第 9 章 b：偏导数与全微分

数学系　　梁卓滨

2017-2018 学年 II

Outline

We are here now...

1. 偏导数

2. 全微分

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？
  1. 固定 y，对 x 求导

  2. 固定 x，对 y 求导

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？
  1. 固定 y，对 x 求导

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \qquad \text{对x偏导数}$$

  2. 固定 x，对 y 求导

偏导数引入

- 对一元函数 $y=f(x)$：导数 $y'=f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z=f(x,y)$：导数？
  1. 固定 y，对 x 求导

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \qquad \text{对}x\text{偏导数}$$

  2. 固定 x，对 y 求导

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \qquad \text{对}y\text{偏导数}$$

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？

1. 固定 y，对 x 求导：$z = f(x, y)$ 关于 x 的变化率

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \qquad \text{对}x\text{偏导数}$$

2. 固定 x，对 y 求导

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \qquad \text{对}y\text{偏导数}$$

偏导数引入

- 对一元函数 $y=f(x)$：导数 $y'=f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z=f(x,y)$：导数？
  1. 固定 y，对 x 求导：$z=f(x,y)$ 关于 x 的变化率

$$\frac{\partial z}{\partial x} \quad 或 \quad z'_x \quad 或 \quad z_x \quad 或 \quad f_x \qquad \text{对}x\text{偏导数}$$

  2. 固定 x，对 y 求导：$z=f(x,y)$ 关于 y 的变化率

$$\frac{\partial z}{\partial y} \quad 或 \quad z'_y \quad 或 \quad z_y \quad 或 \quad f_y \qquad \text{对}y\text{偏导数}$$

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？
  1. 固定 y，对 x 求导：$z = f(x, y)$ 关于 x 的变化率

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \qquad \text{对}x\text{偏导数}$$

  2. 固定 x，对 y 求导：$z = f(x, y)$ 关于 y 的变化率

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \qquad \text{对}y\text{偏导数}$$

例 1 设 $z = f(x, y) = x^2 y + 2x + y + 1$，则

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？
  1. 固定 y，对 x 求导：$z = f(x, y)$ 关于 x 的变化率

$$\frac{\partial z}{\partial x} \quad 或 \quad z'_x \quad 或 \quad z_x \quad 或 \quad f_x$$

对x偏导数

  2. 固定 x，对 y 求导：$z = f(x, y)$ 关于 y 的变化率

$$\frac{\partial z}{\partial y} \quad 或 \quad z'_y \quad 或 \quad z_y \quad 或 \quad f_y$$

对y偏导数

例 1 设 $z = f(x, y) = x^2 y + 2x + y + 1$，则

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

偏导数引入

- 对一元函数 $y=f(x)$：导数 $y'=f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z=f(x,y)$：导数？
  1. 固定 y，对 x 求导：$z=f(x,y)$ 关于 x 的变化率

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \qquad \text{对}x\text{偏导数}$$

  2. 固定 x，对 y 求导：$z=f(x,y)$ 关于 y 的变化率

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \qquad \text{对}y\text{偏导数}$$

例 1 设 $z=f(x,y)=x^2y+2x+y+1$，则

$$\frac{\partial z}{\partial x}=(x^2y+2x+y+1)'_x=$$

$$\frac{\partial z}{\partial y}=$$

偏导数引入

- 对一元函数 $y=f(x)$：导数 $y'=f'(x)\longleftrightarrow$ 变化率
- 对二元函数 $z=f(x,y)$：导数？

1. 固定 y，对 x 求导：$z=f(x,y)$ 关于 x 的变化率

$$\frac{\partial z}{\partial x} \quad 或 \quad z'_x \quad 或 \quad z_x \quad 或 \quad f_x \qquad 对x偏导数$$

2. 固定 x，对 y 求导：$z=f(x,y)$ 关于 y 的变化率

$$\frac{\partial z}{\partial y} \quad 或 \quad z'_y \quad 或 \quad z_y \quad 或 \quad f_y \qquad 对y偏导数$$

例 1 设 $z=f(x,y)=x^2y+2x+y+1$，则

$$\frac{\partial z}{\partial x}=(x^2y+2x+y+1)'_x=2xy+$$

$$\frac{\partial z}{\partial y}=$$

偏导数引入

- 对一元函数 $y=f(x)$：导数 $y'=f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z=f(x,y)$：导数？
  1. 固定 y，对 x 求导：$z=f(x,y)$ 关于 x 的变化率
  $$\frac{\partial z}{\partial x} \quad 或 \quad z'_x \quad 或 \quad z_x \quad 或 \quad f_x \qquad \text{对}x\text{偏导数}$$
  2. 固定 x，对 y 求导：$z=f(x,y)$ 关于 y 的变化率
  $$\frac{\partial z}{\partial y} \quad 或 \quad z'_y \quad 或 \quad z_y \quad 或 \quad f_y \qquad \text{对}y\text{偏导数}$$

例 1 设 $z=f(x,y)=x^2y+2x+y+1$，则
$$\frac{\partial z}{\partial x}=(x^2y+2x+y+1)'_x=2xy+2$$
$$\frac{\partial z}{\partial y}=$$

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？
  1. 固定 y，对 x 求导：$z = f(x, y)$ 关于 x 的变化率

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \qquad \text{对}x\text{偏导数}$$

  2. 固定 x，对 y 求导：$z = f(x, y)$ 关于 y 的变化率

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \qquad \text{对}y\text{偏导数}$$

例 1 设 $z = f(x, y) = x^2y + 2x + y + 1$，则

$$\frac{\partial z}{\partial x} = (x^2y + 2x + y + 1)'_x = 2xy + 2$$

$$\frac{\partial z}{\partial y} = (x^2y + 2x + y + 1)'_y =$$

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？
  1. 固定 y，对 x 求导：$z = f(x, y)$ 关于 x 的变化率

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \qquad \text{对}x\text{偏导数}$$

  2. 固定 x，对 y 求导：$z = f(x, y)$ 关于 y 的变化率

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \qquad \text{对}y\text{偏导数}$$

例 1 设 $z = f(x, y) = x^2y + 2x + y + 1$，则

$$\frac{\partial z}{\partial x} = (x^2y + 2x + y + 1)'_x = 2xy + 2$$

$$\frac{\partial z}{\partial y} = (x^2y + 2x + y + 1)'_y = x^2 +$$

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？
  1. 固定 y，对 x 求导：$z = f(x, y)$ 关于 x 的变化率

$$\frac{\partial z}{\partial x} \quad 或 \quad z'_x \quad 或 \quad z_x \quad 或 \quad f_x \qquad \text{对}x\text{偏导数}$$

  2. 固定 x，对 y 求导：$z = f(x, y)$ 关于 y 的变化率

$$\frac{\partial z}{\partial y} \quad 或 \quad z'_y \quad 或 \quad z_y \quad 或 \quad f_y \qquad \text{对}y\text{偏导数}$$

例 1 设 $z = f(x, y) = x^2y + 2x + y + 1$，则

$$\frac{\partial z}{\partial x} = (x^2y + 2x + y + 1)'_x = 2xy + 2$$

$$\frac{\partial z}{\partial y} = (x^2y + 2x + y + 1)'_y = x^2 + 1$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

例 2 设 $z=f(x,y)=e^{xy}+2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=$$

$$\frac{\partial z}{\partial y}=$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x =$$

$$\frac{\partial z}{\partial y} =$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x =$$

$$\frac{\partial z}{\partial y} =$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} +$$

$$\frac{\partial z}{\partial y} =$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} =$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y =$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y =$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} +$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设 $z = f(x, y) = 2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设 $z = f(x, y) = 2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial y}$$

例 2 设 $z=f(x,y)=e^{xy}+2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$\frac{\partial z}{\partial y}=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

例 3 设 $z=f(x,y)=2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=(2y\sin(3x))'_x=$$

$$\frac{\partial z}{\partial y}$$

例 2 设 $z=f(x,y)=e^{xy}+2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$
$$\frac{\partial z}{\partial y}=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

例 3 设 $z=f(x,y)=2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=(2y\sin(3x))'_x=2y(\sin(3x))'_x=$$
$$\frac{\partial z}{\partial y}$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设 $z = f(x, y) = 2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y\sin(3x))'_x = 2y(\sin(3x))'_x = 2y \cdot 3\cos(3x) =$$

$$\frac{\partial z}{\partial y}$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设 $z = f(x, y) = 2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y\sin(3x))'_x = 2y(\sin(3x))'_x = 2y \cdot 3\cos(3x) = 6y\cos(3x)$$

$$\frac{\partial z}{\partial y}$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$
$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设 $z = f(x, y) = 2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y\sin(3x))'_x = 2y(\sin(3x))'_x = 2y \cdot 3\cos(3x) = 6y\cos(3x)$$
$$\frac{\partial z}{\partial y} = (2y\sin(3x))'_y =$$

例 2 设 $z=f(x,y)=e^{xy}+2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$
$$\frac{\partial z}{\partial y}=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

例 3 设 $z=f(x,y)=2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=(2y\sin(3x))'_x=2y(\sin(3x))'_x=2y\cdot 3\cos(3x)=6y\cos(3x)$$
$$\frac{\partial z}{\partial y}=(2y\sin(3x))'_y=(2y)'_y\cdot\sin(3x)=$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设 $z = f(x, y) = 2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y\sin(3x))'_x = 2y(\sin(3x))'_x = 2y \cdot 3\cos(3x) = 6y\cos(3x)$$

$$\frac{\partial z}{\partial y} = (2y\sin(3x))'_y = (2y)'_y \cdot \sin(3x) = 2\sin(3x)$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解 $u_x =$

$u_y =$

$u_z =$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x =$$

$$u_y =$$

$$u_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x =$$

$$u_y =$$

$$u_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz$$

$$u_y =$$

$$u_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y =$$

$$u_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解 $$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y =$$

$$u_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y =$$

$$u_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z =$$

暨南大學 JINAN UNIVERSITY

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z = xy$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解
$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z = xy + \frac{1}{x}$$

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 x 的偏增量：

$$f(x_0 + \Delta x, y_0) - f(x_0, y_0)$$

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 x 的偏增量:

$$\Delta_x z = f(x_0 + \Delta x, y_0) - f(x_0, y_0)$$

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 x 的偏增量：（x 方向的改变量）
$$\Delta_x z = f(x_0 + \Delta x, y_0) - f(x_0, y_0)$$

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 x 的偏增量：（x 方向的改变量）
$$\Delta_x z = f(x_0 + \Delta x, y_0) - f(x_0, y_0)$$
- $z = f(x, y)$ 在点 (x_0, y_0) 关于 x 的偏导数：

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 x 的偏增量：（x 方向的改变量）
$$\Delta_x z = f(x_0 + \Delta x, y_0) - f(x_0, y_0)$$

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 x 的偏导数：
$$\lim_{\Delta x \to 0} \frac{\Delta_x z}{\Delta x}$$

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 x 的偏增量：（x 方向的改变量）
$$\Delta_x z = f(x_0 + \Delta x, y_0) - f(x_0, y_0)$$

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 x 的偏导数：
$$\lim_{\Delta x \to 0} \frac{\Delta_x z}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$

偏导数准确定义

- $z=f(x,y)$ 在点 (x_0,y_0) 关于 x 的偏增量：（x 方向的改变量）
$$\Delta_x z = f(x_0+\Delta x, y_0) - f(x_0, y_0)$$

- $z=f(x,y)$ 在点 (x_0,y_0) 关于 x 的偏导数：
$$\lim_{\Delta x\to 0}\frac{\Delta_x z}{\Delta x} = \lim_{\Delta x\to 0}\frac{f(x_0+\Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$
$$= \frac{d}{dx}[f(x,y_0)]\bigg|_{x=x_0}$$

