第 11 章 e: 对坐标的曲面积分

数学系 梁卓滨

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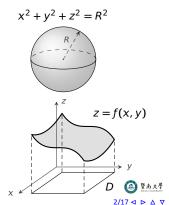
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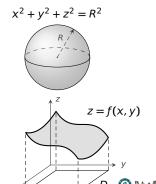
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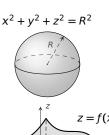
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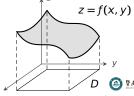
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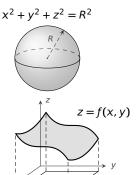
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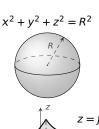


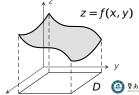
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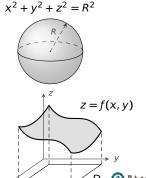


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 - 以上侧为正向的定向函数图形

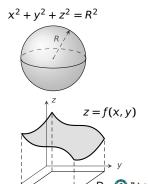


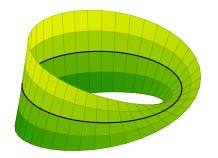
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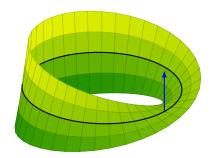
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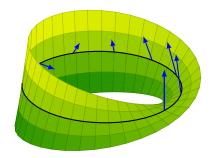
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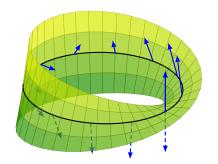
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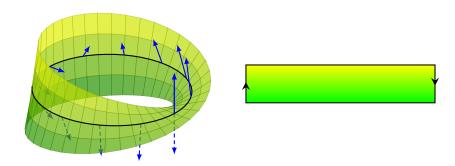




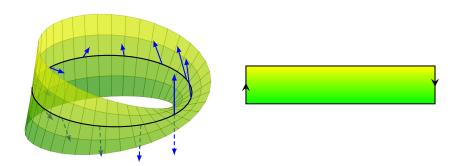








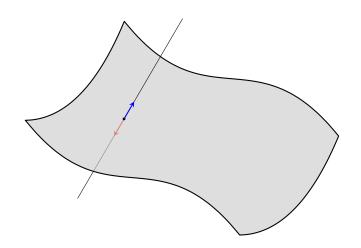
制作方法 将纸带旋转半周,再把两端粘合(如图,使得两端箭头重合)



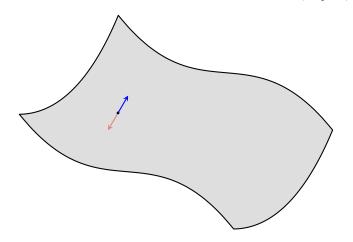
制作方法 将纸带旋转半周,再把两端粘合(如图,使得两端箭头重合)注 如无特殊说明,下面出现的曲面都是可定向的曲面



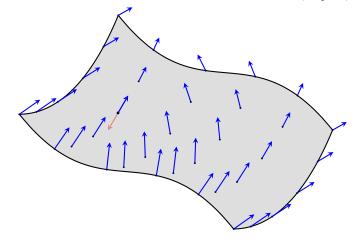
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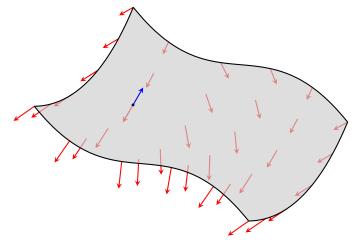
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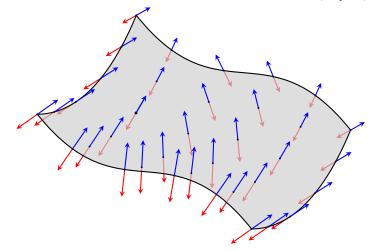
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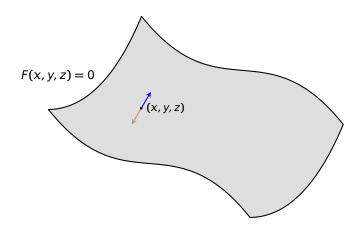


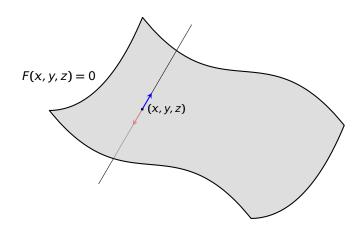
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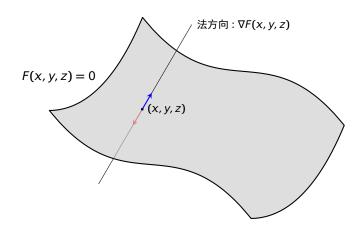


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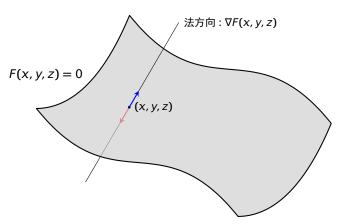




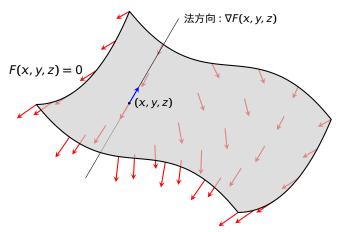


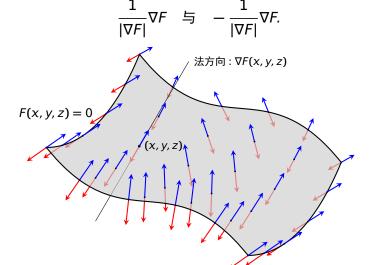


$$\frac{1}{|\nabla F|}\nabla F \quad \leftrightarrows \quad -\frac{1}{|\nabla F|}\nabla F.$$

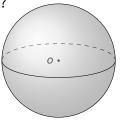


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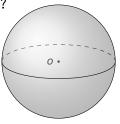




指向外侧,哪个指向内侧?

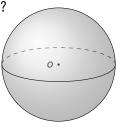


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,则球面方程改写为 $F = 0$ 。

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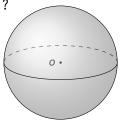


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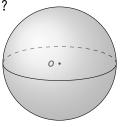
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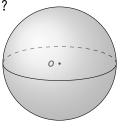


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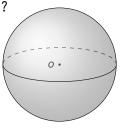


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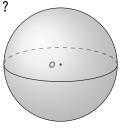


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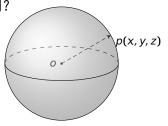


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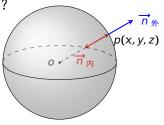


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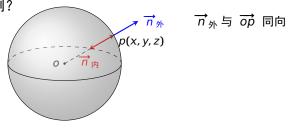


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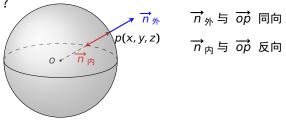
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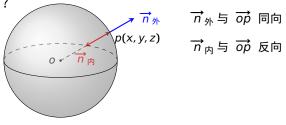


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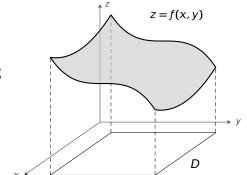
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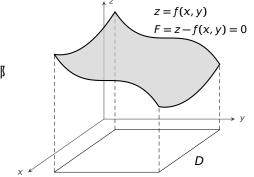
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前一个指向外侧,后一个指向内侧。

