§1.1 二阶三阶行列式

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2017 - 2018 学年 I

教学要求

掌握求解：

- ◇ 二阶行列式计算
- ♣ 三阶行列式计算

- 行列式的概念来源于线性方程组的求解问题
- 17 世纪末由日本数学家关孝和及德国数学家莱布尼茨引入

$$\begin{array}{ccc} \text{二阶行列式} & \longleftrightarrow & \text{二元线性方程组} \\ \text{三阶行列式} & \longleftrightarrow & \text{三元线性方程组} \\ \vdots & & \vdots \\ n\ \text{阶行列式} & \longleftrightarrow & n\ \text{元线性方程组} \\ \vdots & & \vdots \end{array}$$

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & \quad (1) \\ a_{21}x + a_{22}y = b_2 & \quad (2) \end{cases}$$

用消元法求解：

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & \quad (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & \quad (2) \times a_{12} \end{cases}$$

用消元法求解：$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \quad \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

用消元法求解：$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

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$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1)\times a_{22} & \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2)\times a_{12} & \Rightarrow a_{21}a_{12}x + a_{22}a_{12}y = b_2a_{12} \end{cases}$$

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用消元法求解：$(1)\times a_{22} - (2)\times a_{12}$，消去 y，得：

$$x = \underline{\qquad\qquad\qquad}$$

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1)\times a_{22} \quad \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1 a_{22} \\ a_{21}x + a_{22}y = b_2 & (2)\times a_{12} \quad \Rightarrow a_{21}a_{12}x + a_{22}a_{12}y = b_2 a_{12} \end{cases}$$

用消元法求解：$(1)\times a_{22} - (2)\times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{\phantom{a_{11}a_{22} - a_{12}a_{21}}}$$

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$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} & \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} & \Rightarrow a_{21}a_{12}x + a_{22}a_{12}y = b_2a_{12} \end{cases}$$

用消元法求解：$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

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二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \end{cases}$$

用消元法求解：$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

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用消元法求解：$(1)\times a_{22}-(2)\times a_{12}$，消去 y，得：

$$x = \frac{b_1a_{22}-a_{12}b_2}{a_{11}a_{22}-a_{12}a_{21}}$$

$(2)\times a_{11}-(1)\times a_{21}$，消去 x，得：

$$y = \frac{a_{11}b_2-b_1a_{21}}{\phantom{a_{11}a_{22}-a_{12}a_{21}}}$$

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$$\begin{cases} a_{11}x+a_{12}y=b_1 & (1)\times a_{21} & \Rightarrow a_{11}a_{21}x+a_{12}a_{21}y=b_1a_{21} \\ a_{21}x+a_{22}y=b_2 & (2)\times a_{11} & \Rightarrow a_{21}a_{11}x+a_{22}a_{11}y=b_2a_{11} \end{cases}$$

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- 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$

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- 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$，称为二阶行列式

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$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

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练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $x =$, $y =$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $x =$, $y =$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $x = \dfrac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \underline{\qquad}$, $y =$

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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三元线性方程组

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规律

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$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{l} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}$$

规律 不同行不同列的 3 个元素乘积，共 $3! = 6$ 个，并且：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{l} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}$$

规律 不同行不同列的 3 个元素乘积，共 $3! = 6$ 个，并且：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \qquad +$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

为表示三元方程组的解，定义三阶行列式：

$$
\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{l} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}
$$

规律 不同行不同列的 3 个元素乘积，共 $3! = 6$ 个，并且：

$$
\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad + \qquad\qquad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}
$$

为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{l} a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32}-a_{12}a_{21}a_{33}-a_{13}a_{22}a_{31} \end{array}$$

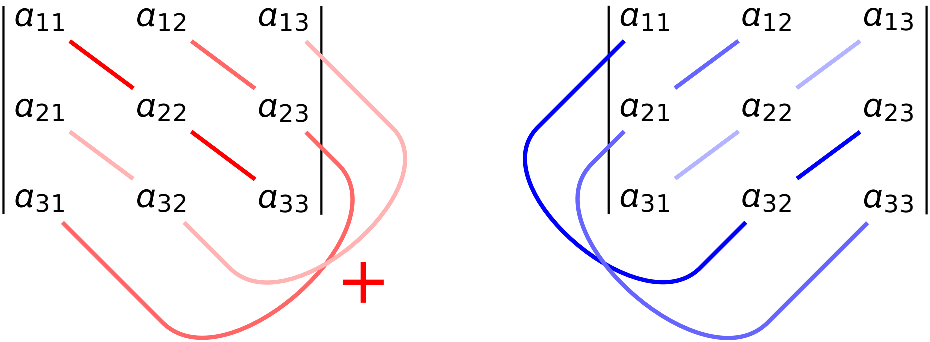
规律 不同行不同列的 3 个元素乘积，共 $3!=6$ 个，并且：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad + \qquad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{l} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}$$

