



# Outline

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## 1. 偏导数

## 2. 全微分

# We are here now...

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## 1. 偏导数

## 2. 全微分

# 偏导数引入

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$$u_y =$$

$$u_z =$$

例 求三元函数  $u = xyz + \frac{z}{x}$  的全部一阶偏导数

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# 偏导数准确定义

- $z = f(x, y)$  在点  $(x_0, y_0)$  关于  $x$  的偏增量：
$$f(x_0 + \Delta x, y_0) - f(x_0, y_0)$$

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例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

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$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

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所以  $f(x, 1) = 2x$

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解 
$$f_x(0, 0) = \left. \frac{d}{dx}[f(x, 0)] \right|_{x=0} = \left. \frac{d}{dx}[0] \right|_{x=0} = 0,$$

$$f_y(0, 0) = \left. \frac{d}{dy}[f(0, y)] \right|_{y=0} = \left. \frac{d}{dy}[0] \right|_{y=0} = 0,$$

注 偏导数存在  $\nRightarrow$  连续

例 设  $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ , 求  $f_x(0, 0), f_y(0, 0)$

解 
$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)] \Big|_{x=0} = \frac{d}{dx}[0] \Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)] \Big|_{y=0} = \frac{d}{dy}[0] \Big|_{y=0} = 0,$$

注 偏导数存在  $\nRightarrow$  连续

(上述  $f(x, y)$  在  $(0, 0)$  处存在偏导数  $f_x(0, 0)$  和  $f_y(0, 0)$ , 但在  $(0, 0)$  处不连续)

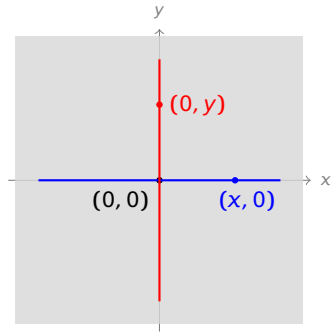
例 设  $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ , 求  $f_x(0, 0), f_y(0, 0)$

解 
$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)] \Big|_{x=0} = \frac{d}{dx}[0] \Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)] \Big|_{y=0} = \frac{d}{dy}[0] \Big|_{y=0} = 0,$$

注 偏导数存在  $\nRightarrow$  连续

(上述  $f(x, y)$  在  $(0, 0)$  处存在偏导数  $f_x(0, 0)$  和  $f_y(0, 0)$ , 但在  $(0, 0)$  处不连续)



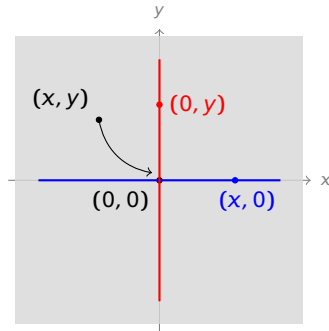
例 设  $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$ , 求  $f_x(0, 0), f_y(0, 0)$

解 
$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)] \Big|_{x=0} = \frac{d}{dx}[0] \Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)] \Big|_{y=0} = \frac{d}{dy}[0] \Big|_{y=0} = 0,$$

注 偏导数存在  $\nRightarrow$  连续

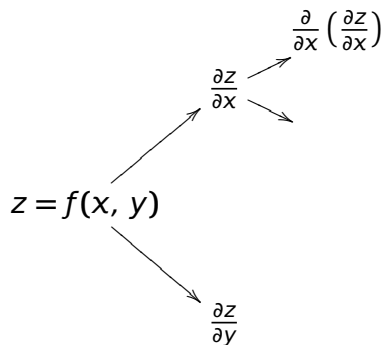
(上述  $f(x, y)$  在  $(0, 0)$  处存在偏导数  $f_x(0, 0)$  和  $f_y(0, 0)$ , 但在  $(0, 0)$  处不连续)



# 二阶偏导数

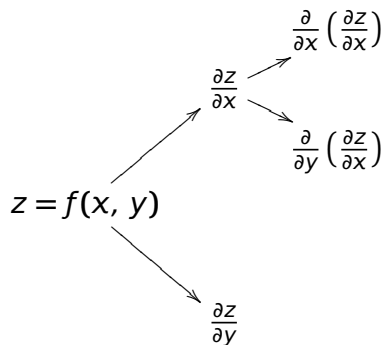
$$\begin{array}{c} \nearrow \frac{\partial z}{\partial x} \\ z = f(x, y) \\ \searrow \frac{\partial z}{\partial y} \end{array}$$

