第 1 章 a：二阶三阶行列式

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2019-2020 学年 I

教学要求

掌握求解:

- ◇ 二阶行列式计算
- ♣ 三阶行列式计算

- **行列式** 可用于表示一些线性方程组的解。

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- 具体地，

二阶行列式 $\longleftrightarrow$ $\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$ (2元2方程)

三阶行列式 $\longleftrightarrow$ $\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$ (3元3方程)

$\vdots$

n阶行列式 $\longleftrightarrow$ n元n方程 的线性方程组

$\vdots$

（ **克莱姆** 法则 ）

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解:

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

用消元法求解：$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \quad \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

用消元法求解：$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

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用消元法求解：$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

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用消元法求解：$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \underline{\qquad\qquad\qquad\qquad}$$

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$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} & \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} & \Rightarrow a_{21}a_{12}x + a_{22}a_{12}y = b_2a_{12} \end{cases}$$

用消元法求解：$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1a_{22} - a_{12}b_2}{\quad}$$

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用消元法求解：$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & \quad (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & \quad (2) \times a_{11} \end{cases}$$

用消元法求解：$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

2 元 2 方程的线性方程组

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用消元法求解：$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

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$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{\qquad}{\qquad}$$

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用消元法求解：$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11}b_2 - b_1a_{21}}{\phantom{a_{11}a_{22} - a_{12}a_{21}}}$$

2 元 2 方程的线性方程组

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用消元法求解：$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

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$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

- 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$

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$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

- 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$，称为 **二阶行列式**

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用消元法求解：$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\qquad}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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小结

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

小结

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

1. 当 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$ 时，

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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1. 当 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$ 时，方程有唯一解：

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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2. 当 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 0$ 时，

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

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2. 当 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 0$ 时，方程或者无解、或者有无穷多的解。

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \quad \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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例 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $x =$ ， $y =$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $x =$ ， $y =$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $x =$, $y =$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $\quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $\quad x = \qquad\qquad , \; y =$

公式:

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例 $\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} \neq 0$ 的充分必要条件是 λ 满足 ________

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$$\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} = \lambda^2 - 3\lambda = \lambda(\lambda - 3)$$

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解 因为

$$\begin{vmatrix} k-1 & 2 \\ 2 & k-1 \end{vmatrix} = (k-1)^2 - 4$$

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所以 $k \neq -1$ 且 $k \neq 3$。

3 元 3 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

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用消元法可解得：

$$x = \frac{\begin{array}{c} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{array}}{\begin{array}{c} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}}$$

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为表示三元方程组的解，定义 **三阶行列式**：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{c} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}$$

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$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{l} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}$$

规律

为表示三元方程组的解，定义**三阶行列式**：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{l} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}$$

规律

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

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规律

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

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规律

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

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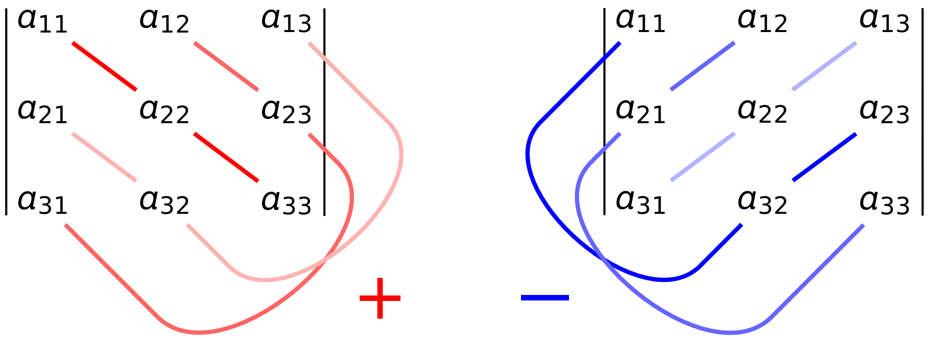
规律



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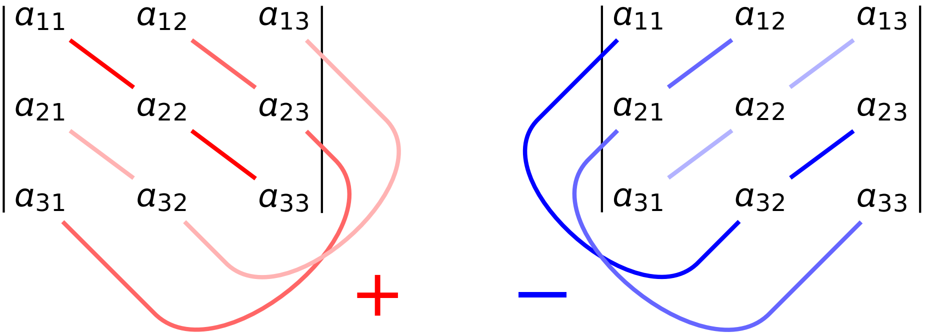
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{l} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}$$

