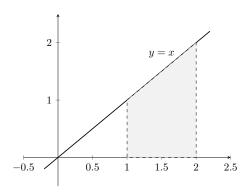
## 第 05 周作业解答

**练习 1.** 用定积分的几何意义计算  $\int_1^2 x dx$  步骤: 1. 确定曲边梯形; 2. 计算曲边梯形的面积

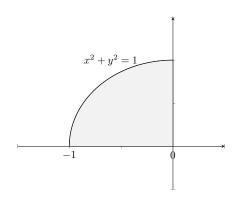
解:



利用梯形面积公式: 梯形面积  $=\frac{1}{2}(1+2)\cdot 1=\frac{3}{2}$ 。所以  $\int_1^2 x dx=\frac{3}{2}$ 。(或者将梯形面积视为两个三角形面积之差)

练习 2. 用定积分表示右图阴影部分面积。

并通过计算面积, 求出该定积分的值。



解: 曲线的方程是

$$y = \sqrt{1 - x^2}.$$

所以阴影部分的面积用定积分表示是

$$\int_{-1}^{0} \sqrt{1 - x^2} dx.$$

注意到该阴影区域的正好是四分之一个半径为 1 的圆盘,所以面积是  $\frac{1}{4}\pi$ ,进而可知

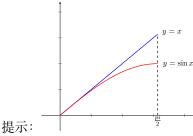
$$\int_{-1}^{0} \sqrt{1 - x^2} dx = \frac{1}{4}\pi.$$

练习 3. 设  $\int_0^1 2f(x)dx = 6$ ,  $\int_1^3 f(x)dx = 8$ ,  $\int_0^3 g(x)dx = 2$ , 求:  $\int_0^3 [5f(x) - 4g(x)]dx$ 

解:

$$\int_{0}^{3} [5f(x) - 4g(x)]dx = 5 \int_{0}^{3} f(x)dx - 4 \int_{0}^{3} g(x)dx$$
$$= 5 \left( \int_{0}^{1} f(x)dx + \int_{1}^{3} f(x)dx \right) - 4 \cdot 2$$
$$= 5(3+8) - 8$$
$$= 47$$

**练习 4.** 根据定积分的性质,比较三组定积分的大小: (1)  $\int_0^1 x^2 dx$  与  $\int_0^1 x^3 dx$ ; (2)  $\int_1^2 e^x dx$  与  $\int_1^2 e^{x^2} dx$ ; (3)  $\int_0^{\frac{\pi}{2}} x dx$  与  $\int_0^{\frac{\pi}{2}} \sin x dx$ .



解:  $(1)\int_0^1 x^2 dx > \int_0^1 x^3 dx$   $(2)\int_1^2 e^x dx < \int_1^2 e^{x^2} dx$   $(3)\int_0^{\frac{\pi}{2}} x dx > \int_0^{\frac{\pi}{2}} \sin x dx$ 

$$(2)\int_{1_{-}}^{2} e^{x} dx < \int_{1_{-}}^{2} e^{x^{2}} dx$$

$$(3)\int_0^{\frac{\pi}{2}} x dx > \int_0^{\frac{\pi}{2}} \sin x dx$$

**练习 5.** 求导数: (1)  $\frac{d}{dx} \int_{x}^{1} \sqrt{1+t^4} dt$ ; (2)  $\frac{d}{dx} \int_{x/x}^{x} e^{-t^2} dt$ 

解: (1)

$$\frac{d}{dx} \int_{x}^{1} \sqrt{1 + t^{4}} dt = -\frac{d}{dx} \int_{1}^{x} \sqrt{1 + t^{4}} dt = -\sqrt{1 + x^{4}}$$

(2)

$$\frac{d}{dx} \int_{\sqrt{x}}^{x} e^{-t^{2}} dt = \frac{d}{dx} \left( \int_{\sqrt{x}}^{0} e^{-t^{2}} dt + \int_{0}^{x} e^{-t^{2}} dt \right) = -\frac{d}{dx} \int_{0}^{\sqrt{x}} e^{-t^{2}} dt + \frac{d}{dx} \int_{0}^{x} e^{-t^{2}} dt$$
$$= -e^{-(\sqrt{x})^{2}} \cdot (\sqrt{x})' + e^{-x^{2}} = -\frac{1}{2\sqrt{x}} e^{-x} + e^{-x^{2}}$$

**练习 6.** 用牛顿—莱布尼茨公式求下列定积分 (1) 
$$\int_0^1 (x^2 + e^x + 100^x) dx$$
; (2)  $\int_9^{16} \frac{\sqrt{x} + 2}{x} dx$ ; (3)  $\int_{\frac{\pi}{2}}^{\pi} (\cos x + \sin x) dx$ 

$$\int_0^1 (x^2 + e^x + 100^x) dx = \left(\frac{1}{3}x^3 + e^x + \frac{100^x}{\ln 100}\right) \Big|_0^1$$
$$= \left(\frac{1}{3} + e + \frac{100}{\ln 100}\right) - \left(0 + 1 + \frac{1}{\ln 100}\right) = -\frac{2}{3} + e + \frac{99}{2\ln 10}$$

(2) 
$$\int_{9}^{16} \frac{\sqrt{x} + 2}{x} dx = \int_{9}^{16} \frac{\sqrt{x}}{x} + \frac{2}{x} dx = \int_{9}^{16} \frac{1}{\sqrt{x}} + \frac{2}{x} dx = \left(2\sqrt{x} + 2\ln|x|\right)\Big|_{9}^{16}$$
$$= \left(2 \cdot \sqrt{16} + 2\ln 16\right) - \left(2 \cdot \sqrt{9} + 2\ln 9\right) = 2 + 4\ln\frac{4}{3}$$

(3) 
$$\int_{\frac{\pi}{2}}^{\pi} (\cos x + \sin x) dx = (\sin x - \cos x) \Big|_{\frac{\pi}{2}}^{\pi} = (\sin \pi - \cos \pi) - (\sin \frac{\pi}{2} - \cos \frac{\pi}{2}) = (0+1) - (1-0) = 0$$