§8.5 多元复合函数与隐函数的求导法

2016-2017 学年 II

教学要求

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Outline of §8.5

1. 多元复合函数的求导法

2. 隐函数的求导法

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2. 隐函数的求导法

二元复合函数求导

设有二元函数 $z = f(u, v)$

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- 设 $u = \varphi(t)$，$v = \psi(t)$，则 $z = f(\varphi(t), \psi(t))$

  问 $\frac{dz}{dt} = ?$

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$$\begin{tikzcd}[row sep=small] & u \arrow[dr] & \\ z \arrow[ur] \arrow[dr] & & t \\ & v \arrow[ur] & \end{tikzcd}$$

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$$

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- 设 $u=\varphi(x,y)$，$v=\psi(x,y)$，则 $z=f(\varphi(x,y),\psi(x,y))$

  问 $\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}=?$

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$$\begin{tikzcd}
 & u \arrow[r] \arrow[dr] & x \\
z \arrow[ur] \arrow[dr] & & \\
 & v & y
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问 $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}=?$

公式 设 $z = f(u, v)$，$u = \varphi(t)$，$v = \psi(t)$，则 $z = f(\varphi(t), \psi(t))$ 的全导数

$$\frac{dz}{dt} =$$

二元复合函数求导公式——中间变量是一元函数

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$$\begin{tikzcd}
 & u \arrow[r] & t \\
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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}$$

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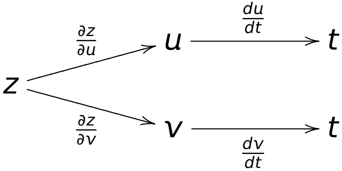
$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}\qquad\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$



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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$

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$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt} + \frac{\partial z}{\partial v} \cdot \frac{dv}{dt}$$

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$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt} + \frac{\partial z}{\partial v} \cdot \frac{dv}{dt}$$
$$= (uv)'_u \cdot$$

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$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial u}\cdot\frac{du}{dt} + \frac{\partial z}{\partial v}\cdot\frac{dv}{dt}\\ &= (uv)'_u\cdot(e^{-t})'_t +\end{aligned}$$

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$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt} + \frac{\partial z}{\partial v} \cdot \frac{dv}{dt}$$

$$= (uv)'_u \cdot (e^{-t})'_t + (uv)'_v \cdot (\sin t)'_t$$

$$= v \cdot (-e^{-t}) +$$

例 设 $z=uv$，而 $u=e^{-t}$，$v=\sin t$，求全导数 $\frac{dz}{dt}$

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$$\begin{aligned}\frac{dz}{dt}&=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}\\&=(uv)'_u\cdot(e^{-t})'_t+(uv)'_v\cdot(\sin t)'_t\\&=v\cdot(-e^{-t})+u\cdot\end{aligned}$$

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&= v \cdot (-e^{-t}) + u \cdot \cos t \\
&= \sin t \cdot (-e^{-t}) + e^{-t} \cdot \cos t \\
&= e^{-t}(\cos t - \sin t)
\end{aligned}
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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$
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解法二

$$\because\quad z=uv=$$

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$$\because \quad z=uv=e^{-t}\cdot\sin t$$

$$\therefore \quad \frac{dz}{dt}=\frac{d}{dt}(e^{-t}\sin t)=(e^{-t})'_t\cdot\sin t+$$

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例 设 $z = uv$，而 $u = e^{-t}$，$v = \sin t$，求全导数 $\frac{dz}{dt}$

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$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial u}\cdot\frac{du}{dt} + \frac{\partial z}{\partial v}\cdot\frac{dv}{dt}\\ &= (uv)'_u\cdot(e^{-t})'_t + (uv)'_v\cdot(\sin t)'_t\\ &= v\cdot(-e^{-t}) + u\cdot\cos t\\ &= \sin t\cdot(-e^{-t}) + e^{-t}\cdot\cos t\\ &= e^{-t}(\cos t - \sin t)\end{aligned}$$

