§5.3 换元积分法

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2017-2018 学年 II

教学要求

◇ 熟练掌握换元积分法：“凑微分”，“变量代换”

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Outline of §5.3

1. 第一类换元积分法：凑微分

2. 第二类换元积分法：变量代换

We are here now...

1. 第一类换元积分法：凑微分

2. 第二类换元积分法：变量代换

第一类换元积分法——“凑微分”法，能干啥？

能够计算如下的不定积分：

$$\int \frac{dx}{2x+1}, \quad \int \cos(\frac{5}{2}x)dx$$

$$\int \frac{x}{\sqrt{3-x^2}}dx, \quad \int x\sin(x^2)dx$$

$$\int \frac{(\ln x)^2}{x}dx, \quad \int e^{\sin x}\cos x dx$$

$$\int \frac{1}{\cos x}dx$$

……

第一类换元积分法（凑微分）原理

- 计算步骤：

$\int$ ”Ì½»§Æ¢È” dx

第一类换元积分法（凑微分）原理

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$$\int \text{”Ì½»§Æ¢È”}\, dx \xlongequal{\quad} \int f(\varphi(x))\varphi'(x)dx$$

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- 计算步骤：

$$\int \text{”Ì½»§Æ¢È”}\, dx \xlongequal{\quad} \int f(\varphi(x))\varphi'(x)dx = \qquad\qquad d\varphi(x)$$

第一类换元积分法（凑微分）原理

- 计算步骤：

$$\int \text{”Ì½»§Æ¢È”}\, dx = \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

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$$\xlongequal{\varphi(x)=u}$$

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$$= F(u) + C$$

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$$\frac{d}{dx}F(\varphi(x)) =$$

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总之 $\int \text{”Ì½»§Æ¢È”}\, dx \xlongequal{\text{凑微分}} \int f(\varphi(x))d\varphi(x)$

$$= \int f(u)du = F(u) + C = F(\varphi(x)) + C$$

凑微分　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

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类型 I: $\int f(ax+b)dx$

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则

$$\int f(ax+b)dx \qquad\qquad d(ax+b)$$

凑微分　　类型 I: $\int f(ax+b)dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx \qquad \qquad \frac{1}{a}d(ax+b)$$

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$$\xlongequal{u=ax+b} \int f(u)\cdot\frac{1}{a}du =$$

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例 1　$\displaystyle\int \frac{1}{1+2x}dx =$

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例 1　$\int \frac{1}{1+2x}dx =$　　　　$d(1+2x)$

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例 1　$$\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x}\cdot\frac{1}{2}d(1+2x) = \frac{1}{2}\int\frac{1}{u}du$$

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$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+2x| + C$$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

凑微分　　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$, $\int \sqrt{3x-1}dx$

解
$$\int \frac{1}{2-3x}dx =$$

$$\int \sqrt{3x-1}dx =$$

凑微分　　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

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凑微分　类型 I: $\int f(ax+b)dx$

例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

解 $$\int \frac{1}{2-3x}dx = \qquad \cdot\left(-\frac{1}{3}\right)d(2-3x)$$

$$\int \sqrt{3x-1}dx =$$

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例 2 求 $\int \frac{1}{2-3x}dx$，$\int \sqrt{3x-1}dx$

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$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x}\cdot(-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$
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$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1}\cdot\frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$

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$$u^{3/2}$$

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$$\frac{2}{3}u^{3/2}$$

凑微分　类型 I: $\int f(ax+b)dx$

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$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1}\cdot\frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$

$$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C$$

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$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1}\cdot\frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$
$$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C = \frac{2}{9}(3x-1)^{3/2} + C$$

凑微分　类型 I: $\int f(ax+b)dx$

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解
$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x}\cdot\left(-\frac{1}{3}\right)d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$
$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$
$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1}\cdot\frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$
$$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C = \frac{2}{9}(3x-1)^{3/2} + C$$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

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解

$$\int \frac{1}{\sqrt{1-5x}}dx =$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \qquad d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \qquad \cdot(-\frac{1}{5})d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　　类型 I: $\int f(ax+b)dx$

