### 第 12 章 e: 傅里叶级数

数学系 梁卓滨

2016-2017 **学年** II



#### Outline

1. 傅里叶级数的概念

2. 周期为 2π 的周期函数的傅里叶级数

3. 一般周期函数的傅里叶级数



#### We are here now...

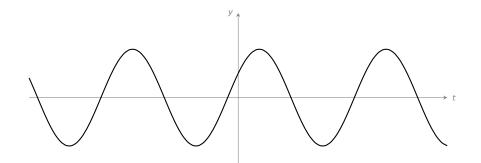
1. 傅里叶级数的概念

2. 周期为 2π 的周期函数的傅里叶级数

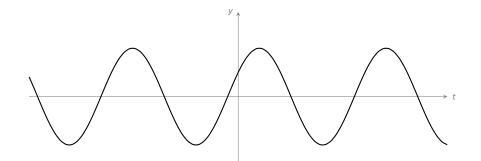
3. 一般周期函数的傅里叶级数



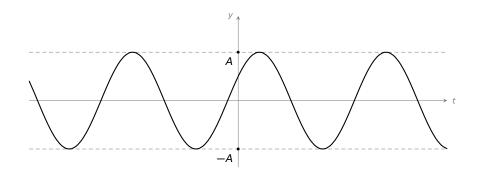
正弦函数  $y = A \sin(\omega t + \varphi)$ 



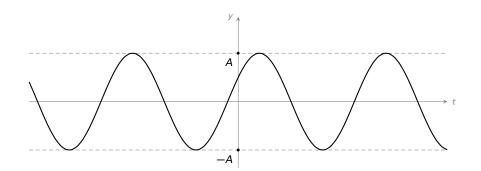
正弦函数 
$$y = A \sin(\omega t + \varphi)$$
 ( $t$ : 时间;



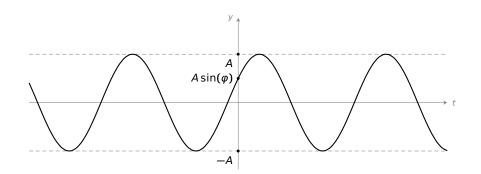
正弦函数 
$$y = A \sin(\omega t + \varphi)$$
 ( $t$ : 时间;  $A$ : 振幅;



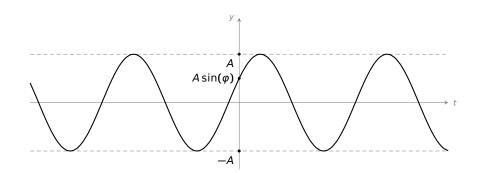
正弦函数  $y = A \sin(\omega t + \varphi)$  (t: 时间; A: 振幅;  $\varphi$ : 初相;



正弦函数  $y = A \sin(\omega t + \varphi)$  (t: 时间; A: 振幅;  $\varphi$ : 初相;

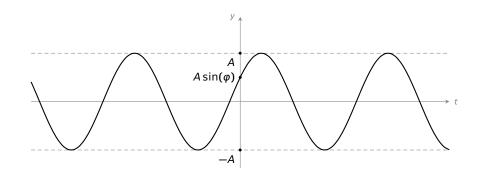


正弦函数  $y = A \sin(\omega t + \varphi)$  (t: 时间; A: 振幅;  $\varphi$ : 初相;  $\omega$ : 频率)



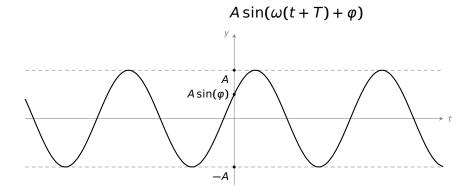
正弦函数  $y = A \sin(\omega t + \varphi)$  (t: 时间; A: 振幅;  $\varphi$ : 初相;  $\omega$ : 频率)

具有周期  $T = \frac{2\pi}{\omega}$ 



正弦函数  $y = A \sin(\omega t + \varphi)$  (t: 时间; A: 振幅;  $\varphi$ : 初相;  $\omega$ : 频率)

具有周期  $T = \frac{2\pi}{\omega}$ ,也就是

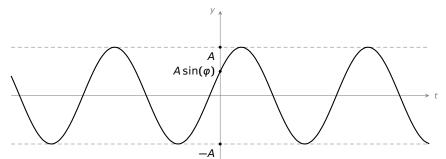




正弦函数  $y = A \sin(\omega t + \varphi)$  (t: 时间; A: 振幅;  $\varphi$ : 初相;  $\omega$ : 频率)

具有周期  $T = \frac{2\pi}{\omega}$ ,也就是

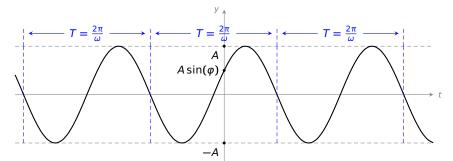
$$A\sin(\omega t + \varphi) = A\sin(\omega(t+T) + \varphi)$$



正弦函数  $y = A \sin(\omega t + \varphi)$  (t: 时间; A: 振幅;  $\varphi$ : 初相;  $\omega$ : 频率)

具有周期  $T = \frac{2\pi}{\omega}$ ,也就是

$$A\sin(\omega t + \varphi) = A\sin(\omega(t+T) + \varphi)$$

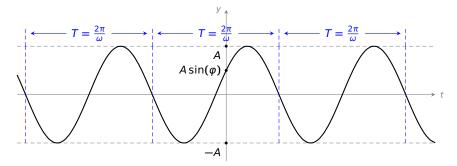




正弦函数  $y = A \sin(\omega t + \varphi)$  (t: 时间; A: 振幅;  $\varphi$ : 初相;  $\omega$ : 频率)

具有周期  $T = \frac{2\pi}{\omega}$ ,也就是

$$A\sin(\omega t + \varphi) = A\sin(\omega(t+T) + \varphi)$$



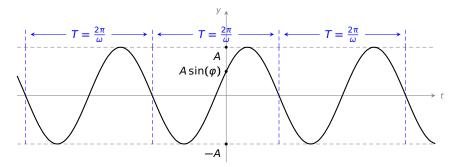
设 n 为正整数,正弦函数  $y = A_n \sin(n\omega t + \varphi_n)$ 



正弦函数  $y = A \sin(\omega t + \varphi)$  (t: 时间; A: 振幅;  $\varphi$ : 初相;  $\omega$ : 频率)

具有周期  $T = \frac{2\pi}{\omega}$ ,也就是

$$A\sin(\omega t + \varphi) = A\sin(\omega(t+T) + \varphi)$$



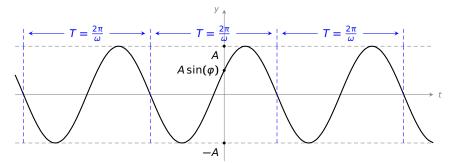
设 n 为正整数,正弦函数  $y = A_n \sin(n\omega t + \varphi_n)$ 的最小周期是  $\frac{2\pi}{n\omega}$ ,



正弦函数  $y = A \sin(\omega t + \varphi)$  (t: 时间; A: 振幅;  $\varphi$ : 初相;  $\omega$ : 频率)

具有周期 
$$T = \frac{2\pi}{\omega}$$
,也就是

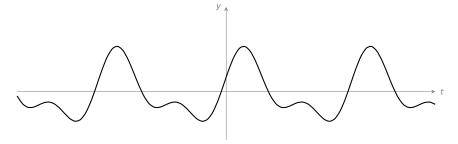
$$A\sin(\omega t + \varphi) = A\sin(\omega(t+T) + \varphi)$$

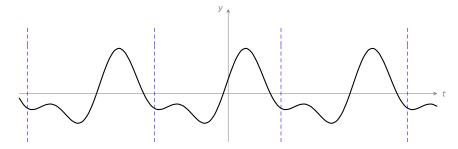


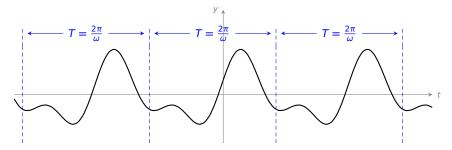
设 n 为正整数,正弦函数  $y = A_n \sin(n\omega t + \varphi_n)$ 的最小周期是  $\frac{2\pi}{n\omega}$ ,显

