

§2.2 矩阵的运算

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2018 - 2019 学年上学期

矩阵的加法运算

定义 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则定义

$$A + B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} + \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$

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$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} + b_{11} \\ \\ \\ \end{pmatrix}_{m \times n}$$

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$$\stackrel{\text{def}}{=} \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & \\ & & & \\ & & & \\ & & & \end{pmatrix}_{m \times n}$$

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称为矩阵 A , B 的**和**。

矩阵的减法运算

矩阵 A , B 的差定义为:

$$A - B = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} - \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}_{m \times n}$$

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例 $A = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix}$, 求 $A + B$ 和 $A - B$

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$$A + B = \begin{pmatrix} 1 & 3 & 5 \\ -1 & 2 & 4 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 3 \\ 7 & 3 & 1 \end{pmatrix} =$$

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矩阵的数乘

定义 设 $A = (a_{ij})_{m \times n}$, k 为数, 则定义

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练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

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$$\begin{aligned} 3A + 2B - 4C &= 3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4 \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix} \end{aligned}$$

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解

$$\begin{aligned} 3A + 2B - 4C &= 3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4 \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix} \end{aligned}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

解

$$\begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

解

$$\begin{aligned} 3A + 2B - 4C &= 3 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + 2 \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 4 \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 6 \\ 9 & 12 \end{pmatrix} + \begin{pmatrix} 6 & 10 \\ 14 & 12 \end{pmatrix} - \begin{pmatrix} 0 & -4 \\ 36 & 12 \end{pmatrix} = \begin{pmatrix} 9 & 20 \\ -13 & 12 \end{pmatrix} \end{aligned}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, 且 $5A + 3X = B$, 求 X

解

$$X = \frac{1}{3}(B - 5A) =$$

$$\begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix}$$

练习 设 $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 \\ 9 & 3 \end{pmatrix}$, 求 $3A + 2B - 4C$

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解

$$\begin{aligned} X &= \frac{1}{3}(B - 5A) = \frac{1}{3} \left(\begin{pmatrix} 3 & 5 \\ 7 & 6 \end{pmatrix} - 5 \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \right) \\ &= \begin{pmatrix} -\frac{2}{3} & -\frac{5}{3} \\ -\frac{8}{3} & -\frac{14}{3} \end{pmatrix} \end{aligned}$$

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解

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注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

$$, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix}, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} \quad , \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} \quad , \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

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性质 设 A, B, C 均是 $m \times n$ 矩阵, k, l 是数, 则

1. $k(A + B) = kA + kB$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, k, l 是数, 则

1. $k(A + B) = kA + kB$
2. $(k + l)A = kA + lA$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

性质 设 A, B, C 均是 $m \times n$ 矩阵, k, l 是数, 则

1. $k(A + B) = kA + kB$
2. $(k + l)A = kA + lA$
3. $(kl)A = k(lA)$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

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3. $(kl)A = k(lA)$
4. $1 \cdot A = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

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$$k(A + B) =$$

$$kA + kB =$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

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$$k(A + B) = k(\quad)_{m \times n}$$

$$kA + kB =$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

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1. $k(A + B) = kA + kB$
2. $(k + l)A = kA + lA$
3. $(kl)A = k(lA)$
4. $1 \cdot A = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$k(A + B) = k(a_{ij} + b_{ij})_{m \times n}$$

$$kA + kB =$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

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2. $(k + l)A = kA + lA$
3. $(kl)A = k(lA)$
4. $1 \cdot A = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$k(A + B) = k(a_{ij} + b_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

$$kA + kB =$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

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1. $k(A + B) = kA + kB$
2. $(k + l)A = kA + lA$
3. $(kl)A = k(lA)$
4. $1 \cdot A = A$

