§1.1 二阶三阶行列式

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教学要求

掌握求解:

- ◇ 二阶行列式计算
- ◆ 三阶行列式计算



• 行列式可用于表示一些线性方程组的解。

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- 具体地,

二阶行列式 ←→ 2元2方程的线性方程组

三阶行列式 ←→ 3元3方程的线性方程组

:

n阶行列式 ←→ n元n方程的线性方程组

:

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解:

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

用消元法求解: $(1) \times \alpha_{22} - (2) \times \alpha_{12}$, 消去 y, 得:

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y, 得:



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用消元法求解: (1) × a_{22} – (2) × a_{12} , 消去 y , 得:

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \Rightarrow a_{21}a_{12}x + a_{22}a_{12}y = b_2a_{12} \end{cases}$$

用消元法求解: $(1) \times \alpha_{22} - (2) \times \alpha_{12}$, 消去 y, 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{12} b_2}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \Rightarrow a_{11}a_{22}x + a_{12}a_{22}y = b_1a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \Rightarrow a_{21}a_{12}x + a_{22}a_{12}y = b_2a_{12} \end{cases}$$

用消元法求解: $(1) \times \alpha_{22} - (2) \times \alpha_{12}$, 消去 y, 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y, 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去 x , 得:



$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \end{cases} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y, 得:

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 $(2) \times a_{11} - (1) \times a_{21}$,消去 x,得:



$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \Rightarrow a_{11}a_{21}x + a_{12}a_{21}y = b_1a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11} \end{cases}$$

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,消去 x ,得:



$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \Rightarrow a_{11}a_{21}x + a_{12}a_{21}y = b_1a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11} \end{cases}$$

用消元法求解: (1) × a_{22} – (2) × a_{12} , 消去 y, 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

 $(2) \times a_{11} - (1) \times a_{21}$,消去 x,得:

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \Rightarrow a_{11}a_{21}x + a_{12}a_{21}y = b_1a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11} \end{cases}$$

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 $(2) \times a_{11} - (1) \times a_{21}$, 消去 x, 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{12}a_{12}}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \Rightarrow a_{11}a_{21}x + a_{12}a_{21}y = b_1a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \Rightarrow a_{21}a_{11}x + a_{22}a_{11}y = b_2a_{11} \end{cases}$$

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 $(2) \times a_{11} - (1) \times a_{21}$, 消去 x, 得:

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$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解: $(1) \times a_{22} - (2) \times a_{12}$, 消去 y, 得:

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$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

• 定义
$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$



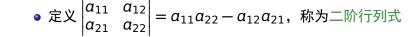
$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

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(2)×α₁₁ – (1)×α₂₁,消去 x,得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$





$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法求解: $(1) \times \alpha_{22} - (2) \times \alpha_{12}$, 消去 y, 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

(2)×α₁₁ – (1)×α₂₁,消去 x,得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{1}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

• 定义 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$,称为二阶行列式



$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

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 $(2) \times a_{11} - (1) \times a_{21}$,消去 x,得:

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$$(2) \times a_{11} - (1) \times a_{21}, \quad \text{iff} x, \quad \text{iff} : \quad |a_{11} \ b_1|$$

 $y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$

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1. 当
$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$$
 时,

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$



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1. 当 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$ 时,方程有唯一解:

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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2. 当 $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 0$ 时,方程或者无解、或者有无穷多的解。



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$



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练习 利用二阶行列式求解下面二元线性方程组

1.
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$$
 $x =$, $y =$

2.
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -- \qquad , \quad y = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}} = --$$

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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练习 利用二阶行列式求解下面二元线性方程组

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$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} \qquad , \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1}$$

2.
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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§1.1 二阶三阶行列式

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - , \quad y = \frac{-20}{1} = -$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$1. \begin{cases} 2x + 5y = 0 \\ x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 1 & 0 \\ 4 & 8 \end{vmatrix}} = \frac{-20}{-20} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ 4 & 8 \end{vmatrix}} = \frac{8}{12} = \frac{8}{12}$$

