第11章e:对坐标的曲面积分

数学系 梁卓滨

2019-2020 学年 II

Outline



定义

• 一个曲面称为可定向,是指该曲面在整体上的具有两侧.



定义

• 一个曲面称为可定向,是指该曲面在整体上的具有两侧.

例

● 球面可定向,有内、外侧之分.

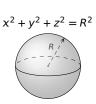
$$x^2 + y^2 + z^2 = R^2$$

定义

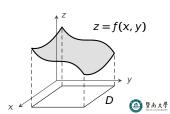
● 一个曲面称为可定向,是指该曲面在整体上的具有两侧.

例

● 球面可定向,有内、外侧之分.



● 二元函数图形可定向,有上、下侧之分.

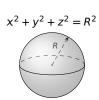


定义

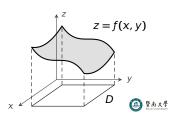
- 一个曲面称为可定向,是指该曲面在整体上的具有两侧.
- 指定可定向曲面的**定向** 是指:指定一侧为正侧,另一侧为负侧.

例

● 球面可定向,有内、外侧之分.



● 二元函数图形可定向,有上、下侧之分.

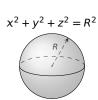


定义

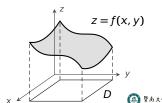
- 一个曲面称为可定向,是指该曲面在整体上的具有两侧。
- 指定可定向曲面的定向是指:指定一侧为正侧,另一侧为负侧。
- 指定了定向的可定向曲面, 称为定向曲面。

例

球面可定向,有内、外侧之分.



二元函数图形可定向,有上、下侧之分.

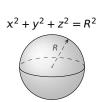


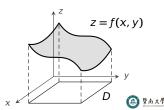
定义

- 一个曲面称为可定向,是指该曲面在整体上的具有两侧.
- 指定可定向曲面的定向是指:指定一侧为正侧,另一侧为负侧.
- 指定了定向的可定向曲面, 称为定向曲面.

- 球面可定向,有内、外侧之分. 两种定向:
 - 以外侧为正向的定向球面



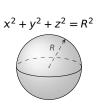


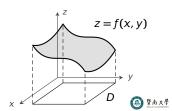


定义

- 一个曲面称为可定向,是指该曲面在整体上的具有两侧.
- 指定可定向曲面的定向是指:指定一侧为正侧,另一侧为负侧.
- 指定了定向的可定向曲面, 称为定向曲面.

- 球面可定向,有内、外侧之分. 两种定向:
 - 以外侧为正向的定向球面
 - 以内侧为正向的定向球面
- 二元函数图形可定向,有上、下侧之分.

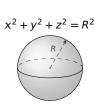


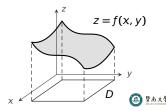


定义

- 一个曲面称为可定向,是指该曲面在整体上的具有两侧.
- 指定可定向曲面的定向是指:指定一侧为正侧,另一侧为负侧.
- 指定了定向的可定向曲面, 称为定向曲面.

- ▼球面可定向,有内、外侧之分. 两种定向:
 - 以外侧为正向的定向球面
 - 以内侧为正向的定向球面
- 二元函数图形可定向,有上、下侧之分. 两种定向:
 - 以上侧为正向的定向函数图形

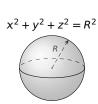


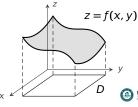


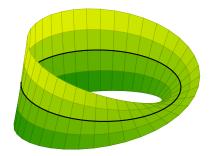
定义

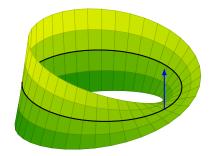
- 一个曲面称为可定向,是指该曲面在整体上的具有两侧。
- 指定可定向曲面的定向是指:指定一侧为正侧,另一侧为负侧。
- 指定了定向的可定向曲面、称为定向曲面。

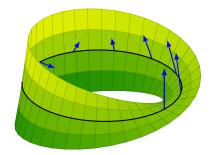
- 球面可定向,有内、外側之分. 两种定向:
 - 以外侧为正向的定向球面
 - 以内侧为正向的定向球面
- 二元函数图形可定向,有上、下侧之分。 两种定向:
 - 以上侧为正向的定向函数图形
 - 以下侧为正向的定向函数图形

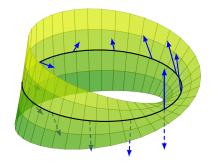


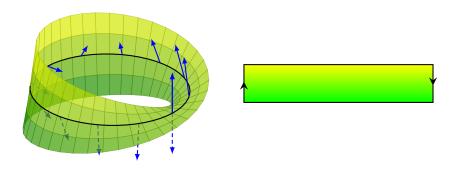






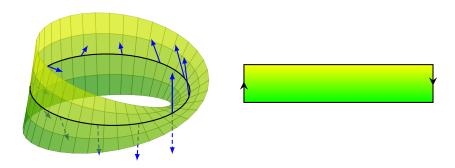






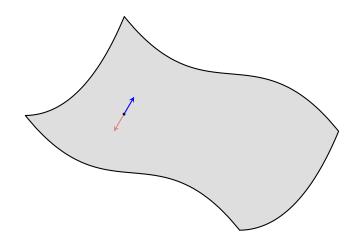
制作方法将纸带旋转半周,再把两端粘合(如图,使得两端箭头重合)



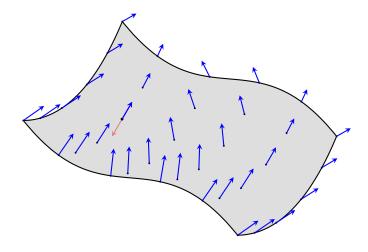




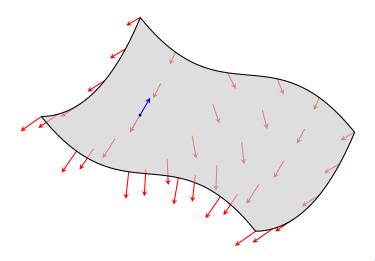
● 曲面上任一点,有两个单位法向量(方向相反),分别指向两侧.



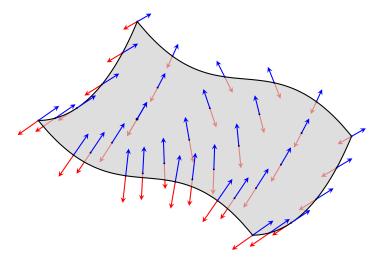
• 曲面上任一点,有两个单位法向量(方向相反),分别指向两侧.



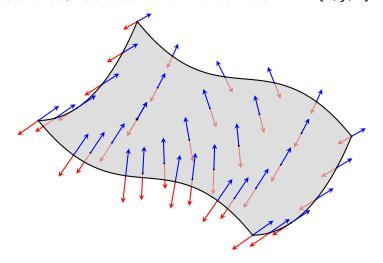
• 曲面上任一点,有两个单位法向量(方向相反),分别指向两侧.

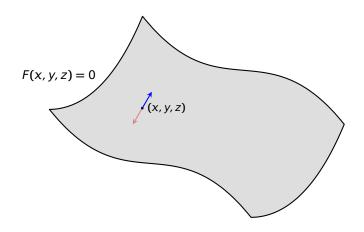


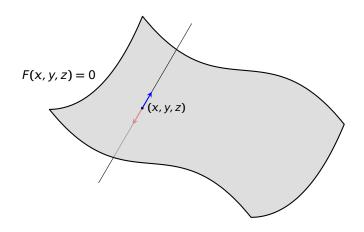
• 曲面上任一点,有两个单位法向量(方向相反),分别指向两侧.

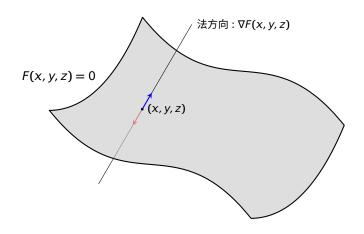


- 曲面上任一点,有两个单位法向量(方向相反),分别指向两侧.
- 给曲面定向,等价于指定其中一个单位法向量场 $\overrightarrow{n}(x, y, z)$.

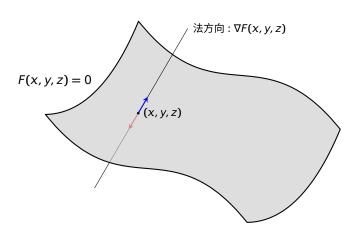




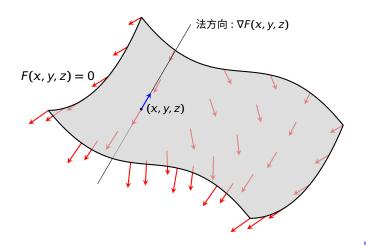




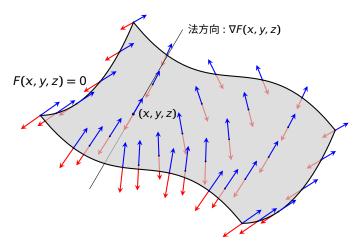
$$\frac{1}{|\nabla F|}\nabla F \quad \leftrightarrows \quad -\frac{1}{|\nabla F|}\nabla F.$$



$$\frac{1}{|\nabla F|}\nabla F \quad = \quad -\frac{1}{|\nabla F|}\nabla F$$



$$\frac{1}{|\nabla F|}\nabla F \quad \leftrightarrows \quad -\frac{1}{|\nabla F|}\nabla F$$



指向外侧,哪个指向内侧?

指向外侧,哪个指向内侧?

$$\mathbf{F} \Leftrightarrow F(x, y, z) = x^2 + y^2 + z^2 - R^2$$
,则球面方程改写为 $F = 0$.

例 1 写出球面 $x^2 + y^2 + z^2 = R^2$ 的两个单位法向量场,并指出哪一个指向外侧,哪个指向内侧?

]?

