## 第 9 章 d: 隐函数的求导公式

数学系 梁卓滨

2016-2017 **学年** II



#### Outline

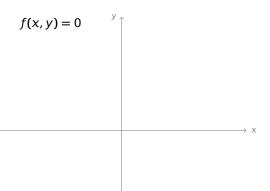
1. 一个方程的情形

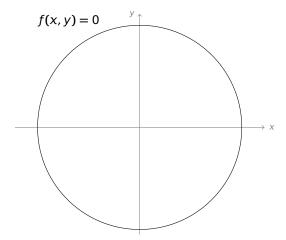
2. 方程组的情形

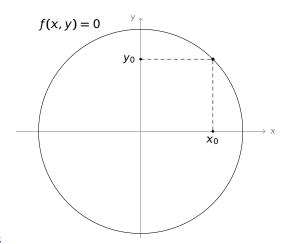
We are here now...

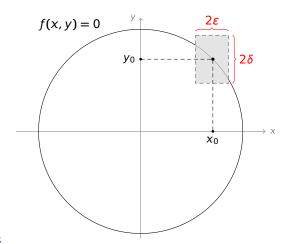
1. 一个方程的情形

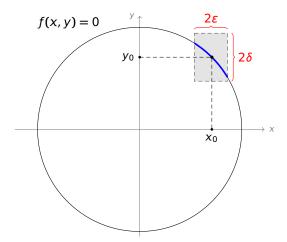
2. 方程组的情形

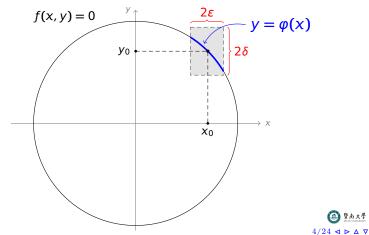


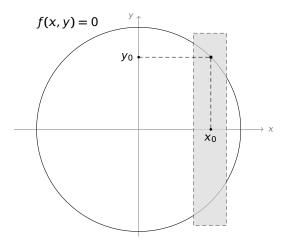


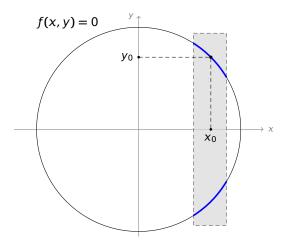


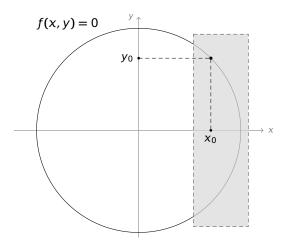


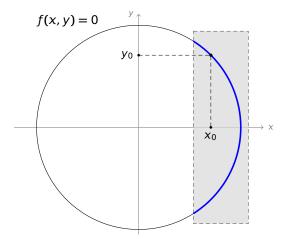


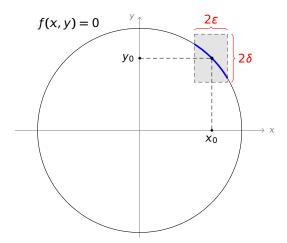


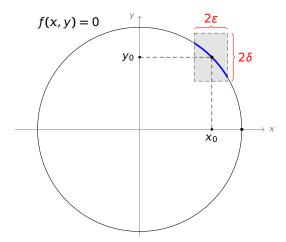


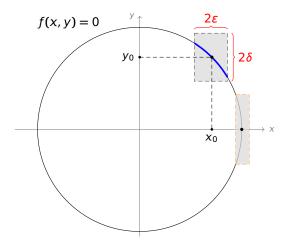


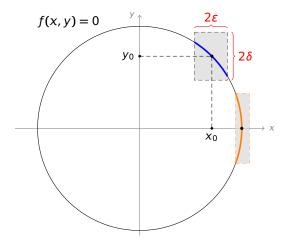


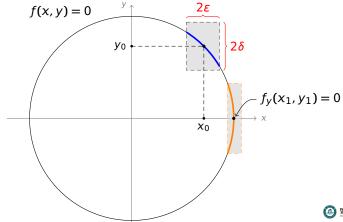


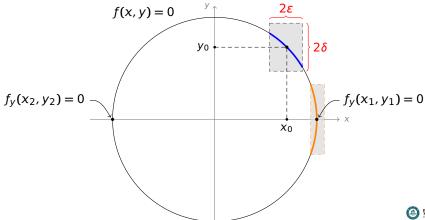


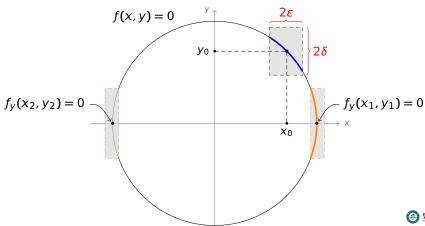


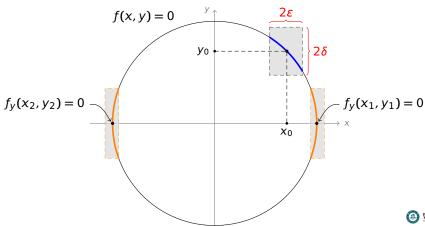


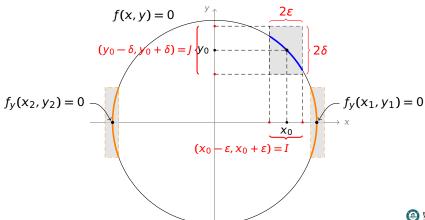




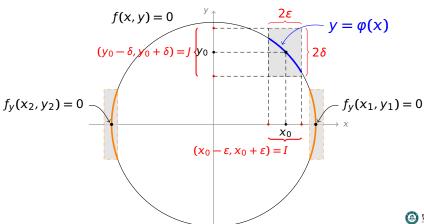






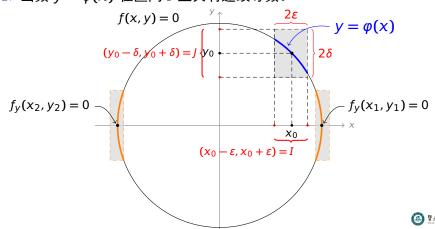






隐函数定理 设 f(x, y) 在点  $p_0(x_0, y_0)$  附近有定义,具有连续偏导;  $f(x_0, y_0) = 0$ ;  $f_y(x_0, y_0) \neq 0$ 。则存在开区间  $I = (x_0 - \varepsilon, x_0 + \varepsilon)$  和  $J = (y_0 - \delta, y_0 + \delta)$ ,使得

- 1. 对任意  $x \in I$ , 方程 f(x,y) = 0 有唯一的解  $y = \varphi(x) \in J$ 。
- 2. 函数  $y = \varphi(x)$  在区间 I 上具有连续导数。



隐函数定理 设 f(x,y) 在点  $p_0(x_0,y_0)$  附近有定义,具有连续偏导;  $f(x_0,y_0)=0$ ;  $f_y(x_0,y_0)\neq 0$ 。则存在开区间  $I=(x_0-\varepsilon,x_0+\varepsilon)$  和  $J=(y_0-\delta,y_0+\delta)$ ,使得

- 1. 对任意  $x \in I$ , 方程 f(x,y) = 0 有唯一的解  $y = \varphi(x) \in J$ 。
- 2. 函数  $y = \varphi(x)$  在区间 I 上具有连续导数。

隐函数定理 设 f(x,y) 在点  $p_0(x_0,y_0)$  附近有定义,具有连续偏导;  $f(x_0,y_0)=0$ ;  $f_y(x_0,y_0)\neq0$ 。则存在开区间  $I=(x_0-\varepsilon,x_0+\varepsilon)$  和  $J=(y_0-\delta,y_0+\delta)$ ,使得

