第 9 章 c：多元复合函数的求导法则

数学系　　梁卓滨

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Outline

二元复合函数求导

设有二元函数 $z = f(u, v)$

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- 设 $u = \varphi(t)$，$v = \psi(t)$，则 $z = f(\varphi(t), \psi(t))$

问 $\frac{dz}{dt} = ?$

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二元复合函数求导公式——中间变量是一元函数

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$$=(uv)'_u\cdot$$

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$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial u}\cdot\frac{du}{dt} + \frac{\partial z}{\partial v}\cdot\frac{dv}{dt} \\ &= (uv)'_u\cdot(e^{-t})'_t + (uv)'_v\cdot(\sin t)'_t \\ &= \end{aligned}$$

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$$\because \quad z = uv =$$

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$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt} + \frac{\partial z}{\partial v} \cdot \frac{dv}{dt}$$
$$= (uv)'_u \cdot (e^{-t})'_t + (uv)'_v \cdot (\sin t)'_t$$
$$= v \cdot (-e^{-t}) + u \cdot \cos t$$
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$$\because \quad z = uv = e^{-t} \cdot \sin t$$
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\end{aligned}
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$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=-\frac{1-e^{2t}}{e^{2t}}\cdot e^t+$$

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z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[dr, "\frac{\partial z}{\partial y}"'] & & \\
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\end{tikzcd}$$

例 设 $z = \frac{y}{x}$，而 $x = e^t$，$y = 1 - e^{2t}$，求全导数 $\frac{dz}{dt}$

解

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} = (\frac{y}{x})'_x \cdot (e^t)'_t + (\frac{y}{x})'_y \cdot (1 - e^{2t})'_t$$

$$= -\frac{y}{x^2} \cdot e^t + \frac{1}{x} \cdot (-2e^{2t}) = -\frac{1 - e^{2t}}{e^{2t}} \cdot e^t + \frac{1}{e^t} \cdot (-2e^{2t})$$

$$=$$

$$\begin{tikzcd}
 & x \arrow[r, "\frac{dx}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[dr, "\frac{\partial z}{\partial y}"'] & & \\
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\end{tikzcd}$$

例 设 $z=\frac{y}{x}$，而 $x=e^t$，$y=1-e^{2t}$，求全导数 $\frac{dz}{dt}$

解

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=\left(\frac{y}{x}\right)'_x\cdot(e^t)'_t+\left(\frac{y}{x}\right)'_y\cdot(1-e^{2t})'_t$$

$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=-\frac{1-e^{2t}}{e^{2t}}\cdot e^t+\frac{1}{e^t}\cdot(-2e^{2t})$$

$$=-e^{-t}-e^t$$

$$\begin{tikzcd} & x \arrow[r, "\frac{dx}{dt}"] & t \\ z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[dr, "\frac{\partial z}{\partial y}"'] & & \\ & y \arrow[r, "\frac{dy}{dt}"'] & t \end{tikzcd}$$

三元复合函数求导公式——中间变量是一元函数

公式 设 $z = f(u, v, w)$，$u = \varphi(t)$，$v = \psi(t)$，$w = \omega(t)$，则 $z = f(\varphi(t), \psi(t), \omega(t))$ 的全导数

$$\frac{dz}{dt} =$$

暨南大学 JINAN UNIVERSITY

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$$\frac{dz}{dt}=$$

$$\begin{tikzcd}
 & u \arrow[r] & t \\
z \arrow[ur] \arrow[r] \arrow[dr] & v \arrow[r] & t \\
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$$\frac{dz}{dt}=$$

$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
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 & w \arrow[r, "\frac{dw}{dt}"] & t
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$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt}$$

$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"] & v \arrow[r, "\frac{dv}{dt}"] & t \\
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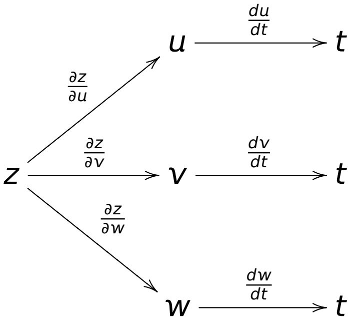
$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}\qquad\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$



