第 4 章 *b*：换元积分法

数学系　　梁卓滨

2019-2020 学年 I

Outline

1. 第一类换元积分法：凑微分

2. 第二类换元积分法：变量代换

We are here now...

1. 第一类换元积分法：凑微分

2. 第二类换元积分法：变量代换

第一类换元积分法——“凑微分”法，能干啥?

能够计算如下的不定积分:

$$\int \frac{dx}{2x+1}, \quad \int \cos(\frac{5}{2}x)dx$$

$$\int \frac{x}{\sqrt{3-x^2}}dx, \quad \int x\sin(x^2)dx$$

$$\int \frac{(\ln x)^2}{x}dx, \quad \int e^{\sin x}\cos x dx$$

$$\int \frac{1}{\cos x}dx$$

$$\cdots\cdots$$

第一类换元积分法（凑微分）原理

计算步骤

$$\int g(x)dx$$

第一类换元积分法（凑微分）原理

计算步骤

$$\int g(x)dx \xlongequal{\quad\quad} \int f(\varphi(x))\varphi'(x)dx$$

第一类换元积分法（凑微分）原理

计算步骤

$$\int g(x)dx = = = \int f(\varphi(x))\varphi'(x)dx$$

第一类换元积分法（凑微分）原理

计算步骤

$$\int g(x)dx = \!=\!= \int f(\varphi(x))\varphi'(x)dx = \qquad\qquad d\varphi(x)$$

第一类换元积分法（凑微分）原理

计算步骤

$$\int g(x)dx \xlongequal{\quad\quad} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

第一类换元积分法（凑微分）原理

计算步骤

$$\int g(x)dx \xlongequal{\text{凑微分}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

第一类换元积分法（凑微分）原理

计算步骤

$$\int g(x)dx \xlongequal{\text{凑微分}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\xlongequal{\varphi(x)=u}$$

第一类换元积分法（凑微分）原理

计算步骤

$$\int g(x)dx \xlongequal{\text{凑微分}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\xlongequal{\varphi(x)=u} \int f(u)du$$

第一类换元积分法（凑微分）原理

计算步骤

$$\int g(x)dx \xlongequal{\text{凑微分}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\xlongequal{\varphi(x)=u} \int f(u)du = F(u) + C$$

第一类换元积分法（凑微分）原理

计算步骤

$$\int g(x)dx \xlongequal{\text{凑微分}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\xlongequal{\varphi(x)=u} \int f(u)du = F(u) + C \xlongequal{u=\varphi(x)} F(\varphi(x)) + C$$

第一类换元积分法（凑微分）原理

计算步骤

$$\int g(x)dx \xlongequal{\text{凑微分}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\xlongequal{\varphi(x)=u} \int f(u)du = F(u) + C \xlongequal{u=\varphi(x)} F(\varphi(x)) + C$$

验证： $F(\varphi(x))$ 确是 $g(x)$ 的原函数.

暨南大學 JINAN UNIVERSITY

第一类换元积分法（凑微分）原理

计算步骤

$$\int g(x)dx \xlongequal{\text{凑微分}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\xlongequal{\varphi(x)=u} \int f(u)du = F(u) + C \xlongequal{u=\varphi(x)} F(\varphi(x)) + C$$

验证： $F(\varphi(x))$ 确是 $g(x)$ 的原函数.

$$\frac{d}{dx}F(\varphi(x)) =$$

第一类换元积分法（凑微分）原理

计算步骤

$$\int g(x)dx \xlongequal{\text{凑微分}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\xlongequal{\varphi(x)=u} \int f(u)du = F(u) + C \xlongequal{u=\varphi(x)} F(\varphi(x)) + C$$

验证： $F(\varphi(x))$ 确是 $g(x)$ 的原函数.

$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) =$$

第一类换元积分法（凑微分）原理

计算步骤

$$\int g(x)dx \xlongequal{\text{凑微分}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\xlongequal{\varphi(x)=u} \int f(u)du = F(u) + C \xlongequal{u=\varphi(x)} F(\varphi(x)) + C$$

验证： $F(\varphi(x))$ 确是 $g(x)$ 的原函数.

$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \cdot \varphi'(x) =$$

暨南大学 JINAN UNIVERSITY

第一类换元积分法（凑微分）原理

计算步骤

$$\int g(x)dx \xlongequal{\text{凑微分}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\xlongequal{\varphi(x)=u} \int f(u)du = F(u) + C \xlongequal{u=\varphi(x)} F(\varphi(x)) + C$$

验证： $F(\varphi(x))$ 确是 $g(x)$ 的原函数.

$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \cdot \varphi'(x) = g(x)$$

第一类换元积分法（凑微分）原理

计算步骤

$$\int g(x)dx \xlongequal{\text{凑微分}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\xlongequal{\varphi(x)=u} \int f(u)du = F(u) + C \xlongequal{u=\varphi(x)} F(\varphi(x)) + C$$

验证： $F(\varphi(x))$ 确是 $g(x)$ 的原函数.

$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \cdot \varphi'(x) = g(x)$$

总之 $$\int g(x)dx \xlongequal{\text{凑微分}} \int f(\varphi(x))d\varphi(x)$$

$$= \int f(u)du = F(u) + C = F(\varphi(x)) + C$$

凑微分　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx$$

凑微分 类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx$$

凑微分　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx \qquad\qquad d(ax+b)$$

凑微分　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx \qquad \frac{1}{a}d(ax+b)$$

凑微分　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a} d(ax+b)$$

凑微分　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a} d(ax+b)$$

$$\overset{u=ax+b}{=\!=\!=\!=}$$

凑微分　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a} d(ax+b)$$

$$\xlongequal{u=ax+b} \int f(u) \cdot \frac{1}{a} du =$$

凑微分 类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx = \int f(ax+b)\cdot\frac{1}{a}d(ax+b)$$

$$\xlongequal{u=ax+b} \int f(u)\cdot\frac{1}{a}du = \quad F(u)$$

凑微分　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx = \int f(ax+b)\cdot\frac{1}{a}d(ax+b)$$

$$\xlongequal{u=ax+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)$$

凑微分　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx = \int f(ax+b)\cdot\frac{1}{a}d(ax+b)$$

$$\xlongequal{u=ax+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u) + C$$

凑微分　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx = \int f(ax+b)\cdot\frac{1}{a}d(ax+b)$$

$$\xlongequal{u=ax+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ax+b) + C$$

凑微分　类型 I: $\int f(ax+b)dx$

假设会算
$$\int f(u)du = F(u) + C$$
则
$$\int f(ax+b)dx = \int f(ax+b)\cdot\frac{1}{a}d(ax+b)$$
$$\xlongequal{u=ax+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ax+b) + C$$

例 1
$$\int \frac{1}{1+2x}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx = \int f(ax+b)\cdot\frac{1}{a}d(ax+b)$$

$$\xlongequal{u=ax+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ax+b) + C$$

例 1

$$\int \frac{1}{1+2x}dx = \qquad\qquad d(1+2x)$$

凑微分　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx = \int f(ax+b)\cdot\frac{1}{a}d(ax+b)$$

$$\xlongequal{u=ax+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ax+b) + C$$

例 1

$$\int\frac{1}{1+2x}dx = \qquad\qquad\qquad \cdot\frac{1}{2}d(1+2x)$$

凑微分　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx = \int f(ax+b)\cdot\frac{1}{a}d(ax+b)$$

$$\xlongequal{u=ax+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ax+b) + C$$

例 1

$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x}\cdot\frac{1}{2}d(1+2x)$$

凑微分　　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx = \int f(ax+b)\cdot\frac{1}{a}d(ax+b)$$

$$\overset{u=ax+b}{=\!=\!=\!=} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ax+b) + C$$

例 1

$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x}\cdot\frac{1}{2}d(1+2x) = \frac{1}{2}\int\frac{1}{u}du$$

凑微分　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx = \int f(ax+b)\cdot\frac{1}{a}d(ax+b)$$

$$\xlongequal{u=ax+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(ax+b)+C$$

