## 第 9 章 d: 隐函数的求导公式

数学系 梁卓滨

2016-2017 **学年** II



## Outline

1. 一个方程的情形

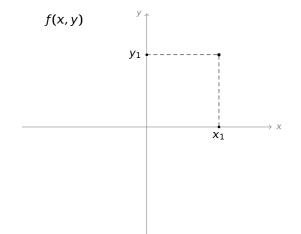
2. 方程组的情形

We are here now...

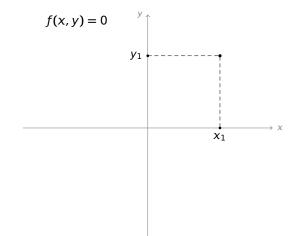
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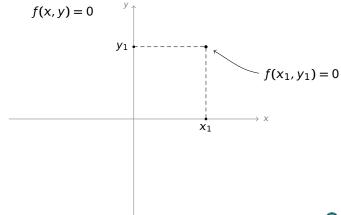
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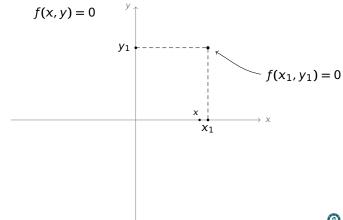
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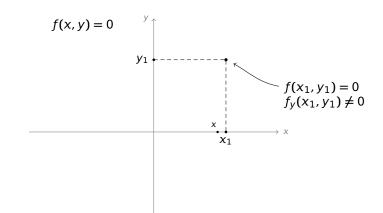
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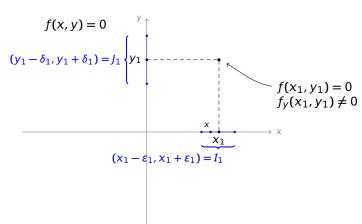


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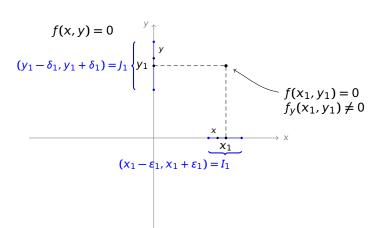


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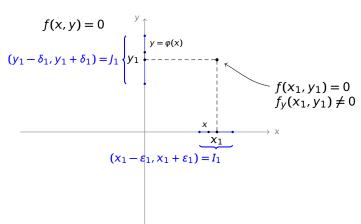




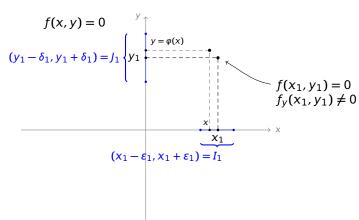
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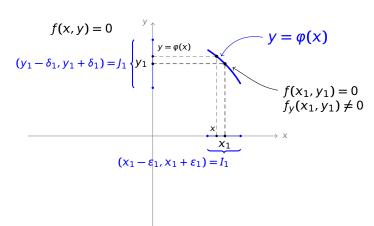
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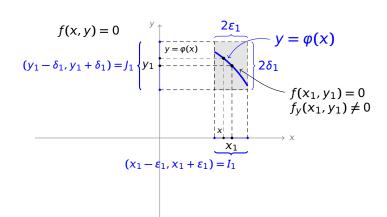
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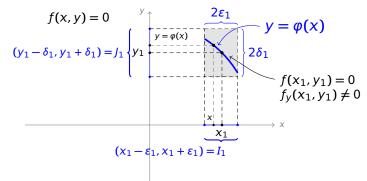
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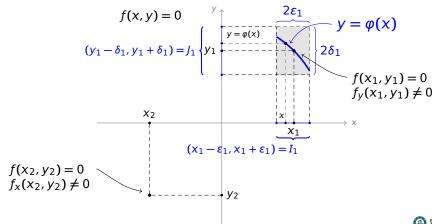
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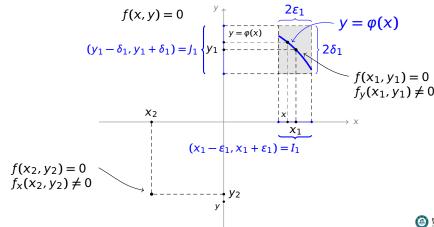
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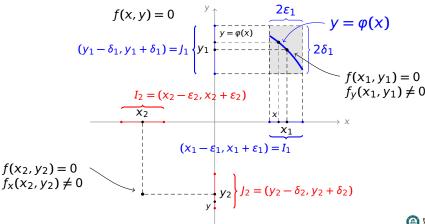
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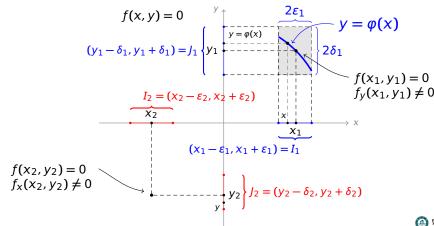