证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$k(A + B) = k(a_{ij} + b_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

$$kA + kB = (ka_{ij})_{m \times n} + (kb_{ij})_{m \times n}$$

注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

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注 区分

$$k \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \begin{vmatrix} k & 2k \\ 3 & 4 \end{vmatrix} \stackrel{\text{or}}{=} \begin{vmatrix} k & 2 \\ 3k & 4 \end{vmatrix} = -2k, \quad k \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} k & 2k \\ 3k & 4k \end{pmatrix}$$

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证明 设 $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{m \times n}$, 则

$$k(A + B) = k(a_{ij} + b_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

$$kA + kB = (ka_{ij})_{m \times n} + (kb_{ij})_{m \times n} = (ka_{ij} + kb_{ij})_{m \times n}$$

所以 $k(A + B) = kA + kB$ 。

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 $(2A - Y) - 2(B + Y) = O$, 求 Y

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 $(2A - Y) - 2(B + Y) = O$, 求 Y

解 $Y = \frac{2}{3}(A - B)$

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 $(2A - Y) - 2(B + Y) = O$, 求 Y

解 $Y = \frac{2}{3}(A - B)$, 所以

$$Y = \frac{2}{3}(A - B) = \frac{2}{3} \left(\begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix} \right)$$

练习 设

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

若 Y 满足 $(2A - Y) - 2(B + Y) = O$, 求 Y

解 $Y = \frac{2}{3}(A - B)$, 所以

$$\begin{aligned} Y &= \frac{2}{3}(A - B) = \frac{2}{3} \left(\begin{pmatrix} 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 3 & 2 & 1 \\ -2 & 1 & -2 & 1 \\ 0 & -1 & 0 & -1 \end{pmatrix} \right) \\ &= \frac{2}{3} \begin{pmatrix} -3 & -1 & -1 & 1 \\ 4 & 0 & 4 & 0 \\ 1 & 3 & 3 & 5 \end{pmatrix} \end{aligned}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$aA + bB + cC =$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$aA + bB + cC = a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} & \end{pmatrix} \end{aligned}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} a + b - c & \\ & \end{pmatrix} \end{aligned}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

假设 $aA + bB + cC = I$, 求数 a, b, c 的值

解

$$\begin{aligned} aA + bB + cC &= a \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} + b \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix} + c \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} a + b - c & b \\ 2a + 3b + c & a - c \end{pmatrix} \end{aligned}$$

练习 设

$$A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 3 & 0 \end{pmatrix}, C = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}, I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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所以

$$\begin{cases} a + b - c = 1 \\ \end{cases}$$

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$$\begin{cases} a + b - c = 1 \\ b = 0 \\ 2a + 3b + c = 0 \\ a - c = 1 \end{cases} \Rightarrow \begin{cases} b = 0 \end{cases}$$

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所以

$$\begin{cases} a + b - c = 1 \\ b = 0 \\ 2a + 3b + c = 0 \\ a - c = 1 \end{cases} \Rightarrow \begin{cases} a = \frac{1}{3} \\ b = 0 \end{cases}$$

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所以

$$\begin{cases} a + b - c = 1 \\ b = 0 \\ 2a + 3b + c = 0 \\ a - c = 1 \end{cases} \Rightarrow \begin{cases} a = \frac{1}{3} \\ b = 0 \\ c = -\frac{2}{3} \end{cases}$$

矩阵的乘积

定义 设 $A = (a_{ik})_{m \times l}$, $B = (b_{kj})_{l \times n}$, 定义矩阵 A , B 的乘积为 $m \times n$ 矩阵:

$$AB = A \cdot B = (a_{ik})_{m \times l} \cdot (b_{kj})_{l \times n} = (c_{ij})_{m \times n}$$

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$$a_{i1} \quad a_{i2} \quad \cdots \quad a_{il}$$

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矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
$$= \begin{pmatrix} c_{11} & \cdots & \cdots & c_{1n} \\ \vdots & & \vdots & \vdots \\ \cdots & c_{ij} & \cdots & \\ \vdots & & \vdots & \vdots \\ c_{m1} & \cdots & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$

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$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
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$$a_{i1} \quad a_{i2} \quad \cdots \quad a_{il}$$

