§8.4 偏导数与全微分

2017-2018 学年 II

Outline of §8.4

1. 二元函数偏导数定义

2. 全微分的定义与计算

We are here now...

1. 二元函数偏导数定义

2. 全微分的定义与计算

偏导数引入

- 对一元函数 $y=f(x)$：导数 $y'=f'(x) \longleftrightarrow$ 变化率

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？
  1. 固定 y，对 x 求导

  2. 固定 x，对 y 求导

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？
  1. 固定 y，对 x 求导

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \qquad \text{对}x\text{偏导数}$$

  2. 固定 x，对 y 求导

偏导数引入

- 对一元函数 $y=f(x)$：导数 $y'=f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z=f(x,y)$：导数？

1. 固定 y，对 x 求导

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \qquad \text{对}x\text{偏导数}$$

2. 固定 x，对 y 求导

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \qquad \text{对}y\text{偏导数}$$

偏导数引入

- 对一元函数 $y=f(x)$：导数 $y'=f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z=f(x,y)$：导数？

1. 固定 y，对 x 求导：$z=f(x,y)$ 关于 x 的变化率

$$\frac{\partial z}{\partial x} \quad 或 \quad z'_x \quad 或 \quad z_x \quad 或 \quad f_x \qquad 对x偏导数$$

2. 固定 x，对 y 求导

$$\frac{\partial z}{\partial y} \quad 或 \quad z'_y \quad 或 \quad z_y \quad 或 \quad f_y \qquad 对y偏导数$$

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？
  1. 固定 y，对 x 求导：$z = f(x, y)$ 关于 x 的变化率

$$\frac{\partial z}{\partial x} \quad 或 \quad z'_x \quad 或 \quad z_x \quad 或 \quad f_x \qquad \text{对}x\text{偏导数}$$

  2. 固定 x，对 y 求导：$z = f(x, y)$ 关于 y 的变化率

$$\frac{\partial z}{\partial y} \quad 或 \quad z'_y \quad 或 \quad z_y \quad 或 \quad f_y \qquad \text{对}y\text{偏导数}$$

偏导数引入

- 对一元函数 $y=f(x)$：导数 $y'=f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z=f(x,y)$：导数？
  1. 固定 y，对 x 求导：$z=f(x,y)$ 关于 x 的变化率

$$\frac{\partial z}{\partial x} \quad 或 \quad z'_x \quad 或 \quad z_x \quad 或 \quad f_x \qquad 对x偏导数$$

  2. 固定 x，对 y 求导：$z=f(x,y)$ 关于 y 的变化率

$$\frac{\partial z}{\partial y} \quad 或 \quad z'_y \quad 或 \quad z_y \quad 或 \quad f_y \qquad 对y偏导数$$

例 1 设 $z=f(x,y)=x^2y+2x+y+1$，则

偏导数引入

- 对一元函数 $y=f(x)$：导数 $y'=f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z=f(x,y)$：导数？
  1. 固定 y，对 x 求导：$z=f(x,y)$ 关于 x 的变化率
     $$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \qquad \text{对}x\text{偏导数}$$
  2. 固定 x，对 y 求导：$z=f(x,y)$ 关于 y 的变化率
     $$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \qquad \text{对}y\text{偏导数}$$

例 1 设 $z=f(x,y)=x^2y+2x+y+1$，则

$$\frac{\partial z}{\partial x}=$$

$$\frac{\partial z}{\partial y}=$$

偏导数引入

- 对一元函数 $y=f(x)$：导数 $y'=f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z=f(x,y)$：导数？
  1. 固定 y，对 x 求导：$z=f(x,y)$ 关于 x 的变化率

$$\frac{\partial z}{\partial x} \quad 或 \quad z'_x \quad 或 \quad z_x \quad 或 \quad f_x$$

对x偏导数

  2. 固定 x，对 y 求导：$z=f(x,y)$ 关于 y 的变化率

$$\frac{\partial z}{\partial y} \quad 或 \quad z'_y \quad 或 \quad z_y \quad 或 \quad f_y$$

对y偏导数

例 1 设 $z=f(x,y)=x^2y+2x+y+1$，则

$$\frac{\partial z}{\partial x}=(x^2y+2x+y+1)'_x=$$

$$\frac{\partial z}{\partial y}=$$

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？

  1. 固定 y，对 x 求导：$z = f(x, y)$ 关于 x 的变化率

  $$\frac{\partial z}{\partial x} \quad 或 \quad z'_x \quad 或 \quad z_x \quad 或 \quad f_x \qquad \text{对}x\text{偏导数}$$

  2. 固定 x，对 y 求导：$z = f(x, y)$ 关于 y 的变化率

  $$\frac{\partial z}{\partial y} \quad 或 \quad z'_y \quad 或 \quad z_y \quad 或 \quad f_y \qquad \text{对}y\text{偏导数}$$

例 1 设 $z = f(x, y) = x^2y + 2x + y + 1$，则

$$\frac{\partial z}{\partial x} = (x^2y + 2x + y + 1)'_x = 2xy +$$

$$\frac{\partial z}{\partial y} =$$

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？
  1. 固定 y，对 x 求导：$z = f(x, y)$ 关于 x 的变化率

     $$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \qquad \text{对}x\text{偏导数}$$

  2. 固定 x，对 y 求导：$z = f(x, y)$ 关于 y 的变化率

     $$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \qquad \text{对}y\text{偏导数}$$

例 1 设 $z = f(x, y) = x^2y + 2x + y + 1$，则

$$\frac{\partial z}{\partial x} = (x^2y + 2x + y + 1)'_x = 2xy + 2$$

$$\frac{\partial z}{\partial y} =$$

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？
  1. 固定 y，对 x 求导：$z = f(x, y)$ 关于 x 的变化率

$$\frac{\partial z}{\partial x} \quad 或 \quad z'_x \quad 或 \quad z_x \quad 或 \quad f_x \qquad \text{对}x\text{偏导数}$$

  2. 固定 x，对 y 求导：$z = f(x, y)$ 关于 y 的变化率

$$\frac{\partial z}{\partial y} \quad 或 \quad z'_y \quad 或 \quad z_y \quad 或 \quad f_y \qquad \text{对}y\text{偏导数}$$

例 1 设 $z = f(x, y) = x^2y + 2x + y + 1$，则

$$\frac{\partial z}{\partial x} = (x^2y + 2x + y + 1)'_x = 2xy + 2$$

$$\frac{\partial z}{\partial y} = (x^2y + 2x + y + 1)'_y =$$

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z = f(x, y)$：导数？
  1. 固定 y，对 x 求导：$z = f(x, y)$ 关于 x 的变化率

$$\frac{\partial z}{\partial x} \quad 或 \quad z'_x \quad 或 \quad z_x \quad 或 \quad f_x \qquad 对x偏导数$$

  2. 固定 x，对 y 求导：$z = f(x, y)$ 关于 y 的变化率

$$\frac{\partial z}{\partial y} \quad 或 \quad z'_y \quad 或 \quad z_y \quad 或 \quad f_y \qquad 对y偏导数$$

例 1 设 $z = f(x, y) = x^2y + 2x + y + 1$，则

$$\frac{\partial z}{\partial x} = (x^2y + 2x + y + 1)'_x = 2xy + 2$$

$$\frac{\partial z}{\partial y} = (x^2y + 2x + y + 1)'_y = x^2 +$$

偏导数引入

- 对一元函数 $y=f(x)$：导数 $y'=f'(x) \longleftrightarrow$ 变化率
- 对二元函数 $z=f(x,y)$：导数？
  1. 固定 y，对 x 求导：$z=f(x,y)$ 关于 x 的变化率

$$\frac{\partial z}{\partial x} \quad 或 \quad z'_x \quad 或 \quad z_x \quad 或 \quad f_x \qquad 对x偏导数$$

  2. 固定 x，对 y 求导：$z=f(x,y)$ 关于 y 的变化率

$$\frac{\partial z}{\partial y} \quad 或 \quad z'_y \quad 或 \quad z_y \quad 或 \quad f_y \qquad 对y偏导数$$

例 1 设 $z=f(x,y)=x^2y+2x+y+1$，则

$$\frac{\partial z}{\partial x}=(x^2y+2x+y+1)'_x=2xy+2$$

$$\frac{\partial z}{\partial y}=(x^2y+2x+y+1)'_y=x^2+1$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

例 2 设 $z=f(x,y)=e^{xy}+2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=(e^{xy}+2xy^2)'_x=$$

$$\frac{\partial z}{\partial y}=$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x =$$

$$\frac{\partial z}{\partial y} =$$

例 2 设 $z=f(x,y)=e^{xy}+2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+$$

$$\frac{\partial z}{\partial y}=$$

例 2 设 $z=f(x,y)=e^{xy}+2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$\frac{\partial z}{\partial y}=$$

