第 5 章 c：定积分的换元积分法与分部积分法

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Outline

- 求定积分 $\int_a^b f(x)dx$ 可分成两步：
  1. 求出不定积分 $\int f(x)dx = F(x) + C$

     （方法：直接积分法、换元积分法、分部积分法（第四章））
  2. $\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$

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  2. $\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$
- 在实际操作中，两步可合成一步：
  - 以换元积分法、分部积分法为例说明

凑微分：例

例 1 计算定积分 $\int_0^{\frac{\pi}{2}} \sin^2 x \cos x dx$

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解法二 $\int_0^{\frac{\pi}{2}} \sin^2 x \cos x dx$

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$$\int_0^{\frac{\pi}{2}} \cos^2 x \sin x dx = -\int_0^{\frac{\pi}{2}} \cos^2 x d\cos x$$

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所以

$$\lim_{x\to 0^+}\frac{\int_0^x \sqrt{x-t}e^t dt}{\sqrt{x^3}} = \lim_{x\to 0^+}\frac{e^x\int_0^x \sqrt{u}e^{-u}du}{\sqrt{x^3}}$$

例 4 求极限 $\lim\limits_{x\to 0^+}\dfrac{\int_0^x \sqrt{x-t}e^t dt}{\sqrt{x^3}}$.

解 注意到

$$\int_0^x \sqrt{x-t}e^t dt \xlongequal{u=x-t} \int_x^0 \sqrt{u}e^{x-u}d(x-u) = \int_x^0 \sqrt{u}e^x e^{-u}(-du)$$

$$= e^x \int_0^x \sqrt{u}e^{-u}du.$$

所以

$$\lim_{x\to 0^+}\frac{\int_0^x \sqrt{x-t}e^t dt}{\sqrt{x^3}} = \lim_{x\to 0^+}\frac{e^x \int_0^x \sqrt{u}e^{-u}du}{\sqrt{x^3}} = \lim_{x\to 0^+}\frac{\int_0^x \sqrt{u}e^{-u}du}{\sqrt{x^3}}$$

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$$= \lim_{x\to 0^+}\frac{\left[\int_0^x \sqrt{u}e^{-u}du\right]'}{(x^{\frac{3}{2}})'}$$

例 4 求极限 $\lim_{x\to0^+}\frac{\int_0^x\sqrt{x-t}e^t dt}{\sqrt{x^3}}$.

解 注意到

$$\int_0^x\sqrt{x-t}e^t dt \xlongequal{u=x-t} \int_x^0\sqrt{u}e^{x-u}d(x-u)=\int_x^0\sqrt{u}e^xe^{-u}(-du)$$
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所以

$$\lim_{x\to0^+}\frac{\int_0^x\sqrt{x-t}e^t dt}{\sqrt{x^3}}=\lim_{x\to0^+}\frac{e^x\int_0^x\sqrt{u}e^{-u}du}{\sqrt{x^3}}=\lim_{x\to0^+}\frac{\int_0^x\sqrt{u}e^{-u}du}{\sqrt{x^3}}$$
$$=\lim_{x\to0^+}\frac{\left[\int_0^x\sqrt{u}e^{-u}du\right]'}{(x^{\frac{3}{2}})'}=\lim_{x\to0^+}\frac{\sqrt{x}e^{-x}}{\frac{3}{2}x^{\frac{1}{2}}}$$

例 4 求极限 $\lim\limits_{x\to0^+}\dfrac{\int_0^x\sqrt{x-t}e^t dt}{\sqrt{x^3}}$.

解 注意到

$$\int_0^x \sqrt{x-t}e^t dt \xlongequal{u=x-t} \int_x^0 \sqrt{u}e^{x-u}d(x-u) = \int_x^0 \sqrt{u}e^x e^{-u}(-du)$$

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分部积分法

- 不定积分的分部积分：

$$\int u dv = uv - \int v du$$

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分部积分法：例

例 1 计算定积分 $\int_0^{\frac{1}{2}} \arcsin x dx$

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$$\int \arcsin x dx =$$

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$$= \left(\frac{1}{2} \cdot \frac{\pi}{6} + \sqrt{3/4}\right) - (0+1) = \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1$$

解法二

$$\int_0^{\frac{1}{2}} \arcsin x dx = x \arcsin x \quad - \int \quad x d \arcsin x$$

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$$= \left(\frac{1}{2} \cdot \frac{\pi}{6} - 0\right)$$

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$$= \left(\frac{1}{2} \cdot \frac{\pi}{6} - 0 \right) \qquad\qquad \frac{1}{\sqrt{1-x^2}} dx$$

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$$= \frac{\pi}{12} + u^{1/2}\Big|_1^{\frac{3}{4}} =$$

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$$\int_0^{\frac{1}{2}} \arcsin x dx = x \arcsin x \Big|_0^{\frac{1}{2}} - \int_0^{\frac{1}{2}} x d \arcsin x$$

$$= \left(\frac{1}{2} \cdot \frac{\pi}{6} - 0\right) - \int_0^{\frac{1}{2}} x \cdot \frac{1}{\sqrt{1-x^2}} dx = \frac{\pi}{12} - \frac{1}{2} \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx^2$$

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例 2 计算定积分 $\int_0^1 xe^{-x}dx$

解
$$\int_0^1 xe^{-x}dx =$$

例 2 计算定积分 $\int_0^1 xe^{-x}dx$

解
$$\int_0^1 xe^{-x}dx = -\int_0^1 xde^{-x} =$$

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$$= -\left([e^{-1} - 0] - \qquad\qquad \right)$$

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$$= -\left([e^{-1} - 0] - (-e^{-x})\Big|_0^1\right)$$

$$= -\left(e^{-1} + e^{-x}\Big|_0^1\right)$$

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$$\int_0^1 xe^{-x}dx = -\int_0^1 xde^{-x} = -\left(xe^{-x}\Big|_0^1 - \int_0^1 e^{-x}dx\right)$$
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$$= -\left(e^{-1}+e^{-x}\Big|_0^1\right) = -\left(e^{-1}+e^{-1}-1\right) = 1-\frac{2}{e}$$

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例 3 计算定积分 $\int_0^{\frac{\pi}{2}} x\sin x dx$

解
$$\int_0^{\frac{\pi}{2}} x\sin x dx =$$

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例 3 计算定积分 $\int_0^{\frac{\pi}{2}} x\sin x dx$

解
$$\int_0^{\frac{\pi}{2}} x\sin x dx = -\int_0^{\frac{\pi}{2}} xd\cos x = -\left(x\cos x \quad - \int \quad \cos x dx\right)$$

例 2 计算定积分 $\int_0^1 xe^{-x}dx$

解 $$\int_0^1 xe^{-x}dx = -\int_0^1 x de^{-x} = -\left(xe^{-x}\Big|_0^1 - \int_0^1 e^{-x}dx\right)$$

$$= -\left([e^{-1} - 0] - (-e^{-x})\Big|_0^1\right)$$

$$= -\left(e^{-1} + e^{-x}\Big|_0^1\right) = -\left(e^{-1} + e^{-1} - 1\right) = 1 - \frac{2}{e}$$

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