§6.4 微积分基本定理

2017-2018 学年 II



教学要求









Outline of §6.4

1. 变上限的定积分

2. 微积分基本定理: 牛顿—莱布尼茨公式

We are here now...

1. 变上限的定积分

2. 微积分基本定理: 牛顿—莱布尼茨公式

$$\int_{a}^{x} f(t)dt, \quad \forall x \in [a, b]$$

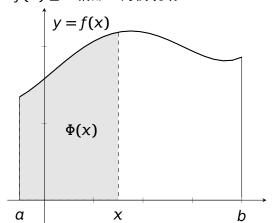
$$\Phi(x) = \int_{a}^{x} f(t)dt, \quad \forall x \in [a, b]$$

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为变上限的定积分。

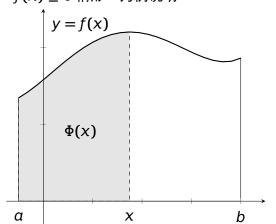
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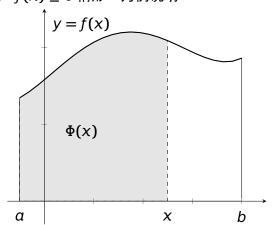
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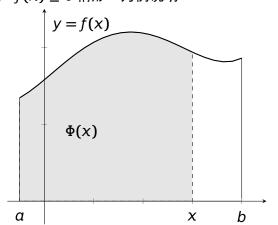
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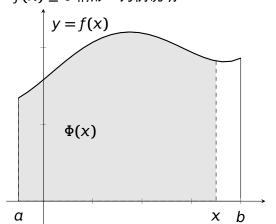
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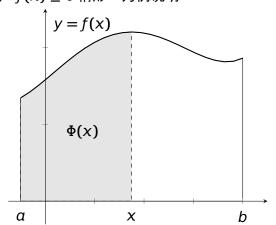
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$$\Phi(x) = \int_{a}^{x} f(t)dt \qquad \forall x \in [a, b]$$



$$\Phi'(x) = \left[\int_a^x f(t)dt\right]' = ? \quad \forall x \in [a, b]$$



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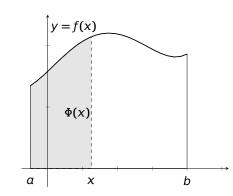
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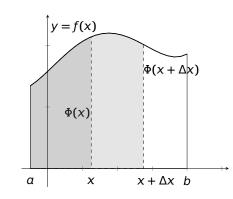
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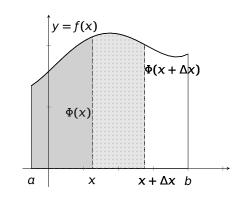
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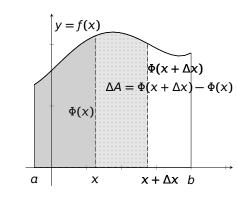
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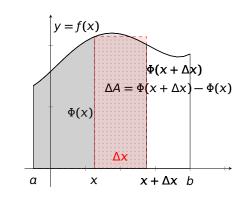
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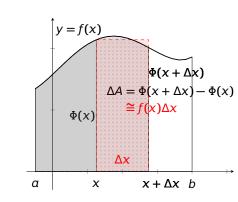
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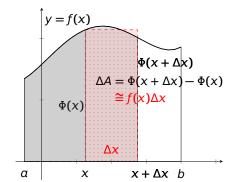
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$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left[\int_{a}^{x + \Delta x} f(t) dt - \int_{a}^{x} f(t) dt \right]$$

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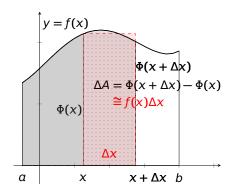
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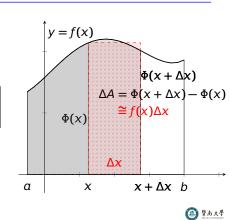
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$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x} (f(\xi) \Delta x)$$



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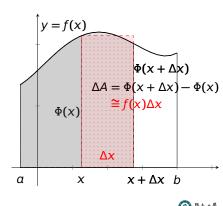
证明

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 $= \lim_{\Delta x \to 0} \frac{1}{\Delta x} (f(\xi) \Delta x) = \lim_{\Delta x \to 0} f(\xi)$



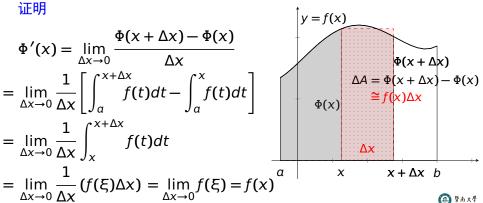
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微积分基本定理 $\Phi'(x) = \left[\int_a^x f(t)dt\right]' = f(x), \quad \forall x \in [a, b]$

例
$$\left[\int_2^x e^{-t} \sin(t^2) dt\right]' =$$

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例
$$\left[\int_{x}^{-2} e^{\sin t} dt\right]' = \underline{\hspace{1cm}}$$

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例
$$\left[\int_{x}^{-2} e^{\sin t} dt \right]' = \underline{-e^{\sin x}}$$

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例
$$\left[\int_{x}^{-2} e^{\sin t} dt\right]' = \underline{-e^{\sin x}}$$

$$\therefore \left[\int_{x}^{-2} e^{\sin t} dt \right]' = \left[-\int_{-2}^{x} e^{\sin t} dt \right]'$$

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$$\left[\int_2^x e^{-t} \sin(t^2) dt\right]' = \underline{e^{-x} \sin(x^2)}.$$

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例
$$\left[\int_{x}^{-2} e^{\sin t} dt\right]' = \underline{-e^{\sin x}}$$

$$\therefore \left[\int_{-\infty}^{-2} e^{\sin t} dt \right]' = \left[- \int_{-\infty}^{\infty} e^{\sin t} dt \right]' = - e^{\sin x}.$$

$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' =$$



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)].$$



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

$$\left[\int_{1}^{x^{2}} \cos t dt\right]' =$$
_____; $\left[\int_{2x}^{-1} \sqrt{1 + t^{2}} dt\right]' =$ _____.



