§2.5 分块矩阵

数学系 梁卓滨

2018 - 2019 学年上学期

分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right)$$

分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right) = \begin{pmatrix} & & \\ & & \end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A=\begin{pmatrix}1&0&0&3\\0&1&0&-1\\0&0&1&0\\0&0&0&1\end{pmatrix}=\left(\begin{array}{ccc:c}1&0&0&3\\0&1&0&-1\\0&0&1&0\\\hdashline 0&0&0&1\end{array}\right)=\begin{pmatrix}I_3&\\&\end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right) = \begin{pmatrix} I_3 & \\ O & \end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A=\begin{pmatrix}1&0&0&3\\0&1&0&-1\\0&0&1&0\\0&0&0&1\end{pmatrix}=\left(\begin{array}{ccc:c}1&0&0&3\\0&1&0&-1\\0&0&1&0\\\hdashline 0&0&0&1\end{array}\right)=\begin{pmatrix}I_3&\\O&I_1\end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A=\begin{pmatrix}1&0&0&3\\0&1&0&-1\\0&0&1&0\\0&0&0&1\end{pmatrix}=\left(\begin{array}{ccc:c}1&0&0&3\\0&1&0&-1\\0&0&1&0\\\hdashline 0&0&0&1\end{array}\right)=\begin{pmatrix}I_3&A_1\\O&I_1\end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right) = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix}$$

$$\stackrel{\text{or}}{=} \left(\begin{array}{cc:cc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right) = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix}$$

$$\overset{\text{or}}{=} \left(\begin{array}{cc:cc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) = \begin{pmatrix} & \\ & \end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A=\begin{pmatrix}1&0&0&3\\0&1&0&-1\\0&0&1&0\\0&0&0&1\end{pmatrix}=\left(\begin{array}{ccc:c}1&0&0&3\\0&1&0&-1\\0&0&1&0\\\hdashline 0&0&0&1\end{array}\right)=\begin{pmatrix}I_3&A_1\\O&I_1\end{pmatrix}$$

$$\overset{\text{or}}{=}\left(\begin{array}{cc:cc}1&0&0&3\\0&1&0&-1\\\hdashline 0&0&1&0\\0&0&0&1\end{array}\right)=\begin{pmatrix}I_2&\\&\end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A=\begin{pmatrix}1&0&0&3\\0&1&0&-1\\0&0&1&0\\0&0&0&1\end{pmatrix}=\left(\begin{array}{ccc:c}1&0&0&3\\0&1&0&-1\\0&0&1&0\\\hdashline 0&0&0&1\end{array}\right)=\begin{pmatrix}I_3&A_1\\O&I_1\end{pmatrix}$$

$$\overset{\text{or}}{=}\left(\begin{array}{cc:cc}1&0&0&3\\0&1&0&-1\\\hdashline 0&0&1&0\\0&0&0&1\end{array}\right)=\begin{pmatrix}I_2&\\&I_2\end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A=\begin{pmatrix}1&0&0&3\\0&1&0&-1\\0&0&1&0\\0&0&0&1\end{pmatrix}=\left(\begin{array}{ccc:c}1&0&0&3\\0&1&0&-1\\0&0&1&0\\ \hdashline 0&0&0&1\end{array}\right)=\begin{pmatrix}I_3&A_1\\O&I_1\end{pmatrix}$$

$$\overset{\text{or}}{=}\left(\begin{array}{cc:cc}1&0&0&3\\0&1&0&-1\\ \hdashline 0&0&1&0\\0&0&0&1\end{array}\right)=\begin{pmatrix}I_2&\\O&I_2\end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A=\begin{pmatrix}1&0&0&3\\0&1&0&-1\\0&0&1&0\\0&0&0&1\end{pmatrix}=\left(\begin{array}{ccc:c}1&0&0&3\\0&1&0&-1\\0&0&1&0\\\hdashline 0&0&0&1\end{array}\right)=\begin{pmatrix}I_3&A_1\\O&I_1\end{pmatrix}$$

$$\overset{\text{or}}{=}\left(\begin{array}{cc:cc}1&0&0&3\\0&1&0&-1\\\hdashline 0&0&1&0\\0&0&0&1\end{array}\right)=\begin{pmatrix}I_2&A_2\\O&I_2\end{pmatrix}$$

分块矩阵引入

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$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right) = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix}$$

$$\overset{\text{or}}{=} \left(\begin{array}{cc:cc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) = \begin{pmatrix} I_2 & A_2 \\ O & I_2 \end{pmatrix}$$

$$\overset{\text{or}}{=} \left(\begin{array}{c:c:c:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

分块矩阵引入

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$$A=\begin{pmatrix}1&0&0&3\\0&1&0&-1\\0&0&1&0\\0&0&0&1\end{pmatrix}=\left(\begin{array}{ccc:c}1&0&0&3\\0&1&0&-1\\0&0&1&0\\ \hdashline 0&0&0&1\end{array}\right)=\begin{pmatrix}I_3&A_1\\O&I_1\end{pmatrix}$$

$$\overset{\text{or}}{=}\left(\begin{array}{cc:cc}1&0&0&3\\0&1&0&-1\\ \hdashline 0&0&1&0\\0&0&0&1\end{array}\right)=\begin{pmatrix}I_2&A_2\\O&I_2\end{pmatrix}$$

$$\overset{\text{or}}{=}\left(\begin{array}{c:c:c:c}1&0&0&3\\0&1&0&-1\\0&0&1&0\\0&0&0&1\end{array}\right)=\begin{pmatrix}\quad\end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right) = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix}$$

$$\overset{\text{or}}{=} \left(\begin{array}{cc|cc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) = \begin{pmatrix} I_2 & A_2 \\ O & I_2 \end{pmatrix}$$

$$\overset{\text{or}}{=} \left(\begin{array}{c|c|c|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) = \begin{pmatrix} \varepsilon_1 & \qquad\qquad\qquad \end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A = \begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \left(\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right) = \begin{pmatrix} I_3 & A_1 \\ O & I_1 \end{pmatrix}$$

$$\overset{\text{or}}{=} \left(\begin{array}{cc:cc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) = \begin{pmatrix} I_2 & A_2 \\ O & I_2 \end{pmatrix}$$

$$\overset{\text{or}}{=} \left(\begin{array}{c:c:c:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & & \end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A=\begin{pmatrix}1&0&0&3\\0&1&0&-1\\0&0&1&0\\0&0&0&1\end{pmatrix}=\left(\begin{array}{ccc:c}1&0&0&3\\0&1&0&-1\\0&0&1&0\\\hdashline 0&0&0&1\end{array}\right)=\begin{pmatrix}I_3&A_1\\O&I_1\end{pmatrix}$$

$$\overset{\text{or}}{\equiv}\left(\begin{array}{cc:cc}1&0&0&3\\0&1&0&-1\\\hdashline 0&0&1&0\\0&0&0&1\end{array}\right)=\begin{pmatrix}I_2&A_2\\O&I_2\end{pmatrix}$$

$$\overset{\text{or}}{\equiv}\left(\begin{array}{c:c:c:c}1&0&0&3\\0&1&0&-1\\0&0&1&0\\0&0&0&1\end{array}\right)=\begin{pmatrix}\varepsilon_1&\varepsilon_2&\varepsilon_3&\end{pmatrix}$$

分块矩阵引入

- 矩阵

$$A=\begin{pmatrix}1&0&0&3\\0&1&0&-1\\0&0&1&0\\0&0&0&1\end{pmatrix}=\left(\begin{array}{ccc|c}1&0&0&3\\0&1&0&-1\\0&0&1&0\\\hline 0&0&0&1\end{array}\right)=\begin{pmatrix}I_3&A_1\\O&I_1\end{pmatrix}$$

$$\overset{\text{or}}{=}\left(\begin{array}{cc|cc}1&0&0&3\\0&1&0&-1\\\hline 0&0&1&0\\0&0&0&1\end{array}\right)=\begin{pmatrix}I_2&A_2\\O&I_2\end{pmatrix}$$