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$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{}{3} , y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{}{3}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$\begin{cases}
2x + 3y = 0 \\
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\end{cases}$$

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$

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$$3x + 8y = 4 \qquad x - \frac{2}{3} = \frac{5}{3} = \frac{1}{3}$$

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解因为

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所以 $\lambda \neq 0$ 且 $\lambda \neq 3$ 。

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所以 $k \neq -1$ 且 $k \neq 3$ 。



$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

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用消元法可解得:

$$x = \frac{b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32}}{-b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3}{a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32}} - a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31}}$$

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为表示三元方程组的解,定义三阶行列式:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$



$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$

规律



$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$

a_{11}	a_{12}	a_{13}	a_{11}	a_{12}	<i>a</i> ₁₃
a ₂₁	a ₂₂	a ₂₃	a_{21}	a ₂₂	a ₂₃
a_{31}	a ₃₂	a ₃₃		a ₃₂	



$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}}$$

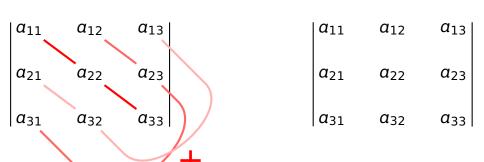
规律不同行不同列的3个元素乘积,共3!=6个,并且:

a_{11}	a ₁₂	a ₁₃	a_{11}	a ₁₂	a ₁₃
a ₂₁	a ₂₂	a ₂₃	a ₂₁	a ₂₂	a ₂₃
a ₃₁	a ₃₂	a ₃₃	a_{31}	a ₃₂	a ₃₃

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{vmatrix}$$

$\begin{vmatrix} a_{11} & a_{12} & a_{13} \end{vmatrix}$	a_{11}	a ₁₂	<i>a</i> ₁₃
a_{21} a_{22} a_{23}	a_{21}	a ₂₂	a ₂₃
$ a_{31} a_{32} a_{33} $	a_{31}	a ₃₂	a ₃₃

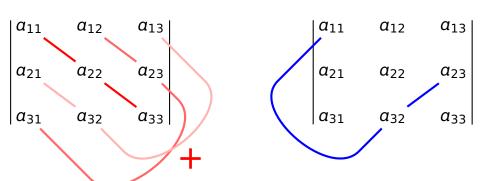
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}}$$



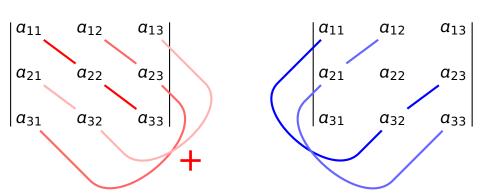


$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}}$$

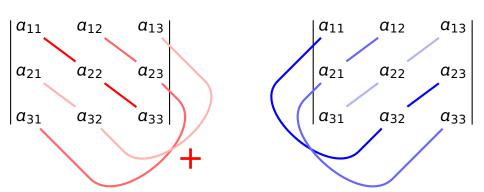
规律不同行不同列的3个元素乘积,共3!=6个,并且:



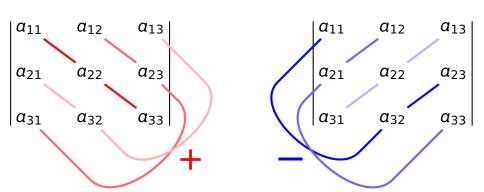
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}}$$

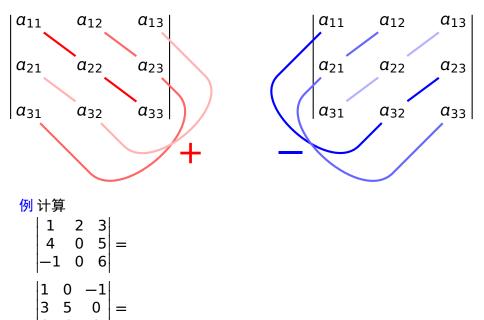


$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}}$$



$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}}$$







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$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$
 $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ 例 计算
$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0$$