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解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$\int \cos(\frac{3}{2}x)dx =$$

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$$= \qquad u^{1/2}$$

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解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}\cdot(-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= \qquad 2u^{1/2}$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

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$$= -\frac{1}{5}\cdot 2u^{1/2} + C$$

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例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

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$$\int \cos(\frac{3}{2}x)dx = \qquad\qquad \cdot\frac{2}{3}d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$，$\int \cos(\frac{3}{2}x)dx$，$\int e^{-\frac{1}{2}x+4}dx$

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$$= \frac{2}{3}\sin(u) + C$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分　类型 I: $\int f(ax+b)dx$

例 3 求 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

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$$\int e^{-\frac{1}{2}x+4}dx = \qquad\qquad \cdot(-2)d(-\frac{1}{2}x+4)$$

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$$= -\frac{1}{5}\cdot 2u^{1/2}+C = -\frac{2}{5}(1-5x)^{1/2}+C$$

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$$= -2e^{u} + C$$

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凑微分　　类型 II: $\int f(ax^2+b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)xdx$$

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凑微分　　类型 II：$\int f(ax^2+b)xdx$

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凑微分　　类型 II: $\int f(ax^2+b)xdx$

假设会算

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则

$$\int f(ax^2+b)xdx \qquad\qquad d(ax^2+b)$$

凑微分　　类型 II: $\int f(ax^2+b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)xdx \qquad \frac{1}{2a}d(ax^2+b)$$

凑微分　　类型 II: $\int f(ax^2+b)xdx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^2+b)xdx = \int f(ax^2+b)\cdot\frac{1}{2a}d(ax^2+b)$$

凑微分　类型 II：$\int f(ax^2+b)xdx$

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$$\overset{u=ax^2+b}{=\!=\!=\!=}$$

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凑微分　类型 II: $\int f(ax^2+b)xdx$

假设会算

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凑微分　　类型 II: $\int f(ax^2+b)xdx$

假设会算

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凑微分　类型 II: $\int f(ax^2+b)xdx$

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凑微分　类型 II: $\int f(ax^2+b)xdx$

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例 1 $\int x\sqrt{1-x^2}dx =$

凑微分　　类型 II: $\int f(ax^2+b)xdx$

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例 1　$\int x\sqrt{1-x^2}dx =$ 　　　　　　　　$d(1-x^2)$

凑微分　类型 II: $\int f(ax^2+b)xdx$

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例 1 $\int x\sqrt{1-x^2}dx =$ $\cdot(-\frac{1}{2})d(1-x^2)$

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例 1　$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2)$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

假设会算

$$\int f(u)du = F(u) + C$$

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例 1　$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$

凑微分　类型 II: $\int f(ax^2+b)xdx$

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$$u^{3/2}$$

凑微分　　类型 II: $\int f(ax^2+b)xdx$

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$$\frac{2}{3}u^{3/2}$$

凑微分　　类型 II: $\int f(ax^2+b)x\,dx$

假设会算

$$\int f(u)du = F(u)+C$$

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例 1 $\displaystyle\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}\cdot\left(-\frac{1}{2}\right)d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C$$

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则

$$\int f(ax^2+b)xdx = \int f(ax^2+b)\cdot\frac{1}{2a}d(ax^2+b)$$

$$\xlongequal{u=ax^2+b} \int f(u)\cdot\frac{1}{2a}du = \frac{1}{2a}F(u)+C = \frac{1}{2a}F(ax^2+b)+C$$

例 1 $$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}\cdot(-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}}+C$$

凑微分　类型 II：$\int f(ax^2+b)x\,dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

凑微分　　类型 II: $\int f(ax^2+b)x dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx =$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$，$\int \frac{x}{1+3x^2}dx$

解

$$\int \frac{x}{\sqrt{3-x^2}}dx = \qquad d(3-x^2)$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分　类型 II: $\int f(ax^2+b)x\,dx$

