

## 第 2 章 $b$ : 矩阵的运算

数学系 梁卓滨

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# 矩阵的加法运算

**定义** 设  $A = (a_{ij})_{m \times n}$ ,  $B = (b_{ij})_{m \times n}$ , 则定义

$$A + B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$

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称为矩阵  $A$ ,  $B$  的**和**。

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例  $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$ ,  $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$ , 求  $A + B$  和  $A - B$

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**定义** 设  $A = (a_{ij})_{m \times n}$ ,  $k$  为数, 则定义

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**练习** 设  $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ ,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ ,  $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$ , 求  $3A + 2B - 4C$

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**练习** 设  $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ ,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且  $5A + 3X = B$ , 求  $X$

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**练习** 设  $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ ,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且  $5A + 3X = B$ , 求  $X$

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**解**

$$X = \frac{1}{3}(B - 5A) =$$

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注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

$$, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix}, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

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**性质** 设  $A, B, C$  均是  $m \times n$  矩阵,  $k, l$  是数, 则

1.  $k(A + B) = kA + kB$



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**证明** 设  $A = (a_{ij})_{m \times n}$ ,  $B = (b_{ij})_{m \times n}$ , 则

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$$kA + kB = (ka_{ij})_{m \times n} + (kb_{ij})_{m \times n}$$



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所以  $k(A + B) = kA + kB$ 。

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若  $Y$  满足  $(2A - Y) - 2(B + Y) = O$ , 求  $Y$

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若  $Y$  满足  $(2A - Y) - 2(B + Y) = O$ , 求  $Y$

解  $Y = \frac{2}{3}(A - B)$

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若  $Y$  满足  $(2A - Y) - 2(B + Y) = O$ , 求  $Y$

解  $Y = \frac{2}{3}(A - B)$ , 所以

$$Y = \frac{2}{3}(A - B) = \frac{2}{3} \left( \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix} \right)$$

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若  $Y$  满足  $(2A - Y) - 2(B + Y) = O$ , 求  $Y$

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$$\begin{aligned} Y &= \frac{2}{3}(A - B) = \frac{2}{3} \left( \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix} \right) \\ &= \frac{2}{3} \begin{pmatrix} -3 & -1 & -1 & 1 \\ 4 & 0 & 4 & 0 \\ 1 & 3 & 3 & 5 \end{pmatrix} \end{aligned}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设  $aA + bB + cC = I$ , 求数  $a, b, c$  的值

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设  $aA + bB + cC = I$ , 求数  $a, b, c$  的值

解

$$aA + bB + cC =$$



练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设  $aA + bB + cC = I$ , 求数  $a, b, c$  的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

练习 设

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练习 设

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所以

{



练习 设

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所以

$$\begin{cases} a + b - c = 1 \\ \end{cases}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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所以

$$\begin{cases} a + b - c = 1 \\ b = 0 \end{cases}$$

练习 设

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所以

$$\begin{cases} a + b - c = 1 \\ b = 0 \\ 2a + 3b + c = 0 \\ a - c = 1 \end{cases} \Rightarrow \begin{cases} b = 0 \end{cases}$$

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所以

$$\begin{cases} a + b - c = 1 \\ b = 0 \\ 2a + 3b + c = 0 \\ a - c = 1 \end{cases} \Rightarrow \begin{cases} a = \frac{1}{3} \\ b = 0 \end{cases}$$

练习 设

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所以

$$\begin{cases} a + b - c = 1 \\ b = 0 \\ 2a + 3b + c = 0 \\ a - c = 1 \end{cases} \Rightarrow \begin{cases} a = \frac{1}{3} \\ b = 0 \\ c = -\frac{2}{3} \end{cases}$$

# 矩阵的乘积

**定义** 设  $A = (a_{ik})_{m \times l}$ ,  $B = (b_{kj})_{l \times n}$ , 定义矩阵  $A$ ,  $B$  的 **乘积** 为  $m \times n$  矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$



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其中

$c_{ij} = A$  第  $i$  行与  $B$  第  $j$  列对应元素乘积的和

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$$a_{i1} \quad a_{i2} \quad \cdots \quad a_{il}$$

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即

$$a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj}$$

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即

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj} = \sum_{k=1}^l a_{ik}b_{kj}$$

# 矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
$$= \begin{pmatrix} c_{11} & \cdots & \cdots & c_{1n} \\ \vdots & & \vdots & \vdots \\ \cdots & c_{ij} & \cdots & \\ \vdots & & \vdots & \vdots \\ c_{m1} & \cdots & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$

