_{y=1}=$$

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$$f(2, y) = 2y + \frac{2}{y} \quad \Rightarrow \quad \frac{d}{dy}[f(2, y)] = 2 - \frac{2}{y^2}$$

$$\Rightarrow \quad \frac{\partial z}{\partial y}\Big|_{\substack{x=2\\y=1}} = \frac{d}{dy}[f(2, y)]\Big|_{y=1} = 0.$$

例 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

例 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x =$$

$$u_y =$$

$$u_z =$$

例 求三元函数 $u=xyz+\frac{z}{x}$ 的全部一阶偏导数

解
$$u_x=(xyz+\frac{z}{x})'_x=$$

$$u_y=$$

$$u_z=$$

例 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x =$$

$$u_y =$$

$$u_z =$$

例 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz$$

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解
$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

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$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$
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$$u_z = (xyz + \frac{z}{x})'_z =$$

例 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z =$$

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解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

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$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z = xy$$

例 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解
$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$
$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$
$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z = xy + \frac{1}{x}$$

二阶偏导数

$$
\begin{tikzcd}[row sep=large]
 & \dfrac{\partial z}{\partial x} \\
z = f(x, y) \arrow[ur] \arrow[dr] & \\
 & \dfrac{\partial z}{\partial y}
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
z=f(x,y) \arrow[ur] \arrow[dr] & & {} \\
 & \frac{\partial z}{\partial y} &
\end{tikzcd}
$$

二阶偏导数

$$\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[dr] & & \\
 & \frac{\partial z}{\partial y} &
\end{tikzcd}$$

二阶偏导数

$$\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & {}
\end{tikzcd}$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small, row sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x \partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x \partial y}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial y}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y\partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): \quad \dfrac{\partial^2 z}{\partial x\partial x}, \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): \quad \dfrac{\partial^2 z}{\partial x\partial y}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): \quad \dfrac{\partial^2 z}{\partial y\partial x}, \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right): \quad \dfrac{\partial^2 z}{\partial y\partial y},
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial y}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial y},
\end{tikzcd}
$$

二阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial y}, \quad z_{xy},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \qquad \frac{\partial^2 z}{\partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \qquad \frac{\partial^2 z}{\partial y \partial y},$$

二阶偏导数

$$z = f(x, y) \to \frac{\partial z}{\partial x}, \quad z = f(x, y) \to \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial y}, \quad z_{xy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \qquad \frac{\partial^2 z}{\partial y \partial x}, \quad z_{yx},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \qquad \frac{\partial^2 z}{\partial y \partial y},$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y \partial x}, \quad z_{yx}, \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y \partial y}, \quad z_{yy},
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right):
\end{tikzcd}
$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \quad f_{xx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial y}, \quad z_{xy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial x}, \quad z_{yx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial y}, \quad z_{yy},$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial x},\quad z_{xx},\quad f_{xx} \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial y},\quad z_{xy},\quad f_{xy} \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y\partial x},\quad z_{yx}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y\partial y},\quad z_{yy},
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial x},\quad z_{xx},\quad f_{xx} \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial y},\quad z_{xy},\quad f_{xy} \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial x},\quad z_{yx},\quad f_{yx} \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial y},\quad z_{yy},
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): \quad \dfrac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \quad f_{xx} \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): \quad \dfrac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \quad f_{xy} \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): \quad \dfrac{\partial^2 z}{\partial y \partial x}, \quad z_{yx}, \quad f_{yx} \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right): \quad \dfrac{\partial^2 z}{\partial y \partial y}, \quad z_{yy}, \quad f_{yy}
\end{tikzcd}
$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x =$$

$$z_y =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} +$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$z_{xx}=$

$z_{xy}=$

$z_{yx}=$

$z_{yy}=$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

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例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

$$z_{xy}=(ye^{xy}+2y^2)'_y=$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

$$z_{xy}=(ye^{xy}+2y^2)'_y=(ye^{xy})'_y+(2y^2)'_y=e^{xy}+xye^{xy}+$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

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$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

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$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

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$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x =$$

$$z_{yy} =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

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$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

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$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

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解

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例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

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$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

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$$z_{yx}=(xe^{xy}+4xy)'_x=(xe^{xy})'_x+(4xy)'_x=e^{xy}+xye^{xy}+4y$$

$$z_{yy}=(xe^{xy}+4xy)'_y=(xe^{xy})'_y+(4xy)'_y=x^2e^{xy}+$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

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$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

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$$z_{yy}=(xe^{xy}+4xy)'_y=(xe^{xy})'_y+(4xy)'_y=x^2e^{xy}+4x$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

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$$z_{yx}=(xe^{xy}+4xy)'_x=(xe^{xy})'_x+(4xy)'_x=e^{xy}+xye^{xy}+4y$$

$$z_{yy}=(xe^{xy}+4xy)'_y=(xe^{xy})'_y+(4xy)'_y=x^2e^{xy}+4x$$

注 此例成立 $z_{xy}=z_{yx}$

三阶偏导数

$$
\begin{tikzcd}
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right) \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right) \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right)
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) & \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z = f(x, y) \arrow[uuur, start anchor=north east] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}$$

三阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right)
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): &
\end{tikzcd}
$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial x}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial y}, \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial x\partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): &
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial x}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial y}, \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial x\partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): &
\end{tikzcd}
$$

三阶偏导数

$$z = f(x, y) \longrightarrow \frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \qquad \frac{\partial^3 z}{\partial x \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \qquad \frac{\partial^3 z}{\partial x \partial x \partial y},$$

$$\frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \qquad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \qquad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$z = f(x, y) \longrightarrow \frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \qquad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \qquad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \qquad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) :$$

