§8.5 多元复合函数与隐函数的求导法则

2017-2018 学年 II

Outline

1. 复合函数的求导法则

2. 隐函数的求导法则

We are here now...

1. 复合函数的求导法则

2. 隐函数的求导法则

二元复合函数求导

设有二元函数 $z = f(u, v)$

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问 $\frac{dz}{dt}$ =?

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二元复合函数求导公式——中间变量是一元函数

公式 设 $z = f(u, v)$，$u = \varphi(t)$，$v = \psi(t)$，则 $z = f(\varphi(t), \psi(t))$ 的全导数

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$$\begin{tikzcd} & u \arrow[r] & t \\ z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[dr, "\frac{\partial z}{\partial v}"'] & & \\ & v \arrow[r] & t \end{tikzcd}$$

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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}$$

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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}\quad\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$

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 & u \arrow[r, "\frac{du}{dt}"] & t \\
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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$

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$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt} + \frac{\partial z}{\partial v} \cdot \frac{dv}{dt}$$
$$= (uv)'_u \cdot$$

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$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial u} \cdot \frac{du}{dt} + \frac{\partial z}{\partial v} \cdot \frac{dv}{dt} \\ &= (uv)'_u \cdot (e^{-t})'_t + (uv)'_v \cdot\end{aligned}$$

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$$\because \quad z=uv=$$

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$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$

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$$\because\quad z=uv=e^{-t}\cdot\sin t$$

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例 设 $z = \frac{y}{x}$，而 $x = e^t$，$y = 1 - e^{2t}$，求全导数 $\frac{dz}{dt}$

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$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=\left(\frac{y}{x}\right)'_x\cdot$$

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$$=$$

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$$=-\frac{y}{x^2}\cdot$$

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$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} = (\frac{y}{x})'_x \cdot (e^t)'_t + (\frac{y}{x})'_y \cdot (1 - e^{2t})'_t$$

$$= -\frac{y}{x^2} \cdot e^t +$$

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$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=$$

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$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=-\frac{1-e^{2t}}{e^{2t}}\cdot e^t+$$

$$\begin{tikzcd}
 & x \arrow[r, "\frac{dx}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[dr, "\frac{\partial z}{\partial y}"'] & & \\
 & y \arrow[r, "\frac{dy}{dt}"'] & t
\end{tikzcd}$$

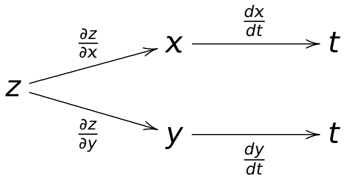
例 设 $z = \frac{y}{x}$，而 $x = e^t$，$y = 1 - e^{2t}$，求全导数 $\frac{dz}{dt}$

解

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} = (\frac{y}{x})'_x \cdot (e^t)'_t + (\frac{y}{x})'_y \cdot (1 - e^{2t})'_t$$

$$= -\frac{y}{x^2} \cdot e^t + \frac{1}{x} \cdot (-2e^{2t}) = -\frac{1 - e^{2t}}{e^{2t}} \cdot e^t + \frac{1}{e^t} \cdot (-2e^{2t})$$

$$=$$



例 设 $z=\frac{y}{x}$，而 $x=e^t$，$y=1-e^{2t}$，求全导数 $\frac{dz}{dt}$

解

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}=(\frac{y}{x})'_x\cdot(e^t)'_t+(\frac{y}{x})'_y\cdot(1-e^{2t})'_t$$

$$=-\frac{y}{x^2}\cdot e^t+\frac{1}{x}\cdot(-2e^{2t})=-\frac{1-e^{2t}}{e^{2t}}\cdot e^t+\frac{1}{e^t}\cdot(-2e^{2t})$$

$$=-e^{-t}-e^t$$

$$\begin{tikzcd}
 & x \arrow[r, "\frac{dx}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial x}"] \arrow[dr, "\frac{\partial z}{\partial y}"'] & & \\
 & y \arrow[r, "\frac{dy}{dt}"'] & t
\end{tikzcd}$$

三元复合函数求导公式——中间变量是一元函数

公式 设 $z=f(u,v,w)$，$u=\varphi(t)$，$v=\psi(t)$，$w=\omega(t)$，则 $z=f(\varphi(t),\psi(t),\omega(t))$ 的全导数

$$\frac{dz}{dt}=$$

三元复合函数求导公式——中间变量是一元函数

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$$\frac{dz}{dt} =$$

$$\begin{tikzcd}
 & u \arrow[r] & t \\
z \arrow[ur] \arrow[r] \arrow[dr] & v \arrow[r] & t \\
 & w \arrow[r] & t
\end{tikzcd}$$

