## §4.2 相似矩阵与矩阵对角化

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定义 设 A, B 是 n 阶方阵。若存在 n 阶可逆矩阵 P, 满足  $P^{-1}AP = B.$ 

则称 A 与 B相似,记为  $A \sim B$ 。

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,  $B$  是  $n$  阶方阵。若存在  $n$  阶可逆矩阵  $P$ , 满足 
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#### 注

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$$A \sim B$$
 ⇔ ∃可逆 $Q$ , 使 $QAQ^{-1} = B$ 



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$$A \sim B \iff \exists \, \text{可逆}Q, \, \text{使}QAQ^{-1} = B$$
 (令  $P := Q^{-1}, \, \text{则} \, P^{-1}AP = B$ )



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2.  $A \sim B \iff B \sim A$ 

$$\underbrace{\begin{pmatrix} 19 & 45 \\ -7 & -17 \end{pmatrix}}_{A} \qquad \underbrace{\begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix}}_{B}$$

$$\left(\begin{array}{cc}1&2\\2&5\end{array}\right)^{-1}\underbrace{\left(\begin{array}{cc}19&45\\-7&-17\end{array}\right)}_{A}\left(\begin{array}{cc}1&2\\2&5\end{array}\right)\quad\underbrace{\left(\begin{array}{cc}3&1\\5&-1\end{array}\right)}_{B}$$



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- 1. "λ 矩阵"的方法,但并不简单的。。。
- 2. 下面只给出两个矩阵相似的必要条件

定理设 $A \sim B$ ,则

- 1. A 与 B 有相同特征值;
- 2. r(A) = r(B);
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证明 存在可逆矩阵 P, 满足

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$$A_{n\times n}$$
 与对角阵  $\Lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{pmatrix}$  相似,则称  $A$ 可对角化

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推论 若方阵  $A_{n \times n}$  有 n 不同特征值,则 A 可对角化。



问题 判断 n 阶方阵 A 是否可以对角化?若能,确定可逆矩阵 P 及对角阵

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步骤

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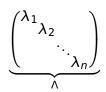
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- 3. 假设存在 n 个线性无关特征向量  $\alpha_1, \alpha_2, \ldots, \alpha_n$ ,记对应特征值为  $\lambda_1, \lambda_2, \ldots, \lambda_n$

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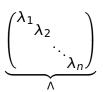




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$$\Rightarrow P^{-1}AP = \Lambda$$



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$$\underbrace{(\alpha_1,\,\alpha_2,\,\ldots,\,\alpha_n)}_{P}$$



- 1. 求出 A 的所有特征值,及相应特征向量
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- 3. 假设存在 n 个线性无关特征向量  $\alpha_1, \alpha_2, \ldots, \alpha_n$ ,记对应特征值为  $\lambda_1, \lambda_2, \ldots, \lambda_n$  (即: $A\alpha_i = \lambda_i \alpha_i$  )则

$$A\underbrace{(\alpha_1, \alpha_2, \ldots, \alpha_n)}_{P} = \underbrace{(\alpha_1, \alpha_2, \ldots, \alpha_n)}_{P} \underbrace{\begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \ddots \\ \lambda_n \end{pmatrix}}_{\Lambda}$$

$$\Rightarrow P^{-1}AP = \Lambda$$



- 1. 求出 A 的所有特征值,及相应特征向量
- 2. 若有 *n* 个线性无关特征向量,则 *A* 可对角化;否则,不能对角化
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$$\Rightarrow AP = P\Lambda \Rightarrow P^{-1}AP = \Lambda$$



例  $A = \begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix}$ 

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$$\bigcirc A = \begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix}$$

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- 特征值  $\lambda_2 = -2$ ,特征向量  $\alpha_2 = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$

$$\bigcirc A = \begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix}$$

- 特征方程:  $0 = |\lambda I A| = (\lambda + 2)(\lambda 4)$
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$$P = ($$
 ) ,  $\wedge = ($ 

$$P^{-1}AP = \Lambda$$



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$$P = (\alpha_1, \alpha_2)$$
 ,  $\Lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}$  ,

$$P^{-1}AP = \Lambda$$



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$$P = (\alpha_1, \alpha_2) = \begin{pmatrix} 1 & 1 \\ 1 & -5 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix},$$

$$P^{-1}AP = \Lambda$$



例 判断 
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
 是否能对角化? 若能, 写出  $P$  和  $\Lambda$ 

特征方程: 0 = |λI − A|



例 判断 
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
 是否能对角化? 若能, 写出  $P$  和  $\Lambda$ 

• 特征方程:  $0 = |\lambda I - A| = (\lambda - 2)^2 (\lambda - 6)$ 

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- 特征值 λ₂ = 6

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- 特征方程:  $0 = |\lambda I A| = (\lambda 2)^2 (\lambda 6)$
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- 特征值  $\lambda_2 = 6$ ,特征向量  $\alpha_3 = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

可见 A 有 3 个线性无关特征向量:  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ 。 所以 A 可以对角化。

• 特征值 
$$\lambda_1 = 2$$
 (二重) ,特征向量  $\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$  , $\alpha_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$  • 特征值  $\lambda_2 = 6$  ,特征向量  $\alpha_3 = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ 

