第 12 章 e: 傅里叶级数

数学系 梁卓滨

2016-2017 **学年** II



Outline

1. 傅里叶级数的概念

2. 周期为 2π 的周期函数的傅里叶级数

3. 一般周期函数的傅里叶级数



We are here now...

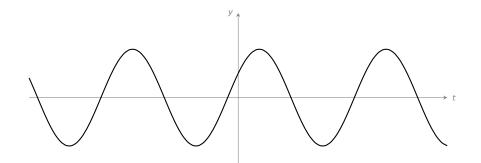
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2. 周期为 2π 的周期函数的傅里叶级数

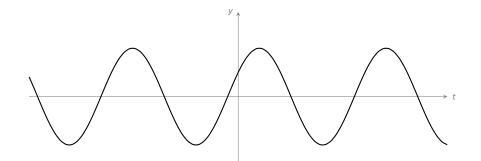
3. 一般周期函数的傅里叶级数



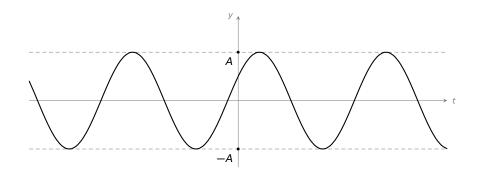
正弦函数 $y = A \sin(\omega t + \varphi)$



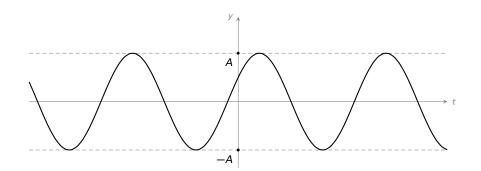
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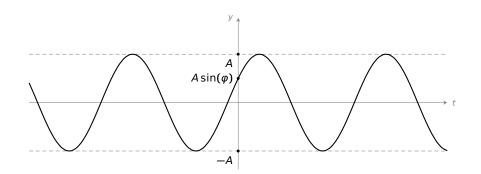
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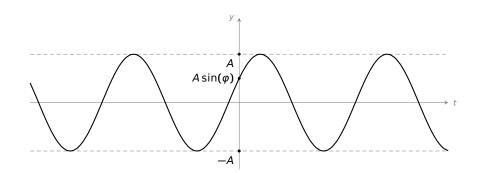
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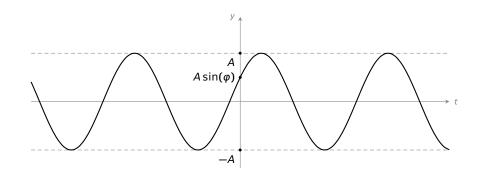


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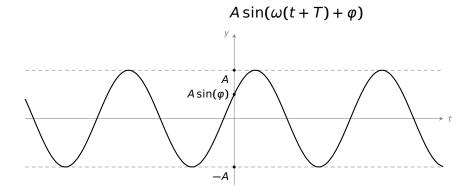
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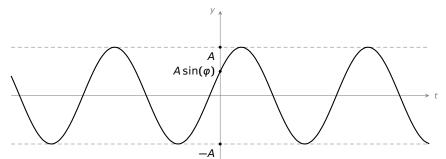




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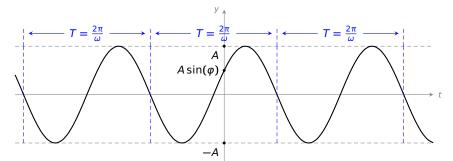
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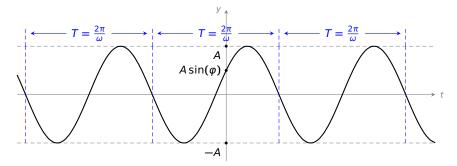