## 二阶偏导数

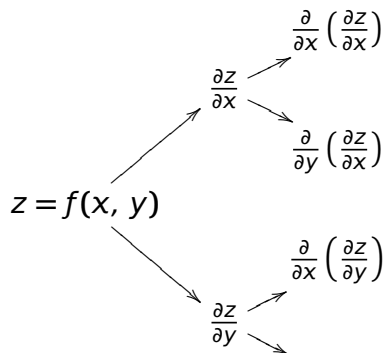




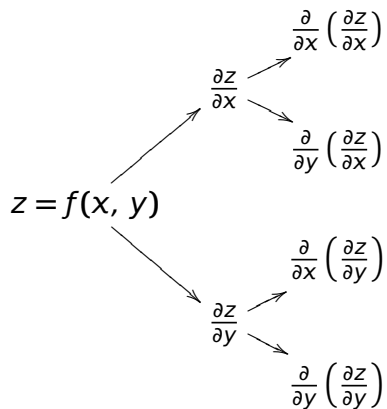
## 二阶偏导数



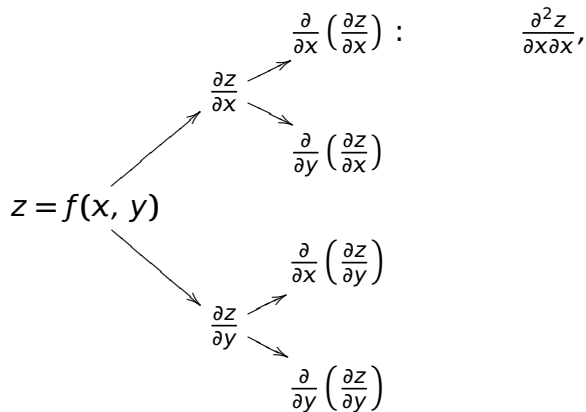
## 二阶偏导数



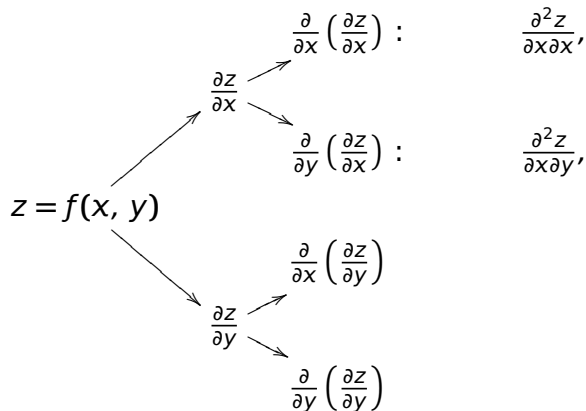
## 二阶偏导数



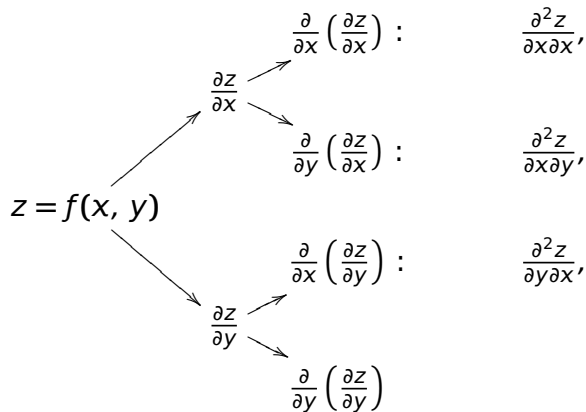
## 二阶偏导数



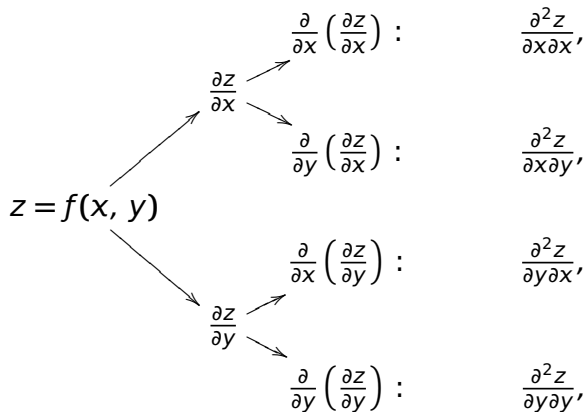
## 二阶偏导数



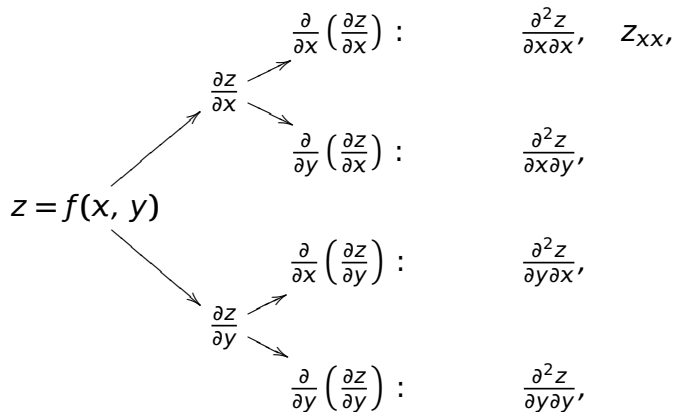
## 二阶偏导数



## 二阶偏导数

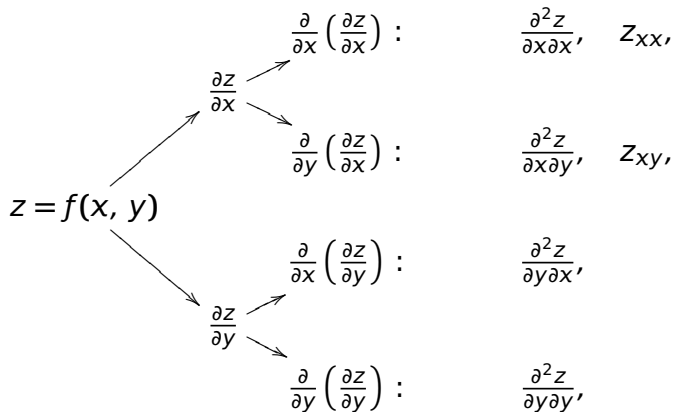


## 二阶偏导数

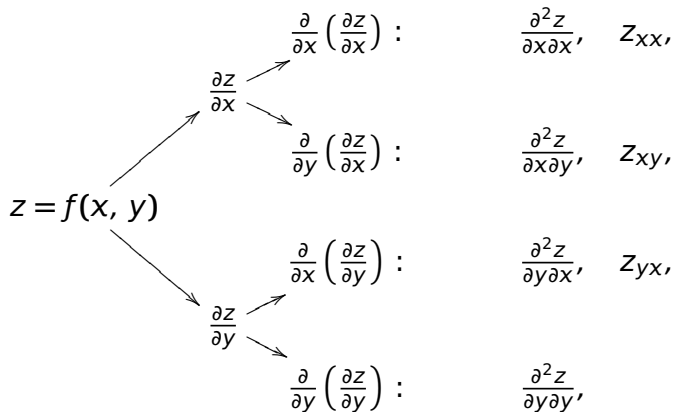




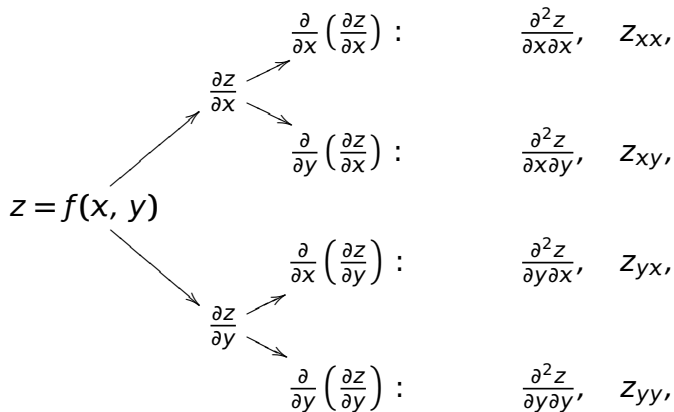
## 二阶偏导数



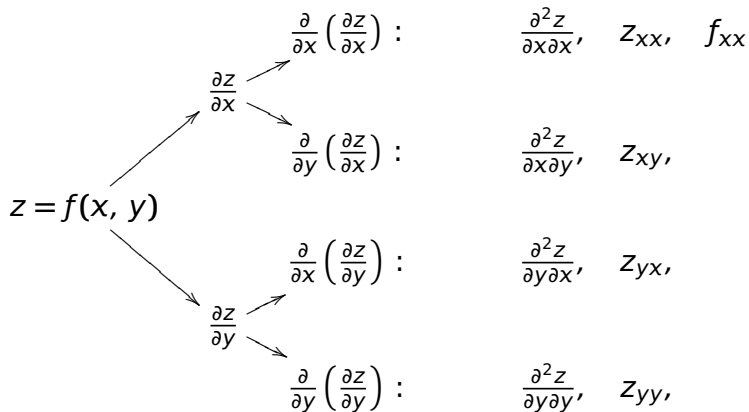
## 二阶偏导数



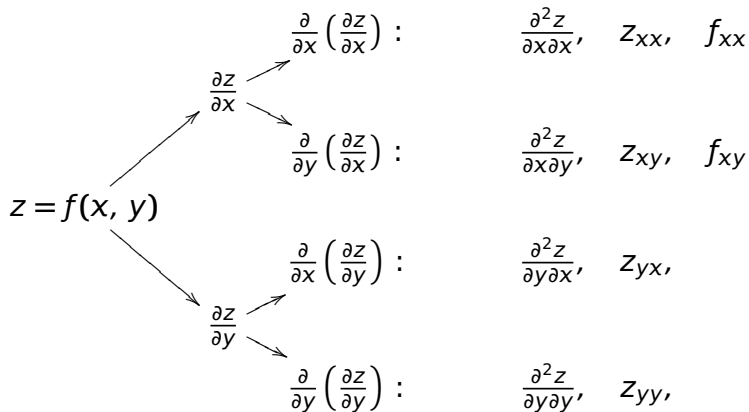
## 二阶偏导数



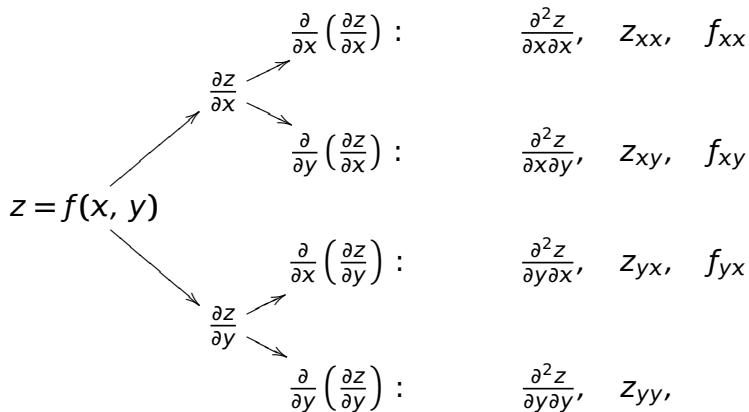
## 二阶偏导数



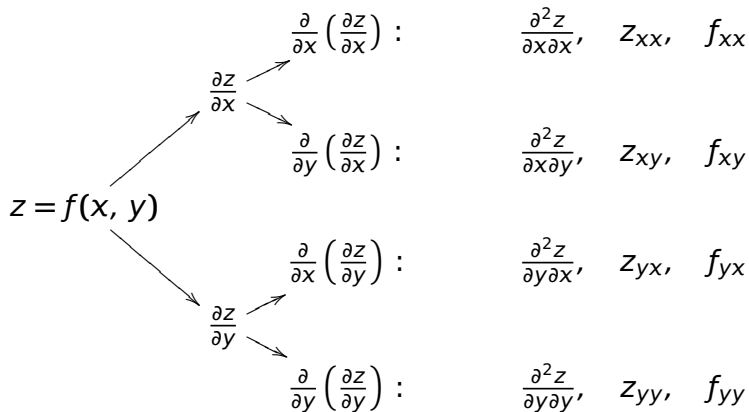
## 二阶偏导数



## 二阶偏导数



## 二阶偏导数



例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解



例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x =$$

$$z_y =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$Z_x =$$

$$Z_y =$$

$$Z_{xx} =$$

$$Z_{xy} =$$

$$Z_{yx} =$$

$$Z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} +$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$



例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} +$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} +$$

$$z_{yx} =$$

$$z_{yy} =$$



例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} +$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

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$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

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$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

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$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

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$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2 e^{xy} +$$



例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2 e^{xy} + 4x$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

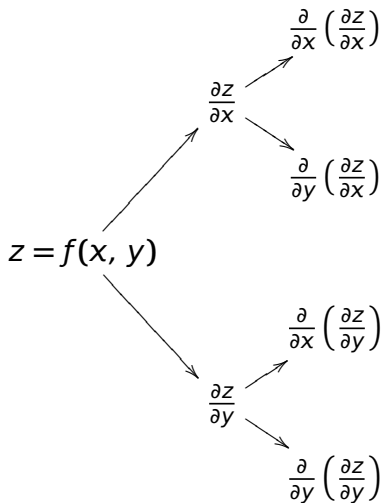
$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

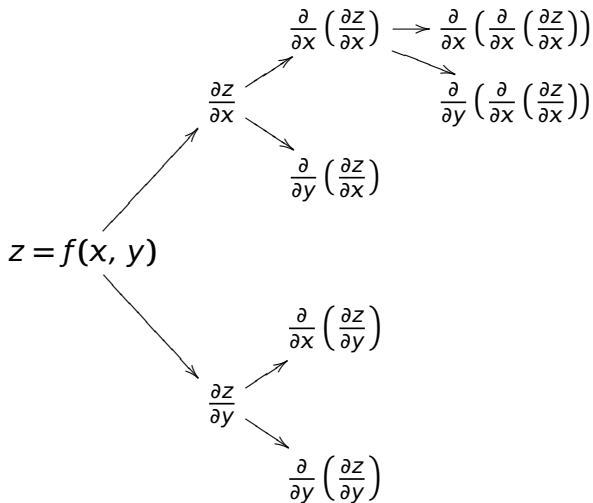
$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2 e^{xy} + 4x$$

