

第 9 章 b: 偏导数与全微分

数学系 梁卓滨

2018-2019 学年 II

We are here now...

1. 偏导数

2. 全微分

偏导数引入

- 对一元函数 $y = f(x)$: 导数 $y' = f'(x) \longleftrightarrow$ 变化率

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$$\begin{aligned} \frac{\partial z}{\partial x} &= (x^2y + 2x + y + 1)'_x = \\ \frac{\partial z}{\partial y} &= \end{aligned}$$

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$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y =$$

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$$\begin{aligned} z(2, y) = 2y + \frac{2}{y} &\Rightarrow \frac{\partial z}{\partial y}(2, 1) = \frac{d}{dy}[z(2, y)]|_{y=1} \\ &= 2 - \frac{2}{y^2} \end{aligned}$$

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例 6 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$, 求 $f_x(0, 0), f_y(0, 0)$

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$$f_x(0, 0)$$

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$$f_x(0, 0) \quad f(x, 0)$$

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解

$$f_x(0, 0) = \left. \frac{d}{dx} [f(x, 0)] \right|_{x=0}$$

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$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)] \Big|_{x=0} = \frac{d}{dx}[0] \Big|_{x=0} = 0,$$

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注 上述 $f(x, y)$ 在 $(0, 0)$ 处存在偏导数 $f_x(0, 0)$ 和 $f_y(0, 0)$,

例 6 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$, 求 $f_x(0, 0), f_y(0, 0)$

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$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)] \Big|_{x=0} = \frac{d}{dx}[0] \Big|_{x=0} = 0,$$

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注 上述 $f(x, y)$ 在 $(0, 0)$ 处存在偏导数 $f_x(0, 0)$ 和 $f_y(0, 0)$, 但可以证明在 $(0, 0)$ 处不连续

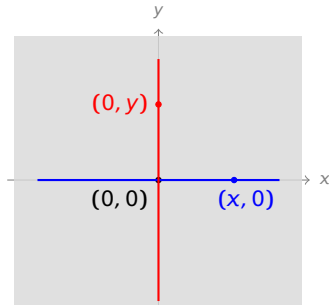
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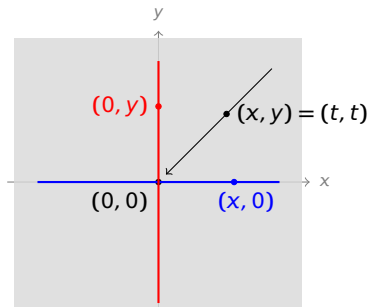
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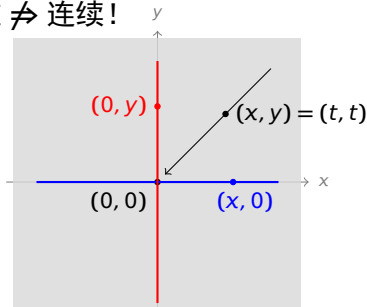
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所以, 偏导数存在 \nRightarrow 连续!



偏导数的极限定义

- 一元函数 $y = f(x)$ 在 $x = x_0$ 处的导数定义为:

$$f'(x_0) =$$

偏导数的极限定义

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$$f'(x_0) = \lim \text{—————}$$

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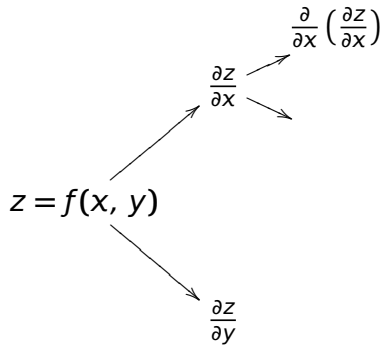
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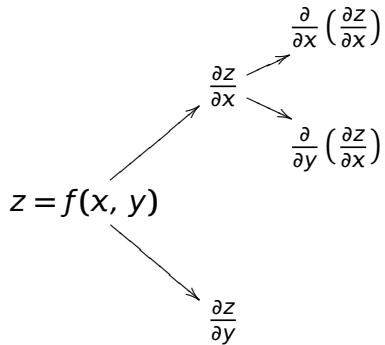
二阶偏导数

$$\begin{array}{ccc} & \nearrow & \frac{\partial z}{\partial x} \\ z = f(x, y) & & \\ & \searrow & \frac{\partial z}{\partial y} \end{array}$$

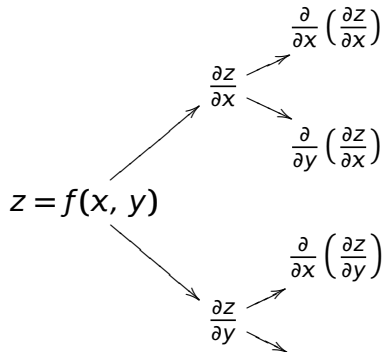
二阶偏导数



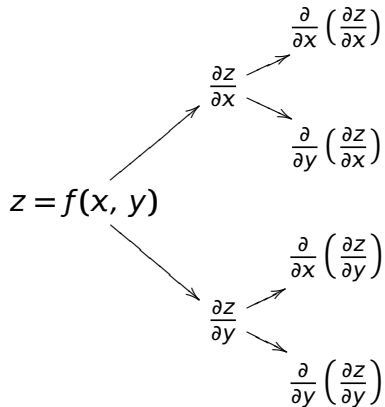
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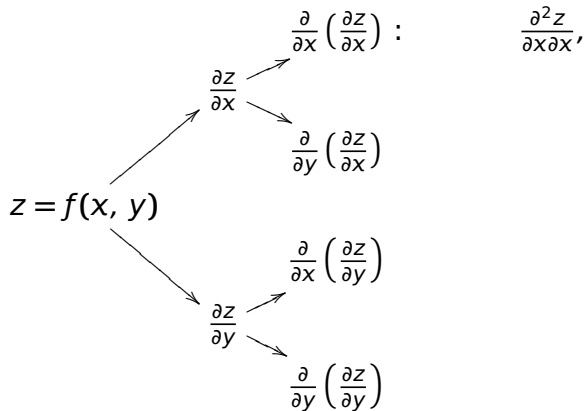
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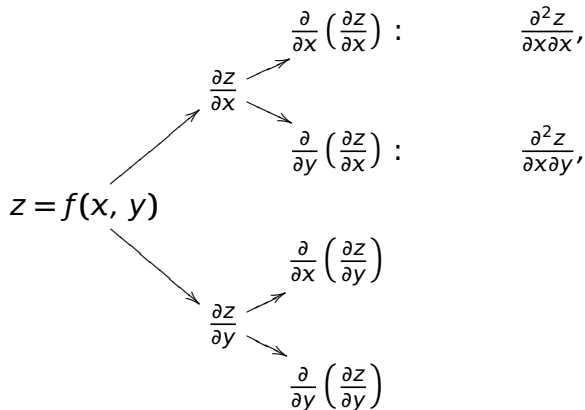
二阶偏导数



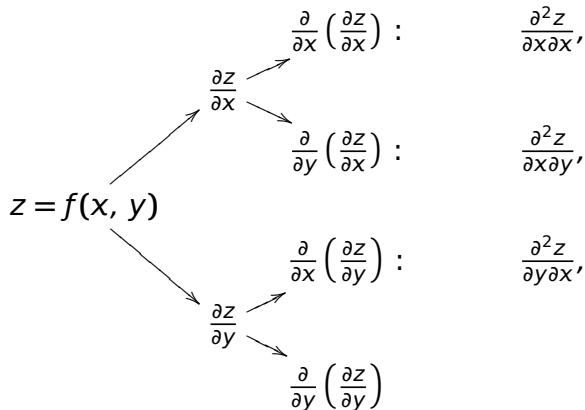
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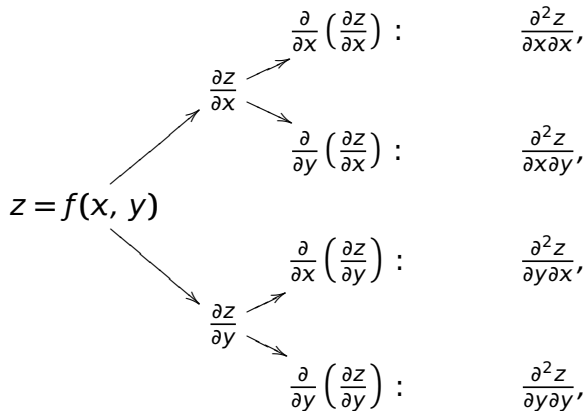
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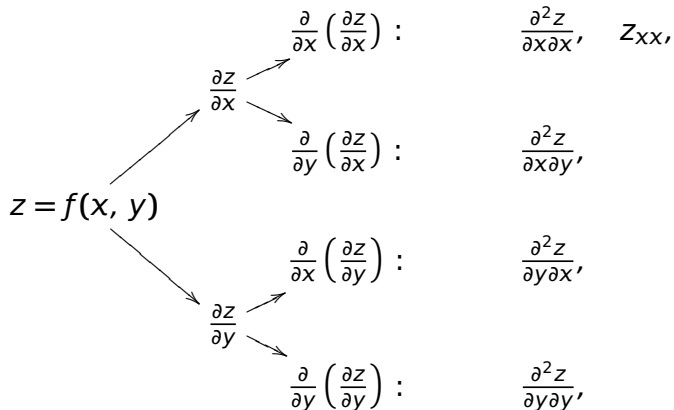
二阶偏导数



