§5.4 分部积分法

数学系　　梁卓滨

2016-2017 学年 II

教学要求

◇ 熟练掌握分部积分法

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Outline of §5.4

分部积分法，能干啥？

能够计算如下的不定积分：

$$\int xe^x dx,\quad \int x^2 \ln x dx,\quad \int x \arctan x dx,\quad \int x \cos x dx$$

$$\int \ln x dx,\quad \int \arcsin x dx,\quad \int \arctan x dx,\quad \int \ln(1+x^2) dx$$

$$\int x^2 e^x dx,\quad \int e^x \cos x dx \qquad \cdots\cdots$$

（可能要结合前面学的换元积分法）

分部积分引入

- 微分公式

$$d(uv) = udv + vdu$$

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$$d(uv) = udv + vdu \quad \Rightarrow \quad udv = d(uv) - vdu$$

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- 微分公式

$$d(uv) = udv + vdu \quad \Rightarrow \quad udv = d(uv) - vdu$$

练习

$$x\cos x dx =$$

$$xe^x dx =$$

$$\ln x dx =$$

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$$d(uv) = udv + vdu \quad \Rightarrow \quad udv = d(uv) - vdu$$

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$$x\cos x dx = x d\sin x =$$

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$$x\cos x dx = xd\sin x = d(x\sin x) - \sin x dx$$

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应用

$$x\cos x dx = \quad xd\sin x = \quad d(x\sin x) - \quad \sin x dx$$

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$$d(uv) = udv + vdu \quad \Rightarrow \quad udv = d(uv) - vdu$$

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$$d(uv) = udv + vdu \quad \Rightarrow \quad udv = d(uv) - vdu$$

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$$x\cos x dx = x d\sin x = d(x\sin x) - \sin x dx$$

$$xe^x dx = x de^x = d(xe^x) - e^x dx$$

$$\ln x dx = d(x\ln x) - x d\ln x = d(x\ln x) - dx$$

应用

$$\int x\cos x dx = \int x d\sin x = \int d(x\sin x) - \int \sin x dx$$

$$= x\sin x + \cos x + C$$

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$$= x\ln x - x + C$$

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$$\boxed{\int udv = uv - \int vdu}$$

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- 两边积分得：

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- 实际应用时的步骤：

$$\int \text{"Ì¡żğÆć"} dx =$$

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- 两边积分得：

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- 实际应用时的步骤：

$$\int \text{"Ì¡żğÆć"} dx = \int uv' dx$$

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- 实际应用时的步骤：

$$\int \text{"Ì¡żğÆć"}\, dx = \int uv'dx$$

$$\xlongequal{\text{凑微分}} \int udv =$$

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例子 求不定积分 $\int x\cos x dx$，$\int xe^{x}dx$，$\int x\sin x dx$

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解

$$\int x\cos x dx =$$

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解

$$\int x\cos x dx = \int x d\sin x = x\sin x - \int \sin x dx = x\sin x + \cos x + C$$

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$$\int x\cos x dx = \int x d\sin x = x\sin x - \int \sin x dx = x\sin x + \cos x + C$$

行不通的做法
$$\int x\cos x dx = \int \cos x \cdot d(\frac{1}{2}x^2)$$

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$$\int x\sin x dx = \int x d(-\cos x) =$$

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$$= \frac{1}{2}x^2\cos x + \underbrace{\int \frac{1}{2}x^2 \sin x dx}_{} = \ldots\ldots$$

例子 求不定积分 $\int x\cos x dx$，$\int xe^x dx$，$\int x\sin x dx$

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$$\int x\sin x dx = \int x d(-\cos x) = x(-\cos x) - \int (-\cos x) dx$$

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$$\int x\sin x dx = \int x d(-\cos x) = x(-\cos x) - \int (-\cos x) dx$$

$$= -x\cos x + \sin x + C$$

行不通的做法

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$$= \frac{1}{2}x^2\cos x + \underbrace{\int \frac{1}{2}x^2\sin x dx} = \ldots\ldots$$

