§1.4 克莱姆法则

数学系 梁卓滨

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Outline of §1.4

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_2 + a_{22}y_2 = b_2 \end{cases}$$



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$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} , \quad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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的解是

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$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$
 称为系数行列式

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• D_i : 将 D 的第 i 列换成常数项 $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$



三元线性方程组 $\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$ 的解是

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系数行列式

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$$\begin{vmatrix} a_{11} & \cdots & a_{1j-1} & a_{1,j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2,j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & & \vdots & & \vdots & & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{n,j} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}$$

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定理(克莱姆法则) 线性方程组

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注 两个前提: (1) 未知元个数 = 方程个数; (2) 系数行列式 $D \neq 0$



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$$D=0$$
情形,以后再说



克莱姆法则证明 (仅验证 $x_j = \frac{D_j}{D}$ 是解,唯一性的证明要用到矩阵知识,略去。)

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验证第 k 条方程成立 ($k = 1, 2, \dots, n$):

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$$x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} a_{11} & a_{1j-1} & a_{1j+1} & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & b_{2} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}}$$

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$$x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} a_{21} & a_{2j+1} & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}}$$

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D

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 $x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \\ a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}} = \frac{b_{1}A_{1j} + b_{2}A_{2j} + \cdots + b_{n}}{D}$



 $x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \\ a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}} = \frac{b_{1}A_{1j} + b_{2}A_{2j} + \cdots + b_{n}}{D}$



 $x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \\ a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}} = \frac{b_{1}A_{1j} + b_{2}A_{2j} + \cdots + b_{n}A_{nj}}{D}$



 $x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} a_{11} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \\ a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}} = \frac{b_{1}A_{1j} + b_{2}A_{2j} + \cdots + b_{n}A_{nj}}{D}$

验证第 k 条方程成立 $(k = 1, 2, \dots, n)$:

$$a_{k1}x_1 + \cdots + a_{kn}x_n =$$



克莱姆法则证明 (仅验证 $x_i = \frac{D_i}{D}$ 是解,唯一性的证明要用到矩阵知识,略去。)

$$x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} a_{11} & \cdots & a_{1j-1} & b_{1} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & b_{2} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}} = \frac{b_{1}A_{1j} + b_{2}A_{2j} + \cdots + b_{n}A_{nj}}{D}$$

$$= \frac{1}{D} \sum_{i=1}^{n} b_{i}A_{ij}$$

验证第
$$k$$
 条方程成立($k = 1, 2, \dots, n$):

 $a_{k1}x_1 + \cdots + a_{kn}x_n =$



克莱姆法则证明 (仅验证 $X_i = \frac{D_i}{D}$ 是解,唯一性的证明要用到矩阵知识,略去。) $\begin{vmatrix} a_{11} & \cdots & a_{1j-1} & b_1 & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & b_2 & a_{2j+1} & \cdots & a_{2n} \end{vmatrix}$

$$x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \\ a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}} = \frac{b_{1}A_{1j} + b_{2}A_{2j} + \cdots + b_{n}A_{nj}}{D}$$

$$= \frac{1}{D} \sum_{i=1}^{n} b_{i}A_{ij}$$

验证第 k 条方程成立 $(k = 1, 2, \dots, n)$:

 $a_{k1}x_1 + \dots + a_{kn}x_n = \sum_{i=1}^n a_{kj}x_j$



克莱姆法则证明 (仅验证 $X_i = \frac{D_i}{D}$ 是解,唯一性的证明要用到矩阵知识,略去。) $a_{11} \cdots a_{1j-1} \xrightarrow{b_1} a_{1j+1} \cdots a_{1n}$ $a_{21} \cdots a_{2j-1} \xrightarrow{b_2} a_{2j+1} \cdots a_{2n}$

$$x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} a_{21} & a_{2j-1} & a_{2} & a_{2j+1} & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}} = \frac{b_{1}A_{1j} + b_{2}A_{2j} + \cdots + b_{n}A_{nj}}{D}$$

$$=\frac{1}{D}\sum_{i=1}^{n}b_{i}A_{ij}$$

验证第 k 条方程成立 $(k = 1, 2, \dots, n)$:

$$a_{k1}x_1 + \dots + a_{kn}x_n = \sum_{j=1}^n a_{kj}x_j = \sum_{j=1}^n a_{kj} \left(\frac{1}{D}\sum_{i=1}^n b_i A_{ij}\right)$$



