第 9 章 c：多元复合函数的求导法则

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Outline

二元复合函数求导

设有二元函数 $z = f(u, v)$

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- 设 $u = \varphi(t)$，$v = \psi(t)$，则 $z = f(\varphi(t), \psi(t))$

问 $\frac{dz}{dt} = ?$

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二元复合函数求导公式——中间变量是一元函数

公式 设 $z = f(u, v)$，$u = \varphi(t)$，$v = \psi(t)$，则 $z = f(\varphi(t), \psi(t))$ 的全导数

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$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt} + \frac{\partial z}{\partial v} \cdot \frac{dv}{dt}$$

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 & u \arrow[r, "\frac{du}{dt}"] & t \\
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$$\because\quad z=uv=$$

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$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} = (\frac{y}{x})'_x \cdot$$

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$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=$$

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$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=-\frac{1-e^{2t}}{e^{2t}}\cdot e^t+$$

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例 设 $z=\frac{y}{x}$，而 $x=e^t$，$y=1-e^{2t}$，求全导数 $\frac{dz}{dt}$

解

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=(\frac{y}{x})'_x\cdot(e^t)'_t+(\frac{y}{x})'_y\cdot(1-e^{2t})'_t$$

$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=-\frac{1-e^{2t}}{e^{2t}}\cdot e^t+\frac{1}{e^t}\cdot(-2e^{2t})$$

$$=$$

$$\begin{tikzcd}
 & x \arrow[r, "\frac{dx}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[dr, "\frac{\partial z}{\partial y}"'] & & \\
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$$=-e^{-t}-e^t$$

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\end{tikzcd}$$

三元复合函数求导公式——中间变量是一元函数

公式 设 $z = f(u, v, w)$，$u = \varphi(t)$，$v = \psi(t)$，$w = \omega(t)$，则 $z = f(\varphi(t), \psi(t), \omega(t))$ 的全导数

$$\frac{dz}{dt} =$$

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$$\begin{tikzcd}
 & u \arrow[r] & t \\
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 & w \arrow[r] & t
\end{tikzcd}$$

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$$\begin{tikzcd}
 & u \arrow[r] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"'] & v \arrow[r] & t \\
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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}$$

$$\begin{tikzcd}
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$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"] & v \arrow[r, "\frac{dv}{dt}"] & t \\
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$$z = f(\varphi(x, y), \psi(x, y))$$

的偏导数是：

$$\frac{\partial z}{\partial x} = \qquad\qquad , \quad \frac{\partial z}{\partial y} =$$

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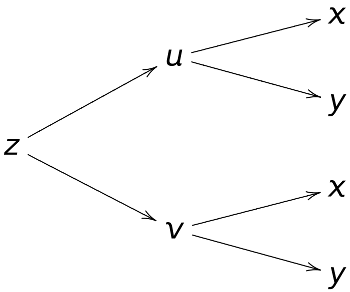
图示



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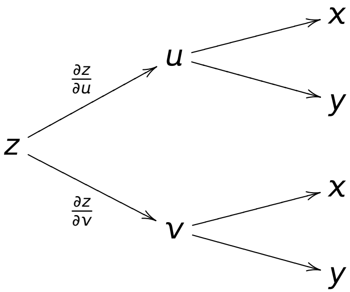

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$$\begin{tikzcd}
 & & x \\
 & u \arrow[ur, "\frac{\partial u}{\partial x}"] \arrow[dr, "\frac{\partial u}{\partial y}"'] & \\
 & & y \\
z \arrow[uur, "\frac{\partial z}{\partial u}"] \arrow[ddr, "\frac{\partial z}{\partial v}"'] & & \\
 & & x \\
 & v \arrow[ur, "\frac{\partial v}{\partial x}"] \arrow[dr, "\frac{\partial v}{\partial y}"'] & \\
 & & y
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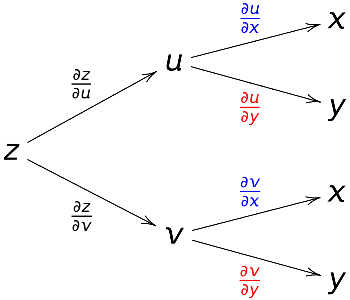

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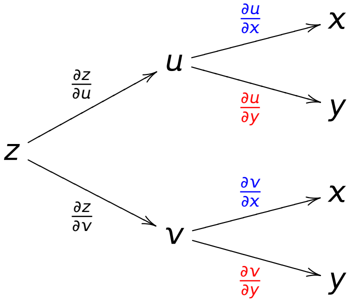
图示



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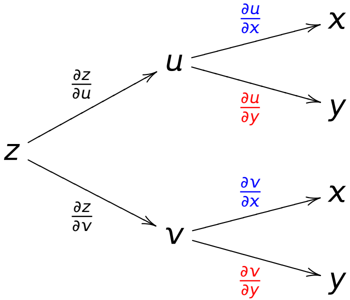