规律 不同行不同列的 3 个元素乘积，共 $3! = 6$ 个，并且：



为表示三元方程组的解，定义三阶行列式：

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{array}{l} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}$$

规律 不同行不同列的 3 个元素乘积，共 $3! = 6$ 个，并且：

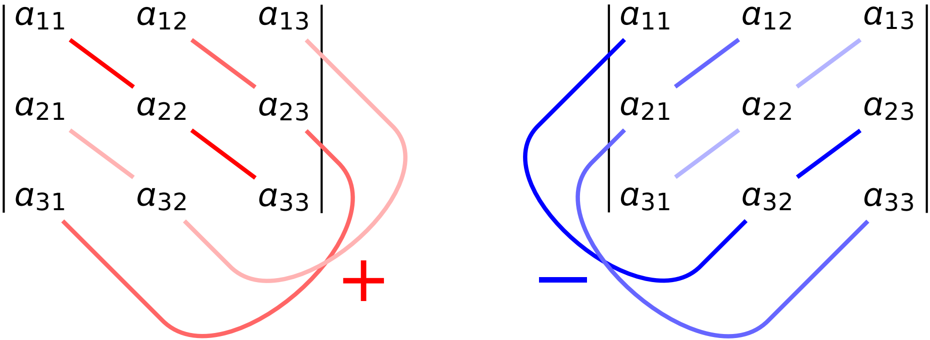
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad + \qquad\qquad - \quad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad + \qquad - \quad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} =$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \quad 1\times 0\times 6 + 2\times 5\times(-1) + 3\times 4\times 0$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad + \qquad - \quad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{array}{c} 1\times 0\times 6 + 2\times 5\times(-1) + 3\times 4\times 0 \\ -1\times 5\times 0 - 2\times 4\times 6 - 3\times 0\times(-1) \end{array}$$

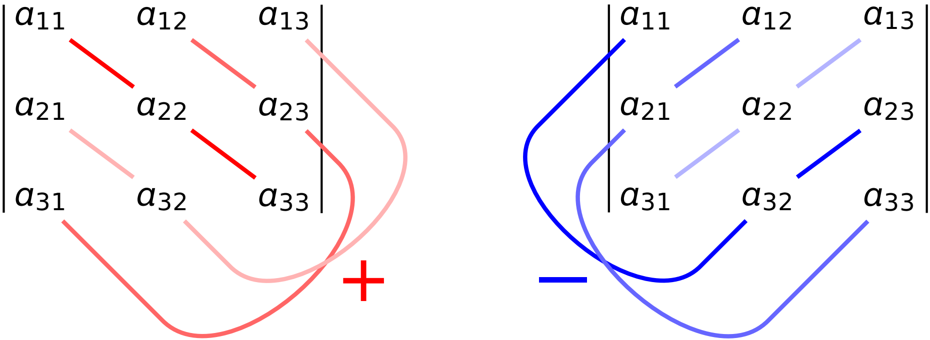
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad + \qquad - \quad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{array}{c} 1\times 0\times 6 + 2\times 5\times(-1) + 3\times 4\times 0 \\ -1\times 5\times 0 - 2\times 4\times 6 - 3\times 0\times(-1) \end{array} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$



例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{matrix} 1\times 0\times 6 + 2\times 5\times(-1) + 3\times 4\times 0 \\ -1\times 5\times 0 - 2\times 4\times 6 - 3\times 0\times(-1) \end{matrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = 1\times 5\times 1 + 0\times 0\times 1 + (-1)\times 3\times 4$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \qquad - \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{matrix} 1\times 0\times 6 + 2\times 5\times(-1) + 3\times 4\times 0 \\ -1\times 5\times 0 - 2\times 4\times 6 - 3\times 0\times(-1) \end{matrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{matrix} 1\times 5\times 1 + 0\times 0\times 1 + (-1)\times 3\times 4 \\ -1\times 0\times 4 - 0\times 3\times 1 - (-1)\times 5\times 1 \end{matrix}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad + \qquad - \quad \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