例 1

$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x}\cdot\frac{1}{2}d(1+2x) = \frac{1}{2}\int\frac{1}{u}du$$

$$= \frac{1}{2}\ln|u| + C$$

凑微分　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx = \int f(ax+b)\cdot\frac{1}{a}d(ax+b)$$

$$\overset{u=ax+b}{=\!=\!=\!=} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ax+b) + C$$

例 1

$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x}\cdot\frac{1}{2}d(1+2x) = \frac{1}{2}\int\frac{1}{u}du$$

$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+2x| + C$$

凑微分　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

凑微分　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx =$$

$$\int \sqrt{3x-1}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx = \qquad\qquad d(2-3x)$$

$$\int \sqrt{3x-1}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx = \qquad\qquad \cdot(-\frac{1}{3})d(2-3x)$$

$$\int \sqrt{3x-1}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot \left(-\frac{1}{3}\right) d(2-3x)$$

$$\int \sqrt{3x-1}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$\int \sqrt{3x-1}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x}\cdot(-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C$$

$$\int \sqrt{3x-1}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx =$$

凑微分　　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x}\cdot(-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \qquad\qquad d(3x-1)$$

凑微分　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x}\cdot(-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \qquad\qquad \cdot\frac{1}{3}d(3x-1)$$

凑微分　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x}\cdot(-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1}\cdot\frac{1}{3}d(3x-1)$$

凑微分　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x}\cdot(-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1}\cdot\frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du$$

凑微分　　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x}\cdot(-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1}\cdot\frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$

凑微分　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x}\cdot(-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1}\cdot\frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$

$$u^{3/2}$$

凑微分　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x}\cdot(-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1}\cdot\frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$

$$\frac{2}{3}u^{3/2}$$

凑微分　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x}\cdot(-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1}\cdot\frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$

$$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C$$

凑微分　　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x}\cdot\left(-\frac{1}{3}\right)d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1}\cdot\frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$

$$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C = \frac{2}{9}(3x-1)^{3/2} + C$$

凑微分　　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x}\cdot(-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1}\cdot\frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$

$$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C = \frac{2}{9}(3x-1)^{3/2} + C$$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

凑微分　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx =$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \qquad d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \qquad\qquad \cdot(-\frac{1}{5})d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= \qquad u^{1/2}$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= \qquad 2u^{1/2}$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2}+C$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \qquad d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \qquad\qquad \cdot \frac{2}{3}d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u\,du$$

$$= \frac{2}{3}\sin(u) + C$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u\,du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2}+C = -\frac{2}{5}(1-5x)^{1/2}+C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u)+C = \frac{2}{3}\sin(\frac{3}{2}x)+C$$

$$\int e^{-\frac{1}{2}x+4}dx = \qquad d(-\frac{1}{2}x+4)$$

凑微分　　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2}+C = -\frac{2}{5}(1-5x)^{1/2}+C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u)+C = \frac{2}{3}\sin(\frac{3}{2}x)+C$$

$$\int e^{-\frac{1}{2}x+4}dx = \qquad\qquad \cdot(-2)d(-\frac{1}{2}x+4)$$

凑微分　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2}+C = -\frac{2}{5}(1-5x)^{1/2}+C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u)+C = \frac{2}{3}\sin(\frac{3}{2}x)+C$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4}\cdot(-2)d(-\frac{1}{2}x+4)$$

凑微分　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4}\cdot(-2)d(-\frac{1}{2}x+4) = -2\int e^u du$$

凑微分　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4}\cdot(-2)d(-\frac{1}{2}x+4) = -2\int e^u du$$

$$= -2e^u + C$$

凑微分　　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5}\cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x\cdot\frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4}\cdot(-2)d(-\frac{1}{2}x+4) = -2\int e^u du$$

$$= -2e^u + C = -2e^{-\frac{1}{2}x+4} + C$$

凑微分　　类型 II：$\int f(ax^2+b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)xdx$$

凑微分　　类型 II: $\int f(ax^2+b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)xdx$$

凑微分　类型 II: $\int f(ax^2+b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)xdx \qquad\qquad d(ax^2+b)$$

凑微分　　类型 II: $\int f(ax^2+b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)xdx \qquad \frac{1}{2a}d(ax^2+b)$$

凑微分 类型 II: $\int f(ax^2+b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)xdx = \int f(ax^2+b)\cdot\frac{1}{2a}d(ax^2+b)$$

凑微分　　类型 II: $\int f(ax^2+b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)xdx = \int f(ax^2+b)\cdot\frac{1}{2a}d(ax^2+b)$$

$$\xlongequal{u=ax^2+b}$$

凑微分　　类型 II: $\int f(ax^2+b)x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)xdx = \int f(ax^2+b)\cdot\frac{1}{2a}d(ax^2+b)$$

$$\xlongequal{u=ax^2+b} \int f(u)\cdot\frac{1}{2a}du =$$

凑微分　　类型 II: $\int f(ax^2+b)x\,dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)x\,dx = \int f(ax^2+b)\cdot\frac{1}{2a}d(ax^2+b)$$

$$\xlongequal{u=ax^2+b} \int f(u)\cdot\frac{1}{2a}du = \qquad F(u)$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)x\,dx = \int f(ax^2+b)\cdot \frac{1}{2a} d(ax^2+b)$$

$$\xlongequal{u=ax^2+b} \int f(u)\cdot\frac{1}{2a}du = \frac{1}{2a}F(u)$$

凑微分　　类型 II: $\int f(ax^2+b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)xdx = \int f(ax^2+b)\cdot\frac{1}{2a}d(ax^2+b)$$

$$\xlongequal{u=ax^2+b} \int f(u)\cdot\frac{1}{2a}du = \frac{1}{2a}F(u) + C$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)x\,dx = \int f(ax^2+b)\cdot\frac{1}{2a}d(ax^2+b)$$

$$\xlongequal{u=ax^2+b} \int f(u)\cdot\frac{1}{2a}du = \frac{1}{2a}F(u)+C = \frac{1}{2a}F(ax^2+b)+C$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)x\,dx = \int f(ax^2+b)\cdot\frac{1}{2a}d(ax^2+b)$$

$$\xlongequal{u=ax^2+b} \int f(u)\cdot\frac{1}{2a}du = \frac{1}{2a}F(u)+C = \frac{1}{2a}F(ax^2+b)+C$$

例 1

$$\int x\sqrt{1-x^2}dx =$$

凑微分　　类型 II: $\int f(ax^2+b)x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)xdx = \int f(ax^2+b)\cdot\frac{1}{2a}d(ax^2+b)$$

$$\xlongequal{u=ax^2+b} \int f(u)\cdot\frac{1}{2a}du = \frac{1}{2a}F(u)+C = \frac{1}{2a}F(ax^2+b)+C$$

例 1

$$\int x\sqrt{1-x^2}dx = \qquad\qquad d(1-x^2)$$

凑微分　类型 II: $\int f(ax^2+b)xdx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(ax^2+b)xdx = \int f(ax^2+b)\cdot\frac{1}{2a}d(ax^2+b)$$

$$\xlongequal{u=ax^2+b}\int f(u)\cdot\frac{1}{2a}du = \frac{1}{2a}F(u)+C = \frac{1}{2a}F(ax^2+b)+C$$

例 1

$$\int x\sqrt{1-x^2}dx = \qquad\qquad \cdot(-\frac{1}{2})d(1-x^2)$$

凑微分　　类型 II: $\int f(ax^2+b)x\,dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)x\,dx = \int f(ax^2+b)\cdot\frac{1}{2a}d(ax^2+b)$$

$$\xlongequal[]{u=ax^2+b} \int f(u)\cdot\frac{1}{2a}du = \frac{1}{2a}F(u)+C = \frac{1}{2a}F(ax^2+b)+C$$

例 1

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2)$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)x\,dx = \int f(ax^2+b)\cdot\frac{1}{2a}d(ax^2+b)$$

$$\xlongequal{u=ax^2+b} \int f(u)\cdot\frac{1}{2a}du = \frac{1}{2a}F(u) + C = \frac{1}{2a}F(ax^2+b) + C$$

例 1 $$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}\cdot\left(-\frac{1}{2}\right)d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

