

Outline

1. 一个方程的情形

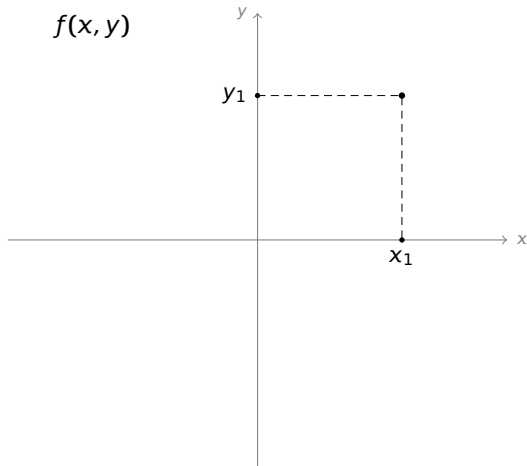
2. 方程组的情形

We are here now...

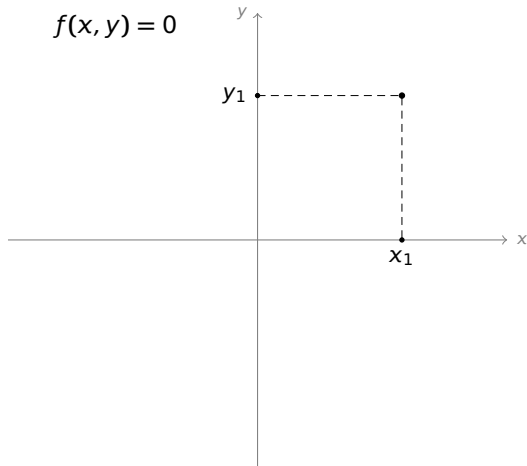
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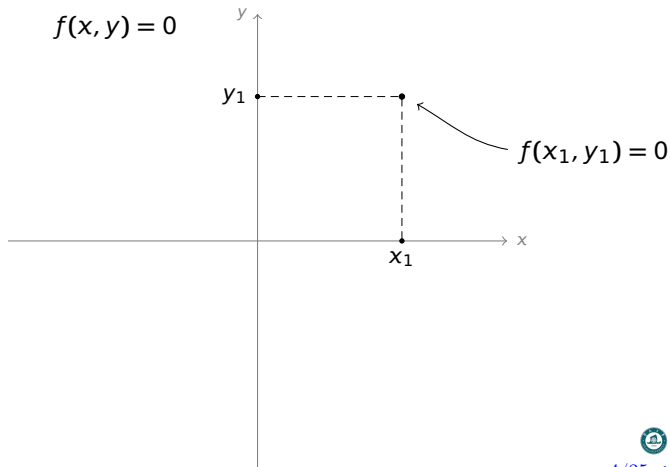
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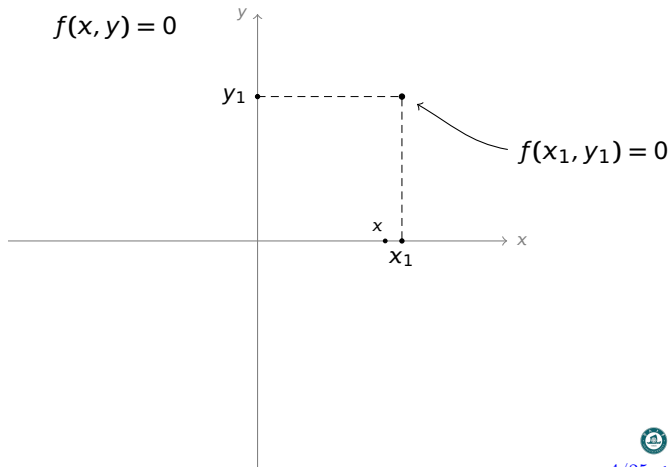
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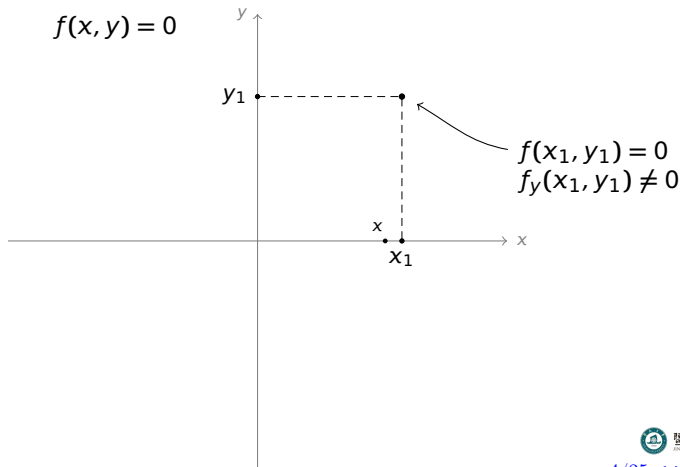
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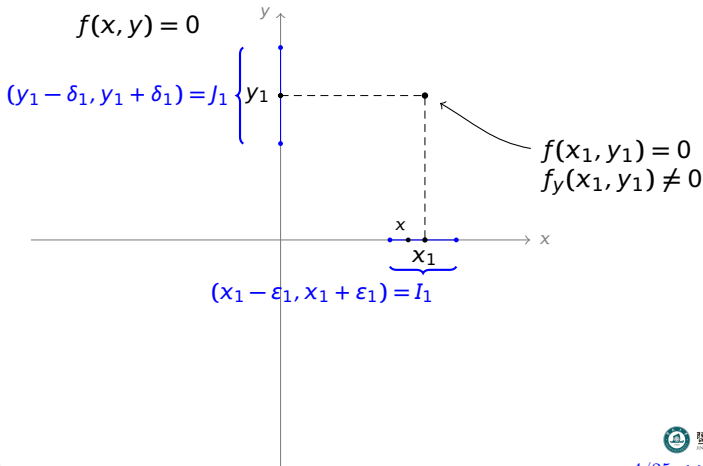
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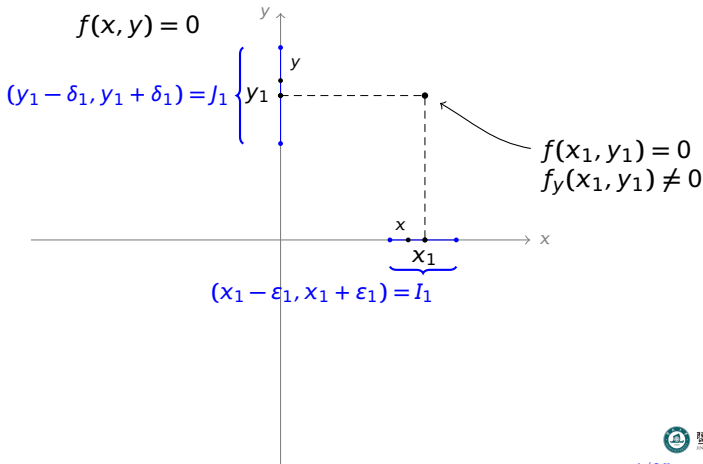