例 2 设 $z=f(x,y)=e^{xy}+2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$\frac{\partial z}{\partial y}=(e^{xy}+2xy^2)'_y=$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y =$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} +$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设 $z = f(x, y) = 2y \sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设 $z = f(x, y) = 2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial y}$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设 $z = f(x, y) = 2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y\sin(3x))'_x =$$

$$\frac{\partial z}{\partial y}$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设 $z = f(x, y) = 2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y\sin(3x))'_x = 2y(\sin(3x))'_x =$$

$$\frac{\partial z}{\partial y}$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设 $z = f(x, y) = 2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y\sin(3x))'_x = 2y(\sin(3x))'_x = 2y \cdot 3\cos(3x) =$$

$$\frac{\partial z}{\partial y}$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设 $z = f(x, y) = 2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y\sin(3x))'_x = 2y(\sin(3x))'_x = 2y \cdot 3\cos(3x) = 6y\cos(3x)$$

$$\frac{\partial z}{\partial y}$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设 $z = f(x, y) = 2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y\sin(3x))'_x = 2y(\sin(3x))'_x = 2y \cdot 3\cos(3x) = 6y\cos(3x)$$

$$\frac{\partial z}{\partial y} = (2y\sin(3x))'_y =$$

例 2 设 $z = f(x, y) = e^{xy} + 2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设 $z = f(x, y) = 2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y\sin(3x))'_x = 2y(\sin(3x))'_x = 2y \cdot 3\cos(3x) = 6y\cos(3x)$$

$$\frac{\partial z}{\partial y} = (2y\sin(3x))'_y = (2y)'_y \cdot \sin(3x) =$$

例 2 设 $z=f(x,y)=e^{xy}+2xy^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$\frac{\partial z}{\partial y}=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

例 3 设 $z=f(x,y)=2y\sin(3x)$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=(2y\sin(3x))'_x=2y(\sin(3x))'_x=2y\cdot 3\cos(3x)=6y\cos(3x)$$

$$\frac{\partial z}{\partial y}=(2y\sin(3x))'_y=(2y)'_y\cdot\sin(3x)=2\sin(3x)$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x =$$

$$u_y =$$

$$u_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x =$$

$$u_y =$$

$$u_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x =$$

$$u_y =$$

$$u_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz$$

$$u_y =$$

$$u_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y =$$

$$u_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y =$$

$$u_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y =$$

$$u_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z = xy$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z = xy + \frac{1}{x}$$

偏导数的极限定义

- 一元函数 $y=f(x)$ 在 $x=x_0$ 处的导数定义为：

$$f'(x_0)=$$

偏导数的极限定义

- 一元函数 $y = f(x)$ 在 $x = x_0$ 处的导数定义为：

$$f'(x_0) = \lim \frac{}{}$$

偏导数的极限定义

- 一元函数 $y=f(x)$ 在 $x=x_0$ 处的导数定义为:

$$f'(x_0)=\lim \frac{f(x_0+\Delta x)-f(x_0)}{}$$

偏导数的极限定义

- 一元函数 $y=f(x)$ 在 $x=x_0$ 处的导数定义为：

$$f'(x_0)=\lim \frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

偏导数的极限定义

- 一元函数 $y = f(x)$ 在 $x = x_0$ 处的导数定义为：

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

偏导数的极限定义

- 一元函数 $y=f(x)$ 在 $x=x_0$ 处的导数定义为：

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

- $z=f(x,y)$ 在点 (x_0,y_0) 处关于 x 的偏导数：

$$\frac{\partial f}{\partial x}(x_0,y_0)=$$

偏导数的极限定义

- 一元函数 $y=f(x)$ 在 $x=x_0$ 处的导数定义为：

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

- $z=f(x,y)$ 在点 (x_0,y_0) 处关于 x 的偏导数：

$$\frac{\partial f}{\partial x}(x_0,y_0)=\lim\ \frac{\qquad\qquad}{\qquad\qquad}$$

偏导数的极限定义

- 一元函数 $y = f(x)$ 在 $x = x_0$ 处的导数定义为：

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$ 在点 (x_0, y_0) 处关于 x 的偏导数：

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{}$$

偏导数的极限定义

- 一元函数 $y=f(x)$ 在 $x=x_0$ 处的导数定义为：

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

- $z=f(x,y)$ 在点 (x_0,y_0) 处关于 x 的偏导数：

$$\frac{\partial f}{\partial x}(x_0,y_0)=\lim\frac{f(x_0+\Delta x,y_0)-f(x_0,y_0)}{\Delta x}$$

偏导数的极限定义

- 一元函数 $y=f(x)$ 在 $x=x_0$ 处的导数定义为：

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

- $z=f(x,y)$ 在点 (x_0,y_0) 处关于 x 的偏导数：

$$\frac{\partial f}{\partial x}(x_0,y_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x,y_0)-f(x_0,y_0)}{\Delta x}$$

偏导数的极限定义

- 一元函数 $y = f(x)$ 在 $x = x_0$ 处的导数定义为:

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$ 在点 (x_0, y_0) 处关于 x 的偏导数:

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} = \qquad f(x, y_0)$$

偏导数的极限定义

- 一元函数 $y=f(x)$ 在 $x=x_0$ 处的导数定义为：

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

- $z=f(x,y)$ 在点 (x_0,y_0) 处关于 x 的偏导数：

$$\frac{\partial f}{\partial x}(x_0,y_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x,y_0)-f(x_0,y_0)}{\Delta x}=\frac{d}{dx}\Big[f(x,y_0)\Big]$$

偏导数的极限定义

- 一元函数 $y=f(x)$ 在 $x=x_0$ 处的导数定义为：

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

- $z=f(x,y)$ 在点 (x_0,y_0) 处关于 x 的偏导数：

$$\frac{\partial f}{\partial x}(x_0,y_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x,y_0)-f(x_0,y_0)}{\Delta x}=\frac{d}{dx}\Big[f(x,y_0)\Big]\Big|_{x=x_0}$$

偏导数的极限定义

- 一元函数 $y = f(x)$ 在 $x = x_0$ 处的导数定义为：

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$ 在点 (x_0, y_0) 处关于 x 的偏导数：

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} = \frac{d}{dx}\Big[f(x, y_0)\Big]\Big|_{x = x_0}$$

- $z = f(x, y)$ 在点 (x_0, y_0) 处关于 y 的偏导数：

$$\frac{\partial f}{\partial y}(x_0, y_0) =$$

偏导数的极限定义

- 一元函数 $y = f(x)$ 在 $x = x_0$ 处的导数定义为：

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$ 在点 (x_0, y_0) 处关于 x 的偏导数：

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} = \frac{d}{dx}\Big[f(x, y_0)\Big]\Big|_{x=x_0}$$

- $z = f(x, y)$ 在点 (x_0, y_0) 处关于 y 的偏导数：

$$\frac{\partial f}{\partial y}(x_0, y_0) = \lim \frac{\qquad\qquad}{\qquad\qquad}$$

偏导数的极限定义

- 一元函数 $y=f(x)$ 在 $x=x_0$ 处的导数定义为：

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

- $z=f(x,y)$ 在点 (x_0,y_0) 处关于 x 的偏导数：

$$\frac{\partial f}{\partial x}(x_0,y_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x,y_0)-f(x_0,y_0)}{\Delta x}=\frac{d}{dx}\Big[f(x,y_0)\Big]\Big|_{x=x_0}$$

- $z=f(x,y)$ 在点 (x_0,y_0) 处关于 y 的偏导数：

$$\frac{\partial f}{\partial y}(x_0,y_0)=\lim\ \frac{f(x_0,y_0+\Delta y)-f(x_0,y_0)}{}$$

偏导数的极限定义

- 一元函数 $y = f(x)$ 在 $x = x_0$ 处的导数定义为：

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$ 在点 (x_0, y_0) 处关于 x 的偏导数：

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} = \frac{d}{dx}\Big[f(x, y_0)\Big]\Big|_{x=x_0}$$

- $z = f(x, y)$ 在点 (x_0, y_0) 处关于 y 的偏导数：

$$\frac{\partial f}{\partial y}(x_0, y_0) = \lim \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$$

暨南大學 JINAN UNIVERSITY

偏导数的极限定义

- 一元函数 $y=f(x)$ 在 $x=x_0$ 处的导数定义为：

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

- $z=f(x,y)$ 在点 (x_0,y_0) 处关于 x 的偏导数：

$$\frac{\partial f}{\partial x}(x_0,y_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x,y_0)-f(x_0,y_0)}{\Delta x}=\frac{d}{dx}\Big[f(x,y_0)\Big]\Big|_{x=x_0}$$

- $z=f(x,y)$ 在点 (x_0,y_0) 处关于 y 的偏导数：

$$\frac{\partial f}{\partial y}(x_0,y_0)=\lim_{\Delta y\to 0}\frac{f(x_0,y_0+\Delta y)-f(x_0,y_0)}{\Delta y}$$

偏导数的极限定义

- 一元函数 $y = f(x)$ 在 $x = x_0$ 处的导数定义为：

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$ 在点 (x_0, y_0) 处关于 x 的偏导数：

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} = \frac{d}{dx}\Big[f(x, y_0)\Big]\Big|_{x = x_0}$$

- $z = f(x, y)$ 在点 (x_0, y_0) 处关于 y 的偏导数：

$$\frac{\partial f}{\partial y}(x_0, y_0) = \lim_{\Delta y \to 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y} = \qquad f(x_0, y)$$

偏导数的极限定义

- 一元函数 $y=f(x)$ 在 $x=x_0$ 处的导数定义为：

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

- $z=f(x,y)$ 在点 (x_0,y_0) 处关于 x 的偏导数：

$$\frac{\partial f}{\partial x}(x_0,y_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x,y_0)-f(x_0,y_0)}{\Delta x}=\frac{d}{dx}\Big[f(x,y_0)\Big]\Big|_{x=x_0}$$

