

§8.4 二元函数偏导数与全微分

2016-2017 学年 II

教学要求



Outline of §8.4

1. 二元函数偏导数定义

3. 全微分的定义与计算

We are here now...

1. 二元函数偏导数定义

3. 全微分的定义与计算

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偏导数准确定义

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- $z = f(x, y)$ 在点 (x_0, y_0) 关于 x 的偏导数:

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta_x z}{\Delta x}$$

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例 设 $z = xy + \frac{x}{y}$, 求 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

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$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=2 \\ y=1}} = \left(x - \frac{x}{y^2} \right) \bigg|_{\substack{x=2 \\ y=1}} = 2 - \frac{2}{1} =$$

例 设 $z = xy + \frac{x}{y}$, 求 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x - \frac{x}{y^2}$$

所以

$$\left. \frac{\partial z}{\partial x} \right|_{\substack{x=2 \\ y=1}} = \left(y + \frac{1}{y} \right) \bigg|_{\substack{x=2 \\ y=1}} = 1 + \frac{1}{1} = 2$$

$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=2 \\ y=1}} = \left(x - \frac{x}{y^2} \right) \bigg|_{\substack{x=2 \\ y=1}} = 2 - \frac{2}{1} = 0$$

例 设 $z = xy + \frac{x}{y}$, 求 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

例 设 $z = xy + \frac{x}{y}$, 求 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0},$$

例 设 $z = xy + \frac{x}{y}$, 求 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = [f(x, y_0)]',$$

例 设 $z = xy + \frac{x}{y}$, 求 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0},$$

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解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx} [f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy} [f(x_0, y)] \Big|_{y=y_0}$$

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$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = [f(x_0, y)]$$

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所以 $f(x, 1)$

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$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

所以

$$f(x, 1) = 2x$$

例 设 $z = xy + \frac{x}{y}$, 求 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx} [f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy} [f(x_0, y)] \Big|_{y=y_0}$$

所以

$$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx} [f(x, 1)] =$$

例 设 $z = xy + \frac{x}{y}$, 求 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

所以

$$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$$

例 设 $z = xy + \frac{x}{y}$, 求 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

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所以

$$f(x, 1) = 2x \Rightarrow \frac{d}{dx}[f(x, 1)] = 2$$

$$\Rightarrow \frac{\partial z}{\partial x} \Big|_{\substack{x=2 \\ y=1}} = \frac{d}{dx}[f(x, 1)] \Big|_{x=2} =$$

例 设 $z = xy + \frac{x}{y}$, 求 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

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所以

$$f(x, 1) = 2x \Rightarrow \frac{d}{dx}[f(x, 1)] = 2$$

$$\Rightarrow \frac{\partial z}{\partial x} \Big|_{\substack{x=2 \\ y=1}} = \frac{d}{dx}[f(x, 1)] \Big|_{x=2} = 2,$$

$$f(2, y)$$

例 设 $z = xy + \frac{x}{y}$, 求 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

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$$f(2, y) = 2y + \frac{2}{y}$$

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$$\Rightarrow \frac{\partial z}{\partial y} \Big|_{\substack{x=2 \\ y=1}} = \frac{d}{dy} [f(2, y)] \Big|_{y=1} = 0.$$

例 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

例 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x =$$

$$u_y =$$

$$u_z =$$

例 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x =$$

$$u_y =$$

$$u_z =$$

例 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解
$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x =$$

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$$u_z =$$

例 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解
$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz$$

$$u_y =$$

$$u_z =$$

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解

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例 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

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$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z =$$

例 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z = xy$$

例 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

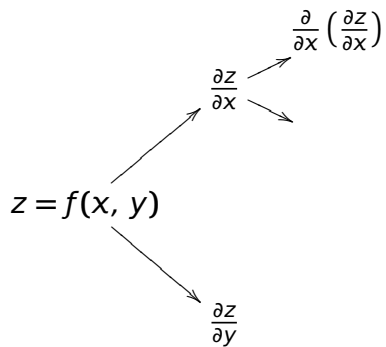
$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z = xy + \frac{1}{x}$$

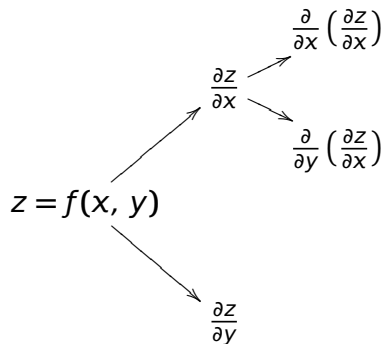
二阶偏导数

$$\begin{array}{c} \nearrow \frac{\partial z}{\partial x} \\ z = f(x, y) \\ \searrow \frac{\partial z}{\partial y} \end{array}$$

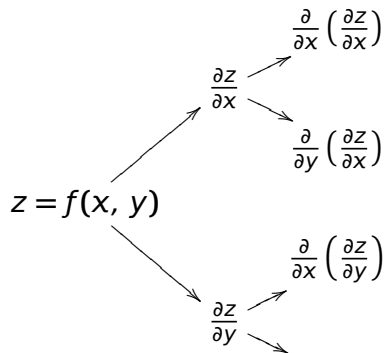
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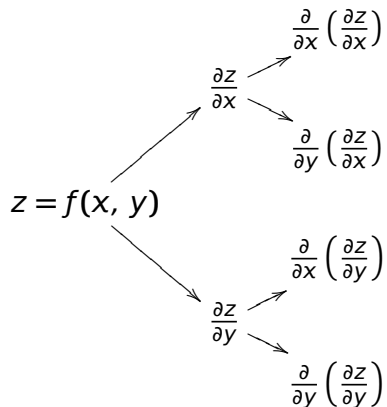
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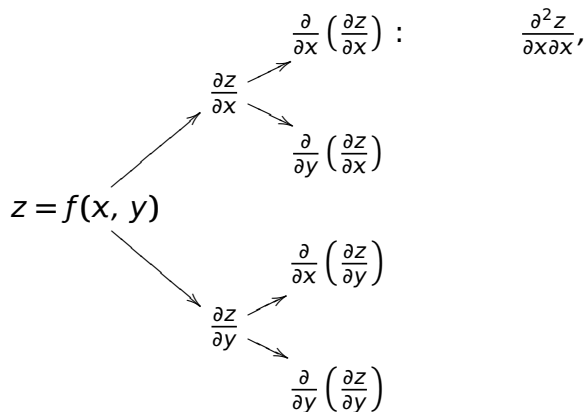
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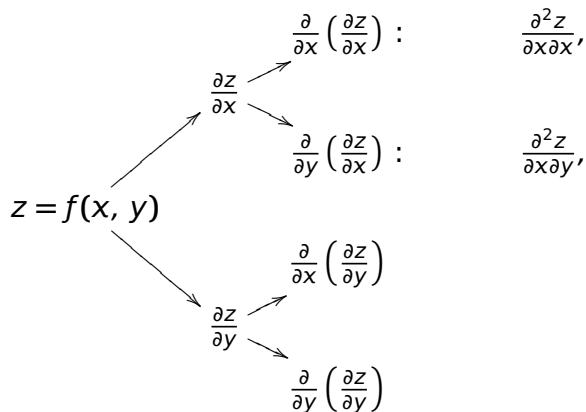
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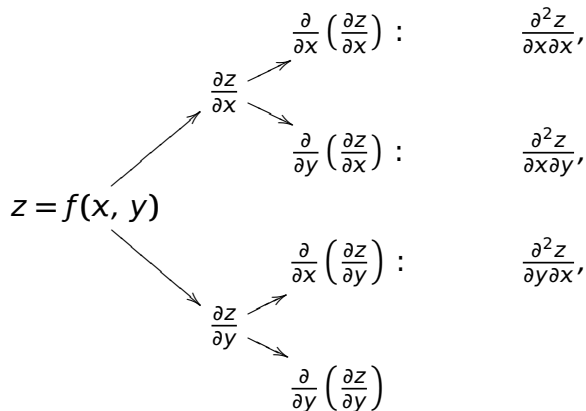
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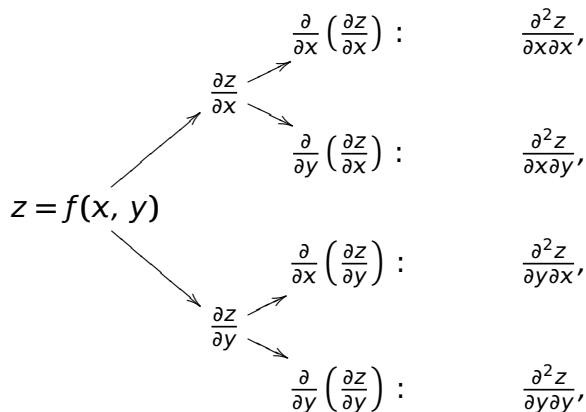
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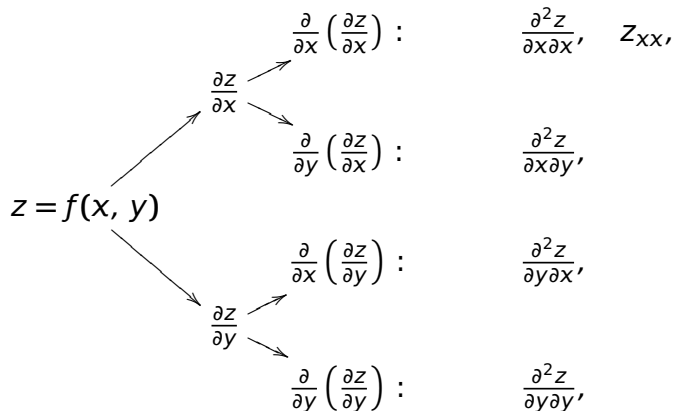
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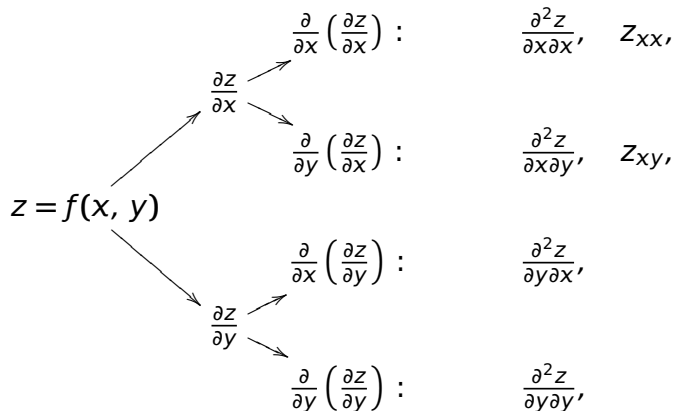
二阶偏导数