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

$$\left[\int_{1}^{x^{2}} \cos t dt\right]' = \underline{\qquad}; \left[\int_{2x}^{-1} \sqrt{1 + t^{2}} dt\right]' = \underline{\qquad}.$$

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$$\left[\int_{1}^{x^{2}} \cos t dt\right]' = \cos(x^{2}).$$



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

$$\left[\int_{1}^{x^{2}} \cos t dt\right]' = \underline{\qquad}; \left[\int_{2x}^{-1} \sqrt{1 + t^{2}} dt\right]' = \underline{\qquad}.$$

$$\left[\int_{1}^{x^{2}} \cos t dt\right]' = \cos(x^{2}) \cdot (x^{2})'$$



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

$$\left[\int_{1}^{x^{2}} \cos t dt\right]' = \underline{\qquad}; \left[\int_{2x}^{-1} \sqrt{1 + t^{2}} dt\right]' = \underline{\qquad}.$$

$$\left[\int_{1}^{x^2} \cos t dt\right]' = \cos(x^2) \cdot (x^2)' = 2x \cos(x^2)$$



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$$\left[\int_{2t}^{-1} \sqrt{1+t^2} dt\right]' =$$



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

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$$\left[\int_{1}^{x^2} \cos t dt\right]' = \cos(x^2) \cdot (x^2)' = 2x \cos(x^2)$$

$$\left[\int_{2x}^{-1} \sqrt{1+t^2} dt \right]' = - \left[\int_{-1}^{2x} \sqrt{1+t^2} dt \right]' =$$



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

$$\left[\int_{1}^{x^{2}} \cos t dt\right]' = \underline{\qquad}; \left[\int_{2x}^{-1} \sqrt{1 + t^{2}} dt\right]' = \underline{\qquad}.$$

$$\left| \int_{1}^{x^{2}} \cos t dt \right|^{2} = \cos(x^{2}) \cdot (x^{2})^{2} = 2x \cos(x^{2})$$

$$\left[\int_{2x}^{-1} \sqrt{1 + t^2} dt \right]' = - \left[\int_{-1}^{2x} \sqrt{1 + t^2} dt \right]' = \sqrt{1 + 4x^2}.$$



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

$$\left[\int_{1}^{x^{2}} \cos t dt\right]' = \underline{\qquad}; \left[\int_{2x}^{-1} \sqrt{1 + t^{2}} dt\right]' = \underline{\qquad}.$$

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$$\left[\int_{2x}^{-1} \sqrt{1+t^2} dt\right]' = -\left[\int_{-1}^{2x} \sqrt{1+t^2} dt\right]' = \sqrt{1+4x^2} \cdot (2x)'$$



$$\left[\int_{a}^{\varphi(x)} f(t)dt\right]' = f[\varphi(x)] \cdot \varphi'(x).$$

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$$= -2\sqrt{1+4x^2}$$



$$\left[\int_{x^3}^{x^2} \ln(1+t)dt\right]'$$



例
$$\left[\int_{x^3}^{x^2} \ln(1+t) dt \right]' =$$

$$\left[\int_{x^3}^{x^2} \ln(1+t)dt \right]' = \left[\int_{x^3}^0 \ln(1+t)dt + \int_0^{x^2} \ln(1+t)dt \right]'$$



例
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$$\left[\int_{x^3}^{x^2} \ln(1+t)dt \right]' = \left[\int_{x^3}^0 \ln(1+t)dt + \int_0^{x^2} \ln(1+t)dt \right]' \\
= \left[-\int_0^{x^3} \ln(1+t)dt + \int_0^{x^2} \ln(1+t)dt \right]'$$



$$\left[\int_{x^3}^{x^2} \ln(1+t)dt \right]' = \left[\int_{x^3}^{0} \ln(1+t)dt + \int_{0}^{x^2} \ln(1+t)dt \right]' \\
= \left[-\int_{0}^{x^3} \ln(1+t)dt + \int_{0}^{x^2} \ln(1+t)dt \right]' \\
= -\left[\int_{0}^{x^3} \ln(1+t)dt \right]' + \left[\int_{0}^{x^2} \ln(1+t)dt \right]' \\
= -\left[\int_{0}^{x^3} \ln(1+t)dt \right]' + \left[\int_{0}^{x^2} \ln(1+t)dt \right]'$$



例
$$\left[\int_{x^3}^{x^2} \ln(1+t)dt\right]' = \underline{\hspace{1cm}}$$

$$\left[\int_{x^3}^{x^2} \ln(1+t)dt \right]' = \left[\int_{x^3}^{0} \ln(1+t)dt + \int_{0}^{x^2} \ln(1+t)dt \right]' \\
= \left[-\int_{0}^{x^3} \ln(1+t)dt + \int_{0}^{x^2} \ln(1+t)dt \right]' \\
= -\left[\int_{0}^{x^3} \ln(1+t)dt \right]' + \left[\int_{0}^{x^2} \ln(1+t)dt \right]' \\
= -\ln(1+x^3).$$