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$$\int \frac{x}{\sqrt{3-x^2}}dx = \qquad \cdot\left(-\frac{1}{2}\right)d(3-x^2)$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分　　类型 II: $\int f(ax^2+b)x\,dx$

例 2 求 $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

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$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

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$$= -\frac{1}{2}\int u^{-1/2}du \qquad\qquad 2u^{1/2}$$

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$$= -\frac{1}{2}\int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C$$

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凑微分　　类型 II：$\int f(ax^2+b)xdx$

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$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2}\cdot\frac{1}{6}d(1+3x^2) = \frac{1}{6}\int \frac{1}{u}du$$

$$= \frac{1}{6}\ln|u|+C = \frac{1}{6}\ln|1+3x^2|+C$$

例 3 求 $\int xe^{x^2}dx$， $\int x\sin(x^2)dx$

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解

$$\int xe^{x^2}dx = \qquad\qquad d(x^2)$$

例 3 求 $\int xe^{x^2}dx$， $\int x\sin(x^2)dx$

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$$\int xe^{x^2}dx = \qquad \frac{1}{2}d(x^2)$$

凑微分　　类型 II：$\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2)$$

凑微分　类型 II：$\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx=\int e^{x^2}\frac{1}{2}d(x^2)=\frac{1}{2}\int e^u du$$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C$$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx=\int e^{x^2}\frac{1}{2}d(x^2)=\frac{1}{2}\int e^u du=\frac{1}{2}e^u+C=\frac{1}{2}e^{x^2}+C$$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

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$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx =$$

凑微分　　类型 II: $\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

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$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

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$$\int x\sin(x^2)dx = \int \sin(x^2)\cdot\frac{1}{2}d(x^2)$$

凑微分　　类型 II：$\int f(ax^2+b)x\,dx$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

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$$\int x\sin(x^2)dx = \int \sin(x^2)\cdot\frac{1}{2}d(x^2) = \frac{1}{2}\int \sin u\,du$$

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解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2)\cdot\frac{1}{2}d(x^2) = \frac{1}{2}\int \sin u\,du$$

$$= -\frac{1}{2}\cos u + C$$

例 3 求 $\int xe^{x^2}dx$，$\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

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$$= -\frac{1}{2}\cos u + C = -\frac{1}{2}\cos(x^2) + C$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx$$

凑微分　　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

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$$\int f(ae^x + b)e^x dx$$

凑微分　　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx \qquad\qquad d(ae^x+b)$$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx \qquad \frac{1}{a}d(ae^x+b)$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx = \int f(ax^2+b)\cdot \frac{1}{a} d(ae^x+b)$$

凑微分　　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx = \int f(ax^2+b)\cdot\frac{1}{a}d(ae^x+b)$$

$$\underline{\underline{u=ae^x+b}}$$

凑微分　　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^x+b)e^x dx = \int f(ax^2+b)\cdot\frac{1}{a}d(ae^x+b)$$

$$\xlongequal{u=ae^x+b} \int f(u)\cdot\frac{1}{a}du =$$

凑微分　　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

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$$\xlongequal{u=ae^x+b} \int f(u)\cdot\frac{1}{a}du = \quad F(u)$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

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凑微分　　类型 III: $\int f(ae^x+b)e^x dx$

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$$\int f(u)du = F(u) + C$$

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$$\int f(ae^x+b)e^x dx = \int f(ax^2+b)\cdot\frac{1}{a}d(ae^x+b)$$

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凑微分　　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

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$$\int f(ae^x+b)e^x dx = \int f(ax^2+b)\cdot \frac{1}{a} d(ae^x+b)$$

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例 1　　$\displaystyle\int \frac{e^x}{1+e^x}dx =$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

$$\int f(u)du = F(u) + C$$

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$$\int f(ae^x+b)e^x dx = \int f(ax^2+b)\cdot\frac{1}{a}d(ae^x+b)$$

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例 1　$\displaystyle\int \frac{e^x}{1+e^x}dx =$　　　　$d(e^x+1)$