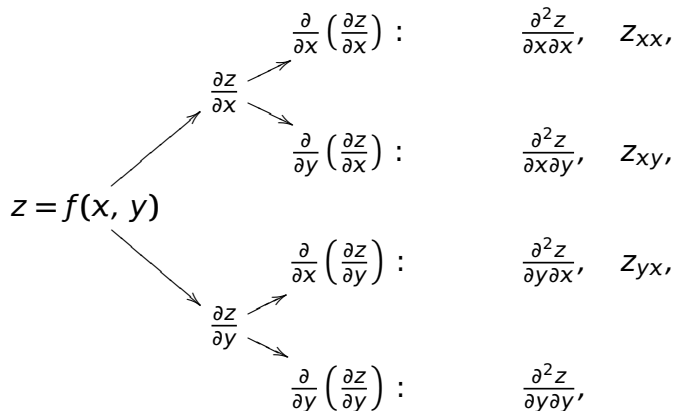
二阶偏导数



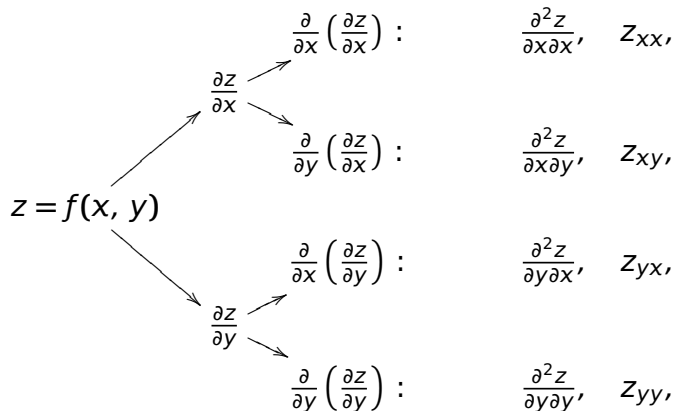
二阶偏导数



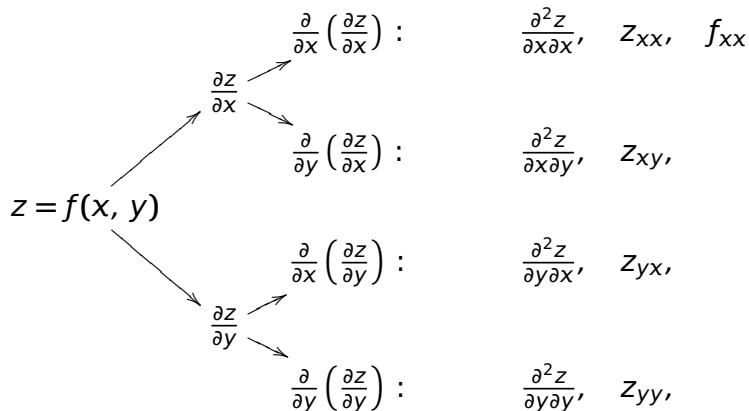
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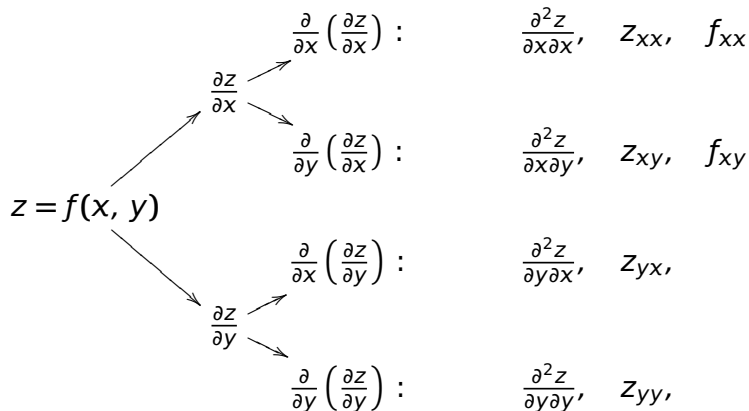
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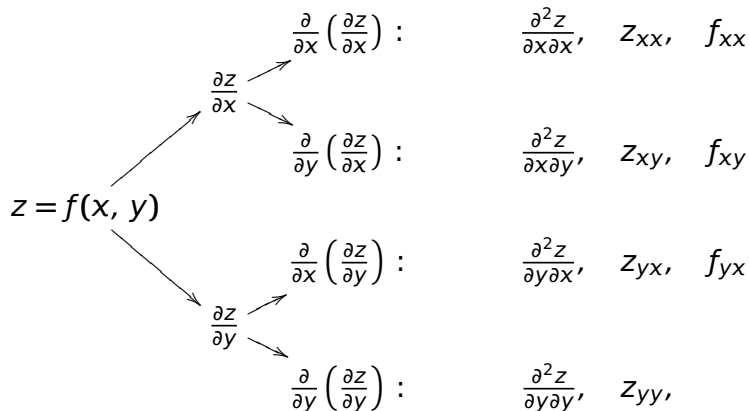
二阶偏导数



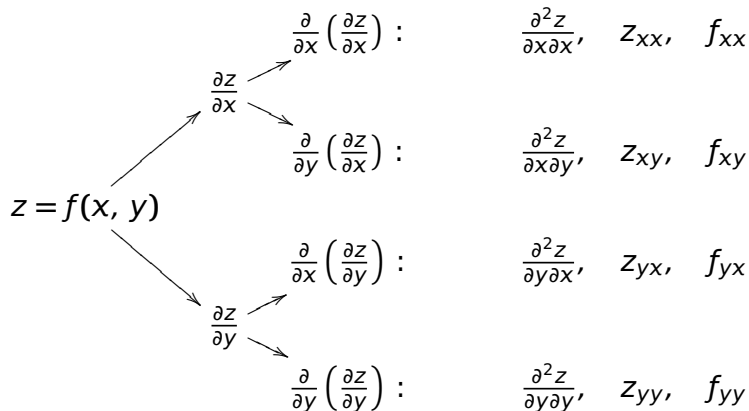
二阶偏导数



二阶偏导数



二阶偏导数



例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x =$$

$$z_y =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} +$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} +$$

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$$z_{xy} =$$

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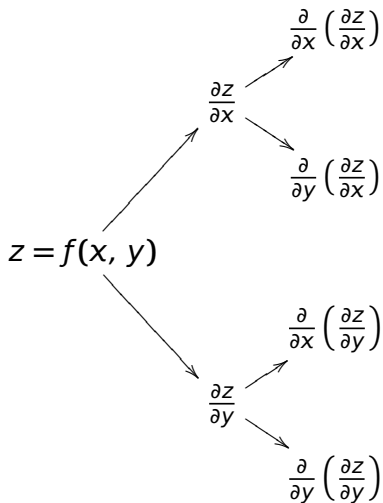
$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