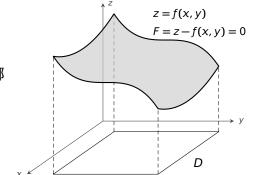






 \mathbf{H} 令 F(x, y, z) = z - f(x, y),则该图形方程改写为 F = 0。



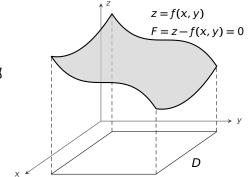


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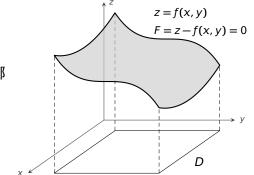


 $\mathbf{w} \Leftrightarrow F(x, y, z) = z - f(x, y)$,则该图形方程改写为 F = 0。计算

$$\nabla F = (-f_X, -f_V, 1), \quad |\nabla F| =$$

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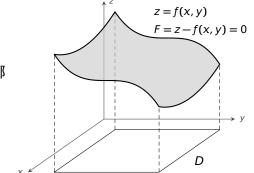


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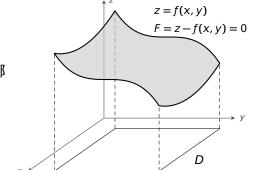


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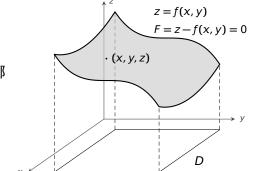


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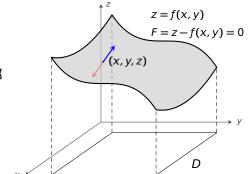
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$$\frac{1}{|\nabla F|} \nabla F = \frac{1}{\sqrt{1 + f_{\times}^2 + f_{\times}^2}} (-f_{X}, -f_{y}, 1), \quad -\frac{1}{|\nabla F|} \nabla F = \frac{1}{\sqrt{1 + f_{\times}^2 + f_{\times}^2}} (f_{X}, f_{y}, -1)$$



例 2 写出二元函数 z = f(x, y) 图 形的两个单位法向量场,并指出哪

一个指向上侧,哪个指向下侧?



解 令 F(x, y, z) = z - f(x, y),则该图形方程改写为 F = 0。计算

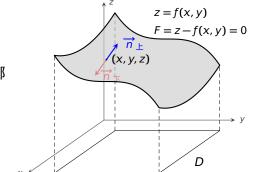
$$\nabla F = (-f_x, -f_y, 1), \qquad |\nabla F| = \sqrt{1 + f_x^2 + f_y^2}$$

所以两个单位法向量场为

$$\frac{1}{|\nabla F|}\nabla F = \frac{1}{\sqrt{1+f_x^2+f_y^2}}(-f_x, -f_y, 1), \quad -\frac{1}{|\nabla F|}\nabla F = \frac{1}{\sqrt{1+f_x^2+f_y^2}}(f_x, f_y, -1)$$



第11章 e: 对坐标的曲面积分



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$$\nabla F = (-f_x, -f_y, 1), \qquad |\nabla F| = \sqrt{1 + f_x^2 + f_y^2}$$

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第11章 e: 对坐标的曲面积分

形的两个单位法向量场, 并指出哪 一个指向上侧,哪个指向下侧?

$$z = f(x, y)$$

$$F = z - f(x, y) = 0$$

$$(x, y, z)$$

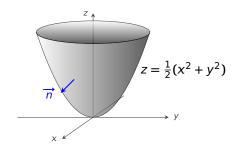
 $\nabla F = (-f_X, -f_y, 1), \qquad |\nabla F| = \sqrt{1 + f_X^2 + f_y^2}$

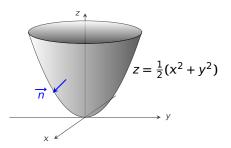
 $\mathbf{H} \Leftrightarrow F(x, y, z) = z - f(x, y)$,则该图形方程改写为 F = 0。计算

 $\frac{1}{|\nabla F|}\nabla F = \frac{1}{\sqrt{1+f_{\nu}^2+f_{\nu}^2}}(-f_{x}, -f_{y}, 1), \quad -\frac{1}{|\nabla F|}\nabla F = \frac{1}{\sqrt{1+f_{\nu}^2+f_{\nu}^2}}(f_{x}, f_{y}, -1)$

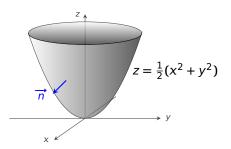
例 2 写出二元函数 z = f(x, y) 图



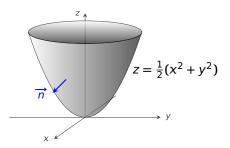




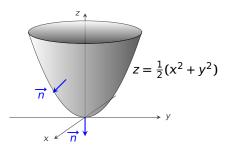
$$\overrightarrow{n} = \frac{1}{\sqrt{1 + z_x^2 + z_y^2}} (z_x, z_y, -1) =$$



$$\overrightarrow{n} = \frac{1}{\sqrt{1 + z_x^2 + z_y^2}} (z_x, z_y, -1) = (x, y, -1)$$



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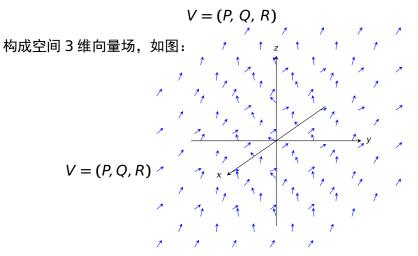
$$\overrightarrow{n} = \frac{1}{\sqrt{1 + z_x^2 + z_y^2}} (z_x, z_y, -1) = \frac{1}{\sqrt{1 + x^2 + y^2}} (x, y, -1)$$

设 P(x, y, z), Q(x, y, z), R(x, y, z) 是三元函数,则

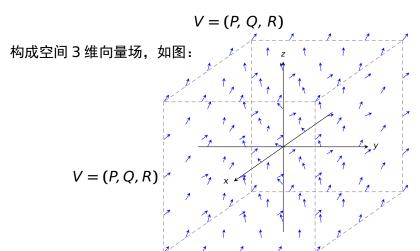
$$V = (P, Q, R)$$

构成空间3维向量场,

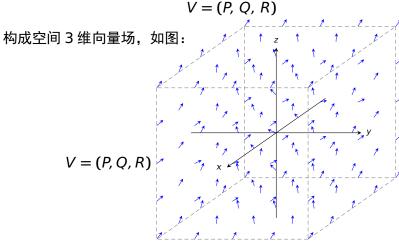
设 P(x, y, z), Q(x, y, z), R(x, y, z) 是三元函数,则



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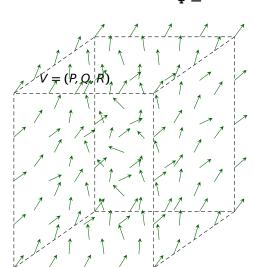