# 矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$

$$= \begin{pmatrix} c_{11} & \cdots & \cdots & c_{1n} \\ \vdots & & \vdots & \vdots \\ \vdots & \cdots & c_{ij} & \cdots \\ \vdots & & \vdots & \vdots \\ c_{m1} & \cdots & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$

$$\begin{matrix} a_{i1} & a_{i2} & \cdots & a_{il} \end{matrix}$$

# 矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
$$= \begin{pmatrix} c_{11} & \cdots & \cdots & c_{1n} \\ \vdots & & \vdots & \vdots \\ \cdots & c_{ij} & \cdots & \\ \vdots & & \vdots & \vdots \\ c_{m1} & \cdots & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$
$$a_{i1}b_{1j} \quad a_{i2}b_{2j} \quad \cdots \quad a_{il}b_{lj}$$

# 矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
$$= \begin{pmatrix} c_{11} & \cdots & \cdots & c_{1n} \\ \vdots & & \vdots & \vdots \\ \cdots & c_{ij} & \cdots & \\ \vdots & & \vdots & \vdots \\ c_{m1} & \cdots & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$
$$a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj}$$

# 矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
$$= \begin{pmatrix} c_{11} & \cdots & \cdots & c_{1n} \\ \vdots & & \vdots & \vdots \\ \cdots & c_{ij} & \cdots & \\ \vdots & & \vdots & \vdots \\ c_{m1} & \cdots & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj}$$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3}$$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \end{pmatrix}$$



# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}_{4 \times 3}$$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$c_{23} =$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21} \quad a_{22}$$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

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例 2 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 计算  $AB$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

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例 2 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 计算  $AB$

解 
$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} =$$



# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

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例 2 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 计算  $AB$

解 
$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

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例 2 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 计算  $AB$

解 
$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} \color{red}{1} & -2 & -3 \\ \color{red}{2} & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} * & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

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解 
$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} \color{red}{1} & -2 & -3 \\ \color{red}{2} & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ \textcolor{red}{a_{21}} & \textcolor{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \textcolor{red}{b_{13}} \\ b_{21} & b_{22} & \textcolor{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \textcolor{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

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例 2 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 计算  $AB$

解 
$$AB = \begin{pmatrix} \textcolor{red}{2} & \textcolor{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & \textcolor{red}{-2} & -3 \\ 2 & \textcolor{red}{-1} & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & \textcolor{red}{*} & \end{pmatrix}_{3 \times 3}$$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

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例 2 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 计算  $AB$

解 
$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & \color{red}{-2} & -3 \\ 2 & \color{red}{-1} & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{pmatrix}_{3 \times 3}$$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ \textcolor{red}{a_{21}} & \textcolor{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \textcolor{red}{b_{13}} \\ b_{21} & b_{22} & \textcolor{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \textcolor{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

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例 2 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 计算  $AB$

解 
$$AB = \begin{pmatrix} \textcolor{red}{2} & \textcolor{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & \textcolor{red}{-3} \\ 2 & -1 & \textcolor{red}{0} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & \textcolor{red}{*} \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

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例 2 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 计算  $AB$

解 
$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & \color{red}{-3} \\ 2 & -1 & \color{red}{0} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

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例 2 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 计算  $AB$

解 
$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$



# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

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例 2 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 计算  $AB$

解 
$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ -3 & 0 & -3 \\ -3 & 0 & -3 \end{pmatrix}_{3 \times 3}$$

# 矩阵的乘积

例 1 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

---

例 2 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 计算  $AB$

解 
$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ -3 & 0 & -3 \\ 5 & -7 & -9 \end{pmatrix}_{3 \times 3}$$

# 矩阵的乘积

例 3 计算  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

# 矩阵的乘积

**例 3** 计算  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

**解**

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} =$$

# 矩阵的乘积

**例 3** 计算  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

**解**

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} & & \\ & & \end{pmatrix}_{2 \times 3}$$

# 矩阵的乘积

**例 3** 计算  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

**解**

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & & \\ & & \end{pmatrix}_{2 \times 3}$$

# 矩阵的乘积

**例 3** 计算  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

**解**

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} \end{pmatrix}_{2 \times 3}$$

# 矩阵的乘积

**例 3** 计算  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

**解**

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \end{pmatrix}_{2 \times 3}$$



# 矩阵的乘积

**例 3** 计算  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

**解**

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \\ 4 & & \end{pmatrix}_{2 \times 3}$$

# 矩阵的乘积

**例 3** 计算  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

**解**

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \\ 4 & -3 & \end{pmatrix}_{2 \times 3}$$

# 矩阵的乘积

**例 3** 计算  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

**解**

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \\ 4 & -3 & -1 \end{pmatrix}_{2 \times 3}$$

**例 4** 设  $A = (1, 2, 3)$ ,  $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ,  $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算  $AB$ ,  $BA$  及  $CB$ 。

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**解**

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

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**解**

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = ( \quad )_{1 \times 1}$$

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**解**

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1}$$

**例 4** 设  $A = (1, 2, 3)$ ,  $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ,  $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算  $AB$ ,  $BA$  及  $CB$ 。

**解**

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$



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# 矩阵的乘积

**例 5** 设  $A = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}$ ,  $B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}$ , 求  $AB$ ,  $BA$

# 矩阵的乘积

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# 矩阵的乘积

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# 矩阵的乘积

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# 矩阵的乘积

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## 矩阵的乘积

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**注**  $AB$  可以存在, 但  $BA$  不一定有意义

## 矩阵的乘积

**例 6** 设  $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$ ,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求  $AB$ ,  $BA$

## 矩阵的乘积

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**解**

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$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & \quad \\ \quad & \quad \end{pmatrix}_{2 \times 2}$$



# 矩阵的乘积

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**注**

1. 即便  $AB$ ,  $BA$  都有意义, 也不一定相等。  
矩阵的乘法不满足交换律!

