第12章e:傅里叶级数

数学系 梁卓滨

2019-2020 学年 II

Outline

1. 周期为 2π 的周期函数的傅里叶级数

3. 一般周期函数的傅里叶级数

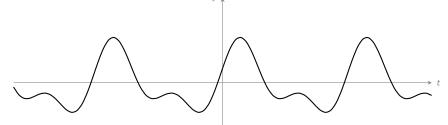


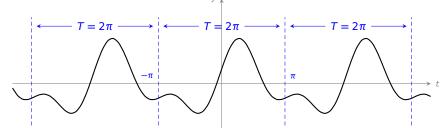
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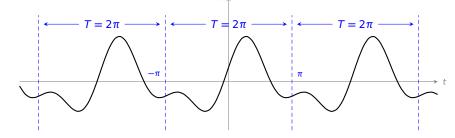
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3. 一般周期函数的傅里叶级数



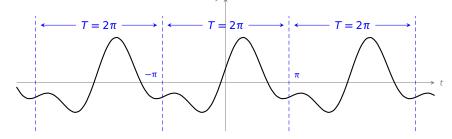






• 注意到三角函数系

 $1, \cos x, \sin x, \cos 2x, \sin 2x, \cdots, \cos nx, \sin nx, \cdots$ 也具有周期 2π

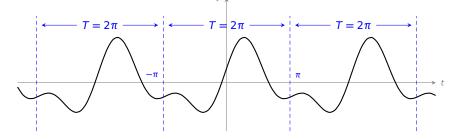


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$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$



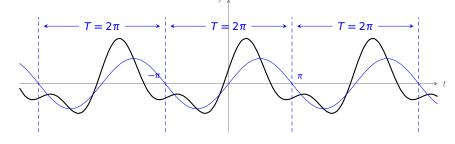


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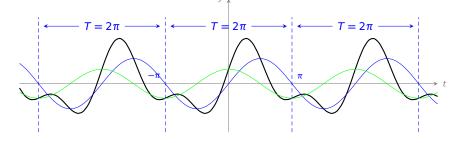


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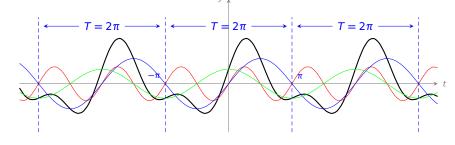


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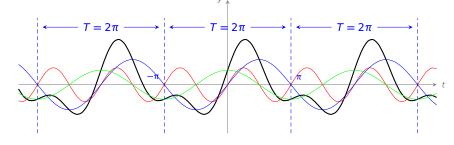


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● 注意到三角函数系

1, $\cos x$, $\sin x$, $\cos 2x$, $\sin 2x$, ..., $\cos nx$, $\sin nx$, ...

也具有周期 2π

问题 是否有如下展开

1. 假设等式成立,则
$$a_n = ?$$

2. 得到 a_n 后,再讨论等式对哪些 x 成立?

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$



1, $\cos x$, $\sin x$, $\cos 2x$, $\sin 2x$, ..., $\cos nx$, $\sin nx$, ...

在区间 $[-\pi, \pi]$ 上正交.

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$$\int_{-\pi}^{\pi} \cos nx dx = 0, \qquad \int_{-\pi}^{\pi} \sin nx dx = 0 \qquad (n = 1, 2, 3, \dots)$$

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$$\int_{-\pi}^{\pi} \cos nx dx = 0, \qquad \int_{-\pi}^{\pi} \sin nx dx = 0 \qquad (n = 1, 2, 3, \dots)$$
$$\int_{-\pi}^{\pi} \sin kx \cdot \cos nx dx = 0 \qquad (k, n = 1, 2, 3, \dots)$$

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在区间 $[-\pi, \pi]$ 上正交. 即上述任意两个相异函数的乘积,在 $[-\pi, \pi]$ 上的积分为零:

$$\int_{-\pi}^{\pi} \cos nx dx = 0, \qquad \int_{-\pi}^{\pi} \sin nx dx = 0 \qquad (n = 1, 2, 3, \dots)$$

$$\int_{-\pi}^{\pi} \sin kx \cdot \cos nx dx = 0 \qquad (k, n = 1, 2, 3, \dots)$$