- $z=f(x,y)$ 在点 (x_0,y_0) 处关于 y 的偏导数：

$$\frac{\partial f}{\partial y}(x_0,y_0)=\lim_{\Delta y\to 0}\frac{f(x_0,y_0+\Delta y)-f(x_0,y_0)}{\Delta y}=\frac{d}{dy}\Big[f(x_0,y)\Big]$$

偏导数的极限定义

- 一元函数 $y=f(x)$ 在 $x=x_0$ 处的导数定义为：

$$f'(x_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x}$$

- $z=f(x,y)$ 在点 (x_0,y_0) 处关于 x 的偏导数：

$$\frac{\partial f}{\partial x}(x_0,y_0)=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x,y_0)-f(x_0,y_0)}{\Delta x}=\frac{d}{dx}\Big[f(x,y_0)\Big]\Big|_{x=x_0}$$

- $z=f(x,y)$ 在点 (x_0,y_0) 处关于 y 的偏导数：

$$\frac{\partial f}{\partial y}(x_0,y_0)=\lim_{\Delta y\to 0}\frac{f(x_0,y_0+\Delta y)-f(x_0,y_0)}{\Delta y}=\frac{d}{dy}\Big[f(x_0,y)\Big]\Big|_{y=y_0}$$

注 求偏导数的值 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$ 有两种方式:

注 求偏导数的值 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$ 有两种方式:

- 先求出 $f(x, y)$ 的偏导数 $f_x(x, y)$ 和 $f_y(x, y)$ 的一般形式,

注 求偏导数的值 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$ 有两种方式：

- 先求出 $f(x, y)$ 的偏导数 $f_x(x, y)$ 和 $f_y(x, y)$ 的一般形式，然后赋值求出 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$。

注 求偏导数的值 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$ 有两种方式：

- 先求出 $f(x, y)$ 的偏导数 $f_x(x, y)$ 和 $f_y(x, y)$ 的一般形式，然后赋值求出 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$。

- $$\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dx}\big[f(x, y_0)\big]\Big|_{x=x_0}$$
  $$\frac{\partial f}{\partial y}(x_0, y_0) = \frac{d}{dy}\big[f(x_0, y)\big]\Big|_{y=y_0}$$

注 求偏导数的值 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$ 有两种方式：

- 先求出 $f(x, y)$ 的偏导数 $f_x(x, y)$ 和 $f_y(x, y)$ 的一般形式，然后赋值求出 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$。

- $\frac{\partial f}{\partial x}(x_0, y_0) = \qquad f(x, y_0)$

  $\frac{\partial f}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$

注 求偏导数的值 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$ 有两种方式：

- 先求出 $f(x, y)$ 的偏导数 $f_x(x, y)$ 和 $f_y(x, y)$ 的一般形式，然后赋值求出 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$。

- $\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dx}\left[f(x, y_0)\right]$

  $\frac{\partial f}{\partial y}(x_0, y_0) = \frac{d}{dy}\left[f(x_0, y)\right]\Big|_{y=y_0}$

注 求偏导数的值 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$ 有两种方式：

- 先求出 $f(x, y)$ 的偏导数 $f_x(x, y)$ 和 $f_y(x, y)$ 的一般形式，然后赋值求出 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$。

- $$\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dx}\left[f(x, y_0)\right]\Big|_{x=x_0}$$
  $$\frac{\partial f}{\partial y}(x_0, y_0) = \frac{d}{dy}\left[f(x_0, y)\right]\Big|_{y=y_0}$$

注 求偏导数的值 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$ 有两种方式：

- 先求出 $f(x, y)$ 的偏导数 $f_x(x, y)$ 和 $f_y(x, y)$ 的一般形式，然后赋值求出 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$。

- $\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dx}\left[f(x, y_0)\right]\Big|_{x=x_0}$

  $\frac{\partial f}{\partial y}(x_0, y_0) = \qquad f(x_0, y)$

注 求偏导数的值 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$ 有两种方式：

- 先求出 $f(x, y)$ 的偏导数 $f_x(x, y)$ 和 $f_y(x, y)$ 的一般形式，然后赋值求出 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$。

- $\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dx}\big[f(x, y_0)\big]\Big|_{x=x_0}$

  $\frac{\partial f}{\partial y}(x_0, y_0) = \frac{d}{dy}\big[f(x_0, y)\big]$

注 求偏导数的值 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$ 有两种方式：

- 先求出 $f(x, y)$ 的偏导数 $f_x(x, y)$ 和 $f_y(x, y)$ 的一般形式，然后赋值求出 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$。

- $$\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dx}\left[f(x, y_0)\right]\Big|_{x=x_0}$$
  $$\frac{\partial f}{\partial y}(x_0, y_0) = \frac{d}{dy}\left[f(x_0, y)\right]\Big|_{y=y_0}$$

注 求偏导数的值 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$ 有两种方式：

- 先求出 $f(x, y)$ 的偏导数 $f_x(x, y)$ 和 $f_y(x, y)$ 的一般形式，然后赋值求出 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$。

- $$\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dx}\left[f(x, y_0)\right]\Big|_{x=x_0}$$
  $$\frac{\partial f}{\partial y}(x_0, y_0) = \frac{d}{dy}\left[f(x_0, y)\right]\Big|_{y=y_0}$$
  （先对无关的变量赋值，然后求导，最后对求导的变量赋值）

注 求偏导数的值 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$ 有两种方式：

- 先求出 $f(x, y)$ 的偏导数 $f_x(x, y)$ 和 $f_y(x, y)$ 的一般形式，然后赋值求出 $\frac{\partial f}{\partial x}(x_0, y_0)$ 和 $\frac{\partial f}{\partial y}(x_0, y_0)$。

- $$\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dx}\left[f(x, y_0)\right]\Big|_{x=x_0}$$
  $$\frac{\partial f}{\partial y}(x_0, y_0) = \frac{d}{dy}\left[f(x_0, y)\right]\Big|_{y=y_0}$$
  （先对无关的变量赋值，然后求导，最后对求导的变量赋值）

两种方式各有优点，要灵活运用

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} =$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x =$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} =$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x =$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} =$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y +$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} =$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} =$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} =$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} =$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z=xy+\frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x}=(xy+\frac{x}{y})'_x=(xy)'_x+(\frac{x}{y})'_x=y+\frac{1}{y}$$

$$\frac{\partial z}{\partial y}=(xy+\frac{x}{y})'_y=$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}}=(y+\frac{1}{y})\Big|_{\substack{x=2\\y=1}}=1+\frac{1}{1}=2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}}=$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y =$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x - \frac{x}{y^2}$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} =$$

例 设 $z=xy+\frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x}=(xy+\frac{x}{y})'_x=(xy)'_x+(\frac{x}{y})'_x=y+\frac{1}{y}$$

$$\frac{\partial z}{\partial y}=(xy+\frac{x}{y})'_y=(xy)'_y+(\frac{x}{y})'_y=x-\frac{x}{y^2}$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}}=\left.(y+\frac{1}{y})\right|_{\substack{x=2\\y=1}}=1+\frac{1}{1}=2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}}=\left.(x-\frac{x}{y^2})\right|_{\substack{x=2\\y=1}}=$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x - \frac{x}{y^2}$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = \left.(y + \frac{1}{y})\right|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} = \left.(x - \frac{x}{y^2})\right|_{\substack{x=2\\y=1}} = 2 - \frac{2}{1} =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x - \frac{x}{y^2}$$

所以

$$\left.\frac{\partial z}{\partial x}\right|_{\substack{x=2\\y=1}} = \left.(y + \frac{1}{y})\right|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\left.\frac{\partial z}{\partial y}\right|_{\substack{x=2\\y=1}} = \left.(x - \frac{x}{y^2})\right|_{\substack{x=2\\y=1}} = 2 - \frac{2}{1} = 0$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(2,1) = \frac{d}{dx}[f(x,1)]\bigg|_{x=2}, \quad \frac{\partial z}{\partial y}(2,1) = \frac{d}{dy}[f(2,y)]\bigg|_{y=1}$$

暨南大学 JINAN UNIVERSITY

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(2,1) = \frac{d}{dx}[f(x,1)]\Big|_{x=2}, \qquad \frac{\partial z}{\partial y}(2,1) = \frac{d}{dy}[f(2,y)]\Big|_{y=1}$$

所以 $f(x,1)$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(2,1) = \frac{d}{dx}[f(x,1)]\Big|_{x=2}, \qquad \frac{\partial z}{\partial y}(2,1) = \frac{d}{dy}[f(2,y)]\Big|_{y=1}$$

所以 $f(x,1) = 2x$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(2, 1) = \frac{d}{dx}[f(x, 1)]\Big|_{x=2}, \qquad \frac{\partial z}{\partial y}(2, 1) = \frac{d}{dy}[f(2, y)]\Big|_{y=1}$$

所以 $\quad f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] =$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(2,1) = \frac{d}{dx}[f(x,1)]\Big|_{x=2}, \qquad \frac{\partial z}{\partial y}(2,1) = \frac{d}{dy}[f(2,y)]\Big|_{y=1}$$

