# 第1章 a: 二阶三阶行列式

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### 教学要求

#### 掌握求解:

- ◇ 二阶行列式计算
- ♣ 三阶行列式计算

• 行列式 可用于表示一些线性方程组的解。

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- 具体地,

( 克莱姆 法则 )

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解:

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \Rightarrow a_{21}a_{12}x + a_{22}a_{12}y = b_2a_{12} \end{cases}$$

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用消元法求解:  $(1) \times a_{22} - (2) \times a_{12}$ , 消去 y, 得:

x = -----

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \Rightarrow a_{21}a_{12}x + a_{22}a_{12}y = b_2a_{12} \end{cases}$$

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{12} a_{12} a_{1$$

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$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \end{cases}$$

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
,消去  $x$ ,得:

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11} \end{cases}$$

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

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,消去 $x$ ,得:

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用消元法求解:  $(1) \times a_{22} - (2) \times a_{12}$ ,消去 y,得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

(2)×a<sub>11</sub>-(1)×a<sub>21</sub>,消去*x*,得:

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \Rightarrow a_{11}a_{21}x + a_{12}a_{21}y = b_1a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11} \end{cases}$$

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
,消去 $x$ ,得:

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \Rightarrow a_{11}a_{21}x + a_{12}a_{21}y = b_1a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11} \end{cases}$$

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}b_2 - b_1a_{21}}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \Rightarrow a_{11}a_{21}x + a_{12}a_{21}y = b_1a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11} \end{cases}$$

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

(2)×
$$a_{11}$$
 – (1)× $a_{21}$ ,消去 $x$ ,得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

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$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

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• 定义 
$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$
,称为二阶行列式

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用消元法求解:  $(1) \times a_{22} - (2) \times a_{12}$ ,消去 y,得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{a_{11} a_{12}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

 $(2) \times a_{11} - (1) \times a_{21}$ ,消去 x,得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{a_{11}a_{12}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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用消元法求解: (1) ×  $\alpha_{22}$  – (2) ×  $\alpha_{12}$  , 消去 y , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

(2)×*a*<sub>11</sub> – (1)×*a*<sub>21</sub>,消去*x*,得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{1}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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1. 当  $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$  时,方程有唯一解:

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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2. 当  $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 0$  时,方程或者无解、或者有无穷多的解。

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x =$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$

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1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = --- , y =$$

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$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - , y =$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

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$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - , y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - \end{cases}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

例 利用二阶行列式求解下面二元线性方程组

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$
2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-3}{3}, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-3}{3}$$

二阶三阶行列式

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} , y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = -$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \ y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = -$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \ y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \ y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3} = -3$$

例  $\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} \neq 0$  的充分必要条件是  $\lambda$  满足 \_\_\_\_\_\_

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$$\begin{vmatrix} \lambda^2 & \lambda \\ 3 & 1 \end{vmatrix} = \lambda^2 - 3\lambda$$

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$$\begin{vmatrix} k-1 & 2 \\ 2 & k-1 \end{vmatrix} \neq 0$$
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解 因为

$$\begin{vmatrix} k-1 & 2 \\ 2 & k-1 \end{vmatrix} = (k-1)^2 - 4 = k^2 - 2k - 3 = (k+1)(k-3)$$

所以 $k \neq -1$ 且 $k \neq 3$ 。

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

### 用消元法可解得:

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32}}{-b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32}} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}$$

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

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## 为表示三元方程组的解,定义**三阶行列式**:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$

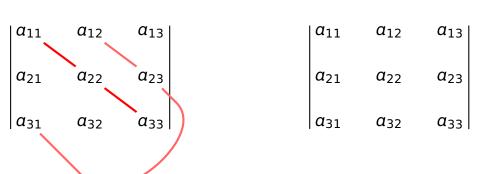
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$

$a_{11}$	$a_{12}$	$a_{13}$	$a_{11}$	$a_{12}$	<i>a</i> <sub>13</sub>
a <sub>21</sub>	$a_{22}$	a <sub>23</sub>	a <sub>21</sub>	a <sub>22</sub>	a <sub>23</sub>
<i>a</i> 21	(Jan	<i>(</i> (2)2)	<i>(</i> 121	(132	022

