

§2.2 矩阵的运算

数学系 梁卓滨

2018 - 2019 学年上学期

矩阵的加法运算

定义 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则定义

$$A + B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$

矩阵的加法运算

定义 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则定义

$$A + B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$
$$\stackrel{\text{def}}{=} \begin{pmatrix} & & & \end{pmatrix}_{m \times n}$$

矩阵的加法运算

定义 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则定义

$$A + B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$
$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} + b_{11} \\ \\ \\ \end{pmatrix}_{m \times n}$$

矩阵的加法运算

定义 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则定义

$$A + B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$
$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} + b_{m1} & a_{m2} + b_{m2} & \cdots & a_{mn} + b_{mn} \end{pmatrix}_{m \times n}$$

矩阵的加法运算

定义 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则定义

$$A + B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$
$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & \\ & & & \\ & & & \\ & & & \end{pmatrix}_{m \times n}$$

矩阵的加法运算

定义 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则定义

$$A + B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$
$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\ & & & \\ & & & \\ & & & \end{pmatrix}_{m \times n}$$

矩阵的加法运算

定义 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则定义

$$A + B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$
$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \\ & & & \\ & & & \end{pmatrix}_{m \times n}$$

矩阵的加法运算

定义 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则定义

$$A + B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$
$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \end{pmatrix}_{m \times n}$$

矩阵的加法运算

定义 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则定义

$$A + B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$
$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} + b_{m1} & a_{m2} + b_{m2} & \cdots & a_{mn} + b_{mn} \end{pmatrix}_{m \times n}$$

矩阵的加法运算

定义 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则定义

$$\begin{aligned} A + B &= \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n} \\ &\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} + b_{m1} & a_{m2} + b_{m2} & \cdots & a_{mn} + b_{mn} \end{pmatrix}_{m \times n} \\ &= (a_{ij} + b_{ij})_{m \times n} \end{aligned}$$

矩阵的加法运算

定义 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则定义

$$\begin{aligned} A + B &= \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n} \\ &\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} + b_{m1} & a_{m2} + b_{m2} & \cdots & a_{mn} + b_{mn} \end{pmatrix}_{m \times n} \\ &= (a_{ij} + b_{ij})_{m \times n} \end{aligned}$$

称为矩阵 A , B 的**和**。

矩阵的减法运算

矩阵 A , B 的差定义为:

$$A - B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} - \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$

矩阵的减法运算

矩阵 A , B 的差定义为:

$$A - B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} - \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$
$$\stackrel{\text{def}}{=} \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}_{m \times n}$$

矩阵的减法运算

矩阵 A , B 的差定义为:

$$A - B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} - \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$
$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} - b_{11} \\ \\ \\ \end{pmatrix}_{m \times n}$$

矩阵的减法运算

矩阵 A , B 的差定义为:

$$A - B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} - \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$
$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} - b_{11} & a_{12} - b_{12} & \cdots & a_{1n} - b_{1n} \\ a_{21} - b_{21} & a_{22} - b_{22} & \cdots & a_{2n} - b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} - b_{m1} & a_{m2} - b_{m2} & \cdots & a_{mn} - b_{mn} \end{pmatrix}_{m \times n}$$

矩阵的减法运算

矩阵 A , B 的差定义为:

$$A - B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} - \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$
$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} - b_{11} & a_{12} - b_{12} & \cdots & \\ & & & \\ & & & \\ & & & \end{pmatrix}_{m \times n}$$

矩阵的减法运算

矩阵 A , B 的差定义为:

$$A - B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} - \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$
$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} - b_{11} & a_{12} - b_{12} & \cdots & a_{1n} - b_{1n} \\ & & & \\ & & & \\ & & & \end{pmatrix}_{m \times n}$$

矩阵的减法运算

矩阵 A , B 的差定义为:

$$A - B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} - \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$
$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} - b_{11} & a_{12} - b_{12} & \cdots & a_{1n} - b_{1n} \\ a_{21} - b_{21} & a_{22} - b_{22} & \cdots & a_{2n} - b_{2n} \\ & & & \\ & & & \end{pmatrix}_{m \times n}$$

矩阵的减法运算

矩阵 A , B 的差定义为:

$$A - B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} - \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$
$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} - b_{11} & a_{12} - b_{12} & \cdots & a_{1n} - b_{1n} \\ a_{21} - b_{21} & a_{22} - b_{22} & \cdots & a_{2n} - b_{2n} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \end{pmatrix}_{m \times n}$$

矩阵的减法运算

矩阵 A , B 的差定义为:

$$A - B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} - \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$
$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} - b_{11} & a_{12} - b_{12} & \cdots & a_{1n} - b_{1n} \\ a_{21} - b_{21} & a_{22} - b_{22} & \cdots & a_{2n} - b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} - b_{m1} & a_{m2} - b_{m2} & \cdots & a_{mn} - b_{mn} \end{pmatrix}_{m \times n}$$

矩阵的减法运算

矩阵 A , B 的差定义为:

$$\begin{aligned} A - B &= \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} - \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n} \\ &\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} - b_{11} & a_{12} - b_{12} & \cdots & a_{1n} - b_{1n} \\ a_{21} - b_{21} & a_{22} - b_{22} & \cdots & a_{2n} - b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} - b_{m1} & a_{m2} - b_{m2} & \cdots & a_{mn} - b_{mn} \end{pmatrix}_{m \times n} \\ &= (a_{ij} - b_{ij})_{m \times n} \end{aligned}$$

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$, 求 $A + B$ 和 $A - B$

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$, 求 $A + B$ 和 $A - B$

解

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} =$$

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$, 求 $A + B$ 和 $A - B$

解

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}_{2 \times 3}$$

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$, 求 $A + B$ 和 $A - B$

解

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & & \\ & & \\ & & \end{pmatrix}_{2 \times 3}$$

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$, 求 $A + B$ 和 $A - B$

解

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & \quad \\ \quad & \quad & \quad \end{pmatrix}_{2 \times 3}$$

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$, 求 $A + B$ 和 $A - B$

解

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 8 \\ 6 & 5 & 5 \end{pmatrix}_{2 \times 3}$$

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$, 求 $A + B$ 和 $A - B$

解

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 8 \\ 6 & 5 & 5 \end{pmatrix}_{2 \times 3}$$

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$, 求 $A + B$ 和 $A - B$

解

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 8 \\ 6 & 5 & 5 \end{pmatrix}_{2 \times 3}$$

$$A - B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} =$$

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$, 求 $A + B$ 和 $A - B$

解

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 8 \\ 6 & 5 & 5 \end{pmatrix}_{2 \times 3}$$

$$A - B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}_{2 \times 3}$$

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$, 求 $A + B$ 和 $A - B$

解

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 8 \\ 6 & 5 & 5 \end{pmatrix}_{2 \times 3}$$

$$A - B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} -1 & & \\ & & \\ & & \end{pmatrix}_{2 \times 3}$$

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$, 求 $A + B$ 和 $A - B$

解

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 8 \\ 6 & 5 & 5 \end{pmatrix}_{2 \times 3}$$

$$A - B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 4 & 2 \\ -8 & -1 & 3 \end{pmatrix}_{2 \times 3}$$

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$, 求 $A + B$ 和 $A - B$

解

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 8 \\ 6 & 5 & 5 \end{pmatrix}_{2 \times 3}$$

$$A - B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 4 & 2 \\ -8 & -1 & 3 \end{pmatrix}_{2 \times 3}$$

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$, 求 $A + B$ 和 $A - B$

解

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 8 \\ 6 & 5 & 5 \end{pmatrix}_{2 \times 3}$$

$$A - B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 4 & 2 \\ -8 & -1 & 3 \end{pmatrix}_{2 \times 3}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

1. $A + B = B + A$

2. $(A + B) + C = A + (B + C)$

3. $A + O = A$

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + O = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$,

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

1. $A + B = B + A$

2. $(A + B) + C = A + (B + C)$

3. $A + O = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$A + B =$$

$$B + A =$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + O = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} =$$

$$B + A =$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + O = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (\quad)_{m \times n},$$

$$B + A =$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + O = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A =$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + O = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} =$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + O = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (\quad)_{m \times n}.$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + O = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (b_{ij} + a_{ij})_{m \times n}.$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + O = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (b_{ij} + a_{ij})_{m \times n}.$$