- 1. 对任意  $x \in I$ , 方程 f(x,y) = 0 有唯一的解  $y = \varphi(x) \in J$ 。
- 2. 函数  $y = \varphi(x)$  在区间 I 上具有连续导数。

隐函数定理"设 f(x,y) 在点  $p_0(x_0,y_0)$  附近有定义,具有连续偏导;  $f(x_0,y_0)=0$ ;  $f_x(x_0,y_0)\neq 0$ 。则存在开区间  $I=(x_0-\varepsilon,x_0+\varepsilon)$  和  $I=(y_0-\delta,y_0+\delta)$ ,使得

- 1. 对任意  $y \in J$ ,方程 f(x, y) = 0 有唯一的解  $x = \psi(y) \in I$ 。
- 2. 函数  $x = \psi(y)$  在区间 J 上具有连续导数。



# 隐函数的求导法Ⅰ

公式 设 y = f(x) 满足 F(x, y) = 0,

公式 设 y = f(x) 满足 F(x, y) = 0, 即 F(x, y(x)) = 0,

## 隐函数的求导法Ⅰ

公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则 
$$\frac{dy}{dx} =$$

# 隐函数的求导法Ⅰ

公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则 
$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则 
$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则 
$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

证明 
$$: F(x, y(x)) = 0$$

公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则 
$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

$$: F(x, y(x)) = 0$$

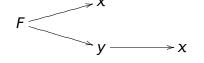
$$0 = \frac{d}{dx}F(x, y(x)) =$$

公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则

$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

证明 
$$: F(x, y(x)) = 0$$

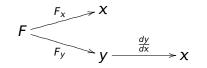
$$\therefore \quad 0 = \frac{d}{dx} F(x, y(x)) =$$



公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则 
$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

$$: F(x, y(x)) = 0$$

$$\therefore \quad 0 = \frac{d}{dx} F(x, y(x)) =$$

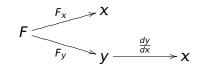


公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则

$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

证明 
$$: F(x, y(x)) = 0$$

$$\therefore 0 = \frac{d}{dx} F(x, y(x)) = F_x +$$

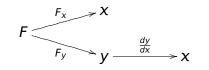


# 隐函数的求导法I

公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则 
$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

证明 
$$:: F(x, y(x)) = 0$$

$$\therefore \quad 0 = \frac{d}{dx} F(x, y(x)) = F_x + F_y \cdot \frac{dy}{dx}$$



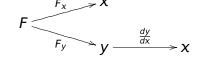
# 隐函数的求导法I

公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则 
$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

$$:: F(x, y(x)) = 0$$

$$\therefore 0 = \frac{d}{dx}F(x, y(x)) = F_x + F_y \cdot \frac{dy}{dx}$$

$$\therefore \quad \frac{dy}{dx} = -\frac{F_x}{F_y}$$





例 设 
$$y = f(x)$$
 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

#### 方法一

$$F(x, y) = 0$$

$$\frac{dy}{dx} = -\frac{r_x}{F_y}$$

例 设 
$$y = f(x)$$
 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

方法一 注意 
$$\sin y + e^x - xy^2 = 0$$

$$F(x, y) = 0$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} =$$

例设 
$$y = f(x)$$
 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

方法一 注意 
$$\sin y + e^x - xy^2 = 0$$
,令  $F(x, y) = \sin y + e^x - xy^2$ ,  
 $F(x, y) = 0$ 

$$\frac{dy}{dx} = -\frac{F_x}{F_y} =$$

方法一 注意  $\sin y + e^x - xy^2 = 0$ , 令  $F(x, y) = \sin y + e^x - xy^2$ , 则

$$F(x,y)=0$$
,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} =$$

方法一 注意 
$$\sin y + e^x - xy^2 = 0$$
, 令  $F(x, y) = \sin y + e^x - xy^2$ , 则

$$F(x,y)=0$$
,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -$$

方法一 注意 
$$\sin y + e^x - xy^2 = 0$$
,令  $F(x, y) = \sin y + e^x - xy^2$ ,则

$$F(x,y)=0$$
,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{(\sin y + e^x - xy^2)_y'}$$

方法一 注意  $\sin y + e^x - xy^2 = 0$ , 令  $F(x, y) = \sin y + e^x - xy^2$ , 则

$$F(x, y) = 0$$
,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法一 注意 
$$\sin y + e^x - xy^2 = 0$$
,令  $F(x, y) = \sin y + e^x - xy^2$ ,则

$$F(x,y)=0$$
,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二



方法一 注意  $\sin y + e^x - xy^2 = 0$ ,令  $F(x, y) = \sin y + e^x - xy^2$ ,则

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 
$$\sin y(x) + e^x - xy(x)^2 = 0$$
,



F(x, y) = 0,所以

方法一 注意  $\sin y + e^x - xy^2 = 0$ ,令  $F(x, y) = \sin y + e^x - xy^2$ ,则

$$F(x, y) = 0$$
,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 
$$\sin y(x) + e^x - xy(x)^2 = 0$$
,所以 
$$0 = (\sin y(x) + e^x - xy(x)^2)_x'$$

例 设 
$$y = f(x)$$
 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

方法一 注意 
$$\sin y + e^x - xy^2 = 0$$
,令  $F(x, y) = \sin y + e^x - xy^2$ ,则  $F(x, y) = 0$ ,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 
$$\sin y(x) + e^x - xy(x)^2 = 0$$
,所以
$$0 = (\sin y(x) + e^x - xy(x)^2)_x'$$

$$= (\sin y(x))_y' + (e^x)_y' - (xy(x)^2)_y'$$

例设 
$$y = f(x)$$
 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

方法一 注意  $\sin y + e^x - xy^2 = 0$ , 令  $F(x, y) = \sin y + e^x - xy^2$ , 则

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 
$$\sin y(x) + e^x - xy(x)^2 = 0$$
,所以
$$0 = (\sin y(x) + e^x - xy(x)^2)_x'$$

$$= (\sin y(x))_x' + (e^x)_x' - (xy(x)^2)_x'$$

$$= \cos y \cdot y'$$



F(x, y) = 0,所以

例 设 
$$y = f(x)$$
 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

方法一 注意  $\sin y + e^x - xy^2 = 0$ , 令  $F(x, y) = \sin y + e^x - xy^2$ , 则

$$F(x, y) = 0$$
,所以
$$dy F_x (\sin y + e^x - xy^2)'_x$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 
$$\sin y(x) + e^x - xy(x)^2 = 0$$
,所以
$$0 = (\sin y(x) + e^x - xy(x)^2)_x'$$

$$= (\sin y(x))_x' + (e^x)_x' - (xy(x)^2)_x'$$

$$= \cos y \cdot y' + e^x$$

例 设 
$$y = f(x)$$
 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

方法一 注意  $\sin y + e^x - xy^2 = 0$ ,令  $F(x, y) = \sin y + e^x - xy^2$ ,则 F(x, y) = 0,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 
$$\sin y(x) + e^x - xy(x)^2 = 0$$
,所以
$$0 = (\sin y(x) + e^x - xy(x)^2)_x'$$

$$= (\sin y(x))_x' + (e^x)_x' - (xy(x)^2)_x'$$

$$= \cos y \cdot y' + e^x - y^2 - 2xy \cdot y'$$



方法一 注意  $\sin y + e^x - xy^2 = 0$ ,令  $F(x, y) = \sin y + e^x - xy^2$ ,则

$$F(x,y)=0$$
,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 
$$\sin y(x) + e^x - xy(x)^2 = 0$$
,所以
$$0 = (\sin y(x) + e^x - xy(x)^2)_x'$$

