§5.4 分部积分法

数学系　　梁卓滨

2017-2018 学年 II

教学要求

- ◇ 熟练掌握分部积分法
- ♣
- ♡
- ♠

Outline of §5.4

分部积分法，能干啥？

能够计算如下的不定积分：

$$\int xe^x dx, \quad \int x^2 \ln x dx, \quad \int x \arctan x dx, \quad \int x \cos x dx$$

$$\int \ln x dx, \quad \int \arcsin x dx, \quad \int \arctan x dx, \quad \int \ln(1+x^2) dx$$

$$\int x^2 e^x dx, \quad \int e^x \cos x dx \qquad \cdots\cdots$$

（可能要结合前面学的换元积分法）

分部积分引入

- 微分公式

$$d(uv) = udv + vdu$$

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$$d(uv) = udv + vdu \quad \Rightarrow \quad udv = d(uv) - vdu$$

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练习

$$x\cos x dx =$$

$$xe^x dx =$$

$$\ln x dx =$$

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应用

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$$\int x\cos x dx = \int xd\sin x = \int d(x\sin x) - \int \sin x dx$$

$$= x\sin x + \cos x + C$$

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- 两边积分得：

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- 实际应用时的步骤：

$$\int \text{”Ì½»§Æ¢”} dx =$$

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- 实际应用时的步骤：

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$$\overset{\text{凑微分}}{=\!=\!=} \int udv =$$

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例 1 求 $\int x\cos x dx$，$\int xe^x dx$，$\int x\sin x dx$

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$$\int x\cos x dx = \int x d\sin x = x\sin x - \int \sin x dx = x\sin x + \cos x + C$$

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$$\int x\cos x dx = \int \cos x \cdot d(\frac{1}{2}x^2)$$

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行不通的做法

$$\int x\cos x dx = \int \cos x \cdot d(\frac{1}{2}x^2) = \frac{1}{2}x^2\cos x - \int \frac{1}{2}x^2 d\cos x$$

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$$\int x\sin x dx = \int x d(-\cos x) =$$

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$$\int x\sin x dx = \int x d(-\cos x) = x(-\cos x) - \int (-\cos x) dx$$

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$$= -x\cos x + \sin x + C$$

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例 2 求 $\int x \ln x dx$，$\int x^2 \ln x dx$

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解
$$\int x\ln x dx = \int \ln x d(\frac{1}{2}x^2) = \frac{1}{2}x^2\cdot\ln x - \int \frac{1}{2}x^2 d\ln x$$

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$$= \frac{1}{2}x^2\cdot\ln x - \int \frac{1}{2}x^2\cdot\frac{1}{x}dx$$

$$\int x^2\ln x dx =$$

例 2 求 $\int x\ln x dx$，$\int x^2\ln x dx$

解
$$\int x\ln x dx=\int \ln x d(\frac{1}{2}x^2)=\frac{1}{2}x^2\cdot\ln x-\int\frac{1}{2}x^2 d\ln x$$
$$=\frac{1}{2}x^2\cdot\ln x-\int\frac{1}{2}x^2\cdot\frac{1}{x}dx$$
$$=\frac{1}{2}x^2\ln x-\frac{1}{4}x^2+C$$
$$\int x^2\ln x dx=$$

例 2 求 $\int x\ln x dx$, $\int x^2\ln x dx$

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$$\int x\ln x dx=\int \ln x d(\frac{1}{2}x^2)=\frac{1}{2}x^2\cdot\ln x-\int\frac{1}{2}x^2 d\ln x$$

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$$=\frac{1}{2}x^2\ln x-\frac{1}{4}x^2+C$$

$$\int x^2\ln x dx=\int \ln x d(\frac{1}{3}x^3)=$$

例 2 求 $\int x\ln x dx$，$\int x^2\ln x dx$

解

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$$=\frac{1}{2}x^2\ln x-\frac{1}{4}x^2+C$$

$$\int x^2\ln x dx=\int \ln x d(\frac{1}{3}x^3)=\frac{1}{3}x^3\cdot\ln x-\int\frac{1}{3}x^3 d\ln x$$

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解

$$\int x\ln x dx = \int \ln x d(\frac{1}{2}x^2) = \frac{1}{2}x^2\cdot\ln x - \int \frac{1}{2}x^2 d\ln x$$

