§2.2　矩阵的运算

数学系　　梁卓滨

2018 - 2019 学年上学期

矩阵的加法运算

定义 设 $A = (a_{ij})_{m \times n}$，$B = (b_{ij})_{m \times n}$，则定义

$$A + B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$

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$$\overset{\text{def}}{=\!=} \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}_{m \times n}$$

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$$= (a_{ij} + b_{ij})_{m \times n}$$

称为矩阵 A，B 的和。

矩阵的减法运算

矩阵 A, B 的差定义为：

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例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$, 求 $A + B$ 和 $A - B$

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解

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} =$$

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解

$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & & \\ & & \end{pmatrix}_{2 \times 3}$$

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$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 8 \end{pmatrix}_{2 \times 3}$$

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$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 8 \\ 6 & 5 & 5 \end{pmatrix}_{2 \times 3}$$

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例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$, 求 $A + B$ 和 $A - B$

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矩阵的数乘

定义 设 $A = (a_{ij})_{m \times n}$，$k$ 为数，则定义

$$kA = k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

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例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$，求 $2A$

矩阵的数乘

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矩阵的数乘

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练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix},$ 求 $3A + 2B - 4C$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \ B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}, \ $ 且 $5A + 3X = B,$ 求 X

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解
$$3A + 2B - 4C = 3\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4\begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$$

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$$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} \qquad \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix},\ B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix},$ 且 $5A + 3X = B,$ 求 X

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$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} \quad \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$

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解
$$3A + 2B - 4C = 3\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4\begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$$
$$= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

解

$$\begin{pmatrix} -\dfrac{2}{3} & -\dfrac{5}{3} \\ -\dfrac{8}{3} & -\dfrac{14}{3} \end{pmatrix}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

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$$3A + 2B - 4C = 3\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4\begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$$

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练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

解

$$X = \frac{1}{3}(B - 5A) =$$

$$\begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix},$ 求 $3A + 2B - 4C$

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解

$$X = \frac{1}{3}(B - 5A) = \frac{1}{3}\left(\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 5\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \right)$$

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$$= \frac{1}{3}\left(\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - \begin{pmatrix} 5 & 10 \\ 15 & 20 \end{pmatrix} \right) = \qquad \begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

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解

$$X = \frac{1}{3}(B - 5A) = \frac{1}{3}\left(\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 5\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\right)$$

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练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}, C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

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$$X = \frac{1}{3}(B - 5A) = \frac{1}{3}\left(\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 5\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\right)$$

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注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \qquad , \qquad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \qquad , \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=\!=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} \qquad , \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

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$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \overset{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} \qquad , \qquad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

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性质 设 A, B, C 均是 $m \times n$ 矩阵，k, l 是数，则

1. $k(A + B) = kA + kB$

注 区分

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1. $k(A + B) = kA + kB$

2. $(k + l)A = kA + lA$

注 区分

$$k\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \overset{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

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4. $1 \cdot A = A$

证明 设 $A = (a_{ij})_{m \times n}$，$B = (b_{ij})_{m \times n}$，则

注 区分

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$$k(A + B) =$$

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$$k(A + B) = k(a_{ij} + b_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

$$kA + kB =$$

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$$kA + kB = (ka_{ij})_{m \times n} + (kb_{ij})_{m \times n}$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \overset{\text{or}}{=\!=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

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$$kA + kB = (ka_{ij})_{m \times n} + (kb_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

所以 $k(A + B) = kA + kB$。

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 $(2A - Y) - 2(B + Y) = O$,求 Y

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 $(2A - Y) - 2(B + Y) = O$,求 Y

解 $Y = \frac{2}{3}(A - B)$

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 $(2A - Y) - 2(B + Y) = O$，求 Y

解 $Y = \frac{2}{3}(A - B)$，所以

$$Y = \frac{2}{3}(A - B) = \frac{2}{3}\left(\begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix} \right)$$

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 $(2A - Y) - 2(B + Y) = O$，求 Y

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$$Y = \frac{2}{3}(A - B) = \frac{2}{3}\left(\begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix} \right)$$

$$= \frac{2}{3} \begin{pmatrix} -3 & -1 & -1 & 1 \\ 4 & 0 & 4 & 0 \\ 1 & 3 & 3 & 5 \end{pmatrix}$$