所以 $A + B = B + A$ 。

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + O = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (b_{ij} + a_{ij})_{m \times n}.$$

所以 $A + B = B + A$ 。另外

$$A + O =$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + O = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (b_{ij} + a_{ij})_{m \times n}.$$

所以 $A + B = B + A$ 。另外

$$A + O = (a_{ij})_{m \times n} + (0)_{m \times n} =$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + O = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (b_{ij} + a_{ij})_{m \times n}.$$

所以 $A + B = B + A$ 。另外

$$A + O = (a_{ij})_{m \times n} + (0)_{m \times n} = (\quad)_{m \times n}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + O = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (b_{ij} + a_{ij})_{m \times n}.$$

所以 $A + B = B + A$ 。另外

$$A + O = (a_{ij})_{m \times n} + (0)_{m \times n} = (a_{ij} + 0)_{m \times n}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + O = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (b_{ij} + a_{ij})_{m \times n}.$$

所以 $A + B = B + A$ 。另外

$$A + O = (a_{ij})_{m \times n} + (0)_{m \times n} = (a_{ij} + 0)_{m \times n} = (a_{ij})_{m \times n}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, O 是 $m \times n$ 零矩阵, 则

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + O = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$A + B = (a_{ij})_{m \times n} + (b_{ij})_{m \times n} = (a_{ij} + b_{ij})_{m \times n},$$

$$B + A = (b_{ij})_{m \times n} + (a_{ij})_{m \times n} = (b_{ij} + a_{ij})_{m \times n}.$$

所以 $A + B = B + A$ 。另外

$$A + O = (a_{ij})_{m \times n} + (0)_{m \times n} = (a_{ij} + 0)_{m \times n} = (a_{ij})_{m \times n} = A.$$

矩阵的数乘

定义 设 $A = (a_{ij})_{m \times n}$, k 为数, 则定义

$$kA = k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

矩阵的数乘

定义 设 $A = (a_{ij})_{m \times n}$, k 为数, 则定义

$$kA = k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix}$$

矩阵的数乘

定义 设 $A = (a_{ij})_{m \times n}$, k 为数, 则定义

$$\begin{aligned} kA &= k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix} \\ &= (ka_{ij})_{m \times n} \end{aligned}$$

矩阵的数乘

定义 设 $A = (a_{ij})_{m \times n}$, k 为数, 则定义

$$kA = k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix} \\ = (ka_{ij})_{m \times n}$$

称为数 k 与矩阵 A 的数乘。

矩阵的数乘

定义 设 $A = (a_{ij})_{m \times n}$, k 为数, 则定义

$$kA = k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix} \\ = (ka_{ij})_{m \times n}$$

称为数 k 与矩阵 A 的数乘。

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, 求 $2A$

矩阵的数乘

定义 设 $A = (a_{ij})_{m \times n}$, k 为数, 则定义

$$\begin{aligned} kA &= k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix} \\ &= (ka_{ij})_{m \times n} \end{aligned}$$

称为数 k 与矩阵 A 的数乘。

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, 求 $2A$

解 $2A = 2 \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} =$

矩阵的数乘

定义 设 $A = (a_{ij})_{m \times n}$, k 为数, 则定义

$$\begin{aligned} kA &= k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix} \\ &= (ka_{ij})_{m \times n} \end{aligned}$$

称为数 k 与矩阵 A 的数乘。

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, 求 $2A$

解 $2A = 2 \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} = \begin{pmatrix} & & \\ & & \end{pmatrix}$

矩阵的数乘

定义 设 $A = (a_{ij})_{m \times n}$, k 为数, 则定义

$$\begin{aligned} kA &= k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix} \\ &= (ka_{ij})_{m \times n} \end{aligned}$$

称为数 k 与矩阵 A 的数乘。

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, 求 $2A$

解 $2A = 2 \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 6 & 10 \\ -2 & 4 & 8 \end{pmatrix}$

矩阵的数乘

定义 设 $A = (a_{ij})_{m \times n}$, k 为数, 则定义

$$\begin{aligned} kA &= k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \stackrel{\text{def}}{=} \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{pmatrix} \\ &= (ka_{ij})_{m \times n} \end{aligned}$$

称为数 k 与矩阵 A 的数乘。

例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, 求 $2A$

解 $2A = 2 \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 6 & 10 \\ -2 & 4 & 8 \end{pmatrix}$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

$$\begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

解

$$\begin{aligned} 3A + 2B - 4C &= 3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4 \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix} \end{aligned}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

解

$$\begin{aligned} 3A + 2B - 4C &= 3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4 \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} \qquad \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix} \end{aligned}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

解

$$\begin{aligned} 3A + 2B - 4C &= 3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4 \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} \qquad \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix} \end{aligned}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

解

$$\begin{aligned} 3A + 2B - 4C &= 3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4 \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix} \end{aligned}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

解

$$\begin{aligned} 3A + 2B - 4C &= 3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4 \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix} \end{aligned}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

解

$$\begin{aligned} 3A + 2B - 4C &= 3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4 \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix} \end{aligned}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

解

$$\begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

解

$$\begin{aligned} 3A + 2B - 4C &= 3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4 \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix} \end{aligned}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

解

$$X = \frac{1}{3}(B - 5A) =$$

$$\begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

解

$$\begin{aligned} 3A + 2B - 4C &= 3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4 \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix} \end{aligned}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

解

$$\begin{aligned} X &= \frac{1}{3}(B - 5A) = \frac{1}{3} \left(\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 5 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \right) \\ &= \begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix} \end{aligned}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

解

$$\begin{aligned} 3A + 2B - 4C &= 3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4 \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix} \end{aligned}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

解

$$\begin{aligned} X &= \frac{1}{3}(B - 5A) = \frac{1}{3} \left(\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 5 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \right) \\ &= \frac{1}{3} \left(\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - \begin{pmatrix} 5 & 10 \\ 15 & 20 \end{pmatrix} \right) = \begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix} \end{aligned}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

解

$$\begin{aligned} 3A + 2B - 4C &= 3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4 \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix} \end{aligned}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

解

$$\begin{aligned} X &= \frac{1}{3}(B - 5A) = \frac{1}{3} \left(\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 5 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \right) \\ &= \frac{1}{3} \left(\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - \begin{pmatrix} 5 & 10 \\ 15 & 20 \end{pmatrix} \right) = \frac{1}{3} \begin{pmatrix} -2 & -5 \\ -8 & -14 \end{pmatrix} = \begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix} \end{aligned}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

解

$$\begin{aligned} 3A + 2B - 4C &= 3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4 \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix} \end{aligned}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

解

$$\begin{aligned} X &= \frac{1}{3}(B - 5A) = \frac{1}{3} \left(\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 5 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \right) \\ &= \frac{1}{3} \left(\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - \begin{pmatrix} 5 & 10 \\ 15 & 20 \end{pmatrix} \right) = \frac{1}{3} \begin{pmatrix} -2 & -5 \\ -8 & -14 \end{pmatrix} = \begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix} \end{aligned}$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

$$, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix}, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} \quad , \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} \quad , \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, k, l 是数, 则

1. $k(A + B) = kA + kB$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, k, l 是数, 则

1. $k(A + B) = kA + kB$
2. $(k + l)A = kA + lA$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, k, l 是数, 则

1. $k(A + B) = kA + kB$
2. $(k + l)A = kA + lA$
3. $(kl)A = k(lA)$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, k, l 是数, 则

1. $k(A + B) = kA + kB$
2. $(k + l)A = kA + lA$
3. $(kl)A = k(lA)$
4. $1 \cdot A = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, k, l 是数, 则

1. $k(A + B) = kA + kB$
2. $(k + l)A = kA + lA$
3. $(kl)A = k(lA)$
4. $1 \cdot A = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$k(A + B) =$$

$$kA + kB =$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, k, l 是数, 则

1. $k(A + B) = kA + kB$
2. $(k + l)A = kA + lA$
3. $(kl)A = k(lA)$
4. $1 \cdot A = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$k(A + B) = k(\quad)_{m \times n}$$

$$kA + kB =$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, k, l 是数, 则

1. $k(A + B) = kA + kB$
2. $(k + l)A = kA + lA$
3. $(kl)A = k(lA)$
4. $1 \cdot A = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$k(A + B) = k(a_{ij} + b_{ij})_{m \times n}$$

$$kA + kB =$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, k, l 是数, 则

1. $k(A + B) = kA + kB$
2. $(k + l)A = kA + lA$
3. $(kl)A = k(lA)$
4. $1 \cdot A = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$k(A + B) = k(a_{ij} + b_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

$$kA + kB =$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, k, l 是数, 则

1. $k(A + B) = kA + kB$
2. $(k + l)A = kA + lA$
3. $(kl)A = k(lA)$
4. $1 \cdot A = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$k(A + B) = k(a_{ij} + b_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

$$kA + kB = (ka_{ij})_{m \times n} + (kb_{ij})_{m \times n}$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, k, l 是数, 则