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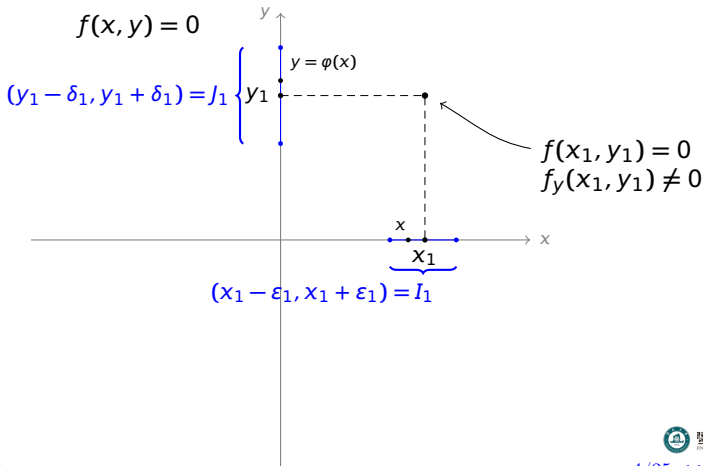
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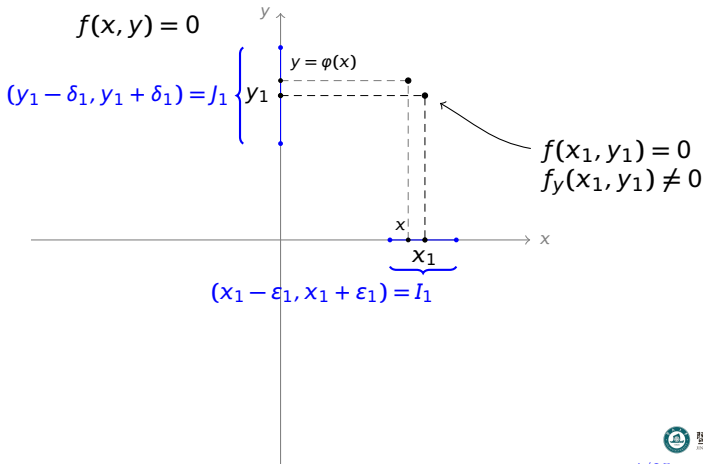
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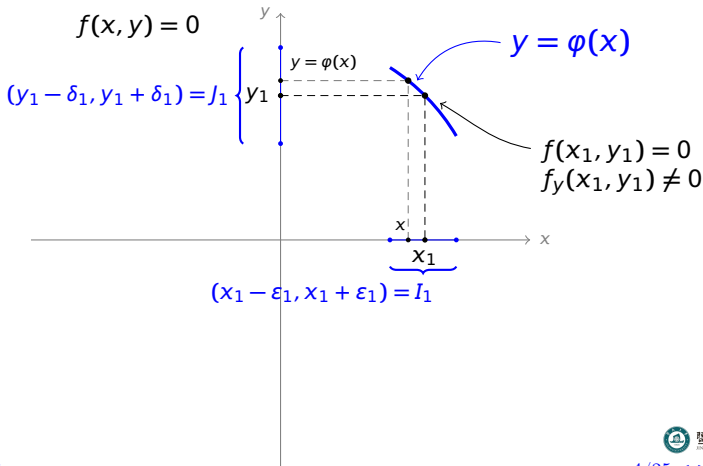


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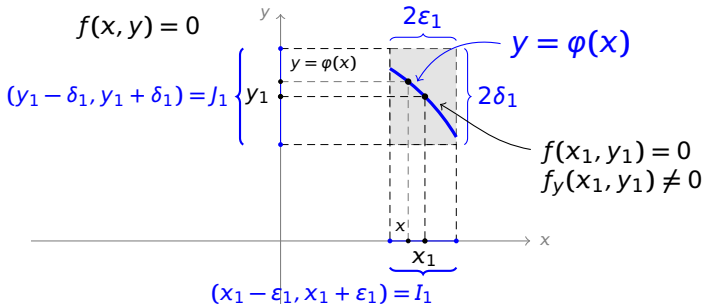


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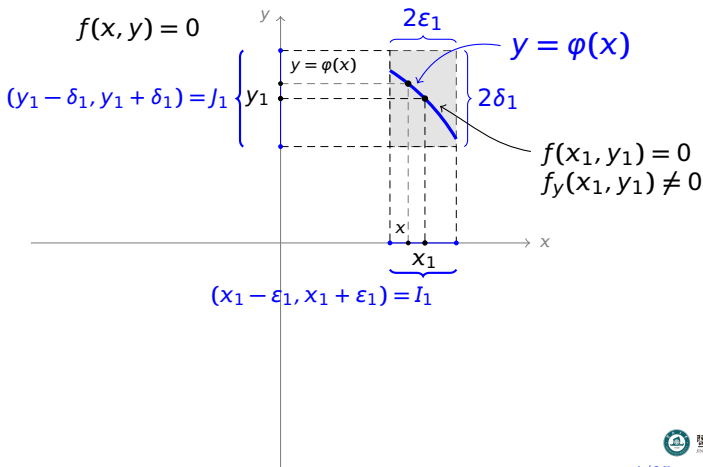
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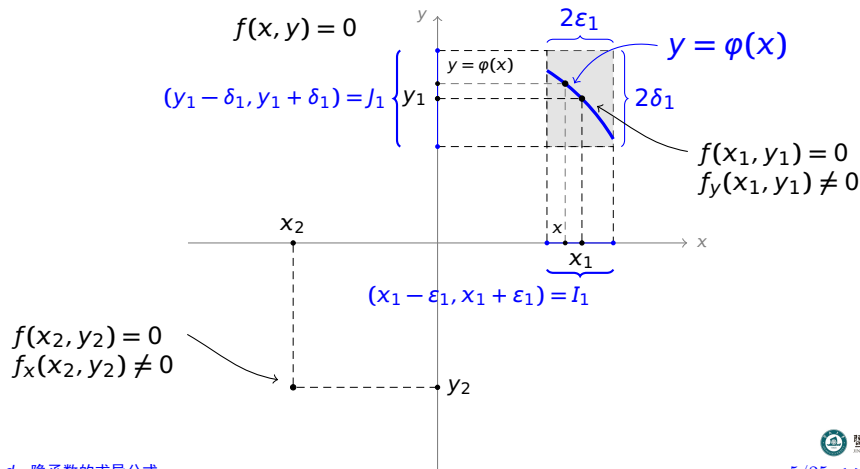