例 计算
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{vmatrix}$$

例 计算
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix} = \begin{vmatrix} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{vmatrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 2 & 5 & -2 \end{vmatrix}$$



例 计算
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{vmatrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 2 & 5 & 0 \end{vmatrix} = 1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4$$

§1.1 二阶三阶行列式

例 计算
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{vmatrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{vmatrix} 1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4 \\ -1 \times 0 \times 4 - 0 \times 3 \times 1 - (-1) \times 5 \times 1 \end{vmatrix}$$

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例 计算
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} 1 \times 0 \times 6 + 2 \times 5 \times (-1) + 3 \times 4 \times 0 \\ -1 \times 5 \times 0 - 2 \times 4 \times 6 - 3 \times 0 \times (-1) \end{vmatrix} = -58$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 5 & 0 \\ 1 & 4 & 1 \end{vmatrix} = \begin{vmatrix} 1 \times 5 \times 1 + 0 \times 0 \times 1 + (-1) \times 3 \times 4 \\ -1 \times 0 \times 4 - 0 \times 3 \times 1 - (-1) \times 5 \times 1 \end{vmatrix} = -2$$

§1.1 二阶三阶行列式

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} =$$

解因为

$$\left| egin{array}{cccc} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{array} \right| = \left| \begin{array}{ccccc} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \end{array} \right|$$

解因为

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{vmatrix}$$

$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{vmatrix} = a^2 + b^2$$



$$\begin{vmatrix} a & b & 0 \\ -b & a & 0 \\ 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} a \times a \times 1 + b \times 0 \times 1 + 0 \times (-b) \times 2 \\ -a \times 0 \times 2 - b \times (-b) \times 1 - 0 \times a \times 1 \end{vmatrix} = a^2 + b^2$$

所以 $\alpha \neq 0$ 或 $b \neq 0$ 。

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:



$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32}}{-b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3} = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}} = \frac{a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}}{-a_{13}a_{22}a_{31}}$$



$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{\begin{array}{c} b_1 a_{22} a_{33} + a_{12} a_{23} b_3 + a_{13} b_2 a_{32} \\ -b_1 a_{23} a_{32} - a_{12} b_2 a_{33} - a_{13} a_{22} b_3 \end{array}}{\begin{array}{c} a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32} \\ -a_{11} a_{23} a_{32} - a_{12} a_{21} a_{33} - a_{13} a_{22} a_{31} \end{array}} = \frac{a_{11}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$



$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{\begin{array}{c} b_{1}a_{22}a_{33} + a_{12}a_{23}b_{3} + a_{13}b_{2}a_{32} \\ -b_{1}a_{23}a_{32} - a_{12}b_{2}a_{33} - a_{13}a_{22}b_{3} \\ -a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}} = \begin{array}{c} b_{1} & a_{12} & a_{13} \\ b_{2} & a_{22} & a_{23} \\ b_{3} & a_{32} & a_{33} \\ \hline a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ \hline \end{array}$$

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

的解可以表示为:

$$x = \frac{\begin{array}{c} b_{1}a_{22}a_{33} + a_{12}a_{23}b_{3} + a_{13}b_{2}a_{32} \\ -b_{1}a_{23}a_{32} - a_{12}b_{2}a_{33} - a_{13}a_{22}b_{3} \\ -a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \\ -a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31} \end{array}} = \frac{\begin{vmatrix} b_{1} & a_{12} & a_{13} \\ b_{2} & a_{22} & a_{23} \\ b_{3} & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

 $y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 & (1) \\ a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$

 a_{21}

 a_{31}

 a_{22}

 a_{32}

 a_{23}

 a_{33}

的解可以表示为:

$$x = \frac{a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}} = \frac{a_{11}}{a_{21}} \begin{vmatrix} a_{11} & b_{1} & a_{13} \\ a_{21} & b_{2} & a_{23} \\ a_{31} & b_{3} & a_{33} \end{vmatrix}}{y = \frac{a_{11}}{a_{11}} \begin{vmatrix} a_{12} & b_{1} \\ a_{21} & a_{22} & b_{2} \\ a_{31} & a_{32} & b_{3} \end{vmatrix}}{a_{11}} , z = \frac{a_{11}}{a_{21}} \begin{vmatrix} a_{12} & a_{12} \\ a_{21} & a_{22} & b_{2} \\ a_{31} & a_{32} & b_{3} \end{vmatrix}}{a_{11}}$$

