§5.3 换元积分法

数学系　　梁卓滨

2017-2018 学年 II

教学要求

$\diamondsuit$ 熟练掌握换元积分法："凑微分"，"变量代换"

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1. 第一类换元积分法：凑微分

第二类换元积分法：变量代换

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第二类换元积分法：变量代换

第一类换元积分法——"凑微分"法，能干啥？

能够计算如下的不定积分：

$$\int \frac{dx}{2x+1}, \quad \int \cos(\frac{5}{2}x)dx$$

$$\int \frac{x}{\sqrt{3-x^2}}dx, \quad \int x\sin(x^2)dx$$

$$\int \frac{(\ln x)^2}{x}dx, \quad \int e^{\sin x}\cos x dx$$

$$\int \frac{1}{\cos x}dx$$

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第一类换元积分法（凑微分）原理

- 计算步骤：

$$\int \text{''ì½»§Æ¢È''}\, dx$$

第一类换元积分法（凑微分）原理

- 计算步骤：

$$\int {}''ì½»§Æ¢È'' \, dx === \int f(\varphi(x))\varphi'(x)dx$$

第一类换元积分法（凑微分）原理

- 计算步骤：

$$\int \text{"ì½»§Æ¢È"} \, dx = \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

第一类换元积分法（凑微分）原理

- 计算步骤：

$$\int ''ì½»§Æ¢È'' \, dx \equiv\!\!\equiv \int f(\varphi(x))\varphi'(x)dx = \int f(\textcolor{red}{\varphi(x)})d\textcolor{red}{\varphi(x)}$$

$$\xlongequal{\varphi(x)=u} \int f(u)du$$

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- 计算步骤：

$$\int \text{"ì½»§Æ¢È"} \, dx \overline{\quad\quad} \int f(\varphi(x))\varphi'(x)dx = \int f({\color{red}\varphi(x)})d{\color{red}\varphi(x)}$$

$$\overline{\underset{\varphi(x)=u}{\quad\quad}} \int f(u)du$$

$$= F(u) + C$$

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$$\underset{\varphi(x)=u}{\overline{\overline{\qquad}}} \int f(u) \, du$$

$$= F(u) + C \overset{u=\varphi(x)}{\overline{\qquad}} F(\varphi(x)) + C$$

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$$\xrightarrow{\varphi(x)=u} \int f(u)du$$

$$= F(u) + C \xrightarrow{u=\varphi(x)} F(\varphi(x)) + C$$

- 验证：$F(\varphi(x))$ 确是 $''\grave{\text{ì}}\frac{1}{2}»§Æ¢\grave{\text{E}}''$ 的原函数！

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$$\frac{d}{dx}F(\varphi(x)) =$$

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$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \cdot \varphi'(x) = ”ì½»§Æ¢È”$$

> 总之 $\quad \int ”ì½»§Æ¢È” \, dx \xrightarrow{\text{凑微分}} \int f(\varphi(x))d\varphi(x)$
>
> $$= \int f(u)du = F(u) + C = F(\varphi(x)) + C$$

例子 求不定积分

$$\int \frac{1}{1+2x}dx, \qquad \int \frac{1}{2-3x}dx, \qquad \int \sqrt{3x-1}dx,$$

$$\int \frac{1}{\sqrt{1-5x}}dx, \qquad \int \cos(\frac{3}{2}x)dx, \qquad \int e^{-\frac{1}{2}x+4}dx.$$

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热身

$$dx = \underline{\qquad} d(2x+1), \qquad dx = \underline{\qquad} d(2-3x),$$

$$dx = \underline{\qquad} d(\frac{3}{2}x), \qquad dx = \underline{\qquad} d(-\frac{1}{2}x+4).$$

例子 求不定积分

$$\int \frac{1}{1+2x}dx, \qquad\qquad \int \frac{1}{2-3x}dx, \qquad\qquad \int \sqrt{3x-1}dx,$$

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$$dx = \underline{\quad\frac{1}{2}\quad}\, d(2x+1), \qquad\qquad dx = \underline{\qquad\qquad}\, d(2-3x),$$

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热身

$$dx = \underline{\ \frac{1}{2}\ }\ d(2x+1), \qquad dx = \underline{\ -\frac{1}{3}\ }\ d(2-3x),$$

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$$dx = \underline{\quad \frac{1}{2} \quad} d(2x+1), \qquad dx = \underline{\quad -\frac{1}{3} \quad} d(2-3x),$$

$$dx = \underline{\quad \frac{2}{3} \quad} d(\frac{3}{2}x), \qquad dx = \underline{\qquad\qquad} d(-\frac{1}{2}x+4).$$

例子 求不定积分

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热身

$$dx = \underline{\quad \frac{1}{2} \quad} d(2x+1), \qquad dx = \underline{\quad -\frac{1}{3} \quad} d(2-3x),$$

$$dx = \underline{\quad \frac{2}{3} \quad} d(\frac{3}{2}x), \qquad dx = \underline{\quad -2 \quad} d(-\frac{1}{2}x+4).$$

例子 求不定积分 $\int \frac{1}{1+2x}dx$, $\int \frac{1}{2-3x}dx$, $\int \sqrt{3x-1}dx$

凑微分 "$\int \text{»}§Æ¢\grave{E}\, dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

例子 求不定积分 $\int \frac{1}{1+2x}dx$, $\int \frac{1}{2-3x}dx$, $\int \sqrt{3x-1}dx$

解 $\displaystyle\int \frac{1}{1+2x}dx =$

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解 $\displaystyle\int \frac{1}{1+2x} dx = \int \frac{1}{1+2x}$

