第 9 章 *b*：偏导数与全微分

数学系　　梁卓滨

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Outline

1. 偏导数

2. 全微分

We are here now...

1. 偏导数

2. 全微分

偏导数引入

- 对一元函数 $y=f(x)$：导数 $y'=f'(x) \longleftrightarrow$ 变化率

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例 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

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偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 x 的偏增量：

$$f(x_0 + \Delta x, y_0) - f(x_0, y_0)$$

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$$\begin{aligned}\lim_{\Delta x\to 0}\frac{\Delta_x z}{\Delta x} &= \lim_{\Delta x\to 0}\frac{f(x_0+\Delta x, y_0) - f(x_0, y_0)}{\Delta x}\\ &= \frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0}\end{aligned}$$

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偏导数记号：

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$$\frac{\partial f}{\partial x}(x_0,y_0), \qquad f'_x(x_0,y_0), \qquad f_x(x_0,y_0)$$

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏增量：

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$$\frac{\partial f}{\partial y}(x_0, y_0), \qquad f'_y(x_0, y_0), \qquad f_y(x_0, y_0)$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

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解法一

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

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解法一

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} =$$

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例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x =$$

$$\frac{\partial z}{\partial y} =$$

所以

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解法一

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解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y +$$

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解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

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解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = \left.(y + \frac{1}{y})\right|_{\substack{x=2\\y=1}} =$$

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例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

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$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} =$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

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$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

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解法一

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$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

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$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z=xy+\frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x}=(xy+\frac{x}{y})'_x=(xy)'_x+(\frac{x}{y})'_x=y+\frac{1}{y}$$

$$\frac{\partial z}{\partial y}=(xy+\frac{x}{y})'_y=(xy)'_y+(\frac{x}{y})'_y=x-\frac{x}{y^2}$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}}=(y+\frac{1}{y})\Big|_{\substack{x=2\\y=1}}=1+\frac{1}{1}=2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}}=$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x - \frac{x}{y^2}$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = \left.(y + \frac{1}{y})\right|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} = \left.(x - \frac{x}{y^2})\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

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$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x - \frac{x}{y^2}$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} = (x - \frac{x}{y^2})\Big|_{\substack{x=2\\y=1}} = 2 - \frac{2}{1} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

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所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

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例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0},$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \quad [f(x, y_0)] \quad ,$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0},$$

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$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \qquad [f(x_0, y)]$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

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$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

例 设 $z=xy+\frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2,1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0},\quad \frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\Big|_{y=y_0}$$

所以

$$f(x,1)$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \qquad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以

$$f(x, 1) = 2x$$

例 设 $z=xy+\frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0)=\frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0)=\frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以

$$f(x, 1)=2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)]=$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以

$$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \qquad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

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$$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$$

$$\Rightarrow \quad \frac{d}{dx}[f(x, 1)]\Big|_{x=2} = 2,$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

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$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \qquad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以

$$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$$

$$\Rightarrow \quad \frac{\partial z}{\partial x}\Big|_{\substack{x=2\\y=1}} = \frac{d}{dx}[f(x, 1)]\Big|_{x=2} = 2,$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

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$$f(2, y) = 2y + \frac{2}{y}$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

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$$\Rightarrow \quad \frac{\partial z}{\partial x}\Big|_{\substack{x=2\\y=1}} = \frac{d}{dx}[f(x, 1)]\Big|_{x=2} = 2,$$

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例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

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$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

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$$\Rightarrow \quad \frac{d}{dy}[f(2, y)]\Big|_{y=1} = 0.$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

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$$\Rightarrow \quad \frac{\partial z}{\partial y}\Big|_{\substack{x=2\\y=1}} = \frac{d}{dy}[f(2, y)]\Big|_{y=1} = 0.$$

例 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

例 设 $f(x, y) = \begin{cases} \dfrac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0)$$

$$f_y(0, 0)$$

例 设 $f(x, y) = \begin{cases} \dfrac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$f_x(0, 0) \qquad f(x, 0)$

