第 9 章 b：偏导数与全微分

数学系　　梁卓滨

2018-2019 学年 II

We are here now...

1. 偏导数

2. 全微分

偏导数引入

- 对一元函数 $y = f(x)$：导数 $y' = f'(x) \longleftrightarrow$ 变化率

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$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z =$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z = xy$$

例 4 求三元函数 $u = xyz + \frac{z}{x}$ 的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z = xy + \frac{1}{x}$$

例 5 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

例 5 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

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解法一

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\frac{\partial z}{\partial x}(2, 1) =$$

$$\frac{\partial z}{\partial y}(2, 1) =$$

例 5 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x =$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\frac{\partial z}{\partial x}(2, 1) =$$

$$\frac{\partial z}{\partial y}(2, 1) =$$

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解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y +$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\frac{\partial z}{\partial x}(2, 1) =$$

$$\frac{\partial z}{\partial y}(2, 1) =$$

例 5 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法一

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所以

$$\frac{\partial z}{\partial x}(2,1) = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} =$$

$$\frac{\partial z}{\partial y}(2,1) =$$

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所以

$$\frac{\partial z}{\partial x}(2,1) = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} =$$

$$\frac{\partial z}{\partial y}(2,1) =$$

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解法一

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$$\frac{\partial z}{\partial y} =$$

所以

$$\frac{\partial z}{\partial x}(2,1) = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\frac{\partial z}{\partial y}(2,1) =$$

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$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x$$

所以

$$\frac{\partial z}{\partial x}(2,1) = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\frac{\partial z}{\partial y}(2,1) =$$

例 5 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

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$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x - \frac{x}{y^2}$$

所以

$$\frac{\partial z}{\partial x}(2, 1) = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

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$$\frac{\partial z}{\partial x}(2, 1) = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

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所以

$$\frac{\partial z}{\partial x}(2, 1) = (y + \frac{1}{y})\Big|_{\substack{x=2\\y=1}} = 1 + \frac{1}{1} = 2$$

$$\frac{\partial z}{\partial y}(2, 1) = (x - \frac{x}{y^2})\Big|_{\substack{x=2\\y=1}} = 2 - \frac{2}{1} =$$

例 5 设 $z=xy+\frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2,1)$ 处的偏导数值

解法一

$$\frac{\partial z}{\partial x}=(xy+\frac{x}{y})'_x=(xy)'_x+(\frac{x}{y})'_x=y+\frac{1}{y}$$

$$\frac{\partial z}{\partial y}=(xy+\frac{x}{y})'_y=(xy)'_y+(\frac{x}{y})'_y=x-\frac{x}{y^2}$$

所以

$$\frac{\partial z}{\partial x}(2,1)=(y+\frac{1}{y})\Big|_{\substack{x=2\\y=1}}=1+\frac{1}{1}=2$$

$$\frac{\partial z}{\partial y}(2,1)=(x-\frac{x}{y^2})\Big|_{\substack{x=2\\y=1}}=2-\frac{2}{1}=0$$

例 5 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二

例 5 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二

求 $\frac{\partial z}{\partial x}(2, 1)$ 时，先对 $z(x, y)$ 取定 $y = 1$，得

例 5 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二

求 $\frac{\partial z}{\partial x}(2, 1)$ 时，先对 $z(x, y)$ 取定 $y = 1$，得

$$z(x, 1) = 2x$$

例 5 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二

求 $\frac{\partial z}{\partial x}(2,1)$ 时，先对 $z(x,y)$ 取定 $y = 1$，得

$$z(x, 1) = 2x \quad \Rightarrow \quad \frac{\partial z}{\partial x}(2,1) = \frac{d}{dx}\left[z(x, 1)\right]\Big|_{x=2}$$

例 5 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

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求 $\frac{\partial z}{\partial y}(2,1)$ 时，

例 5 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

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求 $\frac{\partial z}{\partial y}(2,1)$ 时，先对 $z(x,y)$ 取定 $x=2$，得

$$z(2, y) = 2y + \frac{2}{y}$$

例 5 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 $(2, 1)$ 处的偏导数值

解法二

求 $\frac{\partial z}{\partial x}(2, 1)$ 时，先对 $z(x, y)$ 取定 $y = 1$，得

$$z(x, 1) = 2x \quad \Rightarrow \quad \frac{\partial z}{\partial x}(2, 1) = \frac{d}{dx}\left[z(x, 1)\right]\Big|_{x=2} = 2$$

求 $\frac{\partial z}{\partial y}(2, 1)$ 时，先对 $z(x, y)$ 取定 $x = 2$，得

$$z(2, y) = 2y + \frac{2}{y} \quad \Rightarrow \quad \frac{\partial z}{\partial y}(2, 1) = \frac{d}{dy}\left[z(1, y)\right]\Big|_{y=1}$$

例 5 设 $z = xy + \frac{x}{y}$，求 $\frac{\partial z}{\partial x}$，$\frac{\partial z}{\partial y}$ 和在点 (2, 1) 处的偏导数值

解法二

求 $\frac{\partial z}{\partial x}(2,1)$ 时，先对 $z(x,y)$ 取定 $y=1$，得

$$z(x,\,1) = 2x \quad \Rightarrow \quad \frac{\partial z}{\partial x}(2,1) = \frac{d}{dx}\left[z(x,\,1)\right]\Big|_{x=2} = 2$$

