

第 3 章 f : 函数图形的描绘

数学系 梁卓滨

2019-2020 学年 I

Outline

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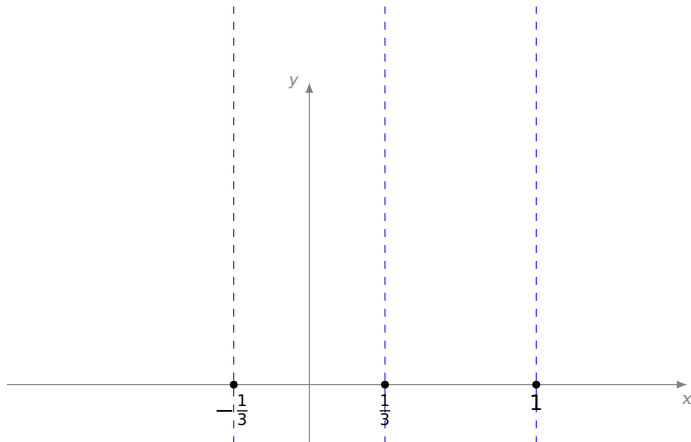
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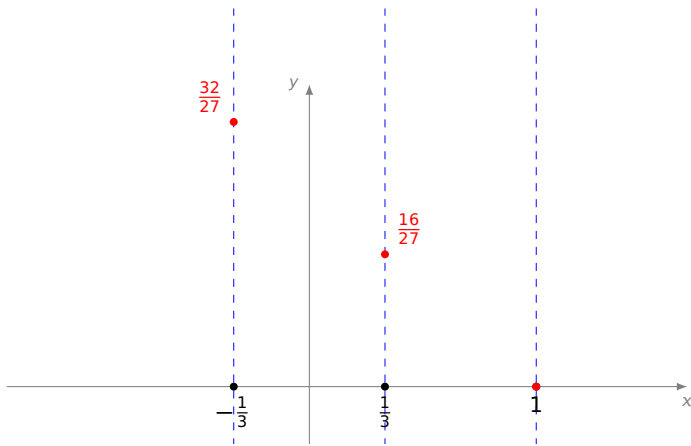
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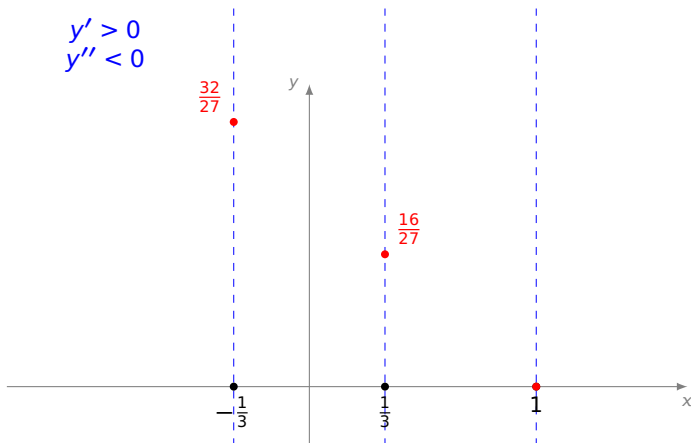
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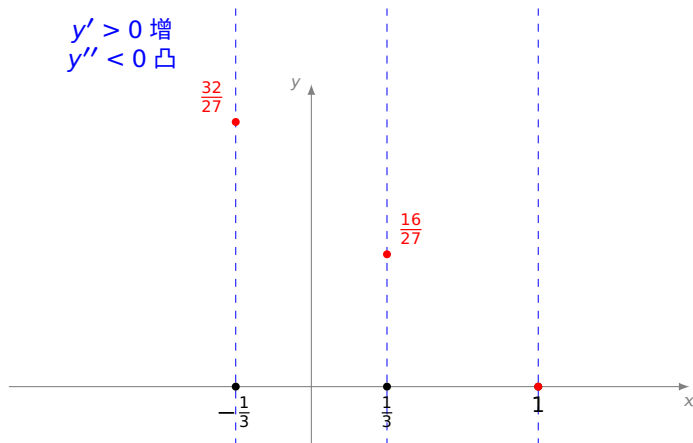
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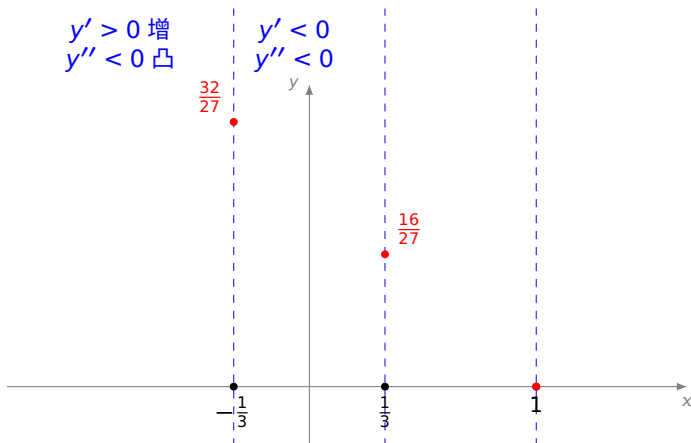
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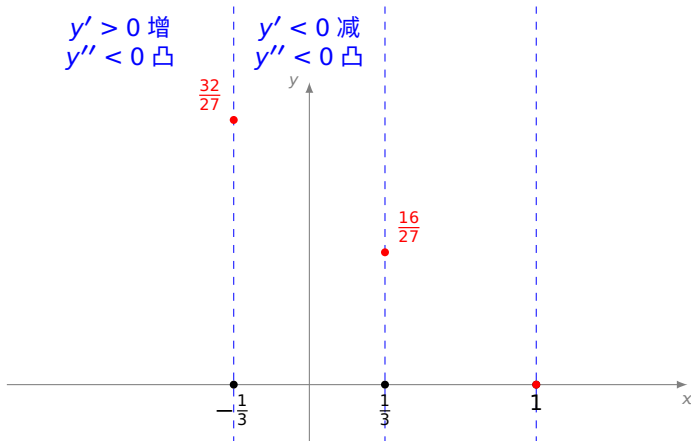