注 此例成立  $z_{xy} = z_{yx}$

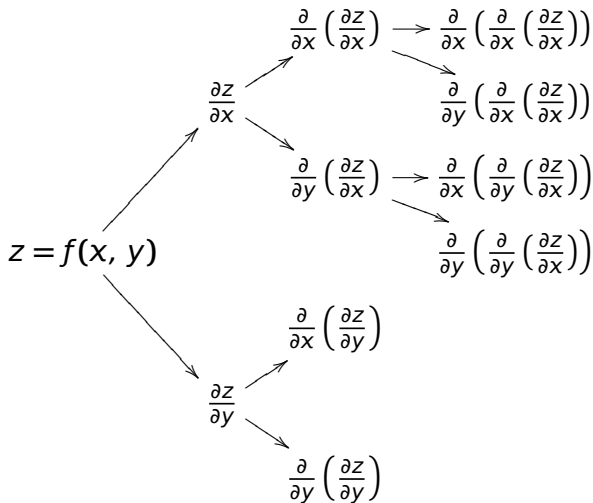
# 三阶偏导数



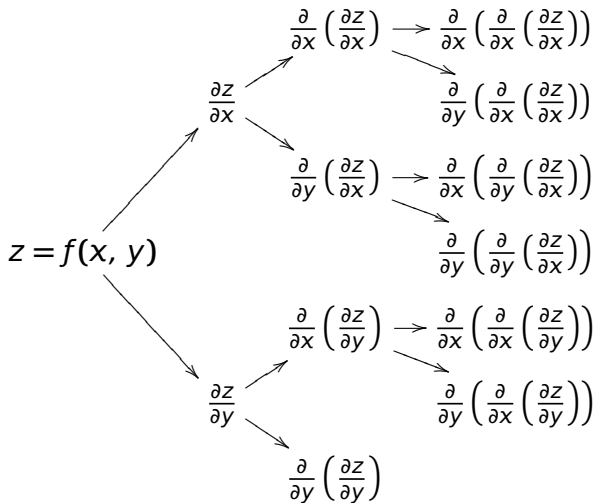
# 三阶偏导数



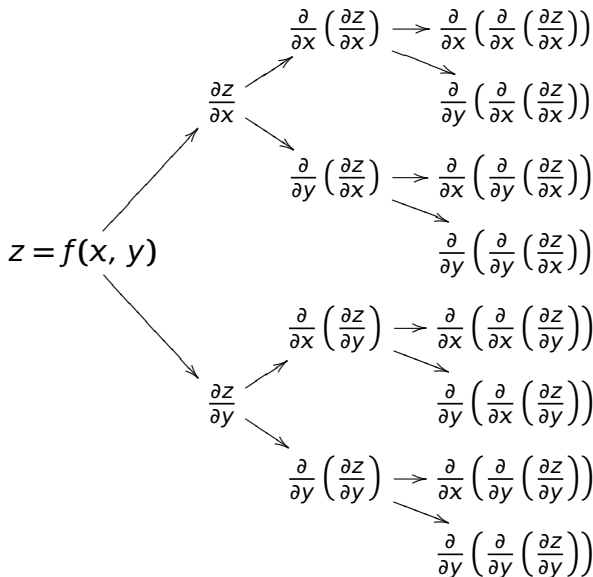
# 三阶偏导数



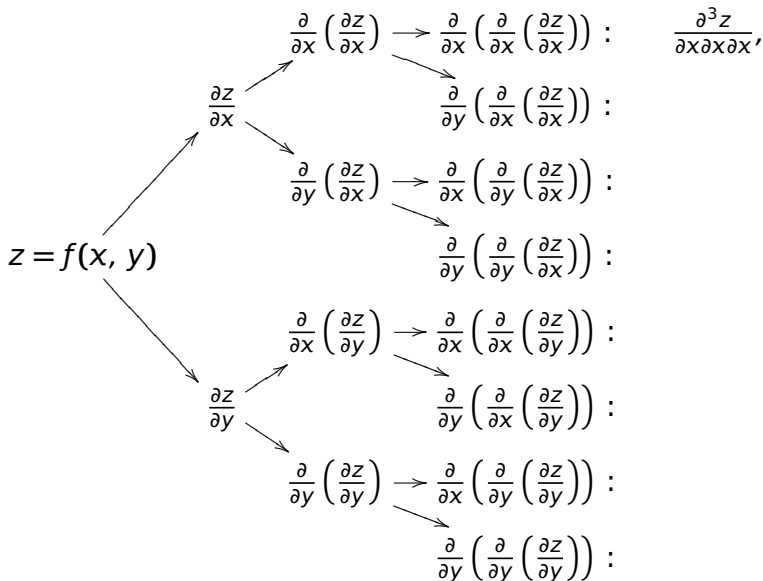
# 三阶偏导数



# 三阶偏导数

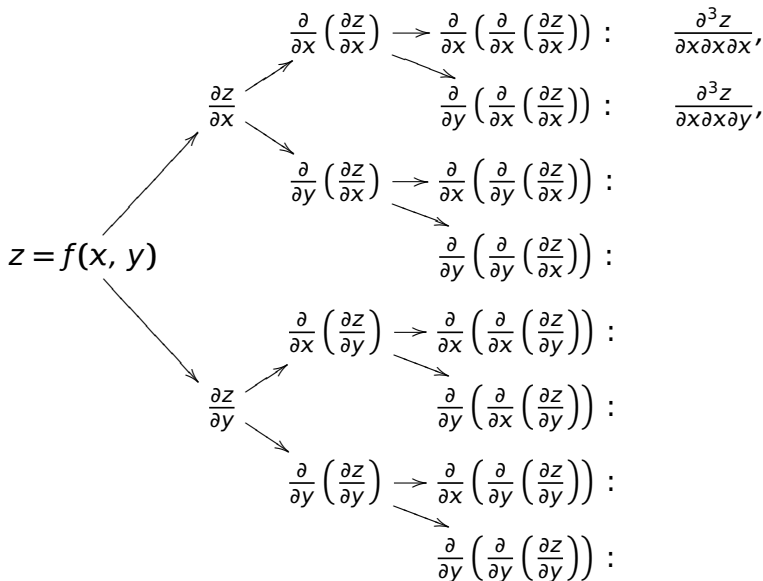


# 三阶偏导数

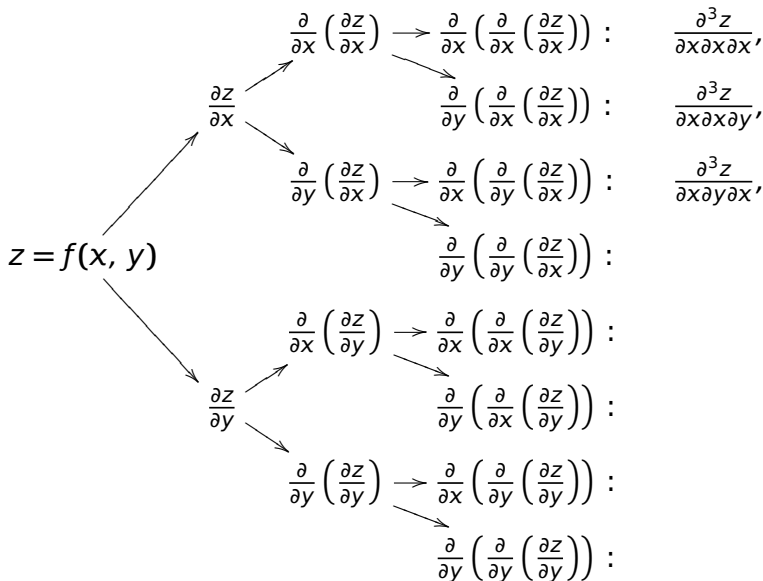




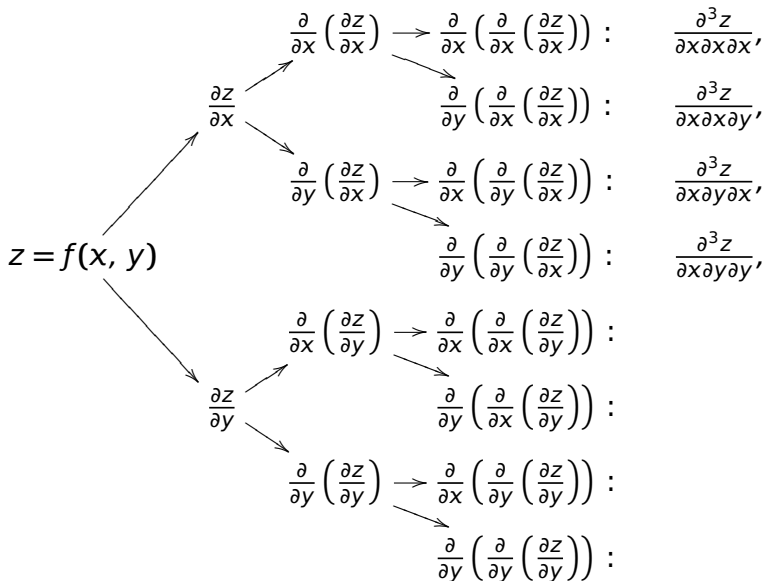
# 三阶偏导数



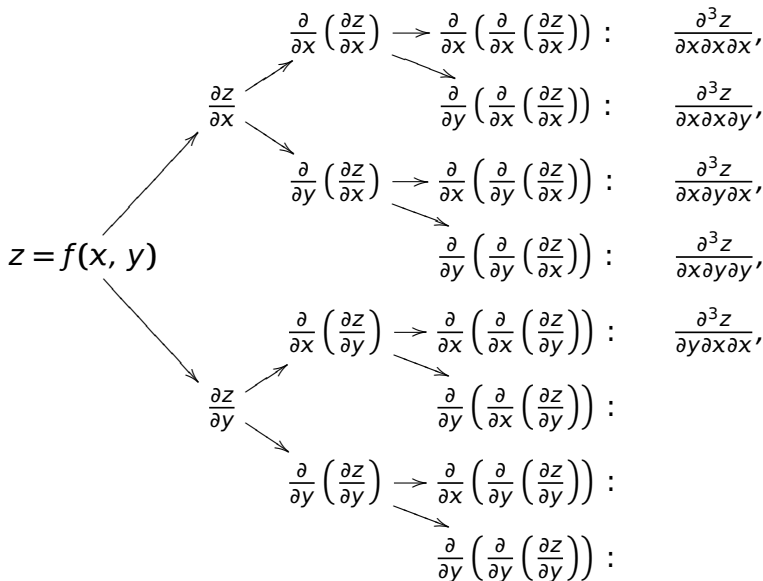
# 三阶偏导数



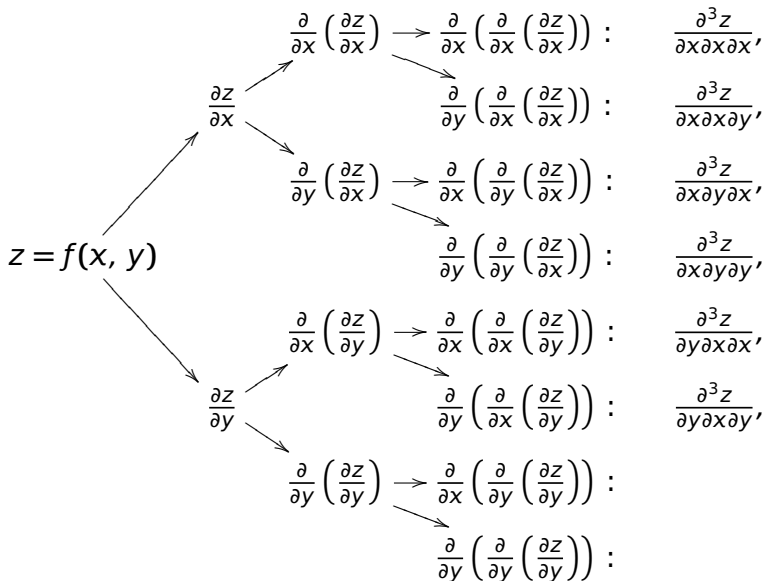
# 三阶偏导数



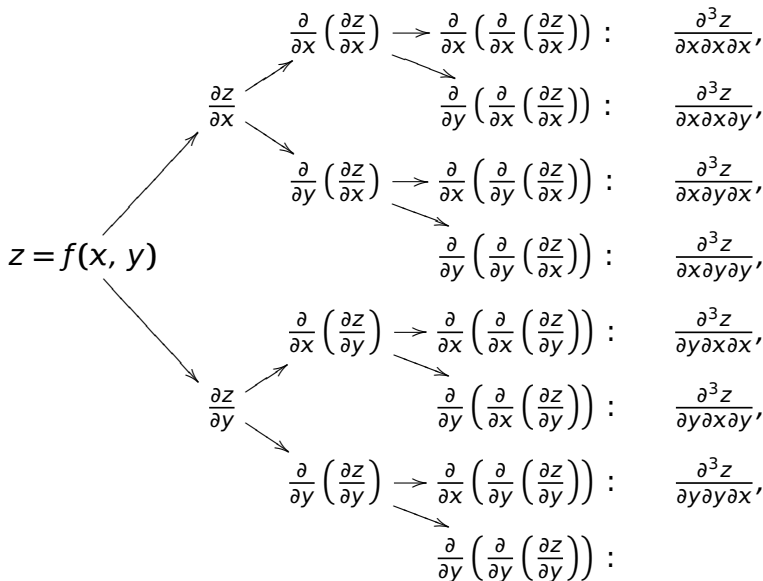
# 三阶偏导数



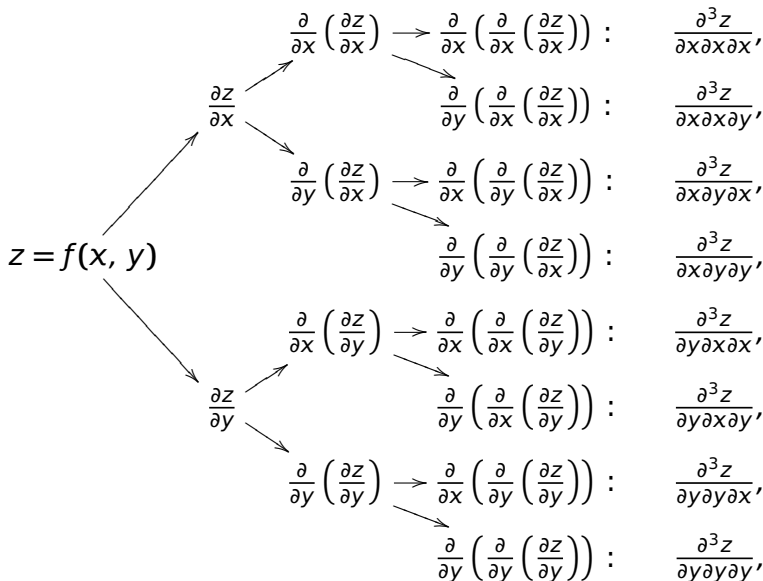
# 三阶偏导数



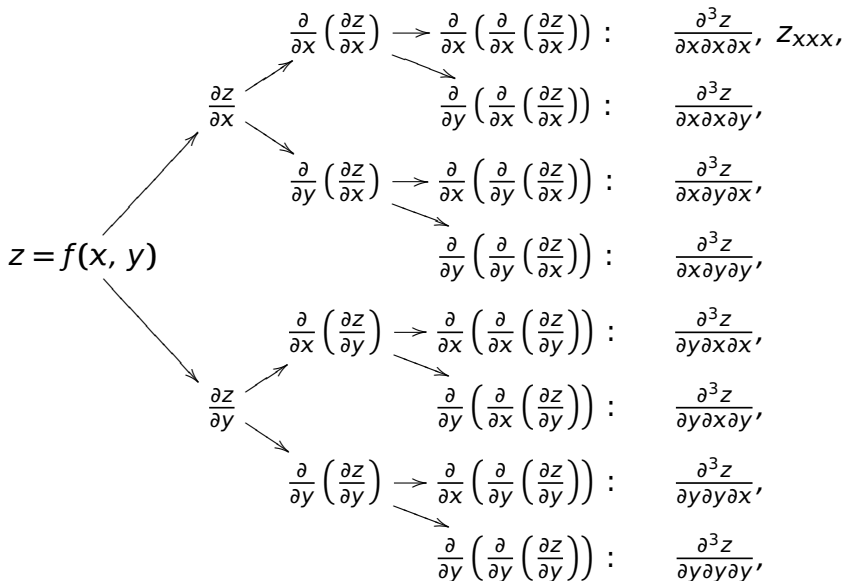
# 三阶偏导数



# 三阶偏导数

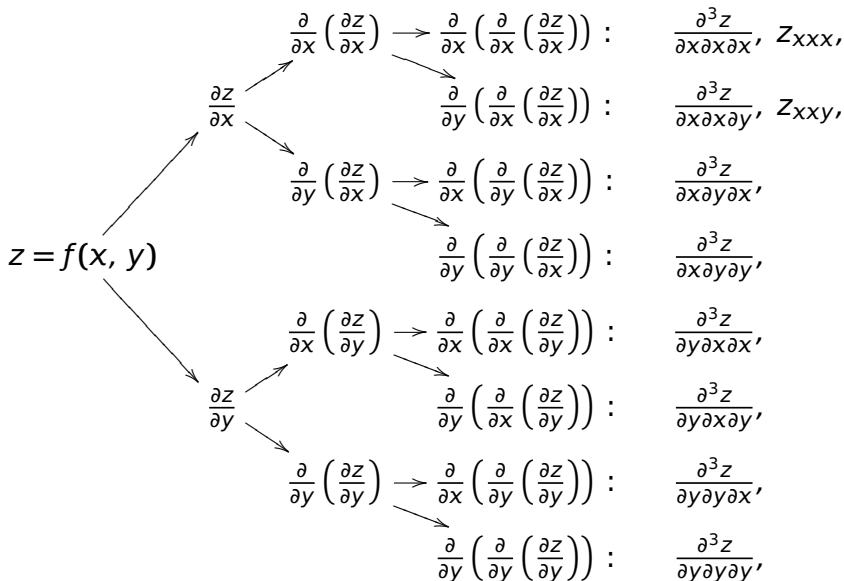


# 三阶偏导数

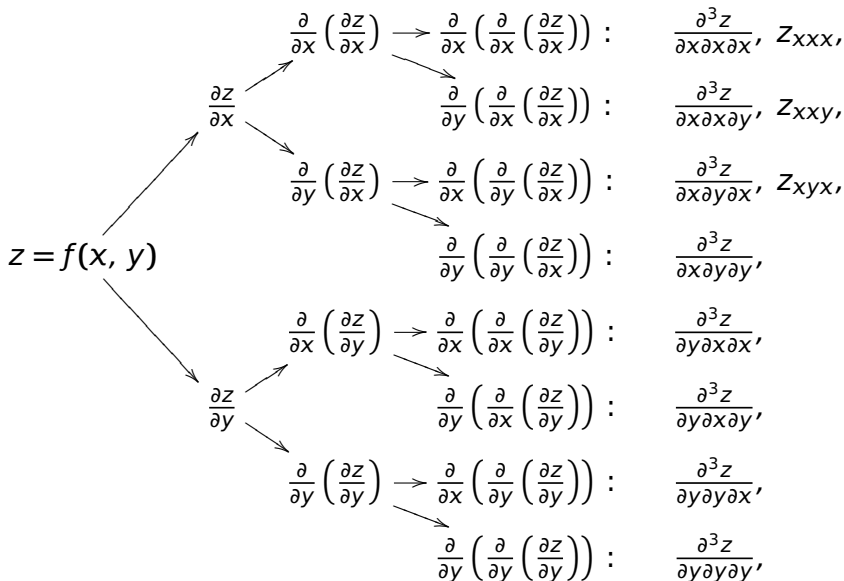




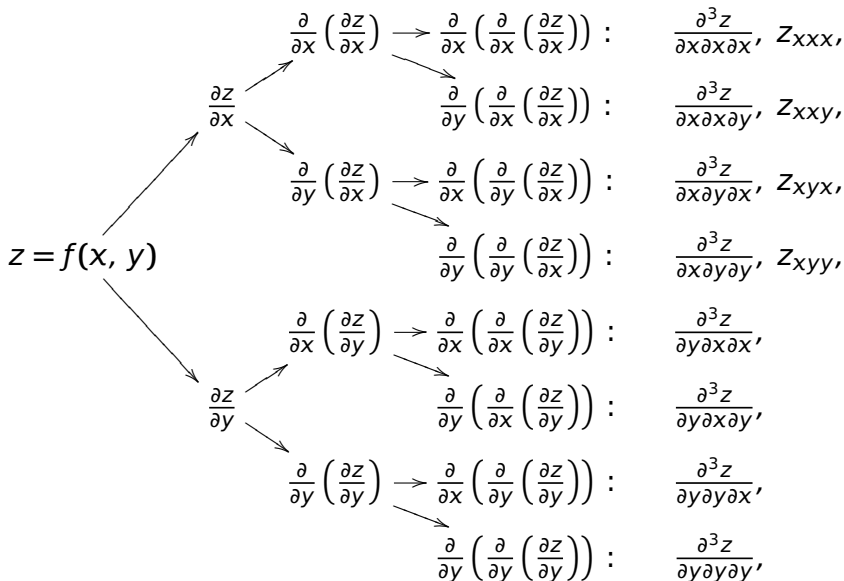
# 三阶偏导数



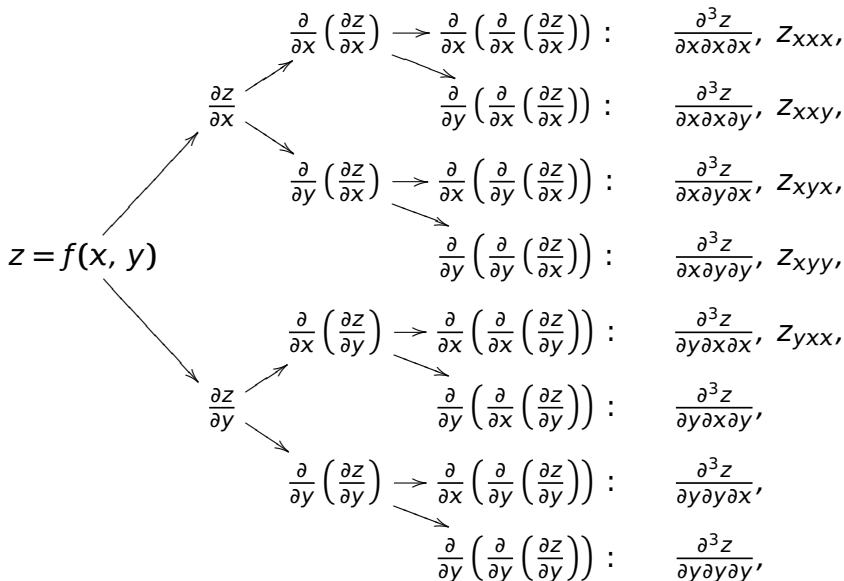