练习 设
$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$，求数 a, b, c 的值

设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$，求数 a, b, c 的值

解

$$aA + bB + cC =$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$，求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$，求数 a, b, c 的值

解

$$aA + bB + cC = a\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b\begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c\begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} & & \\ & & \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$，求数 a, b, c 的值

解

$$aA + bB + cC = a\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b\begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c\begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} a + b - c & \\ & \end{pmatrix}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$，求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} a + b - c & b \\ & \end{pmatrix}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$，求数 a, b, c 的值

解

$$aA + bB + cC = a\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b\begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c\begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & \end{pmatrix}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$，求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix}$$

练习 设

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所以

$$\left\{ \begin{array}{c} \\ \\ \\ \\ \end{array} \right.$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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解

$$aA + bB + cC = a\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b\begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c\begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} a + b - c & b \\ 2a + 3b + c & a - c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

所以

$$\begin{cases} a + b - c = 1 \end{cases}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$，求数 a, b, c 的值

解

$$aA + bB + cC = a\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b\begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c\begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

所以

$$\begin{cases} a+b-c = 1 \\ b = 0 \end{cases}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

所以

$$\begin{cases} a+b-c = 1 \\ b = 0 \\ 2a+3b+c = 0 \end{cases}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$，求数 a, b, c 的值

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$$aA + bB + cC = a\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b\begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c\begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

所以

$$\begin{cases} a+b-c = 1 \\ b = 0 \\ 2a+3b+c = 0 \\ a-c = 1 \end{cases} \Rightarrow \begin{cases} b = 0 \end{cases}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$，求数 a, b, c 的值

解

$$aA + bB + cC = a\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b\begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c\begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

所以

$$\begin{cases} a+b-c=1 \\ b=0 \\ 2a+3b+c=0 \\ a-c=1 \end{cases} \Rightarrow \begin{cases} a=\frac{1}{3} \\ b=0 \end{cases}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$，求数 a, b, c 的值

解

$$aA + bB + cC = a\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b\begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c\begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} a+b-c & b \\ 2a+3b+c & a-c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

所以

$$\begin{cases} a+b-c = 1 \\ b = 0 \\ 2a+3b+c = 0 \\ a-c = 1 \end{cases} \Rightarrow \begin{cases} a = \frac{1}{3} \\ b = 0 \\ c = -\frac{2}{3} \end{cases}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$，$B = (b_{kj})_{l \times n}$，定义矩阵 A，B 的乘积为 $m \times n$ 矩阵：

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

矩阵的乘积

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矩阵的乘积

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$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

$$c_{ij} = A \text{ 第 } i \text{ 行与 } B \text{ 第 } j \text{ 列对应元素乘积的和}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$，$B = (b_{kj})_{l \times n}$，定义矩阵 A，B 的乘积为 $m \times n$ 矩阵：

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其中

$$c_{ij} = A \text{ 第 } i \text{ 行与 } B \text{ 第 } j \text{ 列对应元素乘积的和}$$

即

$$a_{i1} \qquad a_{i2} \qquad \cdots \qquad a_{il}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$，$B = (b_{kj})_{l \times n}$，定义矩阵 A，B 的乘积为 $m \times n$ 矩阵：

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

$$c_{ij} = A \, \text{第} \, i \, \text{行与} \, B \, \text{第} \, j \, \text{列对应元素乘积的和}$$

即

$$a_{i1}b_{1j} \quad a_{i2}b_{2j} \quad \cdots \quad a_{il}b_{lj}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$，$B = (b_{kj})_{l \times n}$，定义矩阵 A，B 的乘积为 $m \times n$ 矩阵：

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即

$$a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$，$B = (b_{kj})_{l \times n}$，定义矩阵 A，B 的**乘积**为 $m \times n$ 矩阵：

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其中

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$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj}$$

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其中

$$c_{ij} = A \text{ 第 } i \text{ 行与 } B \text{ 第 } j \text{ 列对应元素乘积的和}$$

即

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj} \qquad a_{ik}b_{kj}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$，$B = (b_{kj})_{l \times n}$，定义矩阵 A，B 的乘积为 $m \times n$ 矩阵：