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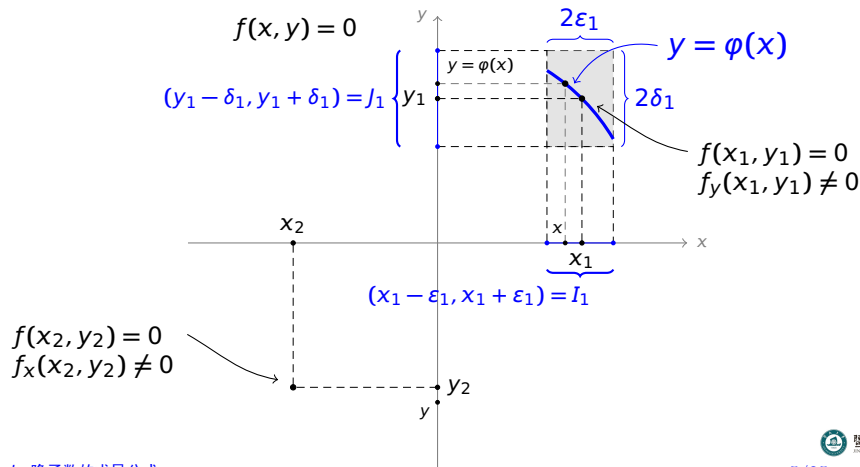
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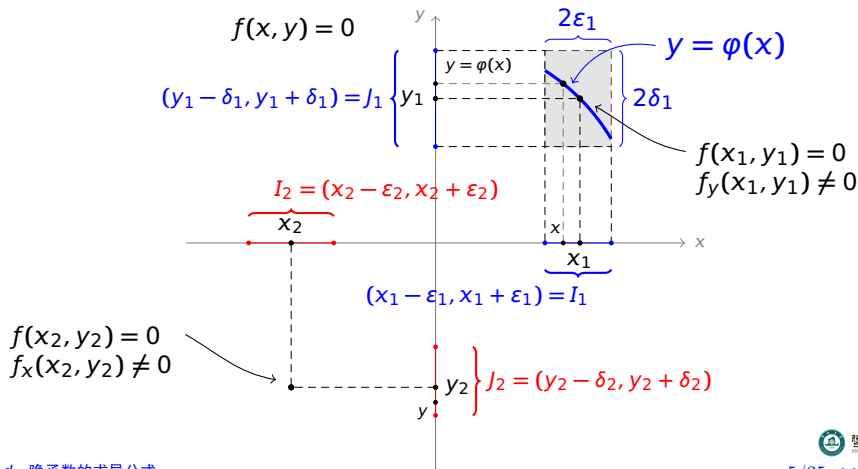
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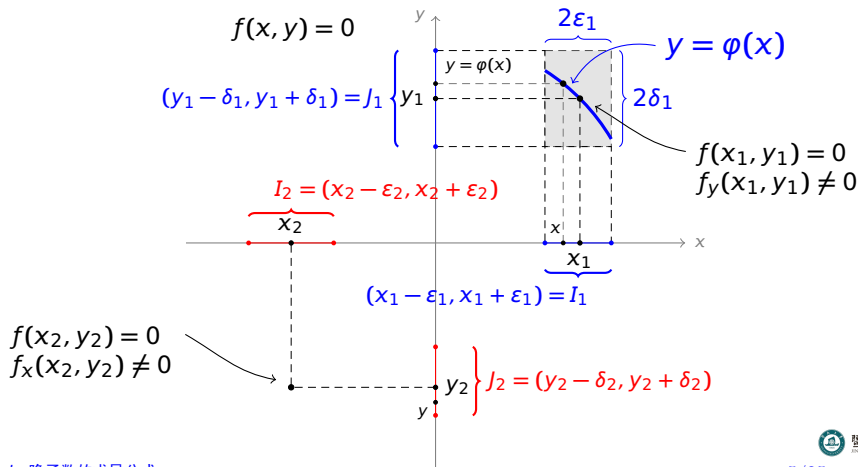