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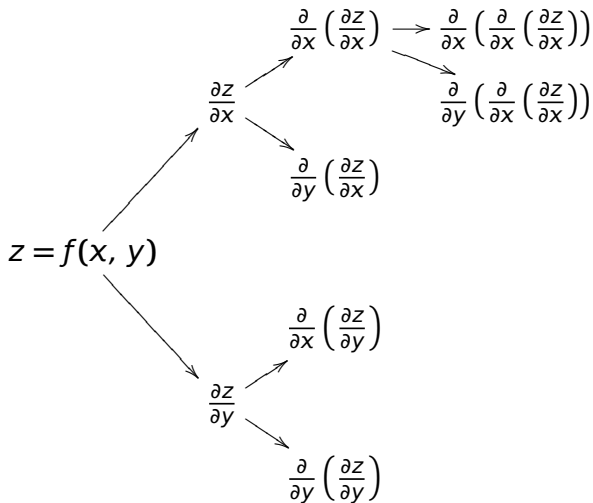
$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2 e^{xy} + 4x$$

注 此例成立 $z_{xy} = z_{yx}$

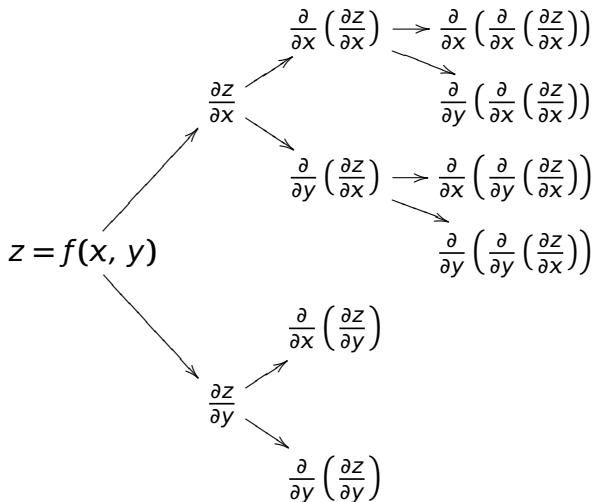
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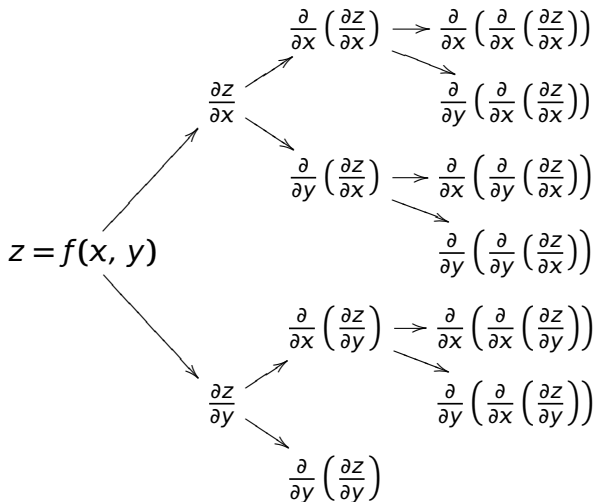
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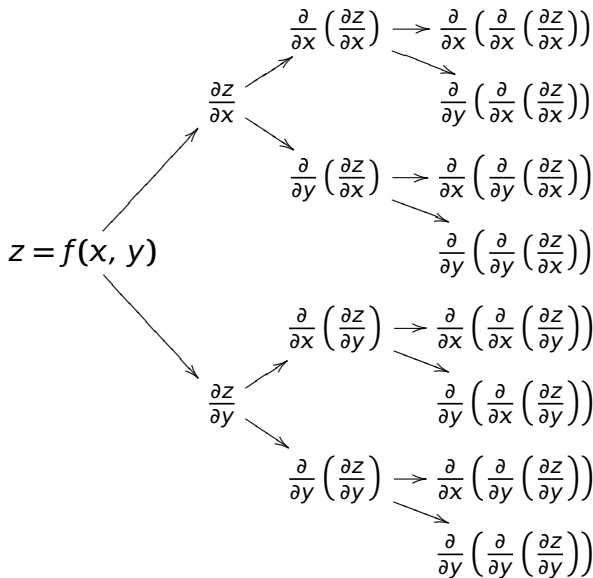
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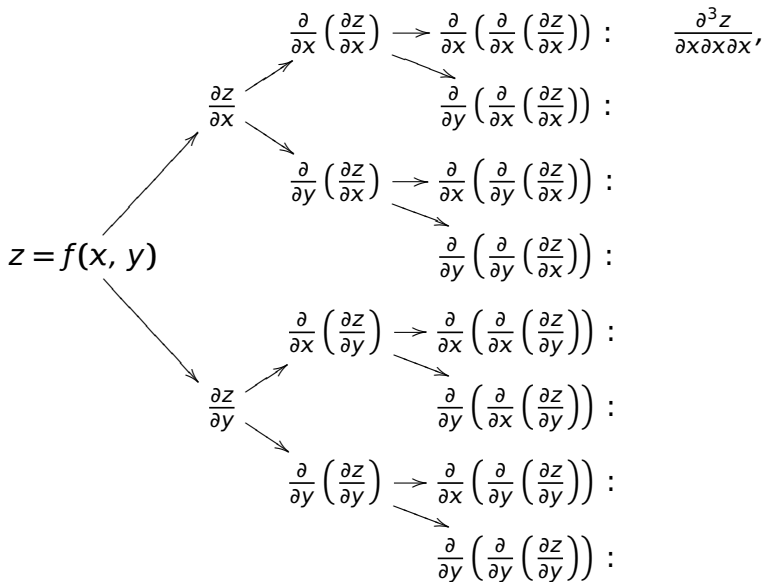
