

§1.1 二阶三阶行列式

数学系 梁卓滨

2018 - 2019 学年上学期

教学要求

掌握求解：

◇ 二阶行列式计算

♣ 三阶行列式计算

- 行列式可用于表示一些线性方程组的解。

- 行列式可用于表示一些线性方程组的解。
- 具体地，

二阶行列式 \longleftrightarrow 2元2方程的线性方程组

三阶行列式 \longleftrightarrow 3元3方程的线性方程组

\vdots

n 阶行列式 \longleftrightarrow n 元 n 方程的线性方程组

\vdots

(克莱姆法则)

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解：

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

用消元法求解：(1) $\times a_{22}$ - (2) $\times a_{12}$ ，消去 y ，得：

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \Rightarrow a_{21}a_{12}x + a_{22}a_{12}y = b_2a_{12} \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \Rightarrow a_{21}a_{12}x + a_{22}a_{12}y = b_2a_{12} \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

$$x = \underline{\hspace{2cm}}$$

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \Rightarrow a_{21}a_{12}x + a_{22}a_{12}y = b_2a_{12} \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \Rightarrow a_{21}a_{12}x + a_{22}a_{12}y = b_2a_{12} \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$, 消去 x , 得:

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \end{cases} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$, 消去 x , 得:

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \Rightarrow a_{11}a_{21}x + a_{12}a_{21}y = b_1a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11} \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$, 消去 x , 得:

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \Rightarrow a_{11}a_{21}x + a_{12}a_{21}y = b_1a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11} \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$, 消去 x , 得:

$$y = \underline{\hspace{2cm}}$$

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \Rightarrow a_{11}a_{21}x + a_{12}a_{21}y = b_1a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11} \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$, 消去 x , 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \Rightarrow a_{11}a_{21}x + a_{12}a_{21}y = b_1a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11} \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$, 消去 x , 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$, 消去 x , 得:

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$, 消去 x , 得:

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

• 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} a_{22} - a_{12} a_{21}$

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$, 消去 x , 得:

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

• 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$, 称为二阶行列式

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \quad \begin{matrix} b_1 \\ b_2 \end{matrix}$$

$(2) \times a_{11} - (1) \times a_{21}$, 消去 x , 得:

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \quad \begin{matrix} b_1 \\ b_2 \end{matrix}$$

• 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$, 称为二阶行列式

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \end{cases}$$

$$\begin{cases} a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$(2) \times a_{11} - (1) \times a_{21}$, 消去 x , 得:

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

• 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$, 称为二阶行列式

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \end{cases}$$

$$\begin{cases} a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$(2) \times a_{11} - (1) \times a_{21}$, 消去 x , 得:

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

• 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$, 称为二阶行列式

小结

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

小结

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

1. 当 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$ 时,

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

小结

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

1. 当 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$ 时, 方程有唯一解:

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

小结

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

1. 当 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$ 时, 方程有唯一解:

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

2. 当 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 0$ 时,

小结

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

1. 当 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$ 时, 方程有唯一解:

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

2. 当 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 0$ 时, 方程或者无解、或者有无穷多的解。

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \quad , \quad y =$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad , \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \text{---} \quad , \quad y =$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad , \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \text{---}, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \text{---}$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \text{---}, \quad y = \text{---}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -\frac{1}{1}, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -\frac{1}{1}, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{1}{1}$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1}, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \bar{1}$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1}, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1}$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1}$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \text{---}, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-17}{-17} = 1, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-8}{-17} = \frac{8}{17}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{3}{3}, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = -$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{1}{3}, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-3}{3}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3}, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-3}{3}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3}, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3} = -3$$

例 $\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} \neq 0$ 的充分必要条件是 λ 满足 _____

例 $\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} \neq 0$ 的充分必要条件是 λ 满足 _____

解 因为

$$\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} =$$

例 $\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} \neq 0$ 的充分必要条件是 λ 满足 _____

解 因为

$$\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} = \lambda^2 - 3\lambda$$

