第 9 章 c：多元复合函数的求导法则

数学系　　梁卓滨

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Outline

二元复合函数求导

设有二元函数 $z = f(u, v)$

二元复合函数求导

设有二元函数 $z = f(u, v)$

- 设 $u = \varphi(t)$，$v = \psi(t)$，则 $z = f(\varphi(t), \psi(t))$

问 $\frac{dz}{dt} = ?$

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设有二元函数 $z = f(u, v)$

- 设 $u = \varphi(t)$，$v = \psi(t)$，则 $z = f(\varphi(t), \psi(t))$

$$\begin{tikzcd}[row sep=small] & u \arrow[dr] & \\ z \arrow[ur] \arrow[dr] & & t \\ & v \arrow[ur] & \end{tikzcd}$$

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$$
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\end{tikzcd}
$$

问 $\frac{dz}{dt}=?$

- 设 $u=\varphi(x,y)$，$v=\psi(x,y)$，则 $z=f(\varphi(x,y),\psi(x,y))$

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$$\begin{tikzcd}[row sep=tiny] & u \arrow[r] \arrow[ddr] & x \\ z \arrow[ur] \arrow[dr] & & \\ & v & y \end{tikzcd}$$

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二元复合函数求导公式——中间变量是一元函数

公式 设 $z = f(u, v)$，$u = \varphi(t)$，$v = \psi(t)$，则 $z = f(\varphi(t), \psi(t))$ 的全导数

$$\frac{dz}{dt} =$$

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$$\frac{dz}{dt}=$$

$$\begin{array}{ccccc} & & u & \longrightarrow & t \\ & \nearrow & & & \\ z & & & & \\ & \searrow & & & \\ & & v & \longrightarrow & t \end{array}$$

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$$\begin{tikzcd}
 & u \arrow[r] & t \\
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 & v \arrow[r] & t
\end{tikzcd}$$

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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}$$

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 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[dr, "\frac{\partial z}{\partial v}"'] & & \\
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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}\quad\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$

$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$

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 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[dr, "\frac{\partial z}{\partial v}"'] & & \\
 & v \arrow[r, "\frac{dv}{dt}"'] & t
\end{tikzcd}$$

例 设 $z=uv$，而 $u=e^{-t}$，$v=\sin t$，求全导数 $\frac{dz}{dt}$

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解法一

$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt} + \frac{\partial z}{\partial v} \cdot \frac{dv}{dt}$$
$$=$$

例 设 $z = uv$，而 $u = e^{-t}$，$v = \sin t$，求全导数 $\frac{dz}{dt}$

解法一

$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt} + \frac{\partial z}{\partial v} \cdot \frac{dv}{dt}$$

$$= (uv)'_u \cdot$$

例 设 $z=uv$，而 $u=e^{-t}$，$v=\sin t$，求全导数 $\frac{dz}{dt}$

解法一

$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$
$$=(uv)'_u\cdot(e^{-t})'_t+$$

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$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}\\ &= (uv)'_u\cdot(e^{-t})'_t+(uv)'_v\cdot\end{aligned}$$

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$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial u} \cdot \frac{du}{dt} + \frac{\partial z}{\partial v} \cdot \frac{dv}{dt} \\ &= (uv)'_u \cdot (e^{-t})'_t + (uv)'_v \cdot (\sin t)'_t \\ &= v \cdot\end{aligned}$$

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解法一

$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}\\ &= (uv)'_u\cdot(e^{-t})'_t+(uv)'_v\cdot(\sin t)'_t\\ &= v\cdot(-e^{-t})+\end{aligned}$$

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$$\begin{aligned}\frac{dz}{dt}&=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}\\&=(uv)'_u\cdot(e^{-t})'_t+(uv)'_v\cdot(\sin t)'_t\\&=v\cdot(-e^{-t})+u\cdot\end{aligned}$$