设 P(x, y, z), Q(x, y, z), R(x, y, z) 是三元函数,则

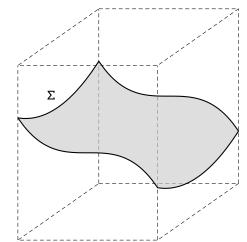


物理应用:向量场 V = (P, Q, R) 可表示流体在任一点处的速度

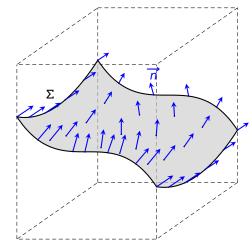


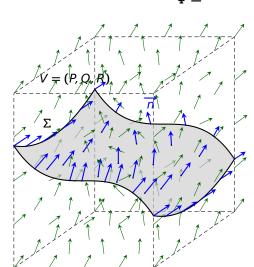


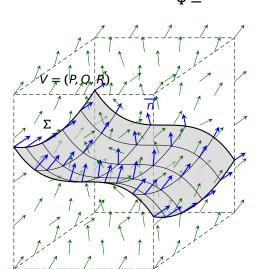


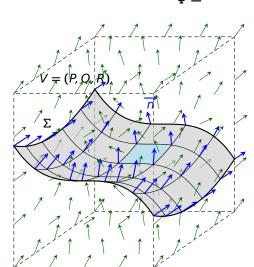


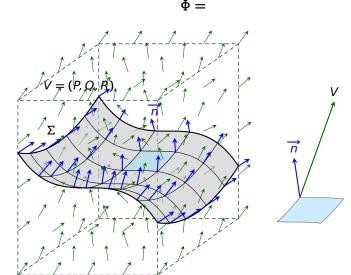


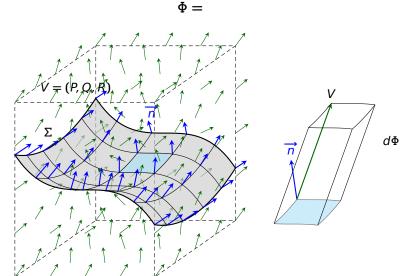


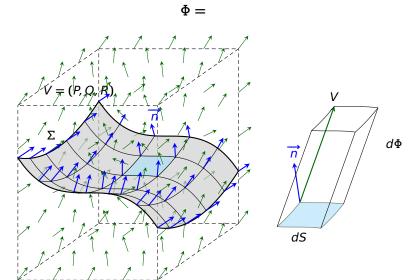


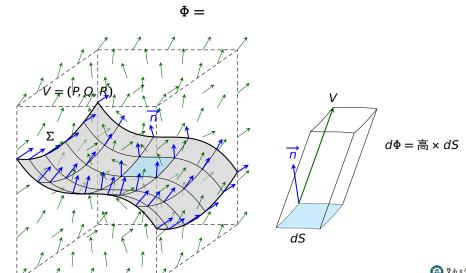


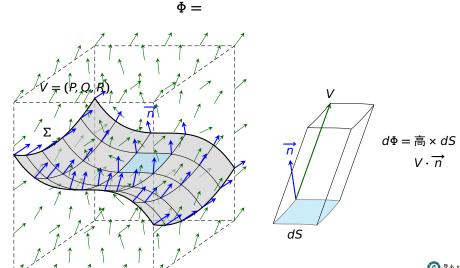


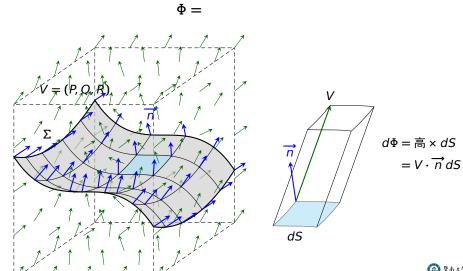




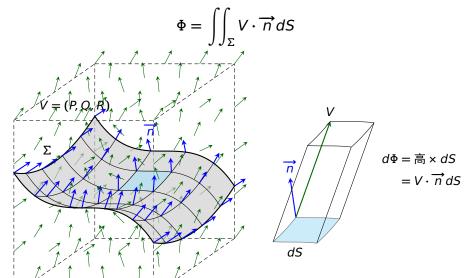








物理应用 流体 V = (P, Q, R) 在单位时间内流过曲面 Σ 一侧(单位法向量 \overrightarrow{n} 所指向的一侧)的流量是:



定义 假设

- V = (P, Q, R) 是空间某区域上的向量场;
- Σ 是定向曲面, \overrightarrow{n} 是 Σ 上指定的单位法向量场;则称

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$$\iint_{\Sigma} V \cdot \overrightarrow{n} \, dS$$

为向量场 V 在定向曲面 Σ 上的曲面积分。

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也记作

$$\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy$$



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(此时也称为对坐标的曲面积分,或第二类曲面积分)



$$\iint_{-\Sigma} Pdydz + Qdzdx + Rdxdy = -\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy$$

$$\iint_{-\Sigma} P dy dz + Q dz dx + R dx dy = -\iint_{\Sigma} P dy dz + Q dz dx + R dx dy$$

物理解释 流过负侧的流量 = - 流过正侧的流量

$$\iint_{-\Sigma} Pdydz + Qdzdx + Rdxdy = -\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy$$

物理解释 流过负侧的流量 = - 流过正侧的流量

证明 设 \overrightarrow{n} 是与 Σ 定向相符的单位法向量场,

$$\iint_{-\Sigma} Pdydz + Qdzdx + Rdxdy = -\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy$$

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$$\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy = \iint_{\Sigma} V \cdot (-\overrightarrow{n}) dS$$



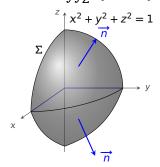
$$\iint_{-\Sigma} Pdydz + Qdzdx + Rdxdy = -\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy$$

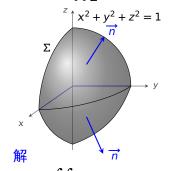
物理解释 流过负侧的流量 = - 流过正侧的流量

证明 设 \overrightarrow{n} 是与 Σ 定向相符的单位法向量场,则 $-\overrightarrow{n}$ 是与 $-\Sigma$ 定向相符的单位法向量场。

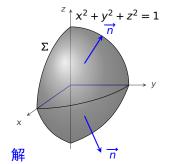
:. 二者数值互为相反数

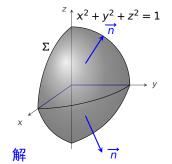




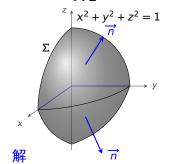


$$m$$
 原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS$

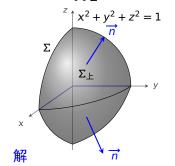




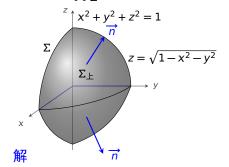
原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} \, dS = \underbrace{V = (0, 0, xyz)}_{\overrightarrow{n} = (x, y, z)}$$



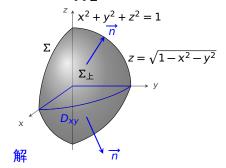
原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS$$



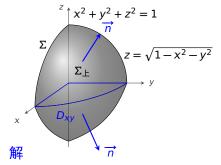
原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$

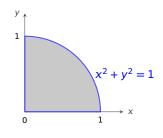


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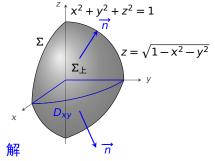


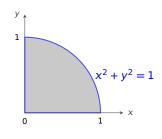
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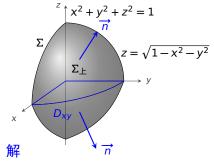


