

§5.3 换元积分法

数学系 梁卓滨

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教学要求

◇ 熟练掌握换元积分法：“凑微分”，“变量代换”



Outline of §5.3

1. 第一类换元积分法：凑微分

第二类换元积分法：变量代换

We are here now...

1. 第一类换元积分法：凑微分

第二类换元积分法：变量代换

第一类换元积分法——“凑微分”法，能干啥？

能够计算如下的不定积分：

$$\int \frac{dx}{2x+1}, \quad \int \cos\left(\frac{5}{2}x\right)dx$$

$$\int \frac{x}{\sqrt{3-x^2}}dx, \quad \int x \sin(x^2)dx$$

$$\int \frac{(\ln x)^2}{x}dx, \quad \int e^{\sin x} \cos x dx$$

$$\int \frac{1}{\cos x}dx$$

.....

第一类换元积分法（凑微分）原理

- 计算步骤:

$$\int f(x) dx$$

第一类换元积分法（凑微分）原理

- 计算步骤:

$$\int f(\varphi(x))\varphi'(x)dx = \int f(u)du$$

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第一类换元积分法（凑微分）原理

- 计算步骤:

$$\int f(\varphi(x))\varphi'(x)dx = \int f(u)du = F(u) + C = F(\varphi(x)) + C$$

第一类换元积分法（凑微分）原理

- 计算步骤:

$$\int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

第一类换元积分法（凑微分）原理

- 计算步骤:

$$\int f(\varphi(x))\varphi'(x)dx \xrightarrow{\text{凑微分}} \int f(\varphi(x))d\varphi(x)$$

第一类换元积分法（凑微分）原理

- 计算步骤:

$$\int f(\varphi(x)) \varphi'(x) dx \xrightarrow[\varphi(x)=u]{\text{凑微分}} \int f(u) du$$

第一类换元积分法（凑微分）原理

- 计算步骤:

$$\int f(\varphi(x))\varphi'(x)dx \xrightarrow{\text{凑微分}} \int f(\varphi(x))d\varphi(x) \\ \xrightarrow{\varphi(x)=u} \int f(u)du$$

第一类换元积分法（凑微分）原理

- 计算步骤:

$$\begin{aligned}\int f(\varphi(x))\varphi'(x)dx &\stackrel{\text{凑微分}}{=} \int f(\varphi(x))d\varphi(x) \\ &\stackrel{\varphi(x)=u}{=} \int f(u)du \\ &= F(u) + C\end{aligned}$$

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- 验证： $F(\varphi(x))$ 确是 $f(x)$ 的原函数！

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$$\begin{aligned}\int f(\varphi(x))\varphi'(x)dx &\stackrel{\text{凑微分}}{=} \int f(\varphi(x))d\varphi(x) \\ &\stackrel{\varphi(x)=u}{=} \int f(u)du \\ &= F(u) + C \stackrel{u=\varphi(x)}{=} F(\varphi(x)) + C\end{aligned}$$

- 验证: $F(\varphi(x))$ 确是 $f(\varphi(x))\varphi'(x)$ 的原函数!

$$\frac{d}{dx}F(\varphi(x)) =$$

第一类换元积分法（凑微分）原理

- 计算步骤:

$$\begin{aligned}\int f(\varphi(x))\varphi'(x)dx &\stackrel{\text{凑微分}}{=} \int f(\varphi(x))d\varphi(x) \\ &\stackrel{\varphi(x)=u}{=} \int f(u)du \\ &= F(u) + C \stackrel{u=\varphi(x)}{=} F(\varphi(x)) + C\end{aligned}$$

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$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) =$$

第一类换元积分法（凑微分）原理

- 计算步骤:

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$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \cdot \varphi'(x) =$$

第一类换元积分法（凑微分）原理

- 计算步骤:

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第一类换元积分法（凑微分）原理

- 计算步骤:

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- 验证: $F(\varphi(x))$ 确是 $f(\varphi(x)) \varphi'(x)$ 的原函数!

$$\frac{d}{dx} F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \varphi'(x)$$

总之

$$\begin{aligned}\int f(\varphi(x)) \varphi'(x) dx &\stackrel{\text{凑微分}}{=} \int f(\varphi(x)) d\varphi(x) \\ &= \int f(u) du = F(u) + C = F(\varphi(x)) + C\end{aligned}$$

凑微分 类型 I: $\int f(ax + b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax + b)dx$$

凑微分 类型 I: $\int f(ax + b)dx$

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凑微分 类型 I: $\int f(ax + b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax + b)dx \qquad d(ax + b)$$

凑微分 类型 I: $\int f(ax + b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax + b)dx \qquad \frac{1}{a}d(ax + b)$$

凑微分 类型 I: $\int f(ax + b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax + b)dx = \int f(ax + b) \cdot \frac{1}{a} d(ax + b)$$

凑微分 类型 I: $\int f(ax + b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax + b)dx = \int f(ax + b) \cdot \frac{1}{a} d(ax + b)$$

$u = ax + b$

凑微分 类型 I: $\int f(ax + b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax + b)dx = \int f(ax + b) \cdot \frac{1}{a}d(ax + b)$$

$$\underline{\underline{u=ax+b}} \int f(u) \cdot \frac{1}{a}du =$$

凑微分 类型 I: $\int f(ax + b)dx$

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$$\int f(u)du = F(u) + C$$

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例 1 $\int \frac{1}{1+2x}dx =$

凑微分 类型 I: $\int f(ax+b)dx$

假设会算

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例 1 $\int \frac{1}{1+2x}dx = \qquad d(1+2x)$

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例 1 $\int \frac{1}{1+2x}dx = \frac{1}{2}d(1+2x)$

凑微分 类型 I: $\int f(ax + b)dx$

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例 1
$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2}d(1+2x) = \frac{1}{2} \int \frac{1}{u}du$$

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凑微分 类型 I: $\int f(ax+b)dx$

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$$\begin{aligned}\int f(ax+b)dx &= \int f(ax+b) \cdot \frac{1}{a}d(ax+b) \\ &\stackrel{u=ax+b}{=} \int f(u) \cdot \frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ax+b) + C\end{aligned}$$

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凑微分 类型 I: $\int f(ax + b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$, $\int \sqrt{3x-1}dx$

凑微分 类型 I: $\int f(ax+b)dx$

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解 $\int \frac{1}{2-3x}dx =$

$$\int \sqrt{3x-1}dx =$$

凑微分 类型 I: $\int f(ax + b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$, $\int \sqrt{3x-1}dx$

解 $\int \frac{1}{2-3x}dx =$ $d(2-3x)$

$$\int \sqrt{3x-1}dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

解 $\int \frac{1}{2-3x} dx = \quad \cdot \left(-\frac{1}{3}\right) d(2-3x)$

$$\int \sqrt{3x-1} dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$, $\int \sqrt{3x-1}dx$

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$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot \left(-\frac{1}{3}\right)d(2-3x)$$