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解 $\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x} \quad d(1+2x)$

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解 $\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2}d(1+2x)$

例子 求不定积分 $\int \frac{1}{1+2x}dx$, $\int \frac{1}{2-3x}dx$, $\int \sqrt{3x-1}dx$

解 $\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2}d(1+2x) = \frac{1}{2}\int \frac{1}{u}du$

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$\qquad\qquad = \frac{1}{2}\ln|u| + C$

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$$\int \frac{1}{2-3x} dx =$$

$$\int \sqrt{3x-1} dx =$$

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$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \qquad d(2-3x)$$

$$\int \sqrt{3x-1} dx =$$

凑微分 "$\int "ì½»§Æ¢È" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例I

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$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3})d(2-3x)$$

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$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$

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$$= -\frac{1}{3}\ln|u| + C$$

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$\int \sqrt{3x-1}dx = \int \sqrt{3x-1}$

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$\displaystyle\qquad\qquad = -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$

$\displaystyle\int \sqrt{3x-1}dx = \int \sqrt{3x-1} \cdot \frac{1}{3}d(3x-1)$

凑微分 "$\int$ ''ì½»§Æ¢È'' $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 1

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凑微分 "$\int$ "ì½»§Æ¢È" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

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解 $\displaystyle\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2}d(1+2x) = \frac{1}{2}\int \frac{1}{u}du$

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$\displaystyle\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$

$\displaystyle\qquad\qquad = -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$

$\displaystyle\int \sqrt{3x-1}dx = \int \sqrt{3x-1} \cdot \frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$

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$\displaystyle\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$\displaystyle\int \sqrt{3x-1}dx = \int \sqrt{3x-1} \cdot \frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$

$$u^{3/2}$$

例子 求不定积分 $\int \frac{1}{1+2x}dx$, $\int \frac{1}{2-3x}dx$, $\int \sqrt{3x-1}dx$

解 $\displaystyle\int \frac{1}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2}d(1+2x) = \frac{1}{2}\int \frac{1}{u}du$

$$= \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|1+2x| + C$$

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3})d(2-3x) = -\frac{1}{3}\int \frac{1}{u}du$$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2-3x| + C$$

$$\int \sqrt{3x-1}dx = \int \sqrt{3x-1} \cdot \frac{1}{3}d(3x-1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$

$$\frac{2}{3}u^{3/2}$$

例子 求不定积分 $\int \frac{1}{1+2x} dx$, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} \, dx$

解 $\displaystyle\int \frac{1}{1+2x} dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x) = \frac{1}{2} \int \frac{1}{u} du$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1+2x| + C$$

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

$$\int \sqrt{3x-1} \, dx = \int \sqrt{3x-1} \cdot \frac{1}{3} d(3x-1) = \frac{1}{3} \int \sqrt{u} \, du = \frac{1}{3} \int u^{1/2} du$$

$$= \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C$$

凑微分 "$\int \text{"ì½»§Æ¢È"} \, dx = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$" 例 I

例子 求不定积分 $\int \frac{1}{1+2x} dx$, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

解 $\displaystyle \int \frac{1}{1+2x} dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x) = \frac{1}{2} \int \frac{1}{u} du$

$\displaystyle \qquad\qquad = \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1+2x| + C$

$\displaystyle \int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$

$\displaystyle \qquad\qquad = -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$

$\displaystyle \int \sqrt{3x-1} dx = \int \sqrt{3x-1} \cdot \frac{1}{3} d(3x-1) = \frac{1}{3} \int \sqrt{u} du = \frac{1}{3} \int u^{1/2} du$

$\displaystyle \qquad\qquad = \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C = \frac{2}{9} (3x-1)^{3/2} + C$

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

凑微分 "$\int f'(\varphi(x))\varphi'(x)\,dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx =$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 "$\int "ì½»§Æ¢È" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2}$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 "$\int "ì½»§Æ¢È" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \qquad d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 "$\int$ "ì½»§Æ¢È" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 "$\int ”ì½»§Æ¢È” dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= \qquad u^{1/2}$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 "$\int$ "ì½»§Æ¢È" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= \quad 2u^{1/2}$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 "$\int$ "ì½»§Æ¢È" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 "$\int ""ì½»§Æ¢È" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \quad d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 "$\int ''i½»§Æ¢È'' dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}} dx$, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

解

$$\int \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x) dx = \int \cos \frac{3}{2}x \cdot \frac{2}{3} d(\frac{3}{2}x) = \frac{2}{3} \int \cos u\, du$$

$$\int e^{-\frac{1}{2}x+4} dx =$$

凑微分 "$\int ”ì½»§Æ¢È” dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u \, du$$

$$= \frac{2}{3}\sin(u) + C$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 "$\int \text{"i½»§Æ¢È"} dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u\,du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4}dx =$$

凑微分 "$\int$ ″ì½»§Æ¢È″ $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u\,du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4}$$

凑微分 "$\int$ "ì½»§Æ¢È" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u\, du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4} \qquad d(-\frac{1}{2}x+4)$$

凑微分 "$\int$ ″ì½»§Æ¢È″ $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 1

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx,\ \int \cos(\frac{3}{2}x)dx,\ \int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u\,du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4} \cdot (-2)d(-\frac{1}{2}x+4)$$

凑微分 "$\int "ì½»§Æ¢È" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u\, du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4} \cdot (-2)d(-\frac{1}{2}x+4) = -2\int e^u du$$