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&=
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&= (uv)'_u \cdot (e^{-t})'_t + (uv)'_v \cdot (\sin t)'_t \\
&= v\cdot(-e^{-t}) + u\cdot\cos t \\
&= \sin t\cdot(-e^{-t}) + e^{-t}\cdot\cos t \\
&= e^{-t}(\cos t - \sin t)
\end{aligned}$$

例 设 $z=uv$，而 $u=e^{-t}$，$v=\sin t$，求全导数 $\frac{dz}{dt}$

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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$
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解法二

$$\because \quad z=uv=$$

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$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial u}\cdot\frac{du}{dt} + \frac{\partial z}{\partial v}\cdot\frac{dv}{dt}\\ &= (uv)'_u\cdot(e^{-t})'_t + (uv)'_v\cdot(\sin t)'_t\\ &= v\cdot(-e^{-t}) + u\cdot\cos t\\ &= \sin t\cdot(-e^{-t}) + e^{-t}\cdot\cos t\\ &= e^{-t}(\cos t - \sin t)\end{aligned}$$

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$$\because \quad z = uv = e^{-t}\cdot\sin t$$

$$\therefore \quad \frac{dz}{dt} = \frac{d}{dt}(e^{-t}\sin t) = (e^{-t})'_t\cdot\sin t +$$

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$$\begin{aligned}\frac{dz}{dt}&=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}\\&=(uv)'_u\cdot(e^{-t})'_t+(uv)'_v\cdot(\sin t)'_t\\&=v\cdot(-e^{-t})+u\cdot\cos t\\&=\sin t\cdot(-e^{-t})+e^{-t}\cdot\cos t\\&=e^{-t}(\cos t-\sin t)\end{aligned}$$

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$$\because\quad z=uv=e^{-t}\cdot\sin t$$

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$$\begin{aligned}\therefore\quad\frac{dz}{dt}&=\frac{d}{dt}(e^{-t}\sin t)=(e^{-t})'_t\cdot\sin t+e^{-t}\cdot(\sin t)'_t\\&=(-e^{-t})\cdot\sin t+e^{-t}\cdot\cos t\end{aligned}$$

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解

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$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=(\frac{y}{x})'_x\cdot$$

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 & x \arrow[r, "\frac{dx}{dt}"] & t \\
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$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=(\frac{y}{x})'_x\cdot(e^t)'_t+$$

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$$=$$

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$$=-\frac{y}{x^2}\cdot$$

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$$=-\frac{y}{x^2}\cdot e^t+$$

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$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=$$

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$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=-\frac{1-e^{2t}}{e^{2t}}\cdot e^t+$$

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解

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=(\frac{y}{x})'_x\cdot(e^t)'_t+(\frac{y}{x})'_y\cdot(1-e^{2t})'_t$$

$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=-\frac{1-e^{2t}}{e^{2t}}\cdot e^t+\frac{1}{e^t}\cdot(-2e^{2t})$$

$$=$$

$$\begin{tikzcd}
 & x \arrow[r, "\frac{dx}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[dr, "\frac{\partial z}{\partial y}"'] & & \\
 & y \arrow[r, "\frac{dy}{dt}"'] & t
\end{tikzcd}$$

例 设 $z=\frac{y}{x}$，而 $x=e^t$，$y=1-e^{2t}$，求全导数 $\frac{dz}{dt}$

解

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=(\frac{y}{x})'_x\cdot(e^t)'_t+(\frac{y}{x})'_y\cdot(1-e^{2t})'_t$$

$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=-\frac{1-e^{2t}}{e^{2t}}\cdot e^t+\frac{1}{e^t}\cdot(-2e^{2t})$$

$$=-e^{-t}-e^t$$

$$\begin{tikzcd}
 & x \arrow[rr, "\frac{dx}{dt}"] & & t \\
z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[dr, "\frac{\partial z}{\partial y}"'] & & & \\
 & y \arrow[rr, "\frac{dy}{dt}"'] & & t
\end{tikzcd}$$