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$$\because \quad z = uv = e^{-t}\cdot\sin t$$

$$\begin{aligned}\therefore \quad \frac{dz}{dt} &= \frac{d}{dt}(e^{-t}\sin t) = (e^{-t})'_t\cdot\sin t + e^{-t}\cdot(\sin t)'_t\\ &= (-e^{-t})\cdot\sin t + e^{-t}\cdot\cos t\end{aligned}$$

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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$

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$$\because\quad z=uv=e^{-t}\cdot\sin t$$

$$\therefore\quad \frac{dz}{dt}=\frac{d}{dt}(e^{-t}\sin t)=(e^{-t})'_t\cdot\sin t+e^{-t}\cdot(\sin t)'_t$$

$$=(-e^{-t})\cdot\sin t+e^{-t}\cdot\cos t=e^{-t}(\cos t-\sin t)$$

例 设 $z = \frac{y}{x}$，而 $x = e^t$，$y = 1 - e^{2t}$，求全导数 $\frac{dz}{dt}$

解

$$\frac{dz}{dt} =$$

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 & y \arrow[r, "\frac{dy}{dt}"'] & t
\end{tikzcd}$$

例 设 $z=\frac{y}{x}$，而 $x=e^t$，$y=1-e^{2t}$，求全导数 $\frac{dz}{dt}$

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$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=(\frac{y}{x})'_x\cdot$$

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解

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=\left(\frac{y}{x}\right)'_x\cdot\left(e^t\right)'_t+$$

$$\begin{tikzcd}
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$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=(\frac{y}{x})'_x\cdot(e^t)'_t+(\frac{y}{x})'_y\cdot$$

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$$=$$

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$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=(\frac{y}{x})'_x\cdot(e^t)'_t+(\frac{y}{x})'_y\cdot(1-e^{2t})'_t$$

$$=-\frac{y}{x^2}\cdot$$

$$\begin{tikzcd} & x \arrow[r, "\frac{dx}{dt}"] & t \\ z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[dr, "\frac{\partial z}{\partial y}"'] & & \\ & y \arrow[r, "\frac{dy}{dt}"'] & t \end{tikzcd}$$

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$$=-\frac{y}{x^2}\cdot e^t+$$

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$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot$$

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$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=$$

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 & x \arrow[r, "\frac{dx}{dt}"] & t \\
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$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=-\frac{1-e^{2t}}{e^{2t}}\cdot e^t+$$

$$\begin{tikzcd}
 & x \arrow[r, "\frac{dx}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[dr, "\frac{\partial z}{\partial y}"'] & & \\
 & y \arrow[r, "\frac{dy}{dt}"'] & t
\end{tikzcd}$$

例 设 $z=\frac{y}{x}$，而 $x=e^t$，$y=1-e^{2t}$，求全导数 $\frac{dz}{dt}$

解

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=(\frac{y}{x})'_x\cdot(e^t)'_t+(\frac{y}{x})'_y\cdot(1-e^{2t})'_t$$

$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=-\frac{1-e^{2t}}{e^{2t}}\cdot e^t+\frac{1}{e^t}\cdot(-2e^{2t})$$

$$=$$

$$\begin{tikzcd}
 & x \arrow[r, "\frac{dx}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[dr, "\frac{\partial z}{\partial y}"'] & & \\
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$$=-e^{-t}-e^t$$

$$\begin{tikzcd}
 & x \arrow[r, "\frac{dx}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[dr, "\frac{\partial z}{\partial y}"'] & & \\
 & y \arrow[r, "\frac{dy}{dt}"'] & t
\end{tikzcd}$$