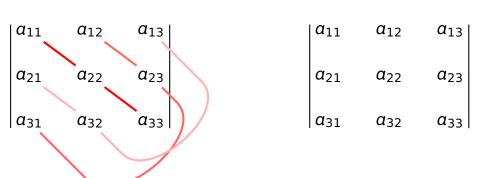
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$

	7 <sub>11</sub>	a <sub>12</sub>	$a_{13}$	$a_{11}$	<i>a</i> <sub>12</sub>	<i>a</i> <sub>13</sub>
0	a <sub>21</sub>	a <sub>22</sub>	a <sub>23</sub>	$a_{21}$	a <sub>22</sub>	a <sub>23</sub>
	<b>X</b> 31	<b>a</b> 32	<i>a</i> 33	<b>a</b> 31	<b>a</b> 32	a33

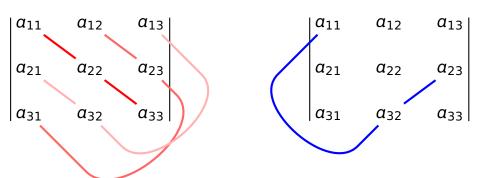
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}}$$



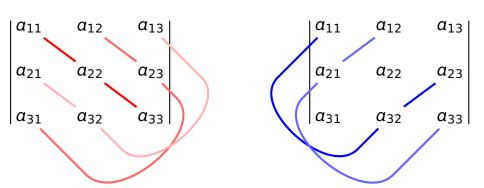
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$



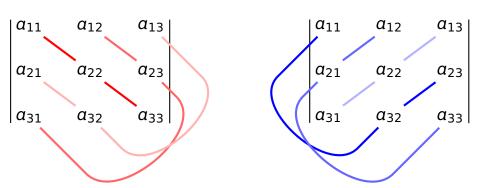
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$



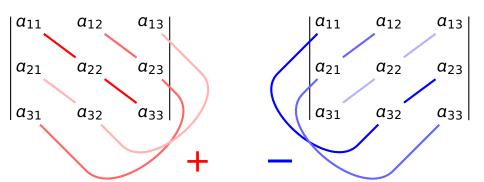
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$



$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$

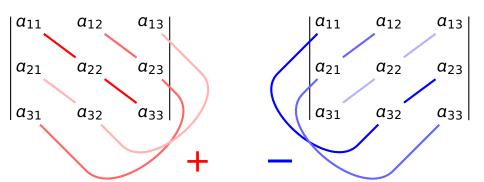


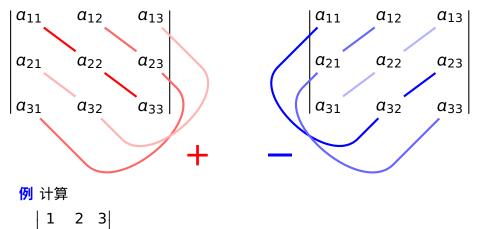
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$



$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$

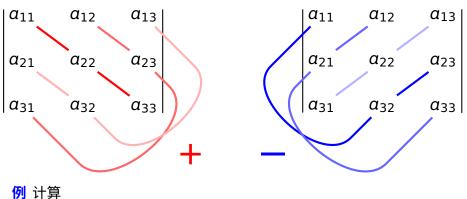
## **规律** (不同行不同列的 3 个元素乘积,共 3! = 6 个)



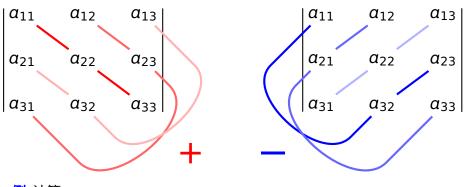


0

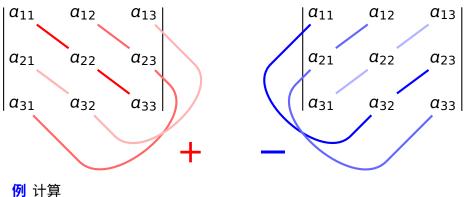
 $\begin{array}{ccc}
0 & -1 \\
5 & 0 \\
4 & 1
\end{array}$ 



$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0$$
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ -1 & 0 & 6 \end{vmatrix} = 0$$