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$$\int_{-\pi}^{\pi} \cos kx \cdot \cos nx dx = 0 \qquad (k, n = 1, 2, 3, \dots, k \neq n)$$

另外

$$\int_{-\pi}^{\pi} \sin^2 nx dx = \int_{-\pi}^{\pi} \cos^2 nx dx = \pi \qquad (n = 1, 2, 3, \dots)$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx, \qquad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx.$$



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形式推导 (1) 当
$$n = 1, 2, 3, \dots$$
 by,
$$\int_{-\pi}^{\pi} f(x) \cos nx dx \qquad \left[\frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_k \cos kx + b_k \sin kx \right) \right] \cos nx$$

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$$\int_{0}^{\pi} f(x) \sin nx dx$$



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(2) 当 $n = 1, 2, 3, \cdots$ 时, $\int_{-\pi}^{\pi} f(x) \sin nx dx = \int_{-\pi}^{\pi} \left[\frac{a_0}{2} + \sum_{k=0}^{\infty} \left(a_k \cos kx + b_k \sin kx \right) \right] \sin nx dx$

$$= \int_{0}^{\pi} b_{n} \sin nx \cdot \sin nx dx$$



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定义 f(x) 的傅里叶级数定义为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

其中

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx, \qquad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx.$$

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问题

- 对哪些 x 傅里叶级数 $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$ 收敛?
- 对哪些 x 成立 $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$?

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定理(收敛定理,狄利克雷充分条件)



12e 傅里叶级数

定理(收敛定理,狄利克雷充分条件) 设 f(x) 是周期为 2π 的周期函

数,如果它满足:

- 1. 在一个周期内连续或只有有限个第一类间断点;
- 2. 在一个周期内至多只有有限个极值点,

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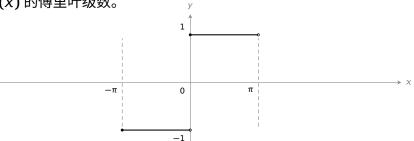
$$\frac{1}{2} \Big[f(x^{-}) + f(x^{+}) \Big] = \frac{a_0}{2} + \sum_{n=1}^{\infty} \Big(a_n \cos nx + b_n \sin nx \Big)$$

$$f(x) = \begin{cases} -1, & -\pi \le x < 0, \\ 1, & 0 \le x < \pi. \end{cases}$$

求出f(x)的傅里叶级数。

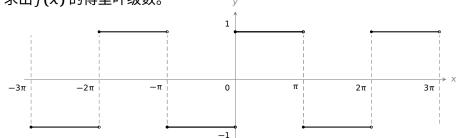
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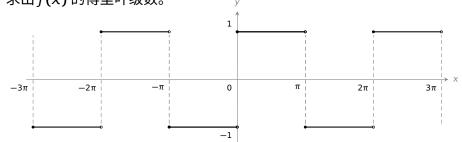
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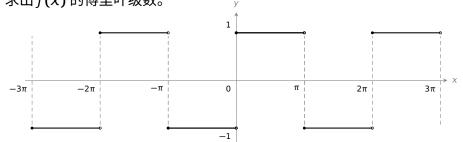
解 计算傅里叶系数如下:

 a_n



$$f(x) = \left\{ \begin{array}{ll} -1, & -\pi \leq x < 0, \\ 1, & 0 \leq x < \pi. \end{array} \right.$$

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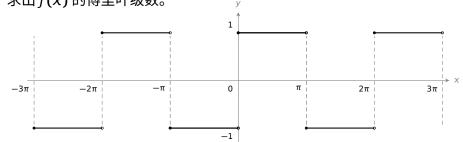
解 计算傅里叶系数如下:

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$



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解 计算傅里叶系数如下:

12e 傅里叶级数

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{5}{10}} 0$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \stackrel{\text{fight}}{=} 0,$$

bn



$$\alpha_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \stackrel{\underline{\hat{\sigma}}(\underline{\mathbf{m}}\underline{\mathbf{m}}}{=} 0,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{\text{fight}}{\pi}} 0,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\text{§fight}} 0,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} \sin nx dx$$



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$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\hat{\sigma}(\underline{R})} 0,$$

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$$= \begin{cases} n = 1, 3, 5, \dots \\ n = 2, 4, 6, \dots \end{cases}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{\partial \mathbb{R}^k}{\pi}} 0,$$

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$$\frac{a_0}{2} + \sum_{n=0}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{\hat{\sigma}(n+1)}{\pi}} 0,$$

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所以傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right) = \sum_{n=1}^{\infty} b_n \sin nx$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{\hat{\sigma}(R)!}{\pi}} 0,$$

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 $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right) = \sum_{n=1}^{\infty} b_n \sin nx$ $= \frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right]$

敛定理分析可知:

• 当
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 是,

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注 2 取
$$x = \frac{\pi}{2}$$
,可得到
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 $\mathbf{\dot{z}}$ 3 当 $x = \frac{\pi}{2}$,傅里叶级数仅仅是条件收敛



f(x) 的傅里叶级数是

$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$$

$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$



$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

$$\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$



$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

考虑部分和

0

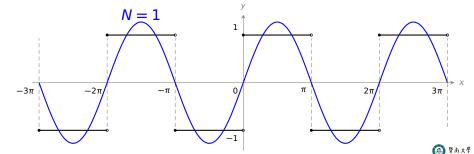
 $\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$

 -2π

 3π

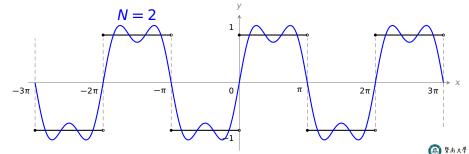
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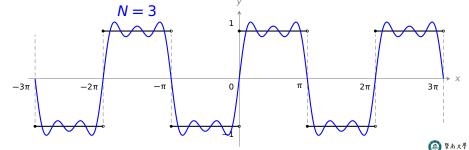
$$\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$



$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

$$\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$

$$N = 3$$



$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

考虑部分和

$$N = 4$$

$$1$$

 $\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$

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3π

$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

 $\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$

考虑部分和

$$N = 5$$

12e 傅里叶级数

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3π

$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

考虑部分和

$$N = 6$$

 $\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$

3π

$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

 $\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$

考虑部分和

$$N = 7$$



3π

$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

 $\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$

考虑部分和

$$N = 8$$

12e 傅里叶级数

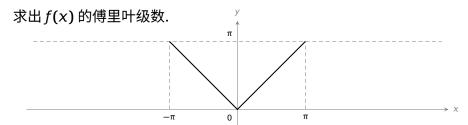
暨南大學

3π

$$f(x) = |x|$$

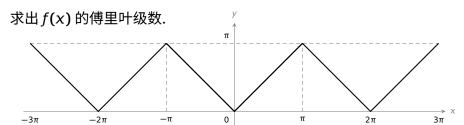
求出 f(x) 的傅里叶级数.

$$f(x) = |x|$$



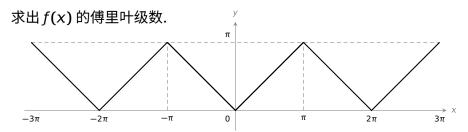


$$f(x) = |x|$$





$$f(x) = |x|$$

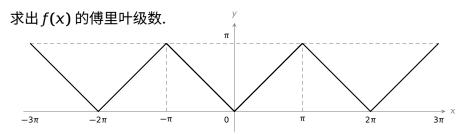


解 计算傅里叶系数如下:

 b_n



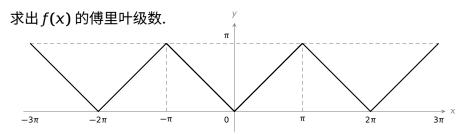
$$f(x) = |x|$$



解 计算傅里叶系数如下:

