第 9 章 d：隐函数的求导公式

数学系　　梁卓滨

2016-2017 学年 II

Outline

1. 一个方程的情形

2. 方程组的情形

We are here now...

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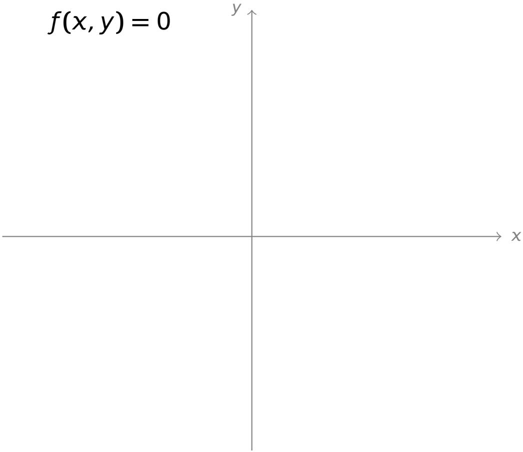
2. 方程组的情形

隐函数定理

$$f(x, y) = 0$$



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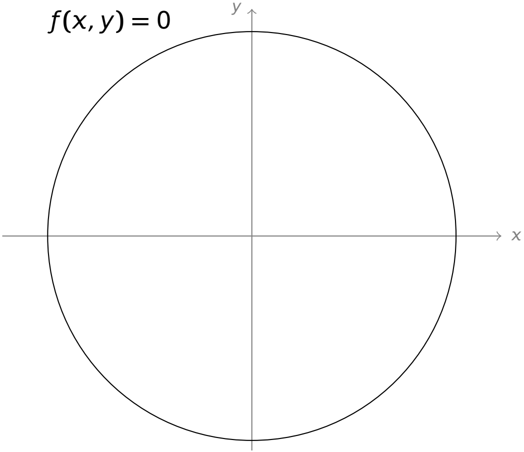

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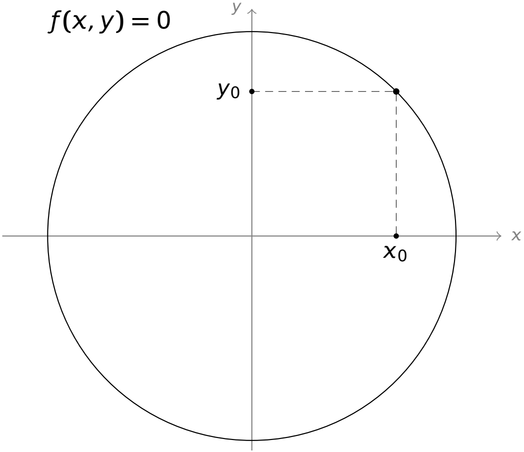

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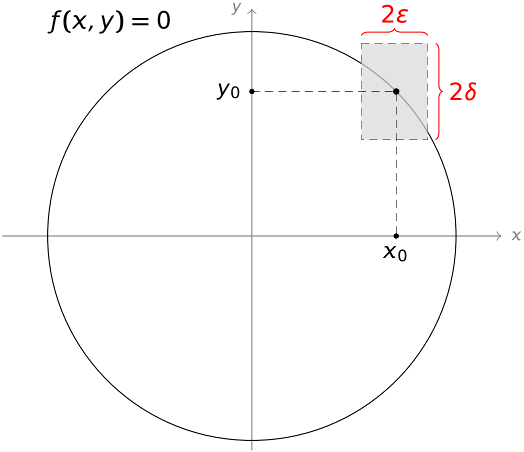

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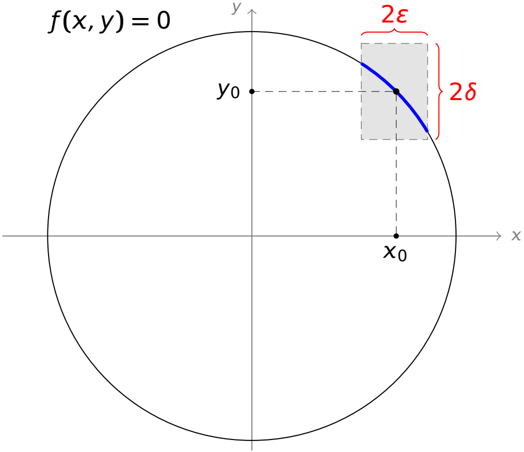

