### 第4章b:换元积分法

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### **Outline**

1. 第一类换元积分法: 凑微分

2. 第二类换元积分法: 变量代换



### We are here now...

1. 第一类换元积分法: 凑微分

2. 第二类换元积分法: 变量代换



### 第一类换元积分法——"凑微分"法,能干啥?

能够计算如下的不定积分:

$$\int \frac{dx}{2x+1}, \quad \int \cos(\frac{5}{2}x)dx$$

$$\int \frac{x}{\sqrt{3-x^2}}dx, \quad \int x\sin(x^2)dx$$

$$\int \frac{(\ln x)^2}{x}dx, \quad \int e^{\sin x}\cos xdx$$

$$\int \frac{1}{\cos x}dx$$
.....

**计算步骤** 
$$\int g(x)dx$$

$$\int g(x)dx = \int f(\varphi(x))\varphi'(x)dx$$

$$\int g(x)dx = \int f(\varphi(x))\varphi'(x)dx$$

$$\int g(x)dx = \int f(\varphi(x))\varphi'(x)dx = d\varphi(x)$$

$$\int g(x)dx = \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\int g(x)dx \xrightarrow{\underline{\text{$\not$ \ensuremath{\not$ \ensuremath{\ensuremath{\not$ \ensuremath{ \ensuremath{\not$ \ensuremath{\ensuremath}\ensuremath{\ensuremath{\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}}}}}}}}}}}}}}}}}}}} }}} } } } \ f(x) \ f(x)} \ f(x) \ f(x) \ f(x)} \ f(x) \ f(x)} \ f(x) \ f(x) \ f(x)} \ f(x) \ f(x) \ f(x)} \ f(x) \ f(x) \ f(x) \ f(x) \ f(x) \ f(x) \ f(x)} \ f(x) \ f(x) \ f(x)} \ f(x) \$$

$$\int g(x)dx = \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\underline{\varphi(x)=u}$$

$$\int g(x)dx \xrightarrow{\underline{\not} \otimes \otimes \otimes} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\xrightarrow{\underline{\varphi(x)=u}} \int f(u)du$$

$$\int g(x)dx \xrightarrow{\frac{k}{2}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\frac{\varphi(x)=u}{u} \int f(u)du = F(u) + C$$

$$\int g(x)dx \xrightarrow{\frac{k}{2}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\frac{\varphi(x)=u}{u} \int f(u)du = F(u) + C \xrightarrow{u=\varphi(x)} F(\varphi(x)) + C$$

#### 计算步骤

$$\int g(x)dx \xrightarrow{\underline{\not = \emptyset}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\underline{\xrightarrow{\varphi(x)=u}} \int f(u)du = F(u) + C \xrightarrow{u=\varphi(x)} F(\varphi(x)) + C$$

#### 计算步骤

$$\int g(x)dx \xrightarrow{\frac{k}{2}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\frac{\varphi(x)=u}{2} \int f(u)du = F(u) + C \xrightarrow{u=\varphi(x)} F(\varphi(x)) + C$$

$$\frac{d}{dx}F(\varphi(x)) =$$

#### 计算步骤

$$\int g(x)dx \xrightarrow{\frac{k}{2}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\frac{\varphi(x)=u}{2} \int f(u)du = F(u) + C \xrightarrow{u=\varphi(x)} F(\varphi(x)) + C$$

$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) =$$

#### 计算步骤

$$\int g(x)dx \xrightarrow{\frac{k}{2}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\frac{\varphi(x)=u}{2} \int f(u)du = F(u) + C \xrightarrow{u=\varphi(x)} F(\varphi(x)) + C$$

$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \cdot \varphi'(x) =$$

#### 计算步骤

$$\int g(x)dx \xrightarrow{\frac{k}{2}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\xrightarrow{\varphi(x)=u} \int f(u)du = F(u) + C \xrightarrow{u=\varphi(x)} F(\varphi(x)) + C$$

$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \cdot \varphi'(x) = g(x)$$

#### 计算步骤

$$\int g(x)dx \xrightarrow{\frac{\pi}{2}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\frac{\varphi(x)=u}{2} \int f(u)du = F(u) + C \xrightarrow{u=\varphi(x)} F(\varphi(x)) + C$$

$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \cdot \varphi'(x) = g(x)$$

总之 
$$\int g(x)dx = \int f(\varphi(x))d\varphi(x)$$
$$= \int f(u)du = F(u) + C = F(\varphi(x)) + C$$

类型 I: 
$$\int f(ax + b)dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax+b)dx$$



类型 I: 
$$\int f(ax + b)dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax+b) dx$$



类型 I: 
$$\int f(ax + b)dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$f(ax+b)dx d(ax+b)$$



类型 I: 
$$\int f(ax + b)dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax+b)dx \qquad \frac{1}{a}d(ax+b)$$



类型 I: 
$$\int f(ax + b)dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a}d(ax+b)$$



类型 I: 
$$\int f(ax+b)dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a}d(ax+b)$$

$$\underline{\underline{u=ax+b}}$$



类型 I: 
$$\int f(ax + b)dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a}d(ax+b)$$

$$\frac{u=ax+b}{a} \int f(u) \cdot \frac{1}{a}du =$$



类型 I: 
$$\int f(ax + b)dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a}d(ax+b)$$

$$\frac{u=ax+b}{a} \int f(u) \cdot \frac{1}{a}du = F(u)$$



类型 I: 
$$\int f(ax + b)dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a}d(ax+b)$$

$$\frac{u=ax+b}{a} \int f(u) \cdot \frac{1}{a}du = \frac{1}{a}F(u)$$



类型 I: 
$$\int f(ax + b)dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a} d(ax+b)$$

$$\xrightarrow{u=ax+b} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C$$



类型 I: 
$$\int f(ax + b)dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a}d(ax+b)$$

$$= \underbrace{\frac{u=ax+b}{a}} \int f(u) \cdot \frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ax+b) + C$$



类型 I: 
$$\int f(ax + b)dx$$

凑微分

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a}d(ax+b)$$

$$\frac{u=ax+b}{a} \int f(u) \cdot \frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ax+b) + C$$

例 1  $\int \frac{1}{1+2x} dx =$ 



类型 I: 
$$\int f(ax + b)dx$$

凑微分

$$\int f(u)du = F(u) + C$$

d(1 + 2x)

∭

$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a} d(ax+b)$$

$$= \underbrace{\frac{u=ax+b}{a}} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(ax+b) + C$$

$$\int \frac{1}{1+2x} dx =$$



类型 I: 
$$\int f(ax + b)dx$$

凑微分

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a}d(ax+b)$$

$$\frac{u=ax+b}{a} \int f(u) \cdot \frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ax+b) + C$$

**例 1** 

$$\int \frac{1}{1+2x} dx = \frac{1}{2} d(1+2x)$$



类型 I: 
$$\int f(ax+b)dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a}d(ax+b)$$

$$\frac{u=ax+b}{a} \int f(u) \cdot \frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ax+b) + C$$

例 1 
$$\int \frac{1}{1+2x} dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x)$$



类型 I:  $\int f(ax + b)dx$ 

假设会算

凑微分

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a}d(ax+b)$$

$$\frac{u=ax+b}{a} \int f(u) \cdot \frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ax+b) + C$$

例 1  $\int \frac{1}{1+2x} dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x) = \frac{1}{2} \int \frac{1}{u} du$ 



类型 I:  $\int f(ax + b)dx$ 凑微分

$$\int f(u)du = F(u) + C$$

$$\int_{f(\alpha \times 1)}$$

则
$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a}d(ax+b)$$

$$\int f(ax+b)dx = \int f(ax+b)dx$$

$$\frac{u=ax+b}{a} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(ax+b) + C$$

 $\int \frac{1}{1+2x} dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x) = \frac{1}{2} \int \frac{1}{u} du$ 

$$\frac{u=ax+b}{a} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u)$$

 $= \frac{1}{2} \ln |u| + C$ 



$$\frac{u=ax+b}{a} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u)$$

$$\int f(u)du = F(u) + C$$

假设会算

$$\int f(ax+b)dx = \int f(ax+b) \cdot \frac{1}{a} d(ax+b)$$

$$\frac{u=ax+b}{a} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(ax+b) + C$$

$$\frac{u=ax+b}{\int} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(ax+b) + C$$

$$\int \frac{1}{1+2x} dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x) = \frac{1}{2} \int \frac{1}{u} du$$

$$\frac{u=ax+b}{=} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + \frac{1}{a} f(u) +$$