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$f(x) = |x|$$



解 计算傅里叶系数如下:

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{sight}} 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{§fight}} 0,$$

$$a_n =$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\underline{\hat{\sigma}(x)}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx$$

 $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\frac{\text{fight}}{\pi}} 0,$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\frac{4}{3}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$



 $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{§fight}} 0,$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx$$

 $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\frac{6}{3}} 0,$ $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{-$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \right]_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \right]_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx$$

$$= \frac{2}{n\pi} \int_0^{\pi} x d\sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$
$$= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \Big|_0^{\pi} \right]$$

 $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{§fight}} 0,$

 $b_n = \frac{1}{\pi} \int_0^{\pi} f(x) \sin nx dx \xrightarrow{\frac{\partial \mathbb{R}^k}{\partial \mathbb{R}^k}} 0,$

 $= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \right]_{0}^{n} = \frac{2}{n^{2}\pi} \left[(-1)^{n} - 1 \right]$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$a_n = -\frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = -\frac{1}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = -\frac{1}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= -\frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = -\frac{2}{n\pi} \left[x \sin nx \right]_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\frac{5}{3}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx dx = \frac{2}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx dx = \frac{2}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx dx dx =$$

 $= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \right]_{0}^{\pi} = \frac{2}{n^{2}\pi} \left[(-1)^{n} - 1 \right] = \begin{cases} n = 1, 3, 5, \dots \\ n = 2, 4, 6, \dots \end{cases}$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \right]_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx$$

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$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\underline{\hat{\sigma}} \text{ (BME)}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$= \frac{2}{n\pi} \int_{-\pi}^{\pi} x d\sin nx = \frac{2}{n\pi} \left[x \sin nx \right]_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx$$

 $= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \right]_{0}^{\pi} = \frac{2}{n^{2}\pi} \left[(-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \cdots \\ n = 2, 4, 6, \cdots \end{cases}$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\frac{6}{(1 + \alpha)^2}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

 $= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \right]_{0}^{\pi} = \frac{2}{n^{2}\pi} \left[(-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \cdots \\ 0, & n = 2, 4, 6, \cdots \end{cases}$

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$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \right]_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx$$

 $= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \right]_{0}^{\pi} = \frac{2}{n^{2}\pi} \left[(-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \cdots \\ 0, & n = 2, 4, 6, \cdots \end{cases}$

 $b_n = \frac{1}{\pi} \int_0^{\pi} f(x) \sin nx dx \xrightarrow{\frac{6}{10}} 0,$

$$a_0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\underline{\delta} \text{(in)}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \right]_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx$$

$$n\pi \int_{0}^{\pi} n\pi \int_{0}^{\pi} n\pi \int_{0}^{\pi} \int_{0}$$

$$n\pi \ln n$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$



 $b_n = \frac{1}{\pi} \int_0^{\pi} f(x) \sin nx dx \xrightarrow{\frac{6}{10}} 0,$ $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$

$$= \frac{2}{n\pi} \int_0^{\pi} x d\sin nx = \frac{2}{n\pi} \left[x \sin nx \right]_0^{\pi} - \int_0^{\pi} \sin nx dx$$

 $= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \right]_{0}^{\pi} = \frac{2}{n^{2}\pi} \left[(-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \cdots \\ 0, & n = 2, 4, 6, \cdots \end{cases}$

 $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx$

$$b_n = \frac{1}{\pi} \int_0^{\pi} f(x) \sin nx dx \xrightarrow{\frac{\partial \mathbb{R}^k}{\partial \mathbb{R}^k}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$\pi \int_{-\pi}^{\pi} \pi \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \right]_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx$$

$$= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \Big|_{0}^{\pi} \right] = \frac{2}{n^{2}\pi} \left[(-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

 $n\pi L n \qquad |_{0}J \qquad n^{2}\pi L \qquad J \qquad (0,$ $a_{0} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} x dx$



