第 9 章 d：隐函数的求导公式

数学系　　梁卓滨

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Outline

1. 隐函数的求导法：一个方程的情形

2. 隐函数的求导法：方程组的情形

3. 隐函数定理

We are here now...

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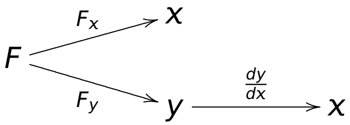

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所以 $y'=-\frac{e^x-y^2}{\cos y-2xy}$

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$$=-\frac{2x+3x^2y+3y^3}{2y+3xy^2+3x^3}$$

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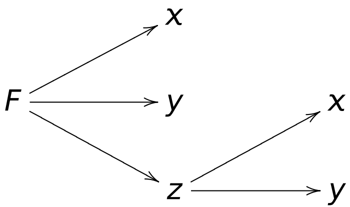

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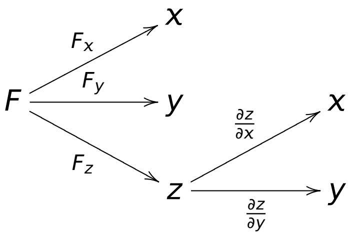

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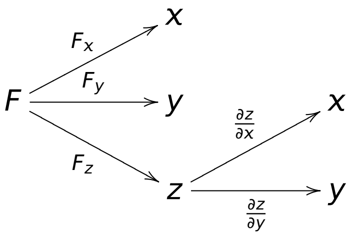

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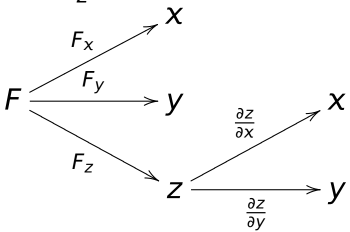
$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z},\qquad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z}\qquad (F_z\neq 0)$$

证明

$$\because\quad F(x,y,z(x,y))=0$$

$$\therefore\quad 0=\frac{\partial}{\partial x}F(x,y,z(x,y))=F_x+F_z\cdot\frac{\partial z}{\partial x}$$

$$\therefore\quad \frac{\partial z}{\partial x}=-\frac{F_x}{F_z},$$



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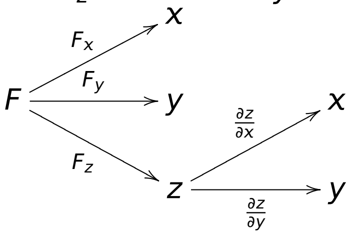

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例 设 $z=f(x,y)$ 满足 $2\sin(x+2y-3z)=x+2y-3z$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解 令 $F(x,y,z)=2\sin(x+2y-3z)-x-2y+3z$，则 $F(x,y,z)=0$，所以

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_x}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

$$=-\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3}=\frac{1}{3}$$

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$$=-\frac{}{-6\cos(x+2y-3z)+3}$$

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$$=-\frac{4\cos(x+2y-3z)}{-6\cos(x+2y-3z)+3}$$

例 设 $z=f(x,y)$ 满足 $2\sin(x+2y-3z)=x+2y-3z$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$=-\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3}=\frac{1}{3}$$

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$$=-\frac{4\cos(x+2y-3z)-2}{-6\cos(x+2y-3z)+3}$$

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解

$$\frac{\partial z}{\partial x}=$$

$$\frac{\partial z}{\partial y}=$$

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例 设 $z=f(x,y)$ 满足 $z-y-x+xe^{z-y-x}=0$，求 dz

解 令 $F(x,y,z)=z-y-x+xe^{z-y-x}$，则 $F(x,y,z)=0$，所以

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=$$

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$$=-\frac{\qquad}{1+xe^{z-y-x}}$$

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$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z-y-x+xe^{z-y-x})'_x}{(z-y-x+xe^{z-y-x})'_z}$$

$$= -\frac{-1}{1+xe^{z-y-x}}$$

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$$= -\frac{-1+e^{z-y-x}-xe^{z-y-x}}{1+xe^{z-y-x}} = \frac{1+(x-1)e^{z-y-x}}{1+xe^{z-y-x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z-y-x+xe^{z-y-x})'_y}{(z-y-x+xe^{z-y-x})'_z} = -\frac{-1-xe^{z-y-x}}{1+xe^{z-y-x}}$$

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$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(z-y-x+xe^{z-y-x})'_y}{(z-y-x+xe^{z-y-x})'_z}=-\frac{-1-xe^{z-y-x}}{1+xe^{z-y-x}}=1$$

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$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=-\frac{1+(x-1)e^{z-y-x}}{1+xe^{z-y-x}}dx+dy$$

例 设 $\Phi(u, v)$ 具有连续偏导数，函数 $z = z(x, y)$ 满足 $\Phi(cx - az, cy - bz) = 0$，证明：

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

$$F_x =$$

$$F_y =$$

$$F_z =$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x$$

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$$F_z =$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y = c\Phi_v$$

$$F_z = \Phi_u \cdot u_z + \Phi_v \cdot v_z = -a\Phi_u - b\Phi_v$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y = c\Phi_v$$

$$F_z = \Phi_u \cdot u_z + \Phi_v \cdot v_z = -a\Phi_u - b\Phi_v$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{c\Phi_u}{a\Phi_u + b\Phi_v}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{c\Phi_v}{a\Phi_u + b\Phi_v}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = \frac{ac\Phi_u}{a\Phi_u + b\Phi_v} + \frac{bc\Phi_v}{a\Phi_u + b\Phi_v}$$

例 设 $\Phi(u, v)$ 具有连续偏导数，函数 $z = z(x, y)$ 满足 $\Phi(cx - az, cy - bz) = 0$，证明：

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x = c\Phi_u$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y = c\Phi_v$$

$$F_z = \Phi_u \cdot u_z + \Phi_v \cdot v_z = -a\Phi_u - b\Phi_v$$

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$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{c\Phi_v}{a\Phi_u + b\Phi_v}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = \frac{ac\Phi_u}{a\Phi_u + b\Phi_v} + \frac{bc\Phi_v}{a\Phi_u + b\Phi_v} = c$$

例 设 $z = f(x, y)$ 满足 $z = x + ye^z$，求 $\frac{\partial^2 z}{\partial x \partial y}$

例 设 $z = f(x, y)$ 满足 $z = x + ye^z$，求 $\frac{\partial^2 z}{\partial x \partial y}$

解 $F(x, y, z) = x + ye^z - z$，则 $F(x, y, z) = 0$

例 设 $z=f(x,y)$ 满足 $z=x+ye^z$，求 $\frac{\partial^2 z}{\partial x\partial y}$

解 $F(x,y,z)=x+ye^z-z$，则 $F(x,y,z)=0$，所以

$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=$$

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$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{(x+ye^z-z)_x}{(x+ye^z-z)_z}=-\frac{\qquad}{ye^z-1}$$

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$$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=\left(-\frac{1}{ye^z-1}\right)'_y$$