例 $\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} \neq 0$ 的充分必要条件是 λ 满足 _____

解 因为

$$\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} = \lambda^2 - 3\lambda = \lambda(\lambda - 3)$$

例 $\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} \neq 0$ 的充分必要条件是 λ 满足 _____

解 因为

$$\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} = \lambda^2 - 3\lambda = \lambda(\lambda - 3)$$

所以 $\lambda \neq 0$ 且 $\lambda \neq 3$ 。

例 $\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} \neq 0$ 的充分必要条件是 λ 满足 _____

解 因为

$$\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} = \lambda^2 - 3\lambda = \lambda(\lambda - 3)$$

所以 $\lambda \neq 0$ 且 $\lambda \neq 3$ 。

练习 行列式 $\begin{vmatrix} k-1 & 2 \\ 2 & k-1 \end{vmatrix} \neq 0$ 的充分必要条件是 k 满足什么条件？

例 $\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} \neq 0$ 的充分必要条件是 λ 满足 _____

解 因为

$$\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} = \lambda^2 - 3\lambda = \lambda(\lambda - 3)$$

所以 $\lambda \neq 0$ 且 $\lambda \neq 3$ 。

练习 行列式 $\begin{vmatrix} k-1 & 2 \\ 2 & k-1 \end{vmatrix} \neq 0$ 的充分必要条件是 k 满足什么条件？

解 因为

$$\begin{vmatrix} k-1 & 2 \\ 2 & k-1 \end{vmatrix} = (k-1)^2 - 4$$

例 $\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} \neq 0$ 的充分必要条件是 λ 满足 _____

解 因为

$$\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} = \lambda^2 - 3\lambda = \lambda(\lambda - 3)$$

所以 $\lambda \neq 0$ 且 $\lambda \neq 3$ 。

练习 行列式 $\begin{vmatrix} k-1 & 2 \\ 2 & k-1 \end{vmatrix} \neq 0$ 的充分必要条件是 k 满足什么条件？

解 因为

$$\begin{vmatrix} k-1 & 2 \\ 2 & k-1 \end{vmatrix} = (k-1)^2 - 4 = k^2 - 2k - 3$$

例 $\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} \neq 0$ 的充分必要条件是 λ 满足 _____

解 因为

$$\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} = \lambda^2 - 3\lambda = \lambda(\lambda - 3)$$

所以 $\lambda \neq 0$ 且 $\lambda \neq 3$ 。

练习 行列式 $\begin{vmatrix} k-1 & 2 \\ 2 & k-1 \end{vmatrix} \neq 0$ 的充分必要条件是 k 满足什么条件？

解 因为

$$\begin{vmatrix} k-1 & 2 \\ 2 & k-1 \end{vmatrix} = (k-1)^2 - 4 = k^2 - 2k - 3 = (k+1)(k-3)$$

例 $\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} \neq 0$ 的充分必要条件是 λ 满足 _____

解 因为

$$\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} = \lambda^2 - 3\lambda = \lambda(\lambda - 3)$$

所以 $\lambda \neq 0$ 且 $\lambda \neq 3$ 。

练习 行列式 $\begin{vmatrix} k-1 & 2 \\ 2 & k-1 \end{vmatrix} \neq 0$ 的充分必要条件是 k 满足什么条件？

解 因为

$$\begin{vmatrix} k-1 & 2 \\ 2 & k-1 \end{vmatrix} = (k-1)^2 - 4 = k^2 - 2k - 3 = (k+1)(k-3)$$

所以 $k \neq -1$ 且 $k \neq 3$ 。

3 元 3 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

3 元 3 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

用消元法可解得：

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} - b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}}$$

3 元 3 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

用消元法可解得：

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} - b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}}$$

3 元 3 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

用消元法可解得：

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} - b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}}$$

为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}$$

为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{matrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{matrix}$$

规律

为表示三元方程组的解, 定义三阶行列式:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{matrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{matrix}$$