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$$\frac{dz}{dt}=$$

$$\begin{tikzcd}
 & u \arrow[r] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"'] & v \arrow[r] & t \\
 & w \arrow[r] & t
\end{tikzcd}$$

三元复合函数求导公式——中间变量是一元函数

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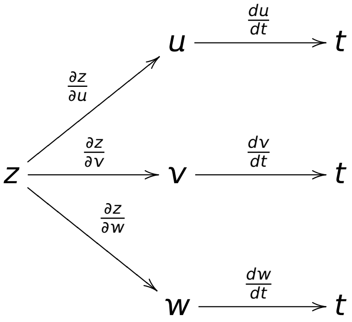
$$\frac{dz}{dt} =$$



三元复合函数求导公式——中间变量是一元函数

公式 设 $z=f(u,v,w)$，$u=\varphi(t)$，$v=\psi(t)$，$w=\omega(t)$，则 $z=f(\varphi(t),\psi(t),\omega(t))$ 的全导数

$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}$$

$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"] & v \arrow[r, "\frac{dv}{dt}"] & t \\
 & w \arrow[r, "\frac{dw}{dt}"] & t
\end{tikzcd}$$

三元复合函数求导公式——中间变量是一元函数

公式 设 $z=f(u,v,w)$，$u=\varphi(t)$，$v=\psi(t)$，$w=\omega(t)$，则 $z=f(\varphi(t),\psi(t),\omega(t))$ 的全导数

$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}\quad\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}$$

$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"] & v \arrow[r, "\frac{dv}{dt}"] & t \\
 & w \arrow[r, "\frac{dw}{dt}"] & t
\end{tikzcd}$$

三元复合函数求导公式——中间变量是一元函数

公式 设 $z=f(u,v,w)$, $u=\varphi(t)$, $v=\psi(t)$, $w=\omega(t)$, 则 $z=f(\varphi(t),\psi(t),\omega(t))$ 的全导数

$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\cdot\frac{du}{dt}\quad\frac{\partial z}{\partial v}\cdot\frac{dv}{dt}\quad\frac{\partial z}{\partial w}\cdot\frac{dw}{dt}$$

$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"] & v \arrow[r, "\frac{dv}{dt}"] & t \\
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\end{tikzcd}$$

三元复合函数求导公式——中间变量是一元函数

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$$\frac{dz}{dt} = \frac{\partial z}{\partial u} \cdot \frac{du}{dt} + \frac{\partial z}{\partial v} \cdot \frac{dv}{dt} + \frac{\partial z}{\partial w} \cdot \frac{dw}{dt}$$

$$\begin{tikzcd}
 & u \arrow[r, "\frac{du}{dt}"] & t \\
z \arrow[ur, "\frac{\partial z}{\partial u}"] \arrow[r, "\frac{\partial z}{\partial v}"] \arrow[dr, "\frac{\partial z}{\partial w}"] & v \arrow[r, "\frac{dv}{dt}"] & t \\
 & w \arrow[r, "\frac{dw}{dt}"] & t
\end{tikzcd}$$

二元复合函数求导公式——中间变量是多元函数

公式 设 $z = f(u, v)$，$u = \varphi(x, y)$，$v = \psi(x, y)$，

二元复合函数求导公式——中间变量是多元函数

公式 设 $z = f(u, v)$，$u = \varphi(x, y)$，$v = \psi(x, y)$，则复合函数

$$z = f(\varphi(x, y), \psi(x, y))$$

的偏导数是：

$$\frac{\partial z}{\partial x} = \qquad\qquad\qquad , \quad \frac{\partial z}{\partial y} =$$

二元复合函数求导公式——中间变量是多元函数

公式 设 $z=f(u,v)$，$u=\varphi(x,y)$，$v=\psi(x,y)$，则复合函数

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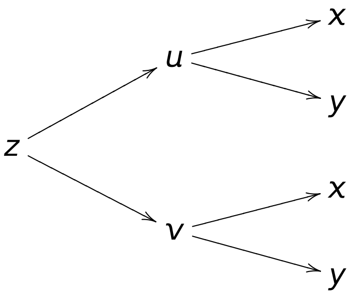
的偏导数是：

$$\frac{\partial z}{\partial x}= \qquad\qquad , \quad \frac{\partial z}{\partial y}=$$