$$\overset{\text{or}}{=}\left(\begin{array}{c|c|c|c}1&0&0&3\\0&1&0&-1\\0&0&1&0\\0&0&0&1\end{array}\right)=\begin{pmatrix}\varepsilon_1&\varepsilon_2&\varepsilon_3&\alpha\end{pmatrix}$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$A = \begin{pmatrix} * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \end{pmatrix}$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$A = \left(\begin{array}{cc|ccc|c|c}
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\
\hline
* & * & * & * & * & \cdots & * \\
\hline
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\hline
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & *
\end{array}\right)$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$
A = \left(\begin{array}{cc:ccc:c:c}
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\
\hdashline
* & * & * & * & * & \cdots & * \\
\hdashline
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\hdashline
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & *
\end{array}\right) = \left(\qquad\qquad\qquad\qquad \right)
$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$A=\left(\begin{array}{cc:ccc:c:c}
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\ \hdashline
* & * & * & * & * & \cdots & * \\ \hdashline
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \hdashline
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & *
\end{array}\right)=\begin{pmatrix} A_{11} & & \\ & & \\ & & \end{pmatrix}$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$
A = \left(\begin{array}{cc:ccc:c:c}
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\
\hdashline
* & * & * & * & * & \cdots & * \\
\hdashline
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\hdashline
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & *
\end{array}\right) = \begin{pmatrix} A_{11} & A_{12} & & \\ & & & \\ & & & \end{pmatrix}
$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$A = \left(\begin{array}{cc:ccc:c:c} * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ \hdashline * & * & * & * & * & \cdots & * \\ \hdashline \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \hdashline * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \end{array}\right) = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{11} \\ & & & \\ & & & \end{pmatrix}$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$A=\left(\begin{array}{cc:ccc:c:c}
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\
\hdashline
* & * & * & * & * & \cdots & * \\
\hdashline
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\hdashline
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & *
\end{array}\right)=\begin{pmatrix}
A_{11} & A_{12} & \cdots & A_{11} \\
A_{21} & A_{22} & \cdots & A_{2t} \\
 & & & \\
 & & &
\end{pmatrix}$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$
A = \left(\begin{array}{cc:ccc:c:c}
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\
\hdashline
* & * & * & * & * & \cdots & * \\
\hdashline
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\hdashline
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & *
\end{array}\right)
= \begin{pmatrix}
A_{11} & A_{12} & \cdots & A_{11} \\
A_{21} & A_{22} & \cdots & A_{2t} \\
\vdots & \vdots & \ddots & \vdots \\
A_{s1} & A_{s2} & \cdots & A_{st}
\end{pmatrix}
$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$
A=\left(\begin{array}{cc:ccc:c:c}
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\ \hdashline
* & * & * & * & * & \cdots & * \\ \hdashline
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \hdashline
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & *
\end{array}\right)
=\begin{pmatrix}
A_{11} & A_{12} & \cdots & A_{11} \\
A_{21} & A_{22} & \cdots & A_{2t} \\
\vdots & \vdots & \ddots & \vdots \\
A_{s1} & A_{s2} & \cdots & A_{st}
\end{pmatrix}
=(A_{pq})
$$

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$A = \left(\begin{array}{cc:ccc:c:c} * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \\ \hdashline * & * & * & * & * & \cdots & * \\ \hdashline \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \hdashline * & * & * & * & * & \cdots & * \\ * & * & * & * & * & \cdots & * \end{array}\right) = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{11} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} = (A_{pq})$$

称为分块矩阵。

分块矩阵

- 一般地，可将任意矩阵 A 作分割成若干子矩阵，例如

$$A=\left(\begin{array}{cc|ccc|c|c}
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & * \\ \hline
* & * & * & * & * & \cdots & * \\ \hline
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \hline
* & * & * & * & * & \cdots & * \\
* & * & * & * & * & \cdots & *
\end{array}\right)=\begin{pmatrix}
A_{11} & A_{12} & \cdots & A_{11} \\
A_{21} & A_{22} & \cdots & A_{2t} \\
\vdots & \vdots & \ddots & \vdots \\
A_{s1} & A_{s2} & \cdots & A_{st}
\end{pmatrix}=(A_{pq})$$

  称为**分块矩阵**。

- 分块矩阵中
  - 每一行的每个子块有相同行数；
  - 每一列的每个子块有相同列数。

分块矩阵的运算：加法

假设矩阵 A, B 同型，且采取相同分块方式：

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{st} \end{pmatrix}$$

分块矩阵的运算：加法

假设矩阵 A, B 同型，且采取相同分块方式：

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{st} \end{pmatrix}$$

$$A + B =$$

分块矩阵的运算：加法

假设矩阵 A, B 同型，且采取相同分块方式：

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{st} \end{pmatrix}$$

$$A + B = \begin{pmatrix} A_{11}+B_{11} & A_{12}+B_{12} & \cdots & A_{1t}+B_{1t} \\ A_{21}+B_{21} & A_{22}+B_{22} & \cdots & A_{2t}+B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1}+B_{s1} & A_{s2}+B_{s2} & \cdots & A_{st}+B_{st} \end{pmatrix}$$

分块矩阵的运算：加法

假设矩阵 A, B 同型，且采取相同分块方式：

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} = (A_{pq}),\ B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{st} \end{pmatrix}$$

则

$$A + B = \begin{pmatrix} A_{11} + B_{11} & A_{12} + B_{12} & \cdots & A_{1t} + B_{1t} \\ A_{21} + B_{21} & A_{22} + B_{22} & \cdots & A_{2t} + B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} + B_{s1} & A_{s2} + B_{s2} & \cdots & A_{st} + B_{st} \end{pmatrix}$$

分块矩阵的运算：加法

假设矩阵 A, B 同型，且采取相同分块方式：

$$A=\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix}=(A_{pq}),\ B=\begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{s1} & B_{s2} & \cdots & B_{st} \end{pmatrix}=(B_{pq})$$

则

$$A+B=\begin{pmatrix} A_{11}+B_{11} & A_{12}+B_{12} & \cdots & A_{1t}+B_{1t} \\ A_{21}+B_{21} & A_{22}+B_{22} & \cdots & A_{2t}+B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1}+B_{s1} & A_{s2}+B_{s2} & \cdots & A_{st}+B_{st} \end{pmatrix}$$

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分块矩阵的运算：加法

例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$，$B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$ 则

$$A + B = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix} =$$

分块矩阵的运算：加法

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$$A+B=\left(\begin{array}{cc|cc}1&0&1&3\\0&1&2&4\\\hline 0&0&-1&0\\0&0&0&-1\end{array}\right)+\begin{pmatrix}1&2&0&0\\2&0&0&0\\6&3&1&0\\0&-2&0&1\end{pmatrix}=$$

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分块矩阵的运算：加法

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分块矩阵的运算：加法

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$$A+B=\left(\begin{array}{cc|cc}1&0&\color{red}{1}&\color{red}{3}\\0&1&\color{red}{2}&\color{red}{4}\\\hline 0&0&-1&0\\0&0&0&-1\end{array}\right)+\left(\begin{array}{cc|cc}1&2&\color{red}{0}&\color{red}{0}\\2&0&\color{red}{0}&\color{red}{0}\\\hline 6&3&1&0\\0&-2&0&1\end{array}\right)=\left(\begin{array}{cc|cc}2&2&\color{red}{1}&\color{red}{3}\\2&1&\color{red}{2}&\color{red}{4}\\\hline &&&\\&&&\end{array}\right)$$

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$$A + B = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) + \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right) = \left(\begin{array}{cc|cc} 2 & 2 & 1 & 3 \\ 2 & 1 & 2 & 4 \\ \hline 6 & 3 & & \\ 0 & -2 & & \end{array}\right)$$

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$$A+B=\left(\begin{array}{cc:cc}1&0&1&3\\0&1&2&4\\ \hdashline 0&0&-1&0\\0&0&0&-1\end{array}\right)+\left(\begin{array}{cc:cc}1&2&0&0\\2&0&0&0\\ \hdashline 6&3&1&0\\0&-2&0&1\end{array}\right)=\left(\begin{array}{cc:cc}2&2&1&3\\2&1&2&4\\ \hdashline 6&3&0&0\\0&-2&0&0\end{array}\right)$$

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或者

$$A + B =$$

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$$A+B=\left(\begin{array}{cc|cc}1&0&1&3\\0&1&2&4\\\hline 0&0&-1&0\\0&0&0&-1\end{array}\right)+\left(\begin{array}{cc|cc}1&2&0&0\\2&0&0&0\\\hline 6&3&1&0\\0&-2&0&1\end{array}\right)=\left(\begin{array}{cc|cc}2&2&1&3\\2&1&2&4\\\hline 6&3&0&0\\0&-2&0&0\end{array}\right)$$

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$$A+B=\left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)+\left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right)=\left(\begin{array}{cc|cc} 2 & 2 & 1 & 3 \\ 2 & 1 & 2 & 4 \\ \hline 6 & 3 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{array}\right)$$

或者

$$A+B=\begin{pmatrix} I & C \\ O & -I \end{pmatrix}+\begin{pmatrix} D & O \\ F & I \end{pmatrix}=\begin{pmatrix} I+D & C \\ F & O \end{pmatrix}=\left(\begin{array}{cc|cc} 2 & 2 & & \\ 2 & 1 & & \\ \hline & & & \\ & & & \end{array}\right)$$

