第 9 章 d：隐函数的求导公式

数学系　　梁卓滨

2016-2017 学年 II

Outline

1. 一个方程的情形

2. 方程组的情形

We are here now...

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2. 方程组的情形

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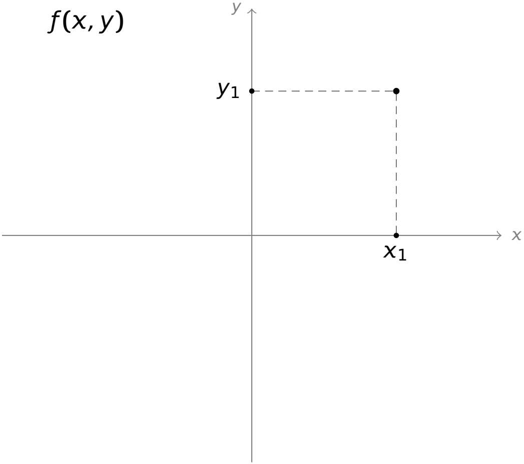

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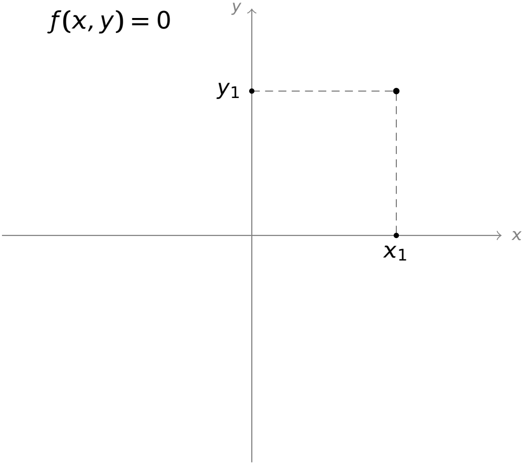

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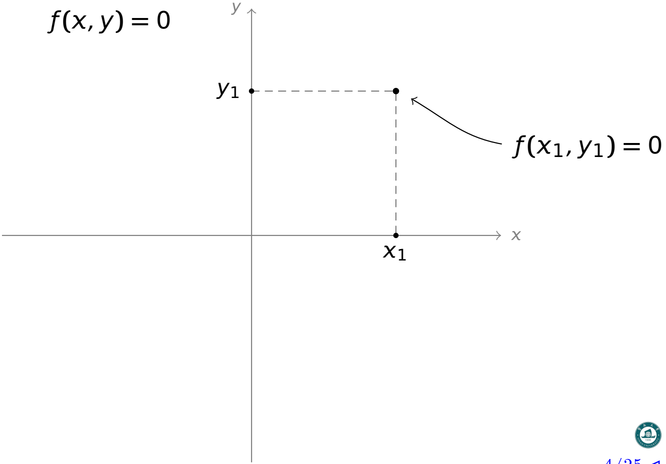

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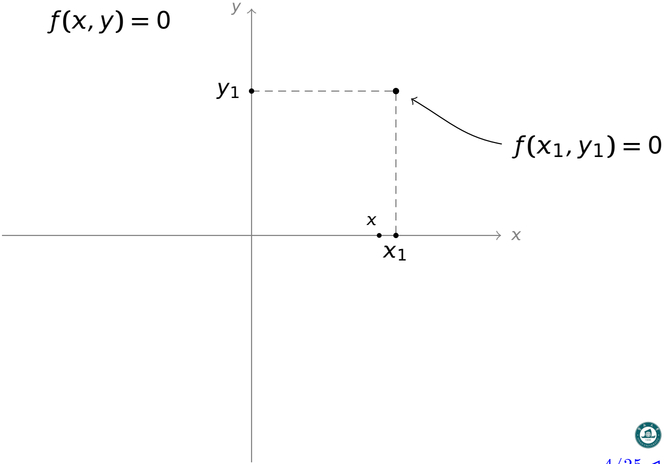

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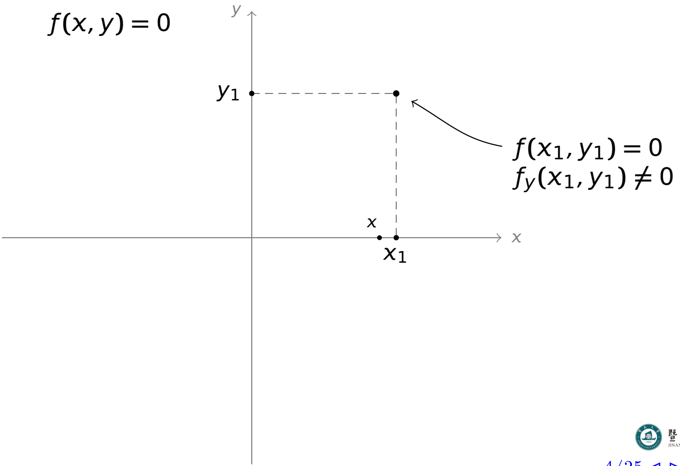

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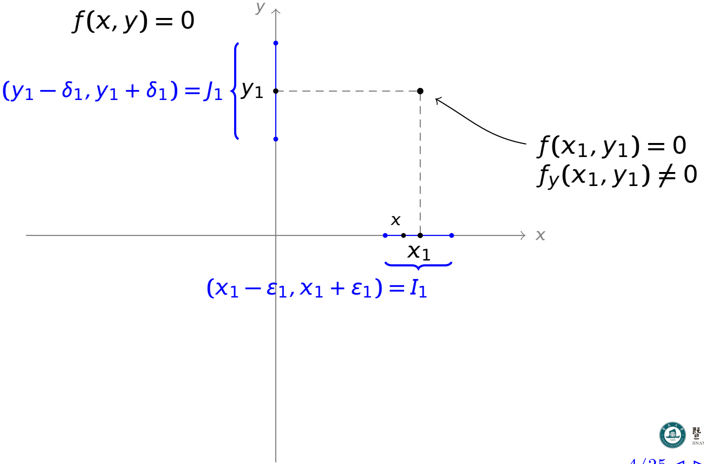

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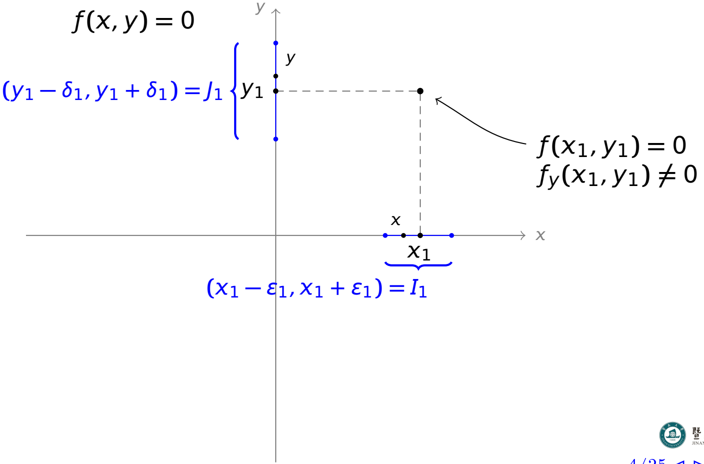

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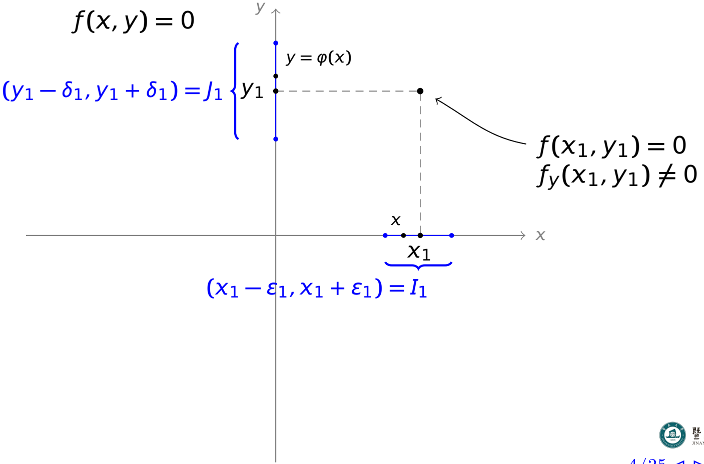

