第 4 章 c: 实对称矩阵的特征值和特征向量

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2019-2020 学年 I

本节内容

- ◇ 向量的内积
- ♣ 正交向量组,施密特正交化方法
- ♡ 正交矩阵
- ♠ 对称矩阵可对角化

对称矩阵 1/33 < ▷ △ ▽

定义
$$\mathbb{R}^n$$
 中两个向量 $\alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix}$ 和 $\beta = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$ 的内积 定义为:

$$\alpha^T \beta =$$

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$$\alpha^{\mathsf{T}}\beta = (\alpha_1 \ \alpha_2 \cdots \alpha_n) \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} =$$

对称矩阵 2/33 < ▷ △ ▽

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对称矩阵 2/33 ◁ ▷ △ 1

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对称矩阵 2/33 ✓ ▶ △

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$$= (-1) \times 2 + 1 \times 0 + 0 \times (-1) + 2 \times 3 = 4$$

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$$\alpha^T \beta = \beta^T \alpha$$

- 2. $(k\alpha)^T\beta = k\alpha^T\beta$, (k是实数)
- 3. $(\alpha + \beta)^T \gamma = \alpha^T \gamma + \beta^T \gamma$
- 4. $\alpha^T \alpha \ge 0$,并且仅当 $\alpha = 0$ 时, $\alpha^T \alpha = 0$

对称矩阵 3/33 マ ▷ Δ ▽

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对称矩阵 4/33 ✓ ▷ △ ▽

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对称矩阵 4/33 マ ▷ △ 5

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村称矩阵 4/33 ✓ ▷ △

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对称矩阵 5/33 < ▷ △ ▽

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对称矩阵 5/33 < ▷ △ ▽

定义

$$||\alpha|| := \sqrt{\alpha^T \alpha} = \sqrt{\alpha_1^2 + \alpha_2^2 + \dots + \alpha_n^2}$$

称为向量的长度或范数。

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例 求向量
$$\alpha = \begin{pmatrix} -4 \\ -5 \\ 6 \end{pmatrix}$$
, $\beta = \begin{pmatrix} -1 \\ 3 \\ 1 \\ 5 \end{pmatrix}$ 的长度。

对称矩阵 6/33 < ▷ △ ▽

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$$||\alpha|| = \sqrt{(-4)^2 + (-5)^2 + 6^2} = \sqrt{16 + 25 + 36} =$$

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长度性质

1. $||\alpha|| \ge 0$,并且仅当 $\alpha = 0$ 时, $||\alpha|| = 0$

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- 3. 对任意向量 α , β , 都成立

$$|\alpha^T \beta| \le ||\alpha|| \cdot ||\beta||$$

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即

$$|a_1b_1 + \dots + a_nb_n| \le \sqrt{a_1^2 + \dots + a_n^2} \cdot \sqrt{b_1^2 + \dots + b_n^2}$$

● 定义 长度为1的向量称为单位向量。

对称矩阵 8/33 < ▷ △ ▽

- 定义 长度为 1 的向量称为单位向量。
- 例 向量

対量
$$\alpha = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \beta = \begin{pmatrix} 1/3 \\ 2/3 \\ -2/3 \end{pmatrix}, \quad \varepsilon_i = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \leftarrow i\text{-th}$$
非単位向量

都是单位向量

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都是单位向量

设 α ≠ 0, 则 ||α|| ≠ 0,

对称矩阵 8/33 < ▷ △ ▽

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对称矩阵 8/33 ✓ ▶ △ ▼

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$$\left\|\frac{1}{||\alpha||}\alpha\right\| = \frac{1}{||\alpha||}||\alpha|| = 1$$

对称矩阵 8/33 < ▷ △ ▽

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都是单位向量

• 设 $\alpha \neq 0$,则 $||\alpha|| \neq 0$,向量 $\frac{1}{||\alpha||} \alpha$ 是单位向量:

$$\left\|\frac{1}{||\alpha||}\alpha\right\| = \frac{1}{||\alpha||}||\alpha|| = 1$$

称 $\frac{1}{||\alpha||}\alpha$ 为 α 的 单位化

$$\alpha = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \beta = \begin{pmatrix} 2 \\ 2 \\ 4 \\ 5 \end{pmatrix}$$

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解

1.
$$||\alpha|| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$
,

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解

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$$||\alpha|| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$
,所以的 α 单位化为:

$$\frac{1}{||\alpha||}\alpha = \frac{1}{\sqrt{14}} \begin{pmatrix} 1\\2\\3 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{14}\\2/\sqrt{14}\\3/\sqrt{14} \end{pmatrix}$$

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2.
$$||\beta|| = \sqrt{2^2 + 2^2 + 4^2 + 5^2} = \sqrt{49} = 7$$
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2.
$$||\beta|| = \sqrt{2^2 + 2^2 + 4^2 + 5^2} = \sqrt{49} = 7$$
,所以的 β 单位化为:

$$\frac{1}{||\beta||}\beta = \frac{1}{7} \begin{pmatrix} 2\\2\\4\\5 \end{pmatrix} = \begin{pmatrix} 2/7\\2/7\\4/7\\5/7 \end{pmatrix}$$

定义 若 $\alpha^T \beta = 0$,则称 α , β 正交 (或垂直)

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例 零向量与任意向量正交:

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例 向量组
$$\varepsilon_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$
, $\varepsilon_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $\varepsilon_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ 中的向量两两正交:

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$$\varepsilon_1^T \varepsilon_2 = 0$$
, $\varepsilon_1^T \varepsilon_3 = 0$, $\varepsilon_2^T \varepsilon_3 = 0$

对称矩阵

定义 若 \mathbb{R}^n 中向量组 $\alpha_1, \alpha_2, \ldots, \alpha_s$ 满足

- 1. 每个向量非零: $\alpha_i \neq 0$, i = 1, 2, ..., s
- 2. 两两正交: $\alpha_i^T \alpha_j = 0$, $i \neq j$

即则称该向量组为正交向量组。

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$$\alpha = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$
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定理 \mathbb{R}^n 中正交向量组 α_1 , α_2 , ..., α_s 一定线性无关。 证明 设

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_s\alpha_s = 0$$

$$k_1 = k_2 = \cdots = k_s = 0$$

证明设

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_s\alpha_s = 0$$

$$0 = \alpha_i^T (k_1 \alpha_1 + k_2 \alpha_2 + \dots + k_i \alpha_i + \dots + k_s \alpha_s)$$

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所以 $k_i = 0$ 。

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$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_s\alpha_s = 0$$

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$$0 = \alpha_i^T (k_1 \alpha_1 + k_2 \alpha_2 + \dots + k_i \alpha_i + \dots + k_s \alpha_s) \xrightarrow{\alpha_i^T \alpha_j = 0 \text{ for } i \neq j} k_i \underbrace{\alpha_i^T \alpha_i}_{\neq 0}$$

所以 $k_i = 0$ 。由 i 的任意性

$$k_1 = k_2 = \cdots = k_s = 0$$

正交化

 $\alpha_1, \alpha_2, \ldots, \alpha_s$ (线性无关) $\longrightarrow \beta_1, \beta_2, \ldots, \beta_s$ (等价,两两正交)

 $\alpha_1, \alpha_2, \ldots, \alpha_s$ (线性无关) $\xrightarrow{\text{正交化}} \beta_1, \beta_2, \ldots, \beta_s$ (等价,两两正交)

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实现正交化步骤(施密特正交化方法):

$$\beta_1 =$$

$$\beta_2 =$$

$$\beta_3 =$$

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$$\beta_3 = \alpha_3 - \frac{1}{||\beta_1||^2} \beta_1 - \frac{\beta_2}{||\beta_1||^2} \beta_1 - \frac{\beta_2}{|\beta_1|}$$

$$\beta_s =$$

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$$\beta_1 = \alpha_1$$

$$\beta_2 = \alpha_2 - \frac{\alpha_2^T \beta_1}{\|\beta_1\|^2} \beta_1$$

$$\beta_3 = \alpha_3 - \frac{1}{||\beta_1||^2} \beta_1 - \frac{1}{||\beta_2||^2} \beta_2$$

:

$$\beta_{5} =$$

对称矩阵

$$\alpha_1, \alpha_2, \ldots, \alpha_s$$
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实现正交化步骤 (施密特正交化方法):

$$\beta_{1} = \alpha_{1}$$

$$\beta_{2} = \alpha_{2} - \frac{\alpha_{2}^{T} \beta_{1}}{||\beta_{1}||^{2}} \beta_{1}$$

$$\beta_{3} = \alpha_{3} - \frac{\alpha_{3}^{T} \beta_{1}}{||\beta_{1}||^{2}} \beta_{1} - \frac{\alpha_{3}^{T} \beta_{1}}{||\beta_{2}||^{2}} \beta_{2}$$

$$\vdots$$

$$\beta_s =$$

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$$\vdots$$

$$\beta_s =$$

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$$\vdots$$

$$\beta_s = \alpha_s - \dots - \beta_1 - \dots - \beta_2 - \dots - \beta_{s-1}$$

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$$\alpha_1, \alpha_2, \ldots, \alpha_s$$
(线性无关) $\xrightarrow{\text{正交化}} \beta_1, \beta_2, \ldots, \beta_s$ (等价,两两正交)

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$$\vdots$$

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$$\vdots$$

$$\beta_{s} = \alpha_{s} - \frac{1}{||\beta_{1}||^{2}} \beta_{1} - \frac{1}{||\beta_{2}||^{2}} \beta_{2} - \dots - \beta_{s-1}$$

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$$\vdots$$

$$\beta_s = \alpha_s - \frac{1}{||\beta_1||^2} \beta_1 - \frac{1}{||\beta_2||^2} \beta_2 - \dots - \frac{1}{||\beta_{s-1}||^2} \beta_{s-1}$$

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$$\vdots$$

$$\beta_s = \alpha_s - \frac{\alpha_s^T \beta_1}{||\beta_1||^2} \beta_1 - \frac{\alpha_s^T \beta_2}{||\beta_2||^2} \beta_2 - \dots - \frac{||\beta_{s-1}||^2}{||\beta_{s-1}||^2} \beta_{s-1}$$

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$$\vdots$$

$$\beta_{s} = \alpha_{s} - \frac{\alpha_{s}^{\mathsf{T}} \beta_{1}}{||\beta_{1}||^{2}} \beta_{1} - \frac{\alpha_{s}^{\mathsf{T}} \beta_{2}}{||\beta_{2}||^{2}} \beta_{2} - \dots - \frac{\alpha_{s}^{\mathsf{T}} \beta_{s-1}}{||\beta_{s-1}||^{2}} \beta_{s-1}$$