例
$$\left[\int_{x^3}^{x^2} \ln(1+t)dt\right]' = \underline{\hspace{1cm}}$$

$$\begin{aligned}
&\text{FF:} \\
&\left[\int_{x^3}^{x^2} \ln(1+t)dt\right]' = \left[\int_{x^3}^{0} \ln(1+t)dt + \int_{0}^{x^2} \ln(1+t)dt\right]' \\
&= \left[-\int_{0}^{x^3} \ln(1+t)dt + \int_{0}^{x^2} \ln(1+t)dt\right]' \\
&= -\left[\int_{0}^{x^3} \ln(1+t)dt\right]' + \left[\int_{0}^{x^2} \ln(1+t)dt\right]' \\
&= -\ln(1+x^3) \cdot (x^3)'
\end{aligned}$$



$$\left[\int_{x^3}^{x^2} \ln(1+t)dt \right]' = \left[\int_{x^3}^{0} \ln(1+t)dt + \int_{0}^{x^2} \ln(1+t)dt \right]' \\
= \left[-\int_{0}^{x^3} \ln(1+t)dt + \int_{0}^{x^2} \ln(1+t)dt \right]' \\
= -\left[\int_{0}^{x^3} \ln(1+t)dt \right]' + \left[\int_{0}^{x^2} \ln(1+t)dt \right]' \\
= -\ln(1+x^3) \cdot (x^3)' + \ln(1+x^2) \cdot$$



$$\begin{aligned}
&\text{if } : \\
&\left[\int_{x^3}^{x^2} \ln(1+t)dt\right]' = \left[\int_{x^3}^{0} \ln(1+t)dt + \int_{0}^{x^2} \ln(1+t)dt\right]' \\
&= \left[-\int_{0}^{x^3} \ln(1+t)dt + \int_{0}^{x^2} \ln(1+t)dt\right]' \\
&= -\left[\int_{0}^{x^3} \ln(1+t)dt\right]' + \left[\int_{0}^{x^2} \ln(1+t)dt\right]' \\
&= -\ln(1+x^3) \cdot (x^3)' + \ln(1+x^2) \cdot (x^2)'
\end{aligned}$$



$$\left[\int_{x^3}^{x^2} \ln(1+t)dt \right]' = \left[\int_{x^3}^0 \ln(1+t)dt + \int_0^{x^2} \ln(1+t)dt \right]'$$

$$= \left[-\int_0^{x^3} \ln(1+t)dt + \int_0^{x^2} \ln(1+t)dt \right]$$
$$= -\left[\int_0^{x^3} \ln(1+t)dt \right]' + \left[\int_0^{x^2} \ln(1+t)dt \right]'$$

 $= -\ln(1+x^3) \cdot (x^3)' + \ln(1+x^2) \cdot (x^2)'$

 $=-3x^2\ln(1+x^3)+2x\ln(1+x^2)$.



We are here now...

1. 变上限的定积分

2. 微积分基本定理: 牛顿—莱布尼茨公式

牛顿一莱布尼茨公式

$$\int_a^b f(x)dx =$$

$$\int_{a}^{b} f(x)dx = F(b) - F(a)$$

$$\int_{a}^{b} f(x)dx = F(b) - F(a) = F(x)\Big|_{a}^{b}.$$

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

$$\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b.$$

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

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$$\int_a^b f(x)dx = F(b) - F(a) = F(x)\Big|_a^b.$$

$$: \Phi(x) = \int_{a}^{x} f(t)dt \mathcal{L}f(x)$$
 的一个原函数

$$F(x) = \Phi(x) + C$$

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

$$\int_{a}^{b} f(x)dx = F(b) - F(a) = F(x)\Big|_{a}^{b}.$$

$$F(x) = \Phi(x) + C$$

$$\therefore F(b) - F(a) = () - ()$$

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

$$\int_{a}^{b} f(x)dx = F(b) - F(a) = F(x)\Big|_{a}^{b}.$$

$$\Phi(x) = \int_{a}^{x} f(t)dt \mathcal{L}f(x)$$
 的一个原函数

$$F(x) = \Phi(x) + C$$

$$\therefore F(b) - F(a) = (\Phi(b) + C) - ($$

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

$$\int_{a}^{b} f(x)dx = F(b) - F(a) = F(x)\Big|_{a}^{b}.$$

$$F(x) = \Phi(x) + C$$

$$\therefore F(b) - F(a) = (\Phi(b) + C) - (\Phi(a) + C)$$

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

$$\int_{a}^{b} f(x)dx = F(b) - F(a) = F(x)\Big|_{a}^{b}.$$

$$F(x) = \Phi(x) + C$$

$$\therefore F(b) - F(a) = (\Phi(b) + C) - (\Phi(a) + C)$$
$$= \Phi(b) - \Phi(a)$$

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

$$\int_{a}^{b} f(x)dx = F(b) - F(a) = F(x)\Big|_{a}^{b}.$$

$$: \Phi(x) = \int_{a}^{x} f(t)dt \mathcal{E}f(x)$$
 的一个原函数

$$\therefore F(x) = \Phi(x) + C$$

$$\therefore F(b) - F(a) = (\Phi(b) + C) - (\Phi(a) + C)$$
$$= \Phi(b) - \Phi(a)$$
$$= \int_{a}^{b} f(t)dt - \int_{a}^{a} f(t)dt$$

设 f(x) 在区间 [a, b] 上连续,F(x) 是 f(x) 任意一个原函数,则

$$\int_{a}^{b} f(x)dx = F(b) - F(a) = F(x)\Big|_{a}^{b}.$$

$$: \Phi(x) = \int_{a}^{x} f(t)dt \mathcal{E}f(x)$$
 的一个原函数

$$F(x) = \Phi(x) + C$$

$$\therefore F(b) - F(a) = (\Phi(b) + C) - (\Phi(a) + C)$$
$$= \Phi(b) - \Phi(a)$$

$$= \int_{a}^{b} f(t)dt - \int_{a}^{a} f(t)dt = \int_{a}^{b} f(t)dt$$



$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1 + x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

$$\iint_0^1 x^2 dx =$$



$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3}$$

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1}$$



$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

$$\iint_0^1 x^2 dx = \frac{1}{3} x^3 \Big|_0^1 = \frac{1}{3} - 0$$



$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$



$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_0^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