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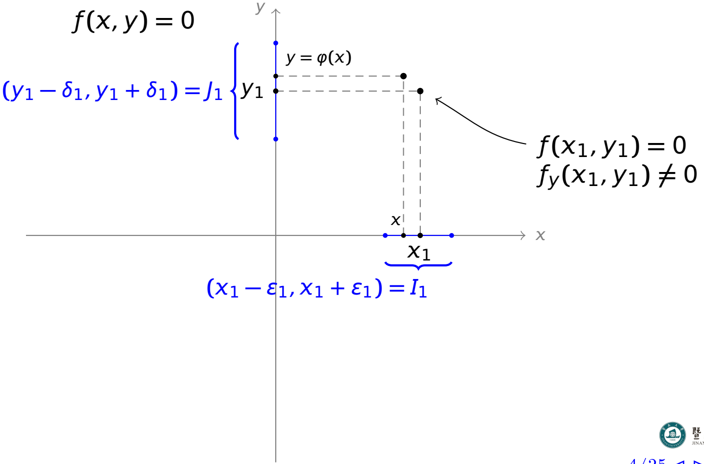

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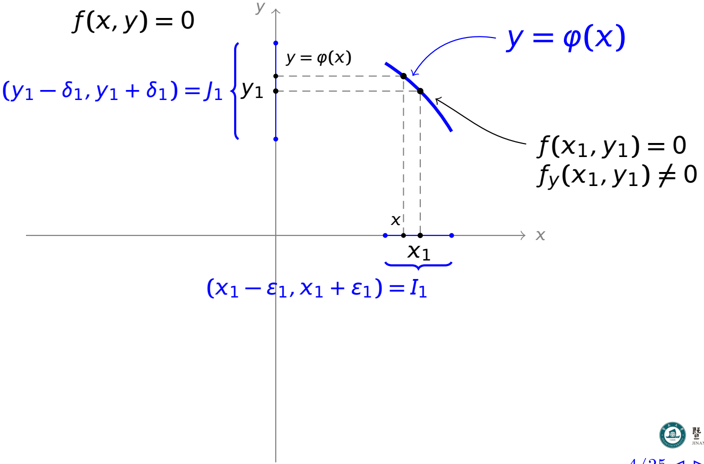

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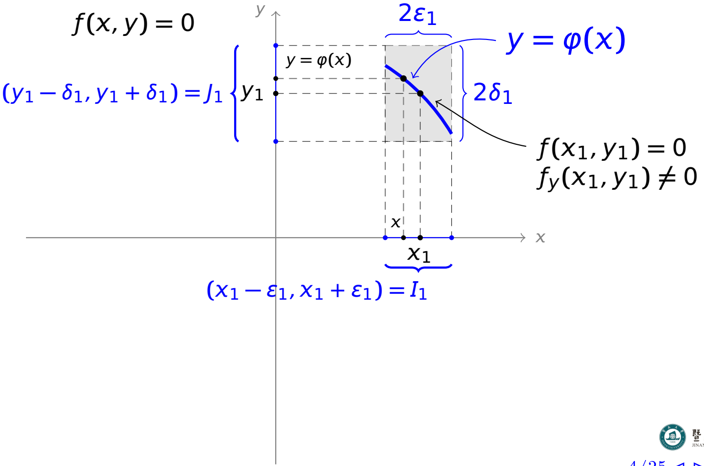

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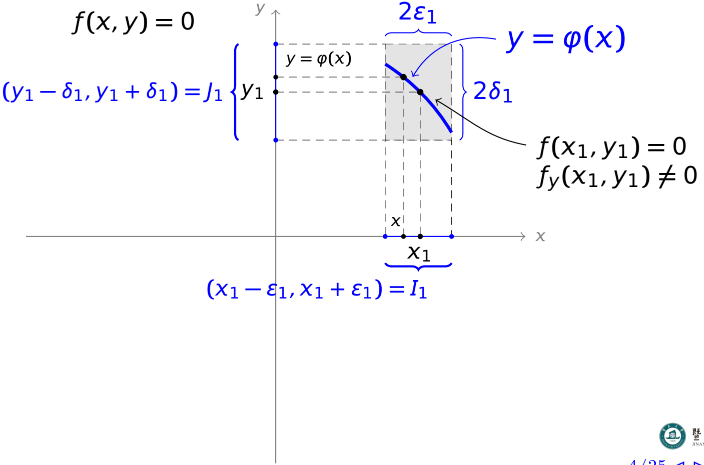

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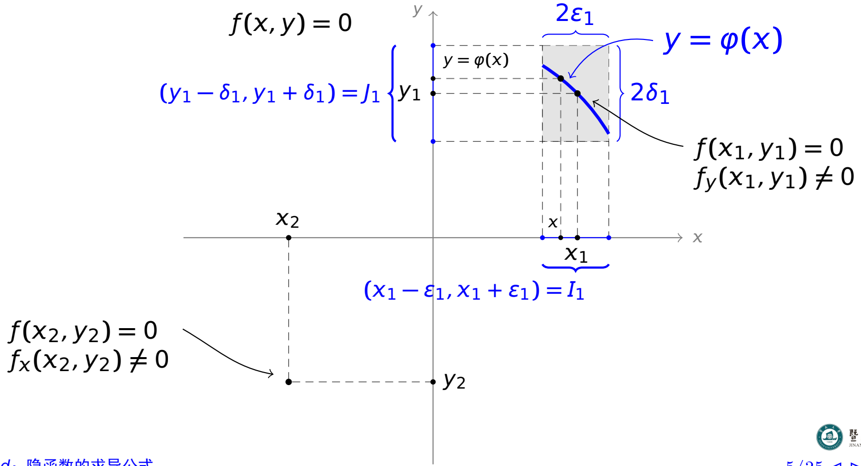

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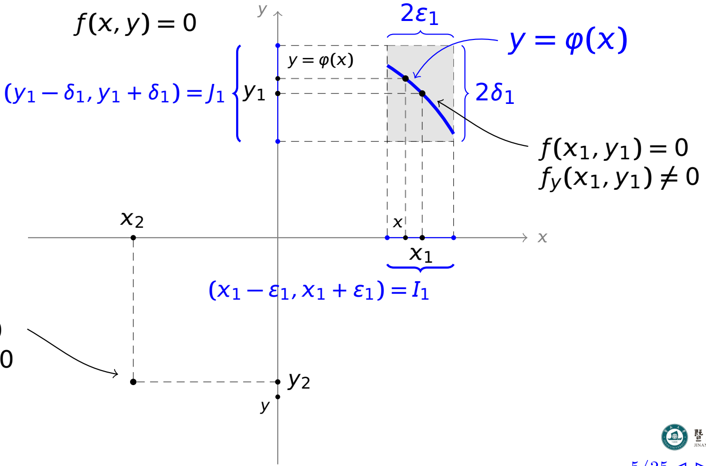

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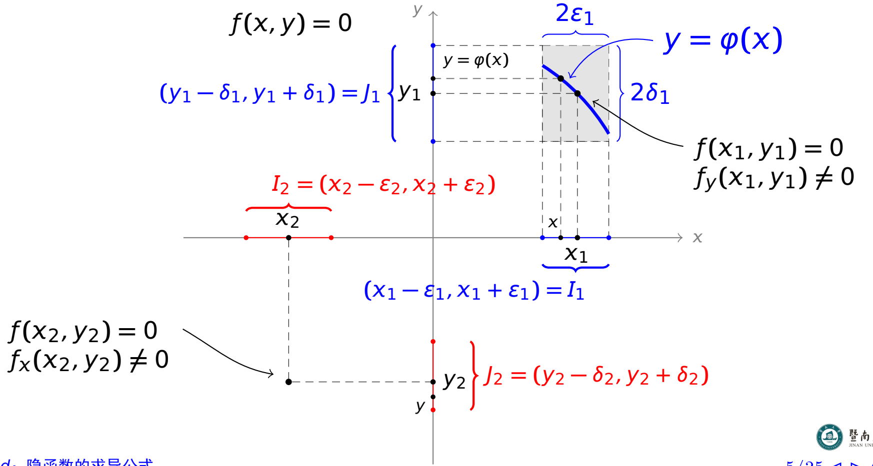