所以

$$f(x,1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x,1)] = 2$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(2, 1) = \frac{d}{dx}[f(x, 1)]\bigg|_{x=2}, \qquad \frac{\partial z}{\partial y}(2, 1) = \frac{d}{dy}[f(2, y)]\bigg|_{y=1}$$

所以 $f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$

$$\Rightarrow \quad \frac{d}{dx}[f(x, 1)]\bigg|_{x=2} = 2,$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(2, 1) = \frac{d}{dx}[f(x, 1)]\Big|_{x=2}, \qquad \frac{\partial z}{\partial y}(2, 1) = \frac{d}{dy}[f(2, y)]\Big|_{y=1}$$

所以

$$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$$

$$\Rightarrow \quad \frac{\partial z}{\partial x}(2, 1) = \frac{d}{dx}[f(x, 1)]\Big|_{x=2} = 2,$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(2,1) = \frac{d}{dx}[f(x,1)]\Big|_{x=2}, \qquad \frac{\partial z}{\partial y}(2,1) = \frac{d}{dy}[f(2,y)]\Big|_{y=1}$$

所以 $f(x,1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x,1)] = 2$

$$\Rightarrow \quad \frac{\partial z}{\partial x}(2,1) = \frac{d}{dx}[f(x,1)]\Big|_{x=2} = 2,$$

$f(2,y)$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(2, 1) = \frac{d}{dx}[f(x, 1)]\Big|_{x=2}, \qquad \frac{\partial z}{\partial y}(2, 1) = \frac{d}{dy}[f(2, y)]\Big|_{y=1}$$

所以

$$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$$

$$\Rightarrow \quad \frac{\partial z}{\partial x}(2, 1) = \frac{d}{dx}[f(x, 1)]\Big|_{x=2} = 2,$$

$$f(2, y) = 2y + \frac{2}{y}$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(2, 1) = \frac{d}{dx}[f(x, 1)]\Big|_{x=2}, \quad \frac{\partial z}{\partial y}(2, 1) = \frac{d}{dy}[f(2, y)]\Big|_{y=1}$$

所以

$$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$$

$$\Rightarrow \quad \frac{\partial z}{\partial x}(2, 1) = \frac{d}{dx}[f(x, 1)]\Big|_{x=2} = 2,$$

$$f(2, y) = 2y + \frac{2}{y} \quad \Rightarrow \quad \frac{d}{dy}[f(2, y)] =$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(2, 1) = \frac{d}{dx}[f(x, 1)]\Big|_{x=2}, \qquad \frac{\partial z}{\partial y}(2, 1) = \frac{d}{dy}[f(2, y)]\Big|_{y=1}$$

所以

$$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$$

$$\Rightarrow \quad \frac{\partial z}{\partial x}(2, 1) = \frac{d}{dx}[f(x, 1)]\Big|_{x=2} = 2,$$

$$f(2, y) = 2y + \frac{2}{y} \quad \Rightarrow \quad \frac{d}{dy}[f(2, y)] = 2 - \frac{2}{y^2}$$

例 设 $z=xy+\frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(2,1)=\frac{d}{dx}[f(x,1)]\Big|_{x=2},\qquad \frac{\partial z}{\partial y}(2,1)=\frac{d}{dy}[f(2,y)]\Big|_{y=1}$$

所以

$$f(x,1)=2x \quad\Rightarrow\quad \frac{d}{dx}[f(x,1)]=2$$

$$\Rightarrow\quad \frac{\partial z}{\partial x}(2,1)=\frac{d}{dx}[f(x,1)]\Big|_{x=2}=2,$$

$$f(2,y)=2y+\frac{2}{y} \quad\Rightarrow\quad \frac{d}{dy}[f(2,y)]=2-\frac{2}{y^2}$$

$$\Rightarrow\quad \frac{d}{dy}[f(2,y)]\Big|_{y=1}=0.$$

例 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(2, 1) = \frac{d}{dx}[f(x, 1)]\Big|_{x=2}, \qquad \frac{\partial z}{\partial y}(2, 1) = \frac{d}{dy}[f(2, y)]\Big|_{y=1}$$

所以

$$f(x, 1) = 2x \quad \Rightarrow \quad \frac{d}{dx}[f(x, 1)] = 2$$

$$\Rightarrow \quad \frac{\partial z}{\partial x}(2, 1) = \frac{d}{dx}[f(x, 1)]\Big|_{x=2} = 2,$$

$$f(2, y) = 2y + \frac{2}{y} \quad \Rightarrow \quad \frac{d}{dy}[f(2, y)] = 2 - \frac{2}{y^2}$$

$$\Rightarrow \quad \frac{\partial z}{\partial y}(2, 1)\frac{d}{dy}[f(2, y)]\Big|_{y=1} = 0.$$

二阶偏导数

$$\begin{tikzcd} & \dfrac{\partial z}{\partial x} \\ z = f(x, y) \arrow[ur] \arrow[dr] & \\ & \dfrac{\partial z}{\partial y} \end{tikzcd}$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
z=f(x,y) \arrow[ur] \arrow[dr] & & {} \\
 & \frac{\partial z}{\partial y} &
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[dr] & & \\
 & \frac{\partial z}{\partial y} &
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & {}
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small, row sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial y}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial x}, \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial y}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y \partial x}, \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial y}, \\
z = f(x, y) \arrow[uuur, phantom] \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial y},
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial y}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial y},
\end{tikzcd}
$$

二阶偏导数

$$\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial x}, \quad z_{xx}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial y}, \quad z_{xy}, \\
z=f(x,y) \arrow[uuur] \arrow[dddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y\partial x}, \\
 & & & \\
 & \frac{\partial z}{\partial y} \arrow[uur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y\partial y},
\end{tikzcd}$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y \partial x}, \quad z_{yx}, \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y \partial y},
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): \quad \dfrac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): \quad \dfrac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): \quad \dfrac{\partial^2 z}{\partial y \partial x}, \quad z_{yx}, \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right): \quad \dfrac{\partial^2 z}{\partial y \partial y}, \quad z_{yy},
\end{tikzcd}
$$

二阶偏导数

$$\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)
\end{tikzcd}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \quad f_{xx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x \partial y}, \quad z_{xy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \qquad \frac{\partial^2 z}{\partial y \partial x}, \quad z_{yx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \qquad \frac{\partial^2 z}{\partial y \partial y}, \quad z_{yy},$$

二阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x\partial x}, \quad z_{xx}, \quad f_{xx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \qquad \frac{\partial^2 z}{\partial x\partial y}, \quad z_{xy}, \quad f_{xy}$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \qquad \frac{\partial^2 z}{\partial y\partial x}, \quad z_{yx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \qquad \frac{\partial^2 z}{\partial y\partial y}, \quad z_{yy},$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x\partial x}, \quad z_{xx}, \quad f_{xx} \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x\partial y}, \quad z_{xy}, \quad f_{xy} \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y\partial x}, \quad z_{yx}, \quad f_{yx} \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y\partial y}, \quad z_{yy},
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}[column sep=small, row sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \quad f_{xx} \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \quad f_{xy} \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y \partial x}, \quad z_{yx}, \quad f_{yx} \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y \partial y}, \quad z_{yy}, \quad f_{yy}
\end{tikzcd}
$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x =$$

$$z_y =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} +$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

$$z_y=(e^{xy}+2xy^2)'_y=(e^{xy})'_y+(2xy^2)'_y=xe^{xy}+4xy$$

$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

$$z_{xy}=(ye^{xy}+2y^2)'_y=(ye^{xy})'_y+(2y^2)'_y=e^{xy}+xye^{xy}+$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} +$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y =$$

暨南大學 JINAN UNIVERSITY

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2e^{xy} +$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2e^{xy} + 4x$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2e^{xy} + 4x$$

注 此例成立 $z_{xy} = z_{yx}$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) & \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right)
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}[column sep=small, row sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : & \frac{\partial^3 z}{\partial x \partial x \partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : & \frac{\partial^3 z}{\partial x \partial x \partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : & \\
 z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : & \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial x}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): &
\end{tikzcd}
$$

三阶偏导数

$$z=f(x,y)\ \to\ \frac{\partial z}{\partial x},\qquad z=f(x,y)\ \to\ \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x}\ \to\ \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\qquad \frac{\partial z}{\partial x}\ \to\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\ \to\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\qquad \frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\ \to\ \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\qquad \frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\ \to\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\qquad \frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\ \to\ \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\qquad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$\frac{\partial z}{\partial y}\ \to\ \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\qquad \frac{\partial z}{\partial y}\ \to\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\ \to\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\ \to\ \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\ \to\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\ \to\ \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$z=f(x,y)\to\frac{\partial z}{\partial x}\to\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial z}{\partial x}\to\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\to\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$z=f(x,y)\to\frac{\partial z}{\partial y}\to\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial z}{\partial y}\to\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\to\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$z = f(x, y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial x \partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial x \partial y},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$z = f(x, y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) :$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$z=f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$z=f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z = f(x, y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x}, \ z_{xxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y}, \ z_{xxy},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$z = f(x, y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$z=f(x,y)\longrightarrow\frac{\partial z}{\partial x}\longrightarrow\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\longrightarrow\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial z}{\partial x}\longrightarrow\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\longrightarrow\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad\frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$z=f(x,y)\longrightarrow\frac{\partial z}{\partial y}\longrightarrow\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\longrightarrow\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial z}{\partial y}\longrightarrow\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\longrightarrow\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad\frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial x\partial x}, \quad z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial x\partial y}, \quad z_{xxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial y\partial x}, \quad z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial y\partial y}, \quad z_{xyy},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y\partial x\partial x}, \quad z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) & : \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) & : \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) & : \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) & : \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy}, \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) & : \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) & : \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) & : \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx}, \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) & : \quad \frac{\partial^3 z}{\partial y\partial y\partial y},
\end{tikzcd}
$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x}, \ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y}, \ z_{xxy},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x}, \ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y}, \ z_{xyy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x}, \ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y}, \ z_{yxy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x}, \ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y}, \ z_{yyy},$$