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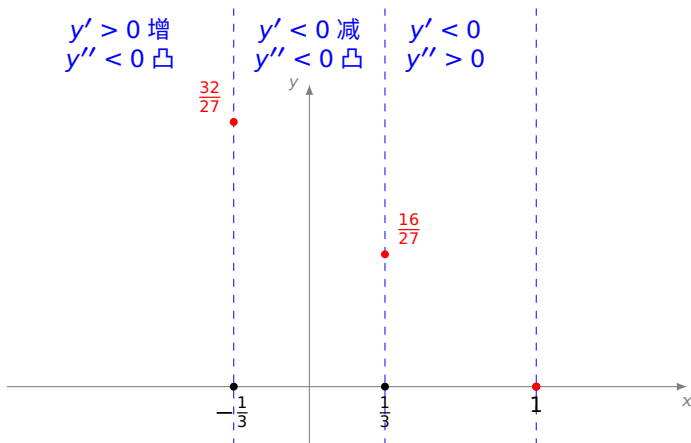
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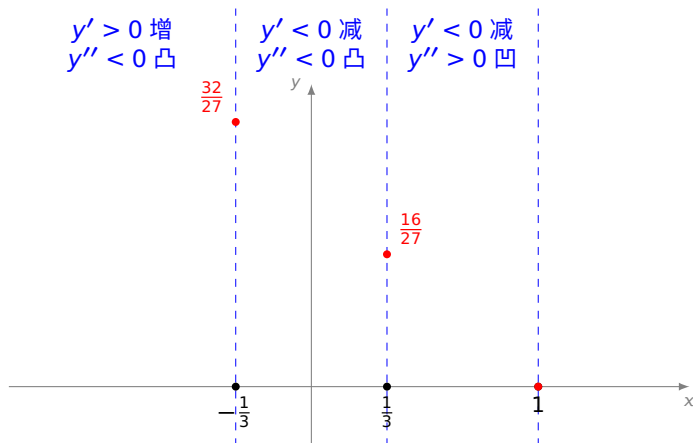
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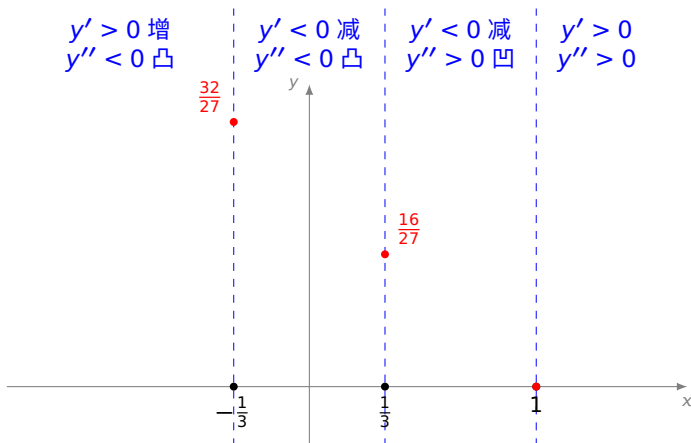
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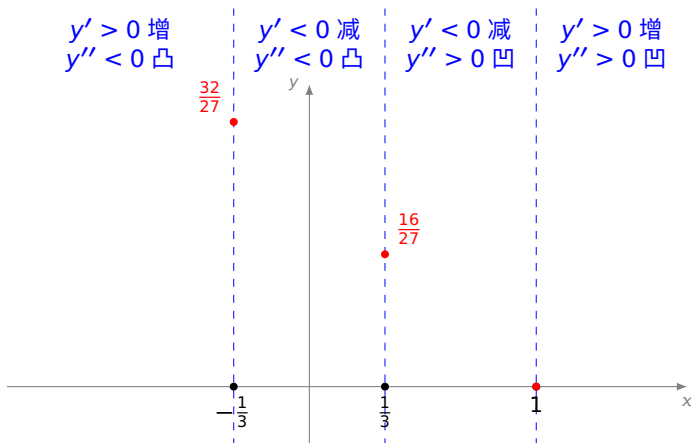
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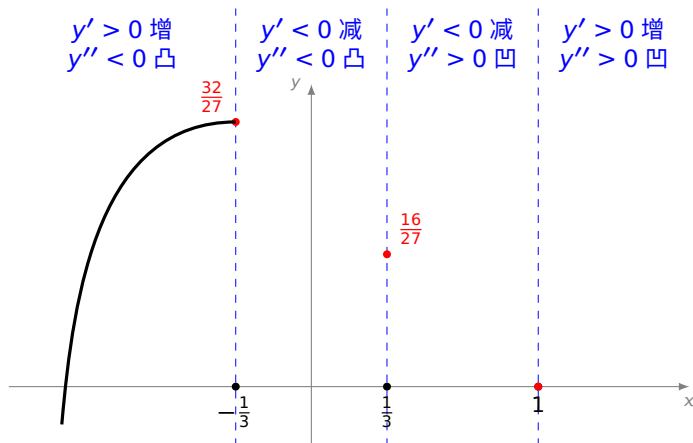
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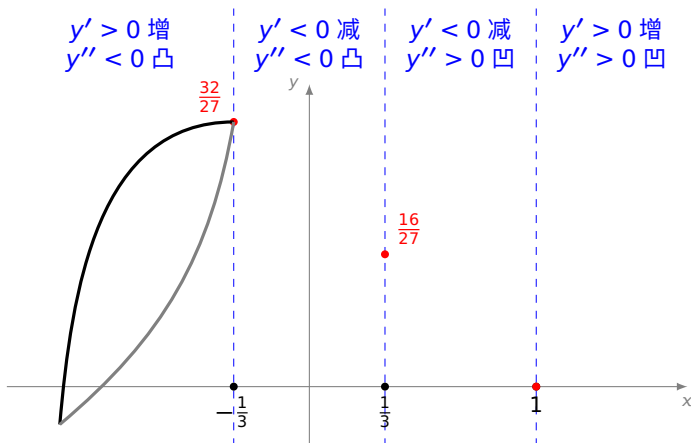