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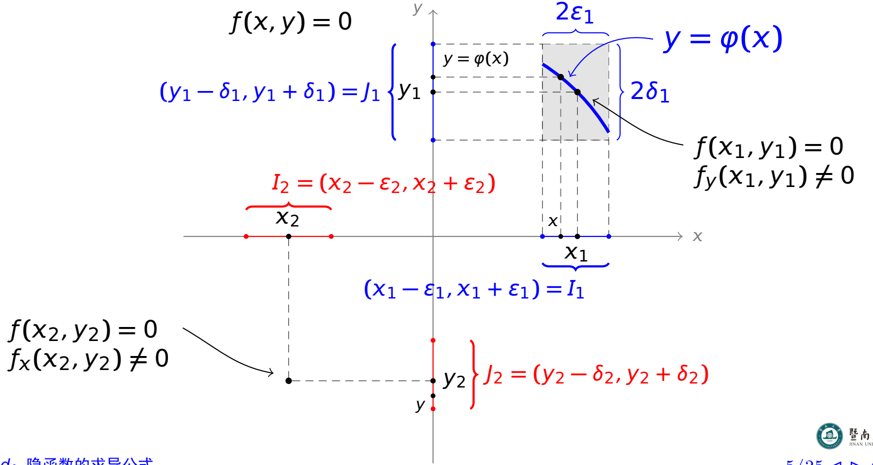

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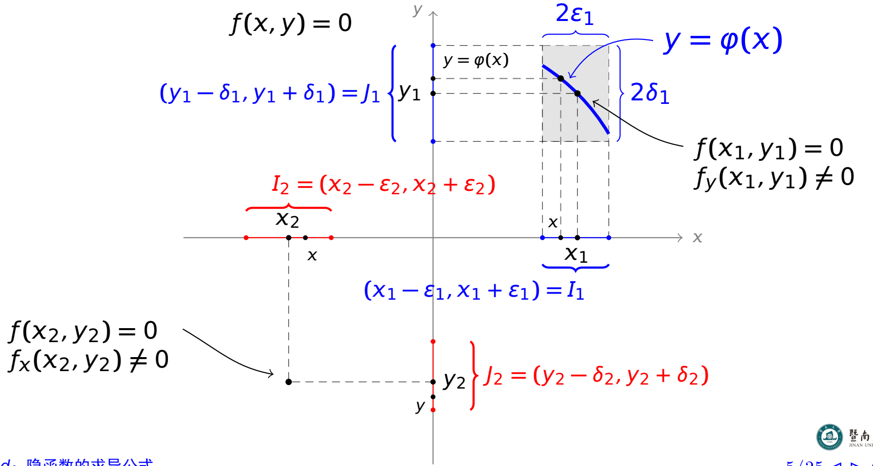

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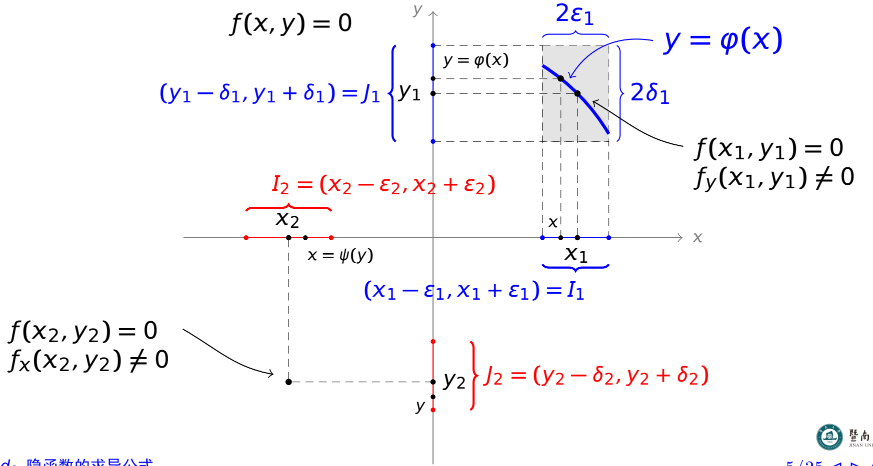

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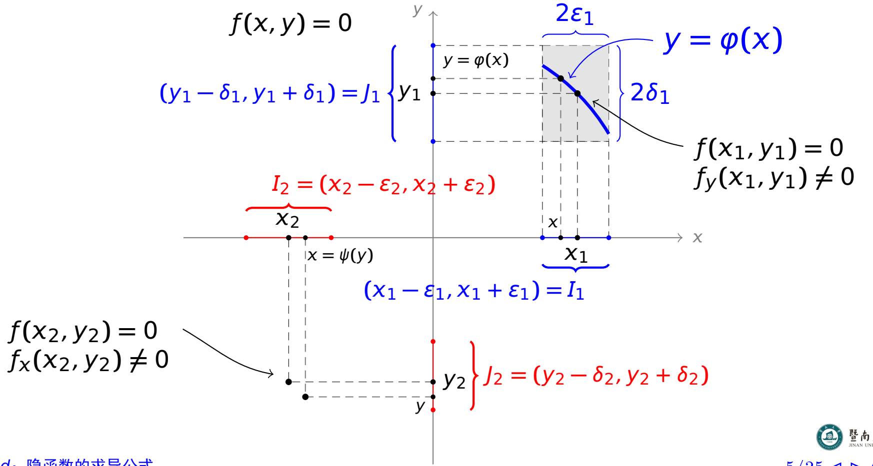

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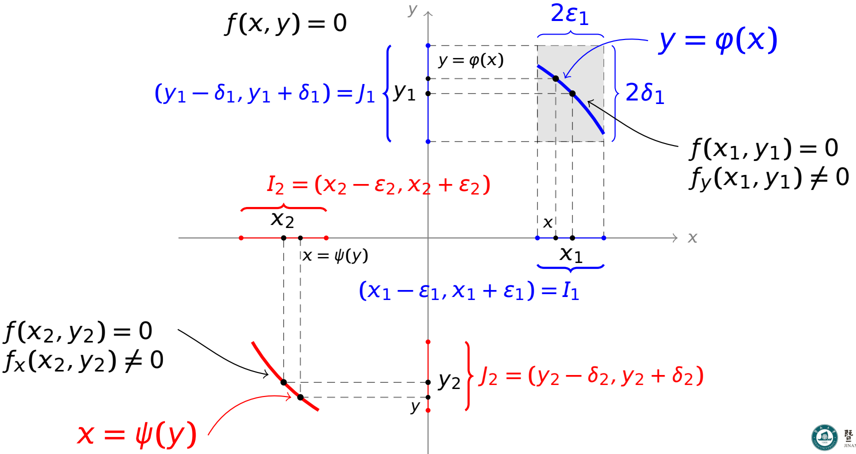

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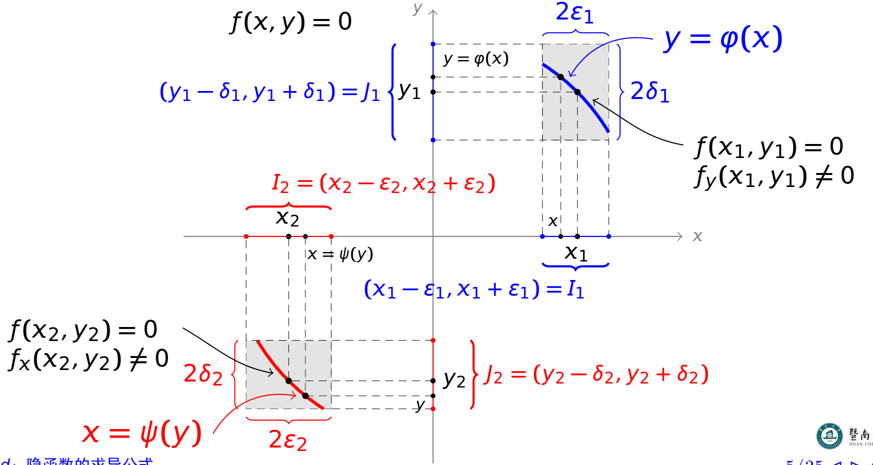

