§1.4　克莱姆法则

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二元线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_2 + a_{22}y_2 = b_2 \end{cases}$$

的解是

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$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \quad , \quad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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注

- $D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ 称为系数行列式

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- $D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ 称为系数行列式，上述公式隐含要求 $D \neq 0$
- D_i：将 D 的第 i 列换成常数项 $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

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系数行列式

对n元线性方程组

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$$\begin{vmatrix} a_{11} & \cdots & a_{1j-1} & a_{1,j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2,j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{n,j} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}$$

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令 $D_j = \begin{vmatrix} a_{11} & \cdots & a_{1\,j-1} & b_1 & a_{1\,j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2\,j-1} & b_2 & a_{2\,j+1} & \cdots & a_{2n} \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & \cdots & a_{n\,j-1} & b_n & a_{n\,j+1} & \cdots & a_{nn} \end{vmatrix}$

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定理（克莱姆法则） 线性方程组

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注 两个前提：(1) 未知元个数 = 方程个数；(2) 系数行列式 $D \neq 0$

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(3) 若 $D = 0$，则方程有无穷多解或无解（以后详说）

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- $\begin{cases} x+y=1 \\ x+y=1 \end{cases}$ ，$D=\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}=0$，实质上只有一条方程 $x+y=1$，显然有无穷多解。

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- $\begin{cases} x+y=1 \\ x+y=0 \end{cases}$，$D=\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}=0$，方程组包含矛盾方程，显然无解。

例 解线性方程组

$$\begin{cases} 2x_1 + \quad x_2 - x_3 = 1 \\ 3x_1 - \quad x_2 - x_3 = -2 \\ -x_1 + 2x_2 + x_3 = 6 \end{cases}$$

练习 解线性方程组

$$\begin{cases} x_1 + x_2 = 90 \\ x_2 + x_3 = 86 \\ x_1 + x_3 = 80 \end{cases}$$

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提示 $D = -5,\ D_1 = -5,\ D_2 = -10,\ D_3 = -15$

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提示 $D = 2,\ D_1 = 84,\ D_2 = 96,\ D_3 = 76$

齐次线性方程组

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$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = 0 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = 0 \end{cases}$$

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证明 $x_1 = x_2 = \cdots = x_n = 0$ 显然是方程组的解

齐次线性方程组

定理 齐次线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = 0 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = 0 \end{cases}$$

当系数行列式 $D \neq 0$ 时，仅有零解（$x_1 = x_2 = \cdots = x_n = 0$）

证明 $x_1 = x_2 = \cdots = x_n = 0$ 显然是方程组的解

另一方面，因为 $D \neq 0$，所以方程组有唯一解（克莱姆法则）

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- 其实是充分必要条件：仅有零解的充分必要条件是 $D \neq 0$

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注

- 其实是充分必要条件：仅有零解的充分必要条件是 $D \neq 0$
- 若 $D = 0$，方程有无穷多的解

例子

例 齐次方程组 $\begin{cases} x_1 - 2x_2 = 0 \\ 2x_1 - 4x_2 = 0 \end{cases}$ 的系数矩阵 $D = \begin{vmatrix} 1 & -2 \\ 2 & -4 \end{vmatrix}$

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例 判断线性方程组
$$\begin{cases} 2x_1 + 3x_2 + 4x_3 + 5x_4 = 0 \\ 3x_1 + 4x_2 + 5x_3 + 5x_4 = 0 \\ 4x_1 + 5x_2 + 6x_3 + 6x_4 = 0 \\ 5x_1 + 6x_2 + 8x_3 + 9x_4 = 0 \end{cases}$$
是否只有零解

解

$$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix}$$

解

$$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix} \overset{r_4 - r_3}{=\!=\!=}$$

解

$$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix} \overset{r_4 - r_3}{=\!=\!=} \begin{vmatrix} & & & \\ & & & \\ & & & \\ 1 & 1 & 2 & 3 \end{vmatrix}$$

解

$$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix} \xlongequal{r_4 - r_3} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$

解

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解

$$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix} \overset{r_4-r_3}{=\!=\!=} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 1 & 1 & 2 & 3 \end{vmatrix} \overset{r_3-r_2}{=\!=\!=} \begin{vmatrix} & & & \\ & & & \\ 1 & 1 & 1 & 1 \\ & & & \end{vmatrix}$$

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$$\xlongequal{r_2-r_1}$$

解

$$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix} \xlongequal{r_4-r_3} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xlongequal{r_3-r_2} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$

$$\xlongequal{r_2-r_1} \begin{vmatrix} 1 & 1 & 1 & 0 \\ & & & \\ & & & \\ & & & \end{vmatrix}$$