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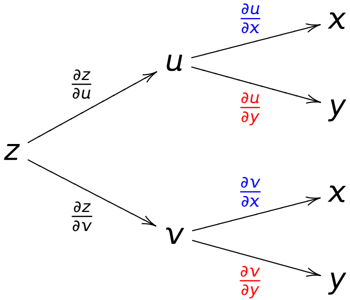
图示



例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

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解
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=$$

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$$\begin{aligned}\frac{\partial z}{\partial x}&=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\&=(e^{2u}\sin v)'_u\cdot\end{aligned}$$

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$$
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&= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot
\end{aligned}
$$

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&= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x \\
&= 2e^{2x^3y}\sin(x^2+y^2)\cdot
\end{aligned}
$$

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解
$$
\begin{aligned}
\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x} \\
&= (e^{2u}\sin v)'_u\cdot(x^3y)'_x + (e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x \\
&= 2e^{2u}\sin v\cdot 3x^2y + e^{2u}\cos v\cdot 2x \\
&= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y + e^{2x^3y}\cos(x^2+y^2)\cdot 2x
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$$= 2e^{2u} \sin v \cdot 3x^2 y + e^{2u} \cos v \cdot 2x$$

$$= 2e^{2x^3 y} \sin(x^2 + y^2) \cdot 3x^2 y + e^{2x^3 y} \cos(x^2 + y^2) \cdot 2x$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y}$$

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$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x$$
$$=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\ &= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\ &= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_y+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_y\\ &=\end{aligned}$$

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例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot (x^2+y^2)'_x$$
$$= 2e^{2u}\sin v \cdot 3x^2 y + e^{2u}\cos v \cdot 2x$$
$$= 2e^{2x^3 y}\sin(x^2+y^2)\cdot 3x^2 y + e^{2x^3 y}\cos(x^2+y^2)\cdot 2x$$

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\end{aligned}
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三元复合函数求导公式：举例

公式 设 $z = f(x, y, u)$，$u = u(x, y)$，

三元复合函数求导公式：举例

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$$z = f(x, y, u(x, y))$$

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$$\frac{\partial z}{\partial x} = \qquad\qquad\qquad , \quad \frac{\partial z}{\partial y} =$$

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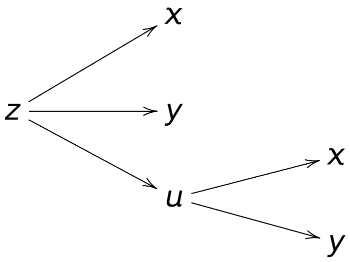
图示



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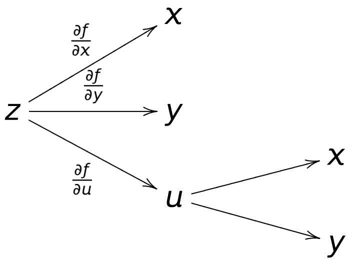

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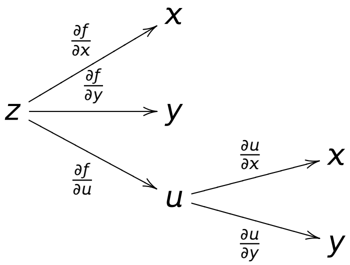
图示



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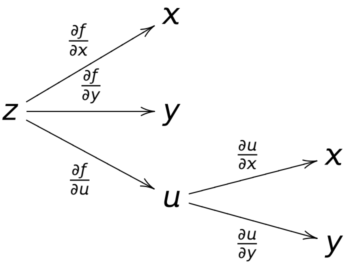
图示



三元复合函数求导公式：举例

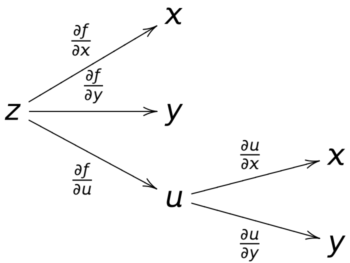
公式 设 $z=f(x,y,u)$，$u=u(x,y)$，则复合函数

$$z=f(x,y,u(x,y))$$

的偏导数是：

$$\frac{\partial z}{\partial x}=\frac{\partial f}{\partial x}+\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x},\qquad \frac{\partial z}{\partial y}=$$

图示



三元复合函数求导公式：举例

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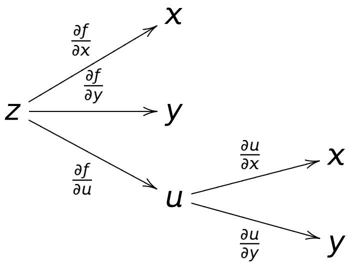
的偏导数是：

$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x}, \qquad \frac{\partial z}{\partial y} = \frac{\partial f}{\partial y} +$$

图示



三元复合函数求导公式：举例

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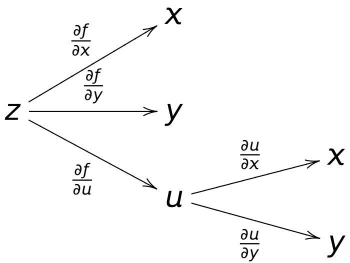
的偏导数是：

