§5.3 换元积分法

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教学要求

◇ 熟练掌握换元积分法: "凑微分", "变量代换"





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Outline of §5.2

1. 第一类换元积分法: 凑微分

第二类换元积分法: 变量代换

We are here now...

1. 第一类换元积分法: 凑微分

第二类换元积分法: 变量代换

第一类换元积分法——"凑微分"法,能干啥?

能够计算如下的不定积分:

$$\int \frac{dx}{2x+1}, \quad \int \cos(\frac{5}{2}x)dx$$

$$\int \frac{x}{\sqrt{3-x^2}}dx, \quad \int x\sin(x^2)dx$$

$$\int \frac{(\ln x)^2}{x}dx, \quad \int e^{\sin x}\cos xdx$$

$$\int \frac{1}{\cos x}dx$$

$$\int "\dot{\mathbf{l}}_{|\dot{\mathbf{z}}|\dot{\mathbf{z}}|\dot{\mathbf{z}}} \mathbf{E} \dot{\mathbf{c}} \dot{\mathbf{E}} \| \, \mathbf{d} \mathbf{x} \|$$

$$\int "\hat{\mathbf{l}}_{i}\dot{\mathbf{z}}\ddot{\mathbf{g}}\mathcal{E}\dot{\mathbf{c}}\dot{\mathbf{E}}"\ d\mathbf{x} = \int f(\varphi(\mathbf{x}))\varphi'(\mathbf{x})d\mathbf{x}$$

$$\int \text{"} \hat{\mathbf{I}}_{i} \dot{\mathbf{z}} \ddot{\mathbf{g}} \mathcal{E} \dot{\mathbf{c}} \dot{\mathbf{E}} \text{"} dx = \int f(\varphi(\mathbf{x})) \varphi'(\mathbf{x}) d\mathbf{x} = \int f(\varphi(\mathbf{x})) d\varphi(\mathbf{x})$$

$$\int \text{"}\tilde{I}_{i}\dot{z}\tilde{g}\tilde{E}\tilde{c}\tilde{E}\text{"}dx = \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$
$$= \frac{\varphi(x)=u}{u} \int f(u)du$$

$$\int \text{"}\hat{I}_{i}\dot{z}\check{g}\mathcal{E}\acute{e}\check{e}\text{"} dx = \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$
$$= \frac{\varphi(x)=u}{m} \int f(u)du$$
$$= F(u) + C$$

$$\int \text{"}\dot{I}_{|\dot{Z}}\ddot{g}\tilde{E}\dot{c}\tilde{E}\text{"} dx = \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$
$$= \frac{\varphi(x)=u}{m} \int f(u)du$$
$$= F(u) + C = \frac{u=\varphi(x)}{m} F(\varphi(x)) + C$$

$$\int \text{"}\tilde{I}_{i}\dot{z}\tilde{g}\tilde{E}\tilde{c}\tilde{E}\text{"} dx \xrightarrow{\frac{\sqrt{2}}{2}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\frac{\varphi(x)=u}{2} \int f(u)du$$

$$= F(u) + C \xrightarrow{u=\varphi(x)} F(\varphi(x)) + C$$

• 计算步骤:

$$\int \text{"}\dot{I}_{i}\dot{z}\ddot{g}\tilde{E}\dot{c}\tilde{E}\text{"} dx \xrightarrow{\underline{k}\otimes d} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\xrightarrow{\underline{\varphi(x)=u}} \int f(u)du$$

$$= F(u) + C \xrightarrow{\underline{u=\varphi(x)}} F(\varphi(x)) + C$$

• 计算步骤:

$$\int \text{"}\dot{I}_{i}\dot{z}\ddot{g}\mathcal{E}\dot{c}\dot{E}\text{"} dx \xrightarrow{\underline{\underline{\xi}\otimes \underline{\mathcal{H}}}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\frac{\underline{\varphi(x)=u}}{\underline{\underline{\mathcal{H}}}} \int f(u)du$$

$$= F(u) + C \xrightarrow{\underline{u=\varphi(x)}} F(\varphi(x)) + C$$

$$\frac{d}{dx}F(\varphi(x)) =$$

• 计算步骤:

$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) =$$

• 计算步骤:

$$\int \text{"}\dot{I}_{i}\dot{z}\ddot{g}\text{E}\dot{c}\dot{E}\text{"} dx \xrightarrow{\underline{\underline{\xi}}\underline{\otimes}\underline{G}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\xrightarrow{\underline{\varphi(x)=u}} \int f(u)du$$

$$= F(u) + C \xrightarrow{\underline{u=\varphi(x)}} F(\varphi(x)) + C$$

$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \cdot \varphi'(x) =$$

• 计算步骤:

$$\int \text{"}\check{I}_{|}\dot{z}\check{g}\check{E}\check{c}\check{E}\text{"} dx \xrightarrow{\underline{\&}\otimes f} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$= \frac{\varphi(x)=u}{} \int f(u)du$$

$$= F(u) + C \xrightarrow{u=\varphi(x)} F(\varphi(x)) + C$$

$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \cdot \varphi'(x) = \text{``l}_{\text{i}} \dot{z} \ddot{g} \cdot \text{E\'e}.$$

• 计算步骤:

$$\int \text{"}\dot{\mathbf{l}}_{\dot{\mathbf{j}}}\dot{\mathbf{z}}\check{\mathbf{g}}\mathcal{E}\dot{\mathbf{c}}\check{\mathbf{E}}\text{"} dx \xrightarrow{\underline{\mathbf{z}}\hat{\mathbf{g}}\hat{\mathbf{f}}} \int f(\varphi(\mathbf{x}))\varphi'(\mathbf{x})d\mathbf{x} = \int f(\varphi(\mathbf{x}))d\varphi(\mathbf{x})$$

$$\frac{\varphi(\mathbf{x})=u}{\underline{\mathbf{g}}}\int f(u)du$$

$$= F(u) + C \xrightarrow{u=\varphi(\mathbf{x})} F(\varphi(\mathbf{x})) + C$$

$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \cdot \varphi'(x) = \text{"}\dot{\mathbf{1}}_{\dot{\mathbf{1}}}\dot{\mathbf{2}}\ddot{\mathbf{g}}\mathcal{E}\dot{\mathbf{c}}\dot{\mathbf{E}}\text{"}$$

总之
$$\int$$
 " $\hat{\mathbf{l}}_{i}\dot{\mathbf{z}}$ ğ \mathbb{E} ć $\hat{\mathbf{E}}$ " $d\mathbf{x} = \frac{\overline{\mathbf{z}}$ $\int f(\boldsymbol{\varphi}(\mathbf{x}))d\boldsymbol{\varphi}(\mathbf{x})$

$$= \int f(u)du = F(u) + C = F(\boldsymbol{\varphi}(\mathbf{x})) + C$$

凑微分 " $\int "\hat{1}_{i}z\check{g}E\hat{c}E" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ "例 I: 热身

例子 求不定积分

$$\int \frac{1}{1+2x} dx, \qquad \int \frac{1}{2-3x} dx, \qquad \int \sqrt{3x-1} dx,$$

$$\int \frac{1}{\sqrt{1-5x}} dx, \qquad \int \cos(\frac{3}{2}x) dx, \qquad \int e^{-\frac{1}{2}x+4} dx.$$

凑微分 " $\int \|\hat{\mathbf{I}}_{izg}\|_{L^2(\dot{\mathbf{E}})} dx = \int f(\varphi(\mathbf{x}))d\varphi(\mathbf{x}) = F(\varphi(\mathbf{x})) + C$ "例 I: 热身