矩阵的乘积

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$$a_{i1}b_{1j} \quad a_{i2}b_{2j} \quad \cdots \quad a_{il}b_{lj}$$

矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
$$= \begin{pmatrix} c_{11} & \cdots & \cdots & c_{1n} \\ \vdots & & \vdots & \vdots \\ \cdots & c_{ij} & \cdots & \\ \vdots & & \vdots & \vdots \\ c_{m1} & \cdots & \cdots & c_{mn} \end{pmatrix}_{m \times n}$$

$$a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj}$$

矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
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$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{il}b_{lj}$$

矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
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矩阵的乘积

$$\begin{pmatrix} a_{11} & \cdots & \cdots & a_{1l} \\ \vdots & & & \vdots \\ a_{i1} & \cdots & \cdots & a_{il} \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{ml} \end{pmatrix}_{m \times l} \cdot \begin{pmatrix} b_{11} & \cdots & b_{1j} & \cdots & b_{1n} \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ b_{l1} & \cdots & b_{lj} & \cdots & b_{ln} \end{pmatrix}_{l \times n}$$
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矩阵的乘积

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矩阵的乘积

例 1 $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3}$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \end{pmatrix}$$

矩阵的乘积

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$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}_{4 \times 3}$$

矩阵的乘积

例 1

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

矩阵的乘积

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$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$c_{23} =$

矩阵的乘积

例 1

$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

$c_{23} = a_{21} \quad a_{22}$

矩阵的乘积

例 1

$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$
$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

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$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

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$$c_{23} = a_{21}b_{13} + a_{22}b_{23}$$

例 2 设 $A = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}$, 计算 AB

矩阵的乘积

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解

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} =$$

矩阵的乘积

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$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

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矩阵的乘积

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$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

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解

$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} \color{red}{1} & -2 & -3 \\ \color{red}{2} & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} * & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

矩阵的乘积

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$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

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解

$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} \color{red}{1} & -2 & -3 \\ \color{red}{2} & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

矩阵的乘积

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$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

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矩阵的乘积

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$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

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矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

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矩阵的乘积

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$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

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$$AB = \begin{pmatrix} \color{red}{2} & \color{red}{3} \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & \color{red}{-3} \\ 2 & -1 & \color{red}{0} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

矩阵的乘积

例 1
$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

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矩阵的乘积

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矩阵的乘积

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$$\begin{pmatrix} a_{11} & a_{12} \\ \color{red}{a_{21}} & \color{red}{a_{22}} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{pmatrix}_{4 \times 2} \cdot \begin{pmatrix} b_{11} & b_{12} & \color{red}{b_{13}} \\ b_{21} & b_{22} & \color{red}{b_{23}} \end{pmatrix}_{2 \times 3} = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & \color{red}{c_{23}} \\ c_{31} & c_{32} & c_{33} \\ c_{41} & c_{42} & c_{43} \end{pmatrix}_{4 \times 3}$$

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解

$$AB = \begin{pmatrix} 2 & 3 \\ 1 & -2 \\ 3 & 1 \end{pmatrix}_{3 \times 2} \cdot \begin{pmatrix} 1 & -2 & -3 \\ 2 & -1 & 0 \end{pmatrix}_{2 \times 3} = \begin{pmatrix} 8 & -7 & -6 \\ -3 & 0 & -3 \\ 5 & -7 & -9 \end{pmatrix}_{3 \times 3}$$

矩阵的乘积

例 3 计算 $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

矩阵的乘积

例 3 计算 $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} =$$

矩阵的乘积

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解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \end{pmatrix}_{2 \times 3}$$

矩阵的乘积

例 3 计算 $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & & \\ & & \end{pmatrix}_{2 \times 3}$$

矩阵的乘积

例 3 计算 $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

解

$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & \quad \\ \quad & \quad & \quad \end{pmatrix}_{2 \times 3}$$

矩阵的乘积

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解

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矩阵的乘积

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$$\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix}_{2 \times 3} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}_{3 \times 3} = \begin{pmatrix} 10 & 4 & -1 \\ 4 & & \end{pmatrix}_{2 \times 3}$$