 $b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32}$

 $-b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3$

 a_{22} a_{23} a_{32} a_{33} a_{12} a_{13} a_{22} a_{23} a_{32} a_{33}

 a_{13}

 a_{12}

 b_2

 b_3

 a_{31} §1.1 二阶三阶行列式

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 a_{21}

 a_{22}

 a_{32}

 a_{23}

 a_{33}

$$\begin{cases} a_{21}x + a_{22}y + a_{23}z = b_2 & (2) \\ a_{31}x + a_{32}y + a_{33}z = b_3 & (3) \end{cases}$$
 示为:

 $a_{11}x + a_{12}y + a_{13}z = b_1$

的解可以表示为:

$$x = \frac{\begin{vmatrix} b_{1}a_{22}a_{33} + a_{12}a_{23}b_{3} + a_{13}b_{2}a_{32} \\ -b_{1}a_{23}a_{32} - a_{12}b_{2}a_{33} - a_{13}a_{22}b_{3} \\ \hline a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \end{vmatrix}}{\begin{vmatrix} b_{1} \\ b_{2} \\ b_{3} \\ \hline a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \end{vmatrix}} = \frac{\begin{vmatrix} b_{1} \\ b_{2} \\ b_{3} \\ \hline a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} \end{vmatrix}}{\begin{vmatrix} a_{11} \\ a_{21} \\ a_{22}a_{33} + a_{23}a_{32} + a_{23}a_{33} + a_{23}a_{21}a_{32} \end{vmatrix}} = \frac{\begin{vmatrix} b_{1} \\ b_{2} \\ b_{3} \\ a_{11}a_{22}a_{33} + a_{23}a_{32} + a_{23}a_{33} + a_{23}a_{21}a_{32} \end{vmatrix}}{\begin{vmatrix} a_{11} \\ a_{21} \\ a_{22}a_{33} + a_{23}a_{33} + a_{23}a_{33} + a_{23}a_{21}a_{32} \end{vmatrix}} = \frac{\begin{vmatrix} b_{1} \\ b_{2} \\ b_{3} \\ a_{21}a_{22}a_{33} + a_{23}a_{33} + a_{23}a_{33} + a_{23}a_{21}a_{32} \end{vmatrix}}{\begin{vmatrix} a_{11} \\ a_{21} \\ a_{22}a_{33} + a_{23}a_{33} + a_{23}a_{33} + a_{23}a_{23}a_{33} + a_{23}a_{23}a_{23}a_{23} + a_{23}a_{23}a_{23}a_{23} + a_{23}a_{23}a_{23} + a_{23}a$$

 $-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$ a_{11} b_1

 a_{21}

 a_{11}

$$a_{13}$$
 a_{23}

$$\begin{array}{c|c} b_2 & a_{23} \\ b_3 & a_{33} \\ \hline a_{12} & a_{13} \\ \end{array}$$

$$\begin{array}{c|cc} b_3 & a_{33} \\ \hline a_{12} & a_{13} \\ a_{22} & a_{23} \\ \end{array}$$



 a_{22}

 a_{32}

 a_{12}

 a_{22}

 a_{32}





 b_2

b₃,

 a_{13}

 a_{23}

 a_{33}

$$a_{21} = a_{31}$$

(1)

$$\frac{a_{33}}{a_{13}}$$

 a_{23}

 a_{33}

 a_{13}

 a_{23}

 a_{12}

 a_{22}

 a_{32}

 a_{12}

 a_{22}

 a_{32}











 a_{11}

 a_{21}

 a_{31}

 a_{11}

 a_{21}

 a_{31}



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的解可以表示为:

$$a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32}$$

$$\begin{vmatrix} b_{1}a_{22}a_{33} + a_{12}a_{23}b_{3} + a_{13}b_{2}a_{32} \\ -b_{1}a_{23}a_{32} - a_{12}b_{2}a_{33} - a_{13}a_{22}b_{3} \\ \frac{11a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}}{a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}} = \begin{vmatrix} b_{1} \\ b_{2} \\ b_{3} \\ a_{11} \\ a_{21} \end{vmatrix}$$