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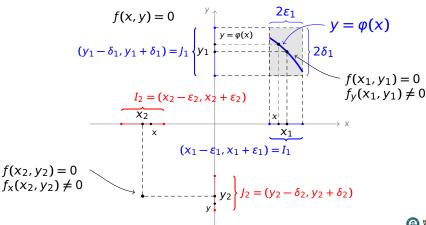


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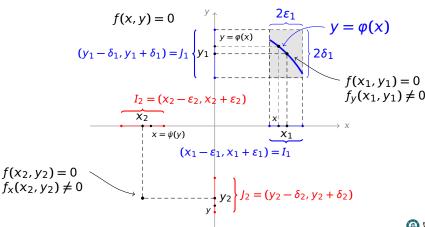
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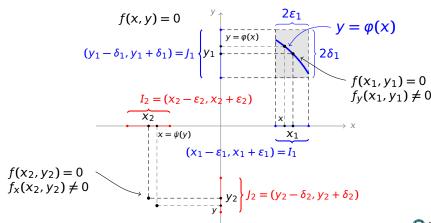
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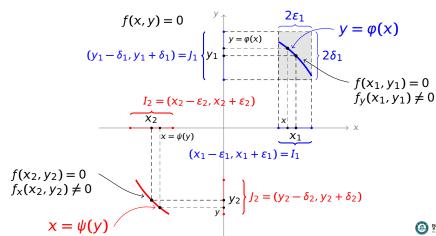


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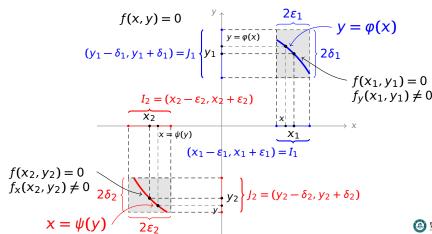
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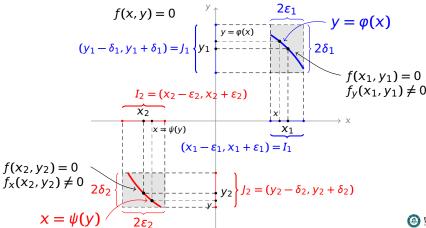
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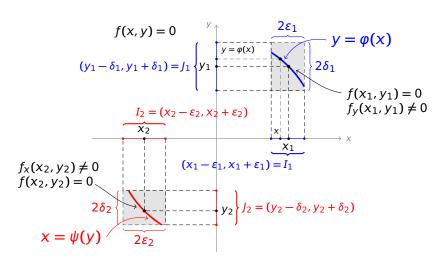
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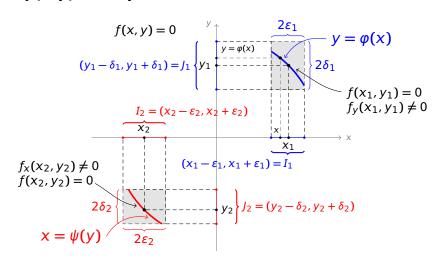


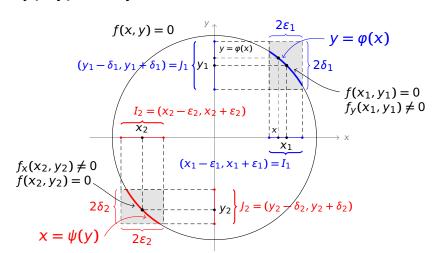
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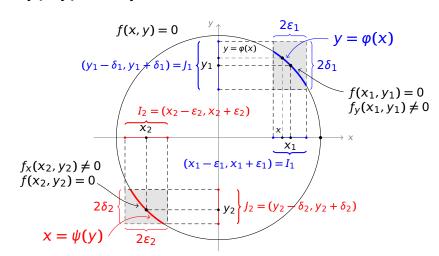
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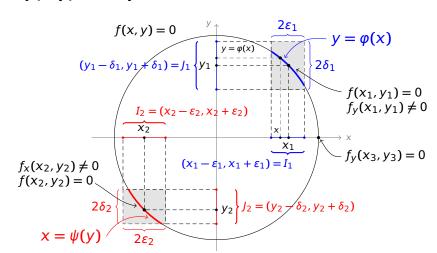


