第 9 章 d：函数

数学系　梁卓滨

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Outline

1. 隐函数的求导法：一个方程的情形

2. 隐函数的求导法：方程组的情形

3. 隐函数定理

We are here now...

1. 隐函数的求导法：一个方程的情形

2. 隐函数的求导法：方程组的情形

3. 隐函数定理

隐函数的求导法 I

问题

给定二元函数 $F(x, y)$ $\Rightarrow$ 考虑方程 $F(x, y) = 0$

隐函数的求导法 I

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给定二元函数 $F(x, y) \;\Rightarrow\;$ 考虑方程 $F(x, y) = 0$

$\Rightarrow\;$ 解出 $y = f(x)$

隐函数的求导法 I

问题

$$
\begin{aligned}
\text{给定二元函数 } F(x,y) \quad &\Rightarrow \quad \text{考虑方程 } F(x,y)=0 \\
&\Rightarrow \quad \text{解出 } y=f(x) \\
&\Rightarrow \quad \frac{dy}{dx}=?
\end{aligned}
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隐函数的求导法 I

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隐函数的求导法 I

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公式

$$\frac{dy}{dx}=-\frac{F_x}{F_y}$$

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$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

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证明

$$F(x,f(x))=0 \quad \Rightarrow$$

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$$F(x,f(x))=0 \;\Rightarrow\; 0=\frac{d}{dx}F(x,f(x))=$$

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证明

$$F(x,f(x))=0 \quad \Rightarrow \quad 0 = \frac{d}{dx}F(x,f(x)) = F_x +$$

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证明

$$F(x,f(x))=0 \;\Rightarrow\; 0=\frac{d}{dx}F(x,f(x))=F_x+F_y\cdot\frac{df}{dx}$$

隐函数的求导法 I

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例 1 设 $y = f(x)$ 满足 $\sin y + e^x = xy^2$，求 $\frac{dy}{dx}$.

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例 1 设 $y=f(x)$ 满足 $\sin y+e^x=xy^2$，求 $\frac{dy}{dx}$.

方法一 注意 $\sin y+e^x-xy^2=0$，令 $F(x,y)=\sin y+e^x-xy^2$，$F(x,y)=0$

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方法二 注意 $\sin y(x)+e^x-xy(x)^2=0$，所以

$$0=\left(\sin y(x)+e^x-xy(x)^2\right)'_x$$

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方法二 注意 $\sin y(x) + e^x - xy(x)^2 = 0$，所以

$$\begin{aligned} 0 &= \left(\sin y(x) + e^x - xy(x)^2\right)'_x \\ &= (\sin y(x))'_x + (e^x)'_x - (xy(x)^2)'_x \end{aligned}$$

例 1 设 $y=f(x)$ 满足 $\sin y+e^x=xy^2$，求 $\frac{dy}{dx}$.

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$$\begin{aligned}0&=\left(\sin y(x)+e^x-xy(x)^2\right)'_x\\&=(\sin y(x))'_x+(e^x)'_x-(xy(x)^2)'_x\\&=\cos y\cdot y'\end{aligned}$$

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所以 $y'=-\frac{e^x-y^2}{\cos y-2xy}$

例 2 设 $y = f(x)$ 满足 $\ln(x^2 + y^2) + 3xy = 4$，求 $\frac{dy}{dx}$.

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解

$F(x, y) = 0$

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例 2 设 $y = f(x)$ 满足 $\ln(x^2 + y^2) + 3xy = 4$，求 $\frac{dy}{dx}$.

解 注意 $\ln(x^2 + y^2) + 3xy - 4 = 0$

$F(x, y) = 0$

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解 注意 $\ln(x^2 + y^2) + 3xy - 4 = 0$，令

$$F(x, y) = \ln(x^2 + y^2) + 3xy - 4$$

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则 $F(x, y) = 0$，所以

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$$= -\frac{\qquad}{\qquad}$$

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$$=-\frac{\frac{2x}{x^2+y^2}+3y}{\frac{2y}{x^2+y^2}+3x}$$

$$=-\frac{2x+3x^2y+3y^3}{2y+3xy^2+3x^3}$$

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隐函数的求导法 II

问题

给定 $F(x, y, z) \Rightarrow$ 考虑方程 $F(x, y, z) = 0$

隐函数的求导法 II

问题

给定 $F(x,y,z)$ $\Rightarrow$ 考虑方程 $F(x,y,z)=0$

$\Rightarrow$ 解出 $z=u(x,y)$

隐函数的求导法 II

问题

$$
\begin{aligned}
\text{给定}\, F(x,y,z) \;&\Rightarrow\; \text{考虑方程}\, F(x,y,z)=0 \\
&\Rightarrow\; \text{解出}\, z=u(x,y) \\
&\Rightarrow\; \frac{\partial z}{\partial x}=?,\quad \frac{\partial z}{\partial y}=?
\end{aligned}
$$

隐函数的求导法 II

问题

给定 $F(x,y,z)$ $\Rightarrow$ 考虑方程 $F(x,y,z)=0$

$\Rightarrow$ ~~解出 $z=u(x,y)$~~ 设 $z=u(x,y)$ 满足 $F(x,y,z)=0$

$$\Rightarrow \quad \frac{\partial z}{\partial x}=?, \qquad \frac{\partial z}{\partial y}=?$$

隐函数的求导法 II

问题

给定 $F(x,y,z)$ $\Rightarrow$ 考虑方程 $F(x,y,z)=0$

$\Rightarrow$ ~~解出 $z=u(x,y)$~~ 设 $z=u(x,y)$ 满足 $F(x,y,z)=0$

$$\Rightarrow \quad \frac{\partial z}{\partial x}=?, \quad \frac{\partial z}{\partial y}=?$$

公式

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}, \quad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z}$$

隐函数的求导法 II

问题

$$
\begin{aligned}
\text{给定 } F(x,y,z) &\Rightarrow \text{考虑方程 } F(x,y,z)=0 \\
&\Rightarrow \cancel{\text{解出 } z=u(x,y)} \quad \text{设 } z=u(x,y) \text{ 满足 } F(x,y,z)=0 \\
&\Rightarrow \frac{\partial z}{\partial x}=?, \quad \frac{\partial z}{\partial y}=?
\end{aligned}
$$

公式

$$
\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}, \quad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z} \qquad (F_z\neq 0)
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隐函数的求导法 II

问题

给定 $F(x,y,z)$ $\Rightarrow$ 考虑方程 $F(x,y,z)=0$

$\Rightarrow$ ~~解出 $z=u(x,y)$~~ 设 $z=u(x,y)$ 满足 $F(x,y,z)=0$

$$\Rightarrow \quad \frac{\partial z}{\partial x}=?, \quad \frac{\partial z}{\partial y}=?$$

公式

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}, \quad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z} \qquad (F_z\neq 0)$$

证明

$F(x,y,u(x,y))=0 \quad \Rightarrow$

隐函数的求导法 II

问题

给定 $F(x,y,z)$ $\Rightarrow$ 考虑方程 $F(x,y,z)=0$

$\Rightarrow$ ~~解出 $z=u(x,y)$~~ 设 $z=u(x,y)$ 满足 $F(x,y,z)=0$

$$\Rightarrow \quad \frac{\partial z}{\partial x}=?, \qquad \frac{\partial z}{\partial y}=?$$

公式

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z} \qquad (F_z\neq 0)$$

证明

$$F(x,y,u(x,y))=0 \quad \Rightarrow \quad 0=\frac{\partial}{\partial x}F(x,y,u(x,y))=$$

隐函数的求导法 II

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给定 $F(x,y,z)$ $\Rightarrow$ 考虑方程 $F(x,y,z)=0$

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$$\Rightarrow \quad \frac{\partial z}{\partial x}=?, \quad \frac{\partial z}{\partial y}=?$$

公式

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}, \quad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

证明

$$F(x,y,u(x,y))=0 \quad \Rightarrow \quad 0=\frac{\partial}{\partial x}F(x,y,u(x,y))=F_x+$$

隐函数的求导法 II

问题

给定 $F(x,y,z)$ $\Rightarrow$ 考虑方程 $F(x,y,z)=0$

$\Rightarrow$ ~~解出 $z=u(x,y)$~~ 设 $z=u(x,y)$ 满足 $F(x,y,z)=0$

$$\Rightarrow \quad \frac{\partial z}{\partial x}=?,\quad \frac{\partial z}{\partial y}=?$$

公式

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z},\quad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z}\qquad (F_z\neq 0)$$

证明

$$F(x,y,u(x,y))=0 \quad\Rightarrow\quad 0=\frac{\partial}{\partial x}F(x,y,u(x,y))=F_x+F_z\cdot\frac{\partial u}{dx}$$

隐函数的求导法 II

问题

$$\text{给定}\, F(x,y,z) \;\Rightarrow\; \text{考虑方程}\, F(x,y,z)=0$$

$$\Rightarrow \cancel{\text{解出}\, z=u(x,y)} \;\; \text{设}\, z=u(x,y)\,\text{满足}\, F(x,y,z)=0$$

$$\Rightarrow \frac{\partial z}{\partial x} = ?, \qquad \frac{\partial z}{\partial y} = ?$$

公式

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

证明

$$F(x,y,u(x,y))=0 \;\Rightarrow\; 0 = \frac{\partial}{\partial x} F(x,y,u(x,y)) = F_x + F_z \cdot \frac{\partial u}{dx}$$

$$\Rightarrow \frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

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给定 $F(x,y,z)$ $\Rightarrow$ 考虑方程 $F(x,y,z)=0$

$\Rightarrow$ ~~解出 $z=u(x,y)$~~ 设 $z=u(x,y)$ 满足 $F(x,y,z)=0$

$$\Rightarrow \quad \frac{\partial z}{\partial x}=?, \qquad \frac{\partial z}{\partial y}=?$$

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$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

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$$F(x,y,u(x,y))=0 \quad \Rightarrow \quad 0=\frac{\partial}{\partial x}F(x,y,u(x,y))=F_x+F_z\cdot\frac{\partial u}{dx}$$

$$\Rightarrow \quad \frac{\partial z}{\partial x}=-\frac{F_x}{F_z}, \quad \text{同理} \quad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z}$$

例 1 设 $z = f(x, y)$ 满足 $x + y + xz = e^z - 1$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$.