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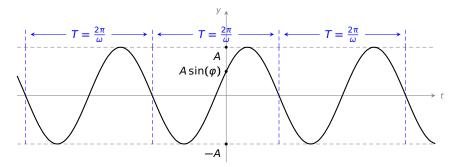
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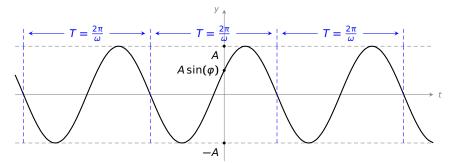
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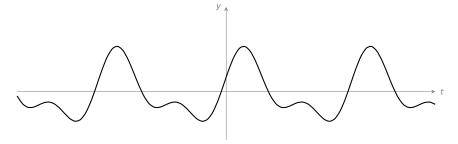
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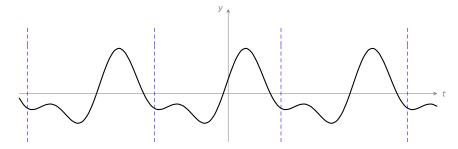


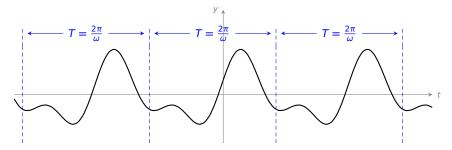
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然 $T = \frac{2\pi}{4}$ 也是周期

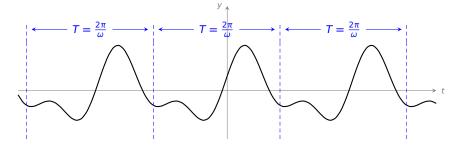








假设 f(t) 是定义域为 \mathbb{R} 的周期函数,周期也是 $T = \frac{2\pi}{\omega}$ 。

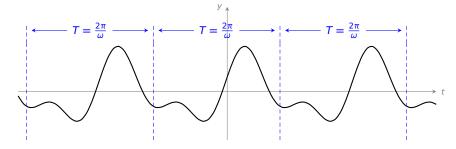


问题 是否有如下展开

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin(n\omega t + \varphi_n)$$



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$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin(n\omega t + \varphi_n)$$

注 在电工学中,上述展开称为谐波分析; A₀ 称为直流分量;

 $A_n \sin(n\omega t + \varphi_n)$ 称为 n 次谐波



设 $T = \frac{2\pi}{\omega} = 2l$,

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注意到
$$\omega = \frac{\pi}{l}$$
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$$A_n \sin(n\omega t + \varphi_n) = A_n \sin(\frac{n\pi t}{l} + \varphi_n)$$

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$$A_n \sin(n\omega t + \varphi_n) = A_n \sin(\frac{n\pi t}{l} + \varphi_n)$$

$$\sin \varphi_n \cos \frac{n\pi t}{l} + \cos \varphi_n \sin \frac{n\pi t}{l}$$

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设 $T = \frac{2\pi}{l} = 2l$,故区间 [-l, l] 是 f(t) 的一个完整周期。

注意到
$$\omega = \frac{\pi}{7}$$
,所以

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$$=: a_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l}$$

这时
$$f(t) = A_0 + \sum_{n=0}^{\infty} A_n \sin(n\omega t + \varphi_n)$$



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$$\omega = \overline{l}$$
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以下不妨先设周期 $T = 2\pi (l = \pi)$ 。 f(x) 的周期区间为 $[-\pi, \pi]$,

设 $T = \frac{2\pi}{4} = 2l$,故区间 [-l, l] 是 f(t) 的一个完整周期。

注意到 $\omega = \frac{\pi}{7}$, 所以

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,凡

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应的展开为

We are here now...

1. 傅里叶级数的概念

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3. 一般周期函数的傅里叶级数



性质 三角函数系

1, $\cos x$, $\sin x$, $\cos 2x$, $\sin 2x$, ..., $\cos nx$, $\sin nx$, ...

