

第 9 章 d: 隐函数的求导公式

数学系 梁卓滨

2018-2019 学年 II

We are here now...

1. 隐函数的求导法：一个方程的情形

2. 隐函数的求导法：方程组的情形

3. 隐函数定理

隐函数的求导法 I

问题

给定二元函数 $F(x, y) \Rightarrow$ 考虑方程 $F(x, y) = 0$

隐函数的求导法 I

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\Rightarrow 解出 $y = f(x)$

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$\Rightarrow \frac{dy}{dx} = ?$

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$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

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$$F(x, f(x)) = 0 \Rightarrow 0 = \frac{d}{dx} F(x, f(x)) = F_x + F_y \cdot \frac{df}{dx}$$

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证明

$$\begin{aligned} F(x, f(x)) = 0 &\Rightarrow 0 = \frac{d}{dx} F(x, f(x)) = F_x + F_y \cdot \frac{df}{dx} \\ &\Rightarrow \frac{df}{dx} = -\frac{F_x}{F_y} \end{aligned}$$

例 1 设 $y = f(x)$ 满足 $\sin y + e^x = xy^2$, 求 $\frac{dy}{dx}$

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方法二

例 1 设 $y = f(x)$ 满足 $\sin y + e^x = xy^2$, 求 $\frac{dy}{dx}$

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$$\begin{aligned} 0 &= (\sin y(x) + e^x - xy(x)^2)'_x \\ &= (\sin y(x))'_x + (e^x)'_x - (xy(x)^2)'_x \end{aligned}$$

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$$\text{所以 } y' = -\frac{e^x - y^2}{\cos y - 2xy}$$

例 2 设 $y = f(x)$ 满足 $\ln(x^2 + y^2) + 3xy = 4$, 求 $\frac{dy}{dx}$

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解

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例 2 设 $y = f(x)$ 满足 $\ln(x^2 + y^2) + 3xy = 4$, 求 $\frac{dy}{dx}$

解 注意 $\ln(x^2 + y^2) + 3xy - 4 = 0$

$$F(x, y) = 0$$

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$$= -\frac{2x}{3y + 2x}$$

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隐函数的求导法 II

问题

给定 $F(x, y, z) \Rightarrow$ 考虑方程 $F(x, y, z) = 0$

隐函数的求导法 II

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给定 $F(x, y, z) \Rightarrow$ 考虑方程 $F(x, y, z) = 0$

\Rightarrow 解出 $z = u(x, y)$

隐函数的求导法 II

问题

给定 $F(x, y, z) \Rightarrow$ 考虑方程 $F(x, y, z) = 0$

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$\Rightarrow \frac{\partial z}{\partial x} = ?, \quad \frac{\partial z}{\partial y} = ?$

隐函数的求导法 II

问题

给定 $F(x, y, z) \Rightarrow$ 考虑方程 $F(x, y, z) = 0$

\Rightarrow ~~解出 $z = u(x, y)$~~ 设 $z = u(x, y)$ 满足 $F(x, y, z) = 0$

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公式

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

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证明 $F(x, y, u(x, y)) = 0 \Rightarrow$

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证明 $F(x, y, u(x, y)) = 0 \Rightarrow 0 = \frac{\partial}{\partial x} F(x, y, u(x, y)) =$

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证明 $F(x, y, u(x, y)) = 0 \Rightarrow 0 = \frac{\partial}{\partial x} F(x, y, u(x, y)) = F_x + F_z \cdot \frac{\partial u}{\partial x}$

隐函数的求导法 II

问题

给定 $F(x, y, z) \Rightarrow$ 考虑方程 $F(x, y, z) = 0$

\Rightarrow ~~解出 $z = u(x, y)$~~ 设 $z = u(x, y)$ 满足 $F(x, y, z) = 0$

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$F(x, y, z) = 0$, 所以

$$\begin{aligned}\frac{\partial z}{\partial x} &= -\frac{F_x}{F_z} = -\frac{(2 \sin(x + 2y - 3z) - x - 2y + 3z)'_x}{(2 \sin(x + 2y - 3z) - x - 2y + 3z)'_z} \\ &= -\frac{2 \cos(x + 2y - 3z)}{-6 \cos(x + 2y - 3z) + 3}\end{aligned}$$

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例 3 设 $z = f(x, y)$ 满足 $z - y - x + xe^{z-y-x} = 0$, 求 dz

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$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z-y-x})'_y}{(z - y - x + xe^{z-y-x})'_z} = -\frac{-1 - xe^{z-y-x}}{1 + xe^{z-y-x}}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$

例 3 设 $z = f(x, y)$ 满足 $z - y - x + xe^{z-y-x} = 0$, 求 dz

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$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = -\frac{1 + (x-1)e^{z-y-x}}{1 + xe^{z-y-x}} dx + dy$$

例 4 设 $\Phi(u, v)$ 具有连续偏导数, 函数 $z = z(x, y)$ 满足 $\Phi(cx - az, cy - bz) = 0$, 证明:

$$a \frac{\partial z}{\partial x} + b \frac{\partial z}{\partial y} = c.$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$, 则

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$, 则

$$\begin{aligned} \frac{\partial z}{\partial x} &= - \frac{F_x}{F_z} = \\ \frac{\partial z}{\partial y} &= - \frac{F_y}{F_z} = \end{aligned}$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$, 则

$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x$$

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$$F_z =$$

$$\frac{\partial z}{\partial x} = - \frac{F_x}{F_z} =$$

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$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x = c\Phi_u$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$, 则

$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x = c \Phi_u$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y$$

$$F_z =$$

$$\frac{\partial z}{\partial x} = - \frac{F_x}{F_z} =$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$, 则

$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x = c\Phi_u$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y = c\Phi_v$$

$$F_z =$$

$$\frac{\partial z}{\partial x} = - \frac{F_x}{F_z} =$$

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例 4 设 $\Phi(u, v)$ 具有连续偏导数, 函数 $z = z(x, y)$ 满足 $\Phi(cx - az, cy - bz) = 0$, 证明:

$$a \frac{\partial z}{\partial x} + b \frac{\partial z}{\partial y} = c.$$

解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$, 则

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We are here now...

1. 隐函数的求导法：一个方程的情形

2. 隐函数的求导法：方程组的情形

3. 隐函数定理

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

回顾：二元线性方程组的求解

二元线性方程组

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用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$ ，消去 y ，得：

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$(2) \times a_{11} - (1) \times a_{21}$ ，消去 x ，得：

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$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}a_{11}x + a_{22}a_{11}y = a_{11}b_2 & (2) \times a_{11} \end{cases}$$

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$(2) \times a_{11} - (1) \times a_{21}$ ，消去 x ，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 \end{cases} \quad (1)$$

$$\begin{cases} a_{21}x + a_{22}y = b_2 \end{cases} \quad (2)$$

用消元法解：

(1) $\times a_{22}$ - (2) $\times a_{12}$ ，消去 y ，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

(2) $\times a_{11}$ - (1) $\times a_{21}$ ，消去 x ，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \end{cases}$$

$$\begin{cases} a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

(1) $\times a_{22}$ - (2) $\times a_{12}$ ，消去 y ，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{b_1 a_{22} - a_{12} b_2}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

(2) $\times a_{11}$ - (1) $\times a_{21}$ ，消去 x ，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{a_{11} b_2 - b_1 a_{21}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \end{cases}$$

$$\begin{cases} a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

(1) $\times a_{22}$ - (2) $\times a_{12}$ ，消去 y ，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

(2) $\times a_{11}$ - (1) $\times a_{21}$ ，消去 x ，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \end{cases}$$

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用消元法解：

(1) $\times a_{22}$ - (2) $\times a_{12}$ ，消去 y ，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

(2) $\times a_{11}$ - (1) $\times a_{21}$ ，消去 x ，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

公式:

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练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \quad , \quad y =$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad , \quad y =$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \text{---} \quad , \quad y =$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad , \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \text{---} \quad , \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad , \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -\frac{1}{1}, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1}, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad , \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \text{---}, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-17}{-17} = 1, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-7}{-17} = \frac{7}{17}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{1}{3}, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = -$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3}, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = -$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = -$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-3}{3}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3} = -3$$

方程组的隐函数求导公式

$$\begin{aligned} F(x, y, u, v) \\ G(x, y, u, v) \end{aligned}$$

方程组的隐函数求导公式

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题：如何计算 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$?