例子 求不定积分 $\int x \ln x dx$，$\int x^2 \ln x dx$

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$$\int x^2 \ln x dx =$$

例子 求不定积分 $\int x\ln x dx$， $\int x^2\ln x dx$

解
$$\int x\ln x dx = \int \ln x d(\frac{1}{2}x^2) =$$

$$\int x^2\ln x dx =$$

例子 求不定积分 $\int x \ln x dx$，$\int x^2 \ln x dx$

解
$$\int x \ln x dx = \int \ln x d(\frac{1}{2}x^2) = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 d \ln x$$

$$\int x^2 \ln x dx =$$

例子 求不定积分 $\int x\ln x dx$，$\int x^2\ln x dx$

解

$$\int x\ln x dx = \int \ln x d(\frac{1}{2}x^2) = \frac{1}{2}x^2\cdot\ln x - \int\frac{1}{2}x^2 d\ln x$$

$$= \frac{1}{2}x^2\cdot\ln x - \int\frac{1}{2}x^2\cdot\frac{1}{x}dx$$

$$\int x^2\ln x dx =$$

例子 求不定积分 $\int x\ln x dx$，$\int x^2\ln x dx$

解

$$\int x\ln x dx = \int \ln x d(\frac{1}{2}x^2) = \frac{1}{2}x^2\cdot\ln x - \int\frac{1}{2}x^2 d\ln x$$

$$= \frac{1}{2}x^2\cdot\ln x - \int\frac{1}{2}x^2\cdot\frac{1}{x}dx$$

$$= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$$

$$\int x^2\ln x dx =$$

例子 求不定积分 $\int x\ln x dx$，$\int x^2\ln x dx$

解
$$\int x\ln x dx = \int \ln x d(\frac{1}{2}x^2) = \frac{1}{2}x^2\cdot\ln x - \int \frac{1}{2}x^2 d\ln x$$
$$= \frac{1}{2}x^2\cdot\ln x - \int \frac{1}{2}x^2\cdot\frac{1}{x}dx$$
$$= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$$
$$\int x^2\ln x dx = \int \ln x d(\frac{1}{3}x^3) =$$

例子 求不定积分 $\int x\ln x dx$，$\int x^2\ln x dx$

解
$$\int x\ln x dx = \int \ln x d(\frac{1}{2}x^2) = \frac{1}{2}x^2\cdot\ln x - \int \frac{1}{2}x^2 d\ln x$$
$$= \frac{1}{2}x^2\cdot\ln x - \int \frac{1}{2}x^2\cdot\frac{1}{x}dx$$
$$= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$$
$$\int x^2\ln x dx = \int \ln x d(\frac{1}{3}x^3) = \frac{1}{3}x^3\cdot\ln x - \int \frac{1}{3}x^3 d\ln x$$

例子 求不定积分 $\int x\ln x dx$，$\int x^2\ln x dx$

解

$$\int x\ln x dx = \int \ln x d(\frac{1}{2}x^2) = \frac{1}{2}x^2\cdot\ln x - \int \frac{1}{2}x^2 d\ln x$$

$$= \frac{1}{2}x^2\cdot\ln x - \int \frac{1}{2}x^2\cdot\frac{1}{x}dx$$

$$= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$$

$$\int x^2\ln x dx = \int \ln x d(\frac{1}{3}x^3) = \frac{1}{3}x^3\cdot\ln x - \int \frac{1}{3}x^3 d\ln x$$

$$= \frac{1}{3}x^3\cdot\ln x - \int \frac{1}{3}x^3\cdot\frac{1}{x}dx$$

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$$= \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$$

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$$= \frac{1}{3}x^3\cdot\ln x - \int\frac{1}{3}x^3\cdot\frac{1}{x}dx$$
$$= \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$$

例子 求不定积分 $\int x \arctan x dx$

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$$\int x \arctan x dx =$$

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例子 求不定积分 $\int x \arctan x dx$