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$$= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$$

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$$= \frac{1}{3}x^3\cdot\ln x - \int \frac{1}{3}x^3\cdot\frac{1}{x}dx$$

$$= \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$$

例 3 求 $\int x \arctan x dx$

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$$\int x\arctan x dx = \int \arctan x d(\frac{1}{2}x^2)$$

例 3 求 $\int x\arctan x dx$

解
$$\int x\arctan x dx = \int \arctan x d(\frac{1}{2}x^2)$$
$$= \frac{1}{2}x^2\arctan x - \int \frac{1}{2}x^2 d\arctan x$$

例 3 求 $\int x\arctan x dx$

解
$$\int x\arctan x dx = \int \arctan x d(\frac{1}{2}x^2)$$
$$= \frac{1}{2}x^2\arctan x - \int \frac{1}{2}x^2 d\arctan x$$
$$= \frac{1}{2}x^2\arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2}dx$$

例 3 求 $\int x\arctan x dx$

解
$$\int x\arctan x dx = \int \arctan x d(\frac{1}{2}x^2)$$
$$= \frac{1}{2}x^2\arctan x - \int \frac{1}{2}x^2 d\arctan x$$
$$= \frac{1}{2}x^2\arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2}dx$$
$$= \frac{1}{2}x^2\arctan x - \frac{1}{2}\int \frac{x^2}{1+x^2}dx$$

例 3 求 $\int x\arctan x dx$

解
$$\int x\arctan x dx = \int \arctan x d(\frac{1}{2}x^2)$$
$$= \frac{1}{2}x^2\arctan x - \int \frac{1}{2}x^2 d\arctan x$$
$$= \frac{1}{2}x^2\arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2}dx$$
$$= \frac{1}{2}x^2\arctan x - \frac{1}{2}\int \frac{x^2}{1+x^2}dx$$
$$= \frac{1}{2}x^2\arctan x - \frac{1}{2}\int \left(1-\frac{1}{1+x^2}\right)dx$$

例 3 求 $\int x\arctan x dx$

解
$$\begin{aligned}\int x\arctan x dx &= \int \arctan x d(\frac{1}{2}x^2)\\ &= \frac{1}{2}x^2\arctan x - \int \frac{1}{2}x^2 d\arctan x\\ &= \frac{1}{2}x^2\arctan x - \int \frac{1}{2}x^2\cdot\frac{1}{1+x^2}dx\\ &= \frac{1}{2}x^2\arctan x - \frac{1}{2}\int \frac{x^2}{1+x^2}dx\\ &= \frac{1}{2}x^2\arctan x - \frac{1}{2}\int \left(1-\frac{1}{1+x^2}\right)dx\\ &= \frac{1}{2}x^2\arctan x - \frac{1}{2}x + \frac{1}{2}\arctan x + C\end{aligned}$$

例 4 求 $\int \ln x dx$，$\int \ln(1+x^2) dx$

例 4 求 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解

$$\int \ln x dx =$$

例 4 求 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解
$$\int \ln x dx = x\ln x - \int x d\ln x =$$

例 4 求 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解

$$\int \ln x dx = x\ln x - \int x d\ln x = x\ln x - \int x \cdot \frac{1}{x} dx$$

例 4 求 $\int \ln x dx$，$\int \ln(1+x^2) dx$

解

$$\int \ln x dx = x\ln x - \int x d\ln x = x\ln x - \int x\cdot\frac{1}{x}dx$$

$$= x\ln x - x + C$$

例 4 求 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解

$$\int \ln x dx = x\ln x - \int x d\ln x = x\ln x - \int x \cdot \frac{1}{x} dx$$

$$= x\ln x - x + C$$

$$\int \ln(1+x^2)dx =$$

例 4 求 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解

$$\int \ln x dx = x\ln x - \int x d\ln x = x\ln x - \int x \cdot \frac{1}{x} dx$$

$$= x\ln x - x + C$$

$$\int \ln(1+x^2)dx = x\ln(1+x^2) - \int x d\ln(1+x^2)$$

例 4 求 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解

$$\int \ln x dx = x\ln x - \int x d\ln x = x\ln x - \int x\cdot\frac{1}{x}dx$$

$$= x\ln x - x + C$$

$$\int \ln(1+x^2)dx = x\ln(1+x^2) - \int x d\ln(1+x^2)$$

$$= x\ln(1+x^2) - \int x\cdot\frac{2x}{1+x^2}dx$$

例 4 求 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解

$$\int \ln x dx = x\ln x - \int x d\ln x = x\ln x - \int x \cdot \frac{1}{x} dx$$