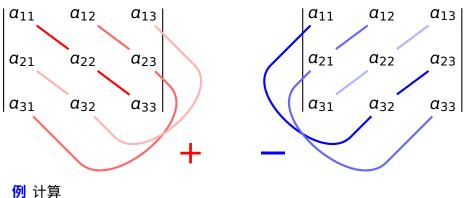


$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{vmatrix} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{vmatrix}$$
$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} =$$



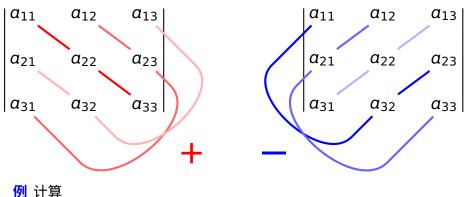
$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{vmatrix} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{vmatrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix}$$



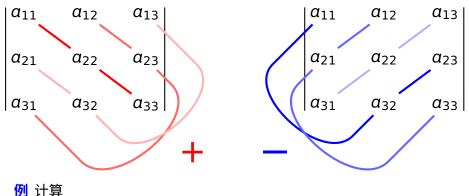
$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{vmatrix} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{vmatrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{vmatrix} 1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4 \end{vmatrix}$$



$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{vmatrix} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{vmatrix} = -5$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{vmatrix} 1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4 \\ -1 \times 0 \times 4 - 0 \times 3 \times 1 - (-1) \times 5 \times 1 \end{vmatrix}$$



$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{vmatrix} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{vmatrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{vmatrix} 1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4 \\ -1 \times 0 \times 4 - 0 \times 3 \times 1 - (-1) \times 5 \times 1 \end{vmatrix} = -2$$

### 解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} =$$

#### 解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2$$

 例
 a
 0
 不为零的充分必要条件是 a, b 满足 \_\_\_\_\_\_

 1
 2
 1

### 解 因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{vmatrix}$$

#### 解因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{vmatrix} = a^2 + b^2$$

#### 解因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{vmatrix} = a^2 + b^2$$

所以 $a \neq 0$ 或 $b \neq 0$ 。

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

#### 的解可以表示为:

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32}}{-b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32}} = -a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}$$

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

#### 的解可以表示为:

$$\begin{array}{c}
b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} \\
-b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3 \\
\hline
a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} \\
-a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}
\end{array} = 
\begin{vmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23}
\end{vmatrix}$$

二阶三阶行列式

 $a_{32}$ 

 $a_{33}$ 

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$A = \frac{\begin{vmatrix} b_{1} & a_{12} & a_{13} \\ b_{1} & a_{22} & a_{33} + a_{12} & a_{23} & b_{3} + a_{13} & b_{2} & a_{32} \\ -b_{1} & a_{23} & a_{32} - a_{12} & b_{2} & a_{33} - a_{13} & a_{22} & b_{3} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ b_{2} & a_{22} & a_{23} \\ b_{3} & a_{32} & a_{33} \end{vmatrix}} = \frac{\begin{vmatrix} b_{1} & a_{12} & a_{13} \\ b_{2} & a_{22} & a_{23} \\ b_{3} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{\begin{array}{c} b_{1}a_{22}a_{33} + a_{12}a_{23}b_{3} + a_{13}b_{2}a_{32} \\ -b_{1}a_{23}a_{32} - a_{12}b_{2}a_{33} - a_{13}a_{22}b_{3} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}}{\begin{array}{c} b_{1} & a_{12} & a_{13} \\ b_{2} & a_{22} & a_{23} \\ b_{3} & a_{32} & a_{33} \end{array}} = \frac{a_{13}a_{22}a_{33}}{a_{21}a_{22}a_{33}} = \frac{a_{11}a_{22}a_{23}a_{23}}{a_{21}a_{22}a_{23}} = \frac{a_{22}a_{23}a_{23}}{a_{22}a_{23}a_{23}} = \frac{a_{22}a_{23}a_{23}}{a_{22}a_{23}a_{23}} = \frac{a_{22}a_{23}a_{23}}{a_{22}a_{23}a_{23}} = \frac{a_{22}a_{23}a_{23}}{a_{22}a_{23}a_{23}} = \frac{a_{22}a_{23}a_{23}a_{23}}{a_{22}a_{23}a_{23}} = \frac{a_{22}a_{23}a_{23}a_{23}a_{23}}{a_{22}a_{23}a_{23}} = \frac{a_{22}a_{23}a_{23}a_{23}a_{23}}{a_{22}a_{23}a_{23}} = \frac{a_{22}a_{23}a_{23}a_{23}a_{23}}{a_{22}a_{23}a_{23}} = \frac{a_{22}a_{23}a_{23}a_{23}a_{23}}{a_{23}a_{23}a_{23}} = \frac{a_{22}a_{23}a_{23}a_{23}a_{23}a_{23}}{a_{23}a_{23}a_{23}} = \frac{a_{22}a_{23}a_{23}a_{23}a_{23}}{a_{23}a_{23}a_{23}} = \frac{a_{22}a_{23}a_{23}a_{23}a_{23}a_{23}}{a_{23}a_{23}a_{23}} = \frac{a_{23}a_{23}a_{23}a_{23}a_{23}a_{23}}{a_{23}a_{23}a_{23}} = \frac{a_{23}a_{23}a_{23}a_{23}a_{23}a_{23}}{a_{23}a_{23}a_{23}} = \frac{a_{23}a_{23}a_{23}a_{23}a_{23}a_{23}a_{23}}{a_{23}a_{23}a_{23}} = \frac{a_{23}a_{23}a_{23}a_{23}a_{23}a_{23}a_{23}a_{23}a_{23}a_{23}}{a_{23}a_{23}a_{23}a_{23}} = \frac{a_{23}a_{23$$