解

$$\int x \arctan x dx = \int \arctan x d(\frac{1}{2}x^2)$$

$$= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 d \arctan x$$

例子 求不定积分 $\int x \arctan x dx$

解
$$\int x \arctan x dx = \int \arctan x d(\frac{1}{2}x^2)$$
$$= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 d \arctan x$$
$$= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2} dx$$

例子 求不定积分 $\int x\arctan x dx$

解

$$\int x\arctan x dx = \int \arctan x d(\frac{1}{2}x^2)$$
$$= \frac{1}{2}x^2\arctan x - \int \frac{1}{2}x^2 d\arctan x$$
$$= \frac{1}{2}x^2\arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2} dx$$
$$= \frac{1}{2}x^2\arctan x - \frac{1}{2}\int \frac{x^2}{1+x^2} dx$$

例子 求不定积分 $\int x \arctan x dx$

解

$$\begin{aligned}\int x \arctan x dx &= \int \arctan x d(\frac{1}{2}x^2) \\ &= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 d \arctan x \\ &= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2} dx \\ &= \frac{1}{2}x^2 \arctan x - \frac{1}{2}\int \frac{x^2}{1+x^2} dx \\ &= \frac{1}{2}x^2 \arctan x - \frac{1}{2}\int \left(1 - \frac{1}{1+x^2}\right) dx\end{aligned}$$

例子 求不定积分 $\int x\arctan x dx$

解
$$\begin{aligned}\int x\arctan x dx &= \int \arctan x d(\frac{1}{2}x^2)\\ &= \frac{1}{2}x^2\arctan x - \int \frac{1}{2}x^2 d\arctan x\\ &= \frac{1}{2}x^2\arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2}dx\\ &= \frac{1}{2}x^2\arctan x - \frac{1}{2}\int \frac{x^2}{1+x^2}dx\\ &= \frac{1}{2}x^2\arctan x - \frac{1}{2}\int \left(1-\frac{1}{1+x^2}\right)dx\\ &= \frac{1}{2}x^2\arctan x - \frac{1}{2}x + \frac{1}{2}\arctan x + C\end{aligned}$$

例子 求不定积分 $\int \ln x dx$，$\int \ln(1+x^2) dx$

例子 求不定积分 $\int \ln x dx$, $\int \ln(1+x^2) dx$

解
$$\int \ln x dx =$$

例子 求不定积分 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解
$$\int \ln x dx = x\ln x - \int x d\ln x =$$

例子 求不定积分 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解
$$\int \ln x dx = x\ln x - \int x d\ln x = x\ln x - \int x\cdot\frac{1}{x}dx$$

例子 求不定积分 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解
$$\int \ln x dx = x\ln x - \int x d\ln x = x\ln x - \int x \cdot \frac{1}{x} dx$$
$$= x\ln x - x + C$$

例子 求不定积分 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解

$$\int \ln x dx = x\ln x - \int x d\ln x = x\ln x - \int x\cdot\frac{1}{x}dx$$
$$= x\ln x - x + C$$

$$\int \ln(1+x^2)dx =$$

例子 求不定积分 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解
$$\int \ln x dx = x\ln x - \int x d\ln x = x\ln x - \int x \cdot \frac{1}{x} dx$$
$$= x\ln x - x + C$$

$$\int \ln(1+x^2)dx = x\ln(1+x^2) - \int x d\ln(1+x^2)$$

例子 求不定积分 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解
$$\int \ln x dx = x\ln x - \int x d\ln x = x\ln x - \int x\cdot\frac{1}{x}dx$$
$$= x\ln x - x + C$$

$$\int \ln(1+x^2)dx = x\ln(1+x^2) - \int x d\ln(1+x^2)$$
$$= x\ln(1+x^2) - \int x\cdot\frac{2x}{1+x^2}dx$$