$$= (\sin y(x))_x' + (e^x)_x' - (xy(x)^2)_x'$$

$$= \cos y \cdot y' + e^x - y^2 - 2xy \cdot y'$$

$$= e^x - y^2 + (\cos y - 2xy)y'$$



方法一 注意  $\sin y + e^x - xy^2 = 0$ ,令  $F(x, y) = \sin y + e^x - xy^2$ ,则

$$F(x, y) = 0, \text{ fill}$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 
$$\sin y(x) + e^x - xy(x)^2 = 0$$
,所以
$$0 = (\sin y(x) + e^x - xy(x)^2)_x'$$

$$= (\sin y(x))_x' + (e^x)_x' - (xy(x)^2)_x'$$

$$= \cos y \cdot y' + e^x - y^2 - 2xy \cdot y'$$

$$= e^x - y^2 + (\cos y - 2xy)y'$$

所以  $y' = -\frac{e^{x}-y^{2}}{\cos y-2xy}$ 

例 设 y = f(x) 满足  $\ln(x^2 + y^2) + 3xy = 4$ , 求  $\frac{dy}{dx}$ 

例 设 
$$y = f(x)$$
 满足  $\ln(x^2 + y^2) + 3xy = 4$ ,求  $\frac{dy}{dx}$ 

解

$$F(x, y) = 0$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = 0$$

例 设 
$$y = f(x)$$
 满足  $\ln(x^2 + y^2) + 3xy = 4$ ,求  $\frac{dy}{dx}$ 解 注意  $\ln(x^2 + y^2) + 3xy - 4 = 0$ 

$$F(x, y) = 0$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = 0$$

例设 
$$y = f(x)$$
 满足  $\ln(x^2 + y^2) + 3xy = 4$ ,求  $\frac{dy}{dx}$ 

解注意 
$$ln(x^2 + y^2) + 3xy - 4 = 0$$
,令

$$F(x, y) = \ln(x^2 + y^2) + 3xy - 4$$

$$F(x, y) = 0$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = 0$$

例 设 
$$y = f(x)$$
 满足  $\ln(x^2 + y^2) + 3xy = 4$ , 求  $\frac{dy}{dx}$ 

解注意 
$$ln(x^2 + y^2) + 3xy - 4 = 0$$
, 令

$$F(x, y) = \ln(x^2 + y^2) + 3xy - 4$$

则 
$$F(x, y) = 0$$
, 所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\ln(x^2 + y^2) + 3xy - 4)_x'}{(\ln(x^2 + y^2) + 3xy - 4)_y'}$$

例 设 
$$y = f(x)$$
 满足  $\ln(x^2 + y^2) + 3xy = 4$ , 求  $\frac{dy}{dx}$ 

解注意 
$$ln(x^2 + y^2) + 3xy - 4 = 0$$
, 令

$$F(x, y) = \ln(x^2 + y^2) + 3xy - 4$$

则 
$$F(x, y) = 0$$
, 所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\ln(x^2 + y^2) + 3xy - 4)_x'}{(\ln(x^2 + y^2) + 3xy - 4)_y'}$$

例 设 
$$y = f(x)$$
 满足  $\ln(x^2 + y^2) + 3xy = 4$ ,求  $\frac{dy}{dx}$ 

解 注意 
$$\ln(x^2 + y^2) + 3xy - 4 = 0$$
,令

$$F(x, y) = \ln(x^2 + y^2) + 3xy - 4$$

则 
$$F(x, y) = 0$$
, 所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\ln(x^2 + y^2) + 3xy - 4)_x'}{(\ln(x^2 + y^2) + 3xy - 4)_y'}$$
$$\frac{2x}{x^2 + y^2} + 3y$$

例 设 
$$y = f(x)$$
 满足  $\ln(x^2 + y^2) + 3xy = 4$ ,求  $\frac{dy}{dx}$ 

解注意 
$$ln(x^2 + y^2) + 3xy - 4 = 0$$
, 令

$$F(x, y) = \ln(x^2 + y^2) + 3xy - 4$$

则 
$$F(x, y) = 0$$
,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\ln(x^2 + y^2) + 3xy - 4)_x'}{(\ln(x^2 + y^2) + 3xy - 4)_y'}$$

$$\frac{2x}{x^2 + y^2} + 3y$$

$$= -\frac{\frac{2x}{x^2 + y^2} + 3y}{\frac{2y}{x^2 + y^2} + 3x}$$

例 设 
$$y = f(x)$$
 满足  $\ln(x^2 + y^2) + 3xy = 4$ ,求  $\frac{dy}{dx}$ 

解注意 
$$ln(x^2 + y^2) + 3xy - 4 = 0$$
, 令

$$F(x, y) = \ln(x^2 + y^2) + 3xy - 4$$

则 
$$F(x, y) = 0$$
, 所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\ln(x^2 + y^2) + 3xy - 4)_x'}{(\ln(x^2 + y^2) + 3xy - 4)_y'}$$

$$= -\frac{\frac{2x}{x^2 + y^2} + 3y}{\frac{2y}{x^2 + y^2} + 3x}$$

$$=\frac{2x+3x^2y+3y^3}{2y+3xy^2+3x^3}$$

公式 设 z = f(x, y) 满足 F(x, y, z) = 0,

公式 设 z = f(x, y) 满足 F(x, y, z) = 0, 即 F(x, y, z(x, y)) = 0,

公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial Z}{\partial X} = \qquad , \qquad \frac{\partial Z}{\partial y} =$$

公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ ,即  $F(x, y, z(x, y)) = 0$ ,则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} =$$

公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ ,即  $F(x, y, z(x, y)) = 0$ ,则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ ,即  $F(x, y, z(x, y)) = 0$ ,则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

$$F(x, y, z(x, y)) = 0$$

公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

$$F(x, y, z(x, y)) = 0$$

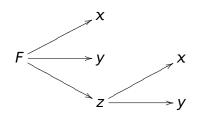
$$\therefore \quad 0 = \frac{\partial}{\partial x} F(x, y, z(x, y)) =$$

公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

$$F(x, y, z(x, y)) = 0$$

$$\therefore \quad 0 = \frac{\partial}{\partial x} F(x, y, z(x, y)) =$$

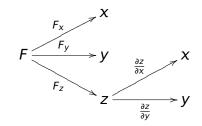


公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

证明 
$$:: F(x, y, z(x, y)) = 0$$

$$\therefore \quad 0 = \frac{\partial}{\partial x} F(x, y, z(x, y)) =$$

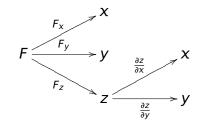


公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

证明 
$$:: F(x, y, z(x, y)) = 0$$

$$\therefore \quad 0 = \frac{\partial}{\partial x} F(x, y, z(x, y)) = F_x + F_z \cdot \frac{\partial z}{\partial x}$$



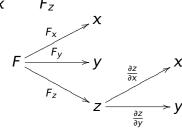
公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

证明 
$$:: F(x, y, z(x, y)) = 0$$

$$\therefore 0 = \frac{\partial}{\partial x} F(x, y, z(x, y)) = F_x + F_z \cdot \frac{\partial z}{\partial x}$$

$$\therefore \frac{\partial z}{\partial x} = -\frac{F_x}{F_z},$$



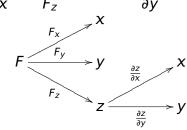
公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial Z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial Z}{\partial y} = -\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

证明 
$$:: F(x, y, z(x, y)) = 0$$

$$\therefore \quad 0 = \frac{\partial}{\partial x} F(x, y, z(x, y)) = F_X + F_Z \cdot \frac{\partial z}{\partial x}$$

∴ 
$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$
,  $= \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$ 



例 设 z = f(x, y) 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

F(x, y, z) = 0

$$\frac{\partial Z}{\partial x} = -\frac{F_x}{F_z} =$$

$$\frac{\partial Z}{\partial y} = -\frac{F_y}{F_z} =$$

解

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

$$\mathbf{R} \diamondsuit F(x, y, z) = x + y + xz - e^z + 1, \qquad F(x, y, z) = 0$$

$$\frac{\partial Z}{\partial x} = -\frac{F_X}{F_Z} =$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} =$$

