

第 2 章 e: 分块矩阵

数学系 梁卓滨

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分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & \begin{matrix} 3 \\ -1 \\ 0 \end{matrix} \\ \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} & 1 \end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & \\ O & \end{pmatrix}$$

分块矩阵引入

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$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & \\ O & I_1 \end{pmatrix}$$

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$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix}$$

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$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix}$$
$$\equiv_{\text{or}} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix}$$
$$\stackrel{\text{or}}{=} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix}$$
$$\stackrel{\text{or}}{=} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_2 & & \\ & I_2 & \end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix}$$
$$\stackrel{\text{or}}{=} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_2 & \\ & I_2 \end{pmatrix}$$

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$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix}$$
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$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix}$$
$$\stackrel{\text{or}}{=} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_2 & A_2 \\ O & I_2 \end{pmatrix}$$

分块矩阵引入

- 矩阵

$$\begin{aligned} A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} &= \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix} \\ &\equiv \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_2 & A_2 \\ O & I_2 \end{pmatrix} \\ &\equiv \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

分块矩阵引入

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$$\begin{aligned} A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} &= \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix} \\ &\stackrel{\text{or}}{=} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_2 & A_2 \\ O & I_2 \end{pmatrix} \\ &\stackrel{\text{or}}{=} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = (\quad) \end{aligned}$$

分块矩阵引入

- 矩阵

$$\begin{aligned} A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} &= \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix} \\ &\stackrel{\text{or}}{=} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_2 & A_2 \\ O & I_2 \end{pmatrix} \\ &\stackrel{\text{or}}{=} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \varepsilon_1 & & & \end{pmatrix} \end{aligned}$$

分块矩阵引入

- 矩阵

$$\begin{aligned} A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} &= \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix} \\ &\stackrel{\text{or}}{=} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_2 & A_2 \\ O & I_2 \end{pmatrix} \\ &\stackrel{\text{or}}{=} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 \end{pmatrix} \end{aligned}$$

分块矩阵引入

- 矩阵

$$\begin{aligned} A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} &= \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix} \\ &\stackrel{\text{or}}{=} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_2 & A_2 \\ O & I_2 \end{pmatrix} \\ &\stackrel{\text{or}}{=} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \end{pmatrix} \end{aligned}$$

分块矩阵引入

- 矩阵

$$\begin{aligned} A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} &= \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix} \\ &\stackrel{\text{or}}{=} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} I_2 & A_2 \\ O & I_2 \end{pmatrix} \\ &\stackrel{\text{or}}{=} \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = (\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_3 \quad \alpha) \end{aligned}$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$A = \begin{pmatrix} * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \end{pmatrix}$$

分块矩阵

- 一般地, 可将任意矩阵 A 作分割成若干子矩阵, 例如

$$A = \begin{pmatrix} * & * & | & * & * & * & | & \cdots & | & * \\ * & * & | & * & * & * & | & \cdots & | & * \\ * & * & | & * & * & * & | & \cdots & | & * \\ \hline * & * & | & * & * & * & | & \cdots & | & * \\ \hline \vdots & \vdots & | & \vdots & \vdots & \vdots & | & \ddots & | & \vdots \\ \hline * & * & | & * & * & * & | & \cdots & | & * \\ * & * & | & * & * & * & | & \cdots & | & * \end{pmatrix}$$

分块矩阵

- 一般地, 可将任意矩阵 A 作分割成若干子矩阵, 例如

$$A = \left(\begin{array}{cc|cc|c|ccc} * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ \hline * & * & * & * & * & \cdots & * \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \hline * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \end{array} \right) = \left(\begin{array}{c} \end{array} \right)$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$A = \begin{pmatrix} * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \end{pmatrix} = \begin{pmatrix} A_{11} & & \end{pmatrix}$$

分块矩阵

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$$A = \begin{pmatrix} * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & & \\ & & & \end{pmatrix}$$

分块矩阵

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$$A = \left(\begin{array}{cc|cc|c|ccc|c} * & * & * & * & * & \cdots & * & \\ * & * & * & * & * & \cdots & * & \\ * & * & * & * & * & \cdots & * & \\ \hline * & * & * & * & * & \cdots & * & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \\ \hline * & * & * & * & * & \cdots & * & \\ * & * & * & * & * & \cdots & * & \end{array} \right) = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1n} \end{pmatrix}$$

分块矩阵

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$$A = \left(\begin{array}{cc|cc|c|ccc|c} * & * & * & * & * & \cdots & * & \\ * & * & * & * & * & \cdots & * & \\ * & * & * & * & * & \cdots & * & \\ \hline * & * & * & * & * & \cdots & * & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \\ \hline * & * & * & * & * & \cdots & * & \\ * & * & * & * & * & \cdots & * & \end{array} \right) = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \end{pmatrix}$$

分块矩阵

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$$A = \left(\begin{array}{cc|cc|c|ccc|c} * & * & * & * & * & \cdots & * & \\ * & * & * & * & * & \cdots & * & \\ * & * & * & * & * & \cdots & * & \\ \hline * & * & * & * & * & \cdots & * & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \\ \hline * & * & * & * & * & \cdots & * & \\ * & * & * & * & * & \cdots & * & \end{array} \right) = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix}$$

分块矩阵

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分块矩阵

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$$A = \left(\begin{array}{cc|cc|c|ccc} * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ \hline * & * & * & * & * & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \hline * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \end{array} \right) = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} = (A_{pq})$$