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其中

$$c_{ij} = A \text{ 第 } i \text{ 行与 } B \text{ 第 } j \text{ 列对应元素乘积的和}$$

即

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj} \qquad \sum_{k=1}^{l} a_{ik}b_{kj}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$，$B = (b_{kj})_{l \times n}$，定义矩阵 A，B 的乘积为 $m \times n$ 矩阵：

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$$c_{ij} = A \, \text{第} \, i \, \text{行与} \, B \, \text{第} \, j \, \text{列对应元素乘积的和}$$

即

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj} = \sum_{k=1}^{l} a_{ik}b_{kj}$$

矩阵的乘积

$$
\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l}
\cdot
\begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}
$$

$$
= \begin{pmatrix} c_{11} & \cdots & & \cdots & c_{1n} \\ \vdots & & \vdots & & \vdots \\ & \cdots & c_{ij} & \cdots & \\ \vdots & & \vdots & & \vdots \\ c_{m1} & \cdots & & \cdots & c_{mn} \end{pmatrix}_{m \times n}
$$

矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$

$$= \begin{pmatrix} c_{11} & \cdots & & \cdots & c_{1n} \\ \vdots & & \vdots & & \vdots \\ & \cdots & c_{ij} & \cdots & \\ \vdots & & \vdots & & \vdots \\ c_{m1} & \cdots & & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$

$$a_{i1} \qquad a_{i2} \qquad \cdots \qquad a_{il}$$

矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$

$$= \begin{pmatrix} c_{11} & \cdots & & \cdots & c_{1n} \\ \vdots & & \vdots & & \vdots \\ & \cdots & c_{ij} & \cdots & \\ \vdots & & \vdots & & \vdots \\ c_{m1} & \cdots & & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$

$$a_{i1}b_{1j} \quad a_{i2}b_{2j} \quad \cdots \quad a_{il}b_{lj}$$

矩阵的乘积

$$
\begin{pmatrix}
a_{11} & \cdots & \cdots & a_{1l} \\
\vdots & & & \vdots \\
a_{i1} & \cdots & \cdots & a_{il} \\
\vdots & & & \vdots \\
a_{m1} & \cdots & \cdots & a_{ml}
\end{pmatrix}_{m \times l}
\cdot
\begin{pmatrix}
b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\
\vdots & & \vdots & & \vdots \\
\vdots & & \vdots & & \vdots \\
b_{l1} & \cdots & b_{lj} & \cdots & b_{ln}
\end{pmatrix}_{l \times n}
$$

$$
=
\begin{pmatrix}
c_{11} & \cdots & & \cdots & c_{1n} \\
\vdots & & \vdots & & \vdots \\
& \cdots & c_{ij} & \cdots & \\
\vdots & & \vdots & & \vdots \\
c_{m1} & \cdots & & \cdots & c_{mn}
\end{pmatrix}_{m \times n}
$$

$$
a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj}
$$

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矩阵的乘积

$$
\begin{pmatrix}
a_{11} & \cdots & \cdots & a_{1l} \\
\vdots & & & \vdots \\
a_{i1} & \cdots & \cdots & a_{il} \\
\vdots & & & \vdots \\
a_{m1} & \cdots & \cdots & a_{ml}
\end{pmatrix}_{m \times l}
\cdot
\begin{pmatrix}
b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\
\vdots & & \vdots & & \vdots \\
\vdots & & \vdots & & \vdots \\
b_{l1} & \cdots & b_{lj} & \cdots & b_{ln}
\end{pmatrix}_{l \times n}
$$

$$
=
\begin{pmatrix}
c_{11} & \cdots & & \cdots & c_{1n} \\
\vdots & & \vdots & & \vdots \\
& \cdots & c_{ij} & \cdots & \\
\vdots & & \vdots & & \vdots \\
c_{m1} & \cdots & & \cdots & c_{mn}
\end{pmatrix}_{m \times n}
$$

$$
c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj}
$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times 3}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times3} = \begin{pmatrix} & & \\ & & \end{pmatrix}$$