例子 求不定积分 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解

$$\int \ln x dx = x\ln x - \int x d\ln x = x\ln x - \int x\cdot\frac{1}{x}dx$$
$$= x\ln x - x + C$$

$$\int \ln(1+x^2)dx = x\ln(1+x^2) - \int x d\ln(1+x^2)$$
$$= x\ln(1+x^2) - \int x\cdot\frac{2x}{1+x^2}dx$$
$$= x\ln(1+x^2) - 2\int\frac{x^2}{1+x^2}dx$$

例子 求不定积分 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解

$$\int \ln x dx = x\ln x - \int x d\ln x = x\ln x - \int x \cdot \frac{1}{x} dx$$
$$= x\ln x - x + C$$

$$\int \ln(1+x^2)dx = x\ln(1+x^2) - \int x d\ln(1+x^2)$$
$$= x\ln(1+x^2) - \int x \cdot \frac{2x}{1+x^2} dx$$
$$= x\ln(1+x^2) - 2\int \frac{x^2}{1+x^2} dx$$
$$= x\ln(1+x^2) - 2\int \left(1 - \frac{1}{1+x^2}\right) dx$$

例子 求不定积分 $\int \ln x dx$，$\int \ln(1+x^2)dx$

解
$$\int \ln x dx = x\ln x - \int x d\ln x = x\ln x - \int x\cdot\frac{1}{x}dx$$
$$= x\ln x - x + C$$

$$\int \ln(1+x^2)dx = x\ln(1+x^2) - \int x d\ln(1+x^2)$$
$$= x\ln(1+x^2) - \int x\cdot\frac{2x}{1+x^2}dx$$
$$= x\ln(1+x^2) - 2\int \frac{x^2}{1+x^2}dx$$
$$= x\ln(1+x^2) - 2\int\left(1-\frac{1}{1+x^2}\right)dx$$
$$= x\ln(1+x^2) - 2x + 2\arctan x + C$$

例子 求不定积分 $\int \arctan x dx$

例子 求不定积分 $\int \arctan x dx$

解

$$\int \arctan x dx =$$

例子 求不定积分 $\int \arctan x dx$

解

$$\int \arctan x dx = x \arctan x - \int x d \arctan x$$

例子 求不定积分 $\int \arctan x dx$

解

$$\int \arctan x dx = x \arctan x - \int x d \arctan x$$
$$= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx$$

例子 求不定积分 $\int \arctan x dx$

解

$$\int \arctan x dx = x \arctan x - \int x d \arctan x$$
$$= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx$$
$$= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} dx^2$$

例子 求不定积分 $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x\arctan x - \int x d\arctan x \\ &= x\arctan x - \int x \cdot \frac{1}{1+x^2} dx \\ &= x\arctan x - \frac{1}{2}\int \frac{1}{1+x^2} dx^2 \\ &= x\arctan x - \frac{1}{2}\int \frac{1}{1+x^2} d(1+x^2)\end{aligned}$$

例子 求不定积分 $\int \arctan x dx$

解
$$\begin{aligned}\int \arctan x dx &= x\arctan x - \int x d\arctan x \\ &= x\arctan x - \int x\cdot\frac{1}{1+x^2}dx \\ &= x\arctan x - \frac{1}{2}\int\frac{1}{1+x^2}dx^2 \\ &= x\arctan x - \frac{1}{2}\int\frac{1}{1+x^2}d(1+x^2)\end{aligned}$$

$$\int\frac{1}{1+x^2}d(1+x^2) =$$

例子 求不定积分 $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x\arctan x - \int x d\arctan x \\ &= x\arctan x - \int x \cdot \frac{1}{1+x^2} dx \\ &= x\arctan x - \frac{1}{2}\int \frac{1}{1+x^2} dx^2 \\ &= x\arctan x - \frac{1}{2}\int \frac{1}{1+x^2} d(1+x^2)\end{aligned}$$