分块矩阵的运算：加法

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或者

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分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA =$$

分块矩阵的运算：数乘

$$A=\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA=\begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

分块矩阵的运算：数乘

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例 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \left(\begin{array}{cc:cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hdashline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)$

分块矩阵的运算：数乘

$$A=\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA=\begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A=\left(\begin{array}{cc:cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hdashline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)=\left(\begin{array}{c:c} I & C \\ \hdashline O & -I \end{array}\right)$

分块矩阵的运算：数乘

$$A=\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA=\begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A=\left(\begin{array}{cc:cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hdashline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)=\left(\begin{array}{c:c} I & C \\ \hdashline O & -I \end{array}\right)$，则

$$kA=$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) = \left(\begin{array}{c|c} I & C \\ \hline O & -I \end{array}\right)$，则

$$kA = k\begin{pmatrix} I & C \\ O & -I \end{pmatrix} =$$

分块矩阵的运算：数乘

$$A=\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA=\begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A=\left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)=\left(\begin{array}{c|c} I & C \\ \hline O & -I \end{array}\right)$，则

$$kA=k\begin{pmatrix} I & C \\ O & -I \end{pmatrix}=\begin{pmatrix} kI & kC \\ O & -kI \end{pmatrix}=$$

分块矩阵的运算：数乘

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA = \begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A = \left(\begin{array}{cc:cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hdashline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) = \left(\begin{array}{c:c} I & C \\ \hdashline O & -I \end{array}\right)$，则

$$kA = k\begin{pmatrix} I & C \\ O & -I \end{pmatrix} = \begin{pmatrix} kI & kC \\ O & -kI \end{pmatrix} = \left(\begin{array}{c:c} & \\ \hdashline & \end{array}\right)$$

分块矩阵的运算：数乘

$$A=\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA=\begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A=\left(\begin{array}{cc:cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hdashline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)=\left(\begin{array}{c:c} I & C \\ \hdashline O & -I \end{array}\right)$，则

$$kA=k\begin{pmatrix} I & C \\ O & -I \end{pmatrix}=\begin{pmatrix} kI & kC \\ O & -kI \end{pmatrix}=\left(\begin{array}{cc:cc} k & 0 & & \\ 0 & k & & \\ \hdashline & & & \\ & & & \end{array}\right)$$

分块矩阵的运算：数乘

$$A=\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA=\begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A=\left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)=\left(\begin{array}{c|c} I & C \\ \hline O & -I \end{array}\right)$，则

$$kA=k\begin{pmatrix} I & C \\ O & -I \end{pmatrix}=\begin{pmatrix} kI & kC \\ O & -kI \end{pmatrix}=\left(\begin{array}{cc|cc} k & 0 & k & 3k \\ 0 & k & 2k & 4k \\ \hline & & & \\ & & & \end{array}\right)$$

分块矩阵的运算：数乘

$$A=\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA=\begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A=\left(\begin{array}{cc:cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hdashline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)=\left(\begin{array}{c:c} I & C \\ \hdashline O & -I \end{array}\right)$，则

$$kA=k\begin{pmatrix} I & C \\ O & -I \end{pmatrix}=\begin{pmatrix} kI & kC \\ O & -kI \end{pmatrix}=\left(\begin{array}{cc:cc} k & 0 & k & 3k \\ 0 & k & 2k & 4k \\ \hdashline 0 & 0 & & \\ 0 & 0 & & \end{array}\right)$$

分块矩阵的运算：数乘

$$A=\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1t} \\ A_{21} & A_{22} & \cdots & A_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{st} \end{pmatrix} \quad \Rightarrow \quad kA=\begin{pmatrix} kA_{11} & kA_{12} & \cdots & kA_{1t} \\ kA_{21} & kA_{22} & \cdots & kA_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ kA_{s1} & kA_{s2} & \cdots & kA_{st} \end{pmatrix}$$

例 设 $A=\left(\begin{array}{cc:cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hdashline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)=\left(\begin{array}{c:c} I & C \\ \hdashline O & -I \end{array}\right)$，则

$$kA=k\begin{pmatrix} I & C \\ O & -I \end{pmatrix}=\begin{pmatrix} kI & kC \\ O & -kI \end{pmatrix}=\left(\begin{array}{cc:cc} k & 0 & k & 3k \\ 0 & k & 2k & 4k \\ \hdashline 0 & 0 & -k & 0 \\ 0 & 0 & 0 & -k \end{array}\right)$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m\times l}$, $B_{l\times n}$ 分块为

$$A=\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix}, \quad B=\begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix}$$

满足：A 的列划分与 B 的行划分方式相同。

分块矩阵的运算：乘积

假设将矩阵 $A_{m\times l}$, $B_{l\times n}$ 分块为

$$A=\begin{array}{c}\begin{array}{cccc} n_1 & n_2 & & n_r \end{array}\\ \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r}\\ A_{21} & A_{22} & \cdots & A_{2r}\\ \vdots & \vdots & \ddots & \vdots\\ A_{s1} & A_{s2} & \cdots & A_{sr}\end{pmatrix}\end{array},\quad B=\begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t}\\ B_{21} & B_{22} & \cdots & B_{2t}\\ \vdots & \vdots & \ddots & \vdots\\ B_{r1} & B_{r2} & \cdots & B_{rt}\end{pmatrix}$$

满足：A 的列划分与 B 的行划分方式相同。

分块矩阵的运算：乘积

假设将矩阵 $A_{m\times l}$, $B_{l\times n}$ 分块为

$$
A=\begin{array}{c}
\begin{array}{cccc} n_1 & n_2 & & n_r \end{array}\\
\begin{pmatrix}
A_{11} & A_{12} & \cdots & A_{1r}\\
A_{21} & A_{22} & \cdots & A_{2r}\\
\vdots & \vdots & \ddots & \vdots\\
A_{s1} & A_{s2} & \cdots & A_{sr}
\end{pmatrix}
\end{array},\quad
B=\begin{array}{cc}
\begin{pmatrix}
B_{11} & B_{12} & \cdots & B_{1t}\\
B_{21} & B_{22} & \cdots & B_{2t}\\
\vdots & \vdots & \ddots & \vdots\\
B_{r1} & B_{r2} & \cdots & B_{rt}
\end{pmatrix}
&
\begin{array}{c} n_1\\ n_2\\ \\ n_r \end{array}
\end{array}
$$

满足：A 的列划分与 B 的行划分方式相同。

分块矩阵的运算：乘积

假设将矩阵 $A_{m\times l}$, $B_{l\times n}$ 分块为

$$A=\begin{array}{c}\begin{array}{cccc} n_1 & n_2 & & n_r \end{array}\\ \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r}\\ A_{21} & A_{22} & \cdots & A_{2r}\\ \vdots & \vdots & \ddots & \vdots\\ A_{s1} & A_{s2} & \cdots & A_{sr}\end{pmatrix}\end{array},\quad B=\begin{array}{cc}\begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t}\\ B_{21} & B_{22} & \cdots & B_{2t}\\ \vdots & \vdots & \ddots & \vdots\\ B_{r1} & B_{r2} & \cdots & B_{rt}\end{pmatrix} & \begin{array}{c} n_1\\ n_2\\ \\ n_r\end{array}\end{array}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB=C=$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m\times l}$, $B_{l\times n}$ 分块为

$$A=\begin{array}{c} \begin{matrix} n_1 & n_2 & & n_r \end{matrix} \\ \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix} \end{array}, \quad B=\begin{array}{cc} \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix} & \begin{matrix} n_1 \\ n_2 \\ \\ n_r \end{matrix} \end{array}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB=C=(C_{pq})$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m\times l}$, $B_{l\times n}$ 分块为

$$A = \begin{array}{c} \begin{array}{cccc} n_1 & n_2 & & n_r \end{array} \\ \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix} \end{array}, \quad B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix} \begin{array}{l} n_1 \\ n_2 \\ \\ n_r \end{array}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中

$$C_{pq} = \qquad\qquad\qquad\qquad .$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m\times l}$, $B_{l\times n}$ 分块为

$$A = \begin{array}{c} \begin{matrix} n_1 & n_2 & & n_r \end{matrix} \\ \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix} \end{array}, \quad B = \begin{array}{cc} \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix} & \begin{matrix} n_1 \\ n_2 \\ \\ n_r \end{matrix} \end{array}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中

$$C_{pq} = A_{p1} \qquad A_{p2} \qquad \cdots \quad A_{pr} \qquad .$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m\times l}$, $B_{l\times n}$ 分块为

$$A = \begin{array}{c} \begin{array}{cccc} n_1 & n_2 & & n_r \end{array} \\ \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix} \end{array}, \quad B = \begin{array}{cc} \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix} & \begin{array}{c} n_1 \\ n_2 \\ \\ n_r \end{array} \end{array}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中

$$C_{pq} = A_{p1}B_{1q} \quad A_{p2}B_{2q} \quad \cdots \quad A_{pr}B_{rq}.$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m\times l}$, $B_{l\times n}$ 分块为

$$A=\begin{array}{c} \begin{array}{cccc} n_1 & n_2 & & n_r \end{array} \\ \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix} \end{array}, \quad B=\begin{array}{cc} \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix} & \begin{array}{c} n_1 \\ n_2 \\ \\ n_r \end{array} \end{array}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB=C=(C_{pq})$$