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$$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=\left(-\frac{1}{ye^z-1}\right)_y'=\frac{(ye^z-1)_y'}{(ye^z-1)^2}$$

$$=\frac{e^z+y(e^z)_y'}{(ye^z-1)^2}$$

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$$=\frac{e^z+y(e^z)'_y}{(ye^z-1)^2}=\frac{e^z+ye^z\cdot\frac{\partial z}{\partial y}}{(ye^z-1)^2}$$

例 设 $z=f(x,y)$ 满足 $z=x+ye^z$，求 $\frac{\partial^2 z}{\partial x\partial y}$

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$$=\frac{e^z+y(e^z)'_y}{(ye^z-1)^2}=\frac{e^z+ye^z\cdot\frac{\partial z}{\partial y}}{(ye^z-1)^2}=\frac{e^z+ye^z\cdot\left(-\frac{e^z}{ye^z-1}\right)}{(ye^z-1)^2}$$

例 设 $z=f(x,y)$ 满足 $z=x+ye^z$，求 $\frac{\partial^2 z}{\partial x\partial y}$

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$$=\frac{e^z+y(e^z)'_y}{(ye^z-1)^2}=\frac{e^z+ye^z\cdot\frac{\partial z}{\partial y}}{(ye^z-1)^2}=\frac{e^z+ye^z\cdot\left(-\frac{e^z}{ye^z-1}\right)}{(ye^z-1)^2}$$

$$=\frac{-e^z}{(ye^z-1)^3}$$

例 设 $z = f(x, y)$ 满足 $z = x + ye^z$，求 $\frac{\partial^2 z}{\partial x \partial y}$

解 $F(x, y, z) = x + ye^z - z$，则 $F(x, y, z) = 0$，所以

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$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)'_y = \frac{(ye^z - 1)'_y}{(ye^z - 1)^2}$$

$$= \frac{e^z + y(e^z)'_y}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \frac{\partial z}{\partial y}}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \left(-\frac{e^z}{ye^z - 1}\right)}{(ye^z - 1)^2}$$

$$= \frac{-e^z}{(ye^z - 1)^3} = \frac{e^z}{(1 + x - z)^3}$$

We are here now...

1. 隐函数的求导法：一个方程的情形

2. 隐函数的求导法：方程组的情形

3. 隐函数定理

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}\,a_{22}x + a_{12}\,a_{22}y = a_{22}b_1 & (1)\times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2)\times a_{12} \end{cases}$$

用消元法解：

$(1)\times a_{22} - (2)\times a_{12}$，消去 y，得：

回顾：二元线性方程组的求解

二元线性方程组

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二元线性方程组

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用消元法解：

$(1)\times a_{22} - (2)\times a_{12}$，消去 y，得：

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

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$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

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回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}\,a_{11}x + a_{22}\,a_{11}y = a_{11}b_2 & (2) \times a_{11} \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}\,a_{21}x + a_{12}\,a_{21}y = a_{21}b_1 & (1)\times a_{21} \\ a_{21}\,a_{11}x + a_{22}\,a_{11}y = a_{11}b_2 & (2)\times a_{11} \end{cases}$$

用消元法解：

$(1)\times a_{22}-(2)\times a_{12}$，消去 y，得：

$$x=\frac{b_1a_{22}-a_{12}b_2}{a_{11}a_{22}-a_{12}a_{21}}$$

$(2)\times a_{11}-(1)\times a_{21}$，消去 x，得：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}\, a_{21} x + a_{12}\, a_{21} y = a_{21} b_1 & (1) \times a_{21} \\ a_{21}\, a_{11} x + a_{22}\, a_{11} y = a_{11} b_2 & (2) \times a_{11} \end{cases}$$

用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

回顾：二元线性方程组的求解

二元线性方程组

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回顾：二元线性方程组的求解

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$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\qquad}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

回顾：二元线性方程组的求解

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$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\quad}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

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$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $\quad x =$ $\qquad\qquad\qquad$, $\quad y =$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $\quad x =$ $\qquad\qquad\qquad$, $y =$

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公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \dfrac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \underline{\qquad}\qquad, \quad y =$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \qquad\qquad\qquad, \; y =$

公式:

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公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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方程组的隐函数求导公式

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

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问题：如何计算 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$?

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求解如下：

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}}$$

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求解如下：

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ \end{cases}$$

方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

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求解如下：

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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求解如下：

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

方程组的隐函数求导公式

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问题：如何计算 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$?

求解如下：

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

$$\Rightarrow u_x = \underline{\qquad\qquad}, \quad v_x = \underline{\qquad\qquad}$$

方程组的隐函数求导公式

假设函数 $u=u(x,y),\ v=v(x,y)$ 满足方程组

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$$=-\frac{1}{J}\frac{\partial(F,G)}{\partial(x,v)}$$

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$$=-\frac{1}{J}\frac{\partial(F,G)}{\partial(x,v)} \qquad\qquad =-\frac{1}{J}\frac{\partial(F,G)}{\partial(u,x)}$$

$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}}$$

$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ \end{cases}$$

$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y+F_u\cdot u_y+F_v\cdot v_y=0 \\ G_y+G_u\cdot u_y+G_v\cdot v_y=0 \end{cases}$$

$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u\cdot u_y + F_v\cdot v_y = -F_y \\ G_u\cdot u_y + G_v\cdot v_y = -G_y \end{cases}$$

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$$\Rightarrow u_y = \underline{\hspace{4cm}}, \quad v_y = \underline{\hspace{4cm}}$$

$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u\cdot u_y+F_v\cdot v_y=-F_y \\ G_u\cdot u_y+G_v\cdot v_y=-G_y \end{cases}$$

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$$\Rightarrow u_y=-\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}},\quad v_y=-\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

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总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

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$u_x =$

$v_x =$

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$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \quad \begin{matrix} \xRightarrow{\frac{\partial}{\partial x}} \\ \\ \xRightarrow{\frac{\partial}{\partial y}} \end{matrix}$$

$u_x =$ $\qquad\qquad$ $v_x =$

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总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\xrightarrow{\frac{\partial}{\partial y}}$$

$u_x =$ $\qquad$ $v_x =$

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$$\xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

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总结 设 $u=u(x,\ y),\ v=v(x,\ y)$ 满足方程组

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所以

$$u_x=-\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \qquad v_x=-\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$u_y= \qquad v_y=$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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所以

$$u_x=-\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}=-\frac{1}{J}\frac{\partial(F,G)}{\partial(x,v)}, \quad v_x=-\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$u_y=-\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \qquad v_y=-\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

总结 设 $u=u(x,y)$, $v=v(x,y)$ 满足方程组

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$$\overset{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_y + F_u\cdot u_y + F_v\cdot v_y = 0 \\ G_y + G_u\cdot u_y + G_v\cdot v_y = 0 \end{cases}$$

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$$u_y = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \qquad v_y = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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$$\overset{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_x = -\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = -\frac{1}{J}\frac{\partial(F, G)}{\partial(x, v)}, \quad v_x = -\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = -\frac{1}{J}\frac{\partial(F, G)}{\partial(u, x)}$$

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总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_x = -\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = -\frac{1}{J}\frac{\partial(F, G)}{\partial(x, v)}, \quad v_x = -\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = -\frac{1}{J}\frac{\partial(F, G)}{\partial(u, x)}$$

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例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \quad \overset{\frac{\partial}{\partial x}}{\Longrightarrow} \qquad \overset{\frac{\partial}{\partial y}}{\Longrightarrow}$$

$u_x =$ $\qquad$ $v_x =$

$u_y =$ $\qquad$ $v_y =$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\xrightarrow{\frac{\partial}{\partial x}}$$

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases}$$

$$\xrightarrow{\frac{\partial}{\partial y}}$$

$u_x=$ $\qquad$ $v_x=$

$u_y=$ $\qquad$ $v_y=$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1 \\ \end{cases}$$

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We are here now...