克莱姆法则证明 (仅验证 $x_j = \frac{D_j}{D}$ 是解,唯一性的证明要用到矩阵知识,略去。) $a_{11} \cdots a_{1j-1} \xrightarrow{b_1} a_{1j+1} \cdots a_{1n}$ $a_{21} \cdots a_{2j-1} \xrightarrow{b_2} a_{2j+1} \cdots a_{2n}$

$$x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} \sum_{i=1}^{n} & \sum_{j=1}^{n} & \sum_$$

$$=\frac{1}{D}\sum_{i=1}^{n}b_{i}A_{ij}$$

验证第 k 条方程成立 $(k = 1, 2, \dots, n)$:

 $a_{k1}x_1 + \dots + a_{kn}x_n = \sum_{i=1}^n a_{kj}x_j = \sum_{i=1}^n a_{kj} \left(\frac{1}{D}\sum_{i=1}^n b_i A_{ij}\right) = \frac{1}{D}\sum_{i=1}^n \sum_{i=1}^n a_{kj}b_i A_{ij}$

 b_k

 $a_{11} \cdots a_{1j-1} \xrightarrow{b_1} a_{1j+1} \cdots a_{1n}$ $a_{21} \cdots a_{2j-1} \xrightarrow{b_2} a_{2j+1} \cdots a_{2n}$

克莱姆法则证明 (仅验证 $X_i = \frac{D_i}{D}$ 是解,唯一性的证明要用到矩阵知识,略去。)

$$x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \\ a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}} = \frac{b_{1}A_{1j} + b_{2}A_{2j} + \cdots + b_{n}A_{nj}}{D}$$

 $=\frac{1}{D}\sum_{i}^{n}b_{i}A_{ij}$ 验证第 k 条方程成立 $(k = 1, 2, \dots, n)$:

 $a_{k1}x_1 + \dots + a_{kn}x_n = \sum_{i=1}^n a_{kj}x_j = \sum_{i=1}^n a_{kj} \left(\frac{1}{D}\sum_{i=1}^n b_i A_{ij}\right) = \frac{1}{D}\sum_{i=1}^n \sum_{i=1}^n a_{kj}b_i A_{ij}$

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 $=\frac{1}{D}\sum_{i=1}\sum_{i=1}a_{kj}b_iA_{ij}$

 $a_{11} \cdots a_{1j-1} \xrightarrow{b_1} a_{1j+1} \cdots a_{1n}$ $a_{21} \cdots a_{2j-1} \xrightarrow{b_2} a_{2j+1} \cdots a_{2n}$ $x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \\ a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}} = \frac{b_{1}A_{1j} + b_{2}A_{2j} + \cdots + b_{n}A_{nj}}{D}$

克莱姆法则证明 (仅验证 $X_i = \frac{D_i}{D}$ 是解,唯一性的证明要用到矩阵知识,略去。)

 $=\frac{1}{D}\sum_{i}^{n}b_{i}A_{ij}$ 验证第 k 条方程成立 $(k = 1, 2, \dots, n)$:

 $a_{k1}x_1 + \dots + a_{kn}x_n = \sum_{i=1}^n a_{kj}x_j = \sum_{i=1}^n a_{kj} \left(\frac{1}{D}\sum_{i=1}^n b_i A_{ij}\right) = \frac{1}{D}\sum_{i=1}^n \sum_{i=1}^n a_{kj}b_i A_{ij}$

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 $a_{11} \cdots a_{1j-1} \xrightarrow{b_1} a_{1j+1} \cdots a_{1n}$ $a_{21} \cdots a_{2j-1} \xrightarrow{b_2} a_{2j+1} \cdots a_{2n}$ $x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \\ a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}} = \frac{b_{1}A_{1j} + b_{2}A_{2j} + \cdots + b_{n}A_{nj}}{D}$

克莱姆法则证明 (仅验证 $X_i = \frac{D_i}{D}$ 是解,唯一性的证明要用到矩阵知识,略去。)

$$\begin{vmatrix} a_{n1} \cdots a_{nj-1} & a_{nj} & a_{nj+1} \cdots a_{nn} \end{vmatrix}$$

$$= \frac{1}{D} \sum_{i=1}^{n} b_i A_{ij}$$

验证第 k 条方程成立 ($k = 1, 2, \dots, n$):