例 计算

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{matrix} 1\times 0\times 6 + 2\times 5\times(-1) + 3\times 4\times 0 \\ -1\times 5\times 0 - 2\times 4\times 6 - 3\times 0\times(-1) \end{matrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{matrix} 1\times 5\times 1 + 0\times 0\times 1 + (-1)\times 3\times 4 \\ -1\times 0\times 4 - 0\times 3\times 1 - (-1)\times 5\times 1 \end{matrix} = -2$$

这时方程组
$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$
的解可以表示为:

,

这时方程组
$$\begin{cases} a_{11}x+a_{12}y+a_{13}z=b_1 & (1) \\ a_{21}x+a_{22}y+a_{23}z=b_2 & (2) \\ a_{31}x+a_{32}y+a_{33}z=b_3 & (3) \end{cases}$$

的解可以表示为:

$$x=\frac{\begin{array}{c} b_1a_{22}a_{33}+a_{12}a_{23}b_3+a_{13}b_2a_{32} \\ -b_1a_{23}a_{32}-a_{12}b_2a_{33}-a_{13}a_{22}b_3 \end{array}}{\begin{array}{c} a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32}-a_{12}a_{21}a_{33}-a_{13}a_{22}a_{31} \end{array}}=$$

,

这时方程组
$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$
的解可以表示为:

$$x = \frac{\begin{array}{c} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{array}}{\begin{array}{c} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}} = \frac{\qquad}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

,

这时方程组

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{\begin{array}{c} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{array}}{\begin{array}{c} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

,

这时方程组
$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$
的解可以表示为:
$$x = \frac{\begin{array}{c} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{array}}{\begin{array}{c} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$
$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} \qquad ,$$

这时方程组
$$\begin{cases} a_{11}x+a_{12}y+a_{13}z=b_1 & (1) \\ a_{21}x+a_{22}y+a_{23}z=b_2 & (2) \\ a_{31}x+a_{32}y+a_{33}z=b_3 & (3) \end{cases}$$

的解可以表示为:

$$x=\frac{\begin{array}{c} b_1a_{22}a_{33}+a_{12}a_{23}b_3+a_{13}b_2a_{32} \\ -b_1a_{23}a_{32}-a_{12}b_2a_{33}-a_{13}a_{22}b_3 \end{array}}{\begin{array}{c} {\color{red}a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32}} \\ {\color{blue}-a_{11}a_{23}a_{32}-a_{12}a_{21}a_{33}-a_{13}a_{22}a_{31}} \end{array}}=\frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

$$y=\frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}, \quad z=\frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

这时方程组
$$\begin{cases} a_{11}x+a_{12}y+a_{13}z=b_1 & (1)\\ a_{21}x+a_{22}y+a_{23}z=b_2 & (2)\\ a_{31}x+a_{32}y+a_{33}z=b_3 & (3)\end{cases}$$

的解可以表示为：

$$x=\frac{\begin{array}{c}b_1a_{22}a_{33}+a_{12}a_{23}b_3+a_{13}b_2a_{32}\\ -b_1a_{23}a_{32}-a_{12}b_2a_{33}-a_{13}a_{22}b_3\end{array}}{\begin{array}{c}a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32}\\ -a_{11}a_{23}a_{32}-a_{12}a_{21}a_{33}-a_{13}a_{22}a_{31}\end{array}}=\frac{\begin{vmatrix} b_1 & a_{12} & a_{13}\\ b_2 & a_{22} & a_{23}\\ b_3 & a_{32} & a_{33}\end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}}=\frac{\quad}{D}$$

$$y=\frac{\begin{vmatrix} a_{11} & b_1 & a_{13}\\ a_{21} & b_2 & a_{23}\\ a_{31} & b_3 & a_{33}\end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}}=\frac{\quad}{D},\qquad z=\frac{\begin{vmatrix} a_{11} & a_{12} & b_1\\ a_{21} & a_{22} & b_2\\ a_{31} & a_{32} & b_3\end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}}=\frac{\quad}{D}$$