然  $T = \frac{2\pi}{4}$  也是周期

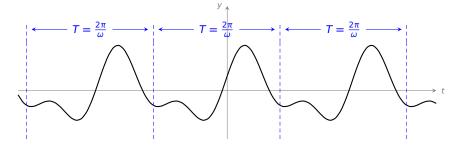








假设 f(t) 是定义域为  $\mathbb{R}$  的周期函数,周期也是  $T = \frac{2\pi}{\omega}$ 。

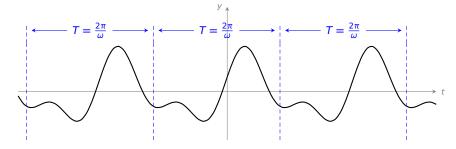


问题 是否有如下展开

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin(n\omega t + \varphi_n)$$



假设 f(t) 是定义域为  $\mathbb{R}$  的周期函数,周期也是  $T = \frac{2\pi}{\omega}$ 。



问题 是否有如下展开

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin(n\omega t + \varphi_n)$$

注 在电工学中,上述展开称为谐波分析; A<sub>0</sub> 称为直流分量;

 $A_n \sin(n\omega t + \varphi_n)$  称为 n 次谐波



设  $T = \frac{2\pi}{\omega} = 2l$ ,

设  $T = \frac{2\pi}{\omega} = 2l$ , 故区间 [-l, l] 是 f(t) 的一个完整周期。

设 
$$T = \frac{2\pi}{\omega} = 2l$$
, 故区间  $[-l, l]$  是  $f(t)$  的一个完整周期。

注意到 
$$\omega = \frac{\pi}{l}$$
,

设 
$$T = \frac{2\pi}{\omega} = 2l$$
,故区间  $[-l, l]$  是  $f(t)$  的一个完整周期。

注意到 
$$\omega = \frac{\pi}{l}$$
,所以

$$A_n \sin(n\omega t + \varphi_n) = A_n \sin(\frac{n\pi t}{l} + \varphi_n)$$

设 
$$T = \frac{2\pi}{\omega} = 2l$$
,故区间  $[-l, l]$  是  $f(t)$  的一个完整周期。

注意到 
$$\omega = \frac{\pi}{l}$$
,所以

$$A_n \sin(n\omega t + \varphi_n) = A_n \sin(\frac{n\pi t}{l} + \varphi_n)$$

$$\sin \varphi_n \cos \frac{n\pi t}{l} + \cos \varphi_n \sin \frac{n\pi t}{l}$$

设 
$$T = \frac{2\pi}{\omega} = 2l$$
,故区间  $[-l, l]$  是  $f(t)$  的一个完整周期。

注意到 
$$\omega = \frac{\pi}{l}$$
,所以

$$A_n \sin(n\omega t + \varphi_n) = A_n \sin(\frac{n\pi t}{l} + \varphi_n)$$

$$= A_n \left[ \sin \varphi_n \cos \frac{n\pi t}{l} + \cos \varphi_n \sin \frac{n\pi t}{l} \right]$$

设 
$$T = \frac{2\pi}{\omega} = 2l$$
,故区间  $[-l, l]$  是  $f(t)$  的一个完整周期。

注意到 
$$\omega = \frac{\pi}{l}$$
,所以

$$A_n \sin(n\omega t + \varphi_n) = A_n \sin(\frac{n\pi t}{l} + \varphi_n)$$

$$= A_n \left[ \sin \varphi_n \cos \frac{n\pi t}{l} + \cos \varphi_n \sin \frac{n\pi t}{l} \right]$$

$$=: a_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l}$$



设  $T = \frac{2\pi}{l} = 2l$ ,故区间 [-l, l] 是 f(t) 的一个完整周期。

注意到 
$$\omega = \frac{\pi}{7}$$
,所以

$$A_n \sin(n\omega t + \varphi_n) = A_n \sin(\frac{n\pi t}{l} + \varphi_n)$$

$$= A_n \left[ \sin \varphi_n \cos \frac{n\pi t}{l} + \cos \varphi_n \sin \frac{n\pi t}{l} \right]$$

$$=: a_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l}$$

这时
$$f(t) = A_0 + \sum_{n=0}^{\infty} A_n \sin(n\omega t + \varphi_n)$$



设  $T = \frac{2\pi}{\Omega} = 2l$ ,故区间 [-l, l] 是 f(t) 的一个完整周期。

 $= A_n \left[ \sin \varphi_n \cos \frac{n\pi t}{t} + \cos \varphi_n \sin \frac{n\pi t}{t} \right]$ 

 $=: a_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l}$ 

 $f(t) = A_0 + \sum_{n=0}^{\infty} A_n \sin(n\omega t + \varphi_n) \qquad \sum_{n=0}^{\infty} \left( a_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l} \right)$ 

注意到 
$$\omega = \frac{\pi}{7}$$
,所以

注息到 
$$\omega = \overline{l}$$
, 所

$$A_n \sin(n\omega t + \varphi_n) = A_n \sin(\frac{n\pi t}{l} + \varphi_n)$$

$$\frac{n}{l}$$
,所



这时

设  $T = \frac{2\pi}{t} = 2l$ ,故区间 [-l, l] 是 f(t) 的一个完整周期。

注意到 
$$\omega = \frac{\pi}{7}$$
,所以

注息到 
$$\omega = \overline{l}$$
, 所

$$A_n \sin(n\omega t + \varphi_n) = A_n \sin(\frac{n\pi t}{l} + \varphi_n)$$

$$= A_n \left[ \sin \varphi_n \cos \frac{n\pi t}{l} + \cos \varphi_n \sin \frac{n\pi t}{l} \right]$$

$$=: \alpha_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l}$$

 $f(t) = A_0 + \sum_{n=0}^{\infty} A_n \sin(n\omega t + \varphi_n) = \frac{a_0}{2} + \sum_{n=0}^{\infty} \left( a_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l} \right)$ 



设  $T = \frac{2\pi}{t} = 2l$ ,故区间 [-l, l] 是 f(t) 的一个完整周期。

注意到 
$$\omega = \frac{\pi}{7}$$
,所以

注息到 
$$\omega = \overline{l}$$
 ,则

$$A_n \sin(n\omega t + \varphi_n) = A_n \sin(\frac{n\pi t}{l} + \varphi_n)$$

$$= A_n \left[ \sin \varphi_n \cos \frac{n\pi t}{l} + \cos \varphi_n \sin \frac{n\pi t}{l} \right]$$

$$=: a_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l}$$

$$=: a_n \cos \frac{1}{l} + b_n \sin \frac{1}{l}$$
 这时

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin(n\omega t + \varphi_n) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l} \right)$$

以下不妨先设周期  $T = 2\pi (l = \pi)$ 。 f(x) 的周期区间为  $[-\pi, \pi]$ ,

设  $T = \frac{2\pi}{4} = 2l$ ,故区间 [-l, l] 是 f(t) 的一个完整周期。

注意到  $\omega = \frac{\pi}{7}$ , 所以

江思到 
$$\omega = \overline{l}$$
,凡

 $A_n \sin(n\omega t + \varphi_n) = A_n \sin(\frac{n\pi t}{t} + \varphi_n)$ 

 $= A_n \left[ \sin \varphi_n \cos \frac{n\pi t}{\iota} + \cos \varphi_n \sin \frac{n\pi t}{\iota} \right]$  $=: a_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l}$ 

 $f(t) = A_0 + \sum_{n=0}^{\infty} A_n \sin(n\omega t + \varphi_n) = \frac{a_0}{2} + \sum_{n=0}^{\infty} \left( a_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l} \right)$ 

- 以下不妨先设周期  $T = 2\pi (l = \pi)$ 。 f(x) 的周期区间为  $[-\pi, \pi]$ ,相  $f(x) = \frac{a_0}{2} + \sum_{n=0}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$

应的展开为

#### We are here now...

1. 傅里叶级数的概念

2. 周期为 2π 的周期函数的傅里叶级数

3. 一般周期函数的傅里叶级数



#### 性质 三角函数系

1,  $\cos x$ ,  $\sin x$ ,  $\cos 2x$ ,  $\sin 2x$ , ...,  $\cos nx$ ,  $\sin nx$ , ...

在区间  $[-\pi, \pi]$  上正交。

1,  $\cos x$ ,  $\sin x$ ,  $\cos 2x$ ,  $\sin 2x$ , ...,  $\cos nx$ ,  $\sin nx$ , ...