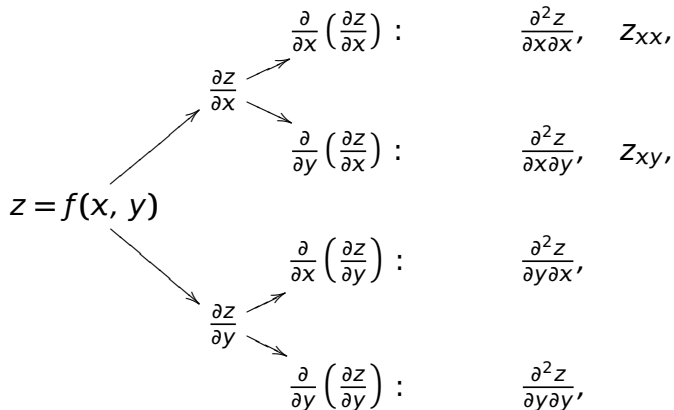
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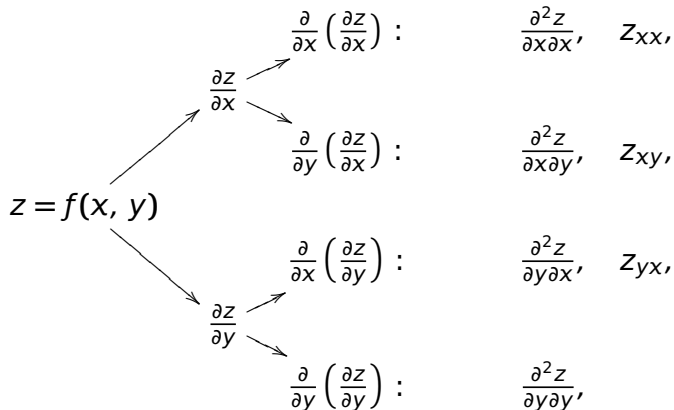
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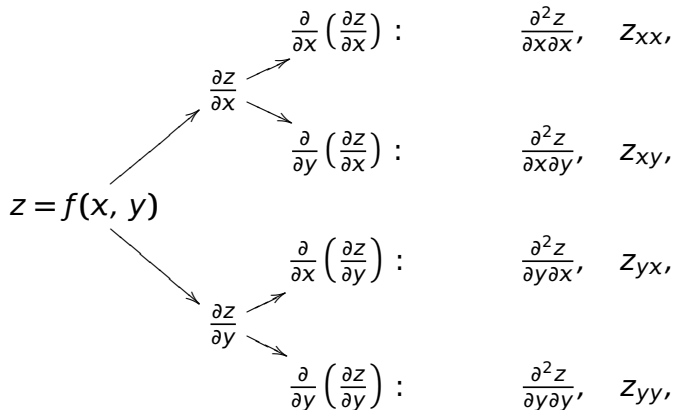
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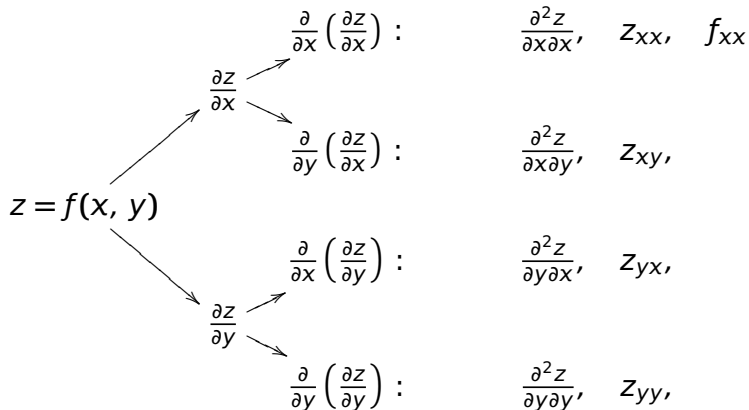
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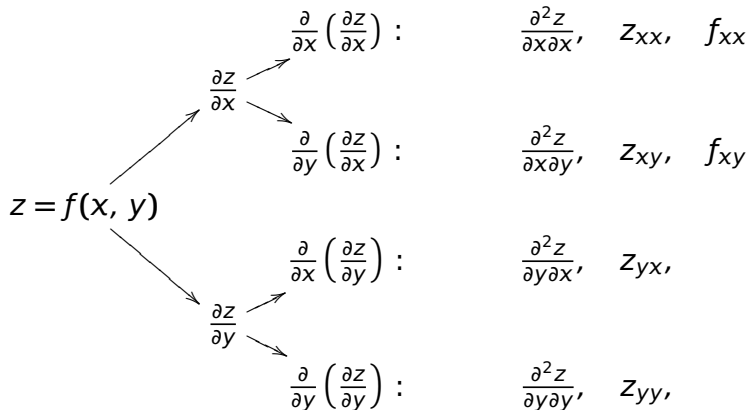
二阶偏导数



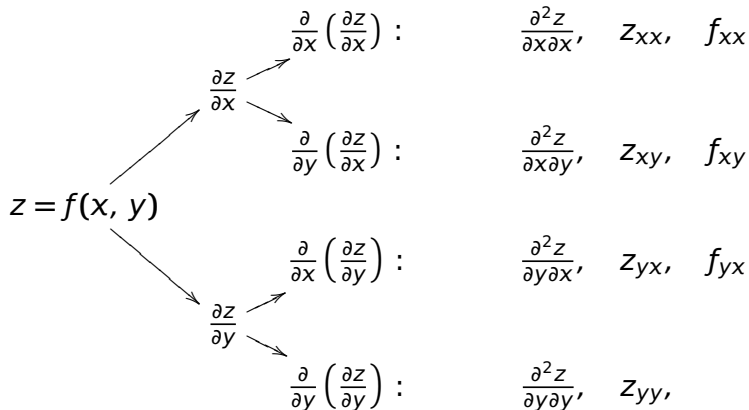
二阶偏导数



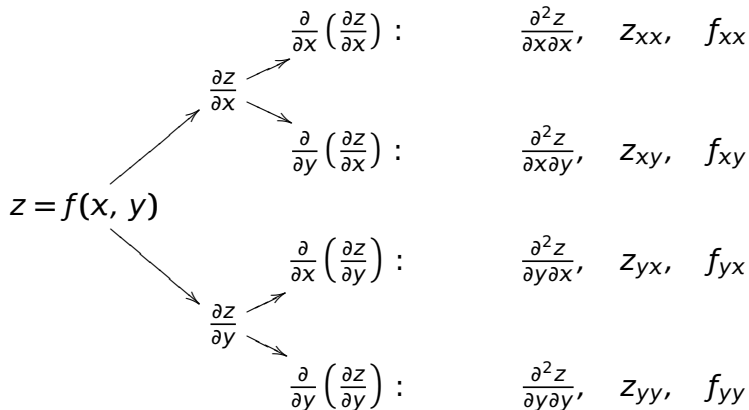
二阶偏导数



二阶偏导数



二阶偏导数



例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

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解

$$z_x =$$

$$z_y =$$

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解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

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$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} +$$

$$z_y =$$

$$z_{xx} =$$

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$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

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$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

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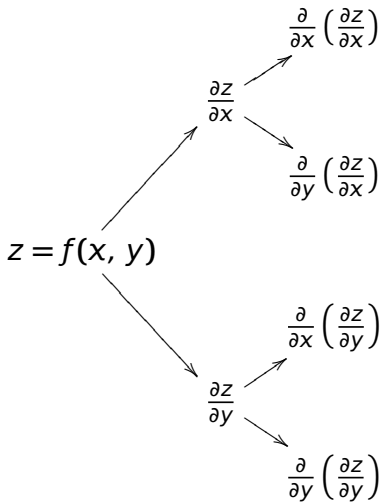
$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

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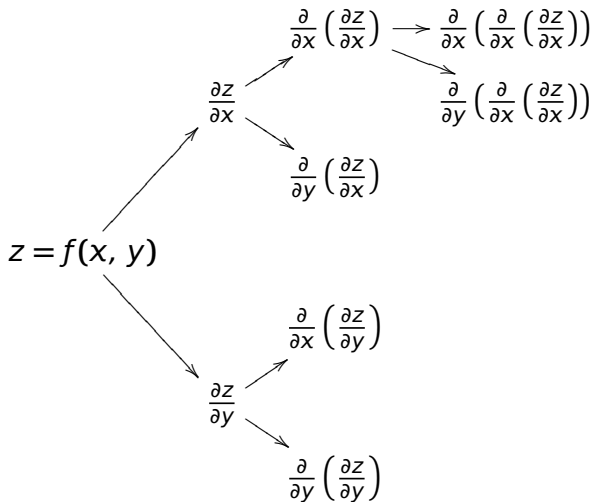
$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2 e^{xy} + 4x$$