这时方程组
$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$
的解可以表示为:
$$x = \frac{\begin{matrix} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{matrix}}{\begin{matrix} {\color{red}a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}} \\ {\color{blue}-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}} \end{matrix}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{D_x}{D}$$
$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\quad}{D}, \qquad z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\quad}{D}$$

这时方程组
$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{\begin{matrix} b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32} \\ -b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3 \end{matrix}}{\begin{matrix} \color{red}{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}} \\ \color{blue}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}} \end{matrix}} = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{D_x}{D}$$

$$y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{D_y}{D}, \quad z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\quad}{D}$$

这时方程组
$$\begin{cases} a_{11}x+a_{12}y+a_{13}z=b_1 & (1)\\ a_{21}x+a_{22}y+a_{23}z=b_2 & (2)\\ a_{31}x+a_{32}y+a_{33}z=b_3 & (3)\end{cases}$$

的解可以表示为:

$$x=\frac{\begin{matrix} b_1a_{22}a_{33}+a_{12}a_{23}b_3+a_{13}b_2a_{32}\\ -b_1a_{23}a_{32}-a_{12}b_2a_{33}-a_{13}a_{22}b_3\end{matrix}}{\begin{matrix} {\color{red}a_{11}a_{22}a_{33}+a_{12}a_{23}a_{31}+a_{13}a_{21}a_{32}}\\ {\color{blue}-a_{11}a_{23}a_{32}-a_{12}a_{21}a_{33}-a_{13}a_{22}a_{31}}\end{matrix}}=\frac{\begin{vmatrix} b_1 & a_{12} & a_{13}\\ b_2 & a_{22} & a_{23}\\ b_3 & a_{32} & a_{33}\end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}}=\frac{D_x}{D}$$

$$y=\frac{\begin{vmatrix} a_{11} & b_1 & a_{13}\\ a_{21} & b_2 & a_{23}\\ a_{31} & b_3 & a_{33}\end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}}=\frac{D_y}{D},\quad z=\frac{\begin{vmatrix} a_{11} & a_{12} & b_1\\ a_{21} & a_{22} & b_2\\ a_{31} & a_{32} & b_3\end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}}=\frac{D_z}{D}$$

例 求解三元线性方程组 $\begin{cases} x+2z=9 \\ 2y+z=8 \\ 4x-3y=-2 \end{cases}$

例 求解三元线性方程组 $\begin{cases} x+2z=9 \\ 2y+z=8 \\ 4x-3y=-2 \end{cases}$

解

$$x=\frac{D_x}{D}=\underline{\qquad\qquad\qquad}$$

$$y=\frac{D_y}{D}=\underline{\qquad\qquad\qquad}$$

$$z=\frac{D_z}{D}=\underline{\qquad\qquad\qquad}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$

解

$$x = \frac{D_x}{D} = \frac{\underline{\hspace{6cm}}}{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}} \qquad y = \frac{D_y}{D} = \underline{\hspace{6cm}}$$

$$z = \frac{D_z}{D} = \underline{\hspace{6cm}}$$

例 求解三元线性方程组 $\begin{cases} x+2z=9 \\ 2y+z=8 \\ 4x-3y=-2 \end{cases}$

解

$$x=\frac{D_x}{D}=\frac{\qquad\qquad}{\begin{vmatrix}1 & 2\\ 2 & 1\\ 4 & -3\end{vmatrix}\quad X}$$

$$y=\frac{D_y}{D}=\underline{\qquad\qquad\qquad}$$

$$z=\frac{D_z}{D}=\underline{\qquad\qquad\qquad}$$

例 求解三元线性方程组 $\begin{cases} x+2z=9 \\ 2y+z=8 \\ 4x-3y=-2 \end{cases}$ （$\begin{cases} x+0y+2z=9 \\ \\ \\ \end{cases}$）

解

$$x=\frac{D_x}{D}=\underline{\qquad\qquad\qquad}$$

$$\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}$$

$$y=\frac{D_y}{D}=\underline{\qquad\qquad\qquad}$$

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解

$$x=\frac{D_x}{D}=\frac{\qquad\qquad\qquad}{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}} \quad {\color{red}X} \qquad\qquad y=\frac{D_y}{D}=\frac{\qquad\qquad\qquad}{}$$

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解

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解

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例 求解三元线性方程组 $\begin{cases} x+2z=9 \\ 2y+z=8 \\ 4x-3y=-2 \end{cases}$ （$\begin{cases} x+0y+2z=9 \\ 0x+2y+z=8 \\ 4x-3y+0z=-2 \end{cases}$ ）