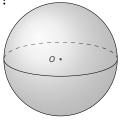
$$\mathbf{F} \Leftrightarrow F(x, y, z) = x^2 + y^2 + z^2 - R^2$$
,则球面方程改写为 $F = 0$. 计算

$$\nabla F = |\nabla F| =$$

$$\frac{1}{|\nabla F|}\nabla F = -\frac{1}{|\nabla F|}\nabla F =$$



指向外侧,哪个指向内侧?



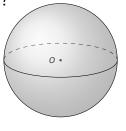
$$\mathbf{F} \Leftrightarrow F(x, y, z) = x^2 + y^2 + z^2 - R^2$$
,则球面方程改写为 $F = 0$. 计算

$$\nabla F = (2x, 2y, 2z), \qquad |\nabla F| =$$

$$\frac{1}{|\nabla F|}\nabla F = -\frac{1}{|\nabla F|}\nabla F =$$



指向外侧,哪个指向内侧?



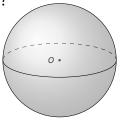
$$\mathbf{F} \Leftrightarrow F(x, y, z) = x^2 + y^2 + z^2 - R^2$$
,则球面方程改写为 $F = 0$. 计算

$$\nabla F = (2x, 2y, 2z), \qquad |\nabla F| = 2\sqrt{x^2 + y^2 + z^2}$$

$$\frac{1}{|\nabla F|}\nabla F = -\frac{1}{|\nabla F|}\nabla F =$$



指向外侧,哪个指向内侧?

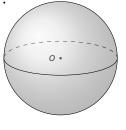


$$\mathbf{F} \Leftrightarrow F(x, y, z) = x^2 + y^2 + z^2 - R^2$$
,则球面方程改写为 $F = 0$. 计算

$$\nabla F = (2x, 2y, 2z), \qquad |\nabla F| = 2\sqrt{x^2 + y^2 + z^2} = 2R$$

$$\frac{1}{|\nabla F|}\nabla F = -\frac{1}{|\nabla F|}\nabla F =$$

指向外侧,哪个指向内侧?



$$\mathbf{F} \Leftrightarrow F(x, y, z) = x^2 + y^2 + z^2 - R^2$$
,则球面方程改写为 $F = 0$. 计算

$$\nabla F = (2x, 2y, 2z), \qquad |\nabla F| = 2\sqrt{x^2 + y^2 + z^2} = 2R$$

$$\frac{1}{|\nabla F|}\nabla F = \frac{1}{R}(x, y, z), \qquad -\frac{1}{|\nabla F|}\nabla F =$$

指向外侧,哪个指向内侧?

解 令
$$F(x, y, z) = x^2 + y^2 + z^2 - R^2$$
,则球面方程改写为 $F = 0$. 计算
$$\nabla F = (2x, 2y, 2z), \qquad |\nabla F| = 2\sqrt{x^2 + y^2 + z^2} = 2R$$

$$\frac{1}{|\nabla F|}\nabla F = \frac{1}{R}(x, y, z), \qquad -\frac{1}{|\nabla F|}\nabla F = -\frac{1}{R}(x, y, z)$$



指向外侧,哪个指向内侧?

解 令
$$F(x, y, z) = x^2 + y^2 + z^2 - R^2$$
,则球面方程改写为 $F = 0$. 计算
$$\nabla F = (2x, 2y, 2z), \qquad |\nabla F| = 2\sqrt{x^2 + y^2 + z^2} = 2R$$

$$\forall F = (2x, 2y, 2z), \quad |\forall F| = 2\sqrt{x^2 + y^2 + z^2} = 2F$$

$$\frac{1}{|\nabla F|}\nabla F = \frac{1}{R}(x, y, z), \qquad -\frac{1}{|\nabla F|}\nabla F = -\frac{1}{R}(x, y, z)$$



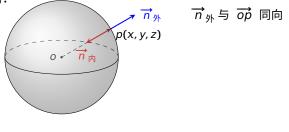
指向外侧,哪个指向内侧?

解 令
$$F(x, y, z) = x^2 + y^2 + z^2 - R^2$$
,则球面方程改写为 $F = 0$. 计算
$$\nabla F = (2x, 2y, 2z), \qquad |\nabla F| = 2\sqrt{x^2 + y^2 + z^2} = 2R$$

$$\frac{1}{|\nabla F|}\nabla F = \frac{1}{R}(x, y, z), \qquad -\frac{1}{|\nabla F|}\nabla F = -\frac{1}{R}(x, y, z)$$



指向外侧,哪个指向内侧?



解 令
$$F(x, y, z) = x^2 + y^2 + z^2 - R^2$$
,则球面方程改写为 $F = 0$. 计算
$$\nabla F = (2x, 2y, 2z), \qquad |\nabla F| = 2\sqrt{x^2 + y^2 + z^2} = 2R$$

$$\frac{1}{|\nabla F|}\nabla F = \frac{1}{R}(x, y, z), \qquad -\frac{1}{|\nabla F|}\nabla F = -\frac{1}{R}(x, y, z)$$



指向外侧,哪个指向内侧?

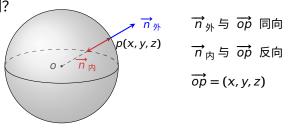
$$\vec{n}_{h}$$
与 \vec{op} 同向 \vec{n}_{h} 与 \vec{op} 反向

解 令
$$F(x, y, z) = x^2 + y^2 + z^2 - R^2$$
,则球面方程改写为 $F = 0$. 计算
$$\nabla F = (2x, 2y, 2z), \qquad |\nabla F| = 2\sqrt{x^2 + y^2 + z^2} = 2R$$

$$\frac{1}{|\nabla F|}\nabla F = \frac{1}{R}(x, y, z), \qquad -\frac{1}{|\nabla F|}\nabla F = -\frac{1}{R}(x, y, z)$$



指向外侧,哪个指向内侧?



$$\mathbf{F} \Leftrightarrow F(x, y, z) = x^2 + y^2 + z^2 - R^2$$
,则球面方程改写为 $F = 0$. 计算
$$\nabla F = (2x, 2y, 2z), \qquad |\nabla F| = 2\sqrt{x^2 + y^2 + z^2} = 2R$$

$$\frac{1}{|\nabla F|}\nabla F = \frac{1}{R}(x, y, z), \qquad -\frac{1}{|\nabla F|}\nabla F = -\frac{1}{R}(x, y, z)$$



例 1 写出球面 $x^2 + y^2 + z^2 = R^2$ 的两个单位法向量场,并指出哪一个 指向外侧,哪个指向内侧?

 $\overrightarrow{n}_{h} = \overrightarrow{op} = (x, y, z)$ $\overrightarrow{n}_{h} = \overrightarrow{op} = (x, y, z)$

解 令
$$F(x, y, z) = x^2 + y^2 + z^2 - R^2$$
,则球面方程改写为 $F = 0$. 计算
$$\nabla F = (2x, 2y, 2z), \qquad |\nabla F| = 2\sqrt{x^2 + y^2 + z^2} = 2R$$

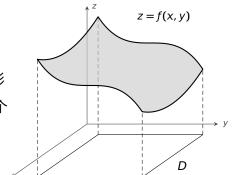
所以两个单位法向量场为

$$\frac{1}{|\nabla F|}\nabla F = \frac{1}{R}(x, y, z), \qquad -\frac{1}{|\nabla F|}\nabla F = -\frac{1}{R}(x, y, z)$$

前一个指向外侧,后一个指向内侧.



例 2 写出二元函数 z = f(x, y) 图形 的两个单位法向量场,并指出哪一个指向上侧,哪个指向下侧?



例 2 写出二元函数 z = f(x, y) 图形 的两个单位法向量场,并指出哪一个 指向上侧,哪个指向下侧?

 \mathbf{H} 令 F(x, y, z) = z - f(x, y),则该图形方程改写为 F = 0.



z = f(x, y)

形 z = f(x, y) F = z - f(x, y) = 0

例 2 写出二元函数 z = f(x, y) 图形的两个单位法向量场,并指出哪一个

 $\mathbf{F} \Leftrightarrow F(x, y, z) = z - f(x, y)$,则该图形方程改写为 F = 0. 计算

$$\nabla F =$$

$$|\nabla F| =$$

所以两个单位法向量场为

指向上侧,哪个指向下侧?

$$\frac{1}{|\nabla F|}\nabla F =$$

$$-\frac{1}{|\nabla F|}\nabla F =$$

z = f(x, y) F = z - f(x, y) = 0

例 2 写出二元函数 z = f(x, y) 图形的两个单位法向量场,并指出哪一个

的两个单位法向量场,并指出哪一个 指向上侧,哪个指向下侧?

 $\mathbf{F} \Leftrightarrow F(x, y, z) = z - f(x, y)$,则该图形方程改写为 F = 0. 计算

$$\nabla F = (-f_x, -f_y, 1), \quad |\nabla F| =$$

$$\frac{1}{|\nabla F|}\nabla F = -\frac{1}{|\nabla F|}\nabla F =$$

F = z - f(x, y) = 0例 2 写出二元函数 z = f(x, y) 图形

的两个单位法向量场,并指出哪一个 指向上侧,哪个指向下侧?

 $\mathbf{F} \Leftrightarrow F(x, y, z) = z - f(x, y)$,则该图形方程改写为 F = 0. 计算

$$\nabla F = (-f_x, -f_y, 1), \qquad |\nabla F| = \sqrt{1 + f_x^2 + f_y^2}$$

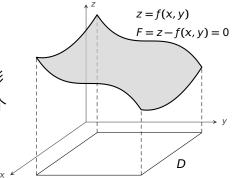
所以两个单位法向量场为

$$\frac{1}{|\nabla F|}\nabla F = -\frac{1}{|\nabla F|}\nabla F =$$



z = f(x, y)

例 2 写出二元函数 z = f(x, y) 图形 的两个单位法向量场,并指出哪一个指向上侧,哪个指向下侧?