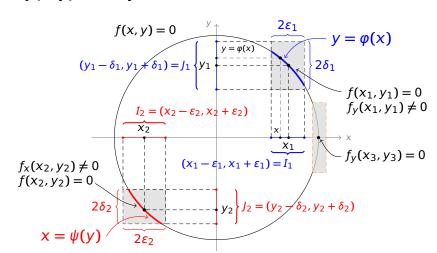


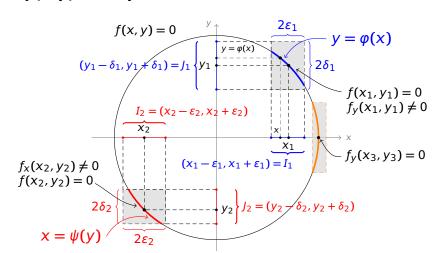


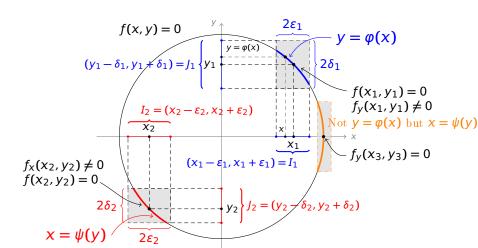


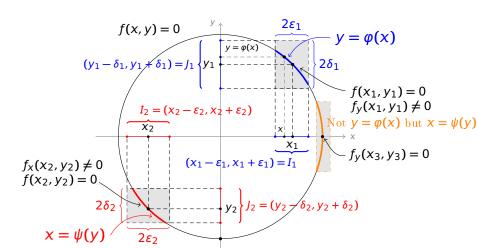


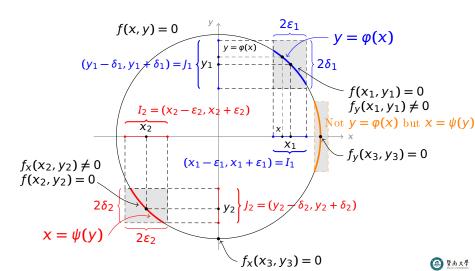


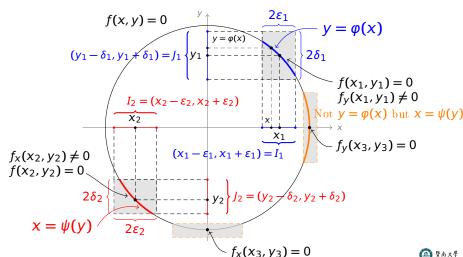






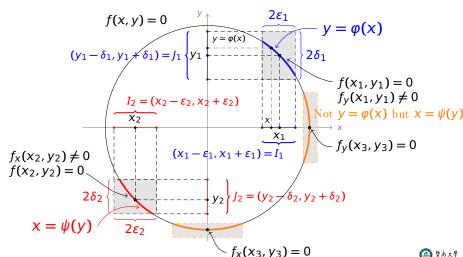






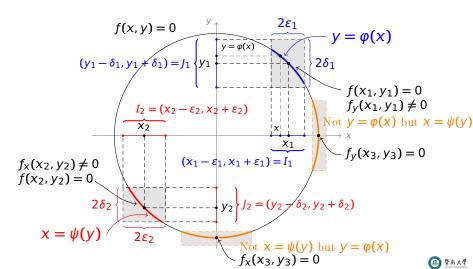
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### 隐函数定理的一个理论应用

性质 设 f(x, y) 具有连续偏导,且  $f_x$ ,  $f_y$  处处不同时为零,若  $E = \{(x, y) | f(x, y) = 0\}$ 

非空,则 E 是平面上"一条曲线"。



隐函数定理 设 f(x, y, z) 在点  $p(x_0, y_0, z_0)$  附近有定义,具有连续偏

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使得:

1. 对任意  $(x, y) \in I_1 \times I_2$ ,方程 f(x, y, z) = 0 有唯一的解  $z = \varphi(x, y)$ 

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- 2. 函数  $z = \varphi(x, y)$  在区域  $I_1 \times I_2$  上具有连续导数。



### 隐函数定理的一个理论应用

性质 设 f(x, y, z) 具有连续偏导,且  $f_x$ ,  $f_y$ ,  $f_z$  处处不同时为零,若  $E = \{(x, y, z) | f(x, y, z) = 0\}$ 

非空,则 E 是空间中"一张曲面"。



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$$:: F(x, y(x)) = 0$$

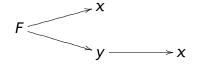
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证明 
$$: F(x, y(x)) = 0$$

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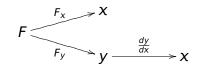


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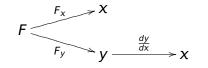


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$$\therefore 0 = \frac{d}{dx} F(x, y(x)) = F_x +$$

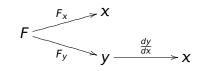


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$$: F(x, y(x)) = 0$$

$$\therefore \quad 0 = \frac{d}{dx} F(x, y(x)) = F_x + F_y \cdot \frac{dy}{dx}$$

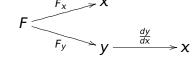


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$$\therefore \quad \frac{dy}{dx} = -\frac{F_x}{F_y}$$





例 设 
$$y = f(x)$$
 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

#### 方法一

$$F(x,\,y)=0$$

$$\frac{dy}{dx} = -\frac{r_x}{F_y}$$

例 设 
$$y = f(x)$$
 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

方法一 注意 
$$\sin y + e^x - xy^2 = 0$$

$$\frac{dy}{dx} = -\frac{F_x}{f}$$

F(x, y) = 0

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$$\sin y + e^x - xy^2 = 0$$
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方法二



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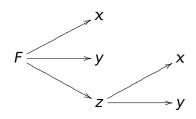
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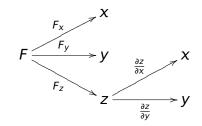


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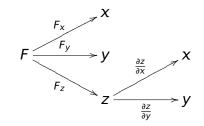


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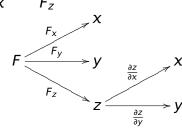
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<u>∂z</u> ∂y