4/21 < ▷ △ ▽



例 2 求  $\int \frac{1}{2-3x} dx$ ,  $\int \sqrt{3x-1} dx$ 

凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 2 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

$$\int \frac{1}{2-3x} dx =$$

$$\int \sqrt{3x - 1} dx =$$



凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 2 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

$$\int \frac{1}{2-3x} dx =$$

$$d(2-3x)$$

$$\int \sqrt{3x-1}dx =$$



凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 2 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

$$\int \frac{1}{2-3x} dx = \cdot \left(-\frac{1}{3}\right) d(2-3x)$$

$$\int \sqrt{3x-1}dx =$$



凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 2 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x)$$

$$\int \sqrt{3x-1}dx =$$



凑微分 类型 I: 
$$\int f(ax + b)dx$$

例 2 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$

$$\int \sqrt{3x-1}dx =$$



凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 2 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$
$$= -\frac{1}{3} \ln|u| + C$$

$$\sqrt{3x-1}dx =$$



凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 2 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$
$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

$$\sqrt{3x-1}dx =$$



凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 2 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$
$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

$$\sqrt{3x-1}dx = d(3x-1)$$



凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 2 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$
$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$
$$\int \sqrt{3x-1} dx = \frac{1}{3} d(3x-1)$$



凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 2 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$
$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$
$$\int \sqrt{3x-1} dx = \int \sqrt{3x-1} \cdot \frac{1}{3} d(3x-1)$$



凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 2 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$
$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$
$$\int \sqrt{3x-1} dx = \int \sqrt{3x-1} \cdot \frac{1}{3} d(3x-1) = \frac{1}{3} \int \sqrt{u} du$$



凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 2 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

$$\int \sqrt{3x-1} dx = \int \sqrt{3x-1} \cdot \frac{1}{2} d(3x-1) = -\frac{1}{2} \int \sqrt{u} du = -\frac{1}{2} \int u^{1/2} du$$

$$\int \sqrt{3x - 1} dx = \int \sqrt{3x - 1} \cdot \frac{1}{3} d(3x - 1) = \frac{1}{3} \int \sqrt{u} du = \frac{1}{3} \int u^{1/2} du$$



**例2** 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

解

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

$$\int \sqrt{3x-1} dx = \int \sqrt{3x-1} \cdot \frac{1}{3} d(3x-1) = \frac{1}{3} \int \sqrt{u} du = \frac{1}{3} \int u^{1/2} du$$

$$u^{3/2}$$



4b 换元积分法

例 2 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

$$\int \sqrt{3x-1} dx = \int \sqrt{3x-1} \cdot \frac{1}{3} d(3x-1) = \frac{1}{3} \int \sqrt{u} du = \frac{1}{3} \int u^{1/2} du$$



例 2 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

$$\int \sqrt{3x-1} dx = \int \sqrt{3x-1} \cdot \frac{1}{3} d(3x-1) = \frac{1}{3} \int \sqrt{u} du = \frac{1}{3} \int u^{1/2} du$$

$$= \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C$$

**例2** 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

$$\int \sqrt{3x-1} dx = \int \sqrt{3x-1} \cdot \frac{1}{3} d(3x-1) = \frac{1}{3} \int \sqrt{u} du = \frac{1}{3} \int u^{1/2} du$$

$$= \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C = \frac{2}{9} (3x-1)^{3/2} + C$$



**例2** 求 
$$\int \frac{1}{2-3x} dx$$
,  $\int \sqrt{3x-1} dx$ 

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$
$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

$$\int 2-3x \qquad \int 2-3x \qquad 3 \qquad 3 \int u$$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2 - 3x| + C$$

$$\int \sqrt{3x - 1} dx = \int \sqrt{3x - 1} \cdot \frac{1}{3} d(3x - 1) = \frac{1}{3} \int \sqrt{u} du = \frac{1}{3} \int u^{1/2} du$$

 $= \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C = \frac{2}{9} (3x - 1)^{3/2} + C$ 

例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 





凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 解

$$\int \frac{1}{\sqrt{1-5x}} dx =$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$  解

$$\mathbf{F} = \frac{1}{\sqrt{1-5x}} dx, \quad \int \cos(\frac{\pi}{2}x) dx, \quad \int e^{-2xx} dx$$

$$\mathbf{F} = \int \frac{1}{\sqrt{1-5x}} dx = d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

$$\mathbf{P} = \frac{1}{\sqrt{1-5x}} dx = \frac{1}{\sqrt{1-5x}} dx = \frac{1}{5} d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4} dx =$$



凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 解

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

例 3 来 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{\pi}{2}x) dx$ ,  $\int e^{-\frac{\pi}{2}x+4} dx$   
解
$$\int \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4} dx =$$



4b 换元积分法

凑微分 类型 I: 
$$\int f(ax + b)dx$$

例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$

$$= u^{1/2}$$

$$\int e^{-\frac{1}{2}x+4} dx =$$

 $\int \cos(\frac{3}{2}x)dx =$ 



凑微分 类型 I: 
$$\int f(ax + b)dx$$

例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

$$\frac{1}{\sqrt{1-5x}}dx, \int \cos(\frac{1}{2}x)dx, \int e^{-2x}dx$$

$$\int \frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du$$

 $2u^{1/2}$ 

 $\int \cos(\frac{3}{2}x)dx =$ 

 $\int e^{-\frac{1}{2}x+4} dx =$ 



4b 换元积分法

凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C$$

$$\int \cos(\frac{3}{2}x) dx =$$

$$\int e^{-\frac{1}{2}x+4} dx =$$



凑微分 类型 I: 
$$\int f(ax + b)dx$$

例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x) dx =$$

$$\int e^{-\frac{1}{2}x+4} dx =$$



凑微分 类型 I: 
$$\int f(ax + b)dx$$

**例3** 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

$$\iint \sqrt{1-5x} \, dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4} dx =$$



凑微分 类型 I: 
$$\int f(ax+b)dx$$

例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

$$\iint \sqrt{1-5x} \, dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x) dx = \frac{2}{3} d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4} dx =$$





凑微分 类型 I: 
$$\int f(ax + b)dx$$

例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

$$\iint \sqrt{1-5x} \, dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{1}{5}(1 - 5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



凑微分 类型 I: 
$$\int f(ax + b)dx$$

例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1 - 5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x) dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3} d(\frac{3}{2}x) = \frac{2}{3} \int \cos u du$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1 - 5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u) + C$$

 $\int e^{-\frac{1}{2}x+4} dx =$ 



**例3** 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1 - 5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

 $\int e^{-\frac{1}{2}x+4}dx =$ 



**例3** 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1 - 5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x) dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3} d(\frac{3}{2}x) = \frac{2}{3} \int \cos u du$$

$$= \frac{2}{3} \sin(u) + C = \frac{2}{3} \sin(\frac{3}{2}x) + C$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4} dx = d(-\frac{1}{2}x+4)$$



**例3** 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{1}{5} (1 - 5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x) dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3} d(\frac{3}{2}x) = \frac{2}{3} \int \cos u du$$

$$= \frac{2}{3} \sin(u) + C = \frac{2}{3} \sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4} dx = \frac{1}{2}(-2)d(-\frac{1}{2}x+4)$$



例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{1}{5}(1 - 5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x + 4} dx = \int e^{-\frac{1}{2}x + 4} \cdot (-2)d(-\frac{1}{2}x + 4)$$





# 类型 I: $\int f(ax+b)dx$

例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$= -\frac{1}{5} \cdot 2u^{3/2} + C = -\frac{1}{5}(1 - 5x)^{3/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x + 4} dx = \int e^{-\frac{1}{2}x + 4} \cdot (-2)d(-\frac{1}{2}x + 4) = -2\int e^{u} du$$





例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

$$\mathbf{P} = \frac{1}{\sqrt{1 - 5x}} dx, \quad \int \cos(\frac{1}{2}x) dx, \quad \int e^{-\frac{1}{2}x} dx$$

$$\mathbf{P} = \int \frac{1}{\sqrt{1 - 5x}} dx = \int (1 - 5x)^{-\frac{1}{2}} \cdot (-\frac{1}{5}) d(1 -$$

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1 - 5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

 $\int e^{-\frac{1}{2}x+4} dx = \int e^{-\frac{1}{2}x+4} \cdot (-2) d(-\frac{1}{2}x+4) = -2 \int e^{u} du$ 

4b 换元积分法

 $= -2e^{u} + C$ 

# 类型 I: $\int f(ax+b)dx$

例 3 求 
$$\int \frac{1}{\sqrt{1-5x}} dx$$
,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{1}{2}x+4} dx$ 

例 
$$3$$
 求  $\int \frac{1}{\sqrt{1-5x}} dx$ ,  $\int \cos(\frac{3}{2}x) dx$ ,  $\int e^{-\frac{7}{2}x+4} dx$ 解

$$\frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = 0$$

$$\frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x)$$

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$
$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

 $= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1 - 5x)^{1/2} + C$ 

 $\int e^{-\frac{1}{2}x+4} dx = \int e^{-\frac{1}{2}x+4} \cdot (-2) d(-\frac{1}{2}x+4) = -2 \int e^{u} du$ 

类型 II: 
$$\int f(ax^2 + b)xdx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax^2+b)xdx$$



类型 II: 
$$\int f(ax^2 + b)xdx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax^2+b)xdx$$



类型 II: 
$$\int f(ax^2 + b)xdx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax^2 + b)xdx \qquad \qquad d(ax^2 + b)$$



类型 II: 
$$\int f(ax^2 + b)xdx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax^2 + b)x dx \qquad \frac{1}{2a}d(ax^2 + b)$$



类型 II: 
$$\int f(ax^2 + b)xdx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a}d(ax^2 + b)$$