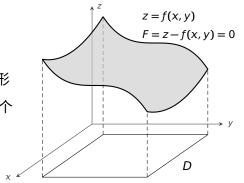
 $\mathbf{F} \Leftrightarrow F(x, y, z) = z - f(x, y)$,则该图形方程改写为 F = 0. 计算

$$\nabla F = (-f_x, -f_y, 1), \qquad |\nabla F| = \sqrt{1 + f_x^2 + f_y^2}$$

$$\tfrac{1}{|\nabla F|}\nabla F=\tfrac{1}{\sqrt{1+f_x^2+f_y^2}}(-f_x,\,-f_y,\,1),\quad -\tfrac{1}{|\nabla F|}\nabla F=$$



例 2 写出二元函数 z = f(x, y) 图形的两个单位法向量场,并指出哪一个指向上侧,哪个指向下侧?



 $\mathbf{p} \Leftrightarrow F(x, y, z) = z - f(x, y)$,则该图形方程改写为 F = 0. 计算

$$\nabla F = (-f_x, -f_y, 1), \qquad |\nabla F| = \sqrt{1 + f_x^2 + f_y^2}$$

$$\frac{1}{|\nabla F|}\nabla F = \frac{1}{\sqrt{1+f_{\nu}^2+f_{\nu}^2}}(-f_{x}, -f_{y}, 1), \quad -\frac{1}{|\nabla F|}\nabla F = \frac{1}{\sqrt{1+f_{\nu}^2+f_{\nu}^2}}(f_{x}, f_{y}, -1)$$



F = z - f(x, y) = 0(x, y, z)例 2 写出二元函数 z = f(x, y) 图形 的两个单位法向量场,并指出哪一个

指向上侧,哪个指向下侧?

 $\mathbf{F} \Leftrightarrow F(x, y, z) = z - f(x, y)$,则该图形方程改写为 F = 0. 计算

$$\nabla F = (-f_x, -f_y, 1), \qquad |\nabla F| = \sqrt{1 + f_x^2 + f_y^2}$$

所以两个单位法向量场为

$$\frac{1}{|\nabla F|}\nabla F = \frac{1}{\sqrt{1+f_{y}^{2}+f_{y}^{2}}}(-f_{x}, -f_{y}, 1), \quad -\frac{1}{|\nabla F|}\nabla F = \frac{1}{\sqrt{1+f_{y}^{2}+f_{y}^{2}}}(f_{x}, f_{y}, -1)$$



z = f(x, y)

z = f(x, y) F = z - f(x, y) = 0 (x, y, z)

例 2 写出二元函数 z = f(x, y) 图形的两个单位法向量场,并指出哪一个指向上侧,哪个指向下侧?

 $\mathbf{F} \Leftrightarrow F(x, y, z) = z - f(x, y)$,则该图形方程改写为 F = 0. 计算

$$\nabla F = (-f_x, -f_y, 1), \qquad |\nabla F| = \sqrt{1 + f_x^2 + f_y^2}$$

$$\frac{1}{|\nabla F|}\nabla F = \frac{1}{\sqrt{1+f_{\vee}^2+f_{\vee}^2}}(-f_X, -f_Y, 1), \quad -\frac{1}{|\nabla F|}\nabla F = \frac{1}{\sqrt{1+f_{\vee}^2+f_{\vee}^2}}(f_X, f_Y, -1)$$



形 z = f(x, y) F = z - f(x, y) = 0

例 2 写出二元函数 z = f(x, y) 图形 的两个单位法向量场,并指出哪一个

 $\mathbf{F} \Leftrightarrow F(x, y, z) = z - f(x, y)$,则该图形方程改写为 F = 0. 计算

$$\nabla F = (-f_x, -f_y, 1), \qquad |\nabla F| = \sqrt{1 + f_x^2 + f_y^2}$$

所以两个单位法向量场为

指向上侧,哪个指向下侧?

$$\frac{1}{|\nabla F|}\nabla F = \frac{1}{\sqrt{1+f_{\vee}^2+f_{\vee}^2}}(-f_X, -f_Y, 1), \quad -\frac{1}{|\nabla F|}\nabla F = \frac{1}{\sqrt{1+f_{\vee}^2+f_{\vee}^2}}(f_X, f_Y, -1)$$



z = f(x, y) F = z - f(x, y) = 0 (x, y, z) D

例 2 写出二元函数 z = f(x, y) 图形的两个单位法向量场,并指出哪一个指向上侧,哪个指向下侧?

 $\mathbf{F} \Leftrightarrow F(x, y, z) = z - f(x, y)$,则该图形方程改写为 F = 0. 计算

$$\nabla F = (-f_x, -f_y, 1), \qquad |\nabla F| = \sqrt{1 + f_x^2 + f_y^2}$$

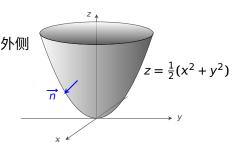
所以两个单位法向量场为

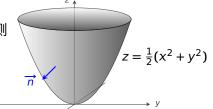
$$\frac{1}{|\nabla F|} \nabla F = \frac{1}{\sqrt{1 + f_{\nu}^2 + f_{\nu}^2}} (-f_{\chi}, -f_{y}, 1), \quad -\frac{1}{|\nabla F|} \nabla F = \frac{1}{\sqrt{1 + f_{\nu}^2 + f_{\nu}^2}} (f_{\chi}, f_{y}, -1)$$

前一个指向上侧,后一个指向下侧.



JINAN UMVERSITY

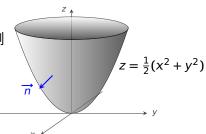




的单位法向量场.

解 该单位法向量场应取为

$$\overrightarrow{n} = \frac{1}{\sqrt{1 + z_{\chi}^2 + z_{y}^2}} (z_{\chi}, z_{y}, -1) =$$

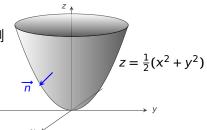


解 该单位法向量场应取为

的单位法向量场.

$$\overrightarrow{n} = \frac{1}{\sqrt{1 + z_x^2 + z_y^2}} (z_x, z_y, -1) = (x, y, -1)$$

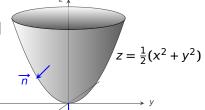




解 该单位法向量场应取为

的单位法向量场.

$$\overrightarrow{n} = \frac{1}{\sqrt{1 + z_x^2 + z_y^2}} (z_x, z_y, -1) = \frac{1}{\sqrt{1 + x^2 + y^2}} (x, y, -1)$$



的单位法向量场.

解 该单位法向量场应取为

$$\overrightarrow{n} = \frac{1}{\sqrt{1 + z_x^2 + z_y^2}} (z_x, z_y, -1) = \frac{1}{\sqrt{1 + x^2 + y^2}} (x, y, -1)$$

设 P(x, y, z), Q(x, y, z), R(x, y, z) 是三元函数,则

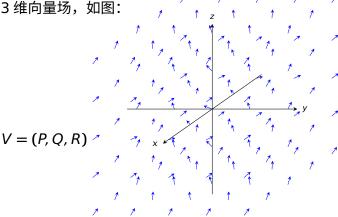
$$V = (P, Q, R)$$

构成空间 3 维向量场,

设 P(x, y, z), Q(x, y, z), R(x, y, z) 是三元函数,则

$$V = (P, Q, R)$$

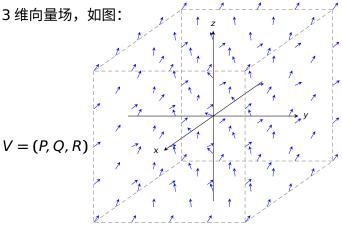
构成空间 3 维向量场,如图:



设 P(x, y, z), Q(x, y, z), R(x, y, z) 是三元函数,则

$$V = (P, Q, R)$$

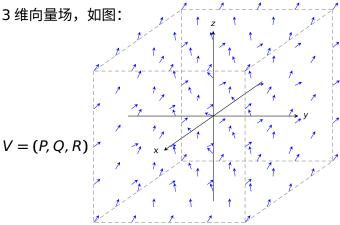
构成空间 3 维向量场,如图:



设 P(x, y, z), Q(x, y, z), R(x, y, z) 是三元函数,则

$$V = (P, Q, R)$$

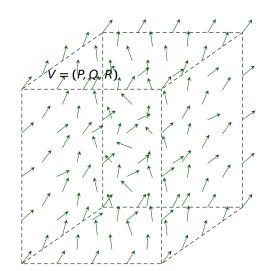
构成空间 3 维向量场,如图:

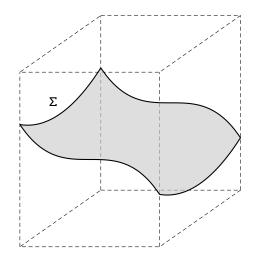


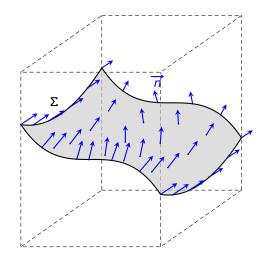
物理应用: 向量场 V = (P, Q, R) 可表示流体在任一点处的速度

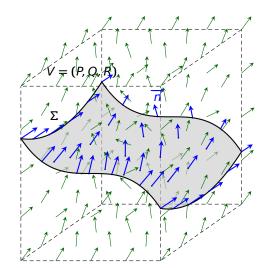


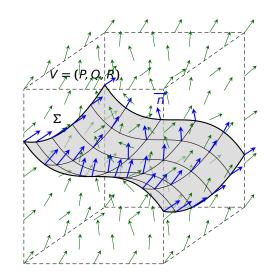
$$\Phi =$$



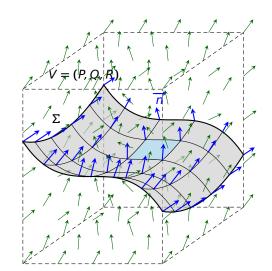




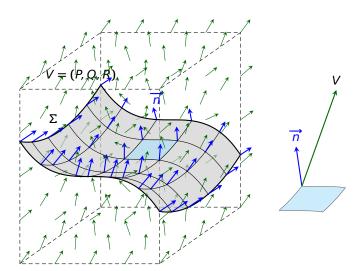




$$\Phi =$$

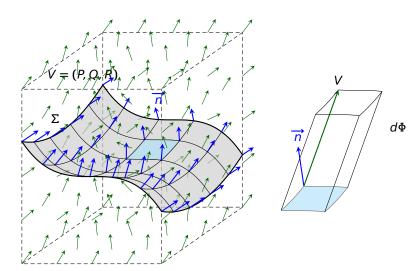


$$\Phi =$$

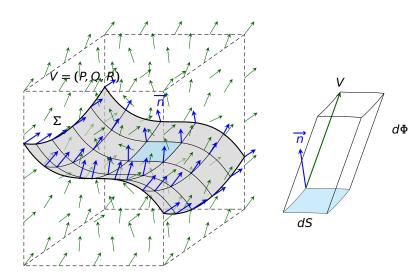




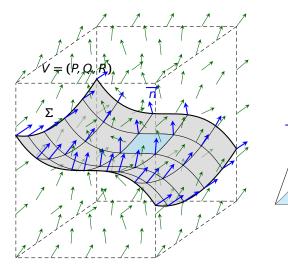
$$\Phi =$$

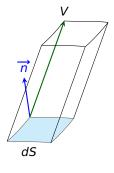


$$\Phi =$$



$$\Phi =$$

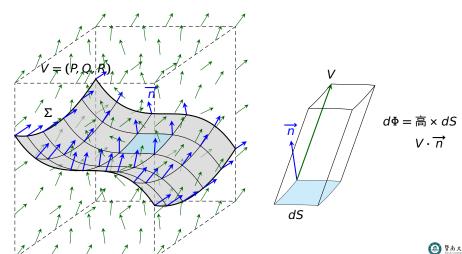




 $d\Phi =$ 高×dS

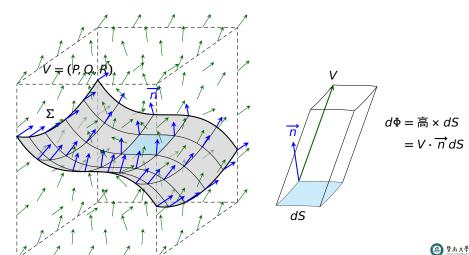
物理应用 流体 V = (P, Q, R) 在单位时间内流过曲面 Σ 一侧(单位法向 量 \overrightarrow{n} 所指向的一侧)的流量是:

$$\Phi =$$



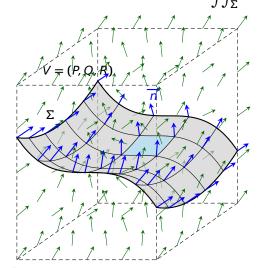
物理应用 流体 V = (P, Q, R) 在单位时间内流过曲面 Σ 一侧(单位法向 量 \overrightarrow{n} 所指向的一侧)的流量是:

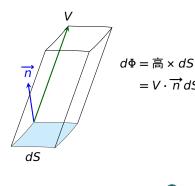
$$\Phi =$$



物理应用 流体 V = (P, Q, R) 在单位时间内流过曲面 Σ 一侧(单位法向 量 \overrightarrow{n} 所指向的一侧)的流量是:

$$\Phi = \iint_{\Sigma} V \cdot \overrightarrow{n} \, dS$$





 $= V \cdot \overrightarrow{n} dS$

定义 假设

- V = (P, Q, R) 是空间某区域上的向量场;
- Σ 是定向曲面, \overrightarrow{n} 是 Σ 上指定的单位法向量场;

则称

定义 假设

- V = (P, Q, R) 是空间某区域上的向量场;
- Σ 是定向曲面, \overrightarrow{n} 是 Σ 上指定的单位法向量场;

则称

$$\iint_{\Sigma} V \cdot \overrightarrow{n} \, dS$$

为向量场 V 在定向曲面 Σ 上的曲面积分 .

定义 假设

- V = (P, Q, R) 是空间某区域上的向量场;
- Σ 是定向曲面, \overrightarrow{n} 是 Σ 上指定的单位法向量场;

则称

$$\iint_{\Sigma} V \cdot \overrightarrow{n} \, dS$$

为向量场 V 在定向曲面 Σ 上的曲面积分 .令 $\overrightarrow{dS} = \overrightarrow{n} dS$,

定义 假设

- V = (P, Q, R) 是空间某区域上的向量场;
- Σ 是定向曲面, \overrightarrow{n} 是 Σ 上指定的单位法向量场;

则称

$$\iint_{\Sigma} V \cdot \overrightarrow{n} \, dS$$

为**向量场 V 在定向曲面 Σ 上的曲面积分** .令 $\overrightarrow{dS} = \overrightarrow{n} dS$,则记作

$$\iint_{\Sigma} V \cdot d\overrightarrow{S}$$

定义 假设

- V = (P, Q, R) 是空间某区域上的向量场;
- Σ 是定向曲面, \overrightarrow{n} 是 Σ 上指定的单位法向量场;

则称

$$\iint_{\Sigma} V \cdot \overrightarrow{n} \, dS$$

为向量场 V 在定向曲面 Σ 上的曲面积分 .令 $\overrightarrow{dS} = \overrightarrow{n} dS$,则记作

$$\iint_{\Sigma} V \cdot d\overrightarrow{S}$$

也记作

$$\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy$$

定义 假设

- V = (P, Q, R) 是空间某区域上的向量场;

则称

$$\iint_{\Sigma} V \cdot \overrightarrow{n} \, dS$$

为向量场 V 在定向曲面 Σ 上的曲面积分 .令 $\overrightarrow{dS} = \overrightarrow{n} dS$,则记作

$$\iint_{\Sigma} V \cdot d\overrightarrow{S}$$

也记作

$$\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy$$

(此时也称为对坐标的曲面积分,或第二类曲面积分)



性质 设 Σ 是定向曲面, $-\Sigma$ 表示与取 Σ 相反侧的有向曲面,则



性质 设 Σ 是定向曲面, $-\Sigma$ 表示与取 Σ 相反侧的有向曲面,则

$$\iint_{-\Sigma} Pdydz + Qdzdx + Rdxdy = -\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy$$

$$\iint_{-\Sigma} Pdydz + Qdzdx + Rdxdy = -\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy$$

物理解释 流过负侧的流量 = - 流过正侧的流量

$$\iint_{-\Sigma} Pdydz + Qdzdx + Rdxdy = -\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy$$

物理解释 流过负侧的流量 = - 流过正侧的流量

证明 设 \overrightarrow{n} 是与 Σ 定向相符的单位法向量场,

$$\iint_{-\Sigma} Pdydz + Qdzdx + Rdxdy = -\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy$$

物理解释 流过负侧的流量 = - 流过正侧的流量

$$\iint_{-\Sigma} Pdydz + Qdzdx + Rdxdy = -\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy$$

物理解释 流过负侧的流量 = - 流过正侧的流量

令
$$V = (P, Q, R)$$
. 则

$$\iint_{-\Sigma} Pdydz + Qdzdx + Rdxdy = -\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy$$

物理解释 流过负侧的流量 = - 流过正侧的流量

令
$$V = (P, Q, R)$$
. 则
$$\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy =$$

$$\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy =$$



$$\iint_{-\Sigma} Pdydz + Qdzdx + Rdxdy = -\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy$$

物理解释 流过负侧的流量 = - 流过正侧的流量

令
$$V = (P, Q, R)$$
. 则
$$\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy = \iint_{\Sigma} V \cdot \overrightarrow{n} dS$$

$$\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy =$$

$$\iint_{-\Sigma} Pdydz + Qdzdx + Rdxdy = -\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy$$

物理解释 流过负侧的流量 = - 流过正侧的流量

令
$$V = (P, Q, R)$$
. 则

$$\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy = \iint_{\Sigma} V \cdot \overrightarrow{n} dS$$

$$\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy = \iint_{\Sigma} V \cdot (-\overrightarrow{n})dS$$

$$\iint_{-\Sigma} Pdydz + Qdzdx + Rdxdy = -\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy$$

物理解释 流过负侧的流量 = - 流过正侧的流量

证明 设 \overrightarrow{n} 是与 Σ 定向相符的单位法向量场,则 $-\overrightarrow{n}$ 是与 $-\Sigma$ 定向相符的单位法向量场.