方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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求解如下：

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} \end{cases}$$

方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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求解如下：

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

方程组的隐函数求导公式

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$$\Rightarrow u_x = \underline{\hspace{2cm}}, \quad v_x = \underline{\hspace{2cm}}$$

方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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$$\Rightarrow u_x = \frac{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_x = \frac{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

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$$\Rightarrow u_x = \frac{\begin{vmatrix} -F_x & F_v \\ -G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_x = \frac{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

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$$\Rightarrow u_x = -\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_x = -\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

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$$\Rightarrow u_y = \text{—————}, \quad v_y = \text{—————}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = \frac{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = \frac{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = \frac{\begin{vmatrix} -F_y & F_v \\ -G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = \frac{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

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$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

$$v_y =$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{matrix} \xRightarrow{\frac{\partial}{\partial x}} \\ \xRightarrow{\frac{\partial}{\partial y}} \end{matrix}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

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$$\xRightarrow{\frac{\partial}{\partial y}}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

$$v_y =$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{matrix} \xRightarrow{\frac{\partial}{\partial x}} \\ \xRightarrow{\frac{\partial}{\partial y}} \end{matrix} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \\ F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

$$v_y =$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{matrix} \xRightarrow{\frac{\partial}{\partial x}} \\ \xRightarrow{\frac{\partial}{\partial y}} \end{matrix} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \\ F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

$$v_y =$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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所以

$$u_x = - \frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$u_y = - \frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$v_x = - \frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$v_y = - \frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{matrix} \xRightarrow{\frac{\partial}{\partial x}} \\ \xRightarrow{\frac{\partial}{\partial y}} \end{matrix} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \\ \\ F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_x = - \frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_x = - \frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$u_y = - \frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = - \frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{aligned} &\xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases} \\ &\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases} \end{aligned}$$

所以

$$u_x = - \frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_x = - \frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(u, x)}$$

$$u_y = - \frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = - \frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{matrix} \xRightarrow{\frac{\partial}{\partial x}} \\ \xRightarrow{\frac{\partial}{\partial y}} \end{matrix} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \\ \\ F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_x = - \frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_x = - \frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(u, x)}$$

$$u_y = - \frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)}, \quad v_y = - \frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{matrix} \xRightarrow{\frac{\partial}{\partial x}} \\ \xRightarrow{\frac{\partial}{\partial y}} \end{matrix} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \\ F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_x = - \frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_x = - \frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(u, x)}$$

$$u_y = - \frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)}, \quad v_y = - \frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(u, y)}$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \begin{array}{l} \xRightarrow{\frac{\partial}{\partial x}} \\ \xRightarrow{\frac{\partial}{\partial y}} \end{array}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

$$v_y =$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \end{cases}$$

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial y}}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

$$v_y =$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u \sin v \cdot v_x = 0 \end{cases}$$
$$\xRightarrow{\frac{\partial}{\partial y}}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

$$v_y =$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \end{cases}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

$$u_x =$$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

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$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u \sin v \cdot v_x = 0 \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

所以 $J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$

$$u_x = \frac{\begin{vmatrix} & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}}{J}$$

$$v_x = \frac{\begin{vmatrix} e^u + \sin v & \\ e^u - \cos v & \end{vmatrix}}{J}$$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

所以 $J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$

$$u_x = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_x = \frac{\begin{vmatrix} & \\ & \end{vmatrix}}{J}$$

$$u_y = \frac{\begin{vmatrix} & \\ & \end{vmatrix}}{J}$$

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$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \begin{aligned} &\xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u \sin v \cdot v_x = 0 \end{cases} \\ &\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases} \end{aligned}$$

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$$u_y = \frac{\begin{vmatrix} & u \cos v \\ & u \sin v \end{vmatrix}}{J}$$

$$v_x = \frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J}$$

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例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

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$$\text{所以 } J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$u_x = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

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$$u_y = \frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}, \quad v_y = \frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J}$$

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$$u_x = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J} = \frac{\sin v}{e^u(\sin v - \cos v) + 1}, \quad v_x = \frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J} = \frac{-e^u + \cos v}{ue^u(\sin v - \cos v) + u}$$

$$u_y = \frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J} \quad v_y = \frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

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$$u_y = \frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J} = \frac{-\cos v}{e^u(\sin v - \cos v) + 1}, \quad v_y = \frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

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$$u_y = \frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J} = \frac{-\cos v}{e^u(\sin v - \cos v) + 1}, \quad v_y = \frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J} = \frac{e^u + \sin v}{ue^u(\sin v - \cos v) + u}$$

We are here now...

1. 隐函数的求导法：一个方程的情形

2. 隐函数的求导法：方程组的情形

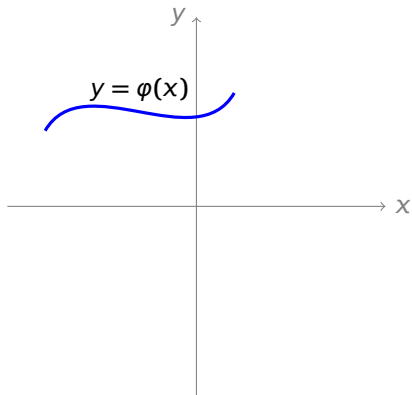
3. 隐函数定理

平面光滑曲线的定义

平面上光滑曲线应该包含：一元光滑函数的图形

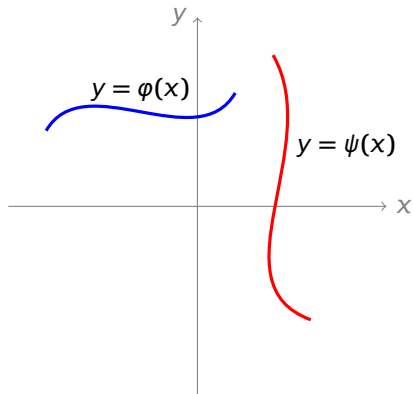
平面光滑曲线的定义

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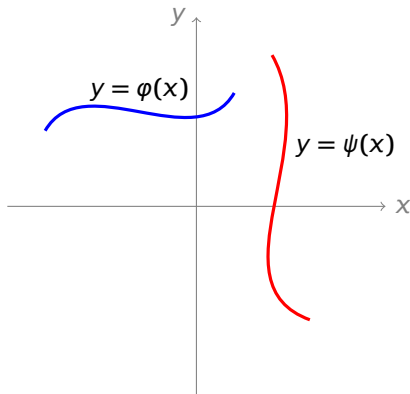
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平面光滑曲线的定义