$$= x\ln x - x + C$$

$$\int \ln(1+x^2)dx = x\ln(1+x^2) - \int x d\ln(1+x^2)$$

$$= x\ln(1+x^2) - \int x \cdot \frac{2x}{1+x^2} dx$$

$$= x\ln(1+x^2) - 2\int \frac{x^2}{1+x^2} dx$$

例 4 求 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解

$$\int \ln x dx = x\ln x - \int x d\ln x = x\ln x - \int x \cdot \frac{1}{x} dx$$

$$= x\ln x - x + C$$

$$\int \ln(1+x^2)dx = x\ln(1+x^2) - \int x d\ln(1+x^2)$$

$$= x\ln(1+x^2) - \int x \cdot \frac{2x}{1+x^2} dx$$

$$= x\ln(1+x^2) - 2\int \frac{x^2}{1+x^2} dx$$

$$= x\ln(1+x^2) - 2\int \left(1 - \frac{1}{1+x^2}\right) dx$$

例 4 求 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解
$$\int \ln x dx = x\ln x - \int x d\ln x = x\ln x - \int x \cdot \frac{1}{x} dx$$
$$= x\ln x - x + C$$

$$\int \ln(1+x^2)dx = x\ln(1+x^2) - \int x d\ln(1+x^2)$$
$$= x\ln(1+x^2) - \int x \cdot \frac{2x}{1+x^2} dx$$
$$= x\ln(1+x^2) - 2\int \frac{x^2}{1+x^2} dx$$
$$= x\ln(1+x^2) - 2\int \left(1 - \frac{1}{1+x^2}\right) dx$$
$$= x\ln(1+x^2) - 2x + 2\arctan x + C$$

例 5 求 $\int \arctan x dx$

例 5 求 $\int \arctan x dx$

解 $$\int \arctan x dx =$$

例 5 求 $\int \arctan x dx$

解 $$\int \arctan x dx = x \arctan x - \int x d \arctan x$$

例 5 求 $\int \arctan x dx$

解
$$\int \arctan x dx = x \arctan x - \int x d \arctan x$$
$$= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx$$

例 5 求 $\int \arctan x dx$

解
$$\int \arctan x dx = x \arctan x - \int x d \arctan x$$
$$= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx$$

$d(1+x^2)$

例 5 求 $\int \arctan x dx$

解

$$\int \arctan x dx = x \arctan x - \int x d \arctan x$$

$$= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx$$

$$\frac{1}{2} d(1+x^2)$$

例 5 求 $\int \arctan x dx$

解 $$\int \arctan x dx = x\arctan x - \int x d\arctan x$$
$$= x\arctan x - \int x \cdot \frac{1}{1+x^2} dx$$
$$= x\arctan x - \int \frac{1}{1+x^2} \cdot \frac{1}{2} d(1+x^2)$$

例 5 求 $\int \arctan x dx$

解
$$
\begin{aligned}
\int \arctan x dx &= x\arctan x - \int x d\arctan x \\
&= x\arctan x - \int x \cdot \frac{1}{1+x^2} dx \\
&= x\arctan x - \int \frac{1}{1+x^2} \cdot \frac{1}{2} d(1+x^2) \\
&= x\arctan x - \frac{1}{2} \int \frac{1}{u} du
\end{aligned}
$$

例 5 求 $\int \arctan x dx$

解
$$
\begin{aligned}
\int \arctan x dx &= x\arctan x - \int x d\arctan x \\
&= x\arctan x - \int x \cdot \frac{1}{1+x^2} dx \\
&= x\arctan x - \int \frac{1}{1+x^2} \cdot \frac{1}{2} d(1+x^2) \\
&= x\arctan x - \frac{1}{2}\int \frac{1}{u} du \\
&= x\arctan x - \frac{1}{2}\ln|u| + C
\end{aligned}
$$