三元复合函数求导公式——中间变量是一元函数

公式 设 $z=f(u,v,w)$，$u=\varphi(t)$，$v=\psi(t)$，$w=\omega(t)$，则 $z=f(\varphi(t),\psi(t),\omega(t))$ 的全导数

$$\frac{dz}{dt}=$$

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$$\frac{dz}{dt}=$$

$$\begin{tikzcd}
 & u \arrow[r] & t \\
z \arrow[ur] \arrow[r] \arrow[dr] & v \arrow[r] & t \\
 & w \arrow[r] & t
\end{tikzcd}$$

三元复合函数求导公式——中间变量是一元函数

公式 设 $z=f(u, v, w)$，$u=\varphi(t)$，$v=\psi(t)$，$w=\omega(t)$，则 $z=f(\varphi(t), \psi(t), \omega(t))$ 的全导数

$$\frac{dz}{dt}=$$

$$\begin{tikzcd}
 & u \arrow[r] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"'] & v \arrow[r] & t \\
 & w \arrow[r] & t
\end{tikzcd}$$

三元复合函数求导公式——中间变量是一元函数

公式 设 $z=f(u,v,w)$，$u=\varphi(t)$，$v=\psi(t)$，$w=\omega(t)$，则 $z=f(\varphi(t),\psi(t),\omega(t))$ 的**全导数**

$$\frac{dz}{dt}=$$

$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"] & v \arrow[r, "\frac{dv}{dt}"] & t \\
 & w \arrow[r, "\frac{dw}{dt}"] & t
\end{tikzcd}$$

三元复合函数求导公式——中间变量是一元函数

公式 设 $z=f(u,v,w)$，$u=\varphi(t)$，$v=\psi(t)$，$w=\omega(t)$，则 $z=f(\varphi(t),\psi(t),\omega(t))$ 的全导数

$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}$$

$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"] & v \arrow[r, "\frac{dv}{dt}"] & t \\
 & w \arrow[r, "\frac{dw}{dt}"] & t
\end{tikzcd}$$

三元复合函数求导公式——中间变量是一元函数

公式 设 $z=f(u,v,w)$，$u=\varphi(t)$，$v=\psi(t)$，$w=\omega(t)$，则 $z=f(\varphi(t),\psi(t),\omega(t))$ 的全导数

$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}\quad\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$

$$\begin{tikzcd} & u \arrow[r, "\frac{du}{dt}"] & t \\ z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"] & v \arrow[r, "\frac{dv}{dt}"] & t \\ & w \arrow[r, "\frac{dw}{dt}"] & t \end{tikzcd}$$

三元复合函数求导公式——中间变量是一元函数

公式 设 $z=f(u,v,w)$，$u=\varphi(t)$，$v=\psi(t)$，$w=\omega(t)$，则 $z=f(\varphi(t),\psi(t),\omega(t))$ 的**全导数**

$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}\quad\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}\quad\frac{\partial z}{\partial w}\cdot\frac{dw}{dt}$$

$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"] & v \arrow[r, "\frac{dv}{dt}"] & t \\
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$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"] & v \arrow[r, "\frac{dv}{dt}"] & t \\
 & w \arrow[r, "\frac{dw}{dt}"] & t
\end{tikzcd}$$

二元复合函数求导公式——中间变量是多元函数

公式 设 $z=f(u,v)$，$u=\varphi(x,y)$，$v=\psi(x,y)$，

二元复合函数求导公式——中间变量是多元函数

公式 设 $z = f(u, v)$，$u = \varphi(x, y)$，$v = \psi(x, y)$，则复合函数

$$z = f(\varphi(x, y), \psi(x, y))$$

的偏导数是:

$$\frac{\partial z}{\partial x} = \qquad\qquad\qquad , \quad \frac{\partial z}{\partial y} =$$

二元复合函数求导公式——中间变量是多元函数

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图示

$$\begin{tikzcd}
 & & x \\
 & u \arrow[ur] \arrow[dr] & \\
 & & y \\
z \arrow[uur] \arrow[ddr] & & \\
 & & x \\
 & v \arrow[ur] \arrow[dr] & \\
 & & y
\end{tikzcd}$$