 $b_n = \frac{1}{\pi} \int_0^{\pi} f(x) \sin nx dx \xrightarrow{\frac{6}{10}} 0,$ $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \Big|_{0}^{\pi} \right] = \frac{2}{n^{2}\pi} \left[(-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

$$a_{0} = \frac{1}{n} \int_{0}^{\pi} f(x) dx = \frac{2}{n^{2}\pi} \int_{0}^{\pi}$$

$$n\pi \ln n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} x dx = \frac{2}{\pi} \cdot \frac{1}{2} x^{2} \Big|_{0}^{\pi}$$



 $b_n = \frac{1}{\pi} \int_0^{\pi} f(x) \sin nx dx \xrightarrow{\frac{6}{10}} 0,$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$
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$$= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \Big|_{0}^{\pi} \right] = \frac{2}{n^{2}\pi} \left[(-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

$$1 \int_{0}^{\pi} 2 \int_{0}^{\pi}$$

 $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} x dx = \frac{2}{\pi} \cdot \frac{1}{2} x^2 \Big|_{0}^{\pi} = \pi.$



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 $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$

$$= \frac{2}{n\pi} \int_0^{\pi} x d\sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \Big|_0^{\pi} \right] = \frac{2}{n^2 \pi} \left[(-1)^n - 1 \right] = \begin{cases} -\frac{4}{n^2 \pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

 $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} x dx = \frac{2}{\pi} \cdot \frac{1}{2} x^2 \Big|_{0}^{\pi} = \pi.$

所以傅里叶级数为
$$\frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos nx$$

 $b_n = \frac{1}{\pi} \int_0^{\pi} f(x) \sin nx dx \xrightarrow{\frac{\partial f(x)}{\partial x}} 0,$



$$\frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos nx = \frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$= \frac{2}{n\pi} \int_0^{\pi} x d\sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \Big|_0^{\pi} \right] = \frac{2}{n^2\pi} \left[(-1)^n - 1 \right] = \begin{cases} -\frac{4}{n^2\pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

$$n\pi \ln |_{0} \int n^{2}\pi \ln |_{0} \int (0, n = 2, 4, 6, 4)$$

$$a_{0} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)dx = \frac{2}{\pi} \int_{0}^{\pi} f(x)dx = \frac{2}{\pi} \int_{0}^{\pi} xdx = \frac{2}{\pi} \cdot \frac{1}{2}x^{2} \Big|_{0}^{\pi} = \pi.$$

 $b_n = \frac{1}{\pi} \int_0^{\pi} f(x) \sin nx dx \xrightarrow{\frac{\partial f(x)}{\partial x}} 0,$

$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$



$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

又因为f(x)是连续函数,



$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

又因为 f(x) 是连续函数,故利用收敛定理分析可知:

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

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$$注 2$$
 取 $x = 0$,可得到

$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

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$$注 2$$
 取 $x = 0$,可得到

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$$

$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

又因为 f(x) 是连续函数,故利用收敛定理分析可知:

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注 2 取 x = 0,可得到

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$$

注 3 偶函数 f(x) 的傅里叶级数是 $\frac{\alpha_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$



$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$



$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$



$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$



$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

考虑部分和

π

2π

 $\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$

3π

 -3π

 -2π

$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$

$$N = 1$$

$$\frac{\pi}{2\pi} = \frac{\pi}{2\pi} = \frac{\pi}{3\pi}$$

$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$

$$N = 2$$

$$\frac{\pi}{2\pi} = \frac{\pi}{2\pi} = \frac{\pi}{3\pi}$$



$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$

$$N = 3$$

$$\frac{\pi}{2\pi} = \frac{\pi}{2\pi} = \frac{\pi}{3\pi}$$



$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$

$$N = 4$$

$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$

$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$

$$N = 5$$

$$\frac{\pi}{2n} = \frac{\pi}{2n} = \frac{\pi}{2n} = \frac{\pi}{3n}$$



$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$

$$N = 6$$

$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$

$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$

$$N = 7$$

$$\frac{\pi}{2\pi} = \frac{\pi}{2\pi} = \frac{\pi}{3\pi}$$

$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \dots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$

$$N = 8$$

$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$



● 若 f(x) 是奇函数,则傅里叶级数为

• 若 f(x) 是偶函数,则傅里叶级数为

• 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx,$$

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● 若 f(x) 是奇函数,则傅里叶级数为

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• 若 f(x) 是偶函数,则傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx,$$