求 $\frac{\partial z}{\partial y}(2,1)$ 时，先对 $z(x,y)$ 取定 $x=2$，得

$$z(2,\,y) = 2y + \frac{2}{y} \quad \Rightarrow \quad \frac{\partial z}{\partial y}(2,1) = \frac{d}{dy}\left[z(1,\,y)\right]\Big|_{y=1}$$

$$= 2 - \frac{2}{y^2}$$

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求 $\frac{\partial z}{\partial y}(2,1)$ 时，先对 $z(x,y)$ 取定 $x = 2$，得

$$z(2,y) = 2y + \frac{2}{y} \quad \Rightarrow \quad \frac{\partial z}{\partial y}(2,1) = \frac{d}{dy}[z(1,y)]\Big|_{y=1}$$

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$$= 2 - \frac{2}{y^2}\Big|_{y=1} = 0$$

例 6 设 $f(x,y)=\begin{cases} \frac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$，求 $f_x(0,0), f_y(0,0)$

例 6 设 $f(x, y) = \begin{cases} \dfrac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0)$$

$$f_y(0, 0)$$

例 6 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$f_x(0, 0)$ $\qquad$ $f(x, 0)$

$f_y(0, 0)$

例 6 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0}$$

$$f_y(0, 0)$$

例 6 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0}$$

$$f_y(0, 0)$$

例 6 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x,0)]\bigg|_{x=0} = \frac{d}{dx}[0]\bigg|_{x=0} = 0,$$

$$f_y(0, 0)$$

例 6 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\bigg|_{x=0} = \frac{d}{dx}[0]\bigg|_{x=0} = 0,$$

$$f_y(0, 0) \qquad f(0, y)$$

例 6 设 $f(x, y) = \begin{cases} \dfrac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x,0)]\bigg|_{x=0} = \frac{d}{dx}[0]\bigg|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0,y)]\bigg|_{x=0}$$

例 6 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x,0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0,y)]\Big|_{x=0} = \frac{d}{dy}[0]\Big|_{y=0}$$

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解

$$f_x(0, 0) = \frac{d}{dx}[f(x,0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

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解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)]\Big|_{x=0} = \frac{d}{dy}[0]\Big|_{y=0} = 0,$$

注 上述 $f(x, y)$ 在 $(0, 0)$ 处存在偏导数 $f_x(0, 0)$ 和 $f_y(0, 0)$,

例 6 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x,0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0,y)]\Big|_{x=0} = \frac{d}{dy}[0]\Big|_{y=0} = 0,$$

注 上述 $f(x, y)$ 在 $(0, 0)$ 处存在偏导数 $f_x(0, 0)$ 和 $f_y(0,0)$，但可以证明在 $(0, 0)$ 处不连续

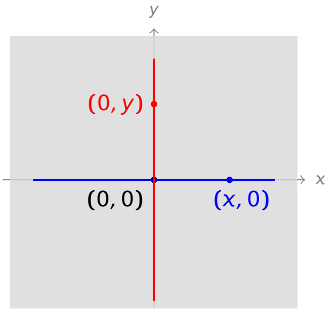
例 6 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)]\Big|_{x=0} = \frac{d}{dy}[0]\Big|_{y=0} = 0,$$

注 上述 $f(x, y)$ 在 $(0, 0)$ 处存在偏导数 $f_x(0, 0)$ 和 $f_y(0, 0)$，但可以证明在 $(0, 0)$ 处不连续



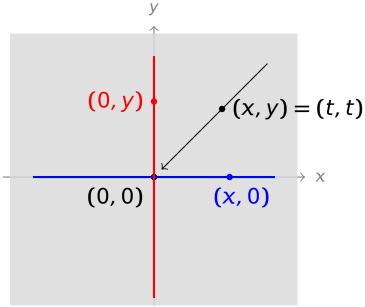
例 6 设 $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x, 0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

$$f_y(0, 0) = \frac{d}{dy}[f(0, y)]\Big|_{x=0} = \frac{d}{dy}[0]\Big|_{y=0} = 0,$$

注 上述 $f(x, y)$ 在 $(0, 0)$ 处存在偏导数 $f_x(0, 0)$ 和 $f_y(0, 0)$，但可以证明在 $(0, 0)$ 处不连续



例 6 设 $f(x, y) = \begin{cases} \dfrac{xy}{x^2+y^2}, & (x,y)\neq(0,0) \\ 0, & (x,y)=(0,0) \end{cases}$，求 $f_x(0, 0), f_y(0, 0)$

解

$$f_x(0, 0) = \frac{d}{dx}[f(x,0)]\Big|_{x=0} = \frac{d}{dx}[0]\Big|_{x=0} = 0,$$

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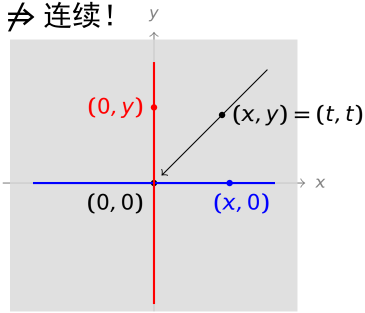
注 上述 $f(x, y)$ 在 $(0, 0)$ 处存在偏导数 $f_x(0, 0)$ 和 $f_y(0,0)$，但可以证明在 $(0, 0)$ 处不连续