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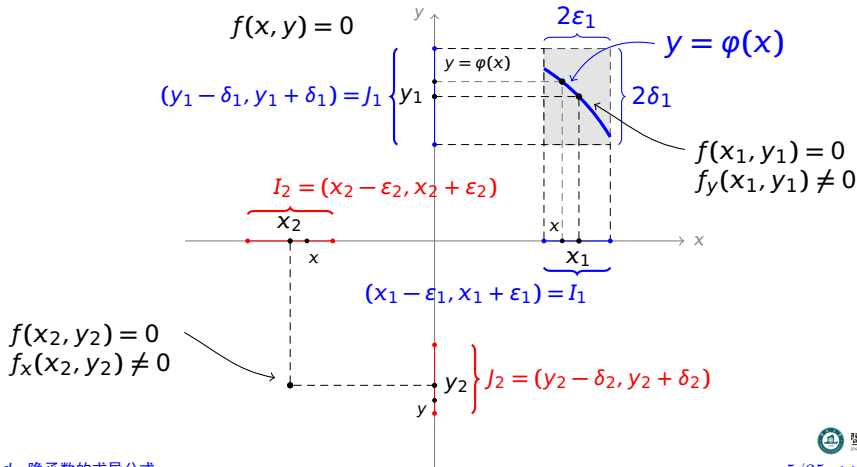
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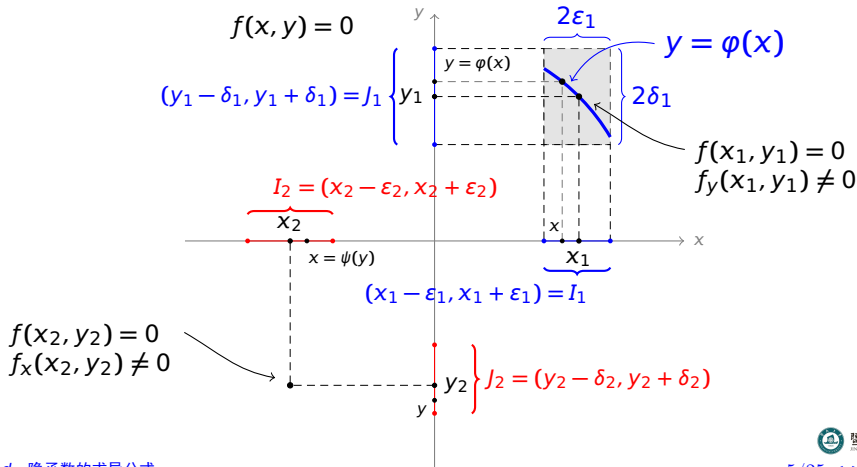
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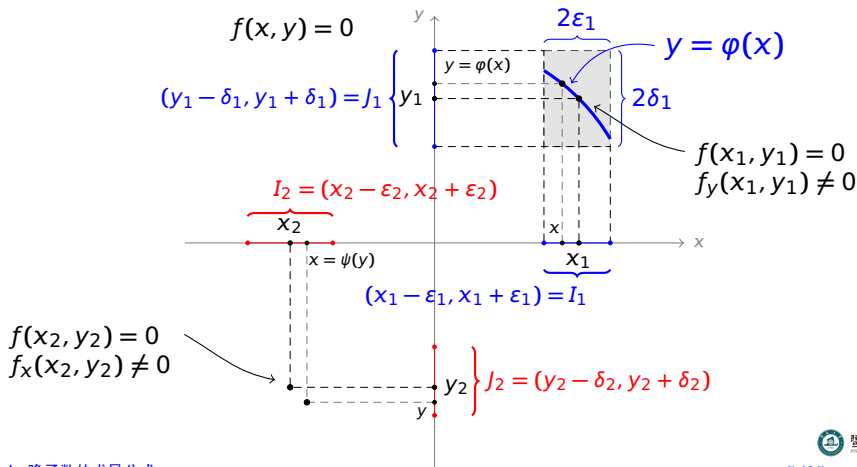
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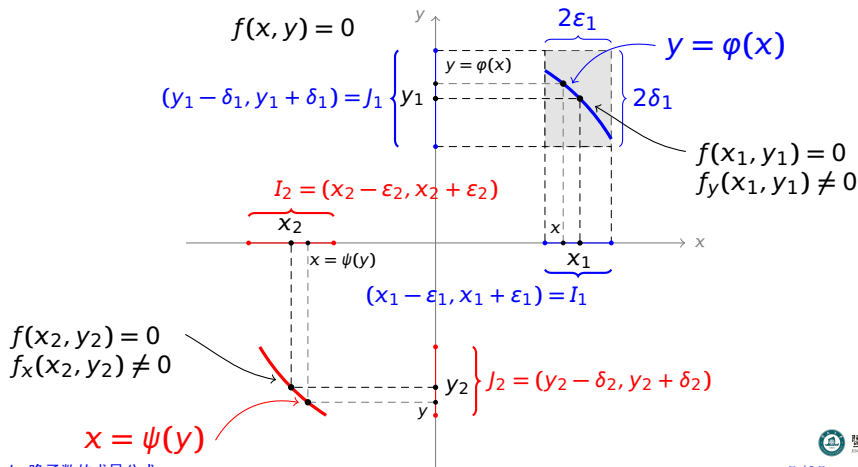
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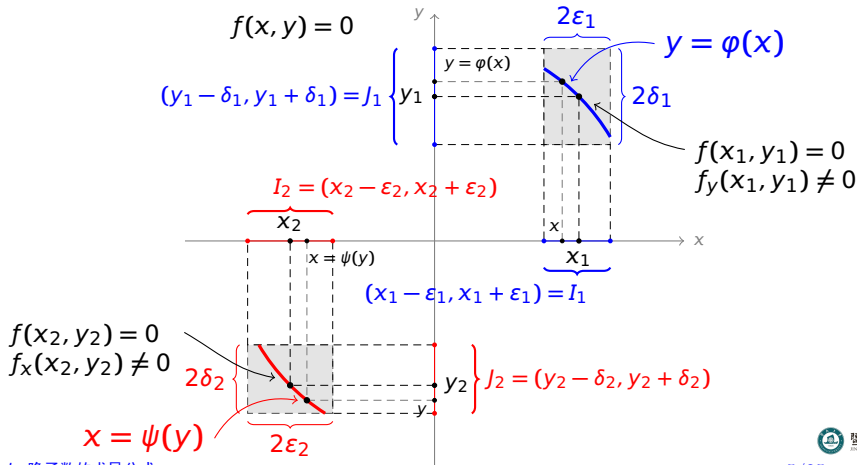
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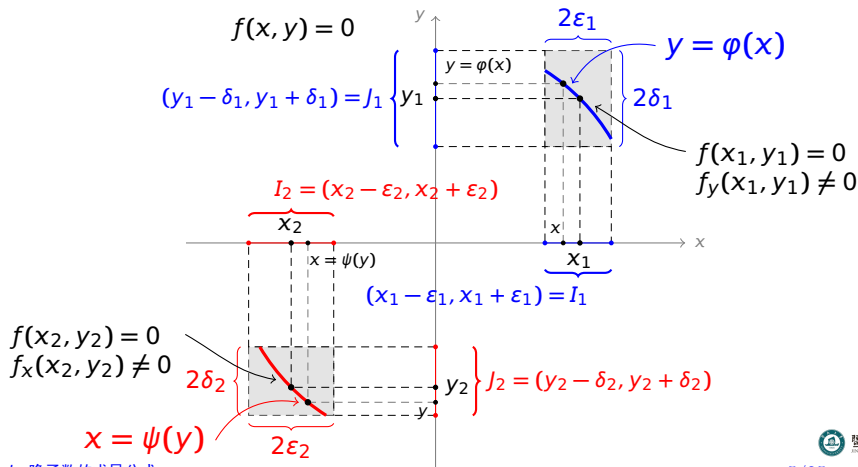
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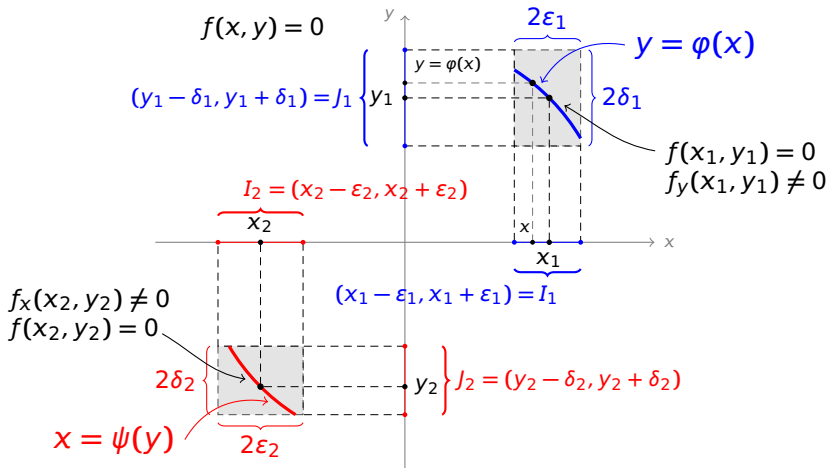


