§2.5　分块矩阵

数学系　　梁卓滨

2018 - 2019 学年上学期

分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array} \right)$$

分块矩阵引入

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$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} & \\ & \end{pmatrix}$$

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$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} I_3 & \\ & \end{pmatrix}$$

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- 矩阵

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$$\underset{or}{=} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

分块矩阵引入

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$$\overset{or}{=} \left(\begin{array}{cc:cc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} I_2 & \\ & \end{pmatrix}$$

分块矩阵引入

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$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix}$$

$$\overset{or}{=\!=} \left(\begin{array}{cc|cc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} I_2 & \\ & I_2 \end{pmatrix}$$

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$$\overset{or}{=\!=\!=} \left(\begin{array}{cc:cc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} I_2 & \\ O & I_2 \end{pmatrix}$$

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$$\overset{or}{=} \left(\begin{array}{cc:cc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} I_2 & A_2 \\ O & I_2 \end{pmatrix}$$

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$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix}$$

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$$\stackrel{or}{=} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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$$\stackrel{or}{=} \left(\begin{array}{c:c:c:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = (\qquad\qquad)$$

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$$\overset{or}{=} \left(\begin{array}{cc:cc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} I_2 & A_2 \\ O & I_2 \end{pmatrix}$$

$$\overset{or}{=} \left(\begin{array}{c:c:c:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} \varepsilon_1 & & & \end{pmatrix}$$

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$$\overset{or}{=} \left(\begin{array}{cc:cc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} I_2 & A_2 \\ O & I_2 \end{pmatrix}$$

$$\overset{or}{=} \left(\begin{array}{c:c:c:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & & \end{pmatrix}$$

分块矩阵引入

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$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix}$$

$$\overset{or}{=} \left(\begin{array}{cc:cc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} I_2 & A_2 \\ O & I_2 \end{pmatrix}$$

$$\overset{or}{=} \left(\begin{array}{c:c:c:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \end{pmatrix}$$

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$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix}$$

$$\stackrel{or}{=} \left(\begin{array}{cc:cc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} I_2 & A_2 \\ O & I_2 \end{pmatrix}$$

$$\stackrel{or}{=} \left(\begin{array}{c:c:c:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \alpha \end{pmatrix}$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$A = \begin{pmatrix} * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \end{pmatrix}$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$
A = \begin{pmatrix}
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & *
\end{pmatrix}
$$

分块矩阵

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$$A = \begin{pmatrix} * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$A = \begin{pmatrix} * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \end{pmatrix} = \begin{pmatrix} A_{11} & & \\ & & \\ & & \end{pmatrix}$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$A = \begin{pmatrix} * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & \\ & & \\ & & \end{pmatrix}$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$A = \begin{pmatrix} * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ & & & \end{pmatrix}$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$A = \begin{pmatrix} * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \end{pmatrix}$$

分块矩阵

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分块矩阵

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分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$A = \begin{pmatrix} * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \cdots & \cdots & \ddots & \cdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} = (A_{pq})$$

称为分块矩阵。

分块矩阵

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$$A = \begin{pmatrix} * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \cdots & \cdots & \ddots & \cdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} = (A_{pq})$$

称为分块矩阵。

- 分块矩阵中
  - 每一行的每个子块有相同行数；
  - 每一列的每个子块有相同列数。

分块矩阵的运算：加法

假设矩阵 A, B 同型，且采取相同分块方式：

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad , \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{st} \end{pmatrix}$$

分块矩阵的运算：加法

假设矩阵 A, B 同型，且采取相同分块方式：

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \qquad , \; B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{st} \end{pmatrix}$$

$$A + B =$$

分块矩阵的运算：加法

假设矩阵 A, B 同型，且采取相同分块方式：

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \qquad , B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{st} \end{pmatrix}$$

$$A + B = \begin{pmatrix} A_{11} + B_{11} & A_{12} + B_{12} & \cdots & A_{1t} + B_{1t} \\ A_{21} + B_{21} & A_{22} + B_{22} & \cdots & A_{2t} + B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} + B_{s1} & A_{s2} + B_{s2} & \cdots & A_{st} + B_{st} \end{pmatrix}$$

分块矩阵的运算：加法

假设矩阵 A, B 同型，且采取相同分块方式：

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} = (A_{pq}),\ B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{st} \end{pmatrix}$$

则

$$A + B = \begin{pmatrix} A_{11} + B_{11} & A_{12} + B_{12} & \cdots & A_{1t} + B_{1t} \\ A_{21} + B_{21} & A_{22} + B_{22} & \cdots & A_{2t} + B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} + B_{s1} & A_{s2} + B_{s2} & \cdots & A_{st} + B_{st} \end{pmatrix}$$