二元复合函数求导公式——中间变量是多元函数

公式 设 $z = f(u, v)$，$u = \varphi(x, y)$，$v = \psi(x, y)$，则复合函数

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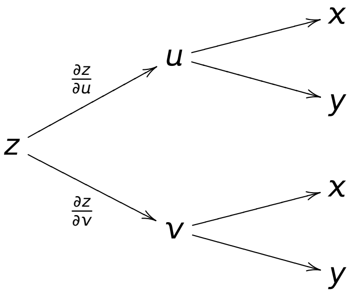
的偏导数是：

$$\frac{\partial z}{\partial x} = \qquad\qquad\qquad\qquad , \quad \frac{\partial z}{\partial y} =$$

图示



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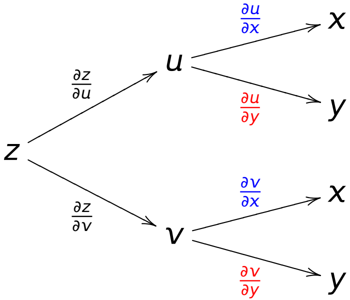
图示



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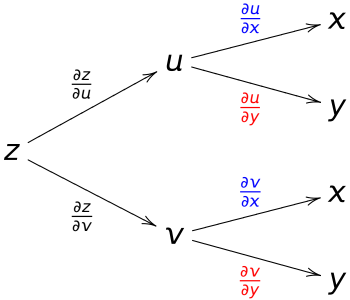
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\qquad\qquad,\quad \frac{\partial z}{\partial y}=$$

图示



二元复合函数求导公式——中间变量是多元函数

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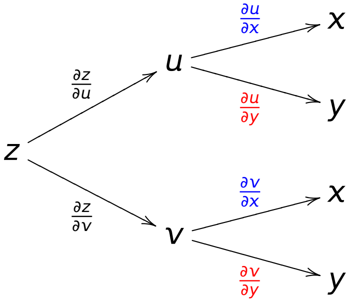
的偏导数是：

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x},\qquad \frac{\partial z}{\partial y}=$$

图示



二元复合函数求导公式——中间变量是多元函数

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的偏导数是：

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x},\qquad \frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+$$

图示

$$\begin{tikzcd}
 & & x \\
 & u \arrow[ur, "\frac{\partial u}{\partial x}"] \arrow[dr, "\frac{\partial u}{\partial y}"'] & \\
 & & y \\
z \arrow[uur, "\frac{\partial z}{\partial u}"] \arrow[ddr, "\frac{\partial z}{\partial v}"'] & & \\
 & & x \\
 & v \arrow[ur, "\frac{\partial v}{\partial x}"] \arrow[dr, "\frac{\partial v}{\partial y}"'] & \\
 & & y
\end{tikzcd}$$

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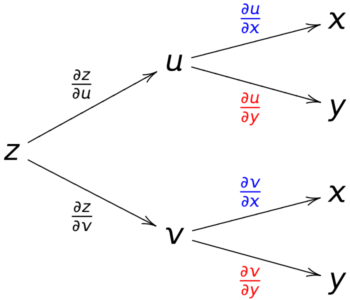
图示



例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$=$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x} \\ &= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x +\end{aligned}$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\ &=\end{aligned}$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$
\begin{aligned}
\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x} \\
&= (e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x \\
&= 2e^{2u}\sin v\cdot
\end{aligned}
$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_x + (e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\ &= 2e^{2u}\sin v\cdot 3x^2y + \end{aligned}$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\begin{aligned}
\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x} \\
&= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot (x^2 + y^2)'_x \\
&= 2e^{2u}\sin v \cdot 3x^2 y + e^{2u}\cos v \cdot 2x \\
&=
\end{aligned}$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\ &= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\ &= 2e^{2x^3y}\sin(x^2+y^2)\cdot\end{aligned}$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\begin{aligned}
\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x} \\
&= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot (x^2 + y^2)'_x \\
&= 2e^{2u}\sin v \cdot 3x^2 y + e^{2u}\cos v \cdot 2x \\
&= 2e^{2x^3 y}\sin(x^2 + y^2)\cdot 3x^2 y +
\end{aligned}$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\begin{aligned}
\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x} \\
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&= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x \\
&= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot
\end{aligned}$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot (x^2+y^2)'_x$$

