第 9 章 c：多元复合函数的求导法则

数学系　　梁卓滨

2019-2020 学年 II

Outline

二元复合函数求导

设有二元函数 $z = f(u, v)$

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- 设 $u = \varphi(t)$，$v = \psi(t)$，则 $z = f(\varphi(t), \psi(t))$

问 $\frac{dz}{dt} =?$

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公式 设 $z = f(u, v)$，$u = \varphi(t)$，$v = \psi(t)$，则 $z = f(\varphi(t), \psi(t))$ 的**全导数**

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$$= -\frac{y}{x^2} \cdot e^t + \frac{1}{x} \cdot (-2e^{2t}) =$$

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$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=-\frac{1-e^{2t}}{e^{2t}}\cdot e^t+$$

$$\begin{tikzcd}
 & x \arrow[r, "\frac{dx}{dt}"] & t \\
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 & y \arrow[r, "\frac{dy}{dt}"'] & t
\end{tikzcd}$$

例 设 $z=\frac{y}{x}$，而 $x=e^t$，$y=1-e^{2t}$，求全导数 $\frac{dz}{dt}$.

解

$$
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&= -\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=-\frac{1-e^{2t}}{e^{2t}}\cdot e^t+\frac{1}{e^t}\cdot(-2e^{2t}) \\
&=
\end{aligned}
$$

$$
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$$=-e^{-t}-e^t$$

$$\begin{tikzcd}
 & x \arrow[r, "\frac{dx}{dt}"] & t \\
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三元复合函数求导公式——中间变量是一元函数

公式 设 $z = f(u, v, w)$，$u = \varphi(t)$，$v = \psi(t)$，$w = \omega(t)$，则 $z = f(\varphi(t), \psi(t), \omega(t))$ 的**全导数**

$$\frac{dz}{dt} =$$

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$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
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二元复合函数求导公式——中间变量是多元函数

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$$z=f(\varphi(x,y),\psi(x,y))$$

的偏导数是：

$$\frac{\partial z}{\partial x}= \qquad\qquad , \quad \frac{\partial z}{\partial y}=$$

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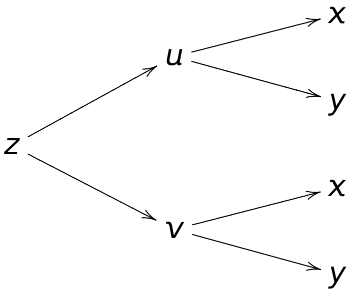
图示



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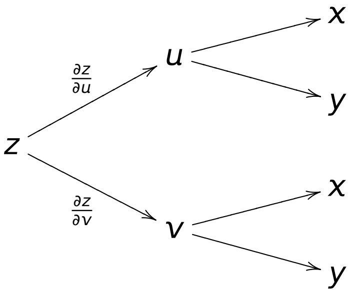

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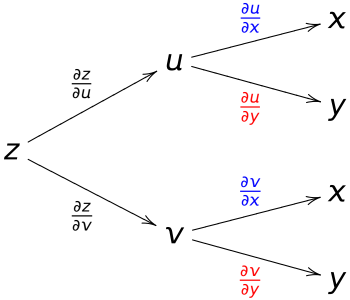

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图示

$$\begin{tikzcd}
 & & x \\
 & u \arrow[ur, "\frac{\partial u}{\partial x}"] \arrow[dr, "\frac{\partial u}{\partial y}"'] & \\
 & & y \\
z \arrow[uur, "\frac{\partial z}{\partial u}"] \arrow[ddr, "\frac{\partial z}{\partial v}"'] & & \\
 & & x \\
 & v \arrow[ur, "\frac{\partial v}{\partial x}"] \arrow[dr, "\frac{\partial v}{\partial y}"'] & \\
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\end{tikzcd}$$

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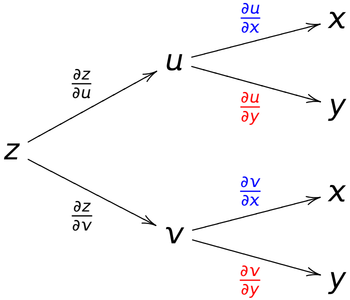

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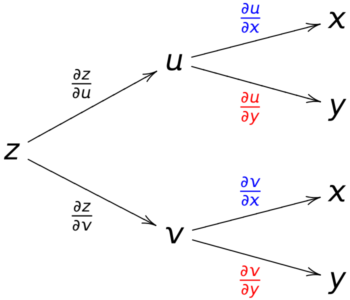