所以，偏导数存在 $\not\Rightarrow$ 连续！



偏导数的极限定义

- 一元函数 $y = f(x)$ 在 $x = x_0$ 处的导数定义为：

$$f'(x_0) =$$

偏导数的极限定义

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$$f'(x_0) = \lim \quad \underline{\hspace{6cm}}$$

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- 一元函数 $y=f(x)$ 在 $x=x_0$ 处的导数定义为：

$$f'(x_0)=\lim \frac{f(x_0+\Delta x)-f(x_0)}{}$$

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二阶偏导数

$$
\begin{tikzcd}
 & \dfrac{\partial z}{\partial x} \\
z = f(x, y) \arrow[ur] \arrow[dr] & \\
 & \dfrac{\partial z}{\partial y}
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
z=f(x,y) \arrow[ur] \arrow[dr] & & {} \\
 & \frac{\partial z}{\partial y} &
\end{tikzcd}
$$

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\begin{tikzcd}
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 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \\
 & \frac{\partial z}{\partial y} &
\end{tikzcd}
$$

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\begin{tikzcd}
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right) \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right) \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & {}
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right) \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right) \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right)
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): \qquad \dfrac{\partial^2 z}{\partial x \partial x}, \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right) \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right)
\end{tikzcd}
$$

二阶偏导数

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\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[r] & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial y}, \\
z = f(x, y) \arrow[ur] \arrow[dr] & & \\
 & \frac{\partial z}{\partial y} \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)
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 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y \partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
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二阶偏导数

$$\begin{tikzcd}
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 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial y}, \\
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 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y\partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
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\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial y}, \\
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 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial x}, \\
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 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x \partial y}, & z_{xy}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y \partial x}, & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & & \\
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\end{tikzcd}
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二阶偏导数

$$z = f(x, y) \longrightarrow \frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx},$$

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 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x\partial y}, & z_{xy}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & & \\
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\begin{tikzcd}[column sep=small]
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \quad f_{xx} \\
 & \dfrac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right): & \dfrac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y \partial x}, \quad z_{yx}, \\
 & \dfrac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right): & \dfrac{\partial^2 z}{\partial y \partial y}, \quad z_{yy},
\end{tikzcd}
$$

二阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x\partial x},\quad z_{xx},\quad f_{xx} \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x\partial y},\quad z_{xy},\quad f_{xy} \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y\partial x},\quad z_{yx}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y\partial y},\quad z_{yy},
\end{tikzcd}
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二阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right):
\end{tikzcd}
$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial x}, \quad z_{xx}, \quad f_{xx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): \quad \frac{\partial^2 z}{\partial x \partial y}, \quad z_{xy}, \quad f_{xy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial x}, \quad z_{yx}, \quad f_{yx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): \quad \frac{\partial^2 z}{\partial y \partial y}, \quad z_{yy},$$

二阶偏导数

$$
\begin{tikzcd}[row sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial x},\quad z_{xx},\quad f_{xx} \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right): & \frac{\partial^2 z}{\partial x\partial y},\quad z_{xy},\quad f_{xy} \\
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 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y\partial x},\quad z_{yx},\quad f_{yx} \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right): & \frac{\partial^2 z}{\partial y\partial y},\quad z_{yy},\quad f_{yy}
\end{tikzcd}
$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x =$$

$$z_y =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+$$

$$z_y=$$

$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

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$$z_{xx}=$$

$$z_{xy}=$$

$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

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$$z_{xx} =$$

$$z_{xy} =$$

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$$z_{yy} =$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

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$$z_{xy} =$$

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解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

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解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

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解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

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$$z_{xx}=(ye^{xy}+2y^2)'_x=(ye^{xy})'_x+(2y^2)'_x=y^2e^{xy}$$

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$$z_{yx}=$$

$$z_{yy}=$$

例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

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解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

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解

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$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

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解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

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例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

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例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

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例 求 $z = e^{xy} + 2xy^2$ 全部二阶偏导数

解

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$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2e^{xy} +$$

例 求 $z=e^{xy}+2xy^2$ 全部二阶偏导数

解

$$z_x=(e^{xy}+2xy^2)'_x=(e^{xy})'_x+(2xy^2)'_x=ye^{xy}+2y^2$$

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解

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$$z_{yy}=(xe^{xy}+4xy)'_y=(xe^{xy})'_y+(4xy)'_y=x^2e^{xy}+4x$$

注 此例成立 $z_{xy}=z_{yx}$

三阶偏导数

$$\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)
\end{tikzcd}$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) & \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[uur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) &
\end{tikzcd}
$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right)
\end{tikzcd}
$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): &
\end{tikzcd}
$$

三阶偏导数

$$\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial x}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): &
\end{tikzcd}$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$
\begin{tikzcd}
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial y}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial x}, \\
z=f(x,y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial y}, \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial x\partial x}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): &
\end{tikzcd}
$$

三阶偏导数

$$z=f(x,y)\ \to\ \frac{\partial z}{\partial x}\ \to\ \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$z=f(x,y)\ \to\ \frac{\partial z}{\partial y}\ \to\ \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \qquad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \qquad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$z=f(x,y) \longrightarrow \frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$z=f(x,y) \longrightarrow \frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):$$