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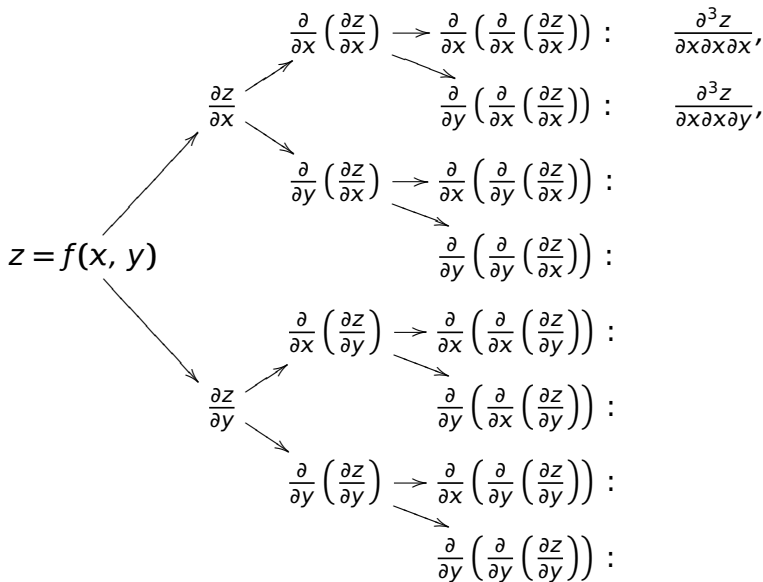
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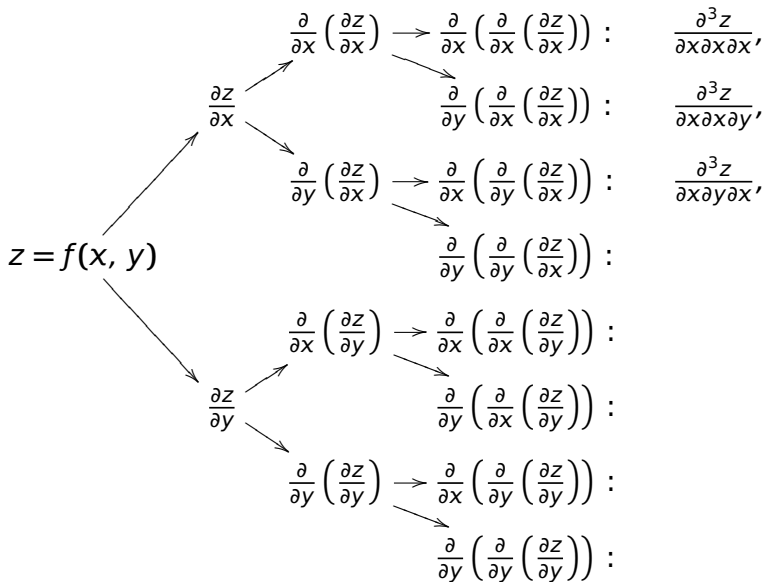
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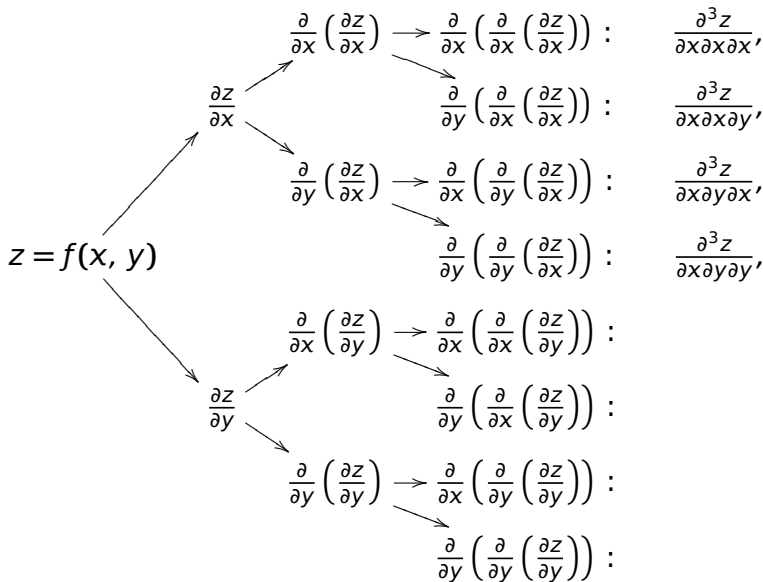
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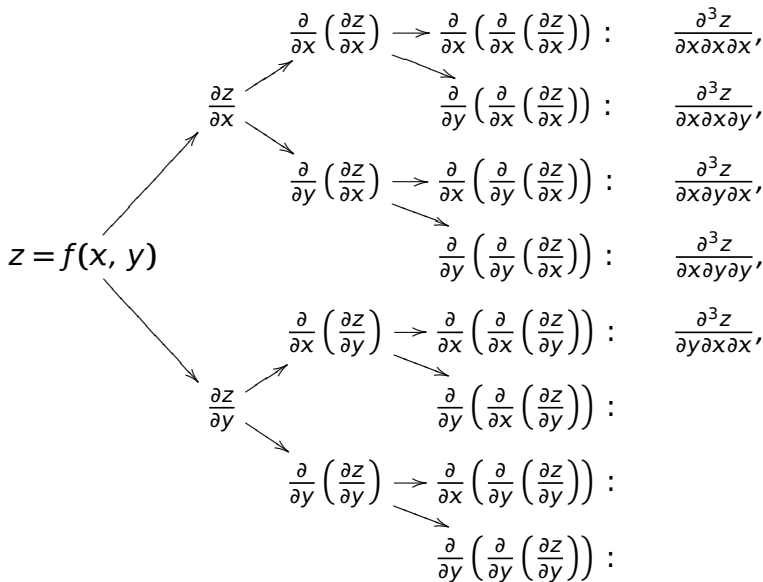
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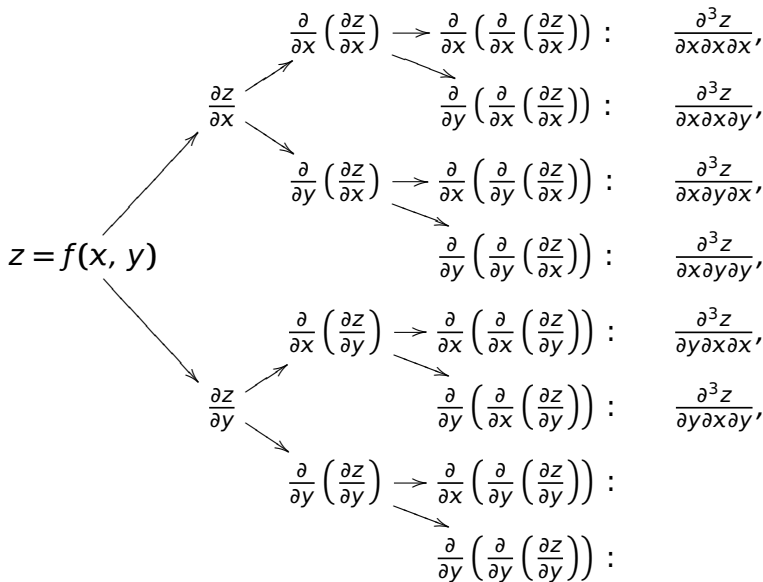
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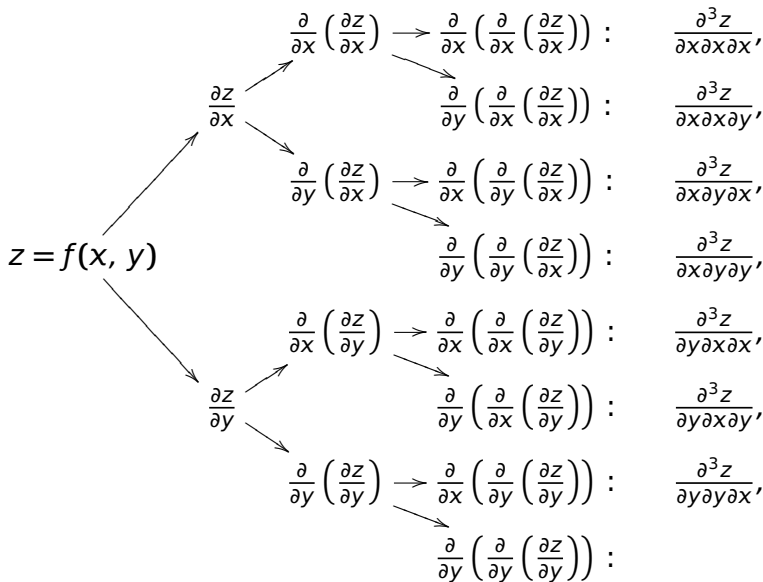