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解 $F(x, y, z) = 0$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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解 令 $F(x, y, z) = x + y + xz - e^z + 1$， $F(x, y, z) = 0$

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解

$F(x,y,z)=0$

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$$= -\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3} = \frac{1}{3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)'_y}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

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例 2 设 $z=f(x,y)$ 满足 $2\sin(x+2y-3z)=x+2y-3z$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$.

解 令 $F(x,\ y,\ z)=2\sin(x+2y-3z)-x-2y+3z$，则 $F(x,\ y,\ z)=0$，所以

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_x}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

$$=-\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3}=\frac{1}{3}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_y}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

$$=-\frac{}{-6\cos(x+2y-3z)+3}$$

例 2 设 $z=f(x,y)$ 满足 $2\sin(x+2y-3z)=x+2y-3z$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$.

解 令 $F(x,y,z)=2\sin(x+2y-3z)-x-2y+3z$，则 $F(x,y,z)=0$，所以

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_x}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

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$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_y}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

$$=-\frac{4\cos(x+2y-3z)}{-6\cos(x+2y-3z)+3}$$

例 2 设 $z = f(x, y)$ 满足 $2\sin(x + 2y - 3z) = x + 2y - 3z$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$.

解 令 $F(x, y, z) = 2\sin(x + 2y - 3z) - x - 2y + 3z$，则 $F(x, y, z) = 0$，所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x + 2y - 3z) - x - 2y + 3z)'_x}{(2\sin(x + 2y - 3z) - x - 2y + 3z)'_z}$$

$$= -\frac{2\cos(x + 2y - 3z) - 1}{-6\cos(x + 2y - 3z) + 3} = \frac{1}{3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x + 2y - 3z) - x - 2y + 3z)'_y}{(2\sin(x + 2y - 3z) - x - 2y + 3z)'_z}$$

$$= -\frac{4\cos(x + 2y - 3z) - 2}{-6\cos(x + 2y - 3z) + 3}$$

例 3 设 $z=f(x,y)$ 满足 $z-y-x+xe^{z-y-x}=0$，求 dz.

例 3 设 $z = f(x, y)$ 满足 $z - y - x + xe^{z-y-x} = 0$，求 dz.

解

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy =$$

例 3 设 $z = f(x, y)$ 满足 $z - y - x + xe^{z-y-x} = 0$，求 dz.

解 令 $F(x, y, z) = z - y - x + xe^{z-y-x}$，则 $F(x, y, z) = 0$

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$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} =$$

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解 令 $F(x,y,z)=z-y-x+xe^{z-y-x}$，则 $F(x,y,z)=0$，所以

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{(z-y-x+xe^{z-y-x})'_x}{(z-y-x+xe^{z-y-x})'_z}$$

$$=-\frac{\qquad\qquad}{\qquad\qquad}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(z-y-x+xe^{z-y-x})'_y}{(z-y-x+xe^{z-y-x})'_z}=-\frac{\qquad\qquad}{\qquad\qquad}$$

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$$=-\frac{\qquad}{1+xe^{z-y-x}}$$

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$$=-\frac{-1}{1+xe^{z-y-x}}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(z-y-x+xe^{z-y-x})'_y}{(z-y-x+xe^{z-y-x})'_z}=-\frac{\qquad}{\qquad}$$

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$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z-y-x})'_x}{(z - y - x + xe^{z-y-x})'_z}$$

$$= -\frac{-1 + e^{z-y-x}}{1 + xe^{z-y-x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z-y-x})'_y}{(z - y - x + xe^{z-y-x})'_z} = -\underline{\qquad\qquad\qquad}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$

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$$=-\frac{-1+e^{z-y-x}-xe^{z-y-x}}{1+xe^{z-y-x}}$$

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$$=-\frac{-1+e^{z-y-x}-xe^{z-y-x}}{1+xe^{z-y-x}}=\frac{1+(x-1)e^{z-y-x}}{1+xe^{z-y-x}}$$

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$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z-y-x})'_x}{(z - y - x + xe^{z-y-x})'_z}$$

$$= -\frac{-1 + e^{z-y-x} - xe^{z-y-x}}{1 + xe^{z-y-x}} = \frac{1 + (x-1)e^{z-y-x}}{1 + xe^{z-y-x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z-y-x})'_y}{(z - y - x + xe^{z-y-x})'_z} = -\frac{\qquad}{1 + xe^{z-y-x}}$$

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例 3 设 $z=f(x,y)$ 满足 $z-y-x+xe^{z-y-x}=0$，求 dz.

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$$=-\frac{-1+e^{z-y-x}-xe^{z-y-x}}{1+xe^{z-y-x}}=\frac{1+(x-1)e^{z-y-x}}{1+xe^{z-y-x}}$$

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$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(z-y-x+xe^{z-y-x})'_y}{(z-y-x+xe^{z-y-x})'_z}=-\frac{-1-xe^{z-y-x}}{1+xe^{z-y-x}}$$

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$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=-\frac{1+(x-1)e^{z-y-x}}{1+xe^{z-y-x}}dx+dy$$

例 4 设 $\Phi(u, v)$ 具有连续偏导数，函数 $z = z(x, y)$ 满足 $\Phi(cx - az, cy - bz) = 0$，证明：

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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$$F_y =$$

$$F_z =$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x$$

$$F_y =$$

$$F_z =$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x = c\Phi_u$$

$$F_y =$$

$$F_z =$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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$$F_z =$$

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$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y = c\Phi_v$$

$$F_z = \Phi_u \cdot u_z + \Phi_v \cdot v_z$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y = c\Phi_v$$

$$F_z = \Phi_u \cdot u_z + \Phi_v \cdot v_z = -a\Phi_u - b\Phi_v$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} =$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

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$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y = c\Phi_v$$

$$F_z = \Phi_u \cdot u_z + \Phi_v \cdot v_z = -a\Phi_u - b\Phi_v$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{c\Phi_u}{a\Phi_u + b\Phi_v}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} =$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} =$$

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$$F_z = \Phi_u \cdot u_z + \Phi_v \cdot v_z = -a\Phi_u - b\Phi_v$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{c\Phi_u}{a\Phi_u + b\Phi_v}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{c\Phi_v}{a\Phi_u + b\Phi_v}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} =$$

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例 4 设 $\Phi(u, v)$ 具有连续偏导数，函数 $z = z(x, y)$ 满足 $\Phi(cx - az, cy - bz) = 0$，证明：

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x = c\Phi_u$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y = c\Phi_v$$

$$F_z = \Phi_u \cdot u_z + \Phi_v \cdot v_z = -a\Phi_u - b\Phi_v$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{c\Phi_u}{a\Phi_u + b\Phi_v}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{c\Phi_v}{a\Phi_u + b\Phi_v}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = \frac{ac\Phi_u}{a\Phi_u + b\Phi_v} + \frac{bc\Phi_v}{a\Phi_u + b\Phi_v}$$

例 4 设 $\Phi(u, v)$ 具有连续偏导数，函数 $z = z(x, y)$ 满足 $\Phi(cx - az, cy - bz) = 0$，证明：

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

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$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{c\Phi_u}{a\Phi_u + b\Phi_v}$$

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$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = \frac{ac\Phi_u}{a\Phi_u + b\Phi_v} + \frac{bc\Phi_v}{a\Phi_u + b\Phi_v} = c$$

例 5 设 $z = f(x, y)$ 满足 $z = x + ye^z$，求 $\frac{\partial^2 z}{\partial x \partial y}$.

例 5 设 $z=f(x,y)$ 满足 $z=x+ye^z$，求 $\frac{\partial^2 z}{\partial x\partial y}$.

解 $F(x,y,z)=x+ye^z-z$，则 $F(x,y,z)=0$

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$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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例 5 设 $z = f(x, y)$ 满足 $z = x + ye^z$，求 $\frac{\partial^2 z}{\partial x \partial y}$.

解 $F(x, y, z) = x + ye^z - z$，则 $F(x, y, z) = 0$ ，所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} =$$

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$$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=\left(-\frac{1}{ye^z-1}\right)_y'$$

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$$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=\left(-\frac{1}{ye^z-1}\right)'_y=\frac{(ye^z-1)'_y}{(ye^z-1)^2}$$

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$$=\frac{e^z+y(e^z)_y'}{(ye^z-1)^2}$$

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$$=\frac{e^z+y(e^z)'_y}{(ye^z-1)^2}=\frac{e^z+ye^z\cdot\frac{\partial z}{\partial y}}{(ye^z-1)^2}$$

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例 5 设 $z = f(x, y)$ 满足 $z = x + ye^z$，求 $\frac{\partial^2 z}{\partial x \partial y}$.