∴ 
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例 设 z = f(x, y) 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

例设 
$$z = f(x, y)$$
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$$F(x,\,y,\,z)=0$$

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解令
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例设 z = f(x, y) 满足  $2\sin(x + 2y - 3z) = x + 2y - 3z$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

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F(x, y, z) = 0

$$\frac{\partial Z}{\partial x} = -\frac{F_X}{F_Z} =$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F} =$$



$$F(x, y, z) = 0$$

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$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

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$$= -\frac{-6\cos(x+2y-3z)}{-6\cos(x+2y-3z)}$$

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解 令 
$$F(x, y, z) = 2 \sin(x + 2y - 3z) - x - 2y + 3z$$
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例 设 z = f(x, y) 满足  $2 \sin(x + 2y - 3z) = x + 2y - 3z$ ,求  $\frac{\partial z}{\partial y}$  和  $\frac{\partial z}{\partial y}$ 

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例 设 
$$z = f(x, y)$$
 满足  $z - y - x + xe^{z-y-x} = 0$ , 求  $dz$ 

解

$$\frac{\partial Z}{\partial X} =$$

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$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

例 设  $\Phi(u, v)$  具有连续偏导数,函数 z = z(x, y) 满足

$$\Phi(cx - az, cy - bz) = 0$$
, 证明: 
$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

解令
$$F(x, y, z) = \Phi(cx - az, cy - bz)$$
,则

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\mathbf{F}(x, y, z) = \Phi(cx - az, cy - bz),$$

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$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\mathbf{F}$$
  $\mathbf{F}(x, y, z) = \Phi(cx - az, cy - bz)$ ,则

 $F_x =$ 

$$F_{y} = F_{z} = \frac{\partial z}{\partial x} = -\frac{F_{x}}{F_{z}} = \frac{\partial z}{\partial z} = F_{y}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

解令
$$F(x, y, z) = \Phi(cx - \alpha z, cy - bz)$$
,则

$$F_X = \Phi_u \cdot u_X + \Phi_V \cdot V_X$$
$$F_V =$$

$$F_y =$$
 $F_z =$ 

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{F_$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

解令
$$F(x, y, z) = \Phi(cx - \alpha z, cy - bz)$$
,则

$$F_X = \Phi_u \cdot u_X + \Phi_v \cdot v_X = c\Phi_u$$

$$F_y = F_z = F_z = F_z$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{F_$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\partial x \partial y \partial y \partial z$$
  
解令  $F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则

$$F_{X} = \Phi_{u} \cdot u_{X} + \Phi_{v} \cdot v_{X} = c\Phi_{u}$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y$$

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$$\frac{\partial Z}{\partial V} =$$



$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\partial x \partial y$$
 解令  $F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则

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$$F_z =$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial x} = -\frac{F_y}{F_z} = -\frac$$

$$\frac{\partial Z}{\partial y} =$$



$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

 $\mathbf{H}$  令  $F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则

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$$\Phi_{u} \cdot u_{y} + \Phi_{v} \cdot v_{y} = c\Phi_{v}$$

$$\Phi_{u} \cdot u_{z} + \Phi_{v} \cdot v_{z}$$

$$F_z = \Phi_u \cdot u_z + \Phi_v \cdot v_z$$

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$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\partial x = \partial y$$
  
解令  $F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则

$$F_X = \Phi_u \cdot u_X + \Phi_V \cdot V_X = c\Phi_u$$

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$$F_{y} = \Phi_{u} \cdot u_{y} + \Phi_{v} \cdot v_{y} = c\Phi_{v}$$

$$F_{z} = \Phi_{u} \cdot u_{z} + \Phi_{v} \cdot v_{z} = -a\Phi_{u} - b\Phi_{v}$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

$$\frac{\partial z}{\partial z} = \frac{F_x}{F_y} = \frac{1}{2}$$

$$\frac{dz}{dy} = -\frac{r_y}{F_z} =$$



$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

 $\mathbf{F} \Rightarrow F(x, y, z) = \Phi(cx - az, cy - bz)$ , 则

$$F_{X} = \Phi_{u} \cdot u_{X} + \Phi_{v} \cdot v_{X} = c\Phi_{u}$$

$$F_{y} = \Phi_{u} \cdot u_{y} + \Phi_{v} \cdot v_{y} = c\Phi_{v}$$

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$$\frac{\partial z}{\partial x} = -\frac{F_{x}}{F_{z}} = \frac{c\Phi_{u}}{a\Phi_{u} + b\Phi_{v}}$$

$$\frac{\partial x}{\partial y} = -\frac{F_y}{F_z} = \frac{\partial z}{\partial z}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\partial x \partial y \partial y \partial z$$
  