类型 II: 
$$\int f(ax^2 + b)xdx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a} d(ax^2 + b)$$

$$u = ax^2 + b$$



假设会算

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax^2 + b)x dx = \int f(ax^2 + b) \cdot \frac{1}{2a} d(ax^2 + b)$$

$$\frac{u = ax^2 + b}{2a} \int f(u) \cdot \frac{1}{2a} du =$$



类型 II: 
$$\int f(ax^2 + b)xdx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax^{2} + b)xdx = \int f(ax^{2} + b) \cdot \frac{1}{2a}d(ax^{2} + b)$$

$$\frac{u=ax^{2}+b}{2a} \int f(u) \cdot \frac{1}{2a}du = F(u)$$



类型 II: 
$$\int f(ax^2 + b)xdx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax^2 + b)x dx = \int f(ax^2 + b) \cdot \frac{1}{2a} d(ax^2 + b)$$

$$\frac{u = ax^2 + b}{2a} \int f(u) \cdot \frac{1}{2a} du = \frac{1}{2a} F(u)$$



类型 II: 
$$\int f(ax^2 + b)xdx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax^2 + b)x dx = \int f(ax^2 + b) \cdot \frac{1}{2a} d(ax^2 + b)$$

$$\frac{u = ax^2 + b}{2a} \int f(u) \cdot \frac{1}{2a} du = \frac{1}{2a} F(u) + C$$



假设会算

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax^{2} + b)xdx = \int f(ax^{2} + b) \cdot \frac{1}{2a}d(ax^{2} + b)$$

$$\frac{u = ax^{2} + b}{2a} \int f(u) \cdot \frac{1}{2a}du = \frac{1}{2a}F(u) + C = \frac{1}{2a}F(ax^{2} + b) + C$$



假设会算

凑微分

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^{2} + b)xdx = \int f(ax^{2} + b) \cdot \frac{1}{2a}d(ax^{2} + b)$$

$$\frac{u=ax^{2}+b}{2a} \int f(u) \cdot \frac{1}{2a}du = \frac{1}{2a}F(u) + C = \frac{1}{2a}F(ax^{2} + b) + C$$

例 1  $\int x\sqrt{1-x^2}dx =$ 



假设会算

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ax^{2} + b)xdx = \int f(ax^{2} + b) \cdot \frac{1}{2a}d(ax^{2} + b)$$

$$\frac{u = ax^{2} + b}{2a} \int f(u) \cdot \frac{1}{2a}du = \frac{1}{2a}F(u) + C = \frac{1}{2a}F(ax^{2} + b) + C$$

例 1 
$$\int x\sqrt{1-x^2}dx =$$

$$d(1-x^2)$$



假设会算

$$\int f(u)du = F(u) + C$$

$$\int f(ax^{2} + b)x dx = \int f(ax^{2} + b) \cdot \frac{1}{2a} d(ax^{2} + b)$$

$$\frac{u = ax^{2} + b}{2a} \int f(u) \cdot \frac{1}{2a} du = \frac{1}{2a} F(u) + C = \frac{1}{2a} F(ax^{2} + b) + C$$

**例 1** 
$$\int x\sqrt{1-x^2}dx =$$

$$dx = \qquad \qquad \cdot (-\frac{1}{2})d(1-x^2)$$



假设会算

凑微分

$$\int f(u)du = F(u) + C$$

则

$$\int f(ax^{2} + b)x dx = \int f(ax^{2} + b) \cdot \frac{1}{2a} d(ax^{2} + b)$$

$$\frac{u = ax^{2} + b}{2a} \int f(u) \cdot \frac{1}{2a} du = \frac{1}{2a} F(u) + C = \frac{1}{2a} F(ax^{2} + b) + C$$

例 1  $\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2)$ 



假设会算

$$\int f(u)du = F(u) + C$$

$$\int f(ax^{2} + b)x dx = \int f(ax^{2} + b) \cdot \frac{1}{2a} d(ax^{2} + b)$$

$$\frac{u = ax^{2} + b}{2a} \int f(u) \cdot \frac{1}{2a} du = \frac{1}{2a} F(u) + C = \frac{1}{2a} F(ax^{2} + b) + C$$

例 1 
$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2} \int u^{\frac{1}{2}}du$$



假设会算

$$\int f(u)du = F(u) + C$$

$$\int f(ax^2 + b)ydy = \int f(ax^2 + b) \cdot \int_{a}^{1} d(ax^2 + b) \cdot \int_{a}^{1} d(ax^2$$

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a}d(ax^2 + b)$$

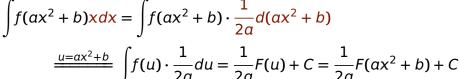
$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2} \int u^{\frac{1}{2}}du$$

113/2

 $\frac{u=ax^2+b}{}$   $\int f(u) \cdot \frac{1}{2a} du = \frac{1}{2a} F(u) + C = \frac{1}{2a} F(ax^2+b) + C$ 



$$\int f(u)du = F(u) + C$$



# 例 1 $\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2} \int u^{\frac{1}{2}}du$ $\frac{2}{3}u^{3/2}$

类型 II:  $\int f(ax^2 + b)xdx$ 凑微分 假设会算

$$\int f(u)du = F(u) + C$$

$$\int f(ax^2 + b)x dx = \int f(ax^2 + b) \cdot \frac{1}{2a} d(ax^2 + b)$$

$$\int f(ax^{2} + b)x dx = \int f(ax^{2} + b) \cdot \frac{1}{2a} d(ax^{2} + b)$$

$$\frac{u = ax^{2} + b}{2a} \int f(u) \cdot \frac{1}{2a} du = \frac{1}{2a} F(u) + C = \frac{1}{2a} F(ax^{2} + b) + C$$

$$\int J(u) \cdot \frac{1}{2a} du = \frac{1}{2a} I(u) + C = \frac{1}{2a} I(ux^2 + b) + C$$

$$\int x \sqrt{1 - x^2} dx = \int (1 - x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2}) d(1 - x^2) = -\frac{1}{2} \int u^{\frac{1}{2}} du$$

$$\frac{u=ax^2+b}{2a}\int f(u)\cdot \frac{1}{2a}du = \frac{1}{2a}F(u)+C$$



假设会算 
$$\int f(u)du = F$$

$$\int f(u)du = F(u) + C$$
则

$$\int_{a}^{b} f(ax^{2} + b)x dx = \int_{a}^{b} f(ax^{2} + b) \cdot \frac{1}{a} d(ax^{2} + b)$$

$$\int f(ax^2 + b)xdx = \int f(ax^2 + b) \cdot \frac{1}{2a}d(ax^2 + b)$$

$$\frac{u=ax^{2}+b}{\int} f(u) \cdot \frac{1}{2a} du = \frac{1}{2a} F(u) + C = \frac{1}{2a} F(ax^{2}+b) + C$$

$$\boxed{\emptyset 1} 1 \qquad \int x\sqrt{1-x^{2}} dx = \int (1-x^{2})^{\frac{1}{2}} \cdot (-\frac{1}{2}) d(1-x^{2}) = -\frac{1}{2} \int u^{\frac{1}{2}} du$$

$$= -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C = -\frac{1}{3} (1 - x^2)^{\frac{3}{2}} + C$$

例 2 求 
$$\int \frac{x}{\sqrt{3-x^2}} dx$$
,  $\int \frac{x}{1+3x^2} dx$ 

凑微分 类型 II: 
$$\int f(ax^2 + b)xdx$$

例 2 求 
$$\int \frac{x}{\sqrt{3-x^2}} dx$$
,  $\int \frac{x}{1+3x^2} dx$ 

$$\int \frac{x}{\sqrt{3-x^2}} dx =$$

$$\int \frac{x}{1+3x^2} dx =$$



凑微分 类型 II: 
$$\int f(ax^2 + b)xdx$$

例 2 求 
$$\int \frac{x}{\sqrt{3-x^2}} dx$$
,  $\int \frac{x}{1+3x^2} dx$ 

$$\int \frac{x}{\sqrt{3-x^2}} dx = d(3-x^2)$$

$$\int \frac{x}{1+3x^2} dx =$$



凑微分 类型 II: 
$$\int f(ax^2 + b)xdx$$

例 2 求 
$$\int \frac{x}{\sqrt{3-x^2}} dx$$
,  $\int \frac{x}{1+3x^2} dx$ 

$$\int \frac{x}{\sqrt{3-x^2}} dx = \cdot (-\frac{1}{2})d(3-x^2)$$

$$\int \frac{x}{1+3x^2} dx =$$



凑微分 类型 II: 
$$\int f(ax^2 + b)xdx$$

例 2 求 
$$\int \frac{x}{\sqrt{3-x^2}} dx$$
,  $\int \frac{x}{1+3x^2} dx$ 

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

$$\int \frac{x}{1+3x^2} dx =$$



凑微分 类型 II: 
$$\int f(ax^2 + b)xdx$$

例 2 求 
$$\int \frac{x}{\sqrt{3-x^2}} dx$$
,  $\int \frac{x}{1+3x^2} dx$ 

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$
$$= -\frac{1}{2} \int u^{-1/2} du$$

$$\int \frac{x}{1+3x^2} dx =$$



凑微分 类型 II: 
$$\int f(ax^2 + b)xdx$$

例 2 求 
$$\int \frac{x}{\sqrt{3-x^2}} dx$$
,  $\int \frac{x}{1+3x^2} dx$ 

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$
$$= -\frac{1}{2} \int u^{-1/2} du \qquad 2u^{1/2}$$
$$\int \frac{x}{1+3x^2} dx =$$