1,  $\cos x$ ,  $\sin x$ ,  $\cos 2x$ ,  $\sin 2x$ , ...,  $\cos nx$ ,  $\sin nx$ , ...

$$\int_{-\pi}^{\pi} \cos nx dx = 0, \qquad \int_{-\pi}^{\pi} \sin nx dx = 0 \qquad (n = 1, 2, 3, \dots)$$

1,  $\cos x$ ,  $\sin x$ ,  $\cos 2x$ ,  $\sin 2x$ , ...,  $\cos nx$ ,  $\sin nx$ , ...

$$\int_{-\pi}^{\pi} \cos nx dx = 0, \qquad \int_{-\pi}^{\pi} \sin nx dx = 0 \qquad (n = 1, 2, 3, \dots)$$

$$\int_{-\pi}^{\pi} \sin kx \cdot \cos nx dx = 0 \qquad (k, n = 1, 2, 3, \dots, k \neq n)$$

1,  $\cos x$ ,  $\sin x$ ,  $\cos 2x$ ,  $\sin 2x$ ,  $\cdots$ ,  $\cos nx$ ,  $\sin nx$ ,  $\cdots$ 

$$\int_{-\pi}^{\pi} \cos nx \, dx = 0, \qquad \int_{-\pi}^{\pi} \sin nx \, dx = 0 \qquad (n = 1, 2, 3, \dots)$$

$$\int_{-\pi}^{\pi} \sin kx \cdot \cos nx \, dx = 0 \qquad (k, n = 1, 2, 3, \dots, k \neq n)$$

$$\int_{-\pi}^{\pi} \sin kx \cdot \sin nx \, dx = 0 \qquad (k, n = 1, 2, 3, \dots, k \neq n)$$

1,  $\cos x$ ,  $\sin x$ ,  $\cos 2x$ ,  $\sin 2x$ , ...,  $\cos nx$ ,  $\sin nx$ , ...

$$\int_{-\pi}^{\pi} \cos nx \, dx = 0, \qquad \int_{-\pi}^{\pi} \sin nx \, dx = 0 \qquad (n = 1, 2, 3, \dots)$$

$$\int_{-\pi}^{\pi} \sin kx \cdot \cos nx \, dx = 0 \qquad (k, n = 1, 2, 3, \dots, k \neq n)$$

$$\int_{-\pi}^{\pi} \sin kx \cdot \sin nx \, dx = 0 \qquad (k, n = 1, 2, 3, \dots, k \neq n)$$

$$\int_{-\pi}^{\pi} \cos kx \cdot \cos nx \, dx = 0 \qquad (k, n = 1, 2, 3, \dots, k \neq n)$$

1,  $\cos x$ ,  $\sin x$ ,  $\cos 2x$ ,  $\sin 2x$ , ...,  $\cos nx$ ,  $\sin nx$ , ...

在区间  $[-\pi, \pi]$  上正交。即上述任意相异两个函数的乘积,在  $[-\pi, \pi]$  上的积分为零:

$$\int_{-\pi}^{\pi} \cos nx dx = 0, \qquad \int_{-\pi}^{\pi} \sin nx dx = 0 \qquad (n = 1, 2, 3, \dots)$$

$$\int_{-\pi}^{\pi} \sin kx \cdot \cos nx dx = 0 \qquad (k, n = 1, 2, 3, \dots, k \neq n)$$

$$\int_{-\pi}^{\pi} \sin kx \cdot \sin nx dx = 0 \qquad (k, n = 1, 2, 3, \dots, k \neq n)$$

$$\int_{-\pi}^{\pi} \cos kx \cdot \cos nx dx = 0 \qquad (k, n = 1, 2, 3, \dots, k \neq n)$$

另外

$$\int_{-\pi}^{\pi} \sin^2 nx dx = \int_{-\pi}^{\pi} \cos^2 nx dx = \pi \qquad (n = 1, 2, 3, \dots)$$



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 1, 2, 3, \dots)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

则

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 1, 2, 3, \dots)$$

"形式推导" (1) 当  $n = 1, 2, 3, \cdots$  时,

$$\int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

则

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 1, 2, 3, \dots)$$

"形式推导" (1) 当  $n = 1, 2, 3, \cdots$  时,

$$\int_{-\pi}^{\pi} f(x) \cos nx dx \qquad \left[ \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos kx + b_k \sin kx \right) \right] \cos nx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

则

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 1, 2, 3, \dots)$$

"形式推导" (1) 当  $n=1, 2, 3, \cdots$  时,

 $\int_{-\pi}^{\pi} f(x) \cos nx dx = \int_{-\pi}^{\pi} \left[ \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos kx + b_k \sin kx \right) \right] \cos nx dx$ 



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

则

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 1, 2, 3, \dots)$$

"形式推导" (1) 当  $n = 1, 2, 3, \cdots$  时,

 $\int_{-\pi}^{\pi} f(x) \cos nx dx = \int_{-\pi}^{\pi} \left[ \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos kx + b_k \sin kx \right) \right] \cos nx dx$ 

$$= \int_{-\pi}^{\pi} a_n \cos nx \cdot \cos nx dx$$



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

则

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \cdots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 1, 2, 3, \cdots)$$

"形式推导" (1) 当  $n = 1, 2, 3, \cdots$  时,

 $\int_{-\pi}^{\pi} f(x) \cos nx dx = \int_{-\pi}^{\pi} \left[ \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos kx + b_k \sin kx \right) \right] \cos nx dx$ 

$$= \int_{-\pi}^{\pi} a_n \cos nx \cdot \cos nx dx = \pi a_n$$



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 1, 2, 3, \dots)$$

"形式推导" (2) 当 
$$n = 1, 2, 3, \cdots$$
 时,

$$\int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 1, 2, 3, \dots)$$

"形式推导" (2) 当 
$$n = 1, 2, 3, \cdots$$
 时,

$$\int_{-\pi}^{\pi} f(x) \sin nx dx \qquad \left[ \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos kx + b_k \sin kx \right) \right] \sin nx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 1, 2, 3, \dots)$$

"形式推导" 
$$(2)$$
 当  $n=1, 2, 3, \cdots$  时,

$$\int_{-\pi}^{\pi} f(x) \sin nx dx = \int_{-\pi}^{\pi} \left[ \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos kx + b_k \sin kx \right) \right] \sin nx dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

则

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 1, 2, 3, \dots)$$

"形式推导" (2) 当  $n=1, 2, 3, \cdots$  时,

$$\int_{-\pi}^{\pi} f(x) \sin nx dx = \int_{-\pi}^{\pi} \left[ \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos kx + b_k \sin kx \right) \right] \sin nx dx$$
$$= \int_{-\pi}^{\pi} b_n \sin nx \cdot \sin nx dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

则

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 1, 2, 3, \dots)$$

"形式推导" (2) 当  $n=1, 2, 3, \cdots$  时,

$$\int_{-\pi}^{\pi} f(x) \sin nx dx = \int_{-\pi}^{\pi} \left[ \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos kx + b_k \sin kx \right) \right] \sin nx dx$$

$$= \int_{-\pi}^{\pi} b_n \sin nx \cdot \sin nx dx = \pi b_n$$



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$\int_{-\pi}^{\pi} f(x) dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

则

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

"形式推导" (3)

$$\int_{-\pi}^{\pi} f(x)dx = \int_{-\pi}^{\pi} \left[ \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos kx + b_k \sin kx \right) \right] dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

则

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

"形式推导" (3)

$$\int_{-\pi}^{\pi} f(x)dx = \int_{-\pi}^{\pi} \left[ \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos kx + b_k \sin kx \right) \right] dx$$
$$= \int_{-\pi}^{\pi} \frac{a_0}{2} dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

则

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

"形式推导" (3)

$$\int_{-\pi}^{\pi} f(x)dx = \int_{-\pi}^{\pi} \left[ \frac{a_0}{2} + \sum_{k=1}^{\infty} \left( a_k \cos kx + b_k \sin kx \right) \right] dx$$
$$= \int_{-\pi}^{\pi} \frac{a_0}{2} dx = \pi a_0$$

## 定义 f(x) 的傅里叶级数定义为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

其中

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$
  $(n = 0, 1, 2, 3, \cdots)$ 

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

## 定义 f(x) 的傅里叶级数定义为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

其中

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$
  $(n = 0, 1, 2, 3, \cdots)$ 

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

问题 何时成立 
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$
?



## 定义 f(x) 的傅里叶级数定义为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

其中

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$
  $(n = 0, 1, 2, 3, \dots)$ 

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

问题 何时成立 
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$
?