例子 求不定积分

$$\int \frac{1}{1+2x} dx, \qquad \int \frac{1}{2-3x} dx, \qquad \int \sqrt{3x-1} dx,$$

$$\int \frac{1}{\sqrt{1-5x}} dx, \qquad \int \cos(\frac{3}{2}x) dx, \qquad \int e^{-\frac{1}{2}x+4} dx.$$

$$dx = d(2x + 1), dx = d(2 - 3x),$$

$$dx = d(\frac{3}{2}x),$$
 $dx = \underline{\qquad} d(-\frac{1}{2}x + 4).$

凑微分 " $\int \|\hat{\mathbf{I}}_{i}\|_{L^2(X)} dx = \int f(\varphi(\mathbf{x})) d\varphi(\mathbf{x}) = F(\varphi(\mathbf{x})) + C$ " 例 I: 热身

$$\int \frac{1}{1+2x} dx, \qquad \int \frac{1}{2-3x} dx, \qquad \int \sqrt{3x-1} dx,$$

$$\int \frac{1}{\sqrt{1-5x}} dx, \qquad \int \cos(\frac{3}{2}x) dx, \qquad \int e^{-\frac{1}{2}x+4} dx.$$

$$dx = \frac{1}{2} d(2x+1), \qquad dx = d(2-3x),$$

$$dx = d(\frac{3}{2}x),$$
 $dx = \underline{\qquad} d(-\frac{1}{2}x + 4).$



凑微分 " $\int \|\hat{\mathbf{I}}_{i}\|_{L^2(X)} dx = \int f(\varphi(\mathbf{x})) d\varphi(\mathbf{x}) = F(\varphi(\mathbf{x})) + C$ " 例 I: 热身

$$\int \frac{1}{1+2x} dx, \qquad \int \frac{1}{2-3x} dx, \qquad \int \sqrt{3x-1} dx,$$

$$\int \frac{1}{\sqrt{1-5x}} dx, \qquad \int \cos(\frac{3}{2}x) dx, \qquad \int e^{-\frac{1}{2}x+4} dx.$$

$$dx = \frac{1}{2} d(2x+1),$$
 $dx = -\frac{1}{3} d(2-3x),$

$$dx = d(\frac{3}{2}x),$$
 $dx = \underline{\qquad} d(-\frac{1}{2}x + 4).$



凑微分 " $\int "\hat{1}_{i}\dot{z}\check{g}Ed\hat{\mathbf{E}}$ " $d\mathbf{x} = \int f(\mathbf{\varphi}(\mathbf{x}))d\mathbf{\varphi}(\mathbf{x}) = F(\mathbf{\varphi}(\mathbf{x})) + C$ " 例 I: 热身

$$\int \frac{1}{1+2x} dx, \qquad \int \frac{1}{2-3x} dx, \qquad \int \sqrt{3x-1} dx,$$

$$\int \frac{1}{\sqrt{1-5x}} dx, \qquad \int \cos(\frac{3}{2}x) dx, \qquad \int e^{-\frac{1}{2}x+4} dx.$$

$$dx = \frac{1}{2} d(2x+1),$$
 $dx = \frac{1}{3} d(2-3x),$

$$dx = \frac{2}{3} d(\frac{3}{2}x),$$
 $dx = \underline{\qquad} d(-\frac{1}{2}x + 4).$



凑微分 " $\int "\hat{1}_{i}\dot{z}\check{g}Ed\hat{\mathbf{E}}$ " $d\mathbf{x} = \int f(\mathbf{\varphi}(\mathbf{x}))d\mathbf{\varphi}(\mathbf{x}) = F(\mathbf{\varphi}(\mathbf{x})) + C$ " 例 I: 热身

$$\int \frac{1}{1+2x} dx, \qquad \int \frac{1}{2-3x} dx, \qquad \int \sqrt{3x-1} dx,$$

$$\int \frac{1}{\sqrt{1-5x}} dx, \qquad \int \cos(\frac{3}{2}x) dx, \qquad \int e^{-\frac{1}{2}x+4} dx.$$

$$dx = \frac{1}{2} d(2x+1),$$
 $dx = \frac{1}{3} d(2-3x),$

$$dx = \frac{2}{3} d(\frac{3}{2}x),$$
 $dx = \underline{-2} d(-\frac{1}{2}x + 4).$

凑微分 " $\int "\hat{I}_{i}z\tilde{g}E\hat{c}E" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分 $\int \frac{1}{1+2x} dx$, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

凑微分 " $["\hat{1}_{izg} \mathbb{E} \hat{c} \mathbb{E} " dx =] f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\mathbf{m} \int \frac{1}{1+2x} dx =$$

凑微分 " $["\hat{1}_{izg} \mathbb{E} \hat{c}] " dx = [f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C " 例 I]$

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\widehat{\mathbf{H}} \int \frac{1}{1+2x} dx = \int \frac{1}{1+2x}$$

凑微分 " $["\hat{1}_{izg} \mathbb{E} \hat{c} \mathbb{E} " dx =] f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\Re \int \frac{1}{1+2x} dx = \int \frac{1}{1$$



凑微分 " $["\hat{1}_{izg} \mathbb{E} \hat{c}] " dx = [f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C " 例 I]$

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\Re \int \frac{1}{1+2x} dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x)$$



凑微分 " $["\hat{1}_{izg} \mathbb{E} \hat{c}] " dx = [f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C " 例 I]$

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\frac{\mathbf{f}}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2}d(1+2x) = \frac{1}{2} \int \frac{1}{u}du$$



凑微分 " $["\hat{1}_{izg} \mathbb{E} \hat{c} \mathbb{E} " dx =] f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

凑微分 " $["\hat{1}_{izg} \mathbb{E} \hat{c} \mathbb{E} " dx =] f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\frac{\mathbf{f}}{\int \frac{1}{1+2x} dx} = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x) = \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1+2x| + C$$

凑微分 " $\int "\hat{1}_{|\hat{z}|} Z \tilde{z} E \hat{c} \tilde{c} dx = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\frac{1}{1+2x}dx = \int \frac{1}{1+2x}dx, \quad \int \frac{1}{2-3x}dx, \quad \int \sqrt{3x-1}dx$$

$$\frac{1}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2}d(1+2x) = \frac{1}{2} \int \frac{1}{u}du$$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1+2x| + C$$

$$\int \frac{1}{2-3x}dx = \frac{1}{2} \ln|x| + C$$

$$\int \sqrt{3x - 1} dx =$$



凑微分 " $\int "\hat{1}_{|\hat{z}|} Z \tilde{z} E \tilde{c} \tilde{c} dx = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\mathbf{H} \int \frac{1}{1+2x} dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x) = \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1+2x| + C$$

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} dx$$

$$\int \sqrt{3x - 1} dx =$$



凑微分 " $\int "\hat{1}_{|\hat{z}|} Z \tilde{z} E \tilde{c} \tilde{c} dx = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 I

$$\int \sqrt{3x - 1} dx =$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\int \sqrt{3x - 1} dx =$$



凑微分 " $\int "\hat{1}_{|\hat{z}|} Z \tilde{z} E \tilde{c} \tilde{c} dx = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$
解 $\int \frac{1}{1+2x} dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x) = \frac{1}{2} \int \frac{1}{u} du$
 $= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1+2x| + C$
 $\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$

$$\int \sqrt{3x - 1} dx =$$



凑微分 " $\int "\hat{1}_{|\hat{z}|} Z \tilde{z} E \tilde{c} \tilde{c} dx = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\frac{1}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2}d(1+2x) = \frac{1}{2} \int \frac{1}{u}du$$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1+2x| + C$$

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3})d(2-3x) = -\frac{1}{3} \int \frac{1}{u}du$$

$$= -\frac{1}{3} \ln|u| + C$$

$$\int \sqrt{3x - 1} dx =$$



凑微分 " $\int "\hat{1}_{|\hat{z}||\hat{z}||E(\hat{E})|} dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\int \sqrt{3x-1}dx =$$



凑微分 " $\int "\hat{1}_{|\hat{z}|} Z \tilde{z} E \tilde{c} \tilde{c} dx = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\int \sqrt{3x - 1} dx = \int \sqrt{3x - 1}$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$d(3x-1)$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

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例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\frac{1}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2}d(1+2x) = \frac{1}{2} \int \frac{1}{u}du$$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1+2x| + C$$

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3})d(2-3x) = -\frac{1}{3} \int \frac{1}{u}du$$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

$$\int \sqrt{3x-1} dx = \int \sqrt{3x-1} \cdot \frac{1}{3} d(3x-1) = \frac{1}{3} \int \sqrt{u} du$$