凑微分 类型 II: 
$$\int f(ax^2 + b)xdx$$

例 2 求 
$$\int \frac{x}{\sqrt{3-x^2}} dx$$
,  $\int \frac{x}{1+3x^2} dx$ 

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$
$$= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C$$
$$\int \frac{x}{1+3x^2} dx =$$

例2 求 
$$\int \frac{x}{\sqrt{3-x^2}} dx$$
,  $\int \frac{x}{1+3x^2} dx$ 

$$\iint \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

$$= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2} dx = \frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

例2 求 
$$\int \frac{x}{\sqrt{3-x^2}} dx$$
,  $\int \frac{x}{1+3x^2} dx$ 

$$\iint \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

$$= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2} dx = d(1+3x^2)$$



例2 求 
$$\int \frac{x}{\sqrt{3-x^2}} dx$$
,  $\int \frac{x}{1+3x^2} dx$ 

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2} dx = \frac{1}{6} d(1+3x^2)$$

例2 求 
$$\int \frac{x}{\sqrt{3-x^2}} dx$$
,  $\int \frac{x}{1+3x^2} dx$ 

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

$$= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2} dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6} d(1+3x^2)$$



凑微分 类型 II: 
$$\int f(ax^2 + b)xdx$$

例2求
$$\int \frac{x}{\sqrt{3-x^2}} dx$$
,  $\int \frac{x}{1+3x^2} dx$ 

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

$$= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2} dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6} d(1+3x^2) = \frac{1}{6} \int \frac{1}{u} du$$



例2 求 
$$\int \frac{x}{\sqrt{3-x^2}} dx$$
,  $\int \frac{x}{1+3x^2} dx$ 

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2} dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6} d(1+3x^2) = \frac{1}{6} \int \frac{1}{u} du$$

$$= \frac{1}{6} \ln|u| + C$$



例2 求 
$$\int \frac{x}{\sqrt{3-x^2}} dx$$
,  $\int \frac{x}{1+3x^2} dx$ 

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2} dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6} d(1+3x^2) = \frac{1}{6} \int \frac{1}{u} du$$

$$= \frac{1}{6} \ln|u| + C = \frac{1}{6} \ln|1+3x^2| + C$$



例2求
$$\int \frac{x}{\sqrt{3-x^2}} dx$$
,  $\int \frac{x}{1+3x^2} dx$ 

$$\mathbf{p}$$

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

$$= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3 - x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1 + 3x^2} dx = \int \frac{1}{1 + 3x^2} \cdot \frac{1}{6} d(1 + 3x^2) = \frac{1}{6} \int \frac{1}{u} du$$

例 3 求 
$$\int xe^{x^2}dx$$
,  $\int x\sin(x^2)dx$ 



 $= \frac{1}{6} \ln|u| + C = \frac{1}{6} \ln|1 + 3x^2| + C$ 

例 3 求 
$$\int xe^{x^2}dx$$
,  $\int x\sin(x^2)dx$ 



例 3 求 
$$\int xe^{x^2}dx$$
,  $\int x\sin(x^2)dx$ 解

$$\int x e^{x^2} dx =$$



例 3 求 
$$\int xe^{x^2}dx$$
,  $\int x\sin(x^2)dx$ 

$$\int x e^{x^2} dx = d(x^2)$$



凑微分 类型 II: 
$$\int f(ax^2 + b)xdx$$

例 3 求 
$$\int xe^{x^2}dx$$
,  $\int x\sin(x^2)dx$ 

$$\int xe^{x^2}dx = \frac{1}{2}d(x^2)$$



凑微分 类型 II: 
$$\int f(ax^2 + b)xdx$$

例 3 求 
$$\int xe^{x^2}dx$$
,  $\int x\sin(x^2)dx$ 

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2)$$



凑微分 类型 II: 
$$\int f(ax^2 + b)xdx$$

例 3 求 
$$\int xe^{x^2}dx$$
,  $\int x\sin(x^2)dx$ 

$$\int x e^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du$$



凑微分 类型 II: 
$$\int f(ax^2 + b)xdx$$

例 3 求 
$$\int xe^{x^2}dx$$
,  $\int x\sin(x^2)dx$ 

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^udu = \frac{1}{2}e^u + C$$



凑微分 类型 II: 
$$\int f(ax^2 + b)xdx$$

**例3** 求 
$$\int xe^{x^2}dx$$
,  $\int x\sin(x^2)dx$ 

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^udu = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$



凑微分 类型 II: 
$$\int f(ax^2 + b)xdx$$

例 3 求 
$$\int xe^{x^2}dx$$
,  $\int x\sin(x^2)dx$ 

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^udu = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x \sin(x^2) dx =$$



例 3 求 
$$\int xe^{x^2}dx$$
,  $\int x\sin(x^2)dx$ 

解

$$\int xe^{x^2}dx = \int e^{x^2} \frac{1}{2}d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = d(x^2)$$

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例 3 求 
$$\int xe^{x^2}dx$$
,  $\int x\sin(x^2)dx$ 

解

$$\int xe^{x^2}dx = \int e^{x^2} \frac{1}{2}d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \frac{1}{2}d(x^2)$$

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凑微分 类型 II: 
$$\int f(ax^2 + b)xdx$$

例 3 求 
$$\int xe^{x^2}dx$$
,  $\int x\sin(x^2)dx$ 

$$\int xe^{x^2}dx = \int e^{x^2} \frac{1}{2}d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$
$$\int x\sin(x^2)dx = \int \sin(x^2) \cdot \frac{1}{2}d(x^2)$$

凑微分 类型 II: 
$$\int f(ax^2 + b)xdx$$

例 3 求 
$$\int xe^{x^2}dx$$
,  $\int x\sin(x^2)dx$ 

$$\int xe^{x^2}dx = \int e^{x^2} \frac{1}{2}d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$
$$\int x\sin(x^2)dx = \int \sin(x^2) \cdot \frac{1}{2}d(x^2) = \frac{1}{2} \int \sin u du$$



例 3 求 
$$\int xe^{x^2}dx$$
,  $\int x\sin(x^2)dx$ 

$$\int xe^{x^2}dx = \int e^{x^2} \frac{1}{2}d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$
$$\int x\sin(x^2)dx = \int \sin(x^2) \cdot \frac{1}{2}d(x^2) = \frac{1}{2} \int \sin u du$$

$$=-\frac{1}{2}\cos u+C$$



例 3 求 
$$\int xe^{x^2}dx$$
,  $\int x\sin(x^2)dx$ 

$$\int xe^{x^2}dx = \int e^{x^2} \frac{1}{2}d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$
$$\int x\sin(x^2)dx = \int \sin(x^2) \cdot \frac{1}{2}d(x^2) = \frac{1}{2} \int \sin u du$$

$$= -\frac{1}{2}\cos u + C = -\frac{1}{2}\cos(x^2) + C$$



凑微分 类型 III: 
$$\int f(ae^x + b)e^x dx$$

$$\int f(u)du = F(u) + C$$

$$\int f(ae^x + b)e^x dx$$



凑微分 类型 III: 
$$\int f(ae^x + b)e^x dx$$

$$\int f(u)du = F(u) + C$$

$$\int f(ae^x + b)e^x dx$$



凑微分 类型 III: 
$$\int f(ae^x + b)e^x dx$$

$$\int f(u)du = F(u) + C$$

$$\int f(ae^x + b)e^x dx \qquad \qquad d(ae^x + b)$$



凑微分 类型 III: 
$$\int f(ae^x + b)e^x dx$$

$$\int f(u)du = F(u) + C$$

$$\int f(ae^{x} + b)e^{x}dx \qquad \frac{1}{a}d(ae^{x} + b)$$



类型 III: 
$$\int f(ae^x + b)e^x dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ae^{x} + b)e^{x}dx = \int f(ae^{x} + b) \cdot \frac{1}{a}d(ae^{x} + b)$$