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$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{\pi/2} \sin x dx; \quad \int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

$$\int_0^{\pi/2} \sin x dx = -\cos x$$

$$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} =$$

$$\int_{-2}^{-1} \frac{dx}{x} =$$

$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{\pi/2} \sin x dx; \quad \int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}}; \quad \int_{2}^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

$$\iint_{0}^{\pi/2} \sin x dx = -\cos x \Big|_{0}^{\pi/2}$$

$$\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2} =$$

$$\int_{-2}^{-1} \frac{dx}{x} =$$



$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{\pi/2} \sin x dx; \quad \int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

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$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{\pi/2} \sin x dx; \quad \int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

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$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

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$$\int_{-2}^{-1} \frac{dx}{x} =$$



$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{\pi/2} \sin x dx; \quad \int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

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$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{\pi/2} \sin x dx; \quad \int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

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$$\int_{-2}^{-1} \frac{dx}{x} =$$

例 计算定积分

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_{-2}^{-1} \frac{dx}{x}$$

 $\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$

$$\int_0^{\pi/2} \sin x dx = -\cos x \Big|_0^{\pi/2} = -(\cos \frac{\pi}{2} - \cos 0) = 1$$

$$\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2} = \arctan x \Big|_{1}^{\sqrt{3}} = \arctan \sqrt{3} - \arctan 1 = \frac{\pi}{3} - \frac{\pi}{4}$$

$$\int_{-2}^{-1} \frac{dx}{x} =$$



例 计算定积分

$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{\pi/2} \sin x dx; \quad \int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}}; \quad \int_{2}^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

 $\int_0^{\pi/2} \sin x \, dx = -\cos x \Big|_0^{\pi/2} = -(\cos \frac{\pi}{2} - \cos 0) = 1$

$$\int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}} = \arctan x \Big|_{1}^{\sqrt{3}} = \arctan \sqrt{3} - \arctan 1 = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

$$\int_{-2}^{-1} \frac{dx}{x} = \frac{1}{3} \left(\frac{1}{3} - \frac{\pi}{4} \right) dx$$



例 计算定积分

$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{\pi/2} \sin x dx; \quad \int_{1}^{\sqrt{3}} \frac{dx}{1 + x^{2}}; \quad \int_{2}^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

 $\int_0^{\pi/2} \sin x \, dx = -\cos x \Big|_0^{\pi/2} = -(\cos \frac{\pi}{2} - \cos 0) = 1$

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$$\int_{1}^{\sqrt{3}} \frac{dx}{1 + x^{2}} = \arctan x \Big|_{1}^{\sqrt{3}} = \arctan \sqrt{3} - \arctan 1 = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

$$\int_{-2}^{-1} \frac{dx}{x} = \ln|x|$$



例 计算定积分

$$\int_{0}^{1} x^{2} dx; \quad \int_{0}^{\pi/2} \sin x dx; \quad \int_{1}^{\sqrt{3}} \frac{dx}{1+x^{2}}; \quad \int_{2}^{-1} \frac{dx}{x}$$

$$\iiint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

 $\int_0^{\pi/2} \sin x \, dx = -\cos x \Big|_0^{\pi/2} = -(\cos \frac{\pi}{2} - \cos 0) = 1$

$$\int_{0}^{\sqrt{3}} \sin x dx = -\cos x \Big|_{0}^{\pi/2} = -(\cos \frac{\pi}{2} - \cos 0) = 1$$

$$\int_{1}^{\sqrt{3}} \frac{dx}{1 + x^{2}} = \arctan x \Big|_{1}^{\sqrt{3}} = \arctan \sqrt{3} - \arctan 1 = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

 $\int_{-2}^{-1} \frac{dx}{x} = \ln|x| \Big|_{-2}^{-1}$



例 计算定积分

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_0^{-1} \frac{dx}{x}$$

$$\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$$

$$\iint_{0}^{\pi/2} \sin x dx = -\cos x \Big|_{0}^{\pi/2} = -(\cos \frac{\pi}{2} - \cos 0) = 1$$

 $\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2} = \arctan x \Big|_{1}^{\sqrt{3}} = \arctan \sqrt{3} - \arctan 1 = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$ $\int_{-2}^{-1} \frac{dx}{x} = \ln|x|\Big|_{-2}^{-1} = \ln|-1| - \ln|-2|$



例 计算定积分

$$\int_0^1 x^2 dx; \quad \int_0^{\pi/2} \sin x dx; \quad \int_1^{\sqrt{3}} \frac{dx}{1+x^2}; \quad \int_0^{-1} \frac{dx}{x}$$

 $\iint_{0}^{1} x^{2} dx = \frac{1}{3} x^{3} \Big|_{0}^{1} = \frac{1}{3} - 0 = \frac{1}{3}$

$$\int_0^{\pi/2} \sin x dx = -\cos x \Big|_0^{\pi/2} = -(\cos \frac{\pi}{2} - \cos 0) = 1$$

 $\int_{1}^{\sqrt{3}} \frac{dx}{1+x^2} = \arctan x \Big|_{1}^{\sqrt{3}} = \arctan \sqrt{3} - \arctan 1 = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$ $\int_{-2}^{-1} \frac{dx}{x} = \ln|x|\Big|_{-2}^{-1} = \ln|-1| - \ln|-2| = -\ln 2$