$f_y(0, 0)$

例 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0}$$

$$f_y(0, 0)$$

例 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\bigg|_{x=0} = \frac{d}{dx}[0]\bigg|_{x=0}$$

$$f_y(0, 0)$$

例 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

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例 设 $f(x,y)=\begin{cases}\frac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$，求 $f_x(0,0), f_y(0,0)$

解

$$f_x(0,0)=\frac{d}{dx}[f(x,0)]\Big|_{x=0}=\frac{d}{dx}[0]\Big|_{x=0}=0,$$

$$f_y(0,0) \qquad f(0,y)$$

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解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)]\Big|_{x=0}$$

例 设 $f(x, y)=\begin{cases}\dfrac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解
$$f_x(0, 0)=\frac{d}{dx}[f(x, 0)]\Big|_{x=0}=\frac{d}{dx}[0]\Big|_{x=0}=0,$$
$$f_y(0, 0)=\frac{d}{dy}[f(0, y)]\Big|_{x=0}=\frac{d}{dy}[0]\Big|_{y=0}$$

例 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)]\Big|_{x=0} = \frac{d}{dy}[0]\Big|_{y=0} = 0,$$

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注 偏导数存在 $\not\Rightarrow$ 连续

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解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)]\Big|_{x=0} = \frac{d}{dy}[0]\Big|_{y=0} = 0,$$

注 偏导数存在 $\not\Rightarrow$ 连续

（上述 $f(x, y)$ 在 $(0, 0)$ 处存在偏导数 $f_x(0, 0)$ 和 $f_y(0, 0)$，但在 $(0, 0)$ 处不连续）

例 设 $f(x, y) = \begin{cases} \dfrac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

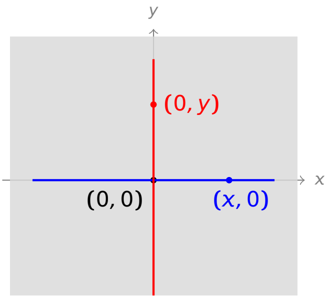
解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\bigg|_{x=0} = \frac{d}{dx}[0]\bigg|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)]\bigg|_{x=0} = \frac{d}{dy}[0]\bigg|_{y=0} = 0,$$

注 偏导数存在 $\not\Rightarrow$ 连续

（上述 $f(x, y)$ 在 $(0, 0)$ 处存在偏导数 $f_x(0, 0)$ 和 $f_y(0, 0)$，但在 $(0, 0)$ 处不连续）



例 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

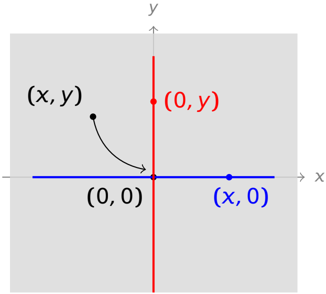
解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\bigg|_{x=0} = \frac{d}{dx}[0]\bigg|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)]\bigg|_{x=0} = \frac{d}{dy}[0]\bigg|_{y=0} = 0,$$

注 偏导数存在 $\not\Rightarrow$ 连续

（上述 $f(x, y)$ 在 $(0, 0)$ 处存在偏导数 $f_x(0, 0)$ 和 $f_y(0, 0)$，但在 $(0, 0)$ 处不连续）



二阶偏导数

$$
\begin{tikzcd}
 & \dfrac{\partial z}{\partial x} \\
z = f(x, y) \arrow[ur] \arrow[dr] & \\
 & \dfrac{\partial z}{\partial y}
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
z = f(x, y) \arrow[ur] \arrow[dr] & & {} \\
 & \frac{\partial z}{\partial y} &
\end{tikzcd}
$$

二阶偏导数

$$\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
z=f(x,y) \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial y} &
\end{tikzcd}$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & {}
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small, row sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right) \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right) \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right) \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right)
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x \partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) & \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial x}, \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial y}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right) & \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

二阶偏导数

$$\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial y}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y\partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}$$

二阶偏导数

$$\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial y}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y\partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y\partial y},
\end{tikzcd}$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): \quad \dfrac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): \quad \dfrac{\partial^2 z}{\partial x \partial y}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): \quad \dfrac{\partial^2 z}{\partial y \partial x}, \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right): \quad \dfrac{\partial^2 z}{\partial y \partial y},
\end{tikzcd}
$$

