

## §3.4 向量组的秩

数学系 梁卓滨

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- $\alpha_{j_1}, \alpha_{j_2}, \dots, \alpha_{j_r}$  与  $\beta_{k_1}, \beta_{k_2}, \dots, \beta_{k_t}$  等价（相互线性表示）；且

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**注**  $r(\alpha_1, \alpha_2, \dots, \alpha_s) \leq s$  且  $\leq m$  (维数)。

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例 设  $\alpha_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ , 则  
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# 秩

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$$A_{m \times n} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

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$$A_{m \times n} = \begin{pmatrix} \alpha_1 & & & \\ a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

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# 秩

设

$$A_{m \times n} = \begin{matrix} & \alpha_1 & \alpha_2 & & \alpha_n \\ \beta_1 & a_{11} & a_{12} & \cdots & a_{1n} \\ \beta_2 & a_{21} & a_{22} & \cdots & a_{2n} \\ & \vdots & \vdots & \ddots & \vdots \\ & a_{m1} & a_{m2} & \cdots & a_{mn} \end{matrix} = (\alpha_1 \alpha_2 \cdots \alpha_n)$$

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# 初等变换求极大无关组

**问题** 给出  $m$  维的向量组  $\alpha_1, \alpha_2, \dots, \alpha_n$ , 如何求出其一组极大无关组?

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4. 通过简化的阶梯型矩阵, 容易看出其余列如何用极大无关组线性表示

例 求向量组  $\alpha_1 = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$  的一个极大无关组；并把其余向量用该极大无关组线性表示。

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解

	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$
$\begin{pmatrix} 2 & 1 & 2 & 3 \\ 4 & 1 & 3 & 5 \\ 2 & 0 & 1 & 2 \end{pmatrix}$				

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$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$
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$$\xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 \end{pmatrix} \rightarrow$$

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**解**  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4$

$$\begin{pmatrix} 2 & 1 & 2 & 3 \\ 4 & 1 & 3 & 5 \\ 2 & 0 & 1 & 2 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

**例** 求向量组  $\alpha_1 = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$  的一个极大无关组; 并把其余向量用该极大无关组线性表示。

**解**  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4$

$$\begin{pmatrix} 2 & 1 & 2 & 3 \\ 4 & 1 & 3 & 5 \\ 2 & 0 & 1 & 2 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\xrightarrow{r_1-r_2}$$

**例** 求向量组  $\alpha_1 = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$  的一个极大无关组; 并把其余向量用该极大无关组线性表示。

**解**  $\alpha_1$   $\alpha_2$   $\alpha_3$   $\alpha_4$

$$\begin{pmatrix} 2 & 1 & 2 & 3 \\ 4 & 1 & 3 & 5 \\ 2 & 0 & 1 & 2 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\xrightarrow{r_1-r_2} \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\xrightarrow{r_1-r_2} \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\frac{1}{2} \times r_1}$$

**例** 求向量组  $\alpha_1 = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$  的一个极大无关组; 并把其余向量用该极大无关组线性表示。

**解**  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4$

$$\begin{pmatrix} 2 & 1 & 2 & 3 \\ 4 & 1 & 3 & 5 \\ 2 & 0 & 1 & 2 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\xrightarrow{r_1-r_2} \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\frac{1}{2} \times r_1} \begin{pmatrix} 1 & 0 & \frac{1}{2} & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 2 & 1 & 2 & 3 \\ 4 & 1 & 3 & 5 \\ 2 & 0 & 1 & 2 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{r_1-r_2} \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\frac{1}{2} \times r_1} \begin{pmatrix} 1 & 0 & \frac{1}{2} & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\xrightarrow{r_1-r_2} \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\frac{1}{2} \times r_1} \begin{pmatrix} 1 & 0 & \frac{1}{2} & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

所以

- $r(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = r(A) = 2;$



**例** 求向量组  $\alpha_1 = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$  的一个极大无关组; 并把其余向量用该极大无关组线性表示。

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$$\begin{pmatrix} 2 & 1 & 2 & 3 \\ 4 & 1 & 3 & 5 \\ 2 & 0 & 1 & 2 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\xrightarrow{r_1-r_2} \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\frac{1}{2} \times r_1} \begin{pmatrix} 1 & 0 & \frac{1}{2} & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 2 & 1 & 2 & 3 \\ 4 & 1 & 3 & 5 \\ 2 & 0 & 1 & 2 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\xrightarrow{r_1-r_2} \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\frac{1}{2} \times r_1} \begin{pmatrix} 1 & 0 & \frac{1}{2} & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

所以

- $r(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = r(A) = 2$ ;
- $\alpha_1, \alpha_2$  是极大无关组;

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**解**  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4$

$$\begin{pmatrix} 2 & 1 & 2 & 3 \\ 4 & 1 & 3 & 5 \\ 2 & 0 & 1 & 2 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$\xrightarrow{r_1-r_2} \begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{\frac{1}{2} \times r_1} \begin{pmatrix} 1 & 0 & \frac{1}{2} & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