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$$\xlongequal{r_2-r_1} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xlongequal{r_1-2r_2}$$

解

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$$\xlongequal{r_2-r_1} \begin{vmatrix}2&3&4&5\\1&1&1&0\\1&1&1&1\\1&1&2&3\end{vmatrix} \xlongequal{r_1-2r_2} \begin{vmatrix}0&1&2&5\\1&1&1&0\\1&1&1&1\\1&1&2&3\end{vmatrix} \xlongequal{r_3-r_2} \begin{vmatrix}0&1&2&5\\1&1&1&0\\0&0&0&1\\1&1&2&3\end{vmatrix}$$

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$$\xlongequal{r_4-2r_2}\begin{vmatrix}0&1&2&5\\1&1&1&0\\0&0&0&1\\0&0&1&3\end{vmatrix}=-\begin{vmatrix}0&1&2\\1&1&1\\0&0&1\end{vmatrix}$$

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$$\xlongequal{r_4 - 2r_2} \begin{vmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{vmatrix} = - \begin{vmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = - \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}$$

解

$$\begin{vmatrix}2&3&4&5\\3&4&5&5\\4&5&6&6\\5&6&8&9\end{vmatrix} \xlongequal{r_4-r_3} \begin{vmatrix}2&3&4&5\\3&4&5&5\\4&5&6&6\\1&1&2&3\end{vmatrix} \xlongequal{r_3-r_2} \begin{vmatrix}2&3&4&5\\3&4&5&5\\1&1&1&1\\1&1&2&3\end{vmatrix}$$

$$\xlongequal{r_2-r_1} \begin{vmatrix}2&3&4&5\\1&1&1&0\\1&1&1&1\\1&1&2&3\end{vmatrix} \xlongequal{r_1-2r_2} \begin{vmatrix}0&1&2&5\\1&1&1&0\\1&1&1&1\\1&1&2&3\end{vmatrix} \xlongequal{r_3-r_2} \begin{vmatrix}0&1&2&5\\1&1&1&0\\0&0&0&1\\1&1&2&3\end{vmatrix}$$

$$\xlongequal{r_4-2r_2} \begin{vmatrix}0&1&2&5\\1&1&1&0\\0&0&0&1\\0&0&1&3\end{vmatrix} = -\begin{vmatrix}0&1&2\\1&1&1\\0&0&1\end{vmatrix} = -\begin{vmatrix}0&1\\1&1\end{vmatrix} = 1 \neq 0$$

解

$$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix} \xlongequal{r_4-r_3} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xlongequal{r_3-r_2} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$

$$\xlongequal{r_2-r_1} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xlongequal{r_1-2r_2} \begin{vmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xlongequal{r_3-r_2} \begin{vmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$

$$\xlongequal{r_4-2r_2} \begin{vmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{vmatrix} = -\begin{vmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = -\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = 1 \neq 0$$

所以齐次线性方程组有唯一解

练习 齐次线性方程组 $\begin{cases} kx_1 + x_4 = 0 \\ x_1 + 2x_2 - x_4 = 0 \\ (k+2)x_1 - x_2 + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 有非零解

的充分必要条件是 k 满足 ____

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的充分必要条件是 k 满足 ____

解

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix}$$

练习 齐次线性方程组 $\begin{cases} kx_1 \qquad\qquad\qquad + \quad x_4 = 0 \\ x_1 + 2x_2 \qquad\quad - \quad x_4 = 0 \\ (k+2)x_1 - \quad x_2 \qquad\quad + 4x_4 = 0 \\ 2x_1 + \quad x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 有非零解的充分必要条件是 k 满足 ____

解

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot$$

练习 齐次线性方程组 $\begin{cases} kx_1 + x_4 = 0 \\ x_1 + 2x_2 - x_4 = 0 \\ (k+2)x_1 - x_2 + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 有非零解的充分必要条件是 k 满足 ____

解

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3\cdot(-1)^{3+4}\begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$

练习 齐次线性方程组 $\begin{cases} kx_1 + x_4 = 0 \\ x_1 + 2x_2 - x_4 = 0 \\ (k+2)x_1 - x_2 + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 有非零解

的充分必要条件是 k 满足 ____

解

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot (-1)^{3+4} \begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$

$$\xlongequal{r_2 + r_1}$$

练习 齐次线性方程组 $\begin{cases} kx_1 + x_4 = 0 \\ x_1 + 2x_2 - x_4 = 0 \\ (k+2)x_1 - x_2 + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 有非零解

的充分必要条件是 k 满足 ____

解

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot (-1)^{3+4} \begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$

$$\overset{r_2+r_1}{=\!=\!=} (-3) \cdot \begin{vmatrix} k & 0 & 1 \\ & & \\ & & \end{vmatrix}$$