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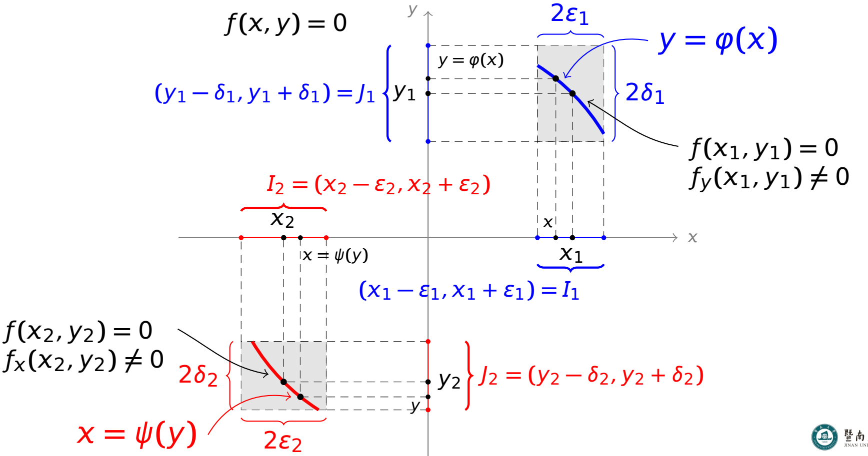

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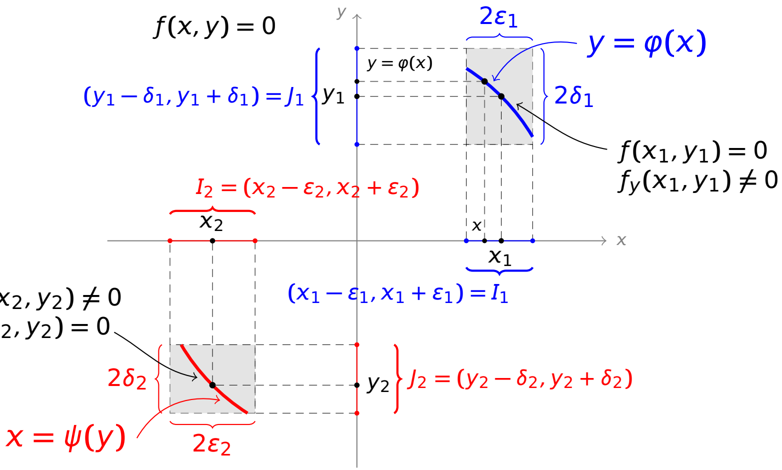

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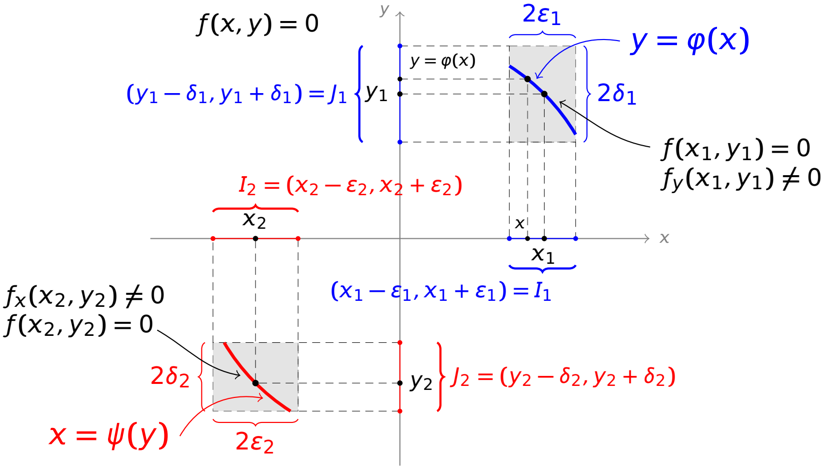

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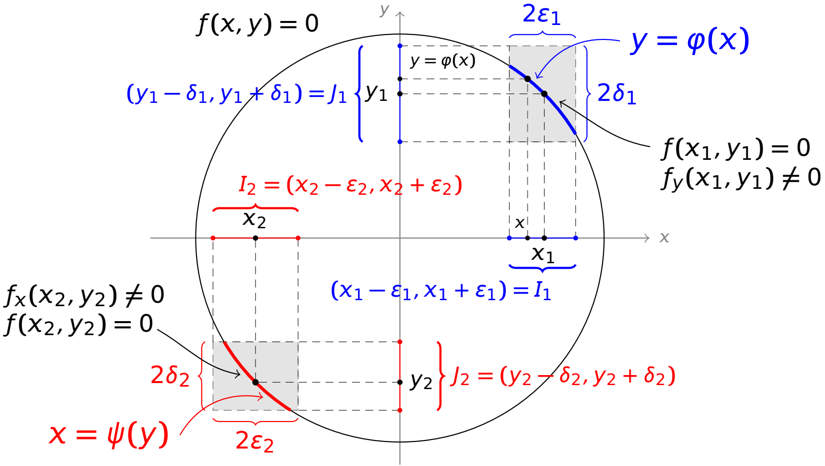

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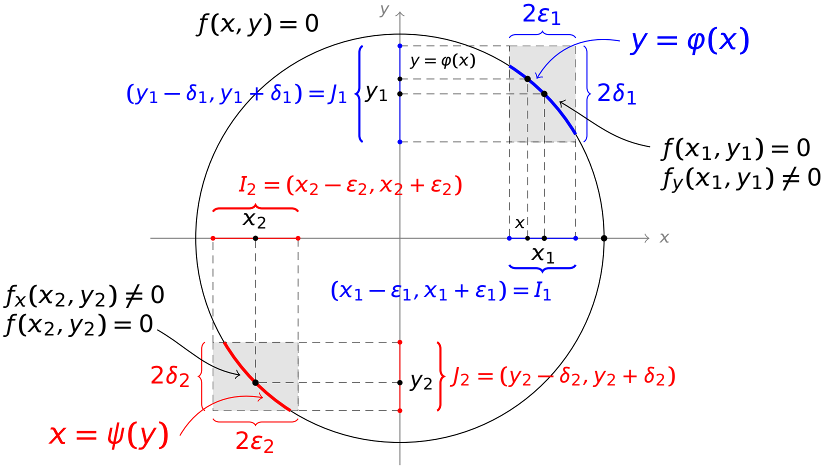

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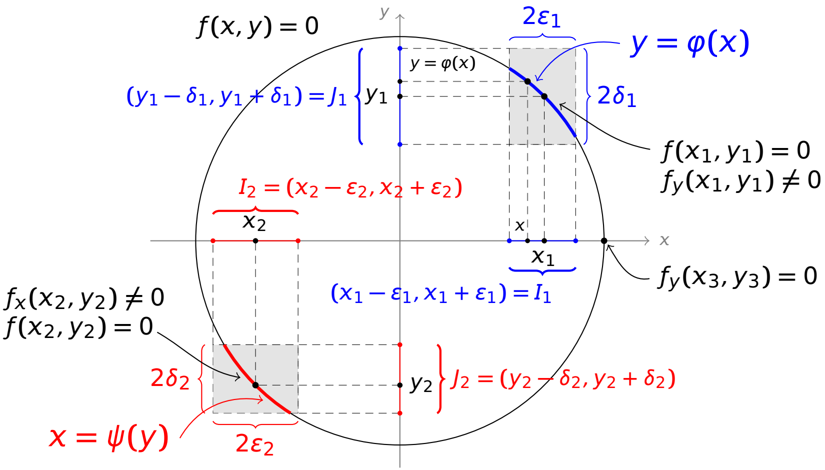