 $a_{k1}x_1 + \dots + a_{kn}x_n = \sum_{i=1}^n a_{kj}x_j = \sum_{i=1}^n a_{kj} \left(\frac{1}{D}\sum_{i=1}^n b_i A_{ij}\right) = \frac{1}{D}\sum_{i=1}^n \sum_{i=1}^n a_{kj}b_i A_{ij}$

 $\sum_{i=1}^{n} a_{kj} b_i A_{ij} = b_i \sum_{i=1}^{n} a_{kj} A_{ij}$

 $a_{11} \cdots a_{1j-1} \xrightarrow{b_1} a_{1j+1} \cdots a_{1n}$ $a_{21} \cdots a_{2j-1} \xrightarrow{b_2} a_{2j+1} \cdots a_{2n}$

克莱姆法则证明 (仅验证 $x_j = \frac{D_j}{D}$ 是解,唯一性的证明要用到矩阵知识,略去。)

$$x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \\ a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}} = \frac{b_{1}A_{1j} + b_{2}A_{2j} + \cdots + b_{n}A_{nj}}{D}$$

$$= \frac{1}{D} \sum_{i=1}^{n} b_{i}A_{ij}$$

验证第 k 条方程成立 ($k = 1, 2, \dots, n$):

 $a_{k1}x_1 + \dots + a_{kn}x_n = \sum_{i=1}^n a_{kj}x_j = \sum_{i=1}^n a_{kj} \left(\frac{1}{D}\sum_{i=1}^n b_i A_{ij}\right) = \frac{1}{D}\sum_{i=1}^n \sum_{i=1}^n a_{kj}b_i A_{ij}$

 $= \frac{1}{D} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{kj} b_i A_{ij} = b_i \sum_{j=1}^{n} a_{kj} A_{ij}$

 $a_{11} \cdots a_{1j-1} \xrightarrow{b_1} a_{1j+1} \cdots a_{1n}$ $a_{21} \cdots a_{2j-1} \xrightarrow{b_2} a_{2j+1} \cdots a_{2n}$ $x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \\ a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}} = \frac{b_{1}A_{1j} + b_{2}A_{2j} + \cdots + b_{n}A_{nj}}{D}$

克莱姆法则证明 (仅验证 $X_i = \frac{D_i}{D}$ 是解,唯一性的证明要用到矩阵知识,略去。)

$$\begin{vmatrix} a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}$$
 $= \frac{1}{D} \sum_{i=1}^{n} b_i A_{ij}$
验证第 k 条方程成立($k = 1, 2, \cdots, n$):

 $a_{k1}x_1 + \dots + a_{kn}x_n = \sum_{i=1}^n a_{kj}x_j = \sum_{i=1}^n a_{kj} \left(\frac{1}{D}\sum_{i=1}^n b_i A_{ij}\right) = \frac{1}{D}\sum_{i=1}^n \sum_{i=1}^n a_{kj}b_i A_{ij}$

 $= \frac{1}{D} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{kj} b_i A_{ij} = \frac{1}{D} \sum_{i=1}^{n} b_i \sum_{j=1}^{n} a_{kj} A_{ij}$

 $x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} a_{11} & \cdots & a_{1j-1} & b_{1} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & b_{2} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}} = \frac{b_{1}A_{1j} + b_{2}A_{2j} + \cdots + b_{n}A_{nj}}{D}$

克莱姆法则证明 (仅验证 $x_j = \frac{D_j}{D}$ 是解,唯一性的证明要用到矩阵知识,略去。)

$$=\frac{1}{D}\sum_{i=1}^{n}b_{i}A_{ij}$$

验证第 k 条方程成立($k=1,2,\cdots,n$):

 $a_{k1}x_1 + \dots + a_{kn}x_n = \sum_{j=1}^n a_{kj}x_j = \sum_{j=1}^n a_{kj} \left(\frac{1}{D}\sum_{i=1}^n b_i A_{ij}\right) = \frac{1}{D}\sum_{j=1}^n \sum_{i=1}^n a_{kj}b_i A_{ij}$ $= \frac{1}{D}\sum_{j=1}^n \sum_{i=1}^n a_{kj}b_i A_{ij} = \frac{1}{D}\sum_{j=1}^n \sum_{i=1}^n a_{kj}b_i A_{ij}$