三元复合函数求导公式——中间变量是一元函数

公式 设 $z=f(u,v,w)$，$u=\varphi(t)$，$v=\psi(t)$，$w=\omega(t)$，则 $z=f(\varphi(t),\psi(t),\omega(t))$ 的全导数

$$\frac{dz}{dt}=$$

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$$\frac{dz}{dt} =$$

$$\begin{tikzcd}
 & u \arrow[r] & t \\
z \arrow[ur] \arrow[r] \arrow[dr] & v \arrow[r] & t \\
 & w \arrow[r] & t
\end{tikzcd}$$

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$$\frac{dz}{dt}=$$

$$\begin{tikzcd}
 & u \arrow[r] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"'] & v \arrow[r] & t \\
 & w \arrow[r] & t
\end{tikzcd}$$

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$$\frac{dz}{dt}=$$

$$\begin{tikzcd}
& u \arrow[r, "\frac{du}{dt}"] & t \\
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& w \arrow[r, "\frac{dw}{dt}"] & t
\end{tikzcd}$$

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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}$$

$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"] & v \arrow[r, "\frac{dv}{dt}"] & t \\
 & w \arrow[r, "\frac{dw}{dt}"] & t
\end{tikzcd}$$

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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}\qquad\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$

$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"] & v \arrow[r, "\frac{dv}{dt}"] & t \\
 & w \arrow[r, "\frac{dw}{dt}"] & t
\end{tikzcd}$$

公式 设 $z = f(u, v, w)$，$u = \varphi(t)$，$v = \psi(t)$，$w = \omega(t)$，则 $z = f(\varphi(t), \psi(t), \omega(t))$ 的全导数

$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt} \qquad \frac{\partial z}{\partial v} \cdot \frac{dv}{dt} \qquad \frac{\partial z}{\partial w} \cdot \frac{dw}{dt}$$

$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ru, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[rd, "\frac{\partial z}{\partial w}"] & v \arrow[r, "\frac{dv}{dt}"] & t \\
 & w \arrow[r, "\frac{dw}{dt}"] & t
\end{tikzcd}$$

三元复合函数求导公式——中间变量是一元函数

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$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt} + \frac{\partial z}{\partial v} \cdot \frac{dv}{dt} + \frac{\partial z}{\partial w} \cdot \frac{dw}{dt}$$

$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"] & v \arrow[r, "\frac{dv}{dt}"] & t \\
 & w \arrow[r, "\frac{dw}{dt}"] & t
\end{tikzcd}$$

二元复合函数求导公式——中间变量是多元函数

公式 设 $z=f(u,v)$，$u=\varphi(x,y)$，$v=\psi(x,y)$，

二元复合函数求导公式——中间变量是多元函数

公式 设 $z=f(u,v)$，$u=\varphi(x,y)$，$v=\psi(x,y)$，则复合函数

$$z=f(\varphi(x,y),\psi(x,y))$$

的偏导数是：

$$\frac{\partial z}{\partial x}= \qquad\qquad , \quad \frac{\partial z}{\partial y}=$$

二元复合函数求导公式——中间变量是多元函数

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$$\frac{\partial z}{\partial x}= \qquad\qquad\qquad , \quad \frac{\partial z}{\partial y}=$$

图示

$$\begin{tikzcd}
& & x \\
& u \arrow[ur] \arrow[dr] & \\
& & y \\
z \arrow[uur] \arrow[ddr] & & \\
& & x \\
& v \arrow[ur] \arrow[dr] & \\
& & y
\end{tikzcd}$$

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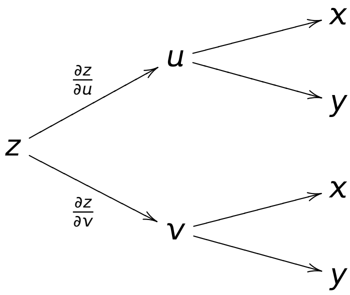
的偏导数是：

$$\frac{\partial z}{\partial x} = \qquad\qquad\qquad\qquad , \quad \frac{\partial z}{\partial y} =$$