$$= 2e^{2u}\sin v \cdot 3x^2 y + e^{2u}\cos v \cdot 2x$$

$$= 2e^{2x^3 y}\sin(x^2+y^2)\cdot 3x^2 y + e^{2x^3 y}\cos(x^2+y^2)\cdot 2x$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot (x^2+y^2)'_x$$

$$= 2e^{2u}\sin v \cdot 3x^2 y + e^{2u}\cos v \cdot 2x$$

$$= 2e^{2x^3 y}\sin(x^2+y^2)\cdot 3x^2 y + e^{2x^3 y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$

$$=$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$
\begin{aligned}
\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\
&=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\
&=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\
&=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\\
&=(e^{2u}\sin v)'_u\cdot
\end{aligned}
$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\ &= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\ &= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_y+\end{aligned}$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$

$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x$$

$$=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$

$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\begin{aligned}
\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x} \\
&= (e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x \\
&= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x \\
&= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x
\end{aligned}$$

$$\begin{aligned}
\frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y} \\
&= (e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y \\
&=
\end{aligned}$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_x + (e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\ &= 2e^{2u}\sin v\cdot 3x^2y + e^{2u}\cos v\cdot 2x\\ &= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y + e^{2x^3y}\cos(x^2+y^2)\cdot 2x\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_y + (e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y\\ &= 2e^{2u}\sin v\cdot\end{aligned}$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x$$
$$=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y$$
$$=2e^{2u}\sin v\cdot x^3+$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\ &= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\ &= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y\\ &= 2e^{2u}\sin v\cdot x^3+e^{2u}\cos v\cdot\end{aligned}$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x$$
$$=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y$$
$$=2e^{2u}\sin v\cdot x^3+e^{2u}\cos v\cdot 2y$$
$$=$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_x + (e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\ &= 2e^{2u}\sin v\cdot 3x^2y + e^{2u}\cos v\cdot 2x\\ &= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y + e^{2x^3y}\cos(x^2+y^2)\cdot 2x\end{aligned}$$

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例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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\end{aligned}$$

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&= 2e^{2u}\sin v\cdot x^3+e^{2u}\cos v\cdot 2y \\
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\end{aligned}$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$
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\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y} \\
&= (e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y \\
&= 2e^{2u}\sin v\cdot x^3+e^{2u}\cos v\cdot 2y \\
&= 2e^{2x^3y}\sin(x^2+y^2)\cdot x^3+e^{2x^3y}\cos(x^2+y^2)\cdot 2y
\end{aligned}
$$

三元复合函数求导公式：举例

公式 设 $z=f(x,y,u)$，$u=u(x,y)$，

三元复合函数求导公式：举例

公式 设 $z = f(x, y, u)$，$u = u(x, y)$，则复合函数

$$z = f(x, y, u(x, y))$$

的偏导数是：

$$\frac{\partial z}{\partial x} = \qquad\qquad , \quad \frac{\partial z}{\partial y} =$$

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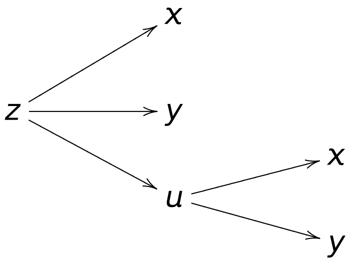
图示



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的偏导数是：

$$\frac{\partial z}{\partial x}= \qquad\qquad\qquad ,\quad \frac{\partial z}{\partial y}=$$

图示

$$\begin{tikzcd}
 & x & \\
z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[r, "\frac{\partial z}{\partial y}"] \arrow[ddr, "\frac{\partial z}{\partial u}"'] & y & \\
 & & x \\
 & u \arrow[ur] \arrow[dr] & \\
 & & y
\end{tikzcd}$$