$$\int \frac{1}{1+x^2} d(1+x^2) = \int \frac{1}{u} du =$$

例子 求不定积分 $\int \arctan x dx$

解
$$\begin{aligned}\int \arctan x dx &= x\arctan x - \int x d\arctan x \\ &= x\arctan x - \int x\cdot\frac{1}{1+x^2}dx \\ &= x\arctan x - \frac{1}{2}\int\frac{1}{1+x^2}dx^2 \\ &= x\arctan x - \frac{1}{2}\int\frac{1}{1+x^2}d(1+x^2)\end{aligned}$$

$$\int\frac{1}{1+x^2}d(1+x^2) = \int\frac{1}{u}du = \ln u + C =$$

例子 求不定积分 $\int \arctan x dx$

解
$$
\begin{aligned}
\int \arctan x dx &= x \arctan x - \int x d \arctan x \\
&= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx \\
&= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} dx^2 \\
&= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} d(1+x^2)
\end{aligned}
$$

$$
\int \frac{1}{1+x^2} d(1+x^2) = \int \frac{1}{u} du = \ln u + C = \ln(1+x^2) + C
$$

例子 求不定积分 $\int \arctan x dx$

解
$$\begin{aligned}\int \arctan x dx &= x\arctan x - \int x d\arctan x \\ &= x\arctan x - \int x \cdot \frac{1}{1+x^2} dx \\ &= x\arctan x - \frac{1}{2}\int \frac{1}{1+x^2} dx^2 \\ &= x\arctan x - \frac{1}{2}\int \frac{1}{1+x^2} d(1+x^2) \\ &= x\arctan x - \frac{1}{2}\ln(1+x^2) + C.\end{aligned}$$

$$\int \frac{1}{1+x^2} d(1+x^2) = \int \frac{1}{u} du = \ln u + C = \ln(1+x^2) + C$$

例子 求不定积分 $\int \arcsin x dx$

例子 求不定积分 $\int \arcsin x dx$

解 $$\int \arcsin x dx =$$

例子 求不定积分 $\int \arcsin x dx$

解 $$\int \arcsin x dx = x \arcsin x - \int x d \arcsin x$$

例子 求不定积分 $\int \arcsin x dx$

解
$$\int \arcsin x dx = x \arcsin x - \int x d \arcsin x$$
$$= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$$

例子 求不定积分 $\int \arcsin x dx$

解
$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\ &= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\ &= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx^2\end{aligned}$$

例子 求不定积分 $\int \arcsin x dx$

解

$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\ &= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\ &= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx^2 \\ &= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} \cdot (-1) d(1-x^2)\end{aligned}$$

例子 求不定积分 $\int \arcsin x dx$

解
$$\int \arcsin x dx = x\arcsin x - \int x d\arcsin x$$
$$= x\arcsin x - \int x\cdot\frac{1}{\sqrt{1-x^2}}dx$$
$$= x\arcsin x - \frac{1}{2}\int\frac{1}{\sqrt{1-x^2}}dx^2$$
$$= x\arcsin x - \frac{1}{2}\int\frac{1}{\sqrt{1-x^2}}\cdot(-1)d(1-x^2)$$

$$\int\frac{1}{\sqrt{1-x^2}}d(1-x^2) =$$

例子 求不定积分 $\int \arcsin x dx$

解
$$\begin{aligned}\int \arcsin x dx &= x\arcsin x - \int x d\arcsin x\\ &= x\arcsin x - \int x\cdot\frac{1}{\sqrt{1-x^2}}dx\\ &= x\arcsin x - \frac{1}{2}\int\frac{1}{\sqrt{1-x^2}}dx^2\\ &= x\arcsin x - \frac{1}{2}\int\frac{1}{\sqrt{1-x^2}}\cdot(-1)d(1-x^2)\end{aligned}$$

$$\int\frac{1}{\sqrt{1-x^2}}d(1-x^2) = \int u^{-\frac{1}{2}}du =$$

例子 求不定积分 $\int \arcsin x dx$

解

$$\begin{aligned}\int \arcsin x dx &= x\arcsin x - \int x d\arcsin x \\ &= x\arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\ &= x\arcsin x - \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}} dx^2 \\ &= x\arcsin x - \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}} \cdot (-1) d(1-x^2)\end{aligned}$$

$$\int \frac{1}{\sqrt{1-x^2}} d(1-x^2) = \int u^{-\frac{1}{2}} du = 2u^{\frac{1}{2}} + C =$$