1. $k(A + B) = kA + kB$
2. $(k + l)A = kA + lA$
3. $(kl)A = k(lA)$
4. $1 \cdot A = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$k(A + B) = k(a_{ij} + b_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

$$kA + kB = (ka_{ij})_{m \times n} + (kb_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, k, l 是数, 则

1. $k(A + B) = kA + kB$
2. $(k + l)A = kA + lA$
3. $(kl)A = k(lA)$
4. $1 \cdot A = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$k(A + B) = k(a_{ij} + b_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

$$kA + kB = (ka_{ij})_{m \times n} + (kb_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

所以 $k(A + B) = kA + kB$ 。

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 $(2A - Y) - 2(B + Y) = O$, 求 Y

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 $(2A - Y) - 2(B + Y) = O$, 求 Y

解 $Y = \frac{2}{3}(A - B)$

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 $(2A - Y) - 2(B + Y) = O$, 求 Y

解 $Y = \frac{2}{3}(A - B)$, 所以

$$Y = \frac{2}{3}(A - B) = \frac{2}{3} \left(\begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix} \right)$$

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 $(2A - Y) - 2(B + Y) = O$, 求 Y

解 $Y = \frac{2}{3}(A - B)$, 所以

$$\begin{aligned} Y &= \frac{2}{3}(A - B) = \frac{2}{3} \left(\begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix} \right) \\ &= \frac{2}{3} \begin{pmatrix} -3 & -1 & -1 & 1 \\ 4 & 0 & 4 & 0 \\ 1 & 3 & 3 & 5 \end{pmatrix} \end{aligned}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$aA + bB + cC =$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} & \end{pmatrix} \end{aligned}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} a + b - c & b \\ 2a + 3b + c & a - c \end{pmatrix} \end{aligned}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} a + b - c & b \\ 2a + 3b + c & a - c \end{pmatrix} \end{aligned}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} a + b - c & b \\ 2a + 3b + c & \end{pmatrix} \end{aligned}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} a + b - c & b \\ 2a + 3b + c & a - c \end{pmatrix} \end{aligned}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} a + b - c & b \\ 2a + 3b + c & a - c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} a + b - c & b \\ 2a + 3b + c & a - c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

所以

{

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} a + b - c & b \\ 2a + 3b + c & a - c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

所以

$$\begin{cases} a + b - c = 1 \\ \end{cases}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} a + b - c & b \\ 2a + 3b + c & a - c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

所以

$$\begin{cases} a + b - c = 1 \\ b = 0 \end{cases}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} a + b - c & b \\ 2a + 3b + c & a - c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

所以

$$\begin{cases} a + b - c = 1 \\ b = 0 \\ 2a + 3b + c = 0 \end{cases}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} a + b - c & b \\ 2a + 3b + c & a - c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

所以

$$\begin{cases} a + b - c = 1 \\ b = 0 \\ 2a + 3b + c = 0 \\ a - c = 1 \end{cases}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} a + b - c & b \\ 2a + 3b + c & a - c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

所以

$$\begin{cases} a + b - c = 1 \\ b = 0 \\ 2a + 3b + c = 0 \\ a - c = 1 \end{cases} \Rightarrow \begin{cases} b = 0 \end{cases}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} a + b - c & b \\ 2a + 3b + c & a - c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

所以

$$\begin{cases} a + b - c = 1 \\ b = 0 \\ 2a + 3b + c = 0 \\ a - c = 1 \end{cases} \Rightarrow \begin{cases} a = \frac{1}{3} \\ b = 0 \end{cases}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} a + b - c & b \\ 2a + 3b + c & a - c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

所以

$$\begin{cases} a + b - c = 1 \\ b = 0 \\ 2a + 3b + c = 0 \\ a - c = 1 \end{cases} \Rightarrow \begin{cases} a = \frac{1}{3} \\ b = 0 \\ c = -\frac{2}{3} \end{cases}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$, $B = (b_{kj})_{l \times n}$, 定义矩阵 A , B 的乘积为 $m \times n$ 矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$, $B = (b_{kj})_{l \times n}$, 定义矩阵 A , B 的乘积为 $m \times n$ 矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$, $B = (b_{kj})_{l \times n}$, 定义矩阵 A , B 的乘积为 $m \times n$ 矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

$c_{ij} = A$ 第 i 行与 B 第 j 列对应元素乘积的和

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$, $B = (b_{kj})_{l \times n}$, 定义矩阵 A , B 的乘积为 $m \times n$ 矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

$c_{ij} = A$ 第 i 行与 B 第 j 列对应元素乘积的和

即

$$a_{i1} \quad a_{i2} \quad \cdots \quad a_{il}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$, $B = (b_{kj})_{l \times n}$, 定义矩阵 A , B 的乘积为 $m \times n$ 矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

$c_{ij} = A$ 第 i 行与 B 第 j 列对应元素乘积的和

即

$$a_{i1}b_{1j} \quad a_{i2}b_{2j} \quad \cdots \quad a_{il}b_{lj}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$, $B = (b_{kj})_{l \times n}$, 定义矩阵 A , B 的乘积为 $m \times n$ 矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

$c_{ij} = A$ 第 i 行与 B 第 j 列对应元素乘积的和

即

$$a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$, $B = (b_{kj})_{l \times n}$, 定义矩阵 A , B 的乘积为 $m \times n$ 矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

$c_{ij} = A$ 第 i 行与 B 第 j 列对应元素乘积的和

即

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$, $B = (b_{kj})_{l \times n}$, 定义矩阵 A , B 的乘积为 $m \times n$ 矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

$c_{ij} = A$ 第 i 行与 B 第 j 列对应元素乘积的和

即

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj} \qquad a_{ik}b_{kj}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$, $B = (b_{kj})_{l \times n}$, 定义矩阵 A , B 的乘积为 $m \times n$ 矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

$c_{ij} = A$ 第 i 行与 B 第 j 列对应元素乘积的和

即

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj} \quad \sum_{k=1}^l a_{ik}b_{kj}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$, $B = (b_{kj})_{l \times n}$, 定义矩阵 A , B 的乘积为 $m \times n$ 矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

c_{ij} = A 第 i 行与 B 第 j 列对应元素乘积的和

即

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj} = \sum_{k=1}^l a_{ik}b_{kj}$$

矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
$$= \begin{pmatrix} c_{11} & \cdots & & \cdots & c_{1n} \\ \vdots & & \vdots & & \vdots \\ & \cdots & c_{ij} & \cdots & \\ \vdots & & \vdots & & \vdots \\ c_{m1} & \cdots & & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$

矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
$$= \begin{pmatrix} c_{11} & \cdots & & \cdots & c_{1n} \\ \vdots & & \vdots & & \vdots \\ & \cdots & c_{ij} & \cdots & \\ \vdots & & \vdots & & \vdots \\ c_{m1} & \cdots & & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$

$a_{i1} \quad a_{i2} \quad \cdots \quad a_{il}$

矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
$$= \begin{pmatrix} c_{11} & \cdots & \cdots & c_{1n} \\ \vdots & & \vdots & \vdots \\ \cdots & c_{ij} & \cdots & \\ \vdots & \vdots & & \vdots \\ c_{m1} & \cdots & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$
$$a_{i1}b_{1j} \quad a_{i2}b_{2j} \quad \cdots \quad a_{il}b_{lj}$$

矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
$$= \begin{pmatrix} c_{11} & \cdots & \cdots & c_{1n} \\ \vdots & & \vdots & \vdots \\ \cdots & c_{ij} & \cdots & \\ \vdots & \vdots & & \vdots \\ c_{m1} & \cdots & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$
$$a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj}$$

矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
$$= \begin{pmatrix} c_{11} & \cdots & \cdots & c_{1n} \\ \vdots & & \vdots & \vdots \\ \cdots & c_{ij} & \cdots & \\ \vdots & \vdots & & \vdots \\ c_{m1} & \cdots & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$
$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj}$$