凑微分 "[" \hat{I}_{izg} $\mathbb{E}(\hat{E})$ " $dx = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2 - 3x| + C$$

$$\int \sqrt{3x - 1} dx = \int \sqrt{3x - 1} \cdot \frac{1}{3}d(3x - 1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1 + 2x| + C$$

$$\int \frac{1}{2 - 3x} dx = \int \frac{1}{2 - 3x} \cdot (-\frac{1}{3}) d(2 - 3x) = -\frac{1}{3} \int \frac{1}{u} du$$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2 - 3x| + C$$

 $\int \sqrt{3x - 1} dx = \int \sqrt{3x - 1} \cdot \frac{1}{3} d(3x - 1) = \frac{1}{3} \int \sqrt{u} du = \frac{1}{3} \int u^{1/2} du$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\Re \int \frac{1}{1+2x} dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x) = \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1 + 2x| + C$$

$$\int \frac{1}{2 - 3x} dx = \int \frac{1}{2 - 3x} \cdot (-\frac{1}{3}) d(2 - 3x) = -\frac{1}{3} \int \frac{1}{u} du$$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2 - 3x| + C$$

 $\frac{2}{3}u^{3/2}$

 $= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2 - 3x| + C$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2 - 3x| + C$$

$$\int \sqrt{3x - 1} dx = \int \sqrt{3x - 1} \cdot \frac{1}{3} d(3x - 1) = \frac{1}{3} \int \sqrt{u} du = \frac{1}{3} \int u^{1/2} du$$

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\Re \int \frac{1}{1+2x} dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x) = \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1 + 2x| + C$$

$$\int \frac{1}{2 - 3x} dx = \int \frac{1}{2 - 3x} \cdot (-\frac{1}{3}) d(2 - 3x) = -\frac{1}{3} \int \frac{1}{u} du$$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2 - 3x| + C$$

 $\int \sqrt{3x - 1} dx = \int \sqrt{3x - 1} \cdot \frac{1}{3} d(3x - 1) = \frac{1}{3} \int \sqrt{u} du = \frac{1}{3} \int u^{1/2} du$

 $=\frac{1}{3}\cdot\frac{2}{3}u^{3/2}+C$

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} dx$$

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\mathbf{H} \int \frac{1}{1+2x} dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x) = \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1 + 2x| + C$$

$$\int \frac{1}{2 - 3x} dx = \int \frac{1}{2 - 3x} \cdot (-\frac{1}{3}) d(2 - 3x) = -\frac{1}{3} \int \frac{1}{u} du$$

$$= -\frac{1}{2} \ln|u| + C = -\frac{1}{2} \ln|2 - 3x| + C$$

 $= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2 - 3x| + C$

 $\int \sqrt{3x - 1} dx = \int \sqrt{3x - 1} \cdot \frac{1}{3} d(3x - 1) = \frac{1}{3} \int \sqrt{u} du = \frac{1}{3} \int u^{1/2} du$ $= \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C = \frac{2}{9} (3x - 1)^{3/2} + C$

凑微分 " $\int \|\hat{I}_{izg} E(\hat{E})\| dx = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}} dx$, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

凑微分 " $\int "\hat{1}_{|\hat{z}|} Z d\hat{z} d\hat{z} d\hat{z} = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\int_{0}^{\frac{\pi}{2}} \frac{1}{\sqrt{1-5x}} dx =$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



凑微分 " $\int \|\hat{\mathbf{I}}_{|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat$

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$$\int \frac{1}{\sqrt{1-5x}} dx$$
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凑微分 " $\int \|\hat{\mathbf{I}}_{|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat$

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$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\int_{1}^{\infty} \frac{1}{\sqrt{1-5x}} dx = \int_{1}^{\infty} (1-5x)^{-1/2} \qquad d(1-5x)$$

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例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\int_{1}^{\infty} \frac{1}{\sqrt{1-5x}} dx = \int_{1}^{\infty} (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



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$$= u^{1/2}$$

$$\int \cos(\frac{3}{2}x)dx =$$

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$$= 2u^{1/2}$$

$$\int \cos(\frac{3}{2}x)dx =$$

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$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1 - 5x)^{1/2} + C$$

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$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x$$

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$$\int \frac{1}{\sqrt{1-5x}} dx$$
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$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1 - 5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \quad d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4} dx =$$



例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$\int e^{-\frac{1}{2}x+4} dx =$$



凑微分 " $["\hat{1}_{izg} \times \hat{C} : dx =] f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C " 例 I$

例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\iint_{\sqrt{1-5x}} \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(-x) dx = \int \cos -x \cdot -d(-x) = -\int \cos u du$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$
$$= \frac{2}{3}\sin(u) + C$$

$$\int e^{-\frac{1}{2}x+4} dx =$$



例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\iint_{0}^{\infty} \frac{1}{\sqrt{1-5x}} dx = \int_{0}^{\infty} (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int_{0}^{\infty} u^{-1/2} du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int_{0}^{\infty} a^{3} dx = \int_{0}^{\infty} a^{3} dx = \int_{0}^{\infty}$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$
$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4} dx =$$



例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$
$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4} dx = \int e^{-\frac{1}{2}x+4}$$



凑微分 " $\int "\tilde{1}_{i}z\tilde{g}Ed\tilde{E}" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$
$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

 $\int e^{-\frac{1}{2}x+4} dx = \int e^{-\frac{1}{2}x+4} \qquad d(-\frac{1}{2}x+4)$



凑微分 " $\int "\tilde{1}_{i}z\tilde{g}Ed\tilde{E}" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$= -\frac{1}{5} \cdot 2u^{3/2} + C = -\frac{1}{5}(1 - 5x)^{3/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4} dx = \int e^{-\frac{1}{2}x+4} \cdot (-2) d(-\frac{1}{2}x+4)$$



凑微分 " $\int "\tilde{1}_{i}z\tilde{g}Ed\tilde{E}" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ "例 I

例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{1}{5}(1 - 5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

 $= \frac{1}{3}\sin(u) + C = \frac{1}{3}\sin(\frac{1}{2}x) + C$ $\int e^{-\frac{1}{2}x+4} dx = \int e^{-\frac{1}{2}x+4} \cdot (-2)d(-\frac{1}{2}x+4) = -2\int e^{u} du$

凑微分 " $\int "\dot{\mathbf{1}}_{\dot{\mathbf{1}}\dot{\mathbf{2}}\dot{\mathbf{g}}}\mathbf{E}\dot{\mathbf{c}}\dot{\mathbf{E}}$ " $d\mathbf{x} = \int f(\mathbf{\varphi}(\mathbf{x}))d\mathbf{\varphi}(\mathbf{x}) = F(\mathbf{\varphi}(\mathbf{x})) + C$ " 例 \mathbf{I}

例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$= -\frac{1}{5} \cdot 2u^{3/2} + C = -\frac{1}{5}(1 - 5x)^{3/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

 $\int e^{-\frac{1}{2}x+4} dx = \int e^{-\frac{1}{2}x+4} \cdot (-2) d(-\frac{1}{2}x+4) = -2 \int e^{u} du$

凑微分 " $\int "\tilde{1}_{i}z\tilde{g}Ed\tilde{E}" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ "例 I

例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1 - 5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

 $= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$

 $\int e^{-\frac{1}{2}x+4} dx = \int e^{-\frac{1}{2}x+4} \cdot (-2) d(-\frac{1}{2}x+4) = -2 \int e^{u} du$ $= -2e^{u} + C = -2e^{-\frac{1}{2}x+4} + C$

例子 求不定积分

$$(1) \int xe^{x^2}dx,$$

$$\int x \sin(x^2) dx,$$

$$(2) \quad \int x\sqrt{1-x^2}dx,$$

(2)
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

凑微分" \int " $\hat{I}_{iz\check{g}}$ $\mathbb{E}(\hat{E})$ " $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ "例 II: 热身