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实现正交化步骤(施密特正交化方法):

$$\beta_1 = \alpha_1$$

$$\boldsymbol{\beta}_1^{\mathsf{T}} \boldsymbol{\beta}_2 = \boldsymbol{\beta}_1^{\mathsf{T}} \left(\alpha_2 - \frac{\alpha_2^{\mathsf{T}} \beta_1}{||\beta_1||^2} \beta_1 \right)$$

$$\beta_3 = \alpha_3 - \frac{\alpha_3^T \beta_1}{||\beta_1||^2} \beta_1 - \frac{\alpha_3^T \beta_2}{||\beta_2||^2} \beta_2$$

:

$$\beta_{s} = \alpha_{s} - \frac{\alpha_{s}^{T} \beta_{1}}{||\beta_{1}||^{2}} \beta_{1} - \frac{\alpha_{s}^{T} \beta_{2}}{||\beta_{2}||^{2}} \beta_{2} - \dots - \frac{\alpha_{s}^{T} \beta_{s-1}}{||\beta_{s-1}||^{2}} \beta_{s-1}$$

对称矩阵 13/33 ✓ ▷ △ ▽

$$\alpha_1, \alpha_2, \ldots, \alpha_s$$
(线性无关) $\xrightarrow{\text{正交化}} \beta_1, \beta_2, \ldots, \beta_s$ (等价,两两正交)

实现正交化步骤(施密特正交化方法):

$$\beta_1 = \alpha_1$$

$$\beta_{1}^{T}\beta_{2} = \beta_{1}^{T} \left(\alpha_{2} - \frac{\alpha_{2}^{T}\beta_{1}}{||\beta_{1}||^{2}}\beta_{1}\right) = \beta_{1}^{T}\alpha_{2} - \frac{\alpha_{2}^{T}\beta_{1}}{||\beta_{1}||^{2}}\beta_{1}^{T}\beta_{1}$$
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$$\beta_{s} = \alpha_{s} - \frac{\alpha_{s}^{\mathsf{T}} \beta_{1}}{||\beta_{1}||^{2}} \beta_{1} - \frac{\alpha_{s}^{\mathsf{T}} \beta_{2}}{||\beta_{2}||^{2}} \beta_{2} - \dots - \frac{\alpha_{s}^{\mathsf{T}} \beta_{s-1}}{||\beta_{s-1}||^{2}} \beta_{s-1}$$

对称矩阵

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对称矩阵 13/33 ✓ ▷ △ ▽

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对称矩阵 13/33 ◁ ▷ △ ▽

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:

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对称矩阵 13/33 ◁ ▷ △ ▽

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对称矩阵 13/33 ✓ ▷ △ ▽

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:

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对称矩阵 13/33 ◁ ▷ △ ▽

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:

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:

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对称矩阵 13/33 ◁ ▷ △ ▽

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$$\vdots$$

$$\beta_{s} = \alpha_{s} - \frac{\alpha_{s}^{T} \beta_{1}}{||\beta_{1}||^{2}} \beta_{1} - \frac{\alpha_{s}^{T} \beta_{2}}{||\beta_{2}||^{2}} \beta_{2} - \dots - \frac{\alpha_{s}^{T} \beta_{s-1}}{||\beta_{s-1}||^{2}} \beta_{s-1}$$

例1 将线性无关组 $\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 3 \\ 3 \\ -1 \\ -1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -2 \\ 0 \\ 6 \\ 8 \end{pmatrix}$ 正交化

对称矩阵

例1 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
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对称矩阵

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对称矩阵 14/33 ✓ ▷ △ ▽

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对称矩阵

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$$\beta_1 = \alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} 3 \\ 3 \\ -1 \\ -1 \end{pmatrix} - \frac{1}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

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$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix} - \frac{4}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

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$$\beta_3 = \alpha_3 - \dots - \beta_1 - \dots - \beta_2$$

$$= \begin{pmatrix} -2 \\ 0 \\ 6 \\ 8 \end{pmatrix} - - - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - - - - \begin{pmatrix} 2 \\ 2 \\ -2 \\ -2 \end{pmatrix}$$

对称矩阵

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$$= \begin{pmatrix} -2 \\ 0 \\ 6 \end{pmatrix} - \frac{1}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \frac{2}{2} \begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix}$$

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$$= \begin{pmatrix} -2 \\ 0 \\ 6 \\ 8 \end{pmatrix} - \frac{12}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \dots - \begin{pmatrix} 2 \\ 2 \\ -2 \\ -2 \end{pmatrix}$$

对称矩阵

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$$\beta_3 = \alpha_3 - \frac{\beta_1 - \beta_2}{\beta_2}$$

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$$= \begin{pmatrix} -2 \\ 0 \\ 6 \\ 8 \end{pmatrix} - \frac{12}{4} \begin{pmatrix} \frac{1}{1} \\ \frac{1}{1} \end{pmatrix} - \frac{2}{16} \begin{pmatrix} \frac{2}{2} \\ -\frac{2}{2} \\ -\frac{2}{2} \end{pmatrix}$$

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$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 3 \\ 3 \\ -1 \\ -1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -2 \\ 0 \\ 6 \\ 8 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \frac{3}{-1} - \frac{4}{4} \begin{pmatrix} 1\\1\\1\\1 \end{pmatrix} = \begin{pmatrix} 2\\2\\-2\\-2 \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \frac{\beta_1 - \beta_2}{\beta_2}$$

$$\beta_3 = \alpha_3 - \frac{\beta_1 - \beta_2}{\beta_2}$$

$$\beta_3 = \alpha_3 - \frac{\beta_1 - \beta_2}{\beta_2}$$

$$= \begin{pmatrix} -2 \\ 0 \\ 6 \\ 8 \end{pmatrix} - \frac{12}{4} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \frac{-32}{16} \begin{pmatrix} 2 \\ 2 \\ -2 \\ -2 \end{pmatrix}$$

例 1 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 3 \\ 3 \\ -1 \\ -1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -2 \\ 0 \\ 6 \\ 8 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} 1\\1\\1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \frac{3}{1} = \begin{pmatrix} 3\\ -1\\ -1 \end{pmatrix} - \frac{4}{4} \begin{pmatrix} 1\\ 1\\ 1 \end{pmatrix} = \begin{pmatrix} 2\\ 2\\ -2\\ -2 \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \dots - \beta_1 - \dots - \beta_2$$

$$= \begin{pmatrix} -2 \\ 0 \\ 6 \\ 8 \end{pmatrix} - \frac{12}{4} \begin{pmatrix} \frac{1}{1} \\ \frac{1}{1} \end{pmatrix} - \frac{-32}{16} \begin{pmatrix} \frac{2}{2} \\ -\frac{2}{2} \\ -\frac{2}{2} \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$$

对称矩阵

例 2 将线性无关组 $\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ 正交化

例 2 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ 正交化

$$\beta_1 =$$

$$\beta_2 =$$

$$\beta_3 =$$

例 2 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1$$

$$\beta_2 =$$

$$\beta_3 =$$

例 2 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1$$

$$\beta_2 = \alpha_2 - \cdots - \beta_1$$

$$\beta_3 =$$

例 2 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1$$

$$\beta_2 = \alpha_2 - \cdots - \beta_1$$

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例 2 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} 1\\1\\0\\1 \end{pmatrix}$$

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例 2 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} 1\\1\\0\\1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} 3\\2\\1\\1 \end{pmatrix} - - \begin{pmatrix} 1\\1\\0\\1 \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \cdots - \beta_1 - \cdots - \beta_2$$

例 2 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} 1\\1\\0\\1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} \frac{3}{2} \\ \frac{1}{1} \end{pmatrix} - \frac{1}{3} \begin{pmatrix} \frac{1}{1} \\ 0 \\ 1 \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \dots - \beta_1 - \dots - \beta_2$$

例 2 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} 1\\1\\0\\1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} \frac{3}{2} \\ \frac{1}{1} \end{pmatrix} - \frac{6}{3} \begin{pmatrix} \frac{1}{1} \\ \frac{0}{1} \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \cdots - \beta_1 - \cdots - \beta_2$$

例 2 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} \frac{1}{1} \\ 0 \\ 1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} 3\\2\\1\\1 \end{pmatrix} - \frac{6}{3} \begin{pmatrix} 1\\1\\0\\1 \end{pmatrix} = \begin{pmatrix} 1\\0\\1\\-1 \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \cdots - \beta_1 - \cdots - \beta_2$$

例 2 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} 1\\1\\0\\1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} \frac{3}{2} \\ \frac{1}{1} \end{pmatrix} - \frac{6}{3} \begin{pmatrix} \frac{1}{1} \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{0} \\ 1 \\ -1 \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \frac{\beta_1 - \beta_2}{\beta_1 - \beta_2}$$

$$= \begin{pmatrix} \frac{2}{1} \\ \frac{1}{2} \end{pmatrix} - - \begin{pmatrix} \frac{1}{1} \\ \frac{1}{2} \end{pmatrix} - - \begin{pmatrix} \frac{1}{1} \\ \frac{1}{2} \end{pmatrix}$$

例 2 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} 1\\1\\0\\1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} \frac{3}{2} \\ \frac{1}{1} \end{pmatrix} - \frac{6}{3} \begin{pmatrix} \frac{1}{1} \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{0} \\ 1 \\ -1 \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \frac{\beta_1 - \beta_2}{\beta_1 - \beta_2}$$

$$= \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \frac{1}{3} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} - - \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

对称矩阵

例 2 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} 1\\1\\0\\1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \frac{\beta_1}{\beta_1} = \begin{pmatrix} \frac{3}{2} \\ \frac{1}{1} \end{pmatrix} - \frac{6}{3} \begin{pmatrix} \frac{1}{1} \\ \frac{1}{0} \\ \frac{1}{1} \end{pmatrix} = \begin{pmatrix} \frac{1}{0} \\ \frac{1}{-1} \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \frac{\beta_1 - \beta_2}{\beta_1 - \beta_2}$$

$$= \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \frac{6}{3} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - - \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

例 2 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} 1\\1\\0\\1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \frac{\beta_1}{\beta_1} = \begin{pmatrix} \frac{3}{2} \\ \frac{1}{1} \end{pmatrix} - \frac{6}{3} \begin{pmatrix} \frac{1}{1} \\ \frac{1}{0} \\ \frac{1}{1} \end{pmatrix} = \begin{pmatrix} \frac{1}{0} \\ \frac{1}{-1} \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \frac{\beta_1 - \beta_2}{\beta_1 - \beta_2}$$

$$= \begin{pmatrix} \frac{2}{1} \\ \frac{1}{2} \end{pmatrix} - \frac{6}{3} \begin{pmatrix} \frac{1}{1} \\ \frac{1}{2} \end{pmatrix} - \frac{1}{3} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