偏导数准确定义

- $z=f(x,y)$ 在点 (x_0,y_0) 关于 x 的偏增量： （x 方向的改变量）

$$\Delta_x z = f(x_0+\Delta x, y_0) - f(x_0, y_0)$$

- $z=f(x,y)$ 在点 (x_0,y_0) 关于 x 的偏导数： （x 方向的导数）

$$\lim_{\Delta x\to 0}\frac{\Delta_x z}{\Delta x} = \lim_{\Delta x\to 0}\frac{f(x_0+\Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$

$$= \frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0}$$

偏导数准确定义

- $z=f(x,y)$ 在点 (x_0,y_0) 关于 x 的偏增量：（x 方向的改变量）

$$\Delta_x z = f(x_0+\Delta x, y_0) - f(x_0, y_0)$$

- $z=f(x,y)$ 在点 (x_0,y_0) 关于 x 的偏导数：（x 方向的导数）

$$\lim_{\Delta x\to 0}\frac{\Delta_x z}{\Delta x} = \lim_{\Delta x\to 0}\frac{f(x_0+\Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$

$$= \frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0}$$

偏导数记号：

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 x 的偏增量：（x 方向的改变量）

$$\Delta_x z = f(x_0 + \Delta x, y_0) - f(x_0, y_0)$$

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 x 的偏导数：（x 方向的导数）

$$\lim_{\Delta x \to 0} \frac{\Delta_x z}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$

$$= \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}$$

偏导数记号：

$$\frac{\partial z}{\partial x} \qquad z'_x \qquad z_x$$

偏导数准确定义

- $z=f(x,y)$ 在点 (x_0,y_0) 关于 x 的偏增量：（x 方向的改变量）

$$\Delta_x z=f(x_0+\Delta x,y_0)-f(x_0,y_0)$$

- $z=f(x,y)$ 在点 (x_0,y_0) 关于 x 的偏导数：（x 方向的导数）

$$\lim_{\Delta x\to 0}\frac{\Delta_x z}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x,y_0)-f(x_0,y_0)}{\Delta x}$$

$$=\frac{d}{dx}[f(x,y_0)]\bigg|_{x=x_0}$$

偏导数记号：

$$\frac{\partial z}{\partial x}\bigg|_{\substack{x=x_0\\y=y_0}},\qquad z'_x\bigg|_{\substack{x=x_0\\y=y_0}},\qquad z_x\bigg|_{\substack{x=x_0\\y=y_0}}$$

偏导数准确定义

- $z=f(x,y)$ 在点 (x_0,y_0) 关于 x 的偏增量：（x 方向的改变量）
$$\Delta_x z=f(x_0+\Delta x,y_0)-f(x_0,y_0)$$
- $z=f(x,y)$ 在点 (x_0,y_0) 关于 x 的偏导数：（x 方向的导数）
$$\lim_{\Delta x\to 0}\frac{\Delta_x z}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x,y_0)-f(x_0,y_0)}{\Delta x}$$
$$=\frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0}$$

偏导数记号：

$$\frac{\partial z}{\partial x}\Big|_{\substack{x=x_0\\y=y_0}},\qquad z'_x\Big|_{\substack{x=x_0\\y=y_0}},\qquad z_x\Big|_{\substack{x=x_0\\y=y_0}}$$

$$\frac{\partial f}{\partial x}\qquad f'_x\qquad f_x$$

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 x 的偏增量：（x 方向的改变量）
$$\Delta_x z = f(x_0 + \Delta x, y_0) - f(x_0, y_0)$$

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 x 的偏导数：（x 方向的导数）
$$\lim_{\Delta x \to 0} \frac{\Delta_x z}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$
$$= \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}$$

偏导数记号：

$$\frac{\partial z}{\partial x}\Big|_{\substack{x=x_0\\y=y_0}}, \qquad z'_x\Big|_{\substack{x=x_0\\y=y_0}}, \qquad z_x\Big|_{\substack{x=x_0\\y=y_0}}$$

$$\frac{\partial f}{\partial x}(x_0, y_0), \qquad f'_x(x_0, y_0), \qquad f_x(x_0, y_0)$$

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏增量:

$$f(x_0, y_0 + \Delta y) - f(x_0, y_0)$$

偏导数准确定义

- $z=f(x,y)$ 在点 (x_0,y_0) 关于 y 的偏增量：

$$\Delta_y z = f(x_0, y_0+\Delta y) - f(x_0, y_0)$$

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏增量：（y 方向的改变量）

$$\Delta_y z = f(x_0, y_0 + \Delta y) - f(x_0, y_0)$$

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏增量：（y 方向的改变量）
$$\Delta_y z = f(x_0, y_0 + \Delta y) - f(x_0, y_0)$$
- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏导数：

偏导数准确定义

- $z=f(x,y)$ 在点 (x_0,y_0) 关于 y 的偏增量：（y 方向的改变量）

$$\Delta_y z = f(x_0, y_0+\Delta y) - f(x_0, y_0)$$

- $z=f(x,y)$ 在点 (x_0,y_0) 关于 y 的偏导数：

$$\lim_{\Delta y \to 0} \frac{\Delta_y z}{\Delta y}$$

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏增量：（y 方向的改变量）
$$\Delta_y z = f(x_0, y_0 + \Delta y) - f(x_0, y_0)$$
- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏导数：
$$\lim_{\Delta y \to 0} \frac{\Delta_y z}{\Delta y} = \lim_{\Delta y \to 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$$

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏增量：（y 方向的改变量）

$$\Delta_y z = f(x_0, y_0 + \Delta y) - f(x_0, y_0)$$

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏导数：

$$\lim_{\Delta y \to 0} \frac{\Delta_y z}{\Delta y} = \lim_{\Delta y \to 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$$

$$= \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏增量：（y 方向的改变量）

$$\Delta_y z = f(x_0, y_0 + \Delta y) - f(x_0, y_0)$$

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏导数：（y 方向的导数）

$$\lim_{\Delta y \to 0} \frac{\Delta_y z}{\Delta y} = \lim_{\Delta y \to 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$$

$$= \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏增量：（y 方向的改变量）
$$\Delta_y z = f(x_0, y_0 + \Delta y) - f(x_0, y_0)$$
- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏导数：（y 方向的导数）
$$\lim_{\Delta y \to 0} \frac{\Delta_y z}{\Delta y} = \lim_{\Delta y \to 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$$
$$= \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

偏导数记号：

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏增量：（y 方向的改变量）
$$\Delta_y z = f(x_0, y_0 + \Delta y) - f(x_0, y_0)$$

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏导数：（y 方向的导数）
$$\lim_{\Delta y \to 0} \frac{\Delta_y z}{\Delta y} = \lim_{\Delta y \to 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$$
$$= \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

偏导数记号：

$$\frac{\partial z}{\partial y} \qquad z'_y \qquad z_y$$

偏导数准确定义

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏增量：（y 方向的改变量）
$$\Delta_y z = f(x_0, y_0 + \Delta y) - f(x_0, y_0)$$

- $z = f(x, y)$ 在点 (x_0, y_0) 关于 y 的偏导数：（y 方向的导数）
$$\lim_{\Delta y \to 0} \frac{\Delta_y z}{\Delta y} = \lim_{\Delta y \to 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$$
$$= \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

偏导数记号：

$$\frac{\partial z}{\partial y}\Big|_{\substack{x=x_0 \\ y=y_0}}, \qquad z'_y\Big|_{\substack{x=x_0 \\ y=y_0}}, \qquad z_y\Big|_{\substack{x=x_0 \\ y=y_0}}$$

偏导数准确定义

- $z=f(x,y)$ 在点 (x_0,y_0) 关于 y 的偏增量：（y 方向的改变量）

$$\Delta_y z=f(x_0,y_0+\Delta y)-f(x_0,y_0)$$

- $z=f(x,y)$ 在点 (x_0,y_0) 关于 y 的偏导数：（y 方向的导数）

$$\lim_{\Delta y\to 0}\frac{\Delta_y z}{\Delta y}=\lim_{\Delta y\to 0}\frac{f(x_0,y_0+\Delta y)-f(x_0,y_0)}{\Delta y}$$

$$=\frac{d}{dy}[f(x_0,y)]\Big|_{y=y_0}$$

偏导数记号：

$$\frac{\partial z}{\partial y}\Big|_{\substack{x=x_0\\y=y_0}},\qquad z'_y\Big|_{\substack{x=x_0\\y=y_0}},\qquad z_y\Big|_{\substack{x=x_0\\y=y_0}}$$

$$\frac{\partial f}{\partial y}\qquad f'_y\qquad f_y$$

偏导数准确定义

- $z=f(x,y)$ 在点 (x_0,y_0) 关于 y 的偏增量：（y 方向的改变量）
$$\Delta_y z=f(x_0,y_0+\Delta y)-f(x_0,y_0)$$

- $z=f(x,y)$ 在点 (x_0,y_0) 关于 y 的偏导数：（y 方向的导数）
$$\lim_{\Delta y\to 0}\frac{\Delta_y z}{\Delta y}=\lim_{\Delta y\to 0}\frac{f(x_0,y_0+\Delta y)-f(x_0,y_0)}{\Delta y}$$
$$=\frac{d}{dy}[f(x_0,y)]\bigg|_{y=y_0}$$

偏导数记号：

$$\frac{\partial z}{\partial y}\bigg|_{\substack{x=x_0\\y=y_0}},\qquad z'_y\bigg|_{\substack{x=x_0\\y=y_0}},\qquad z_y\bigg|_{\substack{x=x_0\\y=y_0}}$$

$$\frac{\partial f}{\partial y}(x_0,y_0),\qquad f'_y(x_0,y_0),\qquad f_y(x_0,y_0)$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} =$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x =$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} =$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x =$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} =$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y +$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} =$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} =$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} =$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} =$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x - \frac{x}{y^2}$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x - \frac{x}{y^2}$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = \left.(y + \frac{1}{y})\right|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} = \left.(x - \frac{x}{y^2})\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x - \frac{x}{y^2}$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = \left.(y + \frac{1}{y})\right|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} = \left.(x - \frac{x}{y^2})\right|_{\substack{x=2\\y=1}} = 2 - \frac{2}{1} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x - \frac{x}{y^2}$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} = (x - \frac{x}{y^2})\Big|_{\substack{x=2\\y=1}} = 2 - \frac{2}{1} = 0$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0},$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \quad [f(x, y_0)] \quad ,$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0},$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \qquad [f(x_0, y)]$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以 $f(x, 1)$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \qquad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以 $\quad f(x, 1) = 2x$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以 $$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \qquad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以 $$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \qquad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以 $f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$

$$\Rightarrow \quad \frac{d}{dx}[f(x, 1)]\Big|_{x=2} = 2,$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \qquad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以 $\quad f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$

$$\Rightarrow \quad \frac{\partial z}{\partial x}\Big|_{\substack{x=2\\y=1}} = \frac{d}{dx}[f(x, 1)]\Big|_{x=2} = 2,$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \qquad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以 $\quad f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$

$$\Rightarrow \quad \frac{\partial z}{\partial x}\Big|_{\substack{x=2\\y=1}} = \frac{d}{dx}[f(x, 1)]\Big|_{x=2} = 2,$$

$f(2, y)$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以 $f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$

$$\Rightarrow \quad \frac{\partial z}{\partial x}\Big|_{\substack{x=2\\y=1}} = \frac{d}{dx}[f(x, 1)]\Big|_{x=2} = 2,$$

$$f(2, y) = 2y + \frac{2}{y}$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \qquad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以 $f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$

$$\Rightarrow \quad \frac{\partial z}{\partial x}\Big|_{\substack{x=2\\y=1}} = \frac{d}{dx}[f(x, 1)]\Big|_{x=2} = 2,$$

$$f(2, y) = 2y + \frac{2}{y} \quad \Rightarrow \quad \frac{d}{dy}[f(2, y)] =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以

$$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$$

$$\Rightarrow \quad \frac{\partial z}{\partial x}\Big|_{\substack{x=2\\y=1}} = \frac{d}{dx}[f(x, 1)]\Big|_{x=2} = 2,$$

$$f(2, y) = 2y + \frac{2}{y} \quad \Rightarrow \quad \frac{d}{dy}[f(2, y)] = 2 - \frac{2}{y^2}$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\bigg|_{x=x_0}, \qquad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\bigg|_{y=y_0}$$

所以

$$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$$

$$\Rightarrow \quad \frac{\partial z}{\partial x}\bigg|_{\substack{x=2\\y=1}} = \frac{d}{dx}[f(x, 1)]\bigg|_{x=2} = 2,$$

$$f(2, y) = 2y + \frac{2}{y} \quad \Rightarrow \quad \frac{d}{dy}[f(2, y)] = 2 - \frac{2}{y^2}$$

$$\Rightarrow \quad \frac{d}{dy}[f(2, y)]\bigg|_{y=1} = 0.$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}, \qquad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

所以

$$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$$

$$\Rightarrow \quad \frac{\partial z}{\partial x}\Big|_{\substack{x=2\\y=1}} = \frac{d}{dx}[f(x, 1)]\Big|_{x=2} = 2,$$

$$f(2, y) = 2y + \frac{2}{y} \quad \Rightarrow \quad \frac{d}{dy}[f(2, y)] = 2 - \frac{2}{y^2}$$

$$\Rightarrow \quad \frac{\partial z}{\partial y}\Big|_{\substack{x=2\\y=1}} = \frac{d}{dy}[f(2, y)]\Big|_{y=1} = 0.$$