凑微分 " $\int$ "ì½»§Æ¢È" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u\,du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4} \cdot (-2)d(-\frac{1}{2}x+4) = -2\int e^u du$$

$$= -2e^u + C$$

凑微分 "$\int "ì½»§Æ¢È" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}}dx$, $\int \cos(\frac{3}{2}x)dx$, $\int e^{-\frac{1}{2}x+4}dx$

解

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5}\int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u\, du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4}dx = \int e^{-\frac{1}{2}x+4} \cdot (-2)d(-\frac{1}{2}x+4) = -2\int e^u du$$

$$= -2e^u + C = -2e^{-\frac{1}{2}x+4} + C$$

例子 求不定积分

(1) $\displaystyle\int x e^{x^2}\,dx,$ $\displaystyle\int x \sin(x^2)\,dx,$

(2) $\displaystyle\int x\sqrt{1-x^2}\,dx,$ $\displaystyle\int \frac{x}{\sqrt{3-x^2}}\,dx,$ $\displaystyle\int \frac{x}{1+3x^2}\,dx$

例子 求不定积分

    (1) $\int xe^{x^2}dx,$ $\int x\sin(x^2)dx,$

    (2) $\int x\sqrt{1-x^2}dx,$ $\int \frac{x}{\sqrt{3-x^2}}dx,$ $\int \frac{x}{1+3x^2}dx$

热身

    $xdx = \underline{\qquad} d(x^2)$ $xdx = \underline{\qquad} d(1-x^2)$

    $xdx = \underline{\qquad} d(3-x^2)$ $xdx = \underline{\qquad} d(1+3x^2)$

例子 求不定积分

(1) $\int x e^{x^2} dx,$　　　　$\int x \sin(x^2) dx,$

(2) $\int x\sqrt{1-x^2}dx,$　　　$\int \frac{x}{\sqrt{3-x^2}}dx,$　　　$\int \frac{x}{1+3x^2}dx$

热身

$x\,dx = \underline{\quad\frac{1}{2}\quad} d(x^2)$　　　　$x\,dx = \underline{\qquad\qquad} d(1-x^2)$

$x\,dx = \underline{\qquad\qquad} d(3-x^2)$　　$x\,dx = \underline{\qquad\quad} d(1+3x^2)$

例子 求不定积分

(1)　　$\int x e^{x^2} dx$,　　　　　$\int x \sin(x^2) dx$,

(2)　　$\int x \sqrt{1-x^2} dx$,　　　$\int \frac{x}{\sqrt{3-x^2}} dx$,　　　$\int \frac{x}{1+3x^2} dx$

热身

$x dx = \underline{\quad \frac{1}{2} \quad} d(x^2)$　　　　　$x dx = \underline{\quad -\frac{1}{2} \quad} d(1-x^2)$

$x dx = \underline{\qquad\qquad} d(3-x^2)$　　$x dx = \underline{\qquad\quad} d(1+3x^2)$

例子 求不定积分

(1) $\quad \int x e^{x^2}\,dx,\qquad\qquad \int x\sin(x^2)\,dx,$

(2) $\quad \int x\sqrt{1-x^2}\,dx,\qquad \int \frac{x}{\sqrt{3-x^2}}\,dx,\qquad \int \frac{x}{1+3x^2}\,dx$

热身

$$xdx = \underline{\quad\frac{1}{2}\quad}\,d(x^2) \qquad\qquad xdx = \underline{\quad-\frac{1}{2}\quad}\,d(1-x^2)$$

$$xdx = \underline{\quad-\frac{1}{2}\quad}\,d(3-x^2) \qquad xdx = \underline{\qquad}\,d(1+3x^2)$$

例子 求不定积分

(1) $\quad \int x e^{x^2} dx,$ $\qquad \int x\sin(x^2)dx,$

(2) $\quad \int x\sqrt{1-x^2}dx,$ $\qquad \int \frac{x}{\sqrt{3-x^2}}dx,$ $\qquad \int \frac{x}{1+3x^2}dx$

热身

$x\,dx = \underline{\quad\frac{1}{2}\quad} d(x^2)$ $\qquad x\,dx = \underline{\quad-\frac{1}{2}\quad} d(1-x^2)$

$x\,dx = \underline{\quad-\frac{1}{2}\quad} d(3-x^2)$ $\qquad x\,dx = \underline{\quad\frac{1}{6}\quad} d(1+3x^2)$

例子 求 $\int xe^{x^2}\,dx$, $\int x\sin(x^2)\,dx$

例子 求 $\int xe^{x^2}dx$, $\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx =$$

例子 求 $\int xe^{x^2}dx$, $\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}$$

例子 求 $\int x e^{x^2} dx$, $\int x \sin(x^2) dx$

解

$$\int x e^{x^2} dx = \int e^{x^2} \, d(x^2)$$

例子 求 $\int xe^{x^2}dx$, $\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2)$$

例子 求 $\int xe^{x^2}dx$, $\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du$$

例子 求 $\int xe^{x^2}dx$, $\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C$$

例子 求 $\int xe^{x^2}dx$, $\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

例子 求 $\int xe^{x^2}dx$, $\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx =$$

例子 求 $\int x e^{x^2} dx$, $\int x \sin(x^2) dx$

解

$$\int x e^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

$$\int x \sin(x^2) dx = \int \sin(x^2)$$

例子 求 $\int xe^{x^2}dx$, $\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2) \quad d(x^2)$$

例子 求 $\int xe^{x^2}dx$, $\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2)\cdot\frac{1}{2}d(x^2)$$

例子 求 $\int x e^{x^2} dx$, $\int x \sin(x^2) dx$

解
$$\int x e^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

$$\int x \sin(x^2) dx = \int \sin(x^2) \cdot \frac{1}{2} d(x^2) = \frac{1}{2} \int \sin u \, du$$

