



# 教学要求

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掌握求解：

◇ 二阶行列式计算

♣ 三阶行列式计算

- 行列式可用于表示一些线性方程组的解。

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- 具体地，

二阶行列式  $\longleftrightarrow$  2元2方程的线性方程组

三阶行列式  $\longleftrightarrow$  3元3方程的线性方程组

$\vdots$

$n$ 阶行列式  $\longleftrightarrow$   $n$ 元 $n$ 方程的线性方程组

$\vdots$

(克莱姆法则)

## 2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解：

## 2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

用消元法求解：(1)  $\times a_{22}$  - (2)  $\times a_{12}$ ，消去  $y$ ，得：

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$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22}$$

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用消元法求解:  $(1) \times a_{22} - (2) \times a_{12}$ , 消去  $y$ , 得:

$$x = \underline{\hspace{2cm}}$$

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$(2) \times a_{11} - (1) \times a_{21}$ , 消去  $x$ , 得:

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$(2) \times a_{11} - (1) \times a_{21}$ , 消去  $x$ , 得:

$$y = \underline{\hspace{2cm}}$$

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• 定义  $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} a_{22} - a_{12} a_{21}$

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## 小结

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

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## 小结

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

1. 当  $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$  时,

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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1. 当  $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$  时, 方程有唯一解:

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2. 当  $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 0$  时,

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2. 当  $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 0$  时, 方程或者无解、或者有无穷多的解。

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \quad , \quad y =$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad , \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \text{---} \quad , \quad y =$$

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公式:

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

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### 3 元 3 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

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用消元法可解得：

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} - b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}}$$

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为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}$$

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规律

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规律

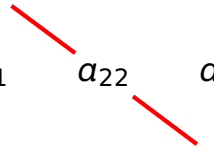
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

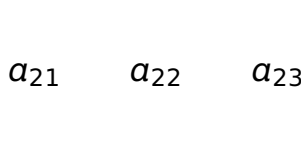
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规律

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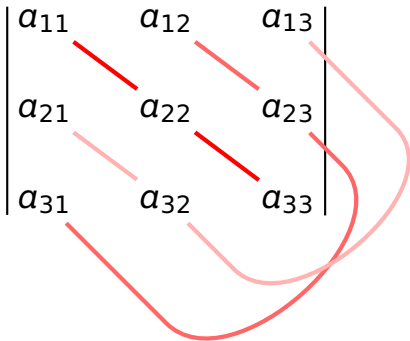
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规律

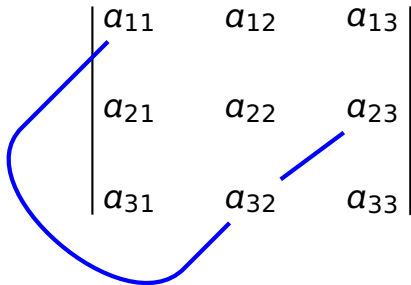
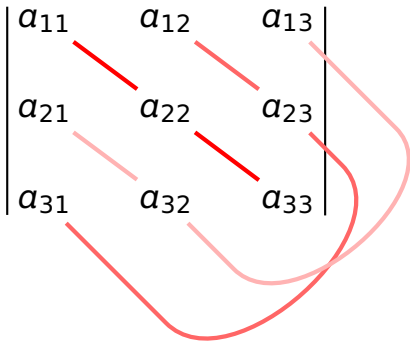


$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

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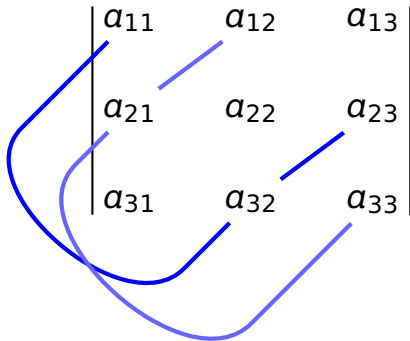
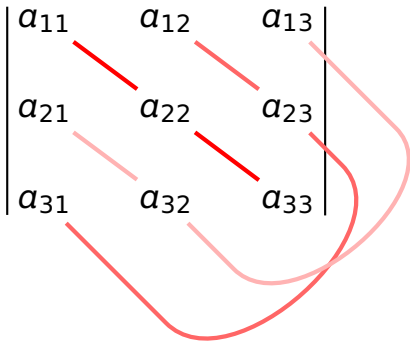
规律



