§6.4 微积分基本定理

2016-2017 学年 II

教学要求

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Outline of §6.4

1. 变上限的定积分

2. 微积分基本定理：牛顿—莱布尼茨公式

We are here now...

1. 变上限的定积分

2. 微积分基本定理：牛顿—莱布尼茨公式

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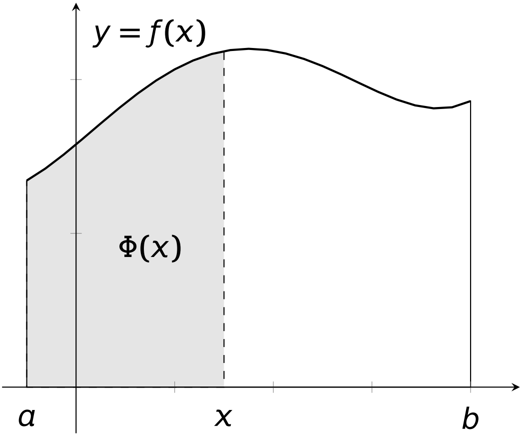

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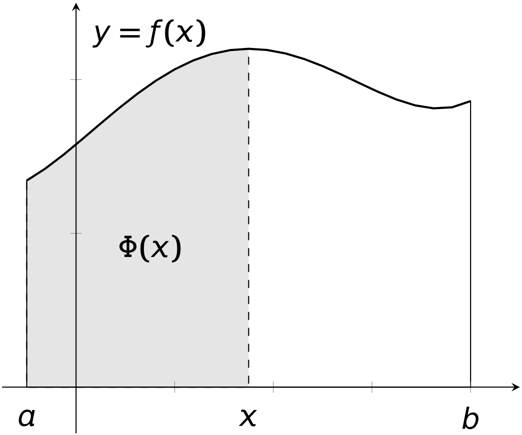

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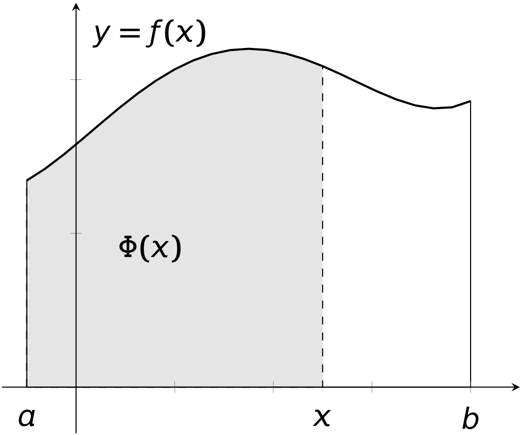

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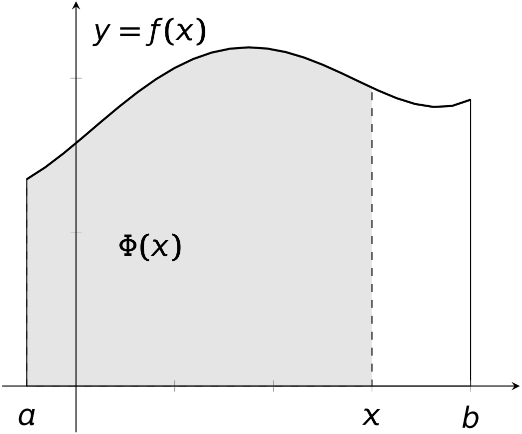

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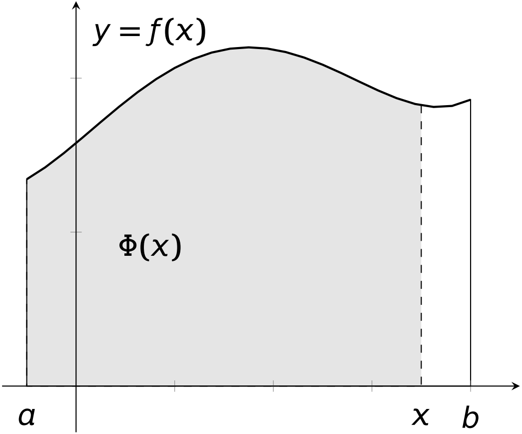