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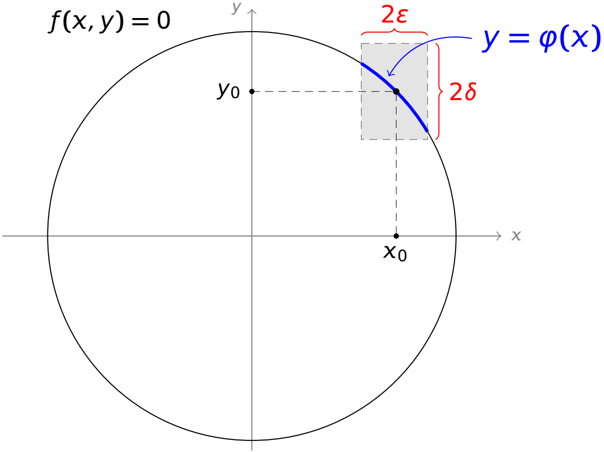

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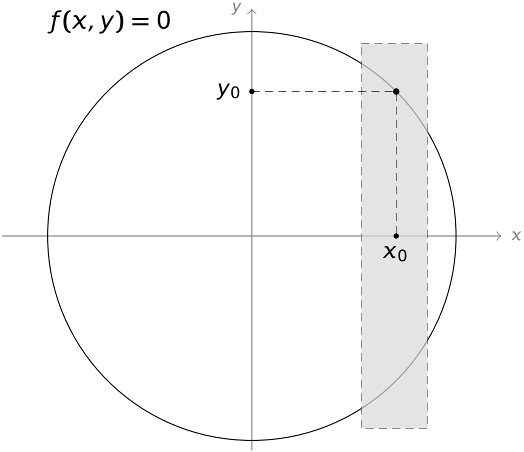

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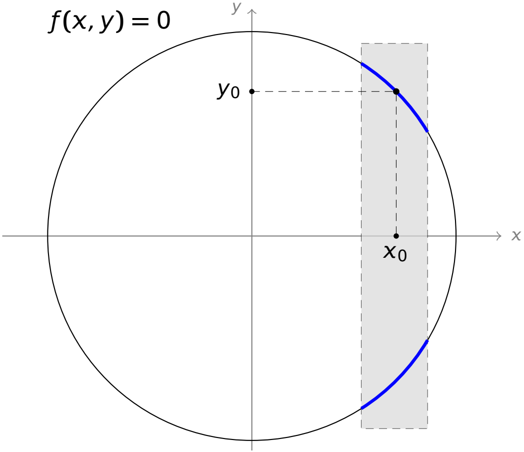

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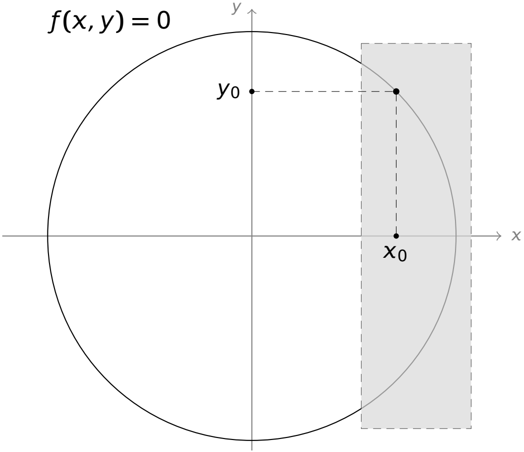

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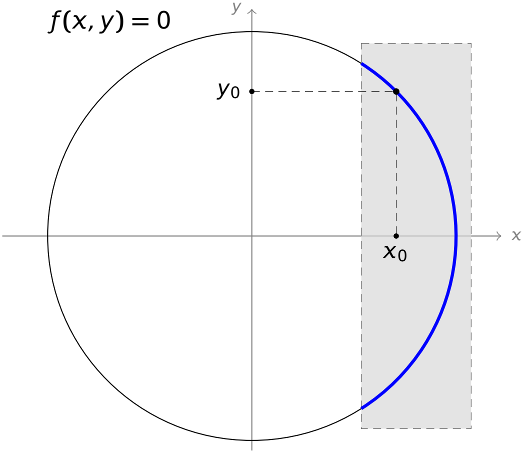

