第 9 章 d：隐函数的求导公式

数学系　　梁卓滨

2016-2017 学年 II

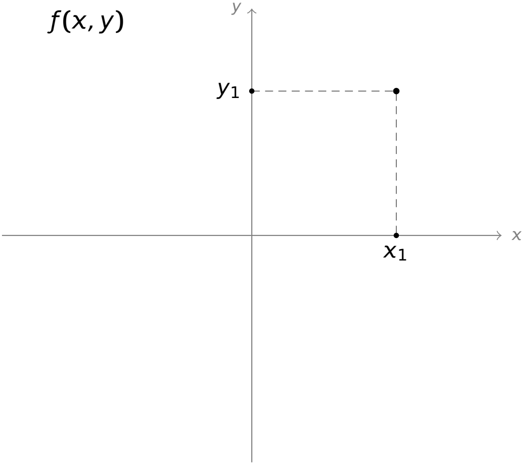
Outline

We are here now...

1. 一个方程的情形

2. 方程组的情形

隐函数定理 设 $f(x,y)$ 在点 $p_1(x_1,y_1)$ 附近有定义，具有连续偏导；



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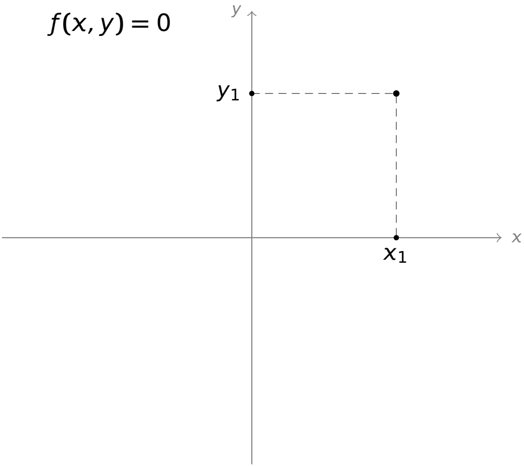

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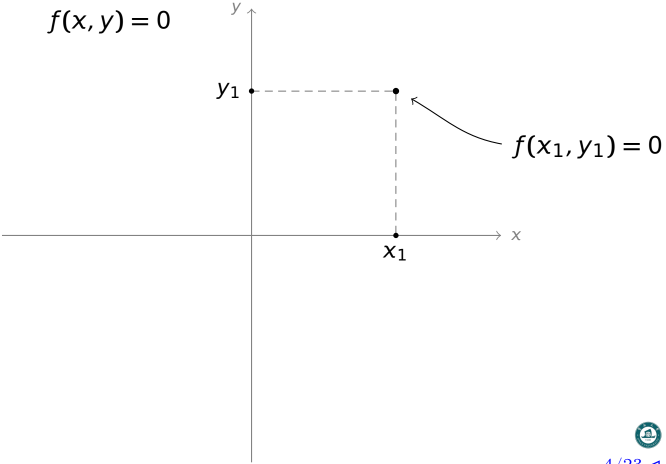

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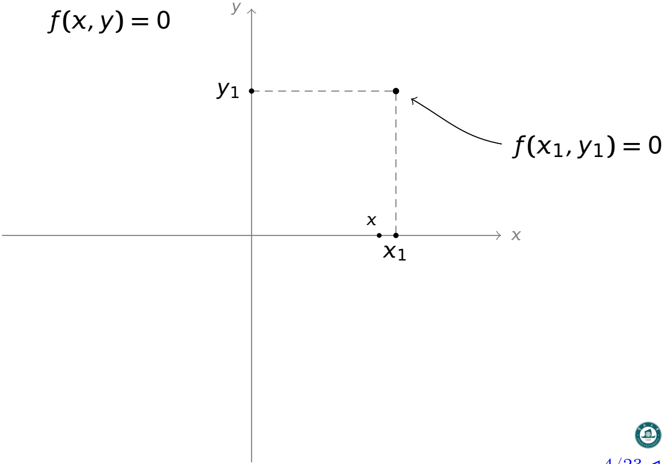

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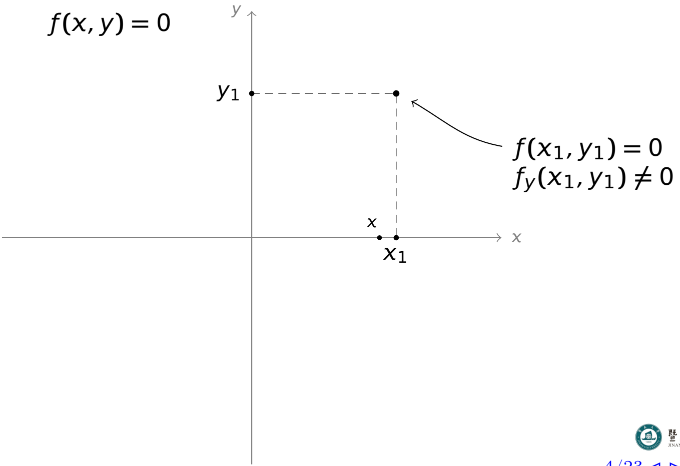

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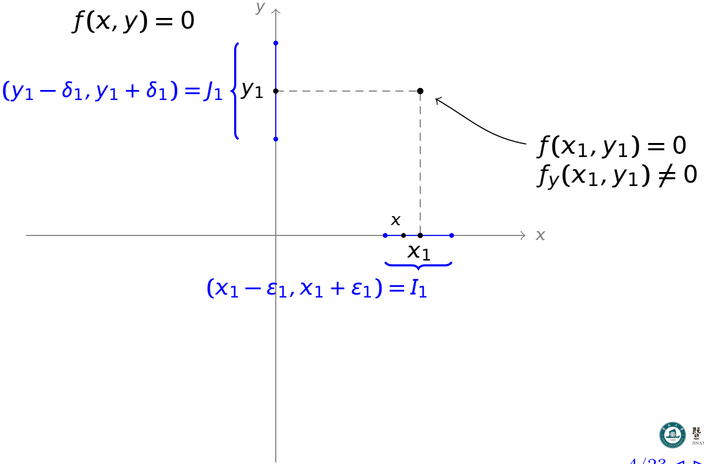

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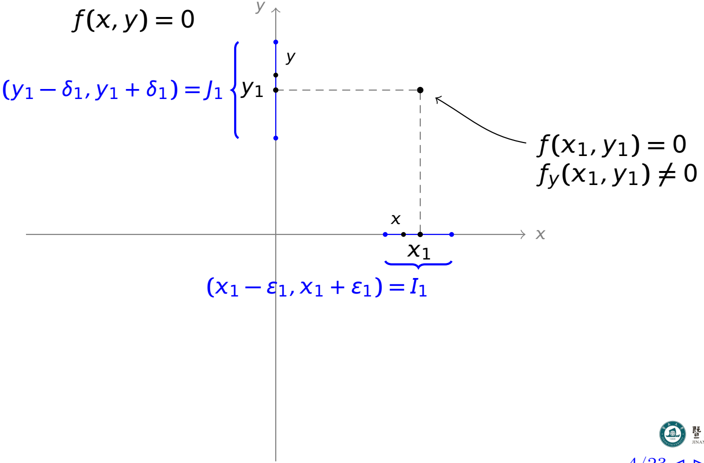