矩阵的乘积

例 1 $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3}$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \end{pmatrix}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \end{pmatrix}_{4 \times 3}$$

矩阵的乘积

例 1

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$c_{23} =$

矩阵的乘积

例 1

$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$c_{23} = a_{21} \quad a_{22}$

矩阵的乘积

例 1

$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$
$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 2 设 $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$, 计算 AB

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 2 设 $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$, 计算 AB

解

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} =$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 2 设 $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$, 计算 AB

解

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 2 设 $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$, 计算 AB

解

$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} \color{red}{1} & -2 & -3 \\ \color{red}{2} & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} * & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 2 设 $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$, 计算 AB

解

$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} \color{red}{1} & -2 & -3 \\ \color{red}{2} & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 2 设 $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$, 计算 AB

解

$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & \color{red}{-2} & -3 \\ 2 & \color{red}{-1} & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & \color{red}{*} & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 2 设 $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$, 计算 AB

解

$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & \color{red}{-2} & -3 \\ 2 & \color{red}{-1} & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 2 设 $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$, 计算 AB

解

$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & \color{red}{-3} \\ 2 & -1 & \color{red}{0} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & \color{red}{*} \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 2 设 $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$, 计算 AB

解

$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & \color{red}{-3} \\ 2 & -1 & \color{red}{0} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 2 设 $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$, 计算 AB

解

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 2 设 $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$, 计算 AB

解

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ -3 & 0 & -3 \\ -3 & 0 & -3 \end{pmatrix}_{3 \times 3}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 2 设 $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$, 计算 AB

解

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ -3 & 0 & -3 \\ 5 & -7 & -9 \end{pmatrix}_{3 \times 3}$$

矩阵的乘积

例 3 计算 $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

矩阵的乘积

例 3 计算 $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} =$$

矩阵的乘积

例 3 计算 $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} & & \\ & & \end{pmatrix}_{2 \times 3}$$

矩阵的乘积

例 3 计算 $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & & \\ & & \end{pmatrix}_{2 \times 3}$$

矩阵的乘积

例 3 计算 $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & \quad \\ \quad & \quad & \quad \end{pmatrix}_{2 \times 3}$$

矩阵的乘积

例 3 计算 $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \end{pmatrix}_{2 \times 3}$$

矩阵的乘积

例 3 计算 $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \\ 4 & & \end{pmatrix}_{2 \times 3}$$

矩阵的乘积

例 3 计算 $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \\ 4 & -3 & \end{pmatrix}_{2 \times 3}$$

矩阵的乘积

例 3 计算 $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \\ 4 & -3 & -1 \end{pmatrix}_{2 \times 3}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (\quad)_{1 \times 1}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) =$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3 \times 3}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3 \times 3}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3 \times 3}$$

$$CB = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3 \times 3}$$

$$CB = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix}_{3 \times 1}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3 \times 3}$$

$$CB = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{3 \times 1}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3 \times 3}$$

$$CB = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{3 \times 1}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3 \times 3}$$

$$CB = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{3 \times 1}$$

矩阵的乘积

例 5 设 $A = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}$, 求 AB , BA

矩阵的乘积

例 5 设 $A = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} =$$

矩阵的乘积

例 5 设 $A = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} = \begin{pmatrix} \quad \quad \quad \end{pmatrix}$$

矩阵的乘积

例 5 设 $A = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} = \begin{pmatrix} \quad \quad \quad \end{pmatrix}_{1 \times 2}$$

矩阵的乘积

例 5 设 $A = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} = \begin{pmatrix} 2 & \end{pmatrix}_{1 \times 2}$$

矩阵的乘积

例 5 设 $A = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} = \begin{pmatrix} 2 & 3 \end{pmatrix}_{1 \times 2}$$

矩阵的乘积

例 5 设 $A = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} = \begin{pmatrix} 2 & 3 \end{pmatrix}_{1 \times 2}$$

$$BA = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3}$$

矩阵的乘积

例 5 设 $A = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} = \begin{pmatrix} 2 & 3 \end{pmatrix}_{1 \times 2}$$

$$BA = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \text{ 没有意义!}$$

矩阵的乘积

例 5 设 $A = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} = \begin{pmatrix} 2 & 3 \end{pmatrix}_{1 \times 2}$$

$$BA = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \text{ 没有意义!}$$

注 AB 可以存在, 但 BA 不一定有意义

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} =$$

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} & \\ & \end{pmatrix}$$

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} & \\ & \end{pmatrix}_{2 \times 2}$$

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & \quad \\ \quad & \quad \end{pmatrix}_{2 \times 2}$$

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ & \end{pmatrix}_{2 \times 2}$$

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & \end{pmatrix}_{2 \times 2}$$

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$

$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} =$$

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$

$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} & \\ & \end{pmatrix}$$

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$

$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} & \\ & \end{pmatrix}_{2 \times 2}$$

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$

$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} 0 & \\ & \end{pmatrix}_{2 \times 2}$$

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$

$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2 \times 2}$$

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$

$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2 \times 2}$$

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$

$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2 \times 2}$$

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$

$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2 \times 2}$$

注

1. 即便 AB , BA 都有意义, 也不一定相等。

矩阵的乘法不满足交换律!

矩阵的乘积

例 6 设 $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$, 求 AB , BA

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$

$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}_{2 \times 2}$$

注

1. 即便 AB , BA 都有意义, 也不一定相等。

矩阵的乘法不满足交换律!

2. $BA = 0$ 不能推出 $B = 0$ 或 $A = 0$

矩阵的乘积

注 即便假设 $A \neq 0$, $BA = CA$ 也推不出 $B = C$ 。如

矩阵的乘积

注 即便假设 $A \neq 0$, $BA = CA$ 也推不出 $B = C$ 。如

$$\underbrace{\begin{pmatrix} 2 & 0 \\ 0 & -6 \end{pmatrix}}_B \underbrace{\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}}_A \quad \underbrace{\begin{pmatrix} 0 & -4 \\ 3 & 0 \end{pmatrix}}_C \underbrace{\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}}_A$$

矩阵的乘积

注 即便假设 $A \neq 0$, $BA = CA$ 也推不出 $B = C$ 。如

$$\underbrace{\begin{pmatrix} 2 & 0 \\ 0 & -6 \end{pmatrix}}_B \underbrace{\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}}_A = \underbrace{\begin{pmatrix} 0 & -4 \\ 3 & 0 \end{pmatrix}}_C \underbrace{\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}}_A$$

矩阵的乘积

注 即便假设 $A \neq 0$, $BA = CA$ 也推不出 $B = C$ 。如

$$\underbrace{\begin{pmatrix} 2 & 0 \\ 0 & -6 \end{pmatrix}}_B \underbrace{\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}}_A = \underbrace{\begin{pmatrix} 0 & -4 \\ 3 & 0 \end{pmatrix}}_C \underbrace{\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}}_A$$

- 总结
1. AB 可以存在, 但 BA 不一定有意义
 2. 即便 AB , BA 都有意义, 也不一定相等。矩阵的乘法不满足交换律! (矩阵相乘要注意顺序)
 3. $BA = 0$ 不能推出 $B = 0$ 或 $A = 0$
 4. 即便假设 $A \neq 0$, $BA = CA$ 也推不出 $B = C$ 。

矩阵乘法的运算法则

设下列各式所涉及的矩阵乘法都是有意义，则

1. $(AB)C = A(BC)$
2. $(A + B)C = AC + BC$
3. $C(A + B) = CA + CB$
4. $k(AB) = (kA)B = A(kB)$

矩阵的转置

定义 将 $m \times n$ 矩阵 A 的行与列互换，得到的 $n \times m$ 矩阵，称为矩阵 A 的转置矩阵，记为 A^T 。

矩阵的转置

定义 将 $m \times n$ 矩阵 A 的行与列互换，得到的 $n \times m$ 矩阵，称为矩阵 A 的**转置矩阵**，记为 A^T 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n}$$

矩阵的转置

定义 将 $m \times n$ 矩阵 A 的行与列互换，得到的 $n \times m$ 矩阵，称为矩阵 A 的**转置矩阵**，记为 A^T 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \Rightarrow A^T = \begin{pmatrix} & & & \end{pmatrix}$$