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图示

$$\begin{tikzcd}
 & & x \\
 & u \arrow[ur, "\frac{\partial u}{\partial x}"] \arrow[dr, "\frac{\partial u}{\partial y}"'] & \\
 & & y \\
z \arrow[uur, "\frac{\partial z}{\partial u}"] \arrow[ddr, "\frac{\partial z}{\partial v}"'] & & \\
 & & x \\
 & v \arrow[ur, "\frac{\partial v}{\partial x}"] \arrow[dr, "\frac{\partial v}{\partial y}"'] & \\
 & & y
\end{tikzcd}$$

例 设 $z = e^{2u} \sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$=$$

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$$= (e^{2u}\sin v)'_u \cdot$$

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解

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$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot (x^2 + y^2)'_x$$
$$= 2e^{2u}\sin v \cdot$$

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&= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\
&= 2e^{2x^3y}\sin(x^2+y^2)\cdot
\end{aligned}
$$

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&= 2e^{2x^3 y}\sin(x^2+y^2)\cdot 3x^2 y + e^{2x^3 y}\cos(x^2+y^2)\cdot
\end{aligned}$$

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解

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$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$

$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x$$

$$=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$

$$=$$

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&= 2e^{2x^3 y}\sin(x^2 + y^2)\cdot 3x^2 y + e^{2x^3 y}\cos(x^2 + y^2)\cdot 2x
\end{aligned}$$

$$\begin{aligned}
\frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y} \\
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&= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y} \\
&= (e^{2u}\sin v)'_u\cdot(x^3y)'_y+
\end{aligned}
$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$

$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x$$

$$=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$

$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot (x^2+y^2)'_x$$

$$= 2e^{2u}\sin v\cdot 3x^2 y + e^{2u}\cos v\cdot 2x$$

$$= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2 y + e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$

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解
$$\begin{aligned}\frac{\partial z}{\partial x}&=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\&=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\&=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\&=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial y}&=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\\&=(e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y\\&=2e^{2u}\sin v\cdot\end{aligned}$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$= (e^{2u}\sin v)'_u \cdot (x^3y)'_x + (e^{2u}\sin v)'_v \cdot (x^2+y^2)'_x$$
$$= 2e^{2u}\sin v \cdot 3x^2y + e^{2u}\cos v \cdot 2x$$
$$= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y + e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

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$$= (e^{2u}\sin v)'_u \cdot (x^3y)'_y + (e^{2u}\sin v)'_v \cdot (x^2+y^2)'_y$$
$$= 2e^{2u}\sin v \cdot x^3 +$$

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$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y$$
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三元复合函数求导公式：举例

公式 设 $z = f(x, y, u)$，$u = u(x, y)$，

三元复合函数求导公式：举例

公式 设 $z = f(x, y, u)$，$u = u(x, y)$，则复合函数

$$z = f(x, y, u(x, y))$$

的偏导数是：

$$\frac{\partial z}{\partial x} = \qquad\qquad\qquad , \quad \frac{\partial z}{\partial y} =$$

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图示

$$\begin{tikzcd}
 & x & \\
z \arrow[ur] \arrow[r] \arrow[dr] & y & \\
 & u \arrow[r] \arrow[dr] & x \\
 & & y
\end{tikzcd}$$

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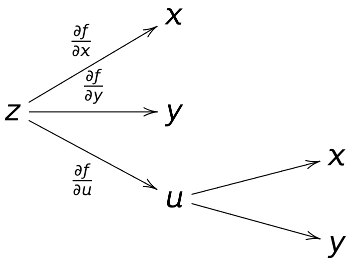
图示



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图示

$$\begin{tikzcd}
 & x & \\
z \arrow[ur, "\frac{\partial f}{\partial x}"] \arrow[r, "\frac{\partial f}{\partial y}"] \arrow[dr, "\frac{\partial f}{\partial u}"'] & y & \\
 & u \arrow[r, "\frac{\partial u}{\partial x}"] \arrow[dr, "\frac{\partial u}{\partial y}"'] & x \\
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\end{tikzcd}$$

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图示

$$\begin{tikzcd}
 & x & \\
z \arrow[ur, "\frac{\partial f}{\partial x}"] \arrow[r, "\frac{\partial f}{\partial y}"] \arrow[dr, "\frac{\partial f}{\partial u}"'] & y & \\
 & u \arrow[r, "\frac{\partial u}{\partial x}"] \arrow[dr, "\frac{\partial u}{\partial y}"'] & x \\
 & & y
\end{tikzcd}$$

