§1.1 二阶三阶行列式

数学系 梁卓滨

2017 - 2018 学年 I



教学要求

掌握求解:

- ◇ 二阶行列式计算
- ◆ 三阶行列式计算



- 行列式的概念来源于线性方程组的求解问题
- 17 世纪末由日本数学家关孝和及德国数学家莱布尼茨引入







二阶行列式 ←→ 二元线性方程组三阶行列式 ←→ 三元线性方程组:n 阶行列式 ←→ n 元线性方程组:::::

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解:

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y, 得:

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y, 得:

 $\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \Rightarrow a_{21}a_{12}x + a_{22}a_{12}y = b_2a_{12} \end{cases}$

用消元法求解: $(1) \times \alpha_{22} - (2) \times \alpha_{12}$, 消去 y, 得:



$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \Rightarrow a_{21}a_{12}x + a_{22}a_{12}y = b_2a_{12} \end{cases}$$
用消元法求解: (1) × a_{22} – (2) × a_{12} , 消去 y , 得:

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \Rightarrow a_{21}a_{12}x + a_{22}a_{12}y = b_2a_{12} \end{cases}$$

用消元法求解: $(1) \times \alpha_{22} - (2) \times \alpha_{12}$, 消去 y, 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{12} b_2}$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \Rightarrow a_{21}a_{12}x + a_{22}a_{12}y = b_2a_{12} \end{cases}$$

用消元法求解: $(1) \times \alpha_{22} - (2) \times \alpha_{12}$, 消去 y, 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y, 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去 x , 得:



$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \end{cases} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y, 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

 $(2) \times a_{11} - (1) \times a_{21}$,消去 x,得:



$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \Rightarrow a_{11}a_{21}x + a_{12}a_{21}y = b_1a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11} \end{cases}$$

用消元法求解: $(1) \times \alpha_{22} - (2) \times \alpha_{12}$, 消去 y, 得:

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, 消去 x , 得:



$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \Rightarrow a_{11}a_{21}x + a_{12}a_{21}y = b_1a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11} \end{cases}$$

用消元法求解: $(1) \times \alpha_{22} - (2) \times \alpha_{12}$, 消去 V, 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

 $(2) \times a_{11} - (1) \times a_{21}$, 消去 x, 得:

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$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \Rightarrow a_{11}a_{21}x + a_{12}a_{21}y = b_1a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11} \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y, 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

 $(2) \times a_{11} - (1) \times a_{21}$,消去 x,得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{12}a_{12}}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \Rightarrow a_{11}a_{21}x + a_{12}a_{21}y = b_1a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11} \end{cases}$$

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 $(2) \times a_{11} - (1) \times a_{21}$,消去 x,得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y, 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去 x , 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

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 $(2) \times a_{11} - (1) \times a_{21}$, 消去 x, 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$



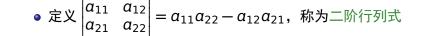
$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解: $(1) \times \alpha_{22} - (2) \times \alpha_{12}$, 消去 y, 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

(2)×α₁₁ – (1)×α₂₁,消去 x,得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$





$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解: $(1) \times \alpha_{22} - (2) \times \alpha_{12}$, 消去 y, 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{a_{11} a_{12}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

 $(2) \times a_{11} - (1) \times a_{21}$,消去 x,得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{1}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

• 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$,称为二阶行列式



$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y, 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$(2) \times a_{11} - (1) \times a_{21}, \quad \text{iff} x, \quad \text{iff} a_{12}$$

 $y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{1}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$

• 定义
$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$
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$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y, 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$(2) \times a_{11} - (1) \times a_{21}, \quad \text{iff } x, \quad \text{iff } a_{11} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & b_2 \end{vmatrix}}$$

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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<mark>练习</mark> 利用二阶行列式求解下面二元线性方程组

1.
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$$
 $x =$, $y =$

2.
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

<mark>练习</mark> 利用二阶行列式求解下面二元线性方程组

1.
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \begin{vmatrix} \begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix} \\ \begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix} = -- \qquad , \quad y = \begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}$$

2.
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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, y =

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$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x =$$

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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, y =

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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,
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§1.1 二阶三阶行列式