三阶偏导数

$$z=f(x,y)\ \to\ \frac{\partial z}{\partial x},\quad \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x}\ \to\ \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y}\ \to\ \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\ \to\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\ \to\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\ \to\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\ \to\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z = f(x, y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},$$

$$z = f(x, y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},\ z_{yxy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z = f(x, y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},\ f_{xyy}$$

$$z = f(x, y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},\ z_{yxy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y) \to \frac{\partial z}{\partial x},\quad z=f(x,y) \to \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\quad \frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\quad \frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},\ f_{yyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y) \to \frac{\partial z}{\partial x},\ \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},\ f_{yyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},\ f_{yyy}$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解 $z_x =$

$z_y =$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解 $z_x =$

$z_y =$

$z_{xx} =$

$z_{xy} =$

$z_{yx} =$

$z_{yy} =$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解 $$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2$$

$$z_y=$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解 $$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解 $$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y$$

$$z_{yx}=$$

$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$
$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$
$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$
$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2$$
$$z_{yy} =$$

$$z_{xxx} =$$

暨南大學 JINAN UNIVERSITY

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$
$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$
$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$
$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2-1$$
$$z_{yy}=$$

$$z_{xxx}=$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解 $$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$

$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2-1$$

$$z_{yy}=(2x^3y-9xy^2-x)'_y=$$

$$z_{xxx}=$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$

$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$

$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$

$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2-1$$

$$z_{yy}=(2x^3y-9xy^2-x)'_y=2x^3$$

$$z_{xxx}=$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} = (6xy^2)'_x =$$

例 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$
$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$
$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$
$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$
$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} = (6xy^2)'_x = 6y^2$$

例 求 $z=x^3y^2-3xy^3-xy+1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x=(x^3y^2-3xy^3-xy+1)'_x=3x^2y^2-3y^3-y$$
$$z_y=(x^3y^2-3xy^3-xy+1)'_y=2x^3y-9xy^2-x$$

$$z_{xx}=(3x^2y^2-3y^3-y)'_x=6xy^2$$
$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$
$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2-1$$
$$z_{yy}=(2x^3y-9xy^2-x)'_y=2x^3-18xy$$

$$z_{xxx}=(6xy^2)'_x=6y^2$$

注 此例成立 $z_{xy}=z_{yx}$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x =$$
$$z_y =$$
$$z_{xx} =$$
$$z_{xy} =$$
$$z_{yx} =$$
$$z_{yy} =$$
$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

暨南大學 JINAN UNIVERSITY

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$
$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$
$$z_{yx} = (3x\cos(3y))'_x =$$
$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$
$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$
$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$
$$z_{yy} =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$

$$z_{yy} = (3x\cos(3y))'_y =$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$
$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$
$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$
$$z_{yy} = (3x\cos(3y))'_y = -9x\sin(3y)$$

$$z_{xyy} =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$
$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$
$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$
$$z_{yy} = (3x\cos(3y))'_y = -9x\sin(3y)$$

$$z_{xyy} = (3\cos(3y))'_y =$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$

$$z_{yy} = (3x\cos(3y))'_y = -9x\sin(3y)$$

$$z_{xyy} = (3\cos(3y))'_y = -9\sin(3y)$$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$
$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$
$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$
$$z_{yy} = (3x\cos(3y))'_y = -9x\sin(3y)$$

$$z_{xyy} = (3\cos(3y))'_y = -9\sin(3y)$$

注 此例成立 $z_{xy} = z_{yx}$

例 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

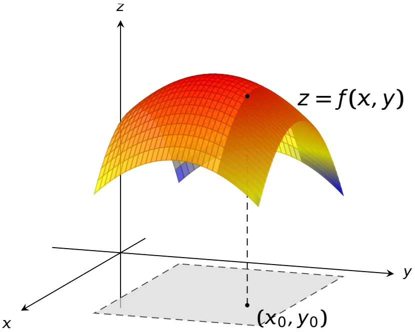
$$z_{xx} = (\sin(3y))'_x = 0$$
$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$
$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$
$$z_{yy} = (3x\cos(3y))'_y = -9x\sin(3y)$$

$$z_{xyy} = (3\cos(3y))'_y = -9\sin(3y)$$

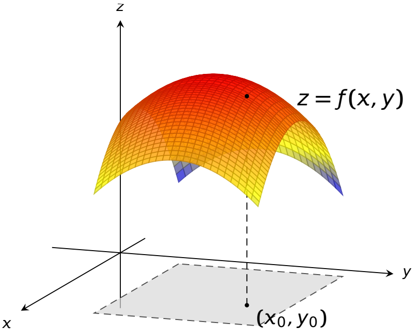
注 此例成立 $z_{xy} = z_{yx}$

性质 设有二元函数 $z = f(x, y)$。若 $\frac{\partial^2 z}{\partial y \partial x}$ 和 $\frac{\partial^2 z}{\partial x \partial y}$ 均连续，则
$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$$

偏导数的几何直观

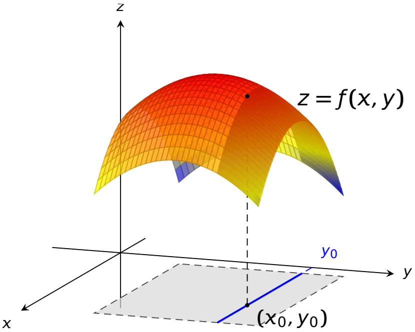


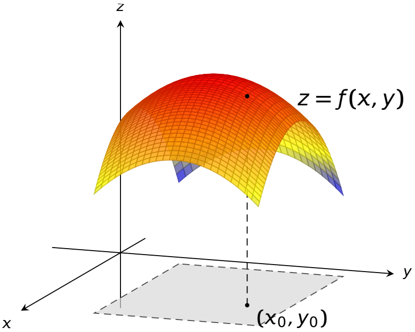
$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0}$$



$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\Big|_{y=y_0}$$

偏导数的几何直观

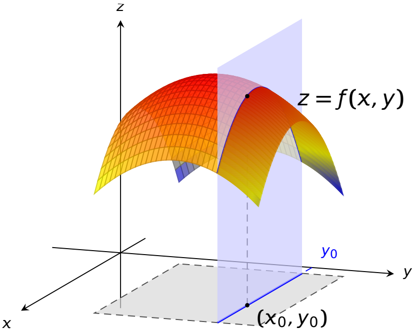


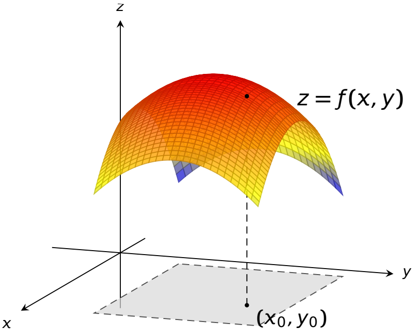


$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\bigg|_{x=x_0}$$

$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\bigg|_{y=y_0}$$

偏导数的几何直观

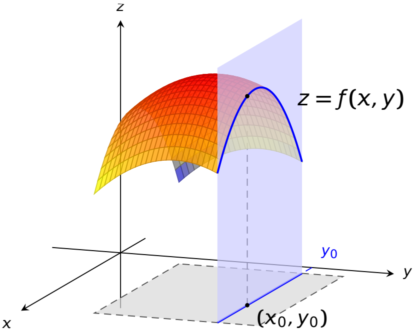




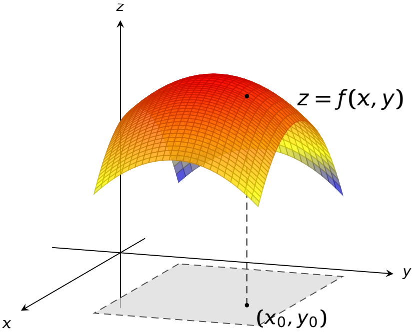
$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0}$$