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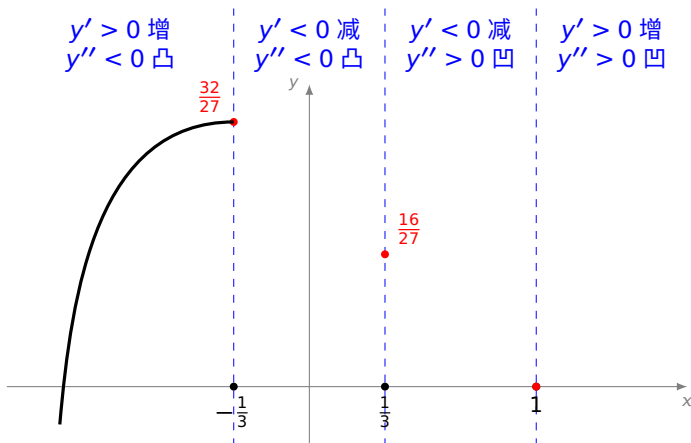
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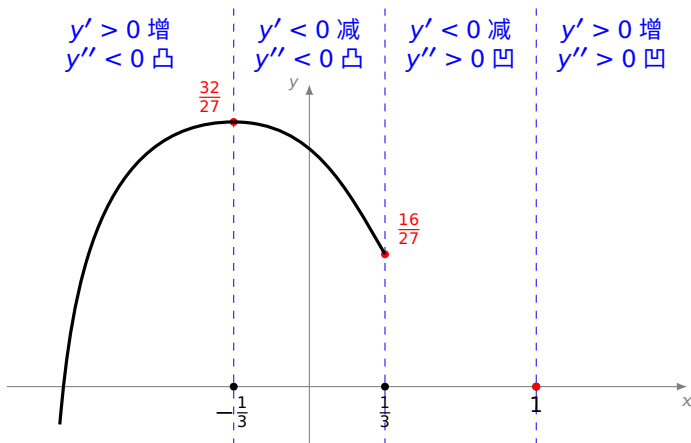
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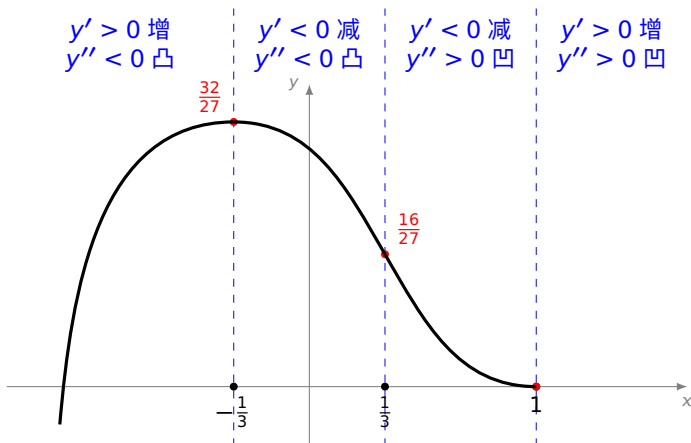
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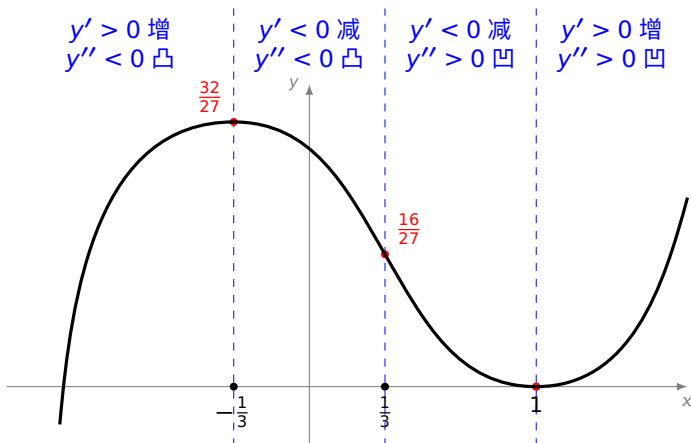
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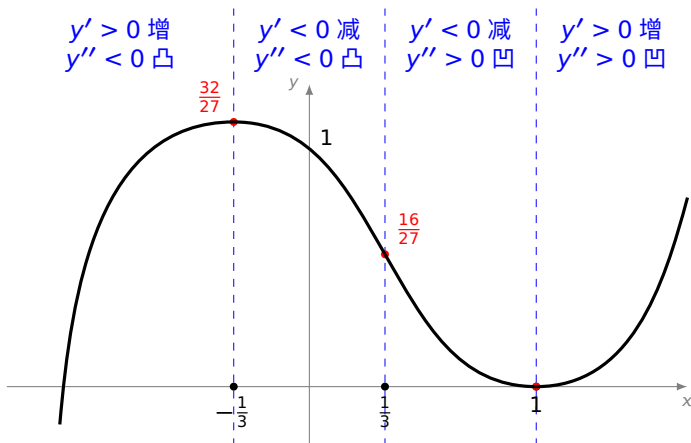
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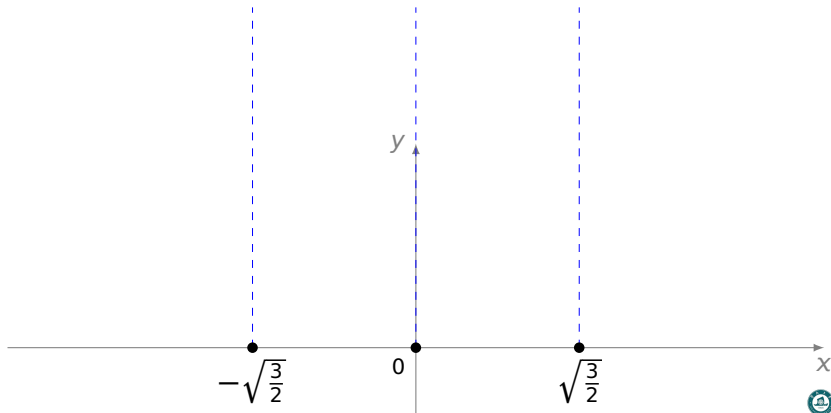