其中

$$C_{pq}=A_{p1}B_{1q}+A_{p2}B_{2q}+\cdots+A_{pr}B_{rq}.$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m\times l}$, $B_{l\times n}$ 分块为

$$A = \begin{array}{c} \begin{matrix} n_1 & n_2 & & n_r \end{matrix} \\ \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix} \end{array}, \quad B = \begin{array}{cc} \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix} & \begin{matrix} n_1 \\ n_2 \\ \\ n_r \end{matrix} \end{array}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中（必然每个子块的乘积有意义）

$$C_{pq} = A_{p1}B_{1q} + A_{p2}B_{2q} + \cdots + A_{pr}B_{rq}.$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m\times l}$, $B_{l\times n}$ 分块为

$$A = \begin{array}{c} \begin{array}{cccc} n_1 & n_2 & & n_r \end{array} \\ \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix} \end{array}, \quad B = \begin{array}{cc} \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix} & \begin{array}{c} n_1 \\ n_2 \\ \\ n_r \end{array} \end{array}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中（必然每个子块的乘积有意义）

$$C_{pq} = A_{p1}B_{1q} + A_{p2}B_{2q} + \cdots + A_{pr}B_{rq}.$$

$$\begin{pmatrix} A_{11} & \cdots\cdots & A_{1r} \\ \vdots & & \vdots \\ A_{p1} & \cdots\cdots & A_{pr} \\ \vdots & & \vdots \\ A_{s1} & \cdots\cdots & A_{sr} \end{pmatrix} \cdot \begin{pmatrix} B_{11} & \cdots & B_{1q} & \cdots & B_{1t} \\ \vdots & & \vdots & & \vdots \\ B_{r1} & \cdots & B_{rq} & \cdots & B_{rt} \end{pmatrix} = \begin{pmatrix} C_{11} & \cdots & & \cdots & C_{1t} \\ \vdots & & \vdots & & \vdots \\ & \cdots & C_{pq} & \cdots & \\ \vdots & & \vdots & & \vdots \\ C_{s1} & \cdots & & \cdots & C_{st} \end{pmatrix}$$

分块矩阵的运算：乘积

假设将矩阵 $A_{m\times l}$, $B_{l\times n}$ 分块为

$$A = \begin{array}{c} \begin{array}{cccc} n_1 & n_2 & & n_r \end{array} \\ \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1r} \\ A_{21} & A_{22} & \cdots & A_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ A_{s1} & A_{s2} & \cdots & A_{sr} \end{pmatrix} \end{array}, \quad B = \begin{array}{cc} \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1t} \\ B_{21} & B_{22} & \cdots & B_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{r1} & B_{r2} & \cdots & B_{rt} \end{pmatrix} & \begin{array}{c} n_1 \\ n_2 \\ \\ n_r \end{array} \end{array}$$

满足：A 的列划分与 B 的行划分方式相同。则

$$AB = C = (C_{pq})$$

其中（必然每个子块的乘积有意义）

$$C_{pq} = A_{p1}B_{1q} + A_{p2}B_{2q} + \cdots + A_{pr}B_{rq}.$$

$$\begin{pmatrix} A_{11} & \cdots\cdots & A_{1r} \\ \vdots & & \vdots \\ A_{p1} & \cdots\cdots & A_{pr} \\ \vdots & & \vdots \\ A_{s1} & \cdots\cdots & A_{sr} \end{pmatrix} \cdot \begin{pmatrix} B_{11} & \cdots & B_{1q} & \cdots & B_{1t} \\ \vdots & & \vdots & & \vdots \\ B_{r1} & \cdots & B_{rq} & \cdots & B_{rt} \end{pmatrix} = \begin{pmatrix} C_{11} & \cdots & & \cdots & C_{1t} \\ \vdots & & \vdots & & \vdots \\ & \cdots & C_{pq} & \cdots & \\ \vdots & & \vdots & & \vdots \\ C_{s1} & \cdots & & \cdots & C_{st} \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \begin{pmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ ，$B = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$

$$AB =$$

分块矩阵的运算：乘积

例 1 设 $A=\left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right)=\left(\begin{array}{c|c} I & C \\ \hline O & -I \end{array}\right)$， $B=\begin{pmatrix} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{pmatrix}$

$$AB=$$

分块矩阵的运算：乘积

例 1 设 $A=\left(\begin{array}{cc|cc}1&0&1&3\\0&1&2&4\\\hline 0&0&-1&0\\0&0&0&-1\end{array}\right)=\left(\begin{array}{c|c}I&C\\\hline O&-I\end{array}\right)$，$B=\left(\begin{array}{cc|cc}1&2&0&0\\2&0&0&0\\\hline 6&3&1&0\\0&-2&0&1\end{array}\right)=\left(\begin{array}{c|c}D&O\\\hline F&I\end{array}\right)$

$$AB=$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) = \left(\begin{array}{c|c} I & C \\ \hline O & -I \end{array}\right)$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right) = \left(\begin{array}{c|c} D & O \\ \hline F & I \end{array}\right)$

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} =$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) = \left(\begin{array}{c|c} I & C \\ \hline O & -I \end{array}\right)$，$B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right) = \left(\begin{array}{c|c} D & O \\ \hline F & I \end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix} \begin{pmatrix} D & O \\ F & I \end{pmatrix} =$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) = \left(\begin{array}{c|c} I & C \\ \hline O & -I \end{array}\right)$，$B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right) = \left(\begin{array}{c|c} D & O \\ \hline F & I \end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}\begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc:cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hdashline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) = \left(\begin{array}{c:c} I & C \\ \hdashline O & -I \end{array}\right)$，$B = \left(\begin{array}{cc:cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hdashline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right) = \left(\begin{array}{c:c} D & O \\ \hdashline F & I \end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}\begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & \\ & \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc:cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hdashline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) = \left(\begin{array}{c:c} I & C \\ \hdashline O & -I \end{array}\right)$, $B = \left(\begin{array}{cc:cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hdashline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right) = \left(\begin{array}{c:c} D & O \\ \hdashline F & I \end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}\begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ & \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A=\left(\begin{array}{cc|cc}1&0&1&3\\0&1&2&4\\\hline 0&0&-1&0\\0&0&0&-1\end{array}\right)=\left(\begin{array}{c|c}I&C\\\hline O&-I\end{array}\right)$，$B=\left(\begin{array}{cc|cc}1&2&0&0\\2&0&0&0\\\hline 6&3&1&0\\0&-2&0&1\end{array}\right)=\left(\begin{array}{c|c}D&O\\\hline F&I\end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix}I&C\\O&-I\end{pmatrix}\begin{pmatrix}D&O\\F&I\end{pmatrix}=\begin{pmatrix}ID+CF&IO+CI\\OD+(-I)F&\end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) = \left(\begin{array}{c|c} I & C \\ \hline O & -I \end{array}\right)$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right) = \left(\begin{array}{c|c} D & O \\ \hline F & I \end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}\begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & OO + (-I)I \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A=\left(\begin{array}{cc|cc}1&0&1&3\\0&1&2&4\\\hline 0&0&-1&0\\0&0&0&-1\end{array}\right)=\left(\begin{array}{c|c}I&C\\\hline O&-I\end{array}\right)$，$B=\left(\begin{array}{cc|cc}1&2&0&0\\2&0&0&0\\\hline 6&3&1&0\\0&-2&0&1\end{array}\right)=\left(\begin{array}{c|c}D&O\\\hline F&I\end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix}I&C\\O&-I\end{pmatrix}\begin{pmatrix}D&O\\F&I\end{pmatrix}=\begin{pmatrix}ID+CF&IO+CI\\OD+(-I)F&OO+(-I)I\end{pmatrix}$$