$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\Big|_{y=y_0}$$

偏导数的几何直观

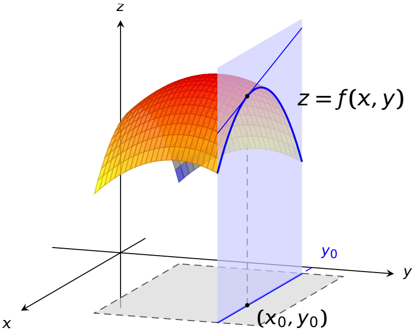


$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0}$$

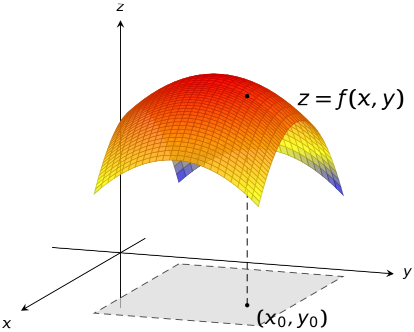


$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\Big|_{y=y_0}$$

偏导数的几何直观

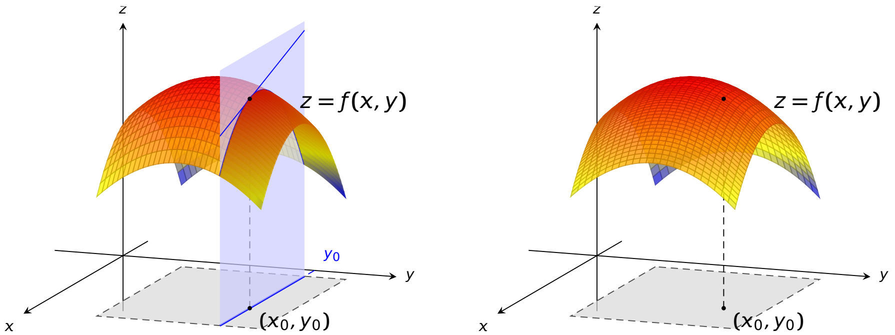


$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]\Big|_{x=x_0}$$



$$\frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]\Big|_{y=y_0}$$

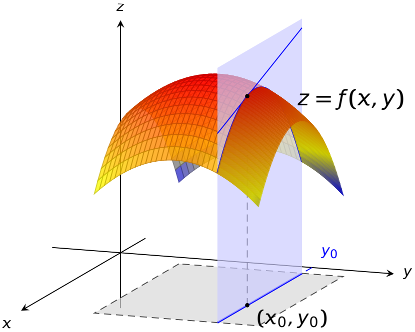
偏导数的几何直观

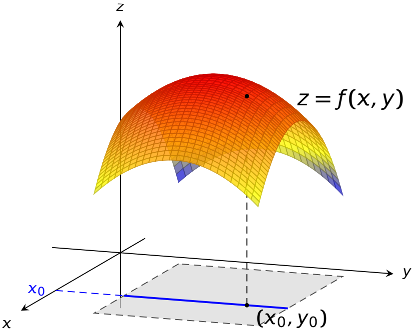


$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0}$$

$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\Big|_{y=y_0}$$

偏导数的几何直观

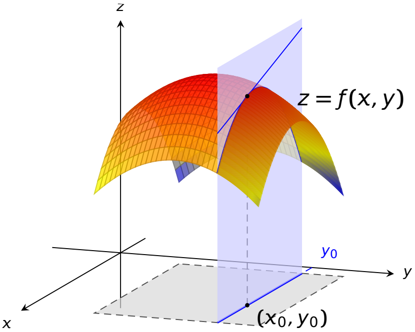




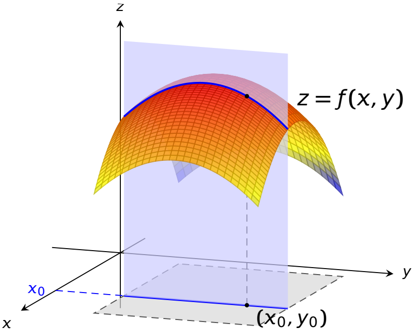
$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\bigg|_{x=x_0}$$

$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\bigg|_{y=y_0}$$

偏导数的几何直观

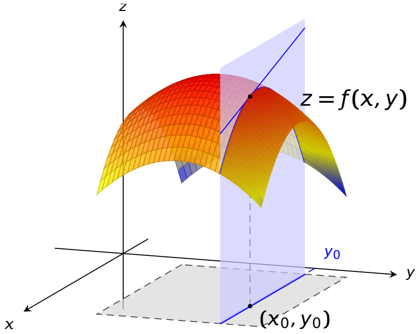


$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\bigg|_{x=x_0}$$

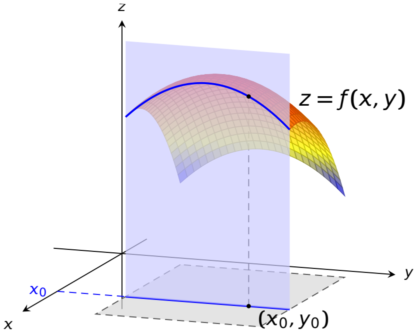


$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\bigg|_{y=y_0}$$

偏导数的几何直观

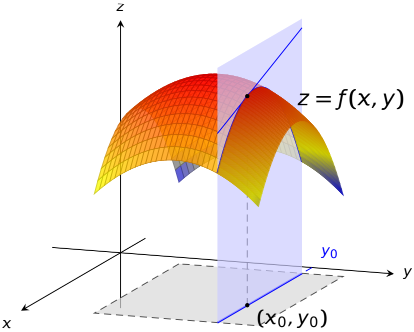


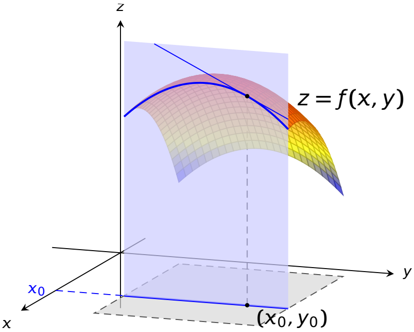
$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\bigg|_{x=x_0}$$



$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\bigg|_{y=y_0}$$

偏导数的几何直观

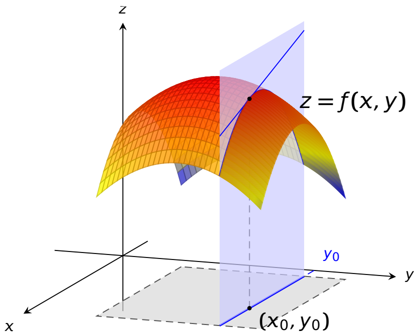


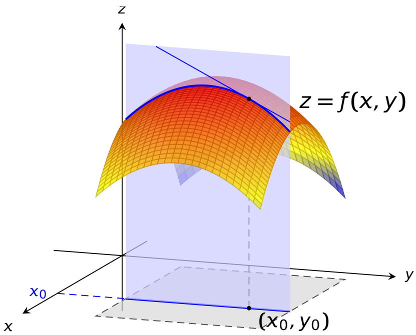


$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\bigg|_{x=x_0}$$

$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\bigg|_{y=y_0}$$

偏导数的几何直观





$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\Big|_{x=x_0}$$

$$\frac{\partial z}{\partial y}(x_0,y_0)=\frac{d}{dy}[f(x_0,y)]\Big|_{y=y_0}$$

We are here now...

1. 二元函数偏导数定义

2. 全微分的定义与计算

回顾一元函数的微分

- 函数 $y = f(x)$ 的增量

$$\Delta y = f(x + \Delta x) - f(x)$$

回顾一元函数的微分

- 函数 $y=f(x)$ 的增量

$$\Delta y=f(x+\Delta x)-f(x)$$

- 若 $y=f(x)$可微，则

$$\Delta y=f(x+\Delta x)-f(x)=A\Delta x+o(\Delta x)$$

回顾一元函数的微分

- 函数 $y = f(x)$ 的增量

$$\Delta y = f(x + \Delta x) - f(x)$$

- 若 $y = f(x)$可微，则

$$\Delta y = f(x + \Delta x) - f(x) = A\Delta x + o(\Delta x) = f'(x)\Delta x + o(\Delta x)$$

回顾一元函数的微分

- 函数 $y=f(x)$ 的增量

$$\Delta y=f(x+\Delta x)-f(x)$$

- 若 $y=f(x)$可微，则

$$\Delta y=f(x+\Delta x)-f(x)=A\Delta x+o(\Delta x)=f'(x)\Delta x+o(\Delta x)$$

此时可用 $f'(x)\Delta x$ 近似代替 Δy,

回顾一元函数的微分

- 函数 $y=f(x)$ 的增量

$$\Delta y=f(x+\Delta x)-f(x)$$

- 若 $y=f(x)$可微，则

$$\Delta y=f(x+\Delta x)-f(x)=A\Delta x+o(\Delta x)=f'(x)\Delta x+o(\Delta x)$$

此时可用 $f'(x)\Delta x$ 近似代替 Δy，称为函数 $y=f(x)$ 的微分，

回顾一元函数的微分

- 函数 $y=f(x)$ 的增量

$$\Delta y=f(x+\Delta x)-f(x)$$

- 若 $y=f(x)$可微，则

$$\Delta y=f(x+\Delta x)-f(x)=A\Delta x+o(\Delta x)=f'(x)\Delta x+o(\Delta x)$$

此时可用 $f'(x)\Delta x$ 近似代替 Δy，称为函数 $y=f(x)$ 的微分，记为：

$$dy=f'(x)dx \quad 或 \quad df=f'(x)dx$$

多元函数的全微分

- 二元函数 $z = f(x, y)$

$$f(x+\Delta x, y+\Delta y) - f(x, y)$$

多元函数的全微分

- 二元函数 $z = f(x, y)$

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

多元函数的全微分

- 二元函数 $z = f(x, y)$的**全增量**

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

多元函数的全微分

- 二元函数 $z = f(x, y)$的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称 $z = f(x, y)$可微是指：