例 2 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$
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$$\beta_1 = \alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} \frac{3}{2} \\ \frac{1}{1} \end{pmatrix} - \frac{6}{3} \begin{pmatrix} \frac{1}{1} \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{0} \\ 1 \\ -1 \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \frac{\beta_1 - \beta_2}{\beta_1 - \beta_2}$$

$$= \begin{pmatrix} \frac{2}{1} \\ \frac{1}{2} \end{pmatrix} - \frac{6}{3} \begin{pmatrix} \frac{1}{1} \\ \frac{1}{2} \end{pmatrix} - \frac{0}{3} \begin{pmatrix} \frac{1}{0} \\ \frac{1}{2} \end{pmatrix}$$

例 2 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 3 \\ 2 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 2 \\ 1 \\ 1 \\ 3 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} 1\\1\\0\\1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \frac{\beta_1}{\beta_1} = \begin{pmatrix} \frac{3}{2} \\ \frac{1}{1} \end{pmatrix} - \frac{6}{3} \begin{pmatrix} \frac{1}{1} \\ \frac{1}{0} \\ \frac{1}{1} \end{pmatrix} = \begin{pmatrix} \frac{1}{0} \\ \frac{1}{-1} \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \frac{\beta_1 - \beta_2}{\beta_1 - \beta_2}$$

$$= \begin{pmatrix} \frac{2}{1} \\ \frac{1}{3} \end{pmatrix} - \frac{6}{3} \begin{pmatrix} \frac{1}{1} \\ 0 \\ 1 \end{pmatrix} - \frac{0}{3} \begin{pmatrix} \frac{1}{0} \\ \frac{1}{-1} \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ \frac{1}{1} \end{pmatrix}$$

例 3 将线性无关组 $\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ 正交化

例 3 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ 正交化

$$\beta_1 =$$

$$\beta_2 =$$

$$\beta_3 =$$

例 3 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
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例 3 将线性无关组
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例 3 将线性无关组
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$$\beta_1 = \alpha_1$$

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$$\beta_1 = \alpha_1 = \begin{pmatrix} \frac{1}{1} \\ \frac{1}{1} \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \cdots - \beta_1$$

$$\beta_3 = \alpha_3 - \cdots - \beta_1 - \cdots - \beta_2$$

例 3 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} \frac{1}{1} \\ \frac{1}{1} \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} 0 \\ \frac{1}{2} \\ 1 \end{pmatrix} - - \begin{pmatrix} \frac{1}{1} \\ \frac{1}{1} \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \cdots - \beta_1 - \cdots - \beta_2$$

例 3 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} \frac{1}{1} \\ \frac{1}{1} \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix} - \frac{1}{4} \begin{pmatrix} \frac{1}{1} \\ \frac{1}{1} \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \cdots - \beta_1 - \cdots - \beta_2$$

例 3 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} \frac{1}{1} \\ \frac{1}{1} \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} 0 \\ \frac{1}{2} \\ 1 \end{pmatrix} - \frac{4}{4} \begin{pmatrix} \frac{1}{1} \\ \frac{1}{1} \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \cdots - \beta_1 - \cdots - \beta_2$$

例 3 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} \frac{1}{1} \\ \frac{1}{1} \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} 0 \\ \frac{1}{2} \\ 1 \end{pmatrix} - \frac{4}{4} \begin{pmatrix} 1 \\ \frac{1}{1} \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ \frac{1}{0} \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \dots - \beta_1 - \dots - \beta_2$$

例 3 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} \frac{1}{1} \\ \frac{1}{1} \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \frac{\beta_1}{\beta_2} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{1} \end{pmatrix} - \frac{4}{4} \begin{pmatrix} 1 \\ \frac{1}{1} \\ \frac{1}{1} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ \frac{1}{0} \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \dots - \beta_1 - \dots - \beta_2$$

$$= \left(\begin{array}{c} -1\\0\\\frac{1}{1}\\1\end{array}\right) - - \left(\begin{array}{c} 1\\1\\1\\1\end{array}\right) - - \left(\begin{array}{c} -1\\0\\1\\0\end{array}\right)$$

例 3 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} 1\\1\\1\\1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \frac{\beta_1}{\beta_2} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{1} \end{pmatrix} - \frac{4}{4} \begin{pmatrix} 1 \\ \frac{1}{1} \\ \frac{1}{1} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ \frac{1}{0} \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \dots - \beta_1 - \dots - \beta_2$$

$$= \begin{pmatrix} -1 \\ 0 \\ \frac{1}{1} \end{pmatrix} - \frac{1}{4} \begin{pmatrix} \frac{1}{1} \\ \frac{1}{1} \end{pmatrix} - - \begin{pmatrix} -1 \\ 0 \\ \frac{1}{0} \end{pmatrix}$$

例 3 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} 1\\1\\1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \frac{\beta_1}{\beta_2} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{1} \end{pmatrix} - \frac{4}{4} \begin{pmatrix} 1 \\ \frac{1}{1} \\ \frac{1}{1} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ \frac{1}{0} \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \frac{\beta_1 - \beta_2}{\beta_1 - \beta_2}$$

$$= \begin{pmatrix} -1 \\ 0 \\ \frac{1}{1} \end{pmatrix} - \frac{1}{4} \begin{pmatrix} \frac{1}{1} \\ \frac{1}{1} \end{pmatrix} - - \begin{pmatrix} -1 \\ 0 \\ \frac{1}{0} \end{pmatrix}$$

例 3 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} \frac{1}{1} \\ \frac{1}{1} \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} 0 \\ \frac{1}{2} \\ 1 \end{pmatrix} - \frac{4}{4} \begin{pmatrix} 1 \\ \frac{1}{1} \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ \frac{1}{0} \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \frac{\beta_1 - \beta_2}{\beta_1 - \beta_2}$$

$$= \left(\begin{array}{c} -1\\0\\1\\1\end{array}\right) - \frac{1}{4} \left(\begin{array}{c} 1\\1\\1\\1\end{array}\right) - \frac{1}{2} \left(\begin{array}{c} -1\\0\\1\\0\end{array}\right)$$

例 3 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} 1\\1\\1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \frac{\beta_1}{\beta_2} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{1} \end{pmatrix} - \frac{4}{4} \begin{pmatrix} 1 \\ \frac{1}{1} \\ \frac{1}{1} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ \frac{1}{0} \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \frac{\beta_1 - \beta_2}{\beta_1 - \beta_2}$$

$$= \begin{pmatrix} -1 \\ 0 \\ \frac{1}{1} \end{pmatrix} - \frac{1}{4} \begin{pmatrix} \frac{1}{1} \\ \frac{1}{1} \end{pmatrix} - \frac{2}{2} \begin{pmatrix} -1 \\ 0 \\ \frac{1}{0} \end{pmatrix}$$

例 3 将线性无关组
$$\alpha_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$ 正交化

$$\beta_1 = \alpha_1 = \begin{pmatrix} 1\\1\\1\\1 \end{pmatrix}$$

$$\beta_2 = \alpha_2 - \frac{\beta_1}{\beta_2} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{1} \end{pmatrix} - \frac{4}{4} \begin{pmatrix} 1 \\ \frac{1}{1} \\ \frac{1}{1} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ \frac{1}{0} \end{pmatrix}$$

$$\beta_3 = \alpha_3 - \dots - \beta_1 - \dots - \beta_2$$

$$= \begin{pmatrix} -1 \\ 0 \\ \frac{1}{1} \end{pmatrix} - \frac{1}{4} \begin{pmatrix} \frac{1}{1} \\ \frac{1}{1} \end{pmatrix} - \frac{2}{2} \begin{pmatrix} -1 \\ 0 \\ \frac{1}{0} \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} \\ -\frac{1}{4} \\ -\frac{1}{4} \end{pmatrix}$$

定义 设 n 阶矩阵 Q 满足 $Q^TQ = I_n$,则称 Q 是正交矩阵。

对称矩阵 17/33 < ▷ △ ▽

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例
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$$Q^{T}Q = \begin{pmatrix} \alpha_{1}^{T} \\ \alpha_{2}^{T} \\ \vdots \\ \alpha_{n}^{T} \end{pmatrix} (\alpha_{1} \alpha_{2} \dots \alpha_{n}) = \begin{pmatrix} \alpha_{1}^{T} \alpha_{1} & \alpha_{1}^{T} \alpha_{2} & \cdots & \alpha_{1}^{T} \alpha_{n} \\ \alpha_{2}^{T} \alpha_{1} & \alpha_{2}^{T} \alpha_{2} & \cdots & \alpha_{2}^{T} \alpha_{n} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{n}^{T} \alpha_{1} & \alpha_{n}^{T} \alpha_{2} & \cdots & \alpha_{n}^{T} \alpha_{n} \end{pmatrix}$$

所以

$$Q^{T}Q = I \quad \Longleftrightarrow \quad \begin{cases} \alpha_{i}^{T}\alpha_{i} = 1, \\ \alpha_{i}^{T}\alpha_{i} = 0, \end{cases}$$

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所以

$$Q^{T}Q = I \iff \begin{cases} \alpha_{i}^{T}\alpha_{i} = 1, & (i = 1, 2, ..., n) \\ \alpha_{i}^{T}\alpha_{j} = 0, & (i \neq j; i, j = 1, 2, ..., n) \end{cases}$$

对称矩阵

$$A_1 = \begin{pmatrix} \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \end{pmatrix}, \quad A_2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \end{pmatrix},$$

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提示 验证: 列向量组是单位正交向量组

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答案 A_1 是正交矩阵

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答案 A_1 是正交矩阵, A_2 不是正交矩阵

- 对任意 n 阶方阵:
 - 1. 一定有n个特征值(计算重数,复数域内),可能有非实数特征值
 - 2. 不一定能对角化

对称矩阵 20/33 ◁ ▷ △ ▽

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 的特征值方程是

$$0 = |\lambda I - A| =$$

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例
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对称矩阵 20/33 < ▷ △ ▽

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- 对实对称矩阵,总成立:
 - 1. 定理 实对称矩阵的特征值都是实数。
 - 2. 定理 实对称矩阵一定可以对角化。

对称矩阵 20/33 < ▶ △ ▽

也就是:设A为实对称矩阵,则一定存在可逆矩阵P,使得

$$P^{-1}AP = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \ddots \\ \lambda_n \end{pmatrix}$$

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对称矩阵 21/33 < ▷ △ ▽

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注 由于正交矩阵满足
$$Q^{-1} = Q^T$$
,上述等价于 $Q^T A Q = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ & \ddots \\ & \lambda_n \end{pmatrix}$