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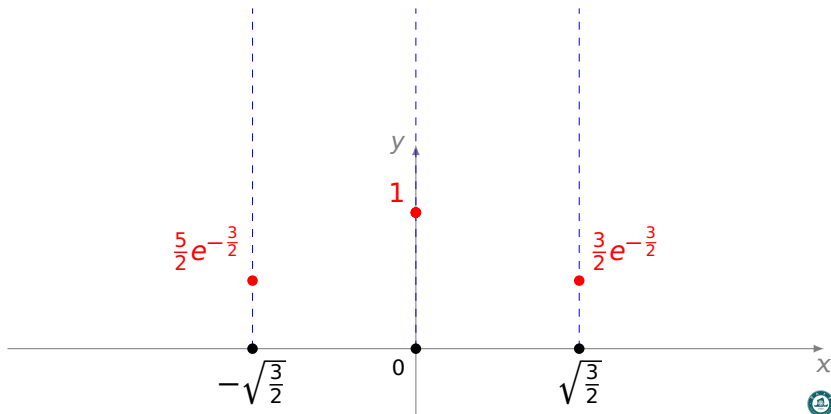
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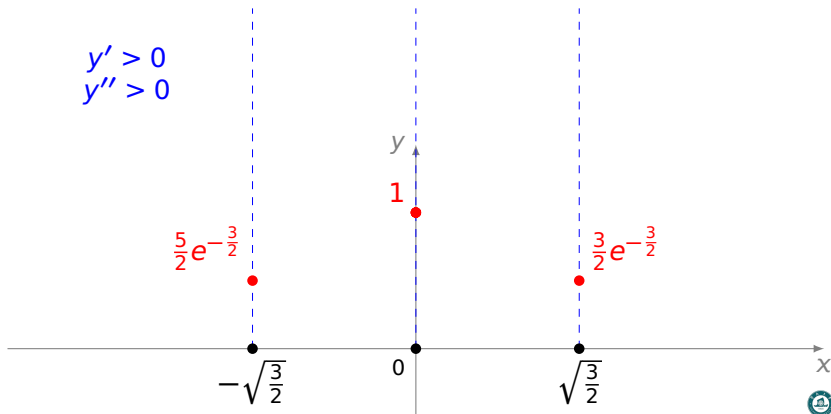
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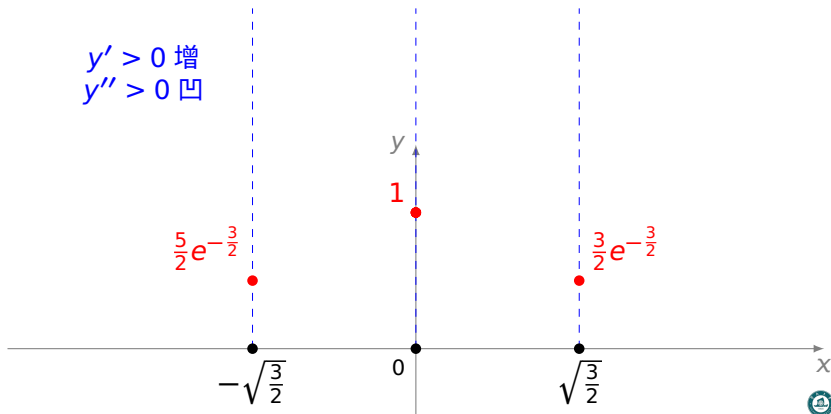
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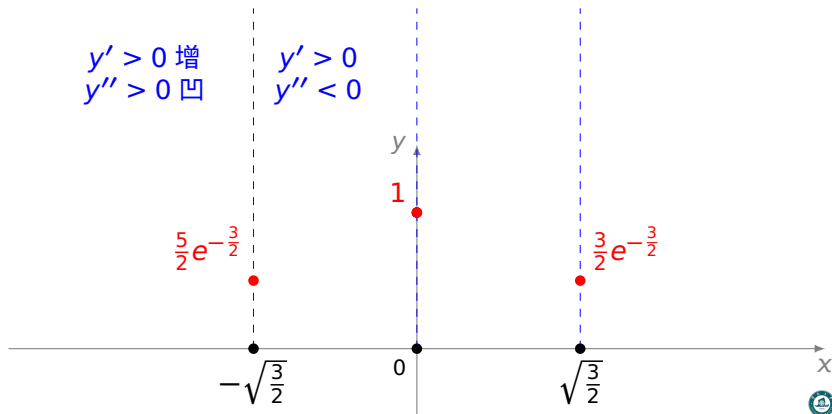
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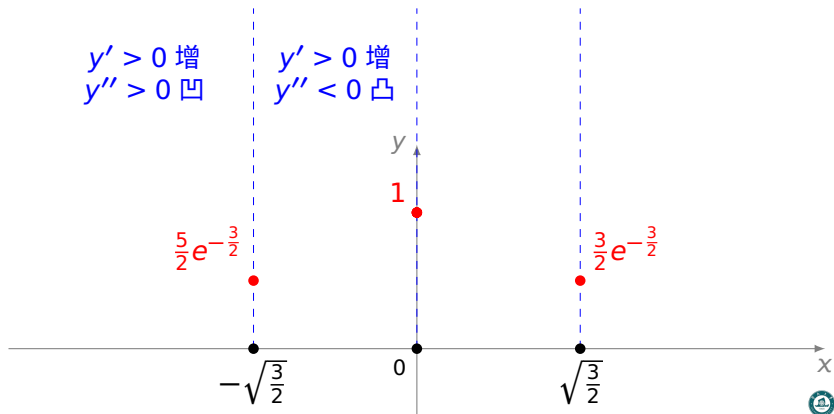
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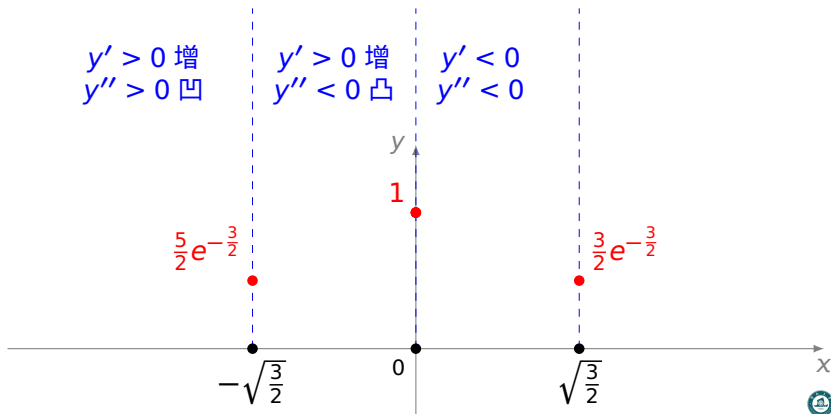