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图示

$$\begin{tikzcd}
 & x & \\
z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[r, "\frac{\partial z}{\partial y}"] \arrow[dr, "\frac{\partial z}{\partial u}"'] & y & \\
 & u \arrow[r, "\frac{\partial u}{\partial x}"] \arrow[dr, "\frac{\partial u}{\partial y}"'] & x \\
 & & y
\end{tikzcd}$$

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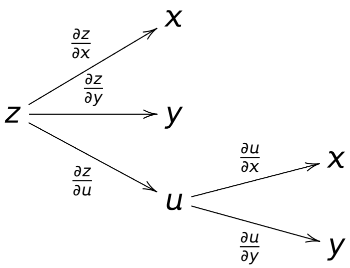
的偏导数是：

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial x} + \qquad\qquad , \quad \frac{\partial z}{\partial y} =$$

图示



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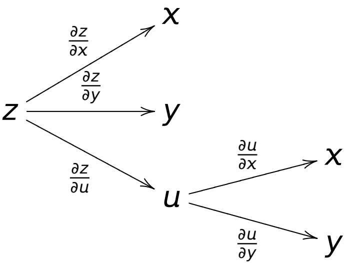
$$z = f(x, y, u(x, y))$$

的偏导数是：

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x}, \qquad \frac{\partial z}{\partial y} =$$

图示



三元复合函数求导公式：举例

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$$z = f(x, y, u(x, y))$$

的偏导数是：

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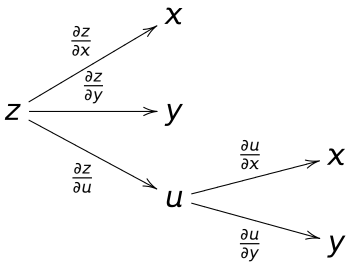
图示



三元复合函数求导公式：举例

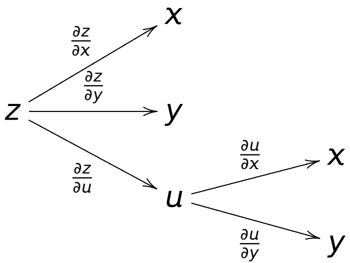
公式 设 $z=f(x,y,u)$，$u=u(x,y)$，则复合函数

$$z=f(x,y,u(x,y))$$

的偏导数是：

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial x}+\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x},\quad \frac{\partial z}{\partial y}=\frac{\partial z}{\partial y}+\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}$$

图示



复合函数的高阶导数

公式 设 $z = f(u, v)$，$u = u(x, y)$，$v = v(x, y)$，则复合函数

$$z = f(u(x, y), v(x, y))$$

的偏导数是：

$$z_x = z_u \cdot u_x + z_v \cdot v_x,$$

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的偏导数是：

$z_x = z_u \cdot u_x + z_v \cdot v_x$,

$z_y = z_u \cdot u_y + z_v \cdot v_y$,

$z_{xx} =$

$z_{xy} =$

$z_{yy} =$

复合函数的高阶导数

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$$z=f(u(x,y),v(x,y))$$

的偏导数是：

$z_x=z_u\cdot u_x+z_v\cdot v_x$,

$z_y=z_u\cdot u_y+z_v\cdot v_y$,

$z_{xx}=(z_x)'_x$

$z_{xy}=$

$z_{yy}=$

复合函数的高阶导数

公式 设 $z = f(u, v)$，$u = u(x, y)$，$v = v(x, y)$，则复合函数

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的偏导数是：

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$$z_y = z_u \cdot u_y + z_v \cdot v_y,$$

$$z_{xx} = (z_x)'_x = (z_u \cdot u_x + z_v \cdot v_x)'_x$$

$$z_{xy} =$$

$$z_{yy} =$$

复合函数的高阶导数

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$$\begin{aligned} z_{xx} &= (z_x)'_x = (z_u \cdot u_x + z_v \cdot v_x)'_x \\ &= (z_u)'_x \cdot u_x + z_u \cdot u_{xx} + (z_v)'_x \cdot v_x + z_v \cdot v_{xx} \end{aligned}$$