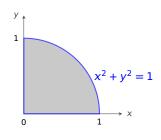


原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$

$$xv(1-x^2-v^2)$$

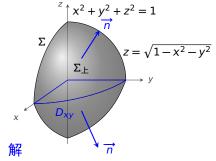


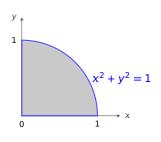




原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$

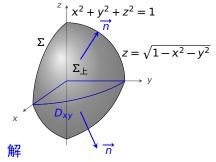
$$xy(1-x^2-y^2)\cdot\sqrt{1+z_x^2+z_y^2}dxdy$$





原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$
$$= \iint_{D_{xyz}} xy(1 - x^2 - y^2) \cdot \sqrt{1 + z_x^2 + z_y^2} dxdy$$





$$x^{2} + y^{2} = 1$$

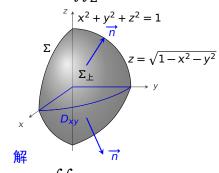
$$0 \qquad 1 \qquad x$$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$

$$= \iint_{D_{vir}} xy(1-x^2-y^2) \cdot \sqrt{1+z_x^2+z_y^2} dxdy$$

$$\cdot \frac{1}{\sqrt{1-x^2-y^2}} dx dx$$





$$x^{2} + y^{2} = 1$$

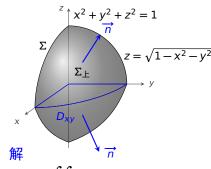
$$0 \qquad 1 \qquad x$$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \underbrace{V = (0, 0, xyz)}_{\overrightarrow{n} = (x, y, z)} = \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$

$$= \iint_{D_{xy}} xy(1 - x^2 - y^2) \cdot \sqrt{1 + z_x^2 + z_y^2} dxdy$$

$$= 2 \iint_{D_{xy}} xy(1 - x^2 - y^2) \cdot \frac{1}{\sqrt{1 - x^2 - y^2}} dxdy$$

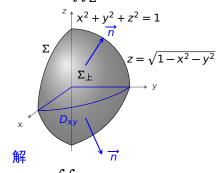




$$x^{2} + y^{2} = 1$$

$$0 \qquad 1 \qquad x$$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$
$$= \iint_{D_{xy}} xy(1 - x^2 - y^2) \cdot \sqrt{1 + z_x^2 + z_y^2} dx dy$$
$$= 2 \iint_{\Sigma} xy\sqrt{1 - x^2 - y^2} dx dy$$



$$x^{2} + y^{2} = 1$$

$$0 \qquad 1 \qquad x$$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$
$$= \iint_{D_{xy}} xy(1 - x^2 - y^2) \cdot \sqrt{1 + z_x^2 + z_y^2} dx dy$$
$$= 2 \iint_{D} xy\sqrt{1 - x^2 - y^2} dx dy \xrightarrow{x = \rho \cos \theta} \cdots$$



原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$

$$\int J\Sigma = \int_{D_{xy}} xy(1-x^2-y^2) \cdot \sqrt{1+z_x^2+z_y^2} dxdy
= 2 \iint_{D_{xy}} xy\sqrt{1-x^2-y^2} dxdy$$

$$\frac{x = \rho \cos \theta}{y = \rho \sin \theta} 2 \iint_{D_{min}} \rho^2 \sin \theta \cos \theta \cdot \sqrt{1 - \rho^2} \cdot \rho d\rho d\theta$$

$$2 \iint_{D_{xy}} \rho^2 \sin \theta \cos \theta \cdot \sqrt{1 - \rho^2 \cdot \rho} d\rho d\theta$$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$

$$= \iint_{D_{xy}} xy(1-x^2-y^2) \cdot \sqrt{1+z_x^2+z_y^2} dx dy$$

$$= 2 \iint_{D_{xy}} xy\sqrt{1-x^2-y^2} dx dy$$

$$\frac{x=\rho\cos\theta}{y=\rho\sin\theta} 2 \iint_{D_{xy}} \rho^2\sin\theta\cos\theta \cdot \sqrt{1-\rho^2} \cdot \rho d\rho d\theta$$

$$= 2 \int \left[\int \sin \theta \cos \theta \rho^3 \sqrt{1 - \rho^2} d\rho \right] d\theta$$



原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$

$$\begin{split} &= \iint_{D_{xy}} xy(1-x^2-y^2) \cdot \sqrt{1+z_x^2+z_y^2} dx dy \\ &= 2 \iint_{D_{xy}} xy\sqrt{1-x^2-y^2} dx dy \\ &= \underbrace{2 \iint_{D_{xy}} xy\sqrt{1-x^2-y^2}}_{y=\rho \sin \theta} 2 \underbrace{\int \int_{D_{xy}} \rho^2 \sin \theta \cos \theta \cdot \sqrt{1-\rho^2} \cdot \rho d\rho d\theta}_{z=\rho \sin \theta} \end{split}$$

$$=2\int_{0}^{\frac{\pi}{2}}\left[\int \sin\theta\cos\theta\rho^{3}\sqrt{1-\rho^{2}}d\rho\right]d\theta$$



原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \underbrace{V = (0, 0, xyz)}_{\overrightarrow{n} = (x, y, z)} = \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$

$$= \iint_{D_{xy}} xy(1-x^2-y^2) \cdot \sqrt{1+z_x^2+z_y^2} dx dy$$

$$= 2 \iint_{D_{xy}} xy\sqrt{1-x^2-y^2} dx dy$$

$$\frac{x=\rho\cos\theta}{y=\rho\sin\theta} 2 \iint_{D_{xy}} \rho^2\sin\theta\cos\theta \cdot \sqrt{1-\rho^2} \cdot \rho d\rho d\theta$$

$$=2\int_{0}^{\frac{\pi}{2}}\left[\int_{0}^{1}\sin\theta\cos\theta\rho^{3}\sqrt{1-\rho^{2}}d\rho\right]d\theta$$



原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma} xyz^2 dS$

$$= \iint_{D_{xy}} xy(1-x^2-y^2) \cdot \sqrt{1+z_x^2+z_y^2} dx dy$$

$$= 2 \iint_{D_{xy}} xy \sqrt{1-x^2-y^2} dx dy$$

$$\frac{x=\rho\cos\theta}{y=\rho\sin\theta} 2 \iint_{D_{xy}} \rho^2 \sin\theta\cos\theta \cdot \sqrt{1-\rho^2} \cdot \rho d\rho d\theta$$

$$= 2 \int_0^{\frac{\pi}{2}} \left[\int_0^1 \sin\theta\cos\theta \rho^3 \sqrt{1-\rho^2} d\rho \right] d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin(2\theta) d\theta \cdot \int_0^1 \rho^2 \sqrt{1-\rho^2} \cdot \rho d\rho$$

$$\frac{1}{1-
ho^2} \cdot
ho d
ho$$



$$= 2 \int_0^{\frac{\pi}{2}} \left[\int_0^1 \sin\theta \cos\theta \rho^3 \sqrt{1 - \rho^2} d\rho \right] d\theta$$
$$= \int_0^{\frac{\pi}{2}} \sin(2\theta) d\theta \cdot \int_0^1 \rho^2 \sqrt{1 - \rho^2} \cdot \rho d\rho$$