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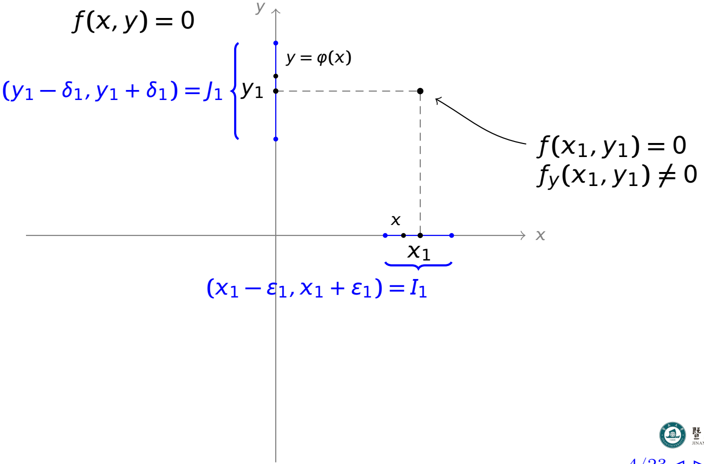

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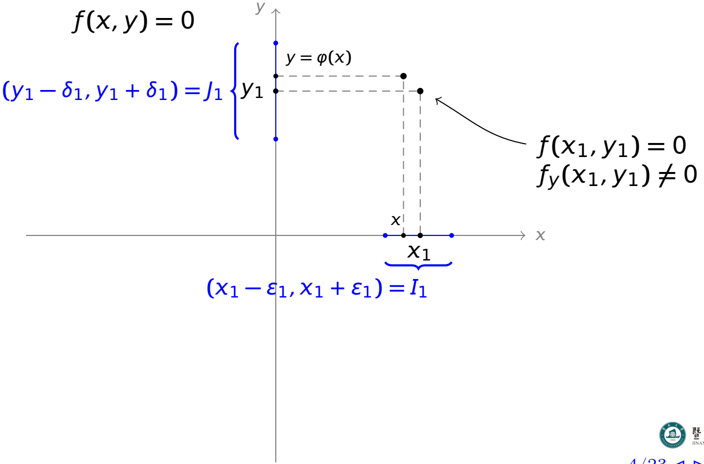

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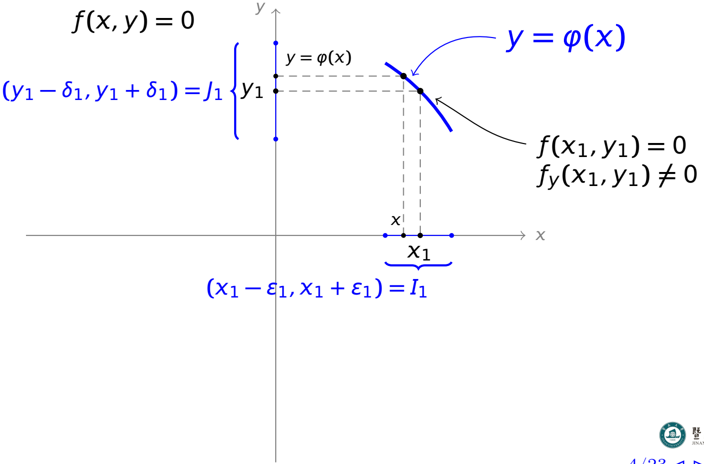

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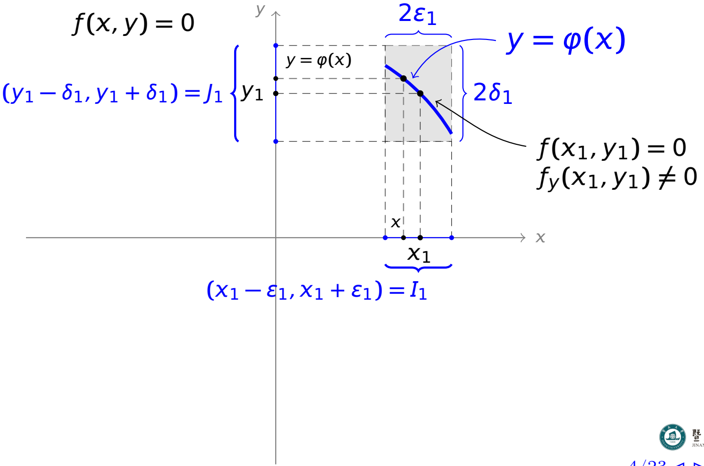

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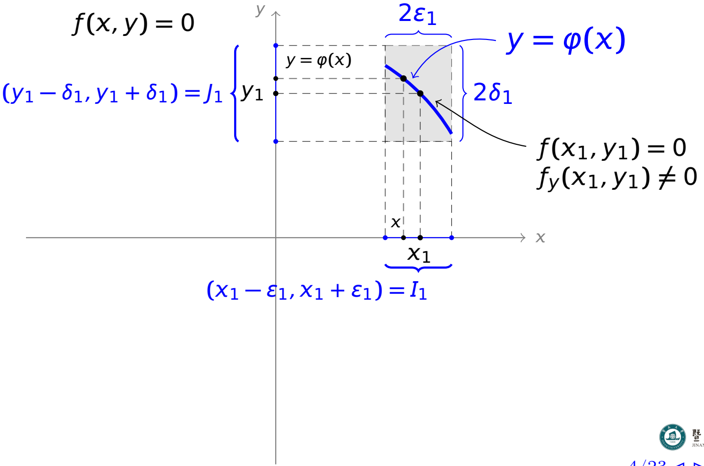

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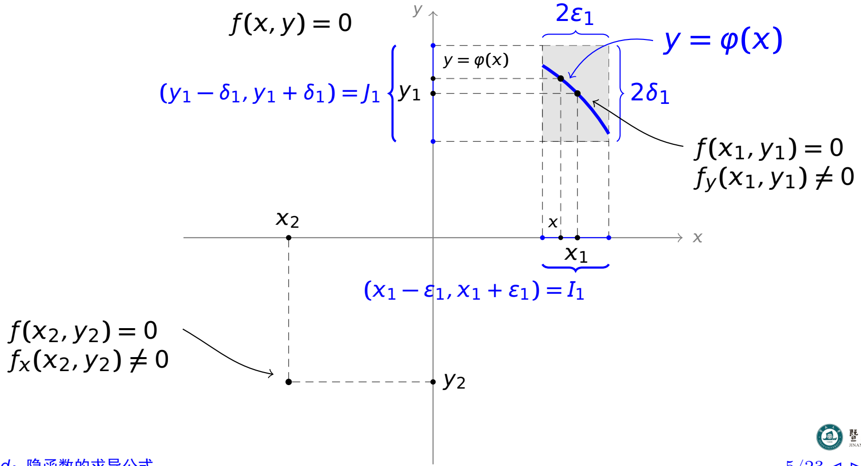