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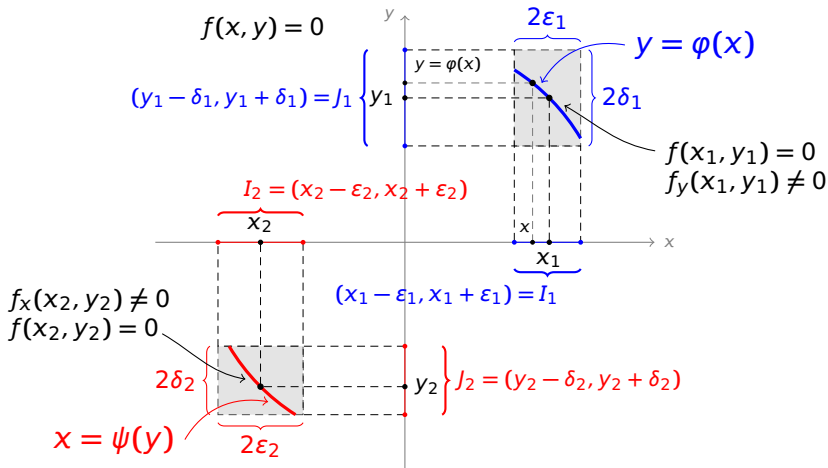