在区间 $[-\pi, \pi]$ 上正交。

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在区间 $[-\pi, \pi]$ 上正交。即上述任意相异两个函数的乘积,在 $[-\pi, \pi]$ 上的积分为零:

$$\int_{-\pi}^{\pi} \cos nx dx = 0, \qquad \int_{-\pi}^{\pi} \sin nx dx = 0 \qquad (n = 1, 2, 3, \dots)$$

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另外

$$\int_{-\pi}^{\pi} \sin^2 nx dx = \int_{-\pi}^{\pi} \cos^2 nx dx = \pi \qquad (n = 1, 2, 3, \dots)$$



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

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"形式推导" (1) 当 $n = 1, 2, 3, \cdots$ 时,

$$\int_{-\pi}^{\pi} f(x) \cos nx dx$$

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$$= \int_{-\pi}^{\pi} a_n \cos nx \cdot \cos nx dx$$



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$$= \int_{-\pi}^{\pi} a_n \cos nx \cdot \cos nx dx = \pi a_n$$



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"形式推导" (2) 当
$$n = 1, 2, 3, \cdots$$
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"形式推导"
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定义 f(x) 的傅里叶级数定义为

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问题 何时成立
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定理(收敛定理, 狄利克雷充分条件)



- 1. 在一个周期内连续或只有有限个第一类间断点;
- 2. 在一个周期内至多只有有限个极值点,

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$$\frac{1}{2} \left[f(x^{-}) + f(x^{+}) \right] = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

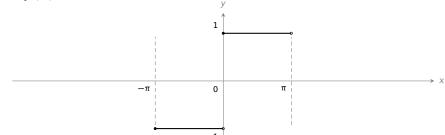


$$f(x) = \begin{cases} -1, & -\pi \le x < 0, \\ 1, & 0 \le x < \pi. \end{cases}$$

求出 f(x) 的傅里叶级数。

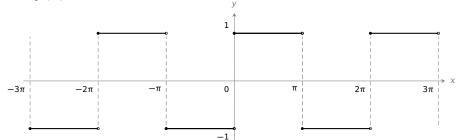
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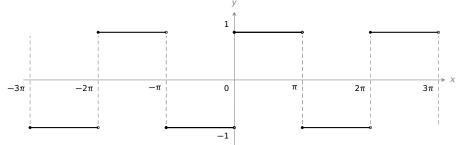
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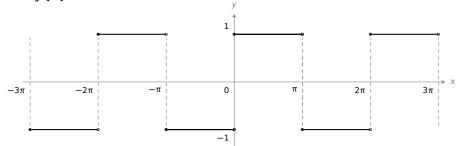
解 计算傅里叶系数如下:

 a_n



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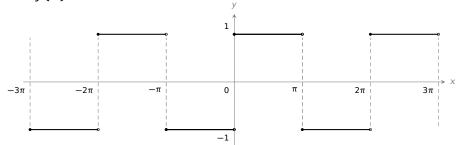


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解 计算傅里叶系数如下:

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{3}} 0$$



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 b_n

$$\alpha_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{\pi} \text{ med}} 0,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{4}} 0,$$

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$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{6}} 0,$$

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$$2 \qquad \cos nx |_{\pi}$$

$$= \frac{2}{\pi} \cdot (-1) \cdot \frac{\cos nx}{n} \Big|_{0}^{\pi}$$



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$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \stackrel{\text{fight}}{===} 0,$$

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$$= \left\{ \begin{array}{c} n = 1, 3, 5, \cdots \\ n = 2, 4, 6, \cdots . \end{array} \right.$$



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所以傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right) = \sum_{n=1}^{\infty} b_n \sin nx$$



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所以傅里叶级数为 $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right) = \sum_{n=1}^{\infty} b_n \sin nx$

$$= \frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$$

R 12 草 e: 傳里叶级委

注 1 f(x) 的傅里叶级数是 $\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$

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(显然,可直接看出当 $x = n\pi$ 时傅里叶级数的值为 0)

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$



注
$$1 f(x)$$
 的傅里叶级数是 $\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$, 利用 收敛定理分析可知:

$$f(x) = \frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$$

• 当 $x = n\pi$ 是,是 f 的间断点,此时

$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right] = \frac{1}{2} \left[f(x^{-}) + f(x^{+}) \right] = 0$$
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$$1 - \frac{1}{2} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots = \frac{\pi}{4}$$



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$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots = \frac{\pi}{4}$$