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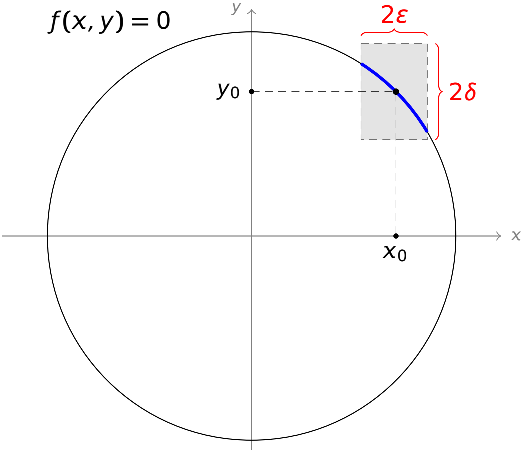

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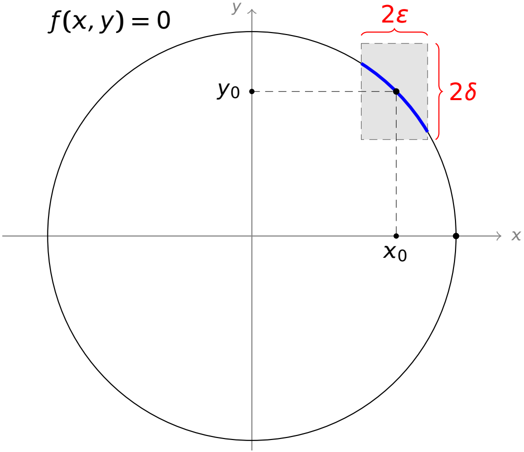

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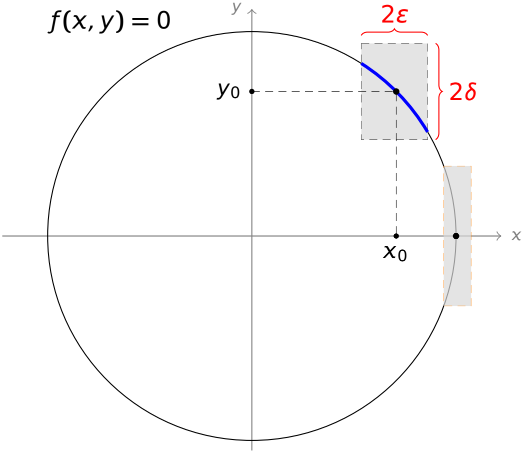

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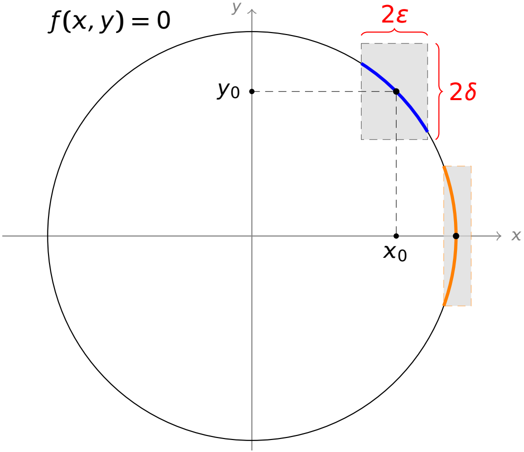

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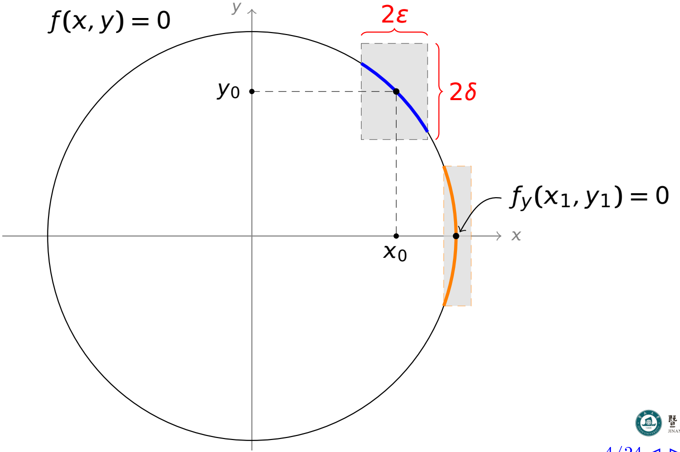