图示



二元复合函数求导公式——中间变量是多元函数

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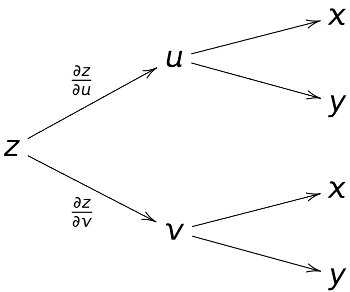
图示



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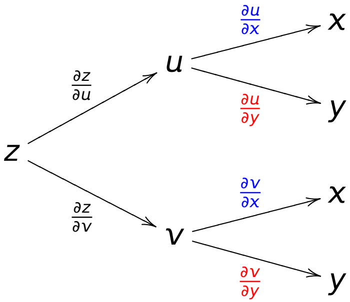
图示



二元复合函数求导公式——中间变量是多元函数

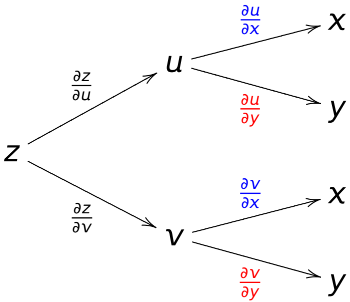
公式 设 $z=f(u,\ v)$，$u=\varphi(x,\ y)$，$v=\psi(x,\ y)$，则复合函数

$$z=f(\varphi(x,\ y),\ \psi(x,\ y))$$

的偏导数是：

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\qquad\qquad,\qquad \frac{\partial z}{\partial y}=$$

图示



二元复合函数求导公式——中间变量是多元函数

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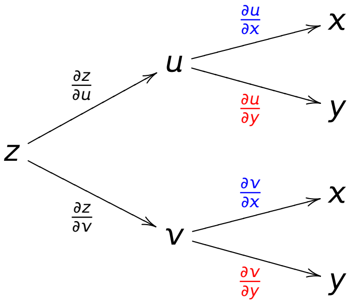
$$z=f(\varphi(x,y),\psi(x,y))$$

的偏导数是：

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x},\qquad \frac{\partial z}{\partial y}=$$

图示



二元复合函数求导公式——中间变量是多元函数

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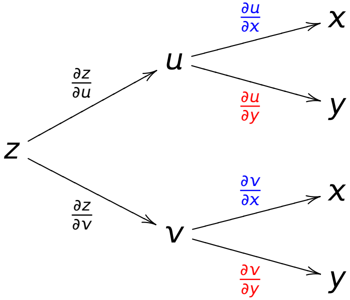
的偏导数是：

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x},\qquad \frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+$$

图示



二元复合函数求导公式——中间变量是多元函数

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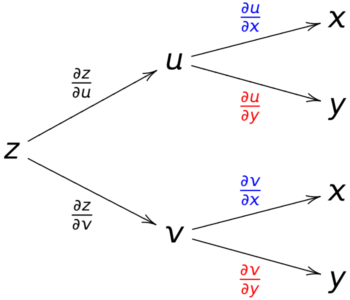
的偏导数是：

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x},\quad \frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$

图示



例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$= (e^{2u}\sin v)'_u \cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$= (e^{2u}\sin v)'_u\cdot(x^3 y)'_x + (e^{2u}\sin v)'_v\cdot$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot (x^2 + y^2)'_x$$
$$=$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$

$$=2e^{2u}\sin v\cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$=2e^{2u}\sin v\cdot 3x^2y+$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$

$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\ &= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\ &=\end{aligned}$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x$$
$$=2e^{2x^3y}\sin(x^2+y^2)\cdot$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\ &= (e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\ &= 2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\ &= 2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+\end{aligned}$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\begin{aligned}\frac{\partial z}{\partial x}&=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\&=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\&=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\&=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot\end{aligned}$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解

$$\begin{aligned}\frac{\partial z}{\partial x}&=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\&=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\&=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\&=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x\end{aligned}$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x$$
$$=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$
$$=$$