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凑微分 类型 I: $\int f(ax+b)dx$

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解
$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot \left(-\frac{1}{3}\right)d(2-3x) = -\frac{1}{3} \int \frac{1}{u}du$$

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$$= -\frac{1}{3} \ln|u| + C$$

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$$\int \sqrt{3x-1}dx = \quad \quad \quad d(3x-1)$$

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$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \quad \cdot \frac{1}{3}d(3x-1)$$

凑微分 类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$, $\int \sqrt{3x-1}dx$

$$\begin{aligned}\text{解 } \int \frac{1}{2-3x}dx &= \int \frac{1}{2-3x} \cdot \left(-\frac{1}{3}\right)d(2-3x) = -\frac{1}{3} \int \frac{1}{u}du \\ &= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C\end{aligned}$$

$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1} \cdot \frac{1}{3}d(3x-1)$$

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$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1} \cdot \frac{1}{3}d(3x-1) = \frac{1}{3} \int \sqrt{u}du = \frac{1}{3} \int u^{1/2}du$$

凑微分 类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$, $\int \sqrt{3x-1}dx$

解
$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot \left(-\frac{1}{3}\right)d(2-3x) = -\frac{1}{3} \int \frac{1}{u}du$$
$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

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$$u^{3/2}$$

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$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

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$$\frac{2}{3}u^{3/2}$$

凑微分 类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$, $\int \sqrt{3x-1}dx$

$$\begin{aligned}\text{解 } \int \frac{1}{2-3x}dx &= \int \frac{1}{2-3x} \cdot \left(-\frac{1}{3}\right)d(2-3x) = -\frac{1}{3} \int \frac{1}{u}du \\ &= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C\end{aligned}$$

$$\begin{aligned}\int \sqrt{3x-1}dx &= \int \sqrt{3x-1} \cdot \frac{1}{3}d(3x-1) = \frac{1}{3} \int \sqrt{u}du = \frac{1}{3} \int u^{1/2}du \\ &= \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C\end{aligned}$$

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$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1} \cdot \frac{1}{3}d(3x-1) = \frac{1}{3} \int \sqrt{u}du = \frac{1}{3} \int u^{1/2}du$$
$$= \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C = \frac{2}{9} (3x-1)^{3/2} + C$$

凑微分 类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$, $\int \sqrt{3x-1}dx$

$$\begin{aligned}\text{解 } \int \frac{1}{2-3x}dx &= \int \frac{1}{2-3x} \cdot \left(-\frac{1}{3}\right)d(2-3x) = -\frac{1}{3} \int \frac{1}{u}du \\ &= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C\end{aligned}$$

$$\begin{aligned}\int \sqrt{3x-1}dx &= \int \sqrt{3x-1} \cdot \frac{1}{3}d(3x-1) = \frac{1}{3} \int \sqrt{u}du = \frac{1}{3} \int u^{1/2}du \\ &= \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C = \frac{2}{9} (3x-1)^{3/2} + C\end{aligned}$$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx =$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解 $\int \frac{1}{\sqrt{1-5x}}dx = \qquad d(1-5x)$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解 $\int \frac{1}{\sqrt{1-5x}}dx = \int \frac{1}{\sqrt{1-5x}} \cdot (-\frac{1}{5})d(1-5x)$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\begin{aligned}\int \frac{1}{\sqrt{1-5x}}dx &= \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du \\ &= u^{1/2}\end{aligned}$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\begin{aligned}\int \frac{1}{\sqrt{1-5x}}dx &= \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du \\ &= 2u^{1/2}\end{aligned}$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\begin{aligned}\int \frac{1}{\sqrt{1-5x}}dx &= \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du \\ &= -\frac{1}{5} \cdot 2u^{1/2} + C\end{aligned}$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\begin{aligned}\int \frac{1}{\sqrt{1-5x}}dx &= \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du \\ &= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C\end{aligned}$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\begin{aligned}\int \frac{1}{\sqrt{1-5x}}dx &= \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du \\ &= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C\end{aligned}$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos(u) \cdot d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\begin{aligned}\int \frac{1}{\sqrt{1-5x}}dx &= \int (1-5x)^{-1/2} \cdot \left(-\frac{1}{5}\right)d(1-5x) = -\frac{1}{5} \int u^{-1/2}du \\ &= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C\end{aligned}$$

$$\int \cos\left(\frac{3}{2}x\right)dx = \frac{2}{3}d\left(\frac{3}{2}x\right)$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\begin{aligned}\int \frac{1}{\sqrt{1-5x}}dx &= \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du \\ &= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C\end{aligned}$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos \frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\begin{aligned}\int \frac{1}{\sqrt{1-5x}}dx &= \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du \\ &= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C\end{aligned}$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos \frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3} \int \cos u du$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\begin{aligned}\int \frac{1}{\sqrt{1-5x}}dx &= \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du \\ &= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C\end{aligned}$$

$$\begin{aligned}\int \cos(\frac{3}{2}x)dx &= \int \cos \frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3} \int \cos u du \\ &= \frac{2}{3} \sin(u) + C\end{aligned}$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\begin{aligned}\int \frac{1}{\sqrt{1-5x}}dx &= \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du \\ &= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C\end{aligned}$$

$$\begin{aligned}\int \cos(\frac{3}{2}x)dx &= \int \cos \frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3} \int \cos u du \\ &= \frac{2}{3} \sin(u) + C = \frac{2}{3} \sin(\frac{3}{2}x) + C\end{aligned}$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\begin{aligned}\int \frac{1}{\sqrt{1-5x}}dx &= \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du \\ &= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C\end{aligned}$$

$$\begin{aligned}\int \cos(\frac{3}{2}x)dx &= \int \cos \frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3} \int \cos u du \\ &= \frac{2}{3} \sin(u) + C = \frac{2}{3} \sin(\frac{3}{2}x) + C\end{aligned}$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4} d(-\frac{1}{2}x+4)$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\begin{aligned}\int \frac{1}{\sqrt{1-5x}}dx &= \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du \\ &= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C\end{aligned}$$

$$\begin{aligned}\int \cos(\frac{3}{2}x)dx &= \int \cos \frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3} \int \cos u du \\ &= \frac{2}{3} \sin(u) + C = \frac{2}{3} \sin(\frac{3}{2}x) + C\end{aligned}$$

$$\int e^{-\frac{1}{2}x+4}dx = \quad \quad \quad \cdot (-2)d(-\frac{1}{2}x+4)$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\begin{aligned}\int \frac{1}{\sqrt{1-5x}}dx &= \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du \\ &= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C\end{aligned}$$

$$\begin{aligned}\int \cos(\frac{3}{2}x)dx &= \int \cos \frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3} \int \cos u du \\ &= \frac{2}{3} \sin(u) + C = \frac{2}{3} \sin(\frac{3}{2}x) + C\end{aligned}$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4} \cdot (-2)d(-\frac{1}{2}x+4)$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\begin{aligned}\int \frac{1}{\sqrt{1-5x}}dx &= \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du \\ &= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C\end{aligned}$$