可见 
$$A$$
 有  $3$  个线性无关特征向量:  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ 。所以  $A$  可以对角化。

$$P=($$
 ) ,  $\Lambda=($  )

$$P^{-1}AP = \Lambda$$



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$$\lambda_1 = 2$$
 (二重),特征向量  $\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ , $\alpha_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$   
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可见 A 有 3 个线性无关特征向量:  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ 。 所以 A 可以对角化。

$$P = (\alpha_1, \alpha_2, \alpha_3) \qquad , \quad \Lambda = \begin{pmatrix} \lambda_1 & \\ & \lambda_2 & \\ & & \lambda_2 \end{pmatrix}$$

则  $P^{-1}AP = \Lambda$ 

- 特征方程:  $0 = |\lambda I A| = (\lambda 2)^2 (\lambda 6)$
- 特征值  $\lambda_1 = 2$  (二重) ,特征向量  $\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$  , $\alpha_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$
- 特征值  $\lambda_2 = 6$ ,特征向量  $\alpha_3 = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ 可见 A 有 3 个线性无关特征向量:  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ 。 所以 A 可以对角化。

今

 $P = (\alpha_1, \alpha_2, \alpha_3) = \begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & -2 \\ 0 & 1 & 3 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}$ 

则  $P^{-1}AP = \Lambda$ 



• 特征方程: 
$$0 = |\lambda I - A| = (\lambda - 2)^2 (\lambda - 6)$$

• 特征值  $\lambda_1 = 2$  (二重) ,特征向量  $\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$  , $\alpha_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ 

• 特征值 
$$\lambda_2 = 6$$
,特征向量  $\alpha_3 = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ 

可见 A 有 3 个线性无关特征向量:  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ 。 所以 A 可以对角化。

 $P = (\alpha_1, \alpha_2, \alpha_3) = \begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & -2 \\ 0 & 1 & 3 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 6 \end{pmatrix},$ 则  $P^{-1}AP = \Lambda$ 

- 特征方程:  $0 = |\lambda I A| = (\lambda 1)(\lambda 2)^2$  Details
- 特征值  $\lambda_1 = 1$ ,特征向量  $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$  Details
- 特征值  $\lambda_2 = 2$ (二重),特征向量  $\alpha_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ , $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$  **Det**

例 判断 
$$A = \begin{pmatrix} 1 & 1 & -1 \\ -2 & 4 & -2 \\ -2 & 2 & 0 \end{pmatrix}$$
 是否能对角化? 若能, 写出  $P$  和  $\Lambda$ 

- 特征方程:  $0 = |\lambda I A| = (\lambda 1)(\lambda 2)^2$  Details
- 特征值  $\lambda_1 = 1$ ,特征向量  $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$  Details
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可见 A 有 3 个线性无关特征向量:  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ 。所以 A 可以对角化。



- 特征方程:  $0 = |\lambda I A| = (\lambda 1)(\lambda 2)^2$  Details
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可见 A 有 3 个线性无关特征向量:  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ 。 所以 A 可以对角化。

$$P=($$
 ,  $\Lambda=\left($ 

则 5-1

$$P^{-1}AP = \Lambda$$
.

- 特征方程:  $0 = |\lambda I A| = (\lambda 1)(\lambda 2)^2$  Details
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今

$$P = (\alpha_1, \alpha_2, \alpha_3)$$
 ,  $\Lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}$ 

则

$$P^{-1}AP = \Lambda$$
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- 特征方程:  $0 = |\lambda I A| = (\lambda 1)(\lambda 2)^2$  Details
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可见 A 有 B 个线性无关特征向量:  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$ 。 所以 A 可以对角化。

令
$$P = (\alpha_1, \alpha_2, \alpha_3) = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix}$$
则

 $P^{-1}AP = \Lambda$ .



- 特征方程:  $0 = |\lambda I A| = (\lambda 1)(\lambda 2)^2$  Details
- 特征值  $\lambda_1 = 1$ ,特征向量  $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$  Details
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$$P = (\alpha_1, \alpha_2, \alpha_3) = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \lambda_1 & \lambda_2 & \\ & \lambda_3 & \end{pmatrix} = \begin{pmatrix} 1 & 2 & \\ & 2 & \end{pmatrix},$$
 则

 $P^{-1}AP = \Lambda$ .



定理 n 阶方阵 A 可对角化的充分必要条件是:每个  $n_i$  重的特征值  $\lambda_i$ ,矩

阵 $\lambda_i I - A$  的秩是 $n - n_i$ 。

图解如下:

不同

特征值

 $\lambda_1$ 

 $n_1$ 

 $(\lambda_i I - A)x = 0$  基础解系 /线性无关特征向量

 $\lambda_2$   $n_2$ 

Ė

$$\lambda_s$$
  $n_s$ 

共 n

 $|\lambda I - A| = (\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \cdots (\lambda - \lambda_s)^{n_s}$ 

 $n_1$   $r(\lambda_1 I - A) = n - n_1$ 

·

图解如下:

不同

特征值

 $\lambda_1$ 

 $(\lambda_i I - A)x = 0$  基础解系数 /线性无关特征向量

$$\lambda_2$$
  $n_2$ 

$$\lambda_s$$
  $n_s$ 

共 n

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图解如下:

特征值

 $\lambda_1$ 

 $n_1$ 

重  $(\lambda_i I - A)x = 0$  基础解系数 /线性无关特征向量

 $r(\lambda_1 I - A) = n - n_1 \implies \alpha_1^{(1)}, \alpha_2^{(1)}, \dots, \alpha_n^{(1)}$ 

$$\lambda_2$$
  $n_2$   $\vdots$   $\vdots$   $\lambda_s$   $n_s$ 

+ n  $|\lambda I - A| = (\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \cdots (\lambda - \lambda_s)^{n_s}$ 

阵  $\lambda_i I - A$  的秩是  $n - n_i$ 。

定理 n 阶方阵 A 可对角化的充分必要条件是:每个  $n_i$  重的特征值  $\lambda_i$  矩

特征值

 $\lambda_1$ 

 $\lambda_s$ 

重 
$$(\lambda_i I - A)x = 0$$
 基础解系数 /线性无关特征向量

 $n_1 r(\lambda_1 I - A) = n - n_1 \Rightarrow \alpha_1^{(1)}, \alpha_2^{(1)}, \dots, \alpha_n^{(1)}$ 

$$\lambda_2$$
 n

$$n_2 \qquad r(\lambda_2 I - A) = n - n_2$$

$$|\lambda I - A| = (\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \cdots (\lambda - \lambda_s)^{n_s}$$

图解如下:

不同

特征值

$$(\lambda_i I - A)x = 0$$
基础解系  
/线性无关特征向量

$$\lambda_1$$
  $\lambda_2$ 

 $\lambda_s$ 

$$n_1 \quad r(\lambda_1 I - A) = n - n_1 \quad \Rightarrow \quad \alpha_1^{(1)}, \, \alpha_2^{(1)}, \, \cdots, \, \alpha_{n_1}^{(1)}$$

$$n_1 \quad r(\lambda_1 I - A) = n - n_1 \quad \Rightarrow \quad \alpha_1 \quad , \alpha_2 \quad , \cdots , \alpha_{n_1}^{(2)}$$
 $n_2 \quad r(\lambda_2 I - A) = n - n_2 \quad \Rightarrow \quad \alpha_1^{(2)}, \alpha_2^{(2)}, \cdots , \alpha_{n_2}^{(2)}$ 

$$\alpha_1^{(2)}, \alpha_2^{(2)}, \cdots, \alpha_{n_2}^{(2)}$$

$$|\lambda I - A| = (\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \cdots (\lambda - \lambda_s)^{n_s}$$

图解如下:

不同

 $(\lambda_i I - A)x = 0$  基础解系 特征值 数 /线性无关特征向量

$$\lambda_{1} \qquad n_{1} \qquad r(\lambda_{1}I - A) = n - n_{1} \quad \Rightarrow \quad \alpha_{1}^{(1)}, \, \alpha_{2}^{(1)}, \, \cdots, \, \alpha_{n_{1}}^{(1)}$$

$$\lambda_{2} \qquad n_{2} \qquad r(\lambda_{2}I - A) = n - n_{2} \quad \Rightarrow \quad \alpha_{1}^{(2)}, \, \alpha_{2}^{(2)}, \, \cdots, \, \alpha_{n_{2}}^{(2)}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\lambda_{s} \qquad n_{s} \qquad r(\lambda_{s}I - A) = n - n_{s}$$

共 n

$$|\lambda I - A| = (\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \cdots (\lambda - \lambda_s)^{n_s}$$



图解如下:

不同

 $(\lambda_i I - A)x = 0$  基础解系 /线性无关特征向量 特征值 数

$$\lambda_{1} \qquad n_{1} \qquad r(\lambda_{1}I - A) = n - n_{1} \quad \Rightarrow \quad \alpha_{1}^{(1)}, \, \alpha_{2}^{(1)}, \, \cdots, \, \alpha_{n_{1}}^{(1)}$$

$$\lambda_{2} \qquad n_{2} \qquad r(\lambda_{2}I - A) = n - n_{2} \quad \Rightarrow \quad \alpha_{1}^{(2)}, \, \alpha_{2}^{(2)}, \, \cdots, \, \alpha_{n_{2}}^{(2)}$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\lambda_{s} \qquad n_{s} \qquad r(\lambda_{s}I - A) = n - n_{s} \quad \Rightarrow \quad \alpha_{1}^{(s)}, \, \alpha_{2}^{(s)}, \, \cdots, \, \alpha_{n_{s}}^{(s)}$$

共 n

 $|\lambda I - A| = (\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \cdots (\lambda - \lambda_s)^{n_s}$ 

图解如下:

不同

特征值

 $\lambda_1$ 

 $\lambda_2$ 

 $\lambda_s$ 

数

 $(\lambda_i I - A)x = 0$  基础解系 /线性无关特征向量

$$n_1 \quad r(\lambda_1 I - A) = n - n_1 \quad \Rightarrow \quad \alpha_1^{(1)}, \, \alpha_2^{(1)}, \, \cdots, \, \alpha_{n_1}^{(1)}$$

$$n_2 \quad r(\lambda_2 I - A) = n - n_2 \quad \Rightarrow \quad \alpha_1^{(2)}, \, \alpha_2^{(2)}, \, \cdots, \, \alpha_{n_2}^{(2)}$$