分块矩阵的运算：加法

假设矩阵 A, B 同型，且采取相同分块方式：

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} = (A_{pq}), \ B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{st} \end{pmatrix} = (B_{pq})$$

则

$$A + B = \begin{pmatrix} A_{11} + B_{11} & A_{12} + B_{12} & \cdots & A_{1t} + B_{1t} \\ A_{21} + B_{21} & A_{22} + B_{22} & \cdots & A_{2t} + B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} + B_{s1} & A_{s2} + B_{s2} & \cdots & A_{st} + B_{st} \end{pmatrix}$$

分块矩阵的运算：加法

假设矩阵 A, B 同型，且采取相同分块方式：

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} = (A_{pq}), \; B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{st} \end{pmatrix} = (B_{pq})$$

则

$$A + B = \begin{pmatrix} A_{11}+B_{11} & A_{12}+B_{12} & \cdots & A_{1t}+B_{1t} \\ A_{21}+B_{21} & A_{22}+B_{22} & \cdots & A_{2t}+B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1}+B_{s1} & A_{s2}+B_{s2} & \cdots & A_{st}+B_{st} \end{pmatrix} = (A_{pq}+B_{pq})$$

分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$ 则

$A + B = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} =$

分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$ 则

$$A + B = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} =$$

分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$ 则

$$A + B = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) + \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right) =$$

分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$ 则

$$A + B = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) + \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right) = \left(\begin{array}{cc|cc} & & & \\ & & & \\ \hline & & & \\ & & & \end{array}\right)$$

分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$ 则

$$A + B = \left(\begin{array}{cc:cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hdashline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) + \left(\begin{array}{cc:cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hdashline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array} \right) = \left(\begin{array}{cc:cc} & & & \\ & & & \\ \hdashline & & & \\ & & & \end{array} \right)$$

分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$ 则

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分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$ 则

$$A + B = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & & \\ 2 & 1 & & \\ & & & \\ & & & \end{pmatrix}$$

分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$ 则

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分块矩阵的运算：加法

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分块矩阵的运算：加法

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分块矩阵的运算：加法

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分块矩阵的运算：加法

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分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$ 则

$$A + B = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 1 & 3 \\ 2 & 1 & 2 & 4 \\ 6 & 3 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}$$

或者

$$A + B =$$

分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$ 则

$$A + B = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 1 & 3 \\ 2 & 1 & 2 & 4 \\ 6 & 3 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}$$

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分块矩阵的运算：加法

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$$A + B = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 1 & 3 \\ 2 & 1 & 2 & 4 \\ 6 & 3 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}$$

或者

$$A + B = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} + \begin{pmatrix} D & O \\ F & I \end{pmatrix} =$$

分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$ 则

$$A + B = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 1 & 3 \\ 2 & 1 & 2 & 4 \\ 6 & 3 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}$$

或者

$$A + B = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} + \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} I+D & C \\ F & O \end{pmatrix} =$$

分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$ 则

$$A + B = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 1 & 3 \\ 2 & 1 & 2 & 4 \\ 6 & 3 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}$$

或者

$$A + B = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} + \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} I+D & C \\ F & O \end{pmatrix} = \begin{pmatrix} & & \\ & & \end{pmatrix}$$

分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$ 则

$$A + B = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 1 & 3 \\ 2 & 1 & 2 & 4 \\ 6 & 3 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}$$

或者

$$A + B = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} + \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} I+D & C \\ F & O \end{pmatrix} = \begin{pmatrix} 2 & 2 & & \\ 2 & 1 & & \\ & & & \\ & & & \end{pmatrix}$$

分块矩阵的运算：加法

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$$A + B = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} + \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} I+D & C \\ F & O \end{pmatrix} = \begin{pmatrix} 2 & 2 & 1 & 3 \\ 2 & 1 & 2 & 4 \\ & & & \\ & & & \end{pmatrix}$$

分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$ 则

$$A + B = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 1 & 3 \\ 2 & 1 & 2 & 4 \\ 6 & 3 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}$$

或者

$$A + B = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} + \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} I+D & C \\ F & O \end{pmatrix} = \begin{pmatrix} 2 & 2 & 1 & 3 \\ 2 & 1 & 2 & 4 \\ 6 & 3 & & \\ 0 & -2 & & \end{pmatrix}$$

分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$ 则

$$A + B = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & 1 & 3 \\ 2 & 1 & 2 & 4 \\ 6 & 3 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}$$