矩阵的乘积

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矩阵的乘积

例 3 计算 $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 3 & 0 & -1 \end{pmatrix}$

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例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (\quad)_{1 \times 1}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) =$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ & & \\ & & \end{pmatrix}_{3 \times 3}$$

例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

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例 4 设 $A = (1, 2, 3)$, $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $C = \begin{pmatrix} 5 & -19 & 11 \\ 6 & -9 & 4 \\ 7 & 1 & -3 \end{pmatrix}$ 。计算 AB , BA 及 CB 。

解

$$AB = (1, 2, 3) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3)_{1 \times 1} = 14$$

$$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (1, 2, 3) = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}_{3 \times 3}$$

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矩阵的乘积

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注

1. 即便 AB , BA 都有意义, 也不一定相等。

矩阵的乘法不满足交换律!

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2. $BA = 0$ 不能推出 $B = 0$ 或 $A = 0$

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注 即便假设 $A \neq 0$, $BA = CA$ 也推不出 $B = C$ 。如

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$$\underbrace{\begin{pmatrix} 2 & 0 \\ 0 & -6 \end{pmatrix}}_B \underbrace{\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}}_A \quad \underbrace{\begin{pmatrix} 0 & -4 \\ 3 & 0 \end{pmatrix}}_C \underbrace{\begin{pmatrix} -2 & 4 \\ 1 & -2 \end{pmatrix}}_A$$

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- 总结**
1. AB 可以存在, 但 BA 不一定有意义
 2. 即便 AB , BA 都有意义, 也不一定相等。矩阵的乘法不满足交换律! (矩阵相乘要注意顺序)
 3. $BA = 0$ 不能推出 $B = 0$ 或 $A = 0$
 4. 即便假设 $A \neq 0$, $BA = CA$ 也推不出 $B = C$ 。

矩阵乘法的运算法则

设下列各式所涉及的矩阵乘法都是有意义，则

1. $(AB)C = A(BC)$
2. $(A + B)C = AC + BC$
3. $C(A + B) = CA + CB$
4. $k(AB) = (kA)B = A(kB)$

矩阵的转置

定义 将 $m \times n$ 矩阵 A 的行与列互换，得到的 $n \times m$ 矩阵，称为矩阵 A 的转置矩阵，记为 A^T 。

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$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n}$$

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$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \Rightarrow A^T = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{pmatrix}_{n \times m}$$

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注

	A	A^T
位置 (i, j) 上的元素		

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$$AA^T = \begin{pmatrix} 2 & 1 & 4 \\ -3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 21 & \\ & \end{pmatrix}_{2 \times 2}$$

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并且

$$\begin{array}{ccc} (AB)^T & AB & B^T A^T \\ (i, j) \text{ 元素} = & (j, i) \text{ 元素} = & (i, j) \text{ 元素} \end{array}$$

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证明 设 $A = A_{m \times l}$, $B = B_{l \times n}$, 则

	AB	$(AB)^T$	B^T	A^T	$B^T A^T$
阶数	$m \times n$	$n \times m$	$n \times l$	$l \times m$	$n \times m$

并且

$$(AB)^T_{(i,j) \text{ 元素}} = AB_{(j,i) \text{ 元素}} = a_{j1}b_{1i} + a_{j2}b_{2i} + \cdots + a_{jl}b_{li} \quad B^T A^T_{(i,j) \text{ 元素}}$$

转置矩阵的性质

1. $(A^T)^T = A$
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A^T 第 j 列元素

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转置矩阵的性质

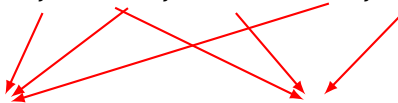
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A^T 第 j 列元素 B^T 第 i 行元素