注 此例成立 $z_{xy} = z_{yx}$

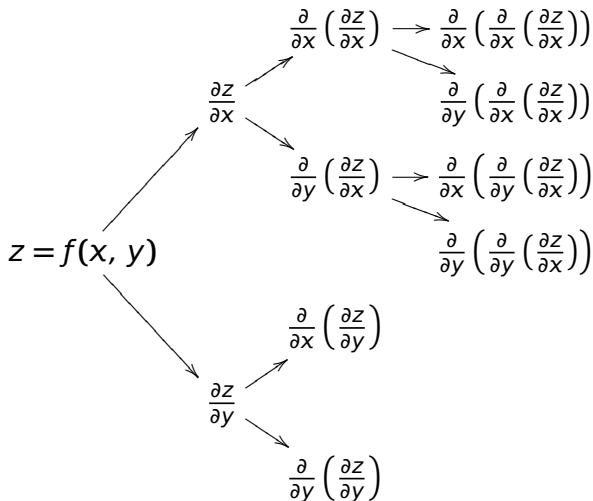
三阶偏导数



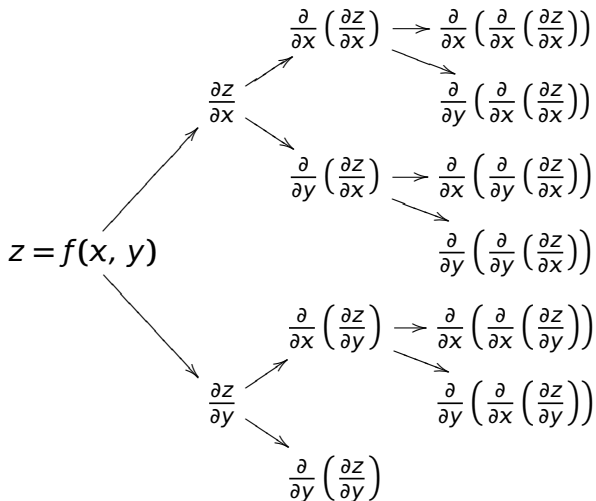
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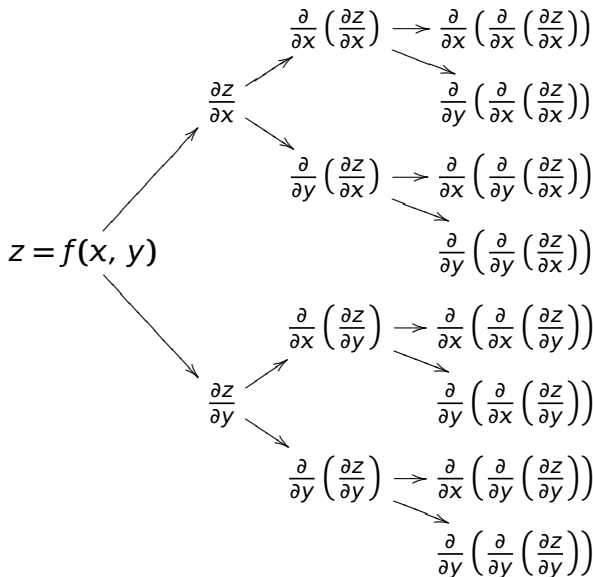
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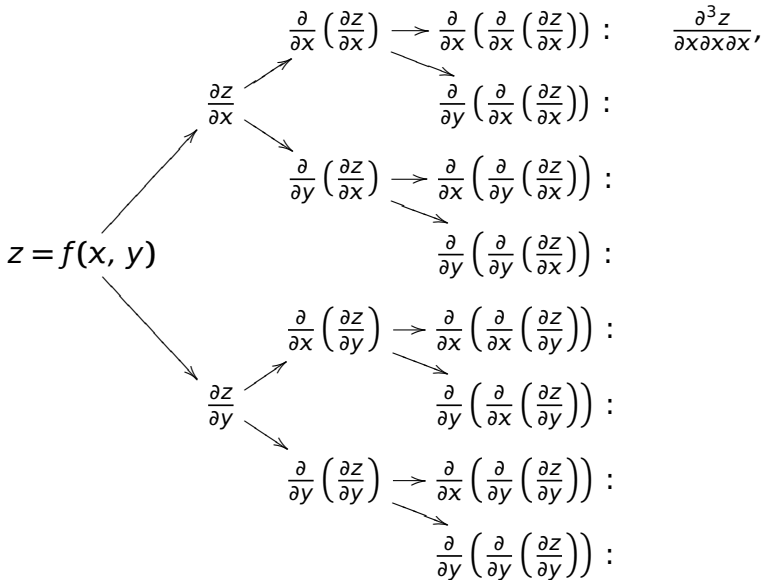
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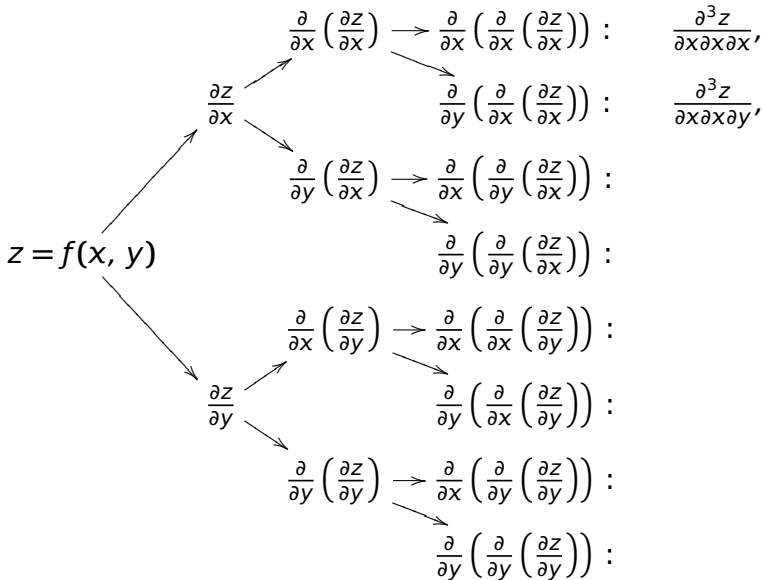
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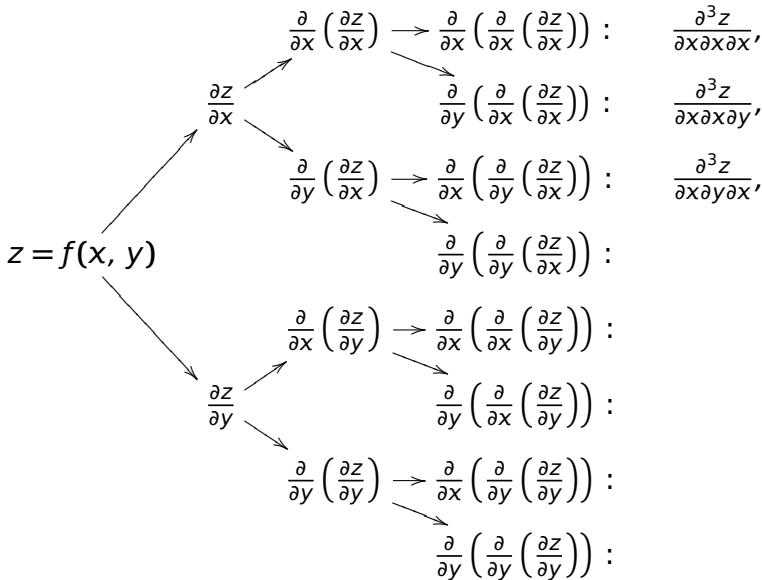
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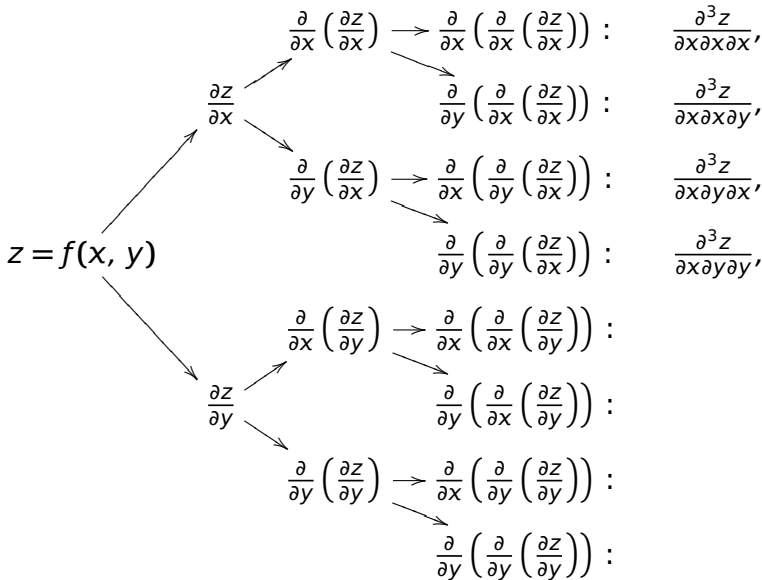