$$=\begin{pmatrix}\quad&\quad\end{pmatrix}=$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) = \left(\begin{array}{c|c} I & C \\ \hline O & -I \end{array}\right)$，$B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right) = \left(\begin{array}{c|c} D & O \\ \hline F & I \end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}\begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & OO + (-I)I \end{pmatrix}$$

$$= \begin{pmatrix} D + CF & \\ & \end{pmatrix} =$$

分块矩阵的运算：乘积

例 1 设 $A=\left(\begin{array}{cc|cc}1&0&1&3\\0&1&2&4\\\hline 0&0&-1&0\\0&0&0&-1\end{array}\right)=\left(\begin{array}{c|c}I&C\\\hline O&-I\end{array}\right)$，$B=\left(\begin{array}{cc|cc}1&2&0&0\\2&0&0&0\\\hline 6&3&1&0\\0&-2&0&1\end{array}\right)=\left(\begin{array}{c|c}D&O\\\hline F&I\end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix}I&C\\O&-I\end{pmatrix}\begin{pmatrix}D&O\\F&I\end{pmatrix}=\begin{pmatrix}ID+CF&IO+CI\\OD+(-I)F&OO+(-I)I\end{pmatrix}$$

$$=\begin{pmatrix}D+CF&C\\ & \end{pmatrix}=$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) = \left(\begin{array}{c|c} I & C \\ \hline O & -I \end{array}\right)$，$B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right) = \left(\begin{array}{c|c} D & O \\ \hline F & I \end{array}\right)$

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$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}\begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & OO + (-I)I \end{pmatrix}$$

$$= \begin{pmatrix} D + CF & C \\ -F & \end{pmatrix} =$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) = \left(\begin{array}{c|c} I & C \\ \hline O & -I \end{array}\right)$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right) = \left(\begin{array}{c|c} D & O \\ \hline F & I \end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}\begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & OO + (-I)I \end{pmatrix}$$

$$= \begin{pmatrix} D + CF & C \\ -F & -I \end{pmatrix} =$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) = \left(\begin{array}{c|c} I & C \\ \hline O & -I \end{array}\right)$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right) = \left(\begin{array}{c|c} D & O \\ \hline F & I \end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}\begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & OO + (-I)I \end{pmatrix}$$

$$= \begin{pmatrix} D + CF & C \\ -F & -I \end{pmatrix} = \left(\begin{array}{c|cc} & 1 & 3 \\ & 2 & 4 \\ \hline & & \\ & & \end{array}\right)$$

分块矩阵的运算：乘积

例 1 设 $A=\left(\begin{array}{cc|cc}1&0&1&3\\0&1&2&4\\\hline 0&0&-1&0\\0&0&0&-1\end{array}\right)=\left(\begin{array}{c|c}I&C\\\hline O&-I\end{array}\right)$，$B=\left(\begin{array}{cc|cc}1&2&0&0\\2&0&0&0\\\hline 6&3&1&0\\0&-2&0&1\end{array}\right)=\left(\begin{array}{c|c}D&O\\\hline F&I\end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix}I&C\\O&-I\end{pmatrix}\begin{pmatrix}D&O\\F&I\end{pmatrix}=\begin{pmatrix}ID+CF&IO+CI\\OD+(-I)F&OO+(-I)I\end{pmatrix}$$

$$=\begin{pmatrix}D+CF&C\\-F&-I\end{pmatrix}=\left(\begin{array}{cc|cc}&&1&3\\&&2&4\\\hline -6&-3&&\\0&2&&\end{array}\right)$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) = \left(\begin{array}{c|c} I & C \\ \hline O & -I \end{array}\right)$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right) = \left(\begin{array}{c|c} D & O \\ \hline F & I \end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}\begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & OO + (-I)I \end{pmatrix}$$

$$= \begin{pmatrix} D + CF & C \\ -F & -I \end{pmatrix} = \left(\begin{array}{cc|cc} & & 1 & 3 \\ & & 2 & 4 \\ \hline -6 & -3 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{array}\right)$$

分块矩阵的运算：乘积

例 1 设 $A=\left(\begin{array}{cc|cc}1&0&1&3\\0&1&2&4\\\hline 0&0&-1&0\\0&0&0&-1\end{array}\right)=\left(\begin{array}{c|c}I&C\\\hline O&-I\end{array}\right)$，$B=\left(\begin{array}{cc|cc}1&2&0&0\\2&0&0&0\\\hline 6&3&1&0\\0&-2&0&1\end{array}\right)=\left(\begin{array}{c|c}D&O\\\hline F&I\end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix}I&C\\O&-I\end{pmatrix}\begin{pmatrix}D&O\\F&I\end{pmatrix}=\begin{pmatrix}ID+CF&IO+CI\\OD+(-I)F&OO+(-I)I\end{pmatrix}$$

$$=\begin{pmatrix}D+CF&C\\-F&-I\end{pmatrix}=\left(\begin{array}{cc|cc}&&1&3\\&&2&4\\\hline -6&-3&-1&0\\0&2&0&-1\end{array}\right)$$

其中

$D+CF=$

分块矩阵的运算：乘积

例 1 设 $A=\left(\begin{array}{cc:cc}1&0&1&3\\0&1&2&4\\\hdashline 0&0&-1&0\\0&0&0&-1\end{array}\right)=\left(\begin{array}{c:c}I&C\\\hdashline O&-I\end{array}\right)$, $B=\left(\begin{array}{cc:cc}1&2&0&0\\2&0&0&0\\\hdashline 6&3&1&0\\0&-2&0&1\end{array}\right)=\left(\begin{array}{c:c}D&O\\\hdashline F&I\end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix}I&C\\O&-I\end{pmatrix}\begin{pmatrix}D&O\\F&I\end{pmatrix}=\begin{pmatrix}ID+CF&IO+CI\\OD+(-I)F&OO+(-I)I\end{pmatrix}$$

$$=\begin{pmatrix}D+CF&C\\-F&-I\end{pmatrix}=\left(\begin{array}{cc:cc}&&1&3\\&&2&4\\\hdashline -6&-3&-1&0\\0&2&0&-1\end{array}\right)$$

其中

$$D+CF=\begin{pmatrix}1&2\\2&0\end{pmatrix}+\begin{pmatrix}1&3\\2&4\end{pmatrix}\begin{pmatrix}6&3\\0&-2\end{pmatrix}=$$

分块矩阵的运算：乘积

例 1 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ \hline 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right) = \left(\begin{array}{c|c} I & C \\ \hline O & -I \end{array}\right)$, $B = \left(\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ \hline 6 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right) = \left(\begin{array}{c|c} D & O \\ \hline F & I \end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & C \\ O & -I \end{pmatrix}\begin{pmatrix} D & O \\ F & I \end{pmatrix} = \begin{pmatrix} ID + CF & IO + CI \\ OD + (-I)F & OO + (-I)I \end{pmatrix}$$

$$= \begin{pmatrix} D + CF & C \\ -F & -I \end{pmatrix} = \left(\begin{array}{cc|cc} & & 1 & 3 \\ & & 2 & 4 \\ \hline -6 & -3 & -1 & 0 \\ 0 & 2 & 0 & -1 \end{array}\right)$$

其中

$$D + CF = \begin{pmatrix} 1 & 2 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 6 & 3 \\ 0 & -2 \end{pmatrix} = \qquad \begin{pmatrix} 6 & -3 \\ 12 & -2 \end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A=\left(\begin{array}{cc|cc}1&0&1&3\\0&1&2&4\\\hline 0&0&-1&0\\0&0&0&-1\end{array}\right)=\left(\begin{array}{c|c}I&C\\\hline O&-I\end{array}\right)$，$B=\left(\begin{array}{cc|cc}1&2&0&0\\2&0&0&0\\\hline 6&3&1&0\\0&-2&0&1\end{array}\right)=\left(\begin{array}{c|c}D&O\\\hline F&I\end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix}I&C\\O&-I\end{pmatrix}\begin{pmatrix}D&O\\F&I\end{pmatrix}=\begin{pmatrix}ID+CF&IO+CI\\OD+(-I)F&OO+(-I)I\end{pmatrix}$$

$$=\begin{pmatrix}D+CF&C\\-F&-I\end{pmatrix}=\left(\begin{array}{cc|cc}&&1&3\\&&2&4\\\hline -6&-3&-1&0\\0&2&0&-1\end{array}\right)$$