例子 求 $\int x e^{x^2} dx$, $\int x \sin(x^2) dx$

解
$$\int x e^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

$$\int x \sin(x^2) dx = \int \sin(x^2) \cdot \frac{1}{2} d(x^2) = \frac{1}{2} \int \sin u \, du$$

$$= -\frac{1}{2} \cos u + C$$

凑微分 "$\int$ "ì½»§Æ¢È" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 III

例子 求 $\int xe^{x^2}dx$, $\int x\sin(x^2)dx$

解

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2)\cdot\frac{1}{2}d(x^2) = \frac{1}{2}\int \sin u\,du$$

$$= -\frac{1}{2}\cos u + C = -\frac{1}{2}\cos(x^2) + C$$

例子 求 $\int x\sqrt{1-x^2}\,dx$, $\int \frac{x}{\sqrt{3-x^2}}\,dx$, $\int \frac{x}{1+3x^2}\,dx$

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx =$$

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}$$

例子 求 $\int x\sqrt{1-x^2}\,dx$, $\int \frac{x}{\sqrt{3-x^2}}\,dx$, $\int \frac{x}{1+3x^2}\,dx$

解

$$\int x\sqrt{1-x^2}\,dx = \int (1-x^2)^{\frac{1}{2}} \qquad d(1-x^2)$$

例子 求 $\int x\sqrt{1-x^2}\,dx$, $\int \frac{x}{\sqrt{3-x^2}}\,dx$, $\int \frac{x}{1+3x^2}\,dx$

解

$$\int x\sqrt{1-x^2}\,dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2)$$

凑微分 "$\int ”ì½»§Æ¢È” \, dx = \int f(\varphi(x)) \, d\varphi(x) = F(\varphi(x)) + C$" 例 IV

例子 求 $\int x\sqrt{1-x^2} \, dx$, $\int \frac{x}{\sqrt{3-x^2}} \, dx$, $\int \frac{x}{1+3x^2} \, dx$

解:

$$\int x\sqrt{1-x^2} \, dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2}) \, d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}} \, du$$

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解:

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$u^{3/2}$$

凑微分 "$\int \text{"ì½»§Æ¢È"} dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$\frac{2}{3}u^{3/2}$$

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C$$

凑微分 "$\int$ "ì½»§Æ¢È" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解:

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

凑微分 "$\int$"ì½»§Æ¢È" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx =$$

$$\int \frac{x}{1+3x^2}dx =$$

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}}$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分 "$\int$ ″ì½»§Æ¢È″ $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \qquad d(3-x^2)$$

$$\int \frac{x}{1+3x^2}dx =$$

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分 "$\int " \text{ì½»§Æ¢È}" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分 "$\int$ "ì½»§Æ¢È" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du \qquad 2u^{1/2}$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分 "$\int$ "ì½»§Æ¢È" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}\,dx$, $\int \frac{x}{\sqrt{3-x^2}}\,dx$, $\int \frac{x}{1+3x^2}\,dx$

解

$$\int x\sqrt{1-x^2}\,dx = \int (1-x^2)^{\frac{1}{2}} \cdot \left(-\frac{1}{2}\right)d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}\,du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}\,dx = \int (3-x^2)^{-\frac{1}{2}} \cdot \left(-\frac{1}{2}\right)d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}\,du = -\frac{1}{2} \cdot 2u^{1/2} + C$$

$$\int \frac{x}{1+3x^2}\,dx =$$

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx =$$

凑微分 "$\int$ "ì½»§Æ¢È" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2}$$

凑微分 "$\int \text{"ì½»§Æ¢È"} \, dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2} \quad d(1+3x^2)$$

凑微分 "$\int "ì½»§Æ¢È" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2}\cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6}d(1+3x^2)$$

凑微分 "$\int ''ì½»§Æ¢È'' \, dx = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}\,dx$, $\int \frac{x}{\sqrt{3-x^2}}\,dx$, $\int \frac{x}{1+3x^2}\,dx$

解

$$\int x\sqrt{1-x^2}\,dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2}) d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}\,du$$

$$= -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}\,dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}\,du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}\,dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6} d(1+3x^2) = \frac{1}{6}\int \frac{1}{u}\,du$$

凑微分 "$\int \text{"}\text{ì½}\text{»§Æ¢È}" \, dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}\,dx$, $\int \frac{x}{\sqrt{3-x^2}}\,dx$, $\int \frac{x}{1+3x^2}\,dx$

解

$$\int x\sqrt{1-x^2}\,dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}\,du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}\,dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}\,du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}\,dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6}d(1+3x^2) = \frac{1}{6}\int \frac{1}{u}\,du$$

$$= \frac{1}{6}\ln|u| + C$$

凑微分 "$\int$ "ï½»§Æ¢È" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 IV

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2}dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6}d(1+3x^2) = \frac{1}{6}\int \frac{1}{u}du$$

$$= \frac{1}{6}\ln|u| + C = \frac{1}{6}\ln|1+3x^2| + C$$

例子 求 $\int e^x \sin(e^x) \, dx$, $\int \frac{e^x}{1+e^x} \, dx$

例子 求 $\int e^x \sin(e^x)dx$, $\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

例子 求 $\int e^x \sin(e^x)dx$, $\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解 $\int e^x \sin(e^x)dx =$

例子 求 $\int e^x \sin(e^x)dx$, $\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解 $$\int e^x \sin(e^x)dx = \int \sin(e^x)$$