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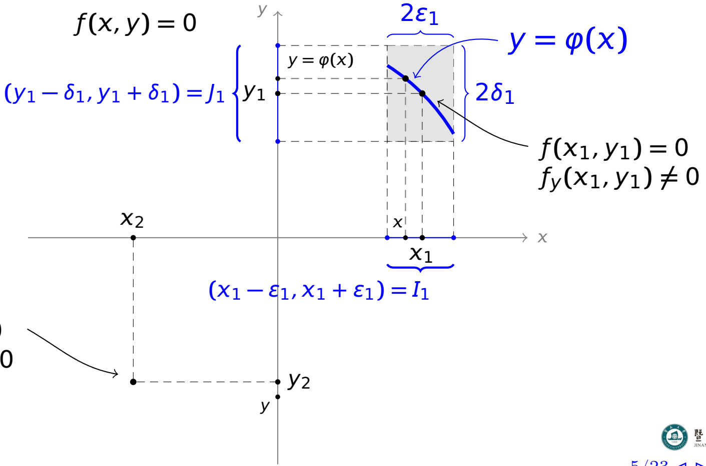

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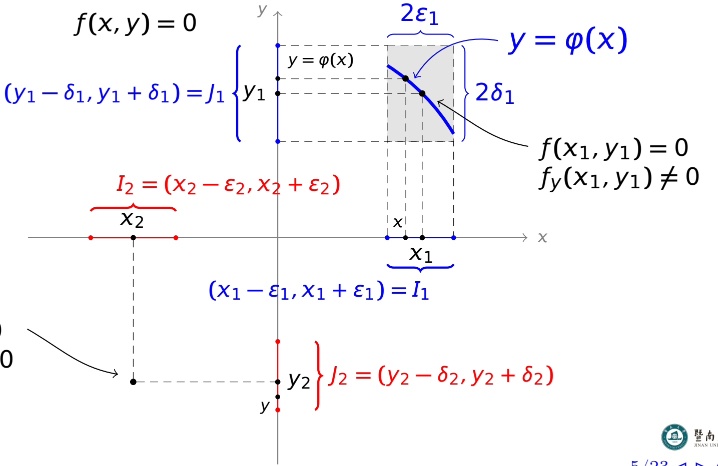

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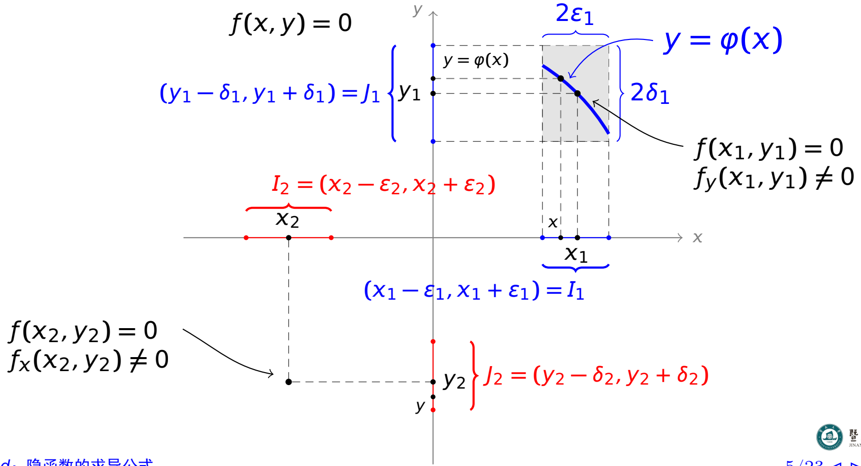

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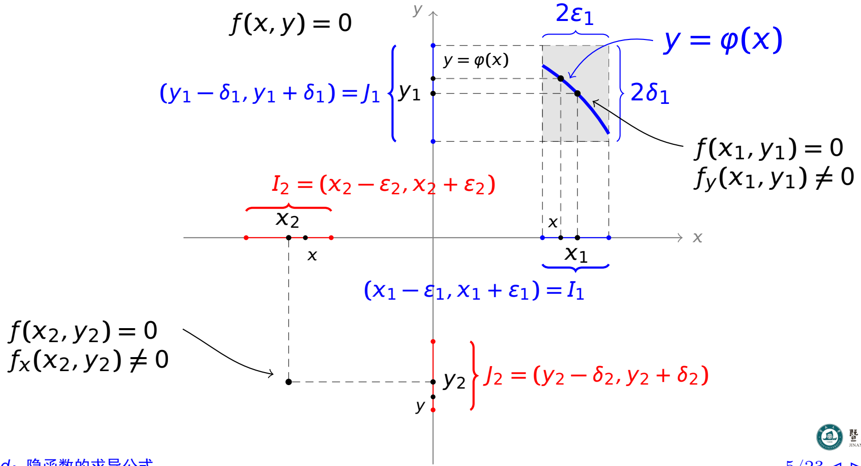

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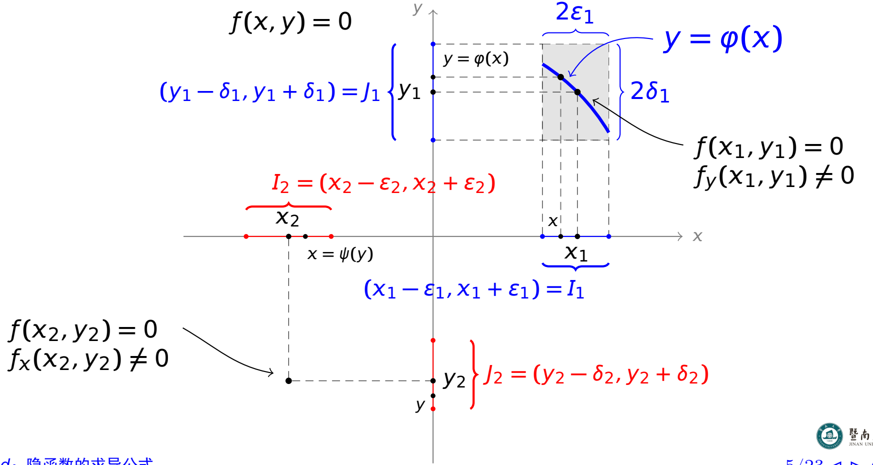

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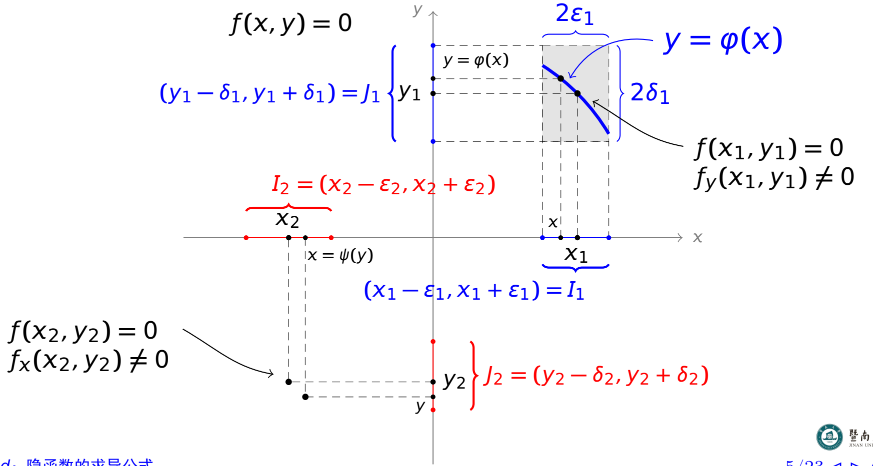