$$\begin{aligned}\int \cos(\frac{3}{2}x)dx &= \int \cos \frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3} \int \cos u du \\ &= \frac{2}{3} \sin(u) + C = \frac{2}{3} \sin(\frac{3}{2}x) + C\end{aligned}$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4} \cdot (-2)d(-\frac{1}{2}x+4) = -2 \int e^u du$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\begin{aligned}\int \frac{1}{\sqrt{1-5x}}dx &= \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du \\ &= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C\end{aligned}$$

$$\begin{aligned}\int \cos(\frac{3}{2}x)dx &= \int \cos \frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3} \int \cos u du \\ &= \frac{2}{3} \sin(u) + C = \frac{2}{3} \sin(\frac{3}{2}x) + C\end{aligned}$$

$$\begin{aligned}\int e^{-\frac{1}{2}x+4}dx &= \int e^{-\frac{1}{2}x+4} \cdot (-2)d(-\frac{1}{2}x+4) = -2 \int e^u du \\ &= -2e^u + C\end{aligned}$$

凑微分 类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\begin{aligned}\int \frac{1}{\sqrt{1-5x}}dx &= \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du \\ &= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C\end{aligned}$$

$$\begin{aligned}\int \cos(\frac{3}{2}x)dx &= \int \cos \frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3} \int \cos u du \\ &= \frac{2}{3} \sin(u) + C = \frac{2}{3} \sin(\frac{3}{2}x) + C\end{aligned}$$

$$\begin{aligned}\int e^{-\frac{1}{2}x+4}dx &= \int e^{-\frac{1}{2}x+4} \cdot (-2)d(-\frac{1}{2}x+4) = -2 \int e^u du \\ &= -2e^u + C = -2e^{-\frac{1}{2}x+4} + C\end{aligned}$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx \qquad d(ax^2 + b)$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx \qquad \frac{1}{2a}d(ax^2 + b)$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a}d(ax^2 + b)$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a}d(ax^2 + b)$$

$$\underline{\underline{u=ax^2+b}}$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a}d(ax^2 + b)$$

$$\underline{\underline{u=ax^2+b}} \int f(u) \cdot \frac{1}{2a}du =$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a} d(ax^2 + b)$$

$$\underline{\underline{u=ax^2+b}} \int f(u) \cdot \frac{1}{2a} du = F(u)$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a}d(ax^2 + b)$$

$$\underline{\underline{u=ax^2+b}} \int f(u) \cdot \frac{1}{2a}du = \frac{1}{2a}F(u)$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a}d(ax^2 + b)$$

$$\underline{\underline{u=ax^2+b}} \int f(u) \cdot \frac{1}{2a}du = \frac{1}{2a}F(u) + C$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a}d(ax^2 + b)$$

$$\underline{\underline{u=ax^2+b}} \int f(u) \cdot \frac{1}{2a}du = \frac{1}{2a}F(u) + C = \frac{1}{2a}F(ax^2 + b) + C$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a}d(ax^2 + b)$$

$$\xrightarrow{u=ax^2+b} \int f(u) \cdot \frac{1}{2a}du = \frac{1}{2a}F(u) + C = \frac{1}{2a}F(ax^2 + b) + C$$

例 1 $\int x\sqrt{1-x^2}dx =$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a}d(ax^2 + b)$$

$$\xrightarrow{u=ax^2+b} \int f(u) \cdot \frac{1}{2a}du = \frac{1}{2a}F(u) + C = \frac{1}{2a}F(ax^2 + b) + C$$

例 1 $\int x\sqrt{1-x^2}dx =$ $d(1-x^2)$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a}d(ax^2 + b)$$

$$\xrightarrow{u=ax^2+b} \int f(u) \cdot \frac{1}{2a}du = \frac{1}{2a}F(u) + C = \frac{1}{2a}F(ax^2 + b) + C$$

例 1 $\int x\sqrt{1-x^2}dx = \quad \cdot \left(-\frac{1}{2}\right)d(1-x^2)$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\begin{aligned}\int f(ax^2 + b)xdx &= \int f(ax^2 + b) \cdot \frac{1}{2a}d(ax^2 + b) \\ &\stackrel{u=ax^2+b}{=} \int f(u) \cdot \frac{1}{2a}du = \frac{1}{2a}F(u) + C = \frac{1}{2a}F(ax^2 + b) + C\end{aligned}$$

例 1 $\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot \left(-\frac{1}{2}\right)d(1-x^2)$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a}d(ax^2 + b)$$

$$\xrightarrow{u=ax^2+b} \int f(u) \cdot \frac{1}{2a}du = \frac{1}{2a}F(u) + C = \frac{1}{2a}F(ax^2 + b) + C$$

例 1 $\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2} \int u^{\frac{1}{2}}du$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a}d(ax^2 + b)$$

$$\xrightarrow{u=ax^2+b} \int f(u) \cdot \frac{1}{2a}du = \frac{1}{2a}F(u) + C = \frac{1}{2a}F(ax^2 + b) + C$$

例 1 $\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2} \int u^{\frac{1}{2}}du$

$$u^{3/2}$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a}d(ax^2 + b)$$

$$\xrightarrow{u=ax^2+b} \int f(u) \cdot \frac{1}{2a}du = \frac{1}{2a}F(u) + C = \frac{1}{2a}F(ax^2 + b) + C$$

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$$= -\frac{2}{3}u^{3/2}$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

假设会算

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$$\begin{aligned}\int x\sqrt{1-x^2}dx &= \int (1-x^2)^{\frac{1}{2}} \cdot \left(-\frac{1}{2}\right)d(1-x^2) = -\frac{1}{2} \int u^{\frac{1}{2}}du \\ &= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C\end{aligned}$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx =$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \qquad d(3-x^2)$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \quad \cdot \left(-\frac{1}{2}\right)d(3-x^2)$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot \left(-\frac{1}{2}\right)d(3-x^2)$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

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$$\begin{aligned}\int \frac{x}{\sqrt{3-x^2}}dx &= \int (3-x^2)^{-\frac{1}{2}} \cdot \left(-\frac{1}{2}\right)d(3-x^2) \\ &= -\frac{1}{2} \int u^{-1/2} du\end{aligned}$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

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$$\int \frac{x}{1+3x^2}dx =$$

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例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

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$$\int \frac{x}{1+3x^2}dx = \qquad d(1+3x^2)$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

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$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot \left(-\frac{1}{2}\right)d(3-x^2)$$

$$= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx = \frac{1}{6}d(1+3x^2)$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\begin{aligned}\int \frac{x}{\sqrt{3-x^2}}dx &= \int (3-x^2)^{-\frac{1}{2}} \cdot \left(-\frac{1}{2}\right)d(3-x^2) \\ &= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C\end{aligned}$$

$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6}d(1+3x^2)$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