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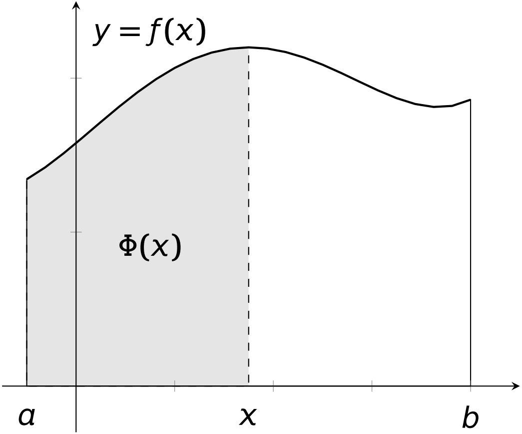

$$\Phi(x) = \int_a^x f(t)dt \qquad \forall x \in [a, b]$$

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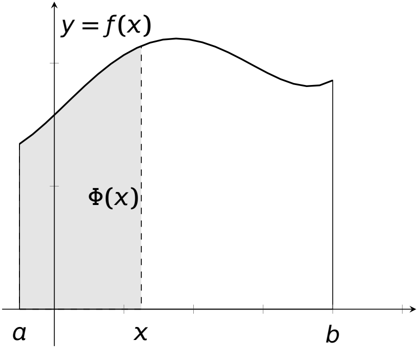

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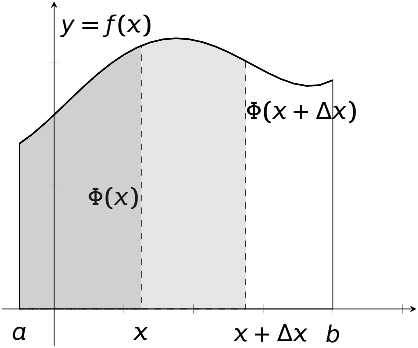

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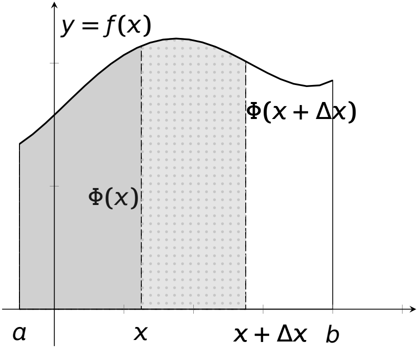

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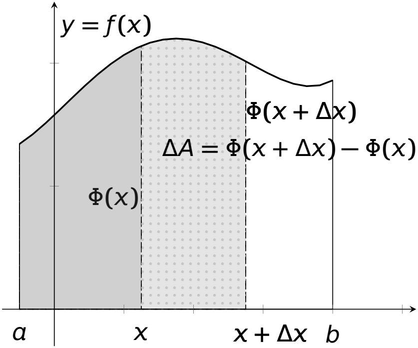

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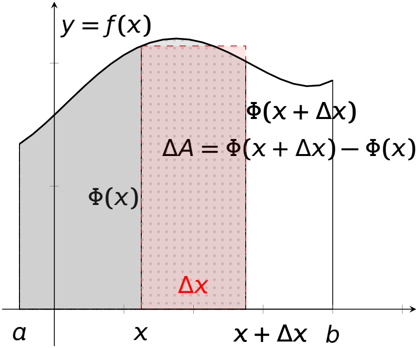

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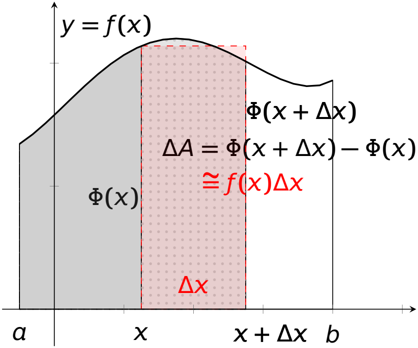

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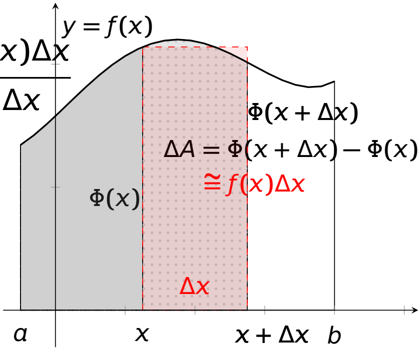

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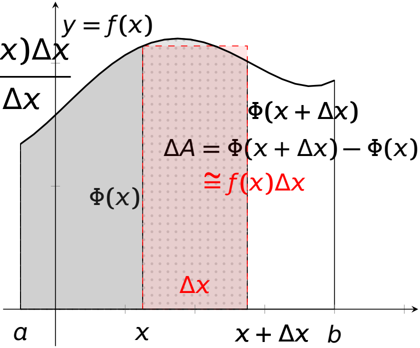