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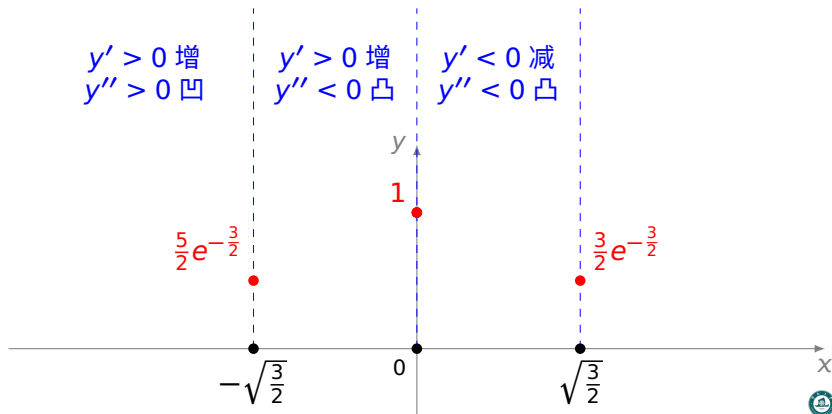
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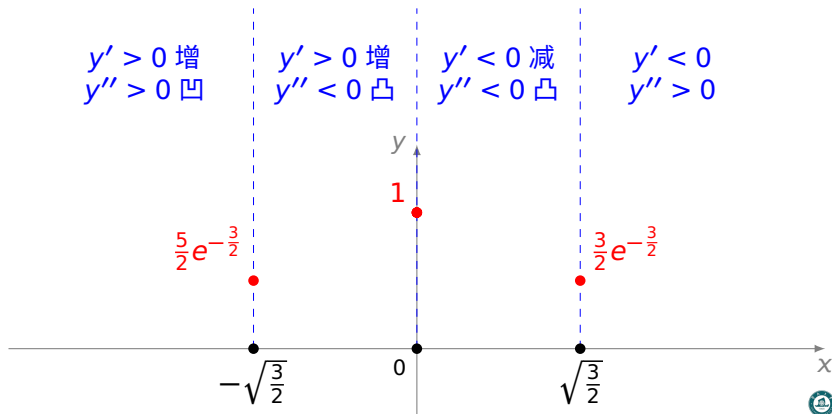
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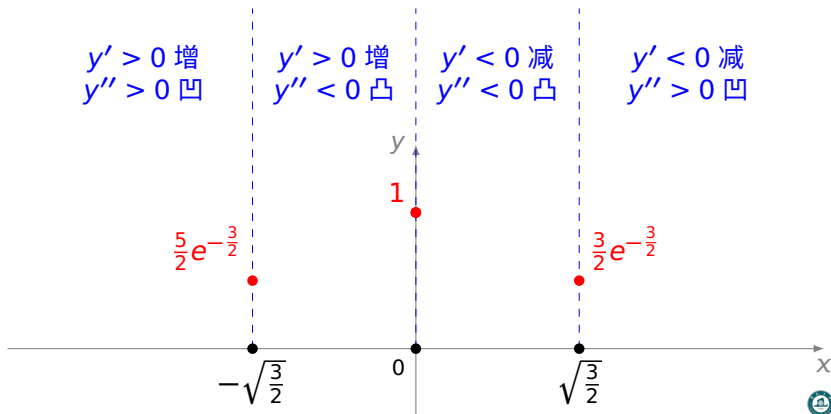
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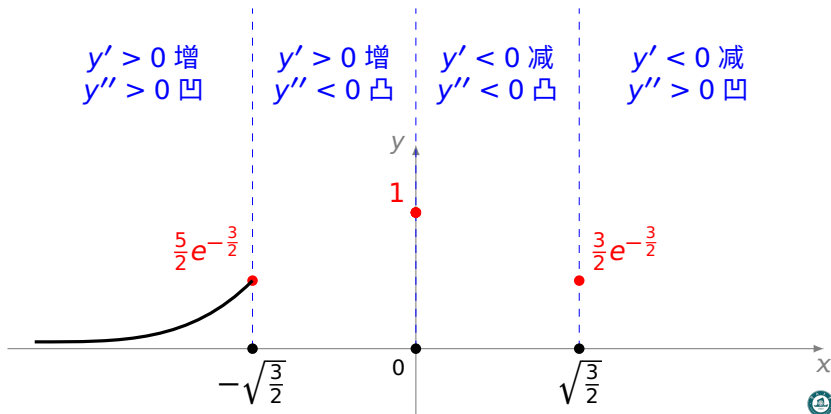
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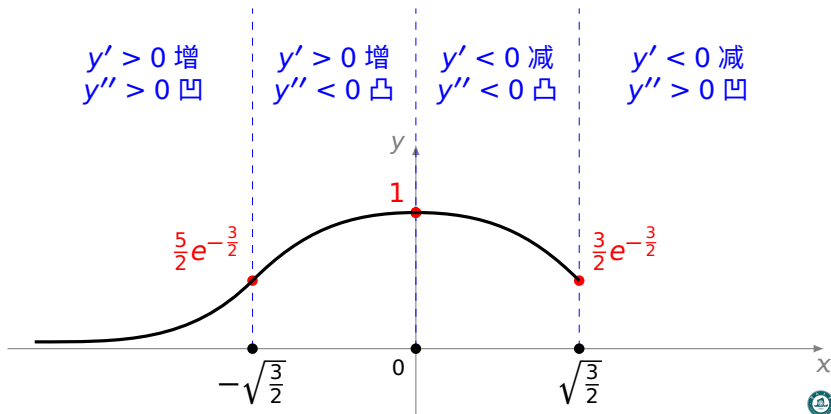
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$$y'' = 2x^2(2x^2 - 3)e^{-x^2} = 0 \Rightarrow x = 0, \pm\sqrt{\frac{3}{2}}$$