定理(收敛定理, 狄利克雷充分条件)



- 1. 在一个周期内连续或只有有限个第一类间断点;
- 2. 在一个周期内至多只有有限个极值点,

- 1. 在一个周期内连续或只有有限个第一类间断点;
- 2. 在一个周期内至多只有有限个极值点,

那么 f(x) 的傅里叶级数收敛,并且

- 1. 在一个周期内连续或只有有限个第一类间断点;
- 2. 在一个周期内至多只有有限个极值点,

那么 f(x) 的傅里叶级数收敛,并且

当 x 是 f(x) 的连续点时,

• 当 x 是 f(x) 的间断点时,

- 1. 在一个周期内连续或只有有限个第一类间断点;
- 2. 在一个周期内至多只有有限个极值点,

那么 f(x) 的傅里叶级数收敛,并且

• 当 x 是 f(x) 的连续点时,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

• 当 x 是 f(x) 的间断点时,

- 1. 在一个周期内连续或只有有限个第一类间断点;
- 2. 在一个周期内至多只有有限个极值点,

那么 f(x) 的傅里叶级数收敛,并且

当 x 是 f(x) 的连续点时,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

• 当 x 是 f(x) 的间断点时,

$$\frac{1}{2} \left[ f(x^{-}) + f(x^{+}) \right] = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

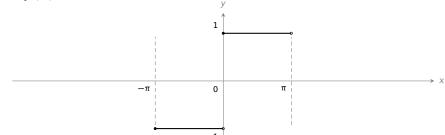


$$f(x) = \begin{cases} -1, & -\pi \le x < 0, \\ 1, & 0 \le x < \pi. \end{cases}$$

求出 f(x) 的傅里叶级数。

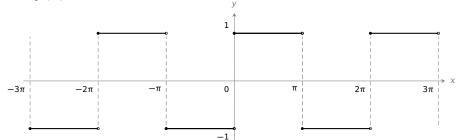
$$f(x) = \begin{cases} -1, & -\pi \le x < 0, \\ 1, & 0 \le x < \pi. \end{cases}$$

求出 f(x) 的傅里叶级数。



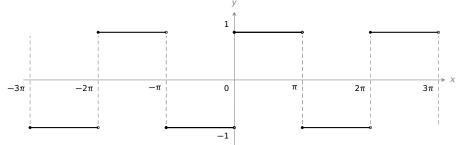
$$f(x) = \begin{cases} -1, & -\pi \le x < 0, \\ 1, & 0 \le x < \pi. \end{cases}$$

求出 f(x) 的傅里叶级数。



$$f(x) = \begin{cases} -1, & -\pi \le x < 0, \\ 1, & 0 \le x < \pi. \end{cases}$$

求出 f(x) 的傅里叶级数。



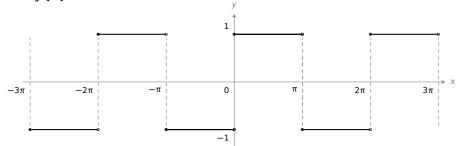
解 计算傅里叶系数如下:

 $a_n$ 



$$f(x) = \begin{cases} -1, & -\pi \le x < 0, \\ 1, & 0 \le x < \pi. \end{cases}$$

求出 f(x) 的傅里叶级数。

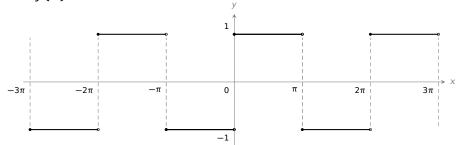


解 计算傅里叶系数如下:

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$f(x) = \begin{cases} -1, & -\pi \le x < 0, \\ 1, & 0 \le x < \pi. \end{cases}$$

求出 f(x) 的傅里叶级数。



解 计算傅里叶系数如下:

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{3}} 0$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{3}} 0,$$

 $b_n$ 

$$\alpha_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{\pi} \text{ med}} 0,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{4}} 0,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{3}} 0,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} \sin nx dx$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{6}} 0,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} \sin nx dx$$

$$2 \qquad \cos nx |_{\pi}$$

$$= \frac{2}{\pi} \cdot (-1) \cdot \frac{\cos nx}{n} \Big|_{0}^{\pi}$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{6}} 0,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} \sin nx dx$$
$$= \frac{2}{\pi} \cdot (-1) \cdot \frac{\cos nx}{n} \Big|_{0}^{\pi} = \frac{2}{n\pi} \Big[ 1 - \cos n\pi \Big]$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \stackrel{\text{fight}}{===} 0,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} \sin nx dx$$
$$= \frac{2}{\pi} \cdot (-1) \cdot \frac{\cos nx}{n} \Big|_{0}^{\pi} = \frac{2}{n\pi} \Big[ 1 - \cos n\pi \Big] = \frac{2}{n\pi} \Big[ 1 - (-1)^n \Big]$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \stackrel{\text{fight}}{===} 0,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} \sin nx dx$$
$$= \frac{2}{\pi} \cdot (-1) \cdot \frac{\cos nx}{n} \Big|_{0}^{\pi} = \frac{2}{n\pi} \Big[ 1 - \cos n\pi \Big] = \frac{2}{n\pi} \Big[ 1 - (-1)^n \Big]$$

$$= \left\{ \begin{array}{c} n = 1, 3, 5, \cdots \\ n = 2, 4, 6, \cdots . \end{array} \right.$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \stackrel{\text{fight}}{===} 0,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} \sin nx dx$$
$$= \frac{2}{\pi} \cdot (-1) \cdot \frac{\cos nx}{n} \Big|_{0}^{\pi} = \frac{2}{n\pi} \Big[ 1 - \cos n\pi \Big] = \frac{2}{n\pi} \Big[ 1 - (-1)^n \Big]$$

$$= \begin{cases} n = 1, 3, 5, \cdots \\ 0, n = 2, 4, 6, \cdots \end{cases}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{6}} 0,$$

$$b_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} \sin nx dx$$

$$= \frac{2}{\pi} \cdot (-1) \cdot \frac{\cos nx}{n} \Big|_{0}^{\pi} = \frac{2}{n\pi} \Big[ 1 - \cos n\pi \Big] = \frac{2}{n\pi} \Big[ 1 - (-1)^{n} \Big]$$

$$= \begin{cases} \frac{4}{n\pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \stackrel{\text{fight}}{===} 0,$$

$$b_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} \sin nx dx$$

$$= \frac{2}{\pi} \cdot (-1) \cdot \frac{\cos nx}{n} \Big|_{0}^{\pi} = \frac{2}{n\pi} \Big[ 1 - \cos n\pi \Big] = \frac{2}{n\pi} \Big[ 1 - (-1)^{n} \Big]$$

$$= \begin{cases} \frac{4}{n\pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

$$\frac{a_0}{2} + \sum_{n=0}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\text{fight}} 0,$$

$$b_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} \sin nx dx$$

$$= \frac{2}{\pi} \cdot (-1) \cdot \frac{\cos nx}{n} \Big|_{0}^{\pi} = \frac{2}{n\pi} \Big[ 1 - \cos n\pi \Big] = \frac{2}{n\pi} \Big[ 1 - (-1)^{n} \Big]$$

$$= \begin{cases} \frac{4}{n\pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

所以傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right) = \sum_{n=1}^{\infty} b_n \sin nx$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{6}} 0,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} \sin nx dx$$
$$= \frac{2}{\pi} \cdot (-1) \cdot \frac{\cos nx}{n} \Big|_{0}^{\pi} = \frac{2}{n\pi} \Big[ 1 - \cos n\pi \Big] = \frac{2}{n\pi} \Big[ 1 - (-1)^n \Big]$$

 $= \begin{cases} \frac{4}{n\pi}, & n = 1, 3, 5, \cdots \\ 0, & n = 2, 4, 6, \cdots \end{cases}$ 

所以傅里叶级数为  $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right) = \sum_{n=1}^{\infty} b_n \sin nx$ 

$$= \frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$$

R 12 草 e: 傳里叶级委

注 1 f(x) 的傅里叶级数是  $\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$ 

注 
$$1 f(x)$$
 的傅里叶级数是  $\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$ , 利用

当 x ≠ nπ 时,

• 当  $x = n\pi$  是,

注 
$$1 f(x)$$
 的傅里叶级数是  $\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$ , 利用

• 当  $x \neq n\pi$  时,是 f 的连续点,

• 当  $x = n\pi$  是,

注 
$$1 f(x)$$
 的傅里叶级数是  $\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$ , 利用

• 当  $x \neq n\pi$  时,是 f 的连续点,此时

$$f(x) = \frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right]$$

• 当  $x = n\pi$  是,

注 
$$1 f(x)$$
 的傅里叶级数是  $\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$ , 利用

• 当  $x \neq n\pi$  时,是 f 的连续点,此时

$$f(x) = \frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right]$$

•  $\exists x = n\pi$  是,是 f 的间断点,

注 
$$1 f(x)$$
 的傅里叶级数是  $\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$ , 利用