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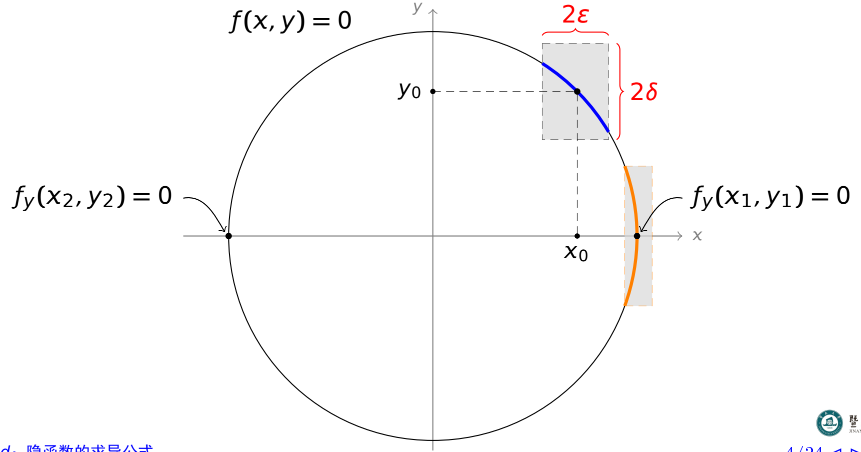

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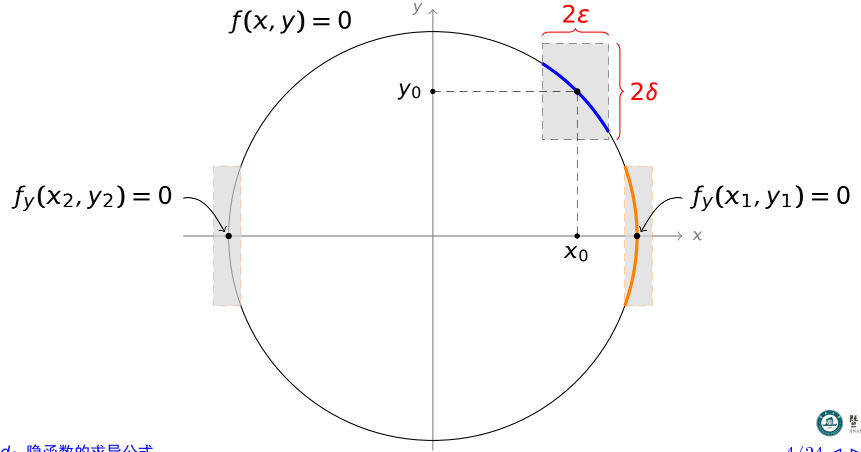

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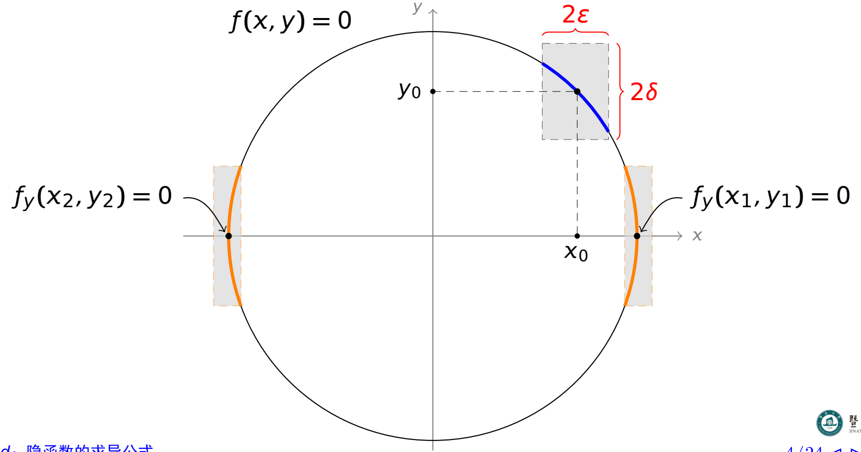

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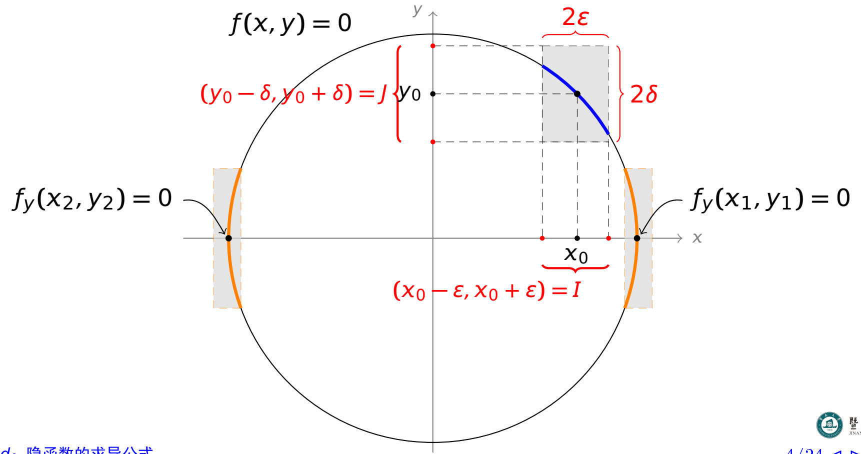