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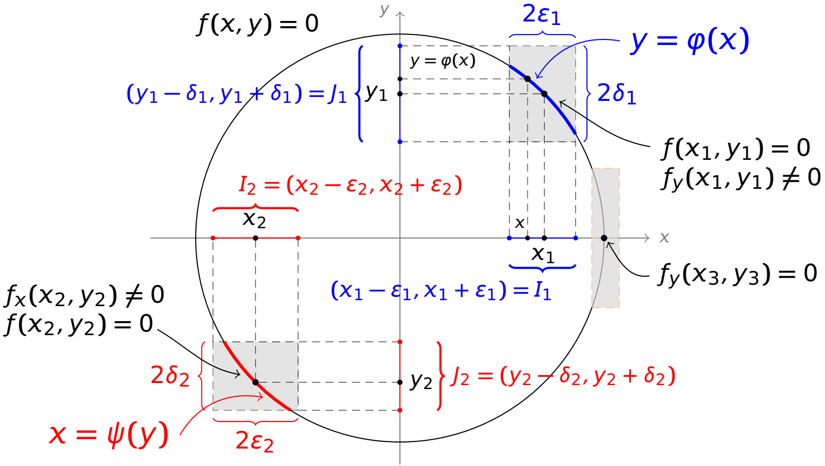

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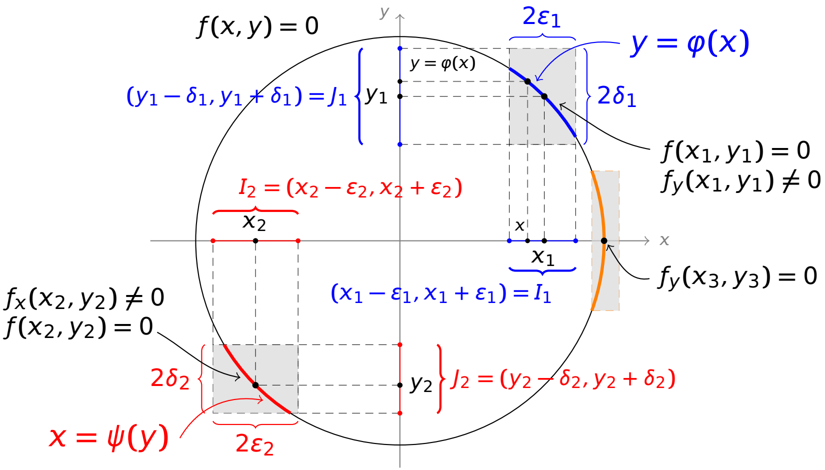

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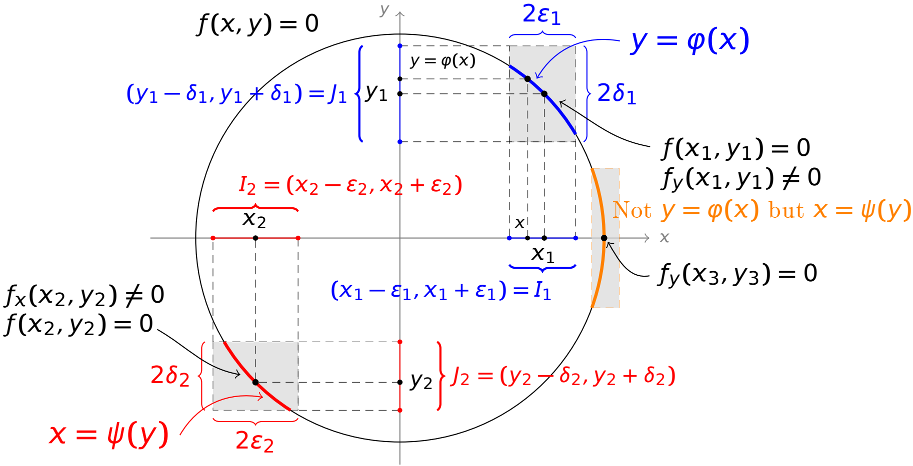

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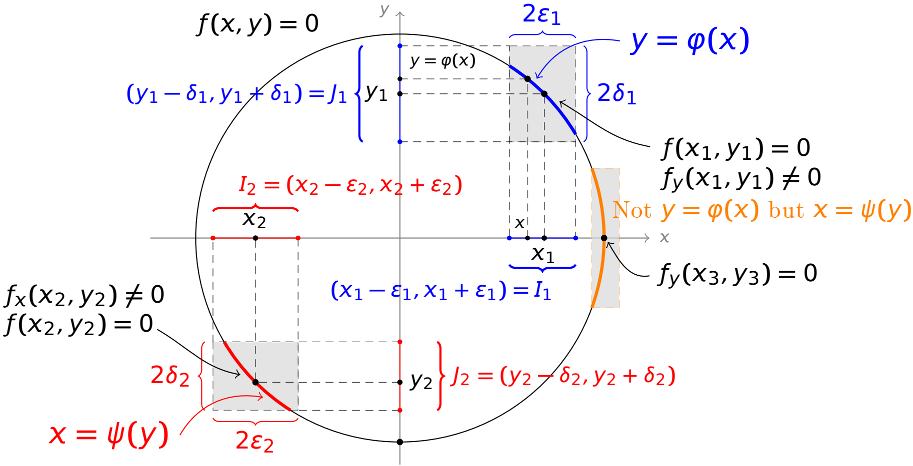

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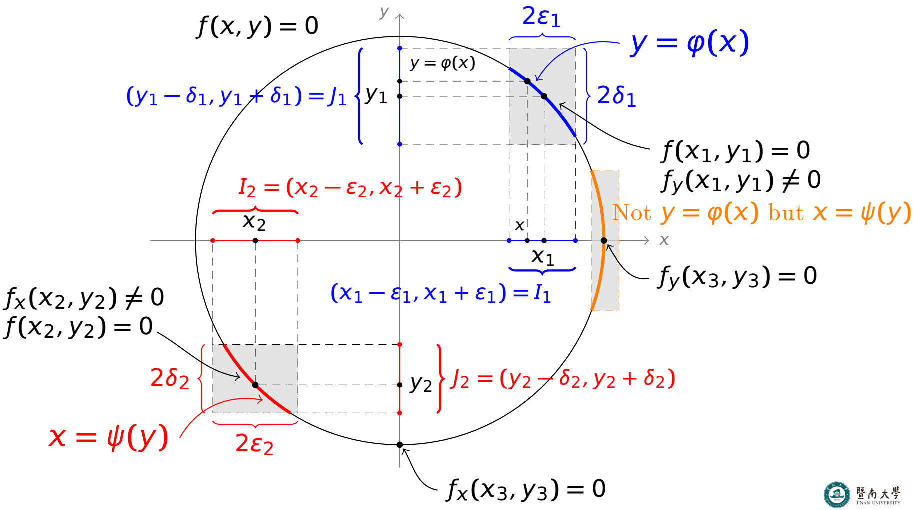

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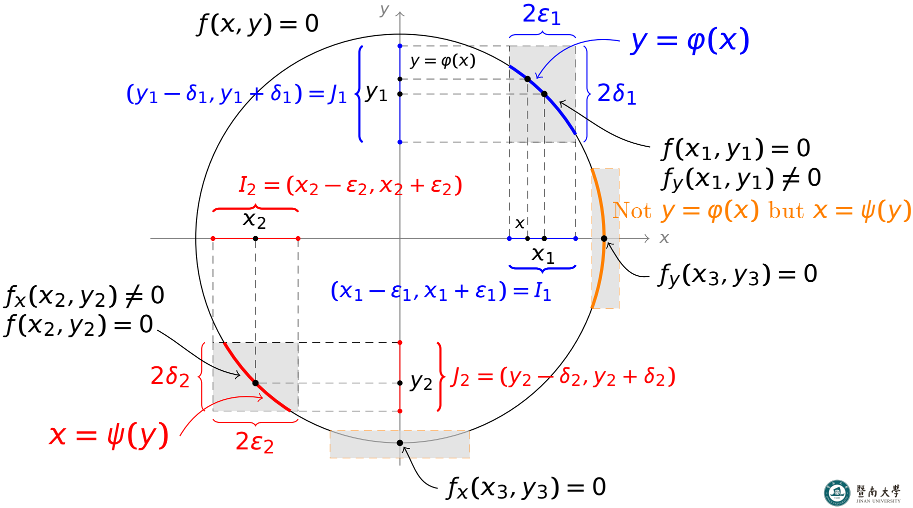

