



# 矩阵的加法运算

定义 设  $A = (a_{ij})_{m \times n}$ ,  $B = (b_{ij})_{m \times n}$ , 则定义

$$A + B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$

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称为矩阵  $A$ ,  $B$  的**和**。

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矩阵  $A$ ,  $B$  的差定义为:

$$A - B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} - \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$

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例  $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$ ,  $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$ , 求  $A + B$  和  $A - B$

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定义 设  $A = (a_{ij})_{m \times n}$ ,  $k$  为数, 则定义

$$kA = k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

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练习 设  $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ ,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ , 且  $5A + 3X = B$ , 求  $X$

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解

$$\begin{aligned} 3A + 2B - 4C &= 3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4 \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} \\ &= \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix} \end{aligned}$$

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解

$$X = \frac{1}{3}(B - 5A) =$$

$$\begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$$

练习 设  $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ ,  $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$ ,  $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$ , 求  $3A + 2B - 4C$

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解

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注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

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注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix}, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

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性质 设  $A, B, C$  均是  $m \times n$  矩阵,  $k, l$  是数, 则

1.  $k(A + B) = kA + kB$



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证明 设  $A = (a_{ij})_{m \times n}$ ,  $B = (b_{ij})_{m \times n}$ , 则

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$$k(A + B) = k(a_{ij} + b_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

$$kA + kB = (ka_{ij})_{m \times n} + (kb_{ij})_{m \times n}$$



## 注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

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$$kA + kB = (ka_{ij})_{m \times n} + (kb_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

所以  $k(A + B) = kA + kB$ 。

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若  $Y$  满足  $(2A - Y) - 2(B + Y) = O$ , 求  $Y$

练习 设

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解  $Y =$

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若  $Y$  满足  $(2A - Y) - 2(B + Y) = O$ , 求  $Y$

解  $Y = \frac{2}{3}(A - B)$

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若  $Y$  满足  $(2A - Y) - 2(B + Y) = O$ , 求  $Y$

解  $Y = \frac{2}{3}(A - B)$ , 所以

$$Y = \frac{2}{3}(A - B) =$$

## 练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

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$$Y = \frac{2}{3}(A - B) = \frac{2}{3} \left( \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix} \right)$$

## 练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

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$$\begin{aligned} Y &= \frac{2}{3}(A - B) = \frac{2}{3} \left( \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix} \right) \\ &= \frac{2}{3} \begin{pmatrix} -3 & -1 & -1 & 1 \\ 4 & 0 & 4 & 0 \\ 1 & 3 & 3 & 5 \end{pmatrix} \end{aligned}$$



练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设  $aA + bB + cC = I$ , 求数  $a, b, c$  的值

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

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所以

{

## 练习 设

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所以

$$\begin{cases} a + b - c = 1 \\ \end{cases}$$

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所以

$$\begin{cases} a + b - c = 1 \\ b = 0 \\ 2a + 3b + c = 0 \\ a - c = 1 \end{cases} \Rightarrow \begin{cases} b = 0 \end{cases}$$

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$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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$$\begin{cases} a + b - c = 1 \\ b = 0 \\ 2a + 3b + c = 0 \\ a - c = 1 \end{cases} \Rightarrow \begin{cases} a = \frac{1}{3} \\ b = 0 \end{cases}$$



## 练习 设

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所以

$$\begin{cases} a + b - c = 1 \\ b = 0 \\ 2a + 3b + c = 0 \\ a - c = 1 \end{cases} \Rightarrow \begin{cases} a = \frac{1}{3} \\ b = 0 \\ c = -\frac{2}{3} \end{cases}$$

# 矩阵的乘积

**定义** 设  $A = (a_{ik})_{m \times l}$ ,  $B = (b_{kj})_{l \times n}$ , 定义矩阵  $A$ ,  $B$  的乘积为  $m \times n$  矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