三阶偏导数

$$
z=f(x,y)\;\rightarrow\;\frac{\partial z}{\partial x}\;\rightarrow\;
\begin{cases}
\dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right)\rightarrow
\begin{cases}
\dfrac{\partial}{\partial x}\left(\dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right)\right): & \dfrac{\partial^3 z}{\partial x\partial x\partial x},\\[2ex]
\dfrac{\partial}{\partial y}\left(\dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial x}\right)\right): & \dfrac{\partial^3 z}{\partial x\partial x\partial y},
\end{cases}\\[6ex]
\dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right)\rightarrow
\begin{cases}
\dfrac{\partial}{\partial x}\left(\dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right)\right): & \dfrac{\partial^3 z}{\partial x\partial y\partial x},\\[2ex]
\dfrac{\partial}{\partial y}\left(\dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial x}\right)\right): & \dfrac{\partial^3 z}{\partial x\partial y\partial y},
\end{cases}
\end{cases}
$$

$$
z=f(x,y)\;\rightarrow\;\frac{\partial z}{\partial y}\;\rightarrow\;
\begin{cases}
\dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right)\rightarrow
\begin{cases}
\dfrac{\partial}{\partial x}\left(\dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right)\right): & \dfrac{\partial^3 z}{\partial y\partial x\partial x},\\[2ex]
\dfrac{\partial}{\partial y}\left(\dfrac{\partial}{\partial x}\left(\dfrac{\partial z}{\partial y}\right)\right): & \dfrac{\partial^3 z}{\partial y\partial x\partial y},
\end{cases}\\[6ex]
\dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right)\rightarrow
\begin{cases}
\dfrac{\partial}{\partial x}\left(\dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right)\right): & \dfrac{\partial^3 z}{\partial y\partial y\partial x},\\[2ex]
\dfrac{\partial}{\partial y}\left(\dfrac{\partial}{\partial y}\left(\dfrac{\partial z}{\partial y}\right)\right): & \dfrac{\partial^3 z}{\partial y\partial y\partial y},
\end{cases}
\end{cases}
$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z = f(x, y)$$

$$z = f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x}, \ z_{xxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y}, \ z_{xxy},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},$$

$$z = f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$z=f(x,y)\ \longrightarrow\ \frac{\partial z}{\partial x}\ \longrightarrow\ \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\ \longrightarrow\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\ \longrightarrow\ \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial z}{\partial x}\ \longrightarrow\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\ \longrightarrow\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\ \longrightarrow\ \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial y},$$

$$z=f(x,y)\ \longrightarrow\ \frac{\partial z}{\partial y}\ \longrightarrow\ \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\ \longrightarrow\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\ \longrightarrow\ \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial z}{\partial y}\ \longrightarrow\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\ \longrightarrow\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\ \longrightarrow\ \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z=f(x,y) \longrightarrow \frac{\partial z}{\partial x},\quad \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial y},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z = f(x, y)$$

$$z = f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},$$

$$z = f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$z=f(x,y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$z=f(x,y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right), \quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},$$

三阶偏导数

$$
\begin{tikzcd}[column sep=small]
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx}, \\
 & \frac{\partial z}{\partial x} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx}, \\
z = f(x, y) \arrow[uur] \arrow[ddr] & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): & \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy}, \\
 & & \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx}, \\
 & \frac{\partial z}{\partial y} \arrow[ur] \arrow[dr] & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy}, \\
 & & \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \arrow[r] \arrow[dr] & \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx}, \\
 & & & \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): & \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},
\end{tikzcd}
$$

三阶偏导数

$$z = f(x, y) \to \frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},$$

$$z = f(x, y) \to \frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial x \partial y},\ z_{yxy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y \partial y \partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)\ \longrightarrow\ \frac{\partial z}{\partial x}\ \longrightarrow\ \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\ \longrightarrow\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\ \longrightarrow\ \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial z}{\partial x}\ \longrightarrow\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\ \longrightarrow\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\ \longrightarrow\ \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$z=f(x,y)\ \longrightarrow\ \frac{\partial z}{\partial y}\ \longrightarrow\ \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\ \longrightarrow\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\ \longrightarrow\ \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial z}{\partial y}\ \longrightarrow\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\ \longrightarrow\ \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\ \longrightarrow\ \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \qquad \frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \qquad \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \qquad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z = f(x, y)$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \rightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \rightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right): \quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \rightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \rightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right): \quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},$$

三阶偏导数

$$z = f(x, y) \to \frac{\partial z}{\partial x},\quad \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \to \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial x \partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial x \partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial y \partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right) : \quad \frac{\partial^3 z}{\partial x \partial y \partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial x \partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial x \partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial y \partial x},\ z_{yyx},\ f_{yyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \to \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right) : \quad \frac{\partial^3 z}{\partial y \partial y \partial y},\ z_{yyy},$$

