第 02 周作业解答

练习 1. 求不定积分

1. $\int \frac{(1+\sqrt{x})^2}{x}dx$
2. $\int(\sqrt{x}+1)(x-\frac{1}{\sqrt{x^3}})dx$

解：1.

$$\int \frac{(1+\sqrt{x})^2}{x}dx = \int \frac{1+2\sqrt{x}+x}{x}dx = \int \frac{1}{x}+2x^{-1/2}+1dx = \ln|x|+4x^{1/2}+x+C = \ln x+4x^{1/2}+x+C.$$

2.

$$\begin{aligned}\int(\sqrt{x}+1)(x-\frac{1}{\sqrt{x^3}})dx &= \int(x^{1/2}+1)(x-x^{-3/2})dx \\ &= \int x^{3/2}+x-x^{-1}-x^{-3/2}dx \\ &= \frac{2}{5}x^{5/2}+\frac{1}{2}x^2-\ln|x|+2x^{-1/2}+C \\ &= \frac{2}{5}x^{5/2}+\frac{1}{2}x^2-\ln x+2x^{-1/2}+C\end{aligned}$$

练习 2. 求不定积分
(1) $\int \frac{x^2}{x^2+1}dx$, (2) $\int \frac{1}{x^2(1+x^2)}dx$.

解：1.

$$\int \frac{x^2}{x^2+1}dx = \int 1-\frac{1}{1+x^2}dx = x-\arctan x+C$$

2.

$$\int \frac{1}{x^2(1+x^2)}dx = \int \frac{1}{x^2}-\frac{1}{1+x^2}dx = \int x^{-2}-\frac{1}{1+x^2}dx = -x^{-1}-\arctan x+C$$

练习 3. 求不定积分
(1) $\int(e^x+5^{-x})2^x dx$, (2) $\int(2^x+3^x)^2 dx$.

解：1.

$$\int(e^x+5^{-x})2^x dx = \int e^x 2^x+(\frac{1}{5})^x 2^x dx = \int(2e)^x+(\frac{2}{5})^x dx = \frac{(2e)^x}{\ln(2e)}+\frac{(\frac{2}{5})^x}{\ln(\frac{2}{5})}+C$$

2.

$$\int(2^x+3^x)^2 dx = \int 2^{2x}+2\cdot 2^x\cdot 3^x+3^{2x}dx = \int 4^x+2\cdot 6^x+9^x dx = \frac{4^x}{\ln 4}+2\cdot\frac{6^x}{\ln 6}+\frac{9^x}{\ln 9}+C$$

练习 4. 求不定积分
(1) $\int(3-2x)^{5/2}dx$, (2) $\int \frac{1}{\sqrt[3]{7-5x}}dx$, (3) $\int \sin(\frac{3}{2}x)dx$

解：1.

$$\begin{aligned}\int (3-2x)^{5/2}dx &= \int (3-2x)^{5/2}\cdot(-\frac{1}{2})d(3-2x) \xlongequal{u=3-2x} -\frac{1}{2}\int u^{5/2}du \\ &= -\frac{1}{7}u^{7/2}+C = -\frac{1}{7}(3-2x)^{7/2}+C\end{aligned}$$

2.

$$\begin{aligned}\int \frac{1}{\sqrt[3]{7-5x}}dx &= \int (7-5x)^{-1/3}dx = \int (7-5x)^{-1/3}\cdot(-\frac{1}{5})d(7-5x) \\ &\xlongequal{u=7-5x} -\frac{1}{5}\int u^{-1/3}du = -\frac{3}{10}u^{2/3}+C = -\frac{3}{10}(7-5x)^{2/3}+C\end{aligned}$$

3.

$$\int \sin(\frac{3}{2}x)dx = \int \sin(\frac{3}{2}x)\cdot\frac{2}{3}d(\frac{3}{2}x) = -\frac{2}{3}\cos(\frac{3}{2}x)+C$$