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其中

$c_{ij} = A$  第  $i$  行与  $B$  第  $j$  列对应元素乘积的和

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即

$$a_{i1} \quad a_{i2} \quad \cdots \quad a_{il}$$

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$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

其中

$c_{ij} = A$  第  $i$  行与  $B$  第  $j$  列对应元素乘积的和

即

$$a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj}$$

# 矩阵的乘积

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$$a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj} = \sum_{k=1}^l a_{ik}b_{kj}$$



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其中

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即

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj} = \sum_{k=1}^l a_{ik}b_{kj}$$

# 矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ \color{red}{a_{i1}} & \cdots & \cdots & \color{red}{a_{il}} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & \color{red}{b_{1j}} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & \color{red}{b_{lj}} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
$$= \begin{pmatrix} c_{11} & \cdots & \cdots & c_{1n} \\ \vdots & & \vdots & \vdots \\ \vdots & \cdots & \color{red}{c_{ij}} & \cdots \\ \vdots & & \vdots & \vdots \\ c_{m1} & \cdots & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$

# 矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ \color{red}{a_{i1}} & \cdots & \cdots & \color{red}{a_{il}} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & \color{red}{b_{1j}} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & \color{red}{b_{lj}} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
$$= \begin{pmatrix} c_{11} & \cdots & \cdots & c_{1n} \\ \vdots & & \vdots & \vdots \\ \vdots & \cdots & \color{red}{c_{ij}} & \cdots \\ \vdots & & \vdots & \vdots \\ c_{m1} & \cdots & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$
$$\begin{matrix} a_{i1} & a_{i2} & \cdots & a_{il} \end{matrix}$$

# 矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ \color{red}{a_{i1}} & \cdots & \cdots & \color{red}{a_{il}} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & \color{red}{b_{1j}} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & \color{red}{b_{lj}} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
$$= \begin{pmatrix} c_{11} & \cdots & \cdots & c_{1n} \\ \vdots & & \vdots & \vdots \\ \cdots & \color{red}{c_{ij}} & \cdots & \\ \vdots & \vdots & & \vdots \\ c_{m1} & \cdots & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$
$$a_{i1}b_{1j} \quad a_{i2}b_{2j} \quad \cdots \quad a_{il}b_{lj}$$

## 矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ \color{red}{a_{i1}} & \cdots & \cdots & \color{red}{a_{il}} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & \color{red}{b_{1j}} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & \color{red}{b_{lj}} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
$$= \begin{pmatrix} c_{11} & \cdots & \cdots & c_{1n} \\ \vdots & & \vdots & \vdots \\ \vdots & \cdots & \color{red}{c_{ij}} & \cdots \\ \vdots & & \vdots & \vdots \\ c_{m1} & \cdots & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$
$$a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj}$$

## 矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ \color{red}{a_{i1}} & \cdots & \cdots & \color{red}{a_{il}} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & \color{red}{b_{1j}} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & \color{red}{b_{lj}} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
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$$a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj} = \sum_{k=1}^l a_{ik}b_{kj}$$

## 矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ \color{red}{a_{i1}} & \cdots & \cdots & \color{red}{a_{il}} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & \color{red}{b_{1j}} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & \color{red}{b_{lj}} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
$$= \begin{pmatrix} c_{11} & \cdots & \cdots & c_{1n} \\ \vdots & & \vdots & \vdots \\ \vdots & \cdots & \color{red}{c_{ij}} & \cdots \\ \vdots & & \vdots & \vdots \\ c_{m1} & \cdots & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj} = \sum_{k=1}^l a_{ik}b_{kj}$$

# 矩阵的乘积

例  $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3}$



# 矩阵的乘积

例  $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \end{pmatrix}$

# 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}_{4 \times 3}$$

# 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

# 矩阵的乘积

例  $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \textcolor{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$

$$c_{23} =$$

# 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21} \quad a_{22}$$

## 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

# 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

## 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求  $AB$



## 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求  $AB$

解

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} =$$

## 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求  $AB$

解

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

## 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求  $AB$

解

$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} \color{red}{1} & -2 & -3 \\ \color{red}{2} & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} * & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