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$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)'_y = \frac{(ye^z - 1)'_y}{(ye^z - 1)^2}$$

$$= \frac{e^z + y(e^z)'_y}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \frac{\partial z}{\partial y}}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \left(-\frac{e^z}{ye^z - 1}\right)}{(ye^z - 1)^2}$$

$$= \frac{-e^z}{(ye^z - 1)^3}$$

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$$= \frac{-e^z}{(ye^z-1)^3} = \frac{e^z}{(1+x-z)^3}$$

We are here now...

1. 隐函数的求导法：一个方程的情形

2. 隐函数的求导法：方程组的情形

3. 隐函数定理

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \qquad \begin{matrix} (1) \times a_{22} \\ (2) \times a_{12} \end{matrix}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}\, a_{22} x + a_{12}\, a_{22} y = a_{22} b_1 & (1) \times a_{22} \\ a_{21} x + a_{22} y = b_2 & (2) \times a_{12} \end{cases}$$

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回顾：二元线性方程组的求解

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$$\begin{cases} a_{11}\,a_{22}x + a_{12}\,a_{22}y = a_{22}b_1 & (1)\times a_{22} \\ a_{21}\,a_{12}x + a_{22}\,a_{12}y = a_{12}b_2 & (2)\times a_{12} \end{cases}$$

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回顾：二元线性方程组的求解

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用消元法解：

$(1)\times a_{22}-(2)\times a_{12}$，消去 y，得：

$$x=\frac{b_1a_{22}-a_{12}b_2}{a_{11}a_{22}-a_{12}a_{21}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \qquad \begin{matrix} (1) \times a_{21} \\ (2) \times a_{11} \end{matrix}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}\,a_{11}x + a_{22}\,a_{11}y = a_{11}b_2 & (2) \times a_{11} \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

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回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}\,a_{21}x + a_{12}\,a_{21}y = a_{21}b_1 & (1)\times a_{21} \\ a_{21}\,a_{11}x + a_{22}\,a_{11}y = a_{11}b_2 & (2)\times a_{11} \end{cases}$$

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$(2)\times a_{11} - (1)\times a_{21}$，消去 x，得：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}\,a_{21}x + a_{12}\,a_{21}y = a_{21}b_1 & (1)\times a_{21} \\ a_{21}\,a_{11}x + a_{22}\,a_{11}y = a_{11}b_2 & (2)\times a_{11} \end{cases}$$

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$(1)\times a_{22} - (2)\times a_{12}$，消去 y，得：

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

$(2)\times a_{11} - (1)\times a_{21}$，消去 x，得：

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

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$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

回顾：二元线性方程组的求解

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$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\qquad}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

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$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \quad \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $x =$, $y =$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $x =$, $y =$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $x = \dfrac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \underline{\qquad}$, $y =$

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公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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练习 利用二阶行列式求解下面二元线性方程组

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1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1}$, $y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -$

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公式:

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练习 利用二阶行列式求解下面二元线性方程组

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练习 利用二阶行列式求解下面二元线性方程组

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2. $$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \_\_\_$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

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公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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方程组的隐函数求导公式

$$F(x, y, u, v)$$
$$G(x, y, u, v)$$

方程组的隐函数求导公式

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

方程组的隐函数求导公式

假设函数 $u=u(x,y)$, $v=v(x,y)$ 满足方程组

$$\begin{cases} F(x,y,u,v)=0, \\ G(x,y,u,v)=0. \end{cases}$$

方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题：如何计算 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$?

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方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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问题：如何计算 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$?

求解如下：

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} \\ \end{cases}$$

方程组的隐函数求导公式

假设函数 $u=u(x,y)$, $v=v(x,y)$ 满足方程组

$$\begin{cases} F(x,y,u,v)=0, \\ G(x,y,u,v)=0. \end{cases}$$

问题： 如何计算 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$?

求解如下：

$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ \end{cases}$$

方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题： 如何计算 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$?

求解如下：

$$\begin{cases} F(x,y,u,v) = 0 \\ G(x,y,u,v) = 0 \end{cases} \overset{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题：如何计算 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$?

求解如下：

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$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}}$$

$$\begin{cases} F(x,y,u,v) = 0 \\ G(x,y,u,v) = 0 \end{cases} \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ \end{cases}$$

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总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

$u_x =$

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$$\left\{\begin{array}{l} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{array}\right. \quad \begin{array}{c} \xrightarrow{\frac{\partial}{\partial x}} \\ \\ \xrightarrow{\frac{\partial}{\partial y}} \end{array}$$

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所以

$$u_x=-\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \qquad v_x=-\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$u_y=-\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \qquad v_y=-\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

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$$u_x=-\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(x,v)},\qquad v_x=-\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_x = -\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = -\frac{1}{J}\frac{\partial(F, G)}{\partial(x, v)}, \quad v_x = -\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = -\frac{1}{J}\frac{\partial(F, G)}{\partial(u, x)}$$

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总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

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例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$.

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$.

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial x}}$$

$$\xRightarrow{\frac{\partial}{\partial y}}$$

$u_x=$ $v_x=$

$u_y=$ $v_y=$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$.

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \overset{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1 \\ \end{cases}$$

$$\overset{\frac{\partial}{\partial y}}{\Longrightarrow}$$

$u_x=$ $\qquad\qquad$ $v_x=$

$u_y=$ $\qquad\qquad$ $v_y=$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$.

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u\sin v \cdot v_x = 0 \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial y}}$$

$u_x =$ $\qquad$ $v_x =$

$u_y =$ $\qquad$ $v_y =$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$.

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases}$$

$$\xrightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u\sin v \cdot v_x = 0 \end{cases}$$

$$\xrightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ \end{cases}$$

$u_x =$ $\qquad$ $v_x =$

$u_y =$ $\qquad$ $v_y =$

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We are here now...

1. 隐函数的求导法：一个方程的情形

2. 隐函数的求导法：方程组的情形

3. 隐函数定理

平面光滑曲线的定义

平面上光滑曲线应该包含：一元光滑函数的图形

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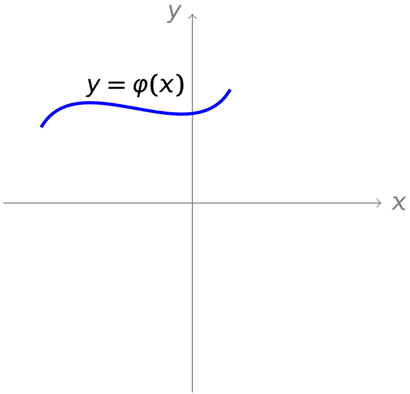

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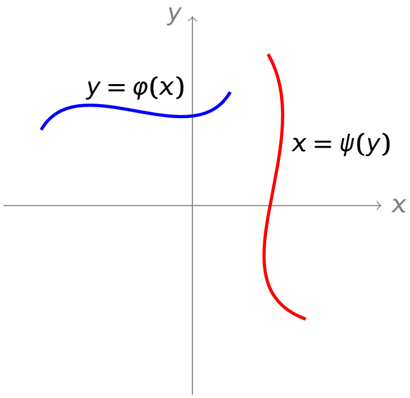

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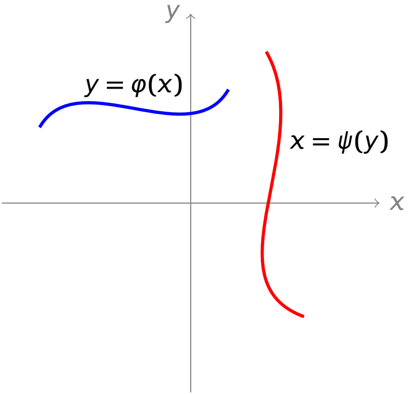

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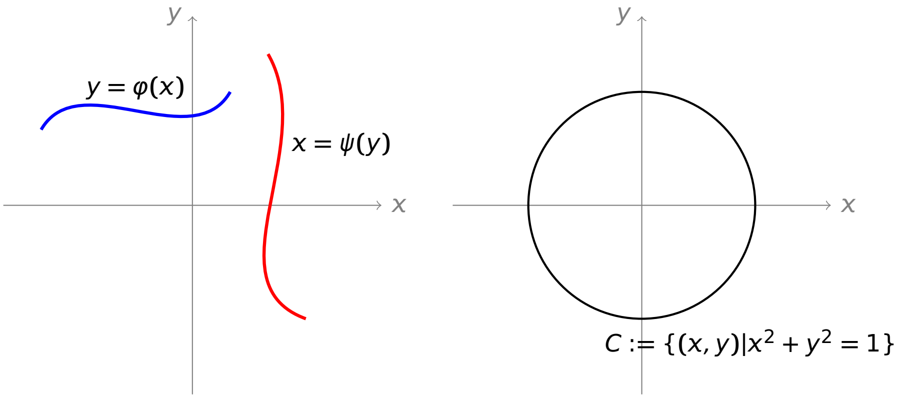