矩阵的乘积

例 1 $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} & & \\ & & \\ & & \\ & & \end{pmatrix}_{4 \times 3}$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times3}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times3}$$

$$c_{23} =$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times 3}$$

$$c_{23} = a_{21} \qquad a_{22}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times 3}$$

$$c_{23} = a_{21}b_{13} \quad a_{22}b_{23}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 2 设 $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$, 计算 AB

矩阵的乘积

例 1

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 2 设 $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$, 计算 AB

解

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3\times2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2\times3} =$$

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解

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3\times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2\times 3} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}_{3\times 3}$$

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$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times 3}$$

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解
$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

矩阵的乘积

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$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

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矩阵的乘积

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$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3\times2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2\times3} = \begin{pmatrix} 8 & -7 & \\ & & \\ & & \end{pmatrix}_{3\times3}$$

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$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times3}$$

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矩阵的乘积

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例 1

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4\times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2\times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4\times 3}$$

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矩阵的乘积

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矩阵的乘积

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矩阵的乘积

例 3 计算 $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

矩阵的乘积

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解

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矩阵的乘积

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解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & & \\ & & \end{pmatrix}_{2 \times 3}$$

矩阵的乘积

例 3 计算 $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2\times3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3\times3} = \begin{pmatrix} 10 & 4 & \\ & & \end{pmatrix}_{2\times3}$$

矩阵的乘积

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矩阵的乘积

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矩阵的乘积

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矩阵的乘积

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解

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例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$。计算 AB,

BA 及 CB。

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$。计算 AB,

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解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

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解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (\qquad\qquad)_{1 \times 1}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$。计算 AB,

BA 及 CB。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1}$$

例 4 设 $A = (1,\ 2,\ 3)$，$B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$，$C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$。计算 AB，

BA 及 CB。

解

$$AB = (1,\ 2,\ 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

例 4 设 $A = (1, 2, 3)$，$B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$，$C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$。计算 AB，BA 及 CB。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) =$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$。计算 AB, BA 及 CB。

解

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$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

例 4 设 $A = (1,\ 2,\ 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$。计算 AB，BA 及 CB。

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解

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$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{pmatrix}_{3 \times 3}$$

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$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3 \times 3}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$。计算 AB, BA 及 CB。

解

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$$CB = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$。计算 AB，BA 及 CB。

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例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$。计算 AB, BA 及 CB。

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$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

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$$CB = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ \\ \end{pmatrix}_{3 \times 1}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$。计算 AB，

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$$AB = (1,\ 2,\ 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1,\ 2,\ 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3 \times 3}$$

$$CB = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}_{3 \times 1}$$

矩阵的乘积

例 5 设 $A = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}$, 求 AB, BA

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注

1. 即便 AB, BA 都有意义, 也不一定相等。

   矩阵的乘法不满足交换律！

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2. $BA = 0$ 不能推出 $B = 0$ 或 $A = 0$

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总结　1. AB 可以存在，但 BA 不一定有意义
2. 即便 AB，BA 都有意义，也不一定相等。矩阵的乘法不满足交换律！（矩阵相乘要注意顺序）
3. $BA = 0$ 不能推出 $B = 0$ 或 $A = 0$
4. 即便假设 $A \neq 0$，$BA = CA$ 也推不出 $B = C$。

矩阵乘法的运算法则

设下列各式所涉及的矩阵乘法都是有意义，则

1. $(AB)C = A(BC)$

2. $(A + B)C = AC + BC$

3. $C(A + B) = CA + CB$

4. $k(AB) = (kA)B = A(kB)$

矩阵的转置

定义 将 $m \times n$ 矩阵 A 的行与列互换，得到的 $n \times m$ 矩阵，称为矩阵 A 的转置矩阵，记为 A^T。

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	A	A^T
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位置 (i, j) 上的元素	A	A^T
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例 3 设 $A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$, 计算 AA^T 及 $A^T A$。

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解

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解

$$AA^T = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} & & \\ & & \end{pmatrix}_{2 \times 2}$$

例 3 设 $A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$，计算 AA^T 及 A^TA。

解

$$AA^T = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & \\ & \end{pmatrix}_{2 \times 2}$$

例 3 设 $A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$，计算 AA^T 及 A^TA。

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$$A^TA = \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

例 3 设 $A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$，计算 AA^T 及 A^TA。

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$$A^TA = \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 13 & 2 & 2 \\ 2 & 1 & 4 \\ 2 & 4 & 20 \end{pmatrix}_{3 \times 3}$$