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + y + xz - e^z + 1)_y'}{(x + y + xz - e^z + 1)_z'}$$

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
= -

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
= -

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{0}{0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
= -

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1}{0+0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{(x+y+xz-e^z+1)_z'}{(x+y+xz-e^z+1)_z'}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
= -

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1}{0+0+x-e^z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
= -

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1}{0+0+x-e^z+0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
= -

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1}{0+0+x-e^z+0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{(x+y+xz-e^z+1)_z'}{(x+y+xz-e^z+0)}$$

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1}{0+0+x-e^z+0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1}{0+0+x-e^z+0}$$

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0}{0+0+x-e^z+0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{(x+y+xz-e^z+1)_z'}{(x+y+xz-e^z+0)}$$



例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0+z}{0+0+x-e^z+0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{(x+y+xz-e^z+1)_z'}{(x+y+xz-e^z+0)}$$

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0+z-0}{0+0+x-e^z+0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{(x+y+xz-e^z+1)_z'}{(x+y+xz-e^z+0)}$$

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0+z-0+0}{0+0+x-e^z+0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{(x+y+xz-e^z+1)_z'}{(x+y+xz-e^z+0)}$$

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0+z-0+0}{0+0+x-e^z+0} = -\frac{1+z}{x-e^z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{(x+y+xz-e^z+1)_z'}{(x+y+xz-e^z+0)}$$

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_X}{F_Z} = -\frac{(x+y+xz-e^z+1)_X'}{(x+y+xz-e^z+1)_Z'}$$
$$= -\frac{1+0+z-0+0}{0+0+x-e^z+0} = -\frac{1+z}{x-e^z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{0}{0+0+x-e^z+0}$$



例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_X}{F_Z} = -\frac{(x+y+xz-e^z+1)_X'}{(x+y+xz-e^z+1)_Z'}$$
$$= -\frac{1+0+z-0+0}{0+0+x-e^z+0} = -\frac{1+z}{x-e^z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{0+1}{0+0+x-e^z+0}$$



例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_X}{F_Z} = -\frac{(x+y+xz-e^z+1)_X'}{(x+y+xz-e^z+1)_Z'}$$
$$= -\frac{1+0+z-0+0}{0+0+x-e^z+0} = -\frac{1+z}{x-e^z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{0+1+0}{0+0+x-e^z+0}$$



例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0+z-0+0}{0+0+x-e^z+0} = -\frac{1+z}{x-e^z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{0+1+0-0}{0+0+x-e^z+0}$$



例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令 
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0+z-0+0}{0+0+x-e^z+0} = -\frac{1+z}{x-e^z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{0+1+0-0+0}{0+0+x-e^z+0} = -\frac{1}{x-e^z}$$



例设 z = f(x, y) 满足  $2\sin(x + 2y - 3z) = x + 2y - 3z$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

例设 z = f(x, y) 满足  $2\sin(x + 2y - 3z) = x + 2y - 3z$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

$$F(x, y, z) = 0$$

$$\frac{\partial Z}{\partial X} = -\frac{F_X}{F_Z} =$$

$$\frac{\partial Z}{\partial y} = -\frac{F_y}{F_z} =$$

例设 z = f(x, y) 满足  $2\sin(x + 2y - 3z) = x + 2y - 3z$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F} =$$



$$F(x, y, z) = 0$$
,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

$$F(x, y, z) = 0$$
,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{(2\sin(x+2y-3z)-x-2y+3z)_z'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$



$$F(x, y, z) = 0$$
,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{-6\cos(x+2y-3z)}{-6\cos(x+2y-3z)}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

● 整角大學

$$F(x, y, z) = 0$$
,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{-6\cos(x+2y-3z)+3}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$



$$F(x, y, z) = 0$$
,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{2\cos(x+2y-3z)}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$



$$F(x, y, z) = 0$$
,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$



例 设 z = f(x, y) 满足  $2 \sin(x + 2y - 3z) = x + 2y - 3z$ ,求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$  解 今  $F(x, y, z) = 2 \sin(x + 2y - 3z) - x - 2y + 3z$ ,则

解 令 
$$F(x, y, z) = 2 \sin(x + 2y - 3z) - x - 2y + 3z$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

$$-6\cos(x+2y-3z)+3$$



解 令 
$$F(x, y, z) = 2 \sin(x + 2y - 3z) - x - 2y + 3z$$
, 则  $F(x, y, z) = 0$ ,所以

例 设 z = f(x, y) 满足  $2 \sin(x + 2y - 3z) = x + 2y - 3z$ ,求  $\frac{\partial z}{\partial y}$  和  $\frac{\partial z}{\partial y}$ 

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{4\cos(x+2y-3z)}{-6\cos(x+2y-3z)+3}$$



解 令 
$$F(x, y, z) = 2 \sin(x + 2y - 3z) - x - 2y + 3z$$
, 则  $F(x, y, z) = 0$ , 所以

例 设 z = f(x, y) 满足  $2 \sin(x + 2y - 3z) = x + 2y - 3z$ ,求  $\frac{\partial z}{\partial y}$  和  $\frac{\partial z}{\partial y}$ 

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{4\cos(x+2y-3z)-2}{-6\cos(x+2y-3z)+3}$$



例 设 
$$z = f(x, y)$$
 满足  $z - y - x + xe^{z-y-x} = 0$ , 求  $dz$ 

解

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$

例 设 
$$z = f(x, y)$$
 满足  $z - y - x + xe^{z-y-x} = 0$ , 求  $dz$ 

解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ 

$$\frac{\partial Z}{\partial X} =$$

$$\frac{\partial Z}{\partial V} =$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$

例 设 
$$z = f(x, y)$$
 满足  $z - y - x + xe^{z - y - x} = 0$ , 求  $dz$ 

解 令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
, 则  $F(x, y, z) = 0$ , 所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

$$\frac{\partial z}{\partial v} = -\frac{F_y}{F_z} =$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



例 设 
$$z = f(x, y)$$
 满足  $z - y - x + xe^{z - y - x} = 0$ ,求  $dz$ 

解 令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})'_y}{(z - y - x + xe^{z - y - x})'_z}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$
$$= -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{1}{(z - y - x + xe^{z - y - x})_z'}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$

$$= -\frac{1 + xe^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{1}{(z - y - x + xe^{z - y - x})_z'}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$
$$= -\frac{1}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$
$$= -\frac{-1 + e^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$
$$= -\frac{-1 + e^{z - y - x} - xe^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{1}{(z - y - x + xe^{z - y - x})_z'}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$

$$= -\frac{-1 + e^{z - y - x} - xe^{z - y - x}}{1 + xe^{z - y - x}} = -\frac{1 + (x - 1)e^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$

$$= -\frac{-1 + e^{z - y - x} - xe^{z - y - x}}{1 + xe^{z - y - x}} = -\frac{1 + (x - 1)e^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{1 + xe^{z - y - x}}{1 + xe^{z - y - x}}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$

$$= -\frac{-1 + e^{z - y - x} - xe^{z - y - x}}{1 + xe^{z - y - x}} = -\frac{1 + (x - 1)e^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{1}{1 + xe^{z - y - x}}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$

$$= -\frac{-1 + e^{z - y - x} - xe^{z - y - x}}{1 + xe^{z - y - x}} = -\frac{1 + (x - 1)e^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{-1 - xe^{z - y - x}}{1 + xe^{z - y - x}}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$

$$= -\frac{-1 + e^{z - y - x} - xe^{z - y - x}}{1 + xe^{z - y - x}} = -\frac{1 + (x - 1)e^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{-1 - xe^{z - y - x}}{1 + xe^{z - y - x}} = 1$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$
$$= -\frac{-1 + e^{z - y - x} - xe^{z - y - x}}{1 + xe^{z - y - x}} = -\frac{1 + (x - 1)e^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{-1 - xe^{z - y - x}}{1 + xe^{z - y - x}} = 1$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy = -\frac{1 + (x - 1)e^{z - y - x}}{1 + xe^{z - y - x}}dx + dy$$