规律　（不同行不同列的 3 个元素乘积，共 $3! = 6$ 个）



$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad + \qquad - \quad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad + \qquad - \qquad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \qquad 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0$$

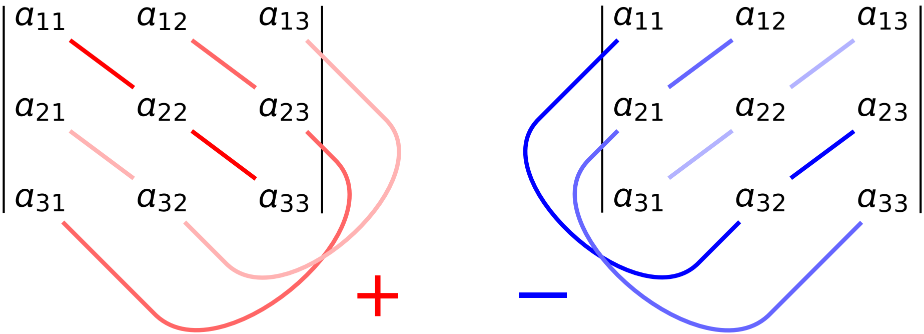
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad + \qquad - \qquad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{array}{c} 1\times 0\times 6 + 2\times 5\times(-1) + 3\times 4\times 0 \\ -1\times 5\times 0 - 2\times 4\times 6 - 3\times 0\times(-1) \end{array}$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{array}{c} 1\times 0\times 6 + 2\times 5\times(-1) + 3\times 4\times 0 \\ -1\times 5\times 0 - 2\times 4\times 6 - 3\times 0\times(-1) \end{array} = -58$$

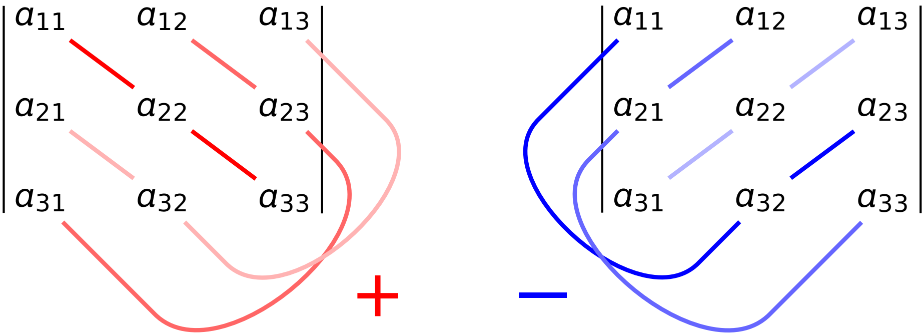
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad + \quad - \quad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{matrix} 1\times 0\times 6 + 2\times 5\times(-1) + 3\times 4\times 0 \\ -1\times 5\times 0 - 2\times 4\times 6 - 3\times 0\times(-1) \end{matrix} = -58$$

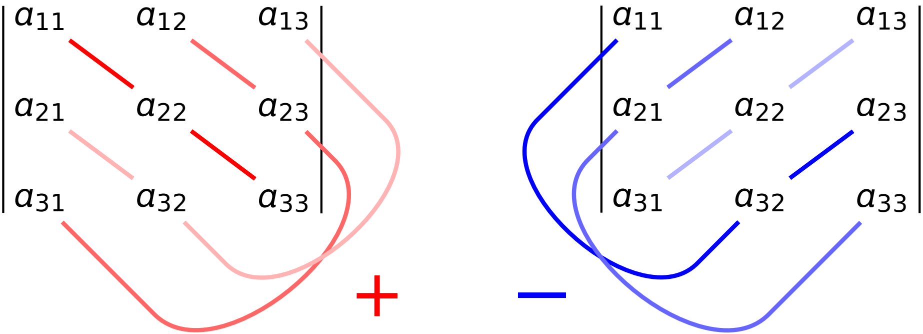
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = 1\times 5\times 1 + 0\times 0\times 1 + (-1)\times 3\times 4$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{matrix} 1\times 0\times 6 + 2\times 5\times(-1) + 3\times 4\times 0 \\ -1\times 5\times 0 - 2\times 4\times 6 - 3\times 0\times(-1) \end{matrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{matrix} 1\times 5\times 1 + 0\times 0\times 1 + (-1)\times 3\times 4 \\ -1\times 0\times 4 - 0\times 3\times 1 - (-1)\times 5\times 1 \end{matrix}$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{array}{c} 1\times0\times6+2\times5\times(-1)+3\times4\times0 \\ -1\times5\times0-2\times4\times6-3\times0\times(-1) \end{array} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{array}{c} 1\times5\times1+0\times0\times1+(-1)\times3\times4 \\ -1\times0\times4-0\times3\times1-(-1)\times5\times1 \end{array} = -2$$