## 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求  $AB$

解

$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} \color{red}{1} & -2 & -3 \\ \color{red}{2} & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

## 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求  $AB$

解

$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & \color{red}{-2} & -3 \\ 2 & \color{red}{-1} & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & \color{red}{*} & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} \end{pmatrix}_{3 \times 3}$$

## 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求  $AB$

解

$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & \color{red}{-2} & -3 \\ 2 & \color{red}{-1} & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} \\ \phantom{0} & \phantom{0} & \phantom{0} \end{pmatrix}_{3 \times 3}$$

## 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求  $AB$

解

$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & \color{red}{-3} \\ 2 & -1 & \color{red}{0} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & \color{red}{*} \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

## 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求  $AB$

解

$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & \color{red}{-3} \\ 2 & -1 & \color{red}{0} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$



## 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求  $AB$

解

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

## 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求  $AB$

解

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ -3 & 0 & -3 \\ -3 & 0 & -3 \end{pmatrix}_{3 \times 3}$$

## 矩阵的乘积

例 
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 设  $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$ ,  $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$ , 求  $AB$

解

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ -3 & 0 & -3 \\ 5 & -7 & -9 \end{pmatrix}_{3 \times 3}$$

# 矩阵的乘积

练习 求  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

# 矩阵的乘积

练习 求  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} =$$

# 矩阵的乘积

练习 求  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} & & \\ & & \end{pmatrix}_{2 \times 3}$$

# 矩阵的乘积

练习 求  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & & \\ & & \end{pmatrix}_{2 \times 3}$$

# 矩阵的乘积

练习 求  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & \quad \\ \quad & \quad & \quad \end{pmatrix}_{2 \times 3}$$



# 矩阵的乘积

练习 求  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \end{pmatrix}_{2 \times 3}$$

# 矩阵的乘积

练习 求  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \\ 4 & & \end{pmatrix}_{2 \times 3}$$

# 矩阵的乘积

练习 求  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \\ 4 & -3 & \end{pmatrix}_{2 \times 3}$$

# 矩阵的乘积

练习 求  $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \\ 4 & -3 & -1 \end{pmatrix}_{2 \times 3}$$

练习  $A = (1, 2, 3)$ ,  $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ,  $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$  计算  $AB$ ,  $BA$  及  $CB$ 。

练习  $A = (1, 2, 3)$ ,  $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ,  $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$  计算  $AB$ ,  $BA$  及  $CB$ 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

练习  $A = (1, 2, 3)$ ,  $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ,  $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$  计算  $AB$ ,  $BA$  及  $CB$ 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = ( \quad )_{1 \times 1}$$

练习  $A = (1, 2, 3)$ ,  $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ,  $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$  计算  $AB$ ,  $BA$  及  $CB$ 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1}$$



练习  $A = (1, 2, 3)$ ,  $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ,  $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$  计算  $AB$ ,  $BA$  及  $CB$ 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

练习  $A = (1, 2, 3)$ ,  $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ,  $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$  计算  $AB$ ,  $BA$  及  $CB$ 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

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练习  $A = (1, 2, 3)$ ,  $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ,  $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$  计算  $AB$ ,  $BA$  及  $CB$ 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

练习  $A = (1, 2, 3)$ ,  $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ,  $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$  计算  $AB$ ,  $BA$  及  $CB$ 。

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练习  $A = (1, 2, 3)$ ,  $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ,  $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$  计算  $AB$ ,  $BA$  及  $CB$ 。

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解

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# 矩阵的乘积

例 设  $A = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}$ ,  $B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}$ , 求  $AB$ ,  $BA$

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例 设  $A = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}$ ,  $B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}$ , 求  $AB$ ,  $BA$