# 三阶偏导数



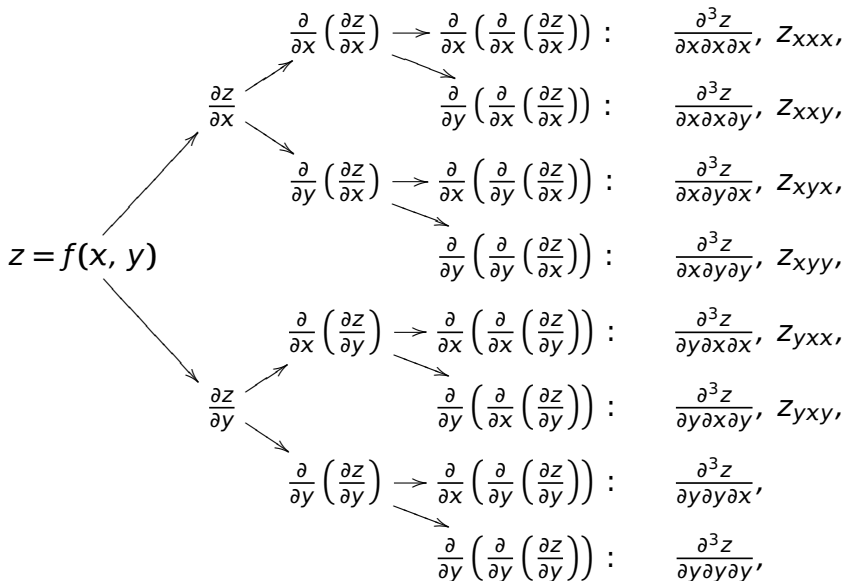
# 三阶偏导数



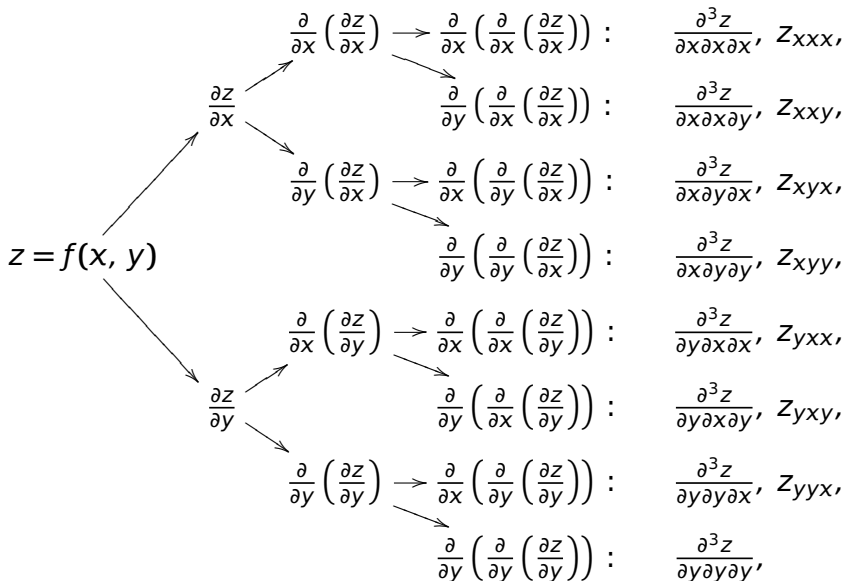
# 三阶偏导数



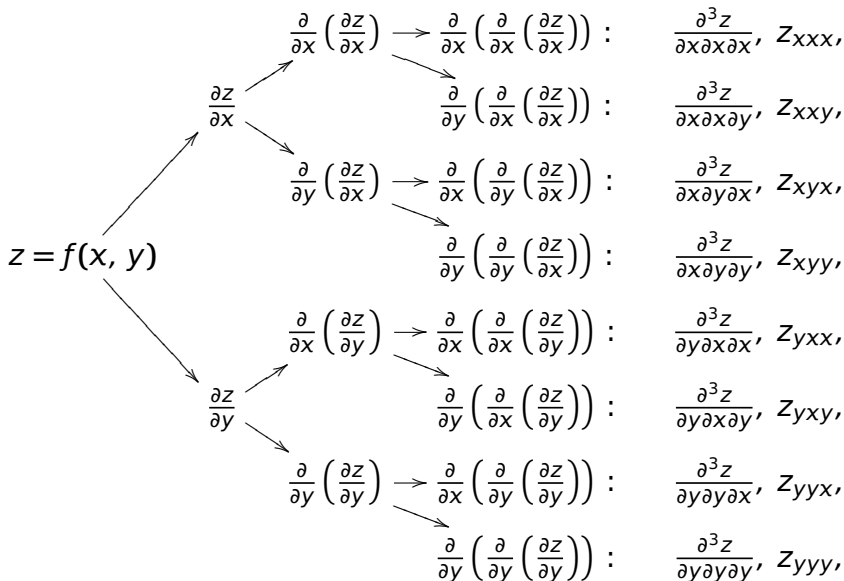
# 三阶偏导数



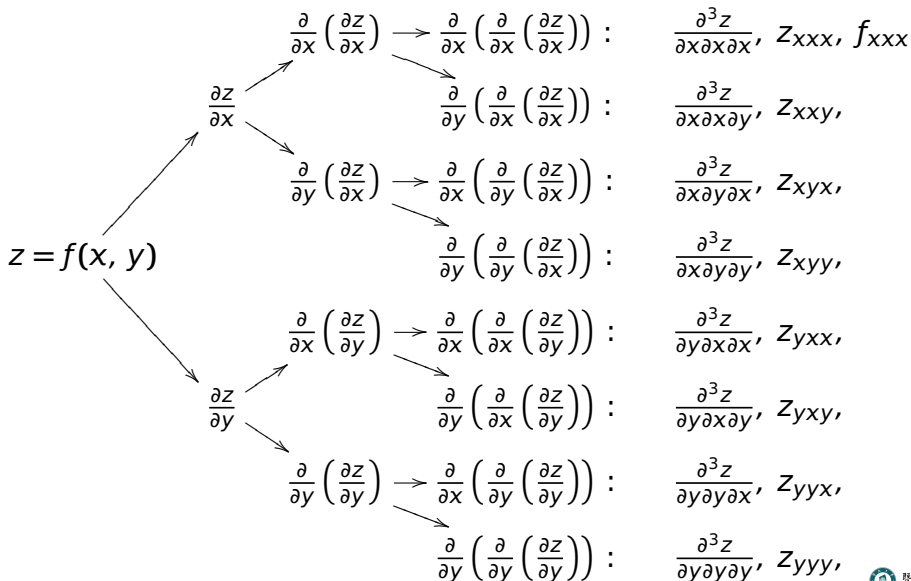
# 三阶偏导数



# 三阶偏导数

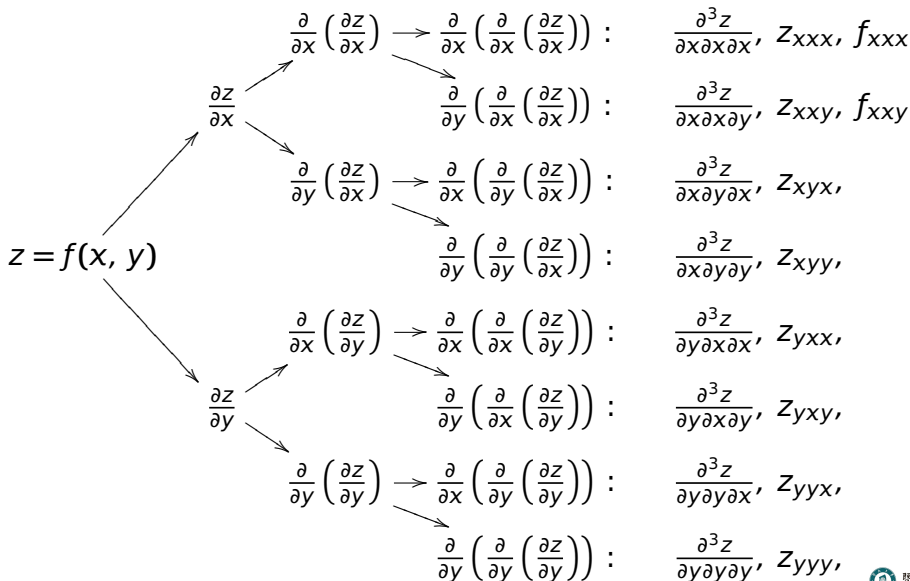


# 三阶偏导数

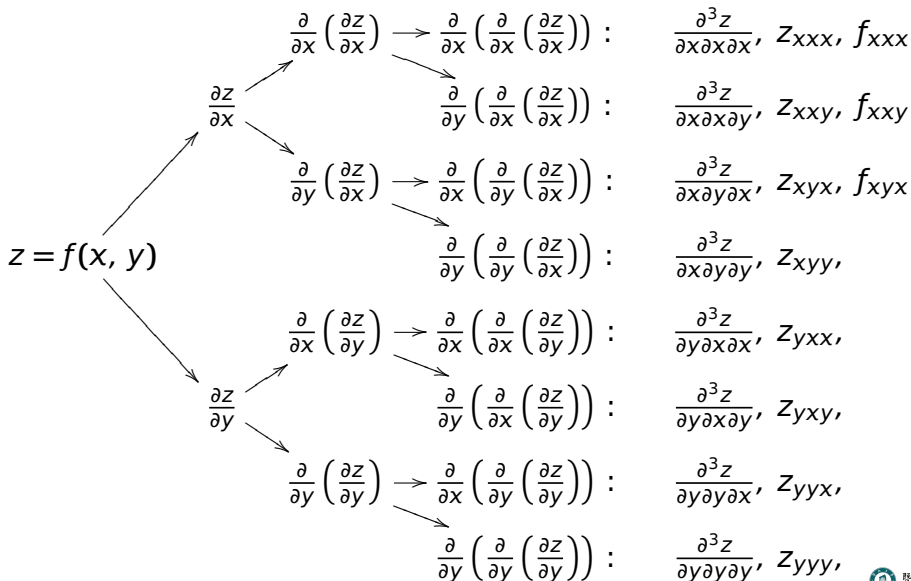




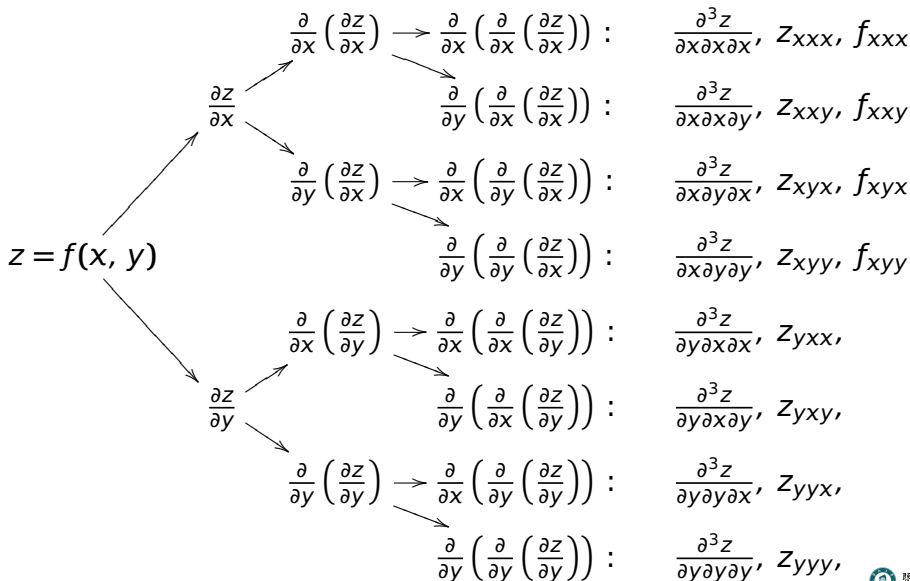
# 三阶偏导数



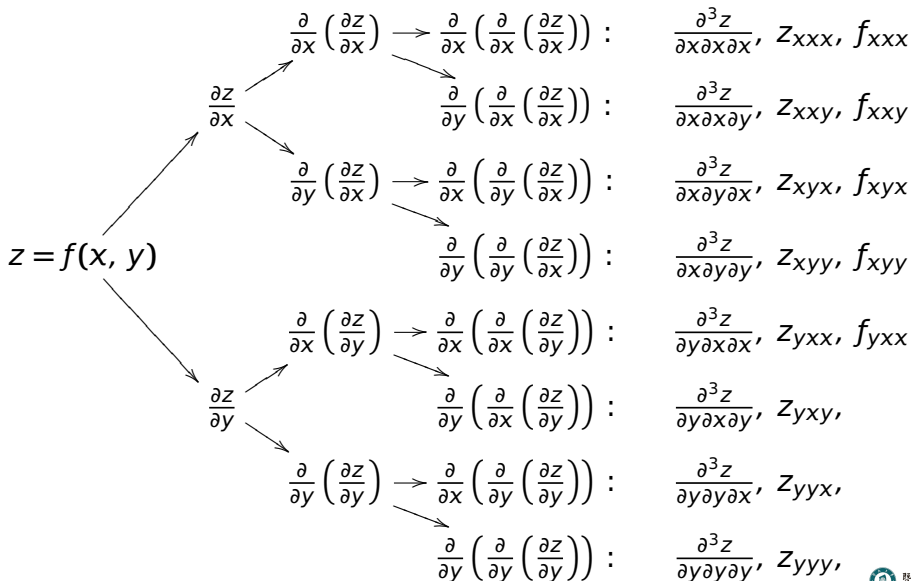
# 三阶偏导数



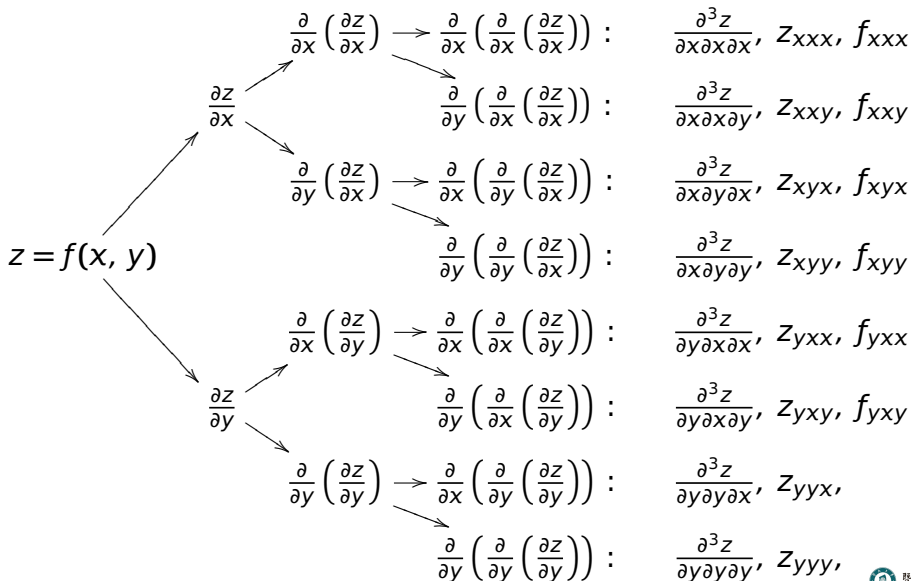
# 三阶偏导数



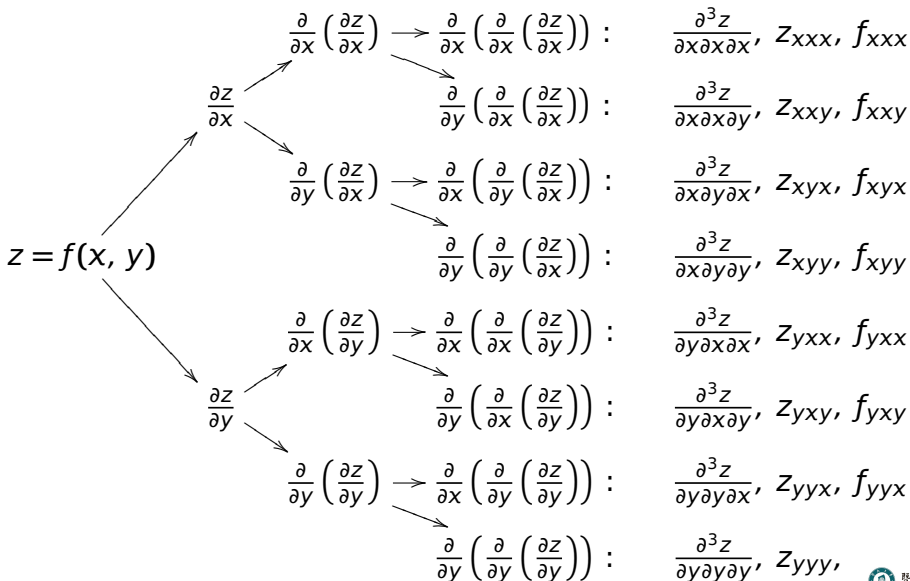
# 三阶偏导数



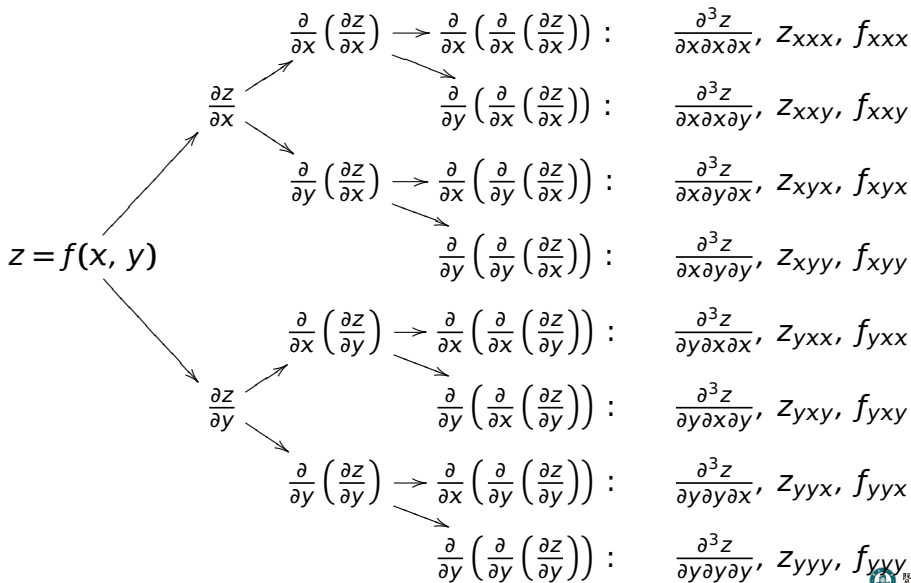
# 三阶偏导数



# 三阶偏导数



# 三阶偏导数



例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解



例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解  $z_x =$