凑微分　类型 II: $\int f(ax^2+b)x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)xdx = \int f(ax^2+b)\cdot \frac{1}{2a}d(ax^2+b)$$

$$\xlongequal{u=ax^2+b} \int f(u)\cdot\frac{1}{2a}du = \frac{1}{2a}F(u)+C = \frac{1}{2a}F(ax^2+b)+C$$

例 1

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$u^{3/2}$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(ax^2+b)x\,dx = \int f(ax^2+b)\cdot\frac{1}{2a}d(ax^2+b)$$

$$\overset{u=ax^2+b}{=\!=\!=} \int f(u)\cdot\frac{1}{2a}du = \frac{1}{2a}F(u)+C = \frac{1}{2a}F(ax^2+b)+C$$

例 1

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$\frac{2}{3}u^{3/2}$$

暨南大學 JINAN UNIVERSITY

凑微分　类型 II: $\int f(ax^2+b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)xdx = \int f(ax^2+b)\cdot\frac{1}{2a}d(ax^2+b)$$

$$\overset{u=ax^2+b}{=\!=\!=} \int f(u)\cdot\frac{1}{2a}du = \frac{1}{2a}F(u)+C = \frac{1}{2a}F(ax^2+b)+C$$

例 1

$$\int x\sqrt{1-x^2}dx = \int(1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C$$

暨南大學 JINAN UNIVERSITY

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)x\,dx = \int f(ax^2+b)\cdot\frac{1}{2a}d(ax^2+b)$$

$$\overset{u=ax^2+b}{=\!=\!=} \int f(u)\cdot\frac{1}{2a}du = \frac{1}{2a}F(u) + C = \frac{1}{2a}F(ax^2+b) + C$$

例 1

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

凑微分　　类型 II：$\int f(ax^2+b)x dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

凑微分　　类型 II: $\int f(ax^2+b)x\,dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx =$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分　　类型 II: $\int f(ax^2+b)x\,dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \qquad\qquad\qquad d(3-x^2)$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分　　类型 II: $\int f(ax^2+b)x\,dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \qquad\qquad \cdot\left(-\frac{1}{2}\right)d(3-x^2)$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分　　类型 II: $\int f(ax^2+b)x\,dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分　　类型 II: $\int f(ax^2+b)x\,dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du \qquad\qquad 2u^{1/2}$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分　　类型 II: $\int f(ax^2+b)x\,dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2} \cdot 2u^{1/2} + C$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx =$$

暨南大學 JINAN UNIVERSITY

凑微分　　类型 II: $\int f(ax^2+b)x\,dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx = \qquad\qquad d(1+3x^2)$$

凑微分　类型 II: $\int f(ax^2+b)x dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx = \qquad \cdot \frac{1}{6} d(1+3x^2)$$

凑微分　　类型 II: $\int f(ax^2+b)xdx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6}d(1+3x^2)$$

暨南大学 JINAN UNIVERSITY

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2}\cdot\frac{1}{6}d(1+3x^2) = \frac{1}{6}\int\frac{1}{u}du$$

暨南大學 JINAN UNIVERSITY

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2}\cdot\frac{1}{6}d(1+3x^2) = \frac{1}{6}\int\frac{1}{u}du$$

$$= \frac{1}{6}\ln|u| + C$$

凑微分　　类型 II: $\int f(ax^2+b)x\,dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6}d(1+3x^2) = \frac{1}{6}\int \frac{1}{u}du$$

$$= \frac{1}{6}\ln|u| + C = \frac{1}{6}\ln|1+3x^2| + C$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2}\cdot\frac{1}{6}d(1+3x^2) = \frac{1}{6}\int \frac{1}{u}du$$

$$= \frac{1}{6}\ln|u| + C = \frac{1}{6}\ln|1+3x^2| + C$$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx =$$

凑微分　　类型 II: $\int f(ax^2+b)xdx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \qquad\qquad d(x^2)$$

凑微分 类型 II: $\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \qquad \frac{1}{2}d(x^2)$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2)$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C$$

凑微分　　类型 II: $\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx =$$

凑微分　　类型 II：$\int f(ax^2+b)x dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \qquad\qquad d(x^2)$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \qquad\qquad\qquad \cdot\frac{1}{2}d(x^2)$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2)\cdot\frac{1}{2}d(x^2)$$

凑微分　类型 II：$\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2)\cdot\frac{1}{2}d(x^2) = \frac{1}{2}\int \sin u\,du$$

暨南大學 JINAN UNIVERSITY

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2)\cdot\frac{1}{2}d(x^2) = \frac{1}{2}\int \sin u\,du$$

$$= -\frac{1}{2}\cos u + C$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2)\cdot\frac{1}{2}d(x^2) = \frac{1}{2}\int \sin u\,du$$

$$= -\frac{1}{2}\cos u + C = -\frac{1}{2}\cos(x^2) + C$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx \qquad\qquad d(ae^x+b)$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx \qquad \frac{1}{a}d(ae^x+b)$$

凑微分　　类型 III：$\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x + b)e^x dx = \int f(ae^x + b) \cdot \frac{1}{a} d(ae^x + b)$$

凑微分　　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx = \int f(ae^x+b)\cdot\frac{1}{a}d(ae^x+b)$$

$$\xlongequal{u=ae^x+b}$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx = \int f(ae^x+b)\cdot\frac{1}{a}d(ae^x+b)$$

$$\xlongequal{u=ae^x+b} \int f(u)\cdot\frac{1}{a}du =$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx = \int f(ae^x+b)\cdot\frac{1}{a}d(ae^x+b)$$

$$\xlongequal{u=ae^x+b} \int f(u)\cdot\frac{1}{a}du = \quad F(u)$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx = \int f(ae^x+b)\cdot \frac{1}{a} d(ae^x+b)$$

$$\xlongequal{u=ae^x+b} \int f(u)\cdot \frac{1}{a} du = \frac{1}{a}F(u)$$

凑微分　　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(ae^x+b)e^x dx = \int f(ae^x+b)\cdot\frac{1}{a}d(ae^x+b)$$

$$\overset{u=ae^x+b}{=\!=\!=\!=} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx = \int f(ae^x+b)\cdot\frac{1}{a}d(ae^x+b)$$

$$\xlongequal{u=ae^x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(ae^x+b)+C$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx = \int f(ae^x+b)\cdot\frac{1}{a}d(ae^x+b)$$

$$\xlongequal{u=ae^x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(ae^x+b)+C$$

例 1

$$\int \frac{e^x}{1+e^x}dx =$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx = \int f(ae^x+b)\cdot\frac{1}{a}d(ae^x+b)$$

$$\xlongequal[\quad]{u=ae^x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(ae^x+b)+C$$

例 1

$$\int \frac{e^x}{1+e^x}dx = \qquad\qquad d(e^x+1)$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx = \int f(ae^x+b)\cdot \frac{1}{a} d(ae^x+b)$$

$$\xlongequal{u=ae^x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(ae^x+b)+C$$

例 1

$$\int \frac{e^x}{1+e^x}dx = \int \frac{1}{1+e^x} d(e^x+1)$$

凑微分 类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx = \int f(ae^x+b)\cdot\frac{1}{a}d(ae^x+b)$$

$$\overset{u=ae^x+b}{=\!=\!=\!=} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ae^x+b) + C$$

例 1

$$\int \frac{e^x}{1+e^x}dx = \int \frac{1}{1+e^x}d(e^x+1)$$

$$= \int \frac{1}{u}du$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx = \int f(ae^x+b)\cdot\frac{1}{a}d(ae^x+b)$$

$$\overset{u=ae^x+b}{=\!=\!=\!=} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(ae^x+b)+C$$

例 1

$$\int \frac{e^x}{1+e^x}dx = \int \frac{1}{1+e^x}d(e^x+1)$$

$$= \int \frac{1}{u}du = \ln|u| + C$$

凑微分　　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx = \int f(ae^x+b)\cdot\frac{1}{a}d(ae^x+b)$$

$$\xlongequal{u=ae^x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ae^x+b) + C$$