矩阵的转置

定义 将 $m \times n$ 矩阵 A 的行与列互换，得到的 $n \times m$ 矩阵，称为矩阵 A 的**转置矩阵**，记为 A^T 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \Rightarrow A^T = \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}_{n \times m}$$

矩阵的转置

定义 将 $m \times n$ 矩阵 A 的行与列互换，得到的 $n \times m$ 矩阵，称为矩阵 A 的**转置矩阵**，记为 A^T 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \Rightarrow A^T = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{n \times m}$$

矩阵的转置

定义 将 $m \times n$ 矩阵 A 的行与列互换，得到的 $n \times m$ 矩阵，称为矩阵 A 的**转置矩阵**，记为 A^T 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \Rightarrow A^T = \begin{pmatrix} a_{11} & a_{21} & & \\ a_{12} & a_{22} & & \\ \vdots & \vdots & & \\ a_{1n} & a_{2n} & & \end{pmatrix}_{n \times m}$$

矩阵的转置

定义 将 $m \times n$ 矩阵 A 的行与列互换，得到的 $n \times m$ 矩阵，称为矩阵 A 的**转置矩阵**，记为 A^T 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \Rightarrow A^T = \begin{pmatrix} a_{11} & a_{21} & \cdots & \\ a_{12} & a_{22} & \cdots & \\ \vdots & \vdots & & \\ a_{1n} & a_{2n} & \cdots & \end{pmatrix}_{n \times m}$$

矩阵的转置

定义 将 $m \times n$ 矩阵 A 的行与列互换，得到的 $n \times m$ 矩阵，称为矩阵 A 的**转置矩阵**，记为 A^T 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \Rightarrow A^T = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{pmatrix}_{n \times m}$$

矩阵的转置

定义 将 $m \times n$ 矩阵 A 的行与列互换，得到的 $n \times m$ 矩阵，称为矩阵 A 的**转置矩阵**，记为 A^T 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \Rightarrow A^T = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{pmatrix}_{n \times m}$$

注

	A	A^T
位置 (i, j) 上的元素		

矩阵的转置

定义 将 $m \times n$ 矩阵 A 的行与列互换，得到的 $n \times m$ 矩阵，称为矩阵 A 的**转置矩阵**，记为 A^T 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \Rightarrow A^T = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{pmatrix}_{n \times m}$$

注

	A	A^T
位置 (i, j) 上的元素	a_{ij}	

矩阵的转置

定义 将 $m \times n$ 矩阵 A 的行与列互换，得到的 $n \times m$ 矩阵，称为矩阵 A 的**转置矩阵**，记为 A^T 。

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \Rightarrow A^T = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{pmatrix}_{n \times m}$$

注

	A	A^T
位置 (i, j) 上的元素	a_{ij}	a_{ji}

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T =$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} & \\ & \\ & \end{pmatrix}$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} & \\ & \\ & \end{pmatrix}_{3 \times 2}$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} 1 & 2 \\ 4 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$

例 2 设 $x = (x_1 \ x_2 \ \cdots \ x_n)$, $y = (y_1 \ y_2 \ \cdots \ y_n)$, 则

$$x^T y =$$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$

例 2 设 $x = (x_1 \ x_2 \ \cdots \ x_n)$, $y = (y_1 \ y_2 \ \cdots \ y_n)$, 则

$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1}$$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$

例 2 设 $x = (x_1 \ x_2 \ \cdots \ x_n)$, $y = (y_1 \ y_2 \ \cdots \ y_n)$, 则

$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1} (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} =$$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$

例 2 设 $x = (x_1 \ x_2 \ \cdots \ x_n)$, $y = (y_1 \ y_2 \ \cdots \ y_n)$, 则

$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1} (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$

例 2 设 $x = (x_1 \ x_2 \ \cdots \ x_n)$, $y = (y_1 \ y_2 \ \cdots \ y_n)$, 则

$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1} (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}_{n \times n}$$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$

例 2 设 $x = (x_1 \ x_2 \ \cdots \ x_n)$, $y = (y_1 \ y_2 \ \cdots \ y_n)$, 则

$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1} (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} = \begin{pmatrix} x_1 y_1 \\ \vdots \\ x_n y_n \end{pmatrix}_{n \times 1}$$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$

例 2 设 $x = (x_1 \ x_2 \ \cdots \ x_n)$, $y = (y_1 \ y_2 \ \cdots \ y_n)$, 则

$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1} (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} = \begin{pmatrix} x_1 y_1 & x_1 y_2 & \cdots & x_1 y_n \\ x_2 y_1 & x_2 y_2 & \cdots & x_2 y_n \\ \vdots & \vdots & \ddots & \vdots \\ x_n y_1 & x_n y_2 & \cdots & x_n y_n \end{pmatrix}_{n \times n}$$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$

例 2 设 $x = (x_1 \ x_2 \ \cdots \ x_n)$, $y = (y_1 \ y_2 \ \cdots \ y_n)$, 则

$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1} (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} = \begin{pmatrix} x_1 y_1 & x_1 y_2 & \cdots \\ & & \end{pmatrix}_{n \times n}$$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$

例 2 设 $x = (x_1 \ x_2 \ \cdots \ x_n)$, $y = (y_1 \ y_2 \ \cdots \ y_n)$, 则

$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1} (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} = \begin{pmatrix} x_1 y_1 & x_1 y_2 & \cdots & x_1 y_n \\ \vdots & \vdots & \ddots & \vdots \\ x_n y_1 & x_n y_2 & \cdots & x_n y_n \end{pmatrix}_{n \times n}$$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$

例 2 设 $x = (x_1 \ x_2 \ \cdots \ x_n)$, $y = (y_1 \ y_2 \ \cdots \ y_n)$, 则

$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1} (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} = \begin{pmatrix} x_1 y_1 & x_1 y_2 & \cdots & x_1 y_n \\ x_2 y_1 & x_2 y_2 & \cdots & x_2 y_n \\ \vdots & \vdots & \ddots & \vdots \\ x_n y_1 & x_n y_2 & \cdots & x_n y_n \end{pmatrix}_{n \times n}$$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$

例 2 设 $x = (x_1 \ x_2 \ \cdots \ x_n)$, $y = (y_1 \ y_2 \ \cdots \ y_n)$, 则

$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1} (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} = \begin{pmatrix} x_1 y_1 & x_1 y_2 & \cdots & x_1 y_n \\ x_2 y_1 & x_2 y_2 & \cdots & x_2 y_n \\ \vdots & \vdots & & \vdots \\ x_n y_1 & x_n y_2 & \cdots & x_n y_n \end{pmatrix}_{n \times n}$$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$

例 2 设 $x = (x_1 \ x_2 \ \cdots \ x_n)$, $y = (y_1 \ y_2 \ \cdots \ y_n)$, 则

$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1} (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} = \begin{pmatrix} x_1 y_1 & x_1 y_2 & \cdots & x_1 y_n \\ x_2 y_1 & x_2 y_2 & \cdots & x_2 y_n \\ \vdots & \vdots & & \vdots \\ x_n y_1 & x_n y_2 & \cdots & x_n y_n \end{pmatrix}_{n \times n}$$

矩阵的转置

例 1 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, 则 $A^T = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix}_{3 \times 2}$

例 2 设 $x = (x_1 \ x_2 \ \cdots \ x_n)$, $y = (y_1 \ y_2 \ \cdots \ y_n)$, 则

$$x^T y = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1} (y_1 \ y_2 \ \cdots \ y_n)_{1 \times n} = \begin{pmatrix} x_1 y_1 & x_1 y_2 & \cdots & x_1 y_n \\ x_2 y_1 & x_2 y_2 & \cdots & x_2 y_n \\ \vdots & \vdots & & \vdots \\ x_n y_1 & x_n y_2 & \cdots & x_n y_n \end{pmatrix}_{n \times n}$$

例 3 设 $A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$, 计算 AA^T 及 $A^T A$.