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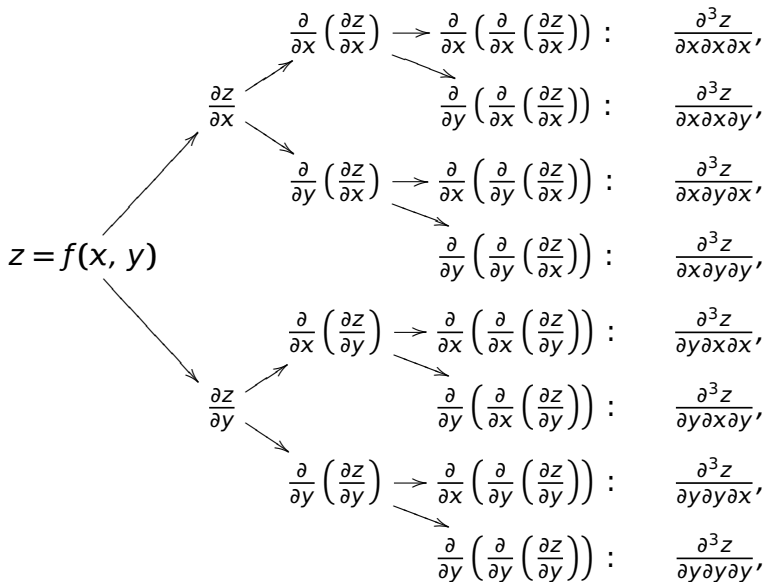
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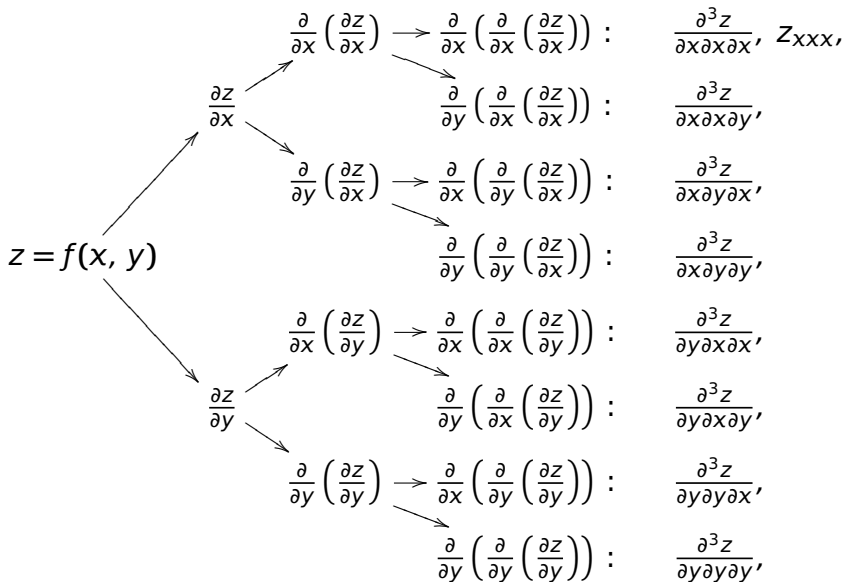
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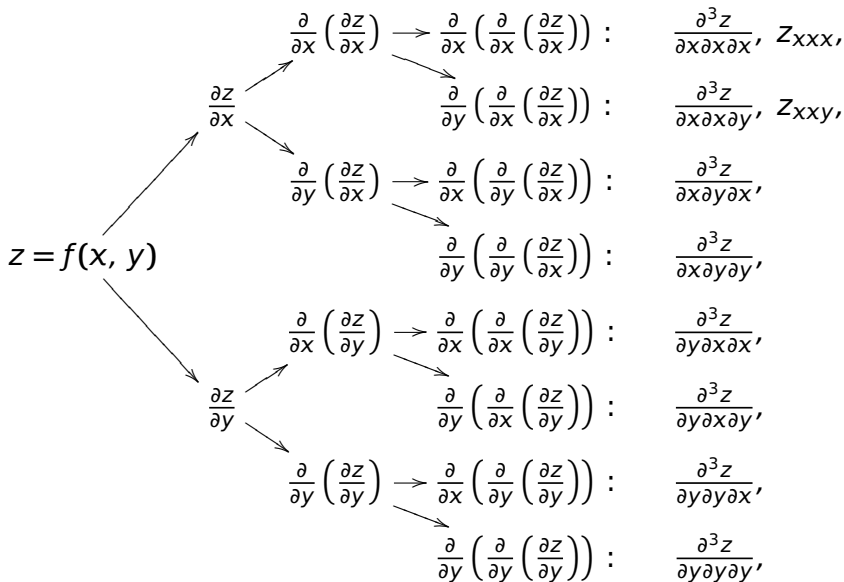
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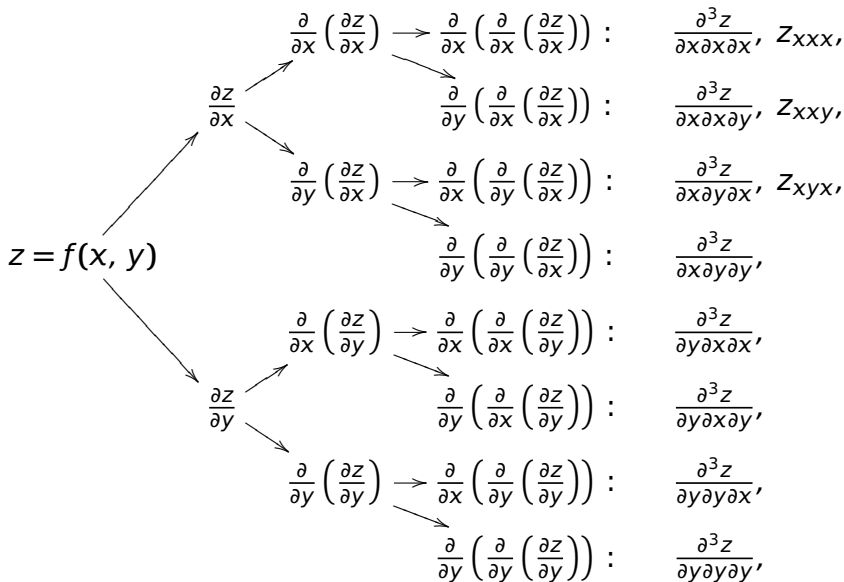
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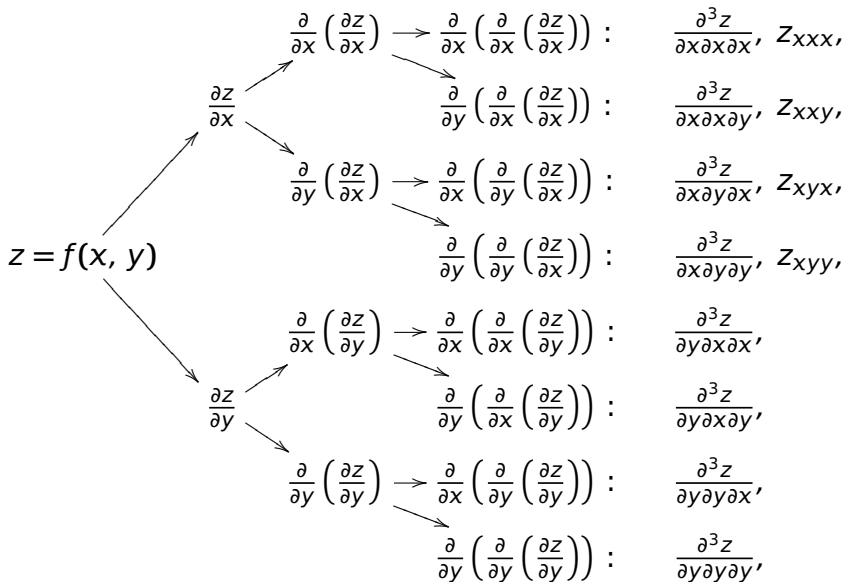
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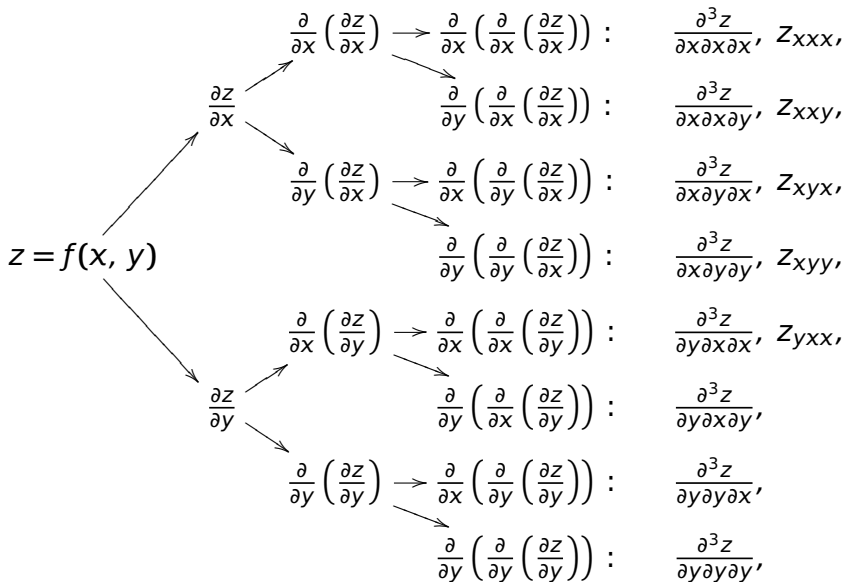