称为**分块矩阵**。

分块矩阵

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称为**分块矩阵**。

- 分块矩阵中
 - 同一行的每个子块有相同行数;
 - 同一列的每个子块有相同列数。

分块矩阵的运算：加法

假设矩阵 A, B 同型，且采取相同分块方式：

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{st} \end{pmatrix}$$

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$$A + B =$$

分块矩阵的运算：加法

假设矩阵 A, B 同型，且采取相同分块方式：

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{st} \end{pmatrix}$$

$$A + B = \begin{pmatrix} A_{11} + B_{11} & A_{12} + B_{12} & \cdots & A_{1t} + B_{1t} \\ A_{21} + B_{21} & A_{22} + B_{22} & \cdots & A_{2t} + B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} + B_{s1} & A_{s2} + B_{s2} & \cdots & A_{st} + B_{st} \end{pmatrix}$$

分块矩阵的运算：加法

假设矩阵 A, B 同型，且采取相同分块方式：

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} = (A_{pq}), \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{st} \end{pmatrix}$$

则

$$A + B = \begin{pmatrix} A_{11} + B_{11} & A_{12} + B_{12} & \cdots & A_{1t} + B_{1t} \\ A_{21} + B_{21} & A_{22} + B_{22} & \cdots & A_{2t} + B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} + B_{s1} & A_{s2} + B_{s2} & \cdots & A_{st} + B_{st} \end{pmatrix}$$

分块矩阵的运算：加法

假设矩阵 A, B 同型，且采取相同分块方式：

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} = (A_{pq}), \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{st} \end{pmatrix} = (B_{pq})$$

则

$$A + B = \begin{pmatrix} A_{11} + B_{11} & A_{12} + B_{12} & \cdots & A_{1t} + B_{1t} \\ A_{21} + B_{21} & A_{22} + B_{22} & \cdots & A_{2t} + B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} + B_{s1} & A_{s2} + B_{s2} & \cdots & A_{st} + B_{st} \end{pmatrix}$$

分块矩阵的运算：加法

假设矩阵 A, B 同型，且采取相同分块方式：

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} = (A_{pq}), \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{st} \end{pmatrix} = (B_{pq})$$

则

$$A + B = \begin{pmatrix} A_{11} + B_{11} & A_{12} + B_{12} & \cdots & A_{1t} + B_{1t} \\ A_{21} + B_{21} & A_{22} + B_{22} & \cdots & A_{2t} + B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} + B_{s1} & A_{s2} + B_{s2} & \cdots & A_{st} + B_{st} \end{pmatrix} = (A_{pq} + B_{pq})$$

分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$

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$$A + B = \left(\begin{array}{cc|cc} \color{red}{1} & \color{red}{0} & 1 & 3 \\ \color{red}{0} & \color{red}{1} & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) + \left(\begin{array}{cc|cc} \color{red}{1} & \color{red}{2} & 0 & 0 \\ \color{red}{2} & \color{red}{0} & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array} \right) = \left(\begin{array}{cc|cc} & & & \\ & & & \\ \hline & & & \\ & & & \end{array} \right)$$

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分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \Rightarrow kA =$$

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例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \Rightarrow kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right)$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \Rightarrow kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \Rightarrow kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, 则

$$kA =$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \Rightarrow kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, 则

$$kA = k \begin{pmatrix} I & C \\ O & -I \end{pmatrix} =$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \Rightarrow kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, 则

$$kA = k \begin{pmatrix} I & C \\ O & -I \end{pmatrix} = \begin{pmatrix} kI & kC \\ kO & -kI \end{pmatrix} =$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \Rightarrow kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, 则

$$kA = k \begin{pmatrix} I & C \\ O & -I \end{pmatrix} = \begin{pmatrix} kI & kC \\ kO & -kI \end{pmatrix} = \left(\begin{array}{cc|cc} & & & \\ & & & \\ \hline & & & \\ & & & \end{array} \right)$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \Rightarrow kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, 则

$$kA = k \begin{pmatrix} I & C \\ O & -I \end{pmatrix} = \begin{pmatrix} kI & kC \\ kO & -kI \end{pmatrix} = \left(\begin{array}{cc|cc} k & 0 & & \\ 0 & k & & \\ \hline & & & \end{array} \right)$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \Rightarrow kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, 则

$$kA = k \begin{pmatrix} I & C \\ O & -I \end{pmatrix} = \begin{pmatrix} kI & kC \\ kO & -kI \end{pmatrix} = \left(\begin{array}{cc|cc} k & 0 & k & 3k \\ 0 & k & 2k & 4k \\ \hline 0 & 0 & -k & 0 \\ 0 & 0 & 0 & -k \end{array} \right)$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \Rightarrow kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, 则

$$kA = k \begin{pmatrix} I & C \\ O & -I \end{pmatrix} = \begin{pmatrix} kI & kC \\ kO & -kI \end{pmatrix} = \left(\begin{array}{cc|cc} k & 0 & k & 3k \\ 0 & k & 2k & 4k \\ \hline 0 & 0 & -k & 0 \\ 0 & 0 & 0 & -k \end{array} \right)$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \Rightarrow kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, 则

$$kA = k \begin{pmatrix} I & C \\ O & -I \end{pmatrix} = \begin{pmatrix} kI & kC \\ kO & -kI \end{pmatrix} = \left(\begin{array}{cc|cc} k & 0 & k & 3k \\ 0 & k & 2k & 4k \\ \hline 0 & 0 & -k & 0 \\ 0 & 0 & 0 & -k \end{array} \right)$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix}$$