或者

$$A + B = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} + \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} I+D & C \\ F & O \end{pmatrix} = \begin{pmatrix} 2 & 2 & 1 & 3 \\ 2 & 1 & 2 & 4 \\ 6 & 3 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA =$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \Rightarrow kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$，则

$$kA =$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \Rightarrow kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, 则

$$kA = k\begin{pmatrix} I & C \\ O & -I \end{pmatrix} =$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, 则

$$kA = k\begin{pmatrix} I & C \\ O & -I \end{pmatrix} = \begin{pmatrix} kI & kC \\ O & -kI \end{pmatrix} =$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \left(\begin{array}{c|c} I & C \\ \hline O & -I \end{array} \right)$, 则

$$kA = k \begin{pmatrix} I & C \\ O & -I \end{pmatrix} = \begin{pmatrix} kI & kC \\ O & -kI \end{pmatrix} = \left(\begin{array}{c|c} & \\ \hline & \end{array} \right)$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \left(\begin{array}{c|c} I & C \\ \hline O & -I \end{array} \right)$，则

$$kA = k \begin{pmatrix} I & C \\ O & -I \end{pmatrix} = \begin{pmatrix} kI & kC \\ O & -kI \end{pmatrix} = \left(\begin{array}{cc|cc} k & 0 & & \\ 0 & k & & \\ \hline & & & \\ & & & \end{array} \right)$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$，则

$$kA = k\begin{pmatrix} I & C \\ O & -I \end{pmatrix} = \begin{pmatrix} kI & kC \\ O & -kI \end{pmatrix} = \begin{pmatrix} k & 0 & k & 3k \\ 0 & k & 2k & 4k \\ & & & \\ & & & \end{pmatrix}$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \left(\begin{array}{cc:cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hdashline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \left(\begin{array}{c:c} I & C \\ \hdashline O & -I \end{array} \right)$, 则

$$kA = k \begin{pmatrix} I & C \\ O & -I \end{pmatrix} = \begin{pmatrix} kI & kC \\ O & -kI \end{pmatrix} = \left(\begin{array}{cc:cc} k & 0 & k & 3k \\ 0 & k & 2k & 4k \\ \hdashline 0 & 0 & & \\ 0 & 0 & & \end{array} \right)$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$，则

$$kA = k\begin{pmatrix} I & C \\ O & -I \end{pmatrix} = \begin{pmatrix} kI & kC \\ O & -kI \end{pmatrix} = \begin{pmatrix} k & 0 & k & 3k \\ 0 & k & 2k & 4k \\ 0 & 0 & -k & 0 \\ 0 & 0 & 0 & -k \end{pmatrix}$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix}$$

满足：A 的列划分与 B 的行划分方式相同。

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB = C =$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中

$$C_{pq} = \qquad\qquad\qquad\qquad\qquad .$$

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中

$$C_{pq} = A_{p1} \qquad A_{p2} \qquad \cdots \qquad A_{pr} \qquad .$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中

$$C_{pq} = A_{p1}B_{1q} \quad A_{p2}B_{2q} \quad \cdots \quad A_{pr}B_{rq}.$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中

$$C_{pq} = A_{p1}B_{1q} + A_{p2}B_{2q} + \cdots + A_{pr}B_{rq}.$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中（**必然每个子块的乘积有意义**）

$$C_{pq} = A_{p1}B_{1q} + A_{p2}B_{2q} + \cdots + A_{pr}B_{rq}.$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中（**必然每个子块的乘积有意义**）

$$C_{pq} = A_{p1}B_{1q} + A_{p2}B_{2q} + \cdots + A_{pr}B_{rq}.$$

$$\begin{pmatrix} A_{11} & \cdots & \cdots & A_{1r} \\ \vdots & & & \vdots \\ A_{p1} & \cdots & \cdots & A_{pr} \\ \vdots & & & \vdots \\ A_{s1} & \cdots & \cdots & A_{sr} \end{pmatrix} \cdot \begin{pmatrix} B_{11} & \cdots & B_{1q} & \cdots & B_{1t} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ B_{r1} & \cdots & B_{rq} & \cdots & B_{rt} \end{pmatrix} = \begin{pmatrix} C_{11} & \cdots & & \cdots & C_{1t} \\ \vdots & & \vdots & & \vdots \\ & \cdots & C_{pq} & \cdots & \\ \vdots & & \vdots & & \vdots \\ C_{s1} & \cdots & & \cdots & C_{st} \end{pmatrix}$$