练习计算定积分

$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

练习计算定积分

$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
$$\int (2x-5)dx$$
, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$



练习计算定积分

$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
$$\int (2x-5)dx$$
, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

解

练习 计算定积分

$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
$$\int (2x-5)dx$$
, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^2 (2x-5)dx =$$

$$\int_{4}^{9} \frac{1}{\sqrt{x}} dx =$$

$$\int_0^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出 $\int (2x-5)dx$, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^2 (2x - 5) dx = (x^2 - 5x)$$

$$\int_{4}^{9} \frac{1}{\sqrt{x}} dx =$$

$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出 $\int (2x-5)dx$, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^2 (2x - 5) dx = (x^2 - 5x) \Big|_0^2$$

$$\int_{4}^{9} \frac{1}{\sqrt{x}} dx =$$

$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
$$\int (2x-5)dx$$
, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^2 (2x-5)dx = (x^2-5x)\big|_0^2 = -6-0$$

$$\int_{4}^{9} \frac{1}{\sqrt{x}} dx =$$

$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^{2}}} =$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
$$\int (2x-5)dx$$
, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^2 (2x - 5) dx = (x^2 - 5x) \Big|_0^2 = -6 - 0 = -6$$

$$\int_0^9 \frac{1}{\sqrt{x}} dx = \frac{1}{\sqrt{x}} dx =$$

$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
$$\int (2x-5)dx$$
, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^2 (2x - 5) dx = (x^2 - 5x) \Big|_0^2 = -6 - 0 = -6$$

$$\int_0^9 \frac{1}{\sqrt{x}} dx = \int_0^9 x^{-1/2} dx = 0$$

$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^{2}}} =$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
$$\int (2x-5)dx$$
, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^2 (2x - 5) dx = (x^2 - 5x) \Big|_0^2 = -6 - 0 = -6$$

$$\int_0^9 \frac{1}{\sqrt{x}} dx = \int_0^9 x^{-1/2} dx = 2\sqrt{x}$$

$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
$$\int (2x-5)dx$$
, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^2 (2x - 5) dx = (x^2 - 5x) \Big|_0^2 = -6 - 0 = -6$$

$$\int_4^9 \frac{1}{\sqrt{x}} dx = \int_4^9 x^{-1/2} dx = 2\sqrt{x} \Big|_4^9$$

$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
$$\int (2x-5)dx$$
, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^2 (2x-5)dx = (x^2-5x)\big|_0^2 = -6-0 = -6$$

$$\int_{4}^{9} \frac{1}{\sqrt{x}} dx = \int_{4}^{9} x^{-1/2} dx = 2\sqrt{x} \Big|_{4}^{9} = 2(3-2)$$

$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
$$\int (2x-5)dx$$
, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^2 (2x - 5) dx = (x^2 - 5x) \Big|_0^2 = -6 - 0 = -6$$

$$\int_4^9 \frac{1}{\sqrt{x}} dx = \int_4^9 x^{-1/2} dx = 2\sqrt{x} \Big|_4^9 = 2(3 - 2) = 2$$

$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}} =$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
$$\int (2x-5)dx$$
, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

解

$$\int_0^2 (2x - 5) dx = (x^2 - 5x) \Big|_0^2 = -6 - 0 = -6$$

$$\int_4^9 \frac{1}{\sqrt{x}} dx = \int_4^9 x^{-1/2} dx = 2\sqrt{x} \Big|_4^9 = 2(3 - 2) = 2$$

 $\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}} = \arcsin x \Big|_{0}^{1/2}$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出
$$\int (2x-5)dx$$
, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^2 (2x - 5) dx = (x^2 - 5x) \Big|_0^2 = -6 - 0 = -6$$

$$\int_4^9 \frac{1}{\sqrt{x}} dx = \int_4^9 x^{-1/2} dx = 2\sqrt{x} \Big|_4^9 = 2(3 - 2) = 2$$

$$\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^{2}}} = \arcsin x \Big|_{0}^{1/2} = \arcsin \frac{1}{2} - \arcsin 0$$



$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出 $\int (2x-5)dx$, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^2 (2x - 5) dx = (x^2 - 5x) \Big|_0^2 = -6 - 0 = -6$$

$$\int_{4}^{9} \frac{1}{\sqrt{x}} dx = \int_{4}^{9} x^{-1/2} dx = 2\sqrt{x} \Big|_{4}^{9} = 2(3-2) = 2$$

$$\int_{4}^{1/2} \frac{dx}{\sqrt{x}} dx = \int_{4}^{1/2} x^{-1/2} dx = 2\sqrt{x} \Big|_{4}^{1} = 2(3-2) = 2$$

$$\int_{0}^{1/2} \frac{dx}{\sqrt{1 - x^{2}}} = \arcsin x \Big|_{0}^{1/2} = \arcsin \frac{1}{2} - \arcsin 0 = \frac{\pi}{6} - 0$$

§6.4 微积分基本定理

$$\int_0^2 (2x-5)dx; \quad \int_4^9 \frac{1}{\sqrt{x}} dx; \quad \int_0^{1/2} \frac{dx}{\sqrt{1-x^2}}$$

提示: 先求出 $\int (2x-5)dx$, $\int \frac{1}{\sqrt{x}}dx$, $\int \frac{dx}{\sqrt{1-x^2}}$

$$\int_0^2 (2x-5)dx = (x^2-5x)\big|_0^2 = -6-0 = -6$$

$$\int_{4}^{9} \frac{1}{\sqrt{x}} dx = \int_{4}^{9} x^{-1/2} dx = 2\sqrt{x} \Big|_{4}^{9} = 2(3-2) = 2$$

$$\int_{4}^{9} \frac{1}{\sqrt{x}} dx = \int_{4}^{9} x^{-1/2} dx = 2\sqrt{x} \Big|_{4}^{9} = 2(3-2) = 2$$