# 矩阵的乘积

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**解**

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$

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**注**

1. 即便  $AB$ ,  $BA$  都有意义, 也不一定相等。  
矩阵的乘法不满足交换律!
2.  $BA = 0$  不能推出  $B = 0$  或  $A = 0$

# 矩阵的乘积

**注** 即便假设  $A \neq 0$ ,  $BA = CA$  也推不出  $B = C$ 。如

# 矩阵的乘积

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- 总结**
1.  $AB$  可以存在, 但  $BA$  不一定有意义
  2. 即便  $AB$ ,  $BA$  都有意义, 也不一定相等。矩阵的乘法不满足交换律! (矩阵相乘要注意顺序)
  3.  $BA = 0$  不能推出  $B = 0$  或  $A = 0$
  4. 即便假设  $A \neq 0$ ,  $BA = CA$  也推不出  $B = C$ 。



# 矩阵乘法的运算法则

设下列各式所涉及的矩阵乘法都是有意义，则

1.  $(AB)C = A(BC)$
2.  $(A + B)C = AC + BC$
3.  $C(A + B) = CA + CB$
4.  $k(AB) = (kA)B = A(kB)$

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**例 3** 设  $A = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix}$ , 计算  $AA^T$  及  $A^TA$ 。

**解**

$$AA^T = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & 2 \\ 2 & 13 \end{pmatrix}_{2 \times 2}$$

$$A^TA = \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 13 & 2 & 2 \\ 2 & 1 & 4 \\ 2 & 4 & 20 \end{pmatrix}_{3 \times 3}$$



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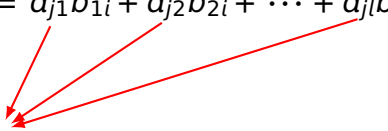
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设  $A = (a_{ij})_{n \times n}$  为  $n$  阶方阵,  $k \in \mathbb{N}$  为自然数, 定义

$$A^k = \underbrace{A \cdot A \cdot \dots \cdot A}_{k \uparrow}$$

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**练习** 设  $A = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$ , 其中  $\lambda$  为常数, 计算  $A^n$ 。

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**解**

$$A^2 =$$

$$A^3 =$$

$$A^4 =$$

$$\vdots$$

$$A^n =$$



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**解**

$$A^2 = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix}$$

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$$A^4 =$$

$$\vdots$$

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$$A^4 =$$

$\vdots$

$$A^n =$$

**练习** 设  $A = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$ , 其中  $\lambda$  为常数, 计算  $A^n$ 。

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$$A^2 = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix}$$

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# 方阵的幂

**注** 设  $A, B$  为  $n$  阶方阵, 一般地

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回忆：对  $n$  阶方阵

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**例** 设  $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$ , 求  $|4A|$

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## 方程组的矩阵表示

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

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$$\parallel$$
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$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

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$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

等价于

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_A \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

系数矩阵

## 方程组的矩阵表示

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等价于

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_A \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} = \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}_b$$

系数矩阵                      常数项矩阵



# 方程组的矩阵表示

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等价于

$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_A \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}}_x = \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}_b$$

系数矩阵                      常数项矩阵

## 方程组的矩阵表示

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系数矩阵                      常数项矩阵

进一步改写成

$$Ax = b$$

# 线性方程组的矩阵表示

例 方程组

$$\begin{cases} x_1 - x_2 + 5x_3 - x_4 = -2 \\ x_1 + x_2 - 2x_3 + 3x_4 = 3 \\ 3x_1 - x_2 + 8x_3 + x_4 = 7 \end{cases}$$

# 线性方程组的矩阵表示

例 方程组

$$\begin{cases} x_1 - x_2 + 5x_3 - x_4 = -2 \\ x_1 + x_2 - 2x_3 + 3x_4 = 3 \\ 3x_1 - x_2 + 8x_3 + x_4 = 7 \end{cases}$$

的矩阵表示  $Ax = b$  是

$$\begin{pmatrix} & & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

# 线性方程组的矩阵表示

例 方程组

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的矩阵表示  $Ax = b$  是

$$\begin{pmatrix} 1 & -1 & 5 & -1 \\ 1 & 1 & -2 & 3 \\ 3 & -1 & 8 & 1 \end{pmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix}$$

# 线性方程组的矩阵表示

例 方程组

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# 线性方程组的矩阵表示

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