图示



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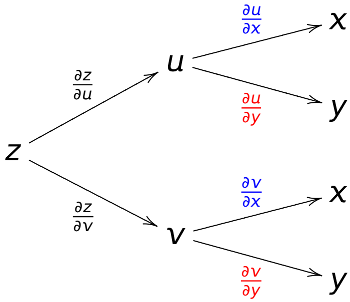
图示



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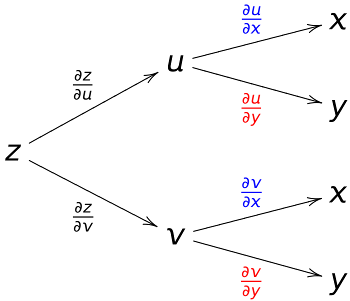
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\qquad\qquad,\qquad \frac{\partial z}{\partial y}=$$

图示



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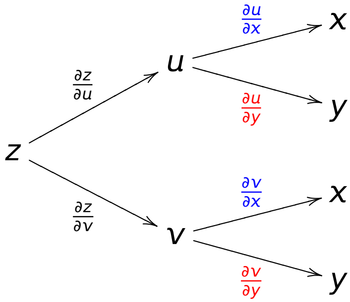
的偏导数是：

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x},\qquad \frac{\partial z}{\partial y}=$$

图示



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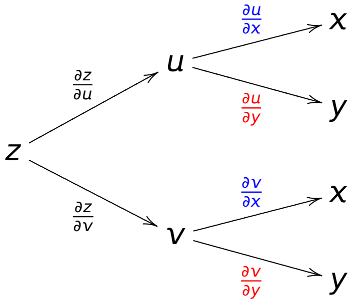
的偏导数是：

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}, \qquad \frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} +$$

图示



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$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x},\qquad \frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$

图示

$$\begin{tikzcd}
 & & x \\
 & u \arrow[ur, "\frac{\partial u}{\partial x}"] \arrow[dr, "\frac{\partial u}{\partial y}"'] & \\
 & & y \\
z \arrow[uur, "\frac{\partial z}{\partial u}"] \arrow[ddr, "\frac{\partial z}{\partial v}"'] & & \\
 & & x \\
 & v \arrow[ur, "\frac{\partial v}{\partial x}"] \arrow[dr, "\frac{\partial v}{\partial y}"'] & \\
 & & y
\end{tikzcd}$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x +$$

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$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$=$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot (x^2 + y^2)'_x$$
$$= 2e^{2u}\sin v \cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\ &=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\ &=2e^{2u}\sin v\cdot 3x^2y+\end{aligned}$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x$$
$$=$$

例 设 $z = e^{2u}\sin v$，$u = x^3y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$= (e^{2u}\sin v)'_u\cdot(x^3y)'_x + (e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$= 2e^{2u}\sin v\cdot 3x^2y + e^{2u}\cos v\cdot 2x$$
$$= 2e^{2x^3y}\sin(x^2+y^2)\cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$
\begin{aligned}
\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\
&=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\
&=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\
&=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+
\end{aligned}
$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$
\begin{aligned}
\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\
&=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\
&=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\
&=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot
\end{aligned}
$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\begin{aligned}\frac{\partial z}{\partial x}&=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\&=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\&=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\&=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x\end{aligned}$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x$$
$$=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$
$$=$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\begin{aligned}\frac{\partial z}{\partial x}&=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\&=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\&=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\&=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial y}&=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\\&=(e^{2u}\sin v)'_u\cdot\end{aligned}$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$= (e^{2u}\sin v)'_u\cdot(x^3y)'_x + (e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$= 2e^{2u}\sin v\cdot 3x^2y + e^{2u}\cos v\cdot 2x$$
$$= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y + e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$
$$= (e^{2u}\sin v)'_u\cdot(x^3y)'_y +$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot (x^2+y^2)'_x$$

$$= 2e^{2u}\sin v \cdot 3x^2 y + e^{2u}\cos v \cdot 2x$$

$$= 2e^{2x^3 y}\sin(x^2+y^2)\cdot 3x^2 y + e^{2x^3 y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$

$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_y + (e^{2u}\sin v)'_v \cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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We are here now...