三阶偏导数

$$z = f(x, y) \longrightarrow \frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$z = f(x, y) \longrightarrow \frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial x \partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial x \partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z=f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},$$

$$z=f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},$$

$$z=f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},\ z_{yxy},$$

$$z=f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \qquad \frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \qquad \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z = f(x, y) \longrightarrow \frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$z = f(x, y) \longrightarrow \frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x}, \; z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y}, \; z_{xxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x}, \; z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y}, \; z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x}, \; z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y}, \; z_{yxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x}, \; z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y}, \; z_{yyy},$$

三阶偏导数

$$\begin{tikzcd}
& & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx} \\
& \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy}, \\
& & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy}, \\
& & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx}, \\
& \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy}, \\
& & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx}, \\
& & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},
\end{tikzcd}$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)\to \frac{\partial z}{\partial x}\to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial z}{\partial x}\to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$z=f(x,y)\to \frac{\partial z}{\partial y}\to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial z}{\partial y}\to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$z=f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)\to\frac{\partial z}{\partial x}\to\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial z}{\partial x}\to\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$z=f(x,y)\to\frac{\partial z}{\partial y}\to\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial z}{\partial y}\to\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},\ f_{yyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y) \longrightarrow \frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$z=f(x,y) \longrightarrow \frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},\ f_{yyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},\ f_{yyy}$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x =$$

$$z_y =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2$$

$$z_y=$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3$$

$$z_y=$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

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注 此例成立 $z_{xy}=z_{yx}$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

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$$z_x =$$

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例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

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$$z_x = (x\sin(3y))'_x =$$

$$z_y =$$

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解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

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$$z_{xy} =$$

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$$z_x = (x\sin(3y))'_x = \sin(3y)$$

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例 求 $z=x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x=(x\sin(3y))'_x=\sin(3y)$$

$$z_y=(x\sin(3y))'_y=3x\cos(3y)$$

$$z_{xx}=(\sin(3y))'_x=0$$

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注 此例成立 $z_{xy} = z_{yx}$

We are here now...

1. 二元函数偏导数定义

3. 全微分的定义与计算

回顾一元函数的微分

- 函数 $y = f(x)$ 的增量

$$\Delta y = f(x+\Delta x) - f(x)$$

回顾一元函数的微分

- 函数 $y=f(x)$ 的增量

$$\Delta y=f(x+\Delta x)-f(x)$$

- 若 $y=f(x)$可微，则

$$\Delta y=f(x+\Delta x)-f(x)=A\Delta x+o(\Delta x)$$

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此时可用 $f'(x)\Delta x$ 近似代替 Δy,

回顾一元函数的微分

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此时可用 $f'(x)\Delta x$ 近似代替 Δy，称为函数 $y=f(x)$ 的微分，记为：

$$dy=f'(x)dx \quad 或 \quad df=f'(x)dx$$

多元函数的全微分

- 二元函数 $z = f(x, y)$

$$f(x+\Delta x, y+\Delta y) - f(x, y)$$

多元函数的全微分

- 二元函数 $z=f(x,y)$

$$\Delta z=f(x+\Delta x,y+\Delta y)-f(x,y)$$

多元函数的全微分

- 二元函数 $z = f(x, y)$的全增量

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- 二元函数 $z = f(x, y)$的**全增量**

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称 $z = f(x, y)$**可微**是指：

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

多元函数的全微分

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- 称 $z=f(x,y)$可微是指：

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$$=A\Delta x+B\Delta y+$$

多元函数的全微分

- 二元函数 $z = f(x, y)$的全增量

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$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

$$= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right)$$

多元函数的全微分

- 二元函数 $z = f(x, y)$的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称 $z = f(x, y)$可微是指:

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \approx A\Delta x + B\Delta y\end{aligned}$$

多元函数的全微分

- 二元函数 $z=f(x,y)$的全增量

$$\Delta z=f(x+\Delta x,y+\Delta y)-f(x,y)$$

- 称 $z=f(x,y)$可微是指：

$$\begin{aligned}\Delta z&=f(x+\Delta x,y+\Delta y)-f(x,y)\\&=A\Delta x+B\Delta y+o\left(\sqrt{(\Delta x)^2+(\Delta y)^2}\right)\approx A\Delta x+B\Delta y\end{aligned}$$

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所以 $z=x^2+y^2$ 可微。

多元函数的全微分 (Cont.)

- 若 $z=f(x,y)$可微，则连续，且存在偏导数 z_x, z_y，还有

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- 若 $z=f(x,y)$ 可微，则 $\Delta z \approx dz$

多元函数的全微分 (Cont.)

- 对三元函数 $u = \varphi(x, y, z)$，其全微分

$$du = u_x dx + u_y dy + u_z dz$$

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$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

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例 计算函数 $z=x^2y+y^2$ 的全微分

解

$$z_x=(x^2y+y^2)_x'=(x^2y)_x'+(y^2)_x'=2xy$$

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例 计算函数 $z = x^2y + y^2$ 的全微分

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例 求 $z = xy$ 在点 $(2, 3)$ 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

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$$dz = 3 \times 0.1 +$$

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将 $(x,y)=(2,3)$ 及 $\Delta x=0.1$、$\Delta y=0.2$ 代入得:
$$dz=3\times 0.1+2\times 0.2=0.7$$

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$$\begin{aligned}\Delta z&=z(2+0.1,3+0.2)-z(2,3)\\&=(2+0.1)\times(3+0.2)-2\times 3\\&=0.72\end{aligned}$$

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$$\begin{aligned}\Delta z&=z(2+0.1,\,3+0.2)-z(2,3)\\&=(2+0.1)\times(3+0.2)-2\times 3\\&=0.72\\&\approx dz\end{aligned}$$