● 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

• 若f(x) 是偶函数,则傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx,$$

● 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

若 f(x) 是偶函数,则傅里叶级数为

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• 若 f(x) 是偶函数,则傅里叶级数为

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证明 (1) 假设 f 为奇函数,则

$$a_n =$$

$$b_n =$$

● 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

● 若 f(x) 是偶函数,则傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \qquad a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx.$$

证明 (1) 假设 f 为奇函数,则

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$
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证明(1)假设f为奇函数,则

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\underline{\hat{S}}(\underline{R})} 0$$

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证明 (2) 假设 f 为偶函数,则

$$b_n =$$

$$a_n =$$

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证明(2)假设 ƒ为偶函数,则

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

● 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

■ 若 f(x) 是偶函数,则傅里叶级数为

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证明 (2) 假设 *f* 为偶函数,则

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{sign}} 0$$

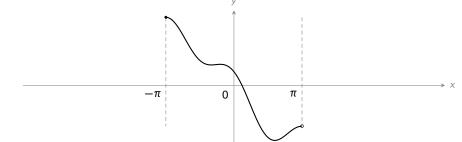
$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\text{sign}} \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx$$



设 f(x) 是定义在区间 $[-\pi, \pi)$ (或 $(-\pi, \pi]$)上的函数,可以对其进行 周期延拓,从而得到定义在 \mathbb{R} 上的周期函数

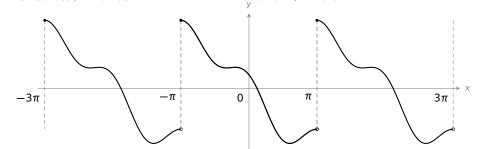


设 f(x) 是定义在区间 $[-\pi, \pi)$ (或 $(-\pi, \pi]$)上的函数,可以对其进行 周期延拓,从而得到定义在 \mathbb{R} 上的周期函数,如图:





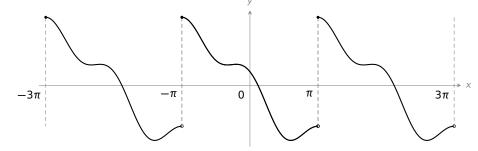
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12e 傅里叶级数

设 f(x) 是定义在区间 $[-\pi, \pi)$ (或 $(-\pi, \pi]$)上的函数,可以对其进行 周期延拓,从而得到定义在 \mathbb{R} 上的周期函数,如图:



延拓后的周期函数任然记为 f(x),此时可以进行傅里叶展开.



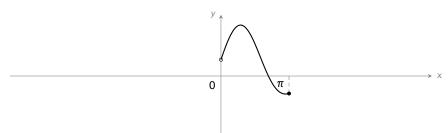
12e傅里叶级数 17/21 < ▶ △ ▼

奇延拓

设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行**奇延拓**,从而得到定义在 \mathbb{R} 上的周期奇函数.

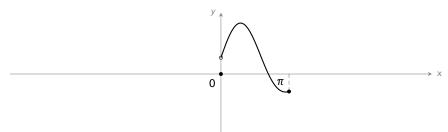


设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行**奇延拓**,从而得到定义在 \mathbb{R} 上的周期奇函数.



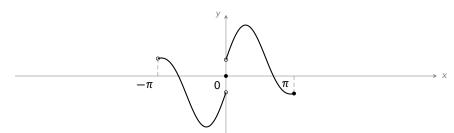


设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行**奇延拓**,从而得到定义在 \mathbb{R} 上的周期奇函数.



