§4.2 相似矩阵与矩阵对角化

数学系 梁卓滨

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定义 设 A, B 是 n 阶方阵。若存在 n 阶可逆矩阵 P, 满足 $P^{-1}AP = B.$

则称 $A 与 B相似, 记为 <math>A \sim B$ 。



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这是:

 $A \sim B \Rightarrow$



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$$A \sim B \Rightarrow P^{-1}AP = B \Rightarrow PBP^{-1} = A$$



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$$A \sim B \ \Rightarrow \ P^{-1}AP = B \ \Rightarrow PBP^{-1} = A \ \xrightarrow{Q:=P^{-1}}$$



定义 设 A, B 是 n 阶方阵。若存在 n 阶可逆矩阵 P, 满足

 $P^{-1}AP = B$.

则称
$$A = B$$
相似,记为 $A \sim B$ 。

$$A \sim B \Rightarrow P^{-1}AP = B \Rightarrow PBP^{-1} = A \xrightarrow{Q:=P^{-1}} Q^{-1}BQ = A$$



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$$A \sim B \Rightarrow P^{-1}AP = B \Rightarrow PBP^{-1} = A \xrightarrow{Q:=P^{-1}} Q^{-1}BQ = A \Rightarrow B \sim A$$



$$\underbrace{\begin{pmatrix} 19 & 45 \\ -7 & -17 \end{pmatrix}}_{A} \qquad \underbrace{\begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix}}_{B}$$

$$\left(\begin{array}{cc}1&2\\2&5\end{array}\right)^{-1}\underbrace{\left(\begin{array}{cc}19&45\\-7&-17\end{array}\right)}_{A}\left(\begin{array}{cc}1&2\\2&5\end{array}\right)\quad\underbrace{\left(\begin{array}{cc}3&1\\5&-1\end{array}\right)}_{B}$$



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所以 $A \sim B$



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- 1. "λ矩阵"的方法,但并不简单的。。。
- 2. 下面只给出两个矩阵相似的必要条件

定理设 $A \sim B$,则

- 1. A 与 B 有相同特征值;
- 2. r(A) = r(B);
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证明 存在可逆矩阵 P,满足

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1. $|\lambda I - B| =$

$$|\lambda I - A|$$

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$$|\lambda I - B| = |\lambda P^{-1}IP - P^{-1}AP| =$$

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$$A_{n\times n}$$
 与对角阵 $\Lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{pmatrix}$ 相似,则称 A 可对角化

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定理 A 可对角化 ⇔ A 有 n 个线性无关的特征向量



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定理 A 可对角化 ⇔ A 有 n 个线性无关的特征向量

推论 若方阵 $A_{n\times n}$ 有 n 不同特征值,则 A 可对角化。



问题 判断 n 阶方阵 A 是否可以对角化? 若能,确定可逆矩阵 P 及对角阵

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步骤

1. 求出 A 的所有特征值,及相应特征向量

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- 2. 若有 n 个线性无关特征向量,则 A 可对角化,否则,不能对角化

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- 3. 假设存在 n 个线性无关特征向量 $\alpha_1, \alpha_2, \ldots, \alpha_n$,记对应特征值为 $\lambda_1, \lambda_2, \ldots, \lambda_n$

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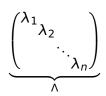
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$$(\alpha_1, \alpha_2, \ldots, \alpha_n)$$



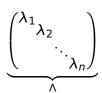


问题 判断 n 阶方阵 A 是否可以对角化?若能,确定可逆矩阵 P 及对角阵 Λ . 使得 $P^{-1}AP = \Lambda$ 。

- 1. 求出 A 的所有特征值,及相应特征向量
- 2. 若有n个线性无关特征向量,则A可对角化;否则,不能对角化
- 3. 假设存在 n 个线性无关特征向量 $\alpha_1, \alpha_2, \ldots, \alpha_n$,记对应特征值为 $\lambda_1, \lambda_2, \ldots, \lambda_n$

$$\underbrace{(\alpha_1,\,\alpha_2,\,\ldots,\,\alpha_n)}_{P}$$

$$\Rightarrow P^{-1}AP = \Lambda$$

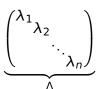


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$$\Rightarrow P^{-1}AP = \Lambda$$



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$$\Rightarrow AP = P\Lambda \Rightarrow P^{-1}AP = \Lambda$$



例 $1A = \begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix}$

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- 特征值 λ₂ = −2,

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$$P = ($$
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$$P = (\alpha_1, \alpha_2)$$
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例 2 判断 $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$ 是否能对角化? 若能,写出 P 和 Λ

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$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
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特征方程: 0 = |λI − A|

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- 特征值 λ₁ = 2 (二重)
- 特征值 λ₂ = 6

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- 特征方程: $0 = |\lambda I A| = (\lambda 2)^2 (\lambda 6)$
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- 特征方程: $0 = |\lambda I A| = (\lambda 2)^2 (\lambda 6)$
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可见 A 有 3 个线性无关特征向量: α_1 , α_2 , α_3 。 所以 A 可以对角化。

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今

$$P = ($$
) , $\Lambda = ($

则 $P^{-1}AP = \Lambda$

• 特征方程:
$$0 = |\lambda I - A| = (\lambda - 2)^2 (\lambda - 6)$$

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$$P = (\alpha_1, \alpha_2, \alpha_3)$$
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- 特征方程: $0 = |\lambda I A| = (\lambda 2)^2 (\lambda 6)$
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 $P = (\alpha_1, \alpha_2, \alpha_3) = \begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & -2 \\ 0 & 1 & 3 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}$