所以

- $r(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = r(A) = 2$ ;
- $\alpha_1, \alpha_2$  是极大无关组;
- $\alpha_3 = \frac{1}{2}\alpha_1 + \alpha_2, \quad \alpha_4 = \alpha_1 + \alpha_2$

例 求向量组  $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 2 \\ 4 \\ 3 \\ 3 \end{pmatrix}$  的一个极大无关组；并把其余向量用该极大无关组线性表示。

例 求向量组  $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 2 \\ 4 \\ 3 \\ 3 \end{pmatrix}$  的一个

极大无关组；并把其余向量用该极大无关组线性表示。

解

$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$
1	0	1	2
2	1	1	4
1	1	0	3
0	2	-2	3

例 求向量组  $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 2 \\ 4 \\ 3 \\ 3 \end{pmatrix}$  的一个

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解

$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$
1	0	1	2
2	1	1	4
1	1	0	3
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$$\xrightarrow[r_3-r_1]{r_2-2r_1}$$

例 求向量组  $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 2 \\ 4 \\ 3 \\ 3 \end{pmatrix}$  的一个

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解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4$

$$\begin{pmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 1 & 1 & 0 & 3 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 3 \end{pmatrix}$$

例 求向量组  $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 2 \\ 4 \\ 3 \\ 3 \end{pmatrix}$  的一个

极大无关组；并把其余向量用该极大无关组线性表示。

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4$

$$\begin{pmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 1 & 1 & 0 & 3 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_4-2r_2]{r_3-r_2}$$



例 求向量组  $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 2 \\ 4 \\ 3 \\ 3 \end{pmatrix}$  的一个

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解  $\alpha_1$   $\alpha_2$   $\alpha_3$   $\alpha_4$

$$\begin{pmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 1 & 1 & 0 & 3 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_4-2r_2]{r_3-r_2} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

例 求向量组  $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 2 \\ 4 \\ 3 \\ 3 \end{pmatrix}$  的一个

极大无关组；并把其余向量用该极大无关组线性表示。

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4$

$$\begin{pmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 1 & 1 & 0 & 3 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_4-2r_2]{r_3-r_2} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$
$$\xrightarrow{r_4-3r_3}$$

例 求向量组  $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 2 \\ 4 \\ 3 \\ 3 \end{pmatrix}$  的一个

极大无关组；并把其余向量用该极大无关组线性表示。

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4$

$$\begin{pmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 1 & 1 & 0 & 3 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_4-2r_2]{r_3-r_2} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$
$$\xrightarrow{r_4-3r_3} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

例 求向量组  $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 2 \\ 4 \\ 3 \\ 3 \end{pmatrix}$  的一个

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解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4$

$$\begin{pmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 1 & 1 & 0 & 3 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_4-2r_2]{r_3-r_2} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$
$$\xrightarrow{r_4-3r_3} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 1 & 1 & 0 & 3 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_4-2r_2]{r_3-r_2} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$
$$\xrightarrow{r_4-3r_3} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

所以

例 求向量组  $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 2 \\ 4 \\ 3 \\ 3 \end{pmatrix}$  的一个

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解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4$

$$\begin{pmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 1 & 1 & 0 & 3 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_4-2r_2]{r_3-r_2} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

$$\xrightarrow{r_4-3r_3} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

所以

- $r(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = r(A) = 3;$

例 求向量组  $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 2 \\ 4 \\ 3 \\ 3 \end{pmatrix}$  的一个

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$$\begin{pmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 1 & 1 & 0 & 3 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_4-2r_2]{r_3-r_2} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$
$$\xrightarrow{r_4-3r_3} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

所以

•  $r(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = r(A) = 3;$

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极大无关组；并把其余向量用该极大无关组线性表示。

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4$

$$\begin{pmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 1 & 1 & 0 & 3 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_4-2r_2]{r_3-r_2} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix} \\ \xrightarrow{r_4-3r_3} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

所以

- $r(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = r(A) = 3$ ;
- $\alpha_1, \alpha_2, \alpha_4$  是极大无关组;



例 求向量组  $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 0 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ -2 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 2 \\ 4 \\ 3 \\ 3 \end{pmatrix}$  的一个

极大无关组；并把其余向量用该极大无关组线性表示。

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4$

$$\begin{pmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 4 \\ 1 & 1 & 0 & 3 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_3-r_1]{r_2-2r_1} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & -2 & 3 \end{pmatrix} \xrightarrow[r_4-2r_2]{r_3-r_2} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$
$$\xrightarrow{r_4-3r_3} \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

所以

- $r(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = r(A) = 3$ ;
- $\alpha_1, \alpha_2, \alpha_4$  是极大无关组;
- $\alpha_3 = \alpha_1 - \alpha_2$

例 求向量组  $\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 4 \\ 5 \\ 6 \end{pmatrix}$ ,  $\alpha_4 = \begin{pmatrix} 4 \\ 5 \\ 6 \\ 7 \end{pmatrix}$  的一个极大无关组；并把其余向量用该极大无关组线性表示。

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解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4$