§6.4 微积分基本定理

 $\int_{0}^{1/2} \frac{dx}{\sqrt{1-x^2}} = \arcsin x \Big|_{0}^{1/2} = \arcsin \frac{1}{2} - \arcsin 0 = \frac{\pi}{6} - 0 = \frac{\pi}{6}$

例 计算定积分
$$\int_0^2 |1-x| dx$$
.
$$\mathbf{M} \int_0^2 |1-x| dx$$

$$\mathbf{m} \int_0^2 |1 - x| dx$$

$$= \int_0^1 |1 - x| dx + \int_1^2 |1 - x| dx$$



$$\mathbf{m} \int_0^2 |1 - x| dx$$

$$= \int_0^1 |1-x| dx + \int_1^2 |1-x| dx = \int_0^1 (1-x) dx +$$



$$\iiint_0^2 |1-x| dx$$

$$= \int_{0}^{1} |1 - x| dx + \int_{1}^{2} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{2} (x - 1) dx$$



例 计算定积分
$$\int_0^2 |1-x| dx$$
.

$$\lim_{x \to \infty} \int_{0}^{2} |1-x| dx$$

$$\begin{aligned}
&= \int_0^1 |1 - x| dx \\
&= \int_0^1 |1 - x| dx + \int_1^2 |1 - x| dx = \int_0^1 (1 - x) dx + \int_1^2 (x - 1) dx \\
&= (x - \frac{1}{2}x^2)
\end{aligned}$$



例 计算定积分
$$\int_0^2 |1-x| dx$$
.

$$\lim_{x \to \infty} \int_0^2 |1-x| dx$$

$$\begin{aligned}
&= \int_0^1 |1 - x| dx \\
&= \int_0^1 |1 - x| dx + \int_1^2 |1 - x| dx = \int_0^1 (1 - x) dx + \int_1^2 (x - 1) dx \\
&= (x - \frac{1}{2}x^2) + (\frac{1}{2}x^2 - x)
\end{aligned}$$

例 计算定积分
$$\int_0^2 |1-x| dx$$
.

$$\lim_{x \to \infty} \int_{0}^{2} |1-x| dx$$

$$\int_{0}^{1} |1-x| dx$$

$$= \int_{0}^{1} |1-x| dx + \int_{1}^{2} |1-x| dx = \int_{0}^{1} (1-x) dx + \int_{1}^{2} (x-1) dx$$

$$= (x - \frac{1}{2}x^{2}) \Big|_{0}^{1} + (\frac{1}{2}x^{2} - x)$$

$$= (x - \frac{1}{2}x^2)\Big|_0^1 + (\frac{1}{2}x^2 - x)^2$$



例 计算定积分
$$\int_0^2 |1-x| dx$$
.

$$\lim_{x \to \infty} \int_{0}^{2} |1-x| dx$$

$$= \int_0^1 |1 - x| dx + \int_1^2 |1 - x| dx = \int_0^1 (1 - x) dx + \int_1^2 (x - 1) dx$$
$$= (x - \frac{1}{x^2}) \Big|_0^1 + (\frac{1}{x^2} - x) \Big|_0^2$$

$$= (x - \frac{1}{2}x^2)\big|_0^1 + (\frac{1}{2}x^2 - x)\big|_1^2$$



例 计算定积分
$$\int_0^2 |1-x| dx$$
.

$$\lim_{x \to \infty} \int_{0}^{2} |1-x| dx$$

$$= \int_0^1 |1 - x| dx + \int_1^2 |1 - x| dx = \int_0^1 (1 - x) dx + \int_1^2 (x - 1) dx$$
$$= (x - \frac{1}{x^2}) \Big|_0^1 + (\frac{1}{x^2} - x) \Big|_1^2 = [\frac{1}{x^2} - 0] + \frac{1}{x^2} + \frac$$

$$= (x - \frac{1}{2}x^2)\big|_0^1 + (\frac{1}{2}x^2 - x)\big|_1^2 = [\frac{1}{2} - 0] +$$



例 计算定积分
$$\int_0^2 |1-x| dx$$
.

$$\mathbf{H} \int_{0}^{2} |1-x| dx$$

$$\int_{0}^{1} |1-x| dx + \int_{1}^{2} |1-x| dx = \int_{0}^{1} (1-x) dx + \int_{1}^{2} (x-1) dx$$
$$= \int_{0}^{1} |1-x| dx + \int_{1}^{2} |1-x| dx = \int_{0}^{1} (1-x) dx + \int_{1}^{2} (x-1) dx$$

$$= (x - \frac{1}{2}x^2)\Big|_0^1 + (\frac{1}{2}x^2 - x)\Big|_1^2 = [\frac{1}{2} - 0] + [0 - (-\frac{1}{2})]$$



例 计算定积分
$$\int_0^2 |1-x| dx$$
.

$$\lim_{x \to \infty} \int_{0}^{2} |1-x| dx$$

$$\int_{0}^{1} |1 - x| dx + \int_{1}^{2} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{2} (x - 1) dx$$

$$= \int_{0}^{1} |1 - x| dx + \int_{1}^{2} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{2} (x - 1) dx$$

$$= (x - \frac{1}{2}x^2)\Big|_0^1 + (\frac{1}{2}x^2 - x)\Big|_1^2 = [\frac{1}{2} - 0] + [0 - (-\frac{1}{2})] = 1$$



$$\lim_{x \to \infty} \int_0^2 |1-x| dx$$

$$\int_{0}^{1} |1 - x| dx$$

$$= \int_{0}^{1} |1 - x| dx + \int_{1}^{2} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{2} (x - 1) dx$$

$$= (x - \frac{1}{2}x^{2}) \Big|_{0}^{1} + (\frac{1}{2}x^{2} - x) \Big|_{1}^{2} = [\frac{1}{2} - 0] + [0 - (-\frac{1}{2})] = 1$$