$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

例 设  $\Phi(u, v)$  具有连续偏导数,函数 z = z(x, y) 满足

$$Φ(cx - az, cy - bz) = 0$$
, 证明:

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\mathbf{F}(x, y, z) = \Phi(cx - az, cy - bz),$$
则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{\partial z}{\partial y} = \frac{F_y}{F_z} = \frac{F_y}{F_z}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\frac{\partial Z}{\partial x} = -\frac{F_X}{F_Z} = \frac{\partial Z}{\partial y} = -\frac{F_Y}{F_Z} = \frac{\partial Z}{\partial y} = \frac{1}{2} = \frac{$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

 $\partial X \qquad \partial Y$ 解令  $F(x, y, z) = \Phi(cx - \alpha z, cy - bz)$ ,则

$$F_{x} = G(x - uz),$$

$$F_V =$$

 $F_7 =$ 

<sub>z</sub> =

$$\frac{\partial Z}{\partial x} = -\frac{F_X}{F_Z} = \frac{\partial Z}{\partial x} = -\frac{F_Y}{F_Z} = \frac{\partial Z}{\partial x} = -\frac{F_Y}{F_Z} = \frac{\partial Z}{\partial x} = \frac{\partial Z}{\partial x}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

 $\mathbf{H}$  令  $F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则

$$F_{X} = \Phi_{u} \cdot u_{X} + \Phi_{V} \cdot \nu_{X}$$

$$F_y =$$

$$F_y = F_z = F_z = F_z$$

$$\frac{\partial z}{\partial x} = -\frac{F_X}{F_Z} =$$

$$\frac{F_y}{F_z} =$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\partial x \qquad \partial y$$
解令  $F(x, y, z) = \Phi(cx - \alpha z, cy - bz)$ ,则

$$F_X = \Phi_u \cdot u_X + \Phi_v \cdot v_X = c\Phi_u$$

$$F_y =$$

$$F_y =$$
 $F_z =$ 

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

$$\frac{F_y}{F_z} =$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\partial x \partial y \partial y \partial z$$
  
解令  $F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则

$$F_X = \Phi_U \cdot U_X + \Phi_V \cdot V_X = c\Phi_U$$

$$F_V = \Phi_U \cdot U_V + \Phi_V \cdot V_V$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y$$
$$F_z =$$

$$\frac{z}{x} = -\frac{F_x}{F_z} = \frac{z}{F_z} = -\frac{F_y}{F_z} = \frac{z}{F_z} =$$

$$\frac{\partial Z}{\partial V} =$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\partial x \partial y \partial y$$
  
解令  $F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则

$$F_{X} = \Phi_{U} \cdot u_{X} + \Phi_{V} \cdot V_{X} = c\Phi_{U}$$

$$F_{V} = \Phi_{U} \cdot u_{V} + \Phi_{V} \cdot V_{V} = c\Phi_{V}$$

$$F_{y} = \Phi_{u} \cdot u_{y} + \Phi_{v} \cdot v_{y} = c\Phi_{v}$$
$$F_{z} =$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{F_$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\mathbf{M}$$
 令  $F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则

$$F_X = \Phi_U \cdot u_X + \Phi_V \cdot V_X = c\Phi_U$$

$$F_X = \Phi_X \cdot U_X + \Phi_X \cdot V_X = c\Phi_X$$

$$F_{y} = \Phi_{u} \cdot u_{y} + \Phi_{v} \cdot v_{y} = c\Phi_{v}$$

$$F_z = \Phi_u \cdot u_z + \Phi_V \cdot V_z$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{F_$$



$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\mathbf{F}(x, y, z) = \Phi(cx - az, cy - bz)$$
,则

$$F_X = \Phi_u \cdot u_X + \Phi_V \cdot V_X = c\Phi_u$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot \nu_y = c\Phi_v$$

$$\Phi_{u} \cdot u_{y} + \Phi_{v} \cdot V_{y} = C\Phi_{v}$$

$$F_z = \Phi_u \cdot u_z + \Phi_v \cdot v_z = -\alpha \Phi_u - b \Phi_v$$
$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

$$\frac{F_y}{F_z} =$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

 $\mathbf{F} \Leftrightarrow F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则  $F_{\mathbf{x}} = \Phi_{U} \cdot u_{\mathbf{x}} + \Phi_{V} \cdot V_{\mathbf{x}} = c\Phi_{U}$ 

$$F_{y} = \Phi_{u} \cdot u_{y} + \Phi_{v} \cdot v_{y} = c\Phi_{v}$$

$$F_{z} = \Phi_{u} \cdot u_{z} + \Phi_{v} \cdot v_{z} = -\alpha\Phi_{u} - b\Phi_{v}$$

$$\frac{\partial z}{\partial x} = -\frac{F_{x}}{F_{z}} = \frac{c\Phi_{u}}{\alpha\Phi_{u} + b\Phi_{v}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_{y}}{F_{z}} = \frac{c\Phi_{v}}{\sigma\Phi_{v} + \sigma_{v}}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\alpha x = \alpha y$$
  
解令  $F(x, y, z) = \Phi(cx - \alpha z, cy - bz)$ ,则

$$F_X = \Phi_u \cdot u_X + \Phi_V \cdot V_X = c\Phi_u$$

$$F_{V} = \Phi_{U} \cdot u_{V} + \Phi_{V} \cdot V_{V} = c\Phi_{V}$$

$$= \Phi_u \cdot u_y + \Phi_v \cdot V_y = C\Phi_v$$

$$F_{z} = \Phi_{u} \cdot u_{z} + \Phi_{v} \cdot v_{z} = -a\Phi_{u} - b\Phi_{v}$$

$$\frac{\partial z}{\partial x} = -\frac{F_{x}}{F_{z}} = \frac{c\Phi_{u}}{a\Phi_{u} + b\Phi_{v}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_{y}}{F_{z}} = \frac{c\Phi_{v}}{a\Phi_{u} + b\Phi_{v}}$$

$$\frac{\partial Z}{\partial y} =$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\mathbf{F} \Leftrightarrow F(x, y, z) = \Phi(cx - az, cy - bz)$$
,则
$$F_{\mathsf{x}} = \Phi_{\mathsf{U}} \cdot \mathsf{U}_{\mathsf{x}} + \Phi_{\mathsf{V}} \cdot \mathsf{V}_{\mathsf{x}} = c\Phi_{\mathsf{U}}$$

$$F_X = \Phi_u \cdot u_X + \Phi_v \cdot V_X = c\Phi_u$$
$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot V_y = c\Phi_v$$

$$F_{z} = \Phi_{u} \cdot u_{z} + \Phi_{v} \cdot v_{z} = -a\Phi_{u} - b\Phi_{v}$$

$$\frac{\partial z}{\partial x} = -\frac{F_{x}}{F_{z}} = \frac{c\Phi_{u}}{a\Phi_{u} + b\Phi_{v}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_{y}}{F_{z}} = \frac{c\Phi_{v}}{a\Phi_{u} + b\Phi_{v}}$$

$$\frac{\partial z}{\partial y} = -\frac{\partial z}{\partial z} = \frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = \frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} + \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} + \frac{\partial z}{\partial y}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