二阶偏导数

$$z = f(x, y) \quad \longrightarrow \quad \frac{\partial z}{\partial x}, \quad \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial x}, \qquad z_{xx},$$

$$\frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial y}, \qquad z_{xy},$$

$$\frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \qquad \frac{\partial^2 z}{\partial y \partial x},$$

$$\frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \qquad \frac{\partial^2 z}{\partial y \partial y},$$

二阶偏导数

$$\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \\
z = f(x, y) \arrow[uuur] \arrow[dddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial x}, \quad z_{yx}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial y},
\end{tikzcd}$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): \quad \dfrac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): \quad \dfrac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): \quad \dfrac{\partial^2 z}{\partial y \partial x}, \quad z_{yx}, \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right): \quad \dfrac{\partial^2 z}{\partial y \partial y}, \quad z_{yy},
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x \partial x},\quad z_{xx},\quad f_{xx} \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x \partial y},\quad z_{xy}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y \partial x},\quad z_{yx}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y \partial y},\quad z_{yy},
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): \quad \dfrac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \quad f_{xx} \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): \quad \dfrac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \quad f_{xy} \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): \quad \dfrac{\partial^2 z}{\partial y \partial x}, \quad z_{yx}, \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right): \quad \dfrac{\partial^2 z}{\partial y \partial y}, \quad z_{yy},
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \quad f_{xx} \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \quad f_{xy} \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y \partial x}, \quad z_{yx}, \quad f_{yx} \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y \partial y}, \quad z_{yy},
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): \quad \dfrac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \quad f_{xx} \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): \quad \dfrac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \quad f_{xy} \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): \quad \dfrac{\partial^2 z}{\partial y \partial x}, \quad z_{yx}, \quad f_{yx} \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right): \quad \dfrac{\partial^2 z}{\partial y \partial y}, \quad z_{yy}, \quad f_{yy}
\end{tikzcd}
$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x =$$

$$z_y =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=$$

$$z_y=$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} +$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} +$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} +$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

$$z_{xy}=(ye^{xy}+2y^2)'_y=(ye^{xy})'_y+(2y^2)'_y=e^{xy}+xye^{xy}+4y$$

$$z_{yx}=(xe^{xy}+4xy)'_x=$$

$$z_{yy}=$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

$$z_{xy}=(ye^{xy}+2y^2)'_y=(ye^{xy})'_y+(2y^2)'_y=e^{xy}+xye^{xy}+4y$$

$$z_{yx}=(xe^{xy}+4xy)'_x=(xe^{xy})'_x+(4xy)'_x=$$

$$z_{yy}=$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

$$z_{xy}=(ye^{xy}+2y^2)'_y=(ye^{xy})'_y+(2y^2)'_y=e^{xy}+xye^{xy}+4y$$

$$z_{yx}=(xe^{xy}+4xy)'_x=(xe^{xy})'_x+(4xy)'_x=e^{xy}+xye^{xy}+$$

$$z_{yy}=$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

$$z_{xy}=(ye^{xy}+2y^2)'_y=(ye^{xy})'_y+(2y^2)'_y=e^{xy}+xye^{xy}+4y$$

$$z_{yx}=(xe^{xy}+4xy)'_x=(xe^{xy})'_x+(4xy)'_x=e^{xy}+xye^{xy}+4y$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2e^{xy} +$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

$$z_{xy}=(ye^{xy}+2y^2)'_y=(ye^{xy})'_y+(2y^2)'_y=e^{xy}+xye^{xy}+4y$$

$$z_{yx}=(xe^{xy}+4xy)'_x=(xe^{xy})'_x+(4xy)'_x=e^{xy}+xye^{xy}+4y$$

$$z_{yy}=(xe^{xy}+4xy)'_y=(xe^{xy})'_y+(4xy)'_y=x^2e^{xy}+4x$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2 e^{xy} + 4x$$

注 此例成立 $z_{xy} = z_{yx}$

三阶偏导数

$$
\begin{tikzcd}[column sep=large, row sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right) \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right) \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right)
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) & \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z = f(x, y) \arrow[uuur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right)
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): &
\end{tikzcd}
$$