1. 隐函数的求导法：一个方程的情形

2. 隐函数的求导法：方程组的情形

3. 隐函数定理

假设 $f(x, y)$ 是光滑的二元函数，其零点集 $\{f = 0\}$ 是平面上点集。

- $\{f = 0\}$ 的形状通常是一条曲线，为什么？
- 如何求曲线 $\{f = 0\}$ 上每一点处的切线？

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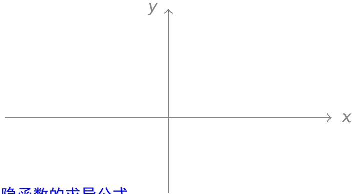
隐函数定理将会解决这些问题

设 $f(x,y)$ 是光滑的二元函数，考察零点集 $\{f=0\}$：

$$f(x,y)=\left(3x^2-y^2\right)^2y^2-\left(x^2+y^2\right)^4$$

$$f(x,y)=\left(x^2+y^2+y\right)^2-\left(x^2+y^2\right)$$

$$f(x,y)=x^2+y^2$$



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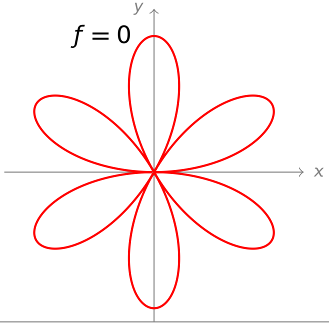

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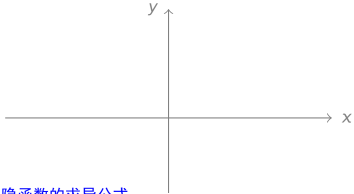

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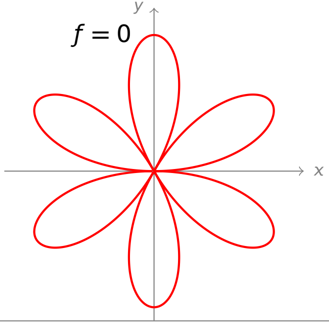

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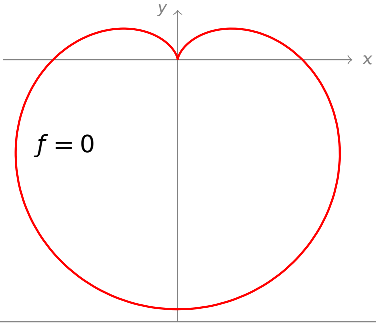

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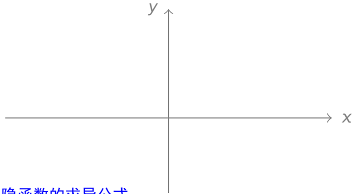

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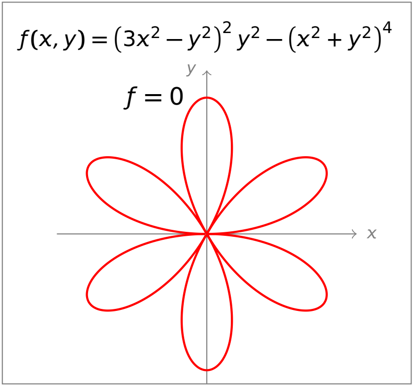

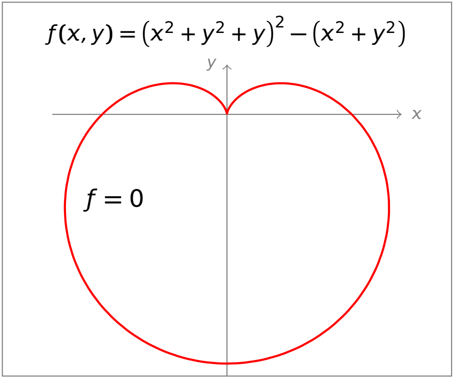

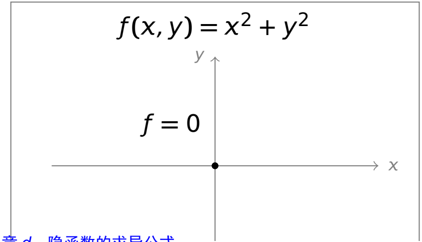

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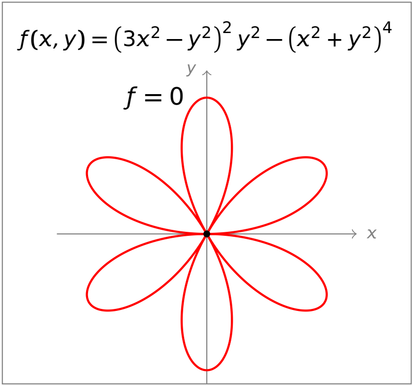

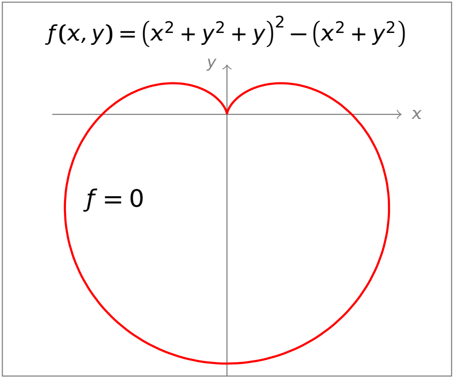

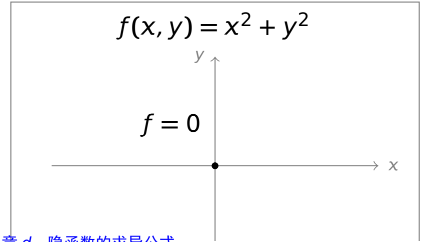

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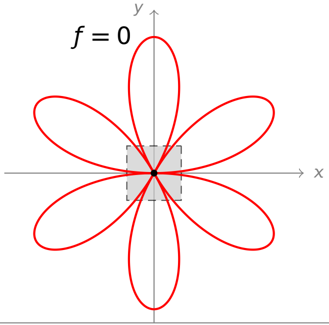
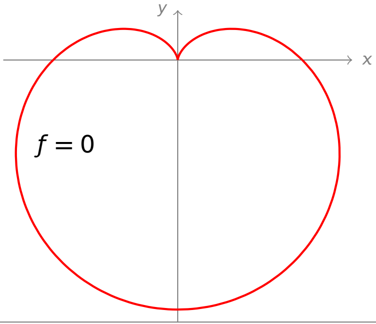


$$f(x,y)=\left(x^2+y^2+y\right)^2-\left(x^2+y^2\right)$$



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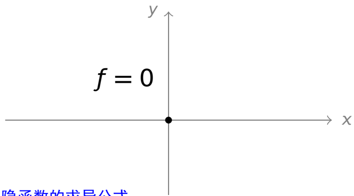