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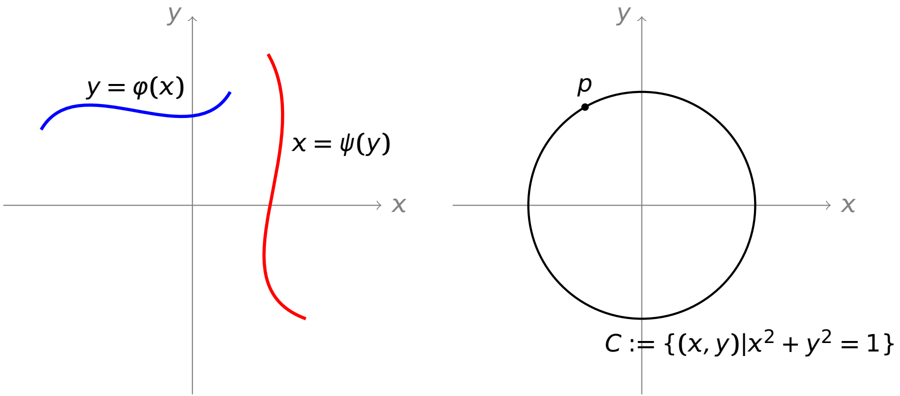

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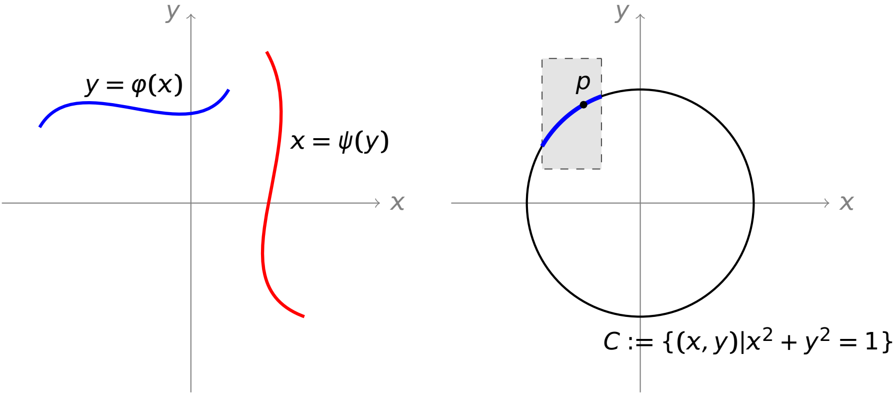

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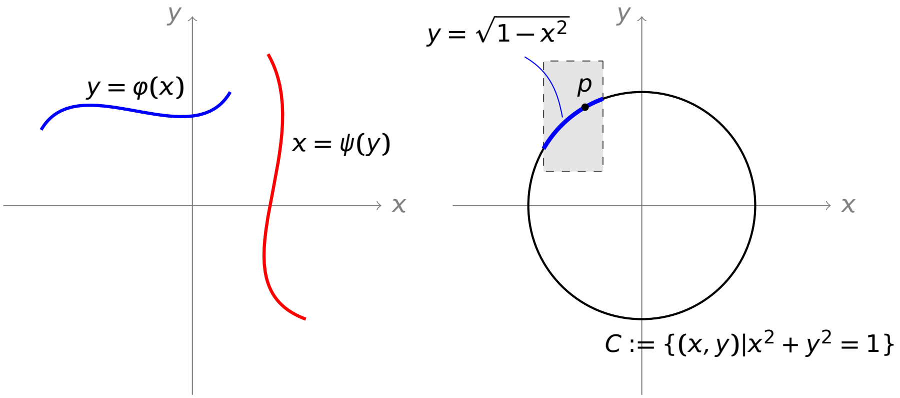

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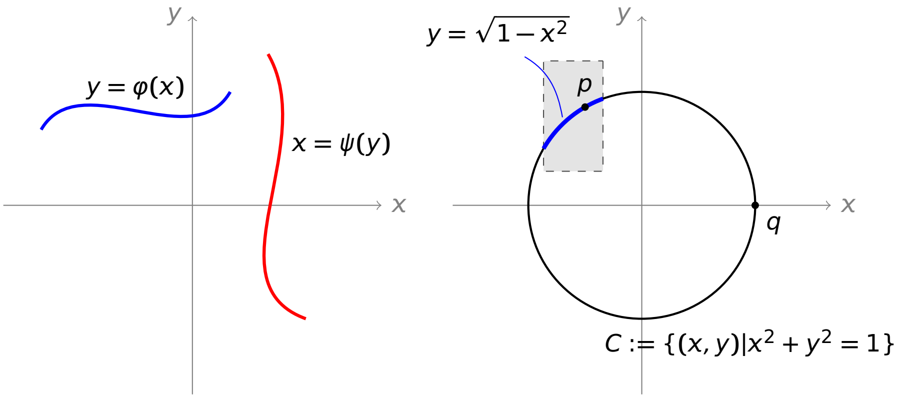

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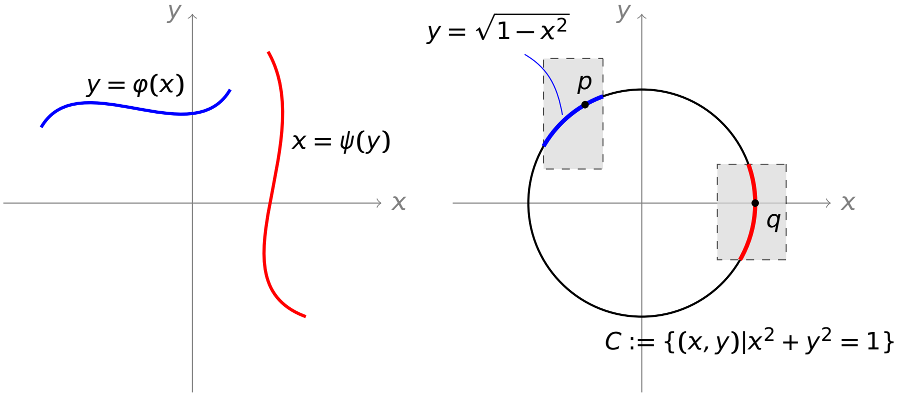

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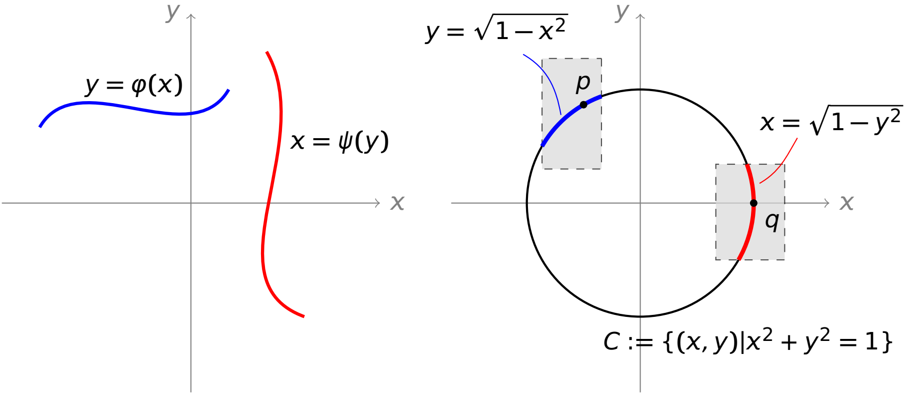

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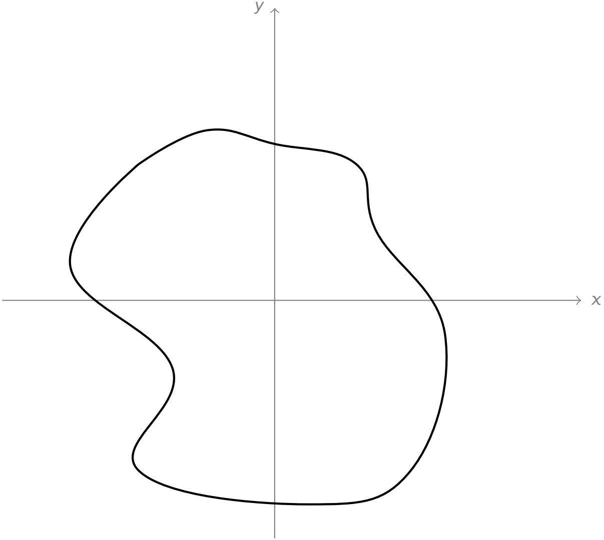

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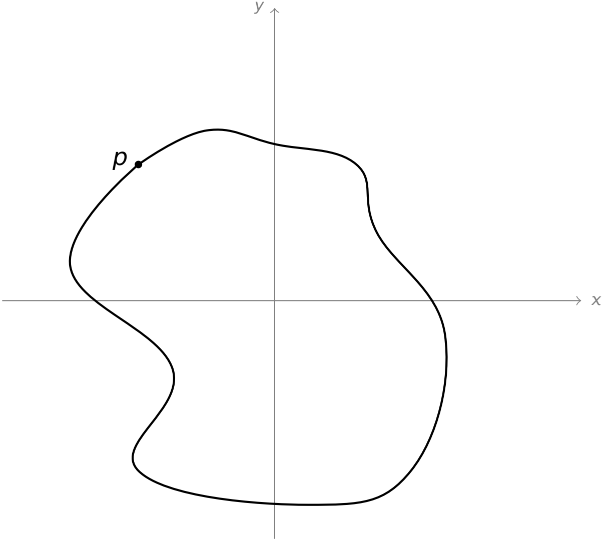