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

多元函数的全微分

- 二元函数 $z=f(x,y)$的全增量

$$\Delta z=f(x+\Delta x,\,y+\Delta y)-f(x,\,y)$$

- 称 $z=f(x,y)$可微是指：

$$\begin{aligned}\Delta z&=f(x+\Delta x,\,y+\Delta y)-f(x,\,y)\\&=A\Delta x+B\Delta y+\end{aligned}$$

多元函数的全微分

- 二元函数 $z = f(x, y)$的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称 $z = f(x, y)$可微是指:

$$\begin{aligned} \Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \end{aligned}$$

多元函数的全微分

- 二元函数 $z = f(x, y)$的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称 $z = f(x, y)$可微是指：

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \approx A\Delta x + B\Delta y\end{aligned}$$

多元函数的全微分

- 二元函数 $z = f(x, y)$的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称 $z = f(x, y)$可微是指:

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \approx A\Delta x + B\Delta y\end{aligned}$$

例 设 $z = f(x, y) = x^2 + y^2$,

多元函数的全微分

- 二元函数 $z = f(x, y)$的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称 $z = f(x, y)$可微是指：

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \approx A\Delta x + B\Delta y\end{aligned}$$

例 设 $z = f(x, y) = x^2 + y^2$，则

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

多元函数的全微分

- 二元函数 $z = f(x, y)$的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称 $z = f(x, y)$可微是指：

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y)\\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \approx A\Delta x + B\Delta y\end{aligned}$$

例 设 $z = f(x, y) = x^2 + y^2$，则

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y)\\ &= \left[(x + \Delta x)^2 + (y + \Delta y)^2\right]\end{aligned}$$

多元函数的全微分

- 二元函数 $z = f(x, y)$的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称 $z = f(x, y)$可微是指：

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \approx A\Delta x + B\Delta y\end{aligned}$$

例 设 $z = f(x, y) = x^2 + y^2$，则

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= \left[(x + \Delta x)^2 + (y + \Delta y)^2\right] - \left[x^2 + y^2\right]\end{aligned}$$

多元函数的全微分

- 二元函数 $z = f(x, y)$的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称 $z = f(x, y)$可微是指：

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \approx A\Delta x + B\Delta y\end{aligned}$$

例 设 $z = f(x, y) = x^2 + y^2$，则

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= \left[(x + \Delta x)^2 + (y + \Delta y)^2\right] - \left[x^2 + y^2\right] \\ &= 2x\Delta x + 2y\Delta y + \left[(\Delta x)^2 + (\Delta y)^2\right]\end{aligned}$$

多元函数的全微分

- 二元函数 $z=f(x,y)$的全增量

$$\Delta z=f(x+\Delta x,y+\Delta y)-f(x,y)$$

- 称 $z=f(x,y)$可微是指：

$$\begin{aligned}\Delta z&=f(x+\Delta x,y+\Delta y)-f(x,y)\\&=A\Delta x+B\Delta y+o\left(\sqrt{(\Delta x)^2+(\Delta y)^2}\right)\approx A\Delta x+B\Delta y\end{aligned}$$

例 设 $z=f(x,y)=x^2+y^2$，则

$$\begin{aligned}\Delta z&=f(x+\Delta x,y+\Delta y)-f(x,y)\\&=\left[(x+\Delta x)^2+(y+\Delta y)^2\right]-\left[x^2+y^2\right]\\&=2x\Delta x+2y\Delta y+\left[(\Delta x)^2+(\Delta y)^2\right]\end{aligned}$$

所以 $z=x^2+y^2$ 可微。

多元函数的全微分 (Cont.)

- 若 $z = f(x, y)$可微，则连续，且存在偏导数 z_x, z_y，还有

$$\Delta z = f(x + \Delta x) - f(x)$$

多元函数的全微分 (Cont.)

- 若 $z = f(x, y)$可微，则连续，且存在偏导数 z_x, z_y，还有

$$\Delta z = f(x + \Delta x) - f(x)$$

$$= z_x(x, y)\Delta x + z_y(x, y)\Delta y +$$

多元函数的全微分 (Cont.)

- 若 $z = f(x, y)$可微，则连续，且存在偏导数 z_x, z_y，还有

$$
\begin{aligned}
\Delta z &= f(x + \Delta x) - f(x) \\
&= z_x(x, y)\Delta x + z_y(x, y)\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right)
\end{aligned}
$$

多元函数的全微分 (Cont.)

- 若 $z = f(x, y)$可微，则连续，且存在偏导数 z_x, z_y，还有

$$\begin{aligned}\Delta z &= f(x+\Delta x) - f(x) \\ &= z_x(x, y)\Delta x + z_y(x, y)\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \\ &\approx z_x(x, y)\Delta x + z_y(x, y)\Delta y\end{aligned}$$

多元函数的全微分 (Cont.)

- 若 $z=f(x,y)$可微，则连续，且存在偏导数 z_x, z_y，还有

$$\begin{aligned}\Delta z &= f(x+\Delta x)-f(x)\\ &= z_x(x,y)\Delta x + z_y(x,y)\Delta y + o\left(\sqrt{(\Delta x)^2+(\Delta y)^2}\right)\\ &\approx z_x(x,y)\Delta x + z_y(x,y)\Delta y\end{aligned}$$

$z=f(x,y)$ 的全微分：$dz = z_x(x,y)dx + z_y(x,y)dy$

多元函数的全微分 (Cont.)

- 若 $z = f(x, y)$可微，则连续，且存在偏导数 z_x, z_y，还有

$$
\begin{aligned}
\Delta z &= f(x+\Delta x) - f(x) \\
&= z_x(x, y)\Delta x + z_y(x, y)\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \\
&\approx z_x(x, y)\Delta x + z_y(x, y)\Delta y
\end{aligned}
$$

  $z = f(x, y)$ 的全微分：$dz = z_x(x, y)dx + z_y(x, y)dy$

- 若 $z = f(x, y)$ 可微，则 $\Delta z \approx dz$

多元函数的全微分 (Cont.)

- 对三元函数 $u = \varphi(x, y, z)$，其全微分

$$du = u_x dx + u_y dy + u_z dz$$

多元函数的全微分 (Cont.)

- 对三元函数 $u=\varphi(x,y,z)$，其全微分

$$du = u_x dx + u_y dy + u_z dz$$

此时

$$\Delta u = \varphi(x+\Delta x, y+\Delta y, z+\Delta z) - \varphi(x,y,z)$$

多元函数的全微分 (Cont.)

- 对三元函数 $u = \varphi(x, y, z)$，其全微分

$$du = u_x dx + u_y dy + u_z dz$$

此时

$$\Delta u = \varphi(x + \Delta x, y + \Delta y, z + \Delta z) - \varphi(x, y, z) \approx du$$

例 1 计算函数 $z = \frac{y}{x}$ 的全微分

解

例 1 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x =$$

$$z_y =$$

例 1 计算函数 $z=\frac{y}{x}$ 的全微分

解

$$z_x =$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 1 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x =$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 1 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 1 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y =$$

$$dz = z_x dx + z_y dy =$$

例 1 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)_x' = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)_y' = \frac{1}{x}$$

$$dz = z_x dx + z_y dy =$$

例 1 计算函数 $z=\frac{y}{x}$ 的全微分

解

$$z_x=\left(\frac{y}{x}\right)'_x=-\frac{y}{x^2}$$

$$z_y=\left(\frac{y}{x}\right)'_y=\frac{1}{x}$$

$$dz=z_xdx+z_ydy=-\frac{y}{x^2}dx+\frac{1}{x}dy$$

例 1 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2}dx + \frac{1}{x}dy$$

例 2 计算函数 $z = x^2y + y^2$ 的全微分

例 1 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)_x' = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)_y' = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2}dx + \frac{1}{x}dy$$

例 2 计算函数 $z = x^2y + y^2$ 的全微分

解 $z_x =$

$z_y =$

例 1 计算函数 $z=\frac{y}{x}$ 的全微分

解

$$z_x=\left(\frac{y}{x}\right)'_x=-\frac{y}{x^2}$$

$$z_y=\left(\frac{y}{x}\right)'_y=\frac{1}{x}$$

$$dz=z_x dx+z_y dy=-\frac{y}{x^2}dx+\frac{1}{x}dy$$

例 2 计算函数 $z=x^2y+y^2$ 的全微分

解 $z_x=$

$z_y=$

$dz=z_x dx+z_y dy=$

例 1 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2}dx + \frac{1}{x}dy$$

例 2 计算函数 $z = x^2y + y^2$ 的全微分

解

$$z_x = (x^2y + y^2)'_x =$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 1 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)_x' = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)_y' = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$

例 2 计算函数 $z = x^2 y + y^2$ 的全微分

解

$$z_x = (x^2 y + y^2)_x' = (x^2 y)_x' + (y^2)_x' =$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 1 计算函数 $z=\frac{y}{x}$ 的全微分