 $|\lambda I - A| = (\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \cdots (\lambda - \lambda_s)^{n_s}$ 

$$n-n_2 \Rightarrow$$

$$n_s$$
  $r(\lambda_s I - A) = n - n_s$   $\Rightarrow$   $\alpha_1^{(s)}, \alpha_2^{(s)}, \dots, \alpha_{n_s}^{(s)}$ 

$$\Rightarrow \qquad \alpha_1^{(s)}$$

$$\Rightarrow \alpha_1^{()}$$

$$^{s)}$$
,  $\alpha_2^{(s)}$ 

共 n 个无关特征向量

$$\cdots$$
,  $\alpha_{n_2}^{(2)}$ 

$$\cdots$$
,  $\alpha_{n_2}^{(2)}$ 

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与 Λ 相似 ⇔

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与  $\Lambda$  相似 ⇔ A 可对角化

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若  $A \subseteq \Lambda$  相似  $\iff$   $A \cap \forall \exists A \cap \lambda_1 = 1$   $\lambda_2 = 2$ 

例 下列哪个矩阵与  $\Lambda = \begin{pmatrix} 100\\ 010\\ 002 \end{pmatrix}$  相似?

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若  $A \subseteq \Lambda$  相似  $\iff$   $A \cap A$  可对角化, $\lambda_1 = 1$  (二重), $\lambda_2 = 2$ 

例 下列哪个矩阵与  $\Lambda = \begin{pmatrix} 100\\ 010\\ 002 \end{pmatrix}$  相似?

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与  $\Lambda$  相似  $\Leftrightarrow$  A 可对角化, $\lambda_1 = 1$  (二重), $\lambda_2 = 2$  (一重)

例 下列哪个矩阵与  $\Lambda = \begin{pmatrix} 100\\ 010\\ 002 \end{pmatrix}$  相似?

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 
$$A$$
 与  $\Lambda$  相似  $\Leftrightarrow$   $A$  可对角化, $\lambda_1 = 1$  (二重), $\lambda_2 = 2$  (一重) 
$$\Leftrightarrow r(I-A) = \qquad \qquad \exists \ r(2I-A) =$$

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与  $\Lambda$  相似  $\Leftrightarrow$  A 可对角化, $\lambda_1 = 1$  (二重), $\lambda_2 = 2$  (一重)  $\Leftrightarrow r(I-A) = 3-2 = 1$  且 r(2I-A) =

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若  $A = \Lambda$  相似  $\iff$   $A = \Lambda$  可对角化, $\lambda_1 = 1$  (二重), $\lambda_2 = 2$  (一重)  $\iff r(I - A) = 3 - 2 = 1 \text{ L} \ r(2I - A) = 3 - 1 = 2$ 

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与 Λ 相似 ⇔ A 可对角化,  $\lambda_1 = 1$  (二重),  $\lambda_2 = 2$  (一重)

解 
$$\Rightarrow r(I-A) = 3-2 = 1 \perp r(2I-A) = 3-1 = 2$$

$$A_1 \qquad A_2 \qquad A_3 \qquad A_4$$

I - A

r(I-A)2I - A

$$r(2I-A)$$



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 $A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ 

例 下列哪个矩阵与  $\Lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$  相似?

提示 若 
$$A$$
 与  $\Lambda$  相似  $\Leftrightarrow$   $A$  可对角化, $\lambda_1 = 1$  (二重), $\lambda_2 = 2$  (一重)

胖	<b>→</b> 1 (1	A) — 3	2 – 1 ± 1 (21	A) = 3 1 = 2
	$A_1$	A <sub>2</sub>	A <sub>3</sub>	<i>A</i> <sub>4</sub>
I – A	$\left(\begin{smallmatrix}0&-1&0\\0&-1&-1\\0&0&0\end{smallmatrix}\right)$			
r(I - A)				

r(2I-A)



$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 
$$A$$
 与  $\Lambda$  相似  $\Leftrightarrow$   $A$  可对角化, $\lambda_1 = 1$  (二重), $\lambda_2 = 2$  (一重)

r(2I-A)相似矩阵与矩阵对角化

$$A_{1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_{2} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_{3} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_{4} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与 Λ 相似 ⇔ A 可对角化,  $\lambda_1 = 1$  (二重),  $\lambda_2 = 2$  (一重)

r(2I-A)

2I - A

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与  $\Lambda$  相似  $\Leftrightarrow$  A 可对角化, $\lambda_1 = 1$  (二重), $\lambda_2 = 2$  (一重)

解 
$$\Rightarrow r(I-A) = 3-2 = 1 \pm r(2I-A) = 3-1 = 2$$

$$A_1 \qquad A_2 \qquad A_3 \qquad A_4$$

$$I-A \qquad \begin{pmatrix} 0-1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$r(I-A)$$

21 – A

r(2I-A)