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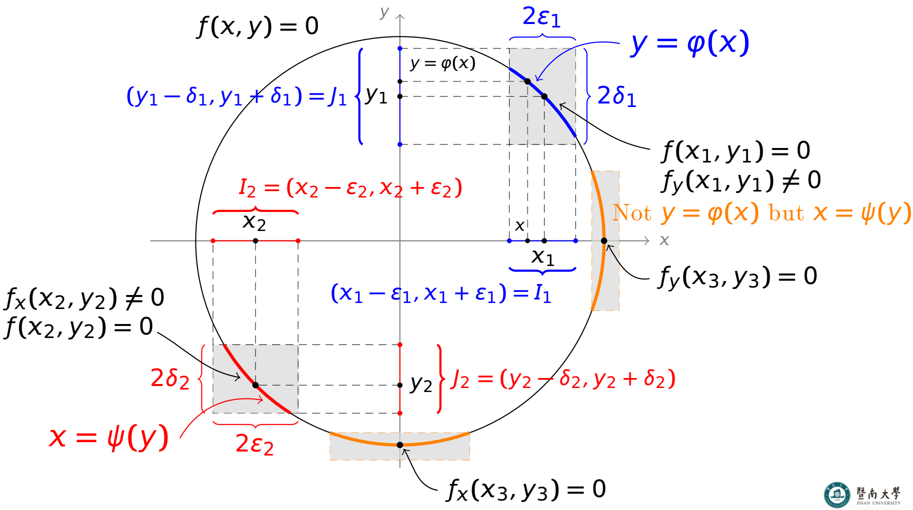

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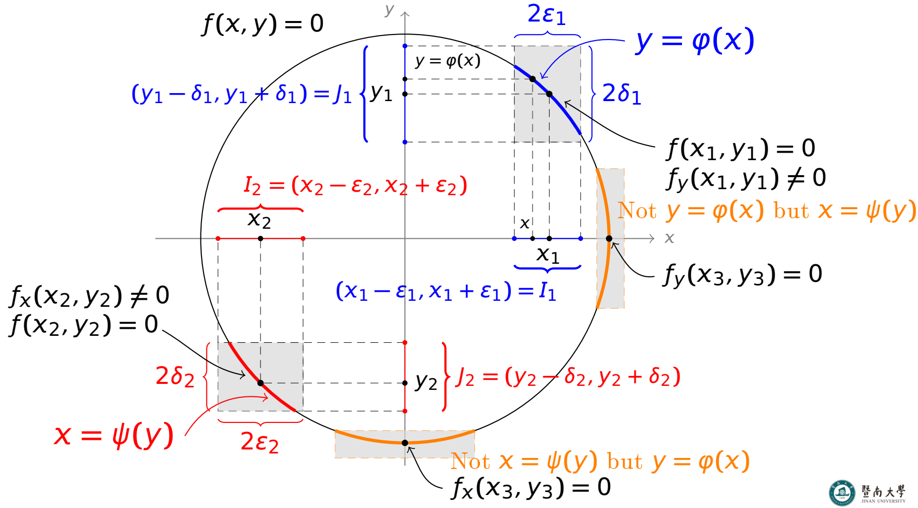

隐函数定理的一个理论应用

性质 设 $f(x, y)$ 具有连续偏导，且 f_x, f_y 处处不同时为零，若

$$E = \{(x, y) | f(x, y) = 0\}$$

非空，则 E 是平面上“一条曲线”。

隐函数定理 设 $f(x, y, z)$ 在点 $p(x_0, y_0, z_0)$ 附近有定义，具有连续偏导；$f(x_0, y_0, z_0) = 0$；

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方程 $f(x,y,z)=0$ 有唯一的解

$z=\varphi(x,y)$

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$$I_1=(x_0-\varepsilon,\ x_0+\varepsilon),\quad I_2=(y_0-\varepsilon,\ y_0+\varepsilon),\quad J=(z_0-\delta,\ z_0+\delta)$$

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2. 函数 $z=\varphi(x,y)$ 在区域 $I_1\times I_2$ 上具有连续导数。

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证明 $\because\quad F(x,y(x))=0$

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方法二

例 设 $y=f(x)$ 满足 $\sin y+e^x=xy^2$，求 $\frac{dy}{dx}$

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方法二 注意 $\sin y(x)+e^x-xy(x)^2=0$，

例 设 $y = f(x)$ 满足 $\sin y + e^x = xy^2$，求 $\frac{dy}{dx}$

方法一 注意 $\sin y + e^x - xy^2 = 0$，令 $F(x, y) = \sin y + e^x - xy^2$，则 $F(x, y) = 0$，所以

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$$0 = \left(\sin y(x) + e^x - xy(x)^2\right)'_x$$

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方法二 注意 $\sin y(x)+e^x-xy(x)^2=0$，所以

$$\begin{aligned}0&=\left(\sin y(x)+e^x-xy(x)^2\right)'_x\\&=(\sin y(x))'_x+(e^x)'_x-(xy(x)^2)'_x\end{aligned}$$

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所以 $y'=-\frac{e^x-y^2}{\cos y-2xy}$

例 设 $y = f(x)$ 满足 $\ln(x^2 + y^2) + 3xy = 4$，求 $\frac{dy}{dx}$

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解

$$F(x, y) = 0$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} =$$

例 设 $y = f(x)$ 满足 $\ln(x^2 + y^2) + 3xy = 4$，求 $\frac{dy}{dx}$

解 注意 $\ln(x^2 + y^2) + 3xy - 4 = 0$

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解 注意 $\ln(x^2+y^2)+3xy-4=0$，令

$$F(x,y)=\ln(x^2+y^2)+3xy-4$$

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解 注意 $\ln(x^2 + y^2) + 3xy - 4 = 0$，令

$$F(x, y) = \ln(x^2 + y^2) + 3xy - 4$$

则 $F(x, y) = 0$，所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\ln(x^2 + y^2) + 3xy - 4)'_x}{(\ln(x^2 + y^2) + 3xy - 4)'_y}$$

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$$=-\frac{\frac{2x}{x^2+y^2}+3y}{}$$

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$$=-\frac{\frac{2x}{x^2+y^2}+3y}{\frac{2y}{x^2+y^2}+3x}$$

$$=\frac{2x+3x^2y+3y^3}{2y+3xy^2+3x^3}$$

隐函数的求导法 II

公式 设 $z = f(x, y)$ 满足 $F(x, y, z) = 0$,

隐函数的求导法 II

公式 设 $z = f(x, y)$ 满足 $F(x, y, z) = 0$，即 $F(x, y, z(x, y)) = 0$，

隐函数的求导法 II

公式 设 $z = f(x, y)$ 满足 $F(x, y, z) = 0$，即 $F(x, y, z(x, y)) = 0$，则

$$\frac{\partial z}{\partial x} = \qquad , \qquad \frac{\partial z}{\partial y} =$$

隐函数的求导法 II

公式 设 $z = f(x, y)$ 满足 $F(x, y, z) = 0$，即 $F(x, y, z(x, y)) = 0$，则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} =$$

隐函数的求导法 II

公式 设 $z = f(x, y)$ 满足 $F(x, y, z) = 0$，即 $F(x, y, z(x, y)) = 0$，则

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$$\begin{tikzcd} & x & \\ F \arrow[ur] \arrow[r] \arrow[dr] & y & x \\ & z \arrow[ur] \arrow[r] & y \end{tikzcd}$$