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$$\begin{aligned}\int \frac{x}{\sqrt{3-x^2}}dx &= \int (3-x^2)^{-\frac{1}{2}} \cdot \left(-\frac{1}{2}\right)d(3-x^2) \\ &= -\frac{1}{2} \int u^{-1/2}du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C\end{aligned}$$

$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6}d(1+3x^2) = \frac{1}{6} \int \frac{1}{u}du$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

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$$\begin{aligned}\int \frac{x}{1+3x^2}dx &= \int \frac{1}{1+3x^2} \cdot \frac{1}{6}d(1+3x^2) = \frac{1}{6} \int \frac{1}{u}du \\ &= \frac{1}{6} \ln|u| + C\end{aligned}$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

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凑微分 类型 II: $\int f(ax^2 + b)xdx$

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$$\begin{aligned}\int \frac{x}{1+3x^2}dx &= \int \frac{1}{1+3x^2} \cdot \frac{1}{6}d(1+3x^2) = \frac{1}{6} \int \frac{1}{u}du \\ &= \frac{1}{6} \ln|u| + C = \frac{1}{6} \ln|1+3x^2| + C\end{aligned}$$

例 3 求 $\int xe^{x^2}dx$, $\int x \sin(x^2)dx$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 3 求 $\int xe^{x^2} dx$, $\int x \sin(x^2) dx$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 3 求 $\int xe^{x^2} dx$, $\int x \sin(x^2) dx$

解

$$\int xe^{x^2} dx =$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 3 求 $\int xe^{x^2} dx$, $\int x \sin(x^2) dx$

解

$$\int xe^{x^2} dx = \int \frac{1}{2} e^{x^2} d(x^2)$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 3 求 $\int xe^{x^2} dx$, $\int x \sin(x^2) dx$

解

$$\int xe^{x^2} dx = \frac{1}{2} d(x^2)$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 3 求 $\int xe^{x^2} dx$, $\int x \sin(x^2) dx$

解

$$\int xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2)$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 3 求 $\int xe^{x^2} dx$, $\int x \sin(x^2) dx$

解

$$\int xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 3 求 $\int xe^{x^2} dx$, $\int x \sin(x^2) dx$

解

$$\int xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

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$$\int xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 3 求 $\int xe^{x^2} dx$, $\int x \sin(x^2) dx$

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$$\int xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$
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凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 3 求 $\int xe^{x^2} dx$, $\int x \sin(x^2) dx$

解

$$\int xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$
$$\int x \sin(x^2) dx = \quad \quad \quad d(x^2)$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 3 求 $\int xe^{x^2} dx$, $\int x \sin(x^2) dx$

解

$$\int xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$
$$\int x \sin(x^2) dx = \quad \cdot \frac{1}{2} d(x^2)$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 3 求 $\int xe^{x^2} dx$, $\int x \sin(x^2) dx$

解

$$\int xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

$$\int x \sin(x^2) dx = \int \sin(x^2) \cdot \frac{1}{2} d(x^2)$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 3 求 $\int xe^{x^2} dx$, $\int x \sin(x^2) dx$

解

$$\int xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

$$\int x \sin(x^2) dx = \int \sin(x^2) \cdot \frac{1}{2} d(x^2) = \frac{1}{2} \int \sin u du$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 3 求 $\int xe^{x^2} dx$, $\int x \sin(x^2) dx$

解

$$\int xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

$$\begin{aligned} \int x \sin(x^2) dx &= \int \sin(x^2) \cdot \frac{1}{2} d(x^2) = \frac{1}{2} \int \sin u du \\ &= -\frac{1}{2} \cos u + C \end{aligned}$$

凑微分 类型 II: $\int f(ax^2 + b)xdx$

例 3 求 $\int xe^{x^2} dx$, $\int x \sin(x^2) dx$

解

$$\int xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

$$\begin{aligned} \int x \sin(x^2) dx &= \int \sin(x^2) \cdot \frac{1}{2} d(x^2) = \frac{1}{2} \int \sin u du \\ &= -\frac{1}{2} \cos u + C = -\frac{1}{2} \cos(x^2) + C \end{aligned}$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x + b)e^x dx$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x + b)e^x dx$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x + b)e^x dx \qquad d(ae^x + b)$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x + b)e^x dx \qquad \frac{1}{a}d(ae^x + b)$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x + b)e^x dx = \int f(ax^2 + b) \cdot \frac{1}{a} d(ae^x + b)$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x + b)e^x dx = \int f(ax^2 + b) \cdot \frac{1}{a} d(ae^x + b)$$

$$\underline{\underline{u=ae^x+b}}$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x + b)e^x dx = \int f(ax^2 + b) \cdot \frac{1}{a} d(ae^x + b)$$

$$\underline{\underline{u=ae^x+b}} \int f(u) \cdot \frac{1}{a} du =$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u) du = F(u) + C$$

则

$$\int f(ae^x + b) \mathbf{e^x dx} = \int f(ax^2 + b) \cdot \frac{1}{a} \mathbf{d(ae^x + b)}$$

$$\underline{\underline{u=ae^x+b}} \int f(u) \cdot \frac{1}{a} du = F(u)$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x + b)e^x dx = \int f(ax^2 + b) \cdot \frac{1}{a} d(ae^x + b)$$

$$\underline{\underline{u=ae^x+b}} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u)$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

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$$\int f(ae^x + b)e^x dx = \int f(ax^2 + b) \cdot \frac{1}{a} d(ae^x + b)$$

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假设会算

$$\int f(u)du = F(u) + C$$

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$$\int f(ae^x + b)e^x dx = \int f(ax^2 + b) \cdot \frac{1}{a} d(ae^x + b)$$

$$\underline{\underline{u=ae^x+b}} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(ae^x + b) + C$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x + b)e^x dx = \int f(ax^2 + b) \cdot \frac{1}{a} d(ae^x + b)$$

$$\underline{\underline{u=ae^x+b}} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(ae^x + b) + C$$

例 1 $\int \frac{e^x}{1 + e^x} dx =$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\begin{aligned}\int f(ae^x + b)e^x dx &= \int f(ax^2 + b) \cdot \frac{1}{a} d(ae^x + b) \\ &\stackrel{u=ae^x+b}{=} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(ae^x + b) + C\end{aligned}$$

例 1 $\int \frac{e^x}{1 + e^x} dx = \int d(e^x + 1)$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x + b)e^x dx = \int f(ax^2 + b) \cdot \frac{1}{a} d(ae^x + b)$$

$$\underline{\underline{u=ae^x+b}} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(ae^x + b) + C$$

例 1 $\int \frac{e^x}{1 + e^x} dx = \int \frac{1}{1 + e^x} d(e^x + 1)$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\begin{aligned}\int f(ae^x + b)e^x dx &= \int f(ax^2 + b) \cdot \frac{1}{a} d(ae^x + b) \\ &\stackrel{\underline{u=ae^x+b}}{=} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(ae^x + b) + C\end{aligned}$$