 $a_{11}x + a_{12}y + a_{13}z = b_1$

 $a_{21}x + a_{22}y + a_{23}z = b_2$

 $a_{31}x + a_{32}y + a_{33}z = b_3$

 $-b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3$ x = $a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}$

 b_3 a_{32}

 a_{12}

 a_{22}

(1)

(2)

(3)

 a_{33} a_{13} a_{23} a_{33}

 a_{13}

 a_{23}

 a_{11} a_{12} $-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$ a_{21} a_{22} a_{31} a_{32} b_1 a_{11} b_1 a_{11} a_{13} a_{12} b_2 a_{21} b_2 a_{21} a_{23} a_{22} b₃, a_{33} a_{31} a_{32}

 a_{11} a_{11} a₁₂ a_{13} a_{12} a_{13} a_{22} a_{21} a_{23} a_{21} a_{22} a_{23} a_{32} a_{33} a_{31} a_{32} a_{33}

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二阶三阶行列式

的解可以表示为: $b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32}$

 a_{11}

 a_{21}

 a_{11}

二阶三阶行列式

$$a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$$

 b_1

 a_{22}

 a_{32}

$$\begin{array}{c|c} b_2 & a_{23} \\ b_3 & a_{33} \\ \hline a_{12} & a_{13} \\ \end{array}$$

 a_{23}

 $-b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3$

 a_{11} a_{12} a_{21} a_{22} b₃1 a_{31} a_{32}

 a_{12}

 a_{22}

 a_{32}

 a_{11}

 a_{21}

 a_{31}

 $a_{11}x + a_{12}y + a_{13}z = b_1$

 $a_{21}x + a_{22}y + a_{23}z = b_2$

 $a_{31}x + a_{32}y + a_{33}z = b_3$

$$b_1$$
 b_2

 a_{13}

 a_{23}

 a_{33}

 b_3 a_{11} a_{21}

 a_{31}

(1)

(2)

(3)

 b_1

- b_2 a_{12}
- a_{12} a_{13} a_{22} a_{32}

 a_{22}

 a_{32}

$$a_{23}$$
 a_{33}

 a_{33}

 a_{13} a_{23}













x =





 a_{23}



 a_{11}

 a_{21}

 a_{11}

 a_{21}

 a_{31}

二阶三阶行列式

这时方程组

的解可以表示为:

 b_1 a_{13}

 b_2 a_{23}

 a_{33}

a₁₂ a_{13}

 a_{22} a_{23} a_{32} a_{33}

 $b_1a_{22}a_{33} + a_{12}a_{23}b_3 + a_{13}b_2a_{32}$

 $-b_1a_{23}a_{32} - a_{12}b_2a_{33} - a_{13}a_{22}b_3$

 $a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}$ $-a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$

 a_{21}

 a_{31}

 a_{11}

 a_{21}

 a_{31}

 $a_{11}x + a_{12}y + a_{13}z = b_1$

 $a_{21}x + a_{22}y + a_{23}z = b_2$

 $a_{31}x + a_{32}y + a_{33}z = b_3$

 a_{11}

 a_{32}

 a_{12}

 a_{22}

 a_{32}

 a_{12} a_{22}

b₃1

 b_2

 a_{13}

 a_{23}

 a_{33}

 b_1

(1)

(2)

