第 01 周作业解答

练习 1. 计算 $\begin{vmatrix} 1 & -2 \\ 3 & -4 \end{vmatrix}$ 和 $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 3 & -2 & 1 \end{vmatrix}$.

解

$$\begin{vmatrix} 1 & -2 \\ 3 & -4 \end{vmatrix} = 1 \times (-4) - (-2) \times 3 = 2$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 1 & -2 \\ 3 & -2 & 1 \end{vmatrix} = \begin{matrix} 1 \cdot 1 \cdot 1 + 2 \cdot (-2) \cdot 3 + 3 \cdot 2 \cdot (-2) \\ -1 \cdot (-2) \cdot (-2) - 2 \cdot 2 \cdot 1 - 3 \cdot 1 \cdot 3 \end{matrix} = -40$$

练习 2. 当 x 为何值时，$\begin{vmatrix} 3 & 1 & x \\ 4 & x & 0 \\ 1 & 0 & x \end{vmatrix} \neq 0$?

解

$$\begin{vmatrix} 3 & 1 & x \\ 4 & x & 0 \\ 1 & 0 & x \end{vmatrix} = x \begin{vmatrix} 3 & 1 & 1 \\ 4 & x & 0 \\ 1 & 0 & 1 \end{vmatrix} = x(3x + 0 + 0 - 0 - 4 - x) = 2x(x-2)$$

所以 $x \neq 0$ 且 $x \neq 2$。

练习 3. 利用公式求解三元线性方程组

$$\begin{cases} x & +y & +z & = 6 \\ x & +2y & -z & = 2 \\ 2x & -3y & -z & = -7 \end{cases}$$

解 (1)

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 2 & -3 & -1 \end{vmatrix} = (-2) + (-2) + (-3) - 3 - (-1) - 4 = -13$$

$$D_x = \begin{vmatrix} 6 & 1 & 1 \\ 2 & 2 & -1 \\ -7 & -3 & -1 \end{vmatrix} = (-12) + 7 + (-6) - 18 - (-2) - (-14) = -13$$

所以 $x = \frac{D_x}{D} = 1$。

(2) 将 $x = 1$ 代入方程 (1)、(2) 得:

$$\begin{cases} y & +z & = 5 \\ 2y & -z & = 1 \end{cases}$$

所以

$$y = \frac{\begin{vmatrix} 5 & 1 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}} = \frac{-6}{-3} = 2, \qquad z = \frac{\begin{vmatrix} 1 & 5 \\ 2 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 7 \end{vmatrix}} = \frac{-9}{-3} = 3$$

总结 $x = 1$, $y = 2$, $z = 3$。

练习 4. 设三阶行列式 $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 6$。利用行列式的性质计算：以下两个行列式分别是多少?

$$D_1 = \begin{vmatrix} a_{11} & a_{13} - 2a_{11} & a_{12} \\ a_{21} & a_{23} - 2a_{21} & a_{22} \\ a_{31} & a_{33} - 2a_{31} & a_{32} \end{vmatrix}, \qquad D_2 = \begin{vmatrix} a_{11} & 2a_{12} & a_{13} \\ 3a_{21} & 6a_{22} & 3a_{23} \\ a_{31} & 2a_{32} & a_{33} \end{vmatrix}$$

解利用行列式的基本性质，可得：

$$D_1 = \begin{vmatrix} a_{11} & a_{13} - 2a_{11} & a_{12} \\ a_{21} & a_{23} - 2a_{21} & a_{22} \\ a_{31} & a_{33} - 2a_{31} & a_{32} \end{vmatrix} \xlongequal{\text{可加性}} \begin{vmatrix} a_{11} & a_{13} & a_{12} \\ a_{21} & a_{23} & a_{22} \\ a_{31} & a_{33} & a_{32} \end{vmatrix} + \underbrace{\cancel{\begin{vmatrix} a_{11} & -2a_{11} & a_{12} \\ a_{21} & -2a_{21} & a_{22} \\ a_{31} & -2a_{31} & a_{32} \end{vmatrix}}}_{\text{两列成比例，故为零}} \xlongequal{\text{交换2, 3列}} - \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -6$$

以及

$$D_2 = \begin{vmatrix} a_{11} & 2a_{12} & a_{13} \\ 3a_{21} & 6a_{22} & 3a_{23} \\ a_{31} & 2a_{32} & a_{33} \end{vmatrix} \xlongequal{\text{数乘性}} 3 \begin{vmatrix} a_{11} & 2a_{12} & a_{13} \\ a_{21} & 2a_{22} & a_{23} \\ a_{31} & 2a_{32} & a_{33} \end{vmatrix} \xlongequal{\text{数乘性}} 3 \cdot 2 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 3 \cdot 2 \cdot 6 = 36$$