转置矩阵的性质

1. $(A^T)^T = A$

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证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$

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阶数	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

并且

$(AB)^T$
(i, j)元素 $=$

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阶数	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

并且

$$\underset{(i, j)元素}{(AB)^T} = \underset{(i, j)元素}{B^T A^T}$$

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并且

$$\begin{array}{c} (AB)^T \\ (i, j)元素 \end{array} = \begin{array}{c} AB \\ (j, i)元素 \end{array} = \qquad\qquad\qquad\qquad \begin{array}{c} B^T A^T \\ (i, j)元素 \end{array}$$

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并且

$$\underset{(i,\,j)\text{元素}}{(AB)^T} = \underset{(j,\,i)\text{元素}}{AB} = a_{j1} \qquad a_{j2} \qquad \cdots \qquad a_{jl} \qquad \underset{(i,\,j)\text{元素}}{B^T A^T}$$

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	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

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$$\begin{matrix} (AB)^T \\ (i, j)\text{元素} \end{matrix} = \begin{matrix} AB \\ (j, i)\text{元素} \end{matrix} = a_{j1}b_{1i} \quad a_{j2}b_{2i} \quad \cdots \quad a_{jl}b_{li} \quad \begin{matrix} B^T A^T \\ (i, j)\text{元素} \end{matrix}$$

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A^T第 j 列元素

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证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

阶数	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
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证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

阶数	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

并且

$$\begin{array}{c} (AB)^T \\ (i, j)\text{元素} \end{array} = \begin{array}{c} AB \\ (j, i)\text{元素} \end{array} = a_{j1}b_{1i} + a_{j2}b_{2i} + \cdots + a_{jl}b_{li} \qquad \begin{array}{c} B^T A^T \\ (i, j)\text{元素} \end{array}$$

A^T第 j 列元素 $\qquad$ B^T第 i 行元素

转置矩阵的性质
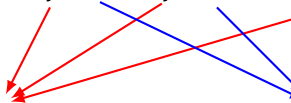
1. $(A^T)^T = A$

2. $(A + B)^T = A^T + B^T$, $(kA)^T = kA^T$

3. $(AB)^T = B^T A^T$

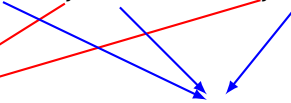证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

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A^T第 j 列元素　　　　　B^T第 i 行元素

方阵的幂

设 $A = (a_{ij})_{n \times n}$ 为 n 阶方阵，$k \in \mathbb{N}$ 为自然数，定义

$$A^k = \underbrace{A \cdot A \cdot \cdots \cdot A}_{k \text{个}}$$

称为方阵 A 的 k 次幂

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方阵的幂的性质 $A^{k_1} A^{k_2} = A^{k_1 + k_2}$，$(A^{k_1})^{k_2} = A^{k_1 k_2}$

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$$(A^{k_1})^{k_2} =$$

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练习 设 $A = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$，其中 λ 为常数，计算 A^n。

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解

$$A^2 =$$

$$A^3 =$$

$$A^4 =$$

$$\vdots$$

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解

$$A^2 = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix}$$

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$$A^4 = A^3 \cdot A = \begin{pmatrix} 1 & 0 \\ 3\lambda & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 4\lambda & 1 \end{pmatrix}$$

$$\vdots$$

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$$\vdots$$

$$A^n = \begin{pmatrix} 1 & 0 \\ n\lambda & 1 \end{pmatrix}$$

方阵的幂

注 设 A, B 为 n 阶方阵，一般地

$$(AB)^k \neq A^k B^k$$

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这是，例如 $k = 2$ 时，

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$$(AB)^2 = (AB) \cdot (AB) = ABAB$$

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注 设 A, B 为 n 阶方阵，一般地

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这是，例如 $k = 2$ 时，

$$(AB)^2 = (AB) \cdot (AB) = AB\textcolor{red}{A}B$$

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但一般地，$AB \neq BA$，

方阵的幂

注 设 A, B 为 n 阶方阵，一般地

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这是，例如 $k = 2$ 时，

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$$A^2 B^2 = (AA) \cdot (BB) = AABB$$