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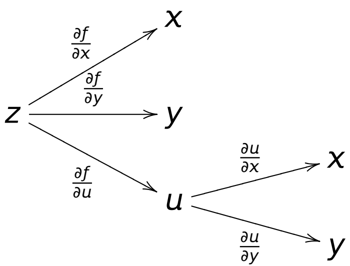
图示



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$$z = f(x, y, u(x, y))$$

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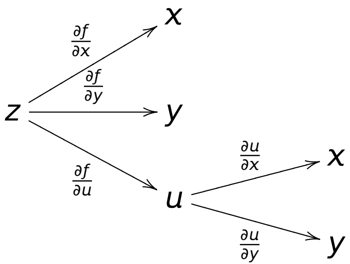
图示



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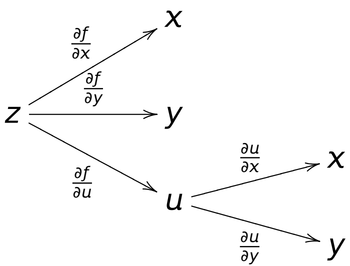
图示



复合函数的高阶导数

公式 设 $z=f(u,v)$，$u=u(x,y)$，$v=v(x,y)$，则复合函数

$$z=f(u(x,y),v(x,y))$$

的偏导数是：

$$z_x=z_u\cdot u_x+z_v\cdot v_x,$$

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$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yy} =$$

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$$\begin{aligned}z_{xx}&=(z_x)'_x=(z_u\cdot u_x+z_v\cdot v_x)'_x\\&=(z_u)'_x\cdot u_x+z_u\cdot u_{xx}+(z_v)'_x\cdot v_x+z_v\cdot v_{xx}\end{aligned}$$

$$z_{xy}=$$

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复合函数的高阶导数

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$$\begin{aligned} z_{xx} &= (z_x)'_x = (z_u \cdot u_x + z_v \cdot v_x)'_x \\ &= (z_u)'_x \cdot u_x + z_u \cdot u_{xx} + (z_v)'_x \cdot v_x + z_v \cdot v_{xx} \\ &= (\qquad\qquad\qquad) \cdot u_x + z_u \cdot u_{xx} + (\qquad\qquad\qquad) \cdot v_x + z_v \cdot v_{xx} \end{aligned}$$

$$z_{xy} =$$

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$$z_{xy}=$$

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$$\begin{aligned}
z_{xx} &= (z_x)'_x = (z_u \cdot u_x + z_v \cdot v_x)'_x \\
&= (z_u)'_x \cdot u_x + z_u \cdot u_{xx} + (z_v)'_x \cdot v_x + z_v \cdot v_{xx} \\
&= (z_{uu}\cdot u_x + z_{uv}\cdot v_x)\cdot u_x + z_u\cdot u_{xx} + (z_{vu}\cdot u_x + z_{vv}\cdot v_x)\cdot v_x + z_v\cdot v_{xx}
\end{aligned}$$

$$z_{xy} =$$

$$z_{yy} =$$

复合函数的高阶导数

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z_{xx}&=(z_x)'_x=(z_u\cdot u_x+z_v\cdot v_x)'_x\\
&=(z_u)'_x\cdot u_x+z_u\cdot u_{xx}+(z_v)'_x\cdot v_x+z_v\cdot v_{xx}\\
&=(z_{uu}\cdot u_x+z_{uv}\cdot v_x)\cdot u_x+z_u\cdot u_{xx}+(z_{vu}\cdot u_x+z_{vv}\cdot v_x)\cdot v_x+z_v\cdot v_{xx}\\
&=z_{uu}u_x^2+2z_{uv}u_xv_x+z_{vv}v_x^2+z_uu_{xx}+z_vv_{xx}
\end{aligned}$$

$$z_{xy}=?$$

$$z_{yy}=?$$

复合函数的高阶导数

公式 设 $z=f(u,v)$，$u=u(x,y)$，$v=v(x,y)$，则复合函数

$$z=f(u(x,y),v(x,y))$$

的偏导数是：

$$z_x=z_u\cdot u_x+z_v\cdot v_x,$$

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$$z_{xy} = (z_x)'_y$$

$$z_{yy} = ?$$

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$$z_{yy}=(z_y)'_y$$

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例 设 $z = f(xy^2, x^2y)$，求 $\frac{\partial^2 z}{\partial x \partial y}$

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$$= e^{x+y} f_w - \cos x \sin y \cdot f_{uv} + \cos x e^{x+y} f_{uw} - \sin y e^{x+y} f_{wv} + e^{2x+2y} f_{ww}$$