例 5 求 $\int \arctan x dx$

解

$$
\begin{aligned}
\int \arctan x dx &= x\arctan x - \int x d\arctan x \\
&= x\arctan x - \int x \cdot \frac{1}{1+x^2} dx \\
&= x\arctan x - \int \frac{1}{1+x^2} \cdot \frac{1}{2} d(1+x^2) \\
&= x\arctan x - \frac{1}{2}\int \frac{1}{u} du \\
&= x\arctan x - \frac{1}{2}\ln|u| + C \\
&= x\arctan x - \frac{1}{2}\ln(1+x^2) + C.
\end{aligned}
$$

例 5 求 $\int \arctan x dx$

解
$$\begin{aligned}\int \arctan x dx &= x\arctan x - \int x d\arctan x \\ &= x\arctan x - \int x \cdot \frac{1}{1+x^2} dx \\ &= x\arctan x - \int \frac{1}{1+x^2} \cdot \frac{1}{2} d(1+x^2) \\ &= x\arctan x - \frac{1}{2}\int \frac{1}{u} du \\ &= x\arctan x - \frac{1}{2}\ln|u| + C \\ &= x\arctan x - \frac{1}{2}\ln(1+x^2) + C.\end{aligned}$$

例 6 求 $\int \arcsin x dx$

例 6 求不定积分 $\int \arcsin x dx$

例 6 求不定积分 $\int \arcsin x dx$

解 $$\int \arcsin x dx =$$

例 6 求不定积分 $\int \arcsin x dx$

解 $$\int \arcsin x dx = x \arcsin x - \int x d \arcsin x$$

例 6 求不定积分 $\int \arcsin x dx$

解
$$\int \arcsin x dx = x \arcsin x - \int x d \arcsin x$$
$$= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$$

例 6 求不定积分 $\int \arcsin x dx$

解
$$\int \arcsin x dx = x\arcsin x - \int x d\arcsin x$$
$$= x\arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$$

$$d(1-x^2)$$

例 6 求不定积分 $\int \arcsin x dx$

解
$$\int \arcsin x dx = x \arcsin x - \int x d \arcsin x$$
$$= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$$
$$-\frac{1}{2} d(1-x^2)$$

例 6 求不定积分 $\int \arcsin x dx$

解
$$\int \arcsin x dx = x \arcsin x - \int x d \arcsin x$$
$$= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$$
$$= x \arcsin x - \int \frac{1}{\sqrt{1-x^2}} \cdot -\frac{1}{2} d(1-x^2)$$

例 6 求不定积分 $\int \arcsin x dx$

解

$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\ &= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\ &= x \arcsin x - \int \frac{1}{\sqrt{1-x^2}} \cdot -\frac{1}{2} d(1-x^2) \\ &= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{u}} \cdot (-1) du\end{aligned}$$

例 6 求不定积分 $\int \arcsin x dx$

解
$$\int \arcsin x dx = x \arcsin x - \int x d \arcsin x$$
$$= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$$
$$= x \arcsin x - \int \frac{1}{\sqrt{1-x^2}} \cdot -\frac{1}{2} d(1-x^2)$$
$$= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{u}} \cdot (-1) du$$

$$2u^{\frac{1}{2}}$$

例 6 求不定积分 $\int \arcsin x dx$

解
$$\int \arcsin x dx = x \arcsin x - \int x d \arcsin x$$
$$= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$$
$$= x \arcsin x - \int \frac{1}{\sqrt{1-x^2}} \cdot -\frac{1}{2} d(1-x^2)$$
$$= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{u}} \cdot (-1) du$$
$$= x \arcsin x + \frac{1}{2} \cdot 2u^{\frac{1}{2}}$$

例 6 求不定积分 $\int \arcsin x dx$

解
$$\begin{aligned}\int \arcsin x dx &= x\arcsin x - \int x d\arcsin x\\ &= x\arcsin x - \int x\cdot\frac{1}{\sqrt{1-x^2}}dx\\ &= x\arcsin x - \int \frac{1}{\sqrt{1-x^2}}\cdot -\frac{1}{2}d(1-x^2)\\ &= x\arcsin x - \frac{1}{2}\int\frac{1}{\sqrt{u}}\cdot(-1)du\\ &= x\arcsin x + \frac{1}{2}\cdot 2u^{\frac{1}{2}}\\ &= x\arcsin x + \sqrt{1-x^2} + C.\end{aligned}$$