(3)

 b_1

 b_2

 b_3

 a_{11}

 a_{21} a_{31} a_{32}

 a_{22}

 a_{12}

 a_{22}

 a_{32} a_{12}

 a_{23} a_{33}

 a_{23}

 a_{13}

 a_{13}







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 $\chi =$

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

$$x = \frac{D_x}{D}, \qquad y = \frac{D_y}{D}, \qquad z = \frac{D_z}{D}$$

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

1. 当 $D \neq 0$ 时,

$$x = \frac{D_x}{D}, \qquad y = \frac{D_y}{D}, \qquad z = \frac{D_z}{D}$$



$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

1. 当 $D \neq 0$ 时,方程有唯一解:

$$x = \frac{D_x}{D}, \qquad y = \frac{D_y}{D}, \qquad z = \frac{D_z}{D}$$

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1. 当 $D \neq 0$ 时,方程有唯一解:

$$x = \frac{D_x}{D}, \qquad y = \frac{D_y}{D}, \qquad z = \frac{D_z}{D}$$

2. 当 D = 0 时,



$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

1. 当 $D \neq 0$ 时,方程有唯一解:

$$x = \frac{D_x}{D}$$
, $y = \frac{D_y}{D}$, $z = \frac{D_z}{D}$

2. 当 D = 0 时, 方程或者无解、或者有无穷多的解。



§1.1 二阶三阶行列式

$$x = \frac{D_x}{D} = ----$$

$$y = \frac{D_y}{D} = -----$$

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix}}$$

$$z = \frac{D_z}{D} = ----$$

$$9$$
 永解二元线性万桂组 $\begin{cases} 2y + z = \\ 4x - 3y = \end{cases}$

$$x = \frac{D_{X}}{D} = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix} X}$$

$$z = \frac{D_z}{D} = -----$$



 $x = \frac{D_X}{D} = \frac{\begin{vmatrix} 1 & 2 \\ 2 & 1 \\ 4 & -3 \end{vmatrix} X}$







例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \end{cases} \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \end{cases} \right) \\ 4x - 3y = -2 \end{cases}$

§1.1 二阶三阶行列式

$$x = \frac{D_x}{D} = \frac{1 \quad 0 \quad 2}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} \qquad y = \frac{D_y}{D} = \frac{1}{D}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \end{cases} \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \end{cases} \right) \\ 4x - 3y = -2 \end{cases}$

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} \qquad y = \frac{D_y}{D} = -----$$



 $x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} \qquad y = \frac{D_y}{D} = \frac{1}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$

D_z	•			
?= <u>_</u> _=	1 0 4	0 2 -3	2 1 0	=

解 $x = \frac{D_X}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$ $y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$

$$= \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

 $z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$



例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

$$z = \frac{D}{D} = \begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}$$

$$z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$

例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

$$I = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$



例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

$$z = \frac{Dz}{D} = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$





解 $\frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1, \ y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$

$$Z = \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$



 $x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \end{vmatrix}} = \frac{-13}{-13} = 1, \ y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13}$

	0	2 -3	1 0	
D _z	1 0 4	0 2 —3	9 8 –2	_
2= <u></u>	1 0 4	0 2 -3	2 1 0	



例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$ 解 $\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \end{vmatrix}$

$$= \frac{D_z}{D} = \frac{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix}} = \frac{-13}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}}$$



 $\begin{aligned}
\mathbf{x} &= \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1, \quad \mathbf{y} = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-26}{-13} = 2
\end{aligned}$

 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

例 求解三元线性方程组

 $x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1, \quad y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-26}{-13} = 2$ $\begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \end{vmatrix}$

 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

解

例 求解三元线性方程组

 $x = \frac{D_x}{D} = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1, \quad y = \frac{D_y}{D} = \frac{\begin{vmatrix} 1 & 9 & 2 \\ 0 & 8 & 1 \\ 4 & -2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-26}{-13} = 2$ $D_z = \begin{vmatrix} 1 & 0 & 9 \\ 0 & 2 & 8 \\ 4 & -3 & -2 \end{vmatrix} = -52$

例 求解三元线性方程组

解

§1.1 二阶三阶行列式

 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases} \left(\begin{cases} x + 0y + 2z = 9 \\ 0x + 2y + z = 8 \\ 4x - 3y + 0z = -2 \end{cases} \right)$