解

$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} =$$

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$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} = \begin{pmatrix} \quad \quad \end{pmatrix}$$

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# 矩阵的乘积

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解

$$AB = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} = \begin{pmatrix} 2 & \end{pmatrix}_{1 \times 2}$$



# 矩阵的乘积

例 设  $A = (3 \ 1 \ 0)$ ,  $B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}$ , 求  $AB$ ,  $BA$

解

$$AB = (3 \ 1 \ 0)_{1 \times 3} \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} = (2 \ 3)_{1 \times 2}$$

# 矩阵的乘积

例 设  $A = \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}$ ,  $B = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}$ , 求  $AB$ ,  $BA$

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$$BA = \begin{pmatrix} 2 & 1 \\ -4 & 0 \\ -3 & 5 \end{pmatrix}_{3 \times 2} \begin{pmatrix} 3 & 1 & 0 \end{pmatrix}_{1 \times 3} \text{ 没有意义!}$$

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注  $AB$  可以存在, 但  $BA$  不一定有意义

# 矩阵的乘积

例 设  $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$ ,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求  $AB$ ,  $BA$

# 矩阵的乘积

例 设  $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$ ,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求  $AB$ ,  $BA$

解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} =$$

## 矩阵的乘积

例 设  $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$ ,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求  $AB$ ,  $BA$

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$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} & \\ & \end{pmatrix}$$

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解

$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & \quad \\ \quad & \quad \end{pmatrix}_{2 \times 2}$$

# 矩阵的乘积

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$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ & \end{pmatrix}_{2 \times 2}$$

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$$BA = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} 0 & \\ & \end{pmatrix}_{2 \times 2}$$



# 矩阵的乘积

例 设  $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$ ,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求  $AB$ ,  $BA$

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注

1. 即便  $AB$ ,  $BA$  都有意义, 也不一定相等。

矩阵的乘法不满足交换律!

# 矩阵的乘积

例 设  $A = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}$ ,  $B = \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}$ , 求  $AB$ ,  $BA$

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$$AB = \begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}_{2 \times 2} \begin{pmatrix} 2 & 4 \\ -3 & -6 \end{pmatrix}_{2 \times 2} = \begin{pmatrix} -16 & -32 \\ 8 & 16 \end{pmatrix}_{2 \times 2}$$

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注

1. 即便  $AB$ ,  $BA$  都有意义, 也不一定相等。

矩阵的乘法不满足交换律!

2.  $BA = 0$  不能推出  $B = 0$  或  $A = 0$

# 矩阵的乘积

注 即便假设  $A \neq 0$ ,  $BA = CA$  也推不出  $B = C$ 。如

# 矩阵的乘积

注 即便假设  $A \neq 0$ ,  $BA = CA$  也推不出  $B = C$ 。如

$$\underbrace{\begin{pmatrix} 2 & 0 \\ 0 & -6 \end{pmatrix}}_B \underbrace{\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}}_A \quad \underbrace{\begin{pmatrix} 0 & -4 \\ 3 & 0 \end{pmatrix}}_C \underbrace{\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}}_A$$

# 矩阵的乘积

注 即便假设  $A \neq 0$ ,  $BA = CA$  也推不出  $B = C$ 。如

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- 总结
1.  $AB$  可以存在, 但  $BA$  不一定有意义
  2. 即便  $AB$ ,  $BA$  都有意义, 也不一定相等。矩阵的乘法不满足交换律! (矩阵相乘要注意顺序)
  3.  $BA = 0$  不能推出  $B = 0$  或  $A = 0$
  4. 即便假设  $A \neq 0$ ,  $BA = CA$  也推不出  $B = C$ 。