规律

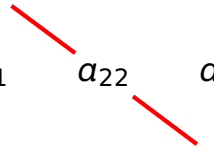
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

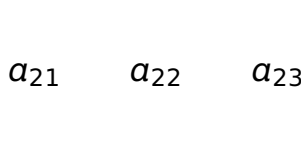
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

为表示三元方程组的解, 定义三阶行列式:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{matrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{matrix}$$

规律

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$


$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$


为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{matrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{matrix}$$

规律

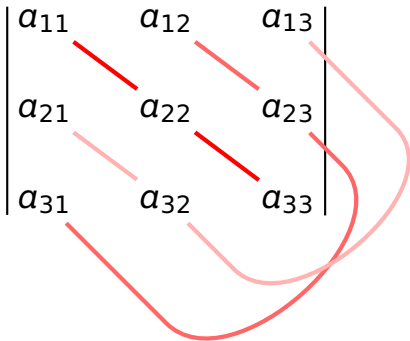
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

为表示三元方程组的解, 定义三阶行列式:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{matrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{matrix}$$

规律

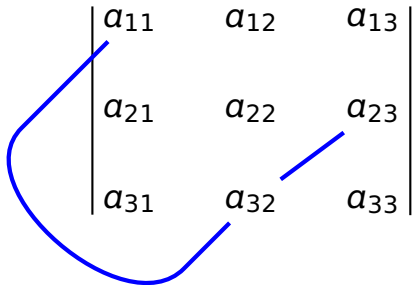
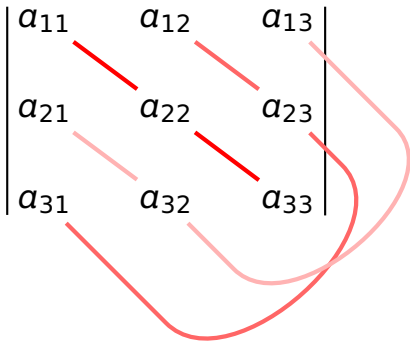


$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{matrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{matrix}$$

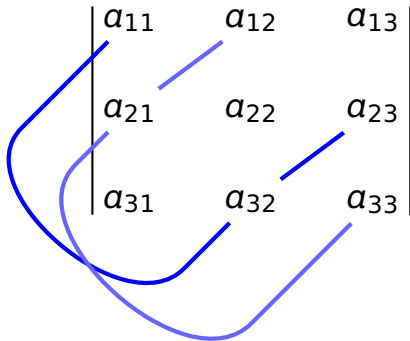
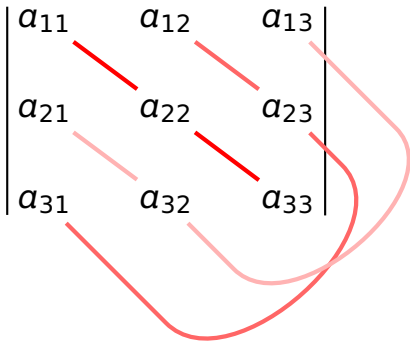
规律



为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{matrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{matrix}$$

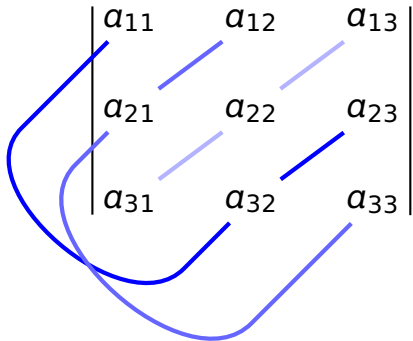
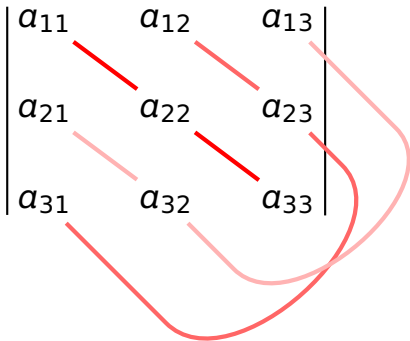
规律