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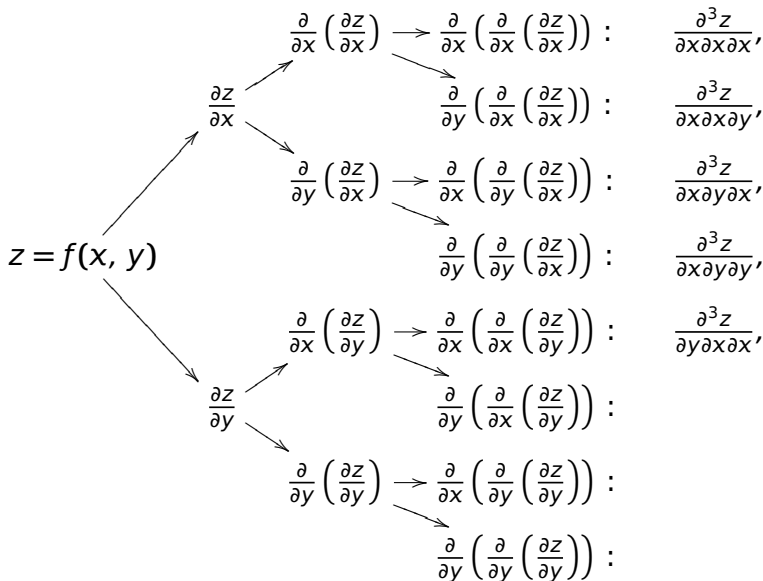
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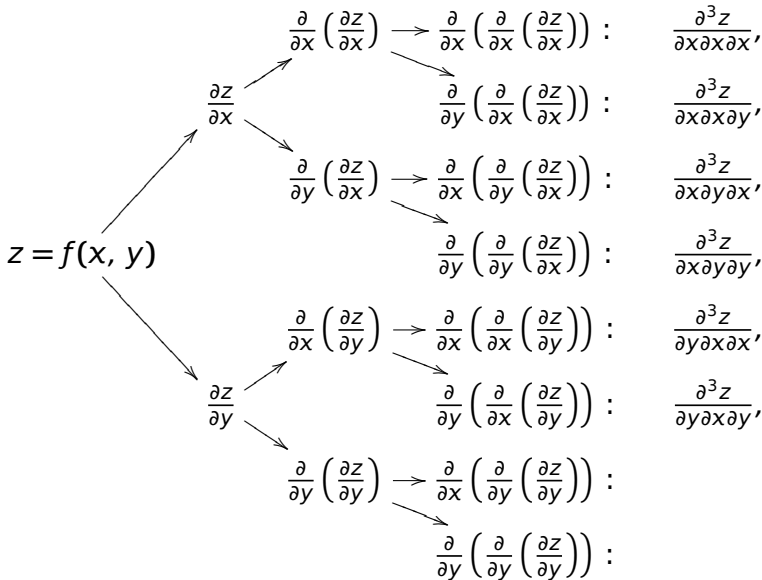
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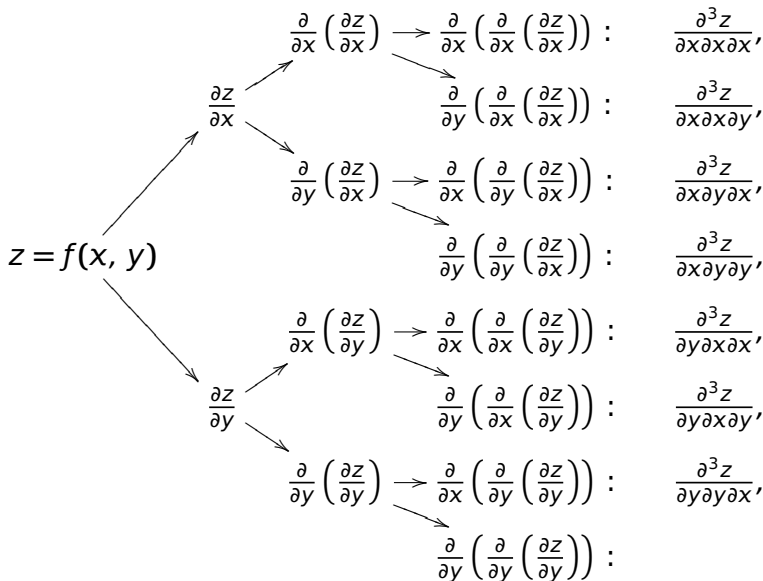
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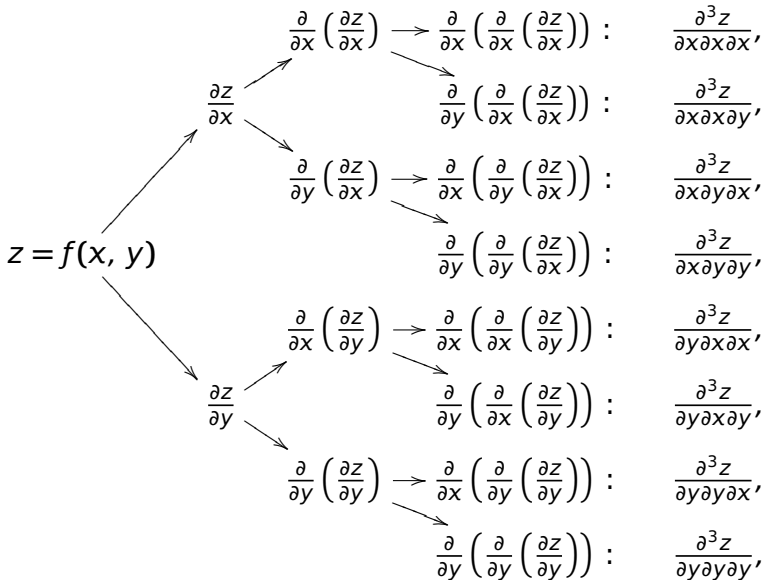
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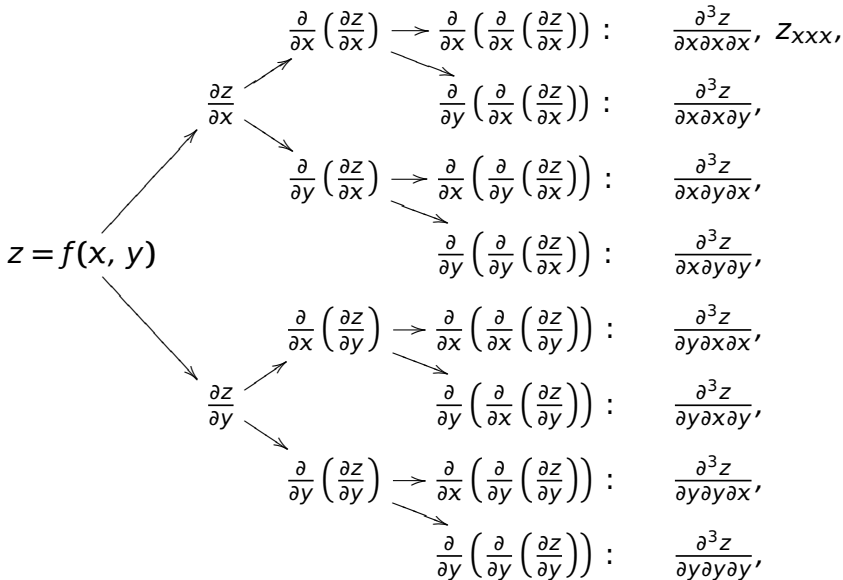
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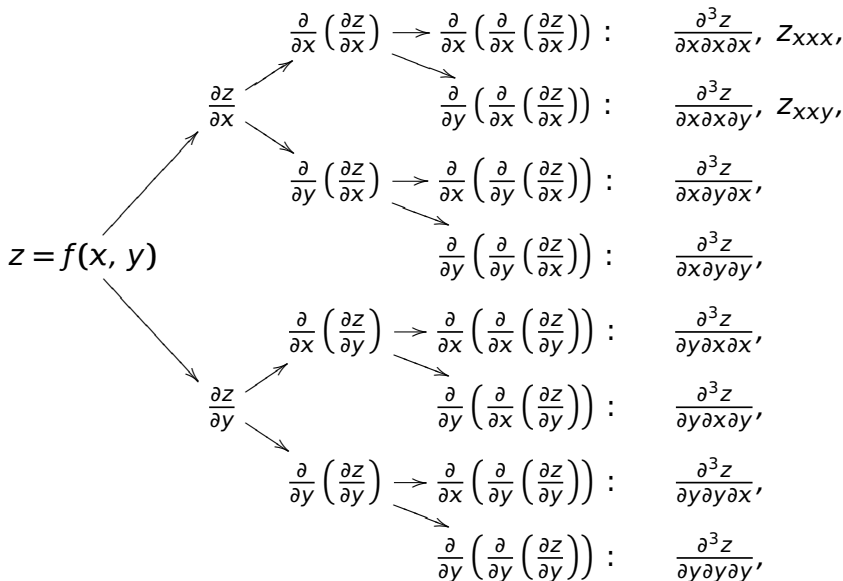
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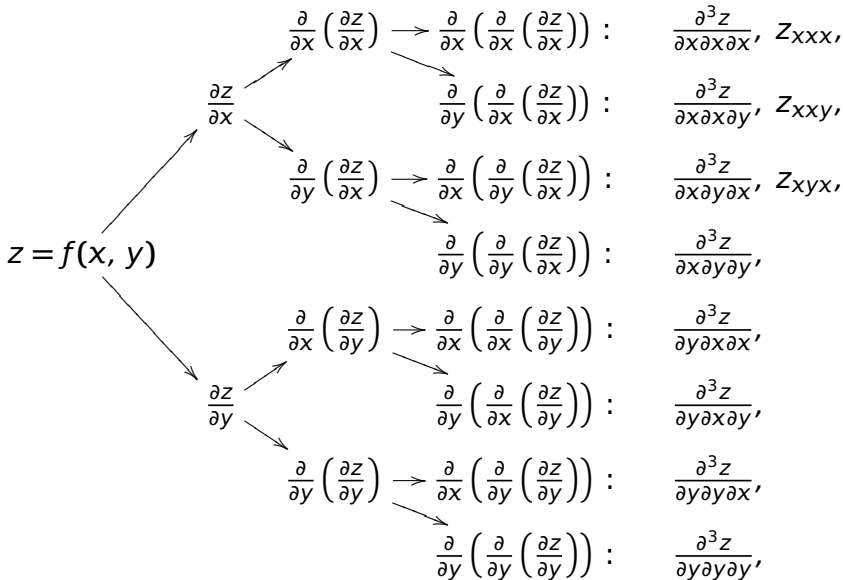