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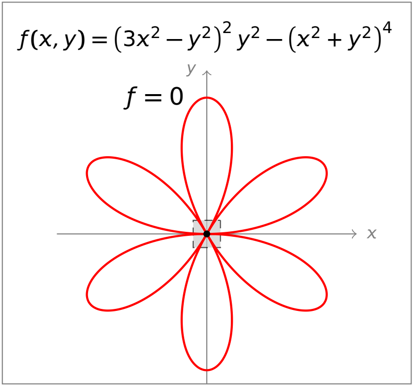

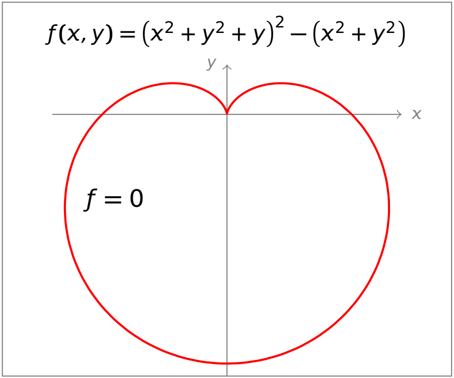

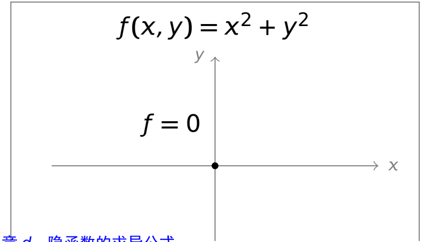

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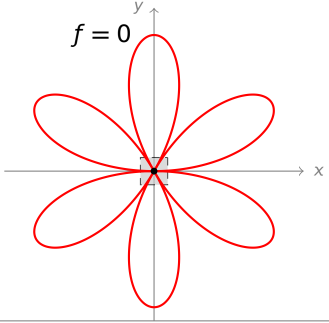

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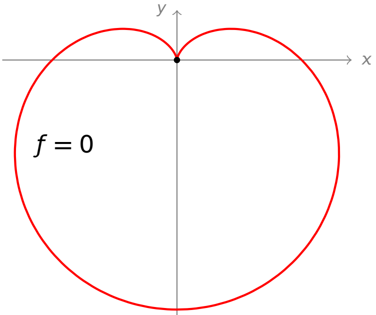

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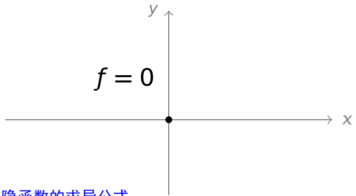

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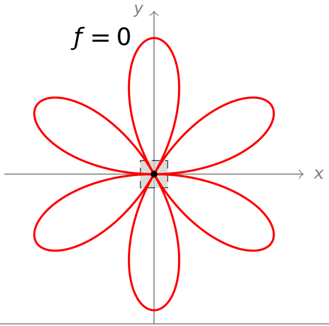

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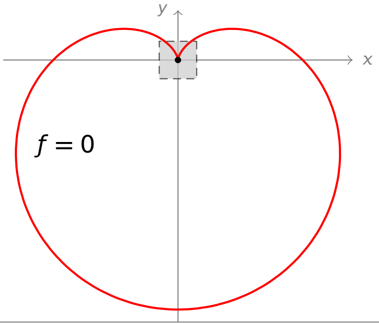

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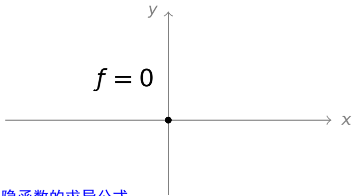

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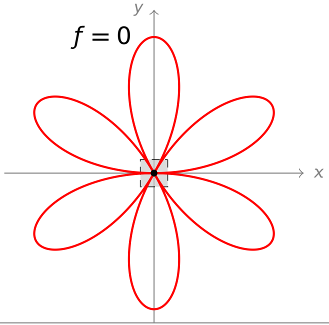

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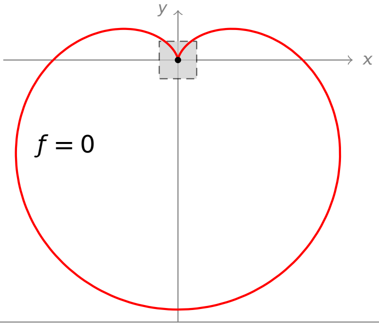

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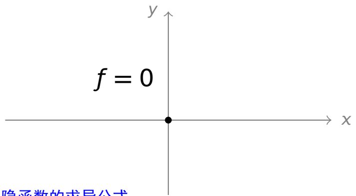

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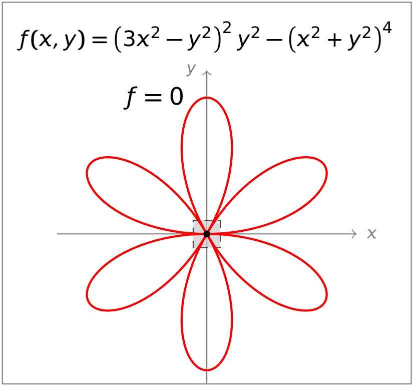

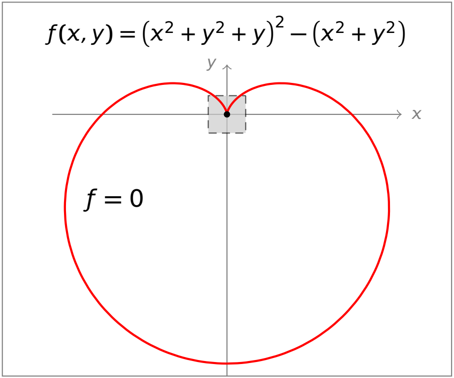

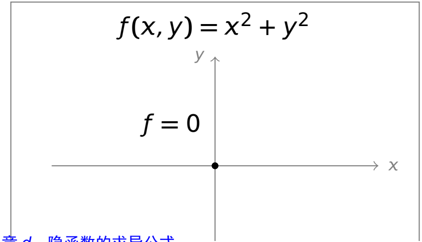

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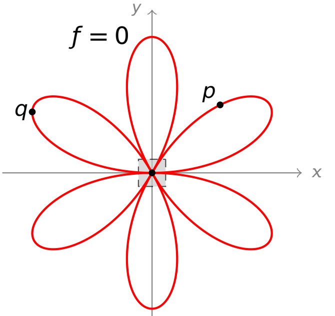