为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{matrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{matrix}$$

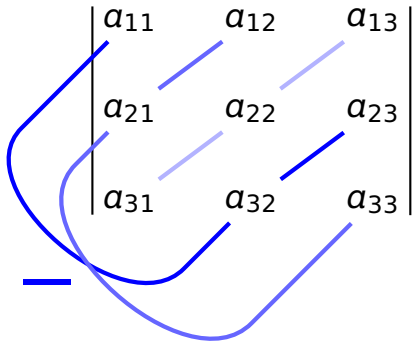
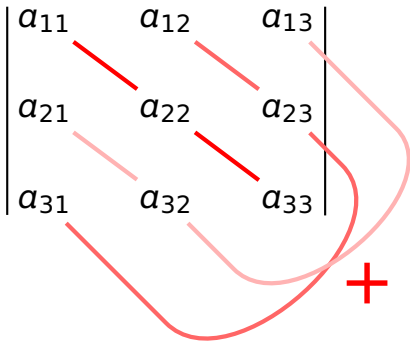
规律



为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{matrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{matrix}$$

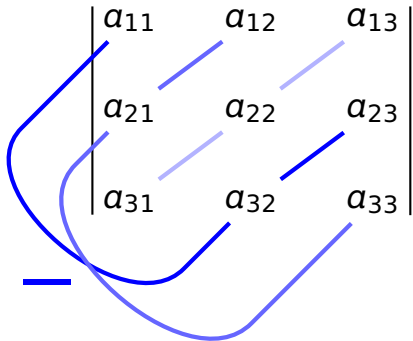
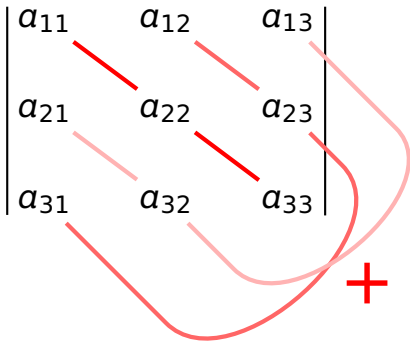
规律

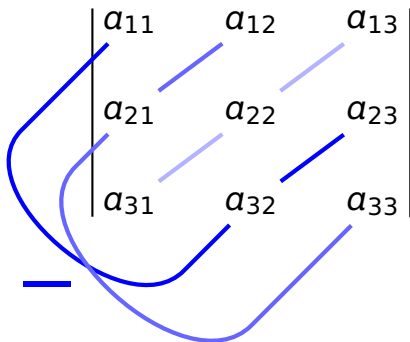
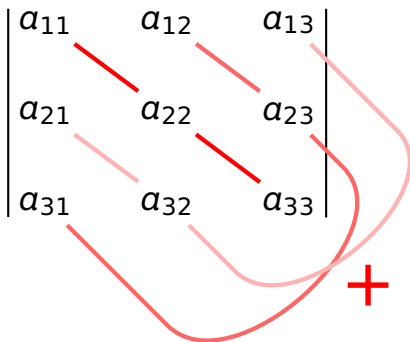


为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{matrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{matrix}$$

规律（不同行不同列的 3 个元素乘积，共 $3! = 6$ 个）

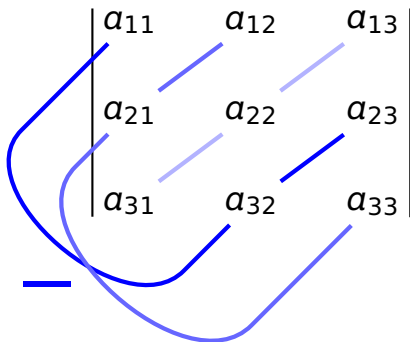
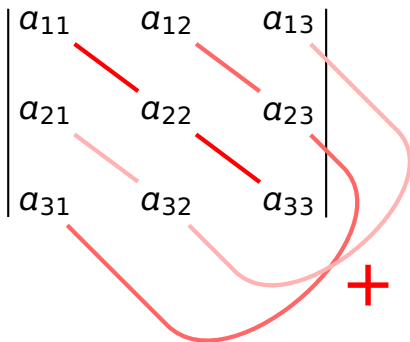