● 当  $x \neq n\pi$  时,是 f 的连续点,此时

$$f(x) = \frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right]$$

• 当  $x = n\pi$  是,是 f 的间断点,此时

$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{1}{2} \left[ f(x^{-}) + f(x^{+}) \right]$$

注 
$$1 f(x)$$
 的傅里叶级数是  $\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$ , 利用 收敛定理分析可知:

•  $\exists x \neq n\pi$  时,是 f 的连续点,此时

$$f(x) = \frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$$

• 当  $x = n\pi$  是,是 f 的间断点,此时

$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{1}{2} \left[ f(x^{-}) + f(x^{+}) \right] = 0$$



注 
$$1 f(x)$$
 的傅里叶级数是  $\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$ , 利用 收敛定理分析可知:

$$f(x) = \frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$$

• 当  $x = n\pi$  是,是 f 的间断点,此时

$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{1}{2} \left[ f(x^{-}) + f(x^{+}) \right] = 0$$

(显然,可直接看出当  $x = n\pi$  时傅里叶级数的值为 0)

注 
$$1 f(x)$$
 的傅里叶级数是  $\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$ , 利用 收敛定理分析可知:

$$f(x) = \frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$$

• 当  $x = n\pi$  是,是 f 的间断点,此时

$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right] = \frac{1}{2} \left[ f(x^{-}) + f(x^{+}) \right] = 0$$
(显然,可直接看出当  $x = n\pi$  时傅里叶级数的值为 0)

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$



注 
$$1 f(x)$$
 的傅里叶级数是  $\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$ , 利用 收敛定理分析可知:

$$f(x) = \frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$$

• 当  $x = n\pi$  是,是 f 的间断点,此时

$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right] = \frac{1}{2} \left[ f(x^{-}) + f(x^{+}) \right] = 0$$
(显然,可直接看出当  $x = n\pi$  时傅里叶级数的值为 0)

$$1 - \frac{1}{2} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots = \frac{\pi}{4}$$



注 
$$1 f(x)$$
 的傅里叶级数是  $\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$ , 利用 收敛定理分析可知:

$$f(x) = \frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right]$$

• 当  $x = n\pi$  是,是 f 的间断点,此时

$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right] = \frac{1}{2} \left[ f(x^{-}) + f(x^{+}) \right] = 0$$
(显然,可直接看出当  $x = n\pi$  时傅里叶级数的值为 0)

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots = \frac{\pi}{4}$$

注 3 奇函数 f(x) 的傅里叶级数是  $\sum_{n=1}^{\infty} b_n \sin nx$ 



$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$$

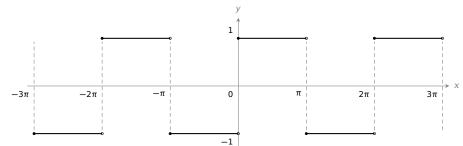
$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

$$\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$

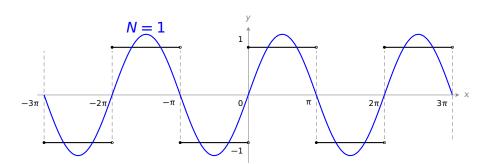
$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

$$\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$



$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

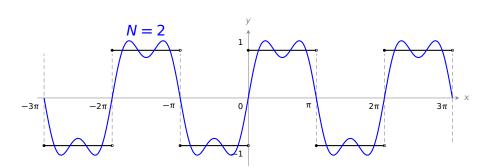
$$\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$





$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

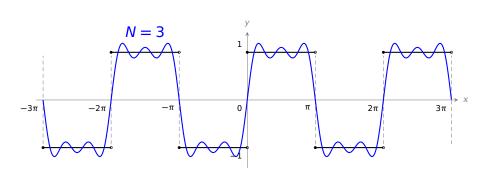
$$\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$





$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

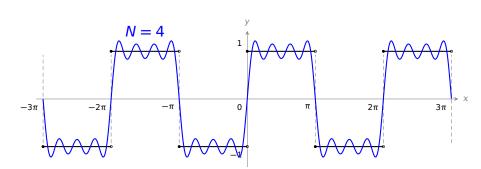
$$\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$





$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

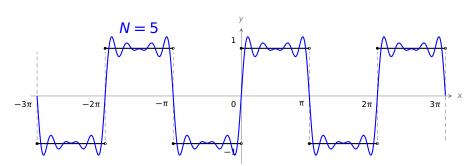
$$\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$





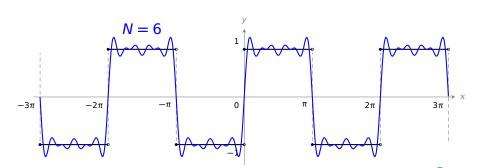
$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

$$\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$



$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

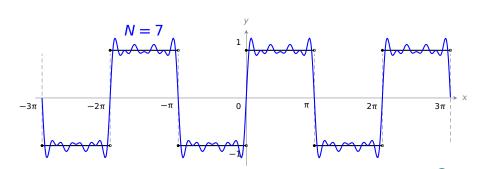
$$\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$





$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

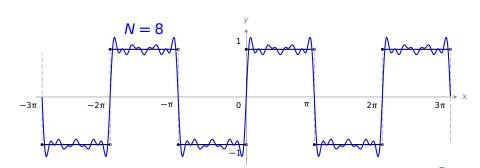
$$\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$





$$\frac{4}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

$$\frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$

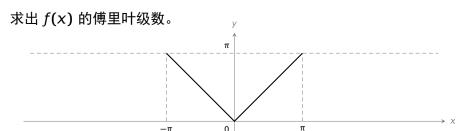




例 设 f(x) 是周期为  $2\pi$  的周期函数,在  $[-\pi, \pi)$  上的表达式为 f(x) = |x|

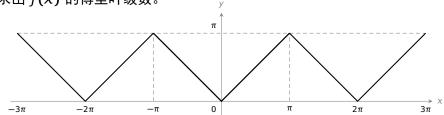
求出 
$$f(x)$$
 的傅里叶级数。

$$f(x) = |x|$$



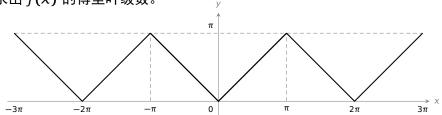
$$f(x) = |x|$$

求出 f(x) 的傅里叶级数。



$$f(x) = |x|$$

求出 f(x) 的傅里叶级数。

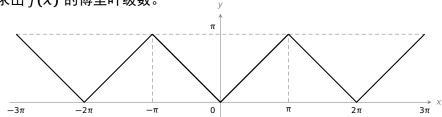


#### 解 计算傅里叶系数如下:

 $b_n$ 

$$f(x) = |x|$$

求出 f(x) 的傅里叶级数。



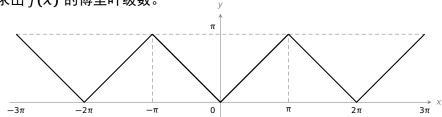
#### 解 计算傅里叶系数如下:

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$



$$f(x) = |x|$$

求出 f(x) 的傅里叶级数。



#### 解 计算傅里叶系数如下:

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{§fight}} 0,$$

$$a_n =$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fille } 0},$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{§fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{§fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$= \frac{2}{n\pi} \int_{0}^{\pi} x d\sin nx = \frac{2}{n\pi} \left[ x \sin nx \right]_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[ x \sin nx \right]_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx$$

$$= \frac{2}{n\pi} \int_{0}^{\pi} xu \sin nx - \frac{1}{n\pi} \left[ x \sin nx \right]_{0}^{\pi}$$

$$= \frac{2}{n\pi} \left[ \frac{1}{n} \cos nx \right]_{0}^{\pi}$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$\frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin$$

$$= \frac{2}{n\pi} \int_0^{\pi} x d \sin nx = \frac{2}{n\pi} \left[ x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$
$$= \frac{2}{n\pi} \left[ \frac{1}{n} \cos nx \Big|_0^{\pi} \right] = \frac{2}{n^2 \pi} \left[ (-1)^n - 1 \right]$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$f(x) \cos nx dx = -\int_{-\pi}^{\pi} f(x) \cos nx dx = -\int_{0}^{\pi} f(x) \cos nx dx = -\int_{0}^{\pi} x \cos nx dx$$
$$= -\int_{0}^{\pi} x d\sin nx = -\int_{0}^{\pi} x \sin nx dx$$

$$\begin{aligned} & = \frac{1}{\pi} \int_{-\pi}^{\pi} (x) \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} (x) \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} x \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} x \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx = \frac{1}{\pi} \int_{0}^{\pi} x d \sin nx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx = \frac{1}{\pi} \int_{0}^{\pi} x d \sin nx = \frac{1}{\pi} \int_{0}^{\pi} x d \sin nx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx = \frac{1}{\pi} \int_{0}^{\pi} x d \sin nx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx = \frac{1}{\pi} \int_{0}^{\pi} x d \sin nx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx dx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx dx dx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx dx dx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx dx dx dx = \frac{1}{\pi$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$f(x) \cos nx dx = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[ x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