例 1

$$\int \frac{e^x}{1+e^x}dx = \int \frac{1}{1+e^x}d(e^x+1)$$

$$= \int \frac{1}{u}du = \ln|u| + C = \ln(e^x+1) + C$$

凑微分　类型 III: $\int f(ae^x + b)e^x dx$

例 2

$$\int e^x \sin(e^x) dx =$$

凑微分　　类型 III: $\int f(ae^x + b)e^x dx$

例 2

$$\int e^x \sin(e^x) dx = \qquad\qquad de^x$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

例 2

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$

凑微分 类型 III: $\int f(ae^x+b)e^x dx$

例 2

$$\int e^x \sin(e^x) dx = \int \sin(e^x) de^x$$
$$= \int \sin u du$$

凑微分　　类型 III: $\int f(ae^x+b)e^x dx$

例 2

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$
$$= \int \sin u du = -\cos u + C$$

凑微分　　类型 III: $\int f(ae^x+b)e^x dx$

例 2

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$
$$= \int \sin u du = -\cos u + C = -\cos(e^x) + C$$

凑微分　类型 IV: $\int f(a\ln x + b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(a\ln x + b)\frac{1}{x}dx$$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx$$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx \qquad\qquad d(a\ln x+b)$$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx \qquad\qquad \frac{1}{a}d(a\ln x+b)$$

凑微分　　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\underline{\underline{u=a\ln x+b}}$$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\xlongequal{u=a\ln x+b} \int f(u)\cdot\frac{1}{a}du =$$

凑微分　　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\xlongequal{u=a\ln x+b} \int f(u)\cdot\frac{1}{a}du = \quad F(u)$$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\xlongequal{u=a\ln x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)$$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\xlongequal{u=a\ln x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C$$

凑微分　　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\xlongequal{u=a\ln x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(a\ln x+b)+C$$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\overset{u=a\ln x+b}{=\!=\!=\!=} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(a\ln x+b)+C$$

例 $$\int \frac{1}{x}\ln x dx =$$

$$\int \frac{1}{x\ln x}dx =$$

凑微分　　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\xlongequal{u=a\ln x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(a\ln x+b)+C$$

例 $\displaystyle\int \frac{1}{x}\ln x dx = \qquad\qquad d\ln x$

$$\int \frac{1}{x\ln x}dx =$$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\xlongequal{u=a\ln x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(a\ln x+b)+C$$

例

$$\int \frac{1}{x}\ln x dx = \int \ln x d\ln x$$

$$\int \frac{1}{x\ln x}dx =$$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\xlongequal{u=a\ln x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(a\ln x+b)+C$$

例 $$\int \frac{1}{x}\ln x dx = \int \ln x d\ln x = \int u du$$

$$\int \frac{1}{x\ln x}dx =$$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\overset{u=a\ln x+b}{=\!=\!=\!=} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(a\ln x+b)+C$$

例 $\int \frac{1}{x}\ln x dx = \int \ln x d\ln x = \int u du = \frac{1}{2}u^2+C$

$\int \frac{1}{x\ln x}dx =$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\xlongequal{u=a\ln x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(a\ln x+b)+C$$

例 $$\int \frac{1}{x}\ln x dx = \int \ln x d\ln x = \int u du = \frac{1}{2}u^2+C = \frac{1}{2}(\ln x)^2+C$$

$$\int \frac{1}{x\ln x}dx =$$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\xlongequal{u=a\ln x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(a\ln x+b)+C$$

例 $\int \frac{1}{x}\ln x dx = \int \ln x d\ln x = \int u du = \frac{1}{2}u^2+C = \frac{1}{2}(\ln x)^2+C$

$\int \frac{1}{x\ln x}dx =$ $\qquad d\ln x$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算
$$\int f(u)du = F(u)+C$$
则
$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$
$$\xlongequal[]{u=a\ln x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(a\ln x+b)+C$$

例
$$\int \frac{1}{x}\ln x dx = \int \ln x d\ln x = \int u du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C$$
$$\int \frac{1}{x\ln x}dx = \int \frac{1}{\ln x}d\ln x$$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\xlongequal{u=a\ln x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(a\ln x+b)+C$$

例

$$\int\frac{1}{x}\ln x dx = \int \ln x d\ln x = \int u du = \frac{1}{2}u^2+C = \frac{1}{2}(\ln x)^2+C$$

$$\int\frac{1}{x\ln x}dx = \int\frac{1}{\ln x}d\ln x = \int\frac{1}{u}du$$

暨南大學 JINAN UNIVERSITY

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\xlongequal{u=a\ln x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(a\ln x+b)+C$$

例

$$\int \frac{1}{x}\ln x dx = \int \ln x d\ln x = \int u du = \frac{1}{2}u^2+C = \frac{1}{2}(\ln x)^2+C$$

$$\int \frac{1}{x\ln x}dx = \int \frac{1}{\ln x}d\ln x = \int \frac{1}{u}du = \ln|u|+C$$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\xlongequal{u=a\ln x+b} \int f(u)\cdot\frac{1}{a}du = \frac{1}{a}F(u)+C = \frac{1}{a}F(a\ln x+b)+C$$

例

$$\int \frac{1}{x}\ln x dx = \int \ln x d\ln x = \int u du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C$$

$$\int \frac{1}{x\ln x}dx = \int \frac{1}{\ln x}d\ln x = \int \frac{1}{u}du = \ln|u| + C = \ln|\ln x| + C$$

凑微分类型 IV: $\int f(\cos x)\sin x dx$, $\int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

凑微分类型 IV: $\int f(\cos x)\sin x dx, \int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx =$$

凑微分类型 IV: $\int f(\cos x)\sin x dx$, $\int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \qquad\qquad\qquad\qquad d\cos x$$

凑微分类型 IV: $\int f(\cos x)\sin x dx, \int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \qquad\qquad (-1)d\cos x$$

凑微分类型 IV: $\int f(\cos x)\sin x dx, \int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1)d\cos x$$

凑微分类型 IV: $\int f(\cos x)\sin x dx, \int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1)d\cos x = -\int e^u du$$

凑微分类型 IV: $\int f(\cos x)\sin x dx$, $\int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1)d\cos x = -\int e^u du$$
$$= -e^u + C$$

凑微分类型 IV: $\int f(\cos x)\sin x dx$, $\int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1)d\cos x = -\int e^u du$$
$$= -e^u + C = -e^{\cos x} + C$$

凑微分类型 IV: $\int f(\cos x)\sin x dx, \int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1)d\cos x = -\int e^u du$$
$$= -e^u + C = -e^{\cos x} + C$$

2. $$\int \frac{\sin x}{1+\cos^2 x}dx =$$

凑微分类型 IV: $\int f(\cos x)\sin x dx$, $\int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1)d\cos x = -\int e^u du$$
$$= -e^u + C = -e^{\cos x} + C$$

2. $$\int \frac{\sin x}{1+\cos^2 x}dx = \qquad (-1)d\cos x$$

凑微分类型 IV: $\int f(\cos x)\sin x dx$, $\int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1)d\cos x = -\int e^{u}du$$
$$= -e^{u} + C = -e^{\cos x} + C$$

2. $$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x$$

凑微分类型 IV: $\int f(\cos x)\sin x dx$, $\int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1)d\cos x = -\int e^u du$$
$$= -e^u + C = -e^{\cos x} + C$$

2. $$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$

凑微分类型 IV: $\int f(\cos x)\sin x dx$, $\int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1.
$$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1)d\cos x = -\int e^u du$$
$$= -e^u + C = -e^{\cos x} + C$$

2.
$$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$
$$= -\arctan u + C$$

凑微分类型 IV: $\int f(\cos x)\sin x dx, \int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1)d\cos x = -\int e^u du$$
$$= -e^u + C = -e^{\cos x} + C$$

2. $$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$
$$= -\arctan u + C = -\arctan(\cos x) + C$$

凑微分类型 IV: $\int f(\cos x)\sin x dx,\ \int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1)d\cos x = -\int e^u du$$
$$= -e^u + C = -e^{\cos x} + C$$

2. $$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$
$$= -\arctan u + C = -\arctan(\cos x) + C$$