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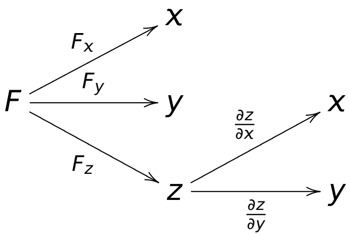

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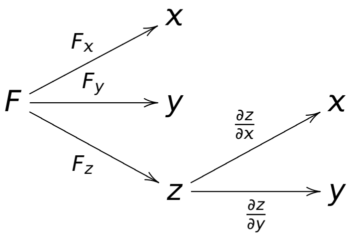

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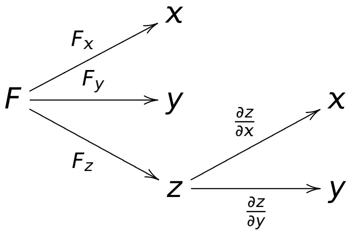

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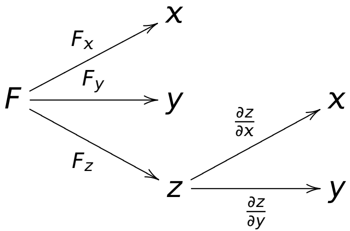

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$F(x,y,z)=0$

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$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_x}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

$$=-\frac{\qquad}{-6\cos(x+2y-3z)}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_y}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

$$=-\frac{\qquad}{\qquad}$$

例 设 $z=f(x,y)$ 满足 $2\sin(x+2y-3z)=x+2y-3z$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$=-\frac{\qquad}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_y}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

$$=-\frac{\qquad}{\qquad}$$

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$$=-\frac{2\cos(x+2y-3z)}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_y}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

$$=-\frac{\quad}{\quad}$$

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$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_x}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

$$=-\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_y}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

$$=-\frac{\qquad}{\qquad}$$

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$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_x}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

$$=-\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_y}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

$$=-\frac{\qquad}{-6\cos(x+2y-3z)+3}$$

例 设 $z = f(x, y)$ 满足 $2\sin(x + 2y - 3z) = x + 2y - 3z$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解 令 $F(x, y, z) = 2\sin(x + 2y - 3z) - x - 2y + 3z$，则
$F(x, y, z) = 0$，所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z) - x - 2y + 3z)'_x}{(2\sin(x+2y-3z) - x - 2y + 3z)'_z}$$

$$= -\frac{2\cos(x+2y-3z) - 1}{-6\cos(x+2y-3z) + 3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z) - x - 2y + 3z)'_y}{(2\sin(x+2y-3z) - x - 2y + 3z)'_z}$$

$$= -\frac{4\cos(x+2y-3z)}{-6\cos(x+2y-3z) + 3}$$

例 设 $z=f(x,y)$ 满足 $2\sin(x+2y-3z)=x+2y-3z$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_x}{(2\sin(x+2y-3z)-x-2y+3z)'_z}=-\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(2\sin(x+2y-3z)-x-2y+3z)'_y}{(2\sin(x+2y-3z)-x-2y+3z)'_z}=-\frac{4\cos(x+2y-3z)-2}{-6\cos(x+2y-3z)+3}$$

例 设 $z=f(x,y)$ 满足 $z-y-x+xe^{z-y-x}=0$，求 dz

例 设 $z = f(x, y)$ 满足 $z - y - x + xe^{z-y-x} = 0$，求 dz

解

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$

例 设 $z=f(x,y)$ 满足 $z-y-x+xe^{z-y-x}=0$，求 dz

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$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(z-y-x+xe^{z-y-x})'_y}{(z-y-x+xe^{z-y-x})'_z}$$

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=$$

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$$=-\frac{\qquad\qquad}{\qquad\qquad}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(z-y-x+xe^{z-y-x})'_y}{(z-y-x+xe^{z-y-x})'_z}=-\frac{\qquad\qquad}{\qquad\qquad}$$

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$$=-\frac{\qquad}{1+xe^{z-y-x}}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(z-y-x+xe^{z-y-x})'_y}{(z-y-x+xe^{z-y-x})'_z}=-\frac{\qquad}{\qquad}$$

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$$=-\frac{-1}{1+xe^{z-y-x}}$$

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$$=-\frac{-1+e^{z-y-x}-xe^{z-y-x}}{1+xe^{z-y-x}}=-\frac{1+(x-1)e^{z-y-x}}{1+xe^{z-y-x}}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(z-y-x+xe^{z-y-x})'_y}{(z-y-x+xe^{z-y-x})'_z}=-\frac{\qquad\qquad}{\qquad\qquad}$$

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$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(z-y-x+xe^{z-y-x})'_y}{(z-y-x+xe^{z-y-x})'_z}=-\frac{\qquad}{1+xe^{z-y-x}}$$

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$$=-\frac{-1+e^{z-y-x}-xe^{z-y-x}}{1+xe^{z-y-x}}=-\frac{1+(x-1)e^{z-y-x}}{1+xe^{z-y-x}}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(z-y-x+xe^{z-y-x})'_y}{(z-y-x+xe^{z-y-x})'_z}=-\frac{-1-xe^{z-y-x}}{1+xe^{z-y-x}}$$

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$$=-\frac{-1+e^{z-y-x}-xe^{z-y-x}}{1+xe^{z-y-x}}=-\frac{1+(x-1)e^{z-y-x}}{1+xe^{z-y-x}}$$

$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(z-y-x+xe^{z-y-x})'_y}{(z-y-x+xe^{z-y-x})'_z}=-\frac{-1-xe^{z-y-x}}{1+xe^{z-y-x}}=1$$

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$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{(z-y-x+xe^{z-y-x})'_y}{(z-y-x+xe^{z-y-x})'_z}=-\frac{-1-xe^{z-y-x}}{1+xe^{z-y-x}}=1$$

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=-\frac{1+(x-1)e^{z-y-x}}{1+xe^{z-y-x}}dx+dy$$

例 设 $\Phi(u, v)$ 具有连续偏导数，函数 $z = z(x, y)$ 满足 $\Phi(cx - az, cy - bz) = 0$，证明：

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

$$F_x =$$

$$F_y =$$

$$F_z =$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} =$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} =$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x$$

$$F_y =$$

$$F_z =$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} =$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x = c\Phi_u$$

$$F_y =$$

$$F_z =$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} =$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} =$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x = c\Phi_u$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y$$

$$F_z =$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} =$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} =$$

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$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x = c\Phi_u$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y = c\Phi_v$$

$$F_z =$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x = c\Phi_u$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y = c\Phi_v$$

$$F_z = \Phi_u \cdot u_z + \Phi_v \cdot v_z$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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例 设 $\Phi(u, v)$ 具有连续偏导数，函数 $z = z(x, y)$ 满足 $\Phi(cx - az, cy - bz) = 0$，证明：

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$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x = c\Phi_u$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y = c\Phi_v$$

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$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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例 设 $\Phi(u, v)$ 具有连续偏导数，函数 $z = z(x, y)$ 满足 $\Phi(cx - az, cy - bz) = 0$，证明：

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$，则

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$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = \frac{ac\Phi_u}{a\Phi_u + b\Phi_v} + \frac{bc\Phi_v}{a\Phi_u + b\Phi_v} = c$$

例 设 $z = f(x, y)$ 满足 $z = x + ye^z$，求 $\frac{\partial^2 z}{\partial x \partial y}$

例 设 $z=f(x,y)$ 满足 $z=x+ye^z$，求 $\frac{\partial^2 z}{\partial x \partial y}$

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例 设 $z=f(x,y)$ 满足 $z=x+ye^z$，求 $\frac{\partial^2 z}{\partial x\partial y}$

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$$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=\left(-\frac{1}{ye^z-1}\right)'_y$$

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例 设 $z = f(x, y)$ 满足 $z = x + ye^z$，求 $\frac{\partial^2 z}{\partial x \partial y}$

解 $F(x, y, z) = x + ye^z - z$，则 $F(x, y, z) = 0$ ，所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)'_y = \frac{(ye^z - 1)'_y}{(ye^z - 1)^2}$$

$$= \frac{e^z + y(e^z)'_y}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \frac{\partial z}{\partial y}}{(ye^z - 1)^2}$$