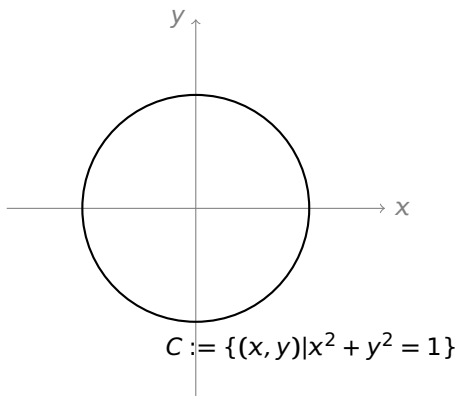
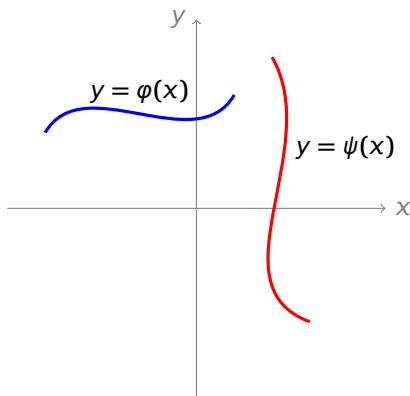
平面上光滑曲线应该包含：一元光滑函数的图形



一般地，平面上一个点集 C 称为光滑曲线，是指该点集“局部”上总可以表示成一元光滑函数的图形。

平面光滑曲线的定义

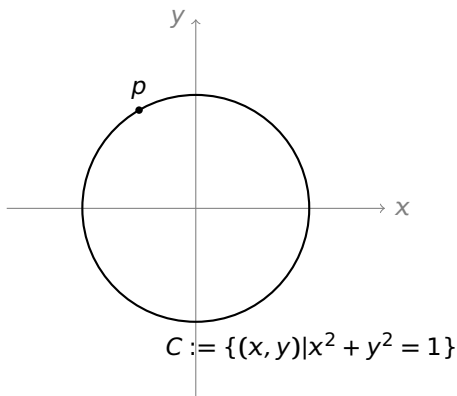
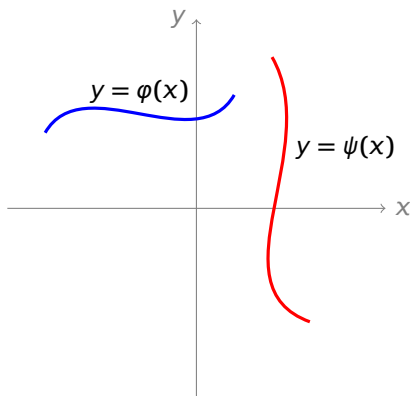
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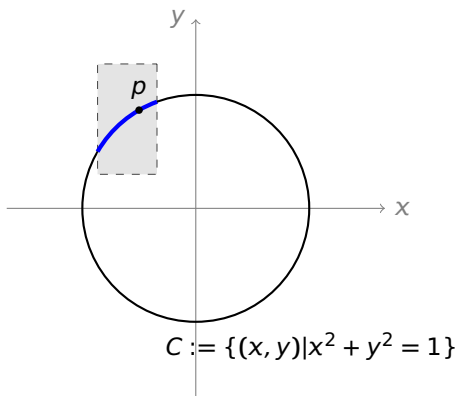
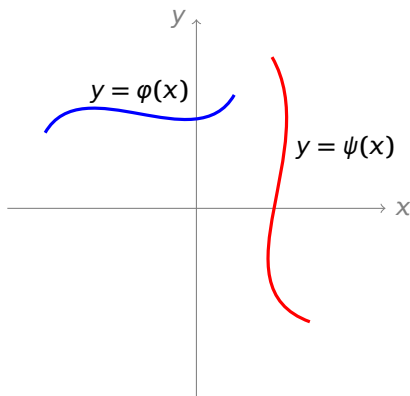
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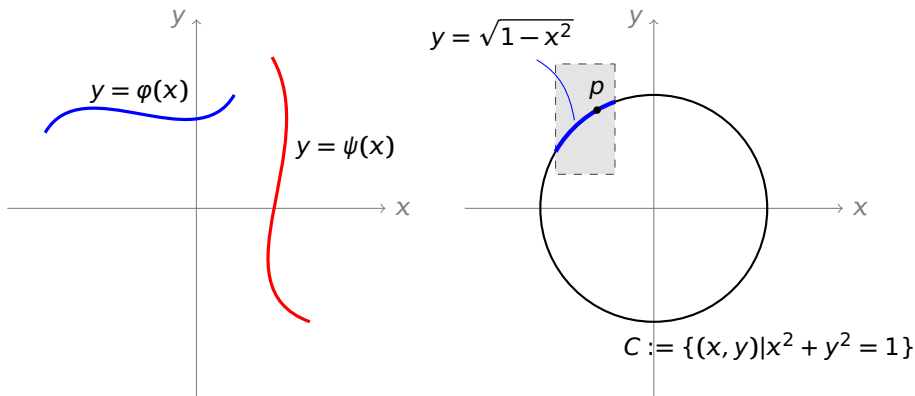
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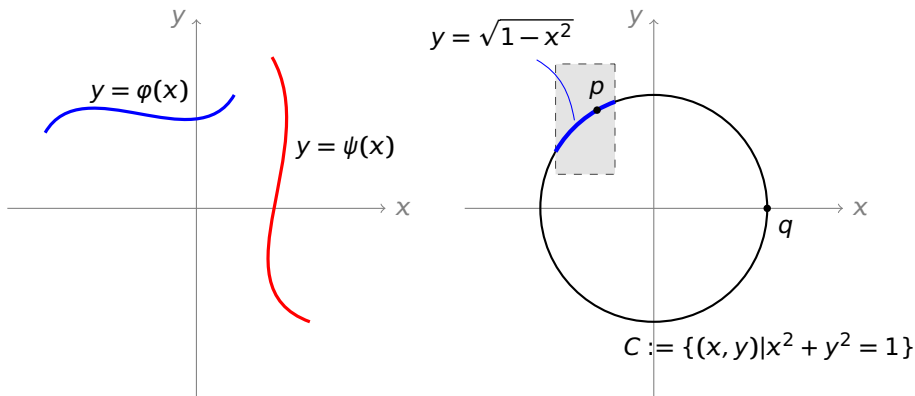
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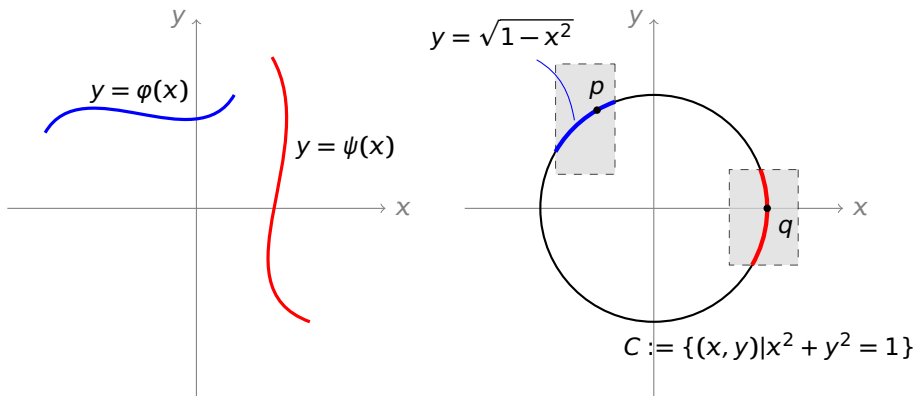
