§6.4 微积分基本定理

2015-2016 学年 II

教学要求

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Outline of §6.4

1. 变上限的定积分

2. 微积分基本定理：牛顿—莱布尼茨公式

We are here now...

1. 变上限的定积分

2. 微积分基本定理：牛顿—莱布尼茨公式

定义 假设 $f(x)$ 是区间 $[a, b]$ 上的连续函数

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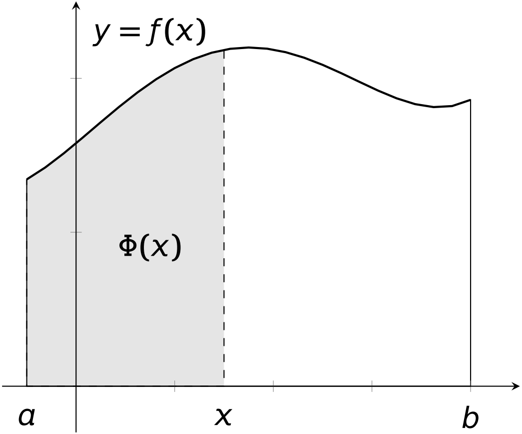

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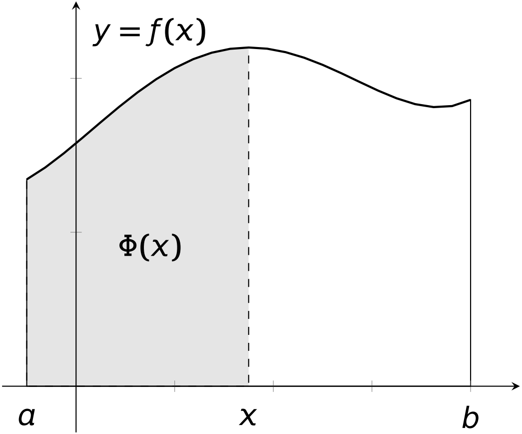

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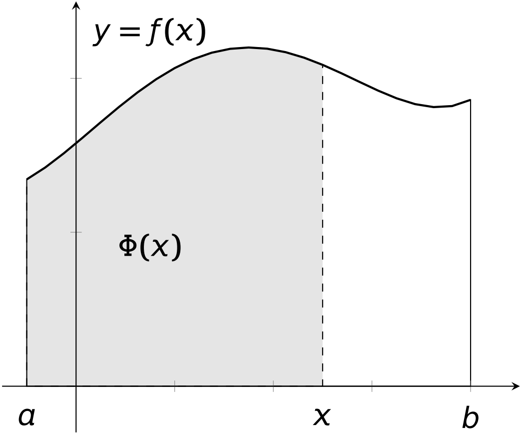

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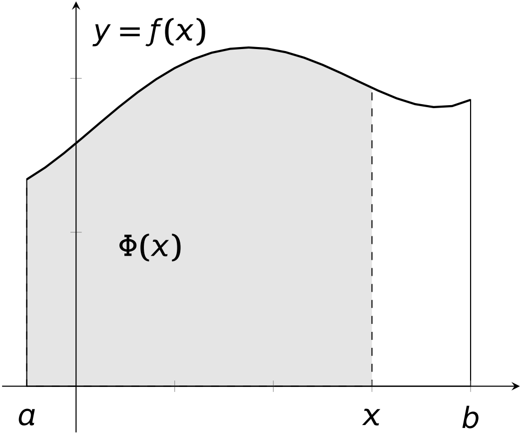

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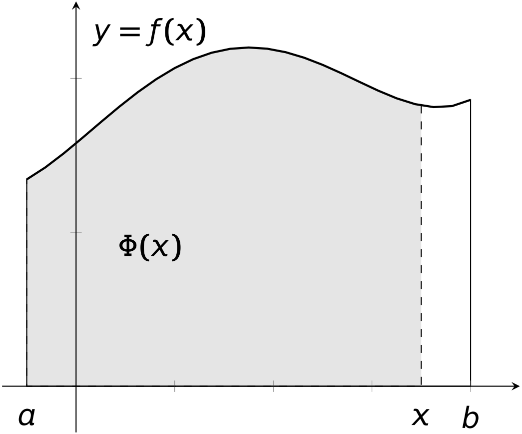

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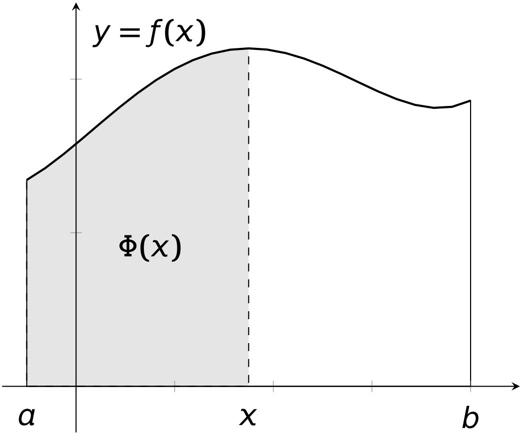