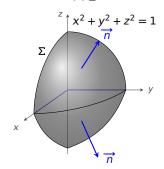
令
$$V = (P, Q, R)$$
. 则

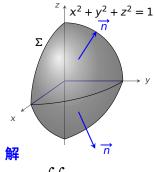
$$\iint_{\Sigma} Pdydz + Qdzdx + Rdxdy = \iint_{\Sigma} V \cdot \overrightarrow{n} dS$$

$$\iint_{-\Sigma} Pdydz + Qdzdx + Rdxdy = \iint_{\Sigma} V \cdot (-\overrightarrow{n})dS$$

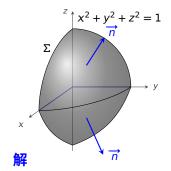
:. 二者数值互为相反数



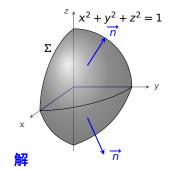




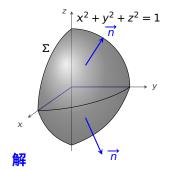
原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS$$



原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \underbrace{V = (0, 0, xyz)}_{V = (0, 0, xyz)}$$

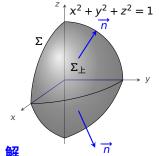


原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \underbrace{V = (0, 0, xyz)}_{\overrightarrow{n} = (x, y, z)}$$



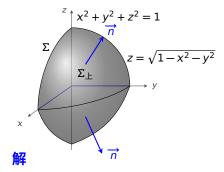
原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS$$





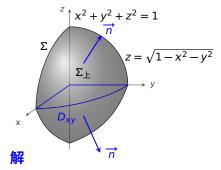
原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \underbrace{V = (0, 0, xyz)}_{\overrightarrow{n} = (x, y, z)} = \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{+}} xyz^2 dS$





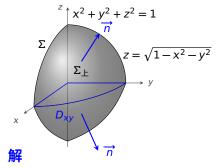
原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\perp}} xyz^2 dS$$

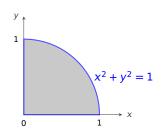




原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \underbrace{V = (0, 0, xyz)}_{\overrightarrow{n} = (x, y, z)} = \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\perp}} xyz^2 dS$$

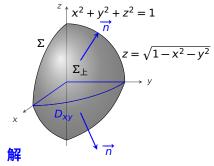


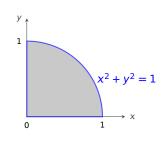




原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$

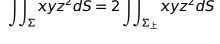


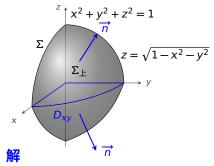


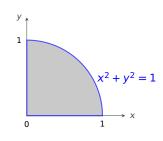


原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$

$$xy(1-x^2-y^2)$$



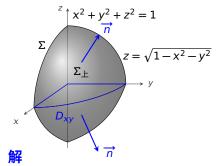


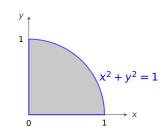


原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$

$$xy(1-x^2-y^2)\cdot\sqrt{1+z_x^2+z_y^2}dxdy$$

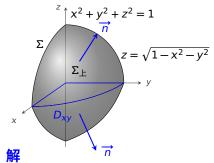


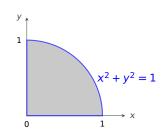




原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$
$$= \iint_{D_{xy}} xy(1 - x^2 - y^2) \cdot \sqrt{1 + z_x^2 + z_y^2} dxdy$$

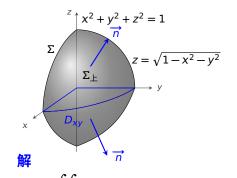


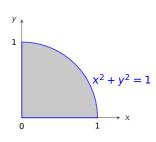




原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$
$$= 2 \iint_{D_{\text{out}}} xy(1 - x^2 - y^2) \cdot \sqrt{1 + z_x^2 + z_y^2} dx dy$$





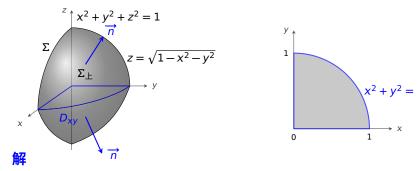


原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$

$$=2\iint_{D_{xy}} xy(1-x^2-y^2) \cdot \sqrt{1+z_x^2+z_y^2} dxdy$$

$$\cdot \frac{1}{\sqrt{1-x^2-y^2}} dxdy$$



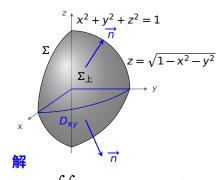


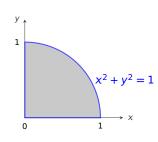
原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$
$$= 2 \iint_{\Sigma} xy(1 - x^2 - y^2) \cdot \sqrt{1 + z_x^2 + z_y^2} dx dy$$

$$=2\iint_{D_{xy}} xy(1-x^2-y^2) \cdot \frac{1}{\sqrt{1-x^2-y^2}} dxdy$$



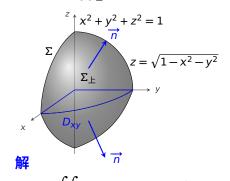


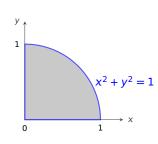




原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$
$$= 2 \iint_{D_{xy}} xy(1 - x^2 - y^2) \cdot \sqrt{1 + z_x^2 + z_y^2} dx dy$$
$$= 2 \iint_{D} xy\sqrt{1 - x^2 - y^2} dx dy$$







原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^{2} dS = 2 \iint_{\Sigma_{\pm}} xyz^{2} dS$$

$$= 2 \iint_{D_{xy}} xy(1 - x^{2} - y^{2}) \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dxdy$$

$$= 2 \iint_{D} xy\sqrt{1 - x^{2} - y^{2}} dxdy \xrightarrow{x = \rho \cos \theta} \cdots$$



原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \underbrace{V = (0, 0, xyz)}_{\overrightarrow{n} = (x, y, z)} = \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma} xyz^2 dS$ $= 2 \iint_{D} xy(1-x^2-y^2) \cdot \sqrt{1+z_x^2+z_y^2} dxdy$ $= 2 \iint_{\mathbb{R}} xy \sqrt{1 - x^2 - y^2} dx dy$

$$\frac{\partial \cos \theta}{\partial \cos \theta} = 2 \iint \rho^2 \sin \theta \cos \theta \cdot \sqrt{1 - \rho^2} \cdot \rho d\rho d\theta$$

$$\frac{x = \rho \cos \theta}{y = \rho \sin \theta} 2 \iint_{D_{xy}} \rho^2 \sin \theta \cos \theta \cdot \sqrt{1 - \rho^2} \cdot \rho d\rho d\theta$$



原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\perp}} xyz^2 dS$$
$$= 2 \iint_{\Sigma} xy(1 - x^2 - y^2) \cdot \sqrt{1 + z_x^2 + z_y^2} dx dy$$

$$= 2 \iint_{D_{xy}} xy(1-x^2-y^2) \cdot \sqrt{1+z_x^2+z_y^2} dxdy$$

$$= 2 \iint_{D_{xy}} xy\sqrt{1-x^2-y^2} dxdy$$

$$\frac{x=\rho\cos\theta}{y=\rho\sin\theta} 2 \iint_{D_{xy}} \rho^2\sin\theta\cos\theta \cdot \sqrt{1-\rho^2} \cdot \rho d\rho d\theta$$

$$= 2 \int \left[\int \sin \theta \cos \theta \rho^3 \sqrt{1 - \rho^2} d\rho \right] d\theta$$



原式 =
$$\iint_{\Sigma} V \cdot \vec{n} \, dS = \frac{V = (0, 0, xyz)}{\vec{n} = (x, y, z)} = \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$

= $2 \iint_{D_{xy}} xy(1 - x^2 - y^2) \cdot \sqrt{1 + z_x^2 + z_y^2} dxdy$
= $2 \iint_{\Sigma_{\pm}} xy\sqrt{1 - x^2 - y^2} dxdy$

$$\frac{x = \rho \cos \theta}{y = \rho \sin \theta} 2 \iint_{D_{avg}} \rho^2 \sin \theta \cos \theta \cdot \sqrt{1 - \rho^2} \cdot \rho d\rho d\theta$$

$$=2\int_{0}^{\frac{\pi}{2}}\left[\int \sin\theta\cos\theta\rho^{3}\sqrt{1-\rho^{2}}d\rho\right]d\theta$$



原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$$
$$= 2 \iint_{D_{xy}} xy(1 - x^2 - y^2) \cdot \sqrt{1 + z_x^2 + z_y^2} dx dy$$
$$= 2 \iint_{\Sigma_{\pm}} xy\sqrt{1 - x^2 - y^2} dx dy$$

$$\frac{x = \rho \cos \theta}{y = \rho \sin \theta} 2 \iint_{D_{TV}} \rho^2 \sin \theta \cos \theta \cdot \sqrt{1 - \rho^2} \cdot \rho d\rho d\theta$$

$$=2\int_{0}^{\frac{\pi}{2}}\left[\int_{0}^{1}\sin\theta\cos\theta\rho^{3}\sqrt{1-\rho^{2}}d\rho\right]d\theta$$



原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma_{\pm}} xyz^2 dS$ $= 2 \iint_{\Sigma} xy(1 - x^2 - y^2) \cdot \sqrt{1 + z_x^2 + z_y^2} dxdy$

$$= 2 \iint_{D_{xy}} xy \sqrt{1 - x^2 - y^2} dx dy$$

$$\frac{x = \rho \cos \theta}{y = \rho \sin \theta} 2 \iint_{D_{xy}} \rho^2 \sin \theta \cos \theta \cdot \sqrt{1 - \rho^2} \cdot \rho d\rho d\theta$$

$$= 2 \int_0^{\frac{\pi}{2}} \left[\int_0^1 \sin \theta \cos \theta \rho^3 \sqrt{1 - \rho^2} d\rho \right] d\theta$$

 $= \int_{0}^{\frac{\pi}{2}} \sin(2\theta) d\theta \cdot \int_{0}^{1} \rho^{2} \sqrt{1 - \rho^{2}} \cdot \rho d\rho$



 $= 2 \iint_{\mathbb{R}^{2}} xy(1-x^{2}-y^{2}) \cdot \sqrt{1+z_{x}^{2}+z_{y}^{2}} dxdy$ $= 2 \iint_{\mathbb{R}} xy \sqrt{1 - x^2 - y^2} dx dy$

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma} xyz^2 dS$

$$\frac{x = \rho \cos \theta}{y = \rho \sin \theta} 2 \iint_{D_{xy}} \rho^2 \sin \theta \cos \theta \cdot \sqrt{1 - \rho^2} \cdot \rho d\rho d\theta$$

$$= 2 \int_0^{\frac{\pi}{2}} \left[\int_0^1 \sin \theta \cos \theta \rho^3 \sqrt{1 - \rho^2} d\rho \right] d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin(2\theta) d\theta \cdot \int_0^1 \rho^2 \sqrt{1 - \rho^2} \cdot \rho d\rho$$