 $\frac{x = \rho \cos \theta}{y = \rho \sin \theta} 2 \iint_{\Omega} \rho^2 \sin \theta \cos \theta \cdot \sqrt{1 - \rho^2} \cdot \rho d\rho d\theta$

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma} xyz^2 dS$

 $= \iint_{D} xy(1-x^2-y^2) \cdot \sqrt{1+z_x^2+z_y^2} dxdy$

 $=2\iint_{\mathbb{R}}xy\sqrt{1-x^2-y^2}dxdy$

$$u = \sqrt{1 - \mu}$$



$$=2\int_0^{\frac{\pi}{2}} \left[\int_0^1 \sin\theta \cos\theta \rho^3 \sqrt{1-\rho^2} d\rho \right] d\theta$$

$$=\int_0^{\frac{\pi}{2}} \sin(2\theta) d\theta \int_0^1 \rho^2 \sqrt{1-\rho^2} d\rho$$

 $\frac{x = \rho \cos \theta}{y = \rho \sin \theta} 2 \iint_{\Omega} \rho^2 \sin \theta \cos \theta \cdot \sqrt{1 - \rho^2} \cdot \rho d\rho d\theta$

 $= \iint_{\mathbb{R}^{n}} xy(1-x^{2}-y^{2}) \cdot \sqrt{1+z_{x}^{2}+z_{y}^{2}} dxdy$

 $= 2 \iint_{\mathbb{R}} xy \sqrt{1 - x^2 - y^2} dx dy$

 $u = \sqrt{1-\rho^2}$

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma} xyz^2 dS$

 $= \int_{0}^{\frac{\pi}{2}} \sin(2\theta) d\theta \cdot \int_{0}^{1} \rho^{2} \sqrt{1 - \rho^{2}} \cdot \rho d\rho$

$$=2\int_0^{\frac{\pi}{2}} \left[\int_0^1 \sin\theta \cos\theta \rho^3 \sqrt{1-\rho^2} d\rho \right] d\theta$$

 $\frac{x = \rho \cos \theta}{y = \rho \sin \theta} 2 \iint_{\Omega} \rho^2 \sin \theta \cos \theta \cdot \sqrt{1 - \rho^2} \cdot \rho d\rho d\theta$

 $= \iiint_{S} xy(1-x^2-y^2) \cdot \sqrt{1+z_x^2+z_y^2} dx dy$

 $= 2 \iint_{\mathbb{R}} xy \sqrt{1 - x^2 - y^2} dx dy$

 $u = \sqrt{1-\rho^2}$

 $= \int_{0}^{\frac{\pi}{2}} \sin(2\theta) d\theta \cdot \int_{0}^{1} \rho^{2} \sqrt{1 - \rho^{2}} \cdot \rho d\rho$

 $(1-u^2)u \cdot (-udu)$

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma} xyz^2 dS$

$$= 2 \iint_{D_{xy}} xy \sqrt{1 - x^2 - y^2} dx dy$$

$$= \frac{x = \rho \cos \theta}{y = \rho \sin \theta} 2 \iint_{D_{xy}} \rho^2 \sin \theta \cos \theta \cdot \sqrt{1 - \rho^2} \cdot \rho d\rho d\theta$$

$$= 2 \int_0^{\frac{\pi}{2}} \left[\int_0^1 \sin \theta \cos \theta \rho^3 \sqrt{1 - \rho^2} d\rho \right] d\theta$$

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma} xyz^2 dS$

 $= \iiint_{S} xy(1-x^2-y^2) \cdot \sqrt{1+z_x^2+z_y^2} dx dy$

 $= \int_{0}^{\frac{\pi}{2}} \sin(2\theta) d\theta \cdot \int_{0}^{1} \rho^{2} \sqrt{1 - \rho^{2}} \cdot \rho d\rho$

 $\frac{u=\sqrt{1-\rho^2}}{\sqrt{1-\rho^2}} \qquad \int_{-1}^{0} (1-u^2)u \cdot (-udu)$

$$\frac{\frac{x=\rho\cos\theta}{y=\rho\sin\theta}}{2} 2 \iint_{D_{xy}} \rho^2 \sin\theta\cos\theta \cdot \sqrt{1-\rho^2} \cdot \rho d\rho d\theta$$
$$= 2 \int_0^{\frac{\pi}{2}} \left[\int_0^1 \sin\theta\cos\theta \rho^3 \sqrt{1-\rho^2} d\rho \right] d\theta$$

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma} xyz^2 dS$

 $= \iint_{\mathbb{R}^{n}} xy(1-x^{2}-y^{2}) \cdot \sqrt{1+z_{x}^{2}+z_{y}^{2}} dxdy$

 $=2\iint_{\mathbb{R}}xy\sqrt{1-x^2-y^2}dxdy$

 $= \int_{0}^{\frac{\pi}{2}} \sin(2\theta) d\theta \cdot \int_{0}^{1} \rho^{2} \sqrt{1 - \rho^{2}} \cdot \rho d\rho$

 $\frac{u=\sqrt{1-\rho^2}}{2} \cdot 1 \cdot \int_{-\infty}^{\infty} (1-u^2)u \cdot (-udu)$

$$= 2 \iint_{D_{xy}} xy \sqrt{1 - x^2 - y^2} dx dy$$

$$\frac{x = \rho \cos \theta}{y = \rho \sin \theta} 2 \iint_{D_{xy}} \rho^2 \sin \theta \cos \theta \cdot \sqrt{1 - \rho^2} \cdot \rho d\rho d\theta$$

 $= \iint_{D} xy(1-x^2-y^2) \cdot \sqrt{1+z_x^2+z_y^2} dxdy$

 $=2\int_{0}^{\frac{\pi}{2}}\left[\int_{0}^{1}\sin\theta\cos\theta\rho^{3}\sqrt{1-\rho^{2}}d\rho\right]d\theta$

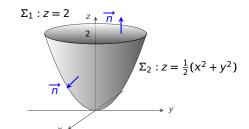
 $= \int_{0}^{\frac{\alpha}{2}} \sin(2\theta) d\theta \cdot \int_{0}^{1} \rho^{2} \sqrt{1 - \rho^{2}} \cdot \rho d\rho$

 $\frac{u = \sqrt{1 - \rho^2}}{1 + \rho^2} \cdot 1 \cdot \int_0^u (1 - u^2) u \cdot (-u du) = \frac{2}{15}$

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma} xyz^2 dS$

例 2 计算 $\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$

其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维 区域的边界,如图:

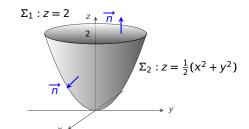


例2计算

$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$

其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维 区域的边界,如图:

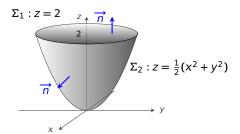
原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS$$



例2计算

$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$

其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维 区域的边界,如图:



原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,



列 2 计算
$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$

其中定向曲面
$$\Sigma = \Sigma_1 \cup \Sigma_2$$
 是三维 区域的边界,如图:

$$\Sigma_1: z = 2 \qquad z \qquad \overrightarrow{n}$$

$$\Sigma_2: z = \frac{1}{2}(x^2 + y^2)$$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