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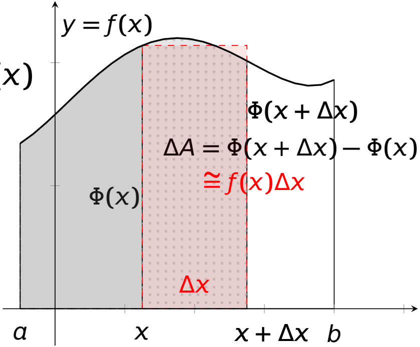

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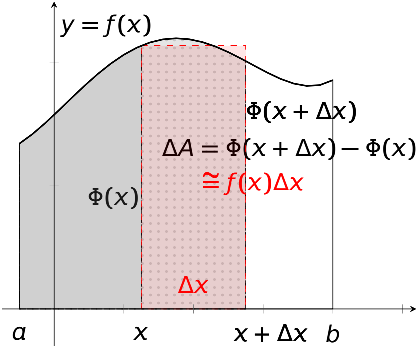

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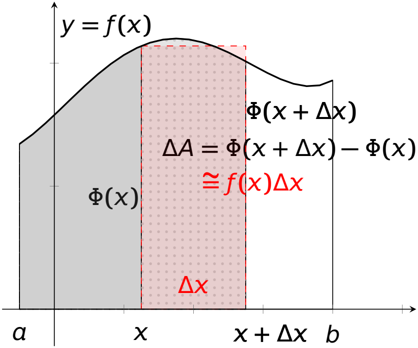

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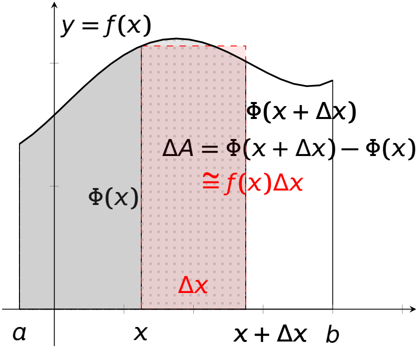

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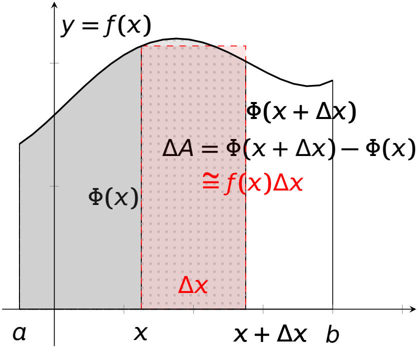

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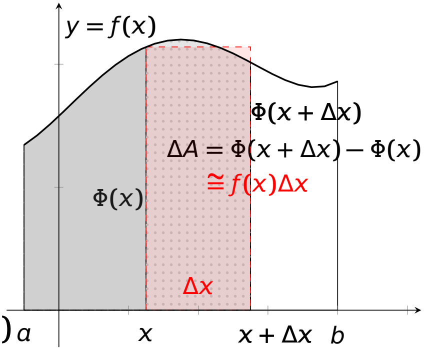

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例 $\left[\int_x^0 e^{-t}\sin(t^2)dt\right]'=$ ____________

解

$$\left[\int_x^0 e^{-t}\sin(t^2)dt\right]'=\left[-\int_0^x e^{-t}\sin(t^2)dt\right]'$$

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$$\because \quad \left[\int_x^{-2} e^{\sin t}dt\right]' = \left[-\int_{-2}^x e^{\sin t}dt\right]'$$

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$$\because\quad \left[\int_x^{-2} e^{\sin t}dt\right]'=\left[-\int_{-2}^x e^{\sin t}dt\right]'=-e^{\sin x}.$$

注：
$$\left[\int_a^{\varphi(x)} f(t)dt\right]' =$$

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例

$\left[\int_1^{x^2}\cos t dt\right]' =$ ____________; $\left[\int_{2x}^{-1}\sqrt{1+t^2}dt\right]' =$ ____________.

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$\left[\int_1^{x^2} \cos t dt\right]' =$ ____________; $\left[\int_{2x}^{-1} \sqrt{1+t^2} dt\right]' =$ ____________.

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解 ：

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$$\left[\int_{2x}^{-1} \sqrt{1+t^2}dt\right]' = -\left[\int_{-1}^{2x} \sqrt{1+t^2}dt\right]' = \quad \sqrt{1+4x^2} \cdot$$

注：

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$\left[\int_1^{x^2}\cos t dt\right]' =$ _______________; $\left[\int_{2x}^{-1}\sqrt{1+t^2}dt\right]' =$ _______________.