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练习 利用二阶行列式求解下面二元线性方程组

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, y =

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

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练习 利用二阶行列式求解下面二元线性方程组

1.
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$
2.
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - , \quad y = \frac{-20}{1} = -1$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \begin{vmatrix} 0 & 5 \\ 4 & 8 \\ 2 & 5 \\ 3 & 8 \end{vmatrix} = \frac{-20}{1} = -20, \quad y = \begin{vmatrix} 2 & 0 \\ 3 & 4 \\ 2 & 5 \\ 3 & 8 \end{vmatrix} = \frac{8}{1} = 8$$

2.
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - , y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = -$$

● 暨南大学

1.1 二阶三阶行列式

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{}{3} , y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = -$

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$(7x + 16v = 1)$$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} , y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{3}{3}$

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1.
$$\begin{cases} 2x + 5y = 0 \\ 2x + 9x & 4 \end{cases} \quad x = \frac{\begin{vmatrix} 4 & 8 \\ 2 & 5 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 2 & 5 \end{vmatrix}} = \frac{1}{2}$$

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$$\begin{vmatrix} 3x + 8y = 4 & 2 & 5 \\ 3 & 8 & 3 \end{vmatrix}$$

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$$\begin{cases} 2x + 3y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{|4 + 6|}{|2 + 5|} = \frac{-20}{1}$$

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$$\begin{vmatrix} 3 & 8 \end{vmatrix} \qquad \begin{vmatrix} 3 & 8 \end{vmatrix}$$
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例 $\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} \neq 0$ 的充分必要条件是 λ 满足 ______

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解因为

$$\begin{vmatrix} k-1 & 2 \\ 2 & k-1 \end{vmatrix} = (k-1)^2 - 4 = k^2 - 2k - 3 = (k+1)(k-3)$$

所以 $k \neq -1$ 且 $k \neq 3$ 。



$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

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用消元法可解得:

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32}}{-b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32}} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}$$

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为表示三元方程组的解,定义三阶行列式:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$



$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$

规律



$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}}$$

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a_{11}	a_{12}	<i>a</i> ₁₃	a_{11}	<i>a</i> ₁₂	<i>a</i> ₁₃
a ₂₁	a ₂₂	a_{23}	a_{21}	a ₂₂	a ₂₃
a_{31}	a ₃₂	a_{33}	a ₃₁	a ₃₂	a ₃₃



$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$

a_{11}	<i>a</i> ₁₂	a ₁₃	a_{11}	<i>a</i> ₁₂	<i>a</i> ₁₃
a ₂₁	a ₂₂	a ₂₃	a_{21}	a ₂₂	a ₂₃
a ₃₁	a ₃₂	a ₃₃	a ₃₁	a ₃₂	a_{33}

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$

规律不同行不同列的3个元素乘积,共3!=6个,并且:

a_{11}	a ₁₂	a ₁₃	a_{11}	a ₁₂	a_{13}
a ₂₁	a ₂₂	a ₂₃	a_{21}	a ₂₂	a ₂₃
a ₃₁	a ₃₂	a ₃₃	a ₃₁	a ₃₂	a ₃₃

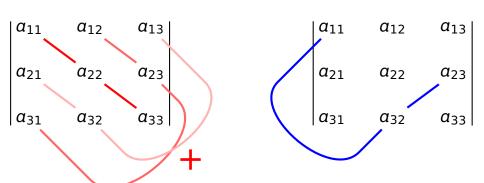
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$

$\begin{vmatrix} a_{11} & a_{12} & a_{13} \end{vmatrix}$	a_{11}	a ₁₂	<i>a</i> ₁₃
a_{21} a_{22} a_{23}	a_{21}	a ₂₂	a ₂₃
$ a_{31} a_{32} a_{33} $	a_{31}	a ₃₂	<i>a</i> ₃₃

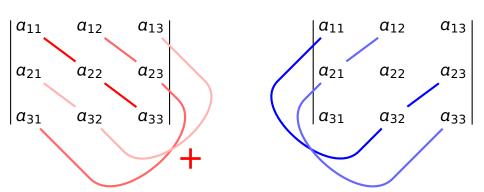
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$\begin{vmatrix} a_{11} & a_{12} & a_{13} \end{vmatrix}$	a_{11}	a ₁₂	<i>a</i> ₁₃
a_{21} a_{22} a_{23}	a_{21}	a ₂₂	a ₂₃
$\begin{vmatrix} a_{31} & a_{32} & a_{33} \end{vmatrix}$	$ a_{31}$	a ₃₂	a ₃₃

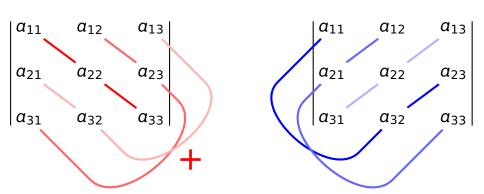
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}}$$