满足： A 的列划分与 B 的行划分方式相同。

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} \overbrace{A_{11} A_{12} \cdots A_{1r}}^{n_1} & \overbrace{A_{12} A_{22} \cdots A_{2r}}^{n_2} & \cdots & \overbrace{A_{1r} A_{2r} \cdots A_{sr}}^{n_r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix}$$

满足： A 的列划分与 B 的行划分方式相同。

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} \overbrace{A_{11} A_{12} \cdots A_{1r}}^{n_1} & \overbrace{A_{21} A_{22} \cdots A_{2r}}^{n_2} & \cdots & \overbrace{A_{s1} A_{s2} \cdots A_{sr}}^{n_r} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix} \begin{matrix} \left[\begin{matrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{matrix} \right] \end{matrix}$$

满足： A 的列划分与 B 的行划分方式相同。

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} \overbrace{A_{11} A_{12} \cdots A_{1r}}^{n_1} & \overbrace{A_{12} A_{22} \cdots A_{2r}}^{n_2} & \overbrace{A_{1r} A_{2r} \cdots A_{sr}}^{n_r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix} \begin{matrix} \left[\begin{matrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{matrix} \right] \end{matrix}$$

满足： A 的列划分与 B 的行划分方式相同。则

$$AB = C =$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} \overbrace{A_{11} A_{12} \cdots A_{1r}}^{n_1} & \overbrace{A_{12} A_{22} \cdots A_{2r}}^{n_2} & \overbrace{A_{1r} A_{2r} \cdots A_{sr}}^{n_r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix} \begin{matrix} \left[\begin{matrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{matrix} \right] \end{matrix}$$

满足： A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} \overbrace{A_{11} A_{12} \cdots A_{1r}}^{n_1} & \overbrace{A_{21} A_{22} \cdots A_{2r}}^{n_2} & \cdots & \overbrace{A_{s1} A_{s2} \cdots A_{sr}}^{n_r} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix} \begin{matrix} \left[\begin{matrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{matrix} \right] \end{matrix}$$

满足： A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中

$$C_{pq} = \sum_{k=1}^r A_{pk} B_{kq}$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} \overbrace{A_{11} A_{12} \cdots A_{1r}}^{n_1} & \overbrace{A_{12} A_{22} \cdots A_{2r}}^{n_2} & \cdots & \overbrace{A_{1r} A_{2r} \cdots A_{sr}}^{n_r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix}$$

$\left. \begin{matrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{matrix} \right\} n$

满足： A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中

$$C_{pq} = A_{p1} \quad A_{p2} \quad \cdots \quad A_{pr} \quad .$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} \overbrace{A_{11} A_{12} \cdots A_{1r}}^{n_1} & \overbrace{A_{21} A_{22} \cdots A_{2r}}^{n_2} & \cdots & \overbrace{A_{s1} A_{s2} \cdots A_{sr}}^{n_r} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix} \begin{matrix} \left[\begin{matrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{matrix} \right] \end{matrix}$$

满足： A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中

$$C_{pq} = A_{p1}B_{1q} \quad A_{p2}B_{2q} \quad \cdots \quad A_{pr}B_{rq}.$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} \overbrace{A_{11} A_{12} \cdots A_{1r}}^{n_1} & \overbrace{A_{12} A_{22} \cdots A_{2r}}^{n_2} & \cdots & \overbrace{A_{1r} A_{2r} \cdots A_{sr}}^{n_r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix}$$

$\left. \begin{matrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{matrix} \right\} n$

满足： A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中

$$C_{pq} = A_{p1}B_{1q} + A_{p2}B_{2q} + \cdots + A_{pr}B_{rq}.$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} \overbrace{A_{11} A_{12} \cdots A_{1r}}^{n_1} & \overbrace{A_{21} A_{22} \cdots A_{2r}}^{n_2} & \cdots & \overbrace{A_{s1} A_{s2} \cdots A_{sr}}^{n_r} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix} \begin{matrix} \left[\begin{matrix} n_1 \\ n_2 \\ \vdots \\ n_r \end{matrix} \right] \end{matrix}$$

满足： A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中（必然每个子块的乘积有意义）

$$C_{pq} = A_{p1}B_{1q} + A_{p2}B_{2q} + \cdots + A_{pr}B_{rq}.$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} \overbrace{A_{11} A_{12} \cdots A_{1r}}^{n_1} & \overbrace{A_{21} A_{22} \cdots A_{2r}}^{n_2} & \overbrace{A_{s1} A_{s2} \cdots A_{sr}}^{n_r} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} B_{12} \cdots B_{1t} \\ B_{21} B_{22} \cdots B_{2t} \\ \vdots \quad \vdots \quad \ddots \quad \vdots \\ B_{r1} B_{r2} \cdots B_{rt} \end{pmatrix} \begin{matrix} \left[\right. n_1 \\ \left[\right. n_2 \\ \left[\right. n_r \end{matrix}$$