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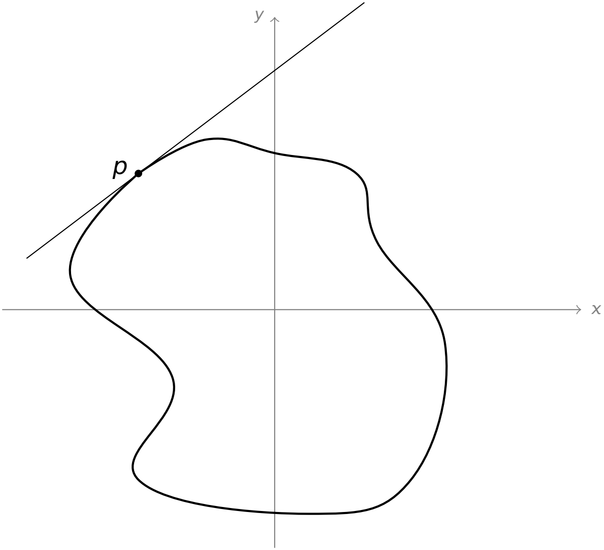

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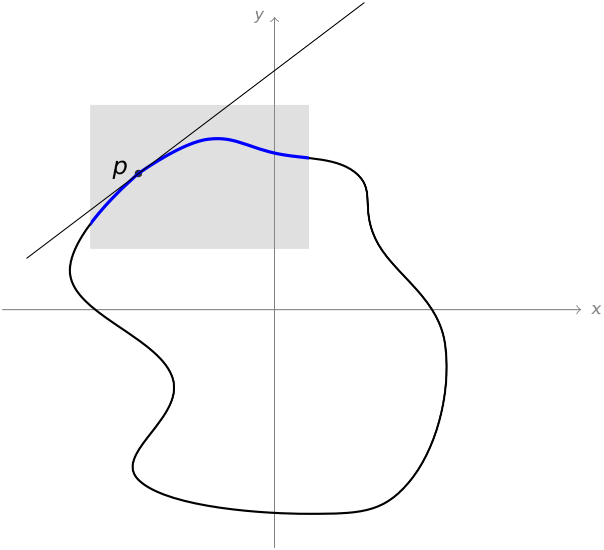

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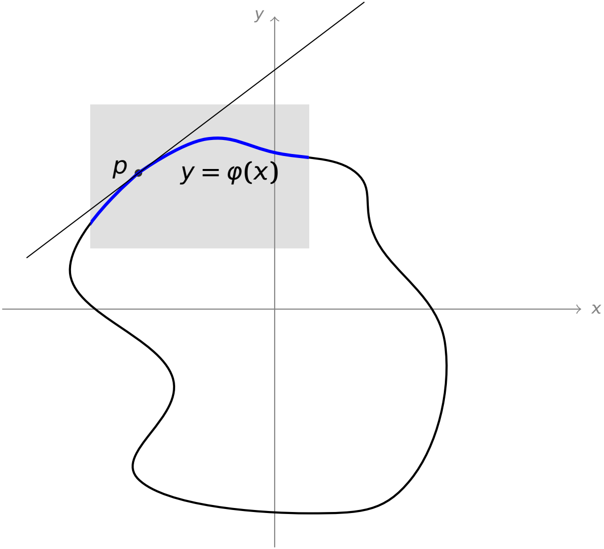

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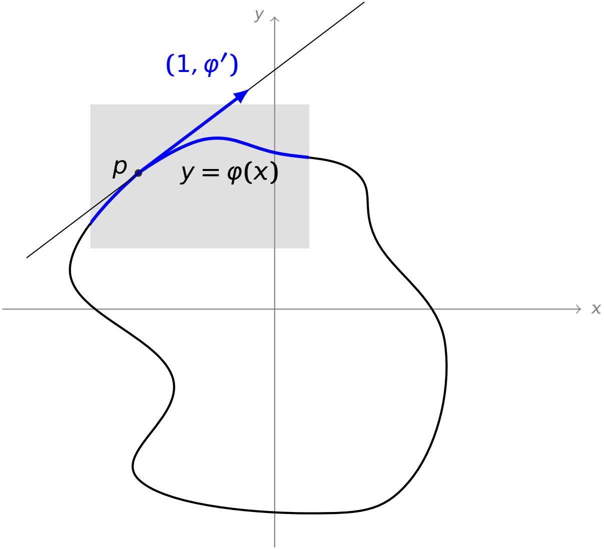

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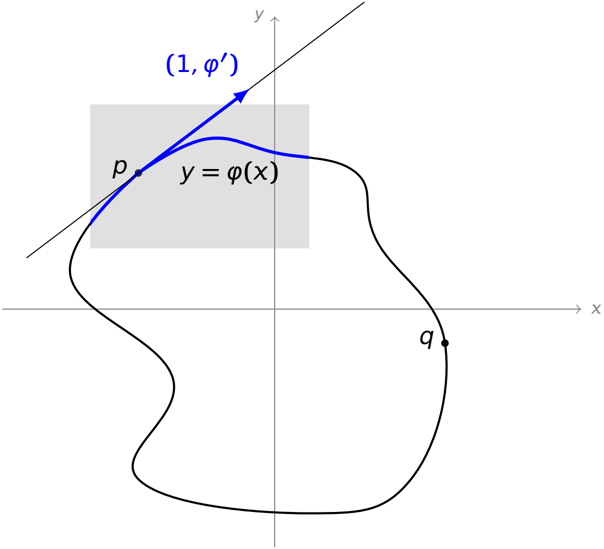

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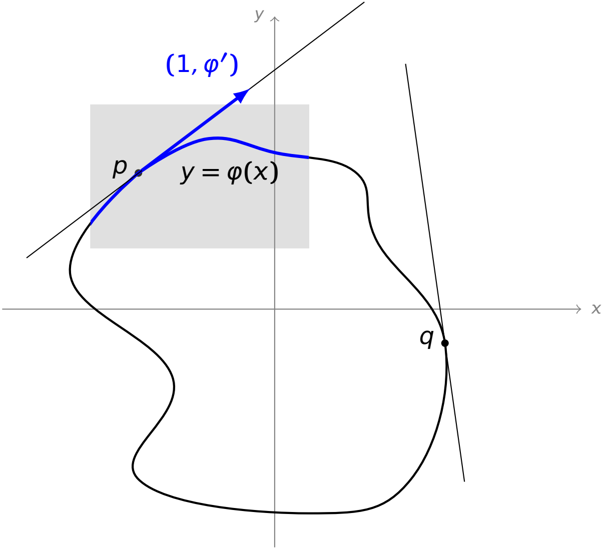

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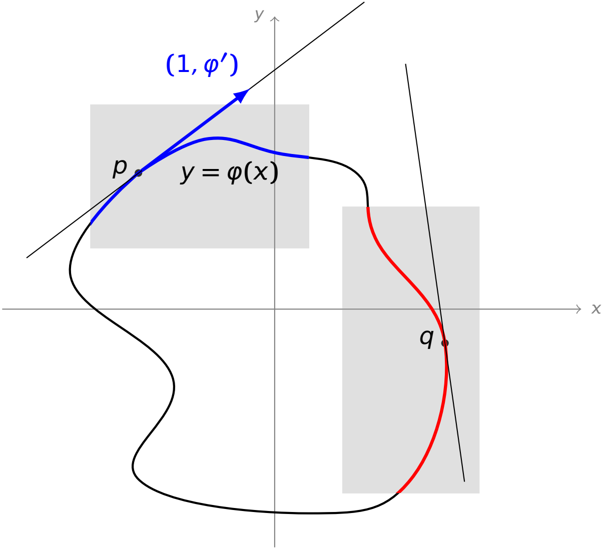

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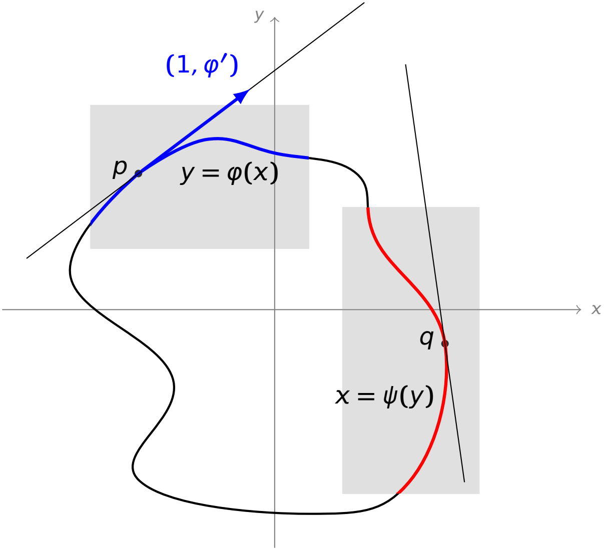

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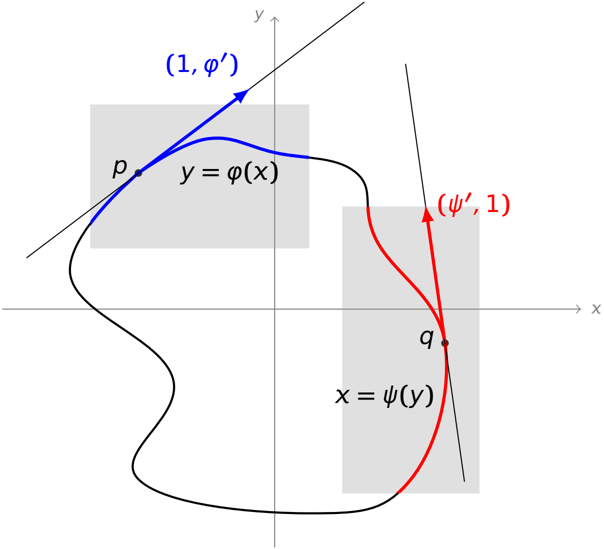

假设 $f(x, y)$ 是光滑的二元函数，其零点集 $\{f = 0\}$ 是平面上点集.