例 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

例 设 $f(x, y) = \begin{cases} \dfrac{xy}{x^2+y^2}, & (x, y)\neq(0, 0) \\ 0, & (x, y)=(0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解 $f_x(0, 0)$

$f_y(0, 0)$

例 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解 $f_x(0, 0)$ $f(x, 0)$

$f_y(0, 0)$

例 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x,0)]\Big|_{x=0}$$

$$f_y(0, 0)$$

例 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x,0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0}$$

$$f_y(0, 0)$$

例 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

$$f_y(0, 0)$$

例 设 $f(x,y)=\begin{cases} \frac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$，求 $f_x(0,0), f_y(0,0)$

解

$$f_x(0,0)=\frac{d}{dx}[f(x,0)]\Big|_{x=0}=\frac{d}{dx}[0]\Big|_{x=0}=0,$$

$$f_y(0,0) \qquad f(0,y)$$

例 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)]\Big|_{x=0}$$

例 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)]\Big|_{x=0} = \frac{d}{dy}[0]\Big|_{y=0}$$

例 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)]\Big|_{x=0} = \frac{d}{dy}[0]\Big|_{y=0} = 0,$$

例 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)]\Big|_{x=0} = \frac{d}{dy}[0]\Big|_{y=0} = 0,$$

注 偏导数存在 $\nRightarrow$ 连续

例 设 $f(x, y) = \begin{cases} \dfrac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)]\Big|_{x=0} = \frac{d}{dy}[0]\Big|_{y=0} = 0,$$

注 偏导数存在 $\not\Rightarrow$ 连续

（上述 $f(x, y)$ 在 $(0, 0)$ 处存在偏导数 $f_x(0, 0)$ 和 $f_y(0, 0)$，但在 $(0, 0)$ 处不连续）

例 设 $f(x,y)=\begin{cases}\frac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$，求 $f_x(0,0), f_y(0,0)$

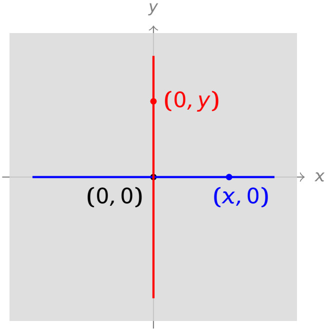
解

$$f_x(0,0)=\frac{d}{dx}[f(x,0)]\Big|_{x=0}=\frac{d}{dx}[0]\Big|_{x=0}=0,$$

$$f_y(0,0)=\frac{d}{dy}[f(0,y)]\Big|_{x=0}=\frac{d}{dy}[0]\Big|_{y=0}=0,$$

注 偏导数存在 $\not\Rightarrow$ 连续

（上述 $f(x,y)$ 在 $(0,0)$ 处存在偏导数 $f_x(0,0)$ 和 $f_y(0,0)$，但在 $(0,0)$ 处不连续）



例 设 $f(x, y) = \begin{cases} \dfrac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

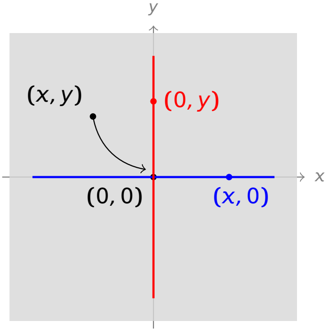
解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)]\Big|_{x=0} = \frac{d}{dy}[0]\Big|_{y=0} = 0,$$

注 偏导数存在 $\not\Rightarrow$ 连续

（上述 $f(x, y)$ 在 $(0, 0)$ 处存在偏导数 $f_x(0, 0)$ 和 $f_y(0, 0)$，但在 $(0, 0)$ 处不连续）



二阶偏导数

$$\begin{tikzcd} & \dfrac{\partial z}{\partial x} \\ z = f(x, y) \arrow[ur] \arrow[dr] & \\ & \dfrac{\partial z}{\partial y} \end{tikzcd}$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
z=f(x,y) \arrow[ur] \arrow[dr] & & {} \\
 & \frac{\partial z}{\partial y} &
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z=f(x,y) \arrow[uur] \arrow[dr] & & \\
 & \frac{\partial z}{\partial y} &
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & {}
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right) : & \dfrac{\partial^2 z}{\partial x \partial x}, \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right) : & \dfrac{\partial^2 z}{\partial x \partial y}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right) & \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small, row sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial x}, \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial y}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y \partial x}, \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

二阶偏导数

$$\begin{tikzcd}[column sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial x}, \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial y}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y \partial x}, \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y \partial y},
\end{tikzcd}$$

二阶偏导数

$$z=f(x,y)\longrightarrow\frac{\partial z}{\partial x}\longrightarrow\begin{cases}\dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x\partial x},\quad z_{xx},\\[2ex]\dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x\partial y},\end{cases}$$

$$z=f(x,y)\longrightarrow\frac{\partial z}{\partial y}\longrightarrow\begin{cases}\dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y\partial x},\\[2ex]\dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y\partial y},\end{cases}$$

二阶偏导数

$$\begin{tikzcd}
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y \partial x}, \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y \partial y},
\end{tikzcd}$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x \partial x}, & z_{xx}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x \partial y}, & z_{xy}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y \partial x}, & z_{yx}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y \partial y}, &
\end{tikzcd}
$$

二阶偏导数

$$\begin{tikzcd}
& & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial x}, \quad z_{xx}, \\
& \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
& & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial y}, \quad z_{xy}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \\
& & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y\partial x}, \quad z_{yx}, \\
& \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
& & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y\partial y}, \quad z_{yy},
\end{tikzcd}$$

二阶偏导数

$$z=f(x,y)\ \to\ \frac{\partial z}{\partial x},\quad \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x}\ \to\ \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y}\ \to\ \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \quad f_{xx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial y}, \quad z_{xy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \qquad \frac{\partial^2 z}{\partial y \partial x}, \quad z_{yx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \qquad \frac{\partial^2 z}{\partial y \partial y}, \quad z_{yy},$$

二阶偏导数

$$z = f(x, y) \quad \to \quad \frac{\partial z}{\partial x} \quad \to \quad \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \quad f_{xx}$$

$$z = f(x, y) \quad \to \quad \frac{\partial z}{\partial x} \quad \to \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \quad f_{xy}$$

$$z = f(x, y) \quad \to \quad \frac{\partial z}{\partial y} \quad \to \quad \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \qquad \frac{\partial^2 z}{\partial y \partial x}, \quad z_{yx},$$

$$z = f(x, y) \quad \to \quad \frac{\partial z}{\partial y} \quad \to \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \qquad \frac{\partial^2 z}{\partial y \partial y}, \quad z_{yy},$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right):
\end{tikzcd}
$$

$$\frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \quad f_{xx}$$

$$\frac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \quad f_{xy}$$

$$\frac{\partial^2 z}{\partial y \partial x}, \quad z_{yx}, \quad f_{yx}$$

$$\frac{\partial^2 z}{\partial y \partial y}, \quad z_{yy},$$

二阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \quad f_{xx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \quad f_{xy}$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \qquad \frac{\partial^2 z}{\partial y \partial x}, \quad z_{yx}, \quad f_{yx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \qquad \frac{\partial^2 z}{\partial y \partial y}, \quad z_{yy}, \quad f_{yy}$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x =$$

$$z_y =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$z_x =$

$z_y =$

$z_{xx} =$

$z_{xy} =$

$z_{yx} =$

$z_{yy} =$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} +$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} +$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} +$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} +$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2e^{xy} +$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

$$z_{xy}=(ye^{xy}+2y^2)'_y=(ye^{xy})'_y+(2y^2)'_y=e^{xy}+xye^{xy}+4y$$

$$z_{yx}=(xe^{xy}+4xy)'_x=(xe^{xy})'_x+(4xy)'_x=e^{xy}+xye^{xy}+4y$$

$$z_{yy}=(xe^{xy}+4xy)'_y=(xe^{xy})'_y+(4xy)'_y=x^2e^{xy}+4x$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2e^{xy} + 4x$$

注 此例成立 $z_{xy} = z_{yx}$

三阶偏导数

$$\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)
\end{tikzcd}$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) & \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right)
\end{tikzcd}
$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$z=f(x,y)\to\frac{\partial z}{\partial x}\to\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\qquad\frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\qquad\frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial z}{\partial x}\to\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$z=f(x,y)\to\frac{\partial z}{\partial y}\to\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial z}{\partial y}\to\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial x}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): &
\end{tikzcd}
$$

三阶偏导数

$$z=f(x,y)$$

$$z=f(x,y)\to\frac{\partial z}{\partial x}\to\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\qquad\frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\qquad\frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial z}{\partial x}\to\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\qquad\frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\qquad\frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$z=f(x,y)\to\frac{\partial z}{\partial y}\to\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial z}{\partial y}\to\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$z=f(x,\,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$z = f(x, y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial x \partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial x \partial y},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$z = f(x, y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial x}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial y}, \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial x\partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial y\partial x}, \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): &
\end{tikzcd}
$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial x \partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y}, \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x}, \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},
\end{tikzcd}
$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x}, \ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y}, \ z_{xxy},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : & \frac{\partial^3 z}{\partial x \partial x \partial x}, \ z_{xxx}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : & \frac{\partial^3 z}{\partial x \partial x \partial y}, \ z_{xxy}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : & \frac{\partial^3 z}{\partial x \partial y \partial x}, \ z_{xyx}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : & \frac{\partial^3 z}{\partial x \partial y \partial y}, \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : & \frac{\partial^3 z}{\partial y \partial x \partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : & \frac{\partial^3 z}{\partial y \partial x \partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : & \frac{\partial^3 z}{\partial y \partial y \partial x}, \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : & \frac{\partial^3 z}{\partial y \partial y \partial y},
\end{tikzcd}
$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \quad \frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \quad \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x}, \ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y}, \ z_{xxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x}, \ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y}, \ z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy}, \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial y\partial x}, \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial y\partial y},
\end{tikzcd}
$$

三阶偏导数

$$z = f(x, y) \nearrow \frac{\partial z}{\partial x}, \qquad z = f(x, y) \searrow \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} \nearrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right), \qquad \frac{\partial z}{\partial x} \searrow \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \nearrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right), \qquad \frac{\partial z}{\partial y} \searrow \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x}, \; z_{xxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \searrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y}, \; z_{xxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x}, \; z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \searrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y}, \; z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x}, \; z_{yxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \searrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y}, \; z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \searrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x}, \; z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y}, \; z_{xxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x}, \; z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y}, \; z_{xyy},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x}, \; z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y}, \; z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x}, \; z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y}, \; z_{yyy},$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x\partial x\partial x}, \; z_{xxx}, \; f_{xxx} \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x\partial x\partial y}, \; z_{xxy}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x\partial y\partial x}, \; z_{xyx}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x\partial y\partial y}, \; z_{xyy}, \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y\partial x\partial x}, \; z_{yxx}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y\partial x\partial y}, \; z_{yxy}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y\partial y\partial x}, \; z_{yyx}, \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y\partial y\partial y}, \; z_{yyy},
\end{tikzcd}
$$

三阶偏导数

$$z=f(x,y) \longrightarrow \frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$z=f(x,y) \longrightarrow \frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x}, \; z_{xxx}, \; f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y}, \; z_{xxy}, \; f_{xxy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x}, \; z_{xyx}, \; f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y}, \; z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x}, \; z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y}, \; z_{yxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x}, \; z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y}, \; z_{yyy},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},\ z_{yyy},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},\ z_{yyy},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},\ z_{yyx},\ f_{yyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},\ z_{yyy},$$

三阶偏导数

$$z = f(x, y) \to \frac{\partial z}{\partial x}, \quad \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial x \partial x}, \ z_{xxx}, \ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial x \partial y}, \ z_{xxy}, \ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial y \partial x}, \ z_{xyx}, \ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial y \partial y}, \ z_{xyy}, \ f_{xyy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial x \partial x}, \ z_{yxx}, \ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial x \partial y}, \ z_{yxy}, \ f_{yxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial y \partial x}, \ z_{yyx}, \ f_{yyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial y \partial y}, \ z_{yyy}, \ f_{yyy}$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解 $z_x =$

$z_y =$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

暨南大學 JINAN UNIVERSITY

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$
$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$
$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$
$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2-1$$
$$z_{yy}=(2x^3y-9xy^2-x)'_y=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$
$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$
$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$
$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2-1$$
$$z_{yy}=(2x^3y-9xy^2-x)'_y=2x^3-18xy$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} = (6xy^2)'_x =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} = (6xy^2)'_x = 6y^2$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} = (6xy^2)'_x = 6y^2$$