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 $P^{-1}AP = \Lambda$

则

- 特征方程: $0 = |\lambda I A| = (\lambda 1)(\lambda 2)^2$ Details
- 特征值 $\lambda_1 = 1$,特征向量 $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ Details
- 特征值 $\lambda_2 = 2$ (二重),特征向量 $\alpha_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ **Det**

例 3 判断
$$A = \begin{pmatrix} 1 & 1 & -1 \\ -2 & 4 & -2 \\ -2 & 2 & 0 \end{pmatrix}$$
 是否能对角化? 若能, 写出 P 和 Λ

- 特征方程: $0 = |\lambda I A| = (\lambda 1)(\lambda 2)^2$ Potails
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可见 A 有 3 个线性无关特征向量: α_1 , α_2 , α_3 。所以 A 可以对角化。

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• 特征方程:
$$0 = |\lambda I - A| = (\lambda - 1)(\lambda - 2)^2$$
 Details

• 特征值
$$\lambda_1 = 1$$
,特征向量 $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ Details

• 特征值
$$\lambda_2 = 2$$
(二重),特征向量 $\alpha_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ Det

可见 A 有 B 个线性无关特征向量: α_1 , α_2 , α_3 。 所以 A 可以对角化。

$$P=($$
 , $\Lambda=\left($

 $P^{-1}AP = \Lambda$.



则

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$$P = (\alpha_1, \alpha_2, \alpha_3)$$
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 $P = \Lambda$.

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$$P = (\alpha_1, \alpha_2, \alpha_3) = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}$$

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例 3 判断
$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$
 是否能对角化? 若能, 写出 P 和 Λ

- 特征方程: $0 = |\lambda I A| = (\lambda + 1)^2 (\lambda 5)$ Details
- 特征值 $\lambda_1=-1$ (二重),特征向量 $\alpha_1=\begin{pmatrix} -1\\1\\0\end{pmatrix}$, $\alpha_2=\begin{pmatrix} -1\\0\\1\end{pmatrix}$ Det
- 特征值 $\lambda_2 = 5$,特征向量 $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ Det



例 3 判断
$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$
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可见 A 有 3 个线性无关特征向量: α_1 , α_2 , α_3 。所以 A 可以对角化。

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则

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则

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可见 A 有 B 个线性无关特征向量: α_1 , α_2 , α_3 。所以 A 可以对角化。

今 $P = (\alpha_1, \alpha_2, \alpha_3) = \begin{pmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 5 \end{pmatrix},$

则
$$P^{-1}AP = \Lambda$$



定理 n 阶方阵 A 可对角化的充分必要条件是:每个 n_i 重的特征值 λ_i ,矩

阵 $\lambda_i I - A$ 的秩是 $n - n_i$ 。

图解如下:

特征值

 λ_1

里 数

 n_1

(λ_iI – A)x = 0 基础解系 /线性无关特征向量

定理 n 阶方阵 A 可对角化的充分必要条件是:每个 n_i 重的特征值 λ_i ,矩

 λ_2 n_2

 \vdots \vdots n_s

n _____

图解如下:

SIMPAN N:			
不同 特征值	重 数		$(\lambda_i I - A)x = 0$ 基础解系 /线性无关特征向量
λ_1	n_1	$r(\lambda_1 I - A) = n - n_1$	

定理 n 阶方阵 A 可对角化的充分必要条件是:每个 n_i 重的特征值 λ_i ,矩

图解如下:

 λ_1

不同 重 特征值 数

 $(\lambda_i I - A)x = 0$ 基础解系 /线性无关特征向量 $r(\lambda_1 I - A) = n - n_1 \implies \alpha_1^{(1)}, \alpha_2^{(1)}, \dots, \alpha_n^{(1)}$

定理 n 阶方阵 A 可对角化的充分必要条件是:每个 n_i 重的特征值 λ_i 矩

$$\lambda_2$$
 n_2
 \vdots \vdots
 λ_s n_s

共n

 n_1

图解如下: 不同

特征值

 λ_1

 λ_s

 $(\lambda_i I - A)x = 0$ 基础解系 /线性无关特征向量

 $n_1 r(\lambda_1 I - A) = n - n_1 \Rightarrow \alpha_1^{(1)}, \alpha_2^{(1)}, \dots, \alpha_n^{(1)}$

定理 n 阶方阵 A 可对角化的充分必要条件是:每个 n_i 重的特征值 λ_i 矩

$$\lambda_2$$
 n_2 $r(\lambda_2 I - A) = n - n_2$
 \vdots \vdots

 $|\lambda I - A| = (\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \cdots (\lambda - \lambda_s)^{n_s}$

相似矩阵与矩阵对角化 11/14 < ▷ △ ▽

定理 n 阶方阵 A 可对角化的充分必要条件是:每个 n_i 重的特征值 λ_i 矩

图解如下:

不同

特征值

 n_{ς}

共 n

 $(\lambda_i I - A)x = 0$ 基础解系 /线性无关特征向量

 λ_1 λ_2

 λ_s

$$n_1 \quad r(\lambda_1 I - A) = n - n_1 \quad \Rightarrow \quad \alpha_1^{(1)}, \, \alpha_2^{(1)}, \, \cdots, \, \alpha_{n_1}^{(1)}$$

$$n_1 \quad r(\lambda_1 I - A) = n_1 \quad \Rightarrow \quad \alpha_1^{(2)}, \, \alpha_2^{(2)}, \, \cdots, \, \alpha_{n_1}^{(2)}$$

$$a_2I-A)=n-$$

$$n_2 \quad r(\lambda_2 I - A) = n - n_2 \quad \Rightarrow \quad \alpha_1^{(2)}, \, \alpha_2^{(2)}, \, \cdots, \, \alpha_{n_2}^{(2)}$$

$$\alpha_{n_2}^{(2)}, \cdots, \alpha_{n_2}^{(2)}$$

定理 n 阶方阵 A 可对角化的充分必要条件是:每个 n_i 重的特征值 λ_i 矩 阵 $\lambda_i I - A$ 的秩是 $n - n_i$ 。

图解如下:

不同

 $(\lambda_i I - A)x = 0$ 基础解系 /线性无关特征向量

$$\lambda_1$$
 n_1 λ_2 n_2

$$r(\lambda_1 I - A) = n - \alpha$$

 n_s $r(\lambda_s I - A) = n - n_s$

 $n_1 r(\lambda_1 I - A) = n - n_1 \Rightarrow \alpha_1^{(1)}, \alpha_2^{(1)}, \dots, \alpha_n^{(1)}$

$$n_2 \quad r(\lambda_2 I - A) = n - n_2 \quad \Rightarrow \quad \alpha_1^{(2)}, \, \alpha_2^{(2)}, \, \cdots, \, \alpha_{n_2}^{(2)}$$

 $|\lambda I - A| = (\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \cdots (\lambda - \lambda_s)^{n_s}$

 λ_s

定理 n 阶方阵 A 可对角化的充分必要条件是:每个 n_i 重的特征值 λ_i 矩 阵 $\lambda_i I - A$ 的秩是 $n - n_i$ 。

图解如下:

不同

 $(\lambda_i I - A)x = 0$ 基础解系 /线性无关特征向量 特征值 数

$$\lambda_1$$
 n_1 $r(\lambda_1 I - A) = n - n_1$ \Rightarrow $\alpha_1^{(1)}, \alpha_2^{(1)}, \dots, \alpha_{n_1}^{(1)}$
 λ_2 n_2 $r(\lambda_2 I - A) = n - n_2$ \Rightarrow $\alpha_1^{(2)}, \alpha_2^{(2)}, \dots, \alpha_{n_2}^{(2)}$

$$\lambda_s$$
 n_s $r(\lambda_s I - A) = n - n_s$ \Rightarrow $\alpha_1^{(s)}, \alpha_2^{(s)}, \cdots, \alpha_{n_s}^{(s)}$

 $|\lambda I - A| = (\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \cdots (\lambda - \lambda_s)^{n_s}$

共 n

定理 n 阶方阵 A 可对角化的充分必要条件是:每个 n_i 重的特征值 λ_i 矩

图解如下: 不同

特征值

 λ_1

 λ_2

 $(\lambda_i I - A)x = 0$ 基础解系 /线性无关特征向量

$$n_1 \quad r(\lambda_1 I - A) = n - n_1 \quad \Rightarrow \quad \alpha_1^{(1)}, \ \alpha_2^{(1)}, \cdots, \ \alpha_{n_1}^{(1)}$$
 $n_2 \quad r(\lambda_2 I - A) = n - n_2 \quad \Rightarrow \quad \alpha_2^{(2)}, \ \alpha_2^{(2)}, \cdots, \ \alpha_{n_2}^{(2)}$

$$\vdots \qquad \vdots \qquad \vdots \\ \lambda_s \qquad n_s \qquad r(\lambda_s I - A)$$

$$n_2 \quad r(\lambda_2 I - A) = n - n_2 \quad \Rightarrow \quad \alpha_1^{(2)}, \, \alpha_2^{(2)}, \, \cdots, \, \alpha_{n_2}^{(2)}$$

$$: \qquad : \qquad : \qquad : \qquad :$$

$$n_s \qquad r(\lambda_s I - A) = n - n_s \quad \Rightarrow \quad \alpha_1^{(s)}, \ \alpha_2^{(s)}, \ \cdots, \ \alpha_{n_s}^{(s)}$$

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

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提示 若 A 与 Λ 相似 ⇔

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与 Λ 相似 ⇔ A 可对角化

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 $A \subseteq \Lambda$ 相似 \iff $A \cap \forall \exists A \cap \lambda_1 = 1$ $\lambda_2 = 2$



例 下列哪个矩阵与 $\Lambda = \begin{pmatrix} 100\\ 010\\ 002 \end{pmatrix}$ 相似?

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 $A \subseteq \Lambda$ 相似 $\iff A$ 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$

例 下列哪个矩阵与 $\Lambda = \begin{pmatrix} 100\\ 010\\ 002 \end{pmatrix}$ 相似?

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 $A \subseteq \Lambda$ 相似 $\Leftrightarrow A \cap \Lambda$ 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重)

例 下列哪个矩阵与 $\Lambda = \begin{pmatrix} 100\\ 010\\ 002 \end{pmatrix}$ 相似?