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- 由隐函数定理可知，如果 $\nabla f\neq 0$，则 $\{f=0\}$ 是一条光滑曲线，且该曲线上任一点 (x,y) 的一个切方向是 $(f_y,-f_x)$（与梯度 ∇f 垂直）.

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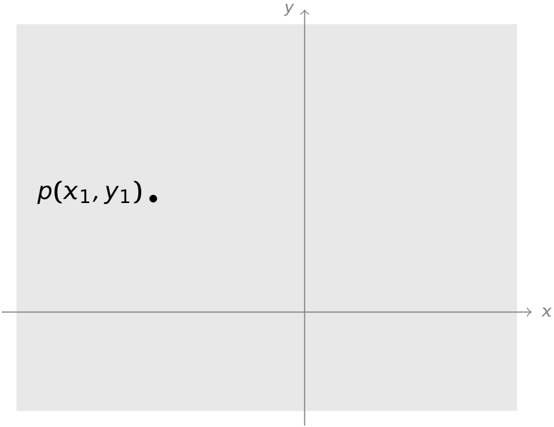

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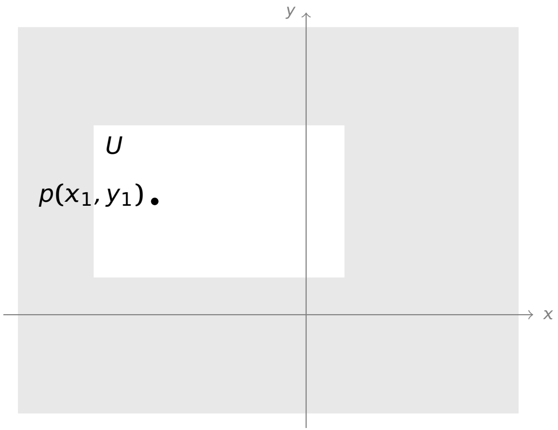

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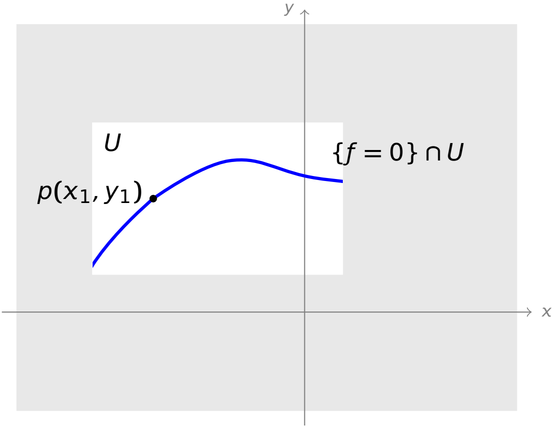

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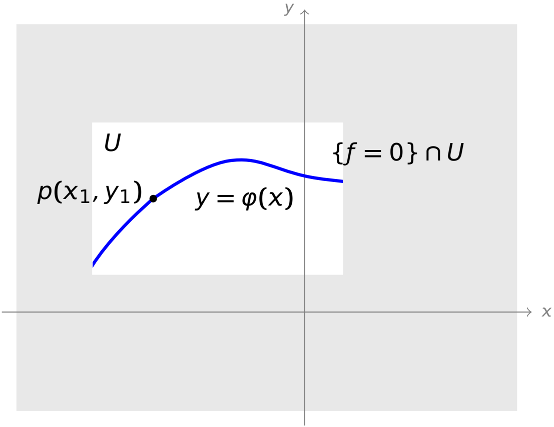

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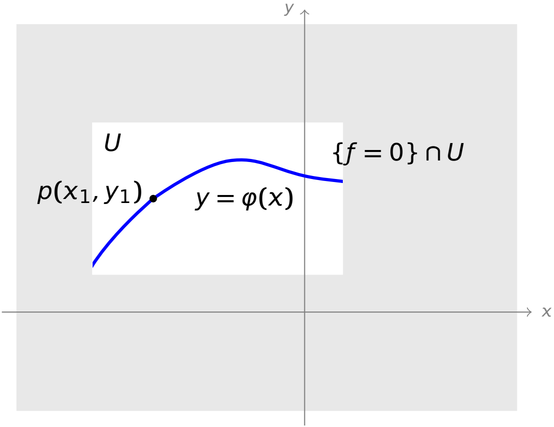

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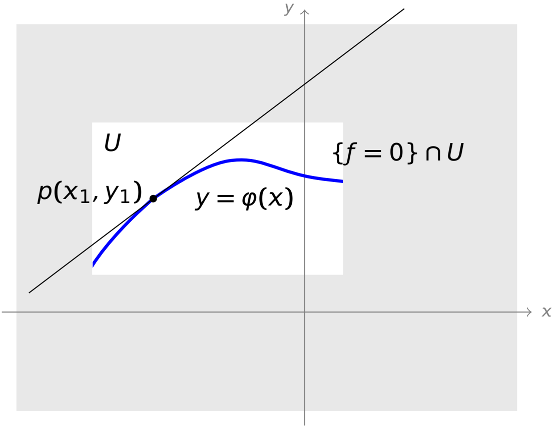

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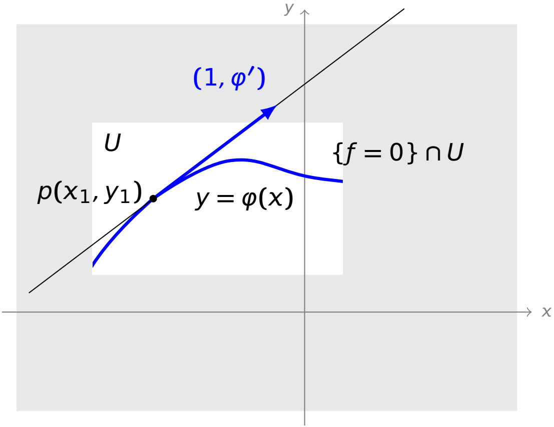

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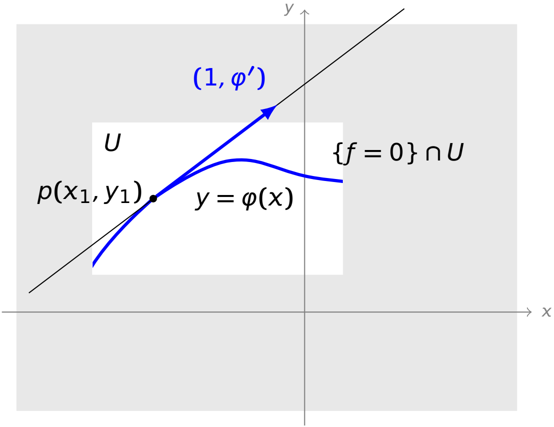

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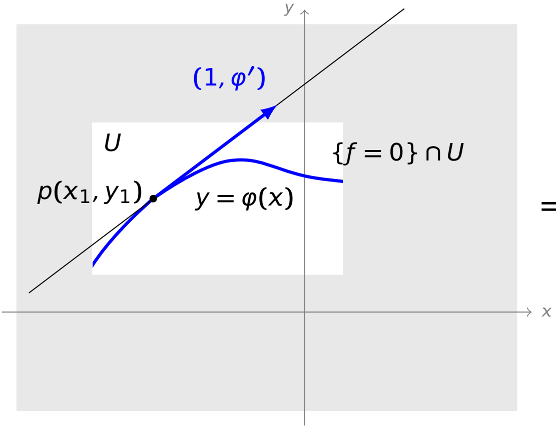

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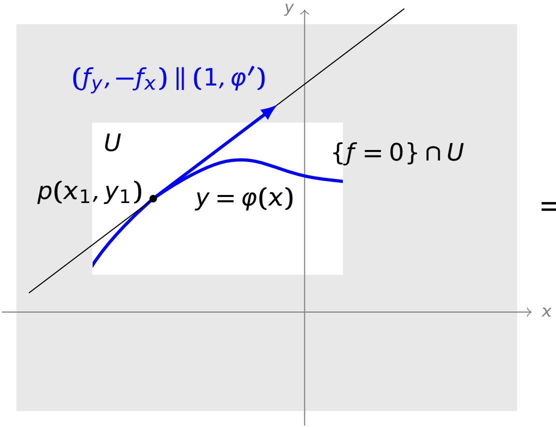

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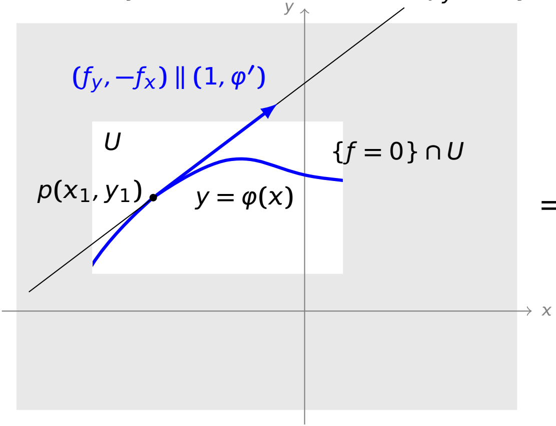