注 此例成立 $z_{xy} = z_{yx}$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$
\begin{aligned}
z_x &= \\
z_y &= \\
z_{xx} &= \\
z_{xy} &= \\
z_{yx} &= \\
z_{yy} &=
\end{aligned}
$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} = (3x\cos(3y))'_x =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$

$$z_{yy} = (3x\cos(3y))'_y =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$

$$z_{yy} = (3x\cos(3y))'_y = -9x\sin(3y)$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$

$$z_{yy} = (3x\cos(3y))'_y = -9x\sin(3y)$$

$$z_{xyy} = (3\cos(3y))'_y =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$

$$z_{yy} = (3x\cos(3y))'_y = -9x\sin(3y)$$

$$z_{xyy} = (3\cos(3y))'_y = -9\sin(3y)$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$

$$z_{yy} = (3x\cos(3y))'_y = -9x\sin(3y)$$

$$z_{xyy} = (3\cos(3y))'_y = -9\sin(3y)$$

注 此例成立 $z_{xy} = z_{yx}$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$

$$z_{yy} = (3x\cos(3y))'_y = -9x\sin(3y)$$

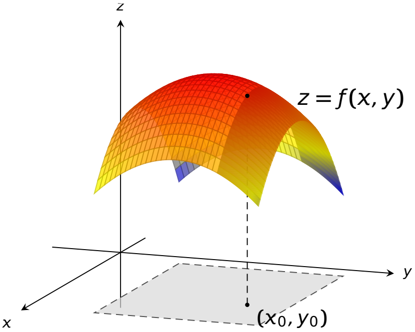
$$z_{xyy} = (3\cos(3y))'_y = -9\sin(3y)$$

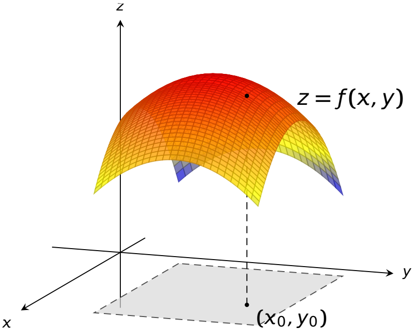
注 此例成立 $z_{xy} = z_{yx}$

性质 设有二元函数 $z = f(x, y)$。若 $\frac{\partial^2 z}{\partial y \partial x}$ 和 $\frac{\partial^2 z}{\partial x \partial y}$ 均连续，则

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$$

偏导数的几何直观

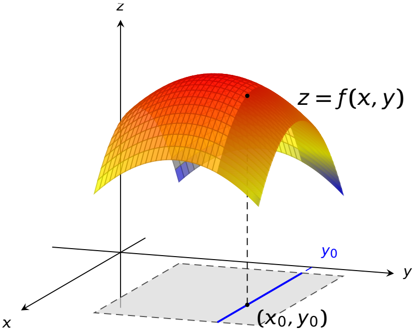




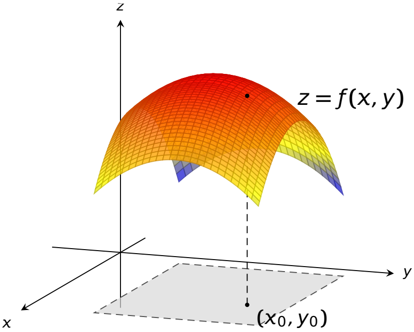
$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0}$$

$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\Big|_{y=y_0}$$

偏导数的几何直观

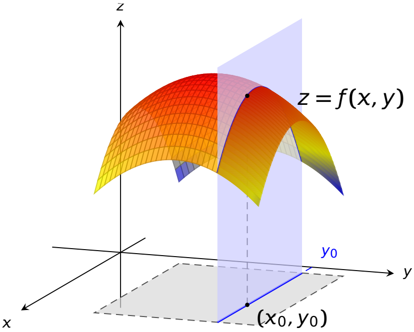


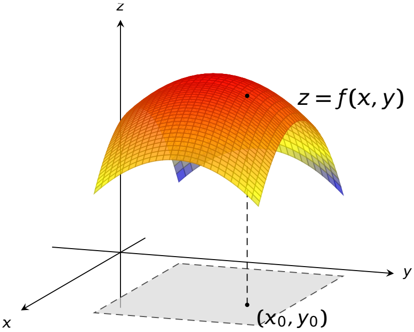
$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0}$$



$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\Big|_{y=y_0}$$

偏导数的几何直观

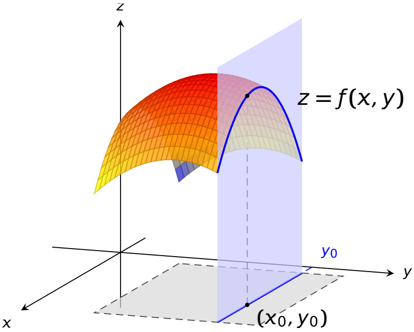




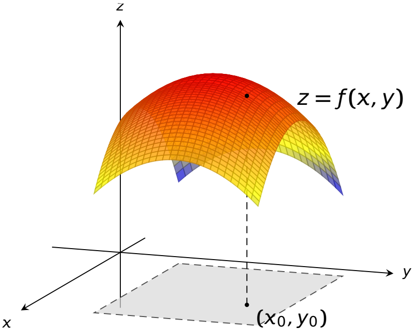
$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\bigg|_{x=x_0}$$

$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\bigg|_{y=y_0}$$

偏导数的几何直观

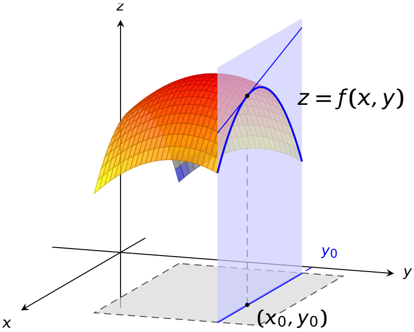


$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\bigg|_{x=x_0}$$

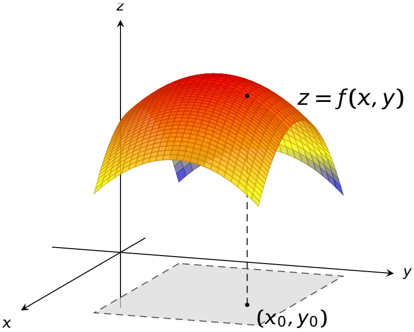


$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\bigg|_{y=y_0}$$

偏导数的几何直观

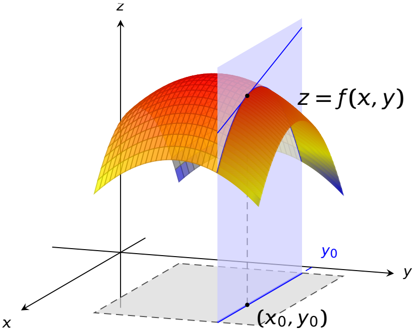


$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0}$$

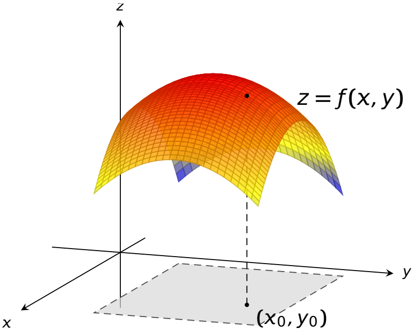


$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\Big|_{y=y_0}$$

偏导数的几何直观

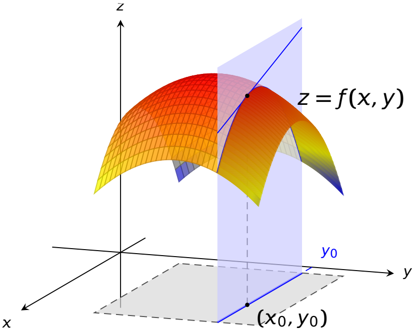


$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0}$$

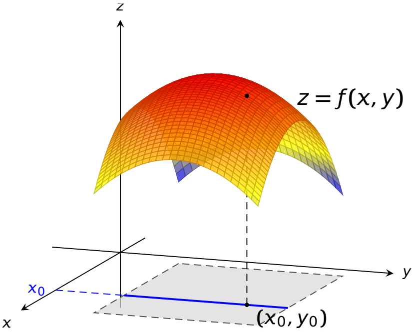


$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\Big|_{y=y_0}$$

偏导数的几何直观

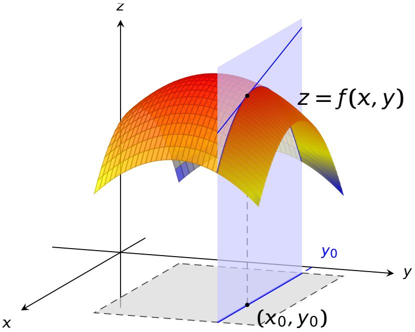


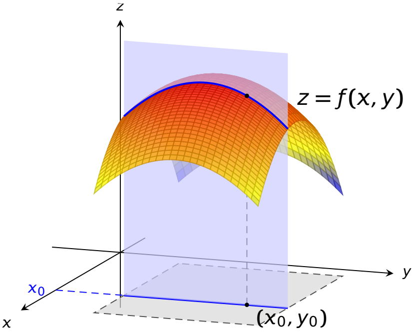
$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\bigg|_{x=x_0}$$



$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\bigg|_{y=y_0}$$

偏导数的几何直观

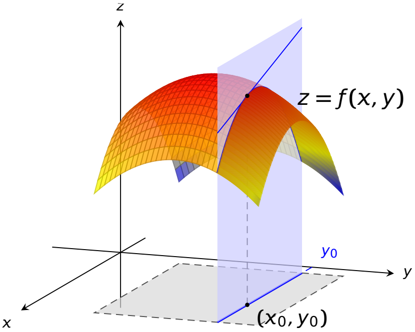


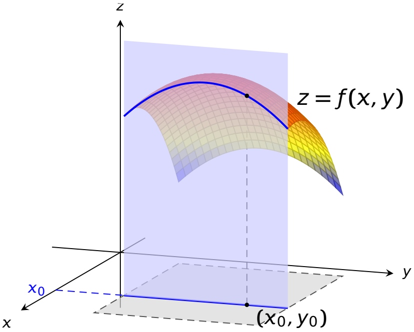


$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0}$$

$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\Big|_{y=y_0}$$

偏导数的几何直观

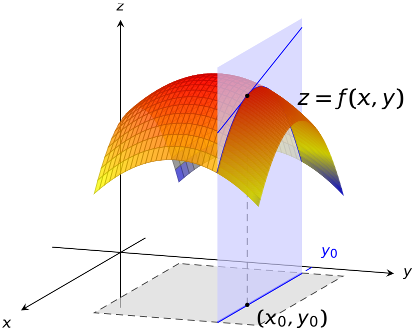




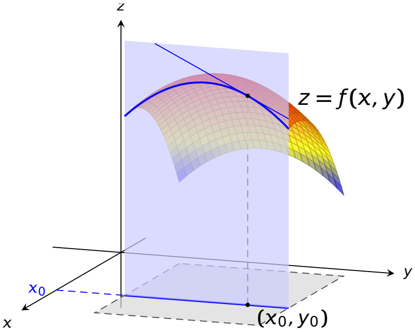
$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0}$$

$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\Big|_{y=y_0}$$

偏导数的几何直观

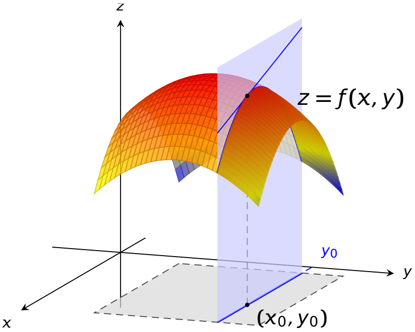


$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0}$$

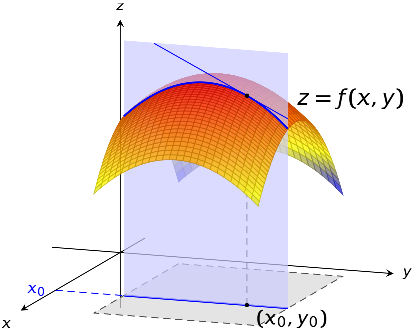


$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\Big|_{y=y_0}$$

偏导数的几何直观



$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\bigg|_{x=x_0}$$



$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\bigg|_{y=y_0}$$

We are here now...