然后判定单调区间，凹凸区间.



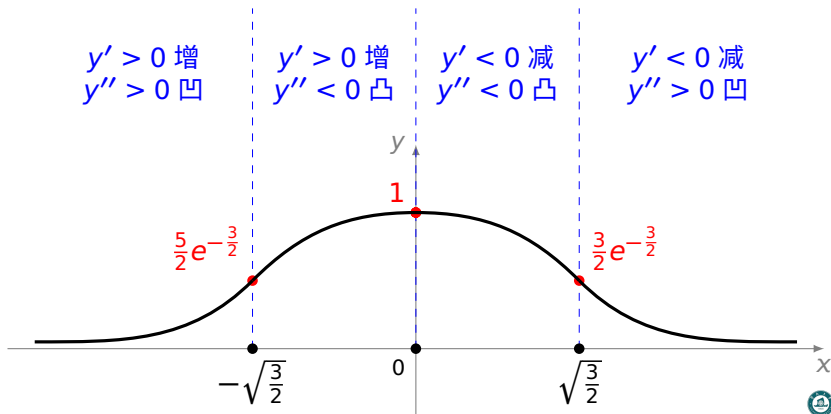
例 2 画出函数 $y = (1 + x^2)e^{-x^2}$ 的图形.

解 先求出驻点、拐点.