例 6 求不定积分 $\int \arcsin x dx$

解
$$\begin{aligned}\int \arcsin x dx &= x\arcsin x - \int x d\arcsin x \\ &= x\arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\ &= x\arcsin x - \int \frac{1}{\sqrt{1-x^2}} \cdot -\frac{1}{2} d(1-x^2) \\ &= x\arcsin x - \frac{1}{2}\int \frac{1}{\sqrt{u}} \cdot (-1) du \\ &= x\arcsin x + \frac{1}{2} \cdot 2u^{\frac{1}{2}} \\ &= x\arcsin x + \sqrt{1-x^2} + C.\end{aligned}$$

例 7 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$

例 6 求不定积分 $\int \arcsin x dx$

解 $$\int \arcsin x dx = x\arcsin x - \int x d\arcsin x$$
$$= x\arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$$
$$= x\arcsin x - \int \frac{1}{\sqrt{1-x^2}} \cdot -\frac{1}{2} d(1-x^2)$$
$$= x\arcsin x - \frac{1}{2}\int \frac{1}{\sqrt{u}} \cdot (-1) du$$
$$= x\arcsin x + \frac{1}{2} \cdot 2u^{\frac{1}{2}}$$
$$= x\arcsin x + \sqrt{1-x^2} + C.$$

例 7 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （提示 两次分部积分）

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例 7 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$

例 7 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$　　（提示 两次分部积分）

例 7 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （提示 两次分部积分）

解 $$\int x^2 e^x dx =$$

例 7 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （提示 两次分部积分）

解 $\displaystyle\int x^2 e^x dx = \int x^2 de^x =$

例 7 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （提示 两次分部积分）

解 $$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 =$$

例 7 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （提示 两次分部积分）

解 $$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2\int e^x x dx$$

例 7 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$　　（提示 两次分部积分）

解 $$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2\int e^x x dx$$

$$= x^2 e^x - 2\left(\int x de^x\right) =$$

例 7 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （提示 两次分部积分）

解 $$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2\int e^x x dx$$

$$= x^2 e^x - 2\left(\int x de^x\right) = x^2 e^x - 2\left(xe^x - \int e^x dx\right)$$

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$ （提示 两次分部积分）

解 $$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2\int e^x x dx$$

$$= x^2 e^x - 2\left(\int x de^x\right) = x^2 e^x - 2\left(x e^x - \int e^x dx\right)$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

例 7 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$　　（提示 两次分部积分）

解 $$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2\int e^x x dx$$

$$= x^2 e^x - 2\left(\int x de^x\right) = x^2 e^x - 2\left(xe^x - \int e^x dx\right)$$

$$= x^2 e^x - 2xe^x + 2e^x + C$$

$$\int x^2 \sin x dx =$$

例 7 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （提示 两次分部积分）

解 $$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2\int e^x x dx$$

$$= x^2 e^x - 2\left(\int x de^x\right) = x^2 e^x - 2\left(xe^x - \int e^x dx\right)$$

$$= x^2 e^x - 2xe^x + 2e^x + C$$

$$\int x^2 \sin x dx = -\int x^2 d\cos x =$$

例 7 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （**提示** 两次分部积分）

解

$$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2\int e^x x dx$$

$$= x^2 e^x - 2\left(\int x de^x\right) = x^2 e^x - 2\left(xe^x - \int e^x dx\right)$$