1. 偏导数

2. 全微分

可微

- 回忆：一元函数 $z = f(x)$ 在 $x = x_0$ 处可微，指

可微

- 回忆：一元函数 $z=f(x)$ 在 $x=x_0$ 处可微，指

  $$\Delta z=f(x_0+\Delta x)-f(x_0)$$

可微

- 回忆：一元函数 $z = f(x)$ 在 $x = x_0$ 处可微，指

$$\Delta z = f(x_0 + \Delta x) - f(x_0) = f'(x_0)\Delta x + o(\quad)$$

可微

- 回忆：一元函数 $z = f(x)$ 在 $x = x_0$ 处可微，指

$$\Delta z = f(x_0 + \Delta x) - f(x_0) = f'(x_0)\Delta x + o(\Delta x)$$

可微

- 回忆：一元函数 $z = f(x)$ 在 $x = x_0$ 处可微，指

$$\Delta z = f(x_0 + \Delta x) - f(x_0) = \underbrace{f'(x_0)\Delta x}_{=dz} + o(\Delta x)$$

可微

- 回忆：一元函数 $z=f(x)$ 在 $x=x_0$ 处可微，指

$$\Delta z=f(x_0+\Delta x)-f(x_0)=\underbrace{f'(x_0)\Delta x}_{=dz}+o(\Delta x)\approx dz$$

可微

- 回忆：一元函数 $z = f(x)$ 在 $x = x_0$ 处可微，指

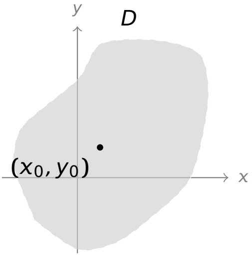
$$\Delta z = f(x_0 + \Delta x) - f(x_0) = \underbrace{f'(x_0)\Delta x}_{=dz} + o(\Delta x) \approx dz$$

- 二元函数 $z = f(x, y)$ 在 (x_0, y_0) 处可微，指

可微

- 回忆：一元函数 $z=f(x)$ 在 $x=x_0$ 处可微，指

$$\Delta z=f(x_0+\Delta x)-f(x_0)=\underbrace{f'(x_0)\Delta x}_{=dz}+o(\Delta x)\approx dz$$

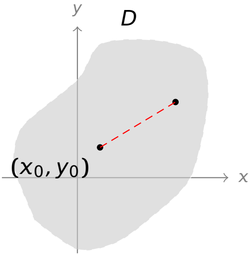


- 二元函数 $z=f(x,y)$ 在 (x_0,y_0) 处可微，指

可微

- 回忆：一元函数 $z = f(x)$ 在 $x = x_0$ 处可微，指

$$\Delta z = f(x_0 + \Delta x) - f(x_0) = \underbrace{f'(x_0)\Delta x}_{=dz} + o(\Delta x) \approx dz$$

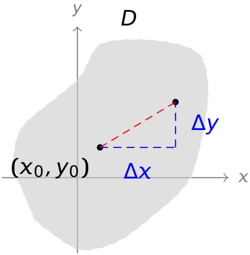


- 二元函数 $z = f(x, y)$ 在 (x_0, y_0) 处可微，指

可微

- 回忆：一元函数 $z=f(x)$ 在 $x=x_0$ 处可微，指

$$\Delta z=f(x_0+\Delta x)-f(x_0)=\underbrace{f'(x_0)\Delta x}_{=dz}+o(\Delta x)\approx dz$$

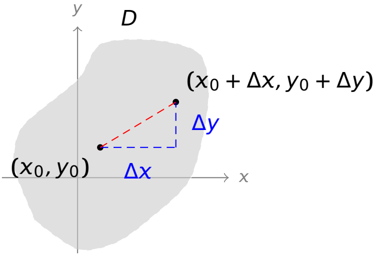


- 二元函数 $z=f(x,y)$ 在 (x_0,y_0) 处可微，指

可微

- 回忆：一元函数 $z = f(x)$ 在 $x = x_0$ 处可微，指

$$\Delta z = f(x_0 + \Delta x) - f(x_0) = \underbrace{f'(x_0)\Delta x}_{=dz} + o(\Delta x) \approx dz$$

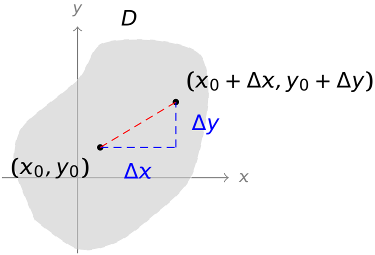


- 二元函数 $z = f(x, y)$ 在 (x_0, y_0) 处可微，指

可微

- 回忆：一元函数 $z=f(x)$ 在 $x=x_0$ 处可微，指

$$\Delta z = f(x_0+\Delta x) - f(x_0) = \underbrace{f'(x_0)\Delta x}_{=dz} + o(\Delta x) \approx dz$$



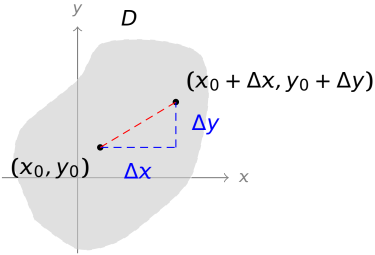
- 二元函数 $z=f(x,y)$ 在 (x_0,y_0) 处可微，指 ∃ 数 A, B 使得：

$$\Delta z = f(x_0+\Delta x, y_0+\Delta y) - f(x_0, y_0)$$

可微

- 回忆：一元函数 $z=f(x)$ 在 $x=x_0$ 处可微，指

$$\Delta z=f(x_0+\Delta x)-f(x_0)=\underbrace{f'(x_0)\Delta x}_{=dz}+o(\Delta x)\approx dz$$



- 二元函数 $z=f(x,y)$ 在 (x_0,y_0) 处可微，指 $\exists$ 数 A, B 使得：

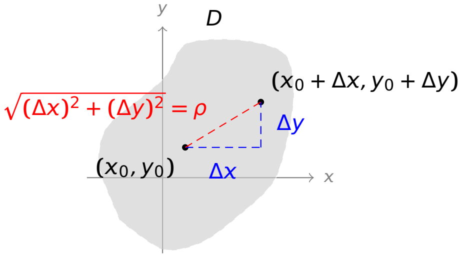
$$\Delta z=f(x_0+\Delta x, y_0+\Delta y)-f(x_0,y_0)$$

$$=A\Delta x+B\Delta y+o(\quad)$$

可微

- 回忆：一元函数 $z = f(x)$ 在 $x = x_0$ 处**可微**，指

$$\Delta z = f(x_0 + \Delta x) - f(x_0) = \underbrace{f'(x_0)\Delta x}_{=dz} + o(\Delta x) \approx dz$$



- 二元函数 $z = f(x, y)$ 在 (x_0, y_0) 处**可微**，指 ∃ 数 A, B 使得：

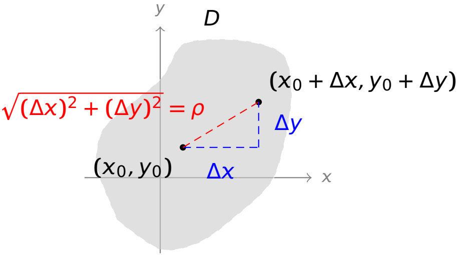
$$\Delta z = f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0, y_0)$$

$$= A\Delta x + B\Delta y + o(\quad)$$

可微

- 回忆：一元函数 $z=f(x)$ 在 $x=x_0$ 处可微，指

$$\Delta z=f(x_0+\Delta x)-f(x_0)=\underbrace{f'(x_0)\Delta x}_{=dz}+o(\Delta x)\approx dz$$



- 二元函数 $z=f(x,y)$ 在 (x_0,y_0) 处可微，指 ∃ 数 A, B 使得：

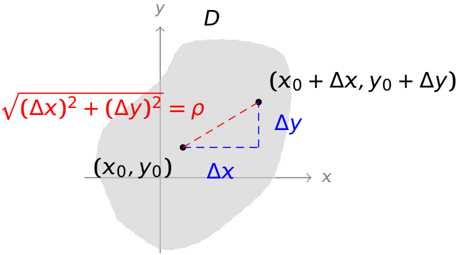
$$\Delta z=f(x_0+\Delta x, y_0+\Delta y)-f(x_0,y_0)$$

$$=A\Delta x+B\Delta y+o(\rho)$$

可微

- 回忆：一元函数 $z=f(x)$ 在 $x=x_0$ 处可微，指

$$\Delta z=f(x_0+\Delta x)-f(x_0)=\underbrace{f'(x_0)\Delta x}_{=dz}+o(\Delta x)\approx dz$$



- 二元函数 $z=f(x,y)$ 在 (x_0,y_0) 处可微，指 ∃ 数 A, B 使得：

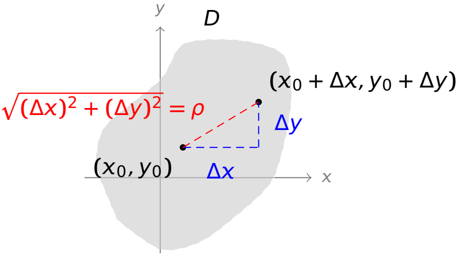
$$\Delta z=f(x_0+\Delta x, y_0+\Delta y)-f(x_0,y_0)$$

$$=\underbrace{A\Delta x+B\Delta y}_{=dz}+o(\rho)$$

可微

- 回忆：一元函数 $z=f(x)$ 在 $x=x_0$ 处可微，指

$$\Delta z=f(x_0+\Delta x)-f(x_0)=\underbrace{f'(x_0)\Delta x}_{=dz}+o(\Delta x)\approx dz$$



- 二元函数 $z=f(x,y)$ 在 (x_0,y_0) 处可微，指 $\exists$ 数 A, B 使得：

$$\Delta z=f(x_0+\Delta x,\, y_0+\Delta y)-f(x_0,\, y_0)$$

$$=\underbrace{A\Delta x+B\Delta y}_{=dz}+o(\rho)\approx dz$$

定理（可微必要条件） 设函数 $z = f(x, y)$ 在该点 (x_0, y_0) 处可微，则

定理（可微必要条件） 设函数 $z = f(x, y)$ 在该点 (x_0, y_0) 处可微，则

1. 函数在该点 (x_0, y_0) 处连续；

定理（可微必要条件） 设函数 $z=f(x,y)$ 在该点 (x_0,y_0) 处可微，则

1. 函数在该点 (x_0,y_0) 处连续；
2. 函数在该点 (x_0,y_0) 处存在偏导数 $\frac{\partial z}{\partial x}(x_0,y_0)$, $\frac{\partial z}{\partial y}(x_0,y_0)$；

定理（可微必要条件） 设函数 $z = f(x, y)$ 在该点 (x_0, y_0) 处可微，则

1. 函数在该点 (x_0, y_0) 处连续；
2. 函数在该点 (x_0, y_0) 处存在偏导数 $\frac{\partial z}{\partial x}(x_0, y_0)$, $\frac{\partial z}{\partial y}(x_0, y_0)$；
3. 函数在该点 (x_0, y_0) 处的全微分为

$$dz = \qquad\qquad \Delta x + \qquad\qquad \Delta y$$

定理（可微必要条件） 设函数 $z=f(x,y)$ 在该点 (x_0,y_0) 处可微，则

1. 函数在该点 (x_0,y_0) 处连续；
2. 函数在该点 (x_0,y_0) 处存在偏导数 $\frac{\partial z}{\partial x}(x_0,y_0)$, $\frac{\partial z}{\partial y}(x_0,y_0)$；
3. 函数在该点 (x_0,y_0) 处的全微分为

$$dz=\frac{\partial z}{\partial x}(x_0,y_0)\Delta x+\qquad\qquad\qquad\Delta y$$

定理（可微必要条件） 设函数 $z=f(x,y)$ 在该点 (x_0,y_0) 处可微，则

1. 函数在该点 (x_0,y_0) 处连续；
2. 函数在该点 (x_0,y_0) 处存在偏导数 $\frac{\partial z}{\partial x}(x_0,y_0)$, $\frac{\partial z}{\partial y}(x_0,y_0)$；
3. 函数在该点 (x_0,y_0) 处的全微分为

$$dz=\frac{\partial z}{\partial x}(x_0,y_0)\Delta x+\frac{\partial z}{\partial y}(x_0,y_0)\Delta y$$