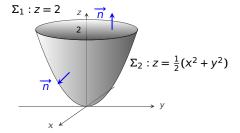
$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} \, dS$$

$$\iint_{\Sigma} V \cdot \overrightarrow{n} \, dS$$



別 2 计算
$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$

其中定向曲面
$$\Sigma = \Sigma_1 \cup \Sigma_2$$
 是三维
区域的边界,如图:



原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS \stackrel{V = (z^2 + x, 0, -z)}{=}$$

$$\iint_{\mathbb{R}} V \cdot \overrightarrow{n} dS \stackrel{V=(z^2+x, 0, -z)}{=}$$

引 2 计算
$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$

其中定向曲面
$$\Sigma = \Sigma_1 \cup \Sigma_2$$
 是三维
区域的边界,如图:

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

 $\Sigma_1 : z = 2$

$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^2 + x, 0, -z)} \overrightarrow{n} = (0, 0, 1)$$

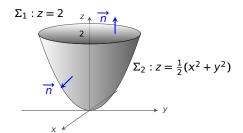
$$\int \int V \cdot \overrightarrow{n} dS \stackrel{V=(z^2+x,0,-z)}{=}$$



 $^{\prime}\Sigma_{2}: z = \frac{1}{2}(x^{2} + y^{2})$

2 计算
$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$

其中定向曲面
$$\Sigma = \Sigma_1 \cup \Sigma_2$$
 是三维 区域的边界,如图:



原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,
$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS = \underbrace{V = (z^2 + x, 0, -z)}_{\overrightarrow{n} = (0, 0, 1)} \iint_{\Sigma_2} -z dS$$

$$\iint_{\mathbb{R}} V \cdot \overrightarrow{n} dS \stackrel{V=(z^2+x,0,-z)}{=}$$

例 2 计算
$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$
 其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

$$\Sigma_1: z = 2$$

$$\Sigma_2: z = \frac{1}{2}(x^2 + y^2)$$

$$\downarrow y$$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,
$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS = \underbrace{\frac{V = (z^2 + x, 0, -z)}{\overrightarrow{n} = (0, 0, 1)}} \iint_{\Sigma_1} -z dS = \iint_{\Sigma_1} -2 dS$$

$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS = \underbrace{\frac{V = (z^2 + x, 0, -z)}{\overrightarrow{n} = (0, 0, 1)}}$$



例 2 计算
$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$
 其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

$$\Sigma_1: z = 2$$

$$z \rightarrow \overrightarrow{n}$$

$$\Sigma_2: z = \frac{1}{2}(x^2 + y^2)$$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,
$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS = \underbrace{\frac{V = (z^2 + x, 0, -z)}{\overrightarrow{n} = (0, 0, 1)}} \iint_{\Sigma_1} -z dS = \iint_{\Sigma_1} -2 dS = -2|\Sigma_1|$$

$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS = \underbrace{\frac{V = (z^2 + x, 0, -z)}{\overrightarrow{n} = (0, 0, 1)}}$$



例 2 计算
$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$
 其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

$$\Sigma_1: z = 2$$

$$z \longrightarrow n$$

$$\Sigma_2: z = \frac{1}{2}(x^2 + y^2)$$

$$y$$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS,$$

$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS = \underbrace{V = (z^2 + x, 0, -z)}_{\overrightarrow{n} = (0, 0, 1)} \iint_{\Sigma_1} -z dS = \iint_{\Sigma_1} -2 dS = -2|\Sigma_1| = -8\pi,$$

$$\iint_{\Sigma_1} \nabla \cdot \overrightarrow{n} \, dS \xrightarrow{V = (z^2 + x, \, 0, -z)}$$



例 2 计算
$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$
 其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

$$\Sigma_1 : z = 2$$

$$\Sigma_2 : z = \frac{1}{2}(x^2 + y^2)$$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS,$$

$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS = \underbrace{\frac{V = (z^2 + x, 0, -z)}{\overrightarrow{n} = (0, 0, 1)}} \iint_{\Sigma_1} -z dS = \iint_{\Sigma_1} -2 dS = -2|\Sigma_1| = -8\pi,$$

$$\iint_{\Sigma_1} \vec{n} = (0, 0, 1)$$

$$\iint_{\Sigma_1} V \cdot \vec{n} \, dS \stackrel{V = (z^2 + x, 0, -z)}{=}$$



例
$$2$$
 计算
$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$
 其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

$$\Sigma_1 : z = 2$$

$$\Sigma_2 : z = \frac{1}{2}(x^2 + y^2)$$

$$\Sigma_2 : z = \frac{1}{2}(x^2 + y^2)$$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS,$$

$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS = \underbrace{\frac{V = (z^2 + x, 0, -z)}{\overrightarrow{n} = (0, 0, 1)}} \iint_{\Sigma_1} -z dS = \iint_{\Sigma_1} -2 dS = -2|\Sigma_1| = -8\pi,$$

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} \, dS \xrightarrow{\overrightarrow{n} = (0, 0, 1)}$$

$$\iint_{\Sigma_{2}} V \cdot \overrightarrow{n} \, dS \xrightarrow{V = (z^{2} + x, 0, -z)} \overrightarrow{\overrightarrow{n}} = \xrightarrow{(x, y, -1)}$$



例
$$2$$
 计算
$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$
 其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

$$\Sigma_1: z = 2$$

$$z \longrightarrow D$$

$$x \longrightarrow D$$

$$y$$

$$D_{XY}$$

解

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

 $\int\!\!\int_{\Sigma_1} V \cdot \overrightarrow{n} \, dS = \frac{V = (z^2 + x, \, 0, -z)}{\overrightarrow{n} = (0, \, 0, \, 1)} \int\!\!\int_{\Sigma_1} -z \, dS = \int\!\!\int_{\Sigma_1} -2 \, dS = -2 |\Sigma_1| = -8\pi,$

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{\overrightarrow{r} = (0, 0, 1)} \iint_{\Sigma_{1}} \iint_{\Sigma_{1}} I_{\Sigma_{1}}$$

$$\iint_{\Sigma_{2}} V \cdot \overrightarrow{n} dS \xrightarrow{\overrightarrow{r} = \frac{(x, y, -1)}{\sqrt{1 + x^{2} + y^{2}}}} \frac{(z^{2} + x)x + z}{\sqrt{1 + x^{2} + y^{2}}}$$

$$(x)x + z$$

 $(x)^2 + v^2$



$$\int \int (z^2 +$$

例 2 计算
$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$
 其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

$$\Sigma_1 : z = 2$$

$$Z \longrightarrow D_{xy}$$

$$\Sigma_2 : z = \frac{1}{2}(x^2 + y^2)$$

解

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

原式 =
$$\iint_{\Sigma} V \cdot \vec{n} \, dS = \iint_{\Sigma_{1}} V \cdot \vec{n} \, dS + \iint_{\Sigma_{2}} V \cdot \vec{n} \, dS,$$

$$\iint_{\Sigma_{1}} V \cdot \vec{n} \, dS = \underbrace{\frac{V = (z^{2} + x, 0, -z)}{\vec{n} = (0, 0, 1)}} \iint_{\Sigma_{1}} -z dS = \iint_{\Sigma_{1}} -2 dS = -2|\Sigma_{1}| = -8\pi,$$