解令  $F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则

$$F_{\mathsf{x}} = \Phi_{\mathsf{u}} \cdot \mathsf{u}_{\mathsf{x}} + \Phi_{\mathsf{v}} \cdot \mathsf{v}_{\mathsf{x}} = c\Phi_{\mathsf{u}}$$

$$-\Phi_{\mathcal{U}} \cdot u_{\mathcal{X}} + \Phi_{\mathcal{V}} \cdot v_{\mathcal{X}} = C\Phi_{\mathcal{U}}$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y = c\Phi_v$$

$$F_z = \Phi_u \cdot u_z + \Phi_v \cdot v_z = -\alpha \Phi_u - b \Phi_v$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{c\Phi_u}{\alpha\Phi_u + b\Phi_v}$$

$$\frac{\partial Z}{\partial y} = -\frac{F_y}{F_z} = \frac{c\Phi_v}{a\Phi_u + b\Phi_v}$$

$$\frac{\partial Z}{\partial V} =$$



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$$\mathbf{F} \Leftrightarrow F(x, y, z) = \Phi(cx - az, cy - bz)$$
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$$F_V = \Phi_U \cdot u_V + \Phi_V \cdot V_V = c\Phi_V$$

$$F_{y} = \Phi_{u} \cdot u_{y} + \Phi_{v} \cdot v_{y} = c\Phi_{v}$$

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$$\frac{\partial Z}{\partial x} = -\frac{F_X}{F_Z} = \frac{c\Phi_u}{a\Phi_u + b\Phi_v}$$

$$\frac{\partial Z}{\partial z} = -\frac{F_y}{F_z} = \frac{c\Phi_v}{\sigma}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{c\Phi_v}{a\Phi_u + b\Phi_v}$$
$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = \frac{ac\Phi_u}{a\Phi_u + b\Phi_v} + \frac{bc\Phi_v}{a\Phi_u + b\Phi_v}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

 $\mathbf{F} \Leftrightarrow F(x, y, z) = \Phi(cx - \alpha z, cy - bz), 则$   $F_{\mathbf{Y}} = \Phi_{\mathbf{Y}} \cdot \mathbf{Y}_{\mathbf{Y}} + \Phi_{\mathbf{Y}} \cdot \mathbf{Y}_{\mathbf{Y}} = c\Phi_{\mathbf{Y}}$ 

$$F_X = \Phi_U \cdot u_X + \Phi_V \cdot V_X = c\Phi_U$$

$$F_Y = \Phi_U \cdot u_Y + \Phi_V \cdot V_Y = c\Phi_V$$

$$F_Z = \Phi_U \cdot u_Z + \Phi_V \cdot V_Z = -a\Phi_U - b\Phi_V$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{c\Phi_u}{a\Phi_u + b\Phi_v}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{c\Phi_v}{a\Phi_u + b\Phi_v}$$

例 设 z = f(x, y) 满足  $z = x + ye^z$ , 求  $\frac{\partial^2 z}{\partial x \partial y}$ 

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$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{1}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \frac{1}{ye^z - 1}$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} =$$

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例 设 
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$$= \frac{e^z + y(e^z)_y'}{(ye^z - 1)^2}$$

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$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)_y' = \frac{(ye^z - 1)_y'}{(ye^z - 1)^2}$$

$$= \frac{e^z + y(e^z)_y'}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \frac{\partial z}{\partial y}}{(ye^z - 1)^2}$$

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$$= \frac{e^z + y(e^z)_y'}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \frac{\partial z}{\partial y}}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \left(-\frac{e^z}{ye^z - 1}\right)}{(ye^z - 1)^2}$$



例 设 z = f(x, y) 满足  $z = x + ye^z$ , 求  $\frac{\partial^2 z}{\partial x \partial y}$ 

$$F(x, y, z) = x + ye^z - z, \ 则 F(x, y, z) = 0, \ 所以$$

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$$= \frac{-e^z}{(ye^z - 1)^2}$$

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$$= \frac{e^z + y(e^z)_y'}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \frac{\partial z}{\partial y}}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \left(-\frac{e^z}{ye^z - 1}\right)}{(ye^z - 1)^2}$$

$$= \frac{-e^z}{(ye^z - 1)^3} = \frac{e^z}{(1 + x - z)^3}$$

We are here now...

1. 一个方程的情形

2. 方程组的情形

#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11} a_{22} x + a_{12} a_{22} y = a_{22} b_1 & (1) \times a_{22} \\ a_{21} x + a_{22} y = b_2 & (2) \times a_{12} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11} a_{22} x + a_{12} a_{22} y = a_{22} b_1 & (1) \times a_{22} \\ a_{21} a_{12} x + a_{22} a_{12} y = a_{12} b_2 & (2) \times a_{12} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11} a_{22} x + a_{12} a_{22} y = a_{22} b_1 & (1) \times a_{22} \\ a_{21} a_{12} x + a_{22} a_{12} y = a_{12} b_2 & (2) \times a_{12} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}a_{11}x + a_{22}a_{11}y = a_{11}b_2 & (2) \times a_{11} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11} a_{21} x + a_{12} a_{21} y = a_{21} b_1 & (1) \times a_{21} \\ a_{21} a_{11} x + a_{22} a_{11} y = a_{11} b_2 & (2) \times a_{11} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11} a_{21} x + a_{12} a_{21} y = a_{21} b_1 & (1) \times a_{21} \\ a_{21} a_{11} x + a_{22} a_{11} y = a_{11} b_2 & (2) \times a_{11} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$