方阵的幂

设 $A = (a_{ij})_{n \times n}$ 为 n 阶方阵, $k \in \mathbb{N}$ 为自然数, 定义

$$A^k = \underbrace{A \cdot A \cdot \dots \cdot A}_{k \text{ 个}}$$

称为方阵 A 的 k 次幂

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$$A^4 =$$

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\vdots

$$A^n =$$

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\vdots

$$A^n = \begin{pmatrix} 1 & 0 \\ n\lambda & 1 \end{pmatrix}$$

方阵的幂

注 设 A, B 为 n 阶方阵, 一般地

$$(AB)^k \neq A^k B^k$$

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这是, 例如 $k = 2$ 时,

$$(AB)^2 =$$

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这是，例如 $k = 2$ 时，

$$(AB)^2 = (AB) \cdot (AB) = A \textcolor{red}{B} A B$$

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但一般地， $AB \neq BA$ ，所以 $(AB)^2 \neq A^2 B^2$

方阵的行列式

回忆：对 n 阶方阵

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

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方阵的行列式

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其行列式规定为

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设 A, B 均是 n 阶方阵, k 为数, 则

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$$|kA| = \left| k \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \right| =$$

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方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

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$$|4A| = 4^3 |A| =$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

解

$$|4A| = 4^3 |A| = 64 \begin{vmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{vmatrix} =$$

方阵行列式的性质

例 设 $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 4 \\ 6 & 15 & 5 \end{pmatrix}$, 求 $|4A|$

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练习 设 A 为三阶方阵, 且 $|A| = -2$, 求 $|A|A^2A^T|$

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$$\begin{aligned} ||A|A^2A^T| &= |A|^3 |A^2A^T| \\ &= |A|^3 |A^2| |A^T| \\ &= |A|^3 |A|^2 |A| \\ &= |A|^6 \end{aligned}$$

方阵行列式的性质

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方程组的矩阵表示

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

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$$\parallel$$
$$\begin{pmatrix} \quad \quad \quad \end{pmatrix}_{m \times n} \begin{pmatrix} \quad \quad \quad \end{pmatrix}_{n \times 1}$$

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$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}_{m \times n} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}_{n \times 1}$$

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系数矩阵

方程组的矩阵表示

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$$\underbrace{\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}}_A \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix} = \underbrace{\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}}_b$$

系数矩阵 常数项矩阵

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系数矩阵 常数项矩阵

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系数矩阵 常数项矩阵

进一步改写成

$$Ax = b$$

线性方程组的矩阵表示

例 方程组

$$\begin{cases} x_1 - x_2 + 5x_3 - x_4 = -2 \\ x_1 + x_2 - 2x_3 + 3x_4 = 3 \\ 3x_1 - x_2 + 8x_3 + x_4 = 7 \end{cases}$$

线性方程组的矩阵表示

例 方程组

$$\begin{cases} x_1 - x_2 + 5x_3 - x_4 = -2 \\ x_1 + x_2 - 2x_3 + 3x_4 = 3 \\ 3x_1 - x_2 + 8x_3 + x_4 = 7 \end{cases}$$

的矩阵表示 $Ax = b$ 是

$$\begin{pmatrix} & & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

线性方程组的矩阵表示

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$$\begin{cases} x_1 - x_2 + 5x_3 - x_4 = -2 \\ x_1 + x_2 - 2x_3 + 3x_4 = 3 \\ 3x_1 - x_2 + 8x_3 + x_4 = 7 \end{cases}$$

的矩阵表示 $Ax = b$ 是

$$\begin{pmatrix} 1 & -1 & 5 & -1 \\ 1 & 1 & -2 & 3 \\ 3 & -1 & 8 & 1 \end{pmatrix} \begin{pmatrix} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} \\ \\ \end{pmatrix}$$

线性方程组的矩阵表示

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$$\begin{cases} x_1 - x_2 + 5x_3 - x_4 = -2 \\ x_1 + x_2 - 2x_3 + 3x_4 = 3 \\ 3x_1 - x_2 + 8x_3 + x_4 = 7 \end{cases}$$

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线性方程组的矩阵表示

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