注 3 奇函数 f(x) 的傅里叶级数是 $\sum_{n=1}^{\infty} b_n \sin nx$



$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$$

$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

$$\sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$

 -2π

$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

考虑部分和

$$\sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$



 3π

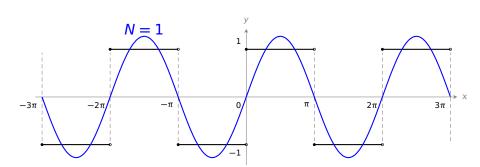


 -3π

 2π

$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

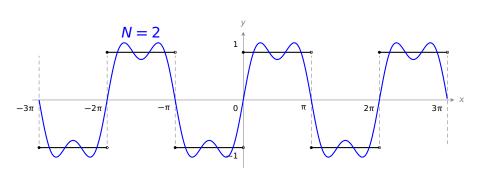
$$\sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$





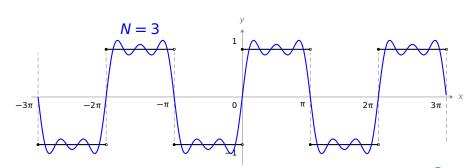
$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

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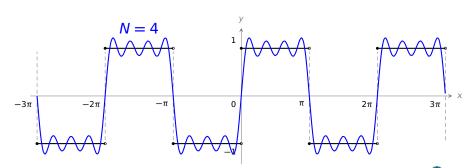
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$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

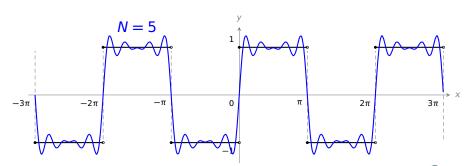
$$\sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$





$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

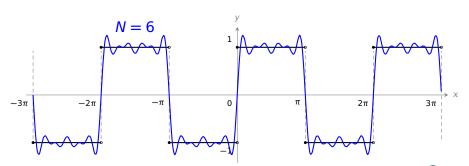
$$\sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$





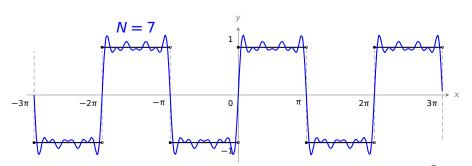
$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

$$\sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$



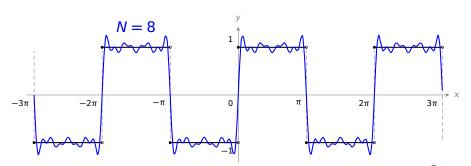
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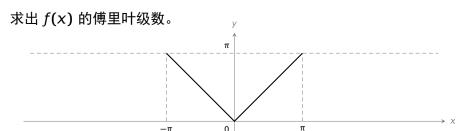




例 设 f(x) 是周期为 2π 的周期函数,在 $[-\pi, \pi)$ 上的表达式为 f(x) = |x|

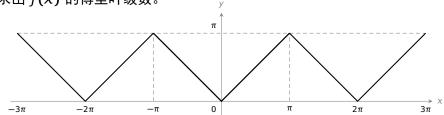
求出
$$f(x)$$
 的傅里叶级数。

$$f(x) = |x|$$



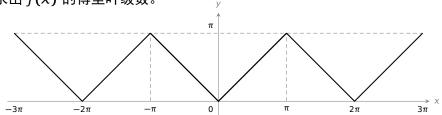
$$f(x) = |x|$$

求出 f(x) 的傅里叶级数。



$$f(x) = |x|$$

求出 f(x) 的傅里叶级数。

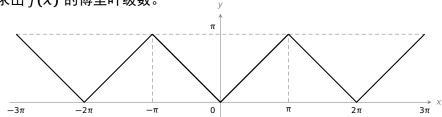


解 计算傅里叶系数如下:

 b_n

$$f(x) = |x|$$

求出 f(x) 的傅里叶级数。



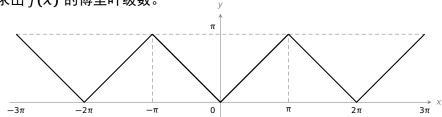
解 计算傅里叶系数如下:

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$



$$f(x) = |x|$$

求出 f(x) 的傅里叶级数。



解 计算傅里叶系数如下:

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{§fight}} 0,$$

$$a_n =$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fille } 0},$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{§fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{§fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$= \frac{2}{n\pi} \int_{0}^{\pi} x d\sin nx = \frac{2}{n\pi} \left[x \sin nx \right]_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \right]_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx$$

$$= \frac{2}{n\pi} \int_{0}^{\pi} xu \sin nx - \frac{1}{n\pi} \left[x \sin nx \right]_{0}^{\pi}$$

$$= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \right]_{0}^{\pi}$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$\frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin$$

$$= \frac{2}{n\pi} \int_0^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$
$$= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \Big|_0^{\pi} \right] = \frac{2}{n^2 \pi} \left[(-1)^n - 1 \right]$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$f(x) \cos nx dx = -\int_{-\pi}^{\pi} f(x) \cos nx dx = -\int_{0}^{\pi} f(x) \cos nx dx = -\int_{0}^{\pi} x \cos nx dx$$
$$= -\int_{0}^{\pi} x d\sin nx = -\int_{0}^{\pi} x \sin nx dx$$

$$\begin{aligned} & = \frac{1}{\pi} \int_{-\pi}^{\pi} (x) \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} (x) \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} x \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} x \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx = \frac{1}{\pi} \int_{0}^{\pi} x d \sin nx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx = \frac{1}{\pi} \int_{0}^{\pi} x d \sin nx = \frac{1}{\pi} \int_{0}^{\pi} x d \sin nx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx = \frac{1}{\pi} \int_{0}^{\pi} x d \sin nx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx = \frac{1}{\pi} \int_{0}^{\pi} x d \sin nx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx dx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx dx dx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx dx dx = \frac{1}{\pi} \int_{0}^{\pi} \sin nx dx dx dx dx = \frac{1}{\pi$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$f(x) \cos nx dx = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

 $= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \Big|_{0}^{\pi} \right] = \frac{2}{n^{2}\pi} \left[(-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \cdots \\ n = 2, 4, 6, \cdots \end{cases}$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} x \cos nx dx$$
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$$\pi \int_{-\pi}^{\pi} \pi \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

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$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

$$n\pi \int_{0}^{\pi} n\pi \int_{0}^{\pi} n\pi \int_{0}^{\pi} \int_{0}$$

 a_0



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

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$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

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$$= \frac{2}{n\pi} \int_0^{\pi} x d\sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$

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$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$\pi \int_{-\pi}^{\pi} \pi \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \Big|_{0}^{\pi} \right] = \frac{2}{n^{2}\pi} \left[(-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

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$$a_{0} = \frac{1}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} x dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} x dx$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{§fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$= \frac{2}{n\pi} \int_0^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]_0^{\pi}$$

$$= \frac{2}{n\pi} \int_0^{\pi} x d\sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$

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 $\frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos nx = \frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$

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$$\dot{x}$$
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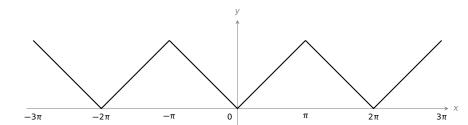
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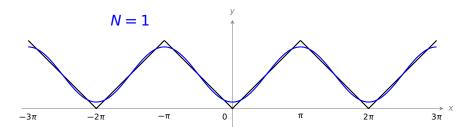
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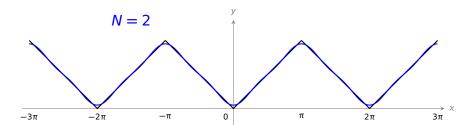
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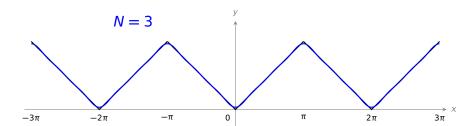
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考虑部分和

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 $\sum_{n=1}^{\infty} \frac{\pi}{2} - \frac{4}{\pi} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$