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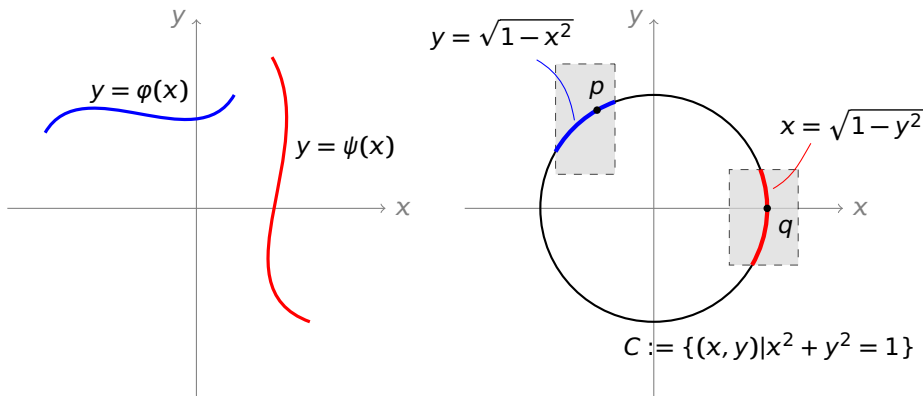
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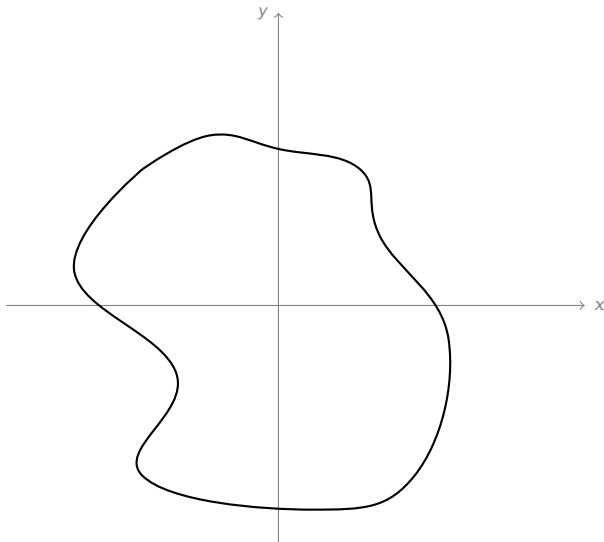
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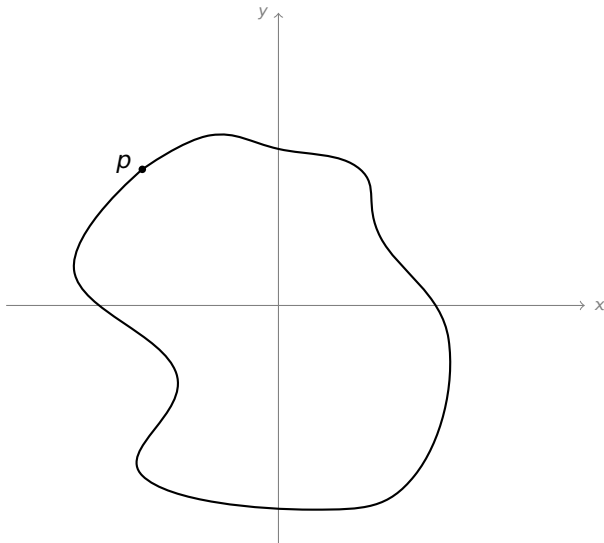
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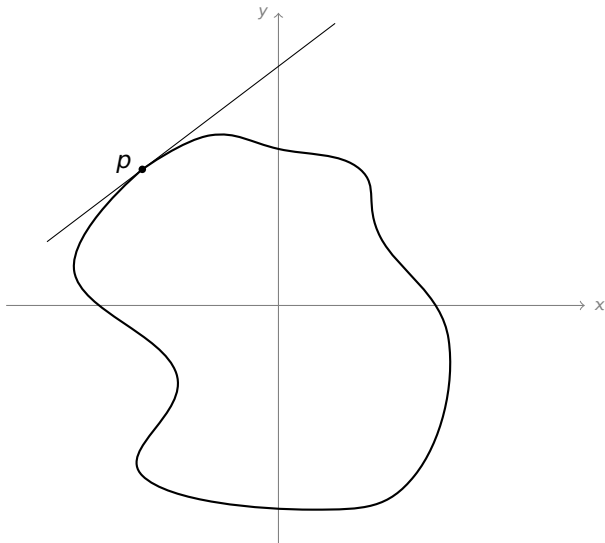
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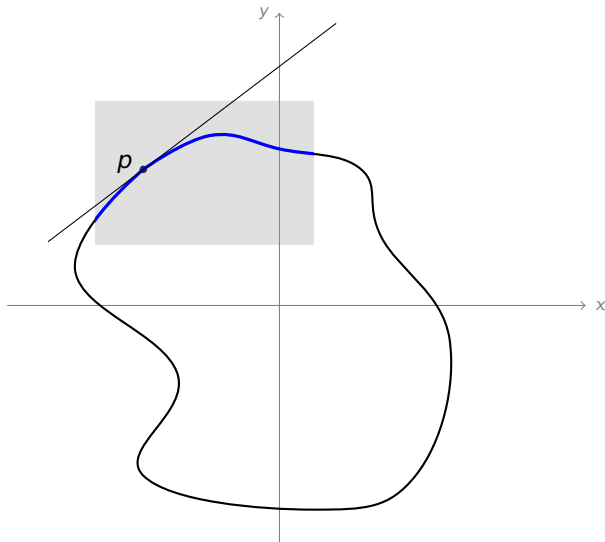
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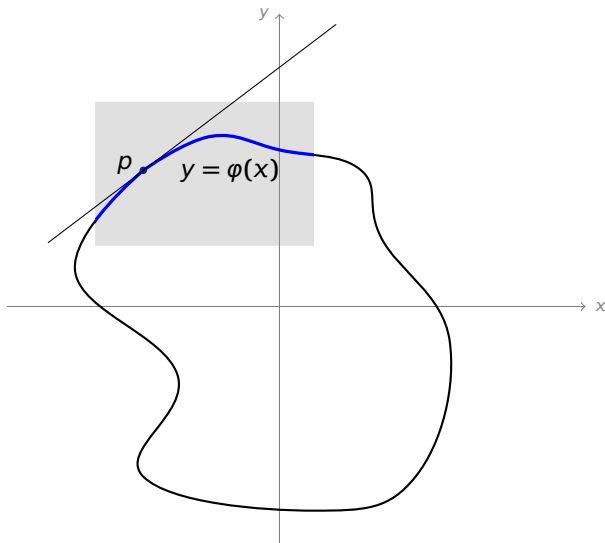