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若
$$A$$
 与 Λ 相似 \Leftrightarrow A 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重)
$$\Leftrightarrow r(I-A) = \qquad \qquad \exists \ r(2I-A) =$$

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与 Λ 相似 \Leftrightarrow A 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重) $\Leftrightarrow r(I-A) = 3-2 = 1 \perp r(2I-A) =$

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 $A = \Lambda$ 相似 \iff $A = \Lambda$ 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重) $\iff r(I - A) = 3 - 2 = 1 \text{ L} \ r(2I - A) = 3 - 1 = 2$

I - Ar(I-A)

例 下列哪个矩阵与 $\Lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ 相似?

$$A_{1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_{2} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_{3} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_{4} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若
$$A = \Lambda$$
 相似 \iff $A = \Lambda$ 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重)

解
$$\Rightarrow r(I-A) = 3-2 = 1$$
且 $r(2I-A) = 3-1 = 2$

$$A_1 \qquad A_2 \qquad A_3 \qquad A_4$$

2I - A

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若
$$A$$
 与 Λ 相似 \Leftrightarrow A 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重) \Leftrightarrow $r(I-A) = 3-2 = 1$ 且 $r(2I-A) = 3-1 = 2$

$$r(2I-A)$$

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与 Λ 相似 ⇔ A 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重) $\Leftrightarrow r(I-A) = 3-2 = 1 \perp r(2I-A) = 3-1 = 2$

$$\frac{A_1}{I-A} = \frac{A_2}{A_3} = \frac{A_3}{A_4}$$

$$I-A = \begin{pmatrix} 0-1 & 0 \\ 0-1 & -1 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0-1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$r(I-A) = 3-2 = 1 \text{ if } r(2I-A) = 3-1 = 2$$

2I - A

r(2I-A)相似矩阵与矩阵对角化



 $A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

例 下列哪个矩阵与 $\Lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ 相似?

提示 若
$$A$$
 与 Λ 相似 \Leftrightarrow A 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重) \Leftrightarrow $r(I-A) = 3-2 = 1$ 且 $r(2I-A) = 3-1 = 2$

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与 Λ 相似 \Leftrightarrow A 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重)

解
$$\Rightarrow r(I-A) = 3-2 = 1 \perp r(2I-A) = 3-1 = 2$$

$$A_1 \qquad A_2 \qquad A_3 \qquad A_4$$

$$I-A \qquad \begin{pmatrix} 0-1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0-1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$r(I-A)$$

2*I* – *A*



$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与 Λ 相似 \Leftrightarrow A 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重)

解
$$\Rightarrow r(I-A) = 3-2 = 1$$
且 $r(2I-A) = 3-1 = 2$

$$A_1 \qquad A_2 \qquad A_3 \qquad A_4$$

$$I-A \qquad \begin{pmatrix} 0-1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$r(I-A) \qquad 2$$

2*I* – A



$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 $A \subseteq \Lambda$ 相似 $\iff A$ 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重)

 $\Leftrightarrow r(I-A) = 3-2 = 1 \perp r(2I-A) = 3-1 = 2$ 解 A_4 $I - A \qquad \begin{pmatrix} 0 - 1 & 0 \\ 0 - 1 - 1 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0 - 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 - 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & -1 \\ 0 - 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$ r(I-A)

2I - A

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与 Λ 相似 ⇔ A 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重)

解	$\Leftrightarrow r(I-A) = 3-2 = 1 \perp r(2I-A) = 3-1 = 2$			
	A_1	A ₂	A ₃	A_4
I-A	$\left(\begin{smallmatrix}0&-1&0\\0&-1&-1\\0&0&0\end{smallmatrix}\right)$	$\begin{pmatrix} 0 - 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$
r(I-A)	2	2	1	

r(2I-A)

2I - A

提示若A与A

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

例 下列哪个矩阵与 $\Lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ 相似?

提示 若 A 与 Λ 相似 \Leftrightarrow A 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重)

解	$\Leftrightarrow r(I-A) = 3-2 = 1 \perp r(2I-A) = 3-1 = 2$			
	A_1	A ₂	A ₃	A_4
I-A	$\left(\begin{smallmatrix}0&-1&0\\0&-1&-1\\0&0&0\end{smallmatrix}\right)$	$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$
r(I-A)	2	2	1	2
			т	

r(2I-A)

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2I - A

例下列哪个矩阵与
$$\Lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$
相似?

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与 Λ 相似 ⇔ A 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重)

解	$\Leftrightarrow r(1-A) = 3-2 = 1 \oplus r(21-A) = 3-1 = 2$			
	A_1	A ₂	A ₃	A_4
I-A	$\left(\begin{smallmatrix}0&-1&0\\0&-1&-1\\0&0&0\end{smallmatrix}\right)$	$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$
r(I-A)	2	2	1	2
2 <i>I</i> – A			$\left(\begin{smallmatrix}1&0&-1\\0&1&0\\0&0&0\end{smallmatrix}\right)$	

r(2I-A)相似矩阵与矩阵对角化

 $A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

例 下列哪个矩阵与 $\Lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ 相似?

提示 若 $A \subseteq \Lambda$ 相似 $\Leftrightarrow A$ 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重) $\Leftrightarrow r(I-A) = 3-2 = 1 \perp r(2I-A) = 3-1 = 2$ 解

r(2I-A)§4.2 相似矩阵与矩阵对角化

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提示 若 A 与 Λ 相似 ⇔ A 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重) $\Leftrightarrow r(I-A) = 3-2 = 1 \perp r(2I-A) = 3-1 = 2$ 6亿

 $A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

卅年	, ,		_ ` '	
	A_1	A ₂	A ₃	A_4
I-A	$\left(\begin{smallmatrix}0&-1&0\\0&-1&-1\\0&0&0\end{smallmatrix}\right)$	$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$
r(I-A)	2	2	1	2
2 <i>I</i> – <i>A</i>	$\begin{pmatrix} 1 - 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix}1-1&0\\0&1&0\\0&0&0\end{pmatrix}$	$\left(\begin{smallmatrix}1&0&-1\\0&1&0\\0&0&0\end{smallmatrix}\right)$	$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

r(2I-A)相似矩阵与矩阵对角化

 $A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

例 下列哪个矩阵与 $\Lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ 相似?