三元复合函数求导公式——中间变量是一元函数

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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}\quad\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}\quad\frac{\partial z}{\partial w}\cdot\frac{dw}{dt}$$

$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"] & v \arrow[r, "\frac{dv}{dt}"] & t \\
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 & u \arrow[r, "\frac{du}{dt}"] & t \\
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二元复合函数求导公式——中间变量是多元函数

公式 设 $z = f(u, v)$，$u = \varphi(x, y)$，$v = \psi(x, y)$，

二元复合函数求导公式——中间变量是多元函数

公式 设 $z=f(u,v)$，$u=\varphi(x,y)$，$v=\psi(x,y)$，则复合函数

$$z=f(\varphi(x,y),\psi(x,y))$$

的偏导数是：

$$\frac{\partial z}{\partial x}= \qquad\qquad , \quad \frac{\partial z}{\partial y}=$$

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图示

$$\begin{tikzcd}
 & & x \\
 & u \arrow[ur] \arrow[dr] & \\
 & & y \\
z \arrow[uur] \arrow[ddr] & & \\
 & & x \\
 & v \arrow[ur] \arrow[dr] & \\
 & & y
\end{tikzcd}$$

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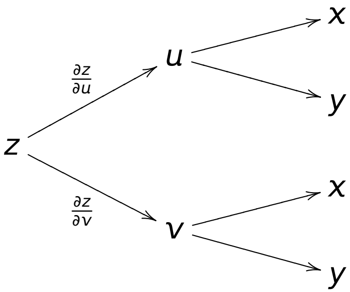
图示



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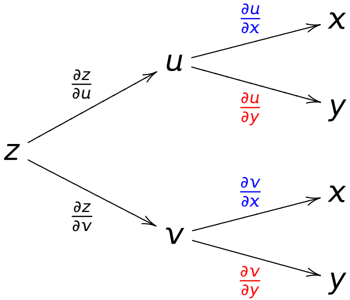
图示



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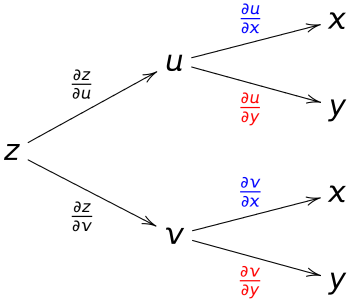
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \qquad\qquad , \quad \frac{\partial z}{\partial y} =$$

图示



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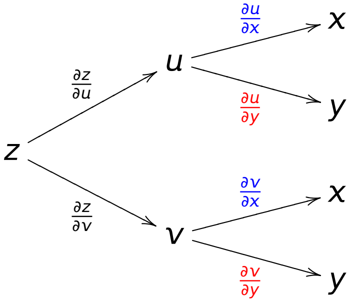
的偏导数是：

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x},\qquad \frac{\partial z}{\partial y}=$$

图示



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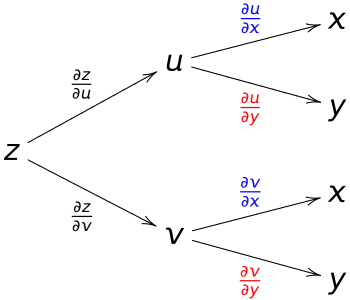
图示



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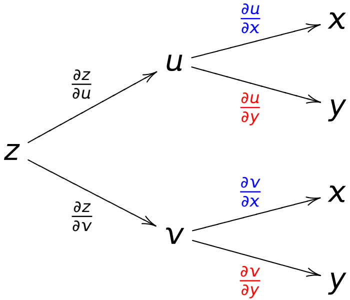
图示