 $u = \sqrt{1-\rho^2}$

 $=2\int_{0}^{\frac{\pi}{2}}\left[\int_{0}^{1}\sin\theta\cos\theta\rho^{3}\sqrt{1-\rho^{2}}d\rho\right]d\theta$ $= \int_{0}^{\frac{\pi}{2}} \sin(2\theta) d\theta \cdot \int_{0}^{1} \rho^{2} \sqrt{1 - \rho^{2}} \cdot \rho d\rho$

 $= 2 \iint_{\mathbb{R}^{2}} xy(1-x^{2}-y^{2}) \cdot \sqrt{1+z_{x}^{2}+z_{y}^{2}} dxdy$ $=2\iint_{\mathbb{R}}xy\sqrt{1-x^2-y^2}dxdy$ $\frac{x = \rho \cos \theta}{y = \rho \sin \theta} 2 \iint_{\Omega} \rho^2 \sin \theta \cos \theta \cdot \sqrt{1 - \rho^2} \cdot \rho d\rho d\theta$

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma} xyz^2 dS$

$$\underbrace{u=\sqrt{1-\rho^2}}_{\text{11e 曲面积分}} \qquad \qquad \cdot (-udu)$$

$$\frac{x = \rho \cos \theta}{y = \rho \sin \theta} 2 \iint_{D_{xy}} \rho^2 \sin \theta \cos \theta \cdot \sqrt{1 - \rho^2} \cdot \rho d\rho d\theta$$

$$= 2 \int_0^{\frac{\pi}{2}} \left[\int_0^1 \sin \theta \cos \theta \rho^3 \sqrt{1 - \rho^2} d\rho \right] d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin(2\theta) d\theta \cdot \int_0^1 \rho^2 \sqrt{1 - \rho^2} \cdot \rho d\rho$$

 $(1-u^2)u \cdot (-udu)$

14/17 < ▶ △ ▽

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma} xyz^2 dS$

 $= 2 \iint_{\mathbb{R}^{2}} xy(1-x^{2}-y^{2}) \cdot \sqrt{1+z_{x}^{2}+z_{y}^{2}} dxdy$

 $=2\iint_{\mathbb{R}}xy\sqrt{1-x^2-y^2}dxdy$

 $u = \sqrt{1-\rho^2}$

11e 曲面积分

$$\frac{\frac{x = \rho \cos \theta}{y = \rho \sin \theta}}{y = \rho \sin \theta} 2 \iint_{D_{xy}} \rho^{2} \sin \theta \cos \theta \cdot \sqrt{1 - \rho^{2}} \cdot \rho d\rho d\theta$$

$$= 2 \int_{0}^{\frac{\pi}{2}} \left[\int_{0}^{1} \sin \theta \cos \theta \rho^{3} \sqrt{1 - \rho^{2}} d\rho \right] d\theta$$

$$= \int_{0}^{\frac{\pi}{2}} \sin(2\theta) d\theta \cdot \int_{0}^{1} \rho^{2} \sqrt{1 - \rho^{2}} \cdot \rho d\rho$$

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma} xyz^2 dS$

 $= 2 \iint_{\mathbb{R}^{2}} xy(1-x^{2}-y^{2}) \cdot \sqrt{1+z_{x}^{2}+z_{y}^{2}} dxdy$

 $= 2 \iint_{\mathbb{R}} xy \sqrt{1 - x^2 - y^2} dx dy$

 $\frac{u=\sqrt{1-\rho^2}}{\sqrt{1-\rho^2}} \qquad \int_{-1}^{0} (1-u^2)u \cdot (-udu)$

$$\frac{\frac{x=\rho\cos\theta}{y=\rho\sin\theta}}{y=\rho\sin\theta} 2 \iint_{D_{xy}} \rho^2 \sin\theta\cos\theta \cdot \sqrt{1-\rho^2} \cdot \rho d\rho d\theta$$

$$= 2 \int_0^{\frac{\pi}{2}} \left[\int_0^1 \sin\theta\cos\theta \rho^3 \sqrt{1-\rho^2} d\rho \right] d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin(2\theta) d\theta \cdot \int_0^1 \rho^2 \sqrt{1-\rho^2} \cdot \rho d\rho$$

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma} xyz^2 dS$

 $= 2 \iint_{\mathbb{R}^{2}} xy(1-x^{2}-y^{2}) \cdot \sqrt{1+z_{x}^{2}+z_{y}^{2}} dxdy$

 $= 2 \iint_{\mathbb{R}} xy \sqrt{1 - x^2 - y^2} dx dy$

 $\frac{u=\sqrt{1-\rho^2}}{2} \cdot 1 \cdot \int_0^0 (1-u^2)u \cdot (-udu)$

$$= 2 \iint_{D_{xy}} xy \sqrt{1 - x^2 - y^2} dx dy$$

$$= \frac{x = \rho \cos \theta}{y = \rho \sin \theta} 2 \iint_{D_{xy}} \rho^2 \sin \theta \cos \theta \cdot \sqrt{1 - \rho^2} \cdot \rho d\rho d\theta$$

$$= 2 \int_0^{\frac{\pi}{2}} \left[\int_0^1 \sin \theta \cos \theta \rho^3 \sqrt{1 - \rho^2} d\rho \right] d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin(2\theta) d\theta \cdot \int_0^1 \rho^2 \sqrt{1 - \rho^2} \cdot \rho d\rho$$

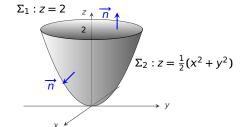
$$= \frac{u = \sqrt{1 - \rho^2}}{1 + \rho^2} \cdot 1 \cdot \int_0^0 (1 - u^2) u \cdot (-u du) = \frac{2}{15}$$

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (0, 0, xyz)} \iint_{\Sigma} xyz^2 dS = 2 \iint_{\Sigma} xyz^2 dS$

 $= 2 \iint_{\mathbb{R}^{2}} xy(1-x^{2}-y^{2}) \cdot \sqrt{1+z_{x}^{2}+z_{y}^{2}} dxdy$

$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$

其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

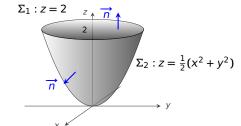




$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$

其中定向曲面 $Σ = Σ_1 \cup Σ_2$ 是三维

$$\mathbf{R}$$
 原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS$



$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$
 其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维 区域的边界,如图:

 $\Sigma_1 : z = 2$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,



$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$

其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

$$\Sigma_1: z = 2$$

$$z \to \overrightarrow{n}$$

$$\Sigma_2: z = \frac{1}{2}(x^2 + y^2)$$

$$x \to y$$

解 原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} \, dS$$

$$\iint_{\Sigma} V \cdot \overrightarrow{n} \, dS$$



$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$

其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

$$\Sigma_1: z = 2$$

$$z \longrightarrow n$$

$$\Sigma_2: z = \frac{1}{2}(x^2 + y^2)$$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)}$$

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)}$$



$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$

其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维 区域的边界,如图:

$$\Sigma_1: z = 2$$

$$Z \longrightarrow D$$

$$\Sigma_2: z = \frac{1}{2}(x^2 + y^2)$$

$$X \longrightarrow Y$$

解 原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} \frac{V \cdot \overrightarrow{n} dS}{\overrightarrow{n} = (0, 0, 1)}$$



$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$
其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

$$\Sigma_1: z = 2$$

$$Z_1 : z = \frac{1}{2}(x^2 + y^2)$$

$$Z_2 : z = \frac{1}{2}(x^2 + y^2)$$

解 原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} \iint_{\Sigma_{1}} -z dS$$

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)}$$



$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$
其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

$$\Sigma_1: z = 2 \qquad z \xrightarrow{n}$$

$$\Sigma_2: z = \frac{1}{2}(x^2 + y^2)$$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} \iint_{\Sigma_{1}} -z dS = \iint_{\Sigma_{1}} -2 dS$$

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)}$$



$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$
其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

 $\Sigma_1: z = 2$ $Z = \frac{1}{2}(x^2 + y^2)$ $Z = \frac{1}{2}(x^2 + y^2)$

解 原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} \iint_{\Sigma_{1}} -z dS = \iint_{\Sigma_{1}} -2 dS = -2|\Sigma_{1}|$$

$$\iint_{\Gamma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} U \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} U \cdot \overrightarrow{n} dS = -2|\Sigma_{1}|$$



$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$

其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

$$\Sigma_1: z = 2 \qquad z \xrightarrow{n}$$

$$\Sigma_2: z = \frac{1}{2}(x^2 + y^2)$$

解 原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,
$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS = \underbrace{V = (z^2 + x, 0, -z)}_{\overrightarrow{n} = (0, 0, 1)} \iint_{\Sigma_2} -z dS = \iint_{\Sigma_1} -2 dS = -2|\Sigma_1| = -8\pi$$
,



$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$

其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

$$\Sigma_1: z = 2$$

$$Z \longrightarrow D$$

$$\Sigma_2: z = \frac{1}{2}(x^2 + y^2)$$

$$X \longrightarrow Y$$

解 原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,
$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS = \underbrace{V = (z^2 + x, 0, -z)}_{\overrightarrow{n} = (0, 0, 1)} \iint_{\Sigma_2} -z dS = \iint_{\Sigma_1} -2 dS = -2|\Sigma_1| = -8\pi$$
,

$$\iint_{\Sigma_{1}} V \cdot \vec{n} \, dS \xrightarrow{\overrightarrow{n} = (0, 0, 1)}$$

$$\iint_{\Sigma_{2}} V \cdot \overrightarrow{n} \, dS \xrightarrow{V = (z^{2} + x, 0, -z)}$$

$$\overrightarrow{\vec{n}} = \underbrace{(x, y, -1)}_{\sqrt{1 + x^{2} + y^{2}}}$$