例 3 设 $A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$, 计算 AA^T 及 A^TA 。

例 3 设 $A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$, 计算 AA^T 及 A^TA 。

解

$$AA^T = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix}$$

例 3 设 $A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$, 计算 AA^T 及 A^TA 。

解

$$AA^T = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}_{2 \times 2}$$

例 3 设 $A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$, 计算 AA^T 及 A^TA 。

解

$$AA^T = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & \\ & \end{pmatrix}_{2 \times 2}$$

例 3 设 $A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$, 计算 AA^T 及 A^TA 。

解

$$AA^T = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & 2 \\ & \end{pmatrix}_{2 \times 2}$$

例 3 设 $A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$, 计算 AA^T 及 A^TA 。

解

$$AA^T = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & 2 \\ 2 & \end{pmatrix}_{2 \times 2}$$

例 3 设 $A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$, 计算 AA^T 及 A^TA 。

解

$$AA^T = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & 2 \\ 2 & 13 \end{pmatrix}_{2 \times 2}$$

例 3 设 $A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$, 计算 AA^T 及 A^TA 。

解

$$AA^T = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & 2 \\ 2 & 13 \end{pmatrix}_{2 \times 2}$$

$$A^TA = \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$$

例 3 设 $A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$, 计算 AA^T 及 A^TA 。

解

$$AA^T = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & 2 \\ 2 & 13 \end{pmatrix}_{2 \times 2}$$

$$A^TA = \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

例 3 设 $A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$, 计算 AA^T 及 A^TA 。

解

$$AA^T = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & 2 \\ 2 & 13 \end{pmatrix}_{2 \times 2}$$

$$A^TA = \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 13 & 2 & 2 \\ 2 & 1 & 4 \\ 2 & 4 & 20 \end{pmatrix}_{3 \times 3}$$

转置矩阵的性质

1. $(A^T)^T = A$

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数					

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$				

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$			

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$		

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

并且

$$(AB)^T$$

(i, j) 元素 =

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

并且

$$\begin{aligned} & (AB)^T \\ & (i, j) \text{元素} = \end{aligned}$$

$$\begin{aligned} & B^T A^T \\ & (i, j) \text{元素} \end{aligned}$$

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

并且

$$\begin{aligned} (AB)^T &= AB \\ (i, j) \text{ 元素} &= (j, i) \text{ 元素} = B^T A^T \\ & \quad (i, j) \text{ 元素} \end{aligned}$$

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

并且

$$(AB)^T \begin{matrix} \text{元素} \\ (i, j) \end{matrix} = \begin{matrix} AB \\ \text{元素} \\ (j, i) \end{matrix} = a_{j1} \quad a_{j2} \quad \cdots \quad a_{jl} \quad \begin{matrix} B^T A^T \\ \text{元素} \\ (i, j) \end{matrix}$$

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

并且

$$(AB)^T \begin{matrix} \text{元素} \\ (i, j) \end{matrix} = \begin{matrix} AB \\ \text{元素} \\ (j, i) \end{matrix} = a_{j1}b_{1i} \quad a_{j2}b_{2i} \quad \cdots \quad a_{jl}b_{li} \quad \begin{matrix} B^T A^T \\ \text{元素} \\ (i, j) \end{matrix}$$

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

并且

$$(AB)^T \begin{matrix} (i, j) \text{ 元素} \end{matrix} = \begin{matrix} AB \\ (j, i) \text{ 元素} \end{matrix} = a_{j1}b_{1i} + a_{j2}b_{2i} + \cdots + a_{jl}b_{li} \quad \begin{matrix} B^T A^T \\ (i, j) \text{ 元素} \end{matrix}$$

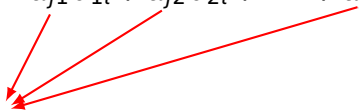
转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

并且

$$(AB)^T \text{ (i, j) 元素} = AB \text{ (j, i) 元素} = a_{j1}b_{1i} + a_{j2}b_{2i} + \cdots + a_{jl}b_{li} \quad B^T A^T \text{ (i, j) 元素}$$


转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

并且

$$(AB)^T \text{ (i, j) 元素} = AB \text{ (j, i) 元素} = a_{j1}b_{1i} + a_{j2}b_{2i} + \cdots + a_{jl}b_{li} \quad B^T A^T \text{ (i, j) 元素}$$

A^T 第 j 列元素

转置矩阵的性质

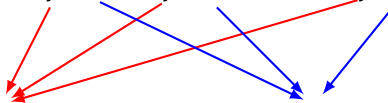
1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

并且

$$(AB)^T \text{ (i, j) 元素} = AB \text{ (j, i) 元素} = a_{j1}b_{1i} + a_{j2}b_{2i} + \cdots + a_{jl}b_{li}$$



A^T 第 j 列元素

$B^T A^T$ (i, j) 元素

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

并且

$$(AB)^T \text{ 的 } (i, j) \text{ 元素} = AB \text{ 的 } (j, i) \text{ 元素} = a_{j1}b_{1i} + a_{j2}b_{2i} + \cdots + a_{jl}b_{li}$$

A^T 第 j 列元素

B^T 第 i 行元素

$B^T A^T \text{ 的 } (i, j) \text{ 元素}$

转置矩阵的性质

1. $(A^T)^T = A$
2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$
3. $(AB)^T = B^T A^T$

证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

并且

$$(AB)^T \text{ 的 } (i, j) \text{ 元素} = AB \text{ 的 } (j, i) \text{ 元素} = a_{j1}b_{1i} + a_{j2}b_{2i} + \cdots + a_{jl}b_{li} = B^T A^T \text{ 的 } (i, j) \text{ 元素}$$

A^T 第 j 列元素 B^T 第 i 行元素

方阵的幂

设 $A = (a_{ij})_{n \times n}$ 为 n 阶方阵, $k \in \mathbb{N}$ 为自然数, 定义

$$A^k = \underbrace{A \cdot A \cdot \dots \cdot A}_{k \text{ 个}}$$

称为方阵 A 的 k 次幂

方阵的幂

设 $A = (a_{ij})_{n \times n}$ 为 n 阶方阵, $k \in \mathbb{N}$ 为自然数, 定义

$$A^k = \underbrace{A \cdot A \cdot \dots \cdot A}_{k \text{ 个}}$$

称为方阵 A 的 k 次幂

方阵的幂的性质 $A^{k_1} A^{k_2} = A^{k_1+k_2}$, $(A^{k_1})^{k_2} = A^{k_1 k_2}$

方阵的幂

设 $A = (a_{ij})_{n \times n}$ 为 n 阶方阵, $k \in \mathbb{N}$ 为自然数, 定义

$$A^k = \underbrace{A \cdot A \cdot \dots \cdot A}_{k \uparrow}$$

称为方阵 A 的 k 次幂

方阵的幂的性质 $A^{k_1} A^{k_2} = A^{k_1+k_2}$, $(A^{k_1})^{k_2} = A^{k_1 k_2}$

这是因为:

$$A^{k_1} A^{k_2} =$$

方阵的幂

设 $A = (a_{ij})_{n \times n}$ 为 n 阶方阵, $k \in \mathbb{N}$ 为自然数, 定义

$$A^k = \underbrace{A \cdot A \cdot \dots \cdot A}_{k \text{ 个}}$$

称为方阵 A 的 k 次幂

方阵的幂的性质 $A^{k_1} A^{k_2} = A^{k_1+k_2}$, $(A^{k_1})^{k_2} = A^{k_1 k_2}$

这是因为:

$$A^{k_1} A^{k_2} = \underbrace{A \cdot A \cdot \dots \cdot A}_{k_1 \text{ 个}} \cdot$$

方阵的幂

设 $A = (a_{ij})_{n \times n}$ 为 n 阶方阵, $k \in \mathbb{N}$ 为自然数, 定义

$$A^k = \underbrace{A \cdot A \cdot \dots \cdot A}_{k \text{ 个}}$$

称为方阵 A 的 k 次幂

方阵的幂的性质 $A^{k_1} A^{k_2} = A^{k_1+k_2}$, $(A^{k_1})^{k_2} = A^{k_1 k_2}$

这是因为:

$$A^{k_1} A^{k_2} = \underbrace{A \cdot A \cdot \dots \cdot A}_{k_1 \text{ 个}} \cdot \underbrace{A \cdot A \cdot \dots \cdot A}_{k_2 \text{ 个}}$$

方阵的幂

设 $A = (a_{ij})_{n \times n}$ 为 n 阶方阵, $k \in \mathbb{N}$ 为自然数, 定义

$$A^k = \underbrace{A \cdot A \cdot \dots \cdot A}_{k \text{ 个}}$$

称为方阵 A 的 k 次幂

方阵的幂的性质 $A^{k_1} A^{k_2} = A^{k_1+k_2}$, $(A^{k_1})^{k_2} = A^{k_1 k_2}$

这是因为:

$$A^{k_1} A^{k_2} = \underbrace{A \cdot A \cdot \dots \cdot A}_{k_1 \text{ 个}} \cdot \underbrace{A \cdot A \cdot \dots \cdot A}_{k_2 \text{ 个}} = A^{k_1+k_2}$$