注 隐函数定理中条件 $f_y(x_1, y_1) \neq 0$ 和 $f_x(x_2, y_2) \neq 0$ 不能去掉, 否则结论可能不成立



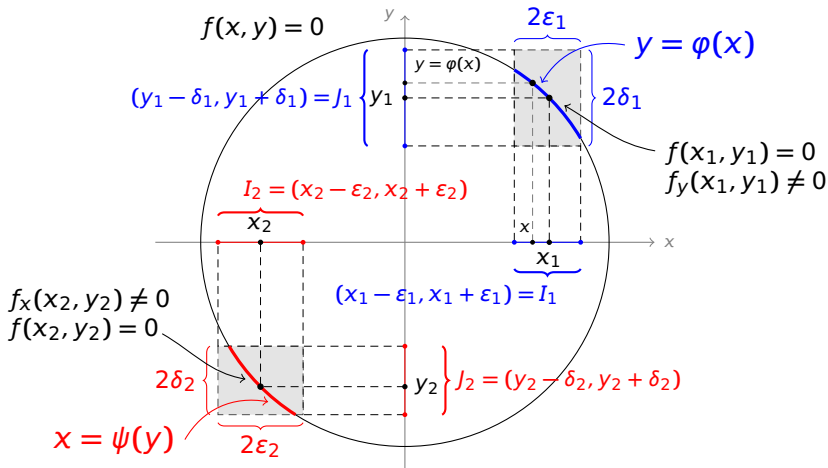
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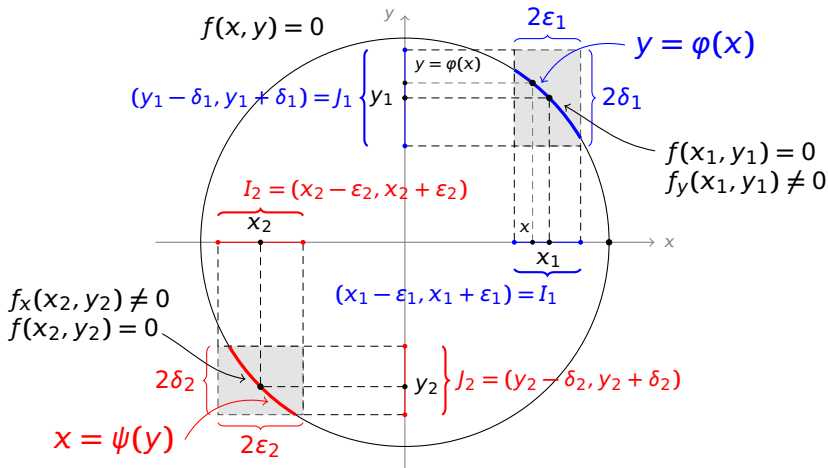
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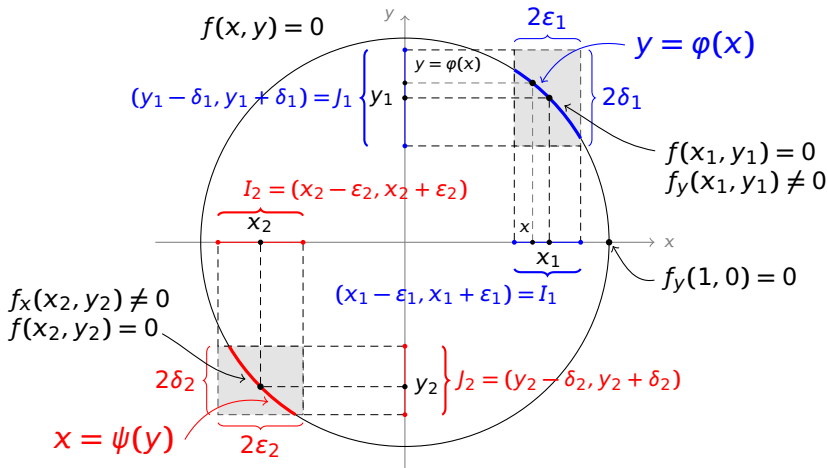
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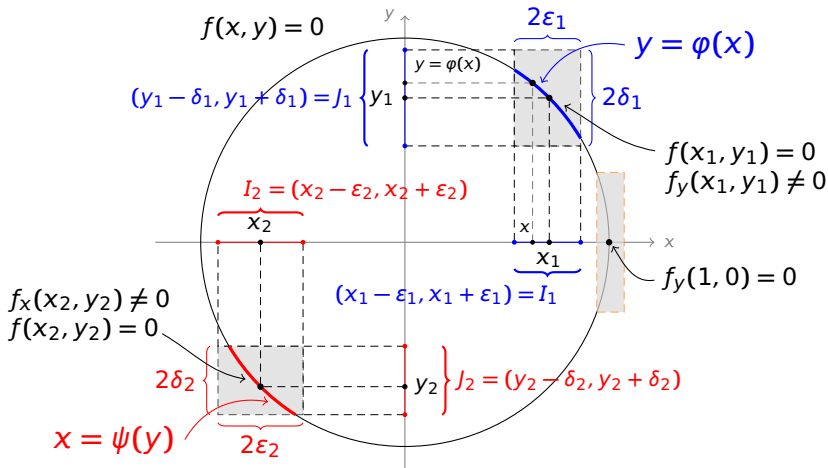
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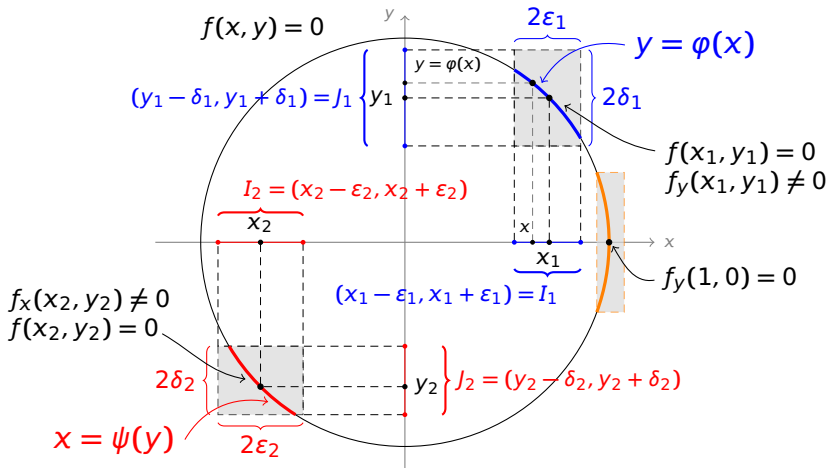
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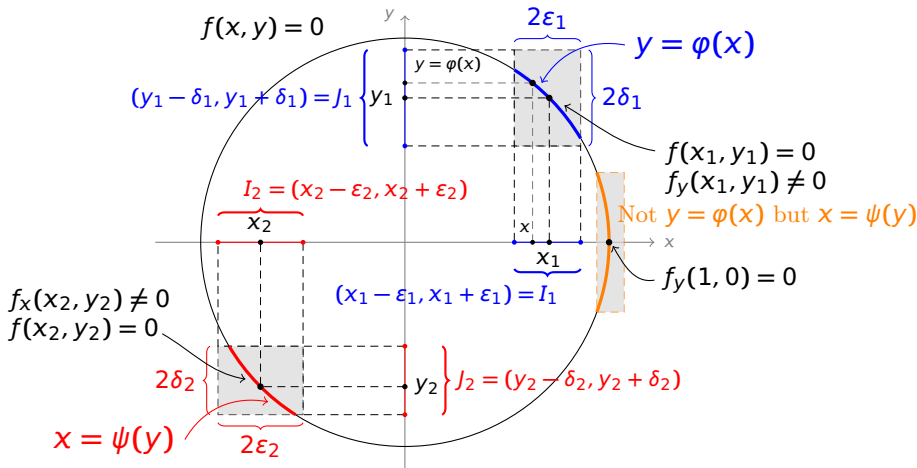
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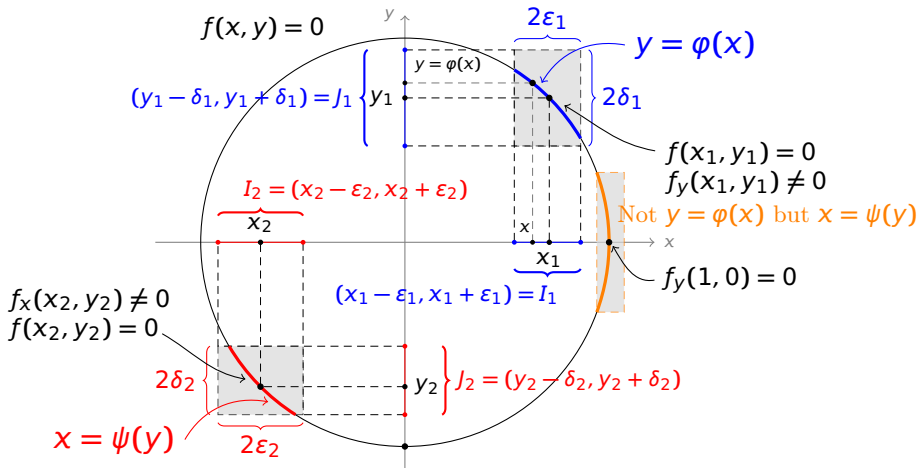
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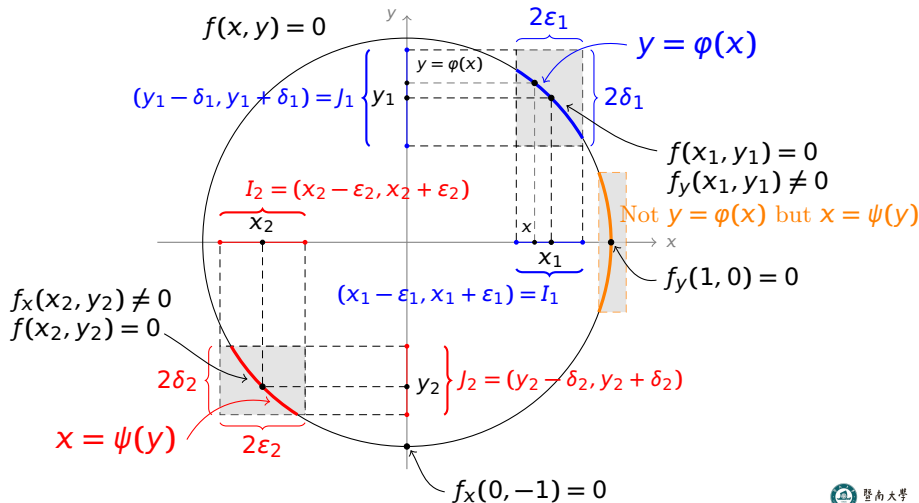
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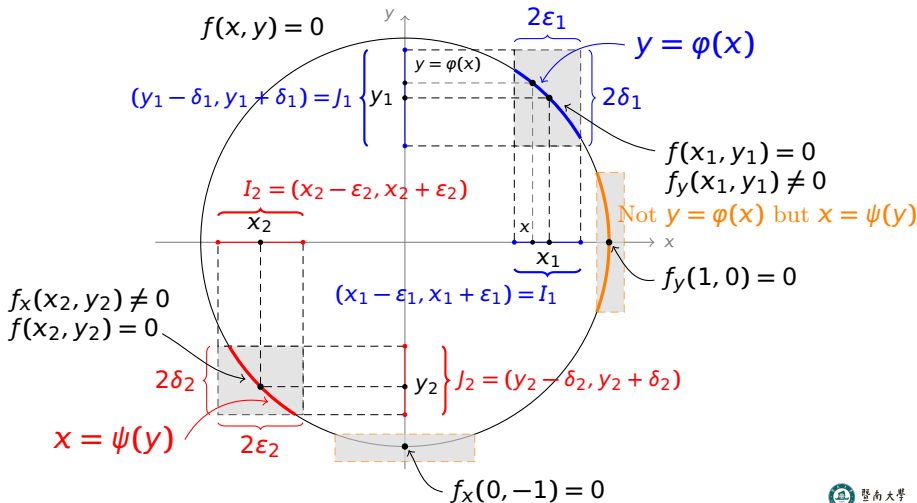
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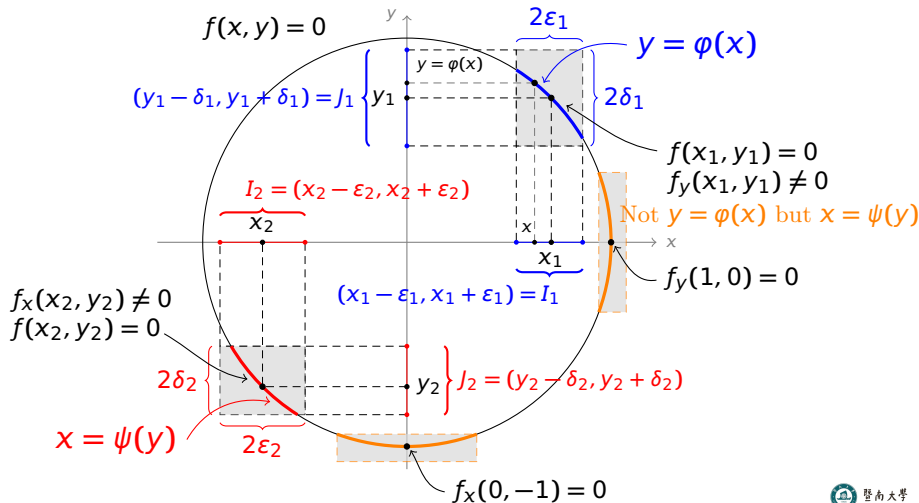
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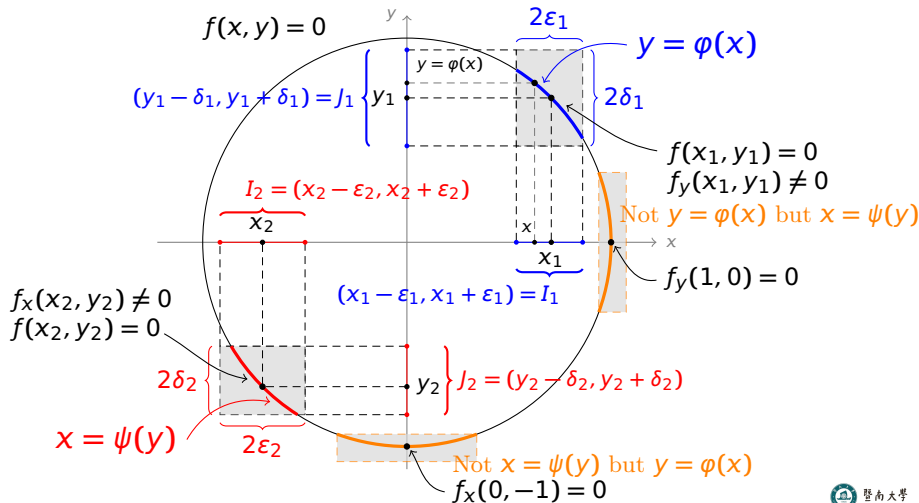
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隐函数定理的一个理论应用