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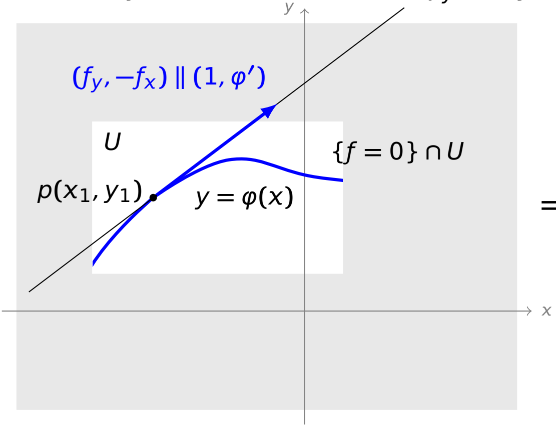

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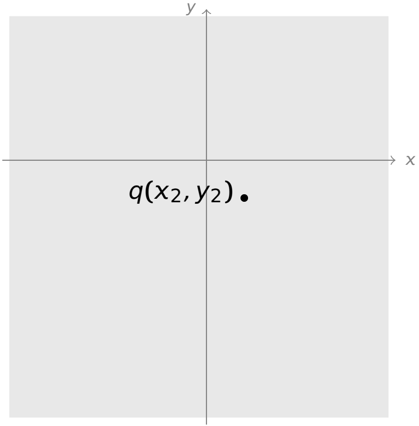

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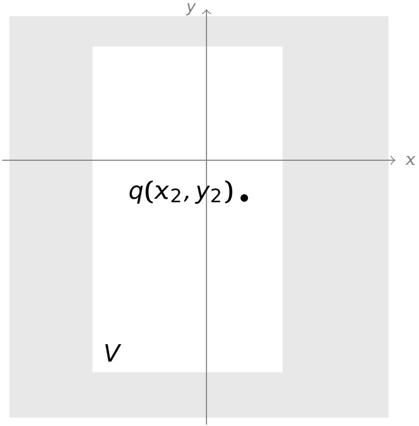

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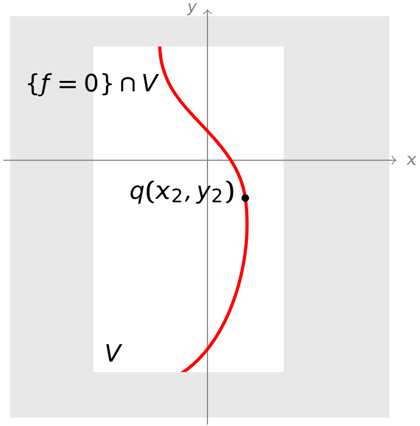

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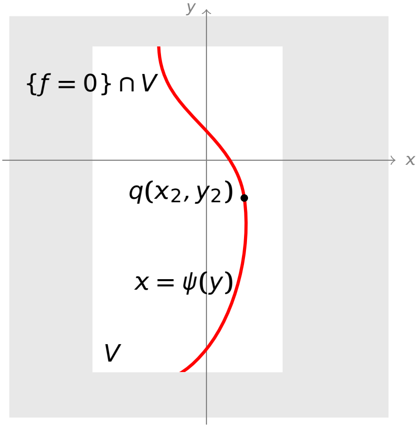

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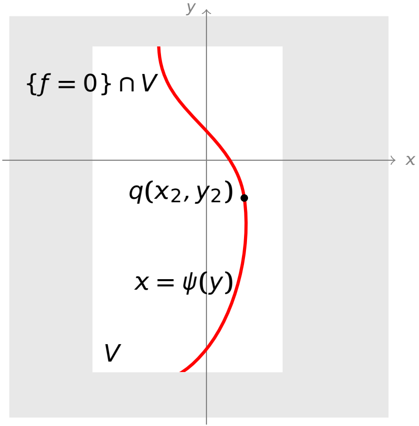

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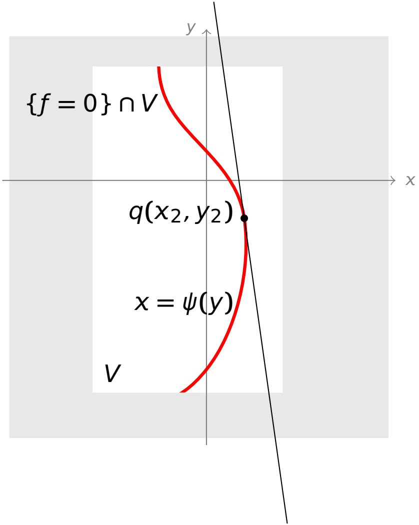

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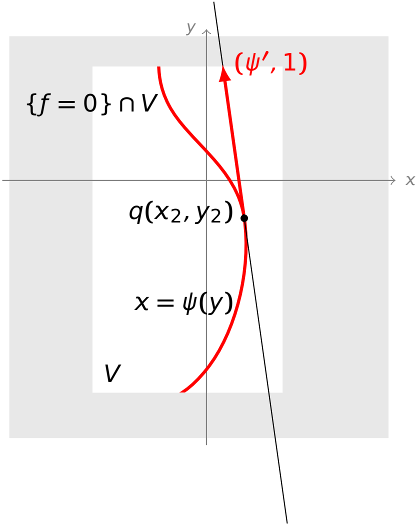

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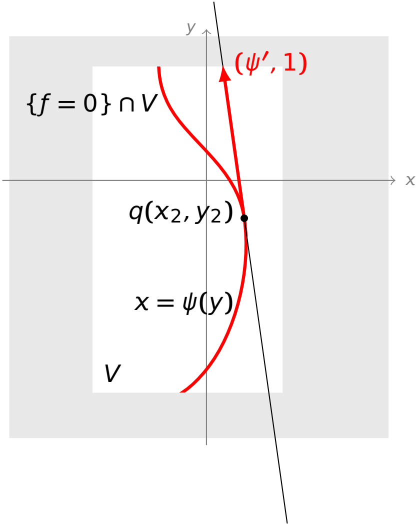

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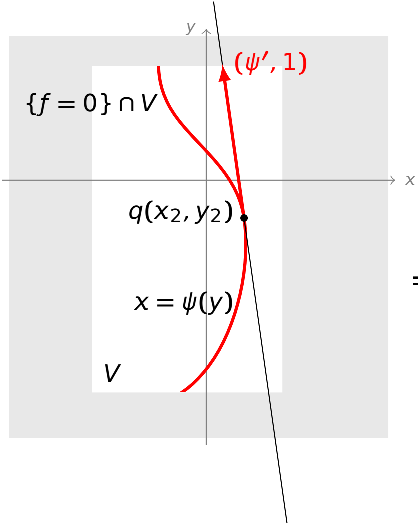

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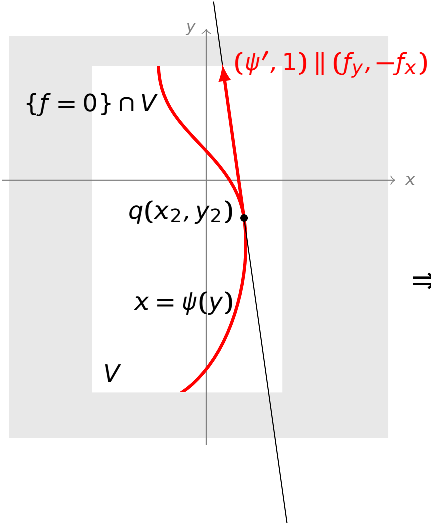

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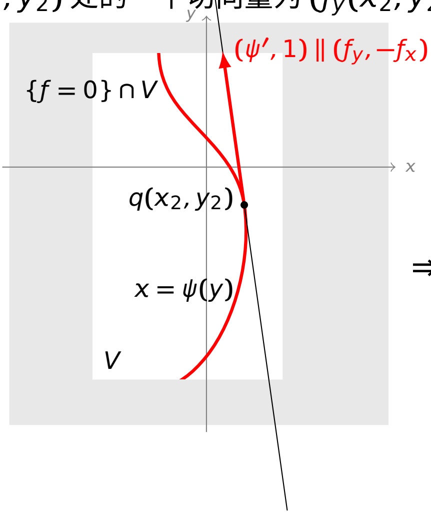

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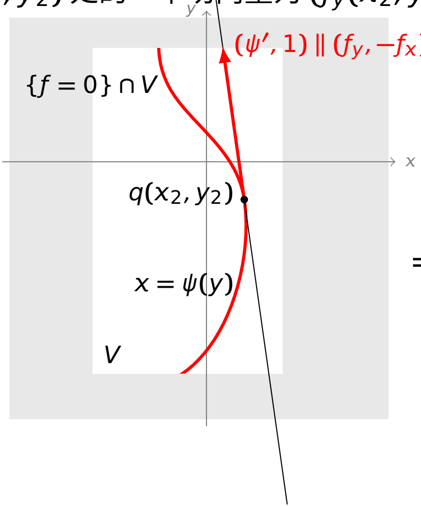

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隐函数定理中的条件 $f_x(x_0, y_0) \neq 0$ 或 $f_y(x_0, y_0) \neq 0$ 不能去掉，如图

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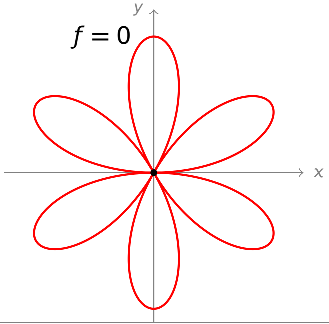