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分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中（**必然每个子块的乘积有意义**）

$$C_{pq} = A_{p1}B_{1q} + A_{p2}B_{2q} + \cdots + A_{pr}B_{rq}.$$

$$\begin{pmatrix} A_{11} & \cdots & \cdots & A_{1r} \\ \vdots & & & \vdots \\ A_{p1} & \cdots & \cdots & A_{pr} \\ \vdots & & & \vdots \\ A_{s1} & \cdots & \cdots & A_{sr} \end{pmatrix} \cdot \begin{pmatrix} B_{11} & \cdots & B_{1q} & \cdots & B_{1t} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ B_{r1} & \cdots & B_{rq} & \cdots & B_{rt} \end{pmatrix} = \begin{pmatrix} C_{11} & \cdots & & \cdots & C_{1t} \\ \vdots & & \vdots & & \vdots \\ & \cdots & C_{pq} & \cdots & \\ \vdots & & \vdots & & \vdots \\ C_{s1} & \cdots & & \cdots & C_{st} \end{pmatrix}$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中（**必然每个子块的乘积有意义**）

$$C_{pq} = A_{p1}B_{1q} + A_{p2}B_{2q} + \cdots + A_{pr}B_{rq}.$$

$$\begin{pmatrix} A_{11} & \cdots & \cdots & A_{1r} \\ \vdots & & & \vdots \\ A_{p1} & \cdots & \cdots & A_{pr} \\ \vdots & & & \vdots \\ A_{s1} & \cdots & \cdots & A_{sr} \end{pmatrix} \cdot \begin{pmatrix} B_{11} & \cdots & B_{1q} & \cdots & B_{1t} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ B_{r1} & \cdots & B_{rq} & \cdots & B_{rt} \end{pmatrix} = \begin{pmatrix} C_{11} & \cdots & & \cdots & C_{1t} \\ \vdots & & \vdots & & \vdots \\ & \cdots & C_{pq} & \cdots & \\ \vdots & & \vdots & & \vdots \\ C_{s1} & \cdots & & \cdots & C_{st} \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$

$$AB =$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$

$AB =$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

$AB =$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}\begin{pmatrix} D & O \\ F & I \end{pmatrix} = $$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} =$$

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} & \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}\begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & \\ & \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ & \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

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$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & \end{pmatrix}$$

分块矩阵的运算：乘积

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$$= \begin{pmatrix} & \\ & \end{pmatrix} =$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

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$$= \begin{pmatrix} D + CF & \\ & \end{pmatrix} =$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

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$$= \begin{pmatrix} D + CF & C \\ & \end{pmatrix} =$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

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分块矩阵的运算：乘积

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分块矩阵的运算：乘积

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$$= \begin{pmatrix} D + CF & C \\ -F & -I \end{pmatrix} = \begin{pmatrix} & & 1 & 3 \\ & & 2 & 4 \\ & & & \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

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$$= \begin{pmatrix} D + CF & C \\ -F & -I \end{pmatrix} = \begin{pmatrix} & & 1 & 3 \\ & & 2 & 4 \\ -6 & -3 & & \\ 0 & 2 & & \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

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分块矩阵的运算：乘积

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$$= \begin{pmatrix} D + CF & C \\ -F & -I \end{pmatrix} = \begin{pmatrix} & & 1 & 3 \\ & & 2 & 4 \\ -6 & -3 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{pmatrix}$$

其中

$$D + CF =$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

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$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & OO + (-I)I \end{pmatrix}$$

$$= \begin{pmatrix} D + CF & C \\ -F & -I \end{pmatrix} = \begin{pmatrix} & & 1 & 3 \\ & & 2 & 4 \\ -6 & -3 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{pmatrix}$$

其中

$$D + CF = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 6 & 3 \\ 0 & -2 \end{pmatrix} =$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

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$$= \begin{pmatrix} D + CF & C \\ -F & -I \end{pmatrix} = \begin{pmatrix} & & 1 & 3 \\ & & 2 & 4 \\ -6 & -3 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{pmatrix}$$

其中

$$D + CF = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 6 & 3 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 6 & -3 \\ 12 & -2 \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

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$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & OO + (-I)I \end{pmatrix}$$

$$= \begin{pmatrix} D + CF & C \\ -F & -I \end{pmatrix} = \begin{pmatrix} & & 1 & 3 \\ & & 2 & 4 \\ -6 & -3 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{pmatrix}$$

其中

$$D + CF = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 6 & 3 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 6 & -3 \\ 12 & -2 \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