例子 求 $\int e^x \sin(e^x)dx$, $\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解 $\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$

例子 求 $\int e^x \sin(e^x)dx$, $\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$

$$= \int \sin u \, du$$

凑微分 "$\int$ ”ì½»§Æ¢È” $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 V

例子 求 $\int e^x \sin(e^x)dx$，$\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$

$$= \int \sin u \, du = -\cos u + C$$

例子 求 $\int e^x \sin(e^x)dx$, $\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解
$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$

$$= \int \sin u\, du = -\cos u + C = -\cos(e^x) + C$$

凑微分 "$\int$ "ì½»§Æ¢È" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 V

例子 求 $\int e^x \sin(e^x)dx$, $\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$

$$= \int \sin u\, du = -\cos u + C = -\cos(e^x) + C$$

$$\int \frac{e^x}{1+e^x}dx$$

例子 求 $\int e^x \sin(e^x) dx$, $\int \frac{e^x}{1+e^x} dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x \sin(e^x) dx = \int \sin(e^x) de^x$$

$$= \int \sin u \, du = -\cos u + C = -\cos(e^x) + C$$

$$\int \frac{e^x}{1+e^x} dx = \int \frac{1}{1+e^x} de^x$$

凑微分 "$\int \text{"ì½»§Æ¢È"} \, dx = \int f(\varphi(x)) \, d\varphi(x) = F(\varphi(x)) + C$" 例 V

例子 求 $\int e^x \sin(e^x) \, dx$, $\int \frac{e^x}{1+e^x} \, dx$

热身
$$e^x \, dx = de^x$$

解
$$\int e^x \sin(e^x) \, dx = \int \sin(e^x) \, de^x$$
$$= \int \sin u \, du = -\cos u + C = -\cos(e^x) + C$$
$$\int \frac{e^x}{1+e^x} \, dx = \int \frac{1}{1+e^x} \, de^x = \int \frac{1}{1+e^x} \, d(e^x + 1)$$

例子 求 $\int e^x \sin(e^x)dx$, $\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$

$$= \int \sin u\,du = -\cos u + C = -\cos(e^x) + C$$

$$\int \frac{e^x}{1+e^x}dx = \int \frac{1}{1+e^x}de^x = \int \frac{1}{1+e^x}d(e^x + 1)$$

$$= \int \frac{1}{u}du$$

例子 求 $\int e^x \sin(e^x)dx$, $\int \frac{e^x}{1+e^x}dx$

热身
$$e^x dx = de^x$$

解
$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$

$$= \int \sin u\, du = -\cos u + C = -\cos(e^x) + C$$

$$\int \frac{e^x}{1+e^x}dx = \int \frac{1}{1+e^x}de^x = \int \frac{1}{1+e^x}d(e^x+1)$$

$$= \int \frac{1}{u}du = \ln|u| + C$$

凑微分 "$\int ''ì½»§Æ¢È'' dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 V

例子 求 $\int e^x \sin(e^x)dx$, $\int \frac{e^x}{1+e^x}dx$

热身

$$e^x dx = de^x$$

解

$$\int e^x \sin(e^x)dx = \int \sin(e^x)de^x$$

$$= \int \sin u\, du = -\cos u + C = -\cos(e^x) + C$$

$$\int \frac{e^x}{1+e^x}dx = \int \frac{1}{1+e^x}de^x = \int \frac{1}{1+e^x}d(e^x+1)$$

$$= \int \frac{1}{u}du = \ln|u| + C = \ln(e^x+1) + C$$

例子 求 $\int \frac{1}{x}\ln x\,dx$, $\int \frac{1}{x\ln x}dx$

例子 求 $\int \frac{1}{x}\ln x\,dx$, $\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

例子 求 $\int \frac{1}{x}\ln x\, dx$, $\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

解

$$\int \frac{1}{x}\ln x\, dx =$$

例子 求 $\int \frac{1}{x}\ln x dx$, $\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

解

$$\int \frac{1}{x}\ln x dx = \int \ln x d\ln x$$

例子 求 $\int \frac{1}{x}\ln x\, dx$, $\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

解

$$\int \frac{1}{x}\ln x\, dx = \int \ln x\, d\ln x = \int u\, du$$

例子 求 $\int \frac{1}{x}\ln x \, dx$, $\int \frac{1}{x\ln x} dx$

热身

$$\frac{1}{x}dx = d\ln x$$

解

$$\int \frac{1}{x}\ln x \, dx = \int \ln x \, d\ln x = \int u \, du$$

$$= \frac{1}{2}u^2 + C$$

例子 求 $\int \frac{1}{x} \ln x \, dx$, $\int \frac{1}{x \ln x} dx$

热身
$$\frac{1}{x} dx = d \ln x$$

解
$$\int \frac{1}{x} \ln x \, dx = \int \ln x \, d \ln x = \int u \, du$$
$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$

凑微分 "$\int$ ”ì½»§Æ¢È” $dx = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$" 例 VI

例子 求 $\int \frac{1}{x} \ln x \, dx$, $\int \frac{1}{x \ln x} dx$

热身
$$\frac{1}{x} dx = d \ln x$$

解
$$\int \frac{1}{x} \ln x \, dx = \int \ln x \, d \ln x = \int u \, du$$
$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$
$$\int \frac{1}{x \ln x} dx =$$

例子 求 $\int \frac{1}{x}\ln x\, dx$, $\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