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与 Λ 相似 ⇔ A 可对角化,  $\lambda_1 = 1$  (二重),  $\lambda_2 = 2$  (一重)

解 
$$\Rightarrow r(I-A) = 3-2 = 1$$
且  $r(2I-A) = 3-1 = 2$ 

$$A_1 \qquad A_2 \qquad A_3 \qquad A_4$$

$$I-A \qquad \begin{pmatrix} 0-1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 0-1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$r(I-A) \qquad 2$$

r(2I-A)

§4.2 相似矩阵与矩阵对角化

2I - A

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与  $\Lambda$  相似  $\Leftrightarrow$  A 可对角化, $\lambda_1 = 1$  (二重), $\lambda_2 = 2$  (一重)

解 
$$\Rightarrow r(I-A) = 3-2 = 1 \pm r(2I-A) = 3-1 = 2$$

$$A_1 \qquad A_2 \qquad A_3 \qquad A_4$$

$$I-A \qquad \begin{pmatrix} 0-1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$r(I-A) \qquad 2 \qquad 2$$

r(2I-A)

2I - A

**● 基本大** 

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若  $A \subseteq \Lambda$  相似  $\Leftrightarrow A$  可对角化, $\lambda_1 = 1$  (二重), $\lambda_2 = 2$  (一重)

解 
$$\Rightarrow r(I-A) = 3-2 = 1$$
且  $r(2I-A) = 3-1 = 2$ 

$$A_1 \qquad A_2 \qquad A_3 \qquad A_4$$

$$I-A \qquad \begin{pmatrix} 0-1 & 0 \\ 0-1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0-1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0-1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$r(I-A) \qquad 2 \qquad 2 \qquad 1$$

r(2I-A)

2I - A

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与  $\Lambda$  相似  $\Leftrightarrow$  A 可对角化, $\lambda_1 = 1$  (二重), $\lambda_2 = 2$  (一重)

解	$\Leftrightarrow r(I-A) = 3-2 = 1 \perp r(2I-A) = 3-1 = 2$			
	$A_1$	A <sub>2</sub>	<b>A</b> <sub>3</sub>	$A_4$
I-A	$\left(\begin{smallmatrix}0&-1&0\\0&-1&-1\\0&0&0\end{smallmatrix}\right)$	$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$
r(I-A)	2	2	1	2

r(2I-A)

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2I - A

例下列哪个矩阵与 
$$\Lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$
 相似?  

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与 Λ 相似 ⇔ A 可对角化,  $\lambda_1 = 1$  (二重),  $\lambda_2 = 2$  (一重)

解	$\Leftrightarrow I(I-A) = 3-2 = I \coprod I(2I-A) = 3-1 = 2$			
	$A_1$	A <sub>2</sub>	<b>A</b> <sub>3</sub>	$A_4$
I-A	$\left(\begin{smallmatrix}0&-1&0\\0&-1&-1\\0&0&0\end{smallmatrix}\right)$	$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$
r(I-A)	2	2	1	2
2 <i>I</i> – A			$\left(\begin{smallmatrix}1&0&-1\\0&1&0\\0&0&0\end{smallmatrix}\right)$	

r(2I-A)

例下列哪个矩阵与 
$$\Lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$
相似?  

$$A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

提示 若 A 与 Λ 相似 ⇔ A 可对角化,  $\lambda_1 = 1$  (二重),  $\lambda_2 = 2$  (一重)  $\Leftrightarrow r(I-A) = 3-2 = 1 \perp r(2I-A) = 3-1 = 2$ 鼦

四十	•	•	,	
	$A_1$	A <sub>2</sub>	<b>A</b> <sub>3</sub>	$A_4$
I-A	$\left(\begin{smallmatrix}0&-1&0\\0&-1&-1\\0&0&0\end{smallmatrix}\right)$	$\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$
r(I-A)	2	2	1	2
2 <i>I</i> – A			$\left(\begin{smallmatrix}1&0&-1\\0&1&0\\0&0&0\end{smallmatrix}\right)$	

例 下列哪个矩阵与  $\Lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$  相似?  $A_1 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_2 = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_3 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_4 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ 

提示 若 A 与 Λ 相似 ⇔ A 可对角化,  $\lambda_1 = 1$  (二重),  $\lambda_2 = 2$  (一重)  $\Leftrightarrow r(I-A) = 3-2 = 1 \exists r(2I-A) = 3-1 = 2$ 47

<b>用牛</b>					
	$A_1$	A <sub>2</sub>	A <sub>3</sub>	$A_4$	
I-A	$\begin{pmatrix} 0 - 1 & 0 \\ 0 - 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 - 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$	
r(I-A)	2	2	1	2	
2 <i>I</i> – <i>A</i>	$\begin{pmatrix} 1 - 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix}1-1&0\\0&1&0\\0&0&0\end{pmatrix}$	$\left(\begin{smallmatrix}1&0&-1\\0&1&0\\0&0&0\end{smallmatrix}\right)$	$\left(\begin{smallmatrix}1&0&-1\\0&0&-1\\0&0&1\end{smallmatrix}\right)$	

r(2I-A)相似矩阵与矩阵对角化

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$$A_{1} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} A_{2} = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_{3} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} A_{4} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$
 提示 若  $A = A$  可对角化, $A_{1} = A$  (二重), $A_{2} = A$  (一重)

例 下列哪个矩阵与  $\Lambda = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$  相似?