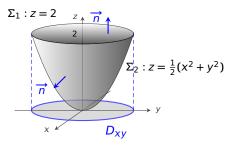
$$\iint_{\Sigma_{2}} V \cdot \vec{n} \, dS = \underbrace{\frac{V = (z^{2} + x, 0, -z)}{\vec{n} = -\frac{(x, y, -1)}{\sqrt{1 + x^{2} + y^{2}}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dx dy$$

 $\frac{(z^2 + x)x + z}{\sqrt{1 + x^2 + y^2}} \cdot \sqrt{1 + z_x^2 + z_y^2} dx dy$



$$\iint (z^2 +$$

例 2 计算
$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$
 其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维



原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

 $\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^2 + x, 0, -z)} \iint_{\Sigma_2} -z dS = \iint_{\Sigma_2} -2 dS = -2|\Sigma_1| = -8\pi,$ $\iint_{\Sigma_2} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^2 + x, 0, -z)} \iint_{D_{xy}} \frac{(z^2 + x)x + z}{\sqrt{1 + x^2 + y^2}} \cdot \sqrt{1 + z_x^2 + z_y^2} dx dy$

$$\int_{\Sigma_{1}} \int_{\Sigma_{1}} \int_{\Sigma_{1}} \frac{z^{2} + x}{\sqrt{1 + x^{2} + y^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dxdy$$



 $\iint_{-} (z^2 + x) dy dz - z dx dy$ 其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维 区域的边界,如图: 解

例2计算

$$\Sigma_1: Z = \frac{1}{2}$$

解
原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS = \underbrace{\frac{V = (z^2 + x, 0, -z)}{\overrightarrow{n} = (0, 0, 1)}} \iint_{\Sigma_1} -z dS = \iint_{\Sigma_1} -2 dS = -2|\Sigma_1| = -8\pi,$$

 $\iint_{\Sigma_2} V \cdot \overrightarrow{n} \, dS \xrightarrow{V = (z^2 + x, \, 0, \, -z)} \iint_{D_{xy}} \frac{(z^2 + x)x + z}{\sqrt{1 + x^2 + y^2}} \cdot \sqrt{1 + z_x^2 + z_y^2} \, dx \, dy$ = $\int_{\Omega} (z^2 + x)x + z dx dy$

例 2 计算 $\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$ 其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维 区域的边界,如图:

$$\Sigma_2 : z = \frac{1}{2}(x^2 + y^2)$$

$$V : \overrightarrow{D}_{XY}$$

$$V : \overrightarrow{D}_{XY}$$

解

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

 $\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} \iint_{\Sigma_{1}} -z dS = \iint_{\Sigma_{1}} -2 dS = -2|\Sigma_{1}| = -8\pi,$ $\iint_{\Sigma_{2}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} \iint_{D_{xy}} \frac{(z^{2} + x)x + z}{\sqrt{1 + x^{2} + y^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dx dy$

e: 对坐标的曲面积分

 $= \iint_{D_{xy}} (z^2 + x)x + z dx dy \xrightarrow{\text{print}} \iint_{D_{xy}} x^2 + z dx dy$

例2计算 $\int \int_{-\infty}^{\infty} (z^2 + x) dy dz - z dx dy$ 其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维 区域的边界,如图:

$$\Sigma_2 : z = \frac{1}{2}(x^2 + x^2)$$

$$\sum_{x \in D_{xy}} z = \sum_{x \in D_{x}} z = \sum_{x \in D_{xy}} z = \sum_{x \in D_{xy}} z = \sum_{x \in D_{xy}} z =$$

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$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,
$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS = \underbrace{\frac{V = (z^2 + x, 0, -z)}{\overrightarrow{n} = (0, 0, 1)}}_{\Sigma_1} \iint_{\Sigma_1} -z dS = \iint_{\Sigma_1} -2 dS = -2|\Sigma_1| = -8\pi$$
,

$$\iint_{\Sigma_{2}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} \iint_{D_{xy}} \frac{(z^{2} + x)x + z}{\sqrt{1 + x^{2} + y^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dx dy$$

$$= \iint_{D_{xy}} (z^{2} + x)x + z dx dy \xrightarrow{\text{phit}} \iint_{D_{xy}} x^{2} + z dx dy = \cdots$$

原式 =
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$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^2 + x, 0, -z)} \iint_{\Sigma_1} -z dS \iint_{\Sigma_1} -2 dS = -2|\Sigma_1| = -8\pi,$$

$$\iint_{\Sigma_{1}} \overrightarrow{n} = (0, 0, 1) \qquad \iint_{\Sigma_{1}} \iint_{\Sigma_{1}} \underbrace{\int \int_{\Sigma_{1}} V \cdot \overrightarrow{n} \, dS} = \underbrace{\frac{V = (z^{2} + x, 0, -z)}{\sqrt{1 + x^{2} + y^{2}}}} \iint_{D_{xy}} \underbrace{\frac{(z^{2} + x)x + z}{\sqrt{1 + x^{2} + y^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} \, dx \, dy}_{= \iint_{D_{xy}} \underbrace{(z^{2} + x)x + z \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}} \frac{1}{2} \iint_{D_{xy}} \underbrace{3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}} \frac{1}{2} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}} \underbrace{1}_{D_{xy}} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}} \underbrace{1}_{D_{xy}} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}} \underbrace{1}_{D_{xy}} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}} \underbrace{1}_{D_{xy}} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}} \underbrace{1}_{D_{xy}} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}} \underbrace{1}_{D_{xy}} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}}} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}}} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}}} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}}} \underbrace{\int \int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}}} \underbrace{\int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}}}} \underbrace{\int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}}}} \underbrace{\int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy}_{= \underbrace{\sum_{z=\frac{1}{2}(x^{2} + y^{2})}}}} \underbrace{\int_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy$$



原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

$$\overrightarrow{n} dS = \frac{V = (z^2 + x, 0, -z)}{\overrightarrow{n} = (0, 0, 1)} \iint_{\Sigma_1} -z dS \iint_{\Sigma_1} -2 dS = -2|\Sigma_1| = -8\pi$$

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{\frac{V = (z^{2} + x, 0, -z)}{\overrightarrow{n} = (0, 0, 1)}} \iint_{\Sigma_{1}} -z dS \iint_{\Sigma_{1}} -2 dS = -2|\Sigma_{1}| = -8\pi,$$

$$\iint_{\Sigma_{2}} V \cdot \overrightarrow{n} dS \xrightarrow{\frac{V = (z^{2} + x, 0, -z)}{\overrightarrow{n} = \frac{(x, y, -1)}{\sqrt{1 + x^{2} + y^{2}}}}} \iint_{D_{xy}} \frac{(z^{2} + x)x + z}{\sqrt{1 + x^{2} + y^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dx dy$$

$$= \iint_{D_{xy}} (z^{2} + x)x + z dx dy \xrightarrow{\underline{\text{mint}}} \iint_{D_{xy}} x^{2} + z dx dy$$

$$\underline{z = \frac{1}{2}(x^{2} + y^{2})} \frac{1}{2} \iint_{D_{xy}} 3x^{2} + y^{2} dx dy \xrightarrow{\underline{\text{mint}}} 2 \iint_{D_{xy}} x^{2} dx dy$$

$$= \iint_{D_{xy}} (z^2 + x)x + z dx dy \xrightarrow{\underline{x} \text{ with }} \iint_{D_{xy}} x^2 + z dx dy$$