证明 设 A 为实对称矩阵, $\lambda_1 \neq \lambda_2$ 为两特征值, α_1 , α_2 为相应特征向量,

$$\alpha_2^T \alpha_1 = 0$$

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注意
$$\alpha_2^T A \alpha_1 = (\alpha_2^T A \alpha_1)^T =$$

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由于 $\lambda_1 \neq \lambda_2$,所以

$$\alpha_2^T \alpha_1 = 0$$

定理 设 A 为实对称矩阵,则存在正交矩阵 Q,使得 $Q^{-1}AQ$ 为对角矩阵。

解释示意图

_	不同 特征值	重 数	正交化	单位化
_	λ_1	n ₁		
	λ_2	n_2		
	:	:		
	λ_{s}	ns		
_		共n		
_				

 $|\lambda I - A| = (\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \cdots (\lambda - \lambda_s)^{n_s}$

4	不同 特征值	重 数	$(\lambda_i I - A)x = 0$ 基础解系	正交化	单位化			
	λ_1	n ₁						
	λ_2	n_2						
	÷	÷						
	λ_{s}	ns						
		共n						
;	$ \lambda I - A = (\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \cdots (\lambda - \lambda_s)^{n_s}$							

_	不同 特征值	重 数	$(\lambda_i I - A)x = 0$ 基础解系	正交化	单位化				
	λ_1	n ₁	$\alpha_1^{(1)}, \cdots, \alpha_{n_1}^{(1)}$						
	λ_2	n_2							
	÷	:							
	λ_{s}	ns							
_		共n							
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	λ_2	n_2	$\alpha_1^{(2)}, \cdots, \alpha_{n_2}^{(2)}$						
	÷	÷							
	λ_s	ns							
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	λ_2	n_2	$\alpha_1^{(2)}, \cdots, \alpha_{n_2}^{(2)}$							
	÷	:	÷							
	λ_s	ns	$\alpha_1^{(s)}, \cdots, \alpha_{n_s}^{(s)}$							
_		共n	共 n 个无关特征向量							
_	$ \lambda I - A = (\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \cdots (\lambda - \lambda_s)^{n_s}$									

不同 特征值	重 数	$(\lambda_i I - A)x = 0$ 基础解系	正交化	单位化						
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:	:	÷								
λ_{s}	ns	$\alpha_1^{(s)}, \cdots, \alpha_{n_s}^{(s)}$								
	共n	共n个无关特征向量								
$ \lambda I - A $ =	$ \lambda I - A = (\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \cdots (\lambda - \lambda_s)^{n_s}$									

•
$$\Leftrightarrow P = (\alpha_1^{(1)}, \dots, \alpha_{n_s}^{(n_s)}), \ \text{III} \ P^{-1}AP = \Lambda_\circ$$

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$$P = (\alpha_1^{(1)}, \dots, \alpha_{n_s}^{(n_s)})$$
,则 $P^{-1}AP = \Lambda$ 。但一般地, P 不是正交 矩阵。

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λ_2	n_2	$\alpha_1^{(2)}, \cdots, \alpha_{n_2}^{(2)}$							
÷	:	÷							
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λ_2	n_2	$\alpha_1^{(2)}, \cdots, \alpha_{n_2}^{(2)}$	\Rightarrow	$\beta_1^{(2)}, \cdots, \beta_{n_2}^{(2)}$					
÷	:	÷							
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解释示意图

不同 特征值	重 数	$(\lambda_i I - A)x = 0$ 基础解系		正交化		单位化
λ_1	n_1	$\alpha_1^{(1)}, \cdots, \alpha_{n_1}^{(1)}$	⇒	$\beta_1^{(1)}, \cdots, \beta_{n_1}^{(1)}$	⇒	$\gamma_1^{(1)}, \cdots, \gamma_{n_1}^{(1)}$
λ_2	n ₂	$\alpha_1^{(2)},\cdots,\alpha_{n_2}^{(2)}$	\Rightarrow	$\beta_1^{(2)}, \cdots, \beta_{n_2}^{(2)}$	⇒	$\gamma_1^{(2)},\cdots,\gamma_{n_2}^{(2)}$
÷	÷	:				:
λ_s	n _s	$\alpha_1^{(s)}, \cdots, \alpha_{n_s}^{(s)}$				
	共n	共 n 个无关特征向量				

• 令
$$P = (\alpha_1^{(1)}, \dots, \alpha_{n_s}^{(n_s)})$$
,则 $P^{-1}AP = \Lambda$ 。但一般地, P 不是正交 矩阵。

 $|\lambda I - A| = (\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \cdots (\lambda - \lambda_s)^{n_s}$

不同 特征值	重 数	$(\lambda_i I - A)x = 0$ 基础解系		正交化		单位化		
λ_1	n_1	$\alpha_1^{(1)}, \cdots, \alpha_{n_1}^{(1)}$	⇒	$\beta_1^{(1)}, \cdots, \beta_{n_1}^{(1)}$	⇒	$\gamma_1^{(1)}, \cdots, \gamma_{n_1}^{(1)}$		
λ_2	n_2	$\alpha_1^{(2)}, \cdots, \alpha_{n_2}^{(2)}$	\Rightarrow	$\beta_1^{(2)}, \cdots, \beta_{n_2}^{(2)}$	\Rightarrow	$\gamma_1^{(2)}, \cdots, \gamma_{n_2}^{(2)}$		
÷	÷	÷ :		÷		:		
$\lambda_{\scriptscriptstyle S}$	ns	$\alpha_1^{(s)}, \cdots, \alpha_{n_s}^{(s)}$	\Rightarrow	$\beta_1^{(s)}, \cdots, \beta_{n_s}^{(s)}$				
	共n	共 n 个无关特征向量						
$ \lambda I - A = (\lambda - \lambda_1)^{n_1} (\lambda - \lambda_2)^{n_2} \cdots (\lambda - \lambda_s)^{n_s}$								

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λ_2	n ₂	$\alpha_1^{(2)}, \cdots, \alpha_{n_2}^{(2)}$	\Rightarrow	$\beta_1^{(2)}, \cdots, \beta_{n_2}^{(2)}$	⇒	$\gamma_1^{(2)}, \cdots, \gamma_{n_2}^{(2)}$		
÷	÷	÷		÷		:		
λ_{s}	ns	$\alpha_1^{(s)}, \cdots, \alpha_{n_s}^{(s)}$	\Rightarrow	$\beta_1^{(s)}, \cdots, \beta_{n_s}^{(s)}$	⇒	$\gamma_1^{(s)}, \cdots, \gamma_{n_s}^{(s)}$		
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解释示意图

	不同 特征值	重 数	$(\lambda_i I - A)x = 0$ 基础解系		正交化		单位化
	λ_1	n_1	$\alpha_1^{(1)}, \cdots, \alpha_{n_1}^{(1)}$	⇒	$\beta_1^{(1)}, \cdots, \beta_{n_1}^{(1)}$	⇒	$\gamma_1^{(1)}, \cdots, \gamma_{n_1}^{(1)}$
	λ_2	n_2	$\alpha_1^{(2)}, \cdots, \alpha_{n_2}^{(2)}$	\Rightarrow	$\beta_1^{(2)}, \cdots, \beta_{n_2}^{(2)}$	\Rightarrow	$\gamma_1^{(2)}, \cdots, \gamma_{n_2}^{(2)}$
	÷	į	÷		÷		:
	$\lambda_{\scriptscriptstyle S}$	ns	$\alpha_1^{(s)}, \cdots, \alpha_{n_s}^{(s)}$	\Rightarrow	$\beta_1^{(s)}, \cdots, \beta_{n_s}^{(s)}$	⇒	$\gamma_1^{(s)}, \cdots, \gamma_{n_s}^{(s)}$
•		共n	共				构成单位正交特征 向量
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• 令 $P = (\alpha_1^{(1)}, \dots, \alpha_{n_s}^{(n_s)})$,则 $P^{-1}AP = \Lambda$ 。但一般地,P 不是正交 矩阵。

正交化

 $(\lambda_i I - A)x = 0$

解释示意图

不同

重

	特征值	数	基础解系				1 1210	
	λ_1	n ₁	$\alpha_1^{(1)}, \cdots, \alpha_{n_1}^{(1)}$	⇒	$\beta_1^{(1)}, \cdots, \beta_{n_1}^{(1)}$	⇒	$\gamma_1^{(1)}, \cdots, \gamma_{n_1}^{(1)}$	
	λ_2	n_2	$\alpha_1^{(2)}, \cdots, \alpha_{n_2}^{(2)}$	\Rightarrow	$\beta_1^{(2)}, \cdots, \beta_{n_2}^{(2)}$	\Rightarrow	$\gamma_1^{(2)},\cdots,\gamma_{n_2}^{(2)}$	
	:	:	:		:		:	
	λ_s	ns	$\alpha_1^{(s)}, \cdots, \alpha_{n_s}^{(s)}$	\Rightarrow	$\beta_1^{(s)}, \cdots, \beta_{n_s}^{(s)}$	\Rightarrow	$\gamma_1^{(s)}, \cdots, \gamma_{n_s}^{(s)}$	
-		共n	共 n 个无关特征向量				构成单位正交特征 向量	
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- 令 $P = (\alpha_1^{(1)}, \dots, \alpha_{n_s}^{(n_s)})$,则 $P^{-1}AP = \Lambda$ 。但一般地,P 不是正交矩阵。
 - $\bullet \Leftrightarrow Q = (\gamma_1^{(1)}, \cdots, \gamma_{n_s}^{(n_s)}),$

单位化

_	不同 特征值	重 数	$(\lambda_i I - A)x = 0$ 基础解系		正交化		单位化	
	λ_1	n ₁	$\alpha_1^{(1)}, \cdots, \alpha_{n_1}^{(1)}$	⇒	$\beta_1^{(1)}, \cdots, \beta_{n_1}^{(1)}$	⇒	$\gamma_1^{(1)}, \cdots, \gamma_{n_1}^{(1)}$	
	λ_2	n_2	$\alpha_1^{(2)}, \cdots, \alpha_{n_2}^{(2)}$	\Rightarrow	$\beta_1^{(2)}, \cdots, \beta_{n_2}^{(2)}$	⇒	$\gamma_1^{(2)},\cdots,\gamma_{n_2}^{(2)}$	
	:	:	÷		÷		:	
	λ_{s}	ns	$\alpha_1^{(s)}, \cdots, \alpha_{n_s}^{(s)}$	⇒	$\beta_1^{(s)}, \cdots, \beta_{n_s}^{(s)}$	\Rightarrow	$\gamma_1^{(s)}, \cdots, \gamma_{n_s}^{(s)}$	
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- 令 $P = (\alpha_1^{(1)}, \dots, \alpha_{n_s}^{(n_s)})$,则 $P^{-1}AP = \Lambda$ 。但一般地,P 不是正交矩阵。
 - 令 $Q = (\gamma_1^{(1)}, \dots, \gamma_{n_c}^{(n_s)})$,则 Q 是正交矩阵,

 $(\lambda_i I - A)x = 0$

基础解系

 $\alpha^{(2)}$... $\alpha^{(2)}$

 $\alpha_1^{(1)}, \cdots, \alpha_n^{(1)}$

正交化

 $\beta_1^{(1)}, \cdots, \beta_{n_1}^{(1)}$

解释示意图

不同

特征值

 λ_1

١.