例 1

$$\begin{aligned}\int \frac{e^x}{1 + e^x} dx &= \int \frac{1}{1 + e^x} d(e^x + 1) \\ &= \int \frac{1}{u} du\end{aligned}$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\begin{aligned}\int f(ae^x + b)e^x dx &= \int f(ax^2 + b) \cdot \frac{1}{a} d(ae^x + b) \\ &\stackrel{\underline{u=ae^x+b}}{=} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(ae^x + b) + C\end{aligned}$$

例 1

$$\begin{aligned}\int \frac{e^x}{1 + e^x} dx &= \int \frac{1}{1 + e^x} d(e^x + 1) \\ &= \int \frac{1}{u} du = \ln |u| + C\end{aligned}$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\begin{aligned}\int f(ae^x + b)e^x dx &= \int f(ax^2 + b) \cdot \frac{1}{a} d(ae^x + b) \\ &\stackrel{\underline{u=ae^x+b}}{=} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(ae^x + b) + C\end{aligned}$$

例 1

$$\begin{aligned}\int \frac{e^x}{1 + e^x} dx &= \int \frac{1}{1 + e^x} d(e^x + 1) \\ &= \int \frac{1}{u} du = \ln |u| + C = \ln(e^x + 1) + C\end{aligned}$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

例 2

$$\int e^x \sin(e^x) dx =$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

例 2

$$\int e^x \sin(e^x) dx = \quad de^x$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

例 2

$$\int e^x \sin(e^x) dx = \int \sin(e^x) de^x$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

例 2

$$\begin{aligned}\int e^x \sin(e^x) dx &= \int \sin(e^x) de^x \\ &= \int \sin u du\end{aligned}$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

例 2

$$\begin{aligned}\int e^x \sin(e^x) dx &= \int \sin(e^x) de^x \\ &= \int \sin u du = -\cos u + C\end{aligned}$$

凑微分 类型 III: $\int f(ae^x + b)e^x dx$

例 2

$$\begin{aligned}\int e^x \sin(e^x) dx &= \int \sin(e^x) de^x \\ &= \int \sin u du = -\cos u + C = -\cos(e^x) + C\end{aligned}$$

凑微分 类型 IV: $\int f(a \ln x + b) \frac{1}{x} dx$

假设会算

$$\int f(u) du = F(u) + C$$

则

$$\int f(a \ln x + b) \frac{1}{x} dx$$

凑微分 类型 IV: $\int f(a \ln x + b) \frac{1}{x} dx$

假设会算

$$\int f(u) du = F(u) + C$$

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假设会算

$$\int f(u) du = F(u) + C$$

则

$$\int f(a \ln x + b) \frac{1}{x} dx$$

$$d(a \ln x + b)$$

凑微分 类型 IV: $\int f(a \ln x + b) \frac{1}{x} dx$

假设会算

$$\int f(u) du = F(u) + C$$

则

$$\int f(a \ln x + b) \frac{1}{x} dx = \frac{1}{a} f(a \ln x + b) d(a \ln x + b)$$

凑微分 类型 IV: $\int f(a \ln x + b) \frac{1}{x} dx$

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$$\int f(u) du = F(u) + C$$

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$$\int f(a \ln x + b) \frac{1}{x} dx = \int f(a \ln x + b) \cdot \frac{1}{a} d(a \ln x + b)$$

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$$\underline{\underline{u = a \ln x + b}}$$

凑微分 类型 IV: $\int f(a \ln x + b) \frac{1}{x} dx$

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$$\int f(u) du = F(u) + C$$

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$$\underline{\underline{u = a \ln x + b}} \int f(u) \cdot \frac{1}{a} du =$$

凑微分 类型 IV: $\int f(a \ln x + b) \frac{1}{x} dx$

假设会算

$$\int f(u) du = F(u) + C$$

则

$$\int f(a \ln x + b) \frac{1}{x} dx = \int f(a \ln x + b) \cdot \frac{1}{a} d(a \ln x + b)$$

$$\underline{\underline{u = a \ln x + b}} \quad \int f(u) \cdot \frac{1}{a} du = F(u)$$

凑微分 类型 IV: $\int f(a \ln x + b) \frac{1}{x} dx$

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$$\xrightarrow{u=a \ln x + b} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(a \ln x + b) + C$$

例 $\int \frac{1}{x} \ln x dx =$

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例 $\int \frac{1}{x} \ln x dx = \int d \ln x$

$$\int \frac{1}{x \ln x} dx =$$

凑微分 类型 IV: $\int f(a \ln x + b) \frac{1}{x} dx$

假设会算

$$\int f(u) du = F(u) + C$$

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例 $\int \frac{1}{x} \ln x dx = \int \ln x d \ln x$

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例 $\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$

$$\int \frac{1}{x \ln x} dx =$$

凑微分 类型 IV: $\int f(a \ln x + b) \frac{1}{x} dx$

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$$\int f(u) du = F(u) + C$$

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例 $\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du = \frac{1}{2} u^2 + C$

$$\int \frac{1}{x \ln x} dx =$$

凑微分 类型 IV: $\int f(a \ln x + b) \frac{1}{x} dx$

假设会算

$$\int f(u) du = F(u) + C$$

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$$\int f(a \ln x + b) \frac{1}{x} dx = \int f(a \ln x + b) \cdot \frac{1}{a} d(a \ln x + b)$$

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例 $\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du = \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$

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$$\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d \ln x = \int \frac{1}{u} du$$

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$$\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d \ln x = \int \frac{1}{u} du = \ln |u| + C$$

凑微分 类型 IV: $\int f(a \ln x + b) \frac{1}{x} dx$

假设会算

$$\int f(u) du = F(u) + C$$

则

$$\int f(a \ln x + b) \frac{1}{x} dx = \int f(a \ln x + b) \cdot \frac{1}{a} d(a \ln x + b)$$

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$$\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d \ln x = \int \frac{1}{u} du = \ln |u| + C = \ln |\ln x| + C$$

凑微分类型 IV: $\int f(\cos x) \sin x dx, \int f(\sin x) \cos x dx$

例子 求 $\int e^{\cos x} \sin x dx, \int \frac{\sin x}{1+\cos^2 x} dx, \int \frac{\cos x}{\sin x} dx$

凑微分类型 IV: $\int f(\cos x) \sin x dx, \int f(\sin x) \cos x dx$

例子 求 $\int e^{\cos x} \sin x dx, \int \frac{\sin x}{1+\cos^2 x} dx, \int \frac{\cos x}{\sin x} dx$

解

1. $\int e^{\cos x} \sin x dx =$

凑微分类型 IV: $\int f(\cos x) \sin x dx, \int f(\sin x) \cos x dx$

例子 求 $\int e^{\cos x} \sin x dx, \int \frac{\sin x}{1+\cos^2 x} dx, \int \frac{\cos x}{\sin x} dx$

解

1. $\int e^{\cos x} \sin x dx = \quad \quad \quad d \cos x$

凑微分类型 IV: $\int f(\cos x) \sin x dx, \int f(\sin x) \cos x dx$

例子 求 $\int e^{\cos x} \sin x dx, \int \frac{\sin x}{1+\cos^2 x} dx, \int \frac{\cos x}{\sin x} dx$