三阶偏导数

$$z=f(x,y)$$

$$\frac{\partial z}{\partial x} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial z}{\partial y} \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right),\quad \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial x},\ z_{xxx},\ f_{xxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial x\partial y},\ z_{xxy},\ f_{xxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial x},\ z_{xyx},\ f_{xyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)\right):\quad \frac{\partial^3 z}{\partial x\partial y\partial y},\ z_{xyy},\ f_{xyy}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial x},\ z_{yxx},\ f_{yxx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial x\partial y},\ z_{yxy},\ f_{yxy}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) \longrightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial x},\ z_{yyx},\ f_{yyx}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right)\right):\quad \frac{\partial^3 z}{\partial y\partial y\partial y},\ z_{yyy},\ f_{yyy}$$

例 1 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

例 1 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x =$$

$$z_y =$$

例 1 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解
$$z_x =$$
$$z_y =$$
$$z_{xx} =$$
$$z_{xy} =$$
$$z_{yx} =$$
$$z_{yy} =$$

例 1 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 1 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 1 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解 $$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 1 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

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$$z_y =$$

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解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y =$$

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$$z_{xx} =$$

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$$z_{yx} =$$

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$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$
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$$z_{xx} =$$

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暨南大学 JINAN UNIVERSITY

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$$z_{yx}=$$

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$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$
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$$z_{yy} =$$

$$z_{xxx} =$$

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$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y =$$

$$z_{xxx} =$$

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$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3$$

$$z_{xxx} =$$

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$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} =$$

例 1 求 $z = x^3y^2 - 3xy^3 - xy + 1$ 全部二阶偏导数及 $\frac{\partial^3 z}{\partial x^3}$

解

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$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

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$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} = (6xy^2)'_x =$$

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$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2-1$$

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$$z_{xxx}=(6xy^2)'_x=6y^2$$

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$$z_{xy}=(3x^2y^2-3y^3-y)'_y=6x^2y-9y^2-1$$
$$z_{yx}=(2x^3y-9xy^2-x)'_x=6x^2y-9y^2-1$$
$$z_{yy}=(2x^3y-9xy^2-x)'_y=2x^3-18xy$$

$$z_{xxx}=(6xy^2)'_x=6y^2$$

注 此例成立 $z_{xy}=z_{yx}$

例 2 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

例 2 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解 $z_x =$

$z_y =$

例 2 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 2 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 2 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 2 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 2 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y =$$
$$z_{xx} =$$
$$z_{xy} =$$
$$z_{yx} =$$
$$z_{yy} =$$
$$z_{xyy} =$$

例 2 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 2 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解

$$z_x = (x\sin(3y))'_x = \sin(3y)$$

$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x =$$

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$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$
$$z_{xx} = (\sin(3y))'_x = 0$$
$$z_{xy} =$$
$$z_{yx} =$$
$$z_{yy} =$$
$$z_{xyy} =$$

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$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$
$$z_{xy} = (\sin(3y))'_y =$$
$$z_{yx} =$$
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$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$
$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$

$$z_{yx} =$$

$$z_{yy} =$$

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$$z_x = (x\sin(3y))'_x = \sin(3y)$$
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$$z_{xx} = (\sin(3y))'_x = 0$$
$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$
$$z_{yx} = (3x\cos(3y))'_x =$$
$$z_{yy} =$$
$$z_{xyy} =$$

例 2 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

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$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$
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$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$
$$z_{yy} =$$
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例 2 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

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$$z_x = (x\sin(3y))'_x = \sin(3y)$$

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$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$

$$z_{yy} = (3x\cos(3y))'_y =$$

$$z_{xyy} =$$

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$$z_x = (x\sin(3y))'_x = \sin(3y)$$

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$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$

$$z_{yy} = (3x\cos(3y))'_y = -9x\sin(3y)$$

$$z_{xyy} =$$

例 2 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

解
$$z_x = (x\sin(3y))'_x = \sin(3y)$$
$$z_y = (x\sin(3y))'_y = 3x\cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$
$$z_{xy} = (\sin(3y))'_y = 3\cos(3y)$$
$$z_{yx} = (3x\cos(3y))'_x = 3\cos(3y)$$
$$z_{yy} = (3x\cos(3y))'_y = -9x\sin(3y)$$

$$z_{xyy} = (3\cos(3y))'_y =$$

例 2 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

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$$z_{yy} = (3x\cos(3y))'_y = -9x\sin(3y)$$

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注 此例成立 $z_{xy} = z_{yx}$

例 2 求 $z = x\sin(3y)$ 全部二阶偏导数及 z_{xyy}

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$$z_x = (x\sin(3y))'_x = \sin(3y)$$
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$$z_{yy} = (3x\cos(3y))'_y = -9x\sin(3y)$$

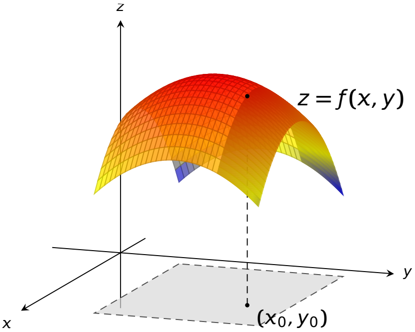
$$z_{xyy} = (3\cos(3y))'_y = -9\sin(3y)$$