类型 III: 
$$\int f(ae^x + b)e^x dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ae^{x} + b)e^{x}dx = \int f(ae^{x} + b) \cdot \frac{1}{a}d(ae^{x} + b)$$

$$\underline{\underline{u=ae^{x}+b}}$$



类型 III: 
$$\int f(ae^x + b)e^x dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ae^{x} + b)e^{x}dx = \int f(ae^{x} + b) \cdot \frac{1}{a}d(ae^{x} + b)$$

$$\frac{u=ae^{x}+b}{a} \int f(u) \cdot \frac{1}{a}du =$$



类型 III: 
$$\int f(ae^x + b)e^x dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ae^{x} + b)e^{x}dx = \int f(ae^{x} + b) \cdot \frac{1}{a}d(ae^{x} + b)$$

$$\frac{u=ae^{x}+b}{a} \int f(u) \cdot \frac{1}{a}du = F(u)$$



类型 III: 
$$\int f(ae^x + b)e^x dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ae^{x} + b)e^{x}dx = \int f(ae^{x} + b) \cdot \frac{1}{a}d(ae^{x} + b)$$

$$\frac{u=ae^{x} + b}{a} \int f(u) \cdot \frac{1}{a}du = \frac{1}{a}F(u)$$



类型 III: 
$$\int f(ae^x + b)e^x dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ae^{x} + b)e^{x}dx = \int f(ae^{x} + b) \cdot \frac{1}{a}d(ae^{x} + b)$$

$$\frac{u=ae^{x}+b}{a} \int f(u) \cdot \frac{1}{a}du = \frac{1}{a}F(u) + C$$



类型 III: 
$$\int f(ae^x + b)e^x dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ae^{x} + b)e^{x}dx = \int f(ae^{x} + b) \cdot \frac{1}{a}d(ae^{x} + b)$$

$$\frac{u=ae^{x}+b}{a} \int f(u) \cdot \frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ae^{x} + b) + C$$



类型 III: 
$$\int f(ae^x + b)e^x dx$$

凑微分

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^{x} + b)e^{x}dx = \int f(ae^{x} + b) \cdot \frac{1}{a}d(ae^{x} + b)$$

$$\frac{u=ae^{x}+b}{a} \int f(u) \cdot \frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ae^{x} + b) + C$$

$$\int \frac{e^x}{1+e^x} dx =$$



类型 III: 
$$\int f(ae^x + b)e^x dx$$

凑微分

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^{x} + b)e^{x}dx = \int f(ae^{x} + b) \cdot \frac{1}{a}d(ae^{x} + b)$$

$$\frac{u=ae^{x}+b}{a} \int f(u) \cdot \frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ae^{x} + b) + C$$

$$\int \frac{e^x}{1+e^x} dx = d(e^x + 1)$$



类型 III: 
$$\int f(ae^x + b)e^x dx$$

凑微分

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^{x} + b)e^{x}dx = \int f(ae^{x} + b) \cdot \frac{1}{a}d(ae^{x} + b)$$

$$\frac{u=ae^{x}+b}{a} \int f(u) \cdot \frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ae^{x} + b) + C$$

$$\int \frac{e^x}{1+e^x} dx = \int \frac{1}{1+e^x} d(e^x + 1)$$



类型 III:  $\int f(ae^x + b)e^x dx$ 

假设会算

凑微分

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^{x} + b)e^{x}dx = \int f(ae^{x} + b) \cdot \frac{1}{a}d(ae^{x} + b)$$

$$\frac{u=ae^{x}+b}{a} \int f(u) \cdot \frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ae^{x} + b) + C$$

$$\int \frac{e^{x}}{1+e^{x}} dx = \int \frac{1}{1+e^{x}} d(e^{x}+1)$$
$$= \int \frac{1}{u} du$$



类型 III:  $\int f(ae^x + b)e^x dx$ 

假设会算

凑微分

$$\int f(u)du = F(u) + C$$

则

$$\int f(ae^{x} + b)e^{x}dx = \int f(ae^{x} + b) \cdot \frac{1}{a}d(ae^{x} + b)$$

$$\frac{u=ae^{x}+b}{a} \int f(u) \cdot \frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ae^{x} + b) + C$$

$$\int \frac{e^x}{1+e^x} dx = \int \frac{1}{1+e^x} d(e^x + 1)$$
$$= \int \frac{1}{u} du = \ln|u| + C$$



**类型 III:**  $\int f(ae^x + b)e^x dx$ 

假设会算

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(ae^{x} + b)e^{x}dx = \int f(ae^{x} + b) \cdot \frac{1}{a}d(ae^{x} + b)$$

$$\frac{u=ae^{x}+b}{a} \int f(u) \cdot \frac{1}{a}du = \frac{1}{a}F(u) + C = \frac{1}{a}F(ae^{x} + b) + C$$

$$\int \frac{e^{x}}{1 + e^{x}} dx = \int \frac{1}{1 + e^{x}} d(e^{x} + 1)$$

$$= \int \frac{1}{u} du = \ln|u| + C = \ln(e^{x} + 1) + C$$

凑微分 类型 III: 
$$\int f(ae^x + b)e^x dx$$

$$\int e^x \sin(e^x) dx =$$



凑微分 类型 III: 
$$\int f(ae^x + b)e^x dx$$

$$\int e^x \sin(e^x) dx = de^x$$



凑微分 类型 III: 
$$\int f(ae^x + b)e^x dx$$

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类型 IV: 
$$\int f(a \ln x + b) \frac{1}{x} dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(a \ln x + b) \frac{1}{x} dx$$



类型 IV: 
$$\int f(a \ln x + b) \frac{1}{x} dx$$

凑微分

$$\int f(u)du = F(u) + C$$

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类型 IV: 
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凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(a \ln x + b) \frac{1}{x} dx \qquad \qquad d(a \ln x + b)$$



类型 IV: 
$$\int f(a \ln x + b) \frac{1}{x} dx$$

凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(a \ln x + b) \frac{1}{x} dx \qquad \frac{1}{a} d(a \ln x + b)$$



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$$\int f(u)du = F(u) + C$$

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$$\underline{u = a \ln x + b}$$



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$$\frac{u = a \ln x + b}{a} \int f(u) \cdot \frac{1}{a} du =$$



类型 IV: 
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$$\frac{u = a \ln x + b}{a} \int f(u) \cdot \frac{1}{a} du = F(u)$$



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凑微分 类型 IV: 
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$$\int \frac{1}{x} \ln x dx = \int \frac{1}{x \ln x} dx = \int \frac{1}{x \ln$$

凑微分 类型 IV: 
$$\int f(a \ln x + b) \frac{1}{x} dx$$

$$\int f(u)du = F(u) + C$$

则
$$\int f(a \ln x + b) \frac{1}{x} dx = \int f(a \ln x + b) \cdot \frac{1}{a} d(a \ln x + b)$$

$$\int \frac{1}{x} \ln x dx = d \ln x$$

$$\int \frac{1}{x \ln x} dx =$$

凑微分 类型 IV: 
$$\int f(a \ln x + b) \frac{1}{x} dx$$

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凑微分 类型 IV: 
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$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$

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$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du = \frac{1}{2}u^2 + C$$

$$\int \frac{1}{x \ln x} dx =$$

类型 IV:  $\int f(a \ln x + b) \frac{1}{y} dx$ 凑微分

假设会算 
$$\int f(u)du = F(u) + C$$

$$\int f(a \ln x + b) \frac{1}{x} dx = \int f(a \ln x + b) \cdot \frac{1}{a} d(a \ln x + b)$$

$$\frac{u=a\ln x+b}{a} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(a\ln x + b) + C$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du = \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$

$$\int \frac{1}{x \ln x} dx = \int \frac{1$$



类型 IV:  $\int f(a \ln x + b) \frac{1}{x} dx$ 凑微分

假设会算 
$$\int f(u)du = F(u) + C$$

$$\int f(a \ln x + b) \frac{1}{x} dx = \int f(a \ln x + b) \cdot \frac{1}{a} d(a \ln x + b)$$

$$\frac{u = a \ln x + b}{a} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(a \ln x + b) + C$$

# $\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du = \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$

 $\int \frac{1}{x \ln x} dx =$ 

 $d \ln x$ 

类型 IV:  $\int f(a \ln x + b) \frac{1}{x} dx$ 凑微分

假设会算 
$$\int f(u)du = F(u) + C$$

$$\int f(a \ln x + b) \frac{1}{x} dx = \int f(a \ln x + b) \cdot \frac{1}{a} d(a \ln x + b)$$

$$\frac{u=a\ln x+b}{\int} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(a\ln x + b) + C$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du = \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$

 $\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d \ln x$ 



类型 IV:  $\int f(a \ln x + b) \frac{1}{y} dx$ 凑微分 假设会算

$$\int f(u)du = F(u) + C$$

则
$$\int f(a|ax|b)^{1} dx = \int f(a|ax|b)^{1} d(a|ax|b)^{1}$$

$$\int f(a \ln x + b) \frac{1}{x} dx = \int f(a \ln x + b) \cdot \frac{1}{a} d(a \ln x + b)$$

 $\frac{u=a\ln x+b}{a} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(a\ln x + b) + C$ 

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du = \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$

$$\frac{u=a\ln x+b}{a}\int f(u)\cdot \frac{1}{a}du = \frac{1}{a}F(u)+C$$

 $\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d \ln x = \int \frac{1}{u} du$ 

类型 IV:  $\int f(a \ln x + b) \frac{1}{y} dx$ 凑微分

假设会算 
$$\int f(u)du = F(u) + C$$

$$\int f(a \ln x + b) \frac{1}{dx} = \int f(a \ln x + b) \cdot \frac{1}{d} (a \ln x + b) \cdot$$

$$\int f(a \ln x + b) \frac{1}{x} dx = \int f(a \ln x + b) \cdot \frac{1}{a} d(a \ln x + b)$$

$$\frac{u=a\ln x+b}{\int} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(a\ln x + b) + C$$

$$\frac{u=a \ln x+b}{x} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = 0$$