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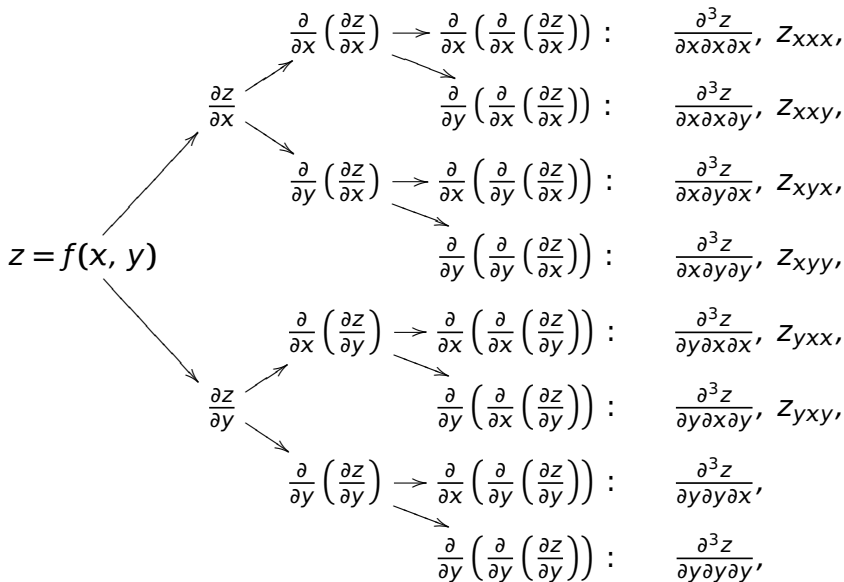
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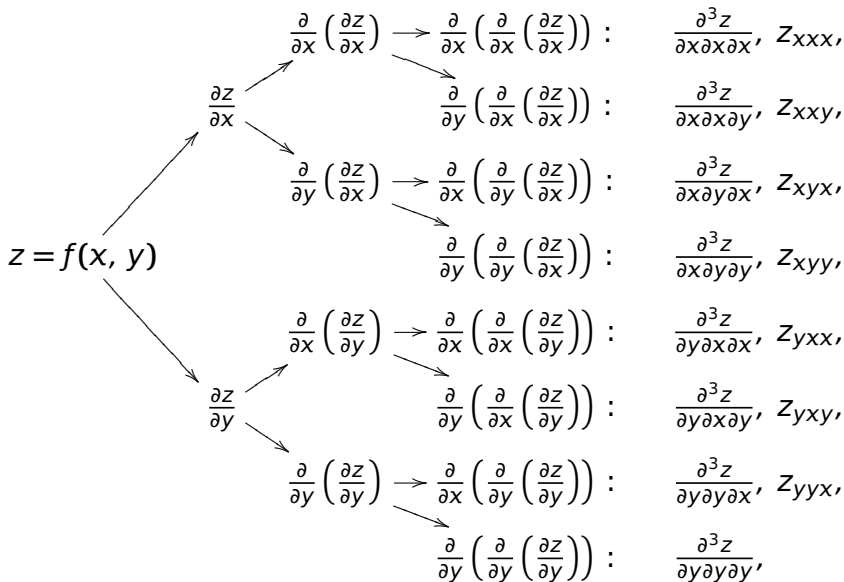
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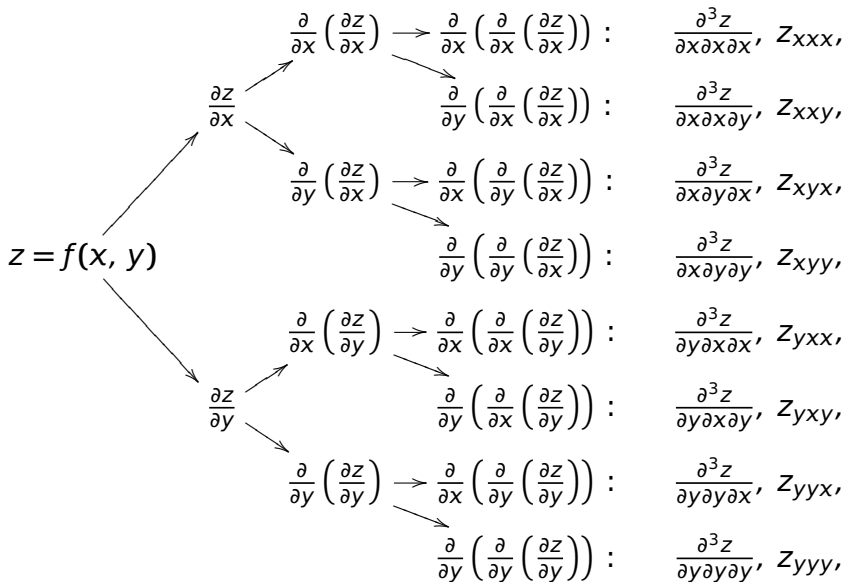
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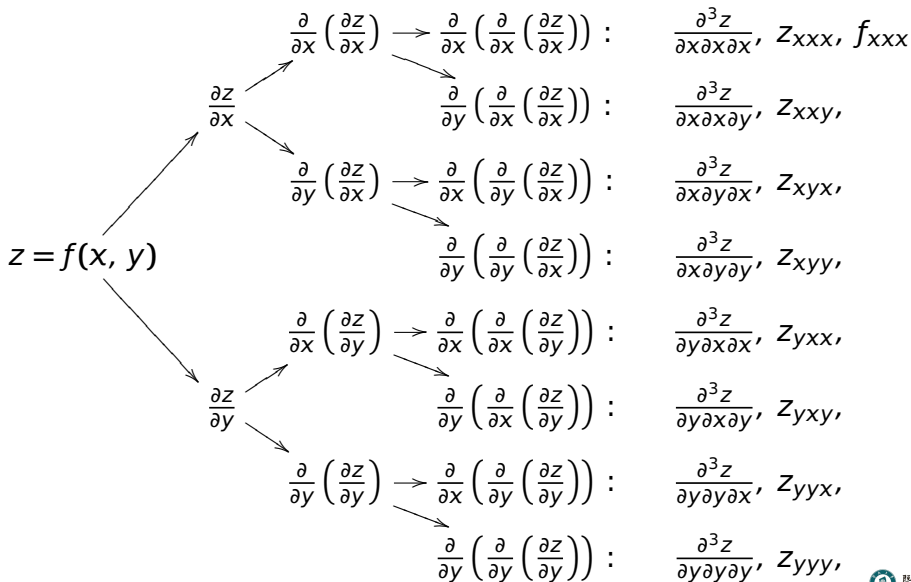
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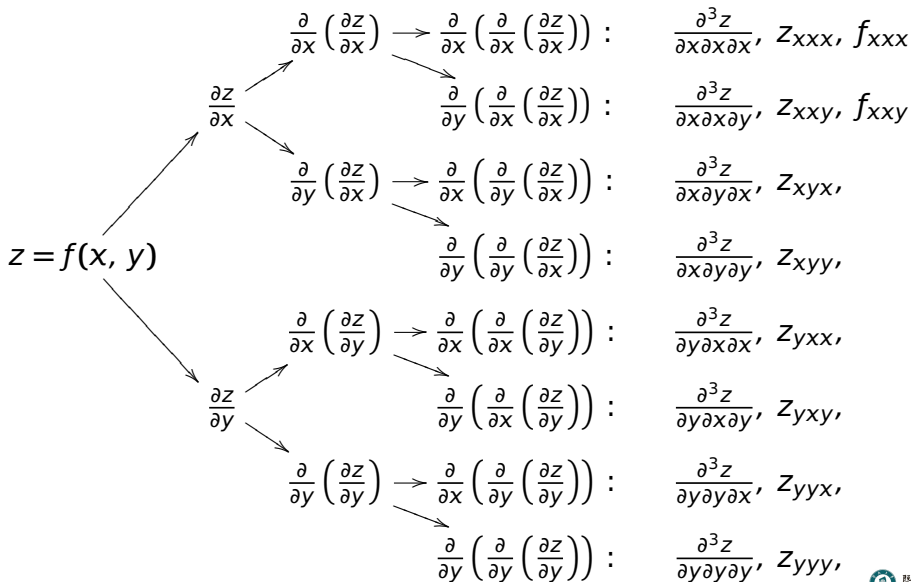