满足： A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中（必然每个子块的乘积有意义）

$$C_{pq} = A_{p1}B_{1q} + A_{p2}B_{2q} + \cdots + A_{pr}B_{rq}.$$

$$\begin{pmatrix} A_{11} & \cdots & A_{1r} \\ \vdots & & \vdots \\ A_{p1} & \cdots & A_{pr} \\ \vdots & & \vdots \\ A_{s1} & \cdots & A_{sr} \end{pmatrix} \cdot \begin{pmatrix} B_{11} & \cdots & B_{1q} & \cdots & B_{1t} \\ \vdots & & \vdots & & \vdots \\ B_{r1} & \cdots & B_{rq} & \cdots & B_{rt} \end{pmatrix} = \begin{pmatrix} C_{11} & \cdots & \cdots & C_{1t} \\ \vdots & & \vdots & \vdots \\ & \cdots & C_{pq} & \cdots \\ \vdots & & \vdots & \vdots \\ C_{s1} & \cdots & \cdots & C_{st} \end{pmatrix}$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m \times l}$, $B_{l \times n}$ 分块为

$$A = \begin{pmatrix} \overbrace{A_{11} A_{12} \cdots A_{1r}}^{n_1} & \overbrace{A_{21} A_{22} \cdots A_{2r}}^{n_2} & \cdots & \overbrace{A_{s1} A_{s2} \cdots A_{sr}}^{n_r} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} B_{12} \cdots B_{1t} \\ B_{21} B_{22} \cdots B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} B_{r2} \cdots B_{rt} \end{pmatrix} \begin{matrix} \left. \begin{matrix} \\ \\ \end{matrix} \right] n_1 \\ \left. \begin{matrix} \\ \\ \end{matrix} \right] n_2 \\ \left. \begin{matrix} \\ \\ \end{matrix} \right] n_r \end{matrix}$$

满足： A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中（必然每个子块的乘积有意义）

$$C_{pq} = A_{p1}B_{1q} + A_{p2}B_{2q} + \cdots + A_{pr}B_{rq}.$$

$$\begin{pmatrix} A_{11} \cdots A_{1r} \\ \vdots \\ A_{p1} \cdots A_{pr} \\ \vdots \\ A_{s1} \cdots A_{sr} \end{pmatrix} \cdot \begin{pmatrix} B_{11} \cdots B_{1t} \\ \vdots \\ B_{r1} \cdots B_{rt} \end{pmatrix} = \begin{pmatrix} C_{11} \cdots C_{1t} \\ \vdots \\ C_{pq} \cdots \\ \vdots \\ C_{s1} \cdots C_{st} \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$

$$AB =$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$

$$AB =$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array} \right) = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

$$AB =$$

分块矩阵的运算：乘积

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(验证: A 的列划分与 B 的行划分方式相同) 则

$$AB =$$

分块矩阵的运算：乘积

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(验证: A 的列划分与 B 的行划分方式相同) 则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} =$$

分块矩阵的运算：乘积

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(验证: A 的列划分与 B 的行划分方式相同) 则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array} \right) = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

(验证: A 的列划分与 B 的行划分方式相同) 则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & \\ & \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array} \right) = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

(验证: A 的列划分与 B 的行划分方式相同) 则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ & \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array} \right) = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

(验证: A 的列划分与 B 的行划分方式相同) 则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array} \right) = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

(验证: A 的列划分与 B 的行划分方式相同) 则

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分块矩阵的运算：乘积

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(验证: A 的列划分与 B 的行划分方式相同) 则

$$\begin{aligned} AB &= \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & OO + (-I)I \end{pmatrix} \\ &= \begin{pmatrix} & \end{pmatrix} \end{aligned}$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array} \right) = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

(验证: A 的列划分与 B 的行划分方式相同) 则

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分块矩阵的运算：乘积

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分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array} \right) = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

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分块矩阵的运算：乘积

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分块矩阵的运算：乘积

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分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array} \right) = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

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其中

$$D + CF =$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array} \right) = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

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其中

$$D + CF = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 6 & 3 \\ 0 & -2 \end{pmatrix} =$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array} \right) = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

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其中

$$D + CF = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 6 & 3 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 6 & -3 \\ 12 & -2 \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array} \right) = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

(验证: A 的列划分与 B 的行划分方式相同) 则

$$\begin{aligned} AB &= \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & OO + (-I)I \end{pmatrix} \\ &= \begin{pmatrix} ID + CF & C \\ -F & -I \end{pmatrix} = \left(\begin{array}{cc|cc} & & 1 & 3 \\ & & 2 & 4 \\ \hline -6 & -3 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{array} \right) \end{aligned}$$

其中

$$D + CF = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 6 & 3 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 6 & -3 \\ 12 & -2 \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array} \right) = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

(验证: A 的列划分与 B 的行划分方式相同) 则

$$\begin{aligned} AB &= \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & OO + (-I)I \end{pmatrix} \\ &= \begin{pmatrix} ID + CF & C \\ -F & -I \end{pmatrix} = \left(\begin{array}{cc|cc} & & 1 & 3 \\ & & 2 & 4 \\ \hline -6 & -3 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{array} \right) \end{aligned}$$

其中

$$D + CF = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 6 & 3 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 6 & -3 \\ 12 & -2 \end{pmatrix} = \begin{pmatrix} 7 & -1 \\ 14 & -2 \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array} \right) = \begin{pmatrix} D & O \\ F & I \end{pmatrix}$

(验证: A 的列划分与 B 的行划分方式相同) 则

$$\begin{aligned} AB &= \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & OO + (-I)I \end{pmatrix} \\ &= \begin{pmatrix} ID + CF & C \\ -F & -I \end{pmatrix} = \left(\begin{array}{cc|cc} 7 & -1 & 1 & 3 \\ -14 & -2 & 2 & 4 \\ \hline -6 & -3 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{array} \right) \end{aligned}$$

其中

$$D + CF = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 6 & 3 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 6 & -3 \\ 12 & -2 \end{pmatrix} = \begin{pmatrix} 7 & -1 \\ 14 & -2 \end{pmatrix}$$

分块矩阵的运算：乘积

例 2 设 $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{pmatrix}$

, $B = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$AB =$$

分块矩阵的运算：乘积

例 2 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array} \right) = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$AB =$$