$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$
其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

$$\Sigma_1: z = 2$$

$$Z \longrightarrow D$$

$$\Sigma_2: z = \frac{1}{2}(x^2 + y^2)$$

$$X \longrightarrow Y$$

解 原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} \iint_{\Sigma_{1}} -z dS = \iint_{\Sigma_{1}} -2 dS = -2|\Sigma_{1}| = -8\pi,$$

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} (z^{2} + x)x + z$$

$$\iint_{\Sigma_2} V \cdot \overrightarrow{n} \, dS = \underbrace{\frac{V = (z^2 + x, \, 0, -z)}{\overrightarrow{n} = \frac{(x, y, -1)}{\sqrt{1 + x^2 + y^2}}}} \qquad \frac{(z^2 + x)x + z}{\sqrt{1 + x^2 + y^2}}$$



例 2 计算
$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$
 其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

$$\Sigma_1: z = 2 \qquad z \xrightarrow{n}$$

$$\Sigma_2: z = \frac{1}{2}(x^2 + y^2)$$

解 原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,
$$\iint_{\Sigma_2} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^2 + x, 0, -z)} \iint_{\Sigma_2} -z dS = \iint_{\Sigma_2} -2 dS = -2 |\Sigma_1| = -8$$

 $\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS \stackrel{V = (z^2 + x, 0, -z)}{\underset{\overrightarrow{n} = (0, 0, 1)}{\longrightarrow}} \iint_{\Sigma_2} -z dS = \iint_{\Sigma_2} -2 dS = -2|\Sigma_1| = -8\pi,$ $\iint_{\Sigma_2} V \cdot \overrightarrow{n} \, dS = \frac{V = (z^2 + x, 0, -z)}{\overrightarrow{n} = \frac{(x, y, -1)}{\sqrt{1 + x^2 + y^2}}} \qquad \frac{(z^2 + x)x + z}{\sqrt{1 + x^2 + y^2}} \cdot \sqrt{1 + z_x^2 + z_y^2} dx dy$

暨南大學

$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$
其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维

$$\Sigma_1: z = 2$$

$$z \longrightarrow n$$

$$\Sigma_2: z = \frac{1}{2}(x^2 + y^2)$$

$$D_{XY}$$

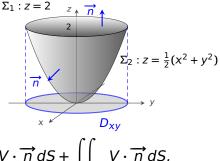
区域的边界,如图:

解 原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

 $\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} \iint_{\Sigma_{2}} -z dS = \iint_{\Sigma_{1}} -2 dS = -2|\Sigma_{1}| = -8\pi,$ $\iint_{\Sigma_2} V \cdot \overrightarrow{n} \, dS \xrightarrow{V = (z^2 + x, \, 0, -z)} \iint_{D_{xy}} \frac{(z^2 + x)x + z}{\sqrt{1 + x^2 + y^2}} \cdot \sqrt{1 + z_x^2 + z_y^2} dx dy$

$$\iint_{\Sigma} (z^2 + x) dy dz - z dx dy$$
 其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三 区域的边界,如图:

其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维 区域的边界,如图: 原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma} V \cdot \overrightarrow{n} dS + \iint_{\Sigma} V \cdot \overrightarrow{n} dS$,



 $\iint_{\Sigma} V \cdot \vec{n} \, dS = \frac{V = (z^2 + x, 0, -z)}{\vec{n} = (0, 0, 1)} \iint_{\Sigma} -z \, dS = \iint_{\Sigma} -2 \, dS = -2 |\Sigma_1| = -8\pi,$ $\iint_{\Sigma_2} V \cdot \overrightarrow{n} \, dS = \frac{V = (z^2 + x, 0, -z)}{\overrightarrow{n} = \frac{(x, y, -1)}{\sqrt{1 + x^2 + y^2}}} \iint_{D_{xy}} \frac{(z^2 + x)x + z}{\sqrt{1 + x^2 + y^2}} \cdot \sqrt{1 + z_x^2 + z_y^2} dx dy$

$$J\Sigma_{2} = \frac{1}{\sqrt{1+x^{2}+y^{2}}} \quad JJD_{xy} \sqrt{1+z}$$
$$= \iint_{D_{xy}} (z^{2}+x)x + zdxdy$$



 $\Sigma_1 : z = 2$ 例2 计算 $\int \int (z^2 + x) dy dz - z dx dy$ 其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维 区域的边界,如图:

 $\iint_{\Sigma_2} V \cdot \overrightarrow{n} \, dS = \frac{V = (z^2 + x, 0, -z)}{\overrightarrow{n} = \frac{(x, y, -1)}{\sqrt{1 + x^2 + y^2}}} \iint_{D_{xy}} \frac{(z^2 + x)x + z}{\sqrt{1 + x^2 + y^2}} \cdot \sqrt{1 + z_x^2 + z_y^2} dx dy$

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} \, dS \xrightarrow{V = (z^{2} + x, \, 0, -z)} \iint_{\Sigma_{1}} -z \, dS = \iint_{\Sigma_{1}} -2 \, dS = -2 |\Sigma_{1}| = -8\pi,$$

$$\iint_{\Sigma_{2}} V \cdot \overrightarrow{n} \, dS \xrightarrow{\frac{V = (z^{2} + x, \, 0, -z)}{\overrightarrow{n} = \frac{(x, y, -1)}{\sqrt{1 + x^{2} + y^{2}}}}} \iint_{D_{xy}} \frac{(z^{2} + x)x + z}{\sqrt{1 + x^{2} + y^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} \, dx \, dy$$

$$= \iint_{D_{xy}} (z^{2} + x)x + z \, dx \, dy \xrightarrow{\text{phots}} \iint_{D_{xy}} x^{2} + z \, dx \, dy$$

 $\Sigma_1 : z = 2$ 例2 计算 $\int \int (z^2 + x) dy dz - z dx dy$ 其中定向曲面 $\Sigma = \Sigma_1 \cup \Sigma_2$ 是三维 区域的边界,如图:

解 原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS,$$

$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS = \underbrace{V = (z^2 + x, 0, -z)}_{\overrightarrow{n} = (0, 0, 1)} \iint_{\Sigma_1} -z dS = \iint_{\Sigma_1} -2 dS = -2|\Sigma_1| = -8\pi,$$

$$\iint_{\Sigma_2} V \cdot \overrightarrow{n} dS = \frac{V = (z^2 + x, 0, -z)}{\overrightarrow{n} = \frac{(x, y, -1)}{\sqrt{1 + x^2 + y^2}}} \iint_{D_{xy}} \frac{(z^2 + x)x + z}{\sqrt{1 + x^2 + y^2}} \cdot \sqrt{1 + z_x^2 + z_y^2} dx dy$$

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} \, dS \xrightarrow{\frac{V - (z^{2} + x, \, 0, -z)}{\overrightarrow{n} = (0, \, 0, \, 1)}} \iint_{\Sigma_{1}} -z \, dS = \iint_{\Sigma_{1}} -2 \, dS = -2|\Sigma_{1}| = -8\pi,$$

$$\iint_{\Sigma_{2}} V \cdot \overrightarrow{n} \, dS \xrightarrow{\frac{V - (z^{2} + x, \, 0, -z)}{\overrightarrow{n} = \frac{(x, y, -1)}{\sqrt{1 + x^{2} + y^{2}}}}} \iint_{D_{xy}} \frac{(z^{2} + x)x + z}{\sqrt{1 + x^{2} + y^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} \, dx \, dy$$

11e 曲面积分

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} \iint_{\Sigma_{1}} -z dS \iint_{\Sigma_{1}} -2 dS = -2|\Sigma_{1}| = -8\pi,$$

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} \iint_{\Sigma_{1}} \frac{(z^{2} + x)x + z}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dx dx$$

$$\iint_{\Sigma_{2}} V \cdot \overrightarrow{n} dS = \frac{V = (z^{2} + x, 0, -z)}{\overrightarrow{n} = \frac{(x, y, -1)}{\sqrt{1 + x^{2} + y^{2}}}} \iint_{D_{xy}} \frac{(z^{2} + x)x + z}{\sqrt{1 + x^{2} + y^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dx dy$$

$$= \iint_{D_{xy}} (z^{2} + x)x + z dx dy \xrightarrow{\text{state}} \iint_{D_{xy}} x^{2} + z dx dy$$

$$\underline{z = \frac{1}{2}(x^{2} + y^{2})} = \frac{1}{2} \iint_{D_{xy}} 3x^{2} + y^{2} dx dy$$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,

$$\iint_{\Sigma_1} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^2 + x, 0, -z)} \iint_{\Sigma_1} -z dS \iint_{\Sigma_1} -2 dS = -2|\Sigma_1| = -8\pi,$$

$$\iint_{\Sigma_{2}} V \cdot \overrightarrow{n} \, dS \xrightarrow{\frac{V = (z^{2} + x, 0, -z)}{\overrightarrow{n}}} \iint_{D_{xy}} \frac{(z^{2} + x)x + z}{\sqrt{1 + x^{2} + y^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} \, dx \, dy$$

$$= \iint_{D_{xy}} (z^{2} + x)x + z \, dx \, dy \xrightarrow{\underline{\text{MWP}}} \iint_{D_{xy}} x^{2} + z \, dx \, dy$$

$$\underline{z = \frac{1}{2}(x^{2} + y^{2})} \frac{1}{2} \iint_{D_{xy}} 3x^{2} + y^{2} \, dx \, dy \xrightarrow{\underline{\text{MWP}}} 2 \iint_{D_{xy}} x^{2} \, dx \, dy$$



$$\iint_{\Sigma} V \cdot \overrightarrow{n} \, dS$$

 $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^2 + x, 0, -z)} \iint_{\Sigma} -z dS \iint_{\Sigma} -2 dS = -2|\Sigma_1| = -8\pi,$