其中

$$D+CF=\begin{pmatrix}1&2\\2&0\end{pmatrix}+\begin{pmatrix}1&3\\2&4\end{pmatrix}\begin{pmatrix}6&3\\0&-2\end{pmatrix}=\begin{pmatrix}1&2\\2&0\end{pmatrix}+\begin{pmatrix}6&-3\\12&-2\end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A=\left(\begin{array}{cc:cc}1&0&1&3\\0&1&2&4\\ \hdashline 0&0&-1&0\\0&0&0&-1\end{array}\right)=\left(\begin{array}{c:c}I&C\\ \hdashline O&-I\end{array}\right)$, $B=\left(\begin{array}{cc:cc}1&2&0&0\\2&0&0&0\\ \hdashline 6&3&1&0\\0&-2&0&1\end{array}\right)=\left(\begin{array}{c:c}D&O\\ \hdashline F&I\end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix}I&C\\O&-I\end{pmatrix}\begin{pmatrix}D&O\\F&I\end{pmatrix}=\begin{pmatrix}ID+CF&IO+CI\\OD+(-I)F&OO+(-I)I\end{pmatrix}$$

$$=\begin{pmatrix}D+CF&C\\-F&-I\end{pmatrix}=\left(\begin{array}{cc:cc}&&1&3\\&&2&4\\ \hdashline -6&-3&-1&0\\0&2&0&-1\end{array}\right)$$

其中

$$D+CF=\begin{pmatrix}1&2\\2&0\end{pmatrix}+\begin{pmatrix}1&3\\2&4\end{pmatrix}\begin{pmatrix}6&3\\0&-2\end{pmatrix}=\begin{pmatrix}1&2\\2&0\end{pmatrix}+\begin{pmatrix}6&-3\\12&-2\end{pmatrix}=\begin{pmatrix}7&-1\\14&-2\end{pmatrix}$$

分块矩阵的运算：乘积

例 1 设 $A=\left(\begin{array}{cc|cc}1&0&1&3\\0&1&2&4\\\hline 0&0&-1&0\\0&0&0&-1\end{array}\right)=\left(\begin{array}{c|c}I&C\\\hline O&-I\end{array}\right)$，$B=\left(\begin{array}{cc|cc}1&2&0&0\\2&0&0&0\\\hline 6&3&1&0\\0&-2&0&1\end{array}\right)=\left(\begin{array}{c|c}D&O\\\hline F&I\end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix}I&C\\O&-I\end{pmatrix}\begin{pmatrix}D&O\\F&I\end{pmatrix}=\begin{pmatrix}ID+CF&IO+CI\\OD+(-I)F&OO+(-I)I\end{pmatrix}$$

$$=\begin{pmatrix}D+CF&C\\-F&-I\end{pmatrix}=\left(\begin{array}{cc|cc}7&-1&1&3\\14&-2&2&4\\\hline -6&-3&-1&0\\0&2&0&-1\end{array}\right)$$

其中

$$D+CF=\begin{pmatrix}1&2\\2&0\end{pmatrix}+\begin{pmatrix}1&3\\2&4\end{pmatrix}\begin{pmatrix}6&3\\0&-2\end{pmatrix}=\begin{pmatrix}1&2\\2&0\end{pmatrix}+\begin{pmatrix}6&-3\\12&-2\end{pmatrix}=\begin{pmatrix}7&-1\\14&-2\end{pmatrix}$$

分块矩阵的运算：乘积

例 2 设 $A=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{pmatrix}$，$B=\begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$AB=$$

分块矩阵的运算：乘积

例 2 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array}\right) = \left(\begin{array}{c|c} I & O \\ \hline A_1 & 2I \end{array}\right)$, $B = \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$AB =$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array}\right)=\left(\begin{array}{c|c} I & O \\ \hline A_1 & 2I \end{array}\right)$, $B=\left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)=\left(\begin{array}{c|c} -I & B_1 \\ \hline O & I \end{array}\right)$

$$AB=$$

分块矩阵的运算：乘积

例 2 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array}\right) = \left(\begin{array}{c|c} I & O \\ \hline A_1 & 2I \end{array}\right)$， $B = \left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) = \left(\begin{array}{c|c} -I & B_1 \\ \hline O & I \end{array}\right)$

$$AB = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix} \begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix} =$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc:cc}1&0&0&0\\0&1&0&0\\ \hdashline 1&3&2&0\\5&2&0&2\end{array}\right)=\left(\begin{array}{c:c}I&O\\ \hdashline A_1&2I\end{array}\right)$, $B=\left(\begin{array}{cc:cc}-1&0&2&1\\0&-1&3&4\\ \hdashline 0&0&1&0\\0&0&0&1\end{array}\right)=\left(\begin{array}{c:c}-I&B_1\\ \hdashline O&I\end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix}I&O\\A_1&2I\end{pmatrix}\begin{pmatrix}-I&B_1\\O&I\end{pmatrix}=$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc:cc}1&0&0&0\\0&1&0&0\\ \hdashline 1&3&2&0\\5&2&0&2\end{array}\right)=\left(\begin{array}{c:c}I&O\\ \hdashline A_1&2I\end{array}\right)$，$B=\left(\begin{array}{cc:cc}-1&0&2&1\\0&-1&3&4\\ \hdashline 0&0&1&0\\0&0&0&1\end{array}\right)=\left(\begin{array}{c:c}-I&B_1\\ \hdashline O&I\end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix}I&O\\A_1&2I\end{pmatrix}\begin{pmatrix}-I&B_1\\O&I\end{pmatrix}=\begin{pmatrix}\quad\end{pmatrix}$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc|cc}1&0&0&0\\0&1&0&0\\\hline 1&3&2&0\\5&2&0&2\end{array}\right)=\left(\begin{array}{c|c}I&O\\\hline A_1&2I\end{array}\right)$, $B=\left(\begin{array}{cc|cc}-1&0&2&1\\0&-1&3&4\\\hline 0&0&1&0\\0&0&0&1\end{array}\right)=\left(\begin{array}{c|c}-I&B_1\\\hline O&I\end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix}I&O\\A_1&2I\end{pmatrix}\begin{pmatrix}-I&B_1\\O&I\end{pmatrix}=\begin{pmatrix}I(-I)+OO& \\ & \end{pmatrix}$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc|cc}1&0&0&0\\0&1&0&0\\\hline 1&3&2&0\\5&2&0&2\end{array}\right)=\left(\begin{array}{c|c}I&O\\\hline A_1&2I\end{array}\right)$，$B=\left(\begin{array}{cc|cc}-1&0&2&1\\0&-1&3&4\\\hline 0&0&1&0\\0&0&0&1\end{array}\right)=\left(\begin{array}{c|c}-I&B_1\\\hline O&I\end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix}I&O\\A_1&2I\end{pmatrix}\begin{pmatrix}-I&B_1\\O&I\end{pmatrix}=\begin{pmatrix}I(-I)+OO&IB_1+OI\\ & \end{pmatrix}$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc:cc}1&0&0&0\\0&1&0&0\\ \hdashline 1&3&2&0\\5&2&0&2\end{array}\right)=\left(\begin{array}{c:c}I&O\\ \hdashline A_1&2I\end{array}\right)$，$B=\left(\begin{array}{cc:cc}-1&0&2&1\\0&-1&3&4\\ \hdashline 0&0&1&0\\0&0&0&1\end{array}\right)=\left(\begin{array}{c:c}-I&B_1\\ \hdashline O&I\end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix}I&O\\A_1&2I\end{pmatrix}\begin{pmatrix}-I&B_1\\O&I\end{pmatrix}=\begin{pmatrix}I(-I)+OO&IB_1+OI\\A_1(-I)+2IO&\end{pmatrix}$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc|cc}1&0&0&0\\0&1&0&0\\\hline 1&3&2&0\\5&2&0&2\end{array}\right)=\left(\begin{array}{c|c}I&O\\\hline A_1&2I\end{array}\right)$，$B=\left(\begin{array}{cc|cc}-1&0&2&1\\0&-1&3&4\\\hline 0&0&1&0\\0&0&0&1\end{array}\right)=\left(\begin{array}{c|c}-I&B_1\\\hline O&I\end{array}\right)$

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分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc:cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hdashline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array}\right)=\left(\begin{array}{c:c} I & O \\ \hdashline A_1 & 2I \end{array}\right)$，$B=\left(\begin{array}{cc:cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)=\left(\begin{array}{c:c} -I & B_1 \\ \hdashline O & I \end{array}\right)$

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$$=\begin{pmatrix} \quad \end{pmatrix}=$$