分块矩阵的运算：乘积

例 2 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array} \right) = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

$$AB =$$

分块矩阵的运算：乘积

例 2 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array} \right) = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

(验证: A 的列划分与 B 的行划分方式相同) 则

$$AB =$$

分块矩阵的运算：乘积

例 2 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array} \right) = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

(验证: A 的列划分与 B 的行划分方式相同) 则

$$AB = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix} \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix} =$$

分块矩阵的运算：乘积

例 2 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array} \right) = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

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$$AB = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix} \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$$

分块矩阵的运算：乘积

例 2 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array} \right) = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

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分块矩阵的运算：乘积

例 2 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array} \right) = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

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分块矩阵的运算：乘积

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分块矩阵的运算：乘积

例 2 设 $A = \begin{pmatrix} 1 & 0 & | & 0 & 0 \\ 0 & 1 & | & 0 & 0 \\ 1 & 3 & | & 2 & 0 \\ 5 & 2 & | & 0 & 2 \end{pmatrix} = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & | & 2 & 1 \\ 0 & -1 & | & 3 & 4 \\ 0 & 0 & | & 1 & 0 \\ 0 & 0 & | & 0 & 1 \end{pmatrix} = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

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分块矩阵的运算：乘积

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分块矩阵的运算：乘积

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$$\begin{aligned} AB &= \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix} \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix} = \begin{pmatrix} I(-I) + OO & IB_1 + OI \\ A_1(-I) + 2IO & A_1B_1 + 2I \end{pmatrix} \\ &= \begin{pmatrix} -I & \\ & \end{pmatrix} \end{aligned}$$

分块矩阵的运算：乘积

例 2 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array} \right) = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

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分块矩阵的运算：乘积

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$$\begin{aligned} AB &= \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix} \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix} = \begin{pmatrix} I(-I) + OO & IB_1 + OI \\ A_1(-I) + 2IO & A_1B_1 + 2I \end{pmatrix} \\ &= \begin{pmatrix} -I & B_1 \\ -A_1 & \end{pmatrix} \end{aligned}$$

分块矩阵的运算：乘积

例 2 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array} \right) = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

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分块矩阵的运算：乘积

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分块矩阵的运算：乘积

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分块矩阵的运算：乘积

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分块矩阵的运算：乘积

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其中

$$A_1B_1 + 2I =$$

分块矩阵的运算：乘积

例 2 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array} \right) = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right) = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

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其中

$$A_1B_1 + 2I = \begin{pmatrix} 1 & 3 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} =$$

分块矩阵的运算：乘积

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其中

$$A_1B_1 + 2I = \begin{pmatrix} 1 & 3 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 11 & 13 \\ 16 & 13 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

分块矩阵的运算：乘积

例 2 设 $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

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其中

$$A_1B_1 + 2I = \begin{pmatrix} 1 & 3 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 11 & 13 \\ 16 & 13 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 13 & 13 \\ 16 & 15 \end{pmatrix}$$

分块矩阵的运算：乘积

例2 设 $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}$

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$$\begin{aligned} AB &= \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix} \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix} = \begin{pmatrix} I(-I) + OO & IB_1 + OI \\ A_1(-I) + 2IO & A_1B_1 + 2I \end{pmatrix} \\ &= \begin{pmatrix} -I & B_1 \\ -A_1 & A_1B_1 + 2I \end{pmatrix} = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ -1 & -3 & 13 & 13 \\ -5 & -2 & 16 & 15 \end{pmatrix} \end{aligned}$$

其中

$$A_1B_1 + 2I = \begin{pmatrix} 1 & 3 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 11 & 13 \\ 16 & 13 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 13 & 13 \\ 16 & 15 \end{pmatrix}$$

例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$$

例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} OO + AA^* & \\ & \end{pmatrix}$$

例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix}$$

例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & \end{pmatrix}$$

例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix}$$

例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{aligned} \begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} &= \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix} \\ &= \begin{pmatrix} & \\ & \end{pmatrix} \end{aligned}$$

例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{aligned} \begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} &= \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix} \\ &= \begin{pmatrix} AA^* & \\ & \end{pmatrix} \end{aligned}$$

例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{aligned} \begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} &= \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix} \\ &= \begin{pmatrix} AA^* & O \\ O & BB^* \end{pmatrix} \end{aligned}$$

例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{aligned} \begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} &= \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix} \\ &= \begin{pmatrix} AA^* & O \\ O & O \end{pmatrix} \end{aligned}$$

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并计算该乘积的行列式。

解

$$\begin{aligned} \begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} &= \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix} \\ &= \begin{pmatrix} AA^* & O \\ O & BB^* \end{pmatrix} \end{aligned}$$

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并计算该乘积的行列式。

解

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例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

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$$\begin{aligned} \begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} &= \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix} \\ &= \begin{pmatrix} AA^* & O \\ O & BB^* \end{pmatrix} = \begin{pmatrix} |A|I_2 & O \\ O & \end{pmatrix} \end{aligned}$$

例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{aligned} \begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} &= \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix} \\ &= \begin{pmatrix} AA^* & O \\ O & BB^* \end{pmatrix} = \begin{pmatrix} |A|I_2 & O \\ O & |B|I_2 \end{pmatrix} \end{aligned}$$

例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{aligned} \begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} &= \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix} \\ &= \begin{pmatrix} AA^* & O \\ O & BB^* \end{pmatrix} = \begin{pmatrix} |A|I_2 & O \\ O & |B|I_2 \end{pmatrix} = \begin{pmatrix} 2I_2 & O \\ O & 3I_2 \end{pmatrix} = \end{aligned}$$