3. $$\int \frac{\cos x}{\sin x}dx =$$

凑微分类型 IV: $\int f(\cos x)\sin x dx$, $\int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1)d\cos x = -\int e^u du$$
$$= -e^u + C = -e^{\cos x} + C$$

2. $$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$
$$= -\arctan u + C = -\arctan(\cos x) + C$$

3. $$\int \frac{\cos x}{\sin x}dx = \qquad d\sin x$$

凑微分类型 IV: $\int f(\cos x)\sin x dx$, $\int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$， $\int \frac{\sin x}{1+\cos^2 x}dx$， $\int \frac{\cos x}{\sin x}dx$

解

1.
$$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1)d\cos x = -\int e^u du$$
$$= -e^u + C = -e^{\cos x} + C$$

2.
$$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$
$$= -\arctan u + C = -\arctan(\cos x) + C$$

3.
$$\int \frac{\cos x}{\sin x}dx = \int \frac{1}{\sin x}d\sin x$$

凑微分类型 IV: $\int f(\cos x)\sin x dx$, $\int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1)d\cos x = -\int e^u du$$
$$= -e^u + C = -e^{\cos x} + C$$

2. $$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$
$$= -\arctan u + C = -\arctan(\cos x) + C$$

3. $$\int \frac{\cos x}{\sin x}dx = \int \frac{1}{\sin x}d\sin x = \int \frac{1}{u}du$$

凑微分类型 IV: $\int f(\cos x)\sin x dx$, $\int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1)d\cos x = -\int e^u du$$
$$= -e^u + C = -e^{\cos x} + C$$

2. $$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$
$$= -\arctan u + C = -\arctan(\cos x) + C$$

3. $$\int \frac{\cos x}{\sin x}dx = \int \frac{1}{\sin x}d\sin x = \int \frac{1}{u}du = \ln|u| + C$$

凑微分类型 IV: $\int f(\cos x)\sin x dx,\ \int f(\sin x)\cos x dx$

例 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1)d\cos x = -\int e^u du$$
$$= -e^u + C = -e^{\cos x} + C$$

2. $$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$
$$= -\arctan u + C = -\arctan(\cos x) + C$$

3. $$\int \frac{\cos x}{\sin x}dx = \int \frac{1}{\sin x}d\sin x = \int \frac{1}{u}du = \ln|u| + C$$
$$= \ln|\sin x| + C$$

凑微分法 "$\int f(\varphi(x))d\varphi(x)$"：例子总结

$$\int \frac{1}{1-3x}dx =$$

$$\int \sqrt{3x-1}dx =$$

$$\int xe^{x^2}dx =$$

$$\int x\sqrt{1-x^2}dx =$$

$$\int \frac{\ln x}{x}dx =$$

$$\int e^{\cos x}\sin x dx =$$

凑微分法 "$\int f(\varphi(x))d\varphi(x)$"：例子总结

$$\int \frac{1}{1-3x}dx = -\frac{1}{3}\int \frac{1}{1-3x}d(1-3x) = -\frac{1}{3}\int \frac{1}{u}du = \cdots$$

$$\int \sqrt{3x-1}dx = \frac{1}{3}\int \sqrt{3x-1}d(3x-1) = \frac{1}{3}\int u^{1/2}du = \cdots$$

$$\int xe^{x^2}dx =$$

$$\int x\sqrt{1-x^2}dx =$$

$$\int \frac{\ln x}{x}dx =$$

$$\int e^{\cos x}\sin x dx =$$

凑微分法 “$\int f(\varphi(x))d\varphi(x)$”：例子总结

$$\int \frac{1}{1-3x}dx = -\frac{1}{3}\int \frac{1}{1-3x}d(1-3x) = -\frac{1}{3}\int \frac{1}{u}du = \cdots$$

$$\int \sqrt{3x-1}dx = \frac{1}{3}\int \sqrt{3x-1}d(3x-1) = \frac{1}{3}\int u^{1/2}du = \cdots$$

$$\int xe^{x^2}dx = \frac{1}{2}\int e^{x^2}dx^2 = \frac{1}{2}\int e^{u}du = \cdots$$

$$\int x\sqrt{1-x^2}dx = -\frac{1}{2}\int \sqrt{1-x^2}d(1-x^2) = -\frac{1}{2}\int u^{1/2}du = \cdots$$

$$\int \frac{\ln x}{x}dx =$$

$$\int e^{\cos x}\sin x dx =$$

凑微分法 “$\int f(\varphi(x))d\varphi(x)$”：例子总结

$$\int \frac{1}{1-3x}dx = -\frac{1}{3}\int \frac{1}{1-3x}d(1-3x) = -\frac{1}{3}\int \frac{1}{u}du = \cdots$$

$$\int \sqrt{3x-1}dx = \frac{1}{3}\int \sqrt{3x-1}d(3x-1) = \frac{1}{3}\int u^{1/2}du = \cdots$$

$$\int xe^{x^2}dx = \frac{1}{2}\int e^{x^2}dx^2 = \frac{1}{2}\int e^u du = \cdots$$

$$\int x\sqrt{1-x^2}dx = -\frac{1}{2}\int \sqrt{1-x^2}d(1-x^2) = -\frac{1}{2}\int u^{1/2}du = \cdots$$

$$\int \frac{\ln x}{x}dx = \int \ln x d(\ln x) = \int u du = \cdots$$

$$\int e^{\cos x}\sin x dx = -\int e^{\cos x}d\cos x = -\int e^u du = \cdots$$

We are here now...

1. 第一类换元积分法：凑微分

2. 第二类换元积分法：变量代换

第二类换元积分法——“变量代换”法，能干啥?

能够计算如下的不定积分:

$$\int x\sqrt{3x-1}dx,\quad \int \frac{x}{\sqrt{x-2}}dx$$

$$\int \frac{1}{1+\sqrt{x}}dx,\quad \int \frac{1}{1+\sqrt[3]{x+1}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

……

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)}$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))d\varphi(t)$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int f(\varphi(t))\varphi'(t)dt$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单，容易求！}} dt$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单，容易求！}} dt$$

$$= G(t) + C$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单，容易求！}} dt$$

$$= G(t) + C \xlongequal{t=\varphi^{-1}(x)}$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单，容易求！}} dt$$

$$= G(t) + C \xlongequal{t=\varphi^{-1}(x)} G\left(\varphi^{-1}(x)\right) + C$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单，容易求！}} dt$$

$$= G(t) + C \xlongequal{t=\varphi^{-1}(x)} G\left(\varphi^{-1}(x)\right) + C$$

- 关键是：如何选取函数 $x = \varphi(t)$?

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单，容易求！}}\,dt$$

$$= G(t) + C \xlongequal{t=\varphi^{-1}(x)} G\left(\varphi^{-1}(x)\right) + C$$

- 关键是：如何选取函数 $x = \varphi(t)$?
在后面的例子中，选取函数 $x = \varphi(t)$ 的方法：

$$\boxed{\begin{array}{c}\text{把被积函数 } f(x) \text{ 中复杂的部分整个设为 } t, \\ \text{从而得到 } x \text{ 与 } t \text{ 的函数关系.}\end{array}}$$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$,