$z_y =$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解  $z_x =$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解  $z_x =$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解  $z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x =$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解  $z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解 
$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解  $z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

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$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$



例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

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$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x + 1$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

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$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

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$$z_{yx} =$$

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$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$



例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} =$$



例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} = (6xy^2)'_x =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

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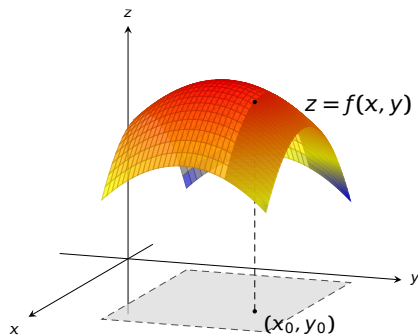
$$z_{xyy} = (3 \cos(3y))'_y = -9 \sin(3y)$$

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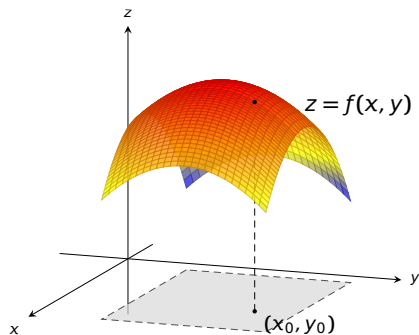
**性质** 设有二元函数  $z = f(x, y)$ 。若  $\frac{\partial^2 z}{\partial y \partial x}$  和  $\frac{\partial^2 z}{\partial x \partial y}$  均连续, 则

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$$

# 偏导数的几何直观



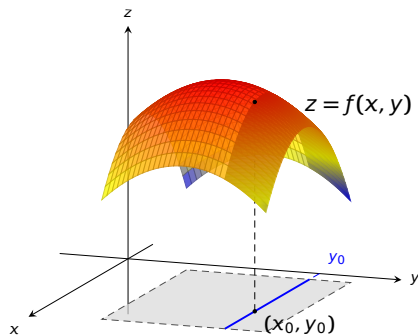
$$\left. \frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \right|_{x=x_0}$$