矩阵。

重数

 n_1

Λ2	112	$\alpha_1, \dots, \alpha_{n_2}^{(1)}$	⇒	$\beta_1, \dots, \beta_{n_2}^{(2)}$	⇒	γ_1 ,, $\gamma_{n_2}^{(2)}$	
÷	÷	:		:		:	
λ_s	ns	$\alpha_1^{(s)}, \cdots, \alpha_{n_s}^{(s)}$	\Rightarrow	$\beta_1^{(s)}, \cdots, \beta_{n_s}^{(s)}$	\Rightarrow	$\gamma_1^{(s)}, \cdots, \gamma_{n_s}^{(s)}$	
	共n	共n个无关特征向量				构成单位正交特征 向量	
$ \lambda I - A =$	$=(\lambda-\lambda_1)$	$^{n_1}(\lambda-\lambda_2)^{n_2}\cdots(\lambda-\lambda_n)^{n_n}$	s) ^{ns}				
• 令 $P = (\alpha_1^{(1)}, \dots, \alpha_{n_s}^{(n_s)})$,则 $P^{-1}AP = \Lambda$ 。但一般地, P 不是正交							

• 令 $Q = (\gamma_1^{(1)}, \dots, \gamma_{n_s}^{(n_s)})$,则 Q 是正交矩阵,且 $Q^{-1}AQ = \Lambda$ 。

对称矩阵

单位化

 $\Rightarrow \gamma_1^{(1)}, \cdots, \gamma_{n_1}^{(1)}$

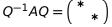
 $g^{(2)} \dots g^{(2)} \rightarrow g^{(2)} \dots g^{(2)}$

$$Q^{-1}AQ = \begin{pmatrix} * \\ * \end{pmatrix}$$

解

特征方程: 0 = |λI − A|

• 特征方程:
$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 2 & -1 \\ -1 & \lambda - 2 \end{vmatrix} =$$



解

• 特征方程: $0 = |\lambda I - A| = \begin{vmatrix} \lambda - 2 & -1 \\ -1 & \lambda - 2 \end{vmatrix} = (\lambda - 2)^2 - 1$

解

• 特征方程: $0 = |\lambda I - A| = \begin{vmatrix} \lambda - 2 & -1 \\ -1 & \lambda - 2 \end{vmatrix} = (\lambda - 1)(\lambda - 3)$

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例1 将矩阵 $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ 正交对角化。

解

• 特征方程: $0 = |\lambda I - A| = \begin{vmatrix} \lambda - 2 & -1 \\ -1 & \lambda - 2 \end{vmatrix} = (\lambda - 1)(\lambda - 3)$ • $\lambda_1 = 1$, 求解 $(\lambda_1 I - A)x = 0$:

•
$$\lambda_2 = 3$$
,求解 $(\lambda_2 I - A)x = 0$:

解

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•
$$\lambda_1 = 1$$
, \overline{x} $\mathbb{R}(\lambda_1 I - A)x = 0$:

$$(1I - A \vdots 0) = \begin{pmatrix} -1 - 1 & 0 \\ -1 - 1 & 0 \end{pmatrix}$$

•
$$\lambda_2 = 3$$
, \overline{x} $\mathbb{R}(\lambda_2 I - A)x = 0$:

 $Q^{-1}AQ = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ 对称矩阵

解

• 特征方程: $0 = |\lambda I - A| = \begin{vmatrix} \lambda - 2 & -1 \\ -1 & \lambda - 2 \end{vmatrix} = (\lambda - 1)(\lambda - 3)$ • $\lambda_1 = 1$,求解 $(\lambda_1 I - A)x = 0$:

$$(1I - A \stackrel{?}{\cdot} 0) = \begin{pmatrix} -1 & -1 & 0 \\ -1 & -1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

•
$$\lambda_2 = 3$$
, \overline{x} M ($\lambda_2 I - A$) $X = 0$:

$$Q^{-1}AQ = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

解

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$$(1I - A \vdots 0) = \begin{pmatrix} -1 & -1 & | & 0 \\ -1 & -1 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix} \qquad \begin{array}{c} x_1 + x_2 = 0 \\ x_1 + x_2 = 0 \\ x_1 + x_2 = 0 \end{array}$$

• $\lambda_2 = 3$, \overline{x} M ($\lambda_2 I - A$)X = 0:

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基础解系:
$$\alpha_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \xrightarrow{\text{单位化}} \gamma_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

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例1 将矩阵
$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$
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• $\lambda_2 = 3$,求解 $(\lambda_2 I - A)x = 0$:

$$(3I - A \stackrel{?}{\cdot} 0) = \begin{pmatrix} 1 & -1 & | & 0 \\ -1 & 1 & | & 0 \end{pmatrix}$$

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. $\forall \mathbf{W} (\lambda_2 I - A) \mathbf{v} = 0$

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 $(3I - A \vdots 0) = \begin{pmatrix} 1 & -1 & | & 0 \\ -1 & 1 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix} \qquad \begin{matrix} x_1 - x_2 & = 0 \\ \downarrow & & \downarrow \end{matrix}$

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基础解系: $\alpha_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \xrightarrow{\text{单位化}} \gamma_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

基础解系: $\alpha_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \xrightarrow{\text{单位化}} \gamma_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

所以取 $Q = \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$,则 $Q^{-1}AQ = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

- 特征方程: $0 = |\lambda I A| = \begin{vmatrix} \lambda 2 & -1 \\ -1 & \lambda 2 \end{vmatrix} = (\lambda 1)(\lambda 3)$

解

特征方程: 0 = |λI − A|

• 特征方程: $0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 \\ -2 & \lambda - 1 \end{vmatrix} =$

例 2 将矩阵
$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$
 正交。

• 特征方程: $0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 \\ -2 & \lambda - 1 \end{vmatrix} = (\lambda - 1)^2 - (-2)^2$

$$Q^{-1}AQ = \left(\begin{array}{c} * \\ * \end{array} \right)$$

例 2 将矩阵
$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$
 正交。

• 特征方程: $0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 \\ -2 & \lambda - 1 \end{vmatrix} = (\lambda + 1)(\lambda - 3)$

$$Q^{-1}AQ = \left(\begin{array}{c} * \\ * \end{array} \right)$$

• 特征方程:
$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 \\ -2 & \lambda - 1 \end{vmatrix} = (\lambda + 1)(\lambda - 3)$$

 \bullet $\lambda_1 = -1$

$$\lambda_2 = 3$$

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$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 \\ -2 & \lambda - 1 \end{vmatrix} = (\lambda + 1)(\lambda - 3)$$

• $\lambda_1 = -1$

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对称矩阵

解

特征方程:
$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 \\ -2 & \lambda - 1 \end{vmatrix} = (\lambda + 1)(\lambda - 3)$$

•
$$\lambda_1 = -1$$
, \overline{X} $\mathbb{R}(\lambda_1 I - A)X = 0$:

•
$$\lambda_2 = 3$$
,求解 $(\lambda_2 I - A)x = 0$:

 $Q^{-1}AQ = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

解

• 特征方程: $0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 \\ -2 & \lambda - 1 \end{vmatrix} = (\lambda + 1)(\lambda - 3)$

• $\lambda_1 = -1$, \overline{x} $\mathbb{R}(\lambda_1 I - A)x = 0$:

$$(-I - A \vdots 0) = \begin{pmatrix} -2 & -2 & | & 0 \\ -2 & -2 & | & 0 \end{pmatrix}$$

• $\lambda_2 = 3$, \overline{x} $\mathbb{R}(\lambda_2 I - A)x = 0$:

 $Q^{-1}AQ = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

解

• 特征方程:
$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 \\ -2 & \lambda - 1 \end{vmatrix} = (\lambda + 1)(\lambda - 3)$$

• $\lambda_1 = -1$, $\Re (\lambda_1 I - A)x = 0$:

$$(-I - A \vdots 0) = \begin{pmatrix} -2 & -2 & | & 0 \\ -2 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix}$$

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解

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• $\lambda_1 = -1$, \vec{x} \vec{x} \vec{x} \vec{y} $\vec{$

$$(-I - A \vdots 0) = \begin{pmatrix} -2 & -2 & | & 0 \\ -2 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix} \qquad \begin{array}{c} x_1 + x_2 = 0 \\ \end{array}$$

• $\lambda_2 = 3$,求解 $(\lambda_2 I - A)x = 0$:

 $Q^{-1}AQ = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

解

• 特征方程:
$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 \\ -2 & \lambda - 1 \end{vmatrix} = (\lambda + 1)(\lambda - 3)$$

• $\lambda_1 = -1$, \vec{x} \vec{x} \vec{y} $(\lambda_1 I - A)x = 0$:

$$(-I - A : 0) = \begin{pmatrix} -2 - 2 & | & 0 \\ -2 - 2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix} \qquad \begin{array}{c} x_1 + x_2 = 0 \\ \downarrow \\ x_1 = -x_2 \end{array}$$

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$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$$
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基础解系: $\alpha_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

• $\lambda_2 = 3$, \vec{x} \vec{x} \vec{x} \vec{x} \vec{y} $\vec{y$

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基础解系:
$$\alpha_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \xrightarrow{\text{单位化}} \gamma_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

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• $\lambda_2 = 3$,求解 $(\lambda_2 I - A)x = 0$:

$$(3I - A \stackrel{?}{\cdot} 0) = \begin{pmatrix} 2 & -2 & | & 0 \\ -2 & 2 & | & 0 \end{pmatrix}$$

 $Q^{-1}AQ = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

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 $(3I - A \vdots 0) = \begin{pmatrix} 2 & -2 & | & 0 \\ -2 & 2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix} \qquad \begin{array}{c} x_1 - x_2 = 0 \\ \end{array}$

•
$$\lambda_2 = 3$$
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•
$$\lambda_2 = 3$$
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 $Q^{-1}AQ = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