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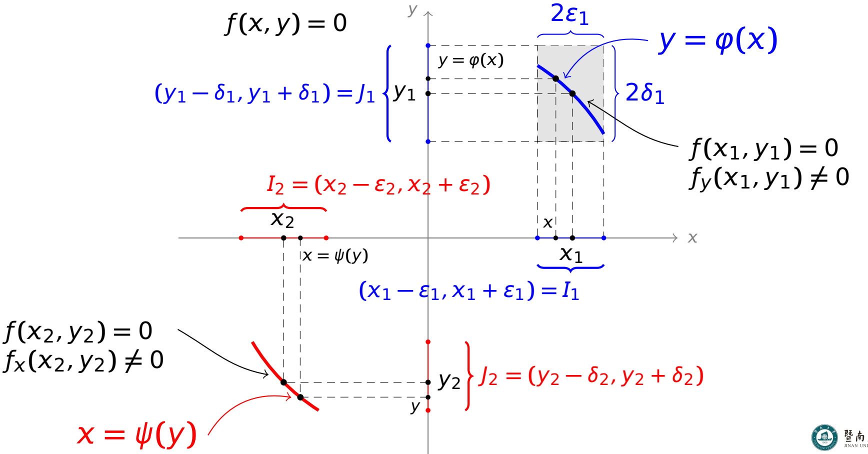

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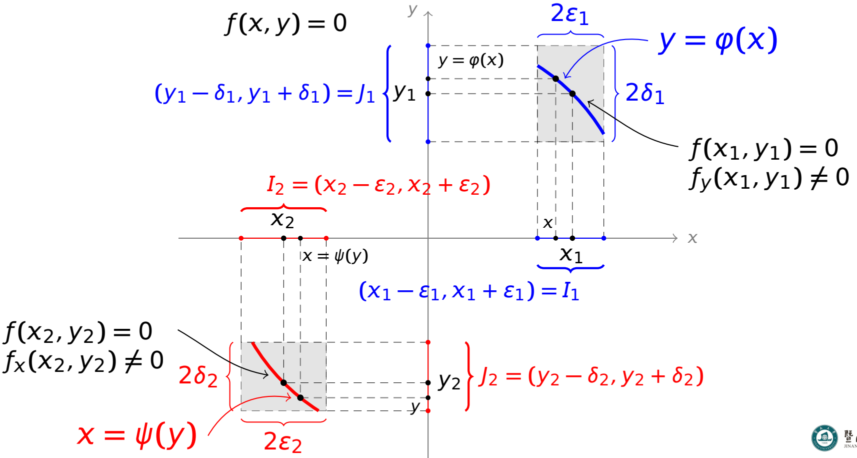

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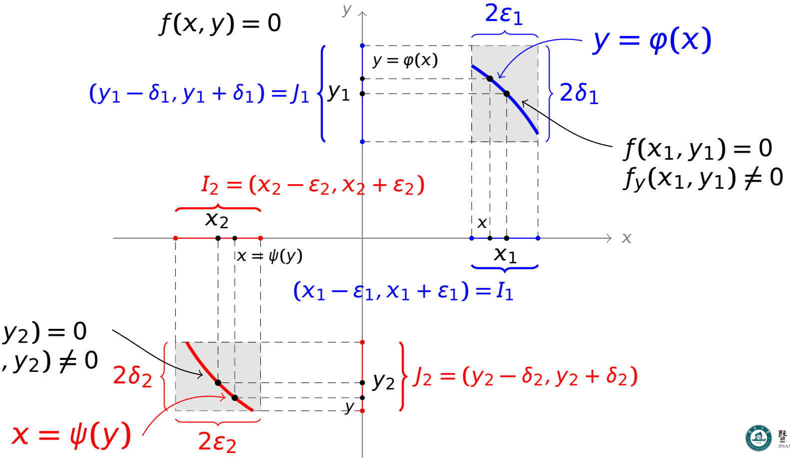

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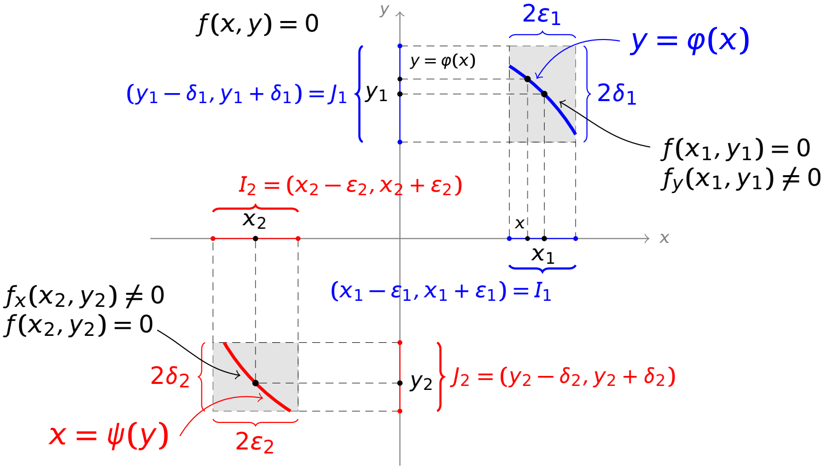

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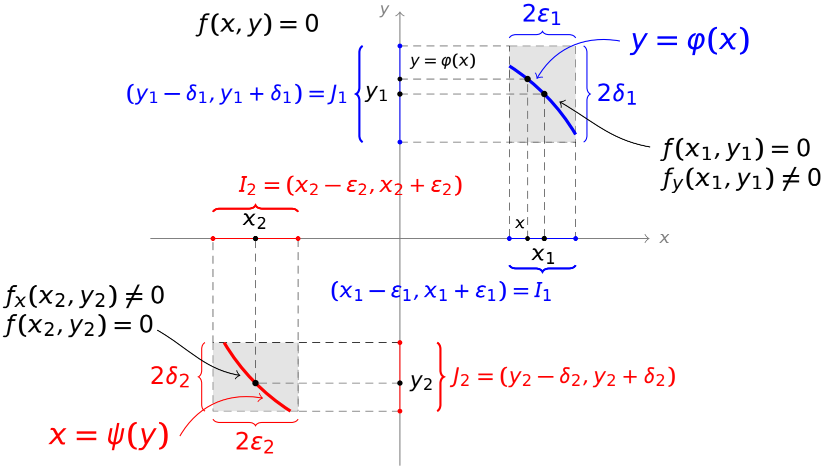