考虑部分和

$$N=4$$

π

3π

 -3π

 -2π

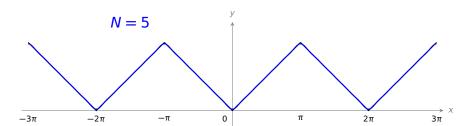
 $-\pi$

2π

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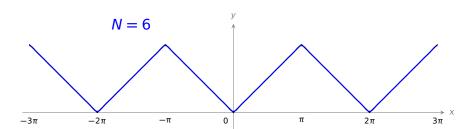




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考虑部分和

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$$N = 7$$

π



3π

 -3π

 -2π

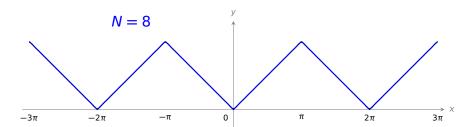
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2π

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$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\frac{\hat{\sigma}(\text{RMt})}{\pi}} \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx$$

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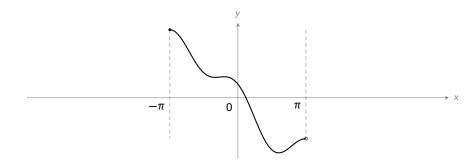
证明 (2) 假设 f 为偶函数,则

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0$$

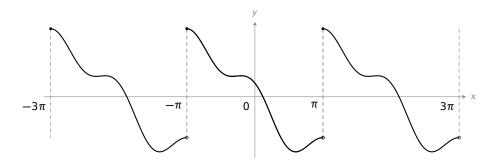
$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\text{fight}} \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx$$

设 f(x) 是定义在区间 $[-\pi, \pi]$ (或 $(-\pi, \pi]$)上的函数,可以对其进行周期延拓,从而得到定义在 \mathbb{R} 上的周期函数

设 f(x) 是定义在区间 $[-\pi, \pi]$ (或 $(-\pi, \pi]$)上的函数,可以对其进行周期延拓,从而得到定义在 \mathbb{R} 上的周期函数,如图:

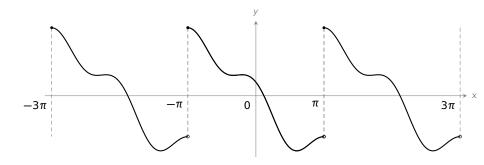


设 f(x) 是定义在区间 $[-\pi, \pi)$ (或 $(-\pi, \pi]$)上的函数,可以对其进行周期延拓,从而得到定义在 \mathbb{R} 上的周期函数,如图:





设 f(x) 是定义在区间 $[-\pi, \pi)$ (或 $(-\pi, \pi]$)上的函数,可以对其进行周期延拓,从而得到定义在 \mathbb{R} 上的周期函数,如图:



延拓后的周期函数任然记为 f(x),此时可以进行傅里叶展开。



设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得到定义在 \mathbb{R} 上的周期奇函数。

设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得到定义在 \mathbb{R} 上的周期奇函数。

设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得到定义在 \mathbb{R} 上的周期奇函数。

奇延拓步骤:

定义 f(0) = 0

设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得到定义在 \mathbb{R} 上的周期奇函数。

奇延拓步骤:

• 定义 f(0) = 0; 当 $x \in (-\pi, 0)$ 时, 定义 f(x) = -f(-x);

设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得到定义在 \mathbb{R} 上的周期奇函数。

奇延拓步骤:

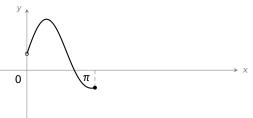
• 定义 f(0) = 0; 当 $x \in (-\pi, 0)$ 时,定义 f(x) = -f(-x); (此时 f 在 $(-\pi, \pi]$ 上有定义,且在 $(-\pi, \pi)$ 上为奇函数)

设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得到定义在 \mathbb{R} 上的周期奇函数。

- 定义 f(0) = 0; 当 $x \in (-\pi, 0)$ 时,定义 f(x) = -f(-x); (此时 f 在 $(-\pi, \pi]$ 上有定义,且在 $(-\pi, \pi)$ 上为奇函数)
- 周期延拓 f 在 $(-\pi, \pi]$ 上的取值。