 $y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$ 

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{\begin{array}{c} b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} \\ -b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3 \\ -a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31} \\ -a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31} \end{array}} = \frac{\begin{array}{c} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \\ \hline a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ \hline \end{array}}$$

 $b_1$  $b_1$  $a_{11}$  $a_{13}$ a<sub>11</sub> **a**<sub>12</sub>  $b_2$ b<sub>2</sub>  $a_{21}$ **a**22  $a_{21}$  $a_{23}$ b<sub>3</sub><sup>1</sup>  $b_3$  $a_{33}$  $a_{31}$  $a_{32}$  $a_{31}$ **a**11  $a_{12}$  $a_{13}$ a11  $a_{12}$  $a_{13}$  $a_{21}$  $a_{22}$  $a_{23}$  $a_{21}$  $a_{22}$  $a_{23}$  $a_{31}$  $a_{32}$  $a_{31}$  $a_{33}$  $a_{32}$  $a_{33}$ 

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:
$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32}}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \\ b_3 & a_{32} \end{vmatrix}}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \\ a_{33} & a_{32} \end{vmatrix}}{a_{11} a_{12} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \\ a_{33} & a_{32} \end{vmatrix}}{a_{11} a_{12} a_{23} a_{33} + a_{13} a_{23} a_{33}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \\ a_{33} & a_{32} \end{vmatrix}}{a_{11} a_{12} a_{23} a_{33} + a_{13} a_{23} a_{33}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \\ a_{33} & a_{32} \end{vmatrix}}{a_{11} a_{12} a_{23} a_{33}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \\ a_{33} & a_{32} \end{vmatrix}}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{33} + a_{13} a_{23} a_{33}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \\ a_{33} & a_{32} \end{vmatrix}}{a_{33} a_{33} a_{33} a_{33} a_{33}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{23} \\ a_{33} & a_{33} \end{vmatrix}}{a_{33} a_{33} a_{33} a_{33} a_{33} a_{33} a_{33}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{23} \\ a_{33} & a_{33} \end{vmatrix}}{a_{33} a_{33} a_{33}$$

 $-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$  $b_1$  $a_{11}$  $a_{13}$  $a_{21}$ b<sub>2</sub>  $a_{23}$ 

> $a_{31}$ **a**11

 $a_{21}$ 

 $a_{31}$ 

 $b_3$ 

 $a_{12}$ 

 $a_{22}$ 

 $a_{32}$ 

 $a_{13}$  $a_{23}$  $a_{33}$ 

 $b_1$ a<sub>11</sub>  $a_{12}$  $a_{21}$  $a_{22}$  $a_{31}$  $a_{32}$ 

 $a_{12}$ 

 $a_{22}$ 

 $a_{32}$ 

**a**11

 $a_{21}$ 

 $a_{31}$ 

$$\begin{vmatrix} b_2 \\ b_3 \end{vmatrix}$$
  $a_{13}$ 

 $a_{23}$ 

 $a_{33}$ 

 $a_{21}$ 

 $a_{31}$ 

 $a_{33}$  $a_{13}$  $a_{23}$ 

 $a_{33}$ 

 $a_{13}$ 

 $a_{23}$ 

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

 $b_1$ 

 $b_1$ 

 $b_2$ 

*b*<sub>3</sub>,

 $a_{13}$ 

 $a_{12}$ 

 $a_{22}$ 

 $a_{32}$ 

 $a_{12}$ 

 $a_{12}$ 

 $a_{13}$ 

的解可以表示为:

 $b_1$ 

b<sub>2</sub>

 $b_3$ 

 $a_{12}$ 

 $a_{13}$ 

 $a_{23}$ 

 $a_{33}$ 

 $a_{13}$ 

$$x = \frac{\begin{array}{c} b_{1}a_{22}a_{33} + a_{12}a_{23}b_{3} + a_{13}b_{2}a_{32} \\ -b_{1}a_{23}a_{32} - a_{12}b_{2}a_{33} - a_{13}a_{22}b_{3} \end{array}}{\begin{array}{c} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}} = \frac{\begin{vmatrix} b_{2} & a_{22} & a_{23} \\ b_{3} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\begin{vmatrix} b_{2} & a_{22} & a_{23} \\ b_{3} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\begin{vmatrix} b_{2} & a_{22} & a_{23} \\ b_{3} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\begin{vmatrix} b_{2} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\begin{vmatrix} b_{2} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\begin{vmatrix} b_{2} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\begin{vmatrix} b_{2} & a_{22} & a_{23} \\ a_{21} & a_{22} & a_{23} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{21} & a_{22} & a_{23} \\ a_{21} & a_{22} & a_{23} \end{vmatrix}}$$

a<sub>11</sub>

 $a_{21}$ 

 $a_{31}$ 

**a**11

 $y = \frac{\begin{vmatrix} a_{11} \\ a_{21} \\ a_{31} \end{vmatrix}}{\begin{vmatrix} a_{11} \\ a_{21} \\ a_{31} \end{vmatrix}}$ 

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

 $b_1$ 

b<sub>2</sub>

 $b_3$ 

 $a_{12}$ 

 $a_{13}$ 

 $a_{23}$ 

 $a_{33}$ 

 $a_{13}$ 

$$x = \frac{\begin{array}{c} b_{1}a_{22}a_{33} + a_{12}a_{23}b_{3} + a_{13}b_{2}a_{32} \\ -b_{1}a_{23}a_{32} - a_{12}b_{2}a_{33} - a_{13}a_{22}b_{3} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}} = \frac{\begin{vmatrix} b_{1} & a_{12} & a_{13} \\ b_{2} & a_{22} & a_{23} \\ b_{3} & a_{32} & a_{33} \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\begin{vmatrix} b_{1} & a_{12} & a_{13} \\ b_{2} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\begin{vmatrix} b_{1} & a_{12} & a_{13} \\ b_{2} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\begin{vmatrix} b_{1} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\begin{vmatrix} b_{1} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

 $a_{11}$  $a_{21}$  $a_{31}$ **a**11

 $a_{21}$  $a_{22}$  $a_{23}$  $a_{21}$  $a_{22}$  $a_{23}$  $a_{33}$  $a_{31}$  $a_{32}$  $a_{31}$  $a_{33}$  $a_{32}$ 11/17 ⊲ ⊳ ∆ ⊽

 $a_{11}$ 

 $a_{21}$ 

 $a_{31}$ 

**a**11

 $b_1$ 

 $b_2$ 

*b*<sub>3</sub>,

 $a_{13}$ 

 $a_{12}$ 

 $a_{22}$ 

 $a_{32}$ 

 $a_{12}$ 

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{\begin{array}{c} b_{1}a_{22}a_{33} + a_{12}a_{23}b_{3} + a_{13}b_{2}a_{32} \\ -b_{1}a_{23}a_{32} - a_{12}b_{2}a_{33} - a_{13}a_{22}b_{3} \\ -a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}} = \frac{\begin{vmatrix} b_{1} & a_{12} & a_{13} \\ b_{2} & a_{22} & a_{23} \\ b_{3} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} = \frac{\begin{vmatrix} b_{1} & a_{12} & a_{13} \\ b_{2} & a_{22} & a_{23} \\ a_{33} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