例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} =$$

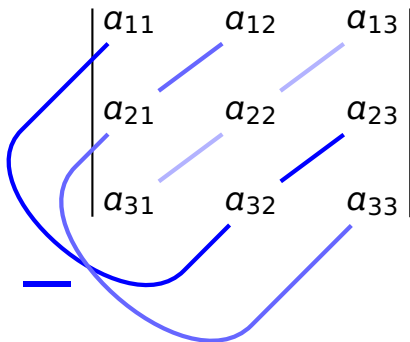
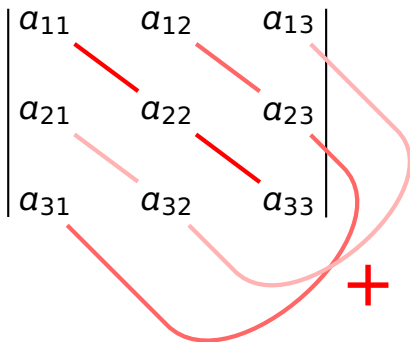
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0$$

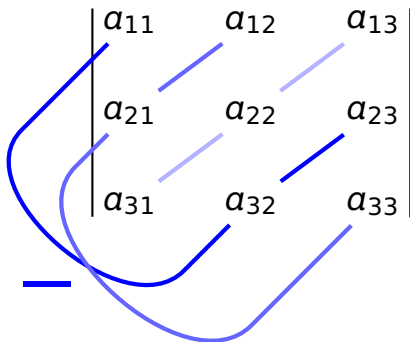
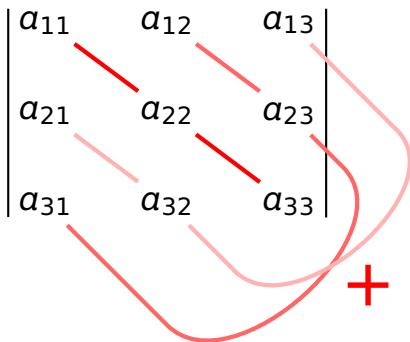
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 - 1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1)$$

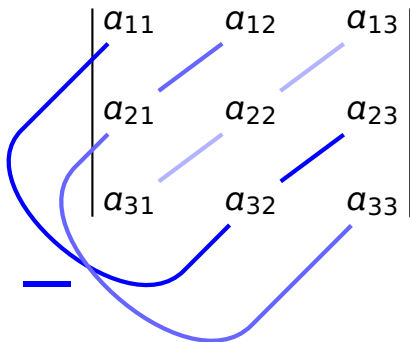
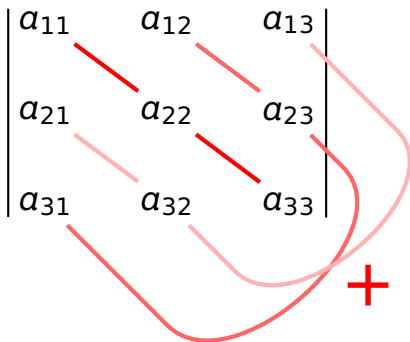
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{aligned} &1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ &- 1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{aligned} = -58$$