$f(x,y)=0$
y
2ε
$(y_0-\delta, y_0+\delta)=J$
y_0
2δ
$f_y(x_2,y_2)=0$
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x
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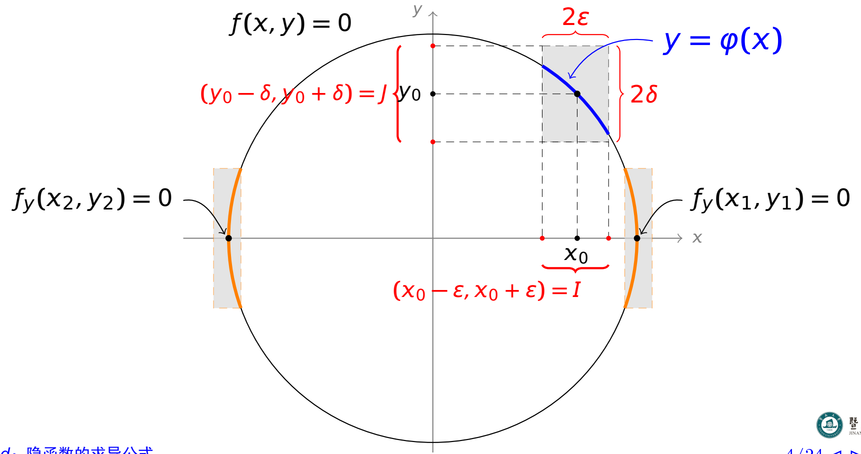

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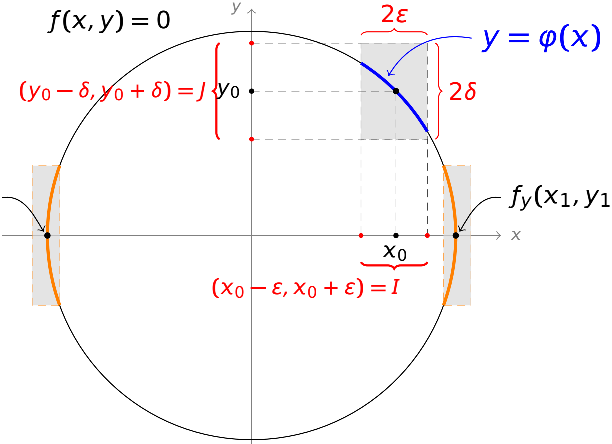
隐函数定理 设 $f(x,y)$ 在点 $p_0(x_0,y_0)$ 附近有定义，具有连续偏导；$f(x_0,\ y_0)=0$；$f_y(x_0,y_0)\neq 0$。则存在开区间 $I=(x_0-\varepsilon,\ x_0+\varepsilon)$ 和 $J=(y_0-\delta,\ y_0+\delta)$，使得

1. 对任意 $x\in I$，方程 $f(x,y)=0$ 有唯一的解 $y=\varphi(x)\in J$。
2. 函数 $y=\varphi(x)$ 在区间 I 上具有连续导数。



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方法二 注意 $\sin y(x)+e^x-xy(x)^2=0$，所以

$$\begin{aligned}0&=\left(\sin y(x)+e^x-xy(x)^2\right)'_x\\&=(\sin y(x))'_x+(e^x)'_x-(xy(x)^2)'_x\end{aligned}$$

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方法一 注意 $\sin y + e^x - xy^2 = 0$，令 $F(x, y) = \sin y + e^x - xy^2$，则 $F(x, y) = 0$，所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)'_x}{(\sin y + e^x - xy^2)'_y} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 $\sin y(x) + e^x - xy(x)^2 = 0$，所以

$$\begin{aligned} 0 &= \left(\sin y(x) + e^x - xy(x)^2\right)'_x \\ &= (\sin y(x))'_x + (e^x)'_x - (xy(x)^2)'_x \\ &= \cos y \cdot y' + e^x - y^2 - 2xy \cdot y' \\ &= e^x - y^2 + (\cos y - 2xy)y' \end{aligned}$$