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$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & OO + (-I)I \end{pmatrix}$$

$$= \begin{pmatrix} D + CF & C \\ -F & -I \end{pmatrix} = \begin{pmatrix} & & 1 & 3 \\ & & 2 & 4 \\ -6 & -3 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{pmatrix}$$

其中

$$D + CF = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 6 & 3 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 6 & -3 \\ 12 & -2 \end{pmatrix} = \begin{pmatrix} 7 & -1 \\ 14 & -2 \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

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$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}\begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & OO + (-I)I \end{pmatrix}$$

$$= \begin{pmatrix} D + CF & C \\ -F & -I \end{pmatrix} = \begin{pmatrix} 7 & -1 & 1 & 3 \\ 14 & -2 & 2 & 4 \\ -6 & -3 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{pmatrix}$$

其中

$$D + CF = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 6 & 3 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 6 & -3 \\ 12 & -2 \end{pmatrix} = \begin{pmatrix} 7 & -1 \\ 14 & -2 \end{pmatrix}$$

分块矩阵的运算：乘积

例 2 设 $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$AB =$

分块矩阵的运算：乘积

例 2 设 $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$AB =$

分块矩阵的运算：乘积

例 2 设 $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

$AB =$

分块矩阵的运算：乘积

例 2 设 $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

$$AB = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix} \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix} =$$

分块矩阵的运算：乘积

例 2 设 $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

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分块矩阵的运算：乘积

例 2 设 $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

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$$AB = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix} \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix} = \begin{pmatrix} & & \\ & & \end{pmatrix}$$

分块矩阵的运算：乘积

例 2 设 $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix} \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix} = \begin{pmatrix} I(-I) + OO & \\ & \end{pmatrix}$$

分块矩阵的运算：乘积

例 2 设 $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix} \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix} = \begin{pmatrix} I(-I) + OO & IB_1 + OI \end{pmatrix}$$

分块矩阵的运算：乘积

例 2 设 $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix} \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix} = \begin{pmatrix} I(-I) + OO & IB_1 + OI \\ A_1(-I) + 2IO & \end{pmatrix}$$

分块矩阵的运算：乘积

例 2 设 $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

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分块矩阵的运算：乘积

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其中

$A_1B_1 + 2I =$

分块矩阵的运算：乘积

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其中

$$A_1 B_1 + 2I = \begin{pmatrix} 1 & 3 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} =$$

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分块矩阵的运算：乘积

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分块矩阵的运算：乘积

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分块矩阵的运算：乘积

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例 3 设 A, B 均为 2 阶方阵，且 $|A| = 2, |B| = 3$，计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

例 3 设 A, B 均为 2 阶方阵，且 $|A| = 2, |B| = 3$，计算分块矩阵的乘积

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并计算该乘积的行列式。

解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} & & \\ & & \end{pmatrix}$$

例 3 设 A, B 均为 2 阶方阵，且 $|A| = 2,\ |B| = 3$，计算分块矩阵的乘积

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解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix}\begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} OO + AA^* & \\ & \end{pmatrix}$$

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解

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$$= \begin{pmatrix} \quad \quad \end{pmatrix}$$

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解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix}$$

$$= \begin{pmatrix} AA^* & O \\ O & BB^* \end{pmatrix} = \begin{pmatrix} |A|I_2 & O \\ O & \end{pmatrix}$$

例 3 设 A, B 均为 2 阶方阵，且 $|A| = 2, |B| = 3$，计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix}$$

$$= \begin{pmatrix} AA^* & O \\ O & BB^* \end{pmatrix} = \begin{pmatrix} |A|I_2 & O \\ O & |B|I_2 \end{pmatrix}$$

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并计算该乘积的行列式。

解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix}\begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix}$$

$$= \begin{pmatrix} AA^* & O \\ O & BB^* \end{pmatrix} = \begin{pmatrix} |A|I_2 & O \\ O & |B|I_2 \end{pmatrix} = \begin{pmatrix} 2I_2 & O \\ O & 3I_2 \end{pmatrix} =$$

例 3 设 A, B 均为 2 阶方阵，且 $|A| = 2$, $|B| = 3$，计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix}$$

$$= \begin{pmatrix} AA^* & O \\ O & BB^* \end{pmatrix} = \begin{pmatrix} |A|I_2 & O \\ O & |B|I_2 \end{pmatrix} = \begin{pmatrix} 2I_2 & O \\ O & 3I_2 \end{pmatrix} = \begin{pmatrix} 2 & & & \\ & 2 & & \\ \hline & & 3 & \\ & & & 3 \end{pmatrix}$$