例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{aligned} \begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} &= \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix} \\ &= \begin{pmatrix} AA^* & O \\ O & BB^* \end{pmatrix} = \begin{pmatrix} |A|I_2 & O \\ O & |B|I_2 \end{pmatrix} = \begin{pmatrix} 2I_2 & O \\ O & 3I_2 \end{pmatrix} = \begin{pmatrix} 2 & & & \\ & 2 & & \\ & & 3 & \\ & & & 3 \end{pmatrix} \end{aligned}$$

例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{aligned} \begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} &= \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix} \\ &= \begin{pmatrix} AA^* & O \\ O & BB^* \end{pmatrix} = \begin{pmatrix} |A|I_2 & O \\ O & |B|I_2 \end{pmatrix} = \begin{pmatrix} 2I_2 & O \\ O & 3I_2 \end{pmatrix} = \begin{pmatrix} 2 & & & \\ & 2 & & \\ & & 3 & \\ & & & 3 \end{pmatrix} \end{aligned}$$

$$\text{所以 } \left| \begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} \right| = \begin{vmatrix} 2 & & & \\ & 2 & & \\ & & 3 & \\ & & & 3 \end{vmatrix} =$$

例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{aligned} \begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} &= \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix} \\ &= \begin{pmatrix} AA^* & O \\ O & BB^* \end{pmatrix} = \begin{pmatrix} |A|I_2 & O \\ O & |B|I_2 \end{pmatrix} = \begin{pmatrix} 2I_2 & O \\ O & 3I_2 \end{pmatrix} = \left(\begin{array}{cc|cc} 2 & & & \\ & 2 & & \\ \hline & & 3 & \\ & & & 3 \end{array} \right) \end{aligned}$$

$$\text{所以 } \left| \begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} \right| = \begin{vmatrix} 2 & & & \\ & 2 & & \\ & & 3 & \\ & & & 3 \end{vmatrix} = 2 \times 2 \times 3 \times 3 =$$

例 3 设 A, B 均为 2 阶方阵, 且 $|A| = 2, |B| = 3$, 计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{aligned} \begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} &= \begin{pmatrix} OO + AA^* & OB^* + AO \\ BO + OA^* & BB^* + OO \end{pmatrix} \\ &= \begin{pmatrix} AA^* & O \\ O & BB^* \end{pmatrix} = \begin{pmatrix} |A|I_2 & O \\ O & |B|I_2 \end{pmatrix} = \begin{pmatrix} 2I_2 & O \\ O & 3I_2 \end{pmatrix} = \left(\begin{array}{cc|cc} 2 & & & \\ & 2 & & \\ \hline & & 3 & \\ & & & 3 \end{array} \right) \end{aligned}$$

$$\text{所以 } \left| \begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} \right| = \begin{vmatrix} 2 & & & \\ & 2 & & \\ & & 3 & \\ & & & 3 \end{vmatrix} = 2 \times 2 \times 3 \times 3 = 36$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1} 。

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解

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} \quad \quad \quad \\ \quad \quad \quad \end{pmatrix}$$

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解

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵, 证明 D 可逆并求出 D^{-1} 。

解

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix}$$

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其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵, 证明 D 可逆并求出 D^{-1} 。

解 若存在矩阵 W, X, Y, Z 使得

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix}$$

则 D 可逆, 且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$ 。

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

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则 D 可逆, 且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$ 。

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵, 证明 D 可逆并求出 D^{-1} 。

解 若存在矩阵 W, X, Y, Z 使得

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix} = \begin{pmatrix} AX + CW & \\ & \end{pmatrix}$$

则 D 可逆, 且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$ 。

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵, 证明 D 可逆并求出 D^{-1} 。

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$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix} = \begin{pmatrix} AX + CW & AZ + CY \\ O & O \end{pmatrix}$$

则 D 可逆, 且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$ 。

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵, 证明 D 可逆并求出 D^{-1} 。

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则 D 可逆, 且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$ 。

例 4 设分块矩阵

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则 D 可逆, 且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$ 。

例 4 设分块矩阵

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则 D 可逆, 且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$ 。由上式得

$$\begin{cases} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{cases}$$

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则 D 可逆, 且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$ 。由上式得

$$\begin{cases} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{cases} \Rightarrow \begin{cases} \end{cases}$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵, 证明 D 可逆并求出 D^{-1} 。

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则 D 可逆, 且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$ 。由上式得

$$\begin{cases} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{cases} \Rightarrow \begin{cases} \\ \\ \\ Y = B^{-1} \end{cases}$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵, 证明 D 可逆并求出 D^{-1} 。

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$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix} = \begin{pmatrix} AX + CW & AZ + CY \\ BW & BY \end{pmatrix}$$

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$$\begin{cases} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{cases} \Rightarrow \begin{cases} Z = -A^{-1}CY \\ W = O \\ Y = B^{-1} \end{cases}$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵, 证明 D 可逆并求出 D^{-1} 。

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则 D 可逆, 且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$ 。由上式得

$$\begin{cases} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{cases} \Rightarrow \begin{cases} Z = -A^{-1}CY = -A^{-1}CB^{-1} \\ W = O \\ Y = B^{-1} \end{cases}$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵, 证明 D 可逆并求出 D^{-1} 。

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则 D 可逆, 且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$ 。由上式得

$$\begin{cases} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{cases} \Rightarrow \begin{cases} X = A^{-1}(I - CW) \\ Z = -A^{-1}CY = -A^{-1}CB^{-1} \\ W = O \\ Y = B^{-1} \end{cases}$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