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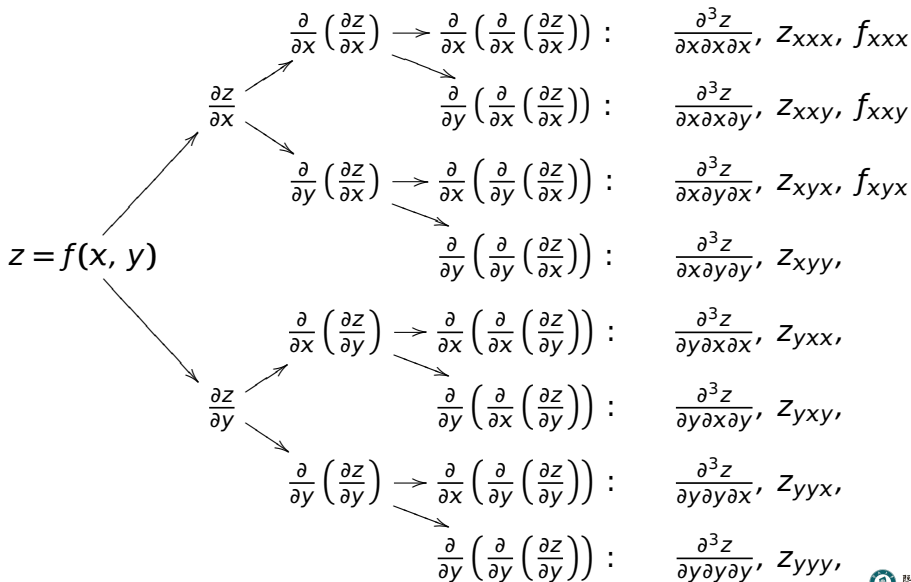
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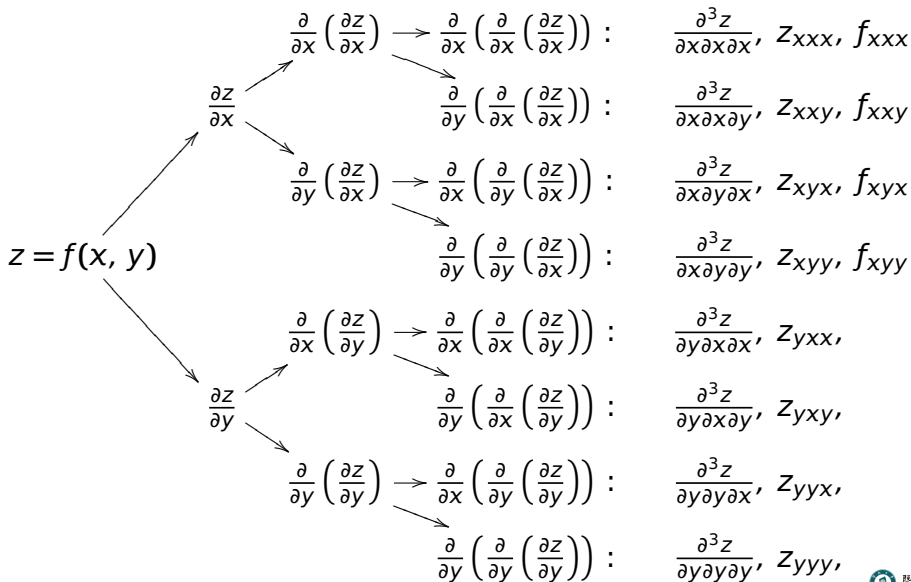
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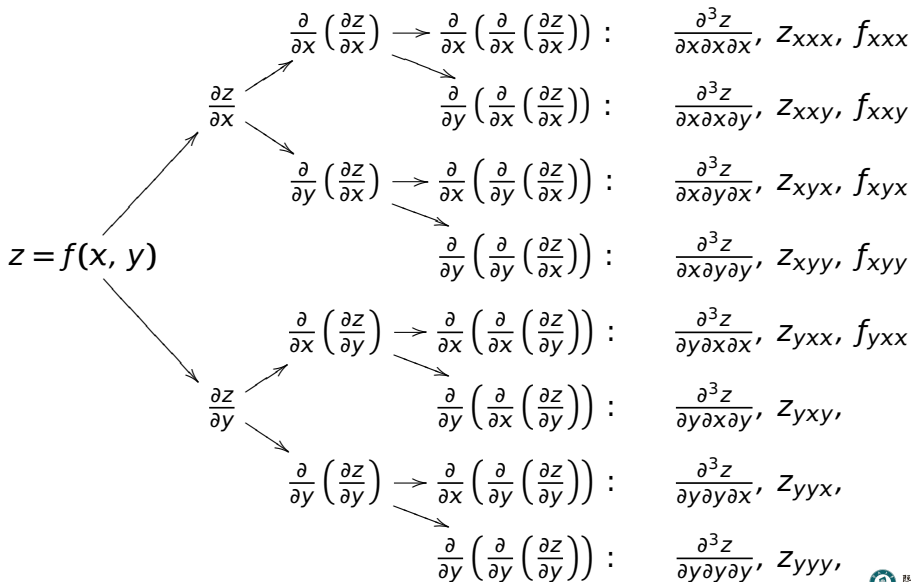
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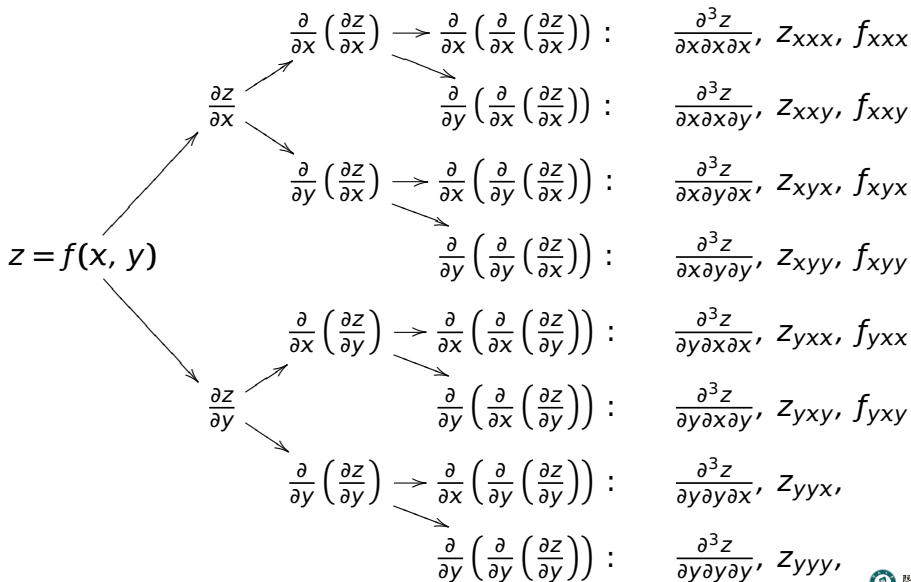
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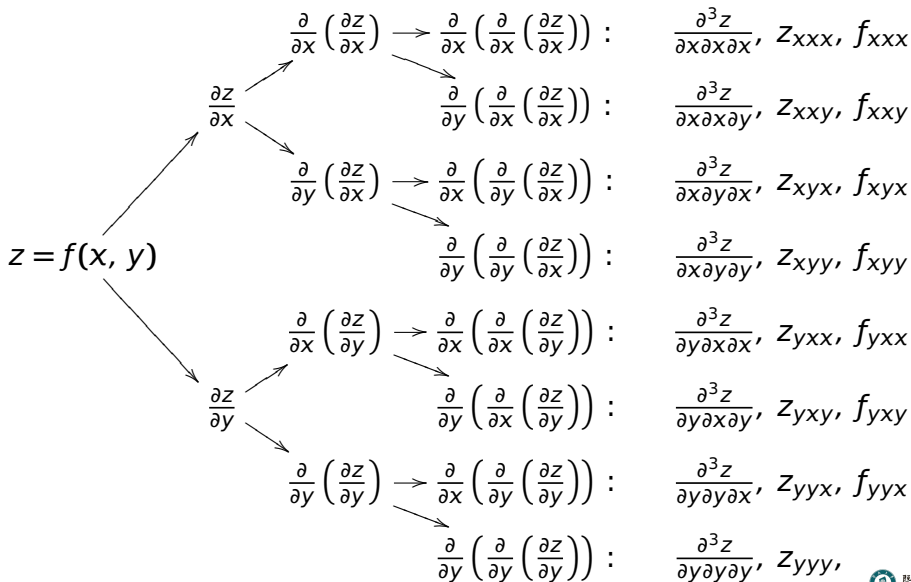
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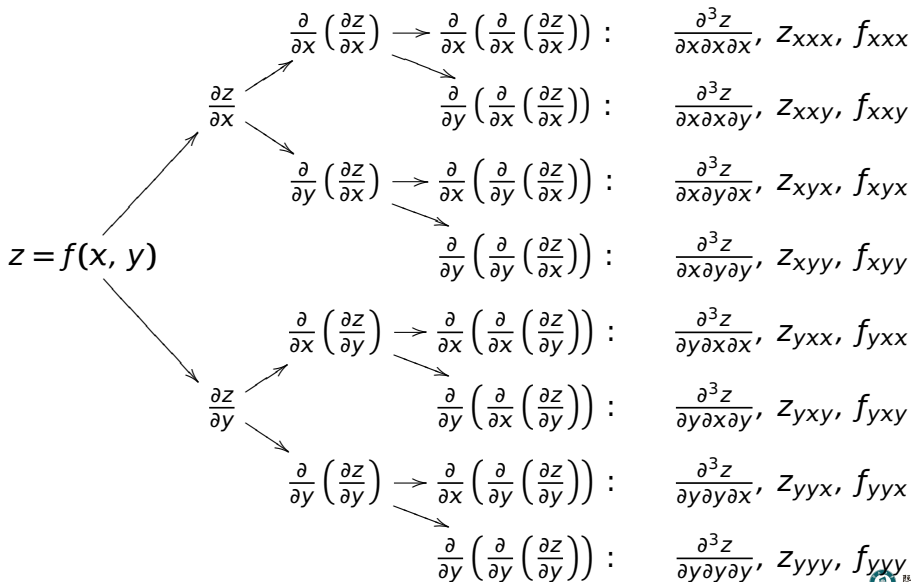
三阶偏导数