例子 求不定积分

(1)
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$,

(2)
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

热身

$$xdx = \underline{\qquad} d(x^2) \qquad \qquad xdx = \underline{\qquad} d(1-x^2)$$

$$xdx = d(3-x^2) \quad xdx = d(1+3x^2)$$

凑微分" \int " $\hat{I}_{iz\bar{g}}$ $\mathbb{E}(\hat{E})$ " $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ "例 II: 热身

例子 求不定积分

(1)
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$,

(2)
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$xdx = \frac{1}{2} d(x^2) \qquad xdx = \underline{\qquad} d(1-x^2)$$

$$xdx = d(3-x^2) \quad xdx = d(1+3x^2)$$

凑微分" $\int "\hat{\mathbf{l}}_{iz\check{\mathbf{g}}}\mathbb{E}\hat{\mathbf{c}}\hat{\mathbf{c}}" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ "例 II:热身

例子 求不定积分

$$(1) \quad \int x e^{x^2} dx,$$

(2)
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

 $\int x \sin(x^2) dx$,

$$xdx = \frac{1}{2} d(x^2)$$
 $xdx = \frac{1}{2} d(1-x^2)$

$$xdx = d(3-x^2) \quad xdx = d(1+3x^2)$$

凑微分" $\int "\hat{\mathbf{l}}_{iz\check{\mathbf{g}}}\mathbb{E}\hat{\mathbf{c}}\hat{\mathbf{c}}" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ "例 II:热身

例子 求不定积分

$$(1) \quad \int x e^{x^2} dx,$$

$$\int x \sin(x^2) dx,$$

$$(2) \quad \int x\sqrt{1-x^2}dx,$$

$$\int \frac{x}{\sqrt{3-x^2}} dx, \qquad \int \frac{x}{1+3x^2} dx$$

$$xdx = \frac{1}{2} d(x^2)$$

$$xdx = -\frac{1}{2} d(1-x^2)$$

$$xdx = -\frac{1}{2} d(3-x^2) \quad xdx = d(1+3x^2)$$

例子 求不定积分

$$(1) \quad \int x e^{x^2} dx,$$

$$\int x \sin(x^2) dx,$$

$$(2) \quad \int x\sqrt{1-x^2}dx,$$

$$\int \frac{x}{\sqrt{3-x^2}} dx, \qquad \int \frac{x}{1+3x^2} dx$$

$$xdx = \frac{1}{2} d(x^2)$$

$$xdx = -\frac{1}{2} d(1-x^2)$$

$$xdx = -\frac{1}{2} d(3-x^2)$$
 $xdx = \frac{1}{6} d(1+3x^2)$

例子 求 $\int xe^{x^2}dx$, $\int x\sin(x^2)dx$

例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int x e^{x^2} dx =$$

例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2}dx = \int e^{x^2}$$

例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int x e^{x^2} dx = \int e^{x^2} d(x^2)$$

例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int x e^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2)$$

例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int x e^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du$$

例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^udu = \frac{1}{2}e^u + C$$



例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^udu = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$



凑微分 " $\int \|\hat{I}_{izg} E(\hat{E})\| dx = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 III

例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2}dx = \int e^{x^2} \frac{1}{2}d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$x \sin(x^2)dx = \int e^{x^2} \frac{1}{2}d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x \sin(x^2) dx =$$



例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\iint xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

$$\int x \sin(x^2) dx = \int \sin(x^2)$$

凑微分 " $\int \|\hat{I}_{izg} E(\hat{E})\| dx = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 III

例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2}dx = \int e^{x^2} \frac{1}{2}d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2) dx = \int \cos(x^2) dx =$$



例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2}dx = \int e^{x^2} \frac{1}{2}d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2) \cdot \frac{1}{2}d(x^2)$$



例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

$$\int x \sin(x^2) dx = \int \sin(x^2) \cdot \frac{1}{2} d(x^2) = \frac{1}{2} \int \sin u du$$



例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

$$\int x \sin(x^2) dx = \int \sin(x^2) \cdot \frac{1}{2} d(x^2) = \frac{1}{2} \int \sin u du$$

$$= -\frac{1}{2} \cos u + C$$

例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

$$\int x \sin(x^2) dx = \int \sin(x^2) \cdot \frac{1}{2} d(x^2) = \frac{1}{2} \int \sin u du$$

$$= -\frac{1}{2} \cos u + C = -\frac{1}{2} \cos(x^2) + C$$

$$os(x^2) + C$$



例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$ 解 $\int x\sqrt{1-x^2}dx =$

例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$ 解
$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}$$

例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$ 解 $\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}$ $d(1-x^2)$

例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2)$$

例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$u^{3/2}$$

例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$\frac{2}{3}u^{3/2}$$

凑微分 " $\int "\hat{1}_{|\hat{z}|} Z d\hat{z} d\hat{x} = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 IV

例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C$$

凑微分 " $\int "\hat{1}_{|\hat{z}|} Z d\hat{z} d\hat{x} = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ " 例 IV

例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

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$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

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$$\int x\sqrt{1-x^2}dx$$
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$$\lim_{x \to \infty} \int x \sqrt{1 - x^2} dx, \quad \int \frac{1}{1 + 3x^2} dx$$

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凑微分 "ʃ"Ì¡żğÆćÈ" dx=ʃf(φ(x))dφ(x)=F(φ(x))+C" 例 IV

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凑微分 " $\int \|\hat{\mathbf{I}}_{|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat$

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$$= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3 - x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1 + 3x^2} dx = \int \frac{1}{1 + 3x^2} \cdot \frac{1}{6} d(1 + 3x^2) = \frac{1}{6} \int \frac{1}{u} du$$



凑微分 " $\int \|\hat{\mathbf{I}}_{|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat{\mathbf{I}}|\hat$

例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\iint_{0}^{\infty} x\sqrt{1-x^{2}} dx = \int_{0}^{\infty} (1-x^{2})^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^{2}) = -\frac{1}{2} \int_{0}^{\infty} u^{\frac{1}{2}} du$$

$$x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2} \int u^{\frac{1}{2}}du$$

$$= \int \frac{1}{2}u^{\frac{3}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2} \int u^{\frac{1}{2}}du$$

$$\int_{0}^{\pi} \frac{2}{1-x^{2}} \frac{2}{1-x^{2}} \frac{2}{1-x^{2}} \frac{1}{1-x^{2}} \frac{1}$$

 $= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3 - x^2)^{\frac{1}{2}} + C$

$$= -\frac{1}{2} \cdot \frac{2}{2} u^{3/2} + C = -\frac{1}{2} (1 - x^2)^{\frac{3}{2}} + C$$

§5.3 换元积分法

 $= -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C = -\frac{1}{3} (1 - x^2)^{\frac{3}{2}} + C$

 $\int \frac{x}{1+3x^2} dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6} d(1+3x^2) = \frac{1}{6} \int \frac{1}{u} du$

 $= \frac{1}{6} \ln |u| + C$

凑微分 " $\int "\hat{1}_{i}\dot{z}\check{g}E\hat{c}E" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 IV 例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\frac{1}{\sqrt{3-x^2}}dx, \quad \int \frac{1}{1+3x^2}dx$$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2} \int u^{\frac{1}{2}}du$$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2} \int u^{\frac{1}{2}}du$$

$$\int \frac{1}{2} \frac{2}{3} dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2} \int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C = -\frac{1}{3} (1 - x^2)^{\frac{3}{2}} + C$$

$$= -\frac{1}{2} \cdot \frac{1}{3} u^{3/2} + C = -\frac{1}{3} (1 - x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{2 - x^2}} dx = \int (3 - x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3 - x^2)$$

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

 $= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3 - x^2)^{\frac{1}{2}} + C$

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

 $= \frac{1}{6} \ln|u| + C = \frac{1}{6} \ln|1 + 3x^2| + C$

 $\int \frac{x}{1+3x^2} dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6} d(1+3x^2) = \frac{1}{6} \int \frac{1}{u} du$

$$= -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C = -\frac{1}{3} (1 - x^2)^{\frac{3}{2}} + C$$

凑微分 " $\int "\tilde{I}_{i}\tilde{z}\tilde{g}\mathbb{E}d\tilde{c}$ " $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 V