提示 若 A 与 Λ 相似 ⇔ A 可对角化, $\lambda_1 = 1$ (二重), $\lambda_2 = 2$ (一重) $\Leftrightarrow r(I-A) = 3-2 = 1 \perp r(2I-A) = 3-1 = 2$ 解

r(2I-A)§4.2 相似矩阵与矩阵对角化

例设
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
, 求 A^n



例设
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,求 A^n

解

$$P^{-1}AP = \begin{pmatrix} 2 & & \\ & 2 & \\ & & 6 \end{pmatrix}$$

例设
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,求 A^n

解

$$P^{-1}AP = \begin{pmatrix} 2 & 2 & \\ & 6 & \end{pmatrix} \Rightarrow A = P \begin{pmatrix} 2 & 2 & \\ & 6 & \end{pmatrix} P^{-1}$$



例设
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,求 A^n

解

$$P^{-1}AP = \begin{pmatrix} 2 & 2 & \\ & 6 & \end{pmatrix} \Rightarrow A = P \begin{pmatrix} 2 & 2 & \\ & 6 & \end{pmatrix} P^{-1} = P \wedge P^{-1}$$



例设
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,求 A^n

解

$$P^{-1}AP = \begin{pmatrix} 2 & 2 & 0 \\ 0 & 6 & 0 \end{pmatrix} \Rightarrow A = P \begin{pmatrix} 2 & 2 & 0 \\ 0 & 6 & 0 \end{pmatrix} P^{-1} = P \wedge P^{-1}$$

所以

$$A^n =$$



例设
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,求 A^n

$$P^{-1}AP = \begin{pmatrix} 2 & 2 & \\ & 2 & \\ & 6 \end{pmatrix} \Rightarrow A = P \begin{pmatrix} 2 & 2 & \\ & 6 \end{pmatrix} P^{-1} = P \wedge P^{-1}$$

所以

$$A^{n} = (P \wedge P^{-1}) \cdot (P \wedge P^{-1})(P \wedge P^{-1}) \cdots (P \wedge P^{-1})(P \wedge P^{-1})$$

例设
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$$P^{-1}AP = \begin{pmatrix} 2 & 2 & \\ & 2 & \\ & 6 \end{pmatrix} \Rightarrow A = P \begin{pmatrix} 2 & 2 & \\ & 6 \end{pmatrix} P^{-1} = P \wedge P^{-1}$$

所以

$$A^{n} = (P \wedge P^{-1}) \cdot (P \wedge P^{-1})(P \wedge P^{-1}) \cdots (P \wedge P^{-1})(P \wedge P^{-1})$$
$$= P \wedge \cdot \wedge \cdots \wedge P^{-1}$$

=

例设
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
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所以

$$A^{n} = (P \wedge P^{-1}) \cdot (P \wedge P^{-1})(P \wedge P^{-1}) \cdots (P \wedge P^{-1})(P \wedge P^{-1})$$
$$= P \wedge \cdot \wedge \cdots \wedge P^{-1}$$
$$= P \wedge^{n} P^{-1}$$

=

例设
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,求 A^n

解

$$P^{-1}AP = \begin{pmatrix} 2 & 2 & \\ & 6 & \end{pmatrix} \quad \Rightarrow \quad A = P \begin{pmatrix} 2 & 2 & \\ & 6 & \end{pmatrix} P^{-1} = P \wedge P^{-1}$$

所以

$$A^{n} = (P \wedge P^{-1}) \cdot (P \wedge P^{-1})(P \wedge P^{-1}) \cdots (P \wedge P^{-1})(P \wedge P^{-1})$$

$$= P \wedge \cdot \wedge \cdots \wedge P^{-1}$$

$$= P \wedge^{n} P^{-1}$$

 $\begin{pmatrix} 2^{n} & & \\ & 2^{n} & \\ & & 6^{n} \end{pmatrix}$



例设
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,求 A^n

$$P^{-1}AP = \begin{pmatrix} 2 & 2 & \\ & 2 & \\ & 6 \end{pmatrix} \Rightarrow A = P \begin{pmatrix} 2 & 2 & \\ & 2 & \\ & 6 \end{pmatrix} P^{-1} = P \wedge P^{-1}$$

所以

$$A^{n} = (P \wedge P^{-1}) \cdot (P \wedge P^{-1})(P \wedge P^{-1}) \cdots (P \wedge P^{-1})(P \wedge P^{-1})$$

$$= P \wedge \cdot \wedge \cdots \wedge P^{-1}$$

$$= P \wedge^{n} P^{-1}$$

$$= \begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & -2 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 2^{n} & 2^{n} \\ 6^{n} \end{pmatrix} \begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & -2 \\ 0 & 1 & 3 \end{pmatrix}^{-1}$$

———The End————

$$0 = |\lambda I - A| =$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$



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$$r_3-r_2$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$

$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -\lambda + 2 & \lambda - 2 \end{vmatrix}$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$
$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -\lambda + 2 & \lambda - 2 \end{vmatrix}$$
$$= (\lambda - 2) \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -1 & 1 \end{vmatrix}$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$
$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -\lambda + 2 & \lambda - 2 \end{vmatrix}$$
$$= (\lambda - 2) \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -1 & 1 \end{vmatrix} \xrightarrow{c_2 + 2c_3}$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$

$$= \frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -\lambda + 2 & \lambda - 2 \end{vmatrix}$$

$$= (\lambda - 2) \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -1 & 1 \end{vmatrix} \stackrel{c_2 + 2c_3}{=} (\lambda - 2) \begin{vmatrix} \lambda - 1 & 0 & 1 \\ 2 & \lambda - 2 & 2 \\ 0 & 0 & 1 \end{vmatrix}$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$

$$= \frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -\lambda + 2 & \lambda - 2 \end{vmatrix}$$

$$= (\lambda - 2) \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -1 & 1 \end{vmatrix} \xrightarrow{c_2 + 2c_3} (\lambda - 2) \begin{vmatrix} \lambda - 1 & 0 & 1 \\ 2 & \lambda - 2 & 2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (\lambda - 2) \begin{vmatrix} \lambda - 1 & 1 \\ 0 & 1 \end{vmatrix}$$