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$,

$$\therefore \int x\sqrt{3x-1}dx=$$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\dfrac{1}{3}(t^2+1)$,

$$\therefore \int x\sqrt{3x-1}dx=$$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$， $\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\dfrac{1}{3}(t^2+1)$, $\quad dx=\dfrac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx=$$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\dfrac{1}{3}(t^2+1)$, $\quad dx=\dfrac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx=\int \frac{1}{3}(t^2+1)$$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\dfrac{1}{3}(t^2+1)$, $\quad dx=\dfrac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx=\int \frac{1}{3}(t^2+1)t\cdot$$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore\ x=\dfrac{1}{3}(t^2+1)$, $\quad dx=\dfrac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx=\int \frac{1}{3}(t^2+1)t\cdot\frac{2}{3}tdt$$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\frac{1}{3}(t^2+1), \quad dx=\frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx=\int \frac{1}{3}(t^2+1)t\cdot\frac{2}{3}tdt=\frac{2}{9}\int t^4+t^2dt$$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\frac{1}{3}(t^2+1)$, $\quad dx=\frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx=\int \frac{1}{3}(t^2+1)t\cdot\frac{2}{3}tdt=\frac{2}{9}\int t^4+t^2dt$$

$$=\frac{2}{45}t^5+\frac{2}{27}t^3+C$$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\dfrac{1}{3}(t^2+1)$, $\quad dx=\dfrac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx=\int \frac{1}{3}(t^2+1)t\cdot\frac{2}{3}tdt=\frac{2}{9}\int t^4+t^2dt$$

$$=\frac{2}{45}t^5+\frac{2}{27}t^3+C=\frac{2}{45}(3x-1)^{\frac{5}{2}}+\frac{2}{27}(3x-1)^{\frac{3}{2}}+C$$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore\ x=\dfrac{1}{3}(t^2+1)$, $\quad dx=\dfrac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx=\int \frac{1}{3}(t^2+1)t\cdot\frac{2}{3}tdt=\frac{2}{9}\int t^4+t^2dt$$

$$=\frac{2}{45}t^5+\frac{2}{27}t^3+C=\frac{2}{45}(3x-1)^{\frac{5}{2}}+\frac{2}{27}(3x-1)^{\frac{3}{2}}+C$$

(2) 设 $t=(x-2)^{\frac{1}{2}}$,

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\frac{1}{3}(t^2+1),\quad dx=\frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx=\int \frac{1}{3}(t^2+1)t\cdot\frac{2}{3}tdt=\frac{2}{9}\int t^4+t^2dt$$

$$=\frac{2}{45}t^5+\frac{2}{27}t^3+C=\frac{2}{45}(3x-1)^{\frac{5}{2}}+\frac{2}{27}(3x-1)^{\frac{3}{2}}+C$$

(2) 设 $t=(x-2)^{\frac{1}{2}}$,

$$\therefore \int \frac{x}{\sqrt{x-2}}dx=$$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\frac{1}{3}(t^2+1)$, $\quad dx=\frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx=\int \frac{1}{3}(t^2+1)t\cdot\frac{2}{3}tdt=\frac{2}{9}\int t^4+t^2dt$$

$$=\frac{2}{45}t^5+\frac{2}{27}t^3+C=\frac{2}{45}(3x-1)^{\frac{5}{2}}+\frac{2}{27}(3x-1)^{\frac{3}{2}}+C$$

(2) 设 $t=(x-2)^{\frac{1}{2}}$, $\quad \therefore x=t^2+2$,

$$\therefore \int \frac{x}{\sqrt{x-2}}dx=$$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\frac{1}{3}(t^2+1), \quad dx=\frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx=\int \frac{1}{3}(t^2+1)t\cdot\frac{2}{3}tdt=\frac{2}{9}\int t^4+t^2dt$$

$$=\frac{2}{45}t^5+\frac{2}{27}t^3+C=\frac{2}{45}(3x-1)^{\frac{5}{2}}+\frac{2}{27}(3x-1)^{\frac{3}{2}}+C$$

(2) 设 $t=(x-2)^{\frac{1}{2}}$, $\quad \therefore x=t^2+2, \quad dx=2tdt$

$$\therefore \int \frac{x}{\sqrt{x-2}}dx=$$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\frac{1}{3}(t^2+1), \quad dx=\frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx=\int \frac{1}{3}(t^2+1)t\cdot\frac{2}{3}tdt=\frac{2}{9}\int t^4+t^2dt$$

$$=\frac{2}{45}t^5+\frac{2}{27}t^3+C=\frac{2}{45}(3x-1)^{\frac{5}{2}}+\frac{2}{27}(3x-1)^{\frac{3}{2}}+C$$

(2) 设 $t=(x-2)^{\frac{1}{2}}$, $\quad \therefore x=t^2+2, \quad dx=2tdt$

$$\therefore \int \frac{x}{\sqrt{x-2}}dx=\int \frac{t^2+2}{t}\cdot 2tdt$$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\dfrac{1}{3}(t^2+1), \quad dx=\dfrac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx=\int \frac{1}{3}(t^2+1)t\cdot\frac{2}{3}tdt=\frac{2}{9}\int t^4+t^2dt$$

$$=\frac{2}{45}t^5+\frac{2}{27}t^3+C=\frac{2}{45}(3x-1)^{\frac{5}{2}}+\frac{2}{27}(3x-1)^{\frac{3}{2}}+C$$

(2) 设 $t=(x-2)^{\frac{1}{2}}$, $\quad \therefore x=t^2+2, \quad dx=2tdt$

$$\therefore \int \frac{x}{\sqrt{x-2}}dx=\int \frac{t^2+2}{t}\cdot 2tdt=2\int t^2+2dt$$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore\ x=\frac{1}{3}(t^2+1), \quad dx=\frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx=\int \frac{1}{3}(t^2+1)t\cdot\frac{2}{3}tdt=\frac{2}{9}\int t^4+t^2dt$$

$$=\frac{2}{45}t^5+\frac{2}{27}t^3+C=\frac{2}{45}(3x-1)^{\frac{5}{2}}+\frac{2}{27}(3x-1)^{\frac{3}{2}}+C$$

(2) 设 $t=(x-2)^{\frac{1}{2}}$, $\quad \therefore\ x=t^2+2, \quad dx=2tdt$

$$\therefore \int \frac{x}{\sqrt{x-2}}dx=\int \frac{t^2+2}{t}\cdot 2tdt=2\int t^2+2dt=\frac{2}{3}t^3+4t+C$$

暨南大學 JINAN UNIVERSITY

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\frac{1}{3}(t^2+1)$, $\quad dx=\frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx=\int \frac{1}{3}(t^2+1)t\cdot\frac{2}{3}tdt=\frac{2}{9}\int t^4+t^2dt$$

$$=\frac{2}{45}t^5+\frac{2}{27}t^3+C=\frac{2}{45}(3x-1)^{\frac{5}{2}}+\frac{2}{27}(3x-1)^{\frac{3}{2}}+C$$

(2) 设 $t=(x-2)^{\frac{1}{2}}$, $\quad \therefore x=t^2+2$, $\quad dx=2tdt$

$$\therefore \int \frac{x}{\sqrt{x-2}}dx=\int \frac{t^2+2}{t}\cdot 2tdt=2\int t^2+2dt=\frac{2}{3}t^3+4t+C$$

$$=\frac{2}{3}(x-2)^{\frac{3}{2}}+4(x-2)^{\frac{1}{2}}+C$$

例 1 求不定积分 $\int x\sqrt{3x-1}dx$，$\int \frac{x}{\sqrt{x-2}}dx$

解

(1) 设 $t=(3x-1)^{\frac{1}{2}}$, $\quad \therefore x=\frac{1}{3}(t^2+1)$, $\quad dx=\frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx=\int \frac{1}{3}(t^2+1)t\cdot\frac{2}{3}tdt=\frac{2}{9}\int t^4+t^2dt$$

$$=\frac{2}{45}t^5+\frac{2}{27}t^3+C=\frac{2}{45}(3x-1)^{\frac{5}{2}}+\frac{2}{27}(3x-1)^{\frac{3}{2}}+C$$

(2) 设 $t=(x-2)^{\frac{1}{2}}$, $\quad \therefore x=t^2+2$, $\quad dx=2tdt$

$$\therefore \int \frac{x}{\sqrt{x-2}}dx=\int \frac{t^2+2}{t}\cdot 2tdt=2\int t^2+2dt=\frac{2}{3}t^3+4t+C$$

$$=\frac{2}{3}(x-2)^{\frac{3}{2}}+4(x-2)^{\frac{1}{2}}+C$$

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

暨南大學 JINAN UNIVERSITY

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}} dx$，$\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$,

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$,

$$\therefore \int \frac{1}{1+\sqrt{x}}dx =$$

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\quad \therefore x = (t-1)^2$,

$$\therefore \int \frac{1}{1+\sqrt{x}}dx =$$

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t=1+x^{\frac{1}{2}}$, $\quad \therefore x=(t-1)^2$, $\quad dx=2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx=$$