解

$$\int \frac{1}{x}\ln x\, dx = \int \ln x\, d\ln x = \int u\, du$$

$$= \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C$$

$$\int \frac{1}{x\ln x}dx = \int \frac{1}{\ln x}d\ln x$$

例子 求 $\int \frac{1}{x} \ln x \, dx$, $\int \frac{1}{x \ln x} dx$

热身

$$\frac{1}{x} dx = d \ln x$$

解

$$\int \frac{1}{x} \ln x \, dx = \int \ln x \, d \ln x = \int u \, du$$

$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$

$$\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d \ln x = \int \frac{1}{u} du$$

例子 求 $\int \frac{1}{x} \ln x \, dx$, $\int \frac{1}{x \ln x} dx$

热身

$$\frac{1}{x} dx = d \ln x$$

解

$$\int \frac{1}{x} \ln x \, dx = \int \ln x \, d \ln x = \int u \, du$$

$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$

$$\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d \ln x = \int \frac{1}{u} du$$

$$= \ln |u| + C$$

例子 求 $\int \frac{1}{x}\ln x\, dx$, $\int \frac{1}{x\ln x}dx$

热身

$$\frac{1}{x}dx = d\ln x$$

解

$$\int \frac{1}{x}\ln x\, dx = \int \ln x\, d\ln x = \int u\, du$$

$$= \frac{1}{2}u^2 + C = \frac{1}{2}(\ln x)^2 + C$$

$$\int \frac{1}{x\ln x}dx = \int \frac{1}{\ln x}d\ln x = \int \frac{1}{u}du$$

$$= \ln|u| + C = \ln|\ln x| + C$$

例子 求 $\int e^{\cos x} \sin x\, dx$, $\int \frac{\sin x}{1 + \cos^2 x}\, dx$, $\int \frac{\cos x}{\sin x}\, dx$

例子 求 $\int e^{\cos x} \sin x \, dx$, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

热身　$\sin x \, dx = d\underline{\hspace{3cm}}$　　　　　, $\cos x \, dx = d\underline{\hspace{2cm}}$

例子 求 $\int e^{\cos x} \sin x\, dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

热身 $\sin x\, dx = d\underline{-\cos x}$, $\cos x\, dx = d\underline{\qquad}$

凑微分 "$\int {}^{\prime\prime}\grave{i}\frac{1}{2}{}_{\text{"}}\S\cancel{E}\cancel{c}\grave{E}{}^{\prime\prime} dx = \int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$" 例 VII

例子 求 $\int e^{\cos x} \sin x\, dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

热身 $\sin x\, dx = d\underline{-\cos x} = -d\cos x$, $\cos x\, dx = d\underline{\hspace{2cm}}$

例子 求 $\int e^{\cos x} \sin x\, dx$, $\int \frac{\sin x}{1+\cos^2 x}\, dx$, $\int \frac{\cos x}{\sin x}\, dx$

热身 $\sin x\, dx = d \underline{-\cos x} = -d\cos x$, $\cos x\, dx = d\,\underline{\sin x}$

例子 求 $\int e^{\cos x}\sin x\,dx,\ \int\frac{\sin x}{1+\cos^2 x}dx,\ \int\frac{\cos x}{\sin x}dx$

热身 $\sin x\,dx = d\underline{-\cos x} = -d\cos x,\quad \cos x\,dx = d\underline{\sin x}$

解

$$\int e^{\cos x}\sin x\,dx =$$

例子 求 $\int e^{\cos x}\sin x\,dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

热身 $\sin x\,dx = d\underline{-\cos x} = -d\cos x$, $\cos x\,dx = d\underline{\sin x}$

解

$$\int e^{\cos x}\sin x\,dx = \int e^{\cos x}(-1)d\cos x$$

例子 求 $\int e^{\cos x}\sin x dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

热身 $\sin x dx = d\underline{-\cos x} = -d\cos x$, $\cos x dx = d\underline{\sin x}$

解

$$\int e^{\cos x}\sin x dx = \int e^{\cos x}(-1)d\cos x = -\int e^u du$$

例子 求 $\int e^{\cos x}\sin x\,dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

热身 $\sin x\,dx = d\underline{-\cos x} = -d\cos x$, $\cos x\,dx = d\underline{\sin x}$

解

$$\int e^{\cos x}\sin x\,dx = \int e^{\cos x}(-1)d\cos x = -\int e^u\,du$$

$$= -e^u + C$$

例子 求 $\int e^{\cos x} \sin x\, dx$, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

热身 $\sin x\, dx = d\underline{-\cos x} = -d\cos x$, $\cos x\, dx = d\underline{\sin x}$

解

$$\int e^{\cos x} \sin x\, dx = \int e^{\cos x}(-1)d\cos x = -\int e^u du$$

$$= -e^u + C = -e^{\cos x} + C$$

凑微分 "$\int$ ''ì½»§Æ¢È'' $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x))+C$" 例 VII

例子 求 $\int e^{\cos x} \sin x \, dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

热身 $\quad \sin x \, dx = d\underline{-\cos x} = -d\cos x, \quad \cos x \, dx = d\underline{\sin x}$

解

$$\int e^{\cos x} \sin x \, dx = \int e^{\cos x}(-1)d\cos x = -\int e^u du$$

$$= -e^u + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1 + \cos^2 x} dx =$$

$$\int \frac{\cos x}{\sin x} dx =$$

例子 求 $\int e^{\cos x}\sin x\,dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

热身 $\sin x\,dx = d\underline{-\cos x} = -d\cos x,$ $\cos x\,dx = d\underline{\sin x}$