$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$

$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -\lambda + 2 & \lambda - 2 \end{vmatrix}$$

$$= (\lambda - 2) \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -1 & 1 \end{vmatrix} \stackrel{c_2 + 2c_3}{=} (\lambda - 2) \begin{vmatrix} \lambda - 1 & 0 & 1 \\ 2 & \lambda - 2 & 2 \\ 0 & 1 \end{vmatrix}$$

$$= (\lambda - 2)(\lambda - 2) \begin{vmatrix} \lambda - 1 & 1 \\ 0 & 1 \end{vmatrix}$$





$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$

$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -\lambda + 2 & \lambda - 2 \end{vmatrix}$$

$$= (\lambda - 2) \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -1 & 1 \end{vmatrix} \stackrel{c_2 + 2c_3}{=} (\lambda - 2) \begin{vmatrix} \lambda - 1 & 0 & 1 \\ 2 & \lambda - 2 & 2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (\lambda - 2)(\lambda - 2) \begin{vmatrix} \lambda - 1 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= (\lambda - 1)(\lambda - 2)^2$$





• $\exists \lambda_1 = 1$, $\forall x \in (\lambda_1 I - A)x = 0$:

$$(1I - A : 0) =$$

$$(1I - A \vdots 0) = \begin{pmatrix} 0 & -1 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 2 & -2 & 1 & | & 0 \end{pmatrix}$$

$$(1I-A:0) = \begin{pmatrix} 0 & -1 & 1 & 0 \\ 2 & -3 & 2 & 0 \\ 2 & -2 & 1 & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & 0 \\ 2 & -3 & 2 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix}$$



$$(1I-A:0) = \begin{pmatrix} 0 & -1 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 2 & -2 & 1 & | & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 0 & -1 & 1 & | & 0 \end{pmatrix}$$

$$r_2-r_1$$

$$(1I - A \vdots 0) = \begin{pmatrix} 0 & -1 & 1 & 0 \\ 2 & -3 & 2 & 0 \\ 2 & -2 & 1 & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & 0 \\ 2 & -3 & 2 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{r_2-r_1} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array} \right)$$



• $\exists \lambda_1 = 1$, $\forall x \in (\lambda_1 I - A)x = 0$:

$$(1I - A \vdots 0) = \begin{pmatrix} 0 & -1 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 2 & -2 & 1 & | & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 0 & -1 & 1 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_2-r_1} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \xrightarrow{r_3-r_2} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$



$$(1I - A : 0) = \begin{pmatrix} 0 & -1 & 1 & 0 \\ 2 & -3 & 2 & 0 \\ 2 & -2 & 1 & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & 0 \\ 2 & -3 & 2 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{r_2 - r_1} \begin{pmatrix} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix} \xrightarrow{r_3 - r_2} \begin{pmatrix} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$



$$(1I - A \vdots 0) = \begin{pmatrix} 0 & -1 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 2 & -2 & 1 & | & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 0 & -1 & 1 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_2-r_1} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \xrightarrow{r_3-r_2} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$(x_1 \quad -\frac{1}{2}x_3 = \frac{1}{2}x_4 = \frac{1}{2}x_5 =$$

所以
$$\begin{cases} x_1 & -\frac{1}{2}x_3 = 0 \end{cases}$$





$$(1I - A \vdots 0) = \begin{pmatrix} 0 & -1 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 2 & -2 & 1 & | & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 0 & -1 & 1 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_2-r_1} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \xrightarrow{r_3-r_2} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\rightarrow \begin{pmatrix}
1 & 0 & -1/2 & | & 0 \\
0 & 1 & -1 & | & 0 \\
0 & 0 & 0 & | & 0
\end{pmatrix}$$

$$\begin{cases}
x_1 & -\frac{1}{2}x_3 = | & 0 \\
0 & 0 & | & 0
\end{cases}$$

所以
$$\begin{cases} x_1 & -\frac{1}{2}x_3 = 0 \\ x_2 - x_3 = 0 \end{cases}$$



• $\exists \lambda_1 = 1$, $\forall M (\lambda_1 I - A) X = 0$:

$$(1I - A \vdots 0) = \begin{pmatrix} 0 & -1 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 2 & -2 & 1 & | & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 0 & -1 & 1 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_2-r_1} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \xrightarrow{r_3-r_2} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

所以
$$\begin{cases} x_1 & -\frac{1}{2}x_3 = 0 \\ x_2 - x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{1}{2}x_3 \\ x_2 = x_3 \end{cases}$$

$$(1I - A : 0) = \begin{pmatrix} 0 & -1 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 2 & -2 & 1 & | & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 0 & -1 & 1 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_2-r_1} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \xrightarrow{r_3-r_2} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

所以 $\begin{cases} x_1 & -\frac{1}{2}x_3 = 0 \\ x_2 - x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{1}{2}x_3 \\ x_2 = x_3 \end{cases}$

基础解系:
$$\alpha_1 = \begin{pmatrix} \\ 2 \end{pmatrix}$$



$$(1I - A:0) = \begin{pmatrix} 0 & -1 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 2 & -2 & 1 & | & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 0 & -1 & 1 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_2-r_1} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \xrightarrow{r_3-r_2} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