方阵的幂

设 $A = (a_{ij})_{n \times n}$ 为 n 阶方阵, $k \in \mathbb{N}$ 为自然数, 定义

$$A^k = \underbrace{A \cdot A \cdots A}_{k \text{ 个}}$$

称为方阵 A 的 k 次幂

方阵的幂的性质 $A^{k_1} A^{k_2} = A^{k_1+k_2}$, $(A^{k_1})^{k_2} = A^{k_1 k_2}$

这是因为:

$$A^{k_1} A^{k_2} = \underbrace{A \cdot A \cdots A}_{k_1 \text{ 个}} \cdot \underbrace{A \cdot A \cdots A}_{k_2 \text{ 个}} = A^{k_1+k_2}$$

$$(A^{k_1})^{k_2} =$$

方阵的幂

设 $A = (a_{ij})_{n \times n}$ 为 n 阶方阵, $k \in \mathbb{N}$ 为自然数, 定义

$$A^k = \underbrace{A \cdot A \cdot \dots \cdot A}_{k \text{ 个}}$$

称为方阵 A 的 k 次幂

方阵的幂的性质 $A^{k_1} A^{k_2} = A^{k_1+k_2}$, $(A^{k_1})^{k_2} = A^{k_1 k_2}$

这是因为:

$$A^{k_1} A^{k_2} = \underbrace{A \cdot A \cdot \dots \cdot A}_{k_1 \text{ 个}} \cdot \underbrace{A \cdot A \cdot \dots \cdot A}_{k_2 \text{ 个}} = A^{k_1+k_2}$$

$$(A^{k_1})^{k_2} = \underbrace{A^{k_1} \cdot A^{k_1} \cdot \dots \cdot A^{k_1}}_{k_2 \text{ 个}}$$

方阵的幂

设 $A = (a_{ij})_{n \times n}$ 为 n 阶方阵, $k \in \mathbb{N}$ 为自然数, 定义

$$A^k = \underbrace{A \cdot A \cdot \dots \cdot A}_{k \text{ 个}}$$

称为方阵 A 的 k 次幂

方阵的幂的性质 $A^{k_1} A^{k_2} = A^{k_1+k_2}$, $(A^{k_1})^{k_2} = A^{k_1 k_2}$

这是因为:

$$A^{k_1} A^{k_2} = \underbrace{A \cdot A \cdot \dots \cdot A}_{k_1 \text{ 个}} \cdot \underbrace{A \cdot A \cdot \dots \cdot A}_{k_2 \text{ 个}} = A^{k_1+k_2}$$

$$(A^{k_1})^{k_2} = \underbrace{A^{k_1} \cdot A^{k_1} \cdot \dots \cdot A^{k_1}}_{k_2 \text{ 个}} = A^{k_1 k_2}$$

练习 设 $A = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$, 其中 λ 为常数, 计算 A^n 。

练习 设 $A = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$, 其中 λ 为常数, 计算 A^n 。

解

$$A^2 =$$

$$A^3 =$$

$$A^4 =$$

$$\vdots$$

$$A^n =$$

练习 设 $A = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$, 其中 λ 为常数, 计算 A^n 。

解

$$A^2 = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$$

$$A^3 =$$

$$A^4 =$$

$$\vdots$$

$$A^n =$$

练习 设 $A = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$, 其中 λ 为常数, 计算 A^n 。

解

$$A^2 = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix}$$

$$A^3 =$$

$$A^4 =$$

$$\vdots$$

$$A^n =$$

练习 设 $A = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$, 其中 λ 为常数, 计算 A^n 。

解

$$A^2 = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A$$

$$A^4 =$$

$$\vdots$$

$$A^n =$$

练习 设 $A = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$, 其中 λ 为常数, 计算 A^n 。

解

$$A^2 = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$$

$$A^4 =$$

$$\vdots$$

$$A^n =$$

练习 设 $A = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$, 其中 λ 为常数, 计算 A^n 。

解

$$A^2 = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 3\lambda & 1 \end{pmatrix}$$

$$A^4 =$$

\vdots

$$A^n =$$

练习 设 $A = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$, 其中 λ 为常数, 计算 A^n 。

解

$$A^2 = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 3\lambda & 1 \end{pmatrix}$$

$$A^4 = A^3 \cdot A$$

$$\vdots$$

$$A^n =$$

练习 设 $A = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$, 其中 λ 为常数, 计算 A^n 。

解

$$A^2 = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 3\lambda & 1 \end{pmatrix}$$

$$A^4 = A^3 \cdot A = \begin{pmatrix} 1 & 0 \\ 3\lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$$

\vdots

$$A^n =$$

练习 设 $A = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$, 其中 λ 为常数, 计算 A^n 。

解

$$A^2 = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 3\lambda & 1 \end{pmatrix}$$

$$A^4 = A^3 \cdot A = \begin{pmatrix} 1 & 0 \\ 3\lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 4\lambda & 1 \end{pmatrix}$$

\vdots

$$A^n =$$

练习 设 $A = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$, 其中 λ 为常数, 计算 A^n 。

解

$$A^2 = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 3\lambda & 1 \end{pmatrix}$$

$$A^4 = A^3 \cdot A = \begin{pmatrix} 1 & 0 \\ 3\lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 4\lambda & 1 \end{pmatrix}$$

\vdots

$$A^n = \begin{pmatrix} 1 & 0 \\ n\lambda & 1 \end{pmatrix}$$

方阵的幂

注 设 A, B 为 n 阶方阵, 一般地

$$(AB)^k \neq A^k B^k$$

方阵的幂

注 设 A, B 为 n 阶方阵，一般地

$$(AB)^k \neq A^k B^k$$

这是，例如 $k = 2$ 时，

$$(AB)^2 =$$

$$A^2 B^2 =$$

方阵的幂

注 设 A, B 为 n 阶方阵，一般地

$$(AB)^k \neq A^k B^k$$

这是，例如 $k = 2$ 时，

$$(AB)^2 = (AB) \cdot (AB) =$$

$$A^2 B^2 =$$

方阵的幂

注 设 A, B 为 n 阶方阵, 一般地

$$(AB)^k \neq A^k B^k$$

这是, 例如 $k = 2$ 时,

$$(AB)^2 = (AB) \cdot (AB) = ABAB$$

$$A^2 B^2 =$$

方阵的幂

注 设 A, B 为 n 阶方阵，一般地

$$(AB)^k \neq A^k B^k$$

这是，例如 $k = 2$ 时，

$$(AB)^2 = (AB) \cdot (AB) = ABAB$$

$$A^2 B^2 = (AA) \cdot (BB) =$$

方阵的幂

注 设 A, B 为 n 阶方阵，一般地

$$(AB)^k \neq A^k B^k$$

这是，例如 $k = 2$ 时，

$$(AB)^2 = (AB) \cdot (AB) = ABAB$$

$$A^2 B^2 = (AA) \cdot (BB) = AABB$$

方阵的幂

注 设 A, B 为 n 阶方阵，一般地

$$(AB)^k \neq A^k B^k$$

这是，例如 $k = 2$ 时，

$$(AB)^2 = (AB) \cdot (AB) = A \textcolor{red}{B} A B$$

$$A^2 B^2 = (AA) \cdot (BB) = A \textcolor{red}{A} B B$$

但一般地， $AB \neq BA$ ，

方阵的幂

注 设 A, B 为 n 阶方阵，一般地

$$(AB)^k \neq A^k B^k$$

这是，例如 $k = 2$ 时，

$$(AB)^2 = (AB) \cdot (AB) = A \textcolor{red}{B} A B$$

$$A^2 B^2 = (AA) \cdot (BB) = A \textcolor{red}{A} B B$$

但一般地， $AB \neq BA$ ，所以 $(AB)^2 \neq A^2 B^2$

方阵的行列式

回忆：对 n 阶方阵

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

方阵的行列式

回忆：对 n 阶方阵

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

其行列式规定为

方阵的行列式

回忆：对 n 阶方阵

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

其行列式规定为

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

方阵的行列式

回忆：对 n 阶方阵

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

其行列式规定为

$$|A| = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

方阵行列式的性质

设 A, B 均是 n 阶方阵, k 为数, 则

1. $|A^T| = |A|$

2. $|kA| = k^n |A|$

3. $|AB| = |A| \cdot |B|$

4. $|AB| = |BA|$

方阵行列式的性质

设 A, B 均是 n 阶方阵, k 为数, 则

1. $|A^T| = |A|$

2. $|kA| = k^n |A|$

3. $|AB| = |A| \cdot |B|$

4. $|AB| = |BA|$

例如

$$|kA| =$$

方阵行列式的性质

设 A, B 均是 n 阶方阵, k 为数, 则

1. $|A^T| = |A|$
2. $|kA| = k^n |A|$
3. $|AB| = |A| \cdot |B|$
4. $|AB| = |BA|$

例如

$$|kA| = \left| k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \right| =$$