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例 求解三元线性方程组 $\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$

例 求解三元线性方程组
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代入方程得

例 求解三元线性方程组
$$\begin{cases} x + 2z = 9\\ 2y + z = 8\\ 4x - 3y = -2 \end{cases}$$

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代入方程得

$$\begin{cases} 1+2z=9\\ 4-3y=-2 \end{cases}$$

例 求解三元线性方程组
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

$$x = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1,$$

代入方程得

$$\begin{cases} 1+2z=9\\ 4-3y=-2 \end{cases} \Rightarrow \begin{cases} z=4\\ y=2 \end{cases}$$



例 求解三元线性方程组
$$\begin{cases} x + 2z = 9 \\ 2y + z = 8 \\ 4x - 3y = -2 \end{cases}$$

$$X = \frac{\begin{vmatrix} 9 & 0 & 2 \\ 8 & 2 & 1 \\ -2 & -3 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 4 & -3 & 0 \end{vmatrix}} = \frac{-13}{-13} = 1,$$

代入方程得
$$\begin{cases} 1 + 2z = 9 \\ 4 - 3v = -2 \end{cases} \Rightarrow \begin{cases} z = 4 \\ v = 2 \end{cases}$$

所以方程的解是 $\begin{cases} x=1 \\ y=2 \\ z=4 \end{cases}$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

$$x_1 = \frac{D_1}{D_1}$$

$$x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

$$x_{1} = \frac{D_{1}}{D} = \frac{1}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_{2} = \frac{D_{2}}{D}, \quad \cdots, \quad x_{n} = \frac{D_{n}}{D}$$



$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

的解可用
$$n$$
 行列式表示:
$$x_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示: (称为克莱姆法则)

的解可用
$$n$$
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$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示: (称为克莱姆法则)

的解刊用
$$n$$
 行列式表示: (林乃兒来姆法则)
$$x_1 = \frac{D_1}{D} = \frac{\begin{vmatrix} b_1 & a_{12} & \cdots & a_{1n} \\ b_2 & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_n & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_2 = \frac{D_2}{D}, \quad \cdots, \quad x_n = \frac{D_n}{D}$$

那么,如何定义行列式,



$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n = b_n \end{cases}$$

的解可用 n 行列式表示: (称为克莱姆法则)

$$x_{1} = \frac{D_{1}}{D} = \frac{\begin{vmatrix} b_{1} & a_{12} & \cdots & a_{1n} \\ b_{2} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}}, \quad x_{2} = \frac{D_{2}}{D}, \quad \cdots, \quad x_{n} = \frac{D_{n}}{D}$$

那么,如何定义行列式,如何快捷计算行列式?



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

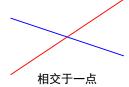
- 每条方程表示平面上的一条直线
- 方程组的解表示两条直线的交点

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

- 每条方程表示平面上的一条直线
- 方程组的解表示两条直线的交点
- 平面上两条直线的位置关系有三种:

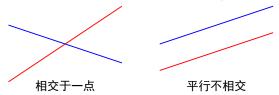
$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

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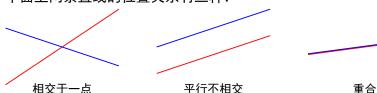
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- 每条方程表示平面上的一条直线
- 方程组的解表示两条直线的交点
- 平面上两条直线的位置关系有三种:



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

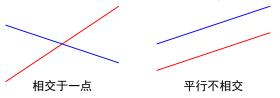
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2元2方程的线性方程组

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• 所以方程组的解有三种情况:

有唯一解、无解、有无穷多的解



重合

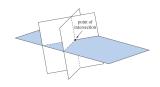
$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

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- 每条方程表示空间上的一个平面
- 方程组的解表示三个平面的交点

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- 每条方程表示空间上的一个平面
- 方程组的解表示三个平面的交点
- 空间上三个平面的位置关系有若干种,例如:

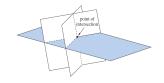




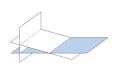


$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

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