 $= \frac{1}{D} \sum_{i=1}^{n} \sum_{j=1}^{n} a_{kj} b_i A_{ij} = \frac{1}{D} \sum_{i=1}^{n} b_i \sum_{j=1}^{n} a_{kj} A_{ij} \qquad b_k \sum_{j=1}^{n} a_{kj} A_{kj}$

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 $\begin{vmatrix} a_{11} & \cdots & a_{1j-1} & b_1 & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & b_2 & a_{2j+1} & \cdots & a_{2n} \end{vmatrix}$ $x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \\ a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}} = \frac{b_{1}A_{1j} + b_{2}A_{2j} + \cdots + b_{n}A_{nj}}{D}$

克莱姆法则证明 (仅验证 $x_j = \frac{D_j}{D}$ 是解,唯一性的证明要用到矩阵知识,略去。)

$$=\frac{1}{D}\sum_{i=1}^{n}b_{i}A_{ij}$$
验证第 k 条方程成立($k=1,2,\cdots,n$):

验证第 k 条方程成立 ($k = 1, 2, \dots, n$):

 $a_{k1}x_1 + \dots + a_{kn}x_n = \sum_{i=1}^n a_{kj}x_j = \sum_{i=1}^n a_{kj} \left(\frac{1}{D}\sum_{i=1}^n b_i A_{ij}\right) = \frac{1}{D}\sum_{i=1}^n \sum_{i=1}^n a_{kj}b_i A_{ij}$

 $= \frac{1}{D} \sum_{i=1}^{n} \sum_{k=1}^{n} a_{kj} b_i A_{ij} = \frac{1}{D} \sum_{i=1}^{n} b_i \sum_{k=1}^{n} a_{kj} A_{ij} = \frac{1}{D} \cdot b_k \sum_{k=1}^{n} a_{kj} A_{kj}$

克莱姆法则证明 (仅验证 $x_j = \frac{D_j}{D}$ 是解,唯一性的证明要用到矩阵知识,略去。) $a_{11} \cdots a_{1j-1} \xrightarrow{b_1} a_{1j+1} \cdots a_{1n}$ $a_{21} \cdots a_{2j-1} \xrightarrow{b_2} a_{2j+1} \cdots a_{2n}$ $x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \\ a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}} = \frac{b_{1}A_{1j} + b_{2}A_{2j} + \cdots + b_{n}A_{nj}}{D}$ $=\frac{1}{D}\sum_{i}^{n}b_{i}A_{ij}$ 验证第 k 条方程成立 ($k = 1, 2, \dots, n$):

$$=rac{1}{D}\sum_{i=1}^{n}b_{i}A_{ij}$$

验证第 k 条方程成立($k=1,2,\cdots,n$):

 $a_{k1}x_1 + \dots + a_{kn}x_n = \sum_{i=1}^n a_{kj}x_j = \sum_{i=1}^n a_{kj} \left(\frac{1}{D}\sum_{i=1}^n b_i A_{ij}\right) = \frac{1}{D}\sum_{i=1}^n \sum_{i=1}^n a_{kj}b_i A_{ij}$

 $=\frac{1}{D}\sum_{i=1}^{n}\sum_{i=1}^{n}a_{kj}b_{i}A_{ij}=\frac{1}{D}\sum_{i=1}^{n}b_{i}\sum_{i=1}^{n}a_{kj}A_{ij}=\frac{1}{D}\cdot b_{k}\sum_{i=1}^{n}a_{kj}A_{kj}=\frac{1}{D}\cdot b_{k}D_{0}b_{k}$