分块矩阵的运算：乘积

例 2 设 $A = \left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array}\right) = \left(\begin{array}{c|c} I & O \\ \hline A_1 & 2I \end{array}\right)$，$B = \left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) = \left(\begin{array}{c|c} -I & B_1 \\ \hline O & I \end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB = \begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}\begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix} = \begin{pmatrix} I(-I) + OO & IB_1 + OI \\ A_1(-I) + 2IO & A_1B_1 + 2I \end{pmatrix}$$

$$= \begin{pmatrix} -I & & \\ & & \end{pmatrix} =$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array}\right)=\left(\begin{array}{c|c} I & O \\ \hline A_1 & 2I \end{array}\right)$, $B=\left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)=\left(\begin{array}{c|c} -I & B_1 \\ \hline O & I \end{array}\right)$

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$$AB=\begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}\begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}=\begin{pmatrix} I(-I)+OO & IB_1+OI \\ A_1(-I)+2IO & A_1B_1+2I \end{pmatrix}$$

$$=\begin{pmatrix} -I & B_1 \\ & \end{pmatrix}=$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc|cc}1&0&0&0\\0&1&0&0\\\hline 1&3&2&0\\5&2&0&2\end{array}\right)=\left(\begin{array}{c|c}I&O\\\hline A_1&2I\end{array}\right)$，$B=\left(\begin{array}{cc|cc}-1&0&2&1\\0&-1&3&4\\\hline 0&0&1&0\\0&0&0&1\end{array}\right)=\left(\begin{array}{c|c}-I&B_1\\\hline O&I\end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix}I&O\\A_1&2I\end{pmatrix}\begin{pmatrix}-I&B_1\\O&I\end{pmatrix}=\begin{pmatrix}I(-I)+OO&IB_1+OI\\A_1(-I)+2IO&A_1B_1+2I\end{pmatrix}$$

$$=\begin{pmatrix}-I&B_1\\-A_1&\end{pmatrix}=$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc:cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hdashline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array}\right)=\left(\begin{array}{c:c} I & O \\ \hdashline A_1 & 2I \end{array}\right)$， $B=\left(\begin{array}{cc:cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)=\left(\begin{array}{c:c} -I & B_1 \\ \hdashline O & I \end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}\begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}=\begin{pmatrix} I(-I)+OO & IB_1+OI \\ A_1(-I)+2IO & A_1B_1+2I \end{pmatrix}$$

$$=\begin{pmatrix} -I & B_1 \\ -A_1 & A_1B_1+2I \end{pmatrix}=$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array}\right)=\left(\begin{array}{c|c} I & O \\ \hline A_1 & 2I \end{array}\right)$，$B=\left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)=\left(\begin{array}{c|c} -I & B_1 \\ \hline O & I \end{array}\right)$

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$$AB=\begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}\begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}=\begin{pmatrix} I(-I)+OO & IB_1+OI \\ A_1(-I)+2IO & A_1B_1+2I \end{pmatrix}$$

$$=\begin{pmatrix} -I & B_1 \\ -A_1 & A_1B_1+2I \end{pmatrix}=\left(\begin{array}{c|c} & \\ \hline & \end{array}\right)$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc:cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hdashline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array}\right)=\left(\begin{array}{c:c} I & O \\ \hdashline A_1 & 2I \end{array}\right)$, $B=\left(\begin{array}{cc:cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hdashline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)=\left(\begin{array}{c:c} -I & B_1 \\ \hdashline O & I \end{array}\right)$

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$$AB=\begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}\begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}=\begin{pmatrix} I(-I)+OO & IB_1+OI \\ A_1(-I)+2IO & A_1B_1+2I \end{pmatrix}$$

$$=\begin{pmatrix} -I & B_1 \\ -A_1 & A_1B_1+2I \end{pmatrix}=\left(\begin{array}{cc:cc} -1 & 0 & & \\ 0 & -1 & & \\ \hdashline & & & \\ & & & \end{array}\right)$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array}\right)=\left(\begin{array}{c|c} I & O \\ \hline A_1 & 2I \end{array}\right)$, $B=\left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)=\left(\begin{array}{c|c} -I & B_1 \\ \hline O & I \end{array}\right)$

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$$=\begin{pmatrix} -I & B_1 \\ -A_1 & A_1B_1+2I \end{pmatrix}=\left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline & & & \\ & & & \end{array}\right)$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc|cc}1&0&0&0\\0&1&0&0\\\hline 1&3&2&0\\5&2&0&2\end{array}\right)=\left(\begin{array}{c|c}I&O\\\hline A_1&2I\end{array}\right)$， $B=\left(\begin{array}{cc|cc}-1&0&2&1\\0&-1&3&4\\\hline 0&0&1&0\\0&0&0&1\end{array}\right)=\left(\begin{array}{c|c}-I&B_1\\\hline O&I\end{array}\right)$

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$$=\begin{pmatrix}-I&B_1\\-A_1&A_1B_1+2I\end{pmatrix}=\left(\begin{array}{cc|cc}-1&0&2&1\\0&-1&3&4\\\hline -1&-3&&\\-5&-2&&\end{array}\right)$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array}\right)=\left(\begin{array}{c|c} I & O \\ \hline A_1 & 2I \end{array}\right)$, $B=\left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)=\left(\begin{array}{c|c} -I & B_1 \\ \hline O & I \end{array}\right)$

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$$=\begin{pmatrix} -I & B_1 \\ -A_1 & A_1B_1+2I \end{pmatrix}=\left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline -1 & -3 & & \\ -5 & -2 & & \end{array}\right)$$

其中

$$A_1B_1+2I=$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc|cc} 1&0&0&0\\ 0&1&0&0\\ \hline 1&3&2&0\\ 5&2&0&2 \end{array}\right)=\left(\begin{array}{c|c} I&O\\ \hline A_1&2I \end{array}\right)$, $B=\left(\begin{array}{cc|cc} -1&0&2&1\\ 0&-1&3&4\\ \hline 0&0&1&0\\ 0&0&0&1 \end{array}\right)=\left(\begin{array}{c|c} -I&B_1\\ \hline O&I \end{array}\right)$

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$$AB=\begin{pmatrix} I&O\\ A_1&2I \end{pmatrix}\begin{pmatrix} -I&B_1\\ O&I \end{pmatrix}=\begin{pmatrix} I(-I)+OO & IB_1+OI\\ A_1(-I)+2IO & A_1B_1+2I \end{pmatrix}$$

$$=\begin{pmatrix} -I & B_1\\ -A_1 & A_1B_1+2I \end{pmatrix}=\left(\begin{array}{cc|cc} -1&0&2&1\\ 0&-1&3&4\\ \hline -1&-3&&\\ -5&-2&& \end{array}\right)$$

其中

$$A_1B_1+2I=\begin{pmatrix} 1&3\\ 5&2 \end{pmatrix}\begin{pmatrix} 2&1\\ 3&4 \end{pmatrix}+2\begin{pmatrix} 1&0\\ 0&1 \end{pmatrix}=$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc|cc}1&0&0&0\\0&1&0&0\\ \hline 1&3&2&0\\5&2&0&2\end{array}\right)=\left(\begin{array}{c|c}I&O\\ \hline A_1&2I\end{array}\right)$，$B=\left(\begin{array}{cc|cc}-1&0&2&1\\0&-1&3&4\\ \hline 0&0&1&0\\0&0&0&1\end{array}\right)=\left(\begin{array}{c|c}-I&B_1\\ \hline O&I\end{array}\right)$

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$$AB=\begin{pmatrix}I&O\\A_1&2I\end{pmatrix}\begin{pmatrix}-I&B_1\\O&I\end{pmatrix}=\begin{pmatrix}I(-I)+OO&IB_1+OI\\A_1(-I)+2IO&A_1B_1+2I\end{pmatrix}$$

$$=\begin{pmatrix}-I&B_1\\-A_1&A_1B_1+2I\end{pmatrix}=\left(\begin{array}{cc|cc}-1&0&2&1\\0&-1&3&4\\ \hline -1&-3&&\\-5&-2&&\end{array}\right)$$

其中

$$A_1B_1+2I=\begin{pmatrix}1&3\\5&2\end{pmatrix}\begin{pmatrix}2&1\\3&4\end{pmatrix}+2\begin{pmatrix}1&0\\0&1\end{pmatrix}=\begin{pmatrix}11&13\\16&13\end{pmatrix}+$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc|cc}1&0&0&0\\0&1&0&0\\\hline 1&3&2&0\\5&2&0&2\end{array}\right)=\left(\begin{array}{c|c}I&O\\\hline A_1&2I\end{array}\right)$, $B=\left(\begin{array}{cc|cc}-1&0&2&1\\0&-1&3&4\\\hline 0&0&1&0\\0&0&0&1\end{array}\right)=\left(\begin{array}{c|c}-I&B_1\\\hline O&I\end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix}I&O\\A_1&2I\end{pmatrix}\begin{pmatrix}-I&B_1\\O&I\end{pmatrix}=\begin{pmatrix}I(-I)+OO&IB_1+OI\\A_1(-I)+2IO&A_1B_1+2I\end{pmatrix}$$

$$=\begin{pmatrix}-I&B_1\\-A_1&A_1B_1+2I\end{pmatrix}=\left(\begin{array}{cc|cc}-1&0&2&1\\0&-1&3&4\\\hline -1&-3&&\\-5&-2&&\end{array}\right)$$