定理（可微必要条件） 设函数 $z=f(x,y)$ 在该点 (x_0,y_0) 处可微，则

1. 函数在该点 (x_0,y_0) 处连续；
2. 函数在该点 (x_0,y_0) 处存在偏导数 $\frac{\partial z}{\partial x}(x_0,y_0)$, $\frac{\partial z}{\partial y}(x_0,y_0)$；
3. 函数在该点 (x_0,y_0) 处的全微分为

$$dz=\frac{\partial z}{\partial x}(x_0,y_0)\Delta x+\frac{\partial z}{\partial y}(x_0,y_0)\Delta y$$

注 通常的记号：$\Delta x=dx$, $\Delta y=dy$。

定理（可微必要条件） 设函数 $z = f(x, y)$ 在该点 (x_0, y_0) 处可微，则

1. 函数在该点 (x_0, y_0) 处连续；
2. 函数在该点 (x_0, y_0) 处存在偏导数 $\frac{\partial z}{\partial x}(x_0, y_0)$, $\frac{\partial z}{\partial y}(x_0, y_0)$；
3. 函数在该点 (x_0, y_0) 处的全微分为

$$dz = \frac{\partial z}{\partial x}(x_0, y_0)\Delta x + \frac{\partial z}{\partial y}(x_0, y_0)\Delta y$$

注 通常的记号：$\Delta x = dx, \Delta y = dy$。这样全微分（存在的话）写成：

定理（可微必要条件） 设函数 $z=f(x,y)$ 在该点 (x_0,y_0) 处可微，则

1. 函数在该点 (x_0,y_0) 处连续；
2. 函数在该点 (x_0,y_0) 处存在偏导数 $\frac{\partial z}{\partial x}(x_0,y_0)$, $\frac{\partial z}{\partial y}(x_0,y_0)$；
3. 函数在该点 (x_0,y_0) 处的全微分为

$$dz=\frac{\partial z}{\partial x}(x_0,y_0)\Delta x+\frac{\partial z}{\partial y}(x_0,y_0)\Delta y$$

注 通常的记号：$\Delta x=dx$, $\Delta y=dy$。这样全微分（存在的话）写成：

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy$$

定理（可微必要条件） 设函数 $z=f(x,y)$ 在该点 (x_0,y_0) 处可微，则

1. 函数在该点 (x_0,y_0) 处连续；
2. 函数在该点 (x_0,y_0) 处存在偏导数 $\frac{\partial z}{\partial x}(x_0,y_0)$, $\frac{\partial z}{\partial y}(x_0,y_0)$；
3. 函数在该点 (x_0,y_0) 处的全微分为

$$dz=\frac{\partial z}{\partial x}(x_0,y_0)\Delta x+\frac{\partial z}{\partial y}(x_0,y_0)\Delta y$$

注 通常的记号：$\Delta x=dx$, $\Delta y=dy$。这样全微分（存在的话）写成：

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy$$

定理（可微充分条件）

定理（可微必要条件） 设函数 $z = f(x, y)$ 在该点 (x_0, y_0) 处可微，则

1. 函数在该点 (x_0, y_0) 处连续；
2. 函数在该点 (x_0, y_0) 处存在偏导数 $\frac{\partial z}{\partial x}(x_0, y_0)$, $\frac{\partial z}{\partial y}(x_0, y_0)$；
3. 函数在该点 (x_0, y_0) 处的全微分为

$$dz = \frac{\partial z}{\partial x}(x_0, y_0)\Delta x + \frac{\partial z}{\partial y}(x_0, y_0)\Delta y$$

注 通常的记号：$\Delta x = dx$, $\Delta y = dy$。这样全微分（存在的话）写成：

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

定理（可微充分条件） 设函数 $z = f(x, y)$ 的偏导数 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ 在点 (x_0, y_0) 连续，

定理（可微必要条件） 设函数 $z=f(x,y)$ 在该点 (x_0,y_0) 处可微，则

1. 函数在该点 (x_0,y_0) 处连续；
2. 函数在该点 (x_0,y_0) 处存在偏导数 $\frac{\partial z}{\partial x}(x_0,y_0)$, $\frac{\partial z}{\partial y}(x_0,y_0)$；
3. 函数在该点 (x_0,y_0) 处的全微分为

$$dz=\frac{\partial z}{\partial x}(x_0,y_0)\Delta x+\frac{\partial z}{\partial y}(x_0,y_0)\Delta y$$

注 通常的记号：$\Delta x=dx$, $\Delta y=dy$。这样全微分（存在的话）写成：

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy$$

定理（可微充分条件） 设函数 $z=f(x,y)$ 的偏导数 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ 在点 (x_0,y_0) 连续，则 $z=f(x,y)$ 在该点 (x_0,y_0) 处可微

定理（可微必要条件） 设函数 $z = f(x, y)$ 在该点 (x_0, y_0) 处可微，则

1. 函数在该点 (x_0, y_0) 处连续；
2. 函数在该点 (x_0, y_0) 处存在偏导数 $\frac{\partial z}{\partial x}(x_0, y_0)$, $\frac{\partial z}{\partial y}(x_0, y_0)$；
3. 函数在该点 (x_0, y_0) 处的全微分为

$$dz = \frac{\partial z}{\partial x}(x_0, y_0)\Delta x + \frac{\partial z}{\partial y}(x_0, y_0)\Delta y$$

注 通常的记号：$\Delta x = dx$, $\Delta y = dy$。这样全微分（存在的话）写成：

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

定理（可微充分条件） 设函数 $z = f(x, y)$ 的偏导数 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ 在点 (x_0, y_0) 连续，则 $z = f(x, y)$ 在该点 (x_0, y_0) 处可微，进而在该点处微分为

定理（可微必要条件） 设函数 $z=f(x,y)$ 在该点 (x_0,y_0) 处可微，则

1. 函数在该点 (x_0,y_0) 处连续；
2. 函数在该点 (x_0,y_0) 处存在偏导数 $\frac{\partial z}{\partial x}(x_0,y_0)$, $\frac{\partial z}{\partial y}(x_0,y_0)$；
3. 函数在该点 (x_0,y_0) 处的全微分为

$$dz=\frac{\partial z}{\partial x}(x_0,y_0)\Delta x+\frac{\partial z}{\partial y}(x_0,y_0)\Delta y$$

注 通常的记号：$\Delta x=dx$, $\Delta y=dy$。这样全微分（存在的话）写成：

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy$$

定理（可微充分条件） 设函数 $z=f(x,y)$ 的偏导数 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ 在点 (x_0,y_0) 连续，则 $z=f(x,y)$ 在该点 (x_0,y_0) 处可微，进而在该点处微分为

$$dz=\frac{\partial z}{\partial x}(x_0,y_0)dx+\frac{\partial z}{\partial y}(x_0,y_0)dy$$

例 设 $z = f(x, y) = x^2 + y^2$，证明函数可微，并计算全微分 dz

例 设 $z=f(x,y)=x^2+y^2$，证明函数可微，并计算全微分 dz

解法一（按定义）

$$\Delta z=f(x+\Delta x,y+\Delta y)-f(x,y)$$

例 设 $z=f(x,y)=x^2+y^2$，证明函数可微，并计算全微分 dz

解法一（按定义）

$$\begin{aligned}\Delta z &= f(x+\Delta x,\, y+\Delta y)-f(x,\, y)\\ &= \left[(x+\Delta x)^2+(y+\Delta y)^2\right]\end{aligned}$$

例 设 $z = f(x, y) = x^2 + y^2$，证明函数可微，并计算全微分 dz

解法一（按定义）

$$\begin{aligned}\Delta z &= f(x+\Delta x, y+\Delta y) - f(x, y) \\ &= \left[(x+\Delta x)^2 + (y+\Delta y)^2\right] - \left[x^2 + y^2\right]\end{aligned}$$

例 设 $z=f(x,y)=x^2+y^2$，证明函数可微，并计算全微分 dz

解法一（按定义）

$$\begin{aligned}\Delta z &= f(x+\Delta x, y+\Delta y)-f(x,y)\\ &= \left[(x+\Delta x)^2+(y+\Delta y)^2\right]-\left[x^2+y^2\right]\\ &= 2x\Delta x+2y\Delta y+\left[(\Delta x)^2+(\Delta y)^2\right]\end{aligned}$$

例 设 $z=f(x,y)=x^2+y^2$，证明函数可微，并计算全微分 dz

解法一（按定义）

$$\begin{aligned}\Delta z &= f(x+\Delta x, y+\Delta y)-f(x,y)\\ &= \left[(x+\Delta x)^2+(y+\Delta y)^2\right]-\left[x^2+y^2\right]\\ &= 2x\Delta x+2y\Delta y+\left[(\Delta x)^2+(\Delta y)^2\right]\\ &= 2x\Delta x+2y\Delta y+\rho^2\end{aligned}$$

例 设 $z=f(x,y)=x^2+y^2$，证明函数可微，并计算全微分 dz

解法一（按定义）

$$\begin{aligned}\Delta z&=f(x+\Delta x,y+\Delta y)-f(x,y)\\&=\left[(x+\Delta x)^2+(y+\Delta y)^2\right]-\left[x^2+y^2\right]\\&=2x\Delta x+2y\Delta y+\left[(\Delta x)^2+(\Delta y)^2\right]\\&=2x\Delta x+2y\Delta y+\rho^2\\&=2x\Delta x+2y\Delta y+o(\rho)\end{aligned}$$

例 设 $z=f(x,y)=x^2+y^2$，证明函数可微，并计算全微分 dz

解法一（按定义）

$$\begin{aligned}\Delta z &= f(x+\Delta x, y+\Delta y)-f(x,y)\\ &=\left[(x+\Delta x)^2+(y+\Delta y)^2\right]-\left[x^2+y^2\right]\\ &=2x\Delta x+2y\Delta y+\left[(\Delta x)^2+(\Delta y)^2\right]\\ &=2x\Delta x+2y\Delta y+\rho^2\\ &=2x\Delta x+2y\Delta y+o(\rho)\end{aligned}$$

所以 $z=x^2+y^2$ 可微，并且 $dz=2xdx+2ydy$

例 设 $z=f(x,y)=x^2+y^2$，证明函数可微，并计算全微分 dz

解法一（按定义）

$$\begin{aligned}\Delta z &= f(x+\Delta x, y+\Delta y)-f(x,y)\\ &= \left[(x+\Delta x)^2+(y+\Delta y)^2\right]-\left[x^2+y^2\right]\\ &= 2x\Delta x+2y\Delta y+\left[(\Delta x)^2+(\Delta y)^2\right]\\ &= 2x\Delta x+2y\Delta y+\rho^2\\ &= 2x\Delta x+2y\Delta y+o(\rho)\end{aligned}$$

所以 $z=x^2+y^2$ 可微，并且 $dz=2xdx+2ydy$

解法二（利用定理）

例 设 $z=f(x,y)=x^2+y^2$，证明函数可微，并计算全微分 dz

解法一（按定义）

$$\begin{aligned}\Delta z &= f(x+\Delta x, y+\Delta y)-f(x,y)\\ &= \left[(x+\Delta x)^2+(y+\Delta y)^2\right]-\left[x^2+y^2\right]\\ &= 2x\Delta x+2y\Delta y+\left[(\Delta x)^2+(\Delta y)^2\right]\\ &= 2x\Delta x+2y\Delta y+\rho^2\\ &= 2x\Delta x+2y\Delta y+o(\rho)\end{aligned}$$

所以 $z=x^2+y^2$ 可微，并且 $dz=2xdx+2ydy$

解法二（利用定理） 先计算偏导数：

$$\frac{\partial z}{\partial x}= \qquad , \qquad \frac{\partial z}{\partial y}=$$

例 设 $z = f(x, y) = x^2 + y^2$，证明函数可微，并计算全微分 dz

解法一（按定义）

$$\begin{aligned}\Delta z &= f(x+\Delta x, y+\Delta y) - f(x, y) \\ &= \left[(x+\Delta x)^2 + (y+\Delta y)^2\right] - \left[x^2 + y^2\right] \\ &= 2x\Delta x + 2y\Delta y + \left[(\Delta x)^2 + (\Delta y)^2\right] \\ &= 2x\Delta x + 2y\Delta y + \rho^2 \\ &= 2x\Delta x + 2y\Delta y + o(\rho)\end{aligned}$$