 $= \frac{2}{n\pi} \left[ \frac{1}{n} \cos nx \Big|_{0}^{\pi} \right] = \frac{2}{n^{2}\pi} \left[ (-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \cdots \\ n = 2, 4, 6, \cdots \end{cases}$ 



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[ x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

 $= \frac{2}{n\pi} \left[ \frac{1}{n} \cos nx \Big|_{0}^{\pi} \right] = \frac{2}{n^{2}\pi} \left[ (-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \cdots \\ 0, & n = 2, 4, 6, \cdots \end{cases}$ 

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$\pi \int_{-\pi}^{\pi} \pi \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[ x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[ x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[ x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[ x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[ x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

$$n\pi \int_{0}^{\pi} n\pi \int_{0}^{\pi} n\pi \int_{0}^{\pi} \int_{0}$$

 $a_0$ 



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$= \frac{2}{n\pi} \int_0^{\pi} x d\sin nx = \frac{2}{n\pi} \left[ x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$

$$= \frac{1}{n\pi} \int_{0}^{\pi} x ds \sin nx = \frac{1}{n\pi} \left[ x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \left[ \frac{1}{n} \cos nx \Big|_{0}^{\pi} \right] = \frac{2}{n^{2}\pi} \left[ (-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

$$= \frac{1}{n\pi} \int_{0}^{\pi} x ds \sin nx = \frac{1}{n\pi} \left[ x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$= \frac{2}{n\pi} \int_0^{\pi} x d\sin nx = \frac{2}{n\pi} \left[ x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \int_0^{\pi} x d\sin nx = \frac{2}{n\pi} \left[ x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \left[ \frac{1}{n} \cos nx \Big|_0^{\pi} \right] = \frac{2}{n^2\pi} \left[ (-1)^n - 1 \right] = \begin{cases} -\frac{4}{n^2\pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$\pi \int_{-\pi}^{\pi} \pi \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[ x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \left[ \frac{1}{n} \cos nx \Big|_{0}^{\pi} \right] = \frac{2}{n^{2}\pi} \left[ (-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

$$= \frac{2}{n\pi} \left[ \frac{1}{n} \cos nx \Big|_{0}^{\pi} \right] = \frac{2}{n^{2}\pi} \left[ (-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

$$a_{0} = \frac{1}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} x dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} x dx$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{§fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$= \frac{2}{n\pi} \int_0^{\pi} x d \sin nx = \frac{2}{n\pi} \left[ x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]_0^{\pi}$$

$$= \frac{2}{n\pi} \int_0^{\pi} x d\sin nx = \frac{2}{n\pi} \left[ x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \left[ \frac{1}{n} \cos nx \Big|_0^{\pi} \right] = \frac{2}{n^2\pi} \left[ (-1)^n - 1 \right] = \begin{cases} -\frac{4}{n^2\pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

$$a_0 = \frac{1}{\pi} \int_0^{\pi} f(x) dx = \frac{2}{\pi} \int_0^{\pi} f(x) dx = \frac{2}{\pi} \int_0^{\pi} x dx = \frac{2}{\pi} \cdot \frac{1}{2} x^2 \Big|_0^{\pi}$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} x dx = \frac{2}{\pi} \cdot \frac{1}{2} x^2 \Big|_{0}^{\pi}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{§fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$= \frac{2}{n\pi} \int_0^{\pi} x d\sin nx = \frac{2}{n\pi} \left[ x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \left[ \frac{1}{n} \cos nx \Big|_{0}^{\pi} \right] = \frac{2}{n^{2}\pi} \left[ (-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

$$= \frac{1}{n\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{n\pi} \int_{0}^{\pi} f$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} x dx = \frac{2}{\pi} \cdot \frac{1}{2} x^2 \Big|_{0}^{\pi} = \pi.$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

 $x = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$ 

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[ x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \left[ \frac{1}{n} \cos nx \Big|_{0}^{\pi} \right] = \frac{2}{n^{2}\pi} \left[ (-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \cdots \\ 0, & n = 2, 4, 6, \cdots \end{cases}$$

 $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} x dx = \frac{2}{\pi} \cdot \frac{1}{2} x^2 \Big|_{0}^{\pi} = \pi.$ 

所以傅里叶级数为 $\frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos nx$ 



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$b_n = -\frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$= \frac{2}{n\pi} \int_{-\pi}^{\pi} x d\sin nx = \frac{2}{n\pi} \left[ x \sin nx \right]_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx$$

$$= \frac{2}{n\pi} \left[ \frac{1}{n} \cos nx \Big|_{0}^{\pi} \right] = \frac{2}{n^{2}\pi} \left[ (-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \cdots \\ 0, & n = 2, 4, 6, \cdots \end{cases}$$

$$a_{0} = \frac{1}{n} \int_{0}^{\pi} f(x) dx = \frac{2}{n^{2}\pi} \int_{0}^{\pi}$$

 $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} x dx = \frac{2}{\pi} \cdot \frac{1}{2} x^2 \Big|_{0}^{\pi} = \pi.$ 所以傅里叶级数为

 $\frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos nx = \frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$ 

## 注 1 f(x) 的傅里叶级数是

$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

## <u>注</u> 1 <math>f(x) 的傅里叶级数是

$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

又因为 f(x) 是连续函数,

### 注 1 f(x) 的傅里叶级数是

$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

又因为 f(x) 是连续函数,故利用收敛定理分析可知:

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

#### 注 1 f(x) 的傅里叶级数是

$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

又因为 f(x) 是连续函数,故利用收敛定理分析可知:

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

### <u>注</u> 1 <math>f(x) 的傅里叶级数是

$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

又因为 f(x) 是连续函数,故利用收敛定理分析可知:

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

$$\dot{x}$$
 2 取  $x = 0$ . 可得到

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$$

#### 注 1 f(x) 的傅里叶级数是

$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

又因为 f(x) 是连续函数,故利用收敛定理分析可知:

$$f(x) = \frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

 $\ge 2$  取 x = 0,可得到

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$$

注 3 偶函数 f(x) 的傅里叶级数是  $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ 



$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$



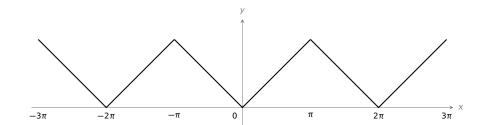
$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{i=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$



$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{i=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

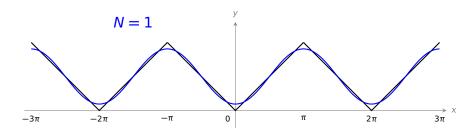
$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$





$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

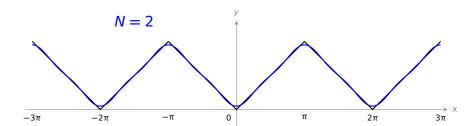
$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$





$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

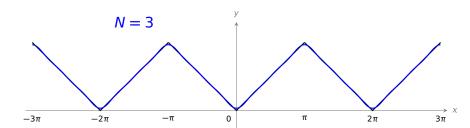
$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$





$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

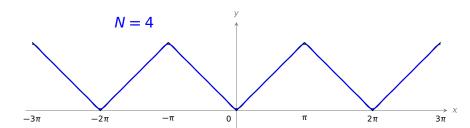
$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$





$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

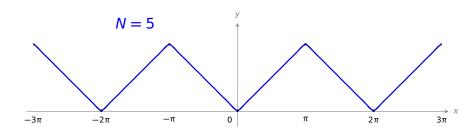
$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$





$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

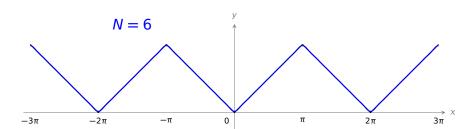
$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$





$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

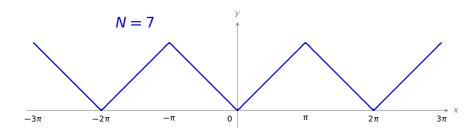
$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$





$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

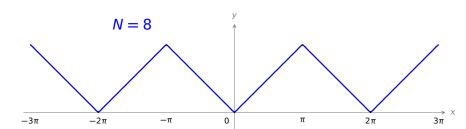
$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$





$$\frac{\pi}{2} - \frac{4}{\pi} \left[ \cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

$$\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{N} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$