1. 多元复合函数的求导法

2. 隐函数的求导法

隐函数的求导法 I

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$$\begin{tikzcd}
 & x & \\
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$$\begin{tikzcd}[column sep=large]
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$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(xy + e^{x+y} - 1)'_x}{(xy + e^{x+y} - 1)'_y} = -\frac{y + e^{x+y}}{x + e^{x+y}}$$

方法二 注意 $xy(x) + e^{x+y(x)} - 1 = 0$，所以

$$\begin{aligned} 0 &= (xy(x) + e^{x+y(x)} - 1)'_x \\ &= (xy(x))'_x + (e^{x+y(x)})'_x \\ &= y + xy' + e^{x+y(x)} \cdot (1 + y') \\ &= (y + e^{x+y}) + y'(x + e^{x+y}) \end{aligned}$$

例 设 $y=f(x)$ 满足 $xy=1-e^{x+y}$，求 $\frac{dy}{dx}$

方法一 注意 $xy+e^{x+y}-1=0$，令 $F(x,y)=xy+e^{x+y}-1$，则 $F(x,y)=0$，所以

$$\frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{(xy+e^{x+y}-1)'_x}{(xy+e^{x+y}-1)'_y}=-\frac{y+e^{x+y}}{x+e^{x+y}}$$

方法二 注意 $xy(x)+e^{x+y(x)}-1=0$，所以

$$\begin{aligned}0&=(xy(x)+e^{x+y(x)}-1)'_x\\&=(xy(x))'_x+(e^{x+y(x)})'_x\\&=y+xy'+e^{x+y(x)}\cdot(1+y')\\&=(y+e^{x+y})+y'(x+e^{x+y})\end{aligned}$$

所以 $y'=-\frac{y+e^{x+y}}{x+e^{x+y}}$

例 设 $y = f(x)$ 满足 $xy + \ln y = \ln x$，求 $\frac{dy}{dx}$

例 设 $y = f(x)$ 满足 $xy + \ln y = \ln x$，求 $\frac{dy}{dx}$

方法一

$F(x, y) = 0$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} =$$

例 设 $y=f(x)$ 满足 $xy+\ln y=\ln x$，求 $\frac{dy}{dx}$

方法一 注意 $xy+\ln y-\ln x=0$

$F(x,y)=0$

$$\frac{dy}{dx}=-\frac{F_x}{F_y}=$$

例 设 $y=f(x)$ 满足 $xy+\ln y=\ln x$，求 $\frac{dy}{dx}$

方法一 注意 $xy+\ln y-\ln x=0$，令 $F(x,y)=xy+\ln y-\ln x$，$F(x,y)=0$

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方法二 注意 $xy(x)+\ln y(x)-\ln x=0$，

例 设 $y = f(x)$ 满足 $xy + \ln y = \ln x$，求 $\frac{dy}{dx}$

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例 设 $y = f(x)$ 满足 $xy + \ln y = \ln x$，求 $\frac{dy}{dx}$

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方法二 注意 $xy(x) + \ln y(x) - \ln x = 0$，所以

$$0 = (xy(x) + \ln y(x) - \ln x)'_x = (xy(x))'_x + (\ln y(x))'_x - (\ln x)'_x$$

$$=$$

例 设 $y=f(x)$ 满足 $xy+\ln y=\ln x$，求 $\frac{dy}{dx}$

方法一 注意 $xy+\ln y-\ln x=0$，令 $F(x,y)=xy+\ln y-\ln x$，则 $F(x,y)=0$，所以

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方法二 注意 $xy(x)+\ln y(x)-\ln x=0$，所以