例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x$$
$$=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x$$
$$=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$
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解
$$\begin{aligned}\frac{\partial z}{\partial x}&=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}\\&=(e^{2u}\sin v)'_u\cdot(x^3y)'_x+(e^{2u}\sin v)'_v\cdot(x^2+y^2)'_x\\&=2e^{2u}\sin v\cdot 3x^2y+e^{2u}\cos v\cdot 2x\\&=2e^{2x^3y}\sin(x^2+y^2)\cdot 3x^2y+e^{2x^3y}\cos(x^2+y^2)\cdot 2x\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial y}&=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}\\&=(e^{2u}\sin v)'_u\cdot(x^3y)'_y+\end{aligned}$$

例 设 $z = e^{2u}\sin v$，$u = x^3 y$，$v = x^2 + y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

解
$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$
$$= (e^{2u}\sin v)'_u \cdot (x^3 y)'_x + (e^{2u}\sin v)'_v \cdot (x^2 + y^2)'_x$$
$$= 2e^{2u}\sin v \cdot 3x^2 y + e^{2u}\cos v \cdot 2x$$
$$= 2e^{2x^3 y}\sin(x^2 + y^2)\cdot 3x^2 y + e^{2x^3 y}\cos(x^2 + y^2)\cdot 2x$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$
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例 设 $z=e^{2u}\sin v$，$u=x^3y$，$v=x^2+y^2$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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三元复合函数求导公式：举例

公式 设 $z = f(x, y, u)$，$u = u(x, y)$，

三元复合函数求导公式：举例

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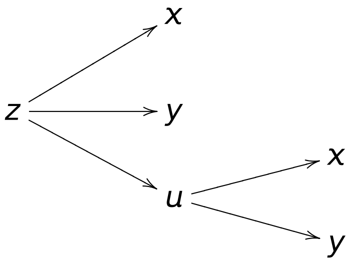
图示



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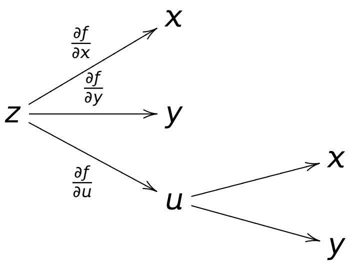

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$$\begin{tikzcd}
 & x & \\
z \arrow[ur, "\frac{\partial f}{\partial x}"] \arrow[r, "\frac{\partial f}{\partial y}"] \arrow[dr, "\frac{\partial f}{\partial u}"'] & y & \\
 & u \arrow[r, "\frac{\partial u}{\partial x}"] \arrow[dr, "\frac{\partial u}{\partial y}"'] & x \\
 & & y
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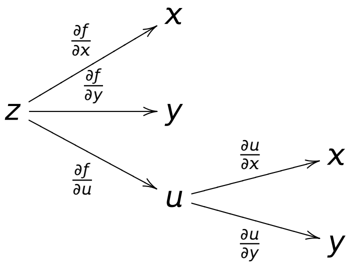
图示



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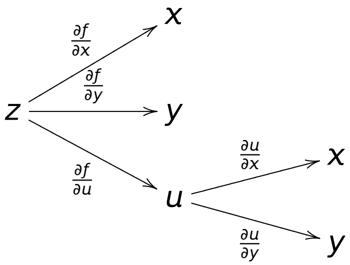
图示



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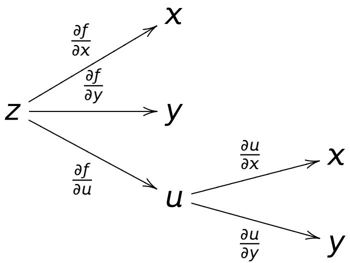

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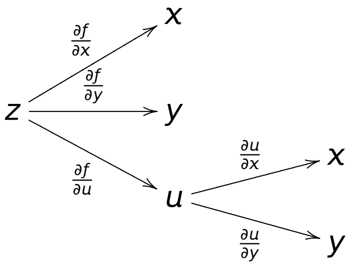

We are here now...

1. 复合函数的求导法则

2. 隐函数的求导法则

隐函数的求导法 I

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隐函数的求导法 I

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$$\begin{tikzcd}[row sep=small] & x & \\ F \arrow[ru] \arrow[rd] & & \\ & y \arrow[r] & x \end{tikzcd}$$

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证明

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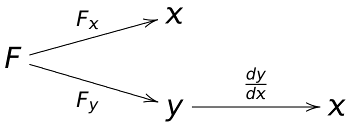

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$$\frac{dy}{dx}=-\frac{F_x}{F_y}=-\frac{(\sin y+e^x-xy^2)'_x}{(\sin y+e^x-xy^2)'_y}=-\frac{e^x-y^2}{\cos y-2xy}$$