性质 设 $f(x, y)$ 具有连续偏导, 且 f_x, f_y 处处不同时为零, 若

$$E = \{(x, y) | f(x, y) = 0\}$$

非空, 则 E 是平面上“一条曲线”。

隐函数定理 设 $f(x, y, z)$ 在点 $p(x_0, y_0, z_0)$ 附近有定义, 具有连续偏导; $f(x_0, y_0, z_0) = 0$;

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方程 $f(x, y, z) = 0$ 有唯一的解

$$z = \varphi(x, y)$$

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方程 $f(x, y, z) = 0$ 有唯一的解

$$z = \varphi(x, y)$$

隐函数定理 设 $f(x, y, z)$ 在点 $p(x_0, y_0, z_0)$ 附近有定义, 具有连续偏导; $f(x_0, y_0, z_0) = 0$; $f_z(x_0, y_0, z_0) \neq 0$ 。则存在开区间

$$I_1 = (x_0 - \varepsilon, x_0 + \varepsilon), \quad I_2 = (y_0 - \varepsilon, y_0 + \varepsilon), \quad J = (z_0 - \delta, z_0 + \delta)$$

使得:

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1. 对任意 $(x, y) \in I_1 \times I_2$, 方程 $f(x, y, z) = 0$ 有唯一的解
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1. 对任意 $(x, y) \in I_1 \times I_2$, 方程 $f(x, y, z) = 0$ 有唯一的解 $z = \varphi(x, y) \in J$
2. 函数 $z = \varphi(x, y)$ 在区域 $I_1 \times I_2$ 上具有连续导数。

隐函数定理的一个理论应用

性质 设 $f(x, y, z)$ 具有连续偏导, 且 f_x, f_y, f_z 处处不同时为零, 若

$$E = \{(x, y, z) | f(x, y, z) = 0\}$$

非空, 则 E 是空间中“一张曲面”。

隐函数的求导法 I

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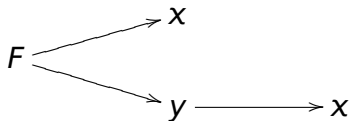
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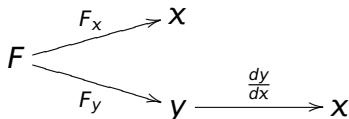
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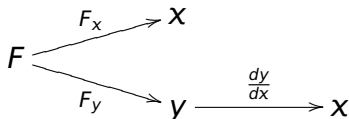
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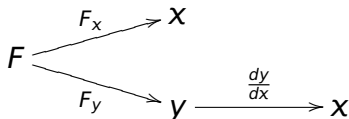
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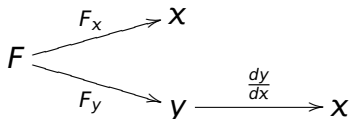
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$$\text{所以 } y' = -\frac{e^x - y^2}{\cos y - 2xy}$$