解

$$x=\frac{D_x}{D}=\frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

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解

$$x=\frac{D_x}{D}=\frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{\qquad}{-13}$$

$$y=\frac{D_y}{D}=\frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

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$$y=\frac{D_y}{D}=\frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

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解

$$x=\frac{D_x}{D}=\frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-13}{-13}=1,\quad y=\frac{D_y}{D}=\frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

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解

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解

$$x=\frac{D_x}{D}=\frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-13}{-13}=1,\quad y=\frac{D_y}{D}=\frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-26}{-13}$$

$$z=\frac{D_z}{D}=\frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{\quad}{-13}$$

例 求解三元线性方程组 $\begin{cases} x+2z=9 \\ 2y+z=8 \\ 4x-3y=-2 \end{cases}$ （$\begin{cases} x+0y+2z=9 \\ 0x+2y+z=8 \\ 4x-3y+0z=-2 \end{cases}$ ）

解

$$x=\frac{D_x}{D}=\frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-13}{-13}=1,\quad y=\frac{D_y}{D}=\frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-26}{-13}=2$$

$$z=\frac{D_z}{D}=\frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{}{-13}$$

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解

$$x=\frac{D_x}{D}=\frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-13}{-13}=1,\quad y=\frac{D_y}{D}=\frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-26}{-13}=2$$

$$z=\frac{D_z}{D}=\frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-52}{-13}$$

例 求解三元线性方程组 $\begin{cases} x+2z=9 \\ 2y+z=8 \\ 4x-3y=-2 \end{cases}$ （$\begin{cases} x+0y+2z=9 \\ 0x+2y+z=8 \\ 4x-3y+0z=-2 \end{cases}$）

解

$$x=\frac{D_x}{D}=\frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-13}{-13}=1,\quad y=\frac{D_y}{D}=\frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-26}{-13}=2$$

$$z=\frac{D_z}{D}=\frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-52}{-13}=4$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$

例 求解三元线性方程组 $\begin{cases} x+2z=9 \\ 2y+z=8 \\ 4x-3y=-2 \end{cases}$

解 先利用公式求出 x

$$x=\frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-13}{-13}=1,$$

例 求解三元线性方程组 $\begin{cases} x+2z=9 \\ 2y+z=8 \\ 4x-3y=-2 \end{cases}$

解 先利用公式求出 x

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代入方程得

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$

解 先利用公式求出 x

$$x = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1,$$

代入方程得

$$\begin{cases} 1 + 2z = 9 \\ 4 - 3y = -2 \end{cases}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$

解 先利用公式求出 x

$$x = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1,$$

代入方程得

$$\begin{cases} 1 + 2z = 9 \\ 4 - 3y = -2 \end{cases} \Rightarrow \begin{cases} z = 4 \\ y = 2 \end{cases}$$

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解 先利用公式求出 x

$$x=\frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}=\frac{-13}{-13}=1,$$

代入方程得

$$\begin{cases} 1+2z=9 \\ 4-3y=-2 \end{cases} \Rightarrow \begin{cases} z=4 \\ y=2 \end{cases}$$

所以方程的解是 $\begin{cases} x=1 \\ y=2 \\ z=4 \end{cases}$

一般地，n 元线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示：

一般地，n 元线性方程组

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的解可用 n 行列式表示：

$$x_1 = \frac{D_1}{D} \qquad x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

一般地，n 元线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示：

$$x_1 = \frac{D_1}{D} = \frac{}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

一般地，n 元线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示：

$$x_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

一般地，n 元线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示：（称为克莱姆法则）

$$x_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

一般地，n 元线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示：（称为克莱姆法则）

$$x_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

那么，如何定义行列式，

一般地，n 元线性方程组

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示：（称为克莱姆法则）

$$x_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

那么，如何定义行列式，如何快捷计算行列式？

练习 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 ____________

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解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} =$$

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解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \quad a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2$$

练习 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 ____________

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{matrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ - a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{matrix}$$

练习 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 ________

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{matrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{matrix} = a^2 + b^2$$

练习 $\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix}$ 不为零的充分必要条件是 a, b 满足 ____________

解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{matrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ - a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{matrix} = a^2 + b^2$$

所以 $a \neq 0$ 或 $b \neq 0$。