所以 $z = x^2 + y^2$ 可微，并且 $dz = 2xdx + 2ydy$

解法二（利用定理） 先计算偏导数：

$$\frac{\partial z}{\partial x} = 2x, \qquad \frac{\partial z}{\partial y} =$$

例 设 $z=f(x,y)=x^2+y^2$，证明函数可微，并计算全微分 dz

解法一（按定义）

$$
\begin{aligned}
\Delta z &= f(x+\Delta x, y+\Delta y)-f(x,y)\\
&=\left[(x+\Delta x)^2+(y+\Delta y)^2\right]-\left[x^2+y^2\right]\\
&=2x\Delta x+2y\Delta y+\left[(\Delta x)^2+(\Delta y)^2\right]\\
&=2x\Delta x+2y\Delta y+\rho^2\\
&=2x\Delta x+2y\Delta y+o(\rho)
\end{aligned}
$$

所以 $z=x^2+y^2$ 可微，并且 $dz=2xdx+2ydy$

解法二（利用定理） 先计算偏导数：

$$\frac{\partial z}{\partial x}=2x,\qquad \frac{\partial z}{\partial y}=2y$$

例 设 $z=f(x,y)=x^2+y^2$，证明函数可微，并计算全微分 dz

解法一（按定义）

$$
\begin{aligned}
\Delta z &= f(x+\Delta x, y+\Delta y)-f(x,y) \\
&= \left[(x+\Delta x)^2+(y+\Delta y)^2\right]-\left[x^2+y^2\right] \\
&= 2x\Delta x+2y\Delta y+\left[(\Delta x)^2+(\Delta y)^2\right] \\
&= 2x\Delta x+2y\Delta y+\rho^2 \\
&= 2x\Delta x+2y\Delta y+o(\rho)
\end{aligned}
$$

所以 $z=x^2+y^2$ 可微，并且 $dz=2xdx+2ydy$

解法二（利用定理） 先计算偏导数：

$$\frac{\partial z}{\partial x}=2x, \qquad \frac{\partial z}{\partial y}=2y$$

可见偏导数存在，且连续。

例 设 $z = f(x, y) = x^2 + y^2$，证明函数可微，并计算全微分 dz

解法一（按定义）

$$\begin{aligned}\Delta z &= f(x+\Delta x, y+\Delta y) - f(x, y) \\ &= \left[(x+\Delta x)^2 + (y+\Delta y)^2\right] - \left[x^2 + y^2\right] \\ &= 2x\Delta x + 2y\Delta y + \left[(\Delta x)^2 + (\Delta y)^2\right] \\ &= 2x\Delta x + 2y\Delta y + \rho^2 \\ &= 2x\Delta x + 2y\Delta y + o(\rho)\end{aligned}$$

所以 $z = x^2 + y^2$ 可微，并且 $dz = 2xdx + 2ydy$

解法二（利用定理） 先计算偏导数：

$$\frac{\partial z}{\partial x} = 2x, \qquad \frac{\partial z}{\partial y} = 2y$$

可见偏导数存在，且连续。所以函数可微，并且全微分为

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

例 设 $z=f(x,y)=x^2+y^2$，证明函数可微，并计算全微分 dz

解法一（按定义）

$$\begin{aligned}\Delta z &= f(x+\Delta x, y+\Delta y)-f(x,y)\\ &=\left[(x+\Delta x)^2+(y+\Delta y)^2\right]-\left[x^2+y^2\right]\\ &=2x\Delta x+2y\Delta y+\left[(\Delta x)^2+(\Delta y)^2\right]\\ &=2x\Delta x+2y\Delta y+\rho^2\\ &=2x\Delta x+2y\Delta y+o(\rho)\end{aligned}$$

所以 $z=x^2+y^2$ 可微，并且 $dz=2xdx+2ydy$

解法二（利用定理） 先计算偏导数：

$$\frac{\partial z}{\partial x}=2x,\qquad \frac{\partial z}{\partial y}=2y$$

可见偏导数存在，且连续。所以函数可微，并且全微分为

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=2xdx+2ydy$$

例 计算函数 $z = e^{\frac{y}{x}}$ 的全微分

例 计算函数 $z = e^{\frac{y}{x}}$ 的全微分

解 先计算偏导数

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial x} =$$

例 计算函数 $z = e^{\frac{y}{x}}$ 的全微分

解 先计算偏导数

$$\frac{\partial z}{\partial x} = \left(e^{\frac{y}{x}}\right)'_x =$$

$$\frac{\partial z}{\partial x} =$$

例 计算函数 $z = e^{\frac{y}{x}}$ 的全微分

解 先计算偏导数

$$\frac{\partial z}{\partial x} = \left(e^{\frac{y}{x}}\right)'_x = e^{\frac{y}{x}} \cdot$$

$$\frac{\partial z}{\partial x} =$$

例 计算函数 $z = e^{\frac{y}{x}}$ 的全微分

解 先计算偏导数

$$\frac{\partial z}{\partial x} = \left(e^{\frac{y}{x}}\right)_x' = e^{\frac{y}{x}} \cdot \left(\frac{y}{x}\right)_x'$$

$$\frac{\partial z}{\partial x} =$$

例 计算函数 $z=e^{\frac{y}{x}}$ 的全微分

解 先计算偏导数

$$\frac{\partial z}{\partial x}=\left(e^{\frac{y}{x}}\right)_x'=e^{\frac{y}{x}}\cdot\left(\frac{y}{x}\right)_x'=-\frac{y}{x^2}e^{\frac{y}{x}}$$

$$\frac{\partial z}{\partial x}=$$

例 计算函数 $z = e^{\frac{y}{x}}$ 的全微分

解 先计算偏导数

$$\frac{\partial z}{\partial x} = \left(e^{\frac{y}{x}}\right)'_x = e^{\frac{y}{x}} \cdot \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2} e^{\frac{y}{x}}$$

$$\frac{\partial z}{\partial x} = \left(e^{\frac{y}{x}}\right)'_y =$$

例 计算函数 $z = e^{\frac{y}{x}}$ 的全微分

解 先计算偏导数

$$\frac{\partial z}{\partial x} = \left(e^{\frac{y}{x}}\right)'_x = e^{\frac{y}{x}} \cdot \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2} e^{\frac{y}{x}}$$

$$\frac{\partial z}{\partial x} = \left(e^{\frac{y}{x}}\right)'_y = e^{\frac{y}{x}} \cdot \left(\frac{y}{x}\right)'_y$$

例 计算函数 $z = e^{\frac{y}{x}}$ 的全微分

解 先计算偏导数

$$\frac{\partial z}{\partial x} = \left(e^{\frac{y}{x}}\right)'_x = e^{\frac{y}{x}} \cdot \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2} e^{\frac{y}{x}}$$

$$\frac{\partial z}{\partial x} = \left(e^{\frac{y}{x}}\right)'_y = e^{\frac{y}{x}} \cdot \left(\frac{y}{x}\right)'_y = \frac{1}{x} e^{\frac{y}{x}}$$

例 计算函数 $z = e^{\frac{y}{x}}$ 的全微分

解 先计算偏导数

$$\frac{\partial z}{\partial x} = \left(e^{\frac{y}{x}}\right)'_x = e^{\frac{y}{x}} \cdot \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2} e^{\frac{y}{x}}$$

$$\frac{\partial z}{\partial x} = \left(e^{\frac{y}{x}}\right)'_y = e^{\frac{y}{x}} \cdot \left(\frac{y}{x}\right)'_y = \frac{1}{x} e^{\frac{y}{x}}$$

可见函数在其自然定义域 $D = \{(x, y) \,|\, x \neq 0\}$ 上存在偏导数且偏导数连续。

例 计算函数 $z=e^{\frac{y}{x}}$ 的全微分

解 先计算偏导数

$$\frac{\partial z}{\partial x}=\left(e^{\frac{y}{x}}\right)'_x=e^{\frac{y}{x}}\cdot\left(\frac{y}{x}\right)'_x=-\frac{y}{x^2}e^{\frac{y}{x}}$$

$$\frac{\partial z}{\partial x}=\left(e^{\frac{y}{x}}\right)'_y=e^{\frac{y}{x}}\cdot\left(\frac{y}{x}\right)'_y=\frac{1}{x}e^{\frac{y}{x}}$$

可见函数在其自然定义域 $D=\{(x,y)\,|\,x\neq 0\}$ 上存在偏导数且偏导数连续。所以函数可微，并且全微分为

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy$$

例 计算函数 $z = e^{\frac{y}{x}}$ 的全微分

解 先计算偏导数

$$\frac{\partial z}{\partial x} = \left(e^{\frac{y}{x}}\right)'_x = e^{\frac{y}{x}} \cdot \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2} e^{\frac{y}{x}}$$

$$\frac{\partial z}{\partial x} = \left(e^{\frac{y}{x}}\right)'_y = e^{\frac{y}{x}} \cdot \left(\frac{y}{x}\right)'_y = \frac{1}{x} e^{\frac{y}{x}}$$

可见函数在其自然定义域 $D = \{(x, y) \,|\, x \neq 0\}$ 上存在偏导数且偏导数连续。所以函数可微，并且全微分为

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = -\frac{y}{x^2} e^{\frac{y}{x}} dx + \frac{1}{x} e^{\frac{y}{x}} dy$$

多元函数的全微分

- 对三元函数 $u=f(x,y,z)$，其全微分

$$du=u_x dx+u_y dy+u_z dz$$

多元函数的全微分

- 对三元函数 $u = f(x, y, z)$，其全微分

$$du = u_x dx + u_y dy + u_z dz$$

此时

$$\Delta u = f(x + \Delta x, y + \Delta y, z + \Delta z) - f(x, y, z)$$

多元函数的全微分

- 对三元函数 $u = f(x, y, z)$，其全微分

$$du = u_x dx + u_y dy + u_z dz$$

此时

$$\Delta u = f(x + \Delta x, y + \Delta y, z + \Delta z) - f(x, y, z)$$

$$= u_x \Delta x + u_y \Delta y + u_z \Delta z + o$$

多元函数的全微分

- 对三元函数 $u = f(x, y, z)$，其全微分

$$du = u_x dx + u_y dy + u_z dz$$

此时

$$\begin{aligned}\Delta u &= f(x+\Delta x, y+\Delta y, z+\Delta z) - f(x, y, z) \\ &= u_x \Delta x + u_y \Delta y + u_z \Delta z + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}\right)\end{aligned}$$

多元函数的全微分

- 对三元函数 $u = f(x, y, z)$，其全微分

$$du = u_x dx + u_y dy + u_z dz$$

此时

$$\begin{aligned}\Delta u &= f(x + \Delta x, y + \Delta y, z + \Delta z) - f(x, y, z) \\ &= u_x \Delta x + u_y \Delta y + u_z \Delta z + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}\right) \\ &\approx du\end{aligned}$$

例 设 $u = x^{yz}$，计算全微分 du

例 设 $u = x^{yz}$，计算全微分 du

例 设 $u = x^{yz}$，计算全微分 du

解 先计算偏导数

$$u_x =$$

$$u_y =$$

$$u_z =$$

例 设 $u = x^{yz}$，计算全微分 du

解 先计算偏导数

$$u_x = (x^{yz})'_x =$$

$$u_y =$$

$$u_z =$$

例 设 $u = x^{yz}$，计算全微分 du

解 先计算偏导数

$$u_x = (x^{yz})'_x = yz \cdot x^{yz-1}$$

$$u_y =$$

$$u_z =$$

例 设 $u = x^{yz}$，计算全微分 du

解 先计算偏导数

$$u_x = (x^{yz})'_x = yz \cdot x^{yz-1}$$

$$u_y = (x^{yz})'_y =$$

$$u_z =$$

例 设 $u = x^{yz}$，计算全微分 du

解 先计算偏导数

$$u_x = (x^{yz})'_x = yz \cdot x^{yz-1}$$

$$u_y = (x^{yz})'_y = x^{yz} \ln(x) \cdot z$$

$$u_z =$$

例 设 $u=x^{yz}$，计算全微分 du

解 先计算偏导数

$$u_x = (x^{yz})'_x = yz \cdot x^{yz-1}$$

$$u_y = (x^{yz})'_y = x^{yz}\ln(x)\cdot z = z\cdot x^{yz}\ln(x)$$

$$u_z =$$

例 设 $u = x^{yz}$，计算全微分 du

解 先计算偏导数

$$u_x = (x^{yz})'_x = yz \cdot x^{yz-1}$$

$$u_y = (x^{yz})'_y = x^{yz}\ln(x) \cdot z = z \cdot x^{yz}\ln(x)$$

$$u_z = (x^{yz})'_z =$$

例 设 $u = x^{yz}$，计算全微分 du

解 先计算偏导数

$$u_x = (x^{yz})'_x = yz \cdot x^{yz-1}$$

$$u_y = (x^{yz})'_y = x^{yz} \ln(x) \cdot z = z \cdot x^{yz} \ln(x)$$

$$u_z = (x^{yz})'_z = x^{yz} \ln(x) \cdot y$$

例 设 $u = x^{yz}$，计算全微分 du

解 先计算偏导数

$$u_x = (x^{yz})'_x = yz \cdot x^{yz-1}$$

$$u_y = (x^{yz})'_y = x^{yz} \ln(x) \cdot z = z \cdot x^{yz} \ln(x)$$

$$u_z = (x^{yz})'_z = x^{yz} \ln(x) \cdot y = y \cdot x^{yz} \ln(x)$$

例 设 $u = x^{yz}$，计算全微分 du

解 先计算偏导数

$$u_x = (x^{yz})'_x = yz \cdot x^{yz-1}$$

$$u_y = (x^{yz})'_y = x^{yz}\ln(x) \cdot z = z \cdot x^{yz}\ln(x)$$

$$u_z = (x^{yz})'_z = x^{yz}\ln(x) \cdot y = y \cdot x^{yz}\ln(x)$$

所以

$$du = u_x dx + u_y dy + u_z dz$$

例 设 $u = x^{yz}$，计算全微分 du

解 先计算偏导数

$$u_x = (x^{yz})'_x = yz \cdot x^{yz-1}$$

$$u_y = (x^{yz})'_y = x^{yz} \ln(x) \cdot z = z \cdot x^{yz} \ln(x)$$

$$u_z = (x^{yz})'_z = x^{yz} \ln(x) \cdot y = y \cdot x^{yz} \ln(x)$$

所以

$$\begin{aligned} du &= u_x dx + u_y dy + u_z dz \\ &= yz \cdot x^{yz-1} dx + z \cdot x^{yz} \ln(x) dy + y \cdot x^{yz} \ln(x) dz \end{aligned}$$