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方法二

例 设 $y = f(x)$ 满足 $\sin y + e^x = xy^2$，求 $\frac{dy}{dx}$

方法一 注意 $\sin y + e^x - xy^2 = 0$，令 $F(x, y) = \sin y + e^x - xy^2$，则 $F(x, y) = 0$，所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)'_x}{(\sin y + e^x - xy^2)'_y} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 $\sin y(x) + e^x - xy(x)^2 = 0$,

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$$\begin{aligned}0&=\left(\sin y(x)+e^x-xy(x)^2\right)'_x\\&=(\sin y(x))'_x+(e^x)'_x-(xy(x)^2)'_x\end{aligned}$$

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所以 $y'=-\frac{e^x-y^2}{\cos y-2xy}$

例 设 $y = f(x)$ 满足 $\ln(x^2 + y^2) + 3xy = 4$，求 $\frac{dy}{dx}$

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例 设 $y = f(x)$ 满足 $\ln(x^2 + y^2) + 3xy = 4$，求 $\frac{dy}{dx}$

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$$F(x, y) = \ln(x^2 + y^2) + 3xy - 4$$

则 $F(x, y) = 0$，所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\ln(x^2 + y^2) + 3xy - 4)'_x}{(\ln(x^2 + y^2) + 3xy - 4)'_y}$$

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$$=-\frac{\frac{2x}{x^2+y^2}+3y}{\frac{2y}{x^2+y^2}+3x}$$

$$=-\frac{2x+3x^2y+3y^3}{2y+3xy^2+3x^3}$$

隐函数的求导法 II

公式 设 $z = z(x, y)$ 满足 $F(x, y, z) = 0$,

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$$\begin{tikzcd}
 & x & \\
F \arrow[ur] \arrow[r] \arrow[dr] & y & x \\
 & z \arrow[ur] \arrow[r] & y
\end{tikzcd}$$

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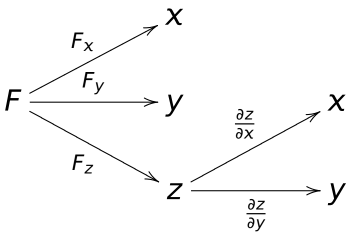

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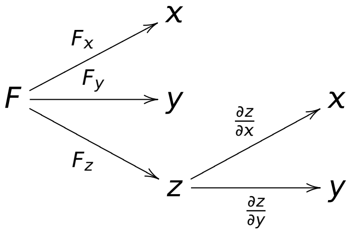

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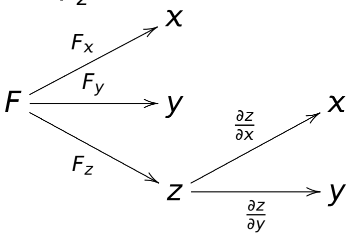

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$$\because\quad F(x,y,z(x,y))=0$$

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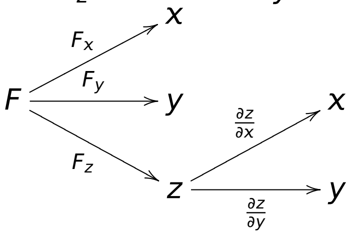
$$\therefore\quad \frac{\partial z}{\partial x}=-\frac{F_x}{F_z},\qquad \text{同理}\quad \frac{\partial z}{\partial y}=-\frac{F_y}{F_z}$$



例 设 $z=f(x, y)$ 满足 $x+y+xz=e^{z}-1$，求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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解 令 $F(x,y,z)=x+y+xz-e^z+1$， $F(x,y,z)=0$

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解 令 $F(x, y, z) = x + y + xz - e^z + 1$，则 $F(x, y, z) = 0$，所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + y + xz - e^z + 1)'_x}{(x + y + xz - e^z + 1)'_z}$$

$$= -\frac{\qquad\qquad}{0 + 0 + x}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + y + xz - e^z + 1)'_y}{(x + y + xz - e^z + 1)'_z}$$

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$$=-\frac{\qquad}{0+0+x-e^z}$$

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$$= -\frac{\qquad}{0 + 0 + x - e^z + 0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + y + xz - e^z + 1)'_y}{(x + y + xz - e^z + 1)'_z}$$

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$$= -\frac{\qquad}{1 + xe^{z-y-x}}$$

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解 令 $F(x,y,z)=z-y-x+xe^{z-y-x}$，则 $F(x,y,z)=0$，所以

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