凑微分　　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

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例 1　$$\int \frac{e^x}{1+e^x}dx = \int \frac{1}{1+e^x}d(e^x+1)$$

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例 1

$$\int \frac{e^x}{1+e^x}dx = \int \frac{1}{1+e^x}d(e^x+1)$$

$$= \int \frac{1}{u}du$$

凑微分　　类型 III: $\int f(ae^x+b)e^x dx$

假设会算

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$$\int \frac{e^x}{1+e^x}dx = \int \frac{1}{1+e^x}d(e^x+1)$$

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凑微分　　类型 III: $\int f(ae^x+b)e^x dx$

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例 1　$$\int\frac{e^x}{1+e^x}dx = \int\frac{1}{1+e^x}d(e^x+1)$$

$$= \int\frac{1}{u}du = \ln|u| + C = \ln(e^x+1)+C$$

例 2

$$\int e^x \sin(e^x)dx =$$

例 2

$$\int e^x \sin(e^x)dx = \qquad\qquad de^x$$

例 2

$$\int e^x \sin(e^x) dx = \int \sin(e^x) de^x$$

例 2

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$
$$= \int \sin u du$$

凑微分　类型 III: $\int f(ae^x+b)e^x dx$

例 2

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$
$$= \int \sin u du = -\cos u + C$$

例 2

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$
$$= \int \sin u du = -\cos u + C = -\cos(e^x) + C$$

凑微分　　类型 IV： $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx$$

凑微分　　类型 IV：$\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx$$

凑微分　　类型 IV：$\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u) + C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx \qquad\qquad d(a\ln x+b)$$

凑微分　　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

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凑微分　　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算

$$\int f(u)du = F(u)+C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

凑微分　　类型 IV：$\int f(a\ln x+b)\frac{1}{x}dx$

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$$\int f(u)du = F(u) + C$$

则

$$\int f(a\ln x+b)\frac{1}{x}dx = \int f(a\ln x+b)\cdot\frac{1}{a}d(a\ln x+b)$$

$$\underline{\underline{u=a\ln x+b}}$$

凑微分　类型 IV: $\int f(a\ln x+b)\frac{1}{x}dx$

假设会算
$$\int f(u)du = F(u) + C$$
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例 $\int\frac{1}{x}\ln x dx=$　　　　$d\ln x$

$\int\frac{1}{x\ln x}dx=$

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凑微分类型 IV： $\int f(\cos x)\sin x dx$, $\int f(\sin x)\cos x dx$

例子 求 $\int e^{\cos x}\sin x dx$，$\int \frac{\sin x}{1+\cos^2 x}dx$，$\int \frac{\cos x}{\sin x}dx$

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解

1. $$\int e^{\cos x}\sin x dx =$$

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解

1. $$\int e^{\cos x}\sin x dx = \qquad\qquad (-1)d\cos x$$

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例子 求 $\int e^{\cos x}\sin x dx$， $\int \frac{\sin x}{1+\cos^2 x}dx$， $\int \frac{\cos x}{\sin x}dx$

解

1. $$\int e^{\cos x}\sin x dx = \int e^{\cos x}\cdot(-1) d\cos x$$

凑微分类型 IV：$\int f(\cos x)\sin x dx, \int f(\sin x)\cos x dx$

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凑微分法 “$\int f(\varphi(x))d\varphi(x)$”：例子总结

$$\int \frac{1}{1-3x}dx =$$

$$\int \sqrt{3x-1}dx =$$

$$\int xe^{x^2}dx =$$

$$\int x\sqrt{1-x^2}dx =$$

$$\int \frac{\ln x}{x}dx =$$

$$\int e^{\cos x}\sin x dx =$$

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凑微分法 “$\int f(\varphi(x))d\varphi(x)$”：例子总结

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$$\int x\sqrt{1-x^2}dx = -\frac{1}{2}\int \sqrt{1-x^2}d(1-x^2) = -\frac{1}{2}\int u^{1/2}du = \cdots$$

$$\int \frac{\ln x}{x}dx = \int \ln x d(\ln x) = \int u du = \cdots$$

$$\int e^{\cos x}\sin x dx = -\int e^{\cos x}d\cos x = -\int e^u du = \cdots$$

We are here now...