 $a_{31}$  $a_{32}$  $b_1$  $b_1$  $a_{11}$  $a_{13}$  $a_{11}$  $a_{12}$  $b_2$  $a_{21}$ b<sub>2</sub>  $a_{21}$  $a_{22}$  $a_{23}$ *b*<sub>3</sub><sup>1</sup>  $b_3$  $a_{33}$  $a_{31}$  $a_{31}$  $a_{32}$ **a**11  $a_{12}$  $a_{13}$ **a**11  $a_{12}$  $a_{13}$ 

 $a_{21}$  $a_{22}$  $a_{23}$  $a_{21}$  $a_{22}$  $a_{23}$  $a_{33}$  $a_{31}$  $a_{32}$  $a_{31}$  $a_{32}$  $a_{33}$ 

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D}$$

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

当 D ≠ 0 时,

$$x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}, \quad z = \frac{D_z}{D}$$

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

1. 当  $D \neq 0$  时,方程有唯一解:

$$x = \frac{D_x}{D}$$
,  $y = \frac{D_y}{D}$ ,  $z = \frac{D_z}{D}$ 

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

1. 当  $D \neq 0$  时,方程有唯一解:

$$x = \frac{D_x}{D}$$
,  $y = \frac{D_y}{D}$ ,  $z = \frac{D_z}{D}$ 

2. 当D = 0时,

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

1. 当  $D \neq 0$  时,方程有唯一解:

$$x = \frac{D_x}{D}$$
,  $y = \frac{D_y}{D}$ ,  $z = \frac{D_z}{D}$ 

2. 当 D = 0 时,方程或者无解、或者有无穷多的解。

例 求解三元线性方程组  $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$ 

例 求解三元线性方程组 
$$\begin{cases} x + 2z = 9\\ 2y + z = 8\\ 4x - 3y = -2 \end{cases}$$
解

$$x = \frac{D_x}{D} = \frac{D_y}{D} =$$

例 求解三元线性方程组 
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

$$x = \frac{D_x}{D} = \frac{1}{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}} \qquad y = \frac{D_y}{D} = \frac{1}{2}$$

$$z = \frac{D_z}{D} = ---$$

例 求解三元线性方程组 
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

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  $y = \frac{D_y}{D} = \frac{1}{2}$ 

$$z = \frac{D_z}{D} = ---$$

$$x = \frac{Dx}{D} = \frac{1}{\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix}}$$

解

$$x = \frac{D_x}{D} = \frac{1}{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}} \qquad y = \frac{D_y}{D} = \frac{1}{2}$$

例 求解三元线性方程组  $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \begin{pmatrix} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \end{pmatrix}$ 

$$x = \frac{D_x}{D} = \frac{1}{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}} \qquad y = \frac{D_y}{D} = \frac{1}{2}$$

解

 $x = \frac{D_X}{D} = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} X}$ 

$$x = \frac{D_x}{D} = \frac{1}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} \qquad y = \frac{D_y}{D} = \frac{1}{D}$$

解

 $x = \frac{D_X}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$ 

$$z = \frac{D_z}{D} = -$$

 $x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ A & -3 & 0 \end{vmatrix}} \qquad y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$  $z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$ 

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二阶三阶行列式

 $\begin{vmatrix} 4 & -3 & 0 \end{vmatrix} = \begin{vmatrix} 2 & 0 & 0 \end{vmatrix}$   $z = \frac{D_z}{D} = \frac{1}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & -3 & 0 \end{vmatrix}}$ 

 $x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} \qquad y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$ 

 $z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$ 

 p</

 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left( \begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$ 

 $Z = \frac{D_Z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$ 

例 求解三元线性方程组

 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left( \begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$ 解

例 求解三元线性方程组 ·

例 求解三元线性方程组  $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \begin{pmatrix} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{pmatrix}$ 解  $\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix} - 13 \qquad D_{V} \begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}$ 

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

二阶三阶行列式

 $\begin{vmatrix} 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix} \qquad \begin{vmatrix} 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}$   $z = \frac{D_z}{A} = \begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix} = ---$ 

例 求解三元线性方程组

解

二阶三阶行列式

 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left( \begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$ 

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 $|4 -3 0| \qquad |4 -3 0|$   $z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \end{vmatrix}} = \frac{-13}{-13}$ 