$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x}, \qquad \frac{\partial z}{\partial y} = \frac{\partial f}{\partial y} + \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial y}$$

图示



例 设 $z=f(x^2-y^2, e^{xy})$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$

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例 设 $z=f(x^2-y^2, e^{xy})$，求 $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$

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例 设 $g=f(\frac{x}{y}, \frac{y}{z})$，求 $\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \frac{\partial g}{\partial z}$

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复合函数的高阶导数

公式 设 $z = f(u, v)$，$u = u(x, y)$，$v = v(x, y)$，则复合函数

$$z = f(u(x, y), v(x, y))$$

的偏导数是：

$$z_x = z_u \cdot u_x + z_v \cdot v_x,$$

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$$z_{xy} =$$

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$$z_{xy}=$$

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$$z_{xy} = ?$$

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$z_{yy}=?$

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$$=(z_u)'_y\cdot u_x+z_u\cdot u_{xy}+(z_v)'_y\cdot v_x+z_v\cdot v_{xy}$$

$$=(\qquad\qquad)\cdot u_x+z_u\cdot u_{xy}+(\qquad\qquad)\cdot v_x+z_v\cdot v_{xy}$$

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$$z_{yy} = ?$$

复合函数的高阶导数

公式 设 $z=f(u,v)$，$u=u(x,y)$，$v=v(x,y)$，则复合函数

$$z=f(u(x,y),v(x,y))$$

的偏导数是：

$$z_x = z_u \cdot u_x + z_v \cdot v_x,$$

$$z_y = z_u \cdot u_y + z_v \cdot v_y,$$

$$z_{xx} = z_{uu}u_x^2 + 2z_{uv}u_x v_x + z_{vv}v_x^2 + z_u u_{xx} + z_v v_{xx}$$

$$\begin{aligned} z_{xy} &= (z_x)'_y = (z_u \cdot u_x + z_v \cdot v_x)'_y \\ &= (z_u)'_y \cdot u_x + z_u \cdot u_{xy} + (z_v)'_y \cdot v_x + z_v \cdot v_{xy} \\ &= (z_{uu}\cdot u_y + z_{uv}\cdot v_y)\cdot u_x + z_u \cdot u_{xy} + (z_{vu}\cdot u_y + z_{vv}\cdot v_y)\cdot v_x + z_v \cdot v_{xy} \end{aligned}$$

$z_{yy} = ?$

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&=(z_u)'_y\cdot u_x+z_u\cdot u_{xy}+(z_v)'_y\cdot v_x+z_v\cdot v_{xy}\\
&=(z_{uu}\cdot u_y+z_{uv}\cdot v_y)\cdot u_x+z_u\cdot u_{xy}+(z_{vu}\cdot u_y+z_{vv}\cdot v_y)\cdot v_x+z_v\cdot v_{xy}\\
&=z_{uu}u_xu_y+z_{uv}(u_xv_y+u_yv_x)+z_{vv}v_xv_y+z_uu_{xy}+z_vv_{xy}
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$$z_{yy} = (z_y)'_y$$

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$$z_{xy}=z_{uu}u_xu_y+z_{uv}(u_xv_y+u_yv_x)+z_{vv}v_xv_y+z_uu_{xy}+z_vv_{xy}$$

$$z_{yy}=(z_y)'_y=(z_u\cdot u_y+z_v\cdot v_y)'_y$$

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$$\begin{aligned} z_{yy} &= (z_y)'_y = (z_u \cdot u_y + z_v \cdot v_y)'_y \\ &= (z_u)'_y \cdot u_y + z_u \cdot u_{yy} + (z_v)'_y \cdot v_y + z_v \cdot v_{yy} \end{aligned}$$

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z_{yy} &= (z_y)'_y = (z_u \cdot u_y + z_v \cdot v_y)'_y \\
&= (z_u)'_y \cdot u_y + z_u \cdot u_{yy} + (z_v)'_y \cdot v_y + z_v \cdot v_{yy} \\
&= (z_{uu} \cdot u_y + z_{uv} \cdot v_y) \cdot u_y + z_u \cdot u_{yy} + (z_{vu} \cdot u_y + z_{vv} \cdot v_y) \cdot v_y + z_v \cdot v_{yy} \\
&= z_{uu}u_y^2 + 2z_{uv}u_y v_y + z_{vv}v_y^2 + z_u u_{yy} + z_v v_{yy}
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$$= 2yf_u + y^2 \cdot (2xyf_{uu} + x^2 f_{uv}) + 2xf_v + 2xy \cdot (2xyf_{vu} + x^2 f_{vv})$$

$$= 2yf_u + 2xf_v + 2xy^3 f_{uu} + x^2y^2 f_{uv} + 4x^2y^2 f_{vu} + 2x^3y f_{vv}$$

$$= 2yf_u + 2xf_v + 2xy^3 f_{uu} + 5x^2y^2 f_{uv} + 2x^3y f_{vv}$$

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解 设 $z=f(u, v, w)$，$u=\sin x$，$v=\cos y$，$w=e^{x+y}$，则

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