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$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{pmatrix} \xrightarrow[r_4-4r_1]{\begin{matrix} r_2-2r_1 \\ r_3-3r_1 \end{matrix}}$$

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$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{pmatrix} \xrightarrow[r_4-4r_1]{\begin{matrix} r_2-2r_1 \\ r_3-3r_1 \end{matrix}} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & -2 & -4 & -6 \\ 0 & -3 & -6 & -9 \end{pmatrix}$$

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解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \alpha_4$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{pmatrix} \xrightarrow[r_4-4r_1]{\begin{matrix} r_2-2r_1 \\ r_3-3r_1 \end{matrix}} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & -2 & -4 & -6 \\ 0 & -3 & -6 & -9 \end{pmatrix}$$

$$\xrightarrow[r_4-3r_2]{r_3-2r_2}$$

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$$\xrightarrow[r_4-3r_2]{r_3-2r_2} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$\xrightarrow[r_4-3r_2]{r_3-2r_2} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$



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$$\xrightarrow[r_4-3r_2]{r_3-2r_2} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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所以

•  $r(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = r(A) = 2;$

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$$\xrightarrow[r_4-3r_2]{r_3-2r_2} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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所以

- $r(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = r(A) = 2$ ;
- $\alpha_1, \alpha_2$  是极大无关组;
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例 假设向量组  $\alpha_1, \alpha_2, \dots, \alpha_s$  可由  $\beta_1, \beta_2, \dots, \beta_t$  线性表示, 则

$$r(\alpha_1, \alpha_2, \dots, \alpha_s) \leq r(\beta_1, \beta_2, \dots, \beta_t).$$

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**证明** 设

$$r_1 = r(\alpha_1, \alpha_2, \dots, \alpha_s),$$

$$r_2 = r(\beta_1, \beta_2, \dots, \beta_t),$$



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证明 设

$$r_1 = r(\alpha_1, \alpha_2, \dots, \alpha_s), \quad \alpha_{i_1}, \alpha_{i_2}, \dots, \alpha_{i_{r_1}} \text{ 是极大无关组}$$

$$r_2 = r(\beta_1, \beta_2, \dots, \beta_t),$$

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$r_2 = r(\beta_1, \beta_2, \dots, \beta_t), \quad \beta_{j_1}, \beta_{j_2}, \dots, \beta_{j_{r_2}}$  是极大无关组

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注意到  $\alpha_{i_1}, \alpha_{i_2}, \dots, \alpha_{i_{r_1}}$  能由  $\beta_{j_1}, \beta_{j_2}, \dots, \beta_{j_{r_2}}$  线性表示,

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$r_1 \leq r_2$ 。

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---

**定理** 设有向量组

(A):  $\alpha_1, \alpha_2, \dots, \alpha_s$

(B):  $\beta_1, \beta_2, \dots, \beta_t$

若它们等价,

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**定理** 设有向量组  $(A): \alpha_1, \alpha_2, \dots, \alpha_s$

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若它们等价, 则  $r(\alpha_1, \alpha_2, \dots, \alpha_s) = r(\beta_1, \beta_2, \dots, \beta_t)$ 。

例 设  $A_{m \times n}$ ,  $B_{n \times s}$  为矩阵, 则  $r(AB) \leq \min\{r(A), r(B)\}$ 。

例 设  $A_{m \times n}, B_{n \times s}$  为矩阵, 则  $r(AB) \leq \min\{r(A), r(B)\}$ 。

证明 设  $AB = C_{m \times s}$



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证明 设  $AB = C_{m \times s}$

$$\underbrace{\begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1s} \\ c_{21} & c_{22} & \cdots & c_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{ms} \end{pmatrix}}_C = \underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_A \underbrace{\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1s} \\ b_{21} & b_{22} & \cdots & b_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{ns} \end{pmatrix}}_B$$

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即

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$$\delta_1 \underbrace{\begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1s} \\ c_{21} & c_{22} & \cdots & c_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{ms} \end{pmatrix}}_C = \underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_A \underbrace{\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1s} \\ b_{21} & b_{22} & \cdots & b_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{ns} \end{pmatrix}}_B \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix}$$

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**例** 设  $A_{m \times n}$ ,  $B_{n \times s}$  为矩阵, 则  $r(AB) \leq \min\{r(A), r(B)\}$ 。

**证明** 设  $AB = C_{m \times s}$

$$\begin{matrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_m \end{matrix} \underbrace{\begin{pmatrix} C_{11} & C_{12} & \cdots & C_{1s} \\ C_{21} & C_{22} & \cdots & C_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ C_{m1} & C_{m2} & \cdots & C_{ms} \end{pmatrix}}_C = \underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_A \underbrace{\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1s} \\ b_{21} & b_{22} & \cdots & b_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{ns} \end{pmatrix}}_B \begin{matrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{matrix}$$

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$$\begin{pmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_m \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix}$$

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$$\Rightarrow \delta_1 = a_{11}\beta_1 + a_{12}\beta_2 + \cdots + a_{1n}\beta_n$$



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