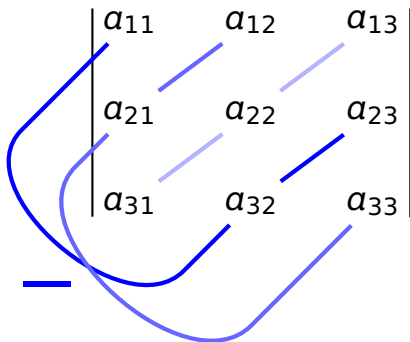
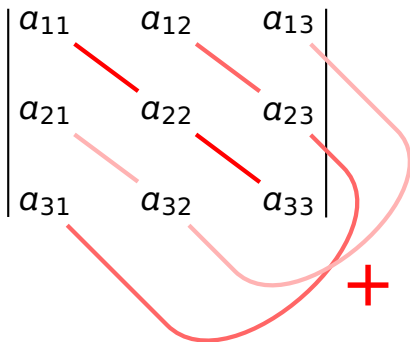
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{aligned} &1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ &- 1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{aligned} = -58$$

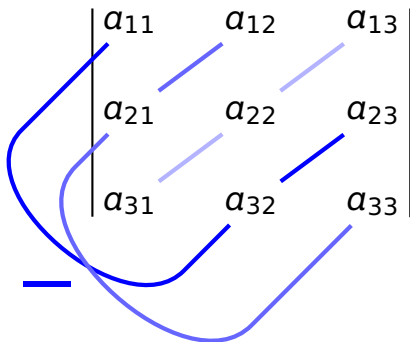
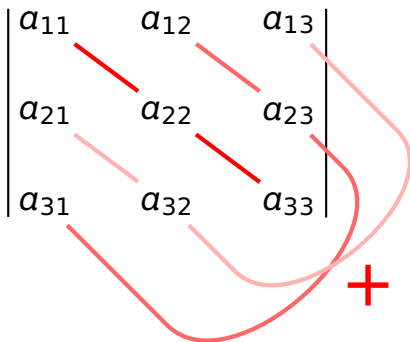
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = 1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{aligned} &1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ &- 1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{aligned} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{aligned} &1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4 \\ &- 1 \times 0 \times 4 - 0 \times 3 \times 1 - (-1) \times 5 \times 1 \end{aligned}$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{aligned} & 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ & - 1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{aligned} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{aligned} & 1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4 \\ & - 1 \times 0 \times 4 - 0 \times 3 \times 1 - (-1) \times 5 \times 1 \end{aligned} = -2$$

练习 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 _____

练习 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 _____

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} =$$

练习 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 _____

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2$$

练习 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 _____

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 - a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1$$

练习 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 _____

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{matrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{matrix} = a^2 + b^2$$

练习 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 _____

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{matrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{matrix} = a^2 + b^2$$

所以 $a \neq 0$ 或 $b \neq 0$ 。

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为：

,

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为：

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} - b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}} =$$

,

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为：

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} - b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{\begin{matrix} a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} \\ - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31} \end{matrix}} = \frac{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

,

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为：

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} - b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

,

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} - b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}},$$

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} - b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}, \quad z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} - b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\quad}{D}$$

$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\quad}{D}, \quad z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\quad}{D}$$

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} - b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{D_x}{D}$$

$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{D'}{D}, \quad z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{D''}{D}$$

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} - b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{D_x}{D}$$

$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{D_y}{D}, \quad z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{D_z}{D}$$

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} - b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{D_x}{D}$$

$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{D_y}{D}, \quad z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{D_z}{D}$$

小结

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D}$$

小结

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

1. 当 $D \neq 0$ 时,

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D}$$

小结

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

1. 当 $D \neq 0$ 时, 方程有唯一解:

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D}$$

小结

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

1. 当 $D \neq 0$ 时, 方程有唯一解:

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D}$$

2. 当 $D = 0$ 时,

小结

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

1. 当 $D \neq 0$ 时, 方程有唯一解:

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D}$$

2. 当 $D = 0$ 时, 方程或者无解、或者有无穷多的解。

例 求解三元线性方程组
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

例 求解三元线性方程组
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

解

$$x = \frac{D_x}{D} = \underline{\hspace{2cm}}$$

$$y = \frac{D_y}{D} = \underline{\hspace{2cm}}$$

$$z = \frac{D_z}{D} = \underline{\hspace{2cm}}$$

例 求解三元线性方程组
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 9 \\ 0 & 1 & 8 \\ 4 & -3 & -2 \end{vmatrix}}$$

$$y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 \\ 2 & 8 \\ 4 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 9 \\ 0 & 1 & 8 \\ 4 & -3 & -2 \end{vmatrix}}$$