练习 计算定积分 $\int_0^3 |2-x| dx$



$$\lim_{x \to \infty} \int_{0}^{2} |1-x| dx$$

$$\int_{0}^{1} |1-x| dx$$

$$= \int_{0}^{1} |1-x| dx + \int_{1}^{2} |1-x| dx = \int_{0}^{1} (1-x) dx + \int_{1}^{2} (x-1) dx$$

$$= (x - \frac{1}{2}x^{2}) \Big|_{0}^{1} + (\frac{1}{2}x^{2} - x) \Big|_{1}^{2} = [\frac{1}{2} - 0] + [0 - (-\frac{1}{2})] = 1$$

练习 计算定积分
$$\int_0^3 |2-x| dx$$

$$\lim_{x \to 0} \int_{0}^{3} |2-x| dx$$



$$\Re \int_{0}^{2} |1-x| dx$$

$$= \int_{0}^{1} |1 - x| dx + \int_{1}^{2} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{2} (x - 1) dx$$
$$= (x - \frac{1}{2}x^{2}) \Big|_{0}^{1} + (\frac{1}{2}x^{2} - x) \Big|_{1}^{2} = [\frac{1}{2} - 0] + [0 - (-\frac{1}{2})] = 1$$

练习 计算定积分 $\int_0^3 |2-x| dx$

$$\mathbf{H} \int_0^3 |2-x| dx$$

$$= \int_{0}^{2} |2 - x| dx + \int_{0}^{3} |2 - x| dx$$



$$\Re \int_{0}^{2} |1-x| dx$$

$$\int_{0}^{1} |1-x| dx$$

$$= \int_{0}^{1} |1-x| dx + \int_{1}^{2} |1-x| dx = \int_{0}^{1} (1-x) dx + \int_{1}^{2} (x-1) dx$$

$$= (x - \frac{1}{2}x^{2}) \Big|_{0}^{1} + (\frac{1}{2}x^{2} - x) \Big|_{1}^{2} = [\frac{1}{2} - 0] + [0 - (-\frac{1}{2})] = 1$$

练习 计算定积分 $\int_0^3 |2-x| dx$

$$\Re$$
 $\int_{0}^{3} |2-x| dx$

$$= \int_{0}^{2} |2-x| dx + \int_{2}^{3} |2-x| dx = \int_{2}^{2} (2-x) dx + \int_{2}^{3} |2-x| dx = \int_{2}^{3} \int_{2}$$



$$\mathbb{H} \int_{0}^{2} |1-x| dx$$

$$\begin{aligned}
&= \int_0^1 |1 - x| dx \\
&= \int_0^1 |1 - x| dx + \int_1^2 |1 - x| dx = \int_0^1 (1 - x) dx + \int_1^2 (x - 1) dx \\
&= (x - \frac{1}{2}x^2) \Big|_0^1 + (\frac{1}{2}x^2 - x) \Big|_1^2 = [\frac{1}{2} - 0] + [0 - (-\frac{1}{2})] = 1
\end{aligned}$$

练习 计算定积分 $\int_0^3 |2-x| dx$

$$\mathbf{H} \int_0^3 |2 - x| dx$$

 $= \int_{0}^{2} |2-x| dx + \int_{3}^{3} |2-x| dx = \int_{0}^{2} (2-x) dx + \int_{3}^{3} (x-2) dx$



§6.4 微积分基本定理

练习 计算定积分 $\int_0^3 |2-x| dx$

$$\lim_{x \to 0} \int_0^2 |1 - x| dx$$

$$\int_0^1 |1-x|dx$$

$$= \int_0^1 |1-x|dx$$

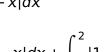
$$= \int_0^1 |1 - x| dx$$

$$= \int_{0}^{1} |1 - x| dx + \int_{1}^{2} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{2} (x - 1) dx$$

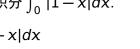
$$= \int_0^1 |1 - x| dx + \int_1^2 |1 - x| dx = \int_0^1 (1 - x) dx + \int_1^2 (x - x) dx + \int_1^2 (x$$

 $\Re \int_{0}^{3} |2-x| dx$

 $=(2x-\frac{1}{2}x^2)$













 $= \int_{0}^{2} |2-x| dx + \int_{0}^{3} |2-x| dx = \int_{0}^{2} (2-x) dx + \int_{0}^{3} (x-2) dx$

$$\int_0^1 |1 - x| d$$

$$= \int_{0}^{1} |1 - x| dx + \int_{1}^{1} |1 - x| dx = \int_{0}^{1} (1 - x) dx + \int_{1}^{1} (x - x)$$

 $\lim_{x \to \infty} \int_{0}^{3} |2-x| dx$

$$= \int_0^1 |1 - x| dx$$

$$= \int_0^1 |1 - x| dx - \frac{1}{2} dx$$

 $= \int_{0}^{1} |1-x| dx + \int_{1}^{2} |1-x| dx = \int_{0}^{1} (1-x) dx + \int_{1}^{2} (x-1) dx$