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• $\lambda_1 = -1$, \overline{x} $\mathbb{R}(\lambda_1 I - A)x = 0$:

$$(-I - A : 0) = \begin{pmatrix} -2 - 2 & | & 0 \\ -2 - 2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix} \qquad \begin{array}{c} x_1 + x_2 = 0 \\ x_1 = -x_2 \end{array}$$

• $\lambda_2 = 3$, \vec{x} \vec{x} \vec{x} \vec{y} $\vec{y$

$$(3I - A : 0) = \begin{pmatrix} 2 & -2 & | & 0 \\ -2 & 2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix} \qquad \begin{cases} x_1 - x_2 = 0 \\ x_1 = x_2 \end{cases}$$

基础解系: $\alpha_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$Q^{-1}AQ = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

解

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$$Q^{-1}AQ = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

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• $\lambda_2 = 3$,求解 $(\lambda_2 I - A)x = 0$:

基础解系: $\alpha_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \xrightarrow{\text{单位化}} \gamma_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

所以取 $Q = \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$,则 $Q^{-1}AQ = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

● $\lambda_1 = -1$, 求解 $(\lambda_1 I - A)x = 0$:

• 特征方程: $0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 \\ -2 & \lambda - 1 \end{vmatrix} = (\lambda + 1)(\lambda - 3)$

 $(-I - A \vdots 0) = \begin{pmatrix} -2 & -2 & | & 0 \\ -2 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix} \qquad \begin{matrix} x_1 + x_2 & = & 0 \\ x_1 & = & -x_2 \end{matrix}$

 $(3I - A : 0) = \begin{pmatrix} 2 & -2 & | & 0 \\ -2 & 2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & | & 0 \\ 0 & 0 & | & 0 \end{pmatrix} \qquad \begin{matrix} x_1 - x_2 & = & 0 \\ & & \downarrow \end{matrix}$

例 2 将矩阵 $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ 正交。

基础解系: $\alpha_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $\xrightarrow{\text{单位化}}$ $\gamma_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

例 3
$$A = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 2 & -2 \\ 0 & -2 & 3 \end{pmatrix}$$

$$Q^{-1}AQ = \left(\begin{array}{c} * \\ * \\ * \end{array}\right)$$

对称矩阵 26/33 < ▷ △ ▽

例 3 $A = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 2 & -2 \\ 0 & -2 & 3 \end{pmatrix}$,特征方程: $0 = |\lambda I - A| = (\lambda + 1)(\lambda - 2)(\lambda - 5)$

$$Q^{-1}AQ = \begin{pmatrix} * & & \\ & * & \\ & & * \end{pmatrix}$$

对称矩阵 26/33 ▽ ▷ □

例 3
$$A = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 2 & -2 \\ 0 & -2 & 3 \end{pmatrix}$$
,特征方程: $0 = |\lambda I - A| = (\lambda + 1)(\lambda - 2)(\lambda - 5)$

$$\bullet \ \lambda_1 = -1,$$

$$\lambda_2 = 2$$

$$\bullet \ \lambda_3 = 5,$$

$$Q^{-1}AQ = \begin{pmatrix} * & & \\ & * & \\ & & * \end{pmatrix}$$

对称矩阵

例 3
$$A = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 2 & -2 \\ 0 & -2 & 3 \end{pmatrix}$$
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$$Q^{-1}AQ = \begin{pmatrix} -1 & 2 & 1 \\ & 2 & 5 \end{pmatrix}$$

对称矩阵

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•
$$\lambda_1 = -1$$
, 特征向量 $\alpha_1 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$

$$\lambda_2 = 2$$
,

$$\bullet \ \lambda_3 = 5,$$

$$Q^{-1}AQ = \begin{pmatrix} -1 & & \\ & 2 & \\ & 5 \end{pmatrix}$$

对称矩阵 26/33 ▽ △ ▽

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 $\bullet \ \lambda_3 = 5,$

$$Q^{-1}AQ = \begin{pmatrix} -1 & & \\ & 2 & \\ & & 5 \end{pmatrix}$$

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$$\lambda_1 = -1$$
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•
$$\lambda_2 = 2$$
, 特征向量 $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$

•
$$\lambda_3 = 5$$
, 特征向量 $\alpha_3 = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

$$Q^{-1}AQ = \begin{pmatrix} -1 & 2 & 1 \\ & 2 & 5 \end{pmatrix}$$

对称矩阵 26/33 ◁ ▷ △ ▽

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$$A = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 2 & -2 \\ 0 & -2 & 3 \end{pmatrix}$$
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•
$$\lambda_1 = -1$$
,特征向量 $\alpha_1 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ 单位化 $\gamma_1 = \begin{pmatrix} 2/3 \\ 2/3 \\ 1/3 \end{pmatrix}$

•
$$\lambda_2 = 2$$
, 特征向量 $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$

•
$$\lambda_3 = 5$$
, 特征向量 $\alpha_3 = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

$$Q^{-1}AQ = \begin{pmatrix} -1 & & \\ & 2 & \\ & & 5 \end{pmatrix}$$

对称矩阵 26/33 ◁ ▷ △ ▽

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$$A = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 2 & -2 \\ 0 & -2 & 3 \end{pmatrix}$$
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,特征向量 $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ $\xrightarrow{\text{单位化}}$ $\gamma_2 = \begin{pmatrix} 2/3 \\ -1/3 \\ -2/3 \end{pmatrix}$

•
$$\lambda_3 = 5$$
, 特征向量 $\alpha_3 = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

$$Q^{-1}AQ = \begin{pmatrix} -1 & & \\ & 2 & \\ & & 5 \end{pmatrix}$$

对称矩阵 26/33 ✓ ▷ △ ▽

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$$A = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 2 & -2 \\ 0 & -2 & 3 \end{pmatrix}$$
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$$\lambda_2 = 2$$
,特征向量 $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ $\xrightarrow{\text{单位化}}$ $\gamma_2 = \begin{pmatrix} 2/3 \\ -1/3 \\ -2/3 \end{pmatrix}$

•
$$\lambda_3 = 5$$
,特征向量 $\alpha_3 = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ 单位化 $\gamma_3 = \begin{pmatrix} 1/3 \\ -2/3 \\ 2/3 \end{pmatrix}$

$$Q^{-1}AQ = \begin{pmatrix} -1 & 2 & 1 \\ & 2 & 5 \end{pmatrix}$$

対称矩阵 26/33 ◁ ▷ △ ▽

例 3
$$A = \begin{pmatrix} 1 & -2 & 0 \\ -2 & 2 & -2 \\ 0 & -2 & 3 \end{pmatrix}$$
,特征方程: $0 = |\lambda I - A| = (\lambda + 1)(\lambda - 2)(\lambda - 5)$

•
$$\lambda_1 = -1$$
,特征向量 $\alpha_1 = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ 单位化 $\gamma_1 = \begin{pmatrix} 2/3 \\ 2/3 \\ 1/3 \end{pmatrix}$

•
$$\lambda_2 = 2$$
,特征向量 $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ 单位化 $\gamma_2 = \begin{pmatrix} 2/3 \\ -1/3 \\ -2/3 \end{pmatrix}$

•
$$\lambda_3 = 5$$
,特征向量 $\alpha_3 = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ 单位化 $\gamma_3 = \begin{pmatrix} 1/3 \\ -2/3 \\ 2/3 \end{pmatrix}$

所以取
$$Q = \underbrace{\begin{pmatrix} 2/3 & 2/3 & 1/3 \\ 2/3 & -1/3 & -2/3 \\ 1/3 & -2/3 & 2/3 \end{pmatrix}}_{Q: 正交阵}$$
,则 $Q^{-1}AQ = \begin{pmatrix} -1 & 2 & 5 \end{pmatrix}$

对称矩阵 26/33 ✓ ▷ △ ▽

例 4
$$A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{pmatrix}$$

$$Q^{-1}AQ = \begin{pmatrix} * \\ * \\ * \end{pmatrix}$$

对称矩阵

例 4
$$A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{pmatrix}$$
,特征方程: $0 = |\lambda I - A| = (\lambda - 1)^2 (\lambda - 10)$

$$Q^{-1}AQ = \begin{pmatrix} * & \\ & * \\ & & * \end{pmatrix}$$

$$\lambda_3 = 10$$

例 4 $A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{pmatrix}$,特征方程: $0 = |\lambda I - A| = (\lambda - 1)^2 (\lambda - 10)$

$$Q^{-1}AQ = \begin{pmatrix} * & & \\ & * & \\ & & * \end{pmatrix}$$

对称矩阵

*λ*₁ = 1 (二重)

$$\bullet$$
 $\lambda_3 = 10$

 $Q^{-1}AQ = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

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例 4 $A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{pmatrix}$,特征方程: $0 = |\lambda I - A| = (\lambda - 1)^2(\lambda - 10)$

*λ*₁ = 1 (二重)

例 4
$$A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{pmatrix}$$
,特征方程: $0 = |\lambda I - A| = (\lambda - 1)^2 (\lambda - 10)$

$$\begin{cases} \alpha_1 = \begin{pmatrix} -2\\1\\0 \end{pmatrix} \\ \alpha_2 = \begin{pmatrix} 2\\0\\1 \end{pmatrix} \end{cases}$$

λ₁ = 1 (二重), 特征向量

• $\lambda_3 = 10$

$$Q^{-1}AQ = \begin{pmatrix} 1 \\ 1 \\ 10 \end{pmatrix}$$

例 4
$$A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{pmatrix}$$
,特征方程: $0 = |\lambda I - A| = (\lambda - 1)^2 (\lambda - 10)$

● $\lambda_1 = 1$ (二重),特征向量

$$\begin{cases}
\alpha_1 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \\
\alpha_2 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}
\end{cases}$$

•
$$\lambda_3 = 10$$
, 特征向量 $\alpha_3 = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

例 4
$$A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{pmatrix}$$
, 特征方程: $0 = |\lambda I - A| = (\lambda - 1)^2 (\lambda - 10)$

● $\lambda_1 = 1$ (二重),特征向量

$$\begin{cases} \alpha_1 = \begin{pmatrix} -2\\1\\0 \end{pmatrix} \xrightarrow{\text{EXM}} \begin{cases} \beta_1 = \begin{pmatrix} -2\\1\\0 \end{pmatrix} \\ \alpha_2 = \begin{pmatrix} 2\\0\\1 \end{pmatrix} \end{cases}$$

• $\lambda_3 = 10$, 特征向量 $\alpha_3 = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

$$Q^{-1}AQ = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

例 4
$$A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{pmatrix}$$
, 特征方程: $0 = |\lambda I - A| = (\lambda - 1)^2 (\lambda - 10)$