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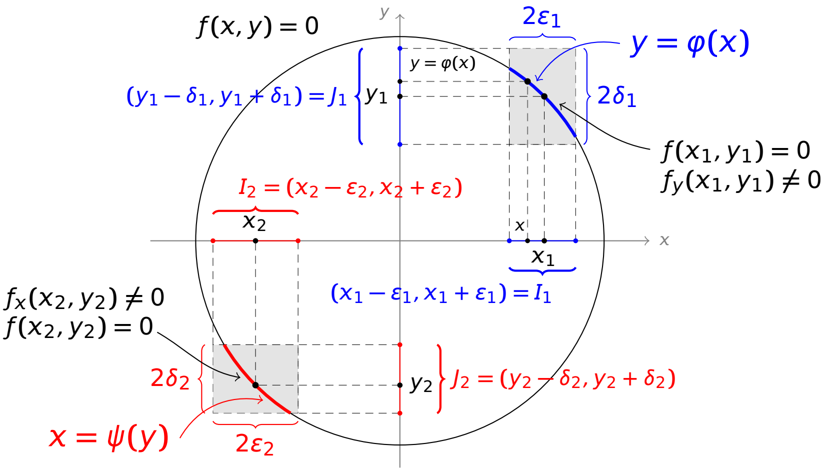

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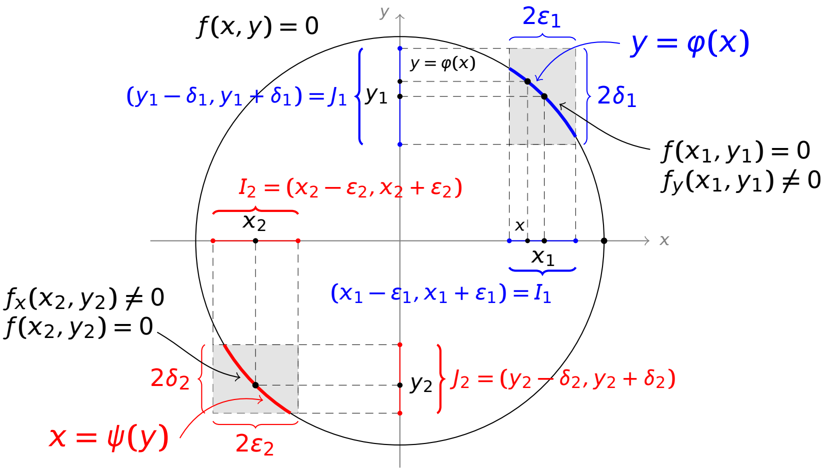

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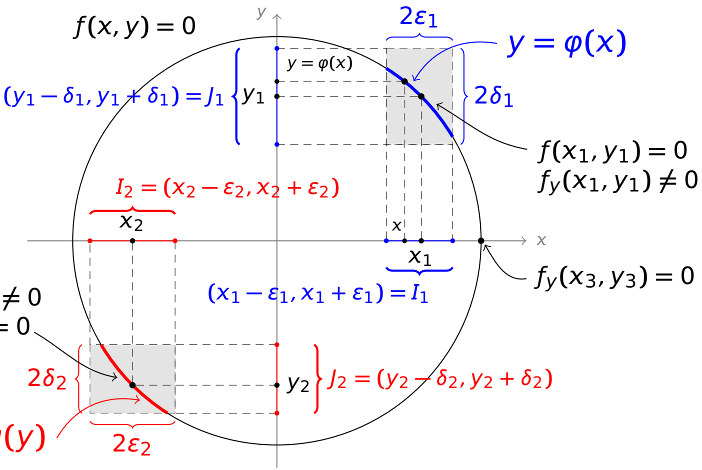

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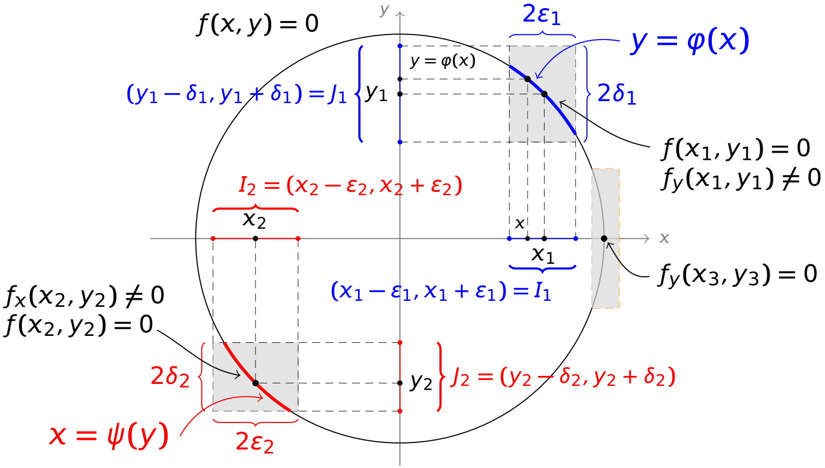

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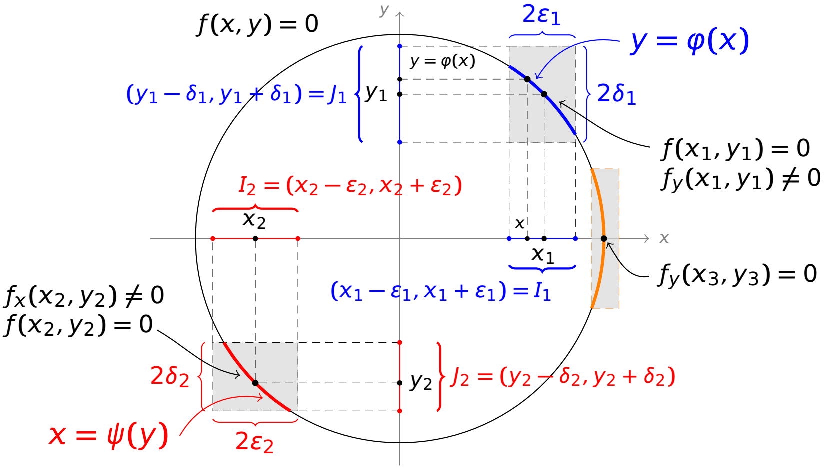

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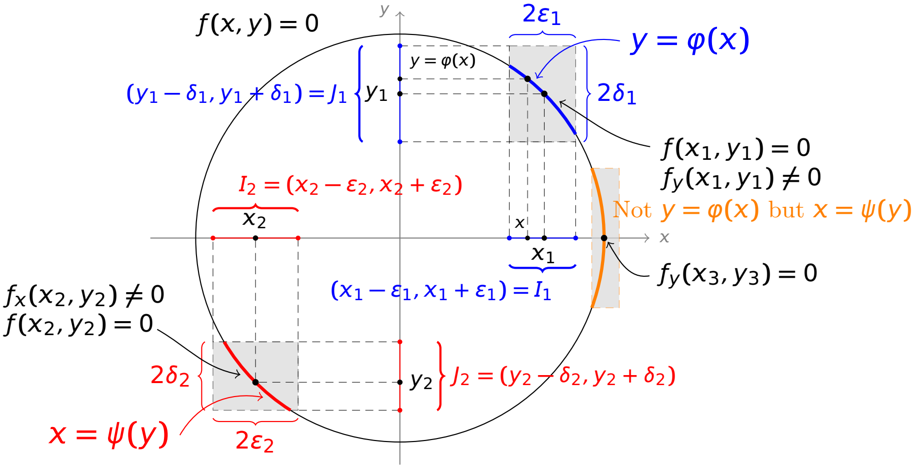