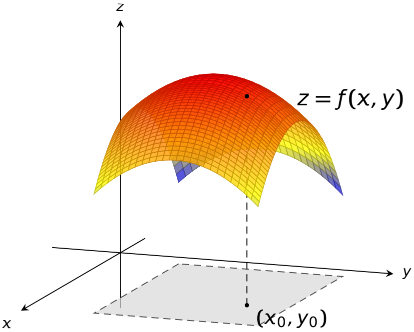
注 此例成立 $z_{xy} = z_{yx}$

性质 设有二元函数 $z = f(x, y)$。若 $\frac{\partial^2 z}{\partial y \partial x}$ 和 $\frac{\partial^2 z}{\partial x \partial y}$ 均连续，则

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$$

偏导数的几何直观





$$\frac{\partial z}{\partial x}(x_0,y_0)=\frac{d}{dx}[f(x,y_0)]\Bigg|_{x=x_0}$$

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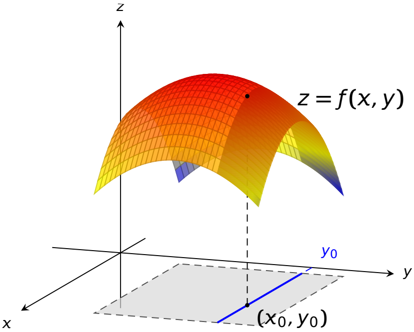

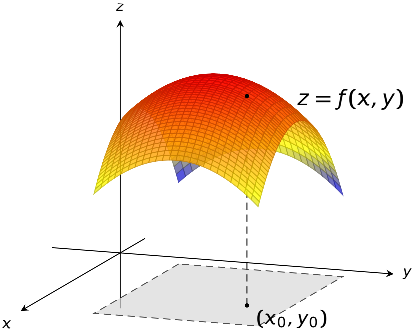

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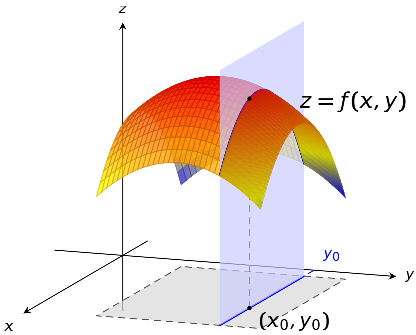

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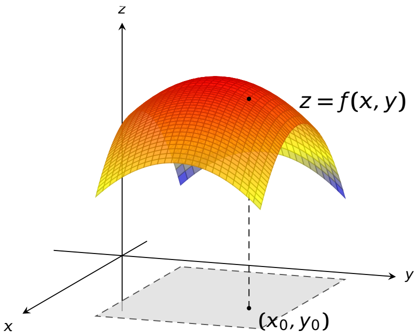

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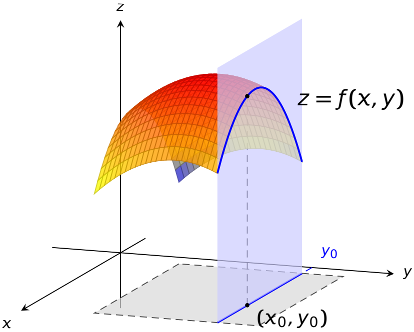

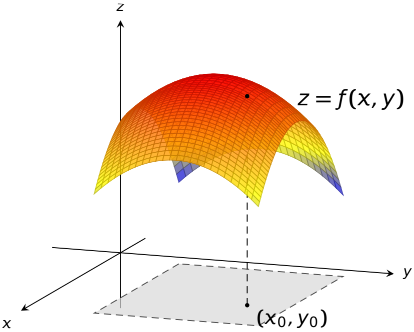

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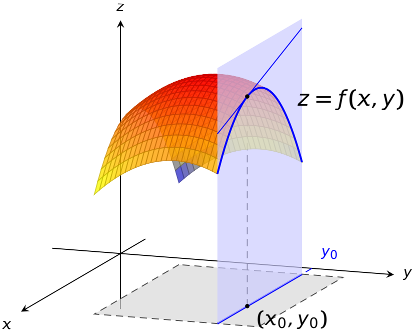

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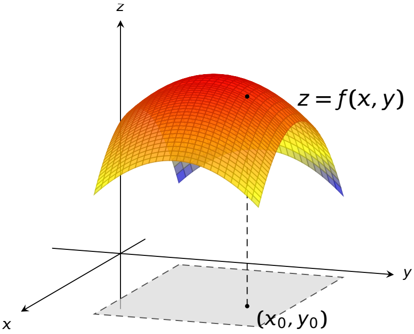

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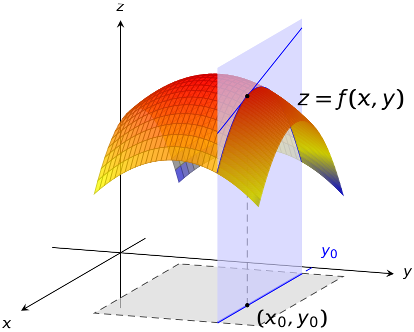

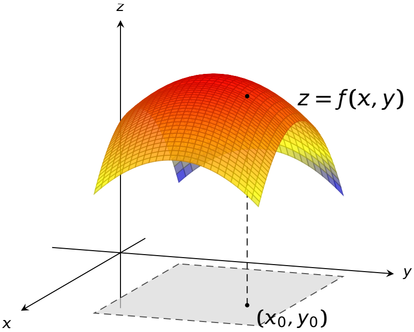

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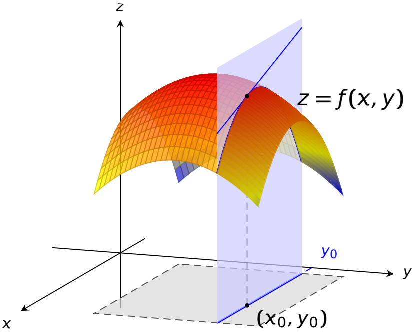