所以 $y' = -\frac{e^x - y^2}{\cos y - 2xy}$

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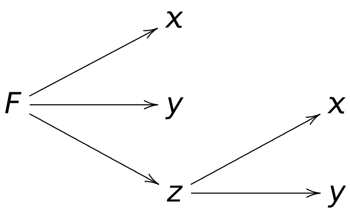

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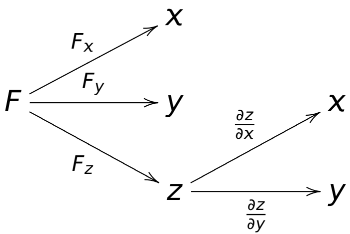

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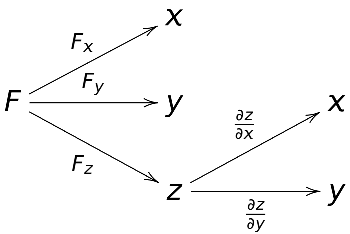

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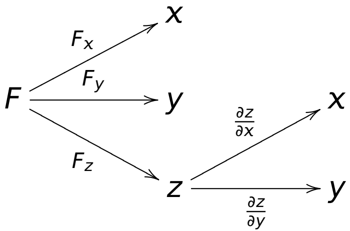

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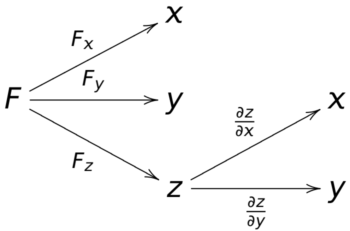

例 设 $z=f(x,y)$ 满足 $x+y+xz=e^z-1$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$=-\frac{1+0+z-0+0}{0+0+x-e^z+0}=-\frac{1+z}{x-e^z}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(x+y+xz-e^z+1)'_y}{(x+y+xz-e^z+1)'_z}$$

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$F(x,y,z)=0$

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$$= -\frac{-1 + e^{z-y-x} - xe^{z-y-x}}{1 + xe^{z-y-x}} = -\frac{1 + (x-1)e^{z-y-x}}{1 + xe^{z-y-x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z-y-x})'_y}{(z - y - x + xe^{z-y-x})'_z} = -\frac{-1 - xe^{z-y-x}}{1 + xe^{z-y-x}} = 1$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy = -\frac{1 + (x-1)e^{z-y-x}}{1 + xe^{z-y-x}}dx + dy$$

例 设 $\Phi(u, v)$ 具有连续偏导数，函数 $z = z(x, y)$ 满足 $\Phi(cx - az, cy - bz) = 0$，证明：

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

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$$F_z=\Phi_u\cdot u_z+\Phi_v\cdot v_z=-a\Phi_u-b\Phi_v$$

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=\frac{c\Phi_u}{a\Phi_u+b\Phi_v}$$

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例 设 $z=f(x,y)$ 满足 $z=x+ye^z$，求 $\frac{\partial^2 z}{\partial x\partial y}$

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We are here now...

1. 一个方程的情形

2. 方程组的情形

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

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用消元法解：

$(1)\times a_{22}-(2)\times a_{12}$，消去 y，得：

$$x = \frac{b_1a_{22}-a_{12}b_2}{a_{11}a_{22}-a_{12}a_{21}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1)\times a_{21} \\ a_{21}\,a_{11}x + a_{22}\,a_{11}y = a_{11}b_2 & (2)\times a_{11} \end{cases}$$

用消元法解：

$(1)\times a_{22} - (2)\times a_{12}$，消去 y，得：

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

$(2)\times a_{11} - (1)\times a_{21}$，消去 x，得：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}\,a_{21}x + a_{12}\,a_{21}y = a_{21}b_1 & (1)\times a_{21} \\ a_{21}\,a_{11}x + a_{22}\,a_{11}y = a_{11}b_2 & (2)\times a_{11} \end{cases}$$

用消元法解：

$(1)\times a_{22} - (2)\times a_{12}$，消去 y，得：

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

$(2)\times a_{11} - (1)\times a_{21}$，消去 x，得：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}\,a_{21}x + a_{12}\,a_{21}y = a_{21}b_1 & (1)\times a_{21} \\ a_{21}\,a_{11}x + a_{22}\,a_{11}y = a_{11}b_2 & (2)\times a_{11} \end{cases}$$