$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}}$$

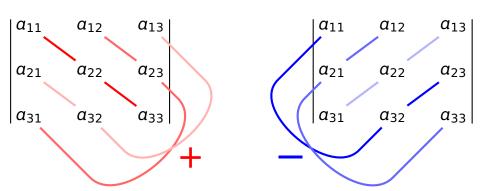


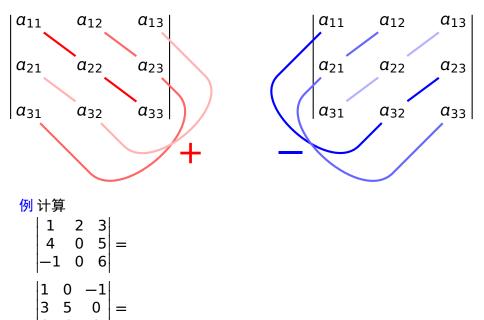
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}}$$



$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}}$$

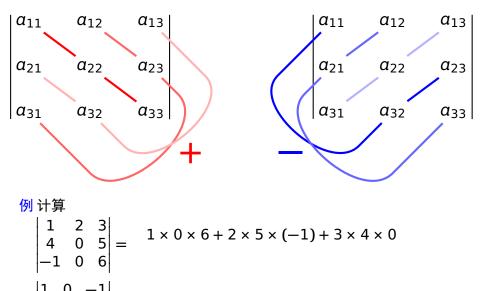
规律不同行不同列的3个元素乘积,共3!=6个,并且:







4



例 计算
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{cases} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{cases}$$

例 计算
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{vmatrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 2 & 5 & 0 \end{vmatrix}$$



例 计算
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{vmatrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 2 & 5 & 0 \end{vmatrix} = 1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4$$



$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$
 $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ 例 计算
$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{vmatrix} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{vmatrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{vmatrix} 1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4 \\ -1 \times 0 \times 4 - 0 \times 3 \times 1 - (-1) \times 5 \times 1 \end{vmatrix}$$
§1.1 二阶三阶行列式

例 计算
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{vmatrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{vmatrix} 1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4 \\ -1 \times 0 \times 4 - 0 \times 3 \times 1 - (-1) \times 5 \times 1 \end{vmatrix} = -2$$

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$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:



$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32}}{-b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3} = \frac{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32}}{-a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}} = \frac{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32}}{-a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}}$$



$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{\begin{array}{c} b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} \\ -b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3 \end{array}}{\begin{array}{c} a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} \\ -a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31} \end{array}} = \frac{a_{11}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$



$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{\begin{array}{c} b_{1}a_{22}a_{33} + a_{12}a_{23}b_{3} + a_{13}b_{2}a_{32} \\ -b_{1}a_{23}a_{32} - a_{12}b_{2}a_{33} - a_{13}a_{22}b_{3} \\ -a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}} = \frac{\begin{vmatrix} b_{1} & a_{12} & a_{13} \\ b_{2} & a_{22} & a_{23} \\ b_{3} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

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$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{\begin{vmatrix} b_{1}a_{22}a_{33} + a_{12}a_{23}b_{3} + a_{13}b_{2}a_{32} \\ -b_{1}a_{23}a_{32} - a_{12}b_{2}a_{33} - a_{13}a_{22}b_{3} \end{vmatrix}}{\begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}} = \frac{\begin{vmatrix} b_{1}a_{12}a_{13} \\ b_{2}a_{22}a_{23} \\ b_{3}a_{32}a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11}a_{12}a_{13} \\ a_{21}a_{22}a_{23} \\ a_{31}a_{32}a_{33} \end{vmatrix}}$$

 a_{11} b_1 a_{13} a_{21} $b_2 \ a_{23}$ a_{11} a_{12} a_{13} a_{21} a_{22} a_{23} a_{31} a_{32} a_{33}



§1.1 二阶三阶行列式

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

 a_{21}

 a_{31}

§1.1 二阶三阶行列式

 a_{22}

 a_{23}

$$x = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}} = \frac{a_{11}}{a_{21}} \begin{vmatrix} a_{11} & b_{1} & a_{13} \\ a_{21} & b_{2} & a_{23} \\ a_{31} & b_{3} & a_{33} \end{vmatrix}}{y = \frac{a_{11}}{a_{11}} \begin{vmatrix} a_{12} & b_{1} \\ a_{21} & a_{22} & b_{2} \\ a_{31} & a_{32} & b_{3} \end{vmatrix}}{a_{11}} , z = \frac{a_{11}}{a_{21}} \begin{vmatrix} a_{12} & a_{12} \\ a_{21} & a_{22} & b_{2} \\ a_{31} & a_{32} & b_{3} \end{vmatrix}}{a_{11}}$$