其中 $A_{r \times r}$ 和 $B_{k \times k}$ 均为可逆方阵, 证明 D 可逆并求出 D^{-1} 。

解 若存在矩阵 W, X, Y, Z 使得

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix} = \begin{pmatrix} AX + CW & AZ + CY \\ BW & BY \end{pmatrix}$$

则 D 可逆, 且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$ 。由上式得

$$\begin{cases} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{cases} \Rightarrow \begin{cases} X = A^{-1}(I - CW) = A^{-1} \\ Z = -A^{-1}CY = -A^{-1}CB^{-1} \\ W = O \\ Y = B^{-1} \end{cases}$$

例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r \times r} & C \\ O & B_{k \times k} \end{pmatrix}$$

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$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} X_{r \times r} & Z_{r \times k} \\ W_{k \times r} & Y_{k \times k} \end{pmatrix} = \begin{pmatrix} AX + CW & AZ + CY \\ BW & BY \end{pmatrix}$$

则 D 可逆, 且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$ 。由上式得

$$\begin{cases} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{cases} \Rightarrow \begin{cases} X = A^{-1}(I - CW) = A^{-1} \\ Z = -A^{-1}CY = -A^{-1}CB^{-1} \\ W = O \\ Y = B^{-1} \end{cases}$$

所以 D 可逆, 且 $D^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}CB^{-1} \\ O & B^{-1} \end{pmatrix}$

$$A, B \text{ 可逆} \Rightarrow \begin{pmatrix} A & C \\ O & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}CB^{-1} \\ O & B^{-1} \end{pmatrix}$$

$$A, B \text{ 可逆} \Rightarrow \begin{pmatrix} A & C \\ O & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}CB^{-1} \\ O & B^{-1} \end{pmatrix}$$

注 特别地, 当 $C = O$ 时,

$$A, B \text{ 可逆} \Rightarrow \begin{pmatrix} A & C \\ O & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}CB^{-1} \\ O & B^{-1} \end{pmatrix}$$

注 特别地, 当 $C = O$ 时,

$$A, B \text{ 可逆} \Rightarrow \begin{pmatrix} A & O \\ O & B \end{pmatrix}^{-1}$$

$$A, B \text{ 可逆} \Rightarrow \begin{pmatrix} A & C \\ O & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}CB^{-1} \\ O & B^{-1} \end{pmatrix}$$

注 特别地, 当 $C = O$ 时,

$$A, B \text{ 可逆} \Rightarrow \begin{pmatrix} A & O \\ O & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & O \\ O & B^{-1} \end{pmatrix}$$

$$A, B \text{ 可逆} \Rightarrow \begin{pmatrix} A & C \\ O & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}CB^{-1} \\ O & B^{-1} \end{pmatrix}$$

注 特别地, 当 $C = O$ 时,

$$A, B \text{ 可逆} \Rightarrow \begin{pmatrix} A & O \\ O & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & O \\ O & B^{-1} \end{pmatrix}$$

当 $A = I_r, B = I_k$ 时,

$$\begin{pmatrix} I & C \\ O & I \end{pmatrix}^{-1}$$

$$A, B \text{ 可逆} \Rightarrow \begin{pmatrix} A & C \\ O & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}CB^{-1} \\ O & B^{-1} \end{pmatrix}$$

注 特别地, 当 $C = O$ 时,

$$A, B \text{ 可逆} \Rightarrow \begin{pmatrix} A & O \\ O & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & O \\ O & B^{-1} \end{pmatrix}$$

当 $A = I_r, B = I_k$ 时,

$$\begin{pmatrix} I & C \\ O & I \end{pmatrix}^{-1} = \begin{pmatrix} I & -C \\ O & I \end{pmatrix}$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

则

$$AI =$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

则

$$AI =$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = (\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_n)$$

则

$$AI =$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = (\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_n)$$

则

$$AI = A(\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_n) =$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = (\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_n)$$

则

$$AI = A(\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_n) = (A\varepsilon_1 \ A\varepsilon_2 \ \cdots \ A\varepsilon_n)$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = (\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_n)$$

则

$$\begin{aligned} AI &= A(\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_n) = (A\varepsilon_1 \ A\varepsilon_2 \ \cdots \ A\varepsilon_n) \\ &= \begin{pmatrix} \vdots & \vdots & \vdots \end{pmatrix} \end{aligned}$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = (\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_n)$$

则

$$\begin{aligned} AI &= A(\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_n) = (A\varepsilon_1 \ A\varepsilon_2 \ \cdots \ A\varepsilon_n) \\ &= \begin{pmatrix} a_{11} & & \\ a_{21} & & \\ \vdots & & \\ a_{m1} & & \end{pmatrix} \end{aligned}$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = (\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_n)$$

则

$$\begin{aligned} AI &= A(\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_n) = (A\varepsilon_1 \ A\varepsilon_2 \ \cdots \ A\varepsilon_n) \\ &= \begin{pmatrix} a_{11} & a_{12} & \vdots & a_{m1} \\ a_{21} & a_{22} & \vdots & a_{m2} \end{pmatrix} \end{aligned}$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = (\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_n)$$

则

$$\begin{aligned} AI &= A(\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_n) = (A\varepsilon_1 \ A\varepsilon_2 \ \cdots \ A\varepsilon_n) \\ &= \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \end{aligned}$$

分块矩阵的运算：乘积

例 5 设

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix} = (\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_n)$$