#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$



二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解:

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{a_{11} a_{12}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

 $(2) \times a_{11} - (1) \times a_{21}$ , 消去 x, 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{a_{11} a_{12}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{a_{11}a_{12}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$



二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

(1) × 
$$a_{22}$$
 – (2) ×  $a_{12}$ , 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \qquad , \quad y =$$

$$\begin{cases}
7x + 16y = 1 \\
2x + 5y = -1
\end{cases} x =$$

$$y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -- \qquad , \quad y = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = --$$

$$\begin{cases}
7x + 16y = 1 \\
2x + 5y = -1
\end{cases} x =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = --- , \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = --$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{1}{1} \qquad , \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -\frac{1}{1}$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

## 练习 利用二阶行列式求解下面二元线性方程组

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{1}{1}$$
, 
$$y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{1}{1}$$

, y =

$$\begin{cases}
7x + 16y = 1 \\
2x + 5y = -1
\end{cases} x =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1}$$
, 
$$y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{1}{1}$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} \quad , \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1}$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1}$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$
2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - , \quad y = \frac{3}{1} = 8$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$
2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - , \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - \end{cases}$$

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$
2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-3}{3}, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-3}{3}$$





$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$\begin{vmatrix} 2x + 5y = 0 & \begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix} = 0$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$\begin{vmatrix} 1 & 8 \\ 2 & 5 \\ 3 & 8 \end{vmatrix} = \frac{-20}{1} = -20, \quad y = \begin{vmatrix} 3 & 4 \\ 2 & 5 \\ 3 & 8 \end{vmatrix} = 1$$

$$1 \quad 16 \mid |7 \quad 1|$$

2.  $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{1}{3} \quad , y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{1}{3}$ 

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

2.  $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} \quad , y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{3}{3}$ 

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$1.$$
  $\begin{cases} 2 \\ 2 \end{cases}$ 

$$\begin{cases}
2x + 5y = 0 \\
3x + 8y = 4
\end{cases}$$

$$\begin{cases} 2x + 3y = 0 \\ 3x + 8y = 2 \end{cases}$$

$$\int 7x + 16y =$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} , y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

1. 
$$\begin{cases} 2x + 5y = \\ 3x + 8y = \end{cases}$$

2.  $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \ y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3}$ 





$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$
2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3} = -3$$

## 方程组的隐函数求导公式

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

## 方程组的隐函数求导公式

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

## 方程组的隐函数求导公式

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$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题:如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \\ G(x, y, u, v) = 0 & \Longrightarrow \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
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$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \\ G(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \end{cases} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \\ G(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \end{cases} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$



假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \\ G(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \end{cases} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \Rightarrow \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

$$\Rightarrow u_x = \begin{vmatrix} -F_x & F_v \\ -G_x & G_v \end{vmatrix}, \quad v_x = \begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}$$



假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

求解如下:
$$\begin{cases}
F(x, y, u, v) = 0 \\
G(x, y, u, v) = 0
\end{cases} \Rightarrow \begin{cases}
F_u \cdot u_x + F_v \cdot v_x = -F_x \\
G_u \cdot u_x + G_v \cdot v_x = -G_x
\end{cases}$$

$$\Rightarrow u_x = \begin{vmatrix}
-F_x & F_v \\
-G_x & G_v
\end{vmatrix}, v_x = \begin{vmatrix}
-F_u & F_x \\
-G_u & G_x
\end{vmatrix}$$

$$\begin{vmatrix}
F_u & F_v \\
G_u & G_v
\end{vmatrix}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

$$\Rightarrow u_x = -\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_x = -\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$



假设函数 u = u(x, y), v = v(x, y) 满足方程组  $\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$ 

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

求解如下:
$$\begin{cases}
F(x, y, u, v) = 0 \\
G(x, y, u, v) = 0
\end{cases} \Rightarrow \begin{cases}
F_{u} \cdot u_{x} + F_{v} \cdot v_{x} = -F_{x} \\
G_{u} \cdot u_{x} + G_{v} \cdot v_{x} = -G_{x}
\end{cases}$$

$$\Rightarrow u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$= -\frac{1}{4} \frac{\partial(F, G)}{\partial(x, x)}$$

假设函数 u = u(x, y), v = v(x, y) 满足方程组  $\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$ 

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

 $\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$   $\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}$ 

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \\ G(x, y, u, v) = 0 & \Longrightarrow \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$



$$\begin{cases} F(x, y, u, v) = 0 & \stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \\ G(x, y, u, v) = 0 & \stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \end{cases} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y =$$
 ——,  $v_y =$  ——



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \\ G(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \end{cases} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = \frac{ }{ \left| \begin{array}{cc} F_u & F_v \\ G_u & G_v \end{array} \right| }, \quad V_y = \frac{ }{ \left| \begin{array}{cc} F_u & F_v \\ G_u & G_v \end{array} \right| }$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = \frac{\begin{vmatrix} -F_y & F_v \\ -G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = \frac{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = \begin{vmatrix} -F_y & F_v \\ -G_y & G_v \end{vmatrix}, \quad v_y = \begin{vmatrix} -F_u & F_y \\ -G_u & G_y \end{vmatrix}$$

$$\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$= -\frac{1}{J} \frac{\partial (F, G)}{\partial (y, v)}$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$= -\frac{1}{J} \frac{\partial (F, G)}{\partial (y, v)} = -\frac{1}{J} \frac{\partial (F, G)}{\partial (u, y)}$$