$$= x^2 e^x - 2xe^x + 2e^x + C$$

$$\int x^2 \sin x dx = -\int x^2 d\cos x = -x^2 \cos x + \int \cos x dx^2$$

例 7 求不定积分 $\int x^2e^xdx$，$\int x^2\sin x dx$ （提示 两次分部积分）

解 $$\int x^2e^xdx=\int x^2de^x=x^2e^x-\int e^xdx^2=x^2e^x-2\int e^xxdx$$

$$=x^2e^x-2\left(\int xde^x\right)=x^2e^x-2\left(xe^x-\int e^xdx\right)$$

$$=x^2e^x-2xe^x+2e^x+C$$

$$\int x^2\sin x dx=-\int x^2d\cos x=-x^2\cos x+\int \cos x dx^2$$

$$=-x^2\cos x+2\int x\cos x dx$$

例 7 求不定积分 $\int x^2e^xdx$，$\int x^2\sin xdx$ （提示 两次分部积分）

解 $$\int x^2e^xdx=\int x^2de^x=x^2e^x-\int e^xdx^2=x^2e^x-2\int e^xxdx$$

$$=x^2e^x-2\left(\int xde^x\right)=x^2e^x-2\left(xe^x-\int e^xdx\right)$$

$$=x^2e^x-2xe^x+2e^x+C$$

$$\int x^2\sin xdx=-\int x^2d\cos x=-x^2\cos x+\int \cos xdx^2$$

$$=-x^2\cos x+2\int x\cos xdx$$

$$=-x^2\cos x+2\int xd\sin x$$

例 7 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （提示 两次分部积分）

解 $\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2\int e^x x dx$

$$= x^2 e^x - 2\left(\int x de^x\right) = x^2 e^x - 2\left(x e^x - \int e^x dx\right)$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

$$\int x^2 \sin x dx = -\int x^2 d\cos x = -x^2 \cos x + \int \cos x dx^2$$

$$= -x^2 \cos x + 2\int x \cos x dx$$

$$= -x^2 \cos x + 2\int x d\sin x$$

$$= -x^2 \cos x + 2\left(x \sin x - \int \sin x dx\right)$$

例 7 求不定积分 $\int x^2e^xdx$，$\int x^2\sin x dx$　　（提示 两次分部积分）

解 $$\int x^2e^xdx=\int x^2de^x=x^2e^x-\int e^xdx^2=x^2e^x-2\int e^xxdx$$

$$=x^2e^x-2\left(\int xde^x\right)=x^2e^x-2\left(xe^x-\int e^xdx\right)$$

$$=x^2e^x-2xe^x+2e^x+C$$

$$\int x^2\sin xdx=-\int x^2d\cos x=-x^2\cos x+\int \cos xdx^2$$

$$=-x^2\cos x+2\int x\cos xdx$$

$$=-x^2\cos x+2\int xd\sin x$$

$$=-x^2\cos x+2\left(x\sin x-\int \sin xdx\right)$$

$$=-x^2\cos x+2x\sin x+2\cos x+C$$

分部积分：例子总结

$$\int xe^x dx =$$

$$\int x\cos x dx =$$

$$\int x^2 \ln x dx =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int xe^x dx = \int x de^x =$$

$$\int x\cos x dx =$$

$$\int x^2 \ln x dx =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int xe^x dx = \int x de^x = xe^x - \int e^x dx = \cdots$$

$$\int x\cos x dx =$$

$$\int x^2 \ln x dx =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int xe^x dx = \int x de^x = xe^x - \int e^x dx = \cdots$$

$$\int x \cos x dx = \int x d \sin x =$$

$$\int x^2 \ln x dx =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int xe^x dx = \int x de^x = xe^x - \int e^x dx = \cdots$$

$$\int x\cos x dx = \int x d\sin x = x\sin x - \int \sin x dx = \cdots$$

$$\int x^2 \ln x dx =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int xe^x dx = \int x de^x = xe^x - \int e^x dx = \cdots$$

$$\int x\cos x dx = \int x d\sin x = x\sin x - \int \sin x dx = \cdots$$

$$\int x^2 \ln x dx = \int \ln x d(\frac{1}{3}x^3) =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int xe^x dx = \int x de^x = xe^x - \int e^x dx = \cdots$$

$$\int x\cos x dx = \int x d\sin x = x\sin x - \int \sin x dx = \cdots$$

$$\int x^2 \ln x dx = \int \ln x d(\frac{1}{3}x^3) = \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^3 d\ln x = \cdots$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int xe^x dx = \int x de^x = xe^x - \int e^x dx = \cdots$$

$$\int x\cos x dx = \int x d\sin x = x\sin x - \int \sin x dx = \cdots$$

$$\int x^2 \ln x dx = \int \ln x d(\frac{1}{3}x^3) = \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^3 d\ln x = \cdots$$

$$\int \ln x dx = x\ln x - \int x d\ln x = \cdots$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int xe^x dx = \int x de^x = xe^x - \int e^x dx = \cdots$$

$$\int x\cos x dx = \int x d\sin x = x\sin x - \int \sin x dx = \cdots$$

$$\int x^2 \ln x dx = \int \ln x d(\frac{1}{3}x^3) = \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^3 d\ln x = \cdots$$

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