例 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 ________________

例 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 ______________

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} =$$

例 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 ________

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \quad a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2$$

例 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$不为零的充分必要条件是 a, b 满足 ________

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{array}{c} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{array}$$

例 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$不为零的充分必要条件是 a, b 满足 ________

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{matrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ - a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{matrix} = a^2 + b^2$$

例 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 ______________

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{array}{c} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{array} = a^2 + b^2$$

所以 $a \neq 0$ 或 $b \neq 0$。

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为：

,

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为：

$$x = \frac{\begin{array}{c} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{array}}{\begin{array}{c} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}} =$$

,

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为：

$$x = \frac{\begin{matrix} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{matrix}}{\begin{matrix} {\color{red}a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}} \\ {\color{blue}-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}} \end{matrix}} = \frac{\qquad}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

,

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

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,

这时方程组
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的解可以表示为:

$$x=\frac{\begin{array}{c}b_1a_{22}a_{33}+a_{12}a_{23}b_3+a_{13}b_2a_{32}\\ -b_1a_{23}a_{32}-a_{12}b_2a_{33}-a_{13}a_{22}b_3\end{array}}{\begin{array}{c}{\color{red}a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32}}\\ {\color{blue}-a_{11}a_{23}a_{32}-a_{12}a_{21}a_{33}-a_{13}a_{22}a_{31}}\end{array}}=\frac{\begin{vmatrix} b_1 & a_{12} & a_{13}\\ b_2 & a_{22} & a_{23}\\ b_3 & a_{32} & a_{33}\end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}}$$

$$y=\frac{\begin{vmatrix} a_{11} & b_1 & a_{13}\\ a_{21} & b_2 & a_{23}\\ a_{31} & b_3 & a_{33}\end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}},$$

这时方程组
$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为：

$$x = \frac{\begin{array}{c} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{array}}{\begin{array}{c} \color{red}{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}} \\ \color{blue}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}} \end{array}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}, \quad z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

这时方程组
$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为：

$$x = \frac{\begin{matrix} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{matrix}}{\begin{matrix} {\color{red}a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}} \\ {\color{blue}-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}} \end{matrix}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\quad}{D}$$

$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\quad}{D}, \quad z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\quad}{D}$$

这时方程组 $\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$

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小结

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

$$x = \frac{D_x}{D}, \qquad y = \frac{D_y}{D}, \qquad z = \frac{D_z}{D}$$

小结

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

1. 当 $D \neq 0$ 时，

$$x = \frac{D_x}{D}, \qquad y = \frac{D_y}{D}, \qquad z = \frac{D_z}{D}$$

小结

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

1. 当 $D \neq 0$ 时，方程有唯一解：

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小结

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1. 当 $D \neq 0$ 时，方程有唯一解：

$$x = \frac{D_x}{D}, \qquad y = \frac{D_y}{D}, \qquad z = \frac{D_z}{D}$$

2. 当 $D = 0$ 时，方程或者无解、或者有无穷多的解。

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$

例 求解三元线性方程组 $\begin{cases} x+2z=9 \\ 2y+z=8 \\ 4x-3y=-2 \end{cases}$

解

$$x=\frac{D_x}{D}=\underline{\qquad\qquad\qquad}$$

$$y=\frac{D_y}{D}=\underline{\qquad\qquad\qquad}$$

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解

$$x=\frac{D_x}{D}=\frac{\qquad\qquad}{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}}$$

$$y=\frac{D_y}{D}=\underline{\qquad\qquad}$$

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例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$

解

$$x = \frac{D_x}{D} = \frac{\qquad\qquad}{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}} \quad \times$$

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例 求解三元线性方程组 $\begin{cases} x+2z=9 \\ 2y+z=8 \\ 4x-3y=-2 \end{cases}$ $\left(\begin{cases} x+0y+2z=9 \\ \\ \\ \end{cases}\right)$

解

$$x=\frac{D_x}{D}=\frac{}{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix} \ \times} \qquad y=\frac{D_y}{D}=\frac{}{}$$