练习 齐次线性方程组 $\begin{cases} kx_1 + x_4 = 0 \\ x_1 + 2x_2 - x_4 = 0 \\ (k+2)x_1 - x_2 + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 有非零解

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解

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot (-1)^{3+4} \begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$

$$\xlongequal{r_2 + r_1} (-3) \cdot \begin{vmatrix} k & 0 & 1 \\ k+1 & 2 & 0 \\ & & \end{vmatrix}$$

练习 齐次线性方程组 $\begin{cases} kx_1 \qquad\qquad\qquad + \ x_4 = 0 \\ x_1 + 2x_2 \qquad\quad - \ x_4 = 0 \\ (k+2)x_1 - \ x_2 \qquad + 4x_4 = 0 \\ 2x_1 + \ x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 有非零解

的充分必要条件是 k 满足 ____

解

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot (-1)^{3+4} \begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$

$$\xlongequal[r_3 - 4r_1]{r_2 + r_1} (-3) \cdot \begin{vmatrix} k & 0 & 1 \\ k+1 & 2 & 0 \\ & & \end{vmatrix}$$

练习 齐次线性方程组 $\begin{cases} kx_1 \qquad\qquad\qquad\quad + \ \ x_4 = 0 \\ \quad x_1 + 2x_2 \qquad\quad - \ \ x_4 = 0 \\ (k+2)x_1 - \ \ x_2 \qquad + 4x_4 = 0 \\ \quad 2x_1 + \ \ x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 有非零解

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$$\xlongequal[r_3-4r_1]{r_2+r_1} (-3)\cdot\begin{vmatrix} k & 0 & 1 \\ k+1 & 2 & 0 \\ -3k+2 & -1 & 0 \end{vmatrix}$$

练习 齐次线性方程组 $\begin{cases} kx_1 + x_4 = 0 \\ x_1 + 2x_2 - x_4 = 0 \\ (k+2)x_1 - x_2 + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 有非零解

的充分必要条件是 k 满足 ____

解

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot (-1)^{3+4} \begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$

$$\xlongequal[r_3-4r_1]{r_2+r_1} (-3) \cdot \begin{vmatrix} k & 0 & 1 \\ k+1 & 2 & 0 \\ -3k+2 & -1 & 0 \end{vmatrix} = (-3) \cdot (-1)^{1+3} \cdot \begin{vmatrix} k+1 & 2 \\ -3k+2 & -1 \end{vmatrix}$$

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解

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3\cdot(-1)^{3+4}\begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$

$$\xlongequal[r_3-4r_1]{r_2+r_1} (-3)\cdot\begin{vmatrix} k & 0 & 1 \\ k+1 & 2 & 0 \\ -3k+2 & -1 & 0 \end{vmatrix} = (-3)\cdot(-1)^{1+3}\cdot\begin{vmatrix} k+1 & 2 \\ -3k+2 & -1 \end{vmatrix}$$

$$= -3(5k-5)$$

练习 齐次线性方程组 $\begin{cases} kx_1 + x_4 = 0 \\ x_1 + 2x_2 - x_4 = 0 \\ (k+2)x_1 - x_2 + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 有非零解

的充分必要条件是 k 满足 ____

解

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot (-1)^{3+4} \begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$

$$\xlongequal[r_3-4r_1]{r_2+r_1} (-3) \cdot \begin{vmatrix} k & 0 & 1 \\ k+1 & 2 & 0 \\ -3k+2 & -1 & 0 \end{vmatrix} = (-3) \cdot (-1)^{1+3} \cdot \begin{vmatrix} k+1 & 2 \\ -3k+2 & -1 \end{vmatrix}$$

$$= -3(5k-5)$$

有非零解当且仅当 $D = 0$,

练习 齐次线性方程组 $\begin{cases} kx_1 \qquad\qquad\qquad + \ x_4 = 0 \\ x_1 + 2x_2 \qquad\quad - \ x_4 = 0 \\ (k+2)x_1 - \ x_2 \qquad + 4x_4 = 0 \\ 2x_1 + \ x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 有非零解

的充分必要条件是 k 满足 ____

解

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3\cdot(-1)^{3+4}\begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$

$$\xlongequal[r_3-4r_1]{r_2+r_1} (-3)\cdot\begin{vmatrix} k & 0 & 1 \\ k+1 & 2 & 0 \\ -3k+2 & -1 & 0 \end{vmatrix} = (-3)\cdot(-1)^{1+3}\cdot\begin{vmatrix} k+1 & 2 \\ -3k+2 & -1 \end{vmatrix}$$

$$= -3(5k-5)$$

有非零解当且仅当 $D=0$，当且仅当 $k=1$