提示 若 A 与 Λ 相似 ⇔ A 可对角化,  $\lambda_1 = 1$  (二重),  $\lambda_2 = 2$  (一重)  $\Leftrightarrow r(I-A) = 3-2 = 1 \perp r(2I-A) = 3-1 = 2$ 

<b>州</b>		` ,		` ,	
		$A_1$	A <sub>2</sub>	<b>A</b> <sub>3</sub>	$A_4$
	I – A	$\left(\begin{smallmatrix}0&-1&0\\0&-1&-1\\0&0&0\end{smallmatrix}\right)$	$\begin{pmatrix} 0 - 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$
_	r(I-A)	2	2	1	2
	2 <i>I</i> – A	$\begin{pmatrix} 1 - 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix}$	$\left(\begin{smallmatrix}1&-1&0\\0&1&0\\0&0&0\end{smallmatrix}\right)$	$\left(\begin{smallmatrix}1&0&-1\\0&1&0\\0&0&0\end{smallmatrix}\right)$	$\left(\begin{smallmatrix}1&0&-1\\0&0&-1\\0&0&1\end{smallmatrix}\right)$

r(2I-A)相似矩阵与矩阵对角化

例 
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,则

$$P^{-1}AP = \begin{pmatrix} 2 & & \\ & 2 & \\ & & 6 \end{pmatrix}$$

例 
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,则

$$P^{-1}AP = \begin{pmatrix} 2 & & \\ & 2 & \\ & & 6 \end{pmatrix} \quad \Rightarrow \quad A = P \begin{pmatrix} 2 & & \\ & 2 & \\ & & 6 \end{pmatrix} P^{-1}$$



例 
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,则

$$P^{-1}AP = \begin{pmatrix} 2 & & \\ & 2 & \\ & 6 \end{pmatrix} \Rightarrow A = P \begin{pmatrix} 2 & & \\ & 2 & \\ & 6 \end{pmatrix} P^{-1} = P \wedge P^{-1}$$

例 
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,则 
$$P^{-1}AP = \begin{pmatrix} 2 & 2 & 0 \\ 0 & 6 & 0 \end{pmatrix} \Rightarrow A = P \begin{pmatrix} 2 & 0 \\ 0 & 6 & 0 \end{pmatrix} P^{-1} = P \wedge P^{-1}$$

这相当对 A 作了一个"好的"分解。



例 
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,则
$$P^{-1}AP = \begin{pmatrix} 2 & 2 & 0 \\ 0 & 6 & 0 \end{pmatrix} \Rightarrow A = P \begin{pmatrix} 2 & 0 \\ 0 & 6 & 0 \end{pmatrix} P^{-1} = P \wedge P^{-1}$$

这相当对 A 作了一个"好的"分解。应用:

$$A^n =$$

例 
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,则
$$P^{-1}AP = \begin{pmatrix} 2 & 2 & 0 \\ 0 & 6 & 0 \end{pmatrix} \Rightarrow A = P \begin{pmatrix} 2 & 0 \\ 0 & 6 & 0 \end{pmatrix} P^{-1} = P \wedge P^{-1}$$

这相当对 
$$A$$
 作了一个"好的"分解。应用:
$$A^{n} = (P \wedge P^{-1}) \cdot (P \wedge P^{-1})(P \wedge P^{-1}) \cdots (P \wedge P^{-1})(P \wedge P^{-1})$$
=

例 
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,则 
$$P^{-1}AP = \begin{pmatrix} 2 & & \\ & 2 & \\ & 6 \end{pmatrix} \implies A = P \begin{pmatrix} 2 & & \\ & 2 & \\ & 6 \end{pmatrix} P^{-1} = P \wedge P^{-1}$$

这相当对 
$$A$$
 作了一个"好的"分解。应用:
$$A^n = (P \wedge P^{-1}) \cdot (P \wedge P^{-1})(P \wedge P^{-1}) \cdot \cdots (P \wedge P^{-1})(P \wedge P^{-1})$$
$$= P \wedge \cdot \wedge \cdots \wedge P^{-1}$$

例 
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,则 
$$P^{-1}AP = \begin{pmatrix} 2 & & \\ & 2 & \\ & 6 \end{pmatrix} \implies A = P \begin{pmatrix} 2 & & \\ & 2 & \\ & 6 \end{pmatrix} P^{-1} = P \wedge P^{-1}$$