例子 求 $\int e^x \sin(e^x) dx$, $\int \frac{e^x}{1+e^x} dx$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^{x}dx = de^{x}$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^x dx = de^x$$

$$\iint e^x \sin(e^x) dx =$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^{x}dx = de^{x}$$

$$\int e^x \sin(e^x) dx = \int \sin(e^x)$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^{x}dx = de^{x}$$

$$\int e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x}$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^{x}dx = de^{x}$$

$$\iint e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x}$$
$$= \int \sin u du$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^{x}dx = de^{x}$$

$$\int e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x}$$

$$= \int \sin u du = -\cos u + C$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^{x}dx = de^{x}$$

$$\iint e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x}$$
$$= \int \sin u du = -\cos u + C = -\cos(e^{x}) + C$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^x dx = de^x$$

$$\iint e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x}$$

$$= \int \sin u du = -\cos u + C = -\cos(e^{x}) + C$$

$$\int \frac{e^{x}}{1 + e^{x}} dx$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^x dx = de^x$$

$$\iint e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x}$$

$$= \int \sin u du = -\cos u + C = -\cos(e^{x}) + C$$

$$\int \frac{e^{x}}{1 + e^{x}} dx = \int \frac{1}{1 + e^{x}} de^{x}$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^x dx = de^x$$

$$\iint e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x}$$

$$= \int \sin u du = -\cos u + C = -\cos(e^{x}) + C$$

$$\int \frac{e^{x}}{1 + e^{x}} dx = \int \frac{1}{1 + e^{x}} de^{x} = \int \frac{1}{1 + e^{x}} d(e^{x} + 1)$$

凑微分 "[" \hat{I}_{izg} ÆćÈ" $dx=[f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$ "例 V

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^{x}dx = de^{x}$$

$$\iint e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x}$$

$$= \int \sin u du = -\cos u + C = -\cos(e^{x}) + C$$

$$\int \frac{e^{x}}{1 + e^{x}} dx = \int \frac{1}{1 + e^{x}} de^{x} = \int \frac{1}{1 + e^{x}} d(e^{x} + 1)$$

$$= \int \frac{1}{u} du$$

凑微分 "[" \hat{I}_{izg} ÆćÈ" $dx=[f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$ "例 V

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^{x}dx = de^{x}$$

$$\iint e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x}$$

$$= \int \sin u du = -\cos u + C = -\cos(e^{x}) + C$$

$$\int \frac{e^{x}}{1 + e^{x}} dx = \int \frac{1}{1 + e^{x}} de^{x} = \int \frac{1}{1 + e^{x}} d(e^{x} + 1)$$

$$= \int \frac{1}{u} du = \ln|u| + C$$



凑微分 "[" \hat{I}_{izg} ÆćÈ" $dx=[f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$ "例 V

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^{x}dx = de^{x}$$

$$\iint e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x}$$
$$= \int \sin u du = -$$

$$= \int \sin u du = -\cos u + C = -\cos(e^{x}) + C$$

$$\int \frac{e^{x}}{1 + e^{x}} dx = \int \frac{1}{1 + e^{x}} de^{x} = \int \frac{1}{1 + e^{x}} d(e^{x} + 1)$$

$$= \int \frac{1}{u} du = \ln|u| + C = \ln(e^{x} + 1) + C$$



凑微分 " $\int "\hat{1}_{i}z\tilde{g}E\hat{c}E" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 VI

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx =$$



例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$
$$= \frac{1}{2} u^2 + C$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$
$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$
$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$
$$\int \frac{1}{x \ln x} dx =$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$
$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$
$$\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d \ln x$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$
$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$
$$\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d \ln x = \int \frac{1}{u} du$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$

$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$

$$\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d \ln x = \int \frac{1}{u} du$$

$$= \ln |u| + C$$



例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$

$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$

$$\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d \ln x = \int \frac{1}{u} du$$

$$= \ln|u| + C = \ln|\ln x| + C$$



凑微分 " $\int "\hat{\mathbf{I}}_{i}\dot{\mathbf{z}}\tilde{\mathbf{g}}\mathbb{H}\hat{\mathbf{c}}\hat{\mathbf{E}}$ " $d\mathbf{x} = \int f(\boldsymbol{\varphi}(\mathbf{x}))d\boldsymbol{\varphi}(\mathbf{x}) = F(\boldsymbol{\varphi}(\mathbf{x})) + C$ " 例 VII

例子 求 $\int e^{\cos x} \sin x dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

凑微分 " $\int "\hat{\mathbf{I}}_{i}\dot{\mathbf{z}}\tilde{\mathbf{g}}\mathbb{H}\hat{\mathbf{c}}\hat{\mathbf{E}}$ " $d\mathbf{x} = \int f(\boldsymbol{\varphi}(\mathbf{x}))d\boldsymbol{\varphi}(\mathbf{x}) = F(\boldsymbol{\varphi}(\mathbf{x})) + C$ " 例 VII

例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$
热身 $\sin x dx = d$, $\cos x dx = d$

凑微分 "ʃ" $\dot{l}_{\dot{l}\dot{z}\ddot{g}}$ $\mathbb{E}\dot{c}\dot{E}$ " $d\mathbf{x} = \int f(\boldsymbol{\varphi}(\mathbf{x}))d\boldsymbol{\varphi}(\mathbf{x}) = F(\boldsymbol{\varphi}(\mathbf{x})) + C$ " 例 VII

例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

热身
$$\sin x dx = d - \cos x$$
 , $\cos x dx = d$

凑微分 " $\int "\hat{\mathbf{I}}_{\dot{\mathbf{I}}\dot{\mathbf{Z}}\ddot{\mathbf{S}}}\mathbb{E}(\hat{\mathbf{E}}) d\mathbf{x} = \int f(\varphi(\mathbf{x}))d\varphi(\mathbf{x}) = F(\varphi(\mathbf{x})) + C$ " 例 VII

例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

热身
$$\sin x dx = d - \cos x = -d \cos x$$
, $\cos x dx = d$ _____

凑微分 " $\int "\hat{\mathbf{I}}_{|\hat{\mathbf{Z}}|\mathbf{Z}} \mathbb{E}(\hat{\mathbf{E}}) d\mathbf{x} = \int f(\boldsymbol{\varphi}(\mathbf{x})) d\boldsymbol{\varphi}(\mathbf{x}) = F(\boldsymbol{\varphi}(\mathbf{x})) + C$ " 例 VII

例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

热身 $\sin x dx = d - \cos x = -d \cos x$, $\cos x dx = d \sin x$

例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$
热身 $\sin x dx = d - \cos x = -d \cos x$, $\cos x dx = d \sin x$
解
$$\int e^{\cos x} \sin x dx =$$

例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$
热身 $\sin x dx = d - \cos x = -d \cos x$, $\cos x dx = d \sin x$
解
$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x$$

例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$
热身 $\sin x dx = d - \cos x = -d \cos x$, $\cos x dx = d \sin x$
解
$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^u du$$

例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$
热身 $\sin x dx = d - \cos x = -d \cos x$, $\cos x dx = d \sin x$
解
$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^u du$$
$$= -e^u + C$$

凑微分 " $\int "\hat{\mathbf{I}}_{i}\dot{\mathbf{Z}}\mathbf{Z}\mathbf{E}\hat{\mathbf{E}}$ " $d\mathbf{x} = \int f(\boldsymbol{\varphi}(\mathbf{x}))d\boldsymbol{\varphi}(\mathbf{x}) = F(\boldsymbol{\varphi}(\mathbf{x})) + C$ " 例 VII

例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$
热身 $\sin x dx = d - \cos x = -d \cos x$, $\cos x dx = d \sin x$
解
$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^u du$$

$$= -e^u + C = -e^{\cos x} + C$$

例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$
热身 $\sin x dx = d - \cos x = -d \cos x$, $\cos x dx = d \sin x$
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$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d \cos x$$

$$\int \frac{\cos x}{\sin x} dx =$$

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$$\int e^{\cos x} \sin x dx$$
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$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d\cos x = -\int \frac{1}{1 + u^2} du$$
$$= -\arctan u + C$$

$$\int \frac{\cos x}{\sin x} dx =$$



凑微分 " $\int "\hat{\mathbf{I}}_{i}\dot{\mathbf{Z}}\check{\mathbf{Z}}\mathcal{E}\dot{\mathbf{E}}$ " $d\mathbf{x} = \int f(\varphi(\mathbf{x}))d\varphi(\mathbf{x}) = F(\varphi(\mathbf{x})) + C$ " 例 VII