例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\begin{aligned}\frac{\partial z}{\partial x}&=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\&=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+\end{aligned}$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x} \\ &= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot\end{aligned}$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x} \\ &= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot (x^2 + y^2)'_x \\ &= \end{aligned}$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$

$$=2e^{2u}\sin v\cdot$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$= (e^{2u}\sin v)'_u\cdot(x^3y)'_x + (e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$= 2e^{2u}\sin v\cdot 3x^2y +$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$
\begin{aligned}
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&= (e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x \\
&= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x \\
&=
\end{aligned}
$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot (x^2 + y^2)'_x$$

$$= 2e^{2u}\sin v \cdot 3x^2 y + e^{2u}\cos v \cdot 2x$$

$$= 2e^{2x^3 y}\sin(x^2 + y^2) \cdot$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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&= (e^{2u}\sin v)'_u \cdot (x^3y)'_x + (e^{2u}\sin v)'_v \cdot (x^2+y^2)'_x \\
&= 2e^{2u}\sin v \cdot 3x^2y + e^{2u}\cos v \cdot 2x \\
&= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y +
\end{aligned}$$

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解
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$$= 2e^{2u}\sin v\cdot 3x^2y + e^{2u}\cos v\cdot 2x$$
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$$
\begin{aligned}
\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x} \\
&= (e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x \\
&= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x \\
&= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x
\end{aligned}
$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\begin{aligned}\frac{\partial z}{\partial x}&=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\&=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\&=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\&=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x\end{aligned}$$

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$$
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&=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\
&=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\
&=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\\
&=(e^{2u}\sin v)'_u\cdot
\end{aligned}
$$

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解
$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\ &= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\ &= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_y+\end{aligned}$$

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$$= 2e^{2u}\sin v\cdot 3x^2 y + e^{2u}\cos v\cdot 2x$$
$$= 2e^{2x^3 y}\sin(x^2+y^2)\cdot 3x^2 y + e^{2x^3 y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$
$$= (e^{2u}\sin v)'_u\cdot(x^3 y)'_y + (e^{2u}\sin v)'_v\cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\ &= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\ &= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y\\ &=\end{aligned}$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot (x^2 + y^2)'_x$$

$$= 2e^{2u}\sin v \cdot 3x^2 y + e^{2u}\cos v \cdot 2x$$

$$= 2e^{2x^3 y}\sin(x^2 + y^2)\cdot 3x^2 y + e^{2x^3 y}\cos(x^2 + y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$

$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_y + (e^{2u}\sin v)'_v \cdot (x^2 + y^2)'_y$$

$$= 2e^{2u}\sin v \cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\begin{aligned}\frac{\partial z}{\partial x}&=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\&=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\&=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\&=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial y}&=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\\&=(e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y\\&=2e^{2u}\sin v\cdot x^3+\end{aligned}$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

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$$= 2e^{2u}\sin v \cdot 3x^2 y + e^{2u}\cos v \cdot 2x$$

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$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$

$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_y + (e^{2u}\sin v)'_v \cdot (x^2 + y^2)'_y$$

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例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$= (e^{2u}\sin v)'_u\cdot(x^3y)'_x + (e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$= 2e^{2u}\sin v\cdot 3x^2y + e^{2u}\cos v\cdot 2x$$
$$= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y + e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

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$$= 2e^{2u}\sin v\cdot x^3 + e^{2u}\cos v\cdot 2y$$
$$=$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\ &= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\ &= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y\\ &= 2e^{2u}\sin v\cdot x^3+e^{2u}\cos v\cdot 2y\\ &= 2e^{2x^3y}\sin(x^2+y^2)\cdot\end{aligned}$$

例 设 $z = e^{2u}\sin v$，$u = x^3y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

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$$= 2e^{2u}\sin v \cdot 3x^2y + e^{2u}\cos v \cdot 2x$$

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$$= 2e^{2u}\sin v \cdot x^3 + e^{2u}\cos v \cdot 2y$$

$$= 2e^{2x^3y}\sin(x^2+y^2)\cdot x^3 +$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x$$
$$=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$
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例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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三元复合函数求导公式：举例

公式 设 $z = f(x, y, u)$，$u = u(x, y)$，

三元复合函数求导公式：举例

公式 设 $z=f(x,y,u)$，$u=u(x,y)$，则复合函数

$$z=f(x,y,u(x,y))$$

的偏导数是：

$$\frac{\partial z}{\partial x}= \qquad , \qquad \frac{\partial z}{\partial y}=$$

