

## 第 06 周作业解答

练习 1. 求下列定积分

1.  $\int_{-2}^2 f(x)dx$ , 其中  $f(x) = \begin{cases} x+1 & x > 1 \\ 2 & x \leq 1 \end{cases}$ .

2.  $\int_0^{2\pi} |\sin x|dx$ .

解: (1)

$$\begin{aligned}\int_{-2}^2 f(x)dx &= \int_{-2}^1 f(x)dx + \int_1^2 f(x)dx = \int_{-2}^1 2dx + \int_1^2 x+1dx \\ &= 2x|_{-2}^1 + (\frac{1}{2}x^2 + x)|_1^2 = 2(1+2) + [(2+2) - (\frac{1}{2}+1)] = \frac{17}{2}\end{aligned}$$

(2)

$$\begin{aligned}\int_0^{2\pi} |\sin x|dx &= \int_0^{\pi} |\sin x|dx + \int_{\pi}^{2\pi} |\sin x|dx = \int_0^{\pi} \sin xdx + \int_{\pi}^{2\pi} -\sin xdx \\ &= -\cos x|_0^{\pi} + \cos x|_{\pi}^{2\pi} = -(-1-1) + [1 - (-1)] = 4\end{aligned}$$

练习 2. 假设  $\int_1^k \frac{1}{\sqrt[3]{x}}dx = \frac{9}{2}$ , 求出数值  $k$ .

解:

$$\int_1^k \frac{1}{\sqrt[3]{x}}dx = \int_1^k x^{-1/3}dx = \frac{3}{2}x^{\frac{2}{3}}|_1^k = \frac{3}{2}(k^{\frac{2}{3}} - 1).$$

所以

$$\frac{3}{2}(k^{\frac{2}{3}} - 1) = \frac{9}{2} \implies k^{\frac{2}{3}} = 4 \implies k = 4^{\frac{3}{2}} = 2^3 = 8.$$

练习 3. 计算不定积分:

(1)  $\int_{-1}^2 4x\sqrt{2x^2+1}dx$ ; (2)  $\int_1^e \frac{2+\ln x}{x}dx$ ; (3)  $\int_1^2 \frac{e^{2t}}{\sqrt{e^{2t}-1}}dt$ ; (4)  $\int_0^{\frac{\pi}{2}} \cos^5 x \sin xdx$

解: (1)

$$\begin{aligned}\int_{-1}^2 4x\sqrt{2x^2+1}dx &= 2 \int_{-1}^2 \sqrt{2x^2+1}d(x^2) = \int_{-1}^2 \sqrt{2x^2+1}d(2x^2+1) \\ &\stackrel{u=1+2x^2}{=} \int_3^9 u^{1/2}du = \frac{2}{3}u^{3/2} = \frac{2}{3}(9^{3/2} - 3^{3/2}) = 18 - 2\sqrt{3}\end{aligned}$$

(2)

$$\int_1^e \frac{2+\ln x}{x}dx = \int_1^e (2+\ln x)d\ln x = \int_1^e (2+\ln x)d(2+\ln x) \stackrel{u=2+\ln x}{=} \int_2^3 udu = \frac{1}{2}u^2|_2^3 = \frac{1}{2}(3^2 - 2^2) = \frac{5}{2}$$

(3)

$$\int_1^2 \frac{e^{2t}}{\sqrt{e^{2t}-1}} dt = \frac{1}{2} \int_1^2 (e^{2t}-1)^{-\frac{1}{2}} d(e^{2t}-1) \stackrel{u=e^{2t}-1}{=} \frac{1}{2} \int_{e^2-1}^{e^4-1} u^{-1/2} du = \sqrt{u} \Big|_{e^2-1}^{e^4-1} = \sqrt{e^4-1} - \sqrt{e^2-1}$$

(4)

$$\int_0^{\frac{\pi}{2}} \cos^5 x \sin x dx = - \int_0^{\frac{\pi}{2}} \cos^5 x d \cos x = -\frac{1}{6} \cos^6 x \Big|_0^{\frac{\pi}{2}} = -\frac{1}{6}(0-1) = \frac{1}{6}$$

**练习 4.** 计算不定积分

$$(1) \int_2^5 \frac{1}{1+\sqrt{x-1}} dx; \quad (2) \int_4^9 \frac{\sqrt{x}}{\sqrt{x}-1} dx; \quad (3) \int_0^1 \frac{1}{1+e^x} dx$$

解: (1) 令  $t = 1 + \sqrt{x-1}$ , 则  $x = (t-1)^2 + 1$ ,  $dx = 2(t-1)dt$  且  $t = 2 \dots 3$ 。所以

$$\int_2^5 \frac{1}{1+\sqrt{x-1}} dx = \int_2^3 \frac{1}{t} \cdot 2(t-1) dt = 2(t - \ln t) \Big|_2^3 = 2 - 2 \ln \frac{3}{2}$$

(2) 令  $t = \sqrt{x} - 1$ , 则  $x = (t+1)^2$ ,  $dx = 2(t+1)dt$  且  $t = 1 \dots 2$ 。所以

$$\int_4^9 \frac{\sqrt{x}}{\sqrt{x}-1} dx = \int_1^2 \frac{t+1}{t} \cdot 2(t+1) dt = 2 \int_1^2 \left(t + 2 + \frac{1}{t}\right) dt = (t^2 + 4t + 2 \ln t) \Big|_1^2 = 7 + 2 \ln 2$$

(3) 令  $t = 1 + e^x$ , 则  $x = \ln(t-1)$ ,  $dx = \frac{1}{t-1} dt$  且  $t = 2 \dots 1+e$ 。所以

$$\int_0^1 \frac{1}{1+e^x} dx = \int_2^{1+e} \frac{1}{t} \cdot \frac{1}{t-1} dt = \int_2^{1+e} \frac{1}{t-1} - \frac{1}{t} dt = \ln \frac{t-1}{t} \Big|_2^{1+e} = \ln \frac{2e}{1+e}$$

另解:

$$\begin{aligned} \int_0^1 \frac{1}{1+e^x} dx &= \int_0^1 \frac{1+e^x - e^x}{1+e^x} dx = \int_0^1 1 dx - \int_0^1 \frac{e^x}{1+e^x} dx = x \Big|_0^1 - \int_0^1 \frac{1}{1+e^x} de^x \\ &= 1 - \ln(1+e^x) \Big|_0^1 = 1 + \ln \frac{2}{1+e} = \ln \frac{2e}{1+e} \end{aligned}$$

**练习 5.** 用分部积分法计算  $\int_0^{\ln 2} x e^{-3x} dx$

解:

$$\begin{aligned} \int_0^{\ln 2} x e^{-3x} dx &= -\frac{1}{3} \int_0^{\ln 2} x d e^{-3x} = -\frac{1}{3} \left( x e^{-3x} \Big|_0^{\ln 2} - \int_0^{\ln 2} e^{-3x} dx \right) \\ &= -\frac{1}{3} \left( \frac{1}{8} \ln 2 + \frac{1}{3} e^{-3x} \Big|_0^{\ln 2} \right) = \frac{1}{24} \left( \frac{7}{3} - \ln 2 \right) \end{aligned}$$