解

$$\int e^{\cos x}\sin x\,dx = \int e^{\cos x}(-1)d\cos x = -\int e^u\,du$$

$$= -e^u + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x$$

$$\int \frac{\cos x}{\sin x}dx =$$

凑微分 "$\int f(\varphi(x))\varphi'(x)\,dx = \int f(\varphi(x))\,d\varphi(x) = F(\varphi(x)) + C$" 例 VII

例子 求 $\int e^{\cos x}\sin x\,dx$, $\int \frac{\sin x}{1+\cos^2 x}\,dx$, $\int \frac{\cos x}{\sin x}\,dx$

热身 $\sin x\,dx = d\underline{-\cos x} = -d\cos x$, $\cos x\,dx = d\underline{\sin x}$

解

$$\int e^{\cos x}\sin x\,dx = \int e^{\cos x}(-1)\,d\cos x = -\int e^u\,du$$

$$= -e^u + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1+\cos^2 x}\,dx = \int \frac{1}{1+\cos^2 x}(-1)\,d\cos x = -\int \frac{1}{1+u^2}\,du$$

$$\int \frac{\cos x}{\sin x}\,dx =$$

凑微分 "$\int$ "ì½»§Æ¢È" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$" 例 VII

例子 求 $\int e^{\cos x} \sin x \, dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

热身 $\sin x \, dx = d \underline{-\cos x} = -d\cos x$, $\cos x \, dx = d\underline{\sin x}$

解

$$\int e^{\cos x} \sin x \, dx = \int e^{\cos x}(-1)d\cos x = -\int e^u du$$

$$= -e^u + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1+\cos^2 x} dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2} du$$

$$= -\arctan u + C$$

$$\int \frac{\cos x}{\sin x} dx =$$

例子 求 $\int e^{\cos x}\sin x\,dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

热身 $\sin x\,dx = d\underline{-\cos x} = -d\cos x$, $\cos x\,dx = d\underline{\sin x}$

解

$$\int e^{\cos x}\sin x\,dx = \int e^{\cos x}(-1)d\cos x = -\int e^u\,du$$

$$= -e^u + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$

$$= -\arctan u + C = -\arctan(\cos x) + C$$

$$\int \frac{\cos x}{\sin x}dx =$$

例子 求 $\int e^{\cos x}\sin x\, dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

热身 $\sin x\, dx = d\,\underline{-\cos x} = -d\cos x$, $\quad \cos x\, dx = d\,\underline{\sin x}$

解

$$\int e^{\cos x}\sin x\, dx = \int e^{\cos x}(-1)d\cos x = -\int e^u du$$

$$= -e^u + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$

$$= -\arctan u + C = -\arctan(\cos x) + C$$

$$\int \frac{\cos x}{\sin x}dx = \int \frac{1}{\sin x}d\sin x$$

例子 求 $\int e^{\cos x} \sin x \, dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

热身 $\sin x \, dx = d\,\underline{-\cos x} = -d\cos x,$ $\cos x \, dx = d\,\underline{\sin x}$

解

$$\int e^{\cos x} \sin x \, dx = \int e^{\cos x}(-1)d\cos x = -\int e^u du$$

$$= -e^u + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1+\cos^2 x} dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2} du$$

$$= -\arctan u + C = -\arctan(\cos x) + C$$

$$\int \frac{\cos x}{\sin x} dx = \int \frac{1}{\sin x} d\sin x = \int \frac{1}{u} du$$

例子 求 $\int e^{\cos x}\sin x\,dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

热身 $\quad \sin x\,dx = d\underline{-\cos x} = -d\cos x, \quad \cos x\,dx = d\,\underline{\sin x}$

解

$$\int e^{\cos x}\sin x\,dx = \int e^{\cos x}(-1)d\cos x = -\int e^u\,du$$

$$= -e^u + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$

$$= -\arctan u + C = -\arctan(\cos x) + C$$

$$\int \frac{\cos x}{\sin x}dx = \int \frac{1}{\sin x}d\sin x = \int \frac{1}{u}du = \ln|u| + C$$

例子 求 $\int e^{\cos x}\sin x\,dx$, $\int \frac{\sin x}{1+\cos^2 x}dx$, $\int \frac{\cos x}{\sin x}dx$

热身 $\sin x\,dx = d\underline{-\cos x} = -d\cos x, \quad \cos x\,dx = d\underline{\sin x}$

解

$$\int e^{\cos x}\sin x\,dx = \int e^{\cos x}(-1)d\cos x = -\int e^u du$$

$$= -e^u + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1+\cos^2 x}dx = \int \frac{1}{1+\cos^2 x}(-1)d\cos x = -\int \frac{1}{1+u^2}du$$

$$= -\arctan u + C = -\arctan(\cos x) + C$$

$$\int \frac{\cos x}{\sin x}dx = \int \frac{1}{\sin x}d\sin x = \int \frac{1}{u}du = \ln|u| + C$$

$$= \ln|\sin x| + C$$

$$\int \frac{1}{1-3x} dx =$$

$$\int \sqrt{3x-1} \, dx =$$

$$\int x e^{x^2} dx =$$

$$\int x \sqrt{1-x^2} \, dx =$$

$$\int \frac{\ln x}{x} dx =$$

$$\int e^{\cos x} \sin x \, dx =$$

凑微分法 "$\int f[\varphi(x)]\varphi'(x)\,dx=\int f(\varphi(x))d\varphi(x)$"：例子总结

$$\int \frac{1}{1-3x}dx = -\frac{1}{3}\int \frac{1}{1-3x}d(1-3x) = -\frac{1}{3}\int \frac{1}{u}du = \cdots$$

$$\int \sqrt{3x-1}\,dx = \frac{1}{3}\int \sqrt{3x-1}\,d(3x-1) = \frac{1}{3}\int u^{1/2}du = \cdots$$