克莱姆法则证明 (仅验证 $x_j = \frac{D_j}{D}$ 是解,唯一性的证明要用到矩阵知识,略去。) $a_{11} \cdots a_{1j-1} \xrightarrow{b_1} a_{1j+1} \cdots a_{1n}$ $a_{21} \cdots a_{2j-1} \xrightarrow{b_2} a_{2j+1} \cdots a_{2n}$ $x_{j} = \frac{D_{j}}{D} = \frac{\begin{vmatrix} \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & b_{n} & a_{nj+1} & \cdots & a_{nn} \\ a_{11} & \cdots & a_{1j-1} & a_{1j} & a_{1j+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2j-1} & a_{2j} & a_{2j+1} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nj-1} & a_{nj} & a_{nj+1} & \cdots & a_{nn} \end{vmatrix}} = \frac{b_{1}A_{1j} + b_{2}A_{2j} + \cdots + b_{n}A_{nj}}{D}$ $=\frac{1}{D}\sum_{i}^{n}b_{i}A_{ij}$ 验证第 k 条方程成立 ($k = 1, 2, \dots, n$):

验证第 k 条方程成立($k = 1, 2, \dots, n$): $a_{k1}x_1 + \dots + a_{kn}x_n = \sum_{j=1}^n a_{kj}x_j = \sum_{j=1}^n a_{kj} \left(\frac{1}{D}\sum_{i=1}^n b_i A_{ij}\right) = \frac{1}{D}\sum_{j=1}^n \sum_{i=1}^n a_{kj}b_i A_{ij}$ $= \frac{1}{D}\sum_{i=1}^n \sum_{j=1}^n a_{kj}b_i A_{ij} = \frac{1}{D}\sum_{i=1}^n b_i \sum_{j=1}^n a_{kj}A_{ij} = \frac{1}{D} \cdot b_k \sum_{i=1}^n a_{kj}A_{kj} = \frac{1}{D} \cdot b_k D = b_k$

1.4 克

例 解线性方程组

$$\begin{cases} 2x_1 + x_2 - x_3 = 1\\ 3x_1 - x_2 - x_3 = -2\\ -x_1 + 2x_2 + x_3 = 6 \end{cases}$$

练习 解线性方程组

$$\begin{cases} x_1 + x_2 = 90 \\ x_2 + x_3 = 86 \\ x_1 + x_3 = 80 \end{cases}$$

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$$\begin{cases} 2x_1 + x_2 - x_3 = 1\\ 3x_1 - x_2 - x_3 = -2\\ -x_1 + 2x_2 + x_3 = 6 \end{cases}$$

提示
$$D = -5$$
, $D_1 = -5$, $D_2 = -10$, $D_3 = -15$

练习 解线性方程组

$$\begin{cases} x_1 + x_2 = 90 \\ x_2 + x_3 = 86 \\ x_1 + x_3 = 80 \end{cases}$$

例 解线性方程组

$$\begin{cases} 2x_1 + x_2 - x_3 = 1\\ 3x_1 - x_2 - x_3 = -2\\ -x_1 + 2x_2 + x_3 = 6 \end{cases}$$

提示 D = -5, $D_1 = -5$, $D_2 = -10$, $D_3 = -15$

练习 解线性方程组

$$\begin{cases} x_1 + x_2 = 90 \\ x_2 + x_3 = 86 \\ x_1 + x_3 = 80 \end{cases}$$

提示 D = 2, $D_1 = 84$, $D_2 = 96$, $D_3 = 76$



定理 齐次线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = 0 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = 0 \end{cases}$$

当系数行列式 $D \neq 0$ 时,仅有零解($x_1 = x_2 = \cdots = x_n = 0$)

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另一方面,因为 $D \neq 0$,所以方程组有唯一解(克莱姆法则)



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所以方程组除 $x_1 = x_2 = \cdots = x_n = 0$ 外,没有其他解



§1.4 克莱姆法则

定理 齐次线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = 0 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = 0 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = 0 \end{cases}$$

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注

其实是充分必要条件:仅有零解的充分必要条件是 D ≠ 0



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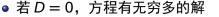
证明
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例 齐次方程组
$$\begin{cases} x_1 - 2x_2 = 0 \\ 2x_1 - 4x_2 = 0 \end{cases}$$
 的系数矩阵 $D = \begin{vmatrix} 1 & -2 \\ 2 & -4 \end{vmatrix}$



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例 判断线性方程组
$$\begin{cases} 2x_1 + 3x_2 + 4x_3 + 5x_4 = 0\\ 3x_1 + 4x_2 + 5x_3 + 5x_4 = 0\\ 4x_1 + 5x_2 + 6x_3 + 6x_4 = 0\\ 5x_1 + 6x_2 + 8x_3 + 9x_4 = 0 \end{cases}$$