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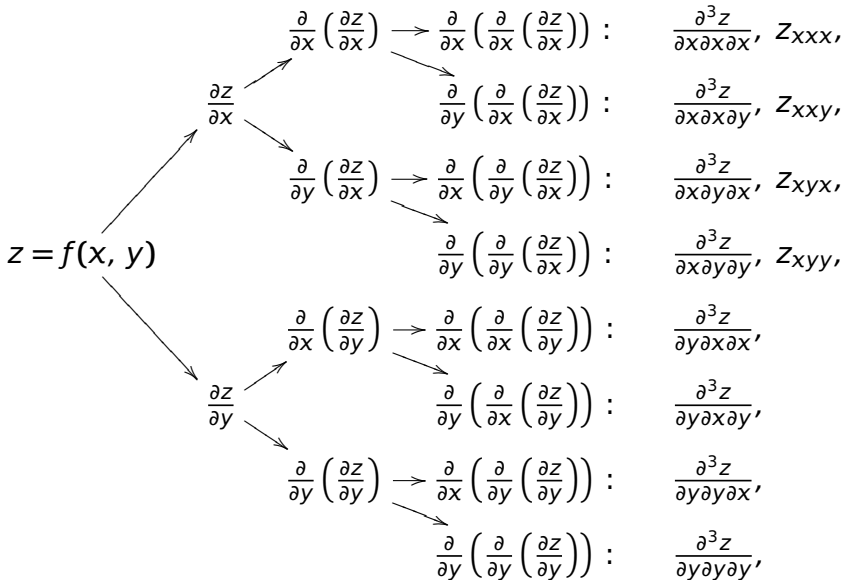
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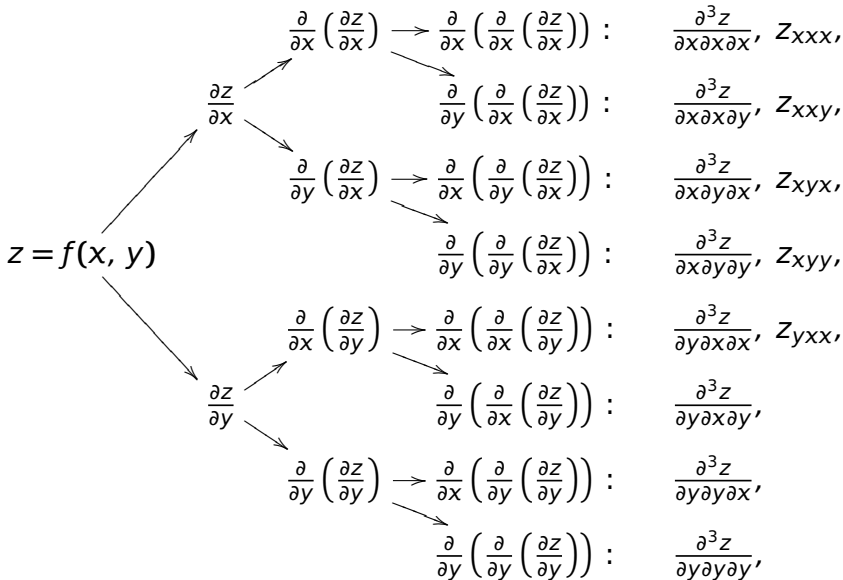
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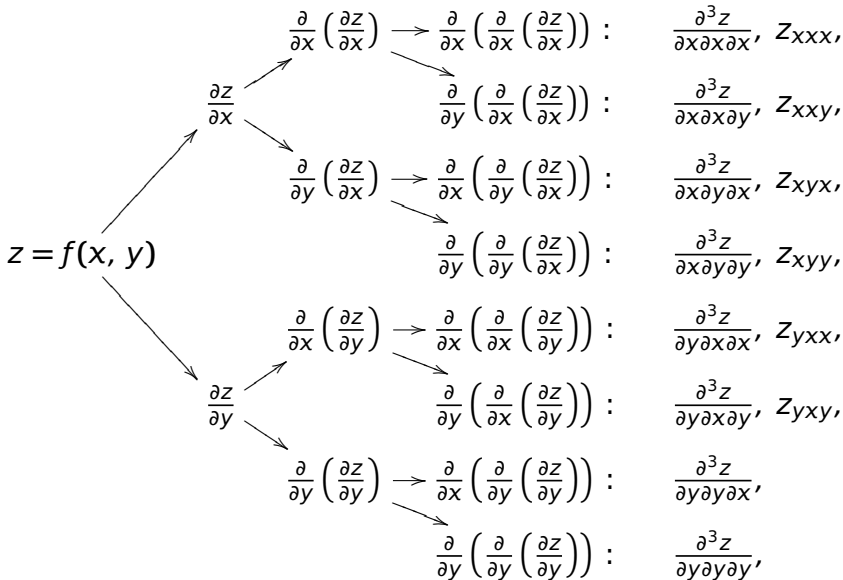
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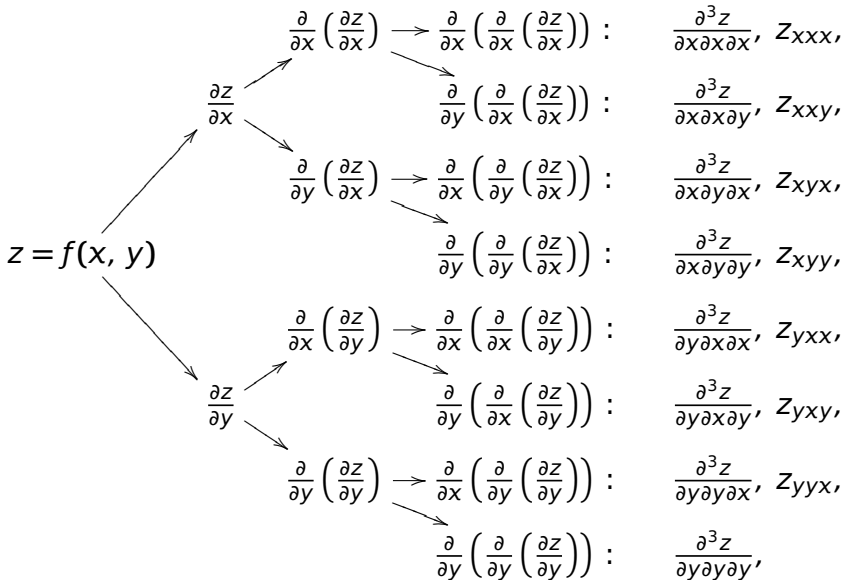
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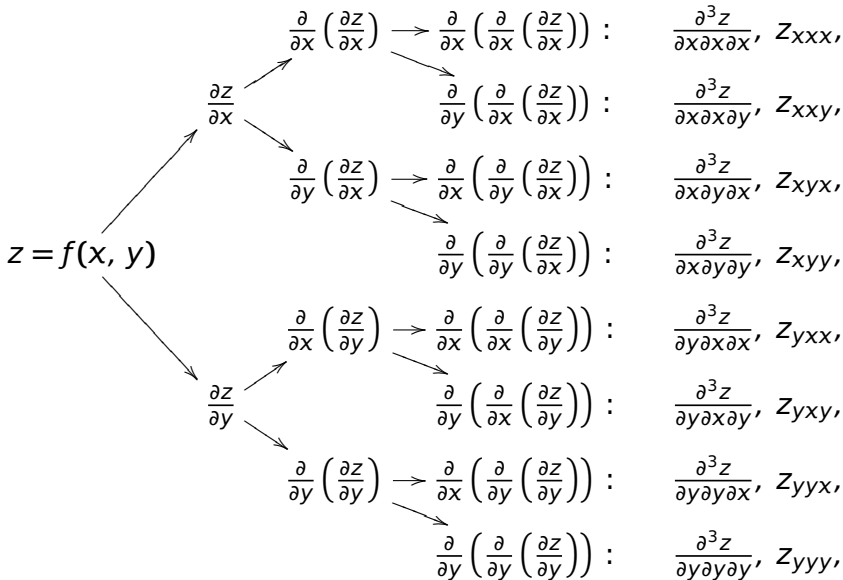
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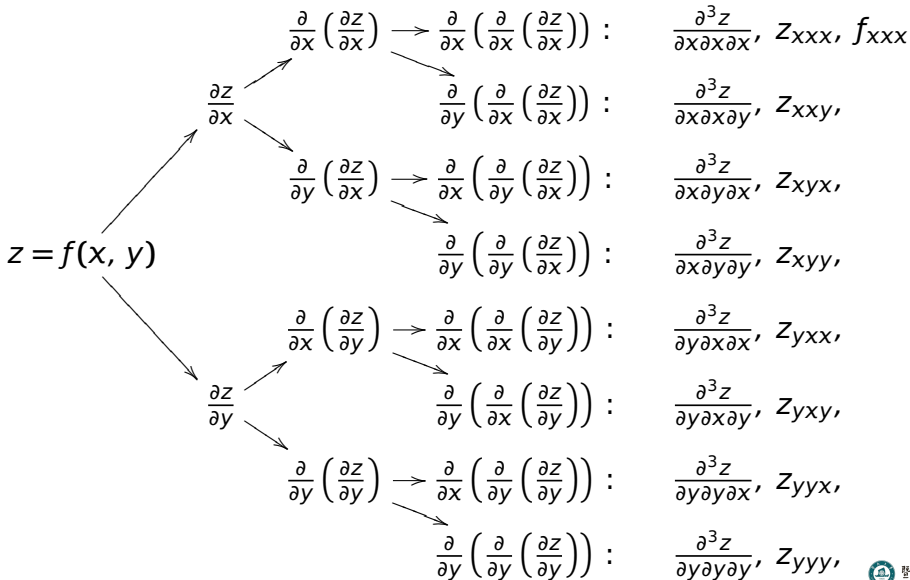
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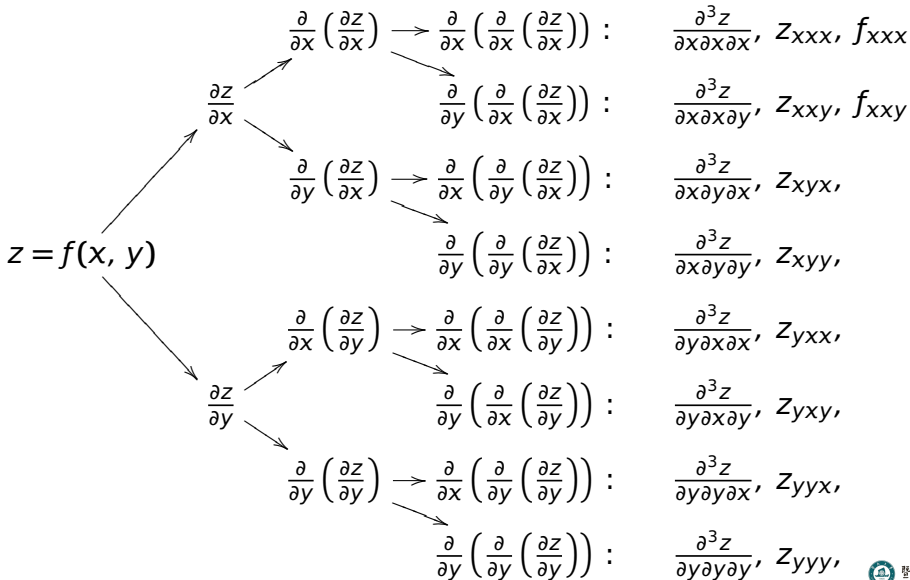