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$$\frac{\partial z}{\partial x}= \qquad\qquad\qquad , \quad \frac{\partial z}{\partial y}=$$

图示

$$\begin{tikzcd}
 & x & \\
z \arrow[ur] \arrow[r] \arrow[ddr] & y & \\
 & & x \\
 & u \arrow[ur] \arrow[dr] & \\
 & & y
\end{tikzcd}$$

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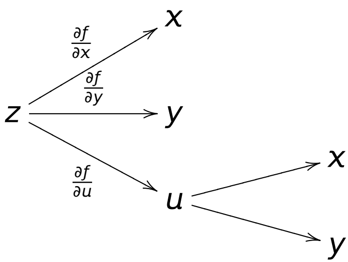
图示



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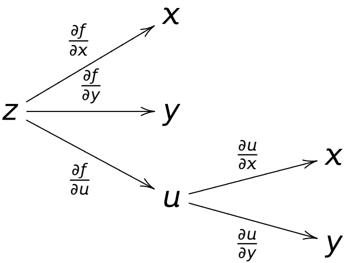
$$z = f(x, y, u(x, y))$$

的偏导数是：

$$\frac{\partial z}{\partial x} = \qquad\qquad\qquad , \quad \frac{\partial z}{\partial y} =$$

图示



三元复合函数求导公式：举例

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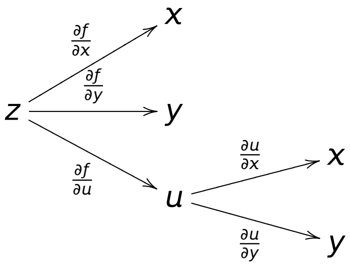
$$z = f(x, y, u(x, y))$$

的偏导数是：

$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} + \qquad\qquad , \quad \frac{\partial z}{\partial y} = $$

图示



三元复合函数求导公式：举例

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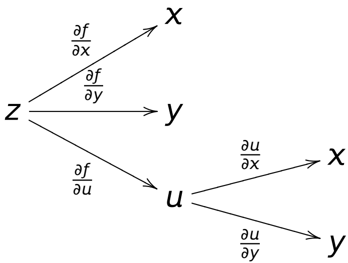
$$z=f(x, y, u(x, y))$$

的偏导数是：

$$\frac{\partial z}{\partial x}=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x}, \qquad \frac{\partial z}{\partial y}=$$

图示



三元复合函数求导公式：举例

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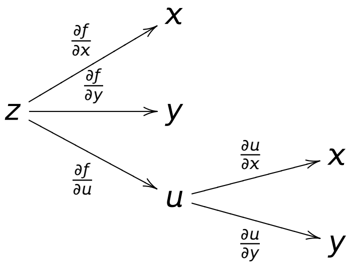
$$z=f(x,y,u(x,y))$$

的偏导数是：

$$\frac{\partial z}{\partial x}=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x},\qquad \frac{\partial z}{\partial y}=\frac{\partial f}{\partial y}+$$

图示



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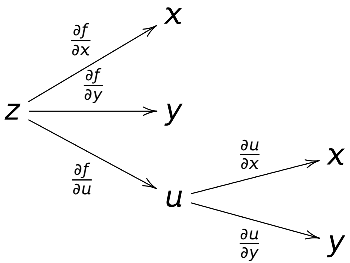
$$z=f(x, y, u(x, y))$$

的偏导数是：

$$\frac{\partial z}{\partial x}=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x}, \quad \frac{\partial z}{\partial y}=\frac{\partial f}{\partial y}+\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y}$$