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\quad \therefore x = (t-1)^2$, $\quad dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx = \int \frac{1}{t} \cdot 2(t-1)dt$$

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t=1+x^{\frac{1}{2}}$，$\quad \therefore x=(t-1)^2$，$\quad dx=2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx=\int \frac{1}{t}\cdot 2(t-1)dt=2\int 1-\frac{1}{t}dt$$

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$，　$\therefore x = (t-1)^2$，　$dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2\int 1 - \frac{1}{t}dt$$

$$= 2t - 2\ln t + C$$

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\quad \therefore x = (t-1)^2$, $\quad dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2\int 1 - \frac{1}{t}dt$$

$$= 2t - 2\ln t + C = 2(1 + x^{\frac{1}{2}}) - 2\ln(1 + x^{\frac{1}{2}}) + C$$

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\quad \therefore x = (t-1)^2$, $\quad dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2\int 1 - \frac{1}{t}dt$$

$$= 2t - 2\ln t + C = 2(1 + x^{\frac{1}{2}}) - 2\ln(1 + x^{\frac{1}{2}}) + C$$

(2) 设 $t = 1 + (1+x)^{\frac{1}{3}}$,

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2\int 1 - \frac{1}{t}dt$$

$$= 2t - 2\ln t + C = 2(1+x^{\frac{1}{2}}) - 2\ln(1+x^{\frac{1}{2}}) + C$$

(2) 设 $t = 1 + (1+x)^{\frac{1}{3}}$,

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}}dx =$$

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\quad \therefore x = (t-1)^2$, $\quad dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2\int 1 - \frac{1}{t}dt$$

$$= 2t - 2\ln t + C = 2(1+x^{\frac{1}{2}}) - 2\ln(1+x^{\frac{1}{2}}) + C$$

(2) 设 $t = 1 + (1+x)^{\frac{1}{3}}$, $\quad \therefore x = (t-1)^3 - 1$,

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}}dx =$$

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t=1+x^{\frac{1}{2}}$, $\therefore x=(t-1)^2$, $dx=2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx=\int \frac{1}{t}\cdot 2(t-1)dt=2\int 1-\frac{1}{t}dt$$

$$=2t-2\ln t+C=2(1+x^{\frac{1}{2}})-2\ln(1+x^{\frac{1}{2}})+C$$

(2) 设 $t=1+(1+x)^{\frac{1}{3}}$, $\therefore x=(t-1)^3-1$, $dx=3(t-1)^2dt$

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}}dx=$$

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\quad \therefore x = (t-1)^2$, $\quad dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2\int 1 - \frac{1}{t}dt$$

$$= 2t - 2\ln t + C = 2(1+x^{\frac{1}{2}}) - 2\ln(1+x^{\frac{1}{2}}) + C$$

(2) 设 $t = 1 + (1+x)^{\frac{1}{3}}$, $\quad \therefore x = (t-1)^3 - 1$, $\quad dx = 3(t-1)^2 dt$

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}}dx = \int \frac{1}{t} \cdot 3(t-1)^2 dt =$$

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t=1+x^{\frac{1}{2}}$, $\quad \therefore x=(t-1)^2$, $\quad dx=2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx=\int \frac{1}{t}\cdot 2(t-1)dt=2\int 1-\frac{1}{t}dt$$

$$=2t-2\ln t+C=2(1+x^{\frac{1}{2}})-2\ln(1+x^{\frac{1}{2}})+C$$

(2) 设 $t=1+(1+x)^{\frac{1}{3}}$, $\quad \therefore x=(t-1)^3-1$, $\quad dx=3(t-1)^2dt$

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}}dx=\int \frac{1}{t}\cdot 3(t-1)^2dt=3\int t-2+\frac{1}{t}dt$$

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t=1+x^{\frac{1}{2}}$, $\quad \therefore x=(t-1)^2$, $\quad dx=2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx=\int \frac{1}{t}\cdot 2(t-1)dt=2\int 1-\frac{1}{t}dt$$

$$=2t-2\ln t+C=2(1+x^{\frac{1}{2}})-2\ln(1+x^{\frac{1}{2}})+C$$

(2) 设 $t=1+(1+x)^{\frac{1}{3}}$, $\quad \therefore x=(t-1)^3-1$, $\quad dx=3(t-1)^2dt$

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}}dx=\int \frac{1}{t}\cdot 3(t-1)^2dt=3\int t-2+\frac{1}{t}dt$$

$$=\frac{3}{2}t^2-6t+3\ln|t|+C$$

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\quad \therefore x = (t-1)^2$, $\quad dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2\int 1 - \frac{1}{t}dt$$

$$= 2t - 2\ln t + C = 2(1+x^{\frac{1}{2}}) - 2\ln(1+x^{\frac{1}{2}}) + C$$

(2) 设 $t = 1 + (1+x)^{\frac{1}{3}}$, $\quad \therefore x = (t-1)^3 - 1$, $\quad dx = 3(t-1)^2 dt$

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}}dx = \int \frac{1}{t} \cdot 3(t-1)^2 dt = 3\int t - 2 + \frac{1}{t}dt$$

$$= \frac{3}{2}t^2 - 6t + 3\ln|t| + C$$

$$= \tfrac{3}{2}(1+(1+x)^{\frac{1}{3}})^2 - 6(1+(1+x)^{\frac{1}{3}}) + 3\ln|1+(1+x)^{\frac{1}{3}}| + C$$

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\quad \therefore x = (t-1)^2$, $\quad dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2\int 1 - \frac{1}{t}dt$$

$$= 2t - 2\ln t + C = 2(1 + x^{\frac{1}{2}}) - 2\ln(1 + x^{\frac{1}{2}}) + C$$

(2) 设 $t = 1 + (1+x)^{\frac{1}{3}}$, $\quad \therefore x = (t-1)^3 - 1$, $\quad dx = 3(t-1)^2dt$

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}}dx = \int \frac{1}{t} \cdot 3(t-1)^2dt = 3\int t - 2 + \frac{1}{t}dt$$

$$= \frac{3}{2}t^2 - 6t + 3\ln|t| + C$$

$$= \tfrac{3}{2}(1 + (1+x)^{\frac{1}{3}})^2 - 6(1 + (1+x)^{\frac{1}{3}}) + 3\ln|1 + (1+x)^{\frac{1}{3}}| + C$$

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

解 设 $t = \sqrt{1 + e^x}$,

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

解 设 $t = \sqrt{1+e^x}$, $\quad \therefore x = \ln(t^2 - 1)$,

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解 设 $t = \sqrt{1+e^x}$, $\quad \therefore x = \ln(t^2-1)$, $\quad dx = \dfrac{2t}{t^2-1}dt$

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解 设 $t = \sqrt{1+e^x}$, $\quad \therefore x = \ln(t^2-1)$, $\quad dx = \dfrac{2t}{t^2-1}dt$

$$\therefore \int \frac{1}{\sqrt{1+e^x}}dx = \int \frac{1}{t} \cdot \frac{2t}{t^2-1}dt$$

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解 设 $t=\sqrt{1+e^x}$, $\quad \therefore x=\ln(t^2-1)$, $\quad dx=\dfrac{2t}{t^2-1}dt$

$$\therefore \int \frac{1}{\sqrt{1+e^x}}dx=\int \frac{1}{t}\cdot\frac{2t}{t^2-1}dt=\int \frac{1}{t-1}-\frac{1}{t+1}dt$$