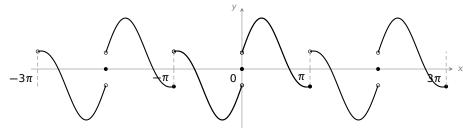


设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行**奇延拓**,从而得到定义在 \mathbb{R} 上的周期奇函数.





设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行**奇延拓**,从而得到定义在 \mathbb{R} 上的周期奇函数.

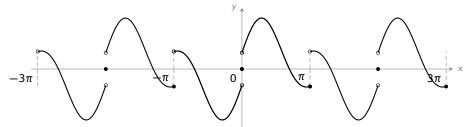




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奇延拓步骤:

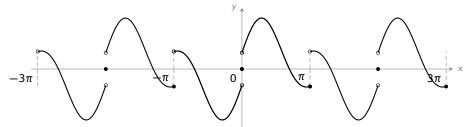
定义 f(0) = 0



设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行**奇延拓**,从而得到定义在 \mathbb{R} 上的周期奇函数.

奇延拓步骤:

• $\mathbb{E} \times f(0) = 0$; $\exists x \in (-\pi, 0)$ 时, $\mathbb{E} \times f(x) = -f(-x)$;



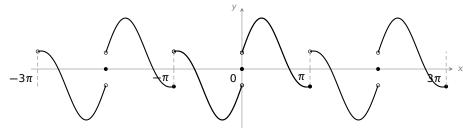


12e 傅里叶级数

设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行**奇延拓**,从而得到定义在 \mathbb{R} 上的周期奇函数.

奇延拓步骤:

• 定义 f(0) = 0; 当 $x \in (-\pi, 0)$ 时,定义 f(x) = -f(-x); (此时 f 在 $(-\pi, \pi]$ 上有定义,且在 $(-\pi, \pi)$ 上为奇函数)

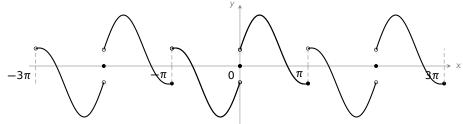




设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行**奇延拓**,从而得到定义在 \mathbb{R} 上的周期奇函数.

奇延拓步骤:

- 定义 f(0) = 0; 当 $x \in (-\pi, 0)$ 时,定义 f(x) = -f(-x); (此时 f 在 $(-\pi, \pi]$ 上有定义,且在 $(-\pi, \pi)$ 上为奇函数)
- 周期延拓 f 在 $(-\pi, \pi]$ 上的取值.



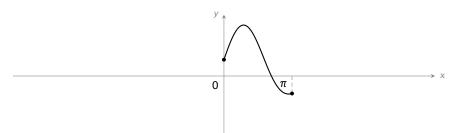


12e 傅里叶级数

设 f(x) 是定义在区间 $[0, \pi]$ 上的函数,可以对其进行**偶延拓**,从而得到定义在 \mathbb{R} 上的周期偶函数.

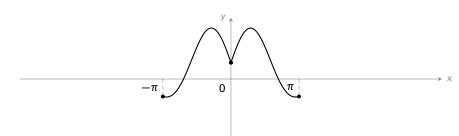


设 f(x) 是定义在区间 $[0, \pi]$ 上的函数,可以对其进行**偶延拓**,从而得到定义在 \mathbb{R} 上的周期偶函数.



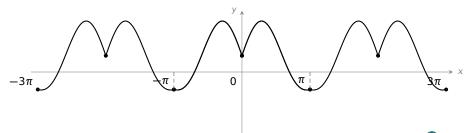


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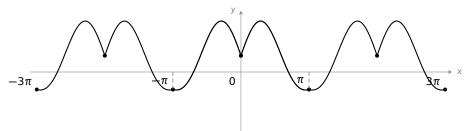




设 f(x) 是定义在区间 $[0, \pi]$ 上的函数,可以对其进行**偶延拓**,从而得到定义在 \mathbb{R} 上的周期偶函数.