 $\mathbf{H}$  令  $F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则  $F_{\mathbf{Y}} = \Phi_{II} \cdot u_{\mathbf{X}} + \Phi_{\mathbf{V}} \cdot \mathbf{V}_{\mathbf{X}} = c\Phi_{II}$ 

$$F_X = \Phi_u \cdot u_X + \Phi_v \cdot v_X = c\Phi_u$$
$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y = c\Phi_v$$

$$F_{y} = \Phi_{u} \cdot u_{y} + \Phi_{v} \cdot v_{y} = c\Phi_{v}$$
  
$$F_{z} = \Phi_{u} \cdot u_{z} + \Phi_{v} \cdot v_{z} = -\alpha\Phi_{u} - b\Phi_{v}$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{c\Phi_u}{a\Phi_u + b\Phi_v}$$
$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{c\Phi_v}{a\Phi_u + b\Phi_v}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = \frac{ac\Phi_{u}}{a\Phi_{u} + b\Phi_{v}} + \frac{bc\Phi_{v}}{a\Phi_{u} + b\Phi_{v}} = c$$

例 设 z = f(x, y) 满足  $z = x + ye^z$ , 求  $\frac{\partial^2 z}{\partial x \partial y}$ 

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ , 求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$ 

例设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{\partial z}{\partial y} = \frac{F_y}{F_z} = \frac{F_y}{F_z}$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ , 求  $\frac{\partial^2 z}{\partial x \partial y}$ 

$$F(x, y, z) = x + ye^z - z, \ 则 \ F(x, y, z) = 0, \ 所以$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z}$$
$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} =$$
$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) =$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = -\frac{\partial}{\partial y}$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{1}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \frac{1}{ye^z - 1}$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} =$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) =$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) =$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) =$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)_y'$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)_y' = \frac{(ye^z - 1)_y'}{(ye^z - 1)^2}$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)_y' = \frac{(ye^z - 1)_y'}{(ye^z - 1)^2}$$

$$= \frac{e^z + y(e^z)_y'}{(ye^z - 1)^2}$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+ye^z - z)_x}{(x+ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+ye^z - z)_y}{(x+ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)_y' = \frac{(ye^z - 1)_y'}{(ye^z - 1)^2}$$

$$= \frac{e^z + y(e^z)_y'}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \frac{\partial z}{\partial y}}{(ye^z - 1)^2}$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)_y' = \frac{(ye^z - 1)_y'}{(ye^z - 1)^2}$$

$$= \frac{e^z + y(e^z)_y'}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \frac{\partial z}{\partial y}}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \left(-\frac{e^z}{ye^z - 1}\right)}{(ye^z - 1)^2}$$



例 设 z = f(x, y) 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

$$F(x, y, z) = x + ye^z - z, \ 则 F(x, y, z) = 0, \ 所以$$

$$\frac{\partial Z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial Z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 Z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial Z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)_y' = \frac{(ye^z - 1)_y'}{(ye^z - 1)^2}$$

$$= \frac{e^z + y(e^z)_y'}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \frac{\partial Z}{\partial y}}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \left(-\frac{e^z}{ye^z - 1}\right)}{(ye^z - 1)^2}$$

$$= \frac{-e^z}{(ye^z - 1)^2}$$

例 设 z = f(x, y) 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^{z} - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)_y' = \frac{(ye^z - 1)_y'}{(ye^z - 1)^2}$$

$$= \frac{e^z + y(e^z)_y'}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \frac{\partial z}{\partial y}}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \left(-\frac{e^z}{ye^z - 1}\right)}{(ye^z - 1)^2}$$

$$= \frac{-e^z}{(ye^z - 1)^3} = \frac{e^z}{(1 + x - z)^3}$$

We are here now...

1. 一个方程的情形

2. 方程组的情形

#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11} a_{22} x + a_{12} a_{22} y = a_{22} b_1 & (1) \times a_{22} \\ a_{21} x + a_{22} y = b_2 & (2) \times a_{12} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11} a_{22} x + a_{12} a_{22} y = a_{22} b_1 & (1) \times a_{22} \\ a_{21} a_{12} x + a_{22} a_{12} y = a_{12} b_2 & (2) \times a_{12} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11} a_{22} x + a_{12} a_{22} y = a_{22} b_1 & (1) \times a_{22} \\ a_{21} a_{12} x + a_{22} a_{12} y = a_{12} b_2 & (2) \times a_{12} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}a_{11}x + a_{22}a_{11}y = a_{11}b_2 & (2) \times a_{11} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11} a_{21} x + a_{12} a_{21} y = a_{21} b_1 & (1) \times a_{21} \\ a_{21} a_{11} x + a_{22} a_{11} y = a_{11} b_2 & (2) \times a_{11} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11} a_{21} x + a_{12} a_{21} y = a_{21} b_1 & (1) \times a_{21} \\ a_{21} a_{11} x + a_{22} a_{11} y = a_{11} b_2 & (2) \times a_{11} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$



#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$



二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解:

(1) × 
$$a_{22}$$
 – (2) ×  $a_{12}$ , 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

 $(2) \times a_{11} - (1) \times a_{21}$ , 消去 x, 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{a_{11}a_{12}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解:

(1)×
$$a_{22}$$
-(2)× $a_{12}$ , 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

 $(2) \times a_{11} - (1) \times a_{21}$ , 消去 x, 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{a_{11}a_{12}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

(1) × 
$$a_{22}$$
 – (2) ×  $a_{12}$ , 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \qquad , \quad y =$$

$$\begin{cases}
7x + 16y = 1 \\
2x + 5y = -1
\end{cases} x =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -- \qquad , \quad y = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = --$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = --- , \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = --$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{1}{1} \qquad , \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -\frac{1}{1}$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{1}{1}$$
, 
$$y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{1}{1}$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1}$$
, 
$$y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{1}{1}$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} \qquad , \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1}$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1}$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$\begin{cases}
7x + 16y = 1 \\
2x + 5y = -1
\end{cases} x =$$

● 整点大·

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$
2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - , \quad y = \frac{3}{1} = 8$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$
2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - , \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - \end{cases}$$

D 整有 ARCNUS

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$
2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-3}{3}, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-3}{3}$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$\begin{vmatrix} 3 & 8 \end{vmatrix} & |3 & 8| \\ 2x + 16y = 1 \\ 2x + 5y = -1 \end{vmatrix} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{3}{3} , y = \frac{\begin{vmatrix} 7 & 1\\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16\\ 2 & 5 \end{vmatrix}} = \frac{3}{3}$$





$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1.  $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$ 

1. 
$$\begin{cases} 2x + 3y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{|4 + 3|}{|2 + 5|} = \frac{-20}{1} = -20, \quad y = \frac{|3 + 4|}{|2 + 5|} = \frac{-20}{3 + 8} = \frac{$$

2.  $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} \quad , y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{3}{3}$ 



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1.  $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$ 

$$\begin{cases} 3x + 8y = 4 \end{cases}$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} , y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3}$$





$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$\begin{cases} 2x + 5y = 0 & |0 \quad 5| \\ 4 \quad 8| \\ -20 & |0 \quad 7| \end{cases}$$

$$\int_{0}^{1} 3x + 8y = 4$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \ y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

2.  $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \ y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3} = -3$ 

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \\ G(x, y, u, v) = 0 & \Longrightarrow \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x,y,u,v) = 0 \\ G(x,y,u,v) = 0 \end{cases} \stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \\ G(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \end{cases} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \\ G(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \end{cases} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \\ G(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \end{cases} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \Rightarrow \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

$$\Rightarrow u_x = \begin{vmatrix} -F_x & F_v \\ -G_x & G_v \end{vmatrix}, \quad v_x = \begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}$$



假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

求解如下:  

$$\begin{cases}
F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \begin{cases}
F_u \cdot u_x + F_v \cdot v_x = -F_x \\
G_u \cdot u_x + G_v \cdot v_x = -G_x
\end{cases}$$

$$\Rightarrow u_x = \begin{vmatrix}
-F_x & F_v \\
-G_x & G_v
\end{vmatrix}, v_x = \begin{vmatrix}
-F_u & F_x \\
-G_u & G_x
\end{vmatrix}$$



假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 & \stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

$$\Rightarrow u_x = -\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_x = -\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$



假设函数 u = u(x, y), v = v(x, y) 满足方程组  $\begin{cases} F(x, y, u, v) = 0, \\ G(x, v, u, v) = 0. \end{cases}$ 

问题:如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial x}$ ?