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例 3 画出函数 $y = \frac{\ln x}{x}$ ($x > 0$) 的图形.

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解 先求出驻点、拐点.

然后判定单调区间, 凹凸区间.

例 3 画出函数 $y = \frac{\ln x}{x}$ ($x > 0$) 的图形.

解 先求出驻点、拐点.

$$y' = \frac{(\ln x)'x - x' \ln x}{x^2}$$

然后判定单调区间，凹凸区间.

例 3 画出函数 $y = \frac{\ln x}{x}$ ($x > 0$) 的图形.

解 先求出驻点、拐点.

$$y' = \frac{(\ln x)'x - x' \ln x}{x^2} = \frac{1 - \ln x}{x^2} = 0$$

然后判定单调区间，凹凸区间.

例 3 画出函数 $y = \frac{\ln x}{x}$ ($x > 0$) 的图形.

解 先求出驻点、拐点.

$$y' = \frac{(\ln x)'x - x' \ln x}{x^2} = \frac{1 - \ln x}{x^2} = 0 \Rightarrow x = e$$

然后判定单调区间，凹凸区间.

例 3 画出函数 $y = \frac{\ln x}{x}$ ($x > 0$) 的图形.

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然后判定单调区间, 凹凸区间.

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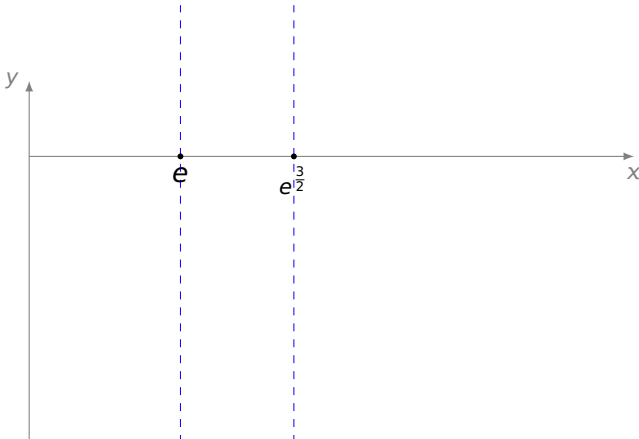