三阶偏导数



三阶偏导数



例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

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解

$$z_x =$$

$$z_y =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

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例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

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$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

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$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

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$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2$$

$$z_y =$$

$$z_{xx} =$$

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例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

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解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

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例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

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$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

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$$z_{xx} =$$

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注 此例成立 $z_{xy} = z_{yx}$

例 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

例 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

例 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

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$$z_y =$$

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解

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$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x \sin(3y))'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x \sin(3y))'_x = \sin(3y)$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x \sin(3y))'_x = \sin(3y)$$

$$z_y = (x \sin(3y))'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

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例 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x \sin(3y))'_x = \sin(3y)$$
$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$

$$z_{xx} =$$

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例 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x \sin(3y))'_x = \sin(3y)$$

$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$

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$$z_{xyy} = (3 \cos(3y))'_y = -9 \sin(3y)$$

注 此例成立 $z_{xy} = z_{yx}$

We are here now...

1. 二元函数偏导数定义

3. 全微分的定义与计算

- 函数 $y = f(x)$ 的增量

$$\Delta y = f(x + \Delta x) - f(x)$$

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$$\Delta y = f(x + \Delta x) - f(x)$$

- 若 $y = f(x)$ 可微, 则

$$\Delta y = f(x + \Delta x) - f(x) = A\Delta x + o(\Delta x)$$

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此时可用 $f'(x)\Delta x$ 近似代替 Δy ,

回顾一元函数的微分

- 函数 $y = f(x)$ 的增量

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此时可用 $f'(x)\Delta x$ 近似代替 Δy , 称为函数 $y = f(x)$ 的微分,

- 函数 $y = f(x)$ 的增量

$$\Delta y = f(x + \Delta x) - f(x)$$

- 若 $y = f(x)$ 可微, 则

$$\Delta y = f(x + \Delta x) - f(x) = A\Delta x + o(\Delta x) = f'(x)\Delta x + o(\Delta x)$$

此时可用 $f'(x)\Delta x$ 近似代替 Δy , 称为函数 $y = f(x)$ 的微分, 记为:

$$dy = f'(x)dx \quad \text{或} \quad df = f'(x)dx$$

多元函数的全微分

- 二元函数 $z = f(x, y)$

$$f(x + \Delta x, y + \Delta y) - f(x, y)$$

多元函数的全微分

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$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right)\end{aligned}$$

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例 设 $z = f(x, y) = x^2 + y^2$,

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多元函数的全微分

- 二元函数 $z = f(x, y)$ 的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

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多元函数的全微分

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例 设 $z = f(x, y) = x^2 + y^2$, 则

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= [(x + \Delta x)^2 + (y + \Delta y)^2] - [x^2 + y^2] \\ &= 2x\Delta x + 2y\Delta y + [(\Delta x)^2 + (\Delta y)^2]\end{aligned}$$

多元函数的全微分

- 二元函数 $z = f(x, y)$ 的全增量

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所以 $z = x^2 + y^2$ 可微。

- 若 $z = f(x, y)$ 可微, 则连续, 且存在偏导数 z_x, z_y , 还有
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$$\Delta z = f(x + \Delta x) - f(x)$$

$$= z_x(x, y)\Delta x + z_y(x, y)\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right)$$

$$\approx z_x(x, y)\Delta x + z_y(x, y)\Delta y$$

$z = f(x, y)$ 的全微分: $dz = z_x(x, y)dx + z_y(x, y)dy$

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$z = f(x, y)$ 的全微分: $dz = z_x(x, y)dx + z_y(x, y)dy$

- 若 $z = f(x, y)$ 可微, 则 $\Delta z \approx dz$

- 对三元函数 $u = \varphi(x, y, z)$, 其全微分

$$du = u_x dx + u_y dy + u_z dz$$

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此时

$$\Delta u = \varphi(x + \Delta x, y + \Delta y, z + \Delta z) - \varphi(x, y, z)$$

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此时

$$\Delta u = \varphi(x + \Delta x, y + \Delta y, z + \Delta z) - \varphi(x, y, z) \approx du$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

解

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$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x =$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

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$$dz = z_x dx + z_y dy =$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

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例 计算函数 $z = x^2y + y^2$ 的全微分

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例 计算函数 $z = x^2y + y^2$ 的全微分

解

$$z_x =$$

$$z_y =$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

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$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

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例 计算函数 $z = x^2y + y^2$ 的全微分

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$$z_x =$$

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$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

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$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$

例 计算函数 $z = x^2y + y^2$ 的全微分

解

$$z_x = (x^2y + y^2)'_x =$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

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例 计算函数 $z = x^2y + y^2$ 的全微分

解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x =$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

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例 计算函数 $z = x^2y + y^2$ 的全微分

解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x = 2xy$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$

例 计算函数 $z = x^2y + y^2$ 的全微分

解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x = 2xy$$

$$z_y = (x^2y + y^2)'_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

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例 计算函数 $z = x^2y + y^2$ 的全微分

解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x = 2xy$$

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$$dz = z_x dx + z_y dy =$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

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例 计算函数 $z = x^2y + y^2$ 的全微分

解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x = 2xy$$

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$$dz = z_x dx + z_y dy =$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$

例 计算函数 $z = x^2y + y^2$ 的全微分

解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x = 2xy$$

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$$dz = z_x dx + z_y dy =$$

例 计算函数 $z = \frac{y}{x}$ 的全微分

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$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

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例 计算函数 $z = x^2y + y^2$ 的全微分

解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x = 2xy$$

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$$dz = z_x dx + z_y dy = 2xy dx + (x^2 + 2y) dy$$

例 求 $z = xy$ 在点 $(2, 3)$ 处, 关于 $\Delta x = 0.1$, $\Delta y = 0.2$ 的全增量 Δz 及全微分 dz 。

解

例 求 $z = xy$ 在点 $(2, 3)$ 处, 关于 $\Delta x = 0.1$, $\Delta y = 0.2$ 的全增量 Δz 及全微分 dz 。

解

$$z_x = \quad , \quad z_y =$$

例 求 $z = xy$ 在点 $(2, 3)$ 处, 关于 $\Delta x = 0.1$, $\Delta y = 0.2$ 的全增量 Δz 及全微分 dz 。

解

$$z_x = \quad , \quad z_y =$$

$$dz = z_x dx + z_y dy =$$

例 求 $z = xy$ 在点 $(2, 3)$ 处, 关于 $\Delta x = 0.1$, $\Delta y = 0.2$ 的全增量 Δz 及全微分 dz 。

解

$$z_x = (xy)'_x = \quad , \quad z_y =$$

$$dz = z_x dx + z_y dy =$$

例 求 $z = xy$ 在点 $(2, 3)$ 处, 关于 $\Delta x = 0.1$, $\Delta y = 0.2$ 的全增量 Δz 及全微分 dz 。

解

$$z_x = (xy)'_x = y, \quad z_y =$$

$$dz = z_x dx + z_y dy =$$

例 求 $z = xy$ 在点 $(2, 3)$ 处, 关于 $\Delta x = 0.1$, $\Delta y = 0.2$ 的全增量 Δz 及全微分 dz 。

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$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y =$$
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例 求 $z = xy$ 在点 $(2, 3)$ 处, 关于 $\Delta x = 0.1$, $\Delta y = 0.2$ 的全增量 Δz 及全微分 dz 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy =$$

例 求 $z = xy$ 在点 $(2, 3)$ 处, 关于 $\Delta x = 0.1$, $\Delta y = 0.2$ 的全增量 Δz 及全微分 dz 。

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$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
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例 求 $z = xy$ 在点 $(2, 3)$ 处, 关于 $\Delta x = 0.1$, $\Delta y = 0.2$ 的全增量 Δz 及全微分 dz 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = ydx + xdy$$

将 $(x, y) = (2, 3)$ 及 $\Delta x = 0.1$ 、 $\Delta y = 0.2$ 代入得:

$$dz =$$

例 求 $z = xy$ 在点 $(2, 3)$ 处, 关于 $\Delta x = 0.1$, $\Delta y = 0.2$ 的全增量 Δz 及全微分 dz 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = ydx + xdy$$

将 $(x, y) = (2, 3)$ 及 $\Delta x = 0.1$ 、 $\Delta y = 0.2$ 代入得:

$$dz = 3 \times 0.1 +$$

例 求 $z = xy$ 在点 $(2, 3)$ 处, 关于 $\Delta x = 0.1$, $\Delta y = 0.2$ 的全增量 Δz 及全微分 dz 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = ydx + xdy$$

将 $(x, y) = (2, 3)$ 及 $\Delta x = 0.1$ 、 $\Delta y = 0.2$ 代入得:

$$dz = 3 \times 0.1 + 2 \times 0.2 =$$

例 求 $z = xy$ 在点 $(2, 3)$ 处, 关于 $\Delta x = 0.1$, $\Delta y = 0.2$ 的全增量 Δz 及全微分 dz 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = ydx + xdy$$

将 $(x, y) = (2, 3)$ 及 $\Delta x = 0.1$ 、 $\Delta y = 0.2$ 代入得:

$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

例 求 $z = xy$ 在点 $(2, 3)$ 处, 关于 $\Delta x = 0.1$, $\Delta y = 0.2$ 的全增量 Δz 及全微分 dz 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = ydx + xdy$$

将 $(x, y) = (2, 3)$ 及 $\Delta x = 0.1$ 、 $\Delta y = 0.2$ 代入得:

$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

而全增量为 $\Delta z =$

例 求 $z = xy$ 在点 $(2, 3)$ 处, 关于 $\Delta x = 0.1$, $\Delta y = 0.2$ 的全增量 Δz 及全微分 dz 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = ydx + xdy$$

将 $(x, y) = (2, 3)$ 及 $\Delta x = 0.1$ 、 $\Delta y = 0.2$ 代入得:

$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

而全增量为

$$\Delta z = z(2 + 0.1, 3 + 0.2) - z(2, 3)$$

例 求 $z = xy$ 在点 $(2, 3)$ 处, 关于 $\Delta x = 0.1$, $\Delta y = 0.2$ 的全增量 Δz 及全微分 dz 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
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$$\begin{aligned} \Delta z &= z(2 + 0.1, 3 + 0.2) - z(2, 3) \\ &= (2 + 0.1) \times (3 + 0.2) - \end{aligned}$$

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