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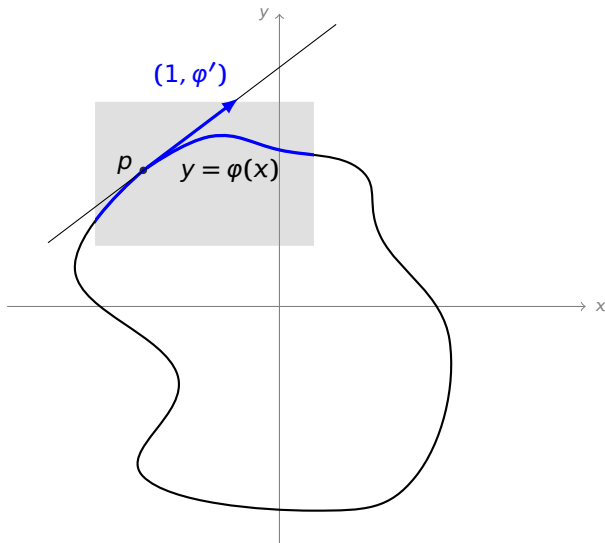
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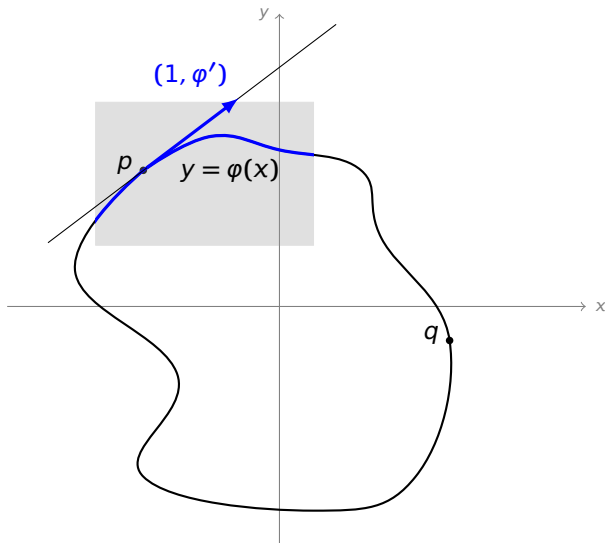
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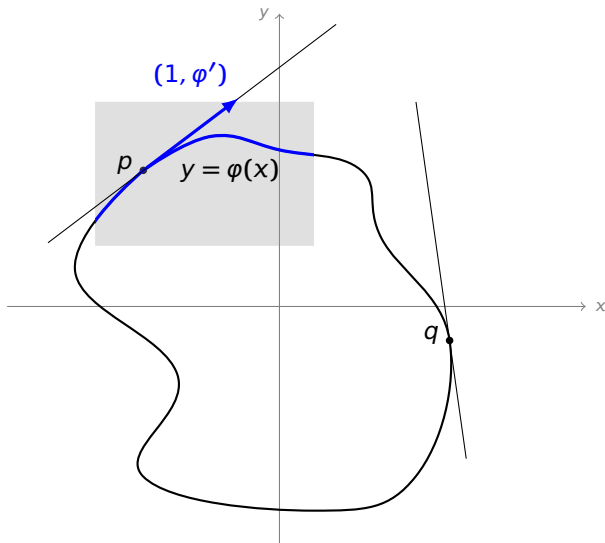
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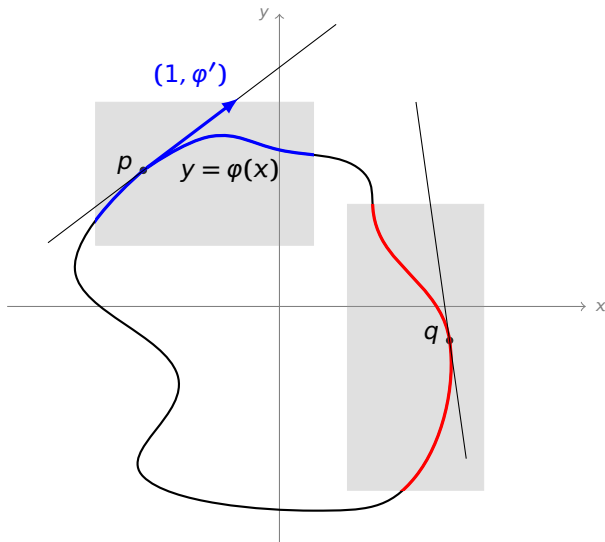
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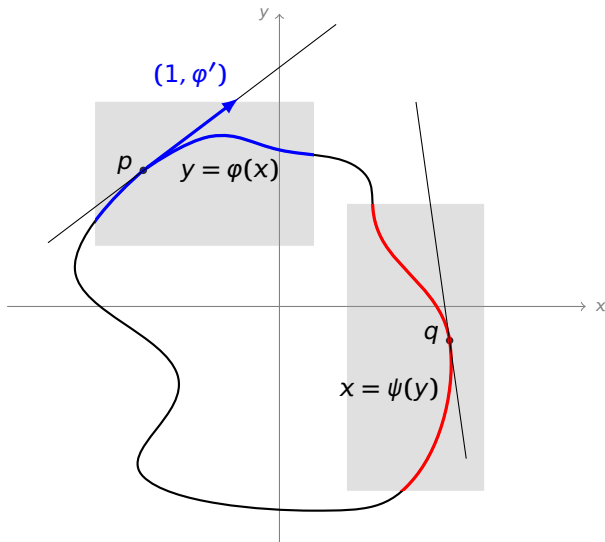
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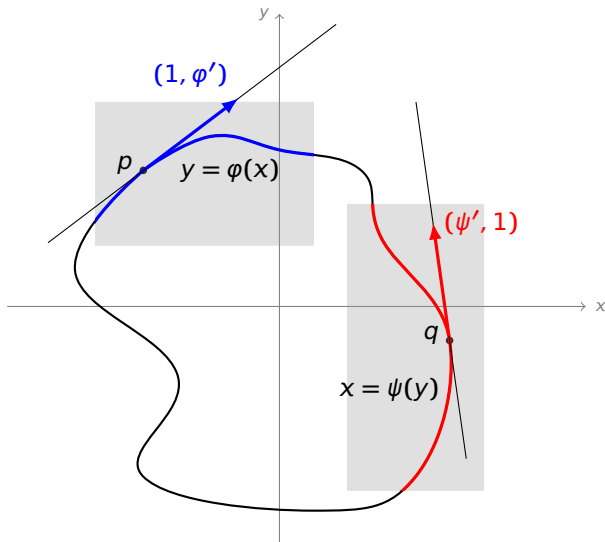
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假设 $f(x, y)$ 是光滑的二元函数，其零点集 $\{f = 0\}$ 是平面上点集。