1. 第一类换元积分法：凑微分

2. 第二类换元积分法：变量代换

第二类换元积分法——“变量代换”法，能干啥？

能够计算如下的不定积分：

$$\int x\sqrt{3x-1}dx, \quad \int \frac{x}{\sqrt{x-2}}dx$$

$$\int \frac{1}{1+\sqrt{x}}dx, \quad \int \frac{1}{1+\sqrt[3]{x+1}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

……

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx$$

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$$\int f(x)dx \xlongequal{x=\varphi(t)}$$

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$$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))d\varphi(t)$$

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- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int f(\varphi(t))\varphi'(t)dt$$

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在后面的例子中，选取函数 $x = \varphi(t)$ 的方法：

把被积函数 $f(x)$ 中复杂的部分整个设为 t，

从而得到 x 与 t 的函数关系！

例 求不定积分 $\int \sqrt{1-x^2}dx$

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注 可见选取合适 $x = \varphi(t)$ 很关键！

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变量代换　例 2

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

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解　(1) 设 $t = 1 + x^{\frac{1}{2}}$,

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解　(1) 设 $t = 1 + x^{\frac{1}{2}}$,

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解　(1) 设 $t=1+x^{\frac{1}{2}}$，　$\therefore x=(t-1)^2$，

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变量代换　例 2

例 2 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$，$\int \frac{1}{1+\sqrt[3]{1+x}}dx$

解　(1) 设 $t = 1 + x^{\frac{1}{2}}$,　$\therefore x = (t-1)^2$,　$dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}}dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2\int 1 - \frac{1}{t}dt$$

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变量代换　例 2

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变量代换　例 2

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变量代换　例 2

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$$= \frac{3}{2}(1 + (1+x)^{\frac{1}{3}})^2 - 6(1 + (1+x)^{\frac{1}{3}}) + 3\ln|1 + (1+x)^{\frac{1}{3}}| + C$$

变量代换　例 3

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

变量代换　例 3

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

解

设 $t = \sqrt{1+e^x}$,

变量代换　例 3

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

解

$$设\ t = \sqrt{1+e^x}, \quad \therefore x = \ln(t^2-1),$$

变量代换　例 3

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解

$$设\ t=\sqrt{1+e^x},\quad \therefore x=\ln(t^2-1),\quad dx=\frac{2t}{t^2-1}dt$$

变量代换　例 3

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解

$$设\ t = \sqrt{1+e^x}, \quad \therefore x = \ln(t^2-1), \quad dx = \frac{2t}{t^2-1}dt$$

$$\therefore \int \frac{1}{\sqrt{1+e^x}}dx = \int \frac{1}{t} \cdot \frac{2t}{t^2-1}dt$$

例 3 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解

设 $t=\sqrt{1+e^x}$, $\quad \therefore x=\ln(t^2-1)$, $\quad dx=\frac{2t}{t^2-1}dt$

$$\therefore \int \frac{1}{\sqrt{1+e^x}}dx=\int \frac{1}{t}\cdot\frac{2t}{t^2-1}dt=\int \frac{1}{t-1}-\frac{1}{t+1}dt$$

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$$=\ln|t-1|-\ln|t+1|+C=\ln\left|\frac{t-1}{t+1}\right|+C$$

$$=\ln\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right)+C$$

变量代换　例 3

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$$= \ln|t-1| - \ln|t+1| + C = \ln\left|\frac{t-1}{t+1}\right| + C$$

$$= \ln\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right) + C$$

$$= 2\ln(\sqrt{1+e^x}-1) - x + C$$

变量代换法：例子总结

$$\int x\sqrt{3x-1}dx$$

$$\int \frac{1}{1+\sqrt{x}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

变量代换法：例子总结

$$\int x\sqrt{3x-1}\,dx \xlongequal{t=\sqrt{3x-1}} \cdots$$

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