解

1. $\int e^{\cos x} \sin x dx = \quad \quad \quad (-1) d \cos x$

凑微分类型 IV: $\int f(\cos x) \sin x dx$, $\int f(\sin x) \cos x dx$

例子 求 $\int e^{\cos x} \sin x dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

解

$$1. \quad \int e^{\cos x} \sin x dx = \int e^{\cos x} \cdot (-1) d \cos x$$

凑微分类型 IV: $\int f(\cos x) \sin x dx$, $\int f(\sin x) \cos x dx$

例子 求 $\int e^{\cos x} \sin x dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

解

$$1. \quad \int e^{\cos x} \sin x dx = \int e^{\cos x} \cdot (-1) d \cos x = - \int e^u du$$

凑微分类型 IV: $\int f(\cos x) \sin x dx$, $\int f(\sin x) \cos x dx$

例子 求 $\int e^{\cos x} \sin x dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

解

$$\begin{aligned} 1. \quad \int e^{\cos x} \sin x dx &= \int e^{\cos x} \cdot (-1) d \cos x = - \int e^u du \\ &= -e^u + C \end{aligned}$$

凑微分类型 IV: $\int f(\cos x) \sin x dx, \int f(\sin x) \cos x dx$

例子 求 $\int e^{\cos x} \sin x dx, \int \frac{\sin x}{1+\cos^2 x} dx, \int \frac{\cos x}{\sin x} dx$

解

$$\begin{aligned} 1. \quad \int e^{\cos x} \sin x dx &= \int e^{\cos x} \cdot (-1) d \cos x = - \int e^u du \\ &= -e^u + C = -e^{\cos x} + C \end{aligned}$$

凑微分类型 IV: $\int f(\cos x) \sin x dx$, $\int f(\sin x) \cos x dx$

例子 求 $\int e^{\cos x} \sin x dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

解

$$\begin{aligned} 1. \quad \int e^{\cos x} \sin x dx &= \int e^{\cos x} \cdot (-1) d \cos x = - \int e^u du \\ &= -e^u + C = -e^{\cos x} + C \end{aligned}$$

$$2. \quad \int \frac{\sin x}{1+\cos^2 x} dx =$$

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$$\begin{aligned} 1. \quad \int e^{\cos x} \sin x dx &= \int e^{\cos x} \cdot (-1) d \cos x = - \int e^u du \\ &= -e^u + C = -e^{\cos x} + C \end{aligned}$$

$$2. \quad \int \frac{\sin x}{1+\cos^2 x} dx = \quad \quad \quad (-1) d \cos x$$

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$$\begin{aligned} 1. \quad \int e^{\cos x} \sin x dx &= \int e^{\cos x} \cdot (-1) d \cos x = - \int e^u du \\ &= -e^u + C = -e^{\cos x} + C \end{aligned}$$

$$2. \quad \int \frac{\sin x}{1+\cos^2 x} dx = \int \frac{1}{1+\cos^2 x} (-1) d \cos x$$

凑微分类型 IV: $\int f(\cos x) \sin x dx$, $\int f(\sin x) \cos x dx$

例子 求 $\int e^{\cos x} \sin x dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

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$$2. \quad \int \frac{\sin x}{1+\cos^2 x} dx = \int \frac{1}{1+\cos^2 x} (-1) d \cos x = - \int \frac{1}{1+u^2} du$$

凑微分类型 IV: $\int f(\cos x) \sin x dx$, $\int f(\sin x) \cos x dx$

例子 求 $\int e^{\cos x} \sin x dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

解

$$\begin{aligned} 1. \quad \int e^{\cos x} \sin x dx &= \int e^{\cos x} \cdot (-1) d \cos x = - \int e^u du \\ &= -e^u + C = -e^{\cos x} + C \end{aligned}$$

$$\begin{aligned} 2. \quad \int \frac{\sin x}{1+\cos^2 x} dx &= \int \frac{1}{1+\cos^2 x} (-1) d \cos x = - \int \frac{1}{1+u^2} du \\ &= -\arctan u + C \end{aligned}$$

凑微分类型 IV: $\int f(\cos x) \sin x dx$, $\int f(\sin x) \cos x dx$

例子 求 $\int e^{\cos x} \sin x dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

解

$$\begin{aligned} 1. \quad \int e^{\cos x} \sin x dx &= \int e^{\cos x} \cdot (-1) d \cos x = - \int e^u du \\ &= -e^u + C = -e^{\cos x} + C \end{aligned}$$

$$\begin{aligned} 2. \quad \int \frac{\sin x}{1+\cos^2 x} dx &= \int \frac{1}{1+\cos^2 x} (-1) d \cos x = - \int \frac{1}{1+u^2} du \\ &= -\arctan u + C = -\arctan(\cos x) + C \end{aligned}$$

凑微分类型 IV: $\int f(\cos x) \sin x dx$, $\int f(\sin x) \cos x dx$

例子 求 $\int e^{\cos x} \sin x dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

解

$$\begin{aligned} 1. \quad \int e^{\cos x} \sin x dx &= \int e^{\cos x} \cdot (-1) d \cos x = - \int e^u du \\ &= -e^u + C = -e^{\cos x} + C \end{aligned}$$

$$\begin{aligned} 2. \quad \int \frac{\sin x}{1+\cos^2 x} dx &= \int \frac{1}{1+\cos^2 x} (-1) d \cos x = - \int \frac{1}{1+u^2} du \\ &= -\arctan u + C = -\arctan(\cos x) + C \end{aligned}$$

$$3. \quad \int \frac{\cos x}{\sin x} dx =$$

凑微分类型 IV: $\int f(\cos x) \sin x dx$, $\int f(\sin x) \cos x dx$

例子 求 $\int e^{\cos x} \sin x dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

解

$$\begin{aligned} 1. \quad \int e^{\cos x} \sin x dx &= \int e^{\cos x} \cdot (-1) d \cos x = - \int e^u du \\ &= -e^u + C = -e^{\cos x} + C \end{aligned}$$

$$\begin{aligned} 2. \quad \int \frac{\sin x}{1+\cos^2 x} dx &= \int \frac{1}{1+\cos^2 x} (-1) d \cos x = - \int \frac{1}{1+u^2} du \\ &= -\arctan u + C = -\arctan(\cos x) + C \end{aligned}$$

$$3. \quad \int \frac{\cos x}{\sin x} dx = \int d \sin x$$

凑微分类型 IV: $\int f(\cos x) \sin x dx$, $\int f(\sin x) \cos x dx$

例子 求 $\int e^{\cos x} \sin x dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