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma} V \cdot \overrightarrow{n} dS + \iint_{\Sigma} V \cdot \overrightarrow{n} dS$,

$$\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{\overrightarrow{r} = (0, 0, 1)} \iint_{\Sigma_{1}} -z dS \iint_{\Sigma_{1}} -2 dS = -2|\Sigma_{1}| = -8\pi,$$

$$\iint_{\Sigma_{2}} V \cdot \overrightarrow{n} dS \xrightarrow{\overrightarrow{r} = \frac{(x, y, -1)}{\sqrt{1 + y^{2} + y^{2}}}} \iint_{D_{xy}} \frac{(z^{2} + x)x + z}{\sqrt{1 + x^{2} + y^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dx dy$$

 $= \iint_{D} (z^{2} + x)x + zdxdy \xrightarrow{\underline{\forall n \nmid t}} \iint_{D} x^{2} + zdxdy$ $\frac{z=\frac{1}{2}(x^2+y^2)}{2} \frac{1}{2} \iint_{\mathbb{R}} 3x^2 + y^2 dx dy \xrightarrow{\text{spate}} 2 \iint_{\mathbb{R}} x^2 dx dy$ $\xrightarrow{\text{yhm}} \iint_{\Omega} x^2 + y^2 dx dy$

$$\int V \cdot \overrightarrow{n} d$$

 $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^2 + x, 0, -z)} \iint_{\Sigma} -z dS \iint_{\Sigma} -2 dS = -2|\Sigma_1| = -8\pi,$

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma} V \cdot \overrightarrow{n} dS + \iint_{\Sigma} V \cdot \overrightarrow{n} dS$,

 $\iint_{\Sigma_2} V \cdot \overrightarrow{n} dS = \underbrace{\frac{V = (z^2 + x, 0, -z)}{\overrightarrow{n} = \frac{(x, y, -1)}{\sqrt{1 + x^2 + y^2}}} \iint_{D_{xy}} \frac{(z^2 + x)x + z}{\sqrt{1 + x^2 + y^2}} \cdot \sqrt{1 + z_x^2 + z_y^2} dx dy$

$$\overrightarrow{n} dS = \frac{\sqrt{=(z^2 + x, 0, -2)}}{\sqrt{1 + x^2 + y^2}} \iint_{D_{xy}} \frac{(z^2 + x)x + z^2}{\sqrt{1 + x^2 + y^2}} \cdot \sqrt{1 + z_x^2 + z_y^2} dxdy$$

$$= \iint_{D_{xy}} (z^2 + x)x + zdxdy \xrightarrow{\underline{\text{MWt}}} \iint_{D_{xy}} x^2 + zdxdy$$

$$\underline{z = \frac{1}{2}(x^2 + y^2)} \frac{1}{2} \iint_{D_{xy}} 3x^2 + y^2 dxdy \xrightarrow{\underline{\text{MWt}}} 2 \iint_{D_{xy}} x^2 dxdy$$

$$\underline{\underline{\text{MWt}}} \iint_{D_{xy}} x^2 + y^2 dxdy = \int_0^{2\pi} \left[\int_0^2 \rho^2 \cdot \rho d\rho \right] d\theta$$

原式 =
$$\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma_1} V \cdot \overrightarrow{n} dS + \iint_{\Sigma_2} V \cdot \overrightarrow{n} dS$$
,
$$\iint_{\Sigma_2} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^2 + x, 0, -z)} \iint_{\Sigma_2} -z dS \iint_{\Sigma_2} -2 dS = -2 |\Sigma_1| = -8\pi,$$

 $\iint_{\Sigma_{1}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} \iint_{\Sigma_{1}} -z dS \iint_{\Sigma_{1}} -2 dS = -2|\Sigma_{1}| = -8\pi,$ $\iint_{\Sigma_{2}} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^{2} + x, 0, -z)} \iint_{D_{xy}} \frac{(z^{2} + x)x + z}{\sqrt{1 + x^{2} + y^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dx dy$

$$= \iint_{D_{xy}} (z^{2} + x)x + z dx dy \xrightarrow{\underline{x} \text{ with}} \iint_{D_{xy}} x^{2} + z dx dy$$

$$= \frac{z = \frac{1}{2}(x^{2} + y^{2})}{2} \frac{1}{2} \iint_{D_{xy}} 3x^{2} + y^{2} dx dy \xrightarrow{\underline{x} \text{ with}} 2 \iint_{D_{xy}} x^{2} dx dy$$

$$= \frac{\underline{x} \text{ with}}{2} \iint_{D_{xy}} x^{2} + y^{2} dx dy = \int_{0}^{2\pi} \left[\int_{0}^{2} \rho^{2} \cdot \rho d\rho \right] d\theta = 8\pi$$

 $= \iint_{D} (z^{2} + x)x + zdxdy \xrightarrow{\underline{\text{symt}}} \iint_{D} x^{2} + zdxdy$

 $\iint_{\Sigma} V \cdot \overrightarrow{n} dS \xrightarrow{V = (z^2 + x, 0, -z)} \iint_{\Sigma} -z dS \iint_{\Sigma} -2 dS = -2|\Sigma_1| = -8\pi,$ $\iint_{\Sigma_2} V \cdot \overrightarrow{n} dS = \underbrace{\frac{V = (z^2 + x, 0, -z)}{\overrightarrow{n} = \frac{(x, y, -1)}{\sqrt{1 + x^2 + y^2}}} \iint_{D_{xy}} \frac{(z^2 + x)x + z}{\sqrt{1 + x^2 + y^2}} \cdot \sqrt{1 + z_x^2 + z_y^2} dx dy$

 $\frac{z=\frac{1}{2}(x^2+y^2)}{2} \frac{1}{2} \iint_{\Omega} 3x^2 + y^2 dx dy \xrightarrow{\text{spate}} 2 \iint_{\Omega} x^2 dx dy$

原式 = $\iint_{\Sigma} V \cdot \overrightarrow{n} dS = \iint_{\Sigma} V \cdot \overrightarrow{n} dS + \iint_{\Sigma} V \cdot \overrightarrow{n} dS$,

原式 = $-8\pi + 8\pi = 0$

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{D_{xy}} R(x, y, z(x, y)) dx dy.$$



$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{D_{xy}} R(x, y, z(x, y)) dx dy.$$

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{\Sigma} V \cdot \overrightarrow{n} dS$$



$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{D_{xy}} R(x, y, z(x, y)) dx dy.$$

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{\Sigma} V \cdot \overrightarrow{n} dS$$

$$\frac{V=(0,0,R)}{\overrightarrow{n} = \frac{1}{\sqrt{1+z_X^2+z_V^2}}(-z_X,-z_Y,1)}$$

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{\Sigma} R(x, y, z(x, y)) dx dy.$$

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{\Sigma} V \cdot \overrightarrow{n} dS$$

$$\frac{V = (0, 0, R)}{\overrightarrow{n} = \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} (-z_{x}, -z_{y}, 1)} \qquad R(x, y, z) \cdot \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}}$$

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{D_{XY}} R(x, y, z(x, y)) dx dy.$$

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{\Sigma} V \cdot \overrightarrow{n} dS$$

$$\frac{V = (0, 0, R)}{\overrightarrow{n} = \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} (-z_{x}, -z_{y}, 1)} \iint_{\Sigma} R(x, y, z) \cdot \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} dS$$



$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{\Sigma} R(x, y, z(x, y)) dx dy.$$

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{\Sigma} V \cdot \overrightarrow{n} dS$$

$$\frac{V = (0, 0, R)}{\overrightarrow{n} = \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} (-z_{x}, -z_{y}, 1)} \iint_{\Sigma} R(x, y, z) \cdot \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} dS$$

$$R(x, y, z(x, y)) \cdot \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}}$$

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{\Sigma} R(x, y, z(x, y)) dx dy.$$

证明 这是:

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{\Sigma} V \cdot \overrightarrow{n} dS$$

$$\frac{V = (0, 0, R)}{\overrightarrow{n} = \frac{1}{\sqrt{1 + z_{X}^{2} + z_{y}^{2}}} (-z_{x}, -z_{y}, 1)} \iint_{\Sigma} R(x, y, z) \cdot \frac{1}{\sqrt{1 + z_{X}^{2} + z_{y}^{2}}} dS$$

$$R(x, y, z(x, y)) \cdot \frac{1}{\sqrt{1 + z_{y}^{2} + z_{y}^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dx dy$$

● 暨南大学

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{D_{x,y}} R(x, y, z(x, y)) dx dy.$$

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{\Sigma} V \cdot \overrightarrow{n} dS$$

$$\frac{V = (0, 0, R)}{\overrightarrow{n} = \frac{1}{\sqrt{1 + z_{X}^{2} + z_{y}^{2}}} (-z_{X}, -z_{y}, 1)} \iint_{\Sigma} R(x, y, z) \cdot \frac{1}{\sqrt{1 + z_{X}^{2} + z_{y}^{2}}} dS$$

$$= \iint_{D_{XY}} R(x, y, z(x, y)) \cdot \frac{1}{\sqrt{1 + z_{X}^{2} + z_{y}^{2}}} \cdot \sqrt{1 + z_{X}^{2} + z_{y}^{2}} dx dy$$



$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{D_{\text{con}}} R(x, y, z(x, y)) dx dy.$$

证明 这是:

$$\iint_{\Sigma} R(x, y, z) dx dy = \iint_{\Sigma} V \cdot \overrightarrow{n} dS$$

$$\frac{V = (0, 0, R)}{\overrightarrow{n} = \frac{1}{\sqrt{1 + z_X^2 + z_y^2}} (-z_x, -z_y, 1)} \iint_{\Sigma} R(x, y, z) \cdot \frac{1}{\sqrt{1 + z_x^2 + z_y^2}} dS$$

 $= \int_{\mathbb{R}} R(x, y, z(x, y)) dxdy$

 $= \iiint_{D_{xy}} R(x, y, z(x, y)) \cdot \frac{1}{\sqrt{1 + z_{x}^{2} + z_{y}^{2}}} \cdot \sqrt{1 + z_{x}^{2} + z_{y}^{2}} dx dy$