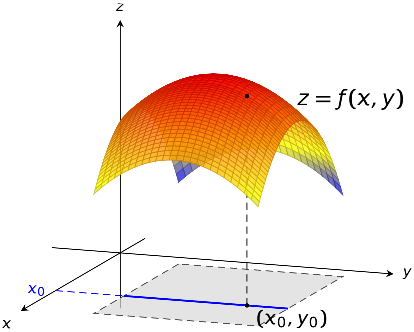

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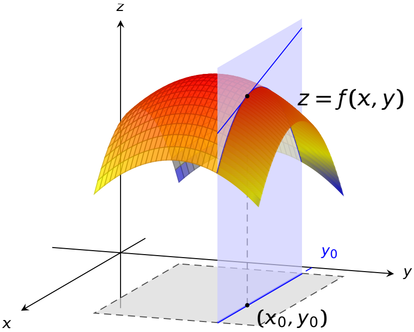

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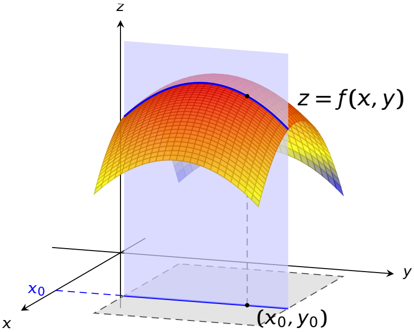

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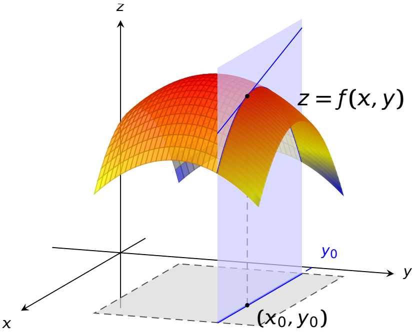

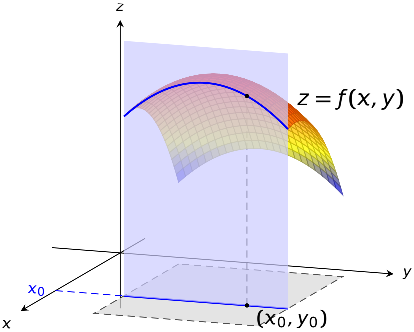

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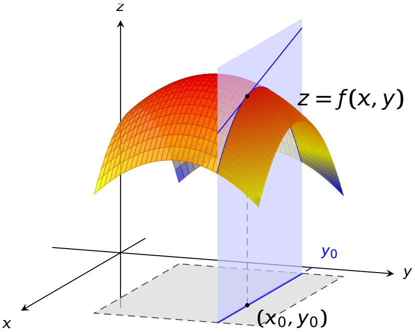

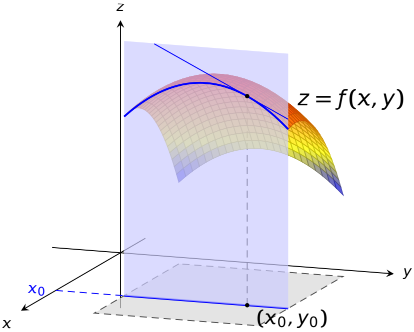

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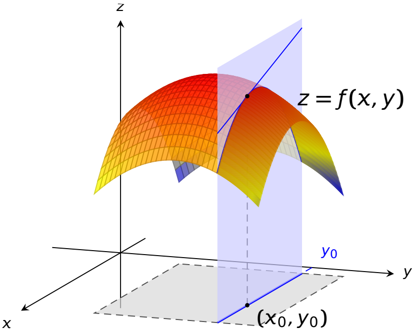

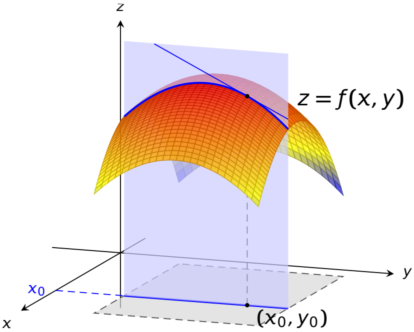

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We are here now...