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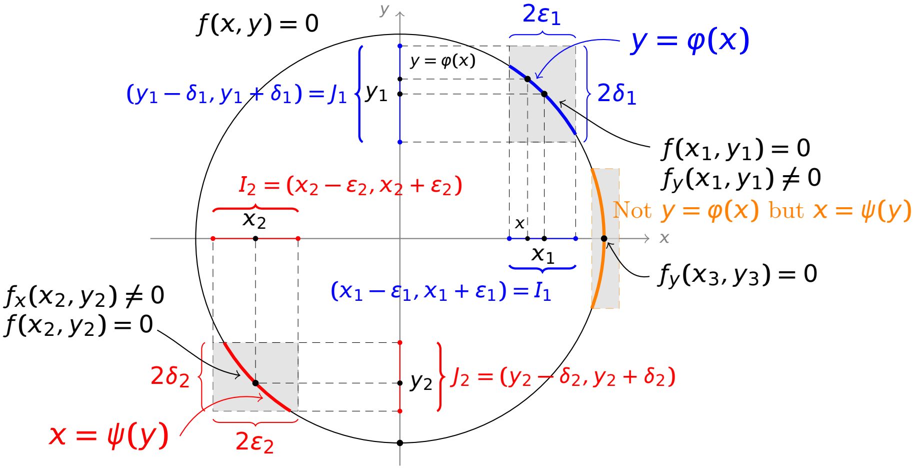

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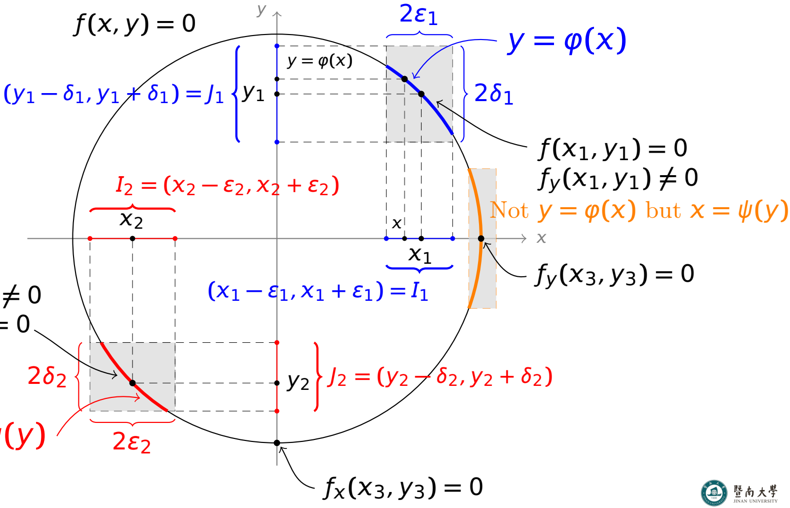

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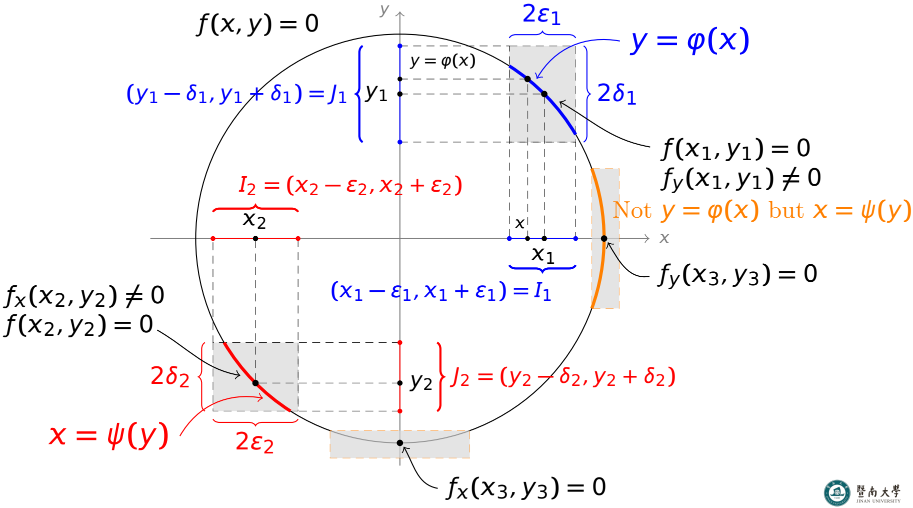

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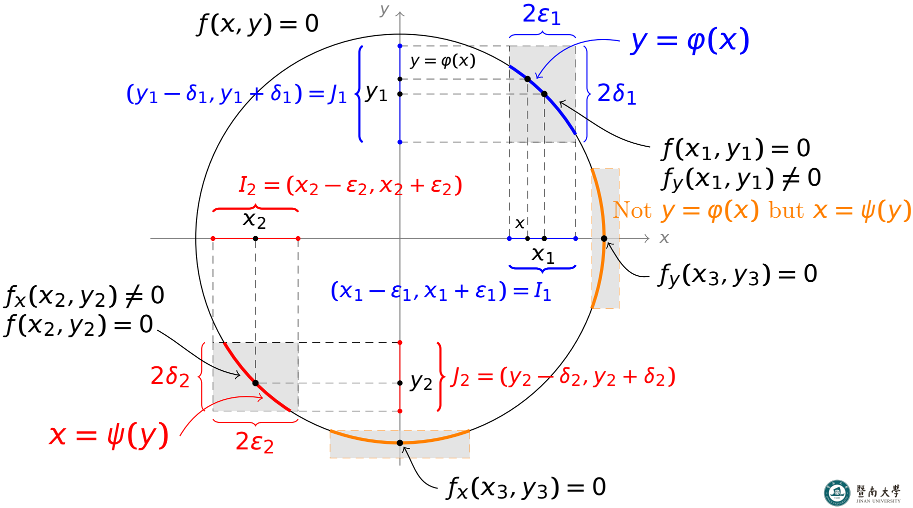