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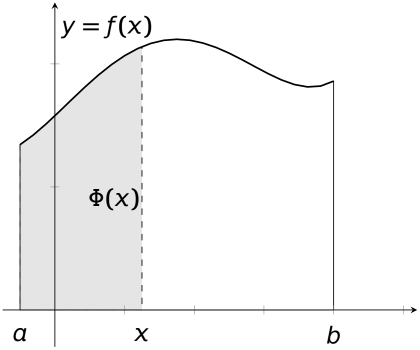

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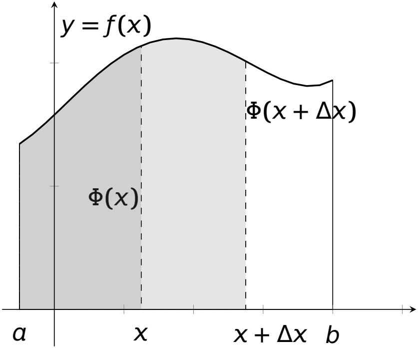

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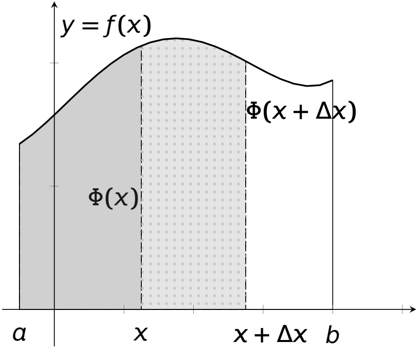

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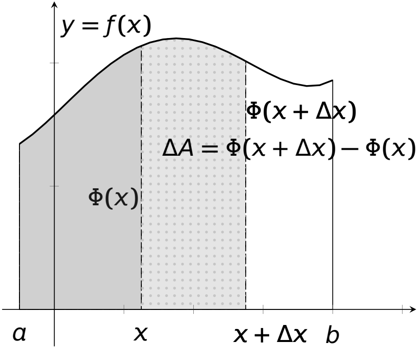

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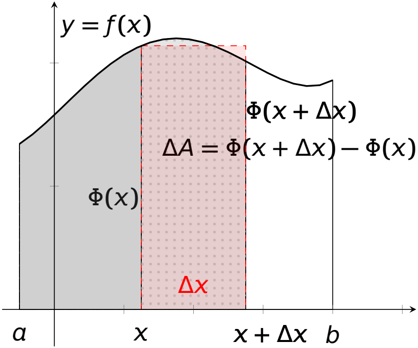

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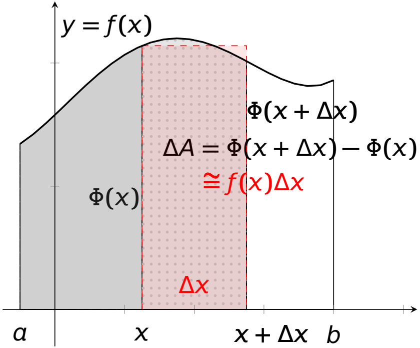

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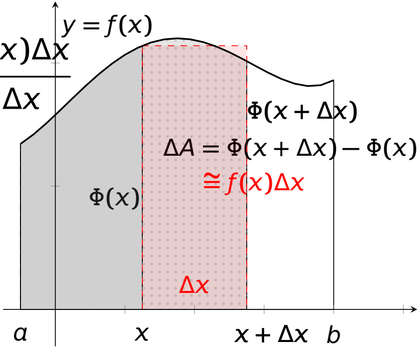

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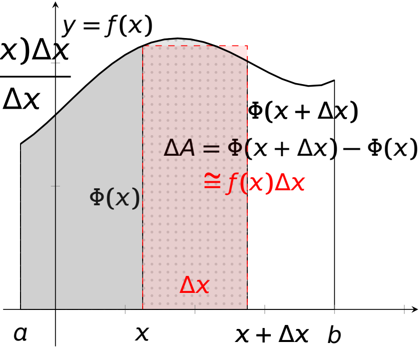

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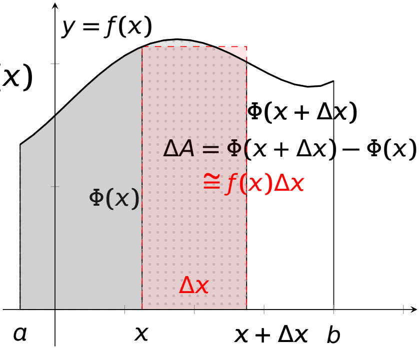

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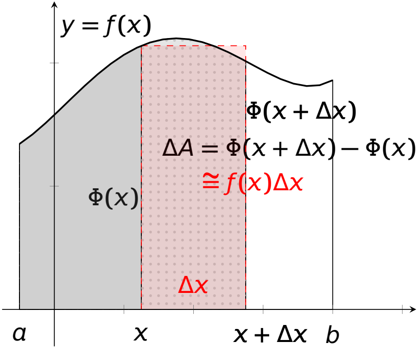