例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$
热身 $\sin x dx = d - \cos x = -d \cos x$, $\cos x dx = d \sin x$

$$\begin{aligned}
&\text{If } e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^u du \\
&= -e^u + C = -e^{\cos x} + C \\
&\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d \cos x = -\int \frac{1}{1 + u^2} du \\
&= -\arctan u + C = -\arctan(\cos x) + C
\end{aligned}$$

$$\int \frac{\cos x}{\sin x} dx = \frac{\cos x}{\sin x} dx = \frac{\cos x}{\sin x} dx = \frac{1}{1 + \cos^2 x} (-1) d \cos x = -\int \frac{1}{1 + u^2} du = -\arctan(\cos x) + C$$

凑微分 " $\int "\hat{\mathbf{I}}_{i}\dot{\mathbf{Z}}\mathbf{Z}\mathbf{E}\hat{\mathbf{E}}$ " $d\mathbf{x} = \int f(\boldsymbol{\varphi}(\mathbf{x}))d\boldsymbol{\varphi}(\mathbf{x}) = F(\boldsymbol{\varphi}(\mathbf{x})) + C$ " 例 VII

例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$
热身 $\sin x dx = d - \cos x = -d \cos x$, $\cos x dx = d \sin x$
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$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^u du$$

$$= -e^{u} + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1 + \cos^{2} x} dx = \int \frac{1}{1 + \cos^{2} x} (-1) d\cos x = -\int \frac{1}{1 + u^{2}} du$$

$$\int 1 + \cos^2 x dx = \int 1 + \cos^2 x dx = \int 1 + u^2$$
$$= -\arctan u + C = -\arctan(\cos x) + C$$

$$\int \frac{\cos x}{\sin x} dx = \int \frac{1}{\sin x} d\sin x$$



凑微分 " $\int "\hat{\mathbf{I}}_{i}\hat{\mathbf{Z}}_{i}^{\mathbf{Z}}\mathbf{E}\hat{\mathbf{C}}\mathbf{E}$ " $d\mathbf{x} = \int f(\varphi(\mathbf{x}))d\varphi(\mathbf{x}) = F(\varphi(\mathbf{x})) + C$ " 例 VII

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$$\int e^{\cos x} \sin x dx$$
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$$\int \frac{\cos x}{\sin x} dx = \int \frac{1}{\sin x} d\sin x = \int \frac{1}{u} du$$



凑微分 " $\int "\hat{\mathbf{I}}_{i}\hat{\mathbf{Z}}_{i}^{\mathbf{Z}}\mathbf{E}\hat{\mathbf{C}}\mathbf{E}$ " $d\mathbf{x} = \int f(\varphi(\mathbf{x}))d\varphi(\mathbf{x}) = F(\varphi(\mathbf{x})) + C$ " 例 VII

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$$\int \frac{\cos x}{\sin x} dx = \int \frac{1}{\sin x} d\sin x = \int \frac{1}{u} du = \ln|u| + C$$



凑微分 " $\int "\hat{I}_{i}\dot{z}\tilde{g}E\hat{c}\tilde{c}$ " $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 VII

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$$\int \frac{\cos x}{\sin x} dx = \int \frac{1}{\sin x} d\sin x = \int \frac{1}{u} du = \ln|u| + C$$

 $= \ln|\sin x| + C$

凑微分法 " $\int \|\hat{\mathbf{i}}_{|\hat{\mathbf{z}}|\mathbf{x}} \mathbb{E}(\hat{\mathbf{c}})\| d\mathbf{x} = \int f(\mathbf{\phi}(\mathbf{x})) d\mathbf{\phi}(\mathbf{x})$ ":例子总结

$$\int \frac{1}{1 - 3x} dx =$$

$$\int \sqrt{3x - 1} dx =$$

$$\int xe^{x^2} dx =$$

$$\int x\sqrt{1 - x^2} dx =$$

$$\int \frac{\ln x}{x} dx =$$

$$\int e^{\cos x} \sin x dx =$$

凑微分法 "ʃ" l_{izg} $Ec\dot{E}$ " $dx = \int f(\varphi(x))d\varphi(x)$ ": 例子总结

$$\int \frac{1}{1-3x} dx = -\frac{1}{3} \int \frac{1}{1-3x} d(1-3x) = -\frac{1}{3} \int \frac{1}{u} du = \cdots$$

$$\int \sqrt{3x-1} dx = \frac{1}{3} \int \sqrt{3x-1} d(3x-1) = \frac{1}{3} \int u^{1/2} du = \cdots$$

$$\int xe^{x^2} dx =$$

$$\int x\sqrt{1-x^2} dx =$$

$$\int \frac{\ln x}{x} dx =$$



 $\int e^{\cos x} \sin x dx =$

凑微分法 " $\int \|\hat{\mathbf{l}}_{iz\breve{\mathbf{g}}} \mathbb{E}(\hat{\mathbf{c}})\| d\mathbf{r} = \int f(\mathbf{r}(\mathbf{r}(\mathbf{x}))) d\mathbf{r}(\mathbf{r})$ ":例子总结

$$\int \frac{1}{1-3x} dx = -\frac{1}{3} \int \frac{1}{1-3x} d(1-3x) = -\frac{1}{3} \int \frac{1}{u} du = \cdots$$

$$\int \sqrt{3x-1} dx = \frac{1}{3} \int \sqrt{3x-1} d(3x-1) = \frac{1}{3} \int u^{1/2} du = \cdots$$

$$\int x e^{x^2} dx = \frac{1}{2} \int e^{x^2} dx^2 = \frac{1}{2} \int e^u du = \cdots$$

$$\int x \sqrt{1-x^2} dx = -\frac{1}{2} \int \sqrt{1-x^2} d(1-x^2) = -\frac{1}{2} \int u^{1/2} du = \cdots$$

$$\int \frac{\ln x}{x} dx =$$

$$\int e^{\cos x} \sin x dx =$$



凑微分法 " $\int \|\hat{\mathbf{I}}_{iz\breve{\mathbf{g}}} \mathbb{E} d\mathbf{x} \| d\mathbf{x} = \int f(\varphi(\mathbf{x})) d\varphi(\mathbf{x})$ ": 例子总结

$$\int \frac{1}{1-3x} dx = -\frac{1}{3} \int \frac{1}{1-3x} d(1-3x) = -\frac{1}{3} \int \frac{1}{u} du = \cdots$$

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$$\int \frac{\ln x}{x} dx = \int \ln x d(\ln x) = \int u du = \cdots$$

$$\int e^{\cos x} \sin x dx = -\int e^{\cos x} d\cos x = -\int e^u du = \cdots$$

We are here now...

1. 第一类换元积分法: 凑微分

第二类换元积分法: 变量代换

第二类换元积分法——"变量代换"法,能干啥?

能够计算如下的不定积分:

$$\int x\sqrt{3x-1}dx, \quad \int \frac{x}{\sqrt{x-2}}dx$$

$$\int \frac{1}{1+\sqrt{x}}dx, \quad \int \frac{1}{1+\sqrt[3]{x+1}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

• 计算步骤: $\int f(x)dx$

$$\int f(x)dx = \frac{x = \varphi(t)}{z}$$

$$\int f(x)dx \stackrel{x=\varphi(t)}{=} \int f(\varphi(t))d\varphi(t)$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int f(\varphi(t))\varphi'(t)dt$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{Emfile}, \text{ α-$gart}} dt$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单, 容易求!}} dt$$
$$= G(t) + C$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单, 容易求!}} dt$$
$$= G(t) + C \xrightarrow{t=\varphi^{-1}(x)}$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{one of }, \text{ as } \text{sp. } t} dt$$
$$= G(t) + C \xrightarrow{t=\varphi^{-1}(x)} G(\varphi^{-1}(x)) + C$$

• 计算步骤:

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{∇m$ fill β, α}, α} dt$$
$$= G(t) + C \xrightarrow{t=\varphi^{-1}(x)} G(\varphi^{-1}(x)) + C$$

• 关键是:如何选取函数 $x = \varphi(t)$?