全微分在近似计算中的应用

设 $z = f(x, y)$，则 $f(x + \Delta x, y + \Delta y) - f(x, y) = dz + o(\rho) \approx dz$

全微分在近似计算中的应用

设 $z=f(x,y)$，则 $f(x+\Delta x, y+\Delta y)-f(x,y)=dz+o(\rho)\approx dz$，

所以

$$f(x+\Delta x, y+\Delta y)\approx f(x,y)+dz$$

全微分在近似计算中的应用

设 $z=f(x,y)$，则 $f(x+\Delta x,y+\Delta y)-f(x,y)=dz+o(\rho)\approx dz$，所以

$$f(x+\Delta x,y+\Delta y)\approx f(x,y)+dz$$

例 计算 $(1.04)^{2.02}$ 的近似值。

全微分在近似计算中的应用

设 $z = f(x, y)$，则 $f(x + \Delta x, y + \Delta y) - f(x, y) = dz + o(\rho) \approx dz$，

所以

$$f(x + \Delta x, y + \Delta y) \approx f(x, y) + dz$$

例 计算 $(1.04)^{2.02}$ 的近似值。

解

$$(1.04)^{2.02} =$$

全微分在近似计算中的应用

设 $z=f(x,y)$，则 $f(x+\Delta x, y+\Delta y)-f(x,y)=dz+o(\rho)\approx dz$，所以

$$f(x+\Delta x, y+\Delta y)\approx f(x,y)+dz$$

例 计算 $(1.04)^{2.02}$ 的近似值。

解

$$(1.04)^{2.02}=(1+0.04)^{2+0.02}$$

全微分在近似计算中的应用

设 $z=f(x,y)$，则 $f(x+\Delta x, y+\Delta y)-f(x,y)=dz+o(\rho)\approx dz$，所以

$$f(x+\Delta x, y+\Delta y)\approx f(x,y)+dz$$

例 计算 $(1.04)^{2.02}$ 的近似值。

解

$$(1.04)^{2.02}=(1+0.04)^{2+0.02}\approx 1^2+dz$$

全微分在近似计算中的应用

设 $z=f(x,y)$，则 $f(x+\Delta x,y+\Delta y)-f(x,y)=dz+o(\rho)\approx dz$，

所以

$$f(x+\Delta x,y+\Delta y)\approx f(x,y)+dz$$

例 计算 $(1.04)^{2.02}$ 的近似值。

解

$$(1.04)^{2.02}=\underbrace{(1+0.04)^{2+0.02}}_{f(x+\Delta x,y+\Delta y)}\approx\underbrace{1^2}_{f(x,y)}+dz$$

全微分在近似计算中的应用

设 $z=f(x,y)$，则 $f(x+\Delta x, y+\Delta y)-f(x,y)=dz+o(\rho)\approx dz$，所以

$$f(x+\Delta x, y+\Delta y)\approx f(x,y)+dz$$

例 计算 $(1.04)^{2.02}$ 的近似值。

解 设 $z=f(x,y)=x^y$，则

$$(1.04)^{2.02}=\underbrace{(1+0.04)^{2+0.02}}_{f(x+\Delta x,y+\Delta y)}\approx\underbrace{1^2}_{f(x,y)}+dz$$

全微分在近似计算中的应用

设 $z = f(x, y)$，则 $f(x+\Delta x, y+\Delta y) - f(x, y) = dz + o(\rho) \approx dz$，所以

$$f(x+\Delta x, y+\Delta y) \approx f(x, y) + dz$$

例 计算 $(1.04)^{2.02}$ 的近似值。

解 设 $z = f(x, y) = x^y$，则

$$(1.04)^{2.02} = \underbrace{(1+0.04)^{2+0.02}}_{f(x+\Delta x, y+\Delta y)} \approx \underbrace{1^2}_{f(x,y)} + dz$$

而

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

全微分在近似计算中的应用

设 $z=f(x,y)$，则 $f(x+\Delta x,y+\Delta y)-f(x,y)=dz+o(\rho)\approx dz$，

所以

$$f(x+\Delta x,y+\Delta y)\approx f(x,y)+dz$$

例 计算 $(1.04)^{2.02}$ 的近似值。

解 设 $z=f(x,y)=x^y$，则

$$(1.04)^{2.02}=\underbrace{(1+0.04)^{2+0.02}}_{f(x+\Delta x,y+\Delta y)}\approx\underbrace{1^2}_{f(x,y)}+dz$$

而

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=yx^{y-1}dx+x^y\ln x\,dy$$

全微分在近似计算中的应用

设 $z=f(x,y)$，则 $f(x+\Delta x, y+\Delta y)-f(x,y)=dz+o(\rho)\approx dz$，

所以

$$f(x+\Delta x, y+\Delta y)\approx f(x,y)+dz$$

例 计算 $(1.04)^{2.02}$ 的近似值。

解 设 $z=f(x,y)=x^y$，则

$$(1.04)^{2.02}=\underbrace{(1+0.04)^{2+0.02}}_{f(x+\Delta x,y+\Delta y)}\approx\underbrace{1^2}_{f(x,y)}+dz$$

而

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=yx^{y-1}dx+x^y\ln x\,dy$$

将 $(x,y)=(1,2)$ 及 $dx=\Delta x=0.04$、$dy=\Delta y=0.02$ 代入得:

全微分在近似计算中的应用

设 $z=f(x,y)$，则 $f(x+\Delta x,y+\Delta y)-f(x,y)=dz+o(\rho)\approx dz$，所以

$$f(x+\Delta x,y+\Delta y)\approx f(x,y)+dz$$

例 计算 $(1.04)^{2.02}$ 的近似值。

解 设 $z=f(x,y)=x^y$，则

$$(1.04)^{2.02}=\underbrace{(1+0.04)^{2+0.02}}_{f(x+\Delta x,y+\Delta y)}\approx\underbrace{1^2}_{f(x,y)}+dz$$

而

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=yx^{y-1}dx+x^y\ln x\,dy$$

将 $(x,y)=(1,2)$ 及 $dx=\Delta x=0.04$、$dy=\Delta y=0.02$ 代入得:

$$dz=$$

全微分在近似计算中的应用

设 $z=f(x,y)$，则 $f(x+\Delta x, y+\Delta y)-f(x,y)=dz+o(\rho)\approx dz$，

所以

$$f(x+\Delta x, y+\Delta y)\approx f(x,y)+dz$$

例 计算 $(1.04)^{2.02}$ 的近似值。

解 设 $z=f(x,y)=x^y$，则

$$(1.04)^{2.02}=\underbrace{(1+0.04)^{2+0.02}}_{f(x+\Delta x,y+\Delta y)}\approx\underbrace{1^2}_{f(x,y)}+dz$$

而

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=yx^{y-1}dx+x^y\ln x\,dy$$

将 $(x,y)=(1,2)$ 及 $dx=\Delta x=0.04$、$dy=\Delta y=0.02$ 代入得：

$$dz=2\cdot 1^1\cdot 0.04+1^2\cdot\ln 1\cdot 0.02$$

全微分在近似计算中的应用

设 $z=f(x,y)$，则 $f(x+\Delta x, y+\Delta y)-f(x,y)=dz+o(\rho)\approx dz$，

所以

$$f(x+\Delta x, y+\Delta y)\approx f(x,y)+dz$$

例 计算 $(1.04)^{2.02}$ 的近似值。

解 设 $z=f(x,y)=x^y$，则

$$(1.04)^{2.02}=\underbrace{(1+0.04)^{2+0.02}}_{f(x+\Delta x,y+\Delta y)}\approx\underbrace{1^2}_{f(x,y)}+dz$$

而

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=yx^{y-1}dx+x^y\ln x\,dy$$

将 $(x,y)=(1,2)$ 及 $dx=\Delta x=0.04$、$dy=\Delta y=0.02$ 代入得：

$$dz=2\cdot1^1\cdot0.04+1^2\cdot\ln1\cdot0.02=0.08$$

全微分在近似计算中的应用

设 $z = f(x, y)$，则 $f(x+\Delta x, y+\Delta y) - f(x, y) = dz + o(\rho) \approx dz$，

所以

$$f(x+\Delta x, y+\Delta y) \approx f(x, y) + dz$$

例 计算 $(1.04)^{2.02}$ 的近似值。

解 设 $z = f(x, y) = x^y$，则

$$(1.04)^{2.02} = \underbrace{(1+0.04)^{2+0.02}}_{f(x+\Delta x, y+\Delta y)} \approx \underbrace{1^2}_{f(x,y)} + dz$$

而

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy = yx^{y-1}dx + x^y \ln x dy$$

将 $(x, y) = (1, 2)$ 及 $dx = \Delta x = 0.04$、$dy = \Delta y = 0.02$ 代入得：

$$dz = 2 \cdot 1^1 \cdot 0.04 + 1^2 \cdot \ln 1 \cdot 0.02 = 0.08$$

所以 $(1.04)^{2.02} \approx dz + 1 = 0.08 + 1 = 1.08$

可微、偏导数存在、连续的区别与联系

设有二元函数 $z = f(x, y)$

可微、偏导数存在、连续的区别与联系

设有二元函数 $z = f(x, y)$

- 在点 (x_0, y_0) 处存在偏导数 $\frac{\partial z}{\partial x}(x_0, y_0)$, $\frac{\partial z}{\partial y}(x_0, y_0)$ $\not\Rightarrow$ f 在点 (x_0, y_0) 处连续

可微、偏导数存在、连续的区别与联系

设有二元函数 $z = f(x, y)$

- 在点 (x_0, y_0) 处存在偏导数 $\frac{\partial z}{\partial x}(x_0, y_0)$, $\frac{\partial z}{\partial y}(x_0, y_0) \not\Rightarrow f$ 在点 (x_0, y_0) 处连续
- 在点 (x_0, y_0) 处存在可微 $\Rightarrow f$ 在点 (x_0, y_0) 处连续，且存在偏导数 $\frac{\partial z}{\partial x}(x_0, y_0)$, $\frac{\partial z}{\partial y}(x_0, y_0)$

可微、偏导数存在、连续的区别与联系

设有二元函数 $z = f(x, y)$

- 在点 (x_0, y_0) 处存在偏导数 $\frac{\partial z}{\partial x}(x_0, y_0)$, $\frac{\partial z}{\partial y}(x_0, y_0) \not\Rightarrow f$ 在点 (x_0, y_0) 处连续

- 在点 (x_0, y_0) 处存在可微 $\Rightarrow f$ 在点 (x_0, y_0) 处连续，且存在偏导数 $\frac{\partial z}{\partial x}(x_0, y_0)$, $\frac{\partial z}{\partial y}(x_0, y_0)$

- 在点 (x_0, y_0) 附近存在偏导数 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$，且偏导数 $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ 在点 (x_0, y_0) 处连续 $\Rightarrow$ 在点 (x_0, y_0) 处可微