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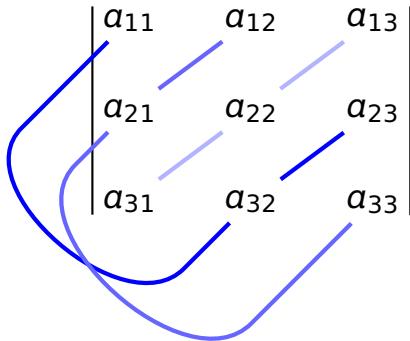
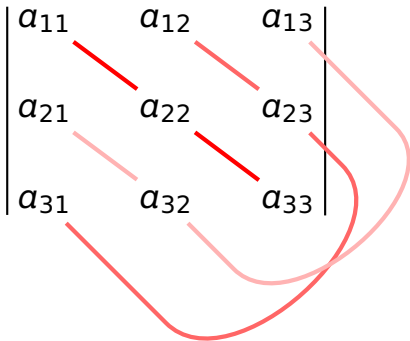
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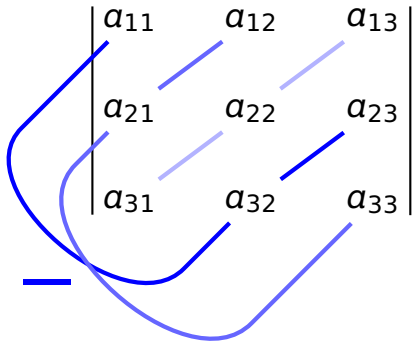
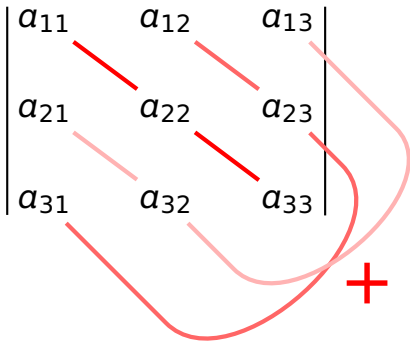
规律



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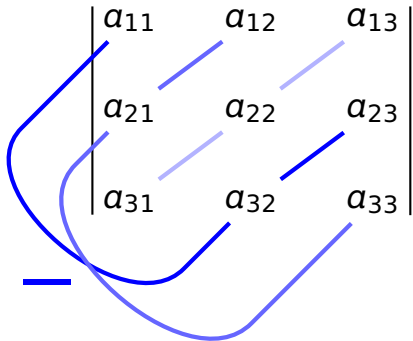
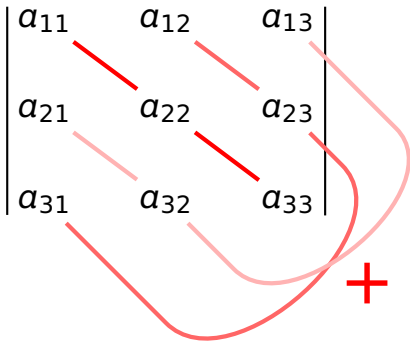
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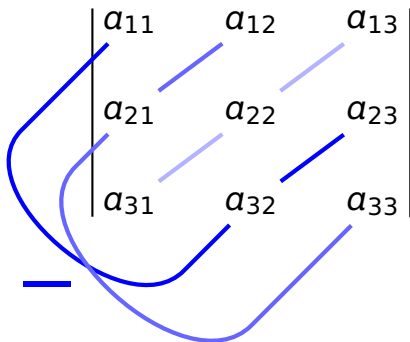
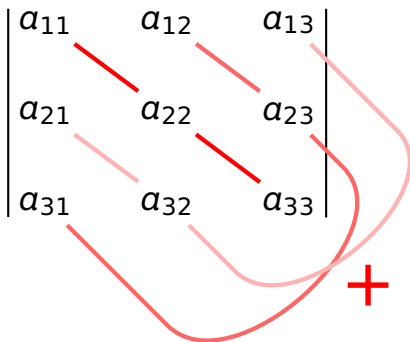