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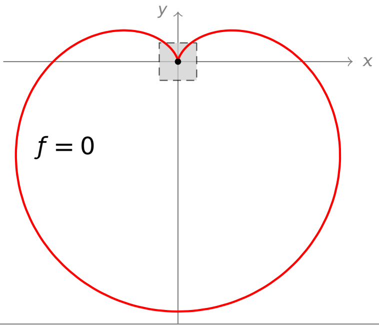

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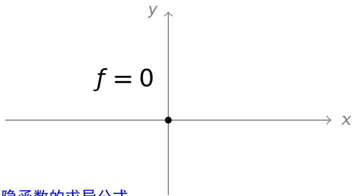

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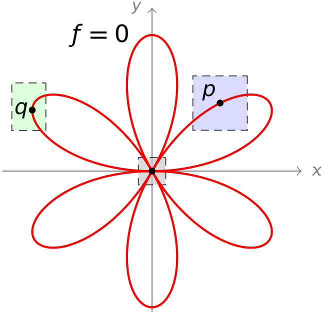

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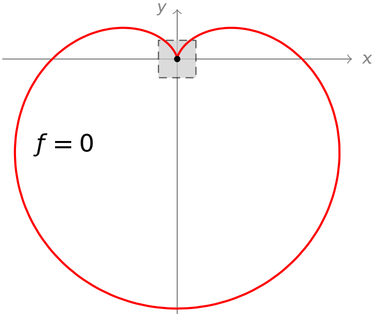

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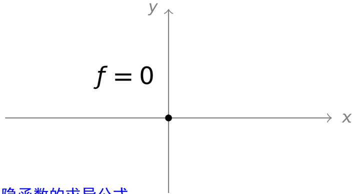

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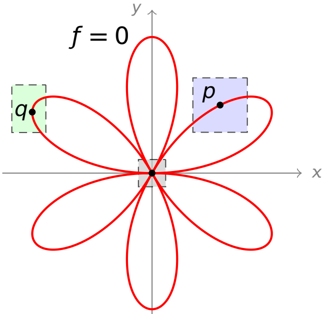

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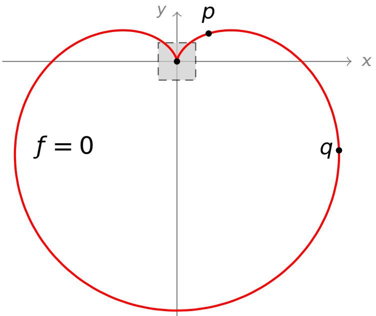

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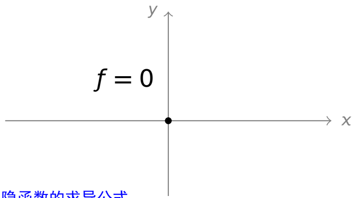

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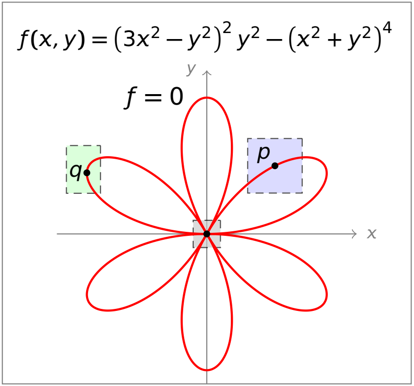

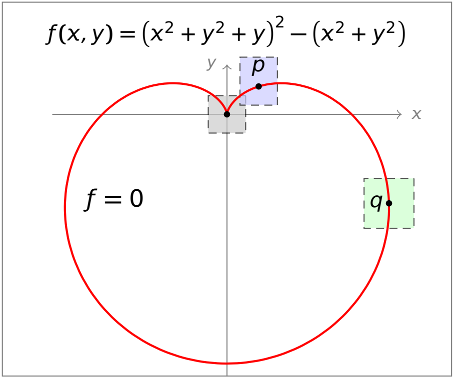

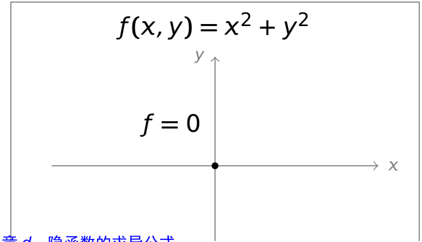

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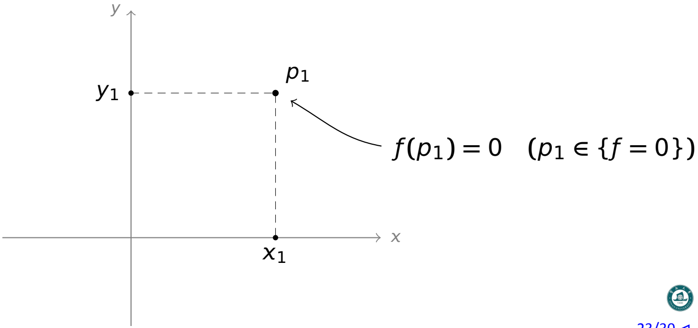
零点集$\{f=0\}$在p_1附近的形状



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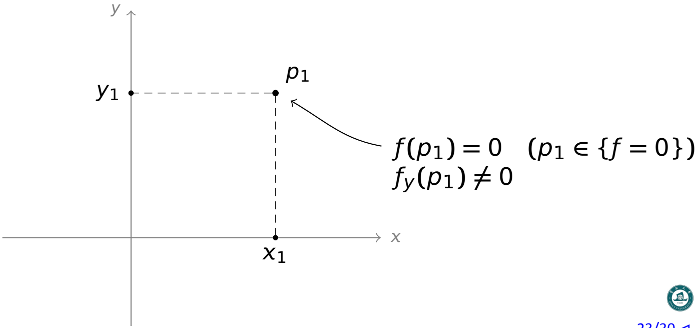
零点集$\{f=0\}$在p_1附近的形状



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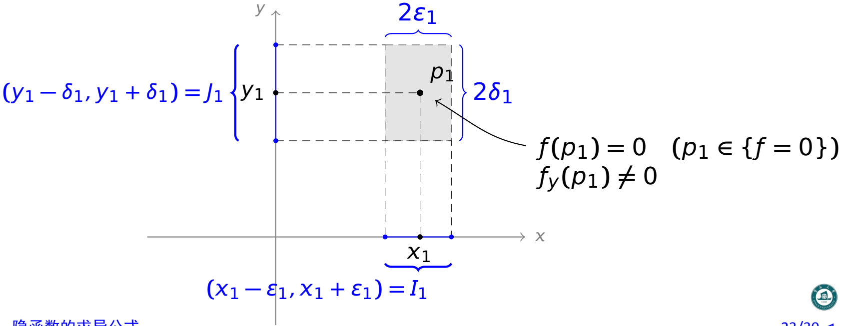
零点集$\{f=0\}$在p_1附近的形状