例子 求不定积分 $\int \arcsin x dx$

解
$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\ &= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\ &= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx^2 \\ &= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} \cdot (-1) d(1-x^2)\end{aligned}$$

$$\int \frac{1}{\sqrt{1-x^2}} d(1-x^2) = \int u^{-\frac{1}{2}} du = 2u^{\frac{1}{2}} + C = 2(1-x^2)^{\frac{1}{2}} + C$$

例子 求不定积分 $\int \arcsin x dx$

解

$$
\begin{aligned}
\int \arcsin x dx &= x\arcsin x - \int x d\arcsin x \\
&= x\arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\
&= x\arcsin x - \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}} dx^2 \\
&= x\arcsin x - \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}} \cdot (-1) d(1-x^2) \\
&= x\arcsin x + \sqrt{1-x^2} + C.
\end{aligned}
$$

$$\int \frac{1}{\sqrt{1-x^2}} d(1-x^2) = \int u^{-\frac{1}{2}} du = 2u^{\frac{1}{2}} + C = 2(1-x^2)^{\frac{1}{2}} + C$$

例子 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$

例子 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （提示 两次分部积分）

例子 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （**提示** 两次分部积分）

解 $\displaystyle\int x^2 e^x dx =$

例子 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （提示 两次分部积分）

解 $$\int x^2 e^x dx = \int x^2 de^x =$$

例子 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （提示 两次分部积分）

解 $\displaystyle\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 =$

例子 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （提示 两次分部积分）

解 $$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2\int e^x x dx$$

例子 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （提示 两次分部积分）

解 $$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2\int e^x x dx$$

$$= x^2 e^x - 2\left(\int x de^x\right) =$$

例子 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$　　（提示 两次分部积分）

解 $$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2\int e^x x dx$$
$$= x^2 e^x - 2\left(\int x de^x\right) = x^2 e^x - 2\left(xe^x - \int e^x dx\right)$$

例子 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （提示 两次分部积分）

解 $$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2\int e^x x dx$$

$$= x^2 e^x - 2\left(\int x de^x\right) = x^2 e^x - 2\left(xe^x - \int e^x dx\right)$$

$$= x^2 e^x - 2xe^x + 2e^x + C$$

例子 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$　　（提示 两次分部积分）

解 $$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2\int e^x x dx$$
$$= x^2 e^x - 2\left(\int x de^x\right) = x^2 e^x - 2\left(xe^x - \int e^x dx\right)$$
$$= x^2 e^x - 2xe^x + 2e^x + C$$

$$\int x^2 \sin x dx =$$

例子 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （**提示** 两次分部积分）

解 $$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2\int e^x x dx$$

$$= x^2 e^x - 2\left(\int x de^x\right) = x^2 e^x - 2\left(xe^x - \int e^x dx\right)$$

$$= x^2 e^x - 2xe^x + 2e^x + C$$

$$\int x^2 \sin x dx = -\int x^2 d\cos x =$$

例子 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$　　（提示 两次分部积分）

解
$$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2\int e^x x dx$$
$$= x^2 e^x - 2\left(\int x de^x\right) = x^2 e^x - 2\left(xe^x - \int e^x dx\right)$$
$$= x^2 e^x - 2xe^x + 2e^x + C$$

$$\int x^2 \sin x dx = -\int x^2 d\cos x = -x^2 \cos x + \int \cos x dx^2$$

例子 求不定积分 $\int x^2e^xdx$，$\int x^2\sin x dx$ （提示 两次分部积分）

解 $$\int x^2e^xdx=\int x^2de^x=x^2e^x-\int e^xdx^2=x^2e^x-2\int e^xxdx$$
$$=x^2e^x-2\left(\int xde^x\right)=x^2e^x-2\left(xe^x-\int e^xdx\right)$$
$$=x^2e^x-2xe^x+2e^x+C$$

$$\int x^2\sin x dx=-\int x^2d\cos x=-x^2\cos x+\int\cos x dx^2$$
$$=-x^2\cos x+2\int x\cos x dx$$

