第 01 周作业解答

练习 1. 化为上三角行列式从而计算四阶行列式
$$D_1 = \begin{bmatrix} -3 & 1 & 4 & -2 \\ 1 & 0 & -1 & 1 \\ 2 & 1 & 0 & -3 \\ 0 & -2 & 1 & 2 \end{bmatrix}$$
 及 $D_2 = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{bmatrix}$

解

$$D_1 = \begin{vmatrix} -3 & 1 & 4 & -2 \\ 1 & 0 & -1 & 1 \\ 2 & 1 & 0 & -3 \\ 0 & -2 & 1 & 2 \end{vmatrix} = \frac{c_3 + c_1}{c_4 - c_1} \begin{vmatrix} -3 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 2 & 1 & 2 & -5 \\ 0 & -2 & 1 & 2 \end{vmatrix} = \frac{\cancel{\text{tx}} \cancel{\text{#x}} - \cancel{\text{Tr}} \cancel{\text{#x}}}{\cancel{\text{#x}}} \cdot (-1)^{2+1} \cdot \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -5 \\ -2 & 1 & 2 \end{vmatrix}$$
$$= \frac{c_2 - c_1}{c_3 - c_1} - \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & -6 \\ -2 & 3 & 4 \end{vmatrix} = -\begin{vmatrix} 1 & -6 \\ 3 & 4 \end{vmatrix} = -22$$

$$D_2 = \begin{vmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{vmatrix} \xrightarrow{\text{$\frac{\kappa \times \Re 2, \, 3, \, 4 \, \Im \ln \Im \Re 1 \, \Im}{1 \, \Im}}} \begin{vmatrix} 6 & 1 & 2 & 3 \\ 6 & 2 & 3 & 0 \\ 6 & 3 & 0 & 1 \\ 6 & 0 & 1 & 2 \end{vmatrix} = 6 \begin{vmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & 3 & 0 \\ 1 & 3 & 0 & 1 \\ 1 & 0 & 1 & 2 \end{vmatrix}$$

$$\xrightarrow{\frac{r_2 - r_1}{r_3 - r_1}} 6 \begin{vmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & -3 \\ 0 & 2 & -2 & -2 \\ 0 & -1 & -1 & -1 \end{vmatrix} = 6 \begin{vmatrix} 1 & 1 & -3 \\ 2 & -2 & -2 \\ -1 & -1 & -1 \end{vmatrix} = 12 \begin{vmatrix} 1 & 1 & -3 \\ 1 & -1 & -1 \\ -1 & -1 & -1 \end{vmatrix}$$

$$\xrightarrow{\frac{r_2 - r_1}{r_3 + r_1}} 12 \begin{vmatrix} 1 & 1 & -3 \\ 0 & -2 & 2 \\ 0 & 0 & -4 \end{vmatrix} = 96$$