$$= \underbrace{\frac{z = \frac{1}{2}(x^2 + y^2)}{2}} \frac{1}{2} \iint_{D_{xy}} 3x^2 + y^2 dx dy \xrightarrow{\underline{x} \text{ with }} 2 \iint_{D_{xy}} x^2 dx dy$$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
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$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^2 + x, 0, -z)} \iint_{\Sigma_1} -z dS \iint_{\Sigma_1} -2 dS = -2|\Sigma_1| = -8\pi,$$

$$\iint_{\Sigma_{2}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} \iint_{D_{xy}} \frac{(z^{2} + x)x + z}{\sqrt{1 + x^{2} + y^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dxdy$$

$$= \iint_{D_{xy}} (z^{2} + x)x + zdxdy \xrightarrow{\underline{\text{Minth}}} \iint_{D_{xy}} x^{2} + zdxdy$$

$$\underline{z = \frac{1}{2}(x^{2} + y^{2})} \frac{1}{2} \iint_{D_{xy}} 3x^{2} + y^{2} dxdy \xrightarrow{\underline{\text{Minth}}} 2 \iint_{D_{xy}} x^{2} dxdy$$

$$\underline{\underline{\text{Minth}}} \iint_{D_{xy}} x^{2} + y^{2} dxdy$$



$$\begin{array}{c}
V \cdot \overrightarrow{n} dS \stackrel{V}{=} \\
V \cdot \overrightarrow{n} dS \stackrel{V}{=} \\
\end{array}$$

 $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^2 + x, 0, -z)} \iint_{\Sigma} -z dS \iint_{\Sigma} -2 dS = -2|\Sigma_1| = -8\pi,$

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma} V \cdot \overrightarrow{n} dS + \iint_{\Sigma} V \cdot \overrightarrow{n} dS$,

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} \, dS \xrightarrow{\overrightarrow{n} = (0, 0, 1)} \iint_{\Sigma_{1}} \sum_{z=0}^{z=1} \int_{\Sigma_{1}} \frac{z}{\sqrt{1 + x^{2} + y^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} \, dx \, dy$$

 $= \iint_{D} (z^{2} + x)x + zdxdy \xrightarrow{\underline{\text{phit}}} \iint_{D} x^{2} + zdxdy$ $\frac{z=\frac{1}{2}(x^2+y^2)}{2} \frac{1}{2} \iint_{\Omega} 3x^2 + y^2 dx dy \xrightarrow{\text{spate}} 2 \iint_{\Omega} x^2 dx dy$ $\frac{\sqrt{3}}{2} \int \partial u du du du = \int \int \int \int \int \partial u du du = \int \int \int \partial u du du = \int \int \partial u du du = \int \partial u du = \partial u du$



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 $= \iint_{D} (z^{2} + x)x + zdxdy \xrightarrow{\underline{\text{phit}}} \iint_{D} x^{2} + zdxdy$

$$\frac{z = \frac{1}{2}(x^2 + y^2)}{2} \frac{1}{2} \iint_{D_{xy}} 3x^2 + y^2 dx dy \xrightarrow{\text{spanse}} 2 \iint_{D_{xy}} x^2 dx dy$$

$$\frac{\text{spanse}}{2} \iint_{D_{xy}} x^2 + y^2 dx dy = \int_{0}^{2\pi} \left[\int_{0}^{2} \rho^2 \cdot \rho d\rho \right] d\theta = 8\pi$$

 $V \cdot n \, dS = \frac{1}{\vec{n} = \frac{(x, y, -1)}{\sqrt{1 + x^2 + y^2}}} \int_{D_{xy}} \frac{1}{\sqrt{1 + x^2 + y^2}} \cdot \sqrt{1 + z_x^2 + z_y^2} \, dx$ $= \int_{D_x} (z^2 + x)x + z dx dy = \frac{x^{\frac{1}{2}}}{\vec{n}} \int_{D_x} x^2 + z dx dy$

 $\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} \iint_{\Sigma_{1}} -z dS \iint_{\Sigma_{1}} -2 dS = -2|\Sigma_{1}| = -8\pi,$ $\iint_{\Sigma_{2}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} \iint_{D_{xy}} \frac{(z^{2} + x)x + z}{\sqrt{1 + x^{2} + y^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dx dy$

 $\frac{z=\frac{1}{2}(x^2+y^2)}{2} \frac{1}{2} \iint_{\Omega} 3x^2 + y^2 dx dy \xrightarrow{\text{spate}} 2 \iint_{\Omega} x^2 dx dy$

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma} V \cdot \overrightarrow{n} dS + \iint_{\Sigma} V \cdot \overrightarrow{n} dS$,

·. 原式 = $-8\pi + 8\pi = 0$



法向量。则

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{D_{xy}} R(x, y, z(x, y)) dx dy.$$

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$$\frac{V=(0,0,R)}{\overrightarrow{n} = \frac{1}{\sqrt{1+z_{y}^{2}+z_{y}^{2}}}(-z_{x},-z_{y},1)}$$

法向量。则

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{D_{xy}} R(x, y, z(x, y)) dx dy.$$

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$$\frac{V = (0, 0, R)}{\overrightarrow{n} = \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} (-z_{x}, -z_{y}, 1)} \qquad R(x, y, z) \cdot \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}}$$

$$\overrightarrow{n} = \frac{1}{\sqrt{1 + z_{0}^{2} + z_{0}^{2}}} (-z_{x}, -z_{y}, 1)$$

$$(2) \cdot \frac{1}{\sqrt{1+z_x^2+z_y^2}}$$

法向量。则

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{D_{XY}} R(x, y, z(x, y)) dx dy.$$

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$$\frac{V = (0, 0, R)}{\overrightarrow{n} = \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} (-z_{x}, -z_{y}, 1)} \iint_{\Sigma} R(x, y, z) \cdot \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} dS$$

法向量。则

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{D_{XY}} R(x, y, z(x, y)) dx dy.$$

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$$R(x, y, z(x, y)) \cdot \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}}$$



法向量。则

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{D_{x,y}} R(x, y, z(x, y)) dx dy.$$

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{\Sigma} V \cdot \overrightarrow{n} dS$$

$$\frac{V = (0, 0, R)}{\overrightarrow{n} = \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} (-z_{x}, -z_{y}, 1)} \iint_{\Sigma} R(x, y, z) \cdot \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} dS$$

$$R(x, y, z(x, y)) \cdot \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dx dy$$

法向量。则

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{\Omega} R(x, y, z(x, y)) dx dy.$$

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{\Sigma} V \cdot \overrightarrow{n} dS$$

$$\frac{V = (0, 0, R)}{\overrightarrow{n} = \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} (-z_{x}, -z_{y}, 1)} \iint_{\Sigma} R(x, y, z) \cdot \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} dS$$

$$= \iint_{D_{XY}} R(x, y, z(x, y)) \cdot \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dx dy$$



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$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{\Sigma} V \cdot \overrightarrow{n} dS$$

$$\frac{V = (0, 0, R)}{\overrightarrow{n} = \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} (-z_{x}, -z_{y}, 1)} \iint_{\Sigma} R(x, y, z) \cdot \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} dS$$

$$= \iint_{D_{xy}} R(x, y, z(x, y)) \cdot \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dx dy$$

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