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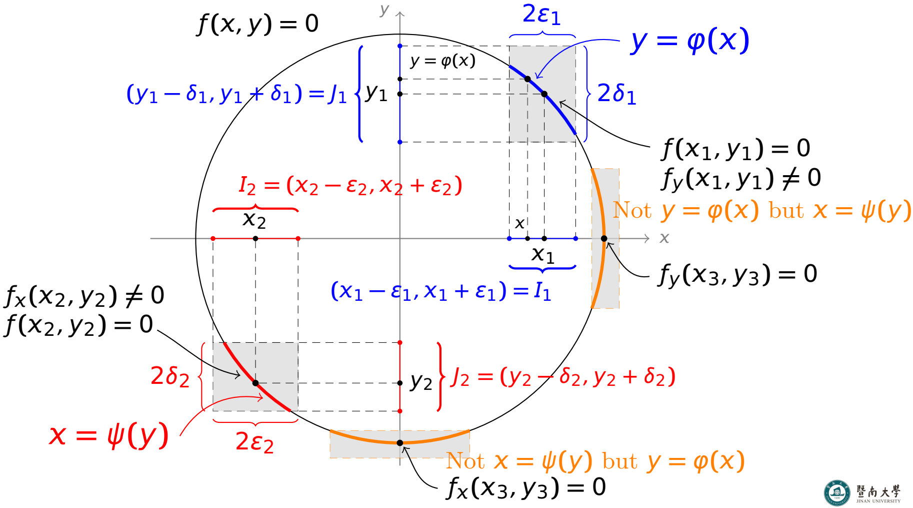

隐函数定理 设 $f(x,y,z)$ 在点 $p(x_0,y_0,z_0)$ 附近有定义，具有连续偏导；$f(x_0,y_0,z_0)=0$；

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方程 $f(x,y,z)=0$ 有唯一的解

$$z=\varphi(x,y)$$

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$$I_1=(x_0-\varepsilon,\ x_0+\varepsilon),\quad I_2=(y_0-\varepsilon,\ y_0+\varepsilon),\quad J=(z_0-\delta,\ z_0+\delta)$$

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$$I_1 = (x_0 - \varepsilon, x_0 + \varepsilon), \quad I_2 = (y_0 - \varepsilon, y_0 + \varepsilon), \quad J = (z_0 - \delta, z_0 + \delta)$$

使得：

1. 对任意 $(x, y) \in I_1 \times I_2$，方程 $f(x, y, z) = 0$ 有唯一的解
   $z = \varphi(x, y)$

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使得：

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2. 函数 $z=\varphi(x,y)$ 在区域 $I_1\times I_2$ 上具有连续导数。

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公式 设 $y = f(x)$ 满足 $F(x, y) = 0$,

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$$\frac{dy}{dx}=-\frac{F_x}{F_y} \qquad (F_y\neq 0)$$

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$$\begin{tikzcd} & x \\ F \arrow[ur] \arrow[dr] & \\ & y \arrow[r] & x \end{tikzcd}$$

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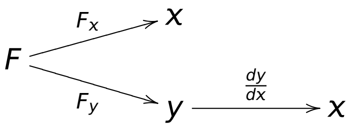

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$$\begin{tikzcd}
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F \arrow[ur, "F_x"] \arrow[dr, "F_y"'] & & \\
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例 设 $y=f(x)$ 满足 $\sin y+e^x=xy^2$，求 $\frac{dy}{dx}$

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方法二 注意 $\sin y(x)+e^x-xy(x)^2=0$，所以

$$\begin{aligned}0&=\left(\sin y(x)+e^x-xy(x)^2\right)'_x\\&=(\sin y(x))'_x+(e^x)'_x-(xy(x)^2)'_x\end{aligned}$$

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方法一 注意 $\sin y + e^x - xy^2 = 0$，令 $F(x, y) = \sin y + e^x - xy^2$，则 $F(x, y) = 0$，所以

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方法二 注意 $\sin y(x) + e^x - xy(x)^2 = 0$，所以

$$\begin{aligned} 0 &= \left(\sin y(x) + e^x - xy(x)^2\right)'_x \\ &= (\sin y(x))'_x + (e^x)'_x - (xy(x)^2)'_x \\ &= \cos y \cdot y' + e^x - y^2 - 2xy \cdot y' \end{aligned}$$

例 设 $y=f(x)$ 满足 $\sin y+e^x=xy^2$，求 $\frac{dy}{dx}$

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方法二 注意 $\sin y(x)+e^x-xy(x)^2=0$，所以

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所以 $y'=-\frac{e^x-y^2}{\cos y-2xy}$

例 设 $y = f(x)$ 满足 $\ln(x^2 + y^2) + 3xy = 4$，求 $\frac{dy}{dx}$

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解

$F(x, y) = 0$

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例 设 $y = f(x)$ 满足 $\ln(x^2 + y^2) + 3xy = 4$，求 $\frac{dy}{dx}$

解 注意 $\ln(x^2 + y^2) + 3xy - 4 = 0$

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$$F(x,y)=\ln(x^2+y^2)+3xy-4$$

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$$F(x,y)=\ln(x^2+y^2)+3xy-4$$

则 $F(x,y)=0$，所以

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$$=-\frac{\frac{2x}{x^2+y^2}+3y}{\qquad}$$

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例 设 $y = f(x)$ 满足 $\ln(x^2 + y^2) + 3xy = 4$，求 $\frac{dy}{dx}$

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$$F(x, y) = \ln(x^2 + y^2) + 3xy - 4$$

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$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\ln(x^2 + y^2) + 3xy - 4)'_x}{(\ln(x^2 + y^2) + 3xy - 4)'_y}$$

$$= -\frac{\frac{2x}{x^2+y^2} + 3y}{\frac{2y}{x^2+y^2} + 3x}$$

$$= \frac{2x + 3x^2y + 3y^3}{2y + 3xy^2 + 3x^3}$$

隐函数的求导法 II

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隐函数的求导法 II

公式 设 $z = f(x, y)$ 满足 $F(x, y, z) = 0$，即 $F(x, y, z(x, y)) = 0$，

隐函数的求导法 II

公式 设 $z = f(x, y)$ 满足 $F(x, y, z) = 0$，即 $F(x, y, z(x, y)) = 0$，则

$$\frac{\partial z}{\partial x} = \qquad , \qquad \frac{\partial z}{\partial y} =$$

隐函数的求导法 II

公式 设 $z = f(x, y)$ 满足 $F(x, y, z) = 0$，即 $F(x, y, z(x, y)) = 0$，则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} =$$

隐函数的求导法 II

公式 设 $z = f(x, y)$ 满足 $F(x, y, z) = 0$，即 $F(x, y, z(x, y)) = 0$，则

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隐函数的求导法 II

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$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z},\qquad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z}\qquad (F_z\neq 0)$$

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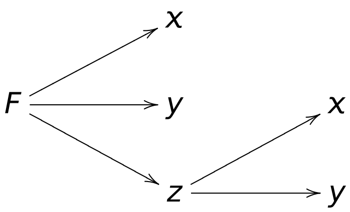

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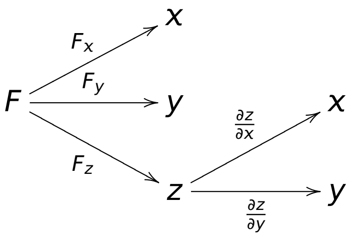

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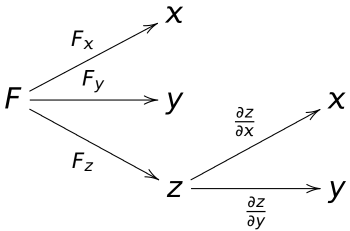