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 1 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 9 \\ 0 & 1 & 8 \\ 4 & -3 & -2 \end{vmatrix}}$$

例 求解三元线性方程组
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}}{D} \quad \text{X}$$

$$y = \frac{D_y}{D} = \frac{\quad}{\quad}$$

$$z = \frac{D_z}{D} = \frac{\quad}{\quad}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$ ($\begin{cases} x + 0y + 2z = 9 \end{cases}$)

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}}{D} \quad \text{X}$$

$$y = \frac{D_y}{D} = \frac{\quad}{\quad}$$

$$z = \frac{D_z}{D} = \frac{\quad}{\quad}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$ $\left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \end{cases} \right)$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}}{D} \quad \text{X}$$

$$y = \frac{D_y}{D} = \frac{\quad}{\quad}$$

$$z = \frac{D_z}{D} = \frac{\quad}{\quad}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$ $\left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}}{D} \quad \text{X}$$

$$y = \frac{D_y}{D} = \frac{\quad}{\quad}$$

$$z = \frac{D_z}{D} = \frac{\quad}{\quad}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$ $\left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}{D}$$

$$y = \frac{D_y}{D} = \underline{\hspace{2cm}}$$

$$z = \frac{D_z}{D} = \underline{\hspace{2cm}}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$ $\left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

$$y = \frac{D_y}{D} = \underline{\hspace{2cm}}$$

$$z = \frac{D_z}{D} = \underline{\hspace{2cm}}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \quad \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

$$y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \quad \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

$$y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \quad \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

$$y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$ $\left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = -13$$

$$y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$ $\left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13}$$

$$y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \quad \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1, \quad y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \quad \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1, \quad y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13}$$

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \quad \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1, \quad y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-26}{-13}$$

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \quad \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1, \quad y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-26}{-13} = 2$$

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \quad \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1, \quad y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-26}{-13} = 2$$

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-52}{-13}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \quad \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1, \quad y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-26}{-13} = 2$$

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-52}{-13} = 4$$

例 求解三元线性方程组
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

例 求解三元线性方程组
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

另解 先利用公式求出 x

$$x = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1,$$

例 求解三元线性方程组
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

另解 先利用公式求出 x

$$x = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1,$$

代入方程得

例 求解三元线性方程组
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

另解 先利用公式求出 x

$$x = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1,$$

代入方程得

$$\begin{cases} 1 + 2z = 9 \\ 4 - 3y = -2 \end{cases}$$

例 求解三元线性方程组
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

另解 先利用公式求出 x

$$x = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1,$$

代入方程得

$$\begin{cases} 1 + 2z = 9 \\ 4 - 3y = -2 \end{cases} \Rightarrow \begin{cases} z = 4 \\ y = 2 \end{cases}$$

例 求解三元线性方程组
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

另解 先利用公式求出 x

$$x = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1,$$

代入方程得

$$\begin{cases} 1 + 2z = 9 \\ 4 - 3y = -2 \end{cases} \Rightarrow \begin{cases} z = 4 \\ y = 2 \end{cases}$$

所以方程的解是
$$\begin{cases} x = 1 \\ y = 2 \\ z = 4 \end{cases}$$

一般地, n 元 n 方程的线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \quad \quad \quad \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示:

$$x_1 = \frac{D_1}{D}$$

$$x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

一般地, n 元 n 方程的线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \quad \quad \quad \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示:

$$x_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{D}, \quad x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

一般地, n 元 n 方程的线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \quad \quad \quad \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示:

$$x_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

一般地, n 元 n 方程的线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \quad \quad \quad \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示: (称为**克莱姆法则**)

$$x_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

一般地, n 元 n 方程的线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \quad \quad \quad \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示: (称为**克莱姆法则**)

$$x_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

那么, 如何**定义行列式**,

一般地, n 元 n 方程的线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \quad \quad \quad \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示: (称为**克莱姆法则**)

$$x_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

那么, 如何**定义行列式**, 如何**快捷计算行列式**?