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规律（不同行不同列的 3 个元素乘积，共  $3! = 6$  个）

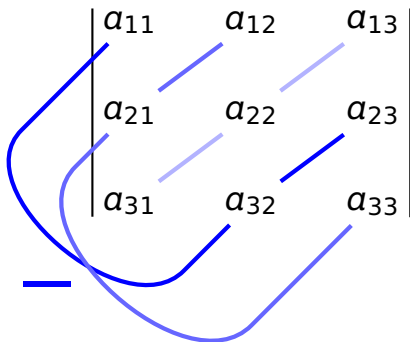
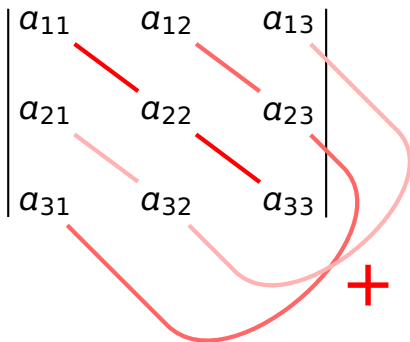




例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$

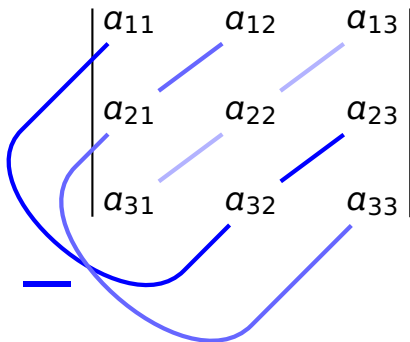
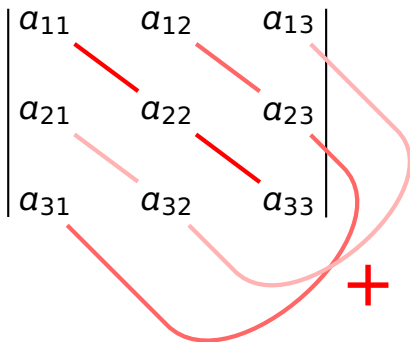


例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$

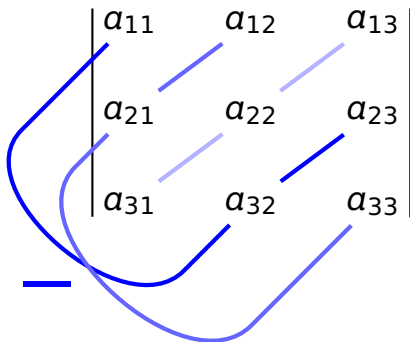
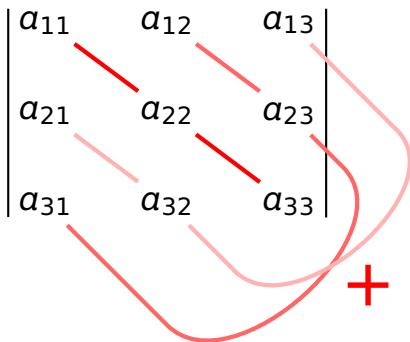




例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 - 1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1)$$