这相当对 
$$A$$
 作了一个"好的"分解。应用:
$$A^{n} = (P\Lambda P^{-1}) \cdot (P\Lambda P^{-1})(P\Lambda P^{-1}) \cdots (P\Lambda P^{-1})(P\Lambda P^{-1})$$

$$= P \wedge \cdot \wedge \cdots \wedge P^{-1}$$

$$= P \wedge^{n} P^{-1}$$

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例 
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,则 
$$P^{-1}AP = \begin{pmatrix} 2 & 2 & 0 \\ 0 & 6 & 0 \end{pmatrix} \Rightarrow A = P \begin{pmatrix} 2 & 0 \\ 0 & 6 & 0 \end{pmatrix} P^{-1} = P \wedge P^{-1}$$

$$A^{n} = (P \wedge P^{-1}) \cdot (P \wedge P^{-1})(P \wedge P^{-1}) \cdots (P \wedge P^{-1})(P \wedge P^{-1})$$

$$= P \wedge \cdot \wedge \cdots \wedge P^{-1}$$

$$= P \wedge^{n} P^{-1}$$

$$= \begin{pmatrix} 2^{n} & & \\ & 6^{n} \end{pmatrix}$$

例 
$$A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 4 & -2 \\ -3 & -3 & 5 \end{pmatrix}$$
,则

$$P^{-1}AP = \begin{pmatrix} 2 & 2 & \\ & 6 & \end{pmatrix} \Rightarrow A = P \begin{pmatrix} 2 & 2 & \\ & 6 & \end{pmatrix} P^{-1} = P \wedge P^{-1}$$

这相当对 A 作了一个"好的"分解。应用:

$$A^{n} = (P \wedge P^{-1}) \cdot (P \wedge P^{-1})(P \wedge P^{-1}) \cdots (P \wedge P^{-1})(P \wedge P^{-1})$$
$$= P \wedge \cdot \wedge \cdots \wedge P^{-1}$$

$$\begin{split} &= P \Lambda^n P^{-1} \\ &= \begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & -2 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 2^n & & \\ & 2^n & \\ & & 6^n \end{pmatrix} \begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & -2 \\ 0 & 1 & 3 \end{pmatrix}^{-1} \end{split}$$

———The End———

$$0 = |\lambda I - A| =$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$

$$r_3-r_2$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$
$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -\lambda + 2 & \lambda - 2 \end{vmatrix}$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$
$$\frac{r_3 - r_2}{=} \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -\lambda + 2 & \lambda - 2 \end{vmatrix}$$
$$= (\lambda - 2) \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -1 & 1 \end{vmatrix}$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$
$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -\lambda + 2 & \lambda - 2 \end{vmatrix}$$
$$= (\lambda - 2) \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -1 & 1 \end{vmatrix} \xrightarrow{c_2 + 2c_3}$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$

$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -\lambda + 2 & \lambda - 2 \end{vmatrix}$$

$$= (\lambda - 2) \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -1 & 1 \end{vmatrix} \stackrel{c_2 + 2c_3}{=} (\lambda - 2) \begin{vmatrix} \lambda - 1 & 0 & 1 \\ 2 & \lambda - 2 & 2 \\ 0 & 0 & 1 \end{vmatrix}$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$

$$= \frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -\lambda + 2 & \lambda - 2 \end{vmatrix}$$

$$= (\lambda - 2) \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -1 & 1 \end{vmatrix} \xrightarrow{c_2 + 2c_3} (\lambda - 2) \begin{vmatrix} \lambda - 1 & 0 & 1 \\ 2 & \lambda - 2 & 2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (\lambda - 2) \begin{vmatrix} \lambda - 1 & 1 \\ 0 & 1 \end{vmatrix}$$





$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$

$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -\lambda + 2 & \lambda - 2 \end{vmatrix}$$

$$= (\lambda - 2) \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -1 & 1 \end{vmatrix} \stackrel{c_2 + 2c_3}{=} (\lambda - 2) \begin{vmatrix} \lambda - 1 & 0 & 1 \\ 2 & \lambda - 2 & 2 \\ 0 & 1 \end{vmatrix}$$

$$= (\lambda - 2)(\lambda - 2) \begin{vmatrix} \lambda - 1 & 1 \\ 0 & 1 \end{vmatrix}$$





$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 2 & -2 & \lambda \end{vmatrix}$$

$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -\lambda + 2 & \lambda - 2 \end{vmatrix}$$

$$= (\lambda - 2) \begin{vmatrix} \lambda - 1 & -1 & 1 \\ 2 & \lambda - 4 & 2 \\ 0 & -1 & 1 \end{vmatrix} \frac{c_2 + 2c_3}{2} (\lambda - 2) \begin{vmatrix} \lambda - 1 & 0 & 1 \\ 2 & \lambda - 2 & 2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (\lambda - 2)(\lambda - 2) \begin{vmatrix} \lambda - 1 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= (\lambda - 1)(\lambda - 2)^2$$



•  $\exists \lambda_1 = 1$ ,  $\forall x \in (\lambda_1 I - A)x = 0$ :

$$(1I - A : 0) =$$

$$(1I - A \vdots 0) = \begin{pmatrix} 0 & -1 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 2 & -2 & 1 & | & 0 \end{pmatrix}$$



$$(1I - A \vdots 0) = \begin{pmatrix} 0 & -1 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 2 & -2 & 1 & | & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 0 & -1 & 1 & | & 0 \end{pmatrix}$$



$$(1I - A \vdots 0) = \begin{pmatrix} 0 & -1 & 1 & 0 \\ 2 & -3 & 2 & 0 \\ 2 & -2 & 1 & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & 0 \\ 2 & -3 & 2 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix}$$

$$r_2-r_1$$

$$(1I - A \vdots 0) = \begin{pmatrix} 0 & -1 & 1 & 0 \\ 2 & -3 & 2 & 0 \\ 2 & -2 & 1 & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & 0 \\ 2 & -3 & 2 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{r_2-r_1} \left( \begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array} \right)$$