图示



复合函数的高阶导数

公式 设 $z=f(u,v)$，$u=u(x,y)$，$v=v(x,y)$，则复合函数

$$z=f(u(x,y),v(x,y))$$

的偏导数是：

$$z_x=z_u\cdot u_x+z_v\cdot v_x,$$

$$z_y=z_u\cdot u_y+z_v\cdot v_y,$$

复合函数的高阶导数

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的偏导数是：

$$z_x = z_u\cdot u_x + z_v\cdot v_x,$$

$$z_y = z_u\cdot u_y + z_v\cdot v_y,$$

$z_{xx} =$

$z_{xy} =$

$z_{yy} =$

复合函数的高阶导数

公式 设 $z=f(u,v)$，$u=u(x,y)$，$v=v(x,y)$，则复合函数

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的偏导数是：

$z_x=z_u\cdot u_x+z_v\cdot v_x,$

$z_y=z_u\cdot u_y+z_v\cdot v_y,$

$z_{xx}=(z_x)'_x$

$z_{xy}=$

$z_{yy}=$

复合函数的高阶导数

公式 设 $z=f(u,v)$，$u=u(x,y)$，$v=v(x,y)$，则复合函数

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的偏导数是：

$z_x=z_u\cdot u_x+z_v\cdot v_x$,

$z_y=z_u\cdot u_y+z_v\cdot v_y$,

$z_{xx}=(z_x)'_x=(z_u\cdot u_x+z_v\cdot v_x)'_x$

$z_{xy}=$

$z_{yy}=$

复合函数的高阶导数

公式 设 $z=f(u,v)$，$u=u(x,y)$，$v=v(x,y)$，则复合函数

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的偏导数是：

$$z_x=z_u\cdot u_x+z_v\cdot v_x,$$

$$z_y=z_u\cdot u_y+z_v\cdot v_y,$$

$$\begin{aligned} z_{xx}&=(z_x)'_x=(z_u\cdot u_x+z_v\cdot v_x)'_x\\ &=(z_u)'_x\cdot u_x+z_u\cdot u_{xx}+(z_v)'_x\cdot v_x+z_v\cdot v_{xx}\end{aligned}$$

$$z_{xy}=$$

$$z_{yy}=$$

复合函数的高阶导数

公式 设 $z=f(u,v)$，$u=u(x,y)$，$v=v(x,y)$，则复合函数

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的偏导数是：

$$z_x = z_u\cdot u_x + z_v\cdot v_x,$$

$$z_y = z_u\cdot u_y + z_v\cdot v_y,$$

$$\begin{aligned} z_{xx} &= (z_x)'_x = (z_u\cdot u_x + z_v\cdot v_x)'_x \\ &= (z_u)'_x\cdot u_x + z_u\cdot u_{xx} + (z_v)'_x\cdot v_x + z_v\cdot v_{xx} \\ &= (\qquad\qquad\qquad)\cdot u_x + z_u\cdot u_{xx} + (\qquad\qquad\qquad)\cdot v_x + z_v\cdot v_{xx} \end{aligned}$$

$$z_{xy} =$$

$$z_{yy} =$$

复合函数的高阶导数

公式 设 $z=f(u,v)$，$u=u(x,y)$，$v=v(x,y)$，则复合函数

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$$z_y=z_u\cdot u_y+z_v\cdot v_y,$$

$$\begin{aligned}z_{xx}&=(z_x)'_x=(z_u\cdot u_x+z_v\cdot v_x)'_x\\&=(z_u)'_x\cdot u_x+z_u\cdot u_{xx}+(z_v)'_x\cdot v_x+z_v\cdot v_{xx}\\&=(z_{uu}\cdot u_x+z_{uv}\cdot v_x)\cdot u_x+z_u\cdot u_{xx}+(\qquad\qquad)\cdot v_x+z_v\cdot v_{xx}\end{aligned}$$

$$z_{xy}=$$

$$z_{yy}=$$

复合函数的高阶导数

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$$\begin{aligned}z_{xx}&=(z_x)'_x=(z_u\cdot u_x+z_v\cdot v_x)'_x\\&=(z_u)'_x\cdot u_x+z_u\cdot u_{xx}+(z_v)'_x\cdot v_x+z_v\cdot v_{xx}\\&=(z_{uu}\cdot u_x+z_{uv}\cdot v_x)\cdot u_x+z_u\cdot u_{xx}+(z_{vu}\cdot u_x+z_{vv}\cdot v_x)\cdot v_x+z_v\cdot v_{xx}\end{aligned}$$