解

$$\begin{aligned} 1. \quad \int e^{\cos x} \sin x dx &= \int e^{\cos x} \cdot (-1) d \cos x = - \int e^u du \\ &= -e^u + C = -e^{\cos x} + C \end{aligned}$$

$$\begin{aligned} 2. \quad \int \frac{\sin x}{1+\cos^2 x} dx &= \int \frac{1}{1+\cos^2 x} (-1) d \cos x = - \int \frac{1}{1+u^2} du \\ &= -\arctan u + C = -\arctan(\cos x) + C \end{aligned}$$

$$3. \quad \int \frac{\cos x}{\sin x} dx = \int \frac{1}{\sin x} d \sin x$$

凑微分类型 IV: $\int f(\cos x) \sin x dx$, $\int f(\sin x) \cos x dx$

例子 求 $\int e^{\cos x} \sin x dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

解

$$\begin{aligned} 1. \quad \int e^{\cos x} \sin x dx &= \int e^{\cos x} \cdot (-1) d \cos x = - \int e^u du \\ &= -e^u + C = -e^{\cos x} + C \end{aligned}$$

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$$\begin{aligned} 3. \quad \int \frac{\cos x}{\sin x} dx &= \int \frac{1}{\sin x} d \sin x = \int \frac{1}{u} du = \ln |u| + C \\ &= \ln |\sin x| + C \end{aligned}$$

凑微分法 “ $\int f(\varphi(x))d\varphi(x)$ ”: 例子总结

$$\int \frac{1}{1-3x} dx =$$

$$\int \sqrt{3x-1} dx =$$

$$\int x e^{x^2} dx =$$

$$\int x \sqrt{1-x^2} dx =$$

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$$\int \frac{1}{1-3x} dx = -\frac{1}{3} \int \frac{1}{1-3x} d(1-3x) = -\frac{1}{3} \int \frac{1}{u} du = \dots$$

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$$\int e^{\cos x} \sin x dx = -\int e^{\cos x} d \cos x = -\int e^u du = \dots$$

We are here now...

1. 第一类换元积分法：凑微分

第二类换元积分法：变量代换

第二类换元积分法——“变量代换”法，能干啥？

能够计算如下的不定积分：

$$\begin{aligned} & \int x\sqrt{3x-1}dx, \quad \int \frac{x}{\sqrt{x-2}}dx \\ & \int \frac{1}{1+\sqrt{x}}dx, \quad \int \frac{1}{1+\sqrt[3]{x+1}}dx \\ & \int \frac{1}{\sqrt{1+e^x}}dx \\ & \dots\dots \end{aligned}$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xrightarrow{x=\varphi(t)}$$

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- 计算步骤：

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t)$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int f(\varphi(t))\varphi'(t)dt$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x) dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t)) d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单, 容易求!}} dt$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x) dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t)) d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单, 容易求!}} dt$$
$$= G(t) + C$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\begin{aligned}\int f(x)dx &\stackrel{x=\varphi(t)}{=} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单, 容易求!}} dt \\ &= G(t) + C \stackrel{t=\varphi^{-1}(x)}{=}\end{aligned}$$

第二类换元积分法（变量代换）原理

- 计算步骤：

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第二类换元积分法（变量代换）原理

- 计算步骤：

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- 关键是：如何选取函数 $x = \varphi(t)$ ？

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$$= G(t) + C \xrightarrow{t=\varphi^{-1}(x)} G(\varphi^{-1}(x)) + C$$

- 关键是：如何选取函数 $x = \varphi(t)$ ？

在后面的例子中，选取函数 $x = \varphi(t)$ 的方法：

把被积函数 $f(x)$ 中复杂的部分整个设为 t ，
从而得到 x 与 t 的函数关系！

变量代换 例 0

例 求不定积分 $\int \sqrt{1-x^2} dx$

变量代换 例 0

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解 $\because -1 \leq x \leq 1$, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$,

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$$\therefore \int \sqrt{1-x^2} dx = \int \sqrt{1-\sin^2 t} d \sin t$$

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变量代换 例 0

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$$\begin{aligned}\therefore \int \sqrt{1-x^2} dx &= \int \sqrt{1-\sin^2 t} d\sin t = \int \cos^2 t dt \\ &= \frac{1}{2} \int \cos 2t + 1 dt\end{aligned}$$

变量代换 例 0

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变量代换 例 0

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变量代换 例 0

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注 可见选取合适 $x = \varphi(t)$ 很关键!

变量代换 例 1

例 1 求不定积分 $\int x\sqrt{3x-1}dx$, $\int \frac{x}{\sqrt{x-2}}dx$

变量代换 例 1

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(1) 设 $t = (3x-1)^{\frac{1}{2}}$,

变量代换 例 1

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$$\therefore \int x\sqrt{3x-1}dx =$$

变量代换 例 1

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解

(1) 设 $t = (3x-1)^{\frac{1}{2}}$, $\therefore x = \frac{1}{3}(t^2 + 1)$,

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变量代换 例 1

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$$(1) \text{ 设 } t = (3x-1)^{\frac{1}{2}}, \quad \therefore x = \frac{1}{3}(t^2 + 1), \quad dx = \frac{2}{3}t dt$$

$$\therefore \int x\sqrt{3x-1}dx =$$

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变量代换 例 1

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变量代换 例 1

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$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}t dt = \frac{2}{9} \int t^4 + t^2 dt$$

变量代换 例 1

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$$(1) \text{ 设 } t = (3x-1)^{\frac{1}{2}}, \quad \therefore x = \frac{1}{3}(t^2 + 1), \quad dx = \frac{2}{3}t dt$$

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$$(2) \text{ 设 } t = (x-2)^{\frac{1}{2}},$$

变量代换 例 1

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$$(2) \text{ 设 } t = (x-2)^{\frac{1}{2}}, \quad \therefore x = t^2 + 2, \quad dx = 2t dt$$

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变量代换 例 1

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$$\therefore \int \frac{x}{\sqrt{x-2}}dx = \int \frac{t^2 + 2}{t} \cdot 2t dt$$

变量代换 例 1

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解

$$(1) \text{ 设 } t = (3x-1)^{\frac{1}{2}}, \quad \therefore x = \frac{1}{3}(t^2 + 1), \quad dx = \frac{2}{3}t dt$$

$$\begin{aligned} \therefore \int x\sqrt{3x-1}dx &= \int \frac{1}{3}(t^2 + 1)t \cdot \frac{2}{3}t dt = \frac{2}{9} \int t^4 + t^2 dt \\ &= \frac{2}{45}t^5 + \frac{2}{27}t^3 + C = \frac{2}{45}(3x-1)^{\frac{5}{2}} + \frac{2}{27}(3x-1)^{\frac{3}{2}} + C \end{aligned}$$

$$(2) \text{ 设 } t = (x-2)^{\frac{1}{2}}, \quad \therefore x = t^2 + 2, \quad dx = 2t dt$$

$$\therefore \int \frac{x}{\sqrt{x-2}}dx = \int \frac{t^2 + 2}{t} \cdot 2t dt = 2 \int t^2 + 2 dt$$

变量代换 例 1

例 1 求不定积分 $\int x\sqrt{3x-1}dx$, $\int \frac{x}{\sqrt{x-2}}dx$

解

$$(1) \text{ 设 } t = (3x-1)^{\frac{1}{2}}, \quad \therefore x = \frac{1}{3}(t^2 + 1), \quad dx = \frac{2}{3}t dt$$