则

$$\begin{aligned} AI &= A(\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_n) = (A\varepsilon_1 \ A\varepsilon_2 \ \cdots \ A\varepsilon_n) \\ &= \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} = A \end{aligned}$$

分块矩阵的运算：乘积

例 6 设 $AB = C$ ，则 C 的每一列均为 A 的所有列的线性组合，系数为 B 的列：

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix}$$

则

$$AB =$$

分块矩阵的运算：乘积

例 6 设 $AB = C$ ，则 C 的每一列均为 A 的所有列的线性组合，系数为 B 的列：

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix}$$

则

$$AB =$$

分块矩阵的运算：乘积

例 6 设 $AB = C$ ，则 C 的每一列均为 A 的所有列的线性组合，系数为 B 的列：

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix} = (\beta_1 \ \beta_2 \ \cdots \ \beta_n)$$

则

$$AB =$$

分块矩阵的运算：乘积

例 6 设 $AB = C$ ，则 C 的每一列均为 A 的所有列的线性组合，系数为 B 的列：

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix} = (\beta_1 \ \beta_2 \ \cdots \ \beta_l)$$

则

$$AB = A(\beta_1, \beta_2, \cdots, \beta_l)$$

分块矩阵的运算：乘积

例 6 设 $AB = C$ ，则 C 的每一列均为 A 的所有列的线性组合，系数为 B 的列：

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix} = (\beta_1 \ \beta_2 \ \cdots \ \beta_l)$$

则

$$AB = A(\beta_1, \beta_2, \cdots, \beta_l) = (A\beta_1, A\beta_2, \cdots, A\beta_l)$$

分块矩阵的运算：乘积

例 6 设 $AB = C$ ，则 C 的每一列均为 A 的所有列的线性组合，系数为 B 的列：

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix} = (\beta_1 \ \beta_2 \ \cdots \ \beta_l)$$

则
$$AB = A(\beta_1, \beta_2, \cdots, \beta_l) = (A\beta_1, A\beta_2, \cdots, A\beta_l)$$

而

$$A\beta_i =$$

分块矩阵的运算：乘积

例 6 设 $AB = C$ ，则 C 的每一列均为 A 的所有列的线性组合，系数为 B 的列：

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix} = (\beta_1 \ \beta_2 \ \cdots \ \beta_l)$$

则

$$AB = A(\beta_1, \beta_2, \cdots, \beta_l) = (A\beta_1, A\beta_2, \cdots, A\beta_l)$$

而

$$A\beta_i = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} b_{1i} \\ b_{2i} \\ \vdots \\ b_{ni} \end{pmatrix}$$

分块矩阵的运算：乘积

例 6 设 $AB = C$ ，则 C 的每一列均为 A 的所有列的线性组合，系数为 B 的列：

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix} = (\beta_1 \ \beta_2 \ \cdots \ \beta_n)$$

则
$$AB = A(\beta_1, \beta_2, \cdots, \beta_l) = (A\beta_1, A\beta_2, \cdots, A\beta_l)$$

而
$$A\beta_i = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} b_{1i} \\ b_{2i} \\ \vdots \\ b_{ni} \end{pmatrix} = (\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n)$$

分块矩阵的运算：乘积

例 6 设 $AB = C$ ，则 C 的每一列均为 A 的所有列的线性组合，系数为 B 的列：

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix} = (\beta_1 \ \beta_2 \ \cdots \ \beta_l)$$

则
$$AB = A(\beta_1, \beta_2, \cdots, \beta_l) = (A\beta_1, A\beta_2, \cdots, A\beta_l)$$

而
$$A\beta_i = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} b_{1i} \\ b_{2i} \\ \vdots \\ b_{ni} \end{pmatrix} = (\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n) \begin{pmatrix} b_{1i} \\ b_{2i} \\ \vdots \\ b_{ni} \end{pmatrix}$$

分块矩阵的运算：乘积

例 6 设 $AB = C$ ，则 C 的每一列均为 A 的所有列的线性组合，系数为 B 的列：

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix} = (\beta_1 \ \beta_2 \ \cdots \ \beta_l)$$

则
$$AB = A(\beta_1, \beta_2, \cdots, \beta_l) = (A\beta_1, A\beta_2, \cdots, A\beta_l)$$

而
$$A\beta_i = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} b_{1i} \\ b_{2i} \\ \vdots \\ b_{ni} \end{pmatrix} = (\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n) \begin{pmatrix} b_{1i} \\ b_{2i} \\ \vdots \\ b_{ni} \end{pmatrix}$$

$$= b_{1i}\alpha_1 + b_{2i}\alpha_2 + \cdots + b_{ni}\alpha_n$$

分块矩阵的运算：乘积

例 6 设 $AB = C$ ，则 C 的每一列均为 A 的所有列的线性组合，系数为 B 的列：

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix} = (\beta_1 \ \beta_2 \ \cdots \ \beta_l)$$

则

$$AB = A(\beta_1, \beta_2, \cdots, \beta_l) = (A\beta_1, A\beta_2, \cdots, A\beta_l)$$

而

$$A\beta_i = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} b_{1i} \\ b_{2i} \\ \vdots \\ b_{ni} \end{pmatrix} = (\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n) \begin{pmatrix} b_{1i} \\ b_{2i} \\ \vdots \\ b_{ni} \end{pmatrix}$$

$$= b_{1i}\alpha_1 + b_{2i}\alpha_2 + \cdots + b_{ni}\alpha_n = b_{1i} \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + b_{2i} \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \cdots + b_{ni} \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix}$$