例 3 设 A, B 均为 2 阶方阵，且 $|A| = 2, |B| = 3$，计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix}$$

$$= \begin{pmatrix} AA^* & O \\ O & BB^* \end{pmatrix} = \begin{pmatrix} |A|I_2 & O \\ O & |B|I_2 \end{pmatrix} = \begin{pmatrix} 2I_2 & O \\ O & 3I_2 \end{pmatrix} = \begin{pmatrix} 2 & & & \\ & 2 & \vdots & \\ \hdashline & & 3 & \\ & & & 3 \end{pmatrix}$$

所以 $\left| \begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} \right| = \begin{vmatrix} 2 & & & \\ & 2 & & \\ & & 3 & \\ & & & 3 \end{vmatrix} =$

例 3 设 A, B 均为 2 阶方阵，且 $|A| = 2$, $|B| = 3$，计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix}\begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix}\begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix}$$

$$= \begin{pmatrix} AA^* & O \\ O & BB^* \end{pmatrix} = \begin{pmatrix} |A|I_2 & O \\ O & |B|I_2 \end{pmatrix} = \begin{pmatrix} 2I_2 & O \\ O & 3I_2 \end{pmatrix} = \begin{pmatrix} 2 & & \vdots & \\ & 2 & \vdots & \\ \cdots & & 3 & \cdots \\ & & & 3 \end{pmatrix}$$

所以 $\left| \begin{pmatrix} O & A \\ B & O \end{pmatrix}\begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} \right| = \begin{vmatrix} 2 & & & \\ & 2 & & \\ & & 3 & \\ & & & 3 \end{vmatrix} = 2 \times 2 \times 3 \times 3 =$

例 3 设 A, B 均为 2 阶方阵，且 $|A| = 2, |B| = 3$，计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix}\begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix}\begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix}$$

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所以 $\begin{vmatrix} O & A \\ B & O \end{vmatrix}\begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}\begin{vmatrix} \\ \end{vmatrix} = \begin{vmatrix} 2 & & & \\ & 2 & & \\ & & 3 & \\ & & & 3 \end{vmatrix} = 2 \times 2 \times 3 \times 3 = 36$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} & & \\ & & \end{pmatrix}$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix}$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解 若存在矩阵 W, X, Y, Z 使得

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix}$$

则 D 可逆，且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r\times r} & C \\ O & B_{k\times k} \end{pmatrix}$$

其中 $A_{r\times r}$ 和 $B_{k\times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解 若存在矩阵 W, X, Y, Z 使得

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r\times r} & Z_{r\times k} \\ W_{k\times r} & Y_{k\times k} \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$$

则 D 可逆，且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解 若存在矩阵 W, X, Y, Z 使得

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix} = \begin{pmatrix} AX + CW & \\ & \end{pmatrix}$$

则 D 可逆，且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解 若存在矩阵 W, X, Y, Z 使得

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix} = \begin{pmatrix} AX + CW & AZ + CY \end{pmatrix}$$

则 D 可逆，且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解 若存在矩阵 W, X, Y, Z 使得

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix} = \begin{pmatrix} AX + CW & AZ + CY \\ BW \end{pmatrix}$$

则 D 可逆，且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解 若存在矩阵 W, X, Y, Z 使得

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix} = \begin{pmatrix} AX + CW & AZ + CY \\ BW & BY \end{pmatrix}$$

则 D 可逆，且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解 若存在矩阵 W, X, Y, Z 使得

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix} = \begin{pmatrix} AX + CW & AZ + CY \\ BW & BY \end{pmatrix}$$

则 D 可逆，且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。由上式得

$$\begin{cases} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{cases}$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵, 证明 D 可逆并求出 D^{-1}。

解 若存在矩阵 W, X, Y, Z 使得

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则 D 可逆, 且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。由上式得

$$\begin{cases} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{cases} \Rightarrow \begin{cases} \\ \\ \\ \end{cases}$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解 若存在矩阵 W, X, Y, Z 使得

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix} = \begin{pmatrix} AX + CW & AZ + CY \\ BW & BY \end{pmatrix}$$

则 D 可逆，且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。由上式得

$$\begin{cases} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{cases} \Rightarrow \begin{cases} \\ \\ \\ Y = B^{-1} \end{cases}$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解 若存在矩阵 W, X, Y, Z 使得

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix} = \begin{pmatrix} AX + CW & AZ + CY \\ BW & BY \end{pmatrix}$$