解

$$z_x=\left(\frac{y}{x}\right)_x'=-\frac{y}{x^2}$$

$$z_y=\left(\frac{y}{x}\right)_y'=\frac{1}{x}$$

$$dz=z_x dx+z_y dy=-\frac{y}{x^2}dx+\frac{1}{x}dy$$

例 2 计算函数 $z=x^2y+y^2$ 的全微分

解

$$z_x=(x^2y+y^2)_x'=(x^2y)_x'+(y^2)_x'=2xy$$

$$z_y=$$

$$dz=z_x dx+z_y dy=$$

例 1 计算函数 $z=\frac{y}{x}$ 的全微分

解

$$z_x=\left(\frac{y}{x}\right)'_x=-\frac{y}{x^2}$$

$$z_y=\left(\frac{y}{x}\right)'_y=\frac{1}{x}$$

$$dz=z_x dx+z_y dy=-\frac{y}{x^2}dx+\frac{1}{x}dy$$

例 2 计算函数 $z=x^2y+y^2$ 的全微分

解

$$z_x=(x^2y+y^2)'_x=(x^2y)'_x+(y^2)'_x=2xy$$

$$z_y=(x^2y+y^2)'_y=$$

$$dz=z_x dx+z_y dy=$$

例 1 计算函数 $z=\frac{y}{x}$ 的全微分

解

$$z_x=\left(\frac{y}{x}\right)'_x=-\frac{y}{x^2}$$

$$z_y=\left(\frac{y}{x}\right)'_y=\frac{1}{x}$$

$$dz=z_x dx+z_y dy=-\frac{y}{x^2}dx+\frac{1}{x}dy$$

例 2 计算函数 $z=x^2y+y^2$ 的全微分

解

$$z_x=(x^2y+y^2)'_x=(x^2y)'_x+(y^2)'_x=2xy$$

$$z_y=(x^2y+y^2)'_y=(x^2y)'_y+(y^2)'_y=$$

$$dz=z_x dx+z_y dy=$$

例 1 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)_x' = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)_y' = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$

例 2 计算函数 $z = x^2 y + y^2$ 的全微分

解

$$z_x = (x^2 y + y^2)_x' = (x^2 y)_x' + (y^2)_x' = 2xy$$

$$z_y = (x^2 y + y^2)_y' = (x^2 y)_y' + (y^2)_y' = x^2$$

$$dz = z_x dx + z_y dy =$$

例 1 计算函数 $z = \frac{y}{x}$ 的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2}dx + \frac{1}{x}dy$$

例 2 计算函数 $z = x^2y + y^2$ 的全微分

解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x = 2xy$$

$$z_y = (x^2y + y^2)'_y = (x^2y)'_y + (y^2)'_y = x^2 + 2y$$

$$dz = z_x dx + z_y dy =$$

例 1 计算函数 $z=\frac{y}{x}$ 的全微分

解

$$z_x=\left(\frac{y}{x}\right)'_x=-\frac{y}{x^2}$$

$$z_y=\left(\frac{y}{x}\right)'_y=\frac{1}{x}$$

$$dz=z_x dx+z_y dy=-\frac{y}{x^2}dx+\frac{1}{x}dy$$

例 2 计算函数 $z=x^2y+y^2$ 的全微分

解

$$z_x=(x^2y+y^2)'_x=(x^2y)'_x+(y^2)'_x=2xy$$

$$z_y=(x^2y+y^2)'_y=(x^2y)'_y+(y^2)'_y=x^2+2y$$

$$dz=z_x dx+z_y dy=2xydx+(x^2+2y)dy$$

例 3 求 $z=xy$ 在点 $(2,3)$ 处，关于 $\Delta x=0.1$，$\Delta y=0.2$ 的全增量 Δz 及全微分 dz。

解

例 3 求 $z = xy$ 在点 $(2, 3)$ 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解 $z_x =$ ，$z_y =$

例 3 求 $z = xy$ 在点 (2, 3) 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解 $z_x =$ ， $z_y =$

$$dz = z_x dx + z_y dy =$$

例 3 求 $z = xy$ 在点 $(2, 3)$ 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解

$$z_x = (xy)'_x = \quad , \qquad z_y =$$

$$dz = z_x dx + z_y dy =$$

例 3 求 $z = xy$ 在点 (2, 3) 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解

$$z_x = (xy)'_x = y\,, \qquad z_y =$$

$$dz = z_x dx + z_y dy =$$

例 3 求 $z = xy$ 在点 (2, 3) 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解 $$z_x = (xy)'_x = y\,, \qquad z_y = (xy)'_y =$$

$$dz = z_x dx + z_y dy =$$

例 3 求 $z = xy$ 在点 (2, 3) 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解
$$z_x = (xy)'_x = y\,, \qquad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy =$$

例 3 求 $z = xy$ 在点 $(2, 3)$ 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解

$$z_x = (xy)'_x = y\,, \qquad z_y = (xy)'_y = x$$

$$dz = z_x dx + z_y dy = y dx + x dy$$

例 3 求 $z = xy$ 在点 $(2, 3)$ 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解

$$z_x = (xy)'_x = y\,, \qquad z_y = (xy)'_y = x$$

$$dz = z_x dx + z_y dy = y dx + x dy$$

将 $(x, y) = (2, 3)$ 及 $\Delta x = 0.1$、$\Delta y = 0.2$ 代入得：

$$dz =$$

例 3 求 $z = xy$ 在点 (2, 3) 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解

$$z_x = (xy)'_x = y\,, \qquad z_y = (xy)'_y = x$$

$$dz = z_x dx + z_y dy = y dx + x dy$$

将 $(x, y) = (2, 3)$ 及 $\Delta x = 0.1$、$\Delta y = 0.2$ 代入得：

$$dz = 3 \times 0.1 +$$

例 3 求 $z = xy$ 在点 $(2, 3)$ 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解

$$z_x = (xy)'_x = y\,, \qquad z_y = (xy)'_y = x$$

$$dz = z_x dx + z_y dy = y dx + x dy$$

将 $(x, y) = (2, 3)$ 及 $\Delta x = 0.1$、$\Delta y = 0.2$ 代入得：

$$dz = 3 \times 0.1 + 2 \times 0.2 =$$

例 3 求 $z = xy$ 在点 $(2, 3)$ 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解
$$z_x = (xy)'_x = y\,, \qquad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = y dx + x dy$$

将 $(x, y) = (2, 3)$ 及 $\Delta x = 0.1$、$\Delta y = 0.2$ 代入得：

$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

例 3 求 $z = xy$ 在点 (2, 3) 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解

$$z_x = (xy)'_x = y\,, \qquad z_y = (xy)'_y = x$$

$$dz = z_x dx + z_y dy = y dx + x dy$$

将 $(x, y) = (2, 3)$ 及 $\Delta x = 0.1$、$\Delta y = 0.2$ 代入得：

$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

而全增量为 $\Delta z =$

例 3 求 $z = xy$ 在点 $(2, 3)$ 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解
$$z_x = (xy)'_x = y\,, \qquad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = y dx + x dy$$

将 $(x, y) = (2, 3)$ 及 $\Delta x = 0.1$、$\Delta y = 0.2$ 代入得：
$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

而全增量为
$$\Delta z = z(2 + 0.1, 3 + 0.2) - z(2, 3)$$

例 3 求 $z = xy$ 在点 (2, 3) 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解
$$z_x = (xy)'_x = y\,, \qquad z_y = (xy)'_y = x$$

$$dz = z_x dx + z_y dy = y dx + x dy$$

将 $(x, y) = (2, 3)$ 及 $\Delta x = 0.1$、$\Delta y = 0.2$ 代入得：

$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

而全增量为
$$\Delta z = z(2 + 0.1, 3 + 0.2) - z(2, 3)$$

$$= (2 + 0.1) \times (3 + 0.2) -$$

例 3 求 $z = xy$ 在点 $(2, 3)$ 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解
$$z_x = (xy)'_x = y\,, \qquad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = y dx + x dy$$

将 $(x, y) = (2, 3)$ 及 $\Delta x = 0.1$、$\Delta y = 0.2$ 代入得：
$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

而全增量为
$$\Delta z = z(2 + 0.1, 3 + 0.2) - z(2, 3)$$
$$= (2 + 0.1) \times (3 + 0.2) - 2 \times 3$$

例 3 求 $z = xy$ 在点 $(2, 3)$ 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解
$$z_x = (xy)'_x = y\,, \qquad z_y = (xy)'_y = x$$

$$dz = z_x dx + z_y dy = y dx + x dy$$

将 $(x, y) = (2, 3)$ 及 $\Delta x = 0.1$、$\Delta y = 0.2$ 代入得：

$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

而全增量为
$$\begin{aligned}\Delta z &= z(2 + 0.1, 3 + 0.2) - z(2, 3) \\ &= (2 + 0.1) \times (3 + 0.2) - 2 \times 3 \\ &= 0.72\end{aligned}$$

例 3 求 $z = xy$ 在点 $(2, 3)$ 处，关于 $\Delta x = 0.1$，$\Delta y = 0.2$ 的全增量 Δz 及全微分 dz。

解

$$z_x = (xy)'_x = y\,, \qquad z_y = (xy)'_y = x$$

$$dz = z_x dx + z_y dy = y dx + x dy$$

将 $(x, y) = (2, 3)$ 及 $\Delta x = 0.1$、$\Delta y = 0.2$ 代入得：

$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

而全增量为

$$\begin{aligned}\Delta z &= z(2 + 0.1, 3 + 0.2) - z(2, 3) \\ &= (2 + 0.1) \times (3 + 0.2) - 2 \times 3 \\ &= 0.72 \\ &\approx dz\end{aligned}$$