$z_{xy}=$

$z_{yy}=$

复合函数的高阶导数

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$$z_y=z_u\cdot u_y+z_v\cdot v_y,$$

$$\begin{aligned}
z_{xx}&=(z_x)'_x=(z_u\cdot u_x+z_v\cdot v_x)'_x\\
&=(z_u)'_x\cdot u_x+z_u\cdot u_{xx}+(z_v)'_x\cdot v_x+z_v\cdot v_{xx}\\
&=(z_{uu}\cdot u_x+z_{uv}\cdot v_x)\cdot u_x+z_u\cdot u_{xx}+(z_{vu}\cdot u_x+z_{vv}\cdot v_x)\cdot v_x+z_v\cdot v_{xx}\\
&=z_{uu}u_x^2+2z_{uv}u_xv_x+z_{vv}v_x^2+z_uu_{xx}+z_vv_{xx}
\end{aligned}$$

$$z_{xy}=$$

$$z_{yy}=$$

复合函数的高阶导数

公式 设 $z = f(u, v)$，$u = u(x, y)$，$v = v(x, y)$，则复合函数

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的偏导数是：

$z_x = z_u \cdot u_x + z_v \cdot v_x$,

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$$\begin{aligned} z_{xx} &= (z_x)'_x = (z_u \cdot u_x + z_v \cdot v_x)'_x \\ &= (z_u)'_x \cdot u_x + z_u \cdot u_{xx} + (z_v)'_x \cdot v_x + z_v \cdot v_{xx} \\ &= (z_{uu} \cdot u_x + z_{uv} \cdot v_x) \cdot u_x + z_u \cdot u_{xx} + (z_{vu} \cdot u_x + z_{vv} \cdot v_x) \cdot v_x + z_v \cdot v_{xx} \\ &= z_{uu} u_x^2 + 2 z_{uv} u_x v_x + z_{vv} v_x^2 + z_u u_{xx} + z_v v_{xx} \end{aligned}$$

$z_{xy} = ?$

$z_{yy} = ?$

复合函数的高阶导数

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$$z_{xx}=z_{uu}u_x^2+2z_{uv}u_xv_x+z_{vv}v_x^2+z_uu_{xx}+z_vv_{xx}$$

$$z_{xy}=$$

$$z_{yy}=?$$

复合函数的高阶导数

公式 设 $z = f(u, v)$，$u = u(x, y)$，$v = v(x, y)$，则复合函数

$$z = f(u(x, y), v(x, y))$$

的偏导数是：

$$z_x = z_u \cdot u_x + z_v \cdot v_x,$$

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z_{xy}&=(z_x)'_y=(z_u\cdot u_x+z_v\cdot v_x)'_y\\
&=(z_u)'_y\cdot u_x+z_u\cdot u_{xy}+(z_v)'_y\cdot v_x+z_v\cdot v_{xy}\\
&=(z_{uu}\cdot u_y+z_{uv}\cdot v_y)\cdot u_x+z_u\cdot u_{xy}+(z_{vu}\cdot u_y+z_{vv}\cdot v_y)\cdot v_x+z_v\cdot v_{xy}\\
&=z_{uu}u_xu_y+z_{uv}(u_xv_y+u_yv_x)+z_{vv}v_xv_y+z_uu_{xy}+z_vv_{xy}
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$$z_{xy}=z_{uu}u_xu_y+z_{uv}(u_xv_y+u_yv_x)+z_{vv}v_xv_y+z_uu_{xy}+z_vv_{xy}$$

$$z_{yy}=(z_y)'_y=(z_u\cdot u_y+z_v\cdot v_y)'_y$$

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例 设 $z=f(xy^2,x^2y)$，求 $\frac{\partial^2 z}{\partial x\partial y}$

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$$=(y^2)'_y\cdot f_u+y^2\cdot(f_u)'_y+(2xy)'_y\cdot f_v+2xy\cdot(f_v)'_y$$

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例 设 $z=f(\sin x, \cos y, e^{x+y})$，求 $\frac{\partial^2 z}{\partial x\partial y}$

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$$= e^{x+y} f_w - \cos x \sin y \cdot f_{uv} + \cos x e^{x+y} f_{uw} - \sin y e^{x+y} f_{wv} + e^{2x+2y} f_{ww}$$