练习 计算定积分 $\int_0^3 |2-x| dx$

 $=(2x-\frac{1}{2}x^2)+(\frac{1}{2}x^2-2x)$

 $= \int_{0}^{2} |2-x| dx + \int_{3}^{3} |2-x| dx = \int_{0}^{2} (2-x) dx + \int_{3}^{3} (x-2) dx$

$$\begin{aligned}
\widehat{\mathbf{H}} & \int_0^2 |1 - x| dx \\
&= \int_0^1 |1 - x| dx
\end{aligned}$$

$$\int_0^{1} |1 - x| a$$

$$= \int_0^1 |1 - x| dx + \int_1^2 |1 - x| dx = \int_0^1 (1 - x) dx + \int_1^2 (x - 1) dx$$
$$= (x - \frac{1}{2}x^2) \Big|_0^1 + (\frac{1}{2}x^2 - x) \Big|_1^2 = [\frac{1}{2} - 0] + [0 - (-\frac{1}{2})] = 1$$

练习 计算定积分 $\int_0^3 |2-x| dx$

练习 计算定积分
$$\int_0^3 |2-x| dx$$

$$\mathbf{H} \int_0^3 |2-x| dx$$

 $= \int_{0}^{2} |2-x| dx + \int_{0}^{3} |2-x| dx = \int_{0}^{2} (2-x) dx + \int_{0}^{3} (x-2) dx$ $= (2x - \frac{1}{2}x^2)\Big|_0^2 + (\frac{1}{2}x^2 - 2x)\Big|_2^3$



$$\int_0^1 |1-x| dx$$

$$= \int_0^1 |1-x| dx + \frac{1}{2} \int_0^1 |1-x| dx + \frac{1}{2}$$

$$= \int_0^1 |1 - x| dx +$$

 $= \int_{0}^{1} |1-x| dx + \int_{1}^{2} |1-x| dx = \int_{0}^{1} (1-x) dx + \int_{1}^{2} (x-1) dx$

$$=\int_0^1 |1-x|dx$$

 $\lim_{x \to \infty} \int_{0}^{3} |2-x| dx$



$$\int_{1}^{1}$$

$$(2-x)|_{1}^{2} = [\frac{1}{2} - \frac{1}{2}]_{2}^{2}$$

 $= \left(x - \frac{1}{2}x^2\right)\Big|_0^1 + \left(\frac{1}{2}x^2 - x\right)\Big|_1^2 = \left[\frac{1}{2} - 0\right] + \left[0 - \left(-\frac{1}{2}\right)\right] = 1$

$$-x)_{1} = [\frac{1}{2} - 0] + [$$

练习 计算定积分
$$\int_0^3 |2-x| dx$$



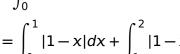
$$= \int_{0}^{2} |2-x| dx + \int_{3}^{3} |2-x| dx = \int_{0}^{2} (2-x) dx + \int_{3}^{3} (x-2) dx$$

$$-x$$
) $dx + \int_{2}^{\infty} (x - 2)^{2}$

$$=\int_0^2 |2|^2$$

 $= (2x - \frac{1}{2}x^2)\Big|_0^2 + (\frac{1}{2}x^2 - 2x)\Big|_2^3 = [2 - 0] +$

$$\int_0^1 |1-x| dx + \int_0^1 |1-x| dx + \int_0^$$



 $= \int_{0}^{1} |1-x| dx + \int_{1}^{2} |1-x| dx = \int_{0}^{1} (1-x) dx + \int_{1}^{2} (x-1) dx$

 $= \left(x - \frac{1}{2}x^2\right)\Big|_0^1 + \left(\frac{1}{2}x^2 - x\right)\Big|_1^2 = \left[\frac{1}{2} - 0\right] + \left[0 - \left(-\frac{1}{2}\right)\right] = 1$

练习 计算定积分 $\int_0^3 |2-x| dx$

 $\lim_{x \to \infty} \int_{0}^{3} |2-x| dx$

 $= \int_{0}^{2} |2-x| dx + \int_{0}^{3} |2-x| dx = \int_{0}^{2} (2-x) dx + \int_{0}^{3} (x-2) dx$ $= (2x - \frac{1}{2}x^2)\Big|_0^2 + (\frac{1}{2}x^2 - 2x)\Big|_2^3 = [2 - 0] + [-\frac{3}{2} - (-2)]$

例 计算定积分
$$\int_0^2 |1-x| dx$$
. 解 $\int_0^2 |x| dx$.

$$\int_0^{\pi} |1 - x| dx$$
$$= \int_0^1 |1 - x| dx + 1$$

$$\int_0^1 |1-x|a$$

$$\int_{1}^{1} |1-x| dx + \int_{1}^{2} |1-x| dx$$

$$=\int_0^1 |1-x|dx + \frac{1}{2}$$

 $= \int_{0}^{1} |1-x| dx + \int_{1}^{2} |1-x| dx = \int_{0}^{1} (1-x) dx + \int_{1}^{2} (x-1) dx$

练习 计算定积分 $\int_0^3 |2-x| dx$

 $\lim_{x \to \infty} \int_{0}^{3} |2-x| dx$

 $= \int_{0}^{2} |2-x| dx + \int_{0}^{3} |2-x| dx = \int_{0}^{2} (2-x) dx + \int_{0}^{3} (x-2) dx$

 $= (2x - \frac{1}{2}x^2)\Big|_0^2 + (\frac{1}{2}x^2 - 2x)\Big|_2^3 = [2 - 0] + [-\frac{3}{2} - (-2)] = \frac{5}{3}$

 $= \left(x - \frac{1}{2}x^2\right)\Big|_0^1 + \left(\frac{1}{2}x^2 - x\right)\Big|_1^2 = \left[\frac{1}{2} - 0\right] + \left[0 - \left(-\frac{1}{2}\right)\right] = 1$

 $\mathbf{m} \int_{0}^{2} |1-x| dx$