例子 求不定积分 $\int x^2e^xdx$，$\int x^2\sin x dx$ （提示 两次分部积分）

解
$$\int x^2e^xdx = \int x^2de^x = x^2e^x - \int e^xdx^2 = x^2e^x - 2\int e^xxdx$$
$$= x^2e^x - 2\left(\int xde^x\right) = x^2e^x - 2\left(xe^x - \int e^xdx\right)$$
$$= x^2e^x - 2xe^x + 2e^x + C$$

$$\int x^2\sin x dx = -\int x^2d\cos x = -x^2\cos x + \int \cos x dx^2$$
$$= -x^2\cos x + 2\int x\cos x dx$$
$$= -x^2\cos x + 2\int xd\sin x$$

例子 求不定积分 $\int x^2e^x dx$，$\int x^2\sin x dx$ （提示 两次分部积分）

解 $$\int x^2e^x dx = \int x^2 de^x = x^2e^x - \int e^x dx^2 = x^2e^x - 2\int e^x x dx$$

$$= x^2e^x - 2\left(\int x de^x\right) = x^2e^x - 2\left(xe^x - \int e^x dx\right)$$

$$= x^2e^x - 2xe^x + 2e^x + C$$

$$\int x^2\sin x dx = -\int x^2 d\cos x = -x^2\cos x + \int \cos x dx^2$$

$$= -x^2\cos x + 2\int x\cos x dx$$

$$= -x^2\cos x + 2\int x d\sin x$$

$$= -x^2\cos x + 2\left(x\sin x - \int \sin x dx\right)$$

例子 求不定积分 $\int x^2 e^x dx$，$\int x^2 \sin x dx$ （提示 两次分部积分）

解 $$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2\int e^x x dx$$

$$= x^2 e^x - 2\left(\int x de^x\right) = x^2 e^x - 2\left(x e^x - \int e^x dx\right)$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

$$\int x^2 \sin x dx = -\int x^2 d\cos x = -x^2 \cos x + \int \cos x dx^2$$

$$= -x^2 \cos x + 2\int x \cos x dx$$

$$= -x^2 \cos x + 2\int x d\sin x$$

$$= -x^2 \cos x + 2\left(x \sin x - \int \sin x dx\right)$$

$$= -x^2 \cos x + 2x \sin x + 2\cos x + C$$

分部积分：例子总结

$$\int xe^x dx =$$

$$\int x\cos x dx =$$

$$\int x^2 \ln x dx =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int xe^x dx = \int x de^x =$$

$$\int x\cos x dx =$$

$$\int x^2 \ln x dx =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int xe^x dx = \int x de^x = xe^x - \int e^x dx = \cdots$$

$$\int x\cos x dx =$$

$$\int x^2 \ln x dx =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int xe^x dx = \int x de^x = xe^x - \int e^x dx = \cdots$$

$$\int x\cos x dx = \int x d\sin x =$$

$$\int x^2 \ln x dx =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int xe^x dx = \int x de^x = xe^x - \int e^x dx = \cdots$$

$$\int x\cos x dx = \int x d\sin x = x\sin x - \int \sin x dx = \cdots$$

$$\int x^2 \ln x dx =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int xe^x dx = \int x de^x = xe^x - \int e^x dx = \cdots$$

$$\int x\cos x dx = \int x d\sin x = x\sin x - \int \sin x dx = \cdots$$

$$\int x^2 \ln x dx = \int \ln x d(\frac{1}{3}x^3) =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int xe^x dx = \int x de^x = xe^x - \int e^x dx = \cdots$$

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = \cdots$$

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