例 设 $z=f(x,y)$ 满足 $z=x+ye^z$，求 $\frac{\partial^2 z}{\partial x \partial y}$

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$$=\frac{e^z+y(e^z)_y'}{(ye^z-1)^2}=\frac{e^z+ye^z\cdot\frac{\partial z}{\partial y}}{(ye^z-1)^2}=\frac{e^z+ye^z\cdot\left(-\frac{e^z}{ye^z-1}\right)}{(ye^z-1)^2}$$

例 设 $z=f(x, y)$ 满足 $z=x+ye^z$，求 $\frac{\partial^2 z}{\partial x\partial y}$

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$$=\frac{e^z+y(e^z)'_y}{(ye^z-1)^2}=\frac{e^z+ye^z\cdot\frac{\partial z}{\partial y}}{(ye^z-1)^2}=\frac{e^z+ye^z\cdot\left(-\frac{e^z}{ye^z-1}\right)}{(ye^z-1)^2}$$

$$=\frac{-e^z}{(ye^z-1)^3}$$

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$$=\frac{e^z+y(e^z)'_y}{(ye^z-1)^2}=\frac{e^z+ye^z\cdot\frac{\partial z}{\partial y}}{(ye^z-1)^2}=\frac{e^z+ye^z\cdot\left(-\frac{e^z}{ye^z-1}\right)}{(ye^z-1)^2}$$

$$=\frac{-e^z}{(ye^z-1)^3}=\frac{e^z}{(1+x-z)^3}$$

We are here now...

1. 一个方程的情形

2. 方程组的情形

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

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$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}\,a_{22}x + a_{12}\,a_{22}y = a_{22}b_1 & (1)\times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2)\times a_{12} \end{cases}$$

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用消元法解：

$(1)\times a_{22} - (2)\times a_{12}$，消去 y，得：

$$x = \frac{b_1a_{22} - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

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二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \end{cases}$$

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$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}\, a_{11}x + a_{22}\, a_{11}y = a_{11}b_2 & (2) \times a_{11} \end{cases}$$

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回顾：二元线性方程组的求解

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回顾：二元线性方程组的求解

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$(2)\times a_{11} - (1)\times a_{21}$，消去 x，得：

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

回顾：二元线性方程组的求解

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用消元法解：

$(1) \times a_{22} - (2) \times a_{12}$，消去 y，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$(2) \times a_{11} - (1) \times a_{21}$，消去 x，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \quad \Rightarrow \quad x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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练习 利用二阶行列式求解下面二元线性方程组

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$ $x =$ ， $y =$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases}$ $x =$ ， $y =$

公式：

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方程组的隐函数求导公式

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

方程组的隐函数求导公式

假设函数 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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问题：如何计算 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ ？

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求解如下：

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}}$$

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求解如下：

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ \end{cases}$$

方程组的隐函数求导公式

假设函数 $u=u(x,y)$, $v=v(x,y)$ 满足方程组

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$$=-\frac{1}{J}\frac{\partial(F,G)}{\partial(x,v)}$$

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$$= -\frac{1}{J}\frac{\partial(F,G)}{\partial(x,v)} \qquad = -\frac{1}{J}\frac{\partial(F,G)}{\partial(u,x)}$$

$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}}$$

$$\begin{cases} F(x,y,u,v)=0 \\ G(x,y,u,v)=0 \end{cases} \xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ \end{cases}$$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

$u_x =$ $\qquad\qquad$ $v_x =$

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所以

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$$u_y = \qquad v_y =$$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_x = -\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = -\frac{1}{J}\frac{\partial(F, G)}{\partial(x, v)}, \quad v_x = -\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = -\frac{1}{J}\frac{\partial(F, G)}{\partial(u, x)}$$

$$u_y = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = -\frac{1}{J}\frac{\partial(F, G)}{\partial(y, v)}, \quad v_y = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = -\frac{1}{J}\frac{\partial(F, G)}{\partial(u, y)}$$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \quad \overset{\frac{\partial}{\partial x}}{\Longrightarrow} \qquad \overset{\frac{\partial}{\partial y}}{\Longrightarrow}$$

$u_x=$ $\qquad$ $v_x=$

$u_y=$ $\qquad$ $v_y=$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \quad \overset{\frac{\partial}{\partial x}}{\Longrightarrow} \quad \overset{\frac{\partial}{\partial y}}{\Longrightarrow}$$

$u_x =$ $\qquad$ $v_x =$

$u_y =$ $\qquad$ $v_y =$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1 \\ \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial y}}$$

$u_x =$ $\qquad$ $v_x =$

$u_y =$ $\qquad$ $v_y =$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

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$$\xRightarrow{\frac{\partial}{\partial y}}$$

$u_x =$ $\qquad$ $v_x =$

$u_y =$ $\qquad$ $v_y =$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ \end{cases}$$

$u_x =$

$v_x =$

$u_y =$

$v_y =$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \overset{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u\sin v \cdot v_x = 0 \end{cases}$$

$$\overset{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u\sin v \cdot v_y = 1 \end{cases}$$

$u_x =$ $\qquad$ $v_x =$

$u_y =$ $\qquad$ $v_y =$

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所以 $J = \begin{vmatrix} e^u + \sin v & u\cos v \\ e^u - \cos v & u\sin v \end{vmatrix}$

$u_x =$ $v_x =$

$u_y =$ $v_y =$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u\sin v \cdot v_x = 0 \end{cases}$$

$$\xrightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u\sin v \cdot v_y = 1 \end{cases}$$

所以 $J = \begin{vmatrix} e^u + \sin v & u\cos v \\ e^u - \cos v & u\sin v \end{vmatrix}$

$$u_x = -\frac{\begin{vmatrix} & \\ & \end{vmatrix}}{J} \qquad v_x = -\frac{\begin{vmatrix} & \\ & \end{vmatrix}}{J}$$

$$u_y = -\frac{\begin{vmatrix} & \\ & \end{vmatrix}}{J} \qquad v_y = -\frac{\begin{vmatrix} & \\ & \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

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所以 $J = \begin{vmatrix} e^u + \sin v & u\cos v \\ e^u - \cos v & u\sin v \end{vmatrix}$

$$u_x = -\frac{\begin{vmatrix} 1 & u\cos v \\ 0 & u\sin v \end{vmatrix}}{J} \qquad v_x = -\frac{\begin{vmatrix} & \\ & \end{vmatrix}}{J}$$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u\sin v \cdot v_y = 1 \end{cases}$$

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$$\xrightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u\sin v \cdot v_y = 1 \end{cases}$$

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例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

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$$\xLongrightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u\sin v \cdot v_x = 0 \end{cases}$$

$$\xLongrightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u\sin v \cdot v_y = 1 \end{cases}$$

所以 $J = \begin{vmatrix} e^u + \sin v & u\cos v \\ e^u - \cos v & u\sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$

$$u_x = -\frac{\begin{vmatrix} 1 & u\cos v \\ 0 & u\sin v \end{vmatrix}}{J} \qquad v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J}$$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u\sin v \cdot v_y = 1 \end{cases}$$

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$$u_x = -\frac{\begin{vmatrix} 1 & u\cos v \\ 0 & u\sin v \end{vmatrix}}{J} = \frac{-\sin v}{e^u(\sin v - \cos v) + 1}, \quad v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J}$$

$$u_y = -\frac{\begin{vmatrix} 0 & u\cos v \\ 1 & u\sin v \end{vmatrix}}{J} \qquad v_y = -\frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

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解

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u\sin v \cdot v_y = 1 \end{cases}$$

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$$u_y = -\frac{\begin{vmatrix} 0 & u\cos v \\ 1 & u\sin v \end{vmatrix}}{J} = \frac{\cos v}{e^u(\sin v - \cos v) + 1}, \quad v_y = -\frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u\sin v \\ y = e^u - u\cos v \end{cases}$，求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u\sin v = x \\ e^u - u\cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u\sin v \cdot v_x = 0 \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u\sin v \cdot v_y = 1 \end{cases}$$

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