

第 02 周作业解答

练习 1. 求不定积分

1. $\int \frac{(1+\sqrt{x})^2}{x} dx$
2. $\int (\sqrt{x} + 1)(x - \frac{1}{\sqrt{x^3}}) dx$

解: 1.

$$\int \frac{(1+\sqrt{x})^2}{x} dx = \int \frac{1+2\sqrt{x}+x}{x} dx = \int \frac{1}{x} + 2x^{-1/2} + 1 dx = \ln|x| + 4x^{1/2} + x + C.$$

2.

$$\begin{aligned} \int (\sqrt{x} + 1)(x - \frac{1}{\sqrt{x^3}}) dx &= \int (x^{1/2} + 1)(x - x^{-3/2}) dx \\ &= \int x^{3/2} + x - x^{-1} - x^{-3/2} dx \\ &= \frac{2}{5} x^{5/2} + \frac{1}{2} x^2 - \ln|x| + 2x^{-1/2} + C \end{aligned}$$

练习 2. 求不定积分

- (1) $\int \frac{x^2}{x^2+1} dx$,
- (2) $\int \frac{1}{x^2(1+x^2)} dx$.

解: 1.

$$\int \frac{x^2}{x^2+1} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$

2.

$$\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx = \int x^{-2} - \frac{1}{1+x^2} dx = -x^{-1} - \arctan x + C$$

练习 3. 求不定积分

- (1) $\int (e^x + 5^{-x})2^x dx$,
- (2) $\int (2^x + 3^x)^2 dx$.

解: 1.

$$\int (e^x + 5^{-x})2^x dx = \int e^x 2^x + (\frac{1}{5})^x 2^x dx = \int (2e)^x + (\frac{2}{5})^x dx = \frac{(2e)^x}{\ln(2e)} + \frac{(\frac{2}{5})^x}{\ln(\frac{2}{5})} + C$$

2.

$$\int (2^x + 3^x)^2 dx = \int 2^{2x} + 2 \cdot 2^x \cdot 3^x + 3^{2x} dx = \int 4^x + 2 \cdot 6^x + 9^x dx = \frac{4^x}{\ln 4} + 2 \cdot \frac{6^x}{\ln 6} + \frac{9^x}{\ln 9} + C$$

练习 4. 求不定积分

- (1) $\int (3-2x)^{5/2} dx$,
- (2) $\int \frac{1}{\sqrt[3]{7-5x}} dx$,
- (3) $\int \sin(\frac{3}{2}x) dx$

解： 1.

$$\begin{aligned}\int (3-2x)^{5/2} dx &= \int (3-2x)^{5/2} \cdot \left(-\frac{1}{2}\right) d(3-2x) \stackrel{u=3-2x}{=} -\frac{1}{2} \int u^{5/2} du \\ &= -\frac{1}{7} u^{7/2} + C = -\frac{1}{7} (3-2x)^{7/2} + C\end{aligned}$$

2.

$$\begin{aligned}\int \frac{1}{\sqrt[3]{7-5x}} dx &= \int (7-5x)^{-1/3} dx = \int (7-5x)^{-1/3} \cdot \left(-\frac{1}{5}\right) d(7-5x) \\ &\stackrel{u=7-5x}{=} -\frac{1}{5} \int u^{-1/3} du = -\frac{3}{10} u^{2/3} + C = -\frac{3}{10} (7-5x)^{2/3} + C\end{aligned}$$

3.

$$\int \sin\left(\frac{3}{2}x\right) dx = \int \sin\left(\frac{3}{2}x\right) \cdot \frac{2}{3} d\left(\frac{3}{2}x\right) = -\frac{2}{3} \cos\left(\frac{3}{2}x\right) + C$$