偶延拓步骤:

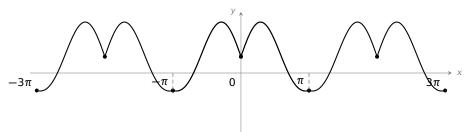
● 当 $x \in [-\pi, 0]$ 时,定义 f(x) = f(-x);



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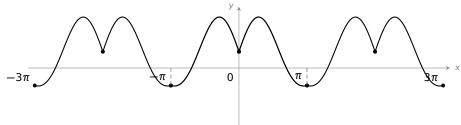
偶延拓步骤:

• 当 $x \in [-\pi, 0]$ 时,定义 f(x) = f(-x); (此时 f 成为定义在 $[-\pi, \pi]$ 上为偶函数)



设 f(x) 是定义在区间 $[0, \pi]$ 上的函数,可以对其进行**偶延拓**,从而得到定义在 \mathbb{R} 上的周期偶函数.

- 当 $x \in [-\pi, 0]$ 时,定义 f(x) = f(-x); (此时 f 成为定义在 $[-\pi, \pi]$ 上为偶函数)
- 周期延拓 f 在 $[-\pi, \pi]$ 上的取值.



We are here now...

1. 周期为 2π 的周期函数的傅里叶级数

3. 一般周期函数的傅里叶级数



假设f(x)是定义在 \mathbb{R} 上周期函数,周期为T=2l,

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中 $a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx$ $(n = 0, 1, 2, 3, \cdots)$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$



$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$\frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n=0,1,2,3)$$

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导"
$$\Rightarrow g(x) = f(\frac{l}{\pi}x),$$



$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

 $a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$ $1 \int_{-l}^{l} n\pi x$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导" 令 $g(x) = f(\frac{l}{\pi}x)$,则 g 是周期为 2π 的周期函数:

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

 $a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$ $1 \int_{-l}^{l} n\pi x$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导" 令 $g(x) = f(\frac{1}{\pi}x)$,则 g 是周期为 2π 的周期函数:

$$q(x+2\pi)$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

 $a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$ 1 $\int_{-l}^{l} n\pi x$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导" 令 $g(x) = f(\frac{l}{\pi}x)$,则 g 是周期为 2π 的周期函数:

$$g(x+2\pi) = f(\frac{l}{\pi}(x+2\pi))$$



$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中 $a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx$ $(n = 0, 1, 2, 3, \cdots)$

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"推导" 令 $g(x) = f(\frac{1}{\pi}x)$,则 g 是周期为 2π 的周期函数:

$$g(x+2\pi) = f(\frac{l}{\pi}(x+2\pi)) = f(\frac{l}{\pi}x+2l)$$



$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中 $a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx$ $(n = 0, 1, 2, 3, \cdots)$

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"推导" 令 $g(x) = f(\frac{l}{\pi}x)$,则 g 是周期为 2π 的周期函数:

$$g(x+2\pi) = f(\frac{l}{\pi}(x+2\pi)) = f(\frac{l}{\pi}x+2l) = f(\frac{l}{\pi}x)$$

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所以 $g(x) = \frac{a_0}{2} + \sum_{n=0}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$



$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

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其中

 b_n



既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

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其中

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz$$

 b_n



$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

所以

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz$$



$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

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$$b_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{-z}) \sin nz dz$$



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$$x = \frac{l}{\pi}z \quad 1 \quad \int_{-\pi}^{l} n\pi x \quad \pi \quad 1 \quad \int_{-\pi}^{l} n\pi z dz$$

$$\frac{x = \frac{l}{\pi} z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l} x) = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi} z) \sin nz dz$$





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 $\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L} d(\frac{\pi}{L}x) = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L} dx,$ $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \sin nz dz$ $\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int f(x) \sin \frac{n\pi x}{l} d(\frac{\pi}{l}x)$



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$$\frac{1}{\pi} \int_{-l}^{\pi} f(x) \cos \frac{n\pi x}{l} dx = \frac{1}{l} \int_{-l}^{\pi} f(x) \cos \frac{n\pi x}{l} dx,$$

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 $\frac{x=\frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L} d(\frac{\pi}{L}x) = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L} dx,$

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