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注：
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$$= -\left[\int_0^{x^3}\ln(1+t)dt\right]' + \left[\int_0^{x^2}\ln(1+t)dt\right]'$$

$$= -\ln(1+x^3)\cdot$$

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$$= -3x^2\ln(1+x^3) + 2x\ln(1+x^2).$$

We are here now...

1. 变上限的定积分

2. 微积分基本定理：牛顿—莱布尼茨公式

牛顿—莱布尼茨公式

$$\int_a^b f(x)dx =$$

牛顿—莱布尼茨公式

$$\int_a^b f(x)dx = F(b) - F(a)$$

牛顿—莱布尼茨公式

$$\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b.$$

牛顿—莱布尼茨公式

设 $f(x)$ 在区间 $[a, b]$ 上连续，$F(x)$ 是 $f(x)$ 任意一个原函数，则

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$$\therefore F(x) = \Phi(x) + C$$

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$$= \Phi(b) - \Phi(a)$$

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牛顿—莱布尼茨公式 $\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b$.

例 计算定积分

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

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$$\int_{-2}^{-1} \frac{dx}{x} =$$

例 计算定积分

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

解

$$\int_0^1 x^2 dx = \frac{1}{3}x^3\Big|_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$$

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练习 计算定积分

$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}}dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

练习 计算定积分

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提示：先求出 $\int (2x-5)dx$，$\int \frac{1}{\sqrt{x}}dx$，$\int \frac{dx}{\sqrt{1-x^2}}$

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解

$$\int_0^2 (2x-5)dx =$$

$$\int_4^9 \frac{1}{\sqrt{x}}dx =$$

$$\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$

练习 计算定积分

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$$\int_0^2 (2x-5)dx = (x^2-5x)$$

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练习 计算定积分

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练习 计算定积分

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练习 计算定积分

$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}}dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

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练习 计算定积分

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练习 计算定积分

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练习 计算定积分

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练习 计算定积分

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例 计算定积分 $\int_0^2 |1-x|dx$.

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解 $$\int_0^2 |1-x|dx$$

例 计算定积分 $\int_0^2 |1-x|dx$.

解 $\displaystyle\int_0^2 |1-x|dx$

$$= \int_0^1 |1-x|dx + \int_1^2 |1-x|dx$$

例 计算定积分 $\int_0^2 |1-x|dx$.

解 $$\int_0^2 |1-x|dx$$

$$= \int_0^1 |1-x|dx + \int_1^2 |1-x|dx = \int_0^1 (1-x)dx +$$

例 计算定积分 $\int_0^2 |1-x|dx$.

解
$$\int_0^2 |1-x|dx$$
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例 计算定积分 $\int_0^2 |1-x|dx$.

解 $\int_0^2 |1-x|dx$

$= \int_0^1 |1-x|dx + \int_1^2 |1-x|dx = \int_0^1 (1-x)dx + \int_1^2 (x-1)dx$

$= (x - \frac{1}{2}x^2)$

例 计算定积分 $\int_0^2 |1-x|dx$.

解 $$\int_0^2 |1-x|dx$$

$$= \int_0^1 |1-x|dx + \int_1^2 |1-x|dx = \int_0^1 (1-x)dx + \int_1^2 (x-1)dx$$

$$= (x - \frac{1}{2}x^2) \quad + (\frac{1}{2}x^2 - x)$$

例 计算定积分 $\int_0^2 |1-x|dx$.

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$$=(x-\frac{1}{2}x^2)\Big|_0^1+(\frac{1}{2}x^2-x)\Big|_1^2=[\frac{1}{2}-0]+$$

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练习 计算定积分 $\int_0^3 |2-x|dx$

例 计算定积分 $\int_0^2 |1-x|dx$.

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练习 计算定积分 $\int_0^3 |2-x|dx$

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练习 计算定积分 $\int_0^3 |2-x|dx$

解 $\displaystyle\int_0^3 |2-x|dx$

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例 计算定积分 $\int_0^2 |1-x|dx$.

解 $\displaystyle\int_0^2 |1-x|dx$

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练习 计算定积分 $\int_0^3 |2-x|dx$

解 $\displaystyle\int_0^3 |2-x|dx$

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例 计算定积分 $\int_0^2 |1-x|dx$.

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练习 计算定积分 $\int_0^3 |2-x|dx$

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练习 计算定积分 $\int_0^3 |2-x|dx$

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暨南大學 JINAN UNIVERSITY