$$0=(xy(x)+\ln y(x)-\ln x)'_x=(xy(x))'_x+(\ln y(x))'_x-(\ln x)'_x$$

$$=y+xy'$$

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隐函数的求导法 II

公式 设 $z = f(x, y)$ 满足 $F(x, y, z) = 0$,

隐函数的求导法 II

公式 设 $z=f(x,y)$ 满足 $F(x,y,z)=0$，即 $F(x,y,z(x,y))=0$，

隐函数的求导法 II

公式 设 $z = f(x, y)$ 满足 $F(x, y, z) = 0$，即 $F(x, y, z(x, y)) = 0$，则

$$\frac{\partial z}{\partial x} = \qquad , \quad \frac{\partial z}{\partial y} =$$

隐函数的求导法 II

公式 设 $z=f(x,y)$ 满足 $F(x,y,z)=0$，即 $F(x,y,z(x,y))=0$，则

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z},\quad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z}$$

隐函数的求导法 II

公式 设 $z=f(x,y)$ 满足 $F(x,y,z)=0$，即 $F(x,y,z(x,y))=0$，

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$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z},\qquad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z}$$

证明 $\because \quad F(x,y,z(x,y))=0$

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证明
$$\because \quad F(x,y,z(x,y))=0$$
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证明

$$\because\quad F(x,y,z(x,y))=0$$

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$$\begin{tikzcd} & x \\ F \arrow[ur] \arrow[r] \arrow[dr] & y & x \\ & z \arrow[ur] \arrow[r] & y \end{tikzcd}$$

隐函数的求导法 II

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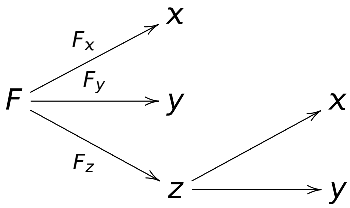
证明

$$\because \quad F(x, y, z(x, y))=0$$

$$\therefore \quad 0=\frac{\partial}{\partial x}F(x, y, z(x, y))=$$



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$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z},\qquad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z}$$

证明

$$\because\quad F(x,y,z(x,y))=0$$

$$\therefore\quad 0=\frac{\partial}{\partial x}F(x,y,z(x,y))=$$

$$\begin{tikzcd}
 & x & \\
F \arrow[ur, "F_x"] \arrow[r, "F_y"] \arrow[dr, "F_z"'] & y & x \\
 & z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[r, "\frac{\partial z}{\partial y}"'] & y
\end{tikzcd}$$

隐函数的求导法 II

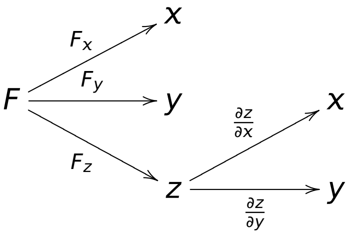
公式 设 $z=f(x,y)$ 满足 $F(x,y,z)=0$，即 $F(x,y,z(x,y))=0$，则

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z},\qquad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z}$$

证明

$$\because\quad F(x,y,z(x,y))=0$$

$$\therefore\quad 0=\frac{\partial}{\partial x}F(x,y,z(x,y))=F_x+F_z\cdot\frac{\partial z}{\partial x}$$



隐函数的求导法 II

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证明

$$\because \quad F(x, y, z(x, y))=0$$

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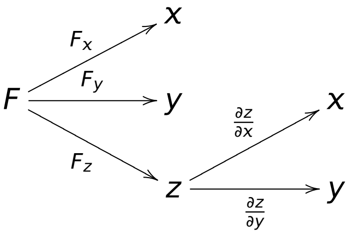

隐函数的求导法 II

公式 设 $z=f(x,y)$ 满足 $F(x,y,z)=0$，即 $F(x,y,z(x,y))=0$，则

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z},\qquad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z}$$