4b 换元积分法

 $\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d \ln x = \int \frac{1}{u} du = \ln |u| + C$ 

类型 IV:  $\int f(a \ln x + b) \frac{1}{y} dx$ 凑微分

$$\int f(u)du = F(u) + C$$

$$\int f(a \ln x + b) \frac{1}{x} dx = \int f(a \ln x + b) \cdot \frac{1}{a} d(a \ln x + b)$$

例	$\int \frac{1}{-} \ln x dx =$	$\int \ln x d \ln x =$	$\int udu = \frac{1}{2}u^2 + C =$	$=\frac{1}{2}(\ln x)^2 + C$

 $\frac{u=a\ln x+b}{a} \int f(u) \cdot \frac{1}{a} du = \frac{1}{a} F(u) + C = \frac{1}{a} F(a\ln x + b) + C$ 

4b 换元积分法

例 求  $\int e^{\cos x} \sin x dx$ ,  $\int \frac{\sin x}{1 + \cos^2 x} dx$ ,  $\int \frac{\cos x}{\sin x} dx$ 



例 求 
$$\int e^{\cos x} \sin x dx$$
,  $\int \frac{\sin x}{1 + \cos^2 x} dx$ ,  $\int \frac{\cos x}{\sin x} dx$ 

1. 
$$\int e^{\cos x} \sin x dx =$$



例 求 
$$\int e^{\cos x} \sin x dx$$
,  $\int \frac{\sin x}{1+\cos^2 x} dx$ ,  $\int \frac{\cos x}{\sin x} dx$ 

1. 
$$\int e^{\cos x} \sin x dx = d\cos x$$



例 求 
$$\int e^{\cos x} \sin x dx$$
,  $\int \frac{\sin x}{1+\cos^2 x} dx$ ,  $\int \frac{\cos x}{\sin x} dx$ 

1. 
$$\int e^{\cos x} \sin x dx = (-1)d \cos x$$



例 求 
$$\int e^{\cos x} \sin x dx$$
,  $\int \frac{\sin x}{1 + \cos^2 x} dx$ ,  $\int \frac{\cos x}{\sin x} dx$ 

1. 
$$\int e^{\cos x} \sin x dx = \int e^{\cos x} \cdot (-1) d \cos x$$



例 求 
$$\int e^{\cos x} \sin x dx$$
,  $\int \frac{\sin x}{1 + \cos^2 x} dx$ ,  $\int \frac{\cos x}{\sin x} dx$ 

1. 
$$\int e^{\cos x} \sin x dx = \int e^{\cos x} \cdot (-1) d \cos x = -\int e^{u} du$$



例 求 
$$\int e^{\cos x} \sin x dx$$
,  $\int \frac{\sin x}{1+\cos^2 x} dx$ ,  $\int \frac{\cos x}{\sin x} dx$ 

1. 
$$\int e^{\cos x} \sin x dx = \int e^{\cos x} \cdot (-1) d \cos x = -\int e^{u} du$$
$$= -e^{u} + C$$



例 求 
$$\int e^{\cos x} \sin x dx$$
,  $\int \frac{\sin x}{1+\cos^2 x} dx$ ,  $\int \frac{\cos x}{\sin x} dx$ 

1. 
$$\int e^{\cos x} \sin x dx = \int e^{\cos x} \cdot (-1) d \cos x = -\int e^{u} du$$
$$= -e^{u} + C = -e^{\cos x} + C$$



例 求 
$$\int e^{\cos x} \sin x dx$$
,  $\int \frac{\sin x}{1 + \cos^2 x} dx$ ,  $\int \frac{\cos x}{\sin x} dx$ 

1. 
$$\int e^{\cos x} \sin x dx = \int e^{\cos x} \cdot (-1) d \cos x = -\int e^{u} du$$
$$= -e^{u} + C = -e^{\cos x} + C$$

$$2. \int \frac{\sin x}{1 + \cos^2 x} dx =$$



例 求 
$$\int e^{\cos x} \sin x dx$$
,  $\int \frac{\sin x}{1 + \cos^2 x} dx$ ,  $\int \frac{\cos x}{\sin x} dx$ 

1. 
$$\int e^{\cos x} \sin x dx = \int e^{\cos x} \cdot (-1) d \cos x = -\int e^{u} du$$
$$= -e^{u} + C = -e^{\cos x} + C$$

$$2. \int \frac{\sin x}{1 + \cos^2 x} dx = (-1)d\cos x$$



例 求 
$$\int e^{\cos x} \sin x dx$$
,  $\int \frac{\sin x}{1 + \cos^2 x} dx$ ,  $\int \frac{\cos x}{\sin x} dx$ 

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$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d\cos x$$



例 求 
$$\int e^{\cos x} \sin x dx$$
,  $\int \frac{\sin x}{1 + \cos^2 x} dx$ ,  $\int \frac{\cos x}{\sin x} dx$ 

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$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d\cos x = -\int \frac{1}{1 + u^2} du$$



例 求 
$$\int e^{\cos x} \sin x dx$$
,  $\int \frac{\sin x}{1+\cos^2 x} dx$ ,  $\int \frac{\cos x}{\sin x} dx$ 

1. 
$$\int e^{\cos x} \sin x dx = \int e^{\cos x} \cdot (-1) d \cos x = -\int e^{u} du$$
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2. 
$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d\cos x = -\int \frac{1}{1 + u^2} du$$
$$= -\arctan u + C$$



例 求 
$$\int e^{\cos x} \sin x dx$$
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例 求 
$$\int e^{\cos x} \sin x dx$$
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$$\int \frac{\cos x}{\sin x} dx =$$



例 求 
$$\int e^{\cos x} \sin x dx$$
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$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d\cos x = -\int \frac{1}{1 + u^2} du$$
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3. 
$$\int \frac{\cos x}{\sin x} dx = d\sin x$$



例 求 
$$\int e^{\cos x} \sin x dx$$
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例 求 
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$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d\cos x = -\int \frac{1}{1 + u^2} du$$
$$= -\arctan u + C = -\arctan(\cos x) + C$$

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$$\int \frac{\cos x}{\sin x} dx = \int \frac{1}{\sin x} d\sin x = \int \frac{1}{u} du$$



例 求 
$$\int e^{\cos x} \sin x dx$$
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$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d\cos x = -\int \frac{1}{1 + u^2} du$$
$$= -\arctan u + C = -\arctan(\cos x) + C$$

3. 
$$\int \frac{\cos x}{\sin x} dx = \int \frac{1}{\sin x} d\sin x = \int \frac{1}{u} du = \ln|u| + C$$



例 求 
$$\int e^{\cos x} \sin x dx$$
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$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d\cos x = -\int \frac{1}{1 + u^2} du$$
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3. 
$$\int \frac{\cos x}{\sin x} dx = \int \frac{1}{\sin x} d\sin x = \int \frac{1}{u} du = \ln|u| + C$$
$$= \ln|\sin x| + C$$



### 凑微分法 " $\int f(\varphi(x))d\varphi(x)$ ":例子总结

$$\int \frac{1}{1 - 3x} dx =$$

$$\int \sqrt{3x - 1} dx =$$

$$\int xe^{x^2} dx =$$

$$\int x\sqrt{1 - x^2} dx =$$

$$\int \frac{\ln x}{x} dx =$$

$$\int e^{\cos x} \sin x dx =$$

## 凑微分法 " $\int f(\varphi(x))d\varphi(x)$ ": 例子总结

$$\int \frac{1}{1-3x} dx = -\frac{1}{3} \int \frac{1}{1-3x} d(1-3x) = -\frac{1}{3} \int \frac{1}{u} du = \cdots$$

$$\int \sqrt{3x-1} dx = \frac{1}{3} \int \sqrt{3x-1} d(3x-1) = \frac{1}{3} \int u^{1/2} du = \cdots$$

$$\int x e^{x^2} dx =$$

$$\int x \sqrt{1-x^2} dx =$$

 $\int e^{\cos x} \sin x dx =$ 

 $\int \frac{\ln x}{x} dx =$ 



# 凑微分法 " $\int f(\varphi(x))d\varphi(x)$ ":例子总结

$$\int \frac{1}{1-3x} dx = -\frac{1}{3} \int \frac{1}{1-3x} d(1-3x) = -\frac{1}{3} \int \frac{1}{u} du = \cdots$$

$$\int \sqrt{3x-1} dx = \frac{1}{3} \int \sqrt{3x-1} d(3x-1) = \frac{1}{3} \int u^{1/2} du = \cdots$$

$$\int xe^{x^2} dx = \frac{1}{2} \int e^{x^2} dx^2 = \frac{1}{2} \int e^u du = \cdots$$

$$\int x\sqrt{1-x^2} dx = -\frac{1}{2} \int \sqrt{1-x^2} d(1-x^2) = -\frac{1}{2} \int u^{1/2} du = \cdots$$