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解

$$x=\frac{D_x}{D}=\frac{\qquad\qquad}{\begin{vmatrix} 1 & 2 & \\ 2 & 1 & \\ 4 & -3 & \end{vmatrix}} \quad X$$

$$y=\frac{D_y}{D}=\underline{\qquad\qquad}$$

$$z=\frac{D_z}{D}=\underline{\qquad\qquad}$$

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解

$$x=\frac{D_x}{D}=\frac{\qquad}{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix} \times}$$

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解

$$x=\frac{D_x}{D}=\frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{\qquad}{-13}$$

$$y=\frac{D_y}{D}=\frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

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$$y=\frac{D_y}{D}=\frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

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解

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例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$ $\left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1, \quad y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-26}{-13}$$

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{}{-13}$$

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解

$$x=\frac{D_x}{D}=\frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-13}{-13}=1,\quad y=\frac{D_y}{D}=\frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-26}{-13}=2$$

$$z=\frac{D_z}{D}=\frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{}{-13}$$

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解

$$x=\frac{D_x}{D}=\frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-13}{-13}=1,\quad y=\frac{D_y}{D}=\frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-26}{-13}=2$$

$$z=\frac{D_z}{D}=\frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-52}{-13}$$

例 求解三元线性方程组 $\begin{cases} x+2z=9 \\ 2y+z=8 \\ 4x-3y=-2 \end{cases}$ （$\begin{cases} x+0y+2z=9 \\ 0x+2y+z=8 \\ 4x-3y+0z=-2 \end{cases}$）

解

$$x=\frac{D_x}{D}=\frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-13}{-13}=1,\quad y=\frac{D_y}{D}=\frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-26}{-13}=2$$

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另解 先利用公式求出 x

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所以方程的解是 $\begin{cases} x=1 \\ y=2 \\ z=4 \end{cases}$

一般地，n 元 n 方程的线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

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问题是，如何定义 n 阶行列式，

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问题是，如何定义 n 阶行列式，如何快捷计算行列式?

补充：方程组的几何理解

2 元 2 方程的线性方程组

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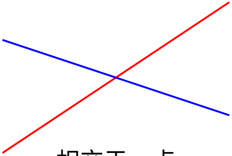

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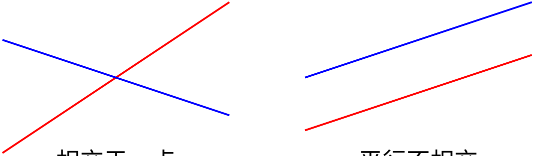

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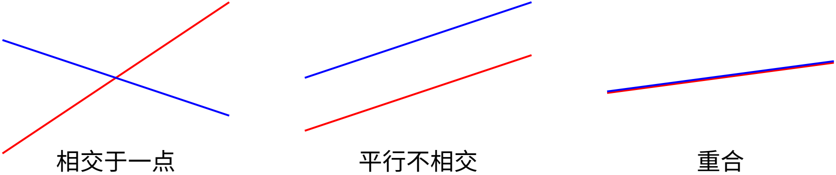

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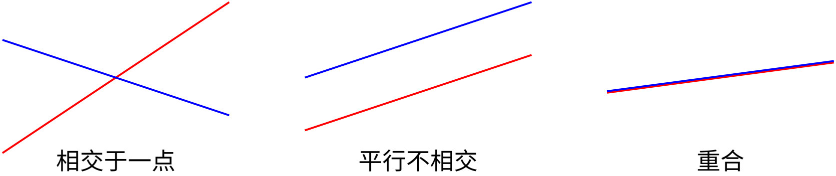

- 所以方程组的解有三种情况：

有唯一解、无解、有无穷多的解

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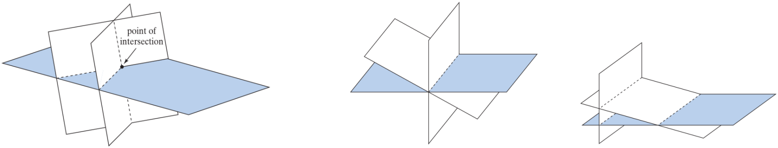

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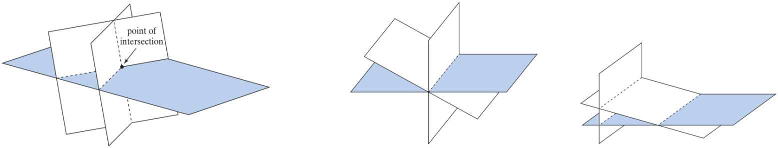

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