三阶偏导数

$$z = f(x, y) \to \frac{\partial z}{\partial x}, \quad \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial x \partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial x \partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial x \partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial x \partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial x \partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial x}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial y}, \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial x\partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): &
\end{tikzcd}
$$

三阶偏导数

$$z=f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$z=f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$z = f(x, y)$$

$$z = f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \qquad \frac{\partial^3 z}{\partial x \partial x \partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \qquad \frac{\partial^3 z}{\partial x \partial x \partial y},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \qquad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \qquad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$z = f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \qquad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \qquad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \qquad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) :$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial x \partial y},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial x}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial y}, \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial x\partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial y\partial x}, \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial y\partial y},
\end{tikzcd}
$$

三阶偏导数

$$z = f(x, y)$$

$$z = f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$z = f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$z = f(x, y) \longrightarrow \frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},$$

$$\frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$z = f(x, y) \longrightarrow \frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$z = f(x, y) \longrightarrow \frac{\partial z}{\partial x},\ \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},$$

$$\frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \qquad \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \qquad \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z = f(x, y) \longrightarrow \frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial x \partial x}, \; z_{xxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial x \partial y}, \; z_{xxy},$$

$$\frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial y \partial x}, \; z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial y \partial y}, \; z_{xyy},$$

$$z = f(x, y) \longrightarrow \frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial x \partial x}, \; z_{yxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial x \partial y}, \; z_{yxy},$$

$$\frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right)
\end{tikzcd}
$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx} \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy}, \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx}, \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},
\end{tikzcd}
$$

三阶偏导数

$$z = f(x, y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},$$

$$z = f(x, y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},\ z_{yxy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y \partial x \partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y \partial y \partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y \partial y \partial y},\ z_{yyy},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},\ z_{yyx},\ f_{yyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},\ z_{yyy},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},\ z_{yyx},\ f_{yyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},\ z_{yyy},\ f_{yyy}$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x =$$

$$z_y =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

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$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

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$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

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$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

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$$z_{xy}=(3x^2y^2-3y^3-y)'_y=$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

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$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

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$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$

$$z_{yx}=(2x^3y-9xy^2-x)'_x=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

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$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

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$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2-1$$

$$z_{yy}=(2x^3y-9xy^2-x)'_y=2x^3-18xy$$

$$z_{xxx}=$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

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$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2-1$$

$$z_{yy}=(2x^3y-9xy^2-x)'_y=2x^3-18xy$$

$$z_{xxx}=(6xy^2)'_x=$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$

$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2-1$$

$$z_{yy}=(2x^3y-9xy^2-x)'_y=2x^3-18xy$$

$$z_{xxx}=(6xy^2)'_x=6y^2$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$
$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$
$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$
$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2-1$$
$$z_{yy}=(2x^3y-9xy^2-x)'_y=2x^3-18xy$$

$$z_{xxx}=(6xy^2)'_x=6y^2$$

注 此例成立 $z_{xy}=z_{yx}$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$
\begin{aligned}
z_x &= (x\sin(3y))'_x = \\
z_y &= \\
z_{xx} &= \\
z_{xy} &= \\
z_{yx} &= \\
z_{yy} &= \\
z_{xyy} &=
\end{aligned}
$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$
\begin{aligned}
z_x &= (x\sin(3y))'_x = \sin(3y) \\
z_y &= \\
z_{xx} &= \\
z_{xy} &= \\
z_{yx} &= \\
z_{yy} &= \\
z_{xyy} &=
\end{aligned}
$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

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性质 设有二元函数 $z=f(x,y)$。若 $\frac{\partial^2 z}{\partial y\partial x}$ 和 $\frac{\partial^2 z}{\partial x\partial y}$ 均连续，则

$$\frac{\partial^2 z}{\partial y\partial x}=\frac{\partial^2 z}{\partial x\partial y}$$

We are here now...