用消元法解：

$(1)\times a_{22} - (2)\times a_{12}$，消去 y，得：

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

$(2)\times a_{11} - (1)\times a_{21}$，消去 x，得：

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\qquad}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\qquad}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\quad}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \quad \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $x =$, $y =$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $x =$, $y =$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{\qquad}{} \qquad , \quad y =$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \qquad\qquad , \ y =$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \quad \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \text{——}$ ， $y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $x =$ ， $y =$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $\quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{\quad}{1}$ $\quad , \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -$

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公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $\quad x = \dfrac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \dfrac{\quad}{1}$ $\quad , \quad y = \dfrac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \dfrac{\quad}{1}$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $\quad x =$ $\quad , \; y =$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1}$, $y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{}{1}$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $x =$, $y =$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $x = \dfrac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1}$, $y = \dfrac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1}$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $x =$, $y =$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $\quad x = \dfrac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \dfrac{-20}{1} = -20, \quad y = \dfrac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \dfrac{8}{1}$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $\quad x =$ $\quad\quad\quad\quad$, $y =$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $\quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $\quad x = \qquad\qquad , \; y =$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \quad \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $\quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$

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公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $\quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$

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公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \quad \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

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2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $\quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{}{3} \quad , \; y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{}{}$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

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2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $\quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{\quad}{3} \quad , \; y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{\quad}{3}$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $\quad x = \dfrac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \dfrac{-20}{1} = -20, \quad y = \dfrac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \dfrac{8}{1} = 8$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $\quad x = \dfrac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \dfrac{21}{3} \qquad , \; y = \dfrac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \dfrac{}{3}$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $\quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $\quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} \quad , \; y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3}$

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方程组的隐函数求导公式

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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问题：如何计算 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ ？

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$$=-\frac{1}{J}\frac{\partial(F,G)}{\partial(x,v)} \qquad =-\frac{1}{J}\frac{\partial(F,G)}{\partial(u,x)}$$

$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}}$$

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总结 设 $u = u(x, y), v = v(x, y)$ 满足方程组

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$u_x =$ $\qquad\qquad$ $v_x =$

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$$\xRightarrow{\frac{\partial}{\partial y}}$$

$u_x =$ $\qquad$ $v_x =$

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所以

$$u_x=-\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \qquad v_x=-\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$u_y= \qquad v_y=$$

总结 设 $u=u(x,y)$, $v=v(x,y)$ 满足方程组

$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \overset{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_x+F_u\cdot u_x+F_v\cdot v_x=0 \\ G_x+G_u\cdot u_x+G_v\cdot v_x=0 \end{cases}$$

$$\overset{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_y+F_u\cdot u_y+F_v\cdot v_y=0 \\ G_y+G_u\cdot u_y+G_v\cdot v_y=0 \end{cases}$$

所以

$$u_x=-\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \qquad v_x=-\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$u_y=-\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \qquad v_y=-\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_x = -\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = -\frac{1}{J}\frac{\partial(F, G)}{\partial(x, v)}, \quad v_x = -\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$u_y = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \qquad v_y = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

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所以

$$u_x=-\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(x,v)}, \quad v_x=-\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(u,x)}$$

$$u_y=-\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \qquad v_y=-\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

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所以

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所以

$$u_x=-\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(x,v)},\quad v_x=-\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(u,x)}$$

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例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \quad \begin{matrix} \xrightarrow{\frac{\partial}{\partial x}} \\ \\ \xrightarrow{\frac{\partial}{\partial y}} \end{matrix}$$

$u_x =$ $\qquad$ $v_x =$

$u_y =$ $\qquad$ $v_y =$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \quad \xRightarrow{\frac{\partial}{\partial x}} \qquad \xRightarrow{\frac{\partial}{\partial y}}$$

$u_x =$ $\qquad$ $v_x =$

$u_y =$ $\qquad$ $v_y =$

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解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1 \\ \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial y}}$$

$u_x =$ $\qquad$ $v_x =$

$u_y =$ $\qquad$ $v_y =$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ \end{cases}$$

$u_x =$ $\qquad$ $v_x =$

$u_y =$ $\qquad$ $v_y =$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

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所以 $J = \begin{vmatrix} e^u + \sin v & u\cos v \\ e^u - \cos v & u\sin v \end{vmatrix}$

$u_x =$ $\qquad$ $v_x =$

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例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