$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

总结 设 
$$u = u(x, y), v = v(x, y)$$
 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

$$u_x =$$

$$v_x =$$

$$u_v =$$

$$\nu_{\scriptscriptstyle Y} =$$

总结 设 
$$u = u(x, y), v = v(x, y)$$
 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

$$u_x = v_x = v_x$$

$$u_{V} = v_{V} = v_{V}$$

总结 设 
$$u = u(x, y), v = v(x, y)$$
 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$u_x =$$

$$v_x =$$

$$u_v =$$

$$v_V =$$

总结 设 
$$u = u(x, y), v = v(x, y)$$
 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

$$u_x = v_x = v_x$$

$$u_{V} = v_{V} = v_{V}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$v_{X} = -\frac{\begin{vmatrix} F_{u} & F_{X} \\ G_{u} & G_{X} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{V} \\ G_{u} & G_{V} \end{vmatrix}}$$

$$u_v =$$

$$v_y =$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial X}}{\Longrightarrow} \begin{cases} F_X + F_U \cdot u_X + F_V \cdot V_X = 0 \\ G_X + G_U \cdot u_X + G_V \cdot V_X = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial Y}}{\Longrightarrow} \begin{cases} F_Y + F_U \cdot u_Y + F_V \cdot V_Y = 0 \\ G_Y + G_U \cdot u_Y + G_V \cdot V_Y = 0 \end{cases}$$

所以

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$
$$u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{v} \\ G_{y} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{y} \end{vmatrix}}$$



$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \Rightarrow \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$
$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{v} \\ G_{y} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{y} \end{vmatrix}}$$

$$v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{y} \end{vmatrix}}$$





$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \end{cases} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以
$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(u, x)}$$

$$u_{y} = -\frac{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

 $v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{y} \end{vmatrix}}$ 



$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \Rightarrow \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$
$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)},$$

$$u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{v} \\ G_{y} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)}, \quad v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

9 章 d: 隐函数的求导公司

**鱼** 暨為

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(u, x)}$$

$$u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{v} \\ G_{y} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)}, \quad v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(u, y)}$$

第 9 章 d: 隐函数的求导公:

● 暨南大学

例设  $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}, \ \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$ 

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}$$

$$\nu_{x}$$
=

$$u_y = v_y = v_y$$

 $u_x =$ 

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}$$

$$\nu_{x}$$
=

$$u_y = v_y = v_y$$

 $u_x =$ 

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^u + u \sin v = x \end{cases}$$

$$\iint e^u + u \sin v = x$$

$$\begin{cases} e^u + u \sin v = x \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\stackrel{\frac{\partial}{\partial y}}{\Longrightarrow}$$

 $\stackrel{\frac{\sigma}{\partial x}}{\Longrightarrow} \left\{ (e^u + \sin v) u_x + u \cos v \cdot v_x = 1 \right.$ 

$$\nu_{x}$$
=

 $u_v =$ 

 $u_x =$ 

$$v_y =$$

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\stackrel{\frac{\sigma}{\partial x}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{x} + u\cos v \cdot v_{x} = 1\\ (e^{u} - \cos v)u_{x} + u\sin v \cdot v_{x} = 0 \end{cases}$$

$$u_x =$$

 $\nu_{\chi} =$ 

$$u_v =$$

 $\nu_y =$ 

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$= x$$

$$= y$$

$$\begin{cases} (e^{u} + \sin v)u_{x} + u\cos v \cdot v_{x} = 1\\ (e^{u} - \cos v)u_{x} + u\sin v \cdot v_{x} = 0\\ &\xrightarrow{\frac{\partial}{\partial y}} \end{cases}$$

$$\begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0 \end{cases}$$

$$u_x =$$

$$\nu_{\chi} =$$

$$u_v =$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{x} + u\cos v \cdot v_{x} = 1\\ (e^{u} - \cos v)u_{x} + u\sin v \cdot v_{x} = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0\\ (e^{u} - \cos v)u_{y} + u\sin v \cdot v_{y} = 1 \end{cases}$$

$$u_x =$$

$$\nu_{\chi} =$$

$$u_v =$$

 $\nu_{v} =$ 

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{=} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$u_x = v_x = v_x$$

$$u_y = v_y = v_y$$

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\cos V$$
,  $\frac{\partial}{\partial x}$ ,  $\frac{\partial}{\partial y}$ ,  $\frac{\partial}{\partial x}$ ,

$$\Longrightarrow$$

$$\stackrel{\frac{\sigma}{\partial x}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{x} + u\cos v \cdot v_{x} = 1\\ (e^{u} - \cos v)u_{x} + u\sin v \cdot v_{x} = 0 \end{cases}$$

$$\stackrel{\frac{\sigma}{\partial y}}{\Longrightarrow} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u\sin v \cdot v_y = 1 \end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$\sin v$$

$$u\sin v$$

$$v_x = -\frac{\left| \frac{1}{J} \right|}{J}$$

$$u_{x} = -\frac{1}{J}$$

$$u_{y} = -\frac{1}{J}$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases} \begin{cases} (e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\ (e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0 \end{cases}$$
$$\xrightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1 \end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