$$\int xe^{x^2}dx =$$

$$\int x\sqrt{1-x^2}\,dx =$$

$$\int \frac{\ln x}{x}dx =$$

$$\int e^{\cos x}\sin x\,dx =$$

$$\int \frac{1}{1-3x}dx = -\frac{1}{3}\int \frac{1}{1-3x}d(1-3x) = -\frac{1}{3}\int \frac{1}{u}du = \cdots$$

$$\int \sqrt{3x-1}\,dx = \frac{1}{3}\int \sqrt{3x-1}\,d(3x-1) = \frac{1}{3}\int u^{1/2}du = \cdots$$

$$\int xe^{x^2}dx = \frac{1}{2}\int e^{x^2}dx^2 = \frac{1}{2}\int e^u du = \cdots$$

$$\int x\sqrt{1-x^2}\,dx = -\frac{1}{2}\int \sqrt{1-x^2}\,d(1-x^2) = -\frac{1}{2}\int u^{1/2}du = \cdots$$

$$\int \frac{\ln x}{x}dx =$$

$$\int e^{\cos x}\sin x\,dx =$$

凑微分法 "$\int \text{"ì½»§Æ¢È"}\, dx = \int f(\varphi(x))d\varphi(x)$"：例子总结

$$\int \frac{1}{1-3x}dx = -\frac{1}{3}\int \frac{1}{1-3x}d(1-3x) = -\frac{1}{3}\int \frac{1}{u}du = \cdots$$

$$\int \sqrt{3x-1}dx = \frac{1}{3}\int \sqrt{3x-1}d(3x-1) = \frac{1}{3}\int u^{1/2}du = \cdots$$

$$\int xe^{x^2}dx = \frac{1}{2}\int e^{x^2}dx^2 = \frac{1}{2}\int e^u du = \cdots$$

$$\int x\sqrt{1-x^2}dx = -\frac{1}{2}\int \sqrt{1-x^2}d(1-x^2) = -\frac{1}{2}\int u^{1/2}du = \cdots$$

$$\int \frac{\ln x}{x}dx = \int \ln x\, d(\ln x) = \int u\, du = \cdots$$

$$\int e^{\cos x}\sin x\, dx = -\int e^{\cos x}d\cos x = -\int e^u du = \cdots$$

1. 第一类换元积分法：凑微分

第二类换元积分法：变量代换

第二类换元积分法——"变量代换"法，能干啥？

能够计算如下的不定积分：

$$\int x\sqrt{3x-1}\,dx, \quad \int \frac{x}{\sqrt{x-2}}\,dx$$

$$\int \frac{1}{1+\sqrt{x}}\,dx, \quad \int \frac{1}{1+\sqrt[3]{x+1}}\,dx$$

$$\int \frac{1}{\sqrt{1+e^x}}\,dx$$

……

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xrightarrow{x=\varphi(t)}$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xlongequal{x=\varphi(t)} \int f(\varphi(t))d\varphi(t)$$

第二类换元积分法（变量代换）原理

- 计算步骤：
$$\int f(x)dx \xLeftarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int f(\varphi(t))\varphi'(t)dt$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单，容易求！}} dt$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xrightarrow{\ x=\varphi(t)\ } \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单，容易求！}}\ dt$$

$$= G(t) + C$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单，容易求！}} dt$$

$$= G(t) + C \xrightarrow{t=\varphi^{-1}(x)}$$

第二类换元积分法（变量代换）原理

- 计算步骤：

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单，容易求！}} dt$$

$$= G(t) + C \xrightarrow{t=\varphi^{-1}(x)} G\left(\varphi^{-1}(x)\right) + C$$

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  在后面的例子中，选取函数 $x = \varphi(t)$ 的方法：

  > 把被积函数 $f(x)$ 中复杂的部分整个设为 t，
  >
  > 从而得到 x 与 t 的函数关系！

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注 可见选取合适 $x = \varphi(t)$ 很关键！

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$$\therefore \int \frac{1}{1+\sqrt{x}}dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2\int 1 - \frac{1}{t}dt$$

$$= 2t - 2\ln t + C = 2(1+x^{\frac{1}{2}}) - 2\ln(1+x^{\frac{1}{2}}) + C$$

(2) 设 $t = 1 + (1+x)^{\frac{1}{3}}$, $\qquad\qquad$,

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变量代换 "$\int f(x)dx \overset{x=\varphi(t)}{=\!=\!=\!=} \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t))+C$" **例 II**

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$$= \frac{3}{2}(1 + (1+x)^{\frac{1}{3}})^2 - 6(1 + (1+x)^{\frac{1}{3}}) + 3\ln|1 + (1+x)^{\frac{1}{3}}| + C$$

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$= \ln|t-1| - \ln|t+1| + C$

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$= \ln\left(\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}\right) + C$

变量代换 "$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t)) + C$" 例 IV

例子 求不定积分 $\int \frac{1}{\sqrt{1+e^x}}dx$

解

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$$\therefore \int \frac{1}{\sqrt{1 + e^x}}dx = \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1}dt = \int \frac{1}{t-1} - \frac{1}{t+1}dt$$

$$= \ln|t-1| - \ln|t+1| + C = \ln\left|\frac{t-1}{t+1}\right| + C$$

$$= \ln\left(\frac{\sqrt{1+e^x} - 1}{\sqrt{1+e^x} + 1}\right) + C$$

$$= 2\ln(\sqrt{1+e^x} - 1) - x + C$$

$$\int x\sqrt{3x-1}\,dx$$

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