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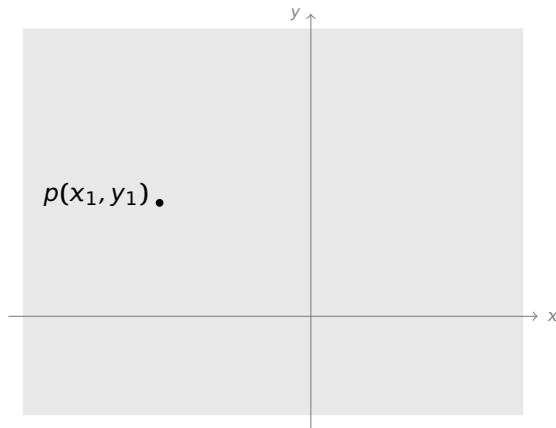
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 - 定义 $\nabla f = (f_x, f_y)$ ，称为 f 的**梯度**。
 - 由**隐函数定理**可知，如果 $(f_x, f_y) \neq (0, 0)$ ，则 $\{f = 0\}$ 是一条光滑曲线，且该曲线上任一点 (x, y) 的一个切方向是 $(f_y, -f_x)$ （与梯度 ∇f 垂直）。

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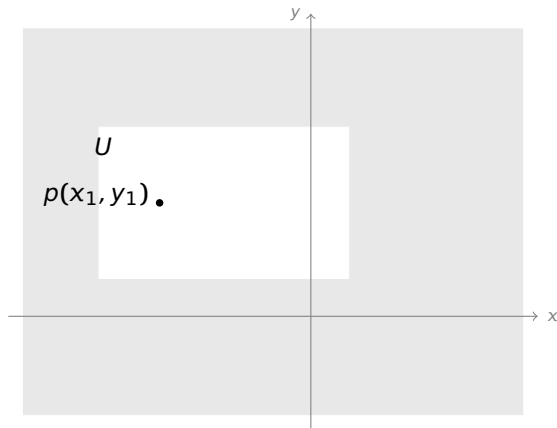
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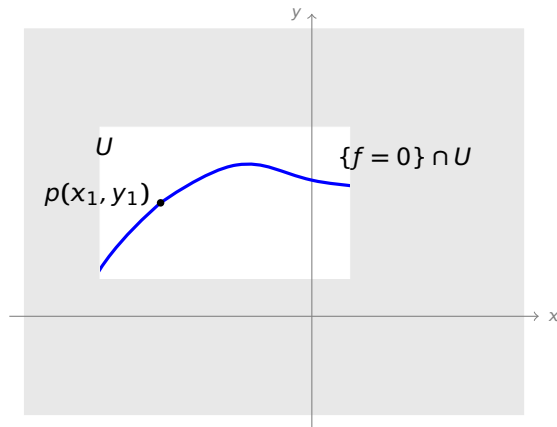
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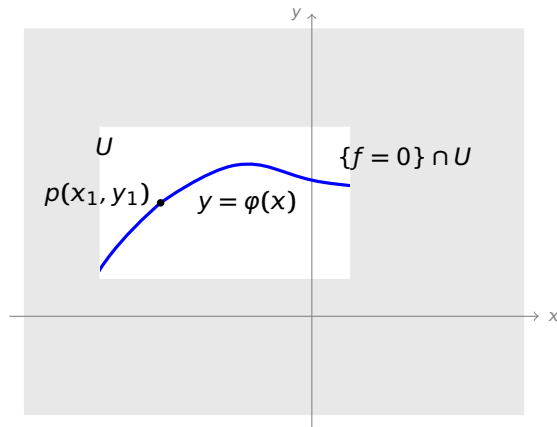
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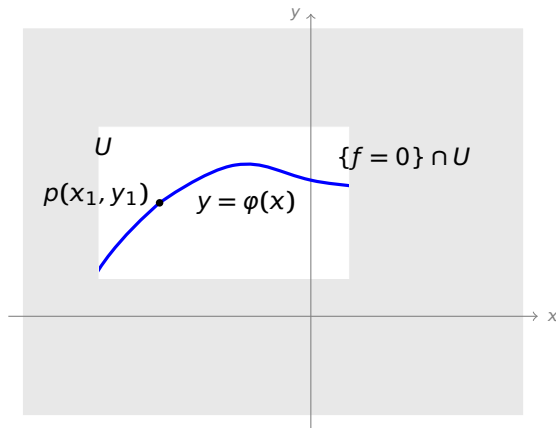
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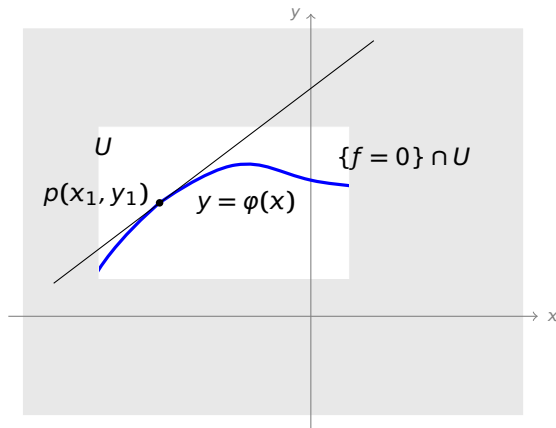
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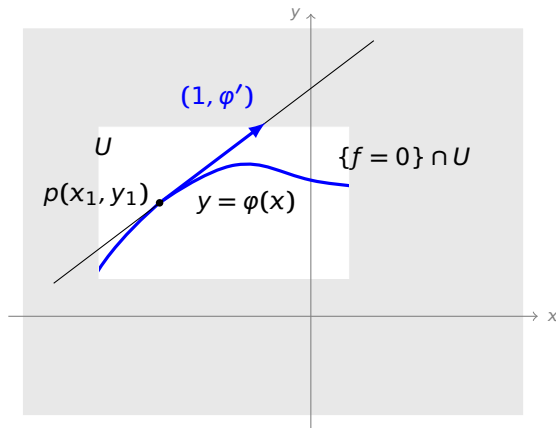
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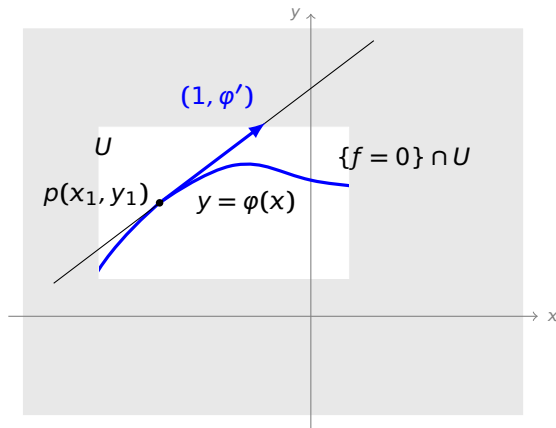
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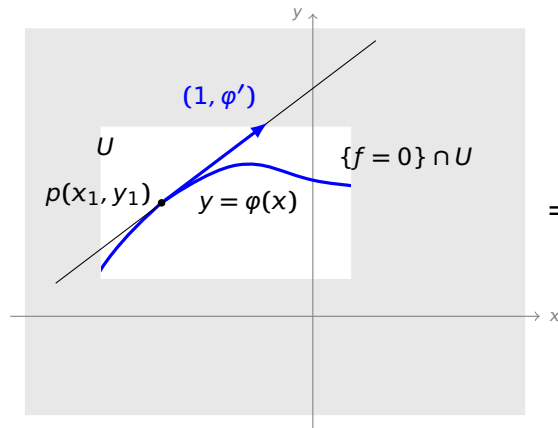


$y = \varphi(x)$ 满足 $f(x, y) = 0$

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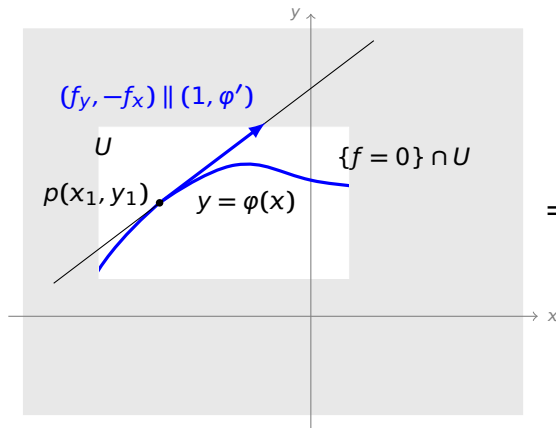


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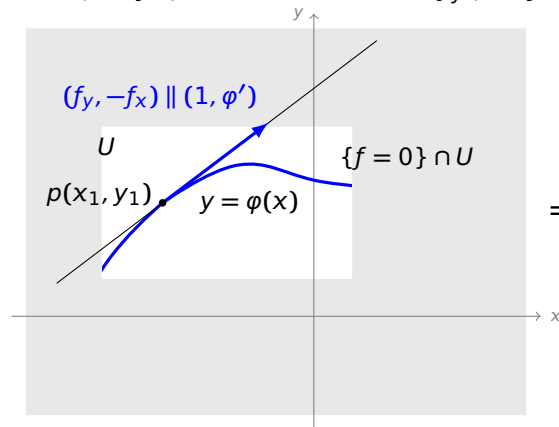
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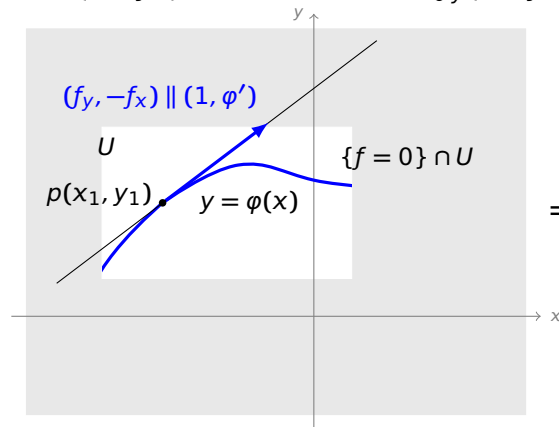
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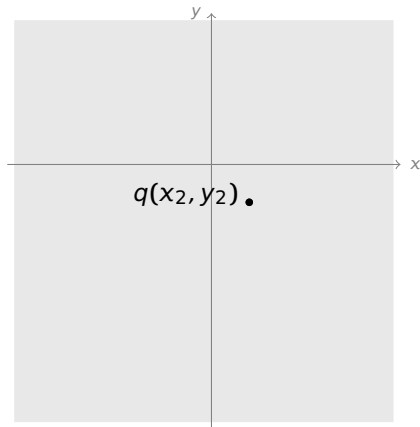