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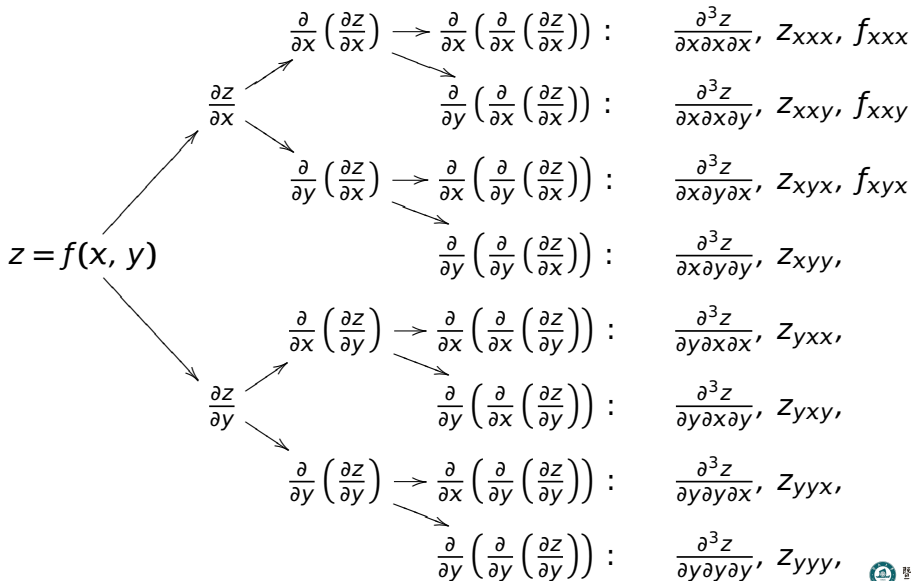
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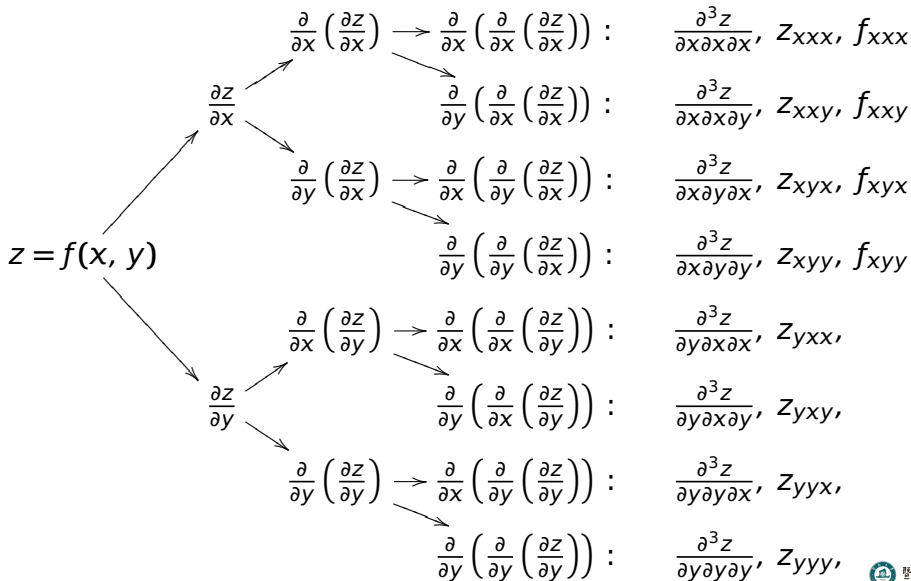
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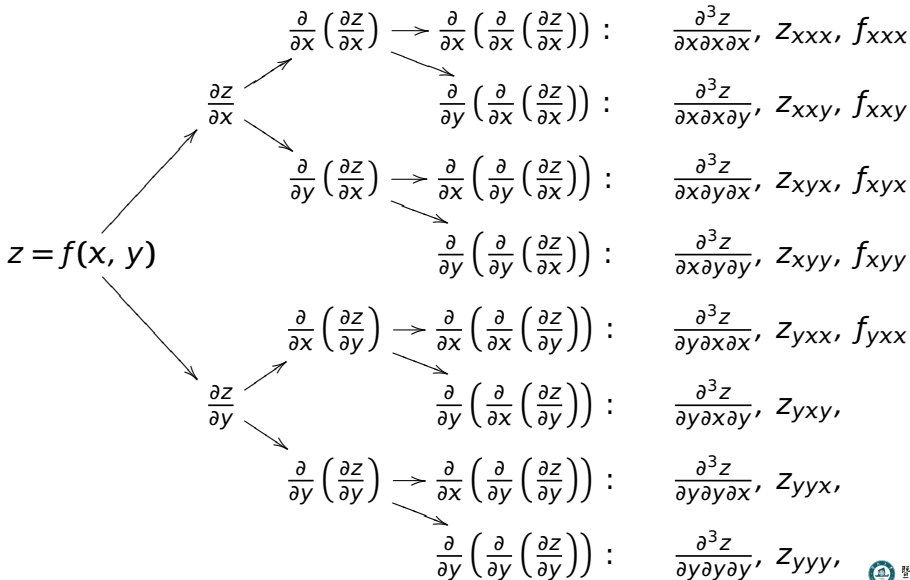
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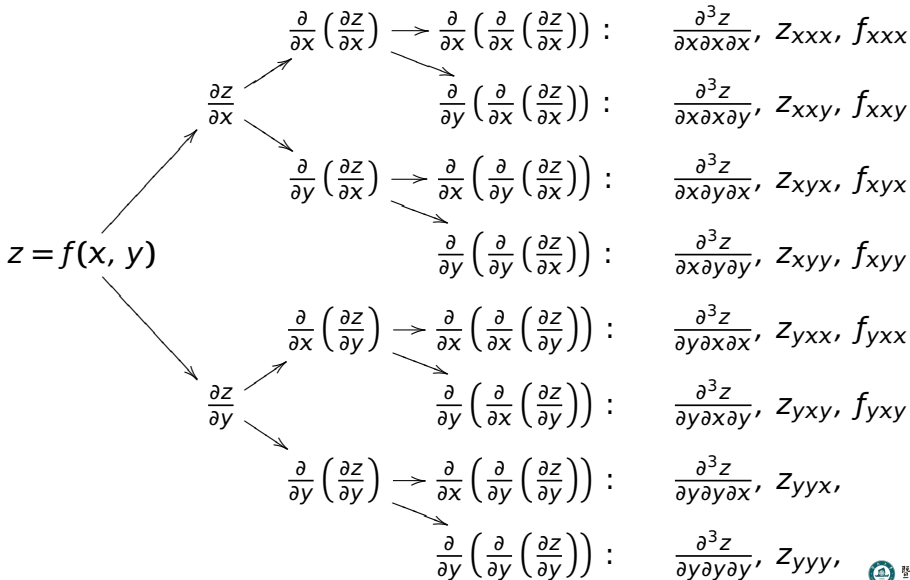
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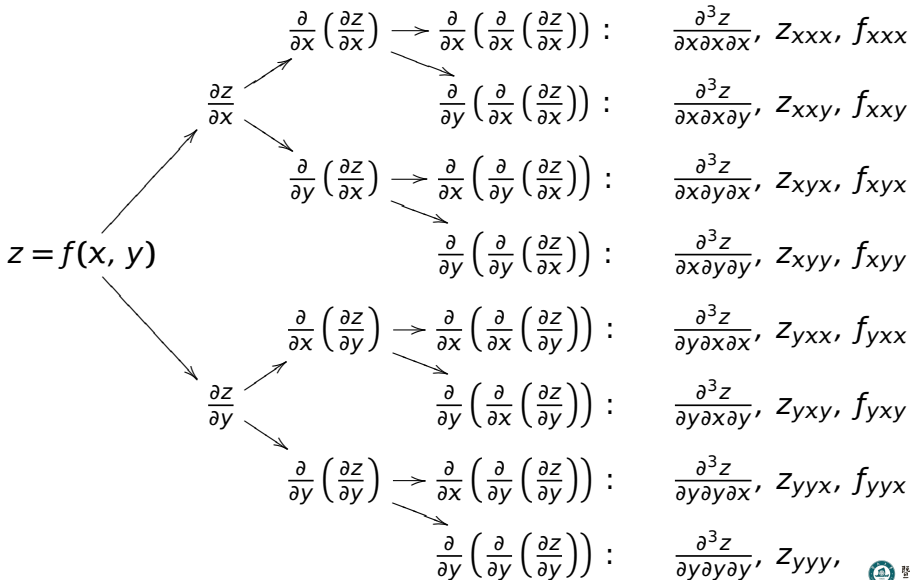
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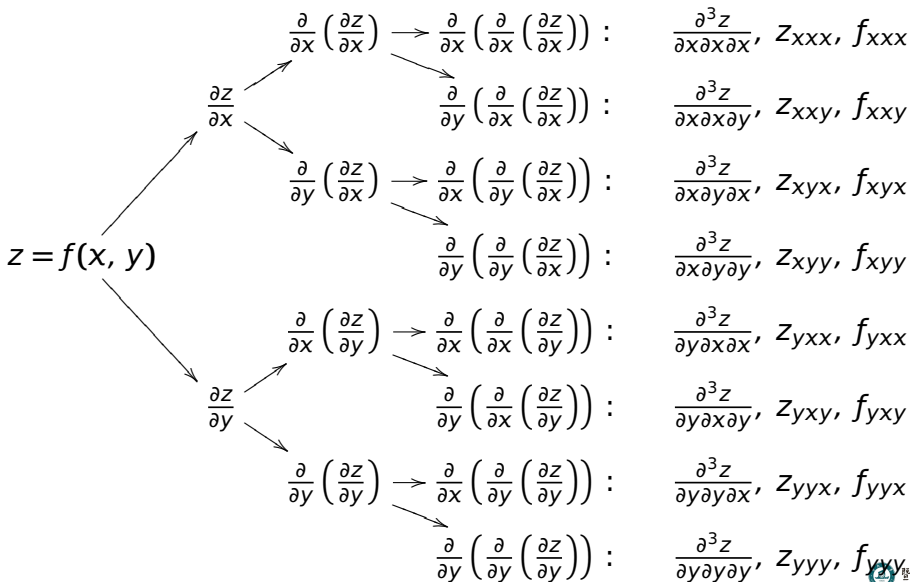
三阶偏导数