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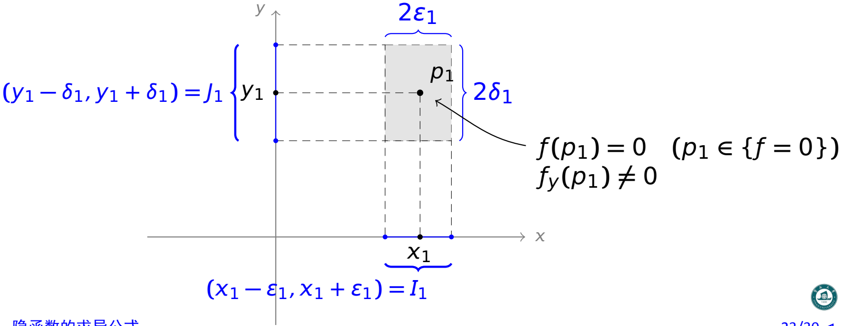
- 区间 $I_1 = (x_1 - \varepsilon, x_1 + \varepsilon)$ 和 $J_1 = (y_1 - \delta, y_1 + \delta)$，

$$\{f = 0\} \cap (I_1 \times J_1) =$$

零点集 $\{f = 0\}$ 在 p_1 附近的形状



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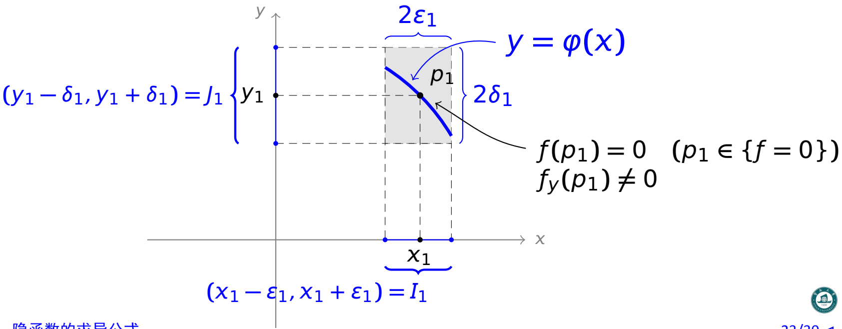
$$\{f=0\}\cap(I_1\times J_1)=$$

零点集 $\{f=0\}$ 在 p_1 附近的形状



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使得

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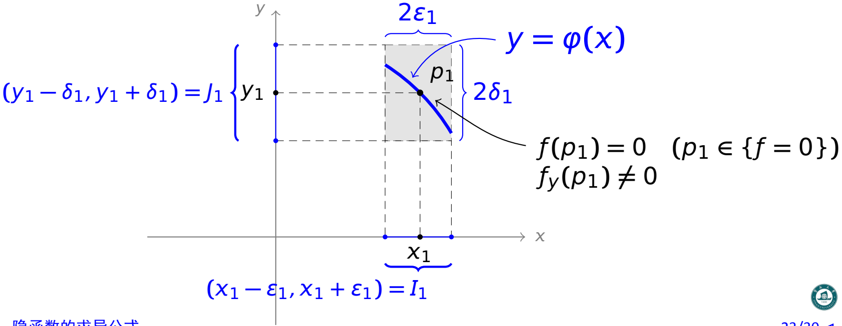
零点集$\{f=0\}$在p_1附近的形状

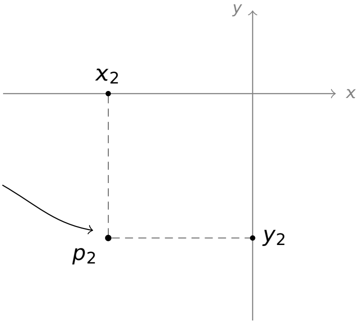


隐函数定理 1.2 设 $f(x,y)$ 在点 $p_2(x_2,y_2)$ 附近有定义，具有连续偏导；$f(x_2,y_2)=0$；$f_x(x_2,y_2)\neq 0$。

零点集$\{f=0\}$在p_1附近的形状

$(p_2\in\{f=0\})$ $\quad f(p_2)=0$

$f_x(p_2)\neq 0$



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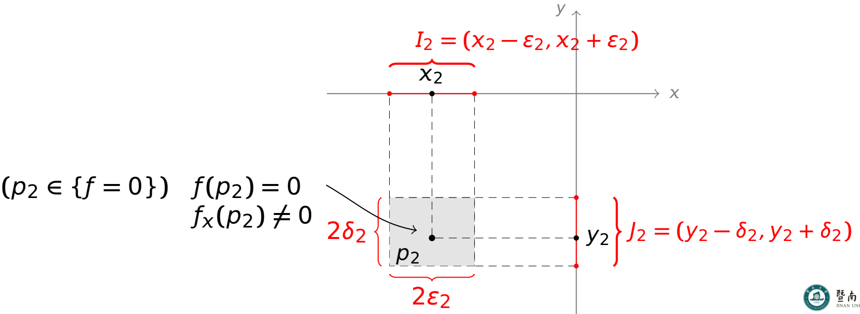
零点集$\{f=0\}$在p_1附近的形状



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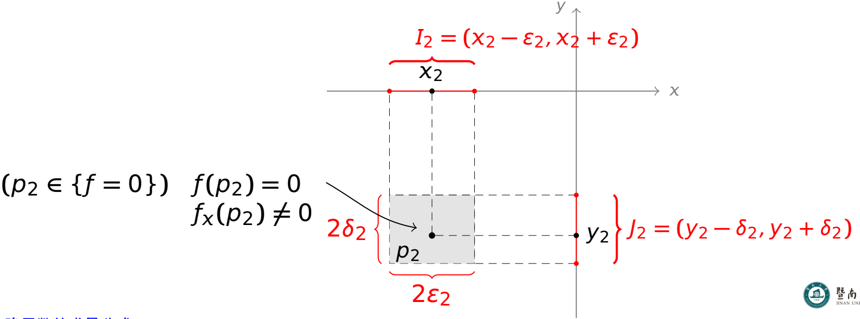
零点集$\{f=0\}$在p_1附近的形状