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$$=-\frac{\qquad\qquad}{-6\cos(x+2y-3z)}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_y}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

$$=-\frac{\qquad\qquad}{\qquad\qquad}$$

例 设 $z=f(x,y)$ 满足 $2\sin(x+2y-3z)=x+2y-3z$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$=-\frac{\qquad}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_y}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

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$$=-\frac{2\cos(x+2y-3z)}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_y}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

$$=-\frac{}{}$$

例 设 $z = f(x, y)$ 满足 $2\sin(x + 2y - 3z) = x + 2y - 3z$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解 令 $F(x, y, z) = 2\sin(x + 2y - 3z) - x - 2y + 3z$，则 $F(x, y, z) = 0$，所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x + 2y - 3z) - x - 2y + 3z)'_x}{(2\sin(x + 2y - 3z) - x - 2y + 3z)'_z}$$

$$= -\frac{2\cos(x + 2y - 3z) - 1}{-6\cos(x + 2y - 3z) + 3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x + 2y - 3z) - x - 2y + 3z)'_y}{(2\sin(x + 2y - 3z) - x - 2y + 3z)'_z}$$

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$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)'_y}{(2\sin(x+2y-3z)-x-2y+3z)'_z} = -\frac{4\cos(x+2y-3z)}{-6\cos(x+2y-3z)+3}$$

例 设 $z=f(x,y)$ 满足 $2\sin(x+2y-3z)=x+2y-3z$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$=-\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_y}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

$$=-\frac{4\cos(x+2y-3z)-2}{-6\cos(x+2y-3z)+3}$$

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$$\frac{\partial z}{\partial y}=$$

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$$=-\frac{\qquad}{1+xe^{z-y-x}}$$

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$$= -\frac{-1+e^{z-y-x}}{1+xe^{z-y-x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z-y-x+xe^{z-y-x})'_y}{(z-y-x+xe^{z-y-x})'_z} = -\frac{\qquad}{\qquad}$$

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$$= -\frac{-1 + e^{z-y-x} - xe^{z-y-x}}{1 + xe^{z-y-x}} = -\frac{1 + (x-1)e^{z-y-x}}{1 + xe^{z-y-x}}$$

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$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(z-y-x+xe^{z-y-x})'_y}{(z-y-x+xe^{z-y-x})'_z}=-\frac{-1-xe^{z-y-x}}{1+xe^{z-y-x}}$$

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$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=-\frac{1+(x-1)e^{z-y-x}}{1+xe^{z-y-x}}dx+dy$$

例 设 $\Phi(u, v)$ 具有连续偏导数，函数 $z = z(x, y)$ 满足 $\Phi(cx - az, cy - bz) = 0$，证明：

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x$$

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$$F_z =$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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$$F_z =$$

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$$F_z = \Phi_u \cdot u_z + \Phi_v \cdot v_z$$

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$$=\frac{-e^z}{(ye^z-1)^3}=\frac{e^z}{(1+x-z)^3}$$

We are here now...

1. 一个方程的情形

2. 方程组的情形

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

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$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}\,a_{22}x + a_{12}\,a_{22}y = a_{22}b_1 & (1)\times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2)\times a_{12} \end{cases}$$

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$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

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回顾：二元线性方程组的求解

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$$y=\frac{a_{11}b_2-b_1a_{21}}{a_{11}a_{22}-a_{12}a_{21}}$$

回顾：二元线性方程组的求解

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$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \quad \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $x =$ ， $y =$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $x =$ ， $y =$

公式：

$$\begin{cases} a_{11}x+a_{12}y=b_1 \\ a_{21}x+a_{22}y=b_2 \end{cases} \Rightarrow \quad x=\frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y=\frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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方程组的隐函数求导公式

$$
\begin{cases}
F(x, y, u, v) = 0, \\
G(x, y, u, v) = 0.
\end{cases}
$$

方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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问题：如何计算 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ ？

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问题：如何计算 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ ？

求解如下：

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}}$$

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假设函数 $u=u(x,y)$, $v=v(x,y)$ 满足方程组

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问题：如何计算 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ ？

求解如下：

$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x+F_u\cdot u_x+F_v\cdot v_x=0 \\ \end{cases}$$

方程组的隐函数求导公式

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$$= -\frac{1}{J}\frac{\partial(F, G)}{\partial(x, v)}$$

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$$= -\frac{1}{J}\frac{\partial(F, G)}{\partial(x, v)} \qquad = -\frac{1}{J}\frac{\partial(F, G)}{\partial(u, x)}$$

$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}}$$

$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ \end{cases}$$

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$$\Rightarrow u_y = \frac{\qquad}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = \frac{\qquad}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

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$$=-\frac{1}{J}\frac{\partial(F,G)}{\partial(y,v)}$$

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$$=-\frac{1}{J}\frac{\partial(F,G)}{\partial(y,v)} \qquad =-\frac{1}{J}\frac{\partial(F,G)}{\partial(u,y)}$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

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$u_x =$

$v_x =$

$u_y =$

$v_y =$

总结 设 $u = u(x, y), v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \quad \begin{matrix} \xRightarrow{\frac{\partial}{\partial x}} \\ \\ \xRightarrow{\frac{\partial}{\partial y}} \end{matrix}$$

$u_x =$ $\qquad$ $v_x =$

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$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\xrightarrow{\frac{\partial}{\partial y}}$$

$u_x =$ $\qquad\qquad$ $v_x =$

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$$\xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y+F_u\cdot u_y+F_v\cdot v_y=0 \\ G_y+G_u\cdot u_y+G_v\cdot v_y=0 \end{cases}$$

所以

$$u_x=-\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \qquad v_x=-\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$u_y= \qquad v_y=$$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y+F_u\cdot u_y+F_v\cdot v_y=0 \\ G_y+G_u\cdot u_y+G_v\cdot v_y=0 \end{cases}$$

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$$u_x=-\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(x,v)}, \quad v_x=-\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$u_y=-\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \qquad v_y=-\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

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总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \quad \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_x = -\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = -\frac{1}{J}\frac{\partial(F, G)}{\partial(x, v)}, \quad v_x = -\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = -\frac{1}{J}\frac{\partial(F, G)}{\partial(u, x)}$$

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总结 设 $u=u(x,y)$, $v=v(x,y)$ 满足方程组

$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x+F_u\cdot u_x+F_v\cdot v_x=0 \\ G_x+G_u\cdot u_x+G_v\cdot v_x=0 \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y+F_u\cdot u_y+F_v\cdot v_y=0 \\ G_y+G_u\cdot u_y+G_v\cdot v_y=0 \end{cases}$$

所以

$$u_x=-\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(x,v)},\quad v_x=-\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(u,x)}$$

$$u_y=-\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(y,v)},\quad v_y=-\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(u,y)}$$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

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解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \quad \overset{\frac{\partial}{\partial x}}{\Longrightarrow} \quad \overset{\frac{\partial}{\partial y}}{\Longrightarrow}$$

$u_x =$ $\qquad$ $v_x =$

$u_y =$ $\qquad$ $v_y =$

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$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1 \\ \end{cases}$$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ \end{cases}$$

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所以 $J = \begin{vmatrix} e^u + \sin v & u\cos v \\ e^u - \cos v & u\sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$

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