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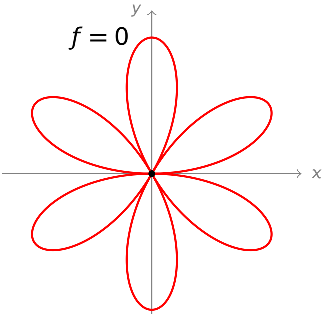

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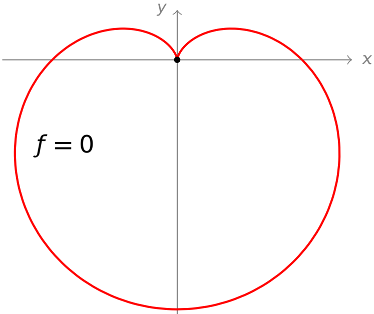

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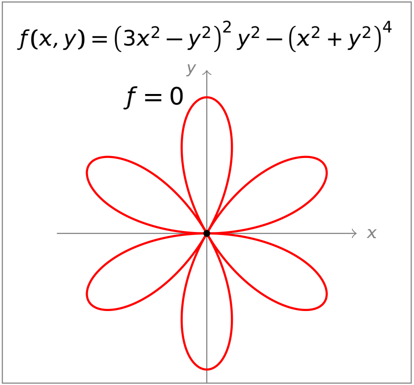

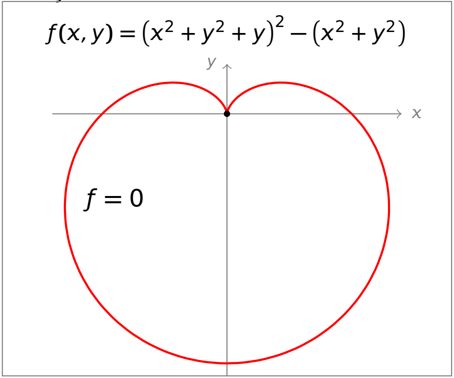

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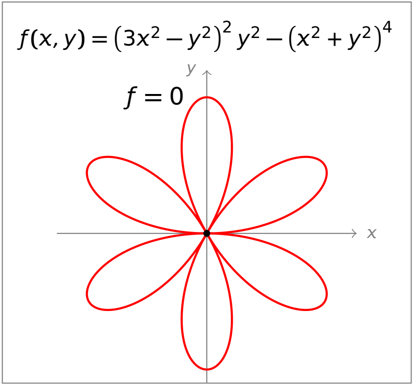

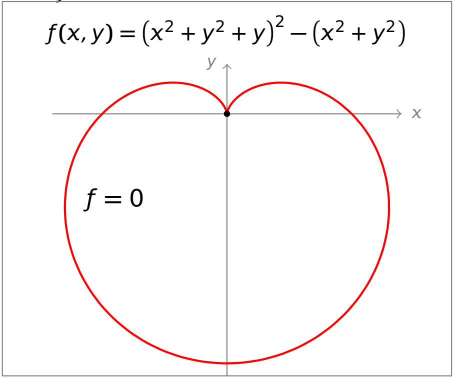

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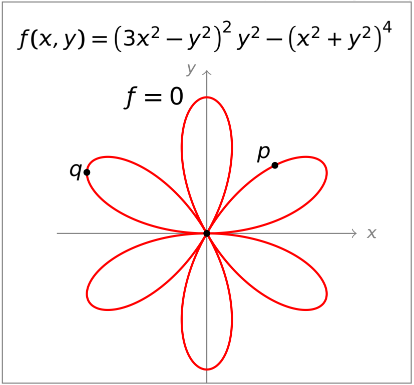

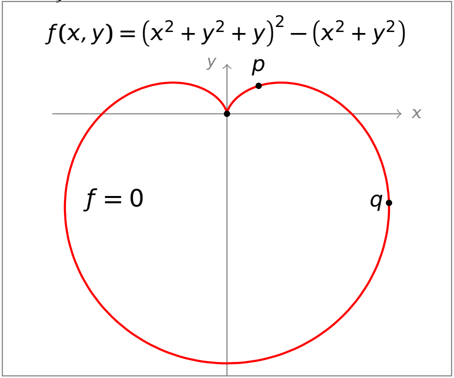

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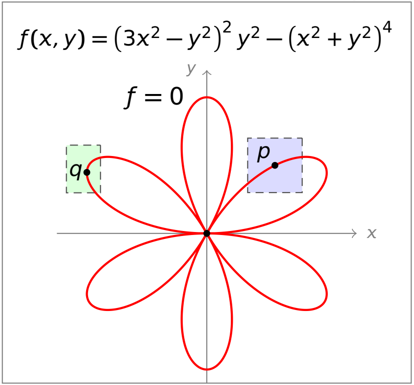

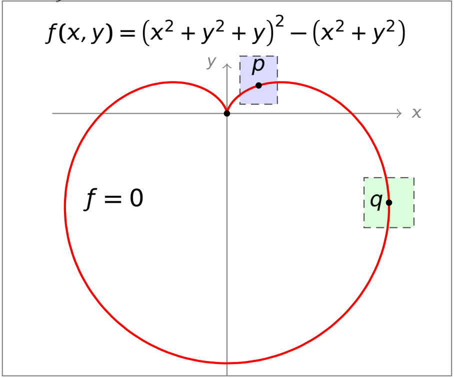

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注 2 等值线 $\{f=c\}$ 可视为空间曲线 $\begin{cases} z=f(x, y) \\ z=c \end{cases}$ 在 xoy 坐标面上的投影.

例 设 $f(x,y) = \left(3x^2 - y^2\right)^2 y^2 - \left(x^2 + y^2\right)^4$

- 在 desmos 上画出等值线 $\{f = c\}$
- 在 CalcPlot3D 上画出曲面 $z = f(x,y)$，平面 $z = c$，及交线空间曲线 $\begin{cases} z = f(x,y) \\ z = c \end{cases}$

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（参考值 $c=-2,\ -0.3,\ 0,\ 0.1$）

空间光滑曲面的定义

空间中光滑曲面应该包含：二元光滑函数的图形，即 $z = \varphi(x, y)$，$y = \psi(x, z)$ 及 $x = \zeta(y, z)$ 的图形

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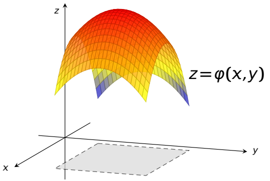

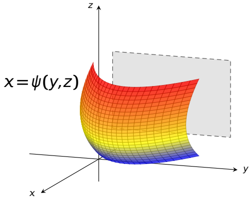

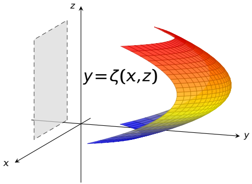

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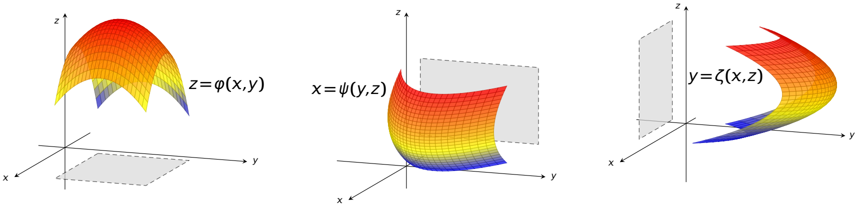

一般地，空间中的点集 S 称为**光滑曲面**，是指 S “局部”上是二元光滑函数的图形。

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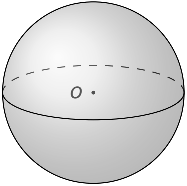

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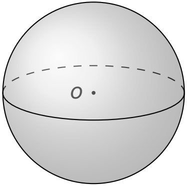

解 这是，球面局部上是如下 6 种二元函数的图形之一：

$$z=\pm\sqrt{1-x^2-y^2},\qquad (\sqrt{x^2+y^2}<1)$$
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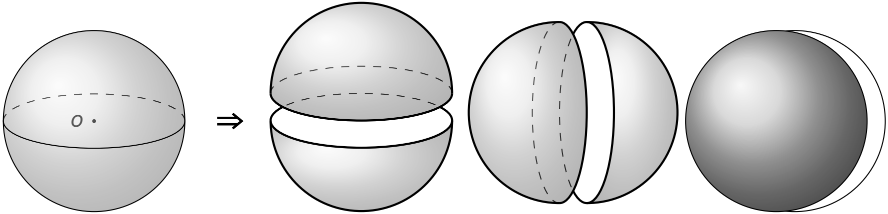

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假设 $f(x,y,z)$ 是光滑的三元函数，其零点集 $\{f=0\}$ 是平面上点集.

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例 设 $f(x,y,z)=(2x^2+y^2+z^2-1)^3-\frac{1}{10}x^2z^3-y^2z^3$

例 设 $f(x,y,z) = (2x^2+y^2+z^2-1)^3 - \frac{1}{10}x^2z^3 - y^2z^3$

- 求出 $\{f=0\}$ 上偏导数全为零的点（临界点）
- 在 CalcPlot3D 上画出曲面 $\{f=0\}$
- 观察临界点附近是否光滑
- 观察曲面哪些部分可以表示成光滑二元函数 $z=\varphi(x,y)$，或 $y=\psi(x,z)$，或 $x=\gamma(y,z)$ 的图形