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所以 $J = \begin{vmatrix} e^u + \sin v & u\cos v \\ e^u - \cos v & u\sin v \end{vmatrix}$

$$u_x = -\frac{\begin{vmatrix} & \\ & \end{vmatrix}}{J} \qquad v_x = -\frac{\begin{vmatrix} & \\ & \end{vmatrix}}{J}$$

$$u_y = -\frac{\begin{vmatrix} & \\ & \end{vmatrix}}{J} \qquad v_y = -\frac{\begin{vmatrix} & \\ & \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

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$$u_x = -\frac{\begin{vmatrix} 1 & u\cos v \\ 0 & u\sin v \end{vmatrix}}{J} \qquad v_x = -\frac{\begin{vmatrix} \quad \end{vmatrix}}{J}$$

$$u_y = -\frac{\begin{vmatrix} \quad \end{vmatrix}}{J} \qquad v_y = -\frac{\begin{vmatrix} \quad \end{vmatrix}}{J}$$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u\sin v \cdot v_y = 1 \end{cases}$$

所以 $J = \begin{vmatrix} e^u + \sin v & u\cos v \\ e^u - \cos v & u\sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$

$$u_x = -\frac{\begin{vmatrix} 1 & u\cos v \\ 0 & u\sin v \end{vmatrix}}{J} \qquad v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J}$$

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例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u\sin v \cdot v_x = 0 \end{cases}$$

$$\xrightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u\sin v \cdot v_y = 1 \end{cases}$$

所以 $J = \begin{vmatrix} e^u + \sin v & u\cos v \\ e^u - \cos v & u\sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$

$$u_x = -\frac{\begin{vmatrix} 1 & u\cos v \\ 0 & u\sin v \end{vmatrix}}{J} = \frac{-\sin v}{e^u(\sin v - \cos v) + 1}, \quad v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J}$$

$$u_y = -\frac{\begin{vmatrix} 0 & u\cos v \\ 1 & u\sin v \end{vmatrix}}{J} \qquad v_y = -\frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u\sin v \cdot v_x = 0 \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u\sin v \cdot v_y = 1 \end{cases}$$

所以 $J = \begin{vmatrix} e^u + \sin v & u\cos v \\ e^u - \cos v & u\sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$

$$u_x = -\frac{\begin{vmatrix} 1 & u\cos v \\ 0 & u\sin v \end{vmatrix}}{J} = \frac{-\sin v}{e^u(\sin v - \cos v) + 1}, \quad v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J} = \frac{e^u - \cos v}{ue^u(\sin v - \cos v) + u}$$

$$u_y = -\frac{\begin{vmatrix} 0 & u\cos v \\ 1 & u\sin v \end{vmatrix}}{J} \qquad v_y = -\frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases}$$

$$\xrightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u\sin v \cdot v_x = 0 \end{cases}$$

$$\xrightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u\sin v \cdot v_y = 1 \end{cases}$$

所以 $J = \begin{vmatrix} e^u + \sin v & u\cos v \\ e^u - \cos v & u\sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$

$$u_x = -\frac{\begin{vmatrix} 1 & u\cos v \\ 0 & u\sin v \end{vmatrix}}{J} = \frac{-\sin v}{e^u(\sin v - \cos v) + 1},\quad v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J} = \frac{e^u - \cos v}{ue^u(\sin v - \cos v) + u}$$

$$u_y = -\frac{\begin{vmatrix} 0 & u\cos v \\ 1 & u\sin v \end{vmatrix}}{J} = \frac{\cos v}{e^u(\sin v - \cos v) + 1},\quad v_y = -\frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u\sin v \cdot v_x = 0 \end{cases}$$

$$\xrightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u\sin v \cdot v_y = 1 \end{cases}$$

所以 $J = \begin{vmatrix} e^u + \sin v & u\cos v \\ e^u - \cos v & u\sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$

$$u_x = -\frac{\begin{vmatrix} 1 & u\cos v \\ 0 & u\sin v \end{vmatrix}}{J} = \frac{-\sin v}{e^u(\sin v - \cos v) + 1}, \quad v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J} = \frac{e^u - \cos v}{ue^u(\sin v - \cos v) + u}$$

$$u_y = -\frac{\begin{vmatrix} 0 & u\cos v \\ 1 & u\sin v \end{vmatrix}}{J} = \frac{\cos v}{e^u(\sin v - \cos v) + 1}, \quad v_y = -\frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J} = -\frac{e^u + \sin v}{ue^u(\sin v - \cos v) + u}$$