•  $\exists \lambda_1 = 1$ ,  $\forall x \in (\lambda_1 I - A)x = 0$ :

$$(1I - A \vdots 0) = \begin{pmatrix} 0 & -1 & 1 & 0 \\ 2 & -3 & 2 & 0 \\ 2 & -2 & 1 & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & 0 \\ 2 & -3 & 2 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{r_2-r_1} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \xrightarrow{r_3-r_2} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$



$$(1I - A \vdots 0) = \begin{pmatrix} 0 & -1 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 2 & -2 & 1 & | & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 0 & -1 & 1 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_2-r_1} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \xrightarrow{r_3-r_2} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$



$$(1I - A \vdots 0) = \begin{pmatrix} 0 & -1 & 1 & 0 \\ 2 & -3 & 2 & 0 \\ 2 & -2 & 1 & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & 0 \\ 2 & -3 & 2 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{r_2-r_1} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \xrightarrow{r_3-r_2} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\longrightarrow \left( \begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$(x_1 \quad -\frac{1}{2}x_3 = \frac{1}{2}x_4 = \frac{1}{2}x_5 =$$

所以 
$$\begin{cases} x_1 & -\frac{1}{2}x_3 = 0 \end{cases}$$



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$$(1I - A \vdots 0) = \begin{pmatrix} 0 & -1 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 2 & -2 & 1 & | & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 0 & -1 & 1 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_2-r_1} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \xrightarrow{r_3-r_2} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\longrightarrow \left( \begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$(x_1 \quad -\frac{1}{2}x_3 = \frac{1}{2}x_3 =$$

所以 
$$\begin{cases} x_1 & -\frac{1}{2}x_3 = 0 \\ x_2 - x_3 = 0 \end{cases}$$

$$\begin{cases} x_1 & 2x_3 = 0 \\ x_2 - x_3 = 0 \end{cases}$$





$$(1I - A \vdots 0) = \begin{pmatrix} 0 & -1 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 2 & -2 & 1 & | & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 0 & -1 & 1 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_2-r_1} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \xrightarrow{r_3-r_2} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

所以 
$$\begin{cases} x_1 & -\frac{1}{2}x_3 = 0 \\ x_2 - x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{1}{2}x_3 \\ x_2 = x_3 \end{cases}$$

$$(1I - A : 0) = \begin{pmatrix} 0 & -1 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 2 & -2 & 1 & | & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 0 & -1 & 1 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_2-r_1} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \xrightarrow{r_3-r_2} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

所以 
$$\begin{cases} x_1 & -\frac{1}{2}x_3 = 0 \\ x_2 - x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{1}{2}x_3 \\ x_2 = x_3 \end{cases}$$

基础解系: 
$$\alpha_1 = \begin{pmatrix} \\ 2 \end{pmatrix}$$



$$(1I - A : 0) = \begin{pmatrix} 0 & -1 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 2 & -2 & 1 & | & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 2 & -2 & 1 & | & 0 \\ 2 & -3 & 2 & | & 0 \\ 0 & -1 & 1 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_2-r_1} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right) \xrightarrow{r_3-r_2} \left(\begin{array}{ccc|c} 2 & -2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

所以  $\begin{cases} x_1 & -\frac{1}{2}x_3 = 0 \\ x_2 - x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{1}{2}x_3 \\ x_2 = x_3 \end{cases}$ 

基础解系: 
$$\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$



•  $\exists \lambda_2 = 2$ ,  $\forall x \in (\lambda_2 I - A)x = 0$ :

$$(2I - A : 0) =$$



•  $\exists \lambda_2 = 2$ ,  $\forall x \in (\lambda_2 I - A)x = 0$ :

$$(2I - A \vdots 0) = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow$$



$$(2I - A \vdots 0) = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$





•  $\exists \lambda_2 = 2$ ,  $\forall M (\lambda_2 I - A) x = 0$ :

$$(2I - A \vdots 0) = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$x_1 - x_2 + x_3 = 0$$



$$(2I - A \vdots 0) = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$x_1 - x_2 + x_3 = 0 \Rightarrow x_1 = x_2 - x_3$$



$$(2I - A \vdots 0) = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$x_1 - x_2 + x_3 = 0 \Rightarrow x_1 = x_2 - x_3$$
  
基础解系:  $\alpha_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ 



$$(2I - A \vdots 0) = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$x_1 - x_2 + x_3 = 0$$
  $\Rightarrow$   $x_1 = x_2 - x_3$  基础解系:  $\alpha_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ 



$$(2I - A \vdots 0) = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$x_1 - x_2 + x_3 = 0 \Rightarrow x_1 = x_2 - x_3$$
  
基础解系:  $\alpha_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ 



$$(2I - A \vdots 0) = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 2 & -2 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$x_1 - x_2 + x_3 = 0 \Rightarrow x_1 = x_2 - x_3$$
  
基础解系:  $\alpha_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ 