三阶偏导数



三阶偏导数



例 1 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

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解 $z_x =$

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解

$$z_x =$$

$$z_y =$$

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$$z_{xy} =$$

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$$z_y =$$

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解
$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2$$

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例 1 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 1 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y =$$

$$z_{xx} =$$

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$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y =$$

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$$z_{xy} =$$

$$z_{yx} =$$

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$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

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$$z_{yx} =$$

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$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

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$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3$$

$$z_{xxx} =$$

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$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} =$$

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$$z_{xxx} = (6xy^2)'_x =$$

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$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} = (6xy^2)'_x = 6y^2$$

注 此例成立 $z_{xy} = z_{yx}$

例 2 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

例 2 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

例 2 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

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$$z_{yy} =$$

例 2 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 2 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x \sin(3y))'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 2 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x \sin(3y))'_x = \sin(3y)$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 2 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

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$$z_x = (x \sin(3y))'_x = \sin(3y)$$

$$z_y = (x \sin(3y))'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

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$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

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$$z_x = (x \sin(3y))'_x = \sin(3y)$$
$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$
$$z_{xx} = (\sin(3y))'_x =$$
$$z_{xy} =$$
$$z_{yx} =$$
$$z_{yy} =$$
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例 2 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

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$$z_x = (x \sin(3y))'_x = \sin(3y)$$

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$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3 \cos(3y)$$

$$z_{yx} =$$

$$z_{yy} =$$

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$$z_{yy} = (3x \cos(3y))'_y =$$

$$z_{xyy} =$$

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$$z_{yy} = (3x \cos(3y))'_y = -9x \sin(3y)$$

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$$z_{xyy} = (3 \cos(3y))'_y = -9 \sin(3y)$$

注 此例成立 $z_{xy} = z_{yx}$

例 2 求 $z = x \sin(3y)$ 全部二阶偏导数及 z_{xyy}

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$$z_{yy} = (3x \cos(3y))'_y = -9x \sin(3y)$$

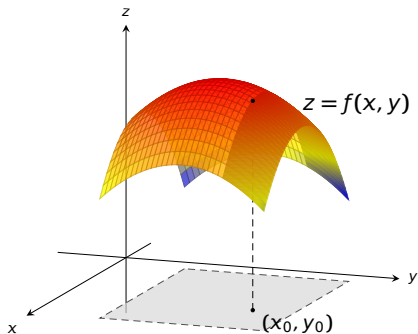
$$z_{xyy} = (3 \cos(3y))'_y = -9 \sin(3y)$$

注 此例成立 $z_{xy} = z_{yx}$

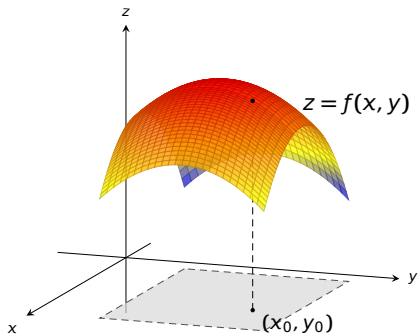
性质 设有二元函数 $z = f(x, y)$ 。若 $\frac{\partial^2 z}{\partial y \partial x}$ 和 $\frac{\partial^2 z}{\partial x \partial y}$ 均连续, 则

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$$

偏导数的几何直观

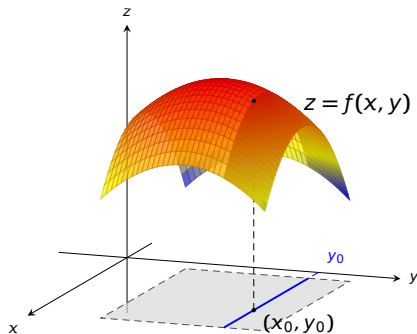


$$\left. \frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \right|_{x=x_0}$$

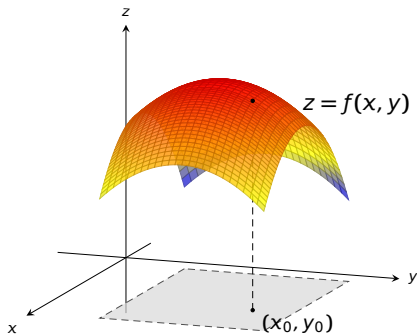


$$\left. \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \right|_{y=y_0}$$

偏导数的几何直观

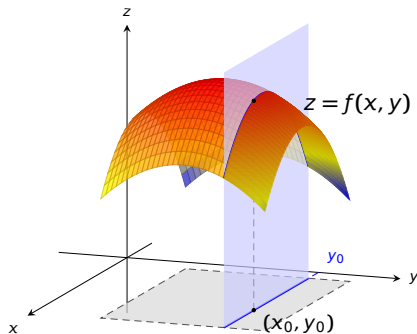


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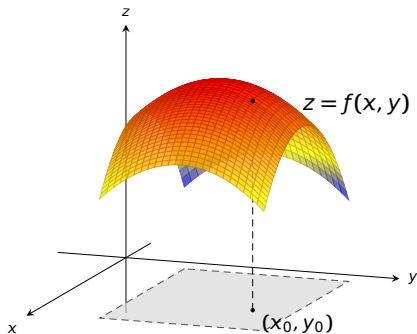


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偏导数的几何直观

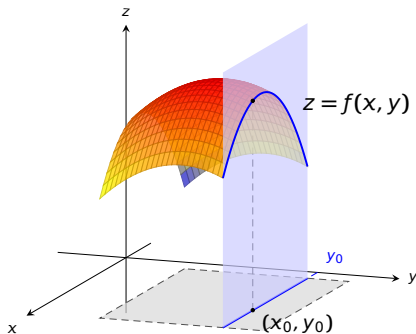


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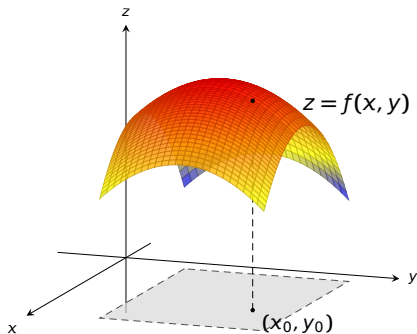


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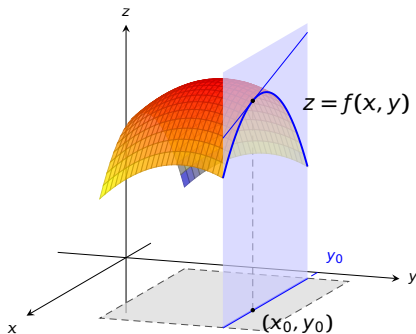


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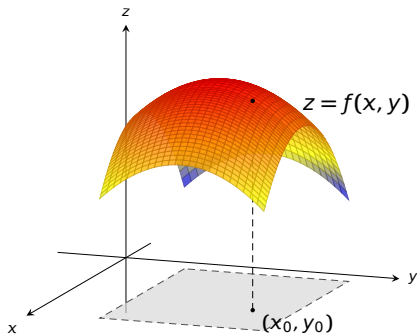


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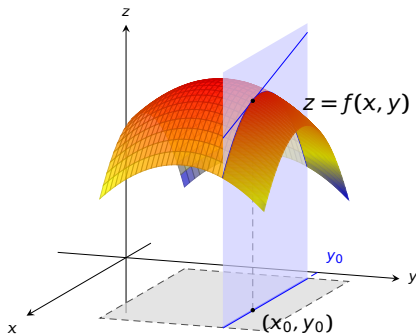