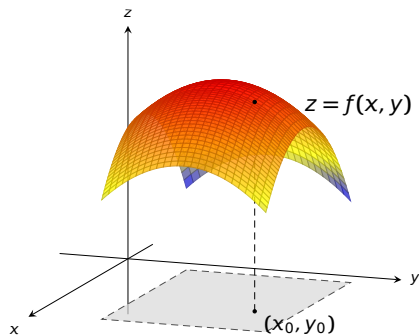
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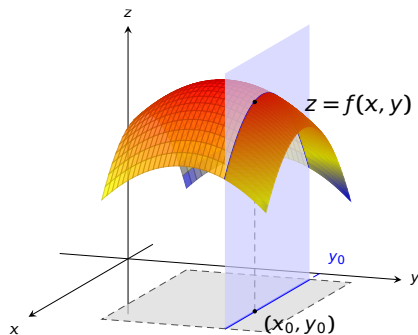


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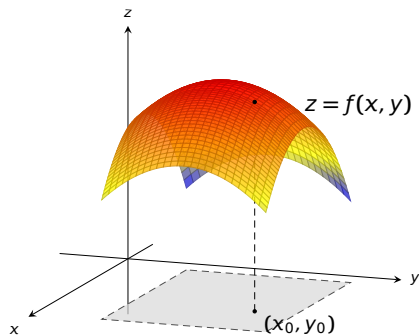


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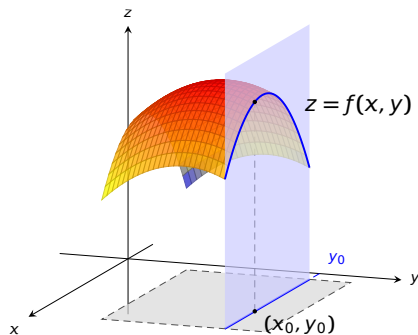


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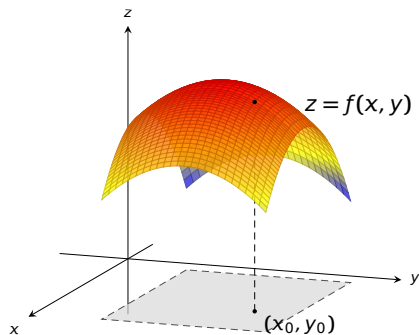


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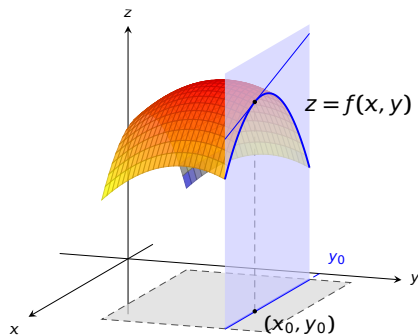


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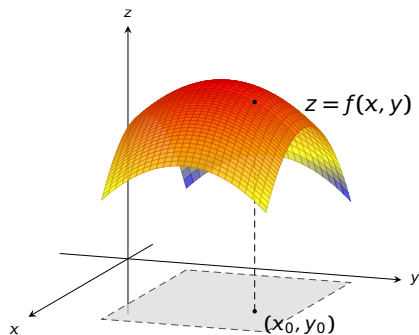


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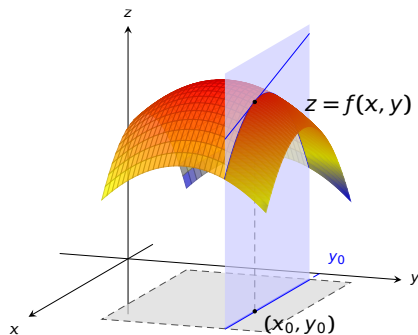


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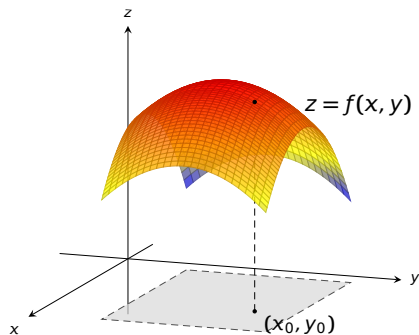