1. 偏导数

2. 全微分

可微

- 回忆：一元函数 $z = f(x)$ 在 $x = x_0$ 处可微，指

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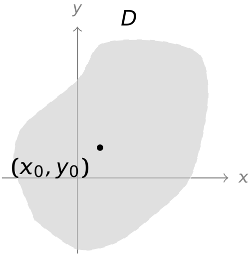

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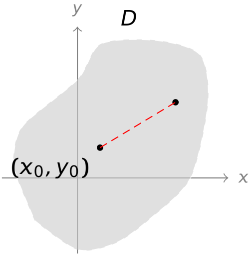

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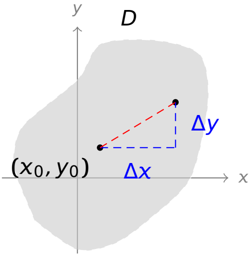

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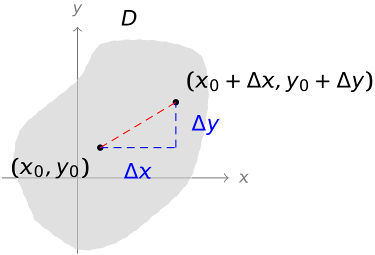

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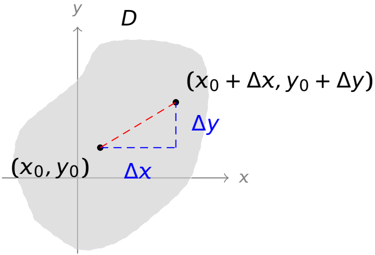

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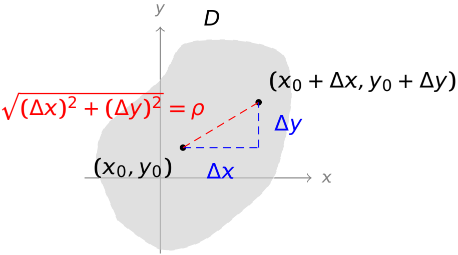

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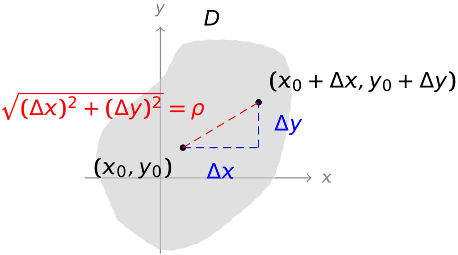

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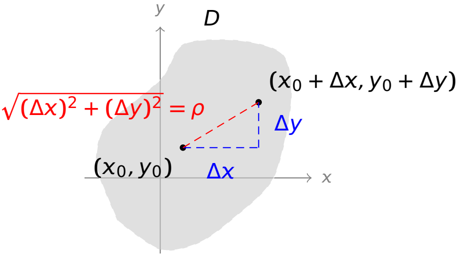

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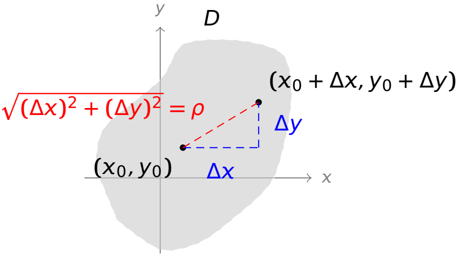

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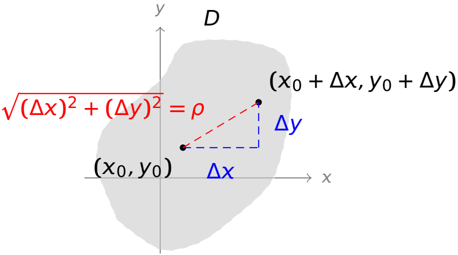

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定理（可微充分条件）

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注 通常的记号：$\Delta x=dx$, $\Delta y=dy$。这样全微分（存在的话）写成：

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全微分在近似计算中的应用

设 $z = f(x, y)$，则 $f(x+\Delta x, y+\Delta y) - f(x, y) = dz + o(\rho) \approx dz$

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所以 $(1.04)^{2.02}\approx dz+1=0.08+1=1.08$

可微、偏导数存在、连续的区别与联系

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