### 求解如下:

 $\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \\ G(x, v, u, v) = 0 & \Longrightarrow \end{cases} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$ 

$$\Rightarrow u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$1 \ \partial(F, G)$$

$$\frac{\partial(F,G)}{\partial(x,v)}$$



假设函数 u = u(x, y), v = v(x, y) 满足方程组  $\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$ 

问题:如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial x}$ ?

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

$$\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}$$

$$\Rightarrow u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$
$$= -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)} \qquad = -\frac{1}{J} \frac{\partial(F, G)}{\partial(u, x)} \underbrace{\bigcirc \underbrace{\mathbb{R} \wedge \lambda^{+}}_{18/24 \ \lor \ b \ \lor \ b}}_{18/24 \ \lor \ b \ \lor \ b} \underbrace{\bigcirc \underbrace{\mathbb{R} \wedge \lambda^{+}}_{18/24 \ \lor \ b \ \lor \ b}}_{18/24 \ \lor \ b \ \lor \ b}$$

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \\ G(x, y, u, v) = 0 & \Longrightarrow \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \\ G(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \end{cases} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \\ G(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \end{cases} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y =$$
 ———,  $v_y =$  ———



$$\begin{cases} F(x, y, u, v) = 0 & \stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G(x, y, u, v) = 0 \end{cases}$$

$$\Rightarrow u_y = \frac{\begin{vmatrix} -F_y & F_v \\ -G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = \frac{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = \begin{vmatrix} -F_y & F_v \\ -G_y & G_v \end{vmatrix}, \quad v_y = \begin{vmatrix} -F_u & F_y \\ -G_u & G_y \end{vmatrix}$$

$$\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$= -\frac{1}{J} \frac{\partial (F, G)}{\partial (y, v)}$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$= -\frac{1}{J} \frac{\partial (F, G)}{\partial (y, v)} \qquad = -\frac{1}{J} \frac{\partial (F, G)}{\partial (u, y)}$$



$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

总结 设 
$$u = u(x, y), v = v(x, y)$$
 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

$$u_x =$$

$$\nu_{x} =$$

$$u_v =$$

$$\nu_y =$$

总结 设 
$$u = u(x, y), v = v(x, y)$$
 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

$$u_x = v_x = v_x$$

$$u_V = v_V = v_V$$

总结 设 
$$u = u(x, y), v = v(x, y)$$
 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$u_x =$$

$$v_x =$$

$$u_v =$$

$$y' = 1$$

总结 设 
$$u = u(x, y), v = v(x, y)$$
 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{\longleftrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

$$u_{x} = v_{x}$$

$$u_{v} = v_{v} = v_{v}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$
$$\xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial X}}{\Longrightarrow} \begin{cases} F_X + F_U \cdot u_X + F_V \cdot v_X = 0 \\ G_X + G_U \cdot u_X + G_V \cdot v_X = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial Y}}{\Longrightarrow} \begin{cases} F_Y + F_U \cdot u_Y + F_V \cdot v_Y = 0 \\ G_Y + G_U \cdot u_Y + G_V \cdot v_Y = 0 \end{cases}$$

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$u_v =$$

$$v_v =$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$
$$u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{v} \\ G_{y} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{y} \end{vmatrix}}$$







$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \Rightarrow \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$
$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{y} \\ G_{x} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, y)}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}$$

$$u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{y} \\ G_{y} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}$$

$$v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}$$





$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(u, x)}$$

$$u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{v} \\ G_{y} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{y} \end{vmatrix}}$$

$$v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{y} \end{vmatrix}}$$



$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)}, \quad v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}$$

第 9 章 d: 隐函数的求导公



$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)},$$

$$u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{v} \\ G_{y} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)}, \quad v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(u, y)}$$

例设  $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}, \ \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$ 

例设 
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}$$

 $\nu_{\nu} =$ 

$$u_X = v_X = v_X$$

 $u_v =$ 

例设 
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}$$

 $\nu_{\nu} =$ 

$$u_X = v_X = v_X$$

 $u_v =$ 

例设 
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x
\end{cases}$$

 $\nu_x =$ 

$$u_x =$$

 $u_v =$ 

$$v_v =$$

 $\stackrel{\frac{\sigma}{\partial x}}{\Longrightarrow} \left\{ (e^u + \sin v) u_x + u \cos v \cdot v_x = 1 \right.$ 

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases} \begin{cases} (e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\ (e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0 \end{cases}$$

$$u_x =$$

 $\nu_x =$ 

$$u_v =$$

 $\nu_{\nu} =$ 

例设 
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$$
, 求  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ 

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$u_x =$$

 $v_{\chi} =$ 

$$u_y =$$

 $\nu_y =$ 

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$= x$$

$$= y$$

$$\begin{cases} (e^{u} + \sin v)u_{x} + u\cos v \cdot v_{x} = 1 \\ (e^{u} - \cos v)u_{x} + u\sin v \cdot v_{x} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u\sin v \cdot v_{y} = 1 \end{cases}$$

$$u_x =$$

$$\nu_{\chi} =$$

$$u_v =$$

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{=} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$u_x = v_x = v_x$$

$$u_y = v_y = v_y$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases} \begin{cases} (e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\ (e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0 \end{cases}$$
$$\stackrel{\frac{\partial}{\partial x}}{\rightleftharpoons} \begin{cases} (e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1 \end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$u_{y}$$
=  $\int$  第 9 章  $d$ : 隐函数的求导公式



例设 
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$$
, 求  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ 

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{=} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^{u} + \sin v & u \cos v \\ e^{u} - \cos v & u \sin v \end{vmatrix}$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$



$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

 $\stackrel{\frac{\sigma}{\partial x}}{\Longrightarrow} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1\\ (e^u - \cos v)u_x + u\sin v \cdot v_x = 0 \end{cases}$  $\stackrel{\frac{\sigma}{\partial y}}{\Longrightarrow} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_v + u\sin v \cdot v_v = 1 \end{cases}$ 

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$u_{y} = -\frac{\begin{vmatrix} u & u & u \\ 0 & u & u \end{vmatrix}}{J}$$

$$v_{x} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{J}$$



例设 
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$$
, 求  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ 

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{=} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_{x} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{J}$$

$$u_{y} = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}$$

$$v_{y} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}$$



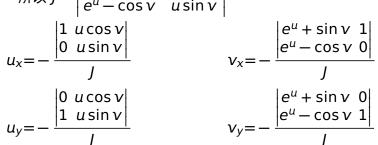
例设 
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$$
, 求  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ 

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}$$

$$\begin{array}{c}
\stackrel{\frac{\partial}{\partial x}}{\Rightarrow} \begin{cases}
(e^{u} + \sin v)u_{x} + u\cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u\sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial y}}{\Rightarrow} \begin{cases}
(e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u\sin v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$





例设 
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$$
, 求  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ 