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$$\begin{cases} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{cases} \Rightarrow \begin{cases} \\ W = O \\ Y = B^{-1} \end{cases}$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解 若存在矩阵 W, X, Y, Z 使得

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix} = \begin{pmatrix} AX + CW & AZ + CY \\ BW & BY \end{pmatrix}$$

则 D 可逆，且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。由上式得

$$\begin{cases} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{cases} \Rightarrow \begin{cases} Z = -A^{-1}CY \\ W = O \\ Y = B^{-1} \end{cases}$$

例 4 设分块矩阵

$$D = \left(\begin{array}{cc} A_{r \times r} & C \\ O & B_{k \times k} \end{array} \right)$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解 若存在矩阵 W, X, Y, Z 使得

$$\left(\begin{array}{cc} I & O \\ O & I \end{array} \right) = \left(\begin{array}{cc} A & C \\ O & B \end{array} \right) \left(\begin{array}{cc} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{array} \right) = \left(\begin{array}{cc} AX + CW & AZ + CY \\ BW & BY \end{array} \right)$$

则 D 可逆，且 $D^{-1} = \left(\begin{array}{cc} X & Z \\ W & Y \end{array} \right)$。由上式得

$$\left\{ \begin{array}{l} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{array} \right. \Rightarrow \left\{ \begin{array}{l} Z = -A^{-1}CY = -A^{-1}CB^{-1} \\ W = O \\ Y = B^{-1} \end{array} \right.$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解 若存在矩阵 W, X, Y, Z 使得

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix} = \begin{pmatrix} AX + CW & AZ + CY \\ BW & BY \end{pmatrix}$$

则 D 可逆，且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。由上式得

$$\begin{cases} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{cases} \Rightarrow \begin{cases} X = A^{-1}(I - CW) \\ Z = -A^{-1}CY = -A^{-1}CB^{-1} \\ W = O \\ Y = B^{-1} \end{cases}$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解 若存在矩阵 W, X, Y, Z 使得

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix} = \begin{pmatrix} AX + CW & AZ + CY \\ BW & BY \end{pmatrix}$$

则 D 可逆，且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。由上式得

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例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

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则 D 可逆，且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。由上式得

$$\begin{cases} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{cases} \Rightarrow \begin{cases} X = A^{-1}(I - CW) = A^{-1} \\ Z = -A^{-1}CY = -A^{-1}CB^{-1} \\ W = O \\ Y = B^{-1} \end{cases}$$

所以 D 可逆，且 $D^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}CB^{-1} \\ O & B^{-1} \end{pmatrix}$

$$A, B \text{ 可逆} \quad \Rightarrow \quad \begin{pmatrix} A & C \\ O & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}CB^{-1} \\ O & B^{-1} \end{pmatrix}$$

$$A, B \text{ 可逆} \quad \Rightarrow \quad \begin{pmatrix} A & C \\ O & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}CB^{-1} \\ O & B^{-1} \end{pmatrix}$$

注 特别地, 当 $C = O$ 时,

A, B 可逆 $\Rightarrow$ $\begin{pmatrix} A & C \\ O & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}CB^{-1} \\ O & B^{-1} \end{pmatrix}$

注 特别地, 当 $C = O$ 时,

A, B 可逆 $\Rightarrow$ $\begin{pmatrix} A & O \\ O & B \end{pmatrix}^{-1}$

A, B 可逆 $\Rightarrow$ $\begin{pmatrix} A & C \\ O & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}CB^{-1} \\ O & B^{-1} \end{pmatrix}$

注 特别地，当 $C = O$ 时,

A, B 可逆 $\Rightarrow$ $\begin{pmatrix} A & O \\ O & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & O \\ O & B^{-1} \end{pmatrix}$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix}$$

分块矩阵的运算：乘积

例 5 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I=\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}=\begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix}$$

则

$$AI$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix}$$

则

$$AI = A \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix} =$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix}$$

则

$$AI = A \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix} = \begin{pmatrix} A\varepsilon_1 & A\varepsilon_2 & \cdots & A\varepsilon_n \end{pmatrix}$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix}$$

则

$$AI = A \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix} = \begin{pmatrix} A\varepsilon_1 & A\varepsilon_2 & \cdots & A\varepsilon_n \end{pmatrix}$$

$$= \begin{pmatrix} & \vdots & \vdots & \vdots & \\ & \vdots & \vdots & \vdots & \\ & \vdots & \vdots & \vdots & \end{pmatrix}$$

分块矩阵的运算：乘积

例 5 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I=\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}=\begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix}$$