• 若 f(x) 是奇函数,则傅里叶级数为

• 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx,$$

• 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx,$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx,$$

• 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx,$$

• 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \qquad a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx.$$

• 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

• 若 f(x) 是偶函数,则傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \qquad a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx.$$

• 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

• 若 f(x) 是偶函数,则傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \qquad a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx.$$

 $\overline{\text{tim}}$  (1) 假设 f 为奇函数,则

$$a_n =$$

$$b_n =$$

• 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

• 若 f(x) 是偶函数,则傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \qquad a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx.$$

 $\overline{\text{tim}}$  (1) 假设 f 为奇函数,则

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$b_n =$$

• 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

• 若 f(x) 是偶函数,则傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \qquad a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx.$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

• 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

• 若 f(x) 是偶函数,则傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \qquad a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx.$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{10}} 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

• 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

• 若 f(x) 是偶函数,则傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \qquad a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx.$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{\hat{\sigma}(\text{RMt})}{\pi}} 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\frac{\hat{\sigma}(\text{RMt})}{\pi}} \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx$$

• 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

• 若 f(x) 是偶函数,则傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \qquad a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx.$$

 $\overline{\text{tr}}$ 明(2) 假设f 为偶函数,则

$$b_n =$$

$$a_n =$$

• 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

• 若 f(x) 是偶函数,则傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \qquad a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx.$$

证明(2)假设f为偶函数,则

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$a_n =$$

• 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

• 若 f(x) 是偶函数,则傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \qquad a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx.$$

证明 (2) 假设 f 为偶函数,则

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$1 \int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

• 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

• 若 f(x) 是偶函数,则傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \qquad a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx.$$

证明(2)假设f为偶函数,则

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\frac{\pi}{\text{millow}}} 0$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

• 若 f(x) 是奇函数,则傅里叶级数为

$$\sum_{n=1}^{\infty} b_n \sin nx, \qquad b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx dx.$$

• 若 f(x) 是偶函数,则傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \qquad a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx.$$

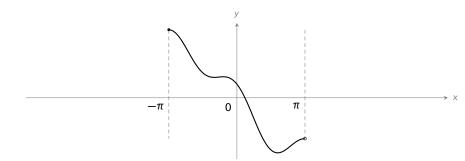
证明 (2) 假设 f 为偶函数,则

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0$$

$$a_n = \frac{1}{\pi} \int_{0}^{\pi} f(x) \cos nx dx \xrightarrow{\text{fight}} \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx$$

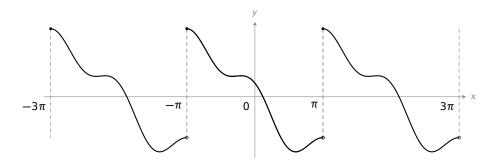
设 f(x) 是定义在区间  $[-\pi, \pi]$ (或  $(-\pi, \pi]$ )上的函数,可以对其进行周期延拓,从而得到定义在  $\mathbb{R}$  上的周期函数

设 f(x) 是定义在区间  $[-\pi, \pi]$ (或  $(-\pi, \pi]$ )上的函数,可以对其进行周期延拓,从而得到定义在  $\mathbb{R}$  上的周期函数,如图:



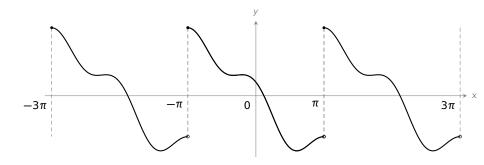


设 f(x) 是定义在区间  $[-\pi, \pi)$ (或  $(-\pi, \pi]$ )上的函数,可以对其进行周期延拓,从而得到定义在  $\mathbb{R}$  上的周期函数,如图:





设 f(x) 是定义在区间  $[-\pi, \pi)$ (或  $(-\pi, \pi]$ )上的函数,可以对其进行周期延拓,从而得到定义在  $\mathbb{R}$  上的周期函数,如图:



延拓后的周期函数任然记为 f(x),此时可以进行傅里叶展开。



设 f(x) 是定义在区间  $(0, \pi]$  上的函数,可以对其进行奇延拓,从而得到定义在  $\mathbb{R}$  上的周期奇函数。

设 f(x) 是定义在区间  $(0, \pi]$  上的函数,可以对其进行奇延拓,从而得到定义在  $\mathbb{R}$  上的周期奇函数。

设 f(x) 是定义在区间  $(0, \pi]$  上的函数,可以对其进行奇延拓,从而得到定义在  $\mathbb{R}$  上的周期奇函数。

## 奇延拓步骤:

定义 f(0) = 0

设 f(x) 是定义在区间  $(0, \pi]$  上的函数,可以对其进行奇延拓,从而得到定义在  $\mathbb{R}$  上的周期奇函数。

### 奇延拓步骤:

• 定义 f(0) = 0; 当  $x \in (-\pi, 0)$  时, 定义 f(x) = -f(-x);

设 f(x) 是定义在区间  $(0, \pi]$  上的函数,可以对其进行奇延拓,从而得到定义在  $\mathbb{R}$  上的周期奇函数。

### 奇延拓步骤:

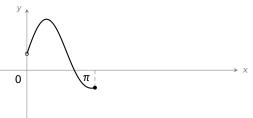
• 定义 f(0) = 0; 当  $x \in (-\pi, 0)$  时,定义 f(x) = -f(-x); (此时 f 在  $(-\pi, \pi]$  上有定义,且在  $(-\pi, \pi)$  上为奇函数)

设 f(x) 是定义在区间  $(0, \pi]$  上的函数,可以对其进行奇延拓,从而得到定义在  $\mathbb{R}$  上的周期奇函数。

- 定义 f(0) = 0; 当  $x \in (-\pi, 0)$  时,定义 f(x) = -f(-x); (此时 f 在  $(-\pi, \pi]$  上有定义,且在  $(-\pi, \pi)$  上为奇函数)
- 周期延拓 f 在  $(-\pi, \pi]$  上的取值。

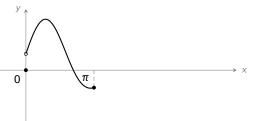
设 f(x) 是定义在区间  $(0, \pi]$  上的函数,可以对其进行奇延拓,从而得到定义在  $\mathbb{R}$  上的周期奇函数。

- 定义 f(0) = 0; 当  $x \in (-\pi, 0)$  时,定义 f(x) = -f(-x); (此时 f 在  $(-\pi, \pi]$  上有定义,且在  $(-\pi, \pi)$  上为奇函数)
- 周期延拓 f 在  $(-\pi, \pi]$  上的取值。



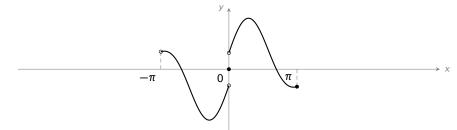
设 f(x) 是定义在区间  $(0, \pi]$  上的函数,可以对其进行奇延拓,从而得到定义在  $\mathbb{R}$  上的周期奇函数。

- 定义 f(0) = 0; 当  $x \in (-\pi, 0)$  时,定义 f(x) = -f(-x); (此时 f 在  $(-\pi, \pi]$  上有定义,且在  $(-\pi, \pi)$  上为奇函数)
- 周期延拓 f 在  $(-\pi, \pi]$  上的取值。



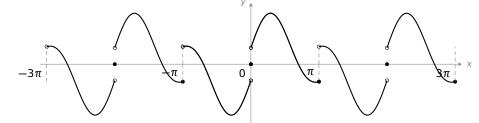
设 f(x) 是定义在区间  $(0, \pi]$  上的函数,可以对其进行奇延拓,从而得到定义在  $\mathbb{R}$  上的周期奇函数。

- 定义 f(0) = 0; 当  $x \in (-\pi, 0)$  时,定义 f(x) = -f(-x); (此时 f 在  $(-\pi, \pi]$  上有定义,且在  $(-\pi, \pi)$  上为奇函数)
- 周期延拓 f 在  $(-\pi, \pi]$  上的取值。



设 f(x) 是定义在区间  $(0, \pi]$  上的函数,可以对其进行奇延拓,从而得到定义在  $\mathbb{R}$  上的周期奇函数。

- 定义 f(0) = 0; 当  $x \in (-\pi, 0)$  时,定义 f(x) = -f(-x); (此时 f 在  $(-\pi, \pi]$  上有定义,且在  $(-\pi, \pi)$  上为奇函数)
- 周期延拓 f 在 (-π, π] 上的取值。