所以 $\begin{cases} x_1 & -\frac{1}{2}x_3 = 0 \\ x_2 - x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{1}{2}x_3 \\ x_2 = x_3 \end{cases}$

基础解系:
$$\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$



• $\exists \lambda_2 = 2$, $\forall x \in (\lambda_2 I - A)x = 0$:

$$(2I - A : 0) =$$



• $\exists \lambda_2 = 2$, $\forall x \in (\lambda_2 I - A)x = 0$:

$$(2I - A \vdots 0) = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow$$



$$(2I - A \vdots 0) = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$





• $\exists \lambda_2 = 2$, $\forall M (\lambda_2 I - A) x = 0$:

$$(2I - A \vdots 0) = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$x_1 - x_2 + x_3 = 0$$



$$(2I - A \vdots 0) = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$x_1 - x_2 + x_3 = 0 \Rightarrow x_1 = x_2 - x_3$$





$$(2I - A \vdots 0) = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$x_1 - x_2 + x_3 = 0$$
 \Rightarrow $x_1 = x_2 - x_3$ 基础解系: $\alpha_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



$$(2I - A \vdots 0) = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$x_1 - x_2 + x_3 = 0$$
 \Rightarrow $x_1 = x_2 - x_3$ 基础解系: $\alpha_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



$$(2I - A \vdots 0) = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$x_1 - x_2 + x_3 = 0 \Rightarrow x_1 = x_2 - x_3$$

基础解系: $\alpha_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



$$(2I - A \vdots 0) = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$x_1 - x_2 + x_3 = 0 \Rightarrow x_1 = x_2 - x_3$$

基础解系: $\alpha_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$



$$0 = |\lambda I - A| =$$

$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$

$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$

$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$
$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -\lambda - 1 & \lambda + 1 \end{vmatrix}$$

$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$
$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -\lambda - 1 & \lambda + 1 \end{vmatrix}$$
$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -1 & 1 \end{vmatrix}$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$

$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -\lambda - 1 & \lambda + 1 \end{vmatrix}$$

$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -1 & 1 \end{vmatrix} \frac{c_2 + c_3}{2}$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$

$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -\lambda - 1 & \lambda + 1 \end{vmatrix}$$

$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -1 & 1 \end{vmatrix} \xrightarrow{c_2 + c_3} (\lambda + 1) \begin{vmatrix} \lambda - 1 & -4 & -2 \\ -2 & \lambda - 3 & -2 \\ 0 & 0 & 1 \end{vmatrix}$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$

$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -\lambda - 1 & \lambda + 1 \end{vmatrix}$$

$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -1 & 1 \end{vmatrix} \stackrel{c_2 + c_3}{=} (\lambda + 1) \begin{vmatrix} \lambda - 1 & -4 & -2 \\ -2 & \lambda - 3 & -2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -4 \\ -2 & \lambda - 3 \end{vmatrix}$$



 $=(\lambda+1)(\lambda^2-4\lambda-5)$

$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$

$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -\lambda - 1 & \lambda + 1 \end{vmatrix}$$

$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -1 & 1 \end{vmatrix} \xrightarrow{\frac{C_2 + C_3}{2}} (\lambda + 1) \begin{vmatrix} \lambda - 1 & -4 & -2 \\ -2 & \lambda - 3 & -2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -4 \\ -2 & \lambda - 3 \end{vmatrix}$$





$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$

$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -\lambda - 1 & \lambda + 1 \end{vmatrix}$$

$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -1 & 1 \end{vmatrix} \frac{c_2 + c_3}{2} (\lambda + 1) \begin{vmatrix} \lambda - 1 & -4 & -2 \\ -2 & \lambda - 3 & -2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -4 \\ -2 & \lambda - 3 \end{vmatrix}$$

$$= (\lambda + 1)^2 (\lambda - 5)$$

 $=(\lambda+1)(\lambda^2-4\lambda-5)$





•
$$\exists \lambda_1 = -1$$
, $\forall M (\lambda_1 I - A) x = 0$:

$$(-I - A : 0) =$$



• $\exists \lambda_1 = -1$, $\forall M (\lambda_1 I - A) X = 0$:

$$(-I-A:0) = \begin{pmatrix} -2 & -2 & -2 & 0 \\ -2 & -2 & -2 & 0 \\ -2 & -2 & -2 & 0 \end{pmatrix} \rightarrow$$





• $\exists \lambda_1 = -1$, $\forall x \in (\lambda_1 I - A)x = 0$:

$$(-I-A:0) = \begin{pmatrix} -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$





• $\exists \lambda_1 = -1$, $\forall x \in (\lambda_1 I - A)x = 0$:

$$(-I-A:0) = \begin{pmatrix} -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$x_1 + x_2 + x_3 = 0$$



• $\exists \lambda_1 = -1$, $\forall x \in (\lambda_1 I - A)x = 0$:

$$(-I - A \vdots 0) = \begin{pmatrix} -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$x_1 + x_2 + x_3 = 0 \Rightarrow x_1 = -x_2 - x_3$$





• $\exists \lambda_1 = -1$, $\forall M (\lambda_1 I - A) x = 0$:

$$(-I-A:0) = \begin{pmatrix} -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$x_1 + x_2 + x_3 = 0 \Rightarrow x_1 = -x_2 - x_3$$

基础解系:
$$\alpha_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



• $\exists \lambda_1 = -1$, $\forall M (\lambda_1 I - A) x = 0$:

$$(-I-A:0) = \begin{pmatrix} -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