• 计算步骤:

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单, }\text{容易求!}} dt$$
$$= G(t) + C \xrightarrow{t=\varphi^{-1}(x)} G(\varphi^{-1}(x)) + C$$

• 关键是:如何选取函数 $x = \varphi(t)$? 在后面的例子中,选取函数 $x = \varphi(t)$ 的方法:

把被积函数 f(x) 中复杂的部分整个设为 t, 从而得到 x 与 t 的函数关系!



例子 求不定积分 $\int \sqrt{1-x^2} dx$

例子 求不定积分
$$\int \sqrt{1-x^2} dx$$

$$x = -1 \le x \le 1$$
, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$,

例子 求不定积分
$$\int \sqrt{1-x^2} dx$$

$$\mathbb{R}$$
 $\because -1 \le x \le 1$, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$,

$$\therefore \int \sqrt{1-x^2} dx = \int \sqrt{1-\sin^2 t} d\sin t$$

例子 求不定积分
$$\int \sqrt{1-x^2} dx$$

$$\mathbb{R}$$
 $:: -1 \le x \le 1$, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\cos t \ge 0$

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$$\int \sqrt{1-x^2} dx$$

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 $:: -1 \le x \le 1$, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\cos t \ge 0$

$$\therefore \int \sqrt{1-x^2} dx = \int \sqrt{1-\sin^2 t} d\sin t = \int \cos^2 t dt$$
$$= \frac{1}{2} \int \cos 2t + 1 dt$$

例子 求不定积分
$$\int \sqrt{1-x^2} dx$$

$$\mathbb{R}$$
 $:: -1 \le x \le 1$, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\cos t \ge 0$

$$\therefore \int \sqrt{1-x^2} dx = \int \sqrt{1-\sin^2 t} d\sin t = \int \cos^2 t dt$$
$$= \frac{1}{2} \int \cos 2t + 1 dt \qquad \frac{1}{2} \sin 2t$$

例子 求不定积分
$$\int \sqrt{1-x^2} dx$$

$$\mathbb{R}$$
 $:: -1 \le x \le 1$, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\cos t \ge 0$

$$\therefore \int \sqrt{1-x^2} dx = \int \sqrt{1-\sin^2 t} d\sin t = \int \cos^2 t dt$$
$$= \frac{1}{2} \int \cos 2t + 1 dt = \frac{1}{2} \cdot \frac{1}{2} \sin 2t + \frac{1}{2} t + C$$

例子 求不定积分
$$\int \sqrt{1-x^2} dx$$

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 $:: -1 \le x \le 1$, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\cos t \ge 0$

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$$= \frac{1}{2} \int \cos 2t + 1 dt = \frac{1}{2} \cdot \frac{1}{2} \sin 2t + \frac{1}{2} t + C$$

$$= \frac{1}{2} \sin t \cos t + \frac{1}{2} t + C$$

例子 求不定积分
$$\int \sqrt{1-x^2} dx$$

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$$= \frac{1}{2} \int \cos 2t + 1 dt = \frac{1}{2} \cdot \frac{1}{2} \sin 2t + \frac{1}{2} t + C$$

$$= \frac{1}{2} \sin t \cos t + \frac{1}{2} t + C$$

$$= \frac{1}{2} x \sqrt{1 - x^2} + \frac{1}{2} \arcsin x + C$$

例子 求不定积分 $\int \sqrt{1-x^2} dx$

$$\mathbb{R}$$
 $\because -1 \le x \le 1$, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\cos t \ge 0$

$$\therefore \int \sqrt{1 - x^2} dx = \int \sqrt{1 - \sin^2 t} d \sin t = \int \cos^2 t dt$$

$$= \frac{1}{2} \int \cos 2t + 1 dt = \frac{1}{2} \cdot \frac{1}{2} \sin 2t + \frac{1}{2} t + C$$

$$= \frac{1}{2} \sin t \cos t + \frac{1}{2} t + C$$

$$= \frac{1}{2} x \sqrt{1 - x^2} + \frac{1}{2} \arcsin x + C$$

 ≥ 1 可见选取合适 $x = \varphi(t)$ 很关键!

例子 求不定积分 $\int x\sqrt{3x-1}dx$, $\int \frac{x}{\sqrt{x-2}}dx$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

解

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

解

$$\therefore \int x\sqrt{3x-1}dx =$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

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例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

(1) 设
$$t = (3x-1)^{\frac{1}{2}}$$
, $\therefore x = \frac{1}{3}(t^2+1)$, $dx = \frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx =$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

(1)
$$\[\] t = (3x-1)^{\frac{1}{2}}, \quad \therefore x = \frac{1}{3}(t^2+1), \quad dx = \frac{2}{3}tdt \]$$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t.$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

(1)
$$\partial_t t = (3x - 1)^{\frac{1}{2}}, \quad \therefore x = \frac{1}{3}(t^2 + 1), \quad dx = \frac{2}{3}tdt$$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$
$$= \frac{2}{45}t^5 + \frac{2}{27}t^3 + C$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
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例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
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$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$
$$= \frac{2}{45}t^5 + \frac{2}{27}t^3 + C = \frac{2}{45}(3x-1)^{\frac{5}{2}} + \frac{2}{27}(3x-1)^{\frac{3}{2}} + C$$

(2)
$$\oplus t = (x-2)^{\frac{1}{2}}$$
,



例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$
$$= \frac{2}{45}t^5 + \frac{2}{27}t^3 + C = \frac{2}{45}(3x-1)^{\frac{5}{2}} + \frac{2}{27}(3x-1)^{\frac{3}{2}} + C$$

$$\therefore \int \frac{x}{\sqrt{x-2}} dx =$$



例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

(1) 设
$$t = (3x-1)^{\frac{1}{2}}$$
, $\therefore x = \frac{1}{3}(t^2+1)$, $dx = \frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$

$$\int 3^{3} + 3 + 3 + 9 \int$$

$$= \frac{2}{45}t^{5} + \frac{2}{27}t^{3} + C = \frac{2}{45}(3x - 1)^{\frac{5}{2}} + \frac{2}{27}(3x - 1)^{\frac{3}{2}} + C$$

(2)
$$\[\psi \] t = (x-2)^{\frac{1}{2}}, \quad \therefore x = t^2 + 2, \]$$

$$\therefore \int \frac{x}{\sqrt{x-2}} dx =$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

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$$\int x\sqrt{3x-1}dx$$
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$$\int x\sqrt{3x-1}dx$$
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例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
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例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
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(2) $\mathfrak{L} = (x-2)^{\frac{1}{2}}$, $\therefore x = t^2 + 2$, $dx = 2tdt$

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$$= \frac{2}{3}(x-2)^{\frac{3}{2}} + 4(x-2)^{\frac{1}{2}} + C$$

例子 求不定积分 $\int \frac{1}{1+\sqrt{x}} dx$, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设
$$t = 1 + x^{\frac{1}{2}}$$
,

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设
$$t = 1 + x^{\frac{1}{2}}$$
,

$$\therefore \int \frac{1}{1+\sqrt{x}} dx =$$



例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设
$$t = 1 + x^{\frac{1}{2}}$$
, $\therefore x = (t-1)^2$,

$$\therefore \int \frac{1}{1+\sqrt{x}} dx =$$

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设
$$t = 1 + x^{\frac{1}{2}}$$
, $x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx =$$

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设
$$t = 1 + x^{\frac{1}{2}}$$
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$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt$$

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2 \int 1 - \frac{1}{t} dt$$

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
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$$= 2t - 2 \ln t + C$$



变量代换 " $\int f(x)dx = \frac{x=\varphi(t)}{f(\varphi(t))\varphi'(t)dt} = G(\varphi^{-1}(t)) + C$ "例 II