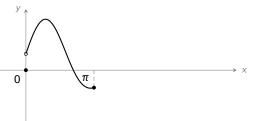
设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得到定义在 \mathbb{R} 上的周期奇函数。

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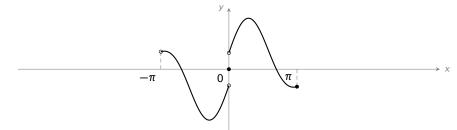
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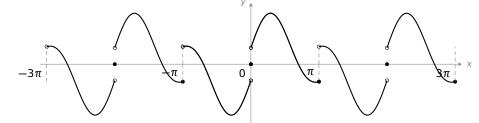
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- 周期延拓 f 在 (-π, π] 上的取值。



设 f(x) 是定义在区间 $[0, \pi]$ 上的函数,可以对其进行偶延拓,从而得到定义在 \mathbb{R} 上的周期偶函数。

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偶延拓步骤:

• 当 $x \in [-\pi, 0]$ 时,定义 f(x) = f(-x);

设 f(x) 是定义在区间 $[0, \pi]$ 上的函数,可以对其进行偶延拓,从而得到定义在 \mathbb{R} 上的周期偶函数。

偶延拓步骤:

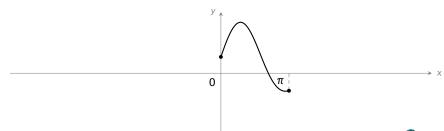
• 当 $x \in [-\pi, 0]$ 时,定义 f(x) = f(-x); (此时 f 成为定义在 $[-\pi, \pi]$ 上为偶函数)

设 f(x) 是定义在区间 $[0, \pi]$ 上的函数,可以对其进行偶延拓,从而得到定义在 \mathbb{R} 上的周期偶函数。

- 当 $x \in [-\pi, 0]$ 时,定义 f(x) = f(-x); (此时 f 成为定义在 $[-\pi, \pi]$ 上为偶函数)
- 周期延拓 f 在 $[-\pi, \pi]$ 上的取值。

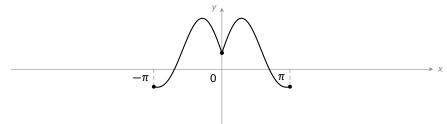
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- 当 $x \in [-\pi, 0]$ 时,定义 f(x) = f(-x); (此时 f 成为定义在 $[-\pi, \pi]$ 上为偶函数)
- 周期延拓 f 在 [-π, π] 上的取值。



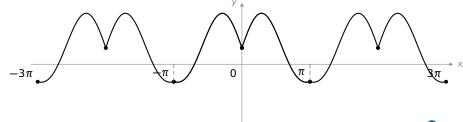
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- 当 $x \in [-\pi, 0]$ 时,定义 f(x) = f(-x); (此时 f 成为定义在 $[-\pi, \pi]$ 上为偶函数)
- 周期延拓 f 在 $[-\pi, \pi]$ 上的取值。



设 f(x) 是定义在区间 $[0, \pi]$ 上的函数,可以对其进行偶延拓,从而得到定义在 \mathbb{R} 上的周期偶函数。

- 当 $x \in [-\pi, 0]$ 时,定义 f(x) = f(-x); (此时 f 成为定义在 $[-\pi, \pi]$ 上为偶函数)
- 周期延拓 f 在 $[-\pi, \pi]$ 上的取值。



We are here now...

1. 傅里叶级数的概念

2. 周期为 2π 的周期函数的傅里叶级数

3. 一般周期函数的傅里叶级数

假设 f(x) 是定义在 \mathbb{R} 上周期函数,周期为 T=2l,

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_{n} = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$1 \int_{-l}^{l} n\pi x$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导" $\Rightarrow q(x) = f(\frac{l}{\pi}x),$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导" 令 $g(x) = f(\frac{1}{\pi}x)$, 则 g 是周期为 2π 的周期函数:



$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导" $\Leftrightarrow g(x) = f(\frac{\iota}{\pi}x)$, 则 g 是周期为 2π 的周期函数:

$$q(x+2\pi)$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导" $\Leftrightarrow g(x) = f(\frac{l}{\pi}x)$, 则 g 是周期为 2π 的周期函数:

$$g(x + 2\pi) = f(\frac{l}{\pi}(x + 2\pi))$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导" $\Leftrightarrow g(x) = f(\frac{l}{\pi}x)$,则 g 是周期为 2π 的周期函数:

$$g(x+2\pi) = f(\frac{l}{\pi}(x+2\pi)) = f(\frac{l}{\pi}x+2l)$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$g(x+2\pi) = f(\frac{l}{\pi}(x+2\pi)) = f(\frac{l}{\pi}x+2l) = f(\frac{l}{\pi}x)$$



$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导" $\Leftrightarrow g(x) = f(\frac{l}{\pi}x)$, 则 g 是周期为 2π 的周期函数:

 $g(x+2\pi) = f(\frac{l}{\pi}(x+2\pi)) = f(\frac{l}{\pi}x+2l) = f(\frac{l}{\pi}x) = g(x)$



假设 f(x) 是定义在 \mathbb{R} 上周期函数,周期为 T=2l,其傅里叶级数应为: $\frac{a_0}{2} + \sum_{i=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$

 $a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx$ $(n = 0, 1, 2, 3, \cdots)$

其中

 $b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx$ $(n = 0, 1, 2, 3, \dots)$

$$b_n = \frac{1}{l} \int_{-l} f(x)$$

"推导" 令 $g(x) = f(\frac{l}{\pi}x)$, 则 g 是周期为 2π 的周期函数: $g(x+2\pi) = f(\frac{l}{\pi}(x+2\pi)) = f(\frac{l}{\pi}x+2l) = f(\frac{l}{\pi}x) = g(x)$

所以
$$g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$



"推守" 令
$$g(x) = f(\frac{1}{\pi}x)$$
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"推导" 令 $g(x) = f(\frac{l}{\pi}x)$, 则 g 是周期为 2π 的周期函数:

$$h_{n} = \frac{1}{n} \int_{0}^{1} f(x) \sin \frac{n\pi x}{n} dx \qquad (n = 0, 1, 2, 3, \dots)$$

 $b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx$ $(n = 0, 1, 2, 3, \dots)$

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

 $f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$

 $\frac{a_0}{2} + \sum_{i=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$

其中

所以

既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$



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其中

 a_n

 b_n



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其中

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz$$

 b_n



既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz$$



既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

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$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$x = \frac{l}{\pi}z$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz$$



既然
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$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

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$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$= \frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos \frac{n\pi x}{l}$$

$$1 \int_{-\pi}^{\pi} f(x) \cos \frac{n\pi x}{l}$$

 $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz$



既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

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$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$= \frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x)$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x)$$

 $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz$

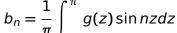


既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

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既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

所以

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

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$$= \frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

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既然
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$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

$$b_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \sin nz dz$$



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$$\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

$$b_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \sin nz dz$$

$$x = \frac{l}{\pi}z$$



既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

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$$\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

 $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(-z) \sin nz dz$ $= \frac{x = \frac{1}{\pi}z}{\pi} \int f(x) \sin \frac{n\pi x}{l}$



既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

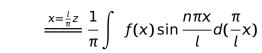
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{-z}) \cos nz dz$$

$$\frac{x = \frac{l}{\pi} z}{\pi} \frac{1}{\pi} \int_{-\pi}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l} x) = \frac{1}{l} \int_{-\pi}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

 $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(-z) \sin nz dz$



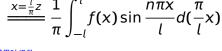


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$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx$,
$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \sin nz dz$	
, , , , ,	





既然 $f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$

所以
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

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$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

 $\frac{x=\frac{l}{\pi}z}{\pi}\frac{1}{\pi}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}d(\frac{\pi}{L}x)=\frac{1}{L}\int_{-L}^{L}f(x)\cos\frac{n\pi x}{L}dx,$ $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(-z) \sin nz dz$

$$\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx.$$

更多内容

傅里叶变换在工程中有许多应用,更多内容可以浏览在"Stanford Engineering Everywhere"中的课程"The Fourier Transform and Its Applications",讲义在 这里。