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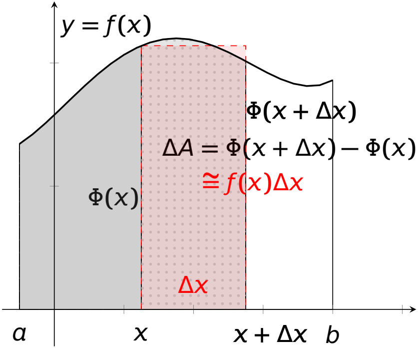

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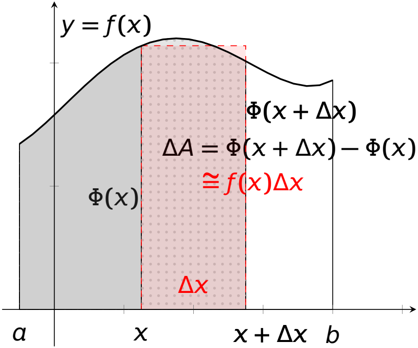

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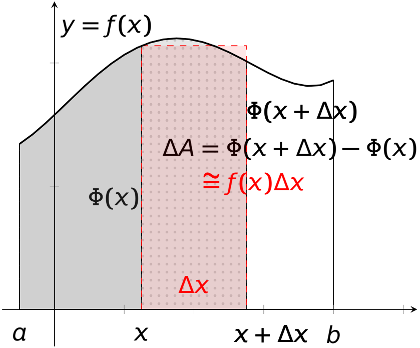

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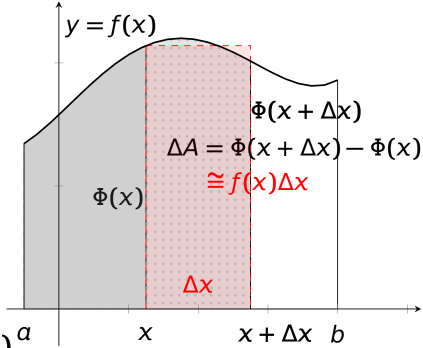

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例 $\left[\int_x^0 e^{-t}\sin(t^2)dt\right]' = \underline{\qquad\qquad\qquad}$

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例 $\left[\int_x^0 e^{-t}\sin(t^2)dt\right]' = \underline{\hspace{4cm}}$

解

$$\left[\int_x^0 e^{-t}\sin(t^2)dt\right]' = \left[-\int_0^x e^{-t}\sin(t^2)dt\right]'$$

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$$\because \quad \left[\int_x^{-2} e^{\sin t}dt\right]' = \left[-\int_{-2}^x e^{\sin t}dt\right]'$$

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注：

$$\left[\int_a^{\varphi(x)} f(t)dt\right]' =$$

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$\left[\int_1^{x^2}\cos t dt\right]' =$ ____________ ; $\left[\int_{2x}^{-1}\sqrt{1+t^2}dt\right]' =$ ____________ .

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We are here now...

1. 变上限的定积分

2. 微积分基本定理：牛顿—莱布尼茨公式

牛顿—莱布尼茨公式

$$\int_a^b f(x)dx =$$

牛顿—莱布尼茨公式

$$\int_a^b f(x)dx = F(b) - F(a)$$

牛顿—莱布尼茨公式

$$\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b.$$

牛顿—莱布尼茨公式

设 $f(x)$ 在区间 $[a, b]$ 上连续，$F(x)$ 是 $f(x)$ 任意一个原函数，则

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$$\begin{aligned}\therefore\ F(b) - F(a) &= (\Phi(b) + C) - (\Phi(a) + C)\\ &= \Phi(b) - \Phi(a)\\ &= \int_a^b f(t)dt - \int_a^a f(t)dt = \int_a^b f(t)dt\end{aligned}$$

例 计算定积分

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

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$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

解

$$\int_0^1 x^2 dx = \frac{1}{3}x^3\Big|_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$$

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练习 计算定积分

$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}}dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

练习 计算定积分

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提示：先求出 $\int(2x-5)dx$，$\int \frac{1}{\sqrt{x}}dx$，$\int \frac{dx}{\sqrt{1-x^2}}$

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例 计算定积分 $\int_0^2 |1-x|dx$.

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例 计算定积分 $\int_0^2 |1-x|dx$.

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$$= \int_0^1 |1-x|dx + \int_1^2 |1-x|dx = \int_0^1 (1-x)dx +$$

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$$= (x - \frac{1}{2}x^2)$$

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练习 计算定积分 $\int_0^3 |2-x|dx$

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解
$$\int_0^3 |2-x|dx$$
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解 $\displaystyle\int_0^2 |1-x|dx$

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练习 计算定积分 $\int_0^3 |2-x|dx$

解 $\displaystyle\int_0^3 |2-x|dx$

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练习 计算定积分 $\int_0^3 |2-x|dx$

解 $\int_0^3 |2-x|dx$

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$$= (2x - \frac{1}{2}x^2) \quad + (\frac{1}{2}x^2 - 2x)$$

例 计算定积分 $\int_0^2 |1-x|dx$.

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