证明

$$\because \quad F(x,y,z(x,y))=0$$

$$\therefore \quad 0=\frac{\partial}{\partial x}F(x,y,z(x,y))=F_x+F_z\cdot\frac{\partial z}{\partial x}$$

$$\therefore \quad \frac{\partial z}{\partial x}=-\frac{F_x}{F_z},$$

同理 $\dfrac{\partial z}{\partial y}=-\dfrac{F_y}{F_z}$

$$\begin{tikzcd}
 & x & \\
F \arrow[ur, "F_x"] \arrow[r, "F_y"] \arrow[dr, "F_z"'] & y & x \\
 & z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[r, "\frac{\partial z}{\partial y}"'] & y
\end{tikzcd}$$

例 设 $z=f(x,y)$ 满足 $x+y+xz=e^z-1$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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解 $F(x,y,z)=0$

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=$$

例 设 $z=f(x,y)$ 满足 $x+y+xz=e^z-1$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解 令 $F(x,y,z)=x+y+xz-e^z+1$，　　$F(x,y,z)=0$

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=$$

例 设 $z=f(x, y)$ 满足 $x+y+xz=e^z-1$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解 令 $F(x, y, z)=x+y+xz-e^z+1$，则 $F(x, y, z)=0$，所以

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{(x+y+xz-e^z+1)'_x}{(x+y+xz-e^z+1)'_z}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(x+y+xz-e^z+1)'_y}{(x+y+xz-e^z+1)'_z}$$

$$=$$

例 设 $z = f(x, y)$ 满足 $x + y + xz = e^z - 1$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解 令 $F(x, y, z) = x + y + xz - e^z + 1$，则 $F(x, y, z) = 0$，所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + y + xz - e^z + 1)'_x}{(x + y + xz - e^z + 1)'_z}$$

$$= -\frac{\qquad}{\qquad}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + y + xz - e^z + 1)'_y}{(x + y + xz - e^z + 1)'_z}$$

$$= -\frac{\qquad}{\qquad}$$

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$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{(x+y+xz-e^z+1)'_x}{(x+y+xz-e^z+1)'_z}$$

$$=-\frac{\qquad\qquad}{0}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(x+y+xz-e^z+1)'_y}{(x+y+xz-e^z+1)'_z}$$

$$=-\frac{\qquad\qquad}{\qquad}$$

例 设 $z=f(x, y)$ 满足 $x+y+xz=e^z-1$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$=-\frac{\qquad\qquad}{0+0}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(x+y+xz-e^z+1)'_y}{(x+y+xz-e^z+1)'_z}$$

$$=-\frac{\qquad\qquad}{\qquad\qquad}$$

例 设 $z = f(x, y)$ 满足 $x + y + xz = e^z - 1$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解 令 $F(x, y, z) = x + y + xz - e^z + 1$，则 $F(x, y, z) = 0$，所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + y + xz - e^z + 1)'_x}{(x + y + xz - e^z + 1)'_z}$$

$$= -\frac{\qquad}{0 + 0 + x}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + y + xz - e^z + 1)'_y}{(x + y + xz - e^z + 1)'_z}$$

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$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{(x+y+xz-e^z+1)'_x}{(x+y+xz-e^z+1)'_z}$$

$$=-\frac{}{0+0+x-e^z}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(x+y+xz-e^z+1)'_y}{(x+y+xz-e^z+1)'_z}$$

$$=-\frac{}{}$$

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$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{(x+y+xz-e^z+1)'_x}{(x+y+xz-e^z+1)'_z}$$

$$=-\frac{\quad}{0+0+x-e^z+0}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(x+y+xz-e^z+1)'_y}{(x+y+xz-e^z+1)'_z}$$

$$=-\frac{\quad}{\quad}$$

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$$=-\frac{\qquad}{0+0+x-e^z+0}$$

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$$= -\frac{1 + 0 + z - 0 + 0}{0 + 0 + x - e^z + 0} = -\frac{1 + z}{x - e^z}$$

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