是否只有零解



2 3 4 5 3 4 5 5 4 5 6 6 5 6 8 9

 $\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix} \xrightarrow{r_4 - r_3}$

2	3	4	5					
3	4	5	5	$r_4 - r_3$	İ			
4	5	6	6		l			
5	6	8	9	r_4-r_3	1	1	2	3

4	5		2	3	4	5
5	5	r_4-r_3	3	4	5	5
6	6		4	5	6	6
8	9		1	1	2	3
	4 5 6 8	4 5 5 5 6 6 8 9	4 5 5 5 6 6 6 8 9	$ \begin{array}{c cccc} 4 & 5 & 2 & 2 \\ 5 & 5 & 7 & 74 - 73 & 3 \\ 6 & 6 & 6 & 1 & 1 \end{array} $	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{vmatrix} 4 & 5 \\ 5 & 5 \\ 6 & 6 \\ 8 & 9 \end{vmatrix} \xrightarrow{r_4 - r_3} \begin{vmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \\ 1 & 1 & 2 \end{vmatrix} $

2	3	4	5		2	3	4	5	
3	4	5	5	r_4-r_3	3	4	5	5	r_3-r_2
4	5	6	6		4	5	6	6	
5	6	8	9	<u>r4-r3</u>	1	1	2	3	

$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix} \xrightarrow{\underline{r_4 - r_3}} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xrightarrow{\underline{r_3 - r_2}} \begin{vmatrix} 1 & 1 & 1 & 1 \end{vmatrix}$



2	3	4	5		2	3	4	5		2	3	4	5
3	4	5	5	r_4-r_3	3	4	5	5	r_3-r_2	3	4	5	5
4	5	6	6		4	5	6	6		1	1	1	1
5	6	8	9		1	1	2	3	<u>r₃-r₂</u>	1	1	2	3



2	3	4	5		2	3	4	5		2	3	4	5
3	4	5	5	r_4-r_3	3	4	5	5	r_3-r_2	3	4	5	5
4	5	6	6		4	5	6	6		1	1	1	1
5	6	8	9	<u>r₄-r₃</u>	1	1	2	3		1	1	2	3

$$r_2-r_1$$



$$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix} \xrightarrow{r_4 - r_3} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xrightarrow{r_3 - r_2} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$

$$\frac{r_2-r_1}{} \begin{vmatrix} 1 & 1 & 1 & 0 \end{vmatrix}$$



$$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix} \xrightarrow{\underline{r_4 - r_3}} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xrightarrow{\underline{r_3 - r_2}} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$

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$$\frac{r_2 - r_1}{-} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xrightarrow{r_1 - 2r_2} \begin{vmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xrightarrow{r_3 - r_2} \begin{vmatrix} 0 & 0 & 0 & 1 \end{vmatrix}$$



$$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix} \xrightarrow{\underline{r_4 - r_3}} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xrightarrow{\underline{r_3 - r_2}} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$



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 $r_4 - 2r_2$



$$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix} \xrightarrow{\underline{r_4 - r_3}} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xrightarrow{\underline{r_3 - r_2}} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$

$$\frac{r_2 - r_1}{2} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix} = \frac{r_1 - 2r_2}{2} \begin{vmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix} = \frac{r_3 - r_2}{2} \begin{vmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$

$$\underline{r_4 - 2r_2} \begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 3 \end{vmatrix} = \frac{r_3 - r_2}{2} \begin{vmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$



$$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix} \xrightarrow{\underline{r_4 - r_3}} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xrightarrow{\underline{r_3 - r_2}} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$

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$$\underline{r_4 - 2r_2} \begin{vmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{vmatrix}$$



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$$\frac{r_2 - r_1}{\begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}} = \frac{r_1 - 2r_2}{\begin{vmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}} = \frac{r_3 - r_2}{\begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}} = \frac{r_3 - r_2}{\begin{vmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}}$$

$$\frac{r_4 - 2r_2}{\begin{array}{c|cccc} \hline \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ \end{array}} = - \begin{vmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$



$$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix} \xrightarrow{\underline{r_4 - r_3}} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xrightarrow{\underline{r_3 - r_2}} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$