λ₁ = 1 (二重) ,特征向量

刈朴矩阵

例 4
$$A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{pmatrix}$$
,特征方程: $0 = |\lambda I - A| = (\lambda - 1)^2 (\lambda - 10)$

λ₁ = 1 (二重) ,特征向量

$$\begin{cases}
\alpha_1 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \xrightarrow{\text{iff}} \begin{cases}
\beta_1 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \xrightarrow{\text{iff}} \begin{cases}
\gamma_1 = \frac{1}{\sqrt{5}} \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \\
\alpha_2 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} & \beta_2 = \begin{pmatrix} 2/5 \\ 4/5 \\ 1 \end{pmatrix}
\end{cases}$$

$$\begin{cases}
\gamma_2 = \frac{5}{3\sqrt{5}} \begin{pmatrix} 2/5 \\ 4/5 \\ 1 \end{pmatrix}$$

•
$$\lambda_3 = 10$$
,特征向量 $\alpha_3 = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ $\xrightarrow{\text{单位化}}$ $\gamma_3 = \begin{pmatrix} 1/3 \\ 2/3 \\ -2/3 \end{pmatrix}$

$$Q^{-1}AQ = \begin{pmatrix} 1 \\ 1 \\ 10 \end{pmatrix}$$

例 4 $A = \begin{pmatrix} 2 & 2 & -2 \\ 2 & 5 & -4 \\ -2 & -4 & 5 \end{pmatrix}$,特征方程: $0 = |\lambda I - A| = (\lambda - 1)^2 (\lambda - 10)$

• $\lambda_1 = 1$ (二重) ,特征向量 $\begin{cases}
\alpha_1 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \xrightarrow{\mathbb{E}^{\chi} \mathbb{K}}
\begin{cases}
\beta_1 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \xrightarrow{\frac{1}{\sqrt{5}}} \begin{pmatrix} \gamma_1 = \frac{1}{\sqrt{5}} \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \\
\beta_2 = \begin{pmatrix} 2/5 \\ 4/5 \\ 1 \end{pmatrix}
\end{cases}
\end{cases}$ $\begin{cases}
\gamma_2 = \frac{5}{3\sqrt{5}} \begin{pmatrix} 2/5 \\ 4/5 \\ 1 \end{pmatrix}
\end{cases}$

•
$$\lambda_3 = 10$$
,特征向量 $\alpha_3 = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ 单位化 $\gamma_3 = \begin{pmatrix} 1/3 \\ 2/3 \\ -2/3 \end{pmatrix}$ 所以取 $Q = \begin{pmatrix} -2/\sqrt{5} & 2/3 & \sqrt{5} & 1/3 \\ 1/\sqrt{5} & 4/3\sqrt{5} & 2/3 \\ 0 & \sqrt{5}/3 & -2/3 \end{pmatrix}$,则 $Q^{-1}AQ = \begin{pmatrix} 1 \\ 1 \\ 10 \end{pmatrix}$

Q: 正交阵

例 5
$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$
,

$$Q^{-1}AQ = \left(\begin{array}{cc} * & \\ & * \\ & \end{array}\right)$$

例 5 $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$,特征方程: $0 = |\lambda I - A| =$

$$Q^{-1}AQ = \begin{pmatrix} * & & \\ & * & \\ & & * \end{pmatrix}$$

例 5 $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$,特征方程: $0 = |\lambda I - A| = (\lambda + 1)^2 (\lambda - 5)$ Det

$$Q^{-1}AQ = \begin{pmatrix} * & & \\ & * & \\ & & * \end{pmatrix}$$

例 5
$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$
,特征方程: $0 = |\lambda I - A| = (\lambda + 1)^2 (\lambda - 5)$ • Det • $\lambda_1 = -1$ (二重)

$$\lambda_2 = 5$$

$$Q^{-1}AQ = \begin{pmatrix} * & & \\ & * & \\ & & * \end{pmatrix}$$

•
$$\lambda_1 = -1$$
 (二重)

例 5 $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$,特征方程: $0 = |\lambda I - A| = (\lambda + 1)^2(\lambda - 5)$ **Det**

$$\lambda_2 = 5$$

$$Q^{-1}AQ = \begin{pmatrix} -1 \\ -1 \\ 5 \end{pmatrix}$$

例 5
$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$
, 特征方程: $0 = |\lambda I - A| = (\lambda + 1)^2(\lambda - 5)$ Det

•
$$\lambda_1 = -1$$
 (二重) ,特征向量:
• Detail
$$\begin{cases} \alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \\ \alpha_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$\lambda_2 = 5$$

$$Q^{-1}AQ = \begin{pmatrix} -1 \\ -1 \\ 5 \end{pmatrix}$$

例 5
$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$
, 特征方程: $0 = |\lambda I - A| = (\lambda + 1)^2(\lambda - 5)$ Det

•
$$\lambda_1 = -1$$
 (二重) ,特征向量:
• Detail
$$\begin{cases} \alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \\ \alpha_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

•
$$\lambda_2 = 5$$
,特征向量: • Det $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$Q^{-1}AQ = \begin{pmatrix} -1 \\ -1 \\ 5 \end{pmatrix}$$

例 5
$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$
,特征方程: $0 = |\lambda I - A| = (\lambda + 1)^2 (\lambda - 5)$ Det

• $\lambda_1 = -1$ (二重),特征向量: • Detail $\begin{cases}
\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} & \xrightarrow{\mathbb{E}^{\mathbb{R}^{\mathbb{R}}}} \\
\alpha_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}
\end{cases}$

•
$$\lambda_2 = 5$$
,特征向量: • Det $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$Q^{-1}AQ = \begin{pmatrix} -1 \\ -1 \\ 5 \end{pmatrix}$$

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例 5
$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$
,特征方程: $0 = |\lambda I - A| = (\lambda + 1)^2 (\lambda - 5)$ Det $\lambda_1 = -1$ (二重) ,特征向量: Detail

$$\lambda_1 = -1 \quad (三重) \quad , \quad \text{特征向量: } \underbrace{ \begin{cases} \alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} }_{\text{正交化}} \quad \begin{cases} \beta_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \end{cases} }_{\text{Q} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}}$$

•
$$\lambda_2 = 5$$
,特征向量: • Det $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$Q^{-1}AQ = \begin{pmatrix} -1 & \\ -1 & \\ 5 \end{pmatrix}$$

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例 5
$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$
,特征方程: $0 = |\lambda I - A| = (\lambda + 1)^2 (\lambda - 5)$ Det $\lambda_1 = -1$ (二重) ,特征向量: Detail

•
$$\lambda_1 = -1$$
 ($\subseteq \underline{1}$), $\gamma_1 \underline{\square} / \gamma_2 \underline{\square}$. $\delta = |\lambda| - |\lambda| -$

•
$$\lambda_2 = 5$$
,特征向量: • Det $\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$$Q^{-1}AQ = \begin{pmatrix} -1 \\ -1 \\ 5 \end{pmatrix}$$

对称矩阵 28/33 < ▷ △ ▽

例 5
$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$
,特征方程: $0 = |\lambda I - A| = (\lambda + 1)^2 (\lambda - 5)$ Det

$$\begin{array}{l} \bullet \ \lambda_1 = -1 \ (= \underline{1} \) \ , \ \text{特征向量:} \\ \left\{ \begin{array}{l} \alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{array} \right\} \xrightarrow{\underline{\text{EX}}\ell} \\ \alpha_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{array} \right\} & \text{Det} \end{array} \right. \begin{cases} \beta_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \xrightarrow{\underline{\text{Pt}}\ell} \\ \beta_2 = \begin{pmatrix} -1/2 \\ -1/2 \\ 1 \end{pmatrix} & \text{Proof } \\ \beta_2 = \begin{pmatrix} -1/2 \\ -1/2 \\ 1 \end{pmatrix} & \text{Proof } \\ \gamma_2 = \begin{pmatrix} -1/\sqrt{6} \\ -1/\sqrt{6} \\ 2/\sqrt{6} \end{pmatrix} \end{cases}$$

•
$$\lambda_2=5$$
,特征向量: $\alpha_3=\begin{pmatrix}1\\1\\1\end{pmatrix} \xrightarrow{\text{单位化}} \gamma_3=\begin{pmatrix}1/\sqrt{3}\\1/\sqrt{3}\\1/\sqrt{3}\end{pmatrix}$
$$Q^{-1}AQ=\begin{pmatrix}-1\\-1\\5\end{pmatrix}$$

对称矩阵 28/33 ✓ ▷ △ ▽

例 5
$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$
, 特征方程: $0 = |\lambda I - A| = (\lambda + 1)^2 (\lambda - 5)$ Det $\lambda_1 = -1$ (二重) ,特征向量: Detail
$$\begin{pmatrix} \alpha_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} & \begin{pmatrix} \beta_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \end{pmatrix} & \begin{pmatrix} \gamma_1 = \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$\begin{array}{l} \bullet \ \lambda_1 = -1 \ (= 1) \ , \ \ \forall \alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \ \xrightarrow{\mathbb{E}^2(\mathbb{R})} \ \begin{cases} \beta_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \ \xrightarrow{\oplus \hat{\alpha} \mathbb{R}} \ \begin{cases} \gamma_1 = \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix} \end{cases} \\ \alpha_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \ \xrightarrow{\bullet \text{ Det}} \ \begin{cases} \beta_2 = \begin{pmatrix} -1/2 \\ -1/2 \\ 1 \end{pmatrix} \ \end{cases} \ \begin{cases} \gamma_2 = \begin{pmatrix} -1/\sqrt{6} \\ -1/\sqrt{6} \\ 2/\sqrt{6} \end{pmatrix} \end{cases}$$

O: 正交阵

_____The End_____

$$0 = |\lambda I - A| =$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$



$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$

$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$

$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -\lambda - 1 & \lambda + 1 \end{vmatrix}$$

$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$

$$\frac{r_3 - r_2}{=} \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -\lambda - 1 & \lambda + 1 \end{vmatrix}$$

$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -1 & 1 \end{vmatrix}$$

$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$
$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -\lambda - 1 & \lambda + 1 \end{vmatrix}$$
$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -1 & 1 \end{vmatrix} \frac{c_2 + c_3}{2}$$

$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$

$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -\lambda - 1 & \lambda + 1 \end{vmatrix}$$

$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -1 & 1 \end{vmatrix} \stackrel{c_2 + c_3}{=} (\lambda + 1) \begin{vmatrix} \lambda - 1 & -4 & -2 \\ -2 & \lambda - 3 & -2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$

$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -\lambda - 1 & \lambda + 1 \end{vmatrix}$$