方阵行列式的性质

设 A, B 均是 n 阶方阵, k 为数, 则

1. $|A^T| = |A|$

2. $|kA| = k^n |A|$

3. $|AB| = |A| \cdot |B|$

4. $|AB| = |BA|$

例如

$$|kA| = \left| k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \right| = \begin{vmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ ka_{n1} & ka_{n2} & \cdots & ka_{nn} \end{vmatrix}$$

方阵行列式的性质

设 A, B 均是 n 阶方阵, k 为数, 则

1. $|A^T| = |A|$
2. $|kA| = k^n |A|$
3. $|AB| = |A| \cdot |B|$
4. $|AB| = |BA|$

例如

$$\begin{aligned} |kA| &= \left| k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \right| = \left| \begin{matrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ ka_{n1} & ka_{n2} & \cdots & ka_{nn} \end{matrix} \right| \\ &= k \cdot k \cdot \cdots \cdot k \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} \end{aligned}$$

方阵行列式的性质

设 A, B 均是 n 阶方阵, k 为数, 则

1. $|A^T| = |A|$
2. $|kA| = k^n |A|$
3. $|AB| = |A| \cdot |B|$
4. $|AB| = |BA|$

例如

$$\begin{aligned} |kA| &= \left| k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \right| = \left| \begin{pmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ ka_{n1} & ka_{n2} & \cdots & ka_{nn} \end{pmatrix} \right| \\ &= k \cdot k \cdot \cdots \cdot k \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = k^n |A| \end{aligned}$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

解

$$|4A| = 4^3 |A| =$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

解

$$|4A| = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} =$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

解

$$|4A| = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} =$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

解

$$|4A| = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 =$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

解

$$|4A| = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 = 192$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

解

$$|4A| = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 = 192$$

练习 设 A 为三阶方阵, 且 $|A| = -2$, 求 $|A|A^2A^T|$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

解

$$|4A| = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 = 192$$

练习 设 A 为三阶方阵, 且 $|A| = -2$, 求 $|A|A^2A^T|$

解

$$|A|A^2A^T| = |A|^3 |A^2A^T|$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

解

$$|4A| = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 = 192$$

练习 设 A 为三阶方阵, 且 $|A| = -2$, 求 $|A|A^2A^T|$

解

$$\begin{aligned} |A|A^2A^T| &= |A|^3 |A^2A^T| \\ &= |A|^3 |A^2| |A^T| \end{aligned}$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

解

$$|4A| = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 = 192$$

练习 设 A 为三阶方阵, 且 $|A| = -2$, 求 $|A|A^2A^T|$

解

$$\begin{aligned} |A|A^2A^T| &= |A|^3 |A^2A^T| \\ &= |A|^3 |A^2| |A^T| \\ &= |A|^3 |A|^2 |A| \end{aligned}$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

解

$$|4A| = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 = 192$$

练习 设 A 为三阶方阵, 且 $|A| = -2$, 求 $|A|A^2A^T|$

解

$$\begin{aligned} |A|A^2A^T| &= |A|^3 |A^2A^T| \\ &= |A|^3 |A^2| |A^T| \\ &= |A|^3 |A|^2 |A| \\ &= |A|^6 \end{aligned}$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

解

$$|4A| = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 = 192$$

练习 设 A 为三阶方阵, 且 $|A| = -2$, 求 $|A|A^2A^T|$

解

$$\begin{aligned} |A|A^2A^T| &= |A|^3 |A^2A^T| \\ &= |A|^3 |A^2| |A^T| \\ &= |A|^3 |A|^2 |A| \\ &= |A|^6 = (-2)^6 \end{aligned}$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

解

$$|4A| = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 = 192$$

练习 设 A 为三阶方阵, 且 $|A| = -2$, 求 $|A|A^2A^T|$

解

$$\begin{aligned} |A|A^2A^T| &= |A|^3 |A^2A^T| \\ &= |A|^3 |A^2| |A^T| \\ &= |A|^3 |A|^2 |A| \\ &= |A|^6 = (-2)^6 = 64 \end{aligned}$$

方程组的矩阵表示

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

方程组的矩阵表示

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

等价于

$$\begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{pmatrix}_{m \times 1} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}_{m \times 1}$$

方程组的矩阵表示

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

等价于

$$\begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{pmatrix}_{m \times 1} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}_{m \times 1}$$
$$\parallel$$
$$\begin{pmatrix} \quad \quad \quad \end{pmatrix}_{m \times n} \begin{pmatrix} \quad \quad \quad \end{pmatrix}_{n \times 1}$$

方程组的矩阵表示

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

等价于

$$\begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{pmatrix}_{m \times 1} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}_{m \times 1}$$
$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1}$$

方程组的矩阵表示

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

等价于

$$\begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{pmatrix}_{m \times 1} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}_{m \times 1}$$
$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1}$$

方程组的矩阵表示

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

等价于

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

方程组的矩阵表示

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

等价于

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_A \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

系数矩阵

方程组的矩阵表示

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

等价于

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_A \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} = \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}_b$$

系数矩阵 常数项矩阵

方程组的矩阵表示

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

等价于

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_A \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}}_x = \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}_b$$

系数矩阵 常数项矩阵

方程组的矩阵表示

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

等价于

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_A \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}}_x = \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}_b$$

系数矩阵 常数项矩阵

进一步改写成

$$Ax = b$$

线性方程组的矩阵表示

例 方程组

$$\begin{cases} x_1 - x_2 + 5x_3 - x_4 = -2 \\ x_1 + x_2 - 2x_3 + 3x_4 = 3 \\ 3x_1 - x_2 + 8x_3 + x_4 = 7 \end{cases}$$

线性方程组的矩阵表示

例 方程组

$$\begin{cases} x_1 - x_2 + 5x_3 - x_4 = -2 \\ x_1 + x_2 - 2x_3 + 3x_4 = 3 \\ 3x_1 - x_2 + 8x_3 + x_4 = 7 \end{cases}$$

的矩阵表示 $Ax = b$ 是

$$\begin{pmatrix} & & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

线性方程组的矩阵表示

例 方程组

$$\begin{cases} x_1 - x_2 + 5x_3 - x_4 = -2 \\ x_1 + x_2 - 2x_3 + 3x_4 = 3 \\ 3x_1 - x_2 + 8x_3 + x_4 = 7 \end{cases}$$

的矩阵表示 $Ax = b$ 是

$$\begin{pmatrix} 1 & -1 & 5 & -1 \\ 1 & 1 & -2 & 3 \\ 3 & -1 & 8 & 1 \end{pmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix}$$

线性方程组的矩阵表示

例 方程组

$$\begin{cases} x_1 - x_2 + 5x_3 - x_4 = -2 \\ x_1 + x_2 - 2x_3 + 3x_4 = 3 \\ 3x_1 - x_2 + 8x_3 + x_4 = 7 \end{cases}$$

的矩阵表示 $Ax = b$ 是

$$\begin{pmatrix} 1 & -1 & 5 & -1 \\ 1 & 1 & -2 & 3 \\ 3 & -1 & 8 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 7 \end{pmatrix}$$

线性方程组的矩阵表示

例 方程组

$$\begin{cases} x_1 - x_2 + 5x_3 - x_4 = -2 \\ x_1 + x_2 - 2x_3 + 3x_4 = 3 \\ 3x_1 - x_2 + 8x_3 + x_4 = 7 \end{cases}$$

的矩阵表示 $Ax = b$ 是

$$\begin{pmatrix} 1 & -1 & 5 & -1 \\ 1 & 1 & -2 & 3 \\ 3 & -1 & 8 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ 7 \end{pmatrix}$$