所以 
$$J = \begin{vmatrix} e^{u} + \sin v & u \cos v \\ e^{u} - \cos v & u \sin v \end{vmatrix}$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_{y} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{=} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

所以 
$$J = \begin{vmatrix} e^u - \cos v & u \sin v \end{vmatrix}$$

$$u_x = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J}$$

$$u_y = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J}$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{=} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_{x} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{J}$$

$$u_{y} = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}$$

$$v_{y} = -\frac{\begin{vmatrix} 1 & u \cos v & 1 \\ 0 & u \cos v & 0 \end{vmatrix}}{J}$$



例设 
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$$
, 求  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ 

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases} \begin{cases} (e^{u} + \sin v)u_{x} + u \sin v \cdot v_{x} = 0 \\ \Rightarrow \end{cases} \begin{cases} (e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1 \end{cases}$$

$$\text{Eff}[U] = \begin{vmatrix} e^{u} + \sin v & u \cos v \end{vmatrix}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix} = \begin{vmatrix} e^u + \sin v \\ 0 & u \sin v \end{vmatrix}$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_{x} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{J}$$

$$u_{y} = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}$$

$$v_{y} = -\frac{\begin{vmatrix} e^{u} + \sin v & 0 \\ e^{u} - \cos v & 1 \end{vmatrix}}{J}$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases} \begin{cases} (e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\ (e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0 \end{cases}$$

 $\stackrel{\frac{\sigma}{\partial y}}{\Longrightarrow} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u\sin v \cdot v_y = 1 \end{cases}$ 

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$u_x = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J}$$

$$u_{x} = -\frac{\begin{vmatrix} 0 & u \sin v \end{vmatrix}}{J}$$

$$u_{y} = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}$$

$$v_y = -\frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{I}$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
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\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{=} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$u_x = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{\int} = \frac{-\sin v}{e^u(\sin v - \cos v) + 1}, v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{\int}$$

$$u_{x} = -\frac{\frac{|o \ u \sin v|}{J}}{\frac{|o \ u \cos v|}{I}} = \frac{\frac{-\sin v}{e^{u}(\sin v - \cos v) + 1}}{\frac{|o \ u \cos v|}{J}}, v_{x} = -\frac{\frac{|o \ u \cos v|}{J}}{\frac{|o \ u \cos v|}{J}}$$

$$v_{y} = -\frac{\frac{|o \ u \cos v|}{I}}{\frac{|o \ u \cos v|}{J}}$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$= x$$

$$= x$$

$$= y$$

$$\begin{cases} (e^{u} + \sin v)u_{x} + u\cos v \cdot v_{x} = 1 \\ (e^{u} - \cos v)u_{x} + u\sin v \cdot v_{x} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u\sin v \cdot v_{y} = 1 \end{cases}$$

所以  $J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$  $u_{x} = -\frac{\begin{vmatrix} 1 & u\cos v \\ 0 & u\sin v \end{vmatrix}}{I} = \frac{-\sin v}{e^{u}(\sin v - \cos v) + 1}, v_{x} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{I} = \frac{e^{u} - \cos v}{ue^{u}(\sin v - \cos v)}$  $v_y = -\frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{t}$ 

## $u_y = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{\cdot}$ d: 隐函数的求导公式



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{=} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{\int} = \frac{-\sin v}{e^{u(\sin v - \cos v) + 1}}, v_{x} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{\int} = \frac{e^{u - \cos v}}{ue^{u(\sin v - \cos v)}}$$
$$u_{y} = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{\int} = \frac{\cos v}{e^{u(\sin v - \cos v) + 1}}, v_{y} = -\frac{\begin{vmatrix} e^{u} + \sin v & 0 \\ e^{u} - \cos v & 1 \end{vmatrix}}{\int}$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \ \dot{x} \ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{=} \begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{y} = 0 \\
e^{u} - \cos v)u_{y} + u \cos v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^{u} + \sin v & u \cos v \\ e^{u} - \cos v & u \sin v \end{vmatrix} = ue^{u}(\sin v - \cos v) + u$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J} = \frac{-\sin v}{e^{u(\sin v - \cos v) + 1}}, v_{x} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{J} = \frac{e^{u} - \cos v}{ue^{u(\sin v - \cos v) + u}}$$

$$u_{y} = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J} = \frac{\cos v}{e^{u(\sin v - \cos v) + 1}}, v_{y} = -\frac{\begin{vmatrix} e^{u} + \sin v & 0 \\ e^{u} - \cos v & 1 \end{vmatrix}}{J} = -\frac{e^{u} + \sin v}{ue^{u(\sin v - \cos v) + u}}$$

 $ue^{u}(\sin v - \cos v) + u$