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$= x$$

$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{x} + u\cos v \cdot v_{x} = 1\\ (e^{u} - \cos v)u_{x} + u\sin v \cdot v_{x} = 0 \end{cases}$$

$$\stackrel{\frac{\sigma}{\sigma}}{\Longrightarrow} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u\sin v \cdot v_y = 1 \end{cases}$$

$$\Longrightarrow \{$$

$$sin v u cos v$$
 $cos v u sin v$ 

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$v_{x} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{\int}$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$u_{y} = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}$$

$$v = -\frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{v}$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{=} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$u_x = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{\int_{-\frac{e^u(\sin v - \cos v) + 1}{e^u(\sin v - \cos v) + 1}} v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{\int_{-\frac{e^u(\sin v - \cos v) + 1}{e^u(\sin v - \cos v) + 1}} v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{\int_{-\frac{e^u(\sin v - \cos v) + 1}{e^u(\sin v - \cos v) + 1}} v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{\int_{-\frac{e^u(\sin v - \cos v) + 1}{e^u(\sin v - \cos v) + 1}} v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{\int_{-\frac{e^u(\sin v - \cos v) + 1}{e^u(\sin v - \cos v) + 1}} v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{\int_{-\frac{e^u(\sin v - \cos v) + 1}{e^u(\sin v - \cos v) + 1}} v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{\int_{-\frac{e^u(\sin v - \cos v) + 1}{e^u(\sin v - \cos v) + 1}} v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{\int_{-\frac{e^u(\sin v - \cos v) + 1}{e^u(\sin v - \cos v) + 1}} v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{\int_{-\frac{e^u(\sin v - \cos v) + 1}{e^u(\sin v - \cos v) + 1}} v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{\int_{-\frac{e^u(\sin v - \cos v) + 1}{e^u(\sin v - \cos v) + 1}} v_x = -\frac{e^u(\sin v - \cos v) + 1}{\int_{-\frac{e^u(\sin v - \cos v) + 1}{e^u(\sin v - \cos v) + 1}} v_x = -\frac{e^u(\sin v - \cos v) + 1}{\int_{-\frac{e^u(\sin v - \cos v) + 1}{e^u(\cos v - \cos v) + 1}} v_x = -\frac{e^u(\cos v - \cos v) + 1}{\int_{-\frac{e^u(\sin v - \cos v) + 1}{e^u(\cos v - \cos v) + 1}} v_x = -\frac{e^u(\cos v - \cos v) + 1}{\int_{-\frac{e^u(\cos v - \cos v) + 1}{e^u(\cos v - \cos v)}} v_x = -\frac{e^u(\cos v - \cos v) + 1}{\int_{-\frac{e^u(\cos v - \cos v) + 1}{e^u(\cos v - \cos v)}} v_x = -\frac{e^u(\cos v - \cos v) + 1}{\int_{-\frac{e^u(\cos v - \cos v) + 1}{e^u(\cos v - \cos v)}} v_x = -\frac{e^u(\cos v - \cos v) + 1}{\int_{-\frac{e^u(\cos v - \cos v) + 1}{e^u(\cos v - \cos v)}} v_x = -\frac{e^u(\cos v - \cos v) + 1}{\int_{-\frac{e^u(\cos v - \cos v) + 1}{e^u(\cos v - \cos v)}} v_x = -\frac{e^u(\cos v - \cos v) + 1}{\int_{-\frac{e^u(\cos v - \cos v) + 1}{e^u(\cos v - \cos v)}} v_x = -\frac{e^u(\cos v - \cos v) + 1}{\int_{-\frac{e^u(\cos v - \cos v) + 1}{e^u(\cos v - \cos v)}} v_x = -\frac{e^u(\cos v - \cos v) + 1}{\int_{-\frac{e^u(\cos v - \cos v) + 1}{e^u(\cos v - \cos v)}} v_x = -\frac{e^u(\cos v - \cos v) + 1}{\int_{-\frac{e^u(\cos v - \cos v) + 1}{e^u(\cos v - \cos v)}} v_x = -\frac{e^u(\cos v - \cos v) + 1}{\int_{-\frac{e^u(\cos v - \cos v) + 1}{e^u(\cos v - \cos v)}} v_x = -\frac{e^u(\cos v - \cos v) + 1}{\int_{-\frac{e^u(\cos v - \cos v) + 1}{e^u(\cos v - \cos v)}} v_x = -\frac{e^u(\cos v - \cos v) + 1}{\int_{-\frac{e^u(\cos v - \cos v$$

$$u_{x} = -\frac{\frac{|o \ u \ sin \ v|}{J}}{\frac{|o \ u \ cos \ v|}{I}} = \frac{\frac{-\sin v}{e^{u}(\sin v - \cos v) + 1}}{v_{x}} = -\frac{\frac{|c \ u \ cos \ v|}{J}}{\frac{|c \ u \ cos \ v|}{I}}$$

$$v_{y} = -\frac{\frac{|e \ u \ cos \ v|}{I}}{\frac{|e^{u} + \sin v \ 0|}{I}}$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\begin{cases}
e^{u} + \sin v \cdot v_{x} = 0 \\
e^{u} - \cos v \cdot v_{y} = 0
\end{cases}$$

$$\begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
e^{u} - \cos v \cdot v_{y} = 1
\end{cases}$$

$$\frac{\partial}{\partial y} \begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u\sin v \cdot v_{y} = 1 \end{cases}$$

$$\text{MRU} J = \begin{vmatrix} e^{u} + \sin v & u\cos v \\ e^{u} - \cos v & u\sin v \end{vmatrix} = ue^{u}(\sin v - \cos v) + u$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u\cos v \\ 0 & u\sin v \end{vmatrix}}{\begin{vmatrix} 1 & u\sin v \end{vmatrix}} = \frac{-\sin v}{e^{u}(\sin v - \cos v) + 1}, \quad v_{x} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{\begin{vmatrix} 1 & u\cos v \\ 0 & u\sin v \end{vmatrix}} = \frac{e^{u} - \cos v}{u\sin v - \cos v} = \frac{e^{u} - \cos v}{u\sin v - \cos v}$$

 $v_y = -\frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{t}$ 

$$\begin{cases} e^{u} - u\cos v = y \\ \stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u\sin v \cdot v_{y} = 1 \end{cases}$$

$$\text{MUD} = \begin{vmatrix} e^{u} + \sin v & u\cos v \\ e^{u} - \cos v & u\sin v \end{vmatrix} = ue^{u}(\sin v - \cos v) + u$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u\cos v \\ 0 & u\sin v \end{vmatrix}}{\int} = \frac{-\sin v}{e^{u}(\sin v - \cos v) + 1}, v_{x} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{\int} = \frac{e^{u} - \cos v}{ue^{u}(\sin v - \cos v) + 1}$$

 $u_y = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{\cdot}$ d: 隐函数的求导公式



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\begin{cases} e^{u} - u \cos v = y \\ \Longrightarrow \end{cases} \begin{cases} (e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1 \end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$|1 \ u \cos v| \qquad |e^u + \sin v \ 1|$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{\int} = \frac{-\sin v}{e^{u(\sin v - \cos v) + 1}}, v_{x} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{\int} = \frac{e^{u - \cos v}}{ue^{u(\sin v - \cos v)}}$$
$$u_{y} = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{\int} = \frac{\cos v}{e^{u(\sin v - \cos v) + 1}}, v_{y} = -\frac{\begin{vmatrix} e^{u} + \sin v & 0 \\ e^{u} - \cos v & 1 \end{vmatrix}}{\int}$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\frac{\partial}{\partial y} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$u_X = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J} = \frac{-\sin v}{e^u(\sin v - \cos v) + 1}, v_X = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J} = \frac{e^u - \cos v}{ue^u(\sin v - \cos v) + u}$$

$$u_y = -\frac{\begin{vmatrix} 0 & u\cos v \\ 1 & u\sin v \end{vmatrix}}{\int} = \frac{\cos v}{e^{u(\sin v - \cos v) + 1}}, v_y = -\frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{\int} = -\frac{e^{u} + \sin v}{\int}$$

 $ue^{u}(\sin v - \cos v) + u$