但一般地，$AB \neq BA$，所以 $(AB)^2 \neq A^2 B^2$

方阵的行列式

回忆：对 n 阶方阵

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

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其行列式规定为

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其行列式规定为

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方阵行列式的性质

设 A, B 均是 n 阶方阵，k 为数，则

1. $|A^T| = |A|$
2. $|kA| = k^n|A|$
3. $|AB| = |A| \cdot |B|$
4. $|AB| = |BA|$

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$$|kA| = \left| k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \right| =$$

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$$= k \cdot k \cdots \cdot k \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

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方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$，求 $|4A|$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$，求 $|4A|$

解

$$|4A| = 4^3 |A| =$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

解

$$|4A| = 4^3|A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} =$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$，求 $\left| 4A \right|$

解

$$\left| 4A \right| = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} =$$

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例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$，求 $|4A|$

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方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$，求 $|4A|$

解

$$|4A| = 4^3|A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 = 192$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$，求 $|4A|$

解

$$|4A| = 4^3|A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} = 64 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 6 & 3 & 5 \end{vmatrix} = 64 \times 3 = 192$$

练习 设 A 为三阶方阵，且 $|A| = -2$，求 $\left| |A|A^2A^T \right|$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

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$$= |A|^3 \left| A^2 \right| \left| A^T \right|$$
$$= |A|^3 |A|^2 |A|$$

方阵行列式的性质

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练习 设 A 为三阶方阵, 且 $|A| = -2$, 求 $\left||A|A^2A^T\right|$

解

$$\left||A|A^2A^T\right| = |A|^3 \left|A^2A^T\right|$$

$$= |A|^3 \left|A^2\right| \left|A^T\right|$$

$$= |A|^3|A|^2|A|$$

$$= |A|^6$$

方阵行列式的性质

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解

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练习 设 A 为三阶方阵, 且 $|A| = -2$, 求 $\left| |A|A^2A^T \right|$

解

$$\begin{aligned}
\left| |A|A^2A^T \right| &= |A|^3 \left| A^2A^T \right| \\
&= |A|^3 \left| A^2 \right| \left| A^T \right| \\
&= |A|^3 |A|^2 |A| \\
&= |A|^6 = (-2)^6
\end{aligned}$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

解

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解

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$$= |A|^3 \left| A^2 \right| \left| A^T \right|$$

$$= |A|^3 |A|^2 |A|$$

$$= |A|^6 = (-2)^6 = 64$$

方程组的矩阵表示

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

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$$\|$$

$$\begin{pmatrix} \quad \\ \quad \end{pmatrix}_{m \times n} \begin{pmatrix} \quad \\ \quad \end{pmatrix}_{n \times 1}$$

方程组的矩阵表示

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$$\parallel$$

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \begin{pmatrix} \\ \\ \\ \end{pmatrix}_{n \times 1}$$

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$$\|$$

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系数矩阵

方程组的矩阵表示

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等价于

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix}}_{\substack{A \\ \text{系数矩阵}}} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} = \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}_{\substack{b \\ \text{常数项矩阵}}}$$

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等价于

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{pmatrix}}_{\substack{A \\ \text{系数矩阵}}} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}_{\substack{b \\ \text{常数项矩阵}}}$$

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进一步改写成

$$Ax = b$$

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线性方程组的矩阵表示

例 方程组

$$\begin{cases} x_1 & -x_2 & +5x_3 & -x_4 & = -2 \\ x_1 & +x_2 & -2x_3 & +3x_4 & = 3 \\ 3x_1 & -x_2 & +8x_3 & +x_4 & = 7 \end{cases}$$

线性方程组的矩阵表示

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的矩阵表示 $Ax = b$ 是

$$\begin{pmatrix} & & & \end{pmatrix} \begin{pmatrix} \\ \end{pmatrix} = \begin{pmatrix} \\ \end{pmatrix}$$

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线性方程组的矩阵表示

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的矩阵表示 $Ax = b$ 是

$$\begin{pmatrix} 1 & -1 & 5 & -1 \\ 1 & 1 & -2 & 3 \\ 3 & -1 & 8 & 1 \end{pmatrix} \begin{pmatrix} \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix}$$

线性方程组的矩阵表示

例 方程组

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