$$\int \frac{\ln x}{x} dx = \frac{1}{2} \int u^{1/2} du = \cdots$$

$$\int e^{\cos x} \sin x dx =$$



# 凑微分法 " $\int f(\varphi(x))d\varphi(x)$ ": 例子总结

$$\int \frac{1}{1-3x} dx = -\frac{1}{3} \int \frac{1}{1-3x} d(1-3x) = -\frac{1}{3} \int \frac{1}{u} du = \cdots$$

$$\int \sqrt{3x-1} dx = \frac{1}{3} \int \sqrt{3x-1} d(3x-1) = \frac{1}{3} \int u^{1/2} du = \cdots$$

$$\int xe^{x^2} dx = \frac{1}{2} \int e^{x^2} dx^2 = \frac{1}{2} \int e^u du = \cdots$$

$$\int x\sqrt{1-x^2} dx = -\frac{1}{2} \int \sqrt{1-x^2} d(1-x^2) = -\frac{1}{2} \int u^{1/2} du = \cdots$$

$$\int \frac{\ln x}{x} dx = \int \ln x d(\ln x) = \int u du = \cdots$$

 $\int e^{\cos x} \sin x dx = -\int e^{\cos x} d\cos x = -\int e^{u} du = \cdots$ 



#### We are here now...

1. 第一类换元积分法: 凑微分

2. 第二类换元积分法: 变量代换



#### 第二类换元积分法——"变量代换"法,能干啥?

能够计算如下的不定积分:

$$\int x\sqrt{3x-1}dx, \quad \int \frac{x}{\sqrt{x-2}}dx$$

$$\int \frac{1}{1+\sqrt{x}}dx, \quad \int \frac{1}{1+\sqrt[3]{x+1}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$
.....

$$\int f(x)dx$$

$$\int f(x)dx \stackrel{x=\varphi(t)}{=}$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t)$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int f(\varphi(t))\varphi'(t)dt$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\nabla m \otimes \mathbb{R}^{\frac{1}{2}}} dt$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单, 容易求!}} dt$$
$$= G(t) + C$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单, 容易求!}} dt$$
$$= G(t) + C \xrightarrow{t=\varphi^{-1}(x)}$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单, 容易求!}} dt$$
$$= G(t) + C \xrightarrow{t=\varphi^{-1}(x)} G(\varphi^{-1}(x)) + C$$

• 计算步骤:

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{\overline{E}\sigma\beta}} dt$$
$$= G(t) + C \xrightarrow{t=\varphi^{-1}(x)} G(\varphi^{-1}(x)) + C$$

• 关键是:如何选取函数 $x = \varphi(t)$ ?

• 计算步骤:

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单, } \text{容易求!}} dt$$
$$= G(t) + C \xrightarrow{t=\varphi^{-1}(x)} G(\varphi^{-1}(x)) + C$$

• 关键是:如何选取函数  $x = \varphi(t)$ ? 在后面的例子中,选取函数  $x = \varphi(t)$  的方法:

把被积函数 f(x) 中复杂的部分整个设为 t,从而得到 x 与 t 的函数关系.



**例1** 求不定积分  $\int x\sqrt{3x-1}dx$ ,  $\int \frac{x}{\sqrt{x-2}}dx$ 

**例1** 求不定积分  $\int x \sqrt{3x-1} dx$ ,  $\int \frac{x}{\sqrt{x-2}} dx$  解 (1) 设  $t = (3x-1)^{\frac{1}{2}}$ ,

**例1** 求不定积分  $\int x\sqrt{3x-1}dx$ ,  $\int \frac{x}{\sqrt{x-2}}dx$ 

$$\therefore \int x\sqrt{3x-1}dx =$$



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(1)  $ightarrow t = (3x - 1)^{\frac{1}{2}}, \quad \therefore x = \frac{1}{3}(t^2 + 1), \quad dx = \frac{2}{3}tdt$  $\therefore \int x\sqrt{3x - 1}dx = \int \frac{1}{3}(t^2 + 1)$ 

**例1** 求不定积分  $\int x\sqrt{3x-1}dx$ ,  $\int \frac{x}{\sqrt{x-2}}dx$ 

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t.$$

**例1** 求不定积分  $\int x\sqrt{3x-1}dx$ ,  $\int \frac{x}{\sqrt{x-2}}dx$ 

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$
$$= \frac{2}{45}t^5 + \frac{2}{27}t^3 + C$$

 $=\frac{2}{45}t^5+\frac{2}{27}t^3+C=\frac{2}{45}(3x-1)^{\frac{5}{2}}+\frac{2}{27}(3x-1)^{\frac{3}{2}}+C$ 

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(2) 设 
$$t = (x-2)^{\frac{1}{2}}$$
, ∴  $x = t^2 + 2$ ,  
∴  $\int \frac{x}{-x} dx = \frac{1}{2}$ 

$$\therefore \int \frac{x}{\sqrt{x-2}} dx =$$

**例 1** 求不定积分  $\int x\sqrt{3x-1}dx$ ,  $\int \frac{x}{\sqrt{x-2}}dx$ 

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$$\int \frac{x}{-x} dx = \int \frac{t^2 + 2}{-x^2} \cdot 2t dt$$

$$\therefore \int \frac{x}{\sqrt{x-2}} dx = \int \frac{t^2+2}{t} \cdot 2t dt$$



**例 1** 求不定积分 
$$\int x\sqrt{3x-1}dx$$
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$$\therefore \int \frac{x}{\sqrt{x-2}} dx = \int \frac{t^2+2}{t} \cdot 2t dt = 2 \int t^2 + 2 dt$$

(2) 设  $t = (x-2)^{\frac{1}{2}}$ , ∴  $x = t^2 + 2$ , dx = 2tdt

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 $=\frac{2}{45}t^5+\frac{2}{27}t^3+C=\frac{2}{45}(3x-1)^{\frac{5}{2}}+\frac{2}{27}(3x-1)^{\frac{3}{2}}+C$ (2) 设  $t = (x-2)^{\frac{1}{2}}$ , ∴  $x = t^2 + 2$ , dx = 2tdt

$$\therefore \int \frac{x}{\sqrt{x-2}} dx = \int \frac{t^2+2}{t} \cdot 2t dt = 2 \int t^2 + 2 dt = \frac{2}{3} t^3 + 4t + C$$



(2)  $\partial t = (x-2)^{\frac{1}{2}}$ , ∴  $x = t^2 + 2$ , dx = 2tdt

4b 换元积分法

 $\therefore \int \frac{x}{\sqrt{x-2}} dx = \int \frac{t^2+2}{t} \cdot 2t dt = 2 \int t^2 + 2 dt = \frac{2}{3} t^3 + 4t + C$  $= \frac{2}{3}(x-2)^{\frac{3}{2}} + 4(x-2)^{\frac{1}{2}} + C$ 

**例 1** 求不定积分  $\int x\sqrt{3x-1}dx$ ,  $\int \frac{x}{\sqrt{x-2}}dx$ 

 $\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$ 

 $=\frac{2}{45}t^5+\frac{2}{27}t^3+C=\frac{2}{45}(3x-1)^{\frac{5}{2}}+\frac{2}{27}(3x-1)^{\frac{3}{2}}+C$ 

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 $\therefore \int \frac{x}{\sqrt{x-2}} dx = \int \frac{t^2+2}{t} \cdot 2t dt = 2 \int t^2 + 2 dt = \frac{2}{3} t^3 + 4t + C$ 

 $= \frac{2}{3}(x-2)^{\frac{3}{2}} + 4(x-2)^{\frac{1}{2}} + C$ 

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**解** (1) 设 
$$t = 1 + x^{\frac{1}{2}}$$
,

$$\mathbf{H} \quad (1) \ \ 0 \ \ t = 1 + x^{\frac{1}{2}},$$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx =$$



解 (1) 设 
$$t = 1 + x^{\frac{1}{2}}$$
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解 (1) 设 
$$t = 1 + x^{\frac{1}{2}}$$
,  $\therefore x = (t-1)^2$ ,  $dx = 2(t-1)dt$ 

$$\therefore \int \frac{1}{1+\sqrt{x}} dx =$$



$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt$$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt = 2 \int 1 - \frac{1}{t} dt$$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2 \int 1 - \frac{1}{t} dt$$
$$= 2t - 2 \ln t + C$$

**解** (1) 设  $t = 1 + x^{\frac{1}{2}}$ .  $\therefore x = (t-1)^2$ . dx = 2(t-1)dt $\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt = 2 \int 1 - \frac{1}{t} dt$  $= 2t - 2\ln t + C = 2(1 + x^{\frac{1}{2}}) - 2\ln(1 + x^{\frac{1}{2}}) + C$ 

(2) 设  $t = 1 + (1 + x)^{\frac{1}{3}}$ .