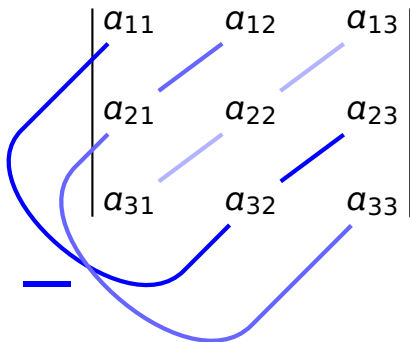
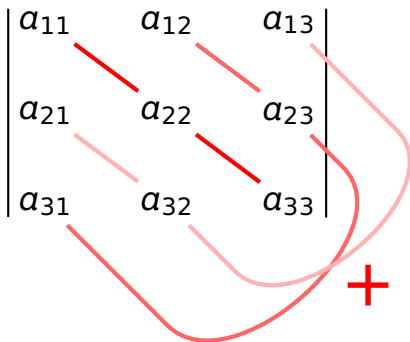
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{aligned} &1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ &- 1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{aligned} = -58$$

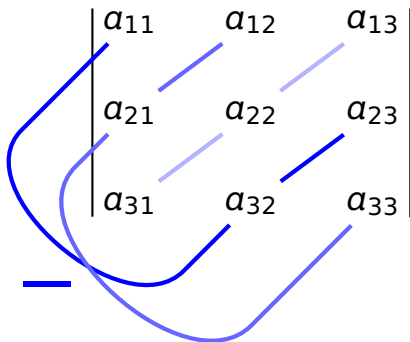
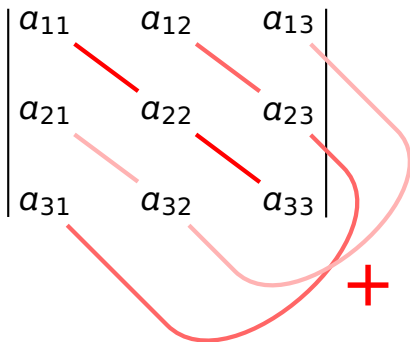
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$



例 计算

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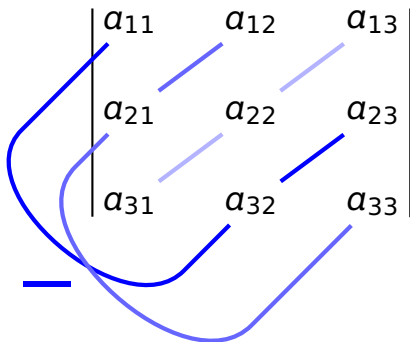
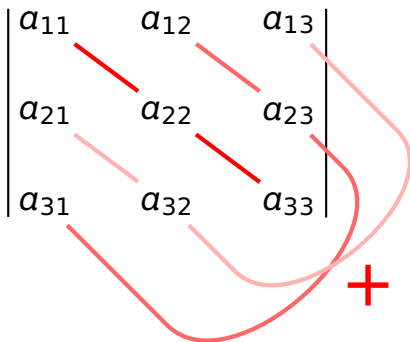
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = 1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{aligned} &1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ &- 1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{aligned} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{aligned} &1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4 \\ &- 1 \times 0 \times 4 - 0 \times 3 \times 1 - (-1) \times 5 \times 1 \end{aligned}$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{aligned} &1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ &- 1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{aligned} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{aligned} &1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4 \\ &- 1 \times 0 \times 4 - 0 \times 3 \times 1 - (-1) \times 5 \times 1 \end{aligned} = -2$$

练习  $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$  不为零的充分必要条件是  $a, b$  满足 \_\_\_\_\_

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解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} =$$

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解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2$$



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解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 - a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1$$

练习  $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$  不为零的充分必要条件是  $a, b$  满足 \_\_\_\_\_

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{matrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{matrix} = a^2 + b^2$$

练习  $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$  不为零的充分必要条件是  $a, b$  满足 \_\_\_\_\_

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所以  $a \neq 0$  或  $b \neq 0$ 。

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为：

,

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的解可以表示为：

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} - b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}} =$$

,

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

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,

这时方程组

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,

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的解可以表示为:

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} - b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}},$$



这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

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$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\quad}{D}, \quad z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\quad}{D}$$

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$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{D'}{D}, \quad z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{D''}{D}$$