 $b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32}$

 $-b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3$

 a_{13}

 a_{23}

 a_{33}

 a_{13}

 a_{23}

 a_{33}

 a_{12}

 a_{22}

 a_{32}

 a_{12}

 a_{22}

 a_{32}

 b_2

 b_3

 a_{32} a_{33} a_{31} a_{32} a_{33} 11/15 < ▶ △ ▼

 a_{22}

 a_{23}

 a_{21}

$$\begin{cases} a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$
示为:

 $a_{11}x + a_{12}y + a_{13}z = b_1$

的解可以表示为:

$$= \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32}}{-b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3} = \frac{\begin{vmatrix} b_1 \\ b_2 \\ b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} \\ -a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31} \end{vmatrix}} = \frac{\begin{vmatrix} b_1 \\ b_2 \\ b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} \\ a_{21} \end{vmatrix}}$$

x =

 a_{11}

 a_{21}

 a_{11}

 a_{21}

 a_{31}

二阶三阶行列式

 b_1 a_{13} b_2

 a_{23} a_{33}

a₁₂

 a_{22}

 a_{32}

 a_{23}

 a_{33}

 a_{13}

 a_{11} a_{21} a_{31}

 a_{11}

 a_{21}

 a_{31}

 a_{22} a_{32}

 a_{12}

 a_{22}

 a_{32}

 a_{12}

b₃, a_{13}

 a_{23}

 a_{33}

 b_1 b_2

(1)

 a_{31}

 a_{22} a_{32} a_{33}

 a_{12}

 a_{22}

 a_{32}

 a_{12} a_{23}

 a_{33} a_{13}

 a_{13}

 a_{23}



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$$\begin{cases} a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$
 示为:

 $a_{11}x + a_{12}y + a_{13}z = b_1$

的解可以表示为: b_1 a_{12} $b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32}$ b_2 a_{22} b_3 $-b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3$ a_{32} a_{11} $a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}$ a_{12}

x = $-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$ a_{11} b_1 a_{11} a_{13}

 a_{21} b_2 a_{33} a₁₂ a_{13}

$$\begin{vmatrix} a_{13}a_{22}a_{31} & a_{21} \\ a_{31} & a_{12} & b_{1} \\ a_{21} & a_{22} & b_{2} \end{vmatrix}$$

 a_{32}

 a_{12}

 a_{22}

 a_{32}

 a_{13}

 a_{23}

 a_{33}

 a_{23}

 a_{31}

 a_{11}

 a_{21}

 a_{31}

 a_{22} a_{23} a_{31} a_{32} a_{33} b₃,

(1)

 a_{13}

 a_{23}

 a_{33}

 a_{13}

 a_{11}









$$= \frac{a_{11}a_{22}a_{33}}{-a_{11}a_{23}a_{3}}$$

 b_1

$$\begin{array}{c} b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} \\ -b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3 \\ \hline a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} \\ -a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31} \end{array} =$$

 $a_{11}x + a_{12}y + a_{13}z = b_1$

 $a_{21}x + a_{22}y + a_{23}z = b_2$ (2)

 $a_{31}x + a_{32}y + a_{33}z = b_3$ (3) a_{12} a_{13} b_2 a_{22} a_{23} bз a_{32} a_{33}

(1)

_		1
a ₁₁ a ₂₁	$a_{12} \\ a_{22}$	a_{13} a_{23}
a_{31}	a_{32}	a ₃₃
		'

 a_{21} b_2 a_{11} a_{12}

 a_{21}

 a_{11}

 a_{23} a_{33} a_{13} a_{22} a_{23}

 a_{13}

 a_{33}

 a_{31} a_{32} $a_{1.3}$ *a*₁₁ a_{12} a_{23} a_{21} a_{22} a_{31} a_{32} a_{33}

 a_{11}

 a_{21}

 a_{12}

 a_{22}

 b_1

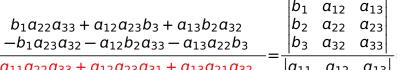
b₂

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 $\begin{cases} a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$ 的解可以表示为:

 $a_{11}x + a_{12}y + a_{13}z = b_1 \quad (1)$

$$b_{1}a_{22}a_{3}$$



$a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}$							a_{11}	a_{12}	a_{13}	
$-a_1$	1 a 23	a ₃₂ –	$-a_{12}a_{21}$	$a_{33} - a_{33}$	1 ₁₃ a ₂	2 a 31		a_{21}		
								a_{31}	a_{32}	a_{33}
$ a_{11} $	b_1	a ₁₃			a_{11}	a_{12}	b_1			
a_{11} a_{21}	b_2	a_{23}			a_{21}	a_{12} a_{22}	b_2	<u>.</u>		
1 .	1.					_	1.	1		

 $y = \frac{\begin{vmatrix} a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{vmatrix}} = \frac{D_y}{D}, \quad z = \frac{\begin{vmatrix} a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{vmatrix}}$

 $y = \frac{D_y}{D} = -$

$$z = \frac{D_z}{D} = -----$$

$$x = \frac{D_x}{D} = \frac{1}{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}} \qquad y = \frac{D_y}{D} = \frac{1}{2}$$



$$x = \frac{D_{x}}{D} = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix} X}$$

$$y = \frac{D_y}{D} = -----$$

解

§1.1 二阶三阶行列式

 $x = \frac{D_X}{D} = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix} X}$

$$=\frac{D}{D}=-$$



D_z		

$$\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix} X$$

$$x = \frac{D_X}{D} = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix} X}$$

$$D_y$$

解

$$D_z$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \end{cases} \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \end{cases} \right) \\ 4x - 3y = -2 \end{cases}$

$$x = \frac{D_{x}}{D} = \frac{1}{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}} X = \frac{D_{y}}{D} = \frac{1}{D}$$

解

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \end{cases} \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \end{cases} \right) \\ 4x - 3y = -2 \end{cases}$

$$y = \frac{D_y}{D} = \frac{D_y}{D}$$



例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} \qquad y = \frac{D_y}{D} = -----$$

$$z = \frac{D_z}{D} = \frac{1}{D}$$



 $x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} \qquad y = \frac{D_y}{D} = \frac{1}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$

D_z				
$z = \frac{1}{D} = \frac{1}{2}$	1	0 2 -3	2	•
	0	2	1 0	
	4	- 3	0	

解 $x = \frac{D_X}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$ $y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$



§1.1 二阶三阶行列式

 $y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$

D	1 0 4	0 2 -3	2 1 0	
D _z	1 0 4	0 2 —3	9 8 -2	
z = = -	1 0 4	0 2 -3	2 1 0	



例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$



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_	0	2 -3	1	
_ D _z _	1 0 4	0 2 —3	9 8 –2	_
- D =	1 0 4	0 2 -3	2 1 0	



 $x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \end{vmatrix}} = \frac{-13}{-13} = 1, \quad y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-26}{-13}$

	0	2 -3	1	
_ D _z _	1 0 4	0 2 —3	9 8 –2	_
	1 0 4	0 2 -3	2 1 0	



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解

例 求解三元线性方程组

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例 求解三元线性方程组

解

§1.1 二阶三阶行列式

 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

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 $= \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \end{vmatrix}} = \frac{1}{2}$

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例 求解三元线性方程组
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

解 先利用公式求出 x

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代入方程得



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 代入方程得

 $\begin{cases} 1+2z=9\\ 4-3v=-2 \end{cases} \Rightarrow \begin{cases} z=4\\ v=2 \end{cases}$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示:

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的解可用 n 行列式表示:

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的解可用 n 行列式表示:

$$x_{1} = \frac{D_{1}}{D} = \frac{1}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_{2} = \frac{D_{2}}{D}, \quad \cdots, \quad x_{n} = \frac{D_{n}}{D}$$



§1.1 二阶三阶行列式

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示:

的解可用
$$n$$
 行列式表示:
$$x_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示: (称为克莱姆法则)

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$$x_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \\ \hline a_{11} & a_{12} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}, \quad x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

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那么,如何定义行列式,



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那么,如何定义行列式,如何快捷计算行列式?



$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} =$$



$$\left| egin{array}{cccc} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{array} \right| = \left| \begin{array}{ccccc} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \end{array} \right|$$

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{vmatrix}$$

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{vmatrix} = a^2 + b^2$$



$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{vmatrix} = a^2 + b^2$$

所以 *α* ≠ 0 或 *b* ≠ 0。