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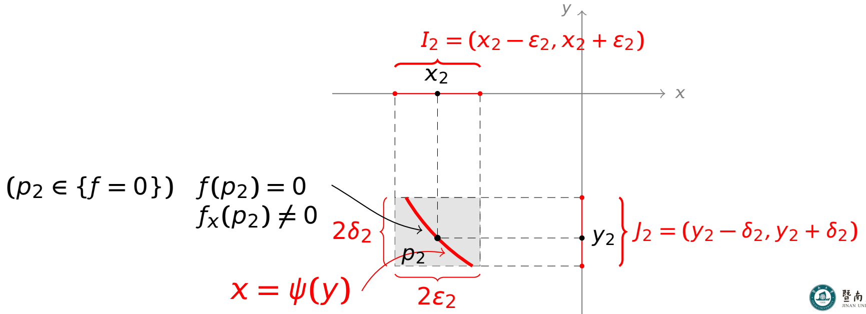
$$\{f=0\}\cap(J_2\times I_2)=$$

零点集$\{f=0\}$在p_1附近的形状



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零点集 $\{f=0\}$ 在 p_1 附近的形状

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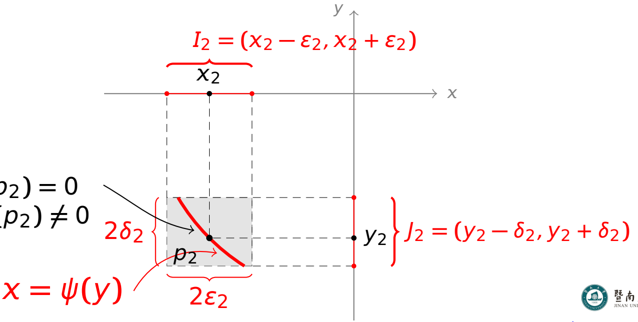

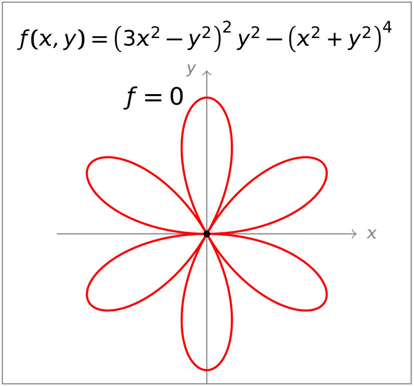
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x
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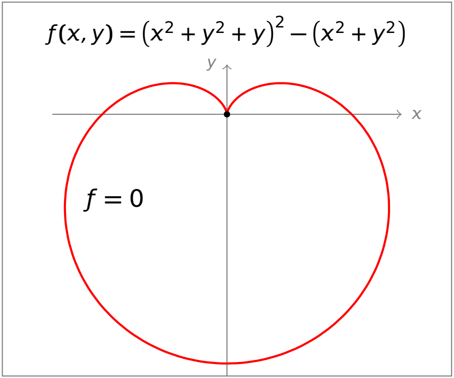
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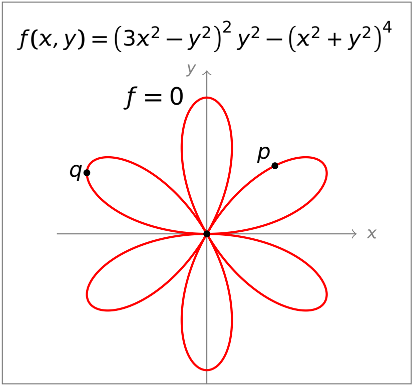
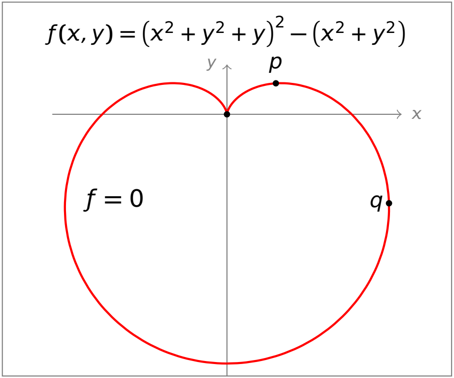
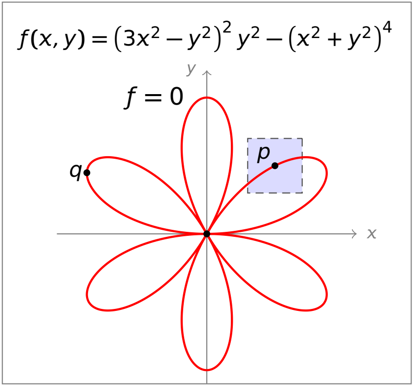
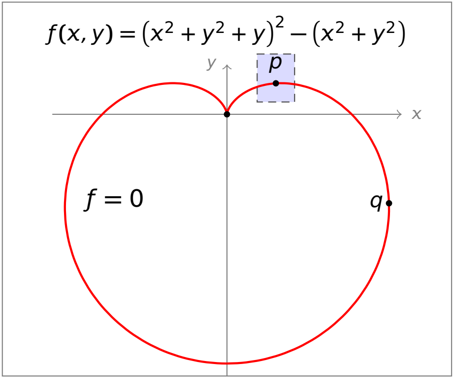
$f=0$

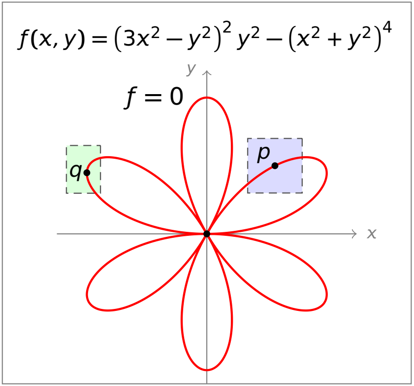

$f(x,y) = (3x^2 - y^2)^2 y^2 - (x^2+y^2)^4$
y
x
$f = 0$
p
q

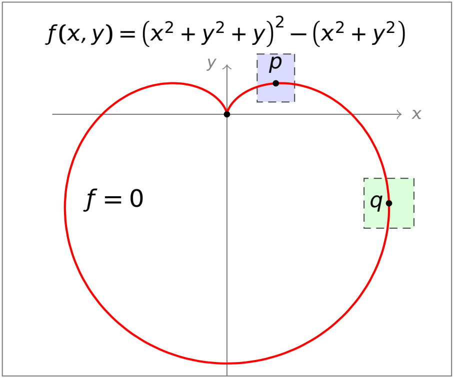

$f(x,y) = (x^2+y^2+y)^2 - (x^2+y^2)$
y
x
p
$f = 0$
q





- 在 p 点附近，$\{f=0\}$ 是函数 $y=\varphi(x)$ 的图形





- 在 p 点附近，$\{f=0\}$ 是函数 $y=\varphi(x)$ 的图形
- 在 q 点附近，$\{f=0\}$ 是函数 $x=\psi(x)$ 的图形

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  - $x = \psi(y)$ 图像的斜率是 $\frac{1}{\psi'(y)}$

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定理 设 $f(x,y)$ 具有连续偏导数，$p(x_0,y_0)$ 满足 $f(x_0,y_0)=0$，且偏导数 $f_x(x_0,y_0)$ 和 $f_y(x_0,y_0)$ 不全为零。则