设 f(x) 是定义在区间  $[0, \pi]$  上的函数,可以对其进行偶延拓,从而得到定义在  $\mathbb{R}$  上的周期偶函数。

设 f(x) 是定义在区间  $[0, \pi]$  上的函数,可以对其进行偶延拓,从而得

到定义在 ℝ 上的周期偶函数。

设 f(x) 是定义在区间  $[0, \pi]$  上的函数,可以对其进行偶延拓,从而得到定义在  $\mathbb{R}$  上的周期偶函数。

偶延拓步骤:

• 当  $x \in [-\pi, 0]$  时,定义 f(x) = f(-x);

设 f(x) 是定义在区间  $[0, \pi]$  上的函数,可以对其进行偶延拓,从而得到定义在  $\mathbb{R}$  上的周期偶函数。

### 偶延拓步骤:

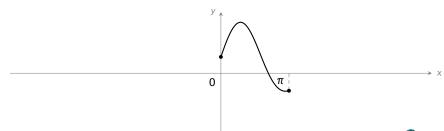
• 当  $x \in [-\pi, 0]$  时,定义 f(x) = f(-x); (此时 f 成为定义在  $[-\pi, \pi]$  上为偶函数)

设 f(x) 是定义在区间  $[0, \pi]$  上的函数,可以对其进行偶延拓,从而得到定义在  $\mathbb{R}$  上的周期偶函数。

- 当  $x \in [-\pi, 0]$  时,定义 f(x) = f(-x); (此时 f 成为定义在  $[-\pi, \pi]$  上为偶函数)
- 周期延拓 f 在  $[-\pi, \pi]$  上的取值。

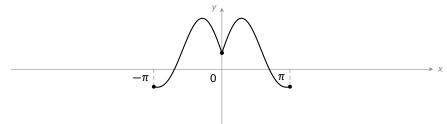
设 f(x) 是定义在区间  $[0, \pi]$  上的函数,可以对其进行偶延拓,从而得到定义在  $\mathbb{R}$  上的周期偶函数。

- 当  $x \in [-\pi, 0]$  时,定义 f(x) = f(-x); (此时 f 成为定义在  $[-\pi, \pi]$  上为偶函数)
- 周期延拓 f 在 [-π, π] 上的取值。



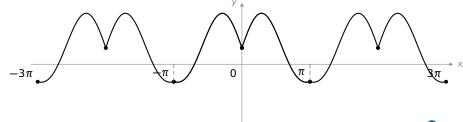
设 f(x) 是定义在区间  $[0, \pi]$  上的函数,可以对其进行偶延拓,从而得到定义在  $\mathbb{R}$  上的周期偶函数。

- 当  $x \in [-\pi, 0]$  时,定义 f(x) = f(-x); (此时 f 成为定义在  $[-\pi, \pi]$  上为偶函数)
- 周期延拓 f 在  $[-\pi, \pi]$  上的取值。



设 f(x) 是定义在区间  $[0, \pi]$  上的函数,可以对其进行偶延拓,从而得到定义在  $\mathbb{R}$  上的周期偶函数。

- 当  $x \in [-\pi, 0]$  时,定义 f(x) = f(-x); (此时 f 成为定义在  $[-\pi, \pi]$  上为偶函数)
- 周期延拓 f 在  $[-\pi, \pi]$  上的取值。



## We are here now...

1. 傅里叶级数的概念

2. 周期为 2π 的周期函数的傅里叶级数

3. 一般周期函数的傅里叶级数

假设 f(x) 是定义在  $\mathbb{R}$  上周期函数,周期为 T=2l,

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_{n} = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$1 \int_{-l}^{l} n\pi x$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导"  $\Rightarrow q(x) = f(\frac{l}{\pi}x),$ 

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导" 令  $g(x) = f(\frac{1}{\pi}x)$ , 则 g 是周期为  $2\pi$  的周期函数:



$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导"  $\Leftrightarrow g(x) = f(\frac{\iota}{\pi}x)$ , 则 g 是周期为  $2\pi$  的周期函数:

$$q(x+2\pi)$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导"  $\Leftrightarrow g(x) = f(\frac{l}{\pi}x)$ , 则 g 是周期为  $2\pi$  的周期函数:

$$g(x + 2\pi) = f(\frac{l}{\pi}(x + 2\pi))$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导"  $\Leftrightarrow g(x) = f(\frac{l}{\pi}x)$ ,则 g 是周期为  $2\pi$  的周期函数:

$$g(x+2\pi) = f(\frac{l}{\pi}(x+2\pi)) = f(\frac{l}{\pi}x+2l)$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$g(x+2\pi) = f(\frac{l}{\pi}(x+2\pi)) = f(\frac{l}{\pi}x+2l) = f(\frac{l}{\pi}x)$$



$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导"  $\Leftrightarrow g(x) = f(\frac{l}{\pi}x)$ , 则 g 是周期为  $2\pi$  的周期函数:

 $g(x+2\pi) = f(\frac{l}{\pi}(x+2\pi)) = f(\frac{l}{\pi}x+2l) = f(\frac{l}{\pi}x) = g(x)$ 



假设 f(x) 是定义在  $\mathbb{R}$  上周期函数,周期为 T=2l,其傅里叶级数应为:  $\frac{a_0}{2} + \sum_{i=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$ 

 $a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx$   $(n = 0, 1, 2, 3, \cdots)$ 

其中

 $b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx$   $(n = 0, 1, 2, 3, \dots)$ 

$$b_n = \frac{1}{l} \int_{-l} f(x)$$

"推导" 令  $g(x) = f(\frac{l}{\pi}x)$ , 则 g 是周期为  $2\pi$  的周期函数:  $g(x+2\pi) = f(\frac{l}{\pi}(x+2\pi)) = f(\frac{l}{\pi}x+2l) = f(\frac{l}{\pi}x) = g(x)$ 

所以
$$g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$



"推守" 令 
$$g(x) = f(\frac{1}{\pi}x)$$
, 则  $g$  是周期为  $2\pi$  的周期函数:
$$g(x+2\pi) = f(\frac{1}{\pi}(x+2\pi)) = f(\frac{1}{\pi}x+2l) = f(\frac{1}{\pi}x) = g(x)$$

"推导" 令  $g(x) = f(\frac{l}{\pi}x)$ , 则 g 是周期为  $2\pi$  的周期函数:

$$h_{n} = \frac{1}{n} \int_{0}^{1} f(x) \sin \frac{n\pi x}{n} dx \qquad (n = 0, 1, 2, 3, \dots)$$

 $b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx$   $(n = 0, 1, 2, 3, \dots)$ 

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

 $f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$ 

 $\frac{a_0}{2} + \sum_{i=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$ 

其中

所以

既然 
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$



既然 
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

所以

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$



状然 
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

所以

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

 $a_n$ 

 $b_n$ 



状然 
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

所以

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz$$

 $b_n$ 



既然 
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz$$



既然 
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz$$



既然 
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

所以

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$x = \frac{l}{\pi}z$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz$$



既然 
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

所以

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$= \frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos \frac{n\pi x}{l}$$

$$1 \int_{-\pi}^{\pi} f(x) \cos \frac{n\pi x}{l}$$

 $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz$ 



既然 
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

所以

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$= \frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x)$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x)$$

 $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz$ 

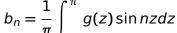


既然 
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$= \frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x)$$





既然 
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$$

所以

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$= \frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz$$



既然 
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

$$b_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \sin nz dz$$



既然 
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

$$b_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \sin nz dz$$

$$x = \frac{l}{\pi}z$$



既然 
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

 $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(-z) \sin nz dz$  $= \frac{x = \frac{1}{\pi}z}{\pi} \int f(x) \sin \frac{n\pi x}{l}$ 



既然 
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

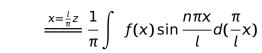
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{-z}) \cos nz dz$$

$$\frac{x = \frac{l}{\pi} z}{\pi} \frac{1}{\pi} \int_{-\pi}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l} x) = \frac{1}{l} \int_{-\pi}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

 $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(-z) \sin nz dz$ 



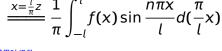


既然 
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx$	,
$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \sin nz dz$	
, , , , ,	





既然  $f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos nx + b_n \sin nx \right)$ 

所以
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

 $\frac{x=\frac{l}{\pi}z}{\pi}\frac{1}{\pi}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}d(\frac{\pi}{L}x)=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}dx,$  $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(-z) \sin nz dz$ 

$$\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx.$$

# 更多内容

傅里叶变换在工程中有许多应用,更多内容可以浏览在"Stanford Engineering Everywhere"中的课程"The Fourier Transform and Its Applications",讲义在 这里。