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$$\frac{r_4 - 2r_2}{\begin{vmatrix}
0 & 1 & 2 & 5 \\
1 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 3
\end{vmatrix} = - \begin{vmatrix}
0 & 1 & 2 \\
1 & 1 & 1 \\
0 & 0 & 1
\end{vmatrix} = - \begin{vmatrix}
0 & 1 \\
1 & 1
\end{vmatrix}$$



$$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix} \xrightarrow{\underline{r_4 - r_3}} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xrightarrow{\underline{r_3 - r_2}} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$

$$\frac{r_2 - r_1}{2} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xrightarrow{r_1 - 2r_2} \begin{vmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xrightarrow{r_3 - r_2} \begin{vmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$

$$\frac{r_4 - 2r_2}{ \begin{vmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{vmatrix} = - \begin{vmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = - \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = 1 \neq 0$$



$$\begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 5 & 6 & 8 & 9 \end{vmatrix} \xrightarrow{\underline{r_4 - r_3}} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 4 & 5 & 6 & 6 \\ 1 & 1 & 2 & 3 \end{vmatrix} \xrightarrow{\underline{r_3 - r_2}} \begin{vmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$

$$= \frac{\underline{r_2 - r_1}}{\begin{vmatrix} 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}} \xrightarrow{\underline{r_1 - 2r_2}} \begin{vmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$

$$\frac{r_4 - 2r_2}{ \begin{vmatrix} 0 & 1 & 2 & 5 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{vmatrix} = - \begin{vmatrix} 0 & 1 & 2 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = - \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = 1 \neq 0$$

所以齐次线性方程组有唯一解



练习 齐次线性方程组 $\begin{cases} kx_1 & + x_4 = 0 \\ x_1 + 2x_2 & - x_4 = 0 \\ (k+2)x_1 - x_2 & + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 有非零解

的充分必要条件是 k 满足 ____



练习 齐次线性方程组
$$\begin{cases} kx_1 + x_4 = 0 \\ x_1 + 2x_2 - x_4 = 0 \\ (k+2)x_1 - x_2 + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$$

的充分必要条件是 k 满足

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix}$$



有非零解

练习 齐次线性方程组
$$\begin{cases} kx_1 & + x_4 = 0 \\ x_1 + 2x_2 & - x_4 = 0 \\ (k+2)x_1 - x_2 & + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$$

的充分必要条件是 k 满足 ____

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3.$$



有非零解

练习 齐次线性方程组
$$\begin{cases} kx_1 + x_4 = 0 \\ x_1 + 2x_2 - x_4 = 0 \\ (k+2)x_1 - x_2 + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$$
 有非零解

的充分必要条件是 k 满足 ____

解

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot (-1)^{3+4} \begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$

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练习 齐次线性方程组
$$\begin{cases} kx_1 & + x_4 = 0 \\ x_1 + 2x_2 & - x_4 = 0 \\ (k+2)x_1 - x_2 & + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$$

的充分必要条件是 k 满足 $_{__}$

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot (-1)^{3+4} \begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$

$$r_2 + r_1$$



练习 齐次线性方程组 $\begin{cases} kx_1 & + x_4 = 0 \\ x_1 + 2x_2 & - x_4 = 0 \\ (k+2)x_1 - x_2 & + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 有非零解

的充分必要条件是 k 满足

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot (-1)^{3+4} \begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$

$$\frac{r_2 + r_1}{k} (-3) \cdot \begin{vmatrix} k & 0 & 1 \\ k & 0 & 1 \end{vmatrix}$$



练习 齐次线性方程组
$$\begin{cases} kx_1 + x_4 = 0 \\ x_1 + 2x_2 - x_4 = 0 \\ (k+2)x_1 - x_2 + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$$
 有非零解

的充分必要条件是 k 满足 ____

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot (-1)^{3+4} \begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$

$$\frac{r_2 + r_1}{k} (-3) \cdot \begin{vmatrix} k & 0 & 1 \\ k+1 & 2 & 0 \end{vmatrix}$$



练习 齐次线性方程组
$$\begin{cases} kx_1 & + x_4 = 0 \\ x_1 + 2x_2 & - x_4 = 0 \\ (k+2)x_1 - x_2 & + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$$
 的充分必要条件 $= k$ 进足