则

$$AI = A\begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix} = \begin{pmatrix} A\varepsilon_1 & A\varepsilon_2 & \cdots & A\varepsilon_n \end{pmatrix}$$

$$=\begin{pmatrix} a_{11} & & & \\ a_{21} & & & \\ \vdots & & & \\ a_{m1} & & & \end{pmatrix}$$

分块矩阵的运算：乘积

例 5 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I=\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}=\begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix}$$

则

$$AI = A\begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix} = \begin{pmatrix} A\varepsilon_1 & A\varepsilon_2 & \cdots & A\varepsilon_n \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} & a_{12} & & \\ a_{21} & a_{22} & & \\ \vdots & \vdots & & \\ a_{m1} & a_{m2} & & \end{pmatrix}$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix}$$

则

$$AI = A \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix} = \begin{pmatrix} A\varepsilon_1 & A\varepsilon_2 & \cdots & A\varepsilon_n \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{1n} \\ \vdots & \vdots & \ddots & a_{1n} \\ a_{m1} & a_{m2} & \cdots & a_{1n} \end{pmatrix}$$

分块矩阵的运算：乘积

例 5 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I=\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}=\begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix}$$

则

$$AI = A\begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix} = \begin{pmatrix} A\varepsilon_1 & A\varepsilon_2 & \cdots & A\varepsilon_n \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{1n} \\ \vdots & \vdots & \ddots & a_{1n} \\ a_{m1} & a_{m2} & \cdots & a_{1n} \end{pmatrix} = \begin{pmatrix} A_1 & A_2 & \cdots & A_n \end{pmatrix}$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix}$$

则

$$AI = A \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix} = \begin{pmatrix} A\varepsilon_1 & A\varepsilon_2 & \cdots & A\varepsilon_n \end{pmatrix}$$

$$= \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{1n} \\ \vdots & \vdots & \ddots & a_{1n} \\ a_{m1} & a_{m2} & \cdots & a_{1n} \end{pmatrix} = \begin{pmatrix} A_1 & A_2 & \cdots & A_n \end{pmatrix} = A$$

分块矩阵的运算：乘积

设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix}$$

分块矩阵的运算：乘积

例 6 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix}$$

分块矩阵的运算：乘积

例 6 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B = \left(\begin{array}{c|c|c|c} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{array}\right) = \begin{pmatrix} \beta_1 & \beta_2 & \cdots & \beta_l \end{pmatrix}$$

分块矩阵的运算：乘积

例 6 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix} = \begin{pmatrix} \beta_1 & \beta_2 & \cdots & \beta_l \end{pmatrix}$$

则

$$AB = A(\beta_1, \beta_2, \cdots, \beta_l)$$

分块矩阵的运算：乘积

例 6 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix} = \begin{pmatrix} \beta_1 & \beta_2 & \cdots & \beta_l \end{pmatrix}$$

则

$$AB = A(\beta_1, \beta_2, \cdots, \beta_l) = (A\beta_1, A\beta_2, \cdots, A\beta_l)$$

分块矩阵的运算：乘积

例 6 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix} = \begin{pmatrix} \beta_1 & \beta_2 & \cdots & \beta_l \end{pmatrix}$$

则

$$AB = A(\beta_1, \beta_2, \cdots, \beta_l) = (A\beta_1, A\beta_2, \cdots, A\beta_l)$$

而

$$A\beta_i =$$

分块矩阵的运算：乘积

例 6 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B = \left(\begin{array}{c|c|c|c} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{array}\right) = \begin{pmatrix} \beta_1 & \beta_2 & \cdots & \beta_l \end{pmatrix}$$

则

$$AB = A(\beta_1, \beta_2, \cdots, \beta_l) = (A\beta_1, A\beta_2, \cdots, A\beta_l)$$

而

$$A\beta_i = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} b_{1i} \\ b_{2i} \\ \vdots \\ b_{ni} \end{pmatrix}$$

分块矩阵的运算：乘积

例 6 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix} = \begin{pmatrix} \beta_1 & \beta_2 & \cdots & \beta_l \end{pmatrix}$$

则

$$AB = A(\beta_1, \beta_2, \cdots, \beta_l) = (A\beta_1, A\beta_2, \cdots, A\beta_l)$$

而

$$A\beta_i = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} b_{1i} \\ b_{2i} \\ \vdots \\ b_{ni} \end{pmatrix} = b_{1i} \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + b_{2i} \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \cdots + b_{ni} \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix}$$