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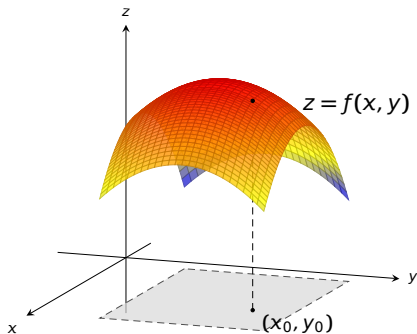


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偏导数的几何直观

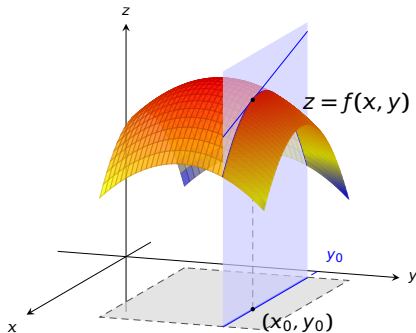


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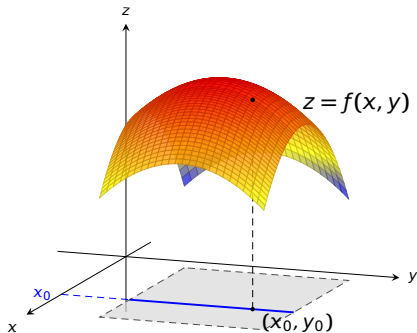


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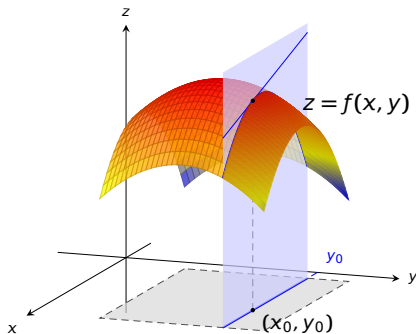


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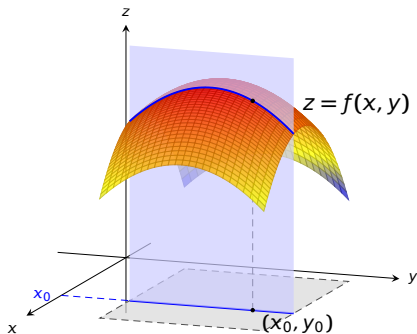


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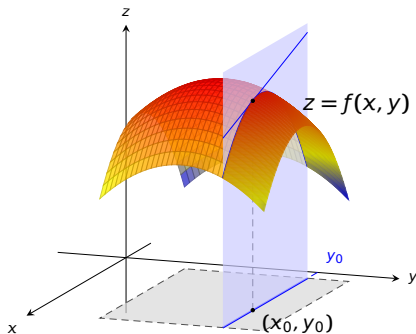


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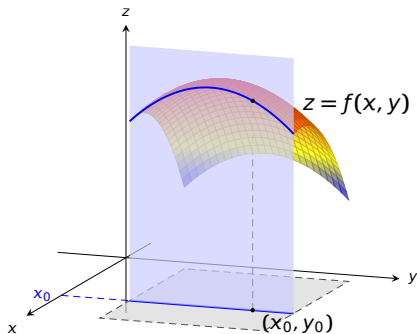


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偏导数的几何直观

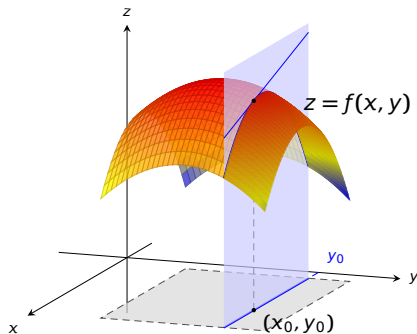


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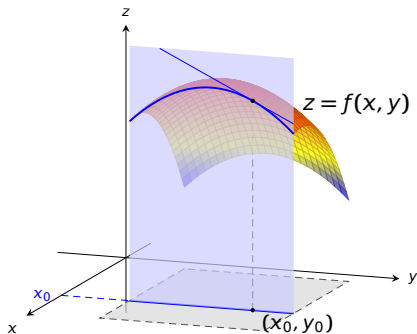


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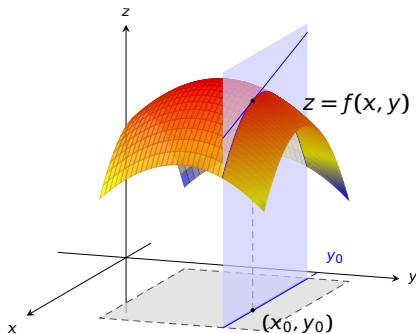


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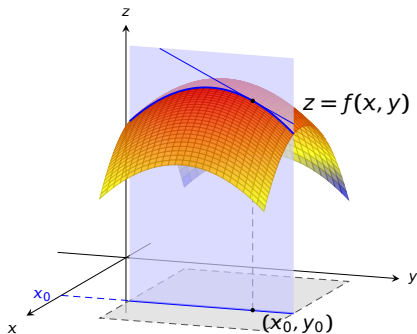


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We are here now...

1. 偏导数

2. 全微分

可微

- 回忆：一元函数 $z = f(x)$ 在 $x = x_0$ 处可微，指

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$$\Delta z = f(x_0 + \Delta x) - f(x_0)$$

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- 回忆：一元函数 $z = f(x)$ 在 $x = x_0$ 处可微，指
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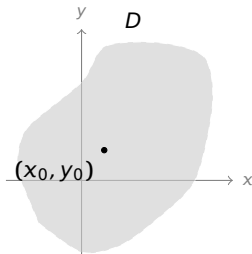
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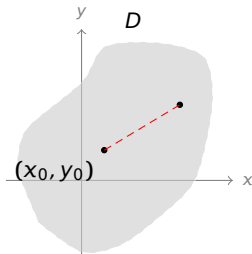


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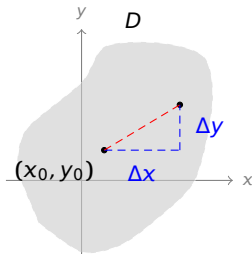


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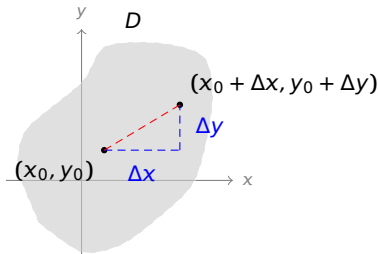


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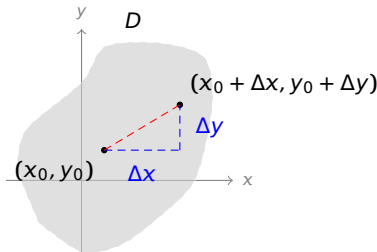


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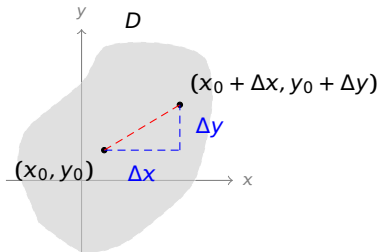
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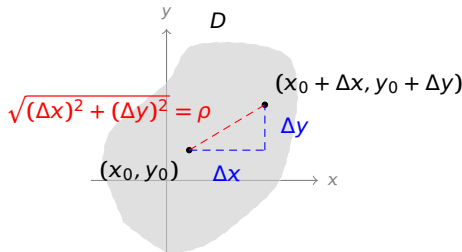
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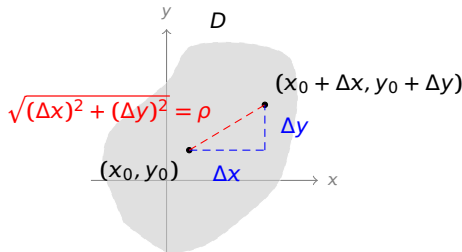
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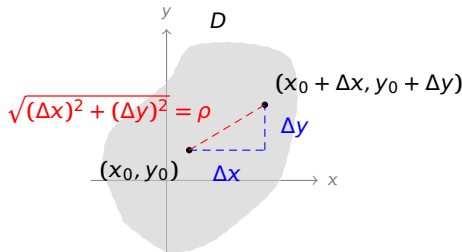
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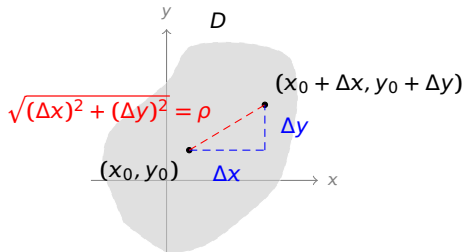
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解
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$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

$$= [(x + \Delta x)^2 + (y + \Delta y)^2] - [x^2 + y^2]$$

$$= 2x\Delta x + 2y\Delta y + [(\Delta x)^2 + (\Delta y)^2]$$

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多元函数的全微分

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全微分在近似计算中的应用

设 $z = f(x, y)$, 则 $f(x + \Delta x, y + \Delta y) - f(x, y) = dz + o(\rho) \approx dz$

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所以 $(1.04)^{2.02} \approx dz + 1 = 0.08 + 1 = 1.08$

可微、偏导数存在、连续的区别与联系

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- 在点 (x_0, y_0) 附近存在偏导数 $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$, 且偏导数 $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ 在点 (x_0, y_0) 处连续 \Rightarrow 在点 (x_0, y_0) 处可微