 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left( \begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$ 

x

解

例 求解三元线性方程组

 $z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13}$ 

 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left( \begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$ 

二阶三阶行列式

解

例 求解三元线性方程组

例 求解三元线性方程组

解

 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left( \begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$ 

 $z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-52}{-13} = 4$ 

**例** 求解三元线性方程组 ·

解

 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left( \begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$ 

例 求解三元线性方程组  $\begin{cases} x + 2z = 9\\ 2y + z = 8\\ 4x - 3y = -2 \end{cases}$ 

例 求解三元线性方程组 
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

### 另解 先利用公式求出 x

$$x = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1$$

例 求解三元线性方程组 
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

### 另解 先利用公式求出 x

$$x = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1$$

代入方程得

例 求解三元线性方程组 
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

#### S解 先利用公式求出 x

$$x = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1,$$

代入方程得

$$\begin{cases} 1+2z=9\\ 4-3y=-2 \end{cases}$$

例 求解三元线性方程组 
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

#### S解 先利用公式求出 x

$$x = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1,$$

代入方程得

$$\begin{cases} 1+2z=9\\ 4-3y=-2 \end{cases} \Rightarrow \begin{cases} z=4\\ y=2 \end{cases}$$

例 求解三元线性方程组  $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$ 

### S解 先利用公式求出 x

$$x = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1,$$

代入方程得

$$\begin{cases} 1+2z=9\\ 4-3y=-2 \end{cases} \Rightarrow \begin{cases} z=4\\ y=2 \end{cases}$$

所以方程的解是  $\begin{cases} x=1 \\ y=2 \\ z=4 \end{cases}$ 

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示:

$$x_1 = \frac{D_1}{D}, \quad x_2 = \frac{D_2}{D}, \quad \dots, \quad x_n = \frac{D_n}{D}$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示:

$$x_{1} = \frac{D_{1}}{D} = \frac{1}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_{2} = \frac{D_{2}}{D}, \quad \cdots, \quad x_{n} = \frac{D_{n}}{D}$$

二阶三阶行列式

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示:

$$x_{1} = \frac{D_{1}}{D} = \frac{\begin{vmatrix} b_{1} & a_{12} & \cdots & a_{1n} \\ b_{2} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_{2} = \frac{D_{2}}{D}, \quad \cdots, \quad x_{n} = \frac{D_{n}}{D}$$

二阶三阶行列式

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示: (称为 克莱姆法则)

$$x_{1} = \frac{D_{1}}{D} = \frac{\begin{vmatrix} b_{1} & a_{12} & \cdots & a_{1n} \\ b_{2} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_{2} = \frac{D_{2}}{D}, \quad \cdots, \quad x_{n} = \frac{D_{n}}{D}$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示: (称为 克莱姆法则)

$$x_{1} = \frac{D_{1}}{D} = \frac{\begin{vmatrix} b_{1} & a_{12} & \cdots & a_{1n} \\ b_{2} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_{2} = \frac{D_{2}}{D}, \quad \cdots, \quad x_{n} = \frac{D_{n}}{D}$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示: (称为 **克莱姆法则**)

$$x_{1} = \frac{D_{1}}{D} = \frac{\begin{vmatrix} b_{1} & a_{12} & \cdots & a_{1n} \\ b_{2} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_{2} = \frac{D_{2}}{D}, \quad \cdots, \quad x_{n} = \frac{D_{n}}{D}$$

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是,如何定义 n 阶行列式,如何快捷计算行列式?

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

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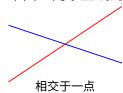
- 每条方程表示平面上的一条直线
- 方程组的解表示两条直线的交点

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2元2方程的线性方程组

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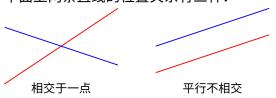


重合

2元2方程的线性方程组

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• 所以方程组的解有三种情况:

有唯一解、无解、有无穷多的解

重合

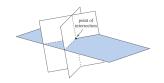
$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

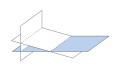
- 每条方程表示空间上的一个平面
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$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

- 每条方程表示空间上的一个平面
- 方程组的解表示三个平面的交点
- 空间上三个平面的位置关系有若干种,例如:

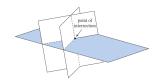




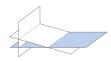


$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

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