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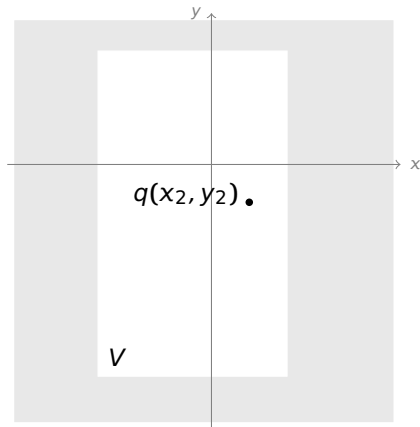
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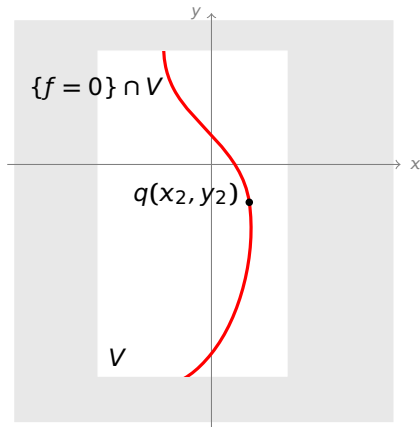
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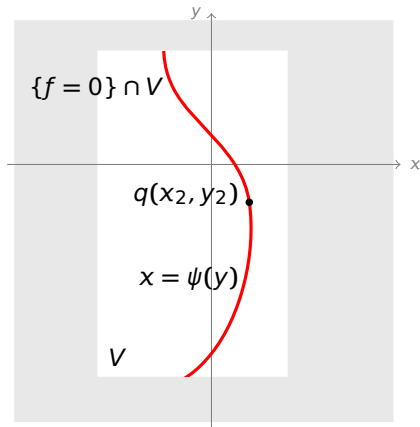
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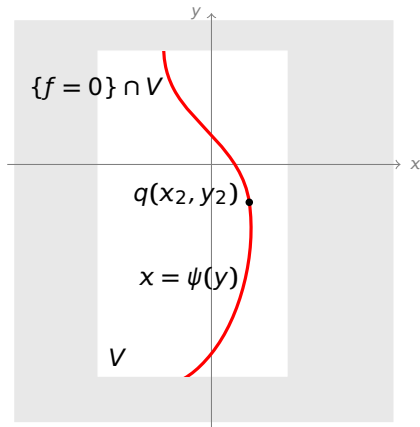
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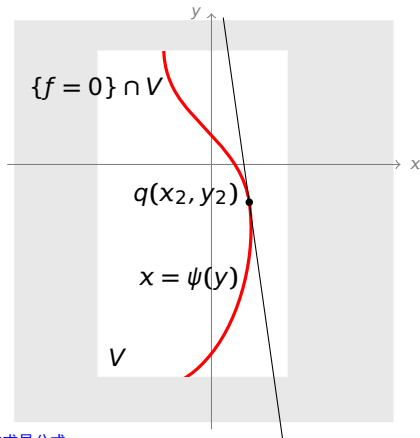
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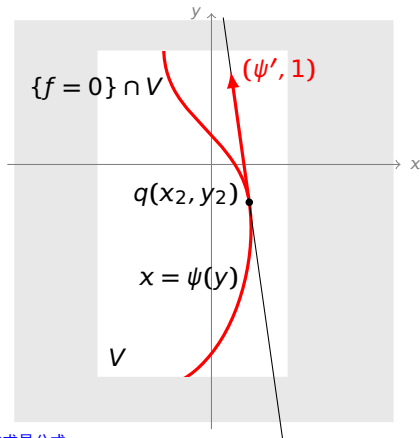
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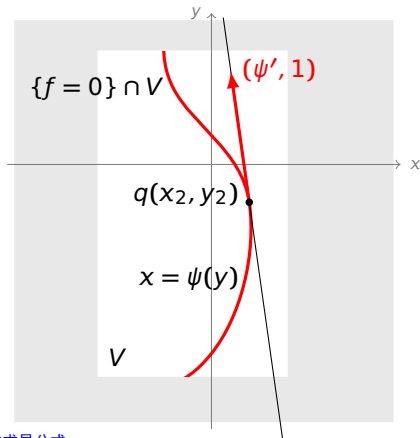
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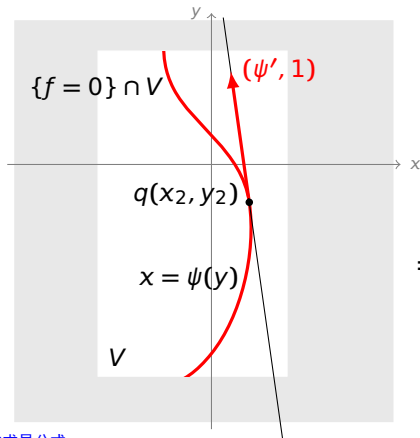


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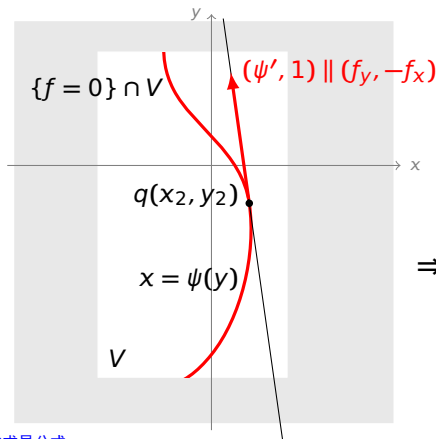


$$\begin{aligned} x = \psi(y) \text{ 满足 } f(x, y) &= 0 \\ \Rightarrow \psi' &= -\frac{f_y}{f_x} \end{aligned}$$

隐函数定理 2

设 $f(x, y)$ 光滑, $f(x_2, y_2) = 0$, $f_x(x_2, y_2) \neq 0$, 则存在光滑函数 $x = \psi(y)$ 使得:

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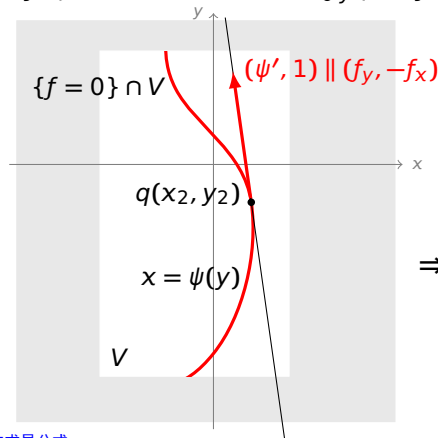
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并且 (x_2, y_2) 处的一个切向量为 $(f_y(x_2, y_2), -f_x(x_2, y_2))$,