- 点集 $\{f=0\}$ 在 p 点附近是光滑曲线；
- 曲线 $\{f=0\}$ 在 p 点处的切线平行于向量 $(f_y,-f_x)$。

设 $f(x,y,z)$ 是三元函数，其零点集 $\{f=0\}$ 是空间中的点集。

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准确来说，就是如下的隐函数定理：

隐函数定理 2.1

设 $f(x,y,z)$ 是三元函数，其零点集 $\{f=0\}$ 是空间中的点集。只要偏导数 f_x, f_y, f_z 不全为零，则 $\{f=0\}$ 是光滑曲面，并且局部上是光滑二元函数的图像。

准确来说，就是如下的隐函数定理：

隐函数定理 2.1 设 $f(x,y,z)$ 在点 $p(x_0,y_0,z_0)$ 附近有定义，具有连续偏导；$f(x_0,y_0,z_0)=0$；$f_z(x_0,y_0,z_0)\neq 0$。

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- 区间
  $$I_1=(x_0-\varepsilon,\, x_0+\varepsilon),\quad I_2=(y_0-\varepsilon,\, y_0+\varepsilon),\quad J=(z_0-\delta,\, z_0+\delta),$$

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$$\{f=0\}\cap(I_1\times I_2\times J)=$$

设 $f(x,y,z)$ 是三元函数，其零点集 $\{f=0\}$ 是空间中的点集。只要偏导数 f_x, f_y, f_z 不全为零，则 $\{f=0\}$ 是光滑曲面，并且局部上是光滑二元函数的图像。

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- 区间

  $I_1=(x_0-\varepsilon,\, x_0+\varepsilon),\quad I_2=(y_0-\varepsilon,\, y_0+\varepsilon),\quad J=(z_0-\delta,\, z_0+\delta)$,
- 函数 $\varphi: I_1\times I_2\to J$，$z=\varphi(x,\,y)$，且具有连续偏导数

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- 区间

  $$I_1=(x_0-\varepsilon,\, x_0+\varepsilon),\quad I_2=(y_0-\varepsilon,\, y_0+\varepsilon),\quad J=(z_0-\delta,\, z_0+\delta),$$
- 函数 $\varphi: I_1\times I_2\to J$，$z=\varphi(x,y)$，且具有连续偏导数

使得

$$\{f=0\}\cap(I_1\times I_2\times J)=\mathrm{Graph}(\varphi).$$

设 $f(x,y,z)$ 是三元函数，其零点集 $\{f=0\}$ 是空间中的点集。只要偏导数 f_x, f_y, f_z 不全为零，则 $\{f=0\}$ 是光滑曲面，并且局部上是光滑二元函数的图像。

准确来说，就是如下的隐函数定理：

隐函数定理 2.2 设 $f(x,y,z)$ 在点 $p(x_0,y_0,z_0)$ 附近有定义，具有连续偏导；$f(x_0,y_0,z_0)=0$；$f_y(x_0,y_0,z_0)\neq 0$。则存在

- 区间

  $I_1=(\qquad\qquad\qquad)$, $\quad I_2=(\qquad\qquad\qquad)$, $\quad J=(\qquad\qquad\qquad)$,

- 函数 $\varphi: I_1\times I_2\to J$, $\qquad\qquad$，且具有连续偏导数

使得

$$\{f=0\}\cap(I_1\times I_2\times J)=\mathrm{Graph}(\varphi).$$

设 $f(x,y,z)$ 是三元函数，其零点集 $\{f=0\}$ 是空间中的点集。只要偏导数 f_x, f_y, f_z 不全为零，则 $\{f=0\}$ 是光滑曲面，并且局部上是光滑二元函数的图像。

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隐函数定理 2.2 设 $f(x,y,z)$ 在点 $p(x_0,y_0,z_0)$ 附近有定义，具有连续偏导；$f(x_0,y_0,z_0)=0$；$f_y(x_0,y_0,z_0)\neq 0$。则存在

- 区间

  $I_1=(\qquad\qquad\qquad)$， $I_2=(\qquad\qquad\qquad)$， $J=(\qquad\qquad\qquad)$，

- 函数 $\varphi: I_1\times I_2\to J$，$y=\varphi(x,z)$，且具有连续偏导数

使得

$$\{f=0\}\cap(I_1\times I_2\times J)=\mathrm{Graph}(\varphi).$$

设 $f(x,y,z)$ 是三元函数，其零点集 $\{f=0\}$ 是空间中的点集。只要偏导数 f_x, f_y, f_z 不全为零，则 $\{f=0\}$ 是光滑曲面，并且局部上是光滑二元函数的图像。

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- 区间

  $I_1=(x_0-\varepsilon,\ x_0+\varepsilon),\quad I_2=(z_0-\varepsilon,\ z_0+\varepsilon),\quad J=(y_0-\delta,\ y_0+\delta)$,
- 函数 $\varphi: I_1\times I_2\to J$，$y=\varphi(x,z)$，且具有连续偏导数

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准确来说，就是如下的隐函数定理：

隐函数定理 2.3 设 $f(x,y,z)$ 在点 $p(x_0,y_0,z_0)$ 附近有定义，具有连续偏导；$f(x_0,y_0,z_0)=0$；$f_x(x_0,y_0,z_0)\neq 0$。则存在

- 区间

  $I_1 = (\qquad\qquad\qquad)$, $\quad I_2 = (\qquad\qquad\qquad)$, $\quad J = (\qquad\qquad\qquad)$,

- 函数 $\varphi : I_1 \times I_2 \to J$, $\qquad\qquad\qquad$ ，且具有连续偏导数

使得

$$\{f=0\} \cap (I_1 \times I_2 \times J) = \mathrm{Graph}(\varphi).$$

设 $f(x,y,z)$ 是三元函数，其零点集 $\{f=0\}$ 是空间中的点集。只要偏导数 f_x, f_y, f_z 不全为零，则 $\{f=0\}$ 是光滑曲面，并且局部上是光滑二元函数的图像。

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- 区间
  $I_1 = (y_0 - \varepsilon, y_0 + \varepsilon), \quad I_2 = (z_0 - \varepsilon, z_0 + \varepsilon), \quad J = (x_0 - \delta, x_0 + \delta)$，
- 函数 $\varphi : I_1 \times I_2 \to J$，$x = \varphi(y, z)$，且具有连续偏导数

使得

$$\{f = 0\} \cap (I_1 \times I_2 \times J) = \mathrm{Graph}(\varphi).$$

例 设 $f(x,y)=(2x^2+y^2+z^2-1)^3-\frac{1}{10}x^2z^3-y^2z^3$

- 求出 $\{f=0\}$ 上偏导数全为零的点（临界点）
- 在 CalcPlot3D 上画出曲面 $\{f=0\}$
- 观察临界点附近是否光滑
- 观察曲面哪些部分可以表示成光滑二元函数 $z=\varphi(x,y)$，或 $y=\psi(x,z)$，或 $x=\gamma(y,z)$ 的图形