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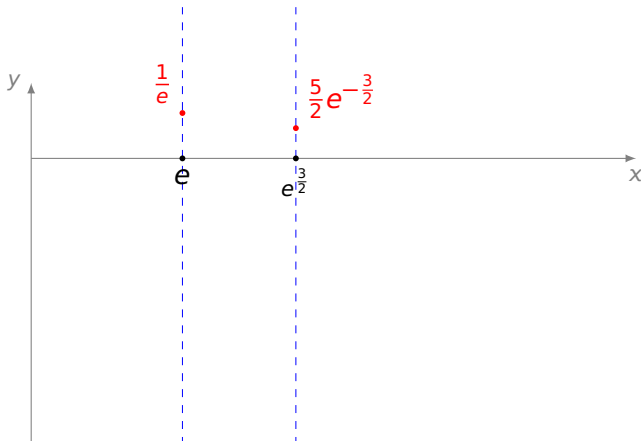
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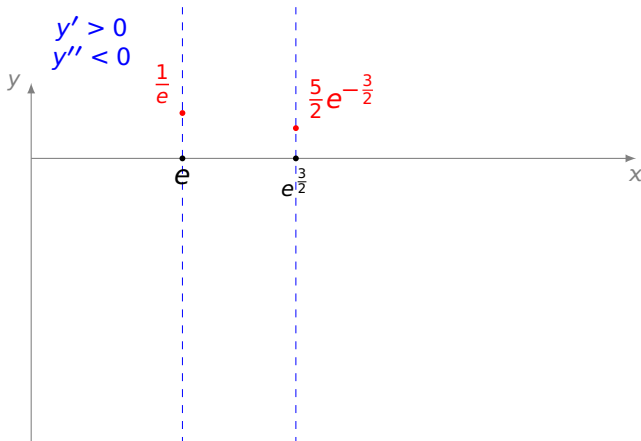
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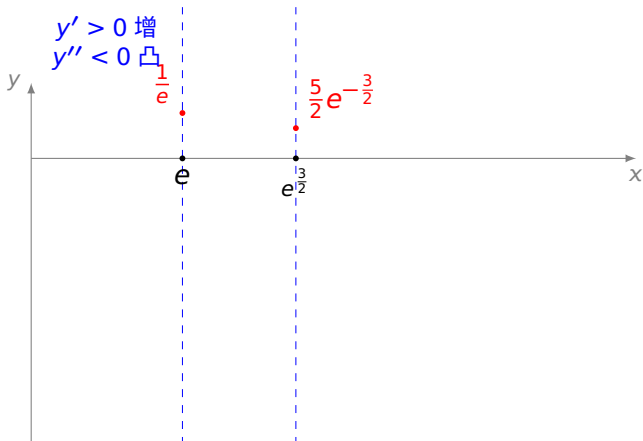
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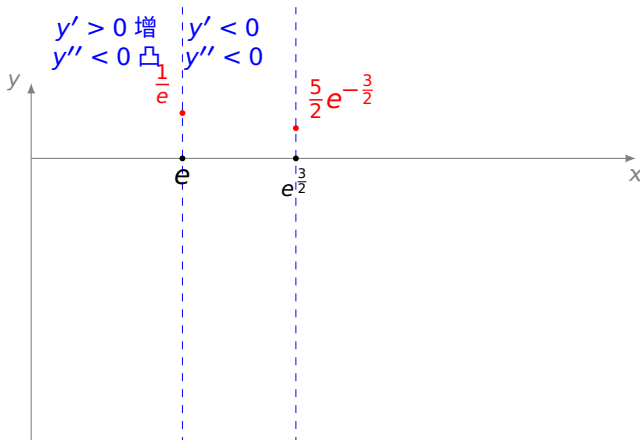
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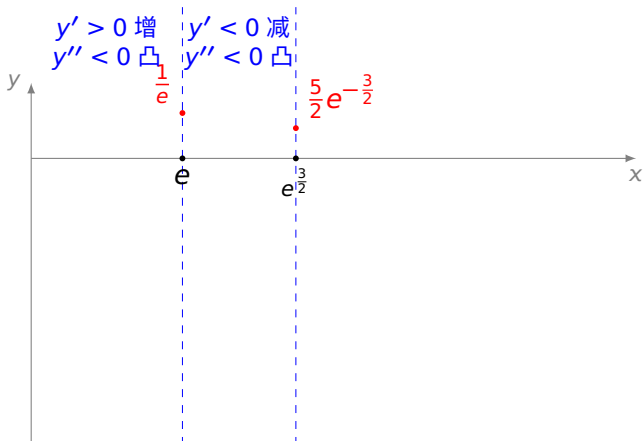
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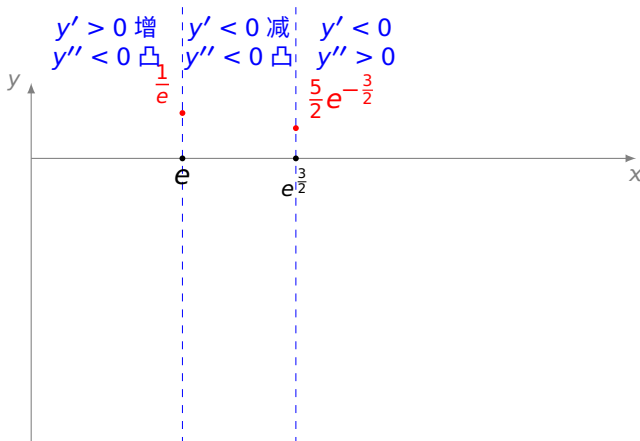
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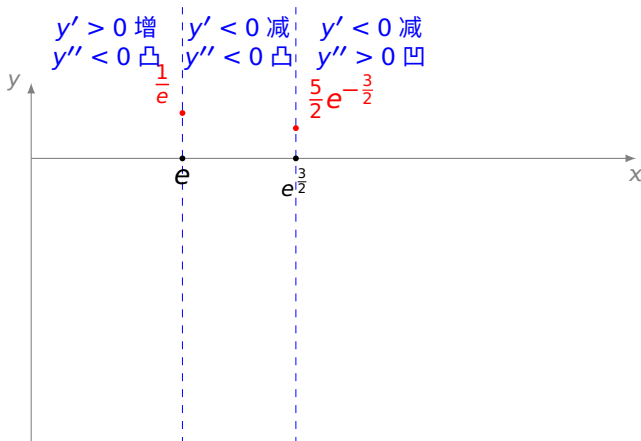
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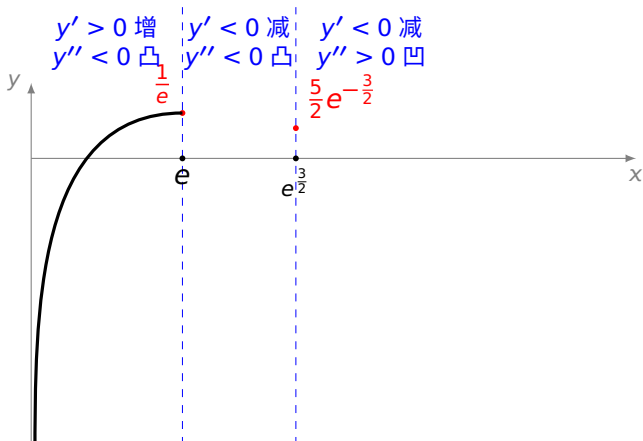
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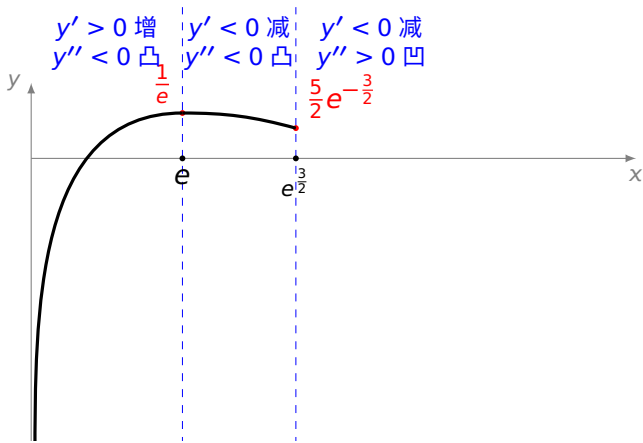
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