的充分必要条件是 k 满足

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot (-1)^{3+4} \begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$

$$\frac{r_2 + r_1}{r_3 - 4r_1} (-3) \cdot \begin{vmatrix} k & 0 & 1 \\ k + 1 & 2 & 0 \end{vmatrix}$$

练习 齐次线性方程组
$$\begin{cases} kx_1 & + x_4 = 0 \\ x_1 + 2x_2 & - x_4 = 0 \\ (k+2)x_1 - x_2 & + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$$
的充分必要条件是 k 满足

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot (-1)^{3+4} \begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$
$$\frac{r_2 + r_1}{r_3 - 4r_1} (-3) \cdot \begin{vmatrix} k & 0 & 1 \\ k+1 & 2 & 0 \\ -3k+2 & -1 & 0 \end{vmatrix}$$



练习 齐次线性方程组 $\begin{cases} kx_1 & + x_4 = 0 \\ x_1 + 2x_2 & - x_4 = 0 \\ (k+2)x_1 - x_2 & + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 有非零解

的充分必要条件是 k 满足 $_{---}$

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot (-1)^{3+4} \begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$

$$\frac{r_2 + r_1}{r_3 - 4r_1} (-3) \cdot \begin{vmatrix} k & 0 & 1 \\ k + 1 & 2 & 0 \\ -3k + 2 & -1 & 0 \end{vmatrix} = (-3) \cdot (-1)^{1+3} \cdot \begin{vmatrix} k + 1 & 2 \\ -3k + 2 & -1 \end{vmatrix}$$



练习 齐次线性方程组 $\begin{cases} kx_1 & + x_4 = 0 \\ x_1 + 2x_2 & - x_4 = 0 \\ (k+2)x_1 - x_2 & + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 的充分必要条件是 k 满足

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot (-1)^{3+4} \begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$
$$\frac{r_2 + r_1}{r_3 - 4r_1} (-3) \cdot \begin{vmatrix} k & 0 & 1 \\ k+1 & 2 & 0 \\ -3k+2 & -1 & 0 \end{vmatrix} = (-3) \cdot (-1)^{1+3} \cdot \begin{vmatrix} k+1 & 2 \\ -3k+2 & -1 \end{vmatrix}$$
$$= -3(5k-5)$$



练习 齐次线性方程组 $\begin{cases} kx_1 & + x_4 = 0 \\ x_1 + 2x_2 & - x_4 = 0 \\ (k+2)x_1 - x_2 & + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 有非零解 的充分必要条件是 k 满足

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot (-1)^{3+4} \begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$
$$\frac{r_2 + r_1}{r_3 - 4r_1} (-3) \cdot \begin{vmatrix} k & 0 & 1 \\ k+1 & 2 & 0 \\ -3k+2 & -1 & 0 \end{vmatrix} = (-3) \cdot (-1)^{1+3} \cdot \begin{vmatrix} k+1 & 2 \\ -3k+2 & -1 \end{vmatrix}$$
$$= -3(5k-5)$$

有非零解当且仅当 D=0.

练习 齐次线性方程组 $\begin{cases} kx_1 & + x_4 = 0 \\ x_1 + 2x_2 & - x_4 = 0 \\ (k+2)x_1 - x_2 & + 4x_4 = 0 \\ 2x_1 + x_2 + 3x_3 + kx_4 = 0 \end{cases}$ 有非零解 的充分必要条件是 k 满足

$$D = \begin{vmatrix} k & 0 & 0 & 1 \\ 1 & 2 & 0 & -1 \\ k+2 & -1 & 0 & 4 \\ 2 & 1 & 3 & k \end{vmatrix} = 3 \cdot (-1)^{3+4} \begin{vmatrix} k & 0 & 1 \\ 1 & 2 & -1 \\ k+2 & -1 & 4 \end{vmatrix}$$
$$\frac{r_2 + r_1}{r_3 - 4r_1} (-3) \cdot \begin{vmatrix} k & 0 & 1 \\ k+1 & 2 & 0 \\ -3k+2 & -1 & 0 \end{vmatrix} = (-3) \cdot (-1)^{1+3} \cdot \begin{vmatrix} k+1 & 2 \\ -3k+2 & -1 \end{vmatrix}$$

有非零解当且仅当 D=0,当且仅当 k=1

=-3(5k-5)