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变量代换 例 1

例 1 求不定积分 $\int x\sqrt{3x-1}dx$, $\int \frac{x}{\sqrt{x-2}}dx$

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变量代换 例 2

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}} dx$, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

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变量代换 例 2

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}} dx$, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$,

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变量代换 例 2

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}} dx$, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$,

$$\therefore \int \frac{1}{1+\sqrt{x}} dx =$$

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变量代换 例 2

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变量代换 例 2

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变量代换 例 2

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变量代换 例 2

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变量代换 例 2

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(2) 设 $t = 1 + (1+x)^{\frac{1}{3}}$, ,

变量代换 例 2

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}} dx$, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

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变量代换 例 2

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}} dx$, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

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(2) 设 $t = 1 + (1+x)^{\frac{1}{3}}$, $\therefore x = (t-1)^3 - 1$,

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变量代换 例 2

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}} dx$, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

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(2) 设 $t = 1 + (1+x)^{\frac{1}{3}}$, $\therefore x = (t-1)^3 - 1$, $dx = 3(t-1)^2 dt$

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}} dx =$$

变量代换 例 2

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}} dx$, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

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$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}} dx = \int \frac{1}{t} \cdot 3(t-1)^2 dt =$$

变量代换 例 2

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}} dx$, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

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$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}} dx = \int \frac{1}{t} \cdot 3(t-1)^2 dt = 3 \int t - 2 + \frac{1}{t} dt$$

变量代换 例 2

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}} dx$, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

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(2) 设 $t = 1 + (1+x)^{\frac{1}{3}}$, $\therefore x = (t-1)^3 - 1$, $dx = 3(t-1)^2 dt$

$$\begin{aligned}\therefore \int \frac{1}{1+\sqrt[3]{1+x}} dx &= \int \frac{1}{t} \cdot 3(t-1)^2 dt = 3 \int t - 2 + \frac{1}{t} dt \\ &= \frac{3}{2}t^2 - 6t + 3 \ln |t| + C\end{aligned}$$

变量代换 例 2

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}} dx$, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

$$\begin{aligned}\therefore \int \frac{1}{1+\sqrt{x}} dx &= \int \frac{1}{t} \cdot 2(t-1)dt = 2 \int 1 - \frac{1}{t} dt \\ &= 2t - 2 \ln t + C = 2(1+x^{\frac{1}{2}}) - 2 \ln(1+x^{\frac{1}{2}}) + C\end{aligned}$$

(2) 设 $t = 1 + (1+x)^{\frac{1}{3}}$, $\therefore x = (t-1)^3 - 1$, $dx = 3(t-1)^2 dt$

$$\begin{aligned}\therefore \int \frac{1}{1+\sqrt[3]{1+x}} dx &= \int \frac{1}{t} \cdot 3(t-1)^2 dt = 3 \int t - 2 + \frac{1}{t} dt \\ &= \frac{3}{2} t^2 - 6t + 3 \ln |t| + C\end{aligned}$$

$$= \frac{3}{2} (1 + (1+x)^{\frac{1}{3}})^2 - 6(1 + (1+x)^{\frac{1}{3}}) + 3 \ln |1 + (1+x)^{\frac{1}{3}}| + C$$

变量代换 例 3

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

变量代换 例 3

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

解

$$\text{设 } t = \sqrt{1+e^x},$$

变量代换 例 3

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

解

$$\text{设 } t = \sqrt{1+e^x}, \quad \therefore x = \ln(t^2 - 1),$$

变量代换 例 3

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

解

$$\text{设 } t = \sqrt{1+e^x}, \quad \therefore x = \ln(t^2 - 1), \quad dx = \frac{2t}{t^2 - 1} dt$$

变量代换 例 3

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

解

$$\text{设 } t = \sqrt{1+e^x}, \quad \therefore x = \ln(t^2 - 1), \quad dx = \frac{2t}{t^2 - 1} dt$$

$$\therefore \int \frac{1}{\sqrt{1+e^x}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1} dt$$

变量代换 例 3

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

解

$$\text{设 } t = \sqrt{1+e^x}, \quad \therefore x = \ln(t^2 - 1), \quad dx = \frac{2t}{t^2 - 1} dt$$

$$\therefore \int \frac{1}{\sqrt{1+e^x}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1} dt = \int \frac{1}{t-1} - \frac{1}{t+1} dt$$

变量代换 例 3

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

解

$$\text{设 } t = \sqrt{1+e^x}, \quad \therefore x = \ln(t^2 - 1), \quad dx = \frac{2t}{t^2 - 1} dt$$

$$\begin{aligned} \therefore \int \frac{1}{\sqrt{1+e^x}} dx &= \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1} dt = \int \frac{1}{t-1} - \frac{1}{t+1} dt \\ &= \ln|t-1| - \ln|t+1| + C \end{aligned}$$

变量代换 例 3

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

解

$$\text{设 } t = \sqrt{1+e^x}, \quad \therefore x = \ln(t^2 - 1), \quad dx = \frac{2t}{t^2 - 1} dt$$

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变量代换 例 3

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

解

$$\begin{aligned}\text{设 } t &= \sqrt{1+e^x}, \quad \therefore x = \ln(t^2 - 1), \quad dx = \frac{2t}{t^2 - 1} dt \\ \therefore \int \frac{1}{\sqrt{1+e^x}} dx &= \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1} dt = \int \frac{1}{t-1} - \frac{1}{t+1} dt \\ &= \ln|t-1| - \ln|t+1| + C = \ln\left|\frac{t-1}{t+1}\right| + C \\ &= \ln\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right) + C\end{aligned}$$

变量代换 例 3

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

解

$$\begin{aligned}\text{设 } t &= \sqrt{1+e^x}, \quad \therefore x = \ln(t^2 - 1), \quad dx = \frac{2t}{t^2 - 1} dt \\ \therefore \int \frac{1}{\sqrt{1+e^x}} dx &= \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1} dt = \int \frac{1}{t-1} - \frac{1}{t+1} dt \\ &= \ln|t-1| - \ln|t+1| + C = \ln\left|\frac{t-1}{t+1}\right| + C \\ &= \ln\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right) + C \\ &= 2\ln(\sqrt{1+e^x}-1) - x + C\end{aligned}$$

变量代换法：例子总结

$$\int x\sqrt{3x-1}dx$$

$$\int \frac{1}{1+\sqrt{x}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

变量代换法：例子总结

$$\int x\sqrt{3x-1}dx \xrightarrow{t=\sqrt{3x-1}} \dots$$

$$\int \frac{1}{1+\sqrt{x}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

变量代换法：例子总结

$$\int x\sqrt{3x-1}dx \xrightarrow{t=\sqrt{3x-1}} \dots$$

$$\int \frac{1}{1+\sqrt{x}}dx \xrightarrow{t=1+\sqrt{x}} \dots$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

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