暨南大學 JINAN UNIVERSITY

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解 设 $t = \sqrt{1+e^x}$, $\quad \therefore x = \ln(t^2-1)$, $\quad dx = \dfrac{2t}{t^2-1}dt$

$$\therefore \int \frac{1}{\sqrt{1+e^x}}dx = \int \frac{1}{t} \cdot \frac{2t}{t^2-1}dt = \int \frac{1}{t-1} - \frac{1}{t+1}dt$$

$$= \ln|t-1| - \ln|t+1| + C$$

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解 设 $t=\sqrt{1+e^x}$, $\quad \therefore x=\ln(t^2-1), \quad dx=\dfrac{2t}{t^2-1}dt$

$$\therefore \int \frac{1}{\sqrt{1+e^x}}dx=\int \frac{1}{t}\cdot\frac{2t}{t^2-1}dt=\int \frac{1}{t-1}-\frac{1}{t+1}dt$$

$$=\ln|t-1|-\ln|t+1|+C=\ln\left|\frac{t-1}{t+1}\right|+C$$

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解 设 $t = \sqrt{1+e^x}$, $\quad \therefore x = \ln(t^2-1)$, $\quad dx = \dfrac{2t}{t^2-1}dt$

$$\therefore \int \frac{1}{\sqrt{1+e^x}}dx = \int \frac{1}{t} \cdot \frac{2t}{t^2-1}dt = \int \frac{1}{t-1} - \frac{1}{t+1}dt$$

$$= \ln|t-1| - \ln|t+1| + C = \ln\left|\frac{t-1}{t+1}\right| + C$$

$$= \ln\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right) + C$$

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解 设 $t=\sqrt{1+e^x}$, $\quad \therefore x=\ln(t^2-1), \quad dx=\dfrac{2t}{t^2-1}dt$

$$\therefore \int \frac{1}{\sqrt{1+e^x}}dx=\int \frac{1}{t}\cdot\frac{2t}{t^2-1}dt=\int \frac{1}{t-1}-\frac{1}{t+1}dt$$

$$=\ln|t-1|-\ln|t+1|+C=\ln\left|\frac{t-1}{t+1}\right|+C$$

$$=\ln\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right)+C$$

$$=2\ln(\sqrt{1+e^x}-1)-x+C$$

变量代换法：例子总结

$$\int x\sqrt{3x-1}dx$$

$$\int \frac{1}{1+\sqrt{x}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

变量代换法：例子总结

$$\int x\sqrt{3x-1}dx \xlongequal{t=\sqrt{3x-1}} \cdots$$

$$\int \frac{1}{1+\sqrt{x}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

变量代换法：例子总结

$$\int x\sqrt{3x-1}\,dx \xlongequal{t=\sqrt{3x-1}} \cdots$$

$$\int \frac{1}{1+\sqrt{x}}\,dx \xlongequal{t=1+\sqrt{x}} \cdots$$

$$\int \frac{1}{\sqrt{1+e^x}}\,dx$$

暨南大學 JINAN UNIVERSITY

变量代换法：例子总结

$$\int x\sqrt{3x-1}dx \xlongequal{t=\sqrt{3x-1}} \cdots$$

$$\int \frac{1}{1+\sqrt{x}}dx \xlongequal{t=1+\sqrt{x}} \cdots$$

$$\int \frac{1}{\sqrt{1+e^x}}dx \xlongequal{t=\sqrt{1+e^x}} \cdots$$

例 4 求不定积分 $\int \sqrt{1-x^2}dx$

例 4 求不定积分 $\int \sqrt{1-x^2}dx$

提示 直接选取合适 $x = \varphi(t)$

例 4 求不定积分 $\int \sqrt{1-x^2}dx$

提示 直接选取合适 $x=\varphi(t)$

解 $\because -1 \le x \le 1$，设 $x=\sin t,\ t\in[-\frac{\pi}{2},\frac{\pi}{2}]$，

例 4 求不定积分 $\int \sqrt{1-x^2}dx$

提示 直接选取合适 $x=\varphi(t)$

解 $\because -1 \le x \le 1$，设 $x=\sin t,\ t\in[-\frac{\pi}{2},\frac{\pi}{2}]$，$\cos t \ge 0$

例 4 求不定积分 $\int \sqrt{1-x^2}dx$

提示 直接选取合适 $x=\varphi(t)$

解 $\because -1 \le x \le 1$，设 $x=\sin t,\ t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$，$\cos t \ge 0$

$$\therefore \int \sqrt{1-x^2}dx = \int \sqrt{1-\sin^2 t}d\sin t$$

例 4 求不定积分 $\int\sqrt{1-x^2}dx$

提示 直接选取合适 $x=\varphi(t)$

解 $\because -1\le x\le 1$，设 $x=\sin t,\ t\in[-\frac{\pi}{2},\frac{\pi}{2}]$，$\cos t\ge 0$

$$\therefore \int\sqrt{1-x^2}dx=\int\sqrt{1-\sin^2 t}d\sin t=\int\cos^2 tdt$$

例 4 求不定积分 $\int \sqrt{1-x^2}dx$

提示 直接选取合适 $x=\varphi(t)$

解 $\because -1 \le x \le 1$，设 $x=\sin t,\ t\in[-\frac{\pi}{2},\ \frac{\pi}{2}]$，$\cos t \ge 0$

$$\therefore \int \sqrt{1-x^2}dx = \int \sqrt{1-\sin^2 t}d\sin t = \int \cos^2 t dt$$

$$= \frac{1}{2}\int \cos 2t + 1 dt$$

例 4 求不定积分 $\int \sqrt{1-x^2}dx$

提示 直接选取合适 $x=\varphi(t)$

解 $\because -1 \le x \le 1$，设 $x=\sin t,\ t\in[-\frac{\pi}{2},\frac{\pi}{2}]$，$\cos t \ge 0$

$$\therefore \int \sqrt{1-x^2}dx = \int \sqrt{1-\sin^2 t}d\sin t = \int \cos^2 t dt$$

$$= \frac{1}{2}\int \cos 2t + 1 dt \qquad \frac{1}{2}\sin 2t$$

例 4 求不定积分 $\int \sqrt{1-x^2}dx$

提示 直接选取合适 $x=\varphi(t)$

解 $\because -1 \le x \le 1$，设 $x=\sin t,\ t\in[-\frac{\pi}{2},\frac{\pi}{2}]$，$\cos t \ge 0$

$$\therefore \int \sqrt{1-x^2}dx = \int \sqrt{1-\sin^2 t}d\sin t = \int \cos^2 t dt$$

$$= \frac{1}{2}\int \cos 2t + 1dt = \frac{1}{2}\cdot\frac{1}{2}\sin 2t$$

例 4 求不定积分 $\int \sqrt{1-x^2}dx$

提示 直接选取合适 $x=\varphi(t)$

解 $\because -1 \le x \le 1$，设 $x=\sin t,\ t\in[-\frac{\pi}{2},\ \frac{\pi}{2}]$，$\cos t \ge 0$

$$\therefore \int \sqrt{1-x^2}dx = \int \sqrt{1-\sin^2 t}d\sin t = \int \cos^2 t dt$$

$$= \frac{1}{2}\int \cos 2t + 1 dt = \frac{1}{2}\cdot\frac{1}{2}\sin 2t + \frac{1}{2}t + C$$

例 4 求不定积分 $\int \sqrt{1-x^2}dx$

提示 直接选取合适 $x=\varphi(t)$

解 $\because -1 \le x \le 1$，设 $x=\sin t,\ t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$，$\cos t \ge 0$

$$\therefore \int \sqrt{1-x^2}dx = \int \sqrt{1-\sin^2 t}d\sin t = \int \cos^2 t dt$$

$$= \frac{1}{2}\int \cos 2t + 1 dt = \frac{1}{2}\cdot\frac{1}{2}\sin 2t + \frac{1}{2}t + C$$

$$= \frac{1}{2}\sin t\cos t + \frac{1}{2}t + C$$

例 4 求不定积分 $\int \sqrt{1-x^2}dx$

提示 直接选取合适 $x=\varphi(t)$

解 $\because -1 \le x \le 1$，设 $x=\sin t,\ t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$，$\cos t \ge 0$

$$\begin{aligned}\therefore \int \sqrt{1-x^2}dx &= \int \sqrt{1-\sin^2 t}d\sin t = \int \cos^2 t dt \\ &= \frac{1}{2}\int \cos 2t + 1 dt = \frac{1}{2}\cdot\frac{1}{2}\sin 2t + \frac{1}{2}t + C \\ &= \frac{1}{2}\sin t\cos t + \frac{1}{2}t + C \\ &= \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\arcsin x + C\end{aligned}$$