# 矩阵乘法的运算法则

设下列各式所涉及的矩阵乘法都是有意义，则

1.  $(AB)C = A(BC)$
2.  $(A + B)C = AC + BC$
3.  $C(A + B) = CA + CB$
4.  $k(AB) = (kA)B = A(kB)$

# 矩阵的转置

**定义** 将  $m \times n$  矩阵  $A$  的行与列互换，得到的  $n \times m$  矩阵，称为矩阵  $A$  的转置矩阵，记为  $A^T$ 。

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证明 设  $A = A_{m \times l}$ ,  $B = B_{l \times n}$

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证明 设  $A = A_{m \times l}$ ,  $B = B_{l \times n}$ , 则

	$AB$	$(AB)^T$	$B^T$	$A^T$	$B^T A^T$
阶数					

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并且

$$(AB)^T \text{ 的 } (i, j) \text{ 元素} =$$

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阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

并且

$$(AB)^T \begin{matrix} (i, j) \text{元素} \end{matrix} = \begin{matrix} AB \\ (j, i) \text{元素} \end{matrix} =$$

$$\begin{matrix} B^T A^T \\ (i, j) \text{元素} \end{matrix}$$

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并且

$$\begin{array}{ccccccc} (AB)^T & & AB & & & & B^T A^T \\ (i, j) \text{元素} & = & (j, i) \text{元素} & = & a_{j1} & a_{j2} & \cdots & a_{jl} & & (i, j) \text{元素} \end{array}$$

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并且

$$\begin{aligned}
 (AB)^T \text{ (i, j) 元素} &= AB \text{ (j, i) 元素} = a_{j1}b_{1i} + a_{j2}b_{2i} + \cdots + a_{jl}b_{li} & B^T A^T \text{ (i, j) 元素} \\
 &\quad \swarrow \quad \swarrow \quad \swarrow \\
 &A^T \text{ 第 } j \text{ 列元素}
 \end{aligned}$$

# 转置矩阵的性质

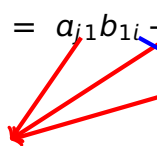
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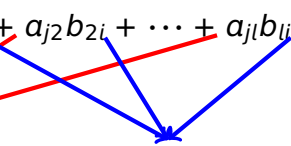
	$AB$	$(AB)^T$	$B^T$	$A^T$	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

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$A^T$  第  $j$  列元素



$B^T$  第  $i$  行元素

$B^T A^T$   
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$A^T$  第  $j$  列元素       $B^T$  第  $i$  行元素

# 方阵的幂

设  $A = (a_{ij})_{n \times n}$  为  $n$  阶方阵,  $k \in \mathbb{N}$  为自然数, 定义

$$A^k = \underbrace{A \cdot A \cdot \dots \cdot A}_{k \text{ 个}}$$

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练习 设  $A = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$ , 其中  $\lambda$  为常数, 计算  $A^n$ 。

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解

$$A^2 =$$

$$A^3 =$$

$$A^4 =$$

$$\vdots$$

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解

$$A^2 = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2\lambda & 1 \end{pmatrix}$$

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$$A^n = \begin{pmatrix} 1 & 0 \\ n\lambda & 1 \end{pmatrix}$$

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注 设  $A, B$  为  $n$  阶方阵, 一般地

$$(AB)^k \neq A^k B^k$$



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回忆：对  $n$  阶方阵

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系数矩阵

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系数矩阵                      常数项矩阵

进一步改写成

$$Ax = b$$



# 线性方程组的矩阵表示

例方程组

$$\begin{cases} x_1 - x_2 + 5x_3 - x_4 = -2 \\ x_1 + x_2 - 2x_3 + 3x_4 = 3 \\ 3x_1 - x_2 + 8x_3 + x_4 = 7 \end{cases}$$

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的矩阵表示  $Ax = b$  是

$$\begin{pmatrix} & & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

# 线性方程组的矩阵表示

例方程组

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的矩阵表示  $Ax = b$  是

$$\begin{pmatrix} 1 & -1 & 5 & -1 \\ 1 & 1 & -2 & 3 \\ 3 & -1 & 8 & 1 \end{pmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix}$$

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