**解** (1) 设  $t = 1 + x^{\frac{1}{2}}$ .  $\therefore x = (t-1)^2$ . dx = 2(t-1)dt

 $\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt = 2 \int 1 - \frac{1}{t} dt$ 

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**解** (1) 设  $t = 1 + x^{\frac{1}{2}}$ .  $\therefore x = (t-1)^2$ . dx = 2(t-1)dt $\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt = 2 \int 1 - \frac{1}{t} dt$  $= 2t - 2\ln t + C = 2(1 + x^{\frac{1}{2}}) - 2\ln(1 + x^{\frac{1}{2}}) + C$ 

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解 (1) 设 
$$t = 1 + x^{\frac{1}{2}}$$
,  $\therefore x = (t-1)^2$ ,  $dx = 2(t-1)dt$   

$$\therefore \int \frac{1}{1 + \sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2 \int 1 - \frac{1}{t} dt$$

$$= 2t - 2 \ln t + C = 2(1 + x^{\frac{1}{2}}) - 2 \ln(1 + x^{\frac{1}{2}}) + C$$

(2) 设 
$$t = 1 + (1 + x)^{\frac{1}{3}}$$
, ∴  $x = (t - 1)^3 - 1$ ,  
∴  $\int \frac{1}{1 + \sqrt[3]{1 + x}} dx =$ 



$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt = 2 \int 1 - \frac{1}{t} dt$$
$$= 2t - 2 \ln t + C = 2(1+x^{\frac{1}{2}}) - 2 \ln(1+x^{\frac{1}{2}})$$

$$= 2t - 2\ln t + C = 2(1 + x^{\frac{1}{2}}) - 2\ln(1 + x^{\frac{1}{2}}) + C$$

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}} dx =$$

m+	$(1) \ \forall \ \ell = 1 + \chi^2,$	$\therefore X = (t-1),$	dx = 2(t-1)dt
	$\therefore \int \frac{1}{1+\sqrt{x}} dx$	$c = \int \frac{1}{t} \cdot 2(t-1)$	$dt = 2\int 1 - \frac{1}{t}dt$

 $(1) \frac{1}{4} + 1 + \sqrt{\frac{1}{2}} + x - (t + 1)^2 + dx - 2(t + 1)dt$ 

$$J = 2t - 2 \ln t + C = 2(1 + x^{\frac{1}{2}}) - 2 \ln(1 + x^{\frac{1}{2}}) + C$$

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}} dx = \int \frac{1}{t} \cdot 3(t-1)^2 dt$$

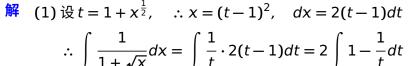
 $\therefore \int \frac{1}{1+\sqrt[3]{1+x}} dx = \int \frac{1}{t} \cdot 3(t-1)^2 dt = 3 \int t - 2 + \frac{1}{t} dt$ 

**解** (1) 设  $t = 1 + x^{\frac{1}{2}}$ ,  $\therefore x = (t-1)^2$ , dx = 2(t-1)dt

 $\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt = 2 \int 1 - \frac{1}{t} dt$ 

(2)  $\partial t = 1 + (1+x)^{\frac{1}{3}}$ ,  $\therefore x = (t-1)^3 - 1$ ,  $dx = 3(t-1)^2 dt$ 

 $= 2t - 2\ln t + C = 2(1 + x^{\frac{1}{2}}) - 2\ln(1 + x^{\frac{1}{2}}) + C$ 



$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2 \int 1 - \frac{1}{t} dt$$
$$= 2t - 2\ln t + C = 2(1+x^{\frac{1}{2}}) - 2\ln(1+x^{\frac{1}{2}}) + C$$

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}} dx = \int \frac{1}{t} \cdot 3(t-1)^2 dt = 3 \int t - 2 + \frac{1}{t} dt$$
$$= \frac{3}{2} t^2 - 6t + 3 \ln|t| + C$$









 $\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt = 2 \int 1 - \frac{1}{t} dt$  $= 2t - 2\ln t + C = 2(1 + x^{\frac{1}{2}}) - 2\ln(1 + x^{\frac{1}{2}}) + C$ 

**例2** 求不定积分  $\int \frac{1}{1+\sqrt{x}} dx$ ,  $\int \frac{1}{1+\sqrt{3}\sqrt{1+x}} dx$ 

(2)  $\[ \] t = 1 + (1+x)^{\frac{1}{3}}, \quad \therefore x = (t-1)^3 - 1, \quad dx = 3(t-1)^2 dt \]$  $\therefore \int \frac{1}{1+\sqrt[3]{1+x}} dx = \int \frac{1}{t} \cdot 3(t-1)^2 dt = 3 \int t - 2 + \frac{1}{t} dt$ 

 $= \frac{3}{2}(1 + (1+x)^{\frac{1}{3}})^2 - 6(1 + (1+x)^{\frac{1}{3}}) + 3\ln|1 + (1+x)^{\frac{1}{3}}| + C$ 



$$\therefore \int \frac{1+\sqrt[3]{1+x}}{1+\sqrt[3]{1+x}} dx = \int \frac{1}{t} \cdot 3(t-1)^2 dt = 3 \int t-2$$

$$= \frac{3}{2}t^2 - 6t + 3\ln|t| + C$$

$$= \frac{3}{2}(1+(1+x)^{\frac{1}{3}})^2 - 6(1+(1+x)^{\frac{1}{3}}) + 3\ln|1+(1+x)^{\frac{1}{3}}| + C$$

 $= \frac{3}{2}t^2 - 6t + 3\ln|t| + C$ 

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 $\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt = 2 \int 1 - \frac{1}{t} dt$ 

**解** (1) 设  $t = 1 + x^{\frac{1}{2}}$ ,  $\therefore x = (t-1)^2$ , dx = 2(t-1)dt



**解** 设 
$$t = \sqrt{1 + e^x}$$
,

解 设 
$$t = \sqrt{1 + e^x}$$
,  $\therefore x = \ln(t^2 - 1)$ ,



解 设 
$$t = \sqrt{1 + e^x}$$
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解 设
$$t = \sqrt{1 + e^x}$$
,  $\therefore x = \ln(t^2 - 1)$ ,  $dx = \frac{2t}{t^2 - 1}dt$ 

$$\therefore \int \frac{1}{\sqrt{1+e^x}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1} dt$$



例 3 求不定积分  $\int \frac{1}{\sqrt{1+\alpha^{X}}} dx$ 

解 设
$$t = \sqrt{1 + e^x}$$
,  $\therefore x = \ln(t^2 - 1)$ ,  $dx = \frac{2t}{t^2 - 1}dt$ 

$$\therefore \int \frac{1}{\sqrt{1+e^{x}}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^{2}-1} dt = \int \frac{1}{t-1} - \frac{1}{t+1} dt$$



例 3 求不定积分  $\int \frac{1}{\sqrt{1+\alpha^{X}}} dx$ 

解 设 
$$t = \sqrt{1 + e^x}$$
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$$= \ln|t-1| - \ln|t+1| + C$$



例 3 求不定积分  $\int \frac{1}{\sqrt{1+\alpha^{X}}} dx$ 

解 设
$$t = \sqrt{1 + e^x}$$
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$$\therefore \int \frac{1}{\sqrt{1+e^{x}}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^{2}-1} dt = \int \frac{1}{t-1} - \frac{1}{t+1} dt$$
$$= \ln|t-1| - \ln|t+1| + C = \ln|\frac{t-1}{t+1}| + C$$



例 3 求不定积分  $\int \frac{1}{\sqrt{1+\alpha^x}} dx$ 

解 设
$$t = \sqrt{1 + e^x}$$
,  $\therefore x = \ln(t^2 - 1)$ ,  $dx = \frac{2t}{t^2 - 1}dt$ 

$$\therefore \int \frac{1}{\sqrt{1+e^{x}}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^{2} - 1} dt = \int \frac{1}{t - 1} - \frac{1}{t + 1} dt$$

$$= \ln|t - 1| - \ln|t + 1| + C = \ln|\frac{t - 1}{t + 1}| + C$$

$$= \ln\left(\frac{\sqrt{1+e^{x}} - 1}{\sqrt{1+e^{x}} + 1}\right) + C$$



**例 3** 求不定积分  $\int \frac{1}{\sqrt{1+\alpha^2}} dx$ 

解 设 
$$t = \sqrt{1 + e^x}$$
,  $\therefore x = \ln(t^2 - 1)$ ,  $dx = \frac{2t}{t^2 - 1}dt$ 

$$\therefore \int \frac{1}{\sqrt{1+e^x}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1} dt = \int \frac{1}{t - 1} - \frac{1}{t + 1} dt$$

$$= \ln|t - 1| - \ln|t + 1| + C = \ln|\frac{t - 1}{t + 1}| + C$$

$$= \ln\left(\frac{\sqrt{1+e^x} - 1}{\sqrt{1+e^x} + 1}\right) + C$$

$$= 2\ln(\sqrt{1+e^x} - 1) - x + C$$

$$= 2 \ln(\sqrt{1 + e^x - 1}) - x + C$$



$$\int x\sqrt{3x-1}dx$$

$$\int \frac{1}{1+\sqrt{x}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$



$$\int x\sqrt{3x-1}dx \xrightarrow{t=\sqrt{3x-1}} \cdots$$

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 :  $-1 \le x \le 1$ , 设  $x = \sin t$ ,  $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ ,



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$$= \frac{1}{2} \sin t \cos t + \frac{1}{2} t + C$$

$$= \frac{1}{2} x \sqrt{1 - x^2} + \frac{1}{2} \arcsin x + C$$