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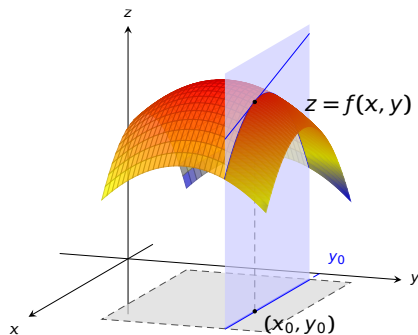


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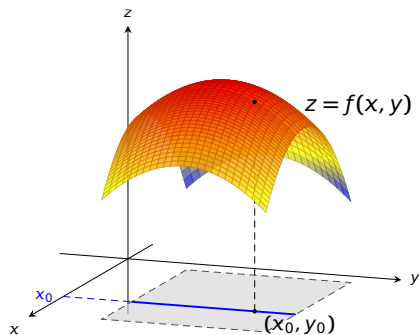


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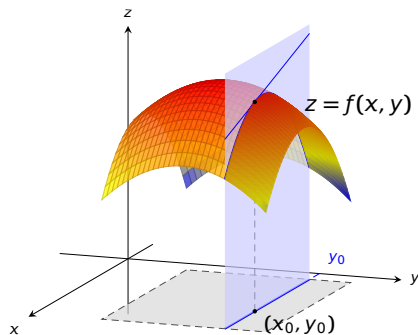


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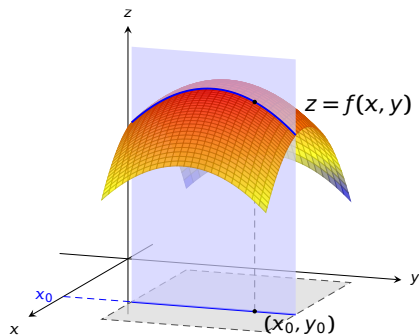


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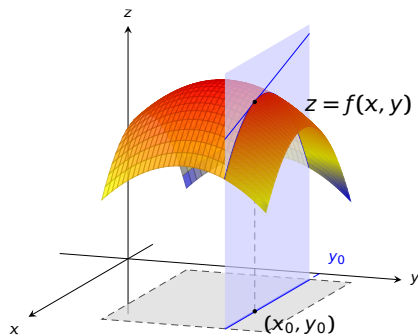


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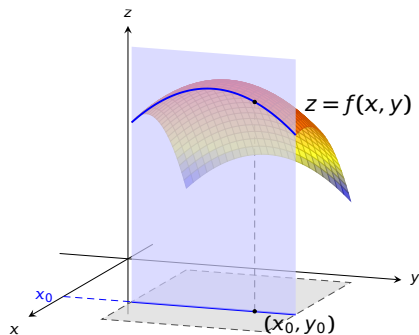


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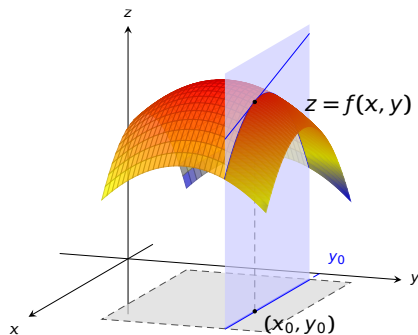
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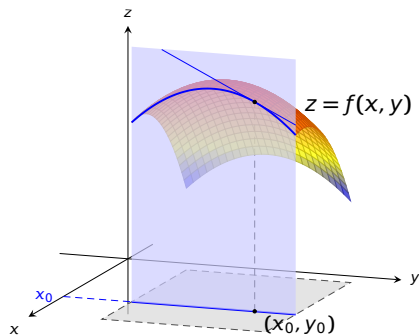
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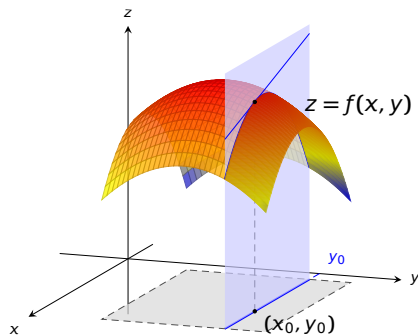


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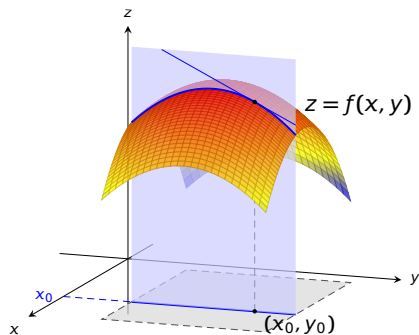


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# We are here now...

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1. 偏导数

2. 全微分

# 可微

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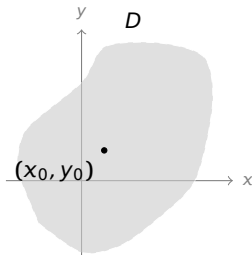
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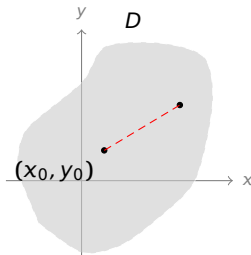


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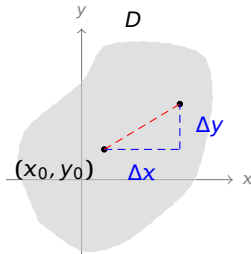


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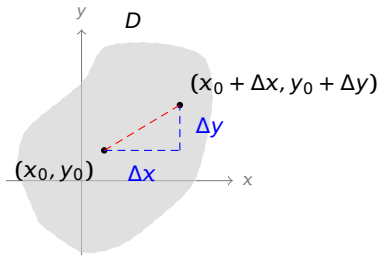


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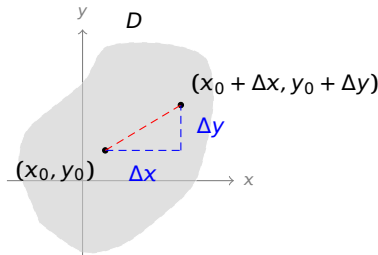


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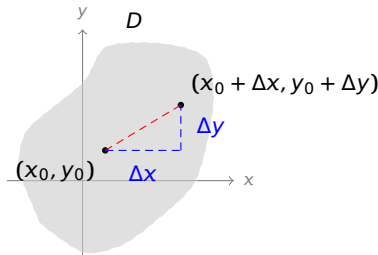
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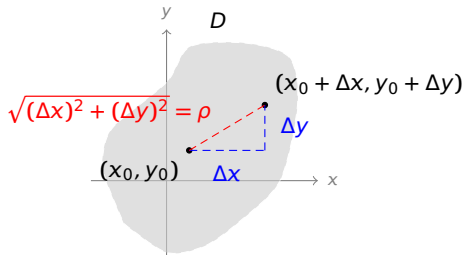
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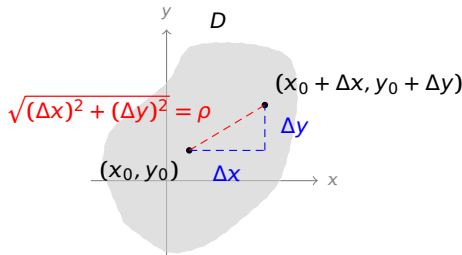
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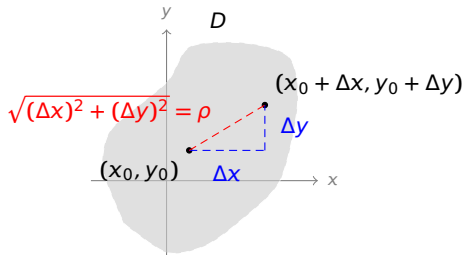
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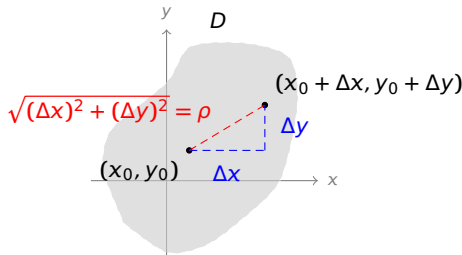
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设  $z = f(x, y)$ , 则  $f(x + \Delta x, y + \Delta y) - f(x, y) = dz + o(\rho) \approx dz$

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$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = yx^{y-1} dx + x^y \ln x dy$$

将  $(x, y) = (1, 2)$  及  $dx = \Delta x = 0.04$ 、 $dy = \Delta y = 0.02$  代入得:

$$dz = 2 \cdot 1^1 \cdot 0.04 + 1^2 \cdot \ln 1 \cdot 0.02 = 0.08$$

所以  $(1.04)^{2.02} \approx dz + 1 = 0.08 + 1 = 1.08$

# 可微、偏导数存在、连续的区别与联系

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设有二元函数  $z = f(x, y)$

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- 在点  $(x_0, y_0)$  处存在可微  $\Rightarrow f$  在点  $(x_0, y_0)$  处连续, 且存在偏导数  $\frac{\partial z}{\partial x}(x_0, y_0), \frac{\partial z}{\partial y}(x_0, y_0)$



# 可微、偏导数存在、连续的区别与联系

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- 在点  $(x_0, y_0)$  附近存在偏导数  $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ , 且偏导数  $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$  在点  $(x_0, y_0)$  处连续  $\Rightarrow$  在点  $(x_0, y_0)$  处可微