$$z_{xy} =$$

$$z_{yy} =$$

复合函数的高阶导数

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$$\begin{aligned}z_{xx}&=(z_x)'_x=(z_u\cdot u_x+z_v\cdot v_x)'_x\\&=(z_u)'_x\cdot u_x+z_u\cdot u_{xx}+(z_v)'_x\cdot v_x+z_v\cdot v_{xx}\\&=(\qquad\qquad\qquad)\cdot u_x+z_u\cdot u_{xx}+(\qquad\qquad\qquad)\cdot v_x+z_v\cdot v_{xx}\end{aligned}$$

$$z_{xy}=$$

$$z_{yy}=$$

复合函数的高阶导数

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$$\begin{aligned} z_{xx} &= (z_x)'_x = (z_u \cdot u_x + z_v \cdot v_x)'_x \\ &= (z_u)'_x \cdot u_x + z_u \cdot u_{xx} + (z_v)'_x \cdot v_x + z_v \cdot v_{xx} \\ &= (z_{uu} \cdot u_x + z_{uv} \cdot v_x) \cdot u_x + z_u \cdot u_{xx} + (\qquad\qquad\qquad) \cdot v_x + z_v \cdot v_{xx} \end{aligned}$$

$$z_{xy} =$$

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$$z_{xy} =$$

$$z_{yy} =$$

复合函数的高阶导数

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$$\begin{aligned}
z_{xx} &= (z_x)'_x = (z_u \cdot u_x + z_v \cdot v_x)'_x \\
&= (z_u)'_x \cdot u_x + z_u \cdot u_{xx} + (z_v)'_x \cdot v_x + z_v \cdot v_{xx} \\
&= (z_{uu}\cdot u_x + z_{uv}\cdot v_x)\cdot u_x + z_u \cdot u_{xx} + (z_{vu}\cdot u_x + z_{vv}\cdot v_x)\cdot v_x + z_v \cdot v_{xx} \\
&= z_{uu}u_x^2 + 2z_{uv}u_x v_x + z_{vv}v_x^2 + z_u u_{xx} + z_v v_{xx}
\end{aligned}$$

$$z_{xy} =$$

$$z_{yy} =$$

复合函数的高阶导数

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$$\begin{aligned}z_{xx}&=(z_x)'_x=(z_u\cdot u_x+z_v\cdot v_x)'_x\\&=(z_u)'_x\cdot u_x+z_u\cdot u_{xx}+(z_v)'_x\cdot v_x+z_v\cdot v_{xx}\\&=(z_{uu}\cdot u_x+z_{uv}\cdot v_x)\cdot u_x+z_u\cdot u_{xx}+(z_{vu}\cdot u_x+z_{vv}\cdot v_x)\cdot v_x+z_v\cdot v_{xx}\\&=z_{uu}u_x^2+2z_{uv}u_xv_x+z_{vv}v_x^2+z_uu_{xx}+z_vv_{xx}\end{aligned}$$

$$z_{xy}=?$$

$$z_{yy}=?$$

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$$z_{xx} = z_{uu} u_x^2 + 2 z_{uv} u_x v_x + z_{vv} v_x^2 + z_u u_{xx} + z_v v_{xx}$$

$$z_{xy} =$$

$$z_{yy} = ?$$

复合函数的高阶导数

公式 设 $z=f(u,v)$，$u=u(x,y)$，$v=v(x,y)$，则复合函数

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例 设 $z = f(xy^2, x^2y)$，求 $\frac{\partial^2 z}{\partial x \partial y}$

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$$=e^{x+y} f_w-\cos x \sin y \cdot f_{uv}+\cos x e^{x+y} f_{uw}-\sin y e^{x+y} f_{wv}+e^{2x+2y} f_{ww}$$