$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -1 & 1 \end{vmatrix} \stackrel{c_2 + c_3}{=} (\lambda + 1) \begin{vmatrix} \lambda - 1 & -4 & -2 \\ -2 & \lambda - 3 & -2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -4 \\ -2 & \lambda - 3 \end{vmatrix}$$

$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$

$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -\lambda - 1 & \lambda + 1 \end{vmatrix}$$

$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -1 & 1 \end{vmatrix} \stackrel{c_2 + c_3}{=} (\lambda + 1) \begin{vmatrix} \lambda - 1 & -4 & -2 \\ -2 & \lambda - 3 & -2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -4 \\ -2 & \lambda - 3 \end{vmatrix}$$

$$= (\lambda + 1)(\lambda^2 - 4\lambda - 5)$$

$$0 = |\lambda I - A| = \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ -2 & -2 & \lambda - 1 \end{vmatrix}$$

$$\frac{r_3 - r_2}{2} \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -\lambda - 1 & \lambda + 1 \end{vmatrix}$$

$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -2 & -2 \\ -2 & \lambda - 1 & -2 \\ 0 & -1 & 1 \end{vmatrix} \stackrel{c_2 + c_3}{=} (\lambda + 1) \begin{vmatrix} \lambda - 1 & -4 & -2 \\ -2 & \lambda - 3 & -2 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (\lambda + 1) \begin{vmatrix} \lambda - 1 & -4 \\ -2 & \lambda - 3 \end{vmatrix}$$

$$= (\lambda + 1)(\lambda^2 - 4\lambda - 5)$$

$$= (\lambda + 1)^2(\lambda - 5)$$

• $\exists \lambda_1 = -1$, \vec{x} \vec{x} \vec{x} \vec{y} \vec{y}

$$(-I - A : 0) =$$



• 当 $\lambda_1 = -1$,求解 $(\lambda_1 I - A)x = 0$:

$$(-I-A:0) = \begin{pmatrix} -2 & -2 & -2 & 0 \\ -2 & -2 & -2 & 0 \\ -2 & -2 & -2 & 0 \end{pmatrix} \rightarrow$$



• 当 $\lambda_1 = -1$,求解 $(\lambda_1 I - A)x = 0$:

$$(-I-A:0) = \begin{pmatrix} -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$



• 当 $\lambda_1 = -1$,求解 $(\lambda_1 I - A)x = 0$:

$$(-I-A:0) = \begin{pmatrix} -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

所以

$$x_1 + x_2 + x_3 = 0$$

→ Back

• $\exists \lambda_1 = -1$, \vec{x} \vec{x} \vec{x} \vec{x} \vec{y} \vec{y}

$$(-I-A:0) = \begin{pmatrix} -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

所以

$$x_1 + x_2 + x_3 = 0 \Rightarrow x_1 = -x_2 - x_3$$

→ Back

• 当 $\lambda_1 = -1$,求解 $(\lambda_1 I - A)x = 0$:

$$(-I-A:0) = \begin{pmatrix} -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

所以

$$x_1 + x_2 + x_3 = 0 \Rightarrow x_1 = -x_2 - x_3$$

基础解系:
$$\alpha_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

→ Back

• 当 $\lambda_1 = -1$,求解 $(\lambda_1 I - A)x = 0$:

$$(-I - A \vdots 0) = \begin{pmatrix} -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

所以

$$x_1 + x_2 + x_3 = 0 \Rightarrow x_1 = -x_2 - x_3$$

基础解系:
$$\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$(-I-A:0) = \begin{pmatrix} -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \\ -2 & -2 & -2 & | & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

所以

$$x_1 + x_2 + x_3 = 0 \Rightarrow x_1 = -x_2 - x_3$$

基础解系:
$$\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

• $\exists \lambda_2 = 5$, \vec{x} \vec{x} \vec{x} \vec{x} \vec{y} \vec

$$(5I - A : 0) =$$

$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & | & 0 \\ -2 & 4 & -2 & | & 0 \\ -2 & -2 & 4 & | & 0 \end{pmatrix}$$

$$(5I-A:0) = \begin{pmatrix} 4 & -2 & -2 & | & 0 \\ -2 & 4 & -2 & | & 0 \\ -2 & -2 & 4 & | & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix}$$

$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & | & 0 \\ -2 & 4 & -2 & | & 0 \\ -2 & -2 & 4 & | & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix}$$

$$r_1 \leftrightarrow r_3$$

$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & 0 \\ -2 & 4 & -2 & 0 \\ -2 & -2 & 4 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -2 & 1 & 0 \\ 2 & -1 & -1 & 0 \end{array} \right)$$

$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & | & 0 \\ -2 & 4 & -2 & | & 0 \\ -2 & -2 & 4 & | & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -2 & 1 & 0 \\ 2 & -1 & -1 & 0 \end{array}\right) \xrightarrow[r_3 - 2r_1]{r_2 - r_1} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -3 & 3 & 0 \end{array}\right)$$

$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & 0 \\ -2 & 4 & -2 & 0 \\ -2 & -2 & 4 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -2 & 1 & 0 \\ 2 & -1 & -1 & 0 \end{array}\right) \xrightarrow[r_3 - 2r_1]{r_2 - r_1} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -3 & 3 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & | & 0 \\ -2 & 4 & -2 & | & 0 \\ -2 & -2 & 4 & | & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -2 & 1 & 0 \\ 2 & -1 & -1 & 0 \end{array}\right) \xrightarrow[r_3 - 2r_1]{r_2 - r_1} \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -3 & 3 & 0 \end{array}\right)$$

$$\longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{r_1 - r_2} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & | & 0 \\ -2 & 4 & -2 & | & 0 \\ -2 & -2 & 4 & | & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 2 & -1 & -1 & | & 0 \end{pmatrix} \xrightarrow{r_2 - r_1} \begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 0 & -3 & 3 & | & 0 \\ 0 & -3 & 3 & | & 0 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \xrightarrow{r_1 - r_2} \begin{pmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

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$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & | & 0 \\ -2 & 4 & -2 & | & 0 \\ -2 & -2 & 4 & | & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 1 & 1 & -2 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 2 & -1 & -1 & | & 0 \end{pmatrix} \xrightarrow{r_2 - r_1} \begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 0 & -3 & 3 & | & 0 \\ 0 & -3 & 3 & | & 0 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \xrightarrow{r_1 - r_2} \begin{pmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$$(x_1, \dots, x_3 = 0)$$

所以
$$\begin{cases} x_1 & -x_3 = 0 \\ x_2 - x_3 = 0 \end{cases}$$

$$f$$
以 $\begin{cases} x_2 - x_3 = 0 \end{cases}$

$$(5I-A:0) = \begin{pmatrix} 4 & -2 & -2 & | & 0 \\ -2 & 4 & -2 & | & 0 \\ -2 & -2 & 4 & | & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 1 & 1 & -2 & | & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 1 & -2 & 1 & | & 0 \\ 2 & -1 & -1 & | & 0 \end{pmatrix} \xrightarrow{r_2 - r_1} \begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 0 & -3 & 3 & | & 0 \\ 0 & -3 & 3 & | & 0 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix} 1 & 1 & -2 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \xrightarrow{r_1 - r_2} \begin{pmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$
所以
$$\begin{cases} x_1 & -x_3 = 0 \\ x_2 - x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = x_3 \\ x_2 = x_3 \end{cases}$$

► Back

$$(5I-A:0) = \begin{pmatrix} 4 & -2 & -2 & 0 \\ -2 & 4 & -2 & 0 \\ -2 & -2 & 4 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 1 & -2 & 1 & 0 \\ 2 & -1 & -1 & 0 \end{pmatrix} \xrightarrow{r_2 - r_1} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -3 & 3 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & -2 & 0 \\ 1 & 1 & -2 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$\longrightarrow \begin{pmatrix}
1 & 1 & -2 & | & 0 \\
0 & 1 & -1 & | & 0 \\
0 & 0 & 0 & | & 0
\end{pmatrix} \xrightarrow{r_1 - r_2} \begin{pmatrix}
1 & 0 & -1 & | & 0 \\
0 & 1 & -1 & | & 0 \\
0 & 0 & 0 & | & 0
\end{pmatrix}$$

$$\int x_1 - x_3 = 0 \qquad \int x_1 = x_3$$

所以
$$\begin{cases} x_1 & -x_3 = 0 \\ x_2 - x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = x_3 \\ x_2 = x_3 \end{cases}$$

基础解系: $\alpha_3 = \begin{pmatrix} \\ \\ 1 \end{pmatrix}$

$$(5I - A \vdots 0) = \begin{pmatrix} 4 & -2 & -2 & 0 \\ -2 & 4 & -2 & 0 \\ -2 & -2 & 4 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 2 & -1 & -1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{pmatrix}$$

$$\xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 1 & -2 & 1 & 0 \\ 2 & -1 & -1 & 0 \end{pmatrix} \xrightarrow{r_2 - r_1} \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & -3 & 3 & 0 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{r_1 - r_2} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

所以 $\begin{cases} x_1 & -x_3 = 0 \\ x_2 - x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = x_3 \\ x_2 = x_3 \end{cases}$

基础解系:
$$\alpha_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

将线性无关组
$$\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ 正交化:



对称矩阵 33/33 □ ▷ △ ▽

将线性无关组
$$\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ 正交化:

$$\beta_1 =$$

$$\beta_2 =$$

将线性无关组
$$\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ 正交化:

$$\beta_1 = \alpha_1$$

$$\beta_2 =$$

将线性无关组
$$\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ 正交化:

$$\beta_1 = \alpha_1$$

$$\beta_2 = \alpha_2 - \dots - \beta_1$$

将线性无关组
$$\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ 正交化:

$$\beta_1 = \alpha_1 = \left(\begin{array}{c} -1\\1\\0\end{array}\right)$$

$$\beta_2 = \alpha_2 - \dots - \beta_1$$

→ Back

将线性无关组
$$\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ 正交化:

$$\beta_1 = \alpha_1 = \left(\begin{array}{c} -1\\1\\0 \end{array}\right)$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - - \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

将线性无关组
$$\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ 正交化:

$$\beta_1 = \alpha_1 = \left(\begin{array}{c} -1\\1\\0 \end{array}\right)$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - \frac{-1}{2} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

将线性无关组
$$\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ 正交化:

$$\beta_1 = \alpha_1 = \left(\begin{array}{c} -1\\1\\0 \end{array}\right)$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

将线性无关组
$$\alpha_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ 正交化:

$$\beta_1 = \alpha_1 = \left(\begin{array}{c} -1\\1\\0\end{array}\right)$$

$$\beta_2 = \alpha_2 - \dots - \beta_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1/2 \\ -1/2 \\ 1 \end{pmatrix}$$