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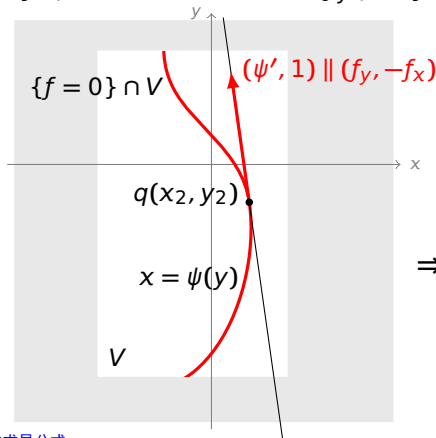
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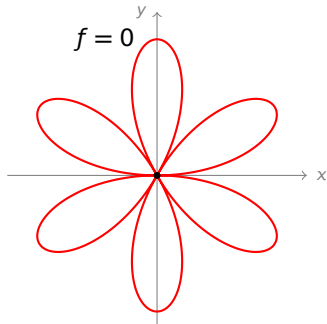
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$$f(x, y) = (3x^2 - y^2)^2 y^2 - (x^2 + y^2)^4$$

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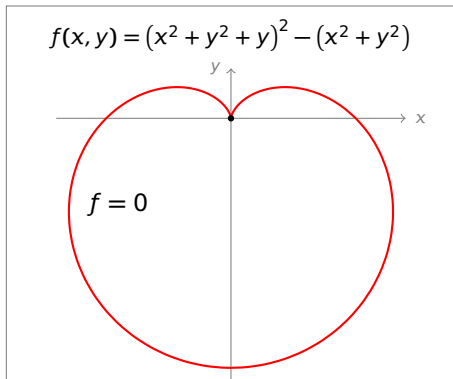
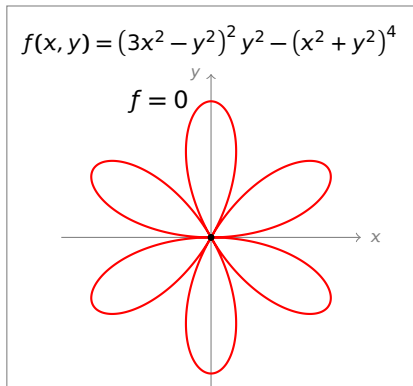
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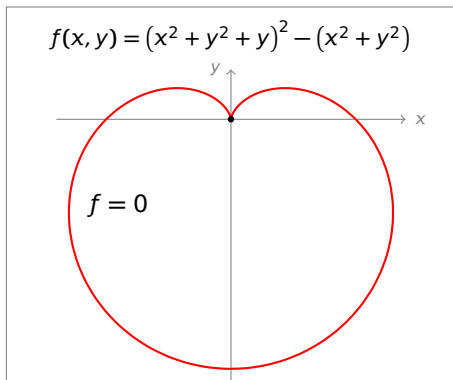
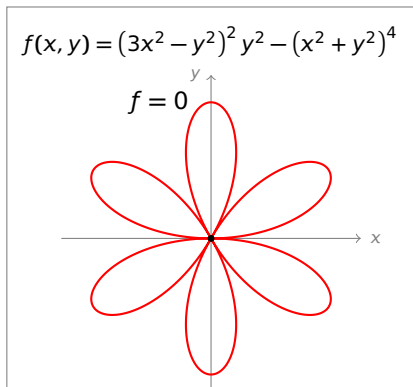


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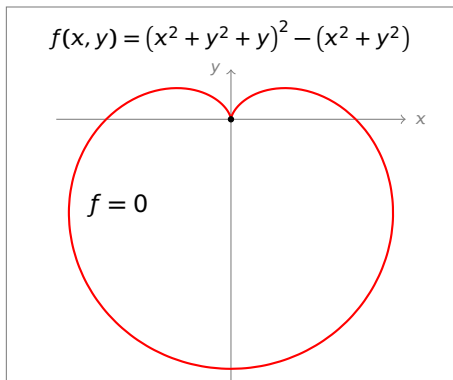
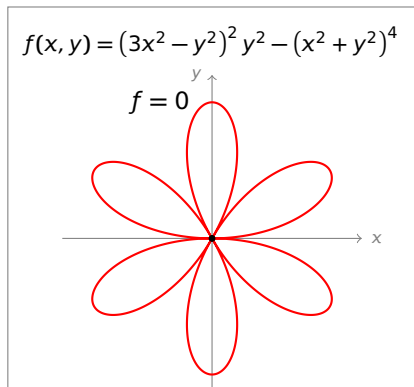


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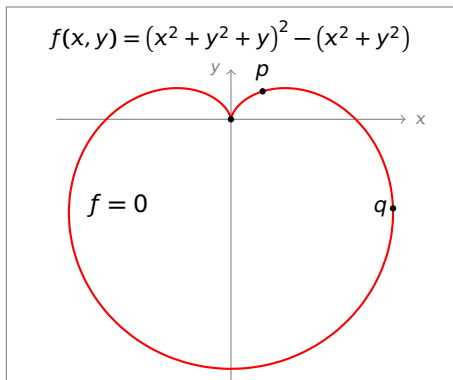
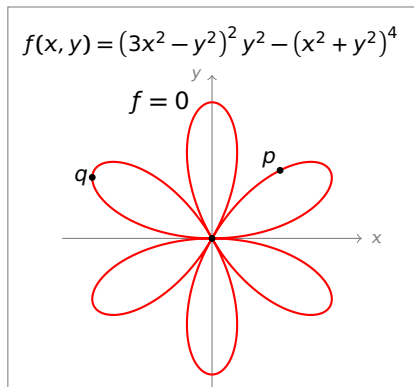
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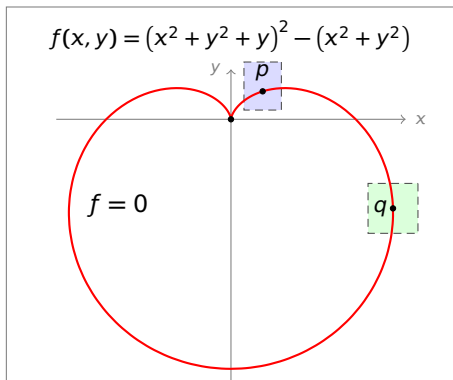
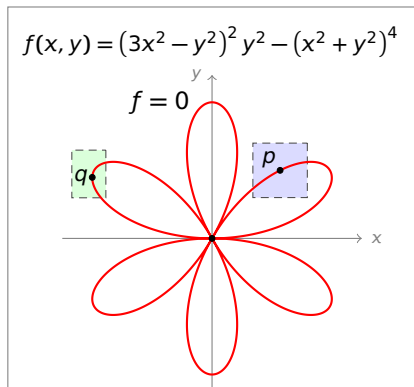
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注 2 等值线 $\{f = c\}$ 可视为空间曲线 $\begin{cases} z = f(x, y) \\ z = c \end{cases}$ 在 xoy 坐标面上的投影。

例 设 $f(x, y) = (3x^2 - y^2)^2 y^2 - (x^2 + y^2)^4$

- 在 **desmos** 上画出等值线 $\{f = c\}$
- 在 **CalcPlot3D** 上画出曲面 $z = f(x, y)$, 平面 $z = c$, 及交线空间曲线 $\begin{cases} z = f(x, y) \\ z = c \end{cases}$

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(参考值 $c = -2, -0.3, 0, 0.1$)

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- 求出 $\{f = 0\}$ 上偏导数全为零的点（临界点）
- 在 CalcPlot3D 上画出曲面 $\{f = 0\}$
- 观察临界点附近是否光滑
- 观察曲面哪些部分可以表示成光滑二元函数 $z = \varphi(x, y)$ ，或 $y = \psi(x, z)$ ，或 $x = \gamma(y, z)$ 的图形

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进一步, 若偏导数处处不全为零, 则 $\{f = c\}$ 是空间中光滑曲面 (称为等值面)。