例 设 $y = f(x)$ 满足 $\ln(x^2 + y^2) + 3xy = 4$, 求 $\frac{dy}{dx}$

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证明 $\because F(x, y, z(x, y)) = 0$

隐函数的求导法 II

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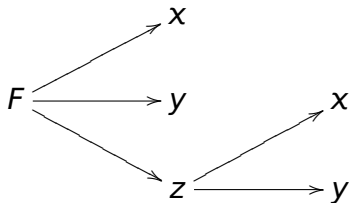
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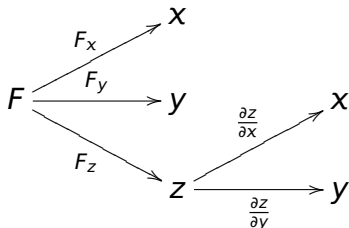
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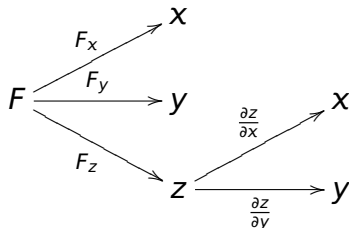
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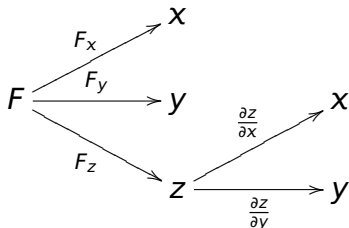
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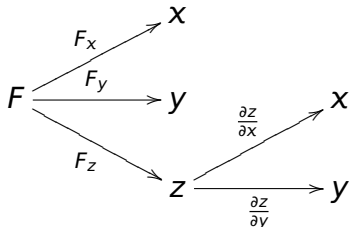
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例 设 $\Phi(u, v)$ 具有连续偏导数, 函数 $z = z(x, y)$ 满足 $\Phi(cx - az, cy - bz) = 0$, 证明:

$$a \frac{\partial z}{\partial x} + b \frac{\partial z}{\partial y} = c.$$

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$$F_y =$$

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We are here now...

1. 一个方程的情形

2. 方程组的情形

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

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回顾：二元线性方程组的求解

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回顾：二元线性方程组的求解

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$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

(1) $\times a_{22}$ - (2) $\times a_{12}$ ，消去 y ，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

(2) $\times a_{11}$ - (1) $\times a_{21}$ ，消去 x ，得：

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$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

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(2) $\times a_{11}$ - (1) $\times a_{21}$ ，消去 x ，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

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用消元法解：

(1) $\times a_{22}$ - (2) $\times a_{12}$ ，消去 y ，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{b_1 a_{22} - a_{12} b_2}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

(2) $\times a_{11}$ - (1) $\times a_{21}$ ，消去 x ，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{a_{11} b_2 - b_1 a_{21}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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(1) $\times a_{22}$ - (2) $\times a_{12}$ ，消去 y ，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

(2) $\times a_{11}$ - (1) $\times a_{21}$ ，消去 x ，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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(1) $\times a_{22}$ - (2) $\times a_{12}$ ，消去 y ，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

(2) $\times a_{11}$ - (1) $\times a_{21}$ ，消去 x ，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \quad , \quad y =$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad , \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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公式:

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \text{---}, \quad y =$$

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - \quad , \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = -$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-17}{3}, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-10}{3}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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公式:

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公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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方程组的隐函数求导公式

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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问题：如何计算 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$?

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$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}}$$

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$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \end{cases}$$

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假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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$$\Rightarrow u_x = \text{—————}, \quad v_x = \text{—————}$$

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$$\Rightarrow u_x = \frac{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_x = \frac{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

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求解如下：

$$\begin{aligned} \begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} &\xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases} \\ \Rightarrow u_x &= \frac{\begin{vmatrix} -F_x & F_v \\ -G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, & v_x &= \frac{\begin{vmatrix} -F_u & F_x \\ -G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \end{aligned}$$

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方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

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$$\Rightarrow u_y = \underline{\hspace{2cm}}, \quad v_y = \underline{\hspace{2cm}}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = \frac{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = \frac{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

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$$\begin{aligned}
 \begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} &\xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases} \\
 \Rightarrow u_y = &\frac{\begin{vmatrix} -F_y & F_v \\ -G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = \frac{\begin{vmatrix} -F_u & F_y \\ -G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}
 \end{aligned}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

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$$= - \frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)}$$

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$$= - \frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)} \quad = - \frac{1}{J} \frac{\partial(F, G)}{\partial(u, y)}$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

$$v_y =$$

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$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{matrix} \xRightarrow{\frac{\partial}{\partial x}} \\ \xRightarrow{\frac{\partial}{\partial y}} \end{matrix}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

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$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$
$$\xRightarrow{\frac{\partial}{\partial y}}$$

$$u_x =$$

$$v_x =$$

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总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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$$u_x =$$

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所以

$$u_x = - \frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$v_x = - \frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$u_y =$$

$$v_y =$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{matrix} \xRightarrow{\frac{\partial}{\partial x}} \\ \xRightarrow{\frac{\partial}{\partial y}} \end{matrix} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \\ \\ F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_x = - \frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$u_y = - \frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$v_x = - \frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$v_y = - \frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{matrix} \xRightarrow{\frac{\partial}{\partial x}} \\ \xRightarrow{\frac{\partial}{\partial y}} \end{matrix} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \\ F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$\begin{aligned} u_x &= - \frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, & v_x &= - \frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \\ u_y &= - \frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, & v_y &= - \frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \end{aligned}$$

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所以

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_x = - \frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_x = - \frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(u, x)}$$

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例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \begin{matrix} \xRightarrow{\frac{\partial}{\partial x}} \\ \xRightarrow{\frac{\partial}{\partial y}} \end{matrix}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

$$v_y =$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

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$$\xRightarrow{\frac{\partial}{\partial y}}$$

$$u_x =$$

$$v_x =$$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \end{cases}$$

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$$u_y = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}, \quad v_y = -\frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J}$$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

$$\text{所以 } J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$u_x = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J} = \frac{-\sin v}{e^u(\sin v - \cos v) + 1}, \quad v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J} = \frac{e^u - \cos v}{ue^u(\sin v - \cos v) + u}$$

$$u_y = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J} = \frac{\cos v}{e^u(\sin v - \cos v) + 1}, \quad v_y = -\frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u \sin v \cdot v_x = 0 \end{cases}$$
$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

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$$u_y = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J} = \frac{\cos v}{e^u(\sin v - \cos v) + 1}, \quad v_y = -\frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J} = -\frac{e^u + \sin v}{ue^u(\sin v - \cos v) + u}$$