所以

$$x_1 + x_2 + x_3 = 0$$
 \Rightarrow $x_1 = -x_2 - x_3$ 基础解系: $\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



• $\exists \lambda_1 = -1$, $\forall M (\lambda_1 I - A) x = 0$:

$$(-I-A:0) = \begin{pmatrix} -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

所以

$$x_1 + x_2 + x_3 = 0$$
 \Rightarrow $x_1 = -x_2 - x_3$ 基础解系: $\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$



• $\exists \lambda_2 = 5$, $\forall M (\lambda_2 I - A) x = 0$:

$$(5I - A : 0) =$$

$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & | & 0 \\ -2 & 4 & -2 & | & 0 \\ -2 & -2 & 4 & | & 0 \end{pmatrix}$$



$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & 0 \\ -2 & 4 & -2 & 0 \\ -2 & -2 & 4 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$



$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & 0 \\ -2 & 4 & -2 & 0 \\ -2 & -2 & 4 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

$$r_1 \leftrightarrow r_3$$





$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & | & 0 \\ -2 & 4 & -2 & | & 0 \\ -2 & -2 & 4 & | & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -2 & 1 & 0 \\ 2 & -1 & -1 & 0 \end{array} \right)$$



$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & | & 0 \\ -2 & 4 & -2 & | & 0 \\ -2 & -2 & 4 & | & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -2 & 1 & 0 \\ 2 & -1 & -1 & 0 \end{array} \right) \xrightarrow[r_3 - 2r_1]{r_2 - r_1} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -3 & 3 & 0 \end{array} \right)$$



$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & | & 0 \\ -2 & 4 & -2 & | & 0 \\ -2 & -2 & 4 & | & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -2 & 1 & 0 \\ 2 & -1 & -1 & 0 \end{array}\right) \xrightarrow[r_3 - 2r_1]{r_2 - r_1} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -3 & 3 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$



$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & | & 0 \\ -2 & 4 & -2 & | & 0 \\ -2 & -2 & 4 & | & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -2 & 1 & 0 \\ 2 & -1 & -1 & 0 \end{array}\right) \xrightarrow[r_3 - 2r_1]{r_2 - r_1} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -3 & 3 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{r_1 - r_2} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$



$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & | & 0 \\ -2 & 4 & -2 & | & 0 \\ -2 & -2 & 4 & | & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -2 & 1 & 0 \\ 2 & -1 & -1 & 0 \end{array}\right) \xrightarrow[r_3 - 2r_1]{r_2 - r_1} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -3 & 3 & 0 \end{array}\right)$$

$$\rightarrow \begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \xrightarrow{r_1 - r_2} \begin{pmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$(x_1 - x_3 = 0)$$

所以
$$\begin{cases} x_1 & -x_3 = 0 \end{cases}$$



● 整点大型

$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & | & 0 \\ -2 & 4 & -2 & | & 0 \\ -2 & -2 & 4 & | & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -2 & 1 & 0 \\ 2 & -1 & -1 & 0 \end{array} \right) \xrightarrow[r_3 - 2r_1]{r_2 - r_1} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -3 & 3 & 0 \end{array} \right)$$

所以
$$\begin{cases} x_1 & -x_3 = 0 \\ x_2 - x_3 = 0 \end{cases}$$





$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & | & 0 \\ -2 & 4 & -2 & | & 0 \\ -2 & -2 & 4 & | & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -2 & 1 & 0 \\ 2 & -1 & -1 & 0 \end{array} \right) \xrightarrow[r_3 - 2r_1]{r_2 - r_1} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -3 & 3 & 0 \end{array} \right)$$

$$\longrightarrow \left(\begin{array}{cc|c} 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{r_1 - r_2} \left(\begin{array}{cc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

所以
$$\begin{cases} x_1 & -x_3 = 0 \\ x_2 - x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = x_3 \\ x_2 = x_3 \end{cases}$$





$$(5I - A : 0) = \begin{pmatrix} 4 & -2 & -2 & 0 \\ -2 & 4 & -2 & 0 \\ -2 & -2 & 4 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 2 & -1 & -1 & | & 0 \end{pmatrix} \xrightarrow{r_2 - r_1} \begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 0 & -3 & 3 & | & 0 \\ 0 & -3 & 3 & | & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & -1 & | & 0 \\ 0 & -3 & 3 & | & 0 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \xrightarrow{r_1 - r_2} \begin{pmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

所以
$$\begin{cases} x_1 & -x_3 = 0 \\ x_2 - x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = x_3 \\ x_2 = x_3 \end{cases}$$

基础解系:
$$\alpha_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$



$$(5I - A : 0) = \begin{pmatrix} 4 & -2 & -2 & 0 \\ -2 & 4 & -2 & 0 \\ -2 & -2 & 4 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 2 & -1 & -1 & | & 0 \end{pmatrix} \xrightarrow{r_2 - r_1} \begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 0 & -3 & 3 & | & 0 \\ 0 & -3 & 3 & | & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & -1 & | & 0 \\ 0 & -1 & | & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix}
1 & 1 & -2 & 0 \\
0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix} \xrightarrow{r_1 - r_2} \begin{pmatrix}
1 & 0 & -1 & 0 \\
0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}$$

$$\begin{cases}
x_1 & -x_3 = 0 & \begin{cases}
x_1 = x_3
\end{cases}$$

所以 $\begin{cases} x_1 & -x_3 = 0 \\ x_2 - x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = x_3 \\ x_2 = x_3 \end{cases}$

基础解系:
$$\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