1. 偏导数

2. 全微分

可微

- 回忆：一元函数 $z = f(x)$ 在 $x = x_0$ 处可微，指

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$$\Delta z = f(x_0+\Delta x) - f(x_0)$$

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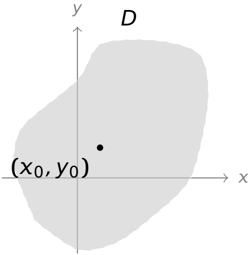

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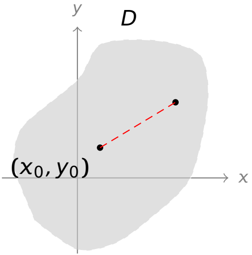

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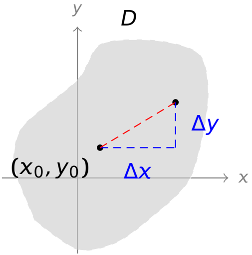

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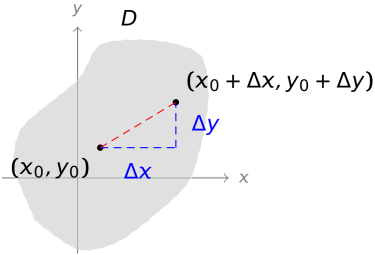

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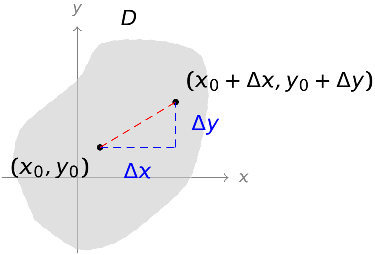

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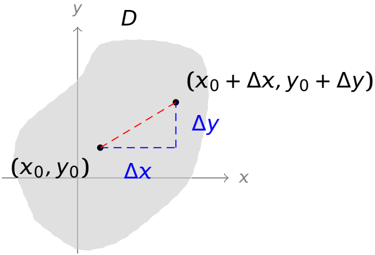

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$$= A\Delta x + B\Delta y + o(\quad)$$

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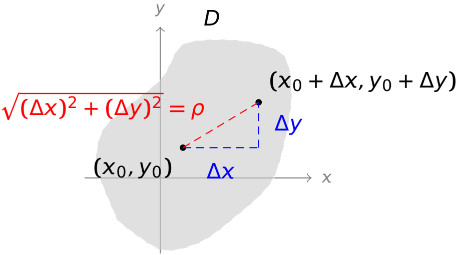

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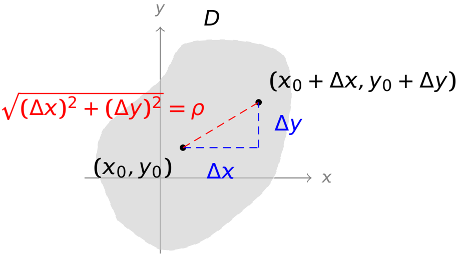

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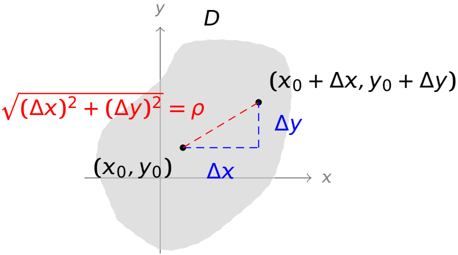

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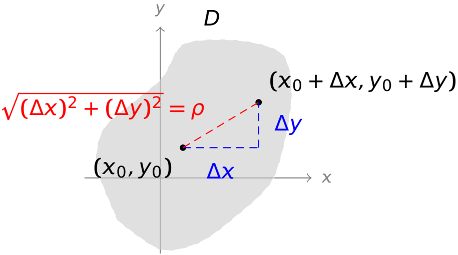

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例 设 $z=f(x,y)=x^2+y^2$，证明函数可微，并计算全微分 dz

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解

$$\Delta z = f(x+\Delta x, y+\Delta y) - f(x,y)$$

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解

$$\begin{aligned}\Delta z&=f(x+\Delta x,y+\Delta y)-f(x,y)\\&=\left[(x+\Delta x)^2+(y+\Delta y)^2\right]\end{aligned}$$

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例 设 $z = f(x, y) = x^2 + y^2$，证明函数可微，并计算全微分 dz

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$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= \left[(x + \Delta x)^2 + (y + \Delta y)^2\right] - \left[x^2 + y^2\right]\end{aligned}$$

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$$\begin{aligned}\Delta z&=f(x+\Delta x,y+\Delta y)-f(x,y)\\&=\left[(x+\Delta x)^2+(y+\Delta y)^2\right]-\left[x^2+y^2\right]\\&=2x\Delta x+2y\Delta y+\left[(\Delta x)^2+(\Delta y)^2\right]\end{aligned}$$

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$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= \left[(x + \Delta x)^2 + (y + \Delta y)^2\right] - \left[x^2 + y^2\right] \\ &= 2x\Delta x + 2y\Delta y + \left[(\Delta x)^2 + (\Delta y)^2\right] \\ &= 2x\Delta x + 2y\Delta y + \rho^2\end{aligned}$$

定义 二元函数 $z=f(x,y)$ 在 (x_0,y_0) 处可微，是指 ∃ 数 A, B 使得：

$$\Delta z=f(x_0+\Delta x,y_0+\Delta y)-f(x_0,y_0)=\underbrace{A\Delta x+B\Delta y}_{=dz}+o(\rho)$$

例 设 $z=f(x,y)=x^2+y^2$，证明函数可微，并计算全微分 dz

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所以 $z = x^2 + y^2$ 可微， 并且 $dz = 2xdx + 2ydy$

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多元函数的全微分

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$$du = u_x dx + u_y dy + u_z dz$$

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所以 $(1.04)^{2.02}\approx dz+1=0.08+1=1.08$

可微、偏导数存在、连续的区别与联系

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