补充：方程组的几何理解

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

补充：方程组的几何理解

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

- 每条方程表示平面上的一条直线
- 方程组的解表示两条直线的交点

补充：方程组的几何理解

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

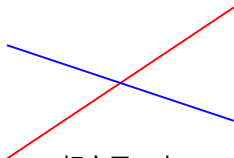
- 每条方程表示平面上的一条直线
- 方程组的解表示两条直线的交点
- 平面上两条直线的位置关系有三种：

补充：方程组的几何理解

2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

- 每条方程表示平面上的一条直线
- 方程组的解表示两条直线的交点
- 平面上两条直线的位置关系有三种：



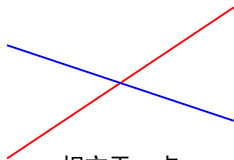
相交于一点

补充：方程组的几何理解

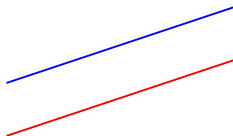
2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

- 每条方程表示平面上的一条直线
- 方程组的解表示两条直线的交点
- 平面上两条直线的位置关系有三种：



相交于一点



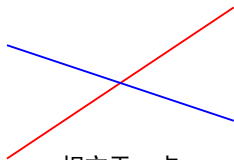
平行不相交

补充：方程组的几何理解

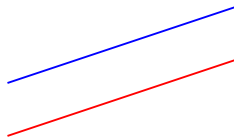
2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

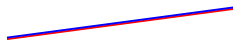
- 每条方程表示平面上的一条直线
- 方程组的解表示两条直线的交点
- 平面上两条直线的位置关系有三种：



相交于一点



平行不相交



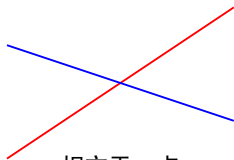
重合

补充：方程组的几何理解

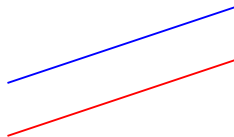
2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

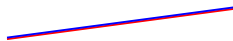
- 每条方程表示平面上的一条直线
- 方程组的解表示两条直线的交点
- 平面上两条直线的位置关系有三种：



相交于一点



平行不相交



重合

- 所以方程组的解有三种情况：

有唯一解、无解、有无穷多的解

3 元 3 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

3 元 3 方程的线性方程组

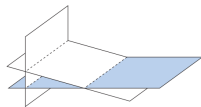
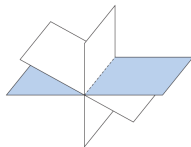
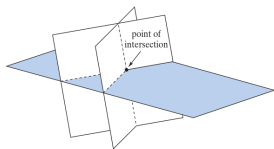
$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

- 每条方程表示空间上的一个平面
- 方程组的解表示三个平面的交点

3 元 3 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

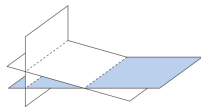
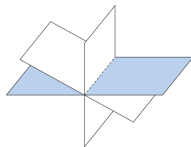
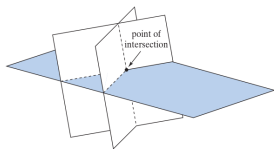
- 每条方程表示空间上的一个平面
- 方程组的解表示三个平面的交点
- 空间上三个平面的位置关系有若干种，例如：



3 元 3 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

- 每条方程表示空间上的一个平面
- 方程组的解表示三个平面的交点
- 空间上三个平面的位置关系有若干种，例如：



- 所以方程组的解有三种情况：

有唯一解、有无穷多的解、无解