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
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$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2 \int 1 - \frac{1}{t} dt$$

$$= 2t - 2\ln t + C = 2(1 + x^{\frac{1}{2}}) - 2\ln(1 + x^{\frac{1}{2}}) + C$$

$$= 2t - 2\ln t + C = 2(1 + x^2) - 2\ln(1 + x^2) + \frac{1}{2}$$

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设
$$t = 1 + x^{\frac{1}{2}}$$
, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2 \int 1 - \frac{1}{t} dt$$

$$\int 1 + \sqrt{x} \qquad \int 1 \qquad \qquad$$

$$= 2(-2)(1+2)$$

(2)
$$\oplus t = 1 + (1+x)^{\frac{1}{3}}$$
,



例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设
$$t = 1 + x^{\frac{1}{2}}$$
, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1 + \sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2 \int 1 - \frac{1}{t} dt$$

$$\int 1 + \sqrt{x} \qquad \int t \qquad \qquad \int \qquad \int$$

$$= 2t - 2 \ln t + C = 2(1 + x^{\frac{1}{2}}) - 2 \ln(1 + x^{\frac{1}{2}}) + C$$

$$= 2t - 2\ln t + C = 2(1 + x^{\frac{1}{2}}) - 2\ln(1 + x^{\frac{1}{2}}) + \frac{1}{2}$$

(2) 设
$$t = 1 + (1 + x)^{\frac{1}{3}}$$
,
∴ $\int \frac{1}{1 + \sqrt[3]{1 + x}} dx =$

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设
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$$\int 1 + \sqrt{x} \qquad \int t \qquad \qquad \int \qquad t$$

$$= 2t - 2 \ln t + C = 2(1 + x^{\frac{1}{2}}) - 2 \ln(1 + x^{\frac{1}{2}}) + C$$

$$= 2t - 2 \ln t + C = 2(1 + x^{\frac{1}{2}}) - 2 \ln(1 + x^{\frac{1}{2}}) + C$$

(2) 设
$$t = 1 + (1 + x)^{\frac{1}{3}}$$
, ∴ $x = (t - 1)^3 - 1$,
∴
$$\int \frac{1}{1 + \sqrt[3]{1 + x}} dx =$$



例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设
$$t = 1 + x^{\frac{1}{2}}$$
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$$\int \frac{1}{1+\sqrt{x}} dx$$
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例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt{3}\sqrt{1+x}} dx$

解 (1) 设
$$t = 1 + x^{\frac{1}{2}}$$
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例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设
$$t = 1 + x^{\frac{1}{2}}$$
, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

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$$= 2t - 2\ln t + C = 2(1 + x^{\frac{1}{2}}) - 2\ln(1 + x^{\frac{1}{2}}) + C$$

变量代换 " $\int f(x)dx = \frac{x=\varphi(t)}{\int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t)) + C}$ "例 II 例子 求不定积分 $\int \frac{1}{1+\sqrt{x}}dx$, $\int \frac{1}{1+\frac{3}{\sqrt{1-x}}}dx$

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$$= 2t - 2\ln t + C = 2(1 + x^{\frac{1}{2}}) - 2\ln(1 + x^{\frac{1}{2}}) + C$$
(2) \mathcal{U} $\$

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}} dx = \int \frac{1}{t} \cdot 3(t-1)^2 dt = 3 \int t - 2 + \frac{1}{t} dt$$
$$= \frac{3}{2} t^2 - 6t + 3 \ln|t| + C$$

 $= \frac{3}{2}(1+(1+x)^{\frac{1}{3}})^2 - 6(1+(1+x)^{\frac{1}{3}}) + 3\ln|1+(1+x)^{\frac{1}{3}}| + C$

.3 换元积

例子 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

例子 求不定积分
$$\int \frac{1}{\sqrt{1+e^x}} dx$$

设
$$t = \sqrt{1 + e^x}$$
,

例子 求不定积分
$$\int \frac{1}{\sqrt{1+e^x}} dx$$

设
$$t = \sqrt{1 + e^x}$$
, $\therefore x = \ln(t^2 - 1)$,

例子 求不定积分
$$\int \frac{1}{\sqrt{1+a^2}} dx$$

设
$$t = \sqrt{1 + e^x}$$
, $\therefore x = \ln(t^2 - 1)$, $dx = \frac{2t}{t^2 - 1} dt$

例子 求不定积分
$$\int \frac{1}{\sqrt{1+a^2}} dx$$

设
$$t = \sqrt{1 + e^x}$$
, $\therefore x = \ln(t^2 - 1)$, $dx = \frac{2t}{t^2 - 1}dt$

$$\therefore \int \frac{1}{\sqrt{1+e^x}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1} dt$$

例子 求不定积分
$$\int \frac{1}{\sqrt{1+a^2}} dx$$

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$$t = \sqrt{1 + e^x}$$
, $\therefore x = \ln(t^2 - 1)$, $dx = \frac{2t}{t^2 - 1}dt$

$$\therefore \int \frac{1}{\sqrt{1+e^{x}}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^{2}-1} dt = \int \frac{1}{t-1} - \frac{1}{t+1} dt$$



例子 求不定积分
$$\int \frac{1}{\sqrt{1+a^{x}}} dx$$

设
$$t = \sqrt{1 + e^x}$$
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$$= \ln|t - 1| - \ln|t + 1| + C$$



例子 求不定积分
$$\int \frac{1}{\sqrt{1+a^{x}}} dx$$

设
$$t = \sqrt{1 + e^x}$$
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$$= \ln|t - 1| - \ln|t + 1| + C = \ln|\frac{t - 1}{t + 1}| + C$$



例子 求不定积分 $\int \frac{1}{\sqrt{1+\alpha^2}} dx$

设
$$t = \sqrt{1 + e^{x}}$$
, $\therefore x = \ln(t^{2} - 1)$, $dx = \frac{2t}{t^{2} - 1}dt$

$$\therefore \int \frac{1}{\sqrt{1 + e^{x}}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^{2} - 1} dt = \int \frac{1}{t - 1} - \frac{1}{t + 1} dt$$

$$= \ln|t - 1| - \ln|t + 1| + C = \ln|\frac{t - 1}{t + 1}| + C$$

$$= \ln\left(\frac{\sqrt{1 + e^{x}} - 1}{\sqrt{1 + e^{x}} + 1}\right) + C$$

例子 求不定积分 $\int \frac{1}{\sqrt{1+a^x}} dx$

解

设
$$t = \sqrt{1 + e^x}$$
, $\therefore x = \ln(t^2 - 1)$, $dx = \frac{2t}{t^2 - 1}dt$

$$\therefore \int \frac{1}{\sqrt{1 + e^x}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1} dt = \int \frac{1}{t - 1} - \frac{1}{t + 1} dt$$

$$= \ln|t - 1| - \ln|t + 1| + C = \ln|\frac{t - 1}{t + 1}| + C$$

$$= \ln\left(\frac{\sqrt{1 + e^x} - 1}{\sqrt{1 + e^x} + 1}\right) + C$$

 $= 2 \ln(\sqrt{1 + e^x} - 1) - x + C$



$$\int x\sqrt{3x-1}dx$$

$$\int \frac{1}{1+\sqrt{x}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

$$\int x\sqrt{3x-1}dx \xrightarrow{\underline{t=\sqrt{3x-1}}} \cdots$$

$$\int \frac{1}{1+\sqrt{x}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

$$\int x\sqrt{3x-1}dx \xrightarrow{t=\sqrt{3x-1}} \cdots$$

$$\int \frac{1}{1+\sqrt{x}}dx \xrightarrow{t=1+\sqrt{x}} \cdots$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

$$\int x\sqrt{3x-1}dx \xrightarrow{\underline{t=\sqrt{3x-1}}} \cdots$$

$$\int \frac{1}{1+\sqrt{x}}dx \xrightarrow{\underline{t=1+\sqrt{x}}} \cdots$$

$$\int \frac{1}{\sqrt{1+e^x}} dx \, \frac{t=\sqrt{1+e^x}}{} \cdots$$