这时方程组

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## 小结

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$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D}$$

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1. 当  $D \neq 0$  时,

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D}$$

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1. 当  $D \neq 0$  时, 方程有唯一解:

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## 小结

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1. 当  $D \neq 0$  时, 方程有唯一解:

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D}$$

2. 当  $D = 0$  时, 方程或者无解、或者有无穷多的解。

例 求解三元线性方程组 
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

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$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

解

$$x = \frac{D_x}{D} = \underline{\hspace{2cm}}$$

$$y = \frac{D_y}{D} = \underline{\hspace{2cm}}$$

$$z = \frac{D_z}{D} = \underline{\hspace{2cm}}$$

例 求解三元线性方程组 
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 0 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

$$y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 0 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

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例 求解三元线性方程组 
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}}{D} \quad \text{X}$$

$$y = \frac{D_y}{D} = \frac{\quad}{\quad}$$

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例 求解三元线性方程组  $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$  (  $\begin{cases} x + 0y + 2z = 9 \end{cases}$  )

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解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

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解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = -13$$

$$y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

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解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13}$$

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$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1, \quad y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

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解

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代入方程得

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所以方程的解是 
$$\begin{cases} x = 1 \\ y = 2 \\ z = 4 \end{cases}$$



一般地,  $n$  元  $n$  方程的线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \quad \quad \quad \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

的解可用  $n$  行列式表示:

$$x_1 = \frac{D_1}{D}$$

$$x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

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那么, 如何**定义行列式**,

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那么, 如何**定义行列式**, 如何**快捷计算行列式**?

# 补充：方程组的几何理解

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2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

# 补充：方程组的几何理解

## 2 元 2 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

- 每条方程表示平面上的一条直线
- 方程组的解表示两条直线的交点

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

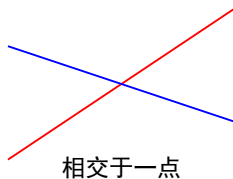
- 每条方程表示平面上的一条直线
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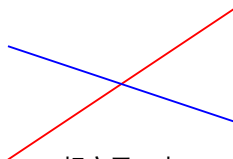


# 补充：方程组的几何理解

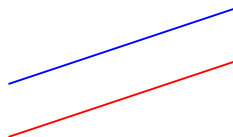
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相交于一点



平行不相交

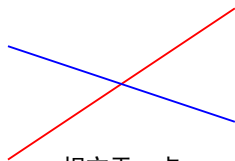


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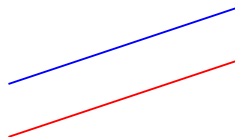
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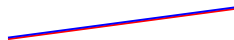
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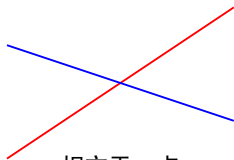
重合

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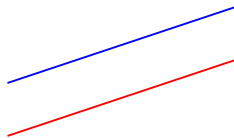
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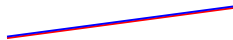
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重合

- 所以方程组的解有三种情况：

有唯一解、无解、有无穷多的解

### 3 元 3 方程的线性方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

### 3 元 3 方程的线性方程组

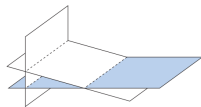
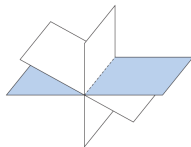
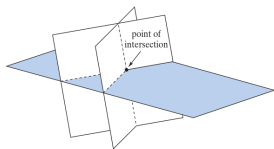
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- 每条方程表示空间上的一个平面
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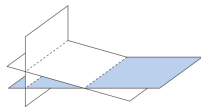
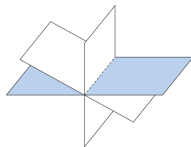
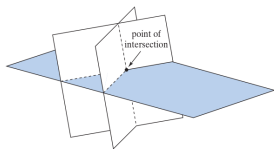
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- 所以方程组的解有三种情况：

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