其中

$$A_1B_1+2I=\begin{pmatrix}1&3\\5&2\end{pmatrix}\begin{pmatrix}2&1\\3&4\end{pmatrix}+2\begin{pmatrix}1&0\\0&1\end{pmatrix}=\begin{pmatrix}11&13\\16&13\end{pmatrix}+\begin{pmatrix}2&0\\0&2\end{pmatrix}$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc|cc}1&0&0&0\\0&1&0&0\\\hline 1&3&2&0\\5&2&0&2\end{array}\right)=\left(\begin{array}{c|c}I&O\\\hline A_1&2I\end{array}\right)$， $B=\left(\begin{array}{cc|cc}-1&0&2&1\\0&-1&3&4\\\hline 0&0&1&0\\0&0&0&1\end{array}\right)=\left(\begin{array}{c|c}-I&B_1\\\hline O&I\end{array}\right)$

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$$AB=\begin{pmatrix}I&O\\A_1&2I\end{pmatrix}\begin{pmatrix}-I&B_1\\O&I\end{pmatrix}=\begin{pmatrix}I(-I)+OO&IB_1+OI\\A_1(-I)+2IO&A_1B_1+2I\end{pmatrix}$$

$$=\begin{pmatrix}-I&B_1\\-A_1&A_1B_1+2I\end{pmatrix}=\left(\begin{array}{cc|cc}-1&0&2&1\\0&-1&3&4\\\hline -1&-3&&\\-5&-2&&\end{array}\right)$$

其中

$$A_1B_1+2I=\begin{pmatrix}1&3\\5&2\end{pmatrix}\begin{pmatrix}2&1\\3&4\end{pmatrix}+2\begin{pmatrix}1&0\\0&1\end{pmatrix}=\begin{pmatrix}11&13\\16&13\end{pmatrix}+\begin{pmatrix}2&0\\0&2\end{pmatrix}=\begin{pmatrix}13&13\\16&15\end{pmatrix}$$

分块矩阵的运算：乘积

例 2 设 $A=\left(\begin{array}{cc|cc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 1 & 3 & 2 & 0 \\ 5 & 2 & 0 & 2 \end{array}\right)=\left(\begin{array}{c|c} I & O \\ \hline A_1 & 2I \end{array}\right)$， $B=\left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)=\left(\begin{array}{c|c} -I & B_1 \\ \hline O & I \end{array}\right)$

（验证：A 的列划分与 B 的行划分方式相同）则

$$AB=\begin{pmatrix} I & O \\ A_1 & 2I \end{pmatrix}\begin{pmatrix} -I & B_1 \\ O & I \end{pmatrix}=\begin{pmatrix} I(-I)+OO & IB_1+OI \\ A_1(-I)+2IO & A_1B_1+2I \end{pmatrix}$$

$$=\begin{pmatrix} -I & B_1 \\ -A_1 & A_1B_1+2I \end{pmatrix}=\left(\begin{array}{cc|cc} -1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 4 \\ \hline -1 & -3 & 13 & 13 \\ -5 & -2 & 16 & 15 \end{array}\right)$$

其中

$$A_1B_1+2I=\begin{pmatrix} 1 & 3 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}+2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}=\begin{pmatrix} 11 & 13 \\ 16 & 13 \end{pmatrix}+\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}=\begin{pmatrix} 13 & 13 \\ 16 & 15 \end{pmatrix}$$

例 3 设 A, B 均为 2 阶方阵，且 $|A| = 2, |B| = 3$，计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

例 3 设 A, B 均为 2 阶方阵，且 $|A| = 2, |B| = 3$，计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix} \begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix}$$

例 3 设 A, B 均为 2 阶方阵，且 $|A|=2$, $|B|=3$，计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix}\begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix}\begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} OO + AA^* & \\ & \end{pmatrix}$$

例 3 设 A, B 均为 2 阶方阵，且 $|A|=2, |B|=3$，计算分块矩阵的乘积

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix}\begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}$$

并计算该乘积的行列式。

解

$$\begin{pmatrix} O & A \\ B & O \end{pmatrix}\begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix} = \begin{pmatrix} OO+AA^* & OB^*+AO \\ & \end{pmatrix}$$

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并计算该乘积的行列式。

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$$= \begin{pmatrix} AA^* & O \\ O & BB^* \end{pmatrix} = \begin{pmatrix} |A|I_2 & O \\ O & |B|I_2 \end{pmatrix} = \begin{pmatrix} 2I_2 & O \\ O & 3I_2 \end{pmatrix} =$$

例 3 设 A, B 均为 2 阶方阵，且 $|A|=2, |B|=3$，计算分块矩阵的乘积

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例 3 设 A, B 均为 2 阶方阵，且 $|A|=2, |B|=3$，计算分块矩阵的乘积

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所以 $\left|\begin{pmatrix} O & A \\ B & O \end{pmatrix}\begin{pmatrix} O & B^* \\ A^* & O \end{pmatrix}\right| = \begin{vmatrix} 2 & & & \\ & 2 & & \\ & & 3 & \\ & & & 3 \end{vmatrix} =$

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例 3 设 A, B 均为 2 阶方阵，且 $|A| = 2$, $|B| = 3$，计算分块矩阵的乘积

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例 4 设分块矩阵

$$D = \begin{pmatrix} A_{r\times r} & C \\ O & B_{k\times k} \end{pmatrix}$$

其中 $A_{r\times r}$ 和 $B_{k\times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

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$$\begin{pmatrix} I & O \\ O & I \end{pmatrix} = \begin{pmatrix} A & C \\ O & B \end{pmatrix} \begin{pmatrix} & \\ & \end{pmatrix}$$

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解 若存在矩阵 W, X, Y, Z 使得

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则 D 可逆，且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。

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则 D 可逆，且 $D^{-1}=\begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。由上式得

$$\begin{cases} AX+CW=I \\ AZ+CY=O \\ BW=O \\ BY=I \end{cases}$$

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则 D 可逆，且 $D^{-1} = \begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。由上式得

$$\begin{cases} AX + CW = I \\ AZ + CY = O \\ BW = O \\ BY = I \end{cases} \Rightarrow \begin{cases} Z = -A^{-1}CY \\ W = O \\ Y = B^{-1} \end{cases}$$

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则 D 可逆，且 $D^{-1}=\begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。由上式得

$$\begin{cases} AX+CW=I \\ AZ+CY=O \\ BW=O \\ BY=I \end{cases} \Rightarrow \begin{cases} X=A^{-1}(I-CW) \\ Z=-A^{-1}CY=-A^{-1}CB^{-1} \\ W=O \\ Y=B^{-1} \end{cases}$$

例 4 设分块矩阵

$$D=\begin{pmatrix} A_{r\times r} & C \\ O & B_{k\times k} \end{pmatrix}$$

其中 $A_{r\times r}$ 和 $B_{k\times k}$ 均为可逆方阵，证明 D 可逆并求出 D^{-1}。

解 若存在矩阵 W, X, Y, Z 使得

$$\begin{pmatrix} I & O \\ O & I \end{pmatrix}=\begin{pmatrix} A & C \\ O & B \end{pmatrix}\begin{pmatrix} X_{r\times r} & Z_{r\times k} \\ W_{k\times r} & Y_{k\times k} \end{pmatrix}=\begin{pmatrix} AX+CW & AZ+CY \\ BW & BY \end{pmatrix}$$

则 D 可逆，且 $D^{-1}=\begin{pmatrix} X & Z \\ W & Y \end{pmatrix}$。由上式得

$$\begin{cases} AX+CW=I \\ AZ+CY=O \\ BW=O \\ BY=I \end{cases} \Rightarrow \begin{cases} X=A^{-1}(I-CW)=A^{-1} \\ Z=-A^{-1}CY=-A^{-1}CB^{-1} \\ W=O \\ Y=B^{-1} \end{cases}$$

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$$A, B\text{ 可逆} \quad \Rightarrow \quad \begin{pmatrix} A & C \\ O & B \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}CB^{-1} \\ O & B^{-1} \end{pmatrix}$$

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分块矩阵的运算：乘积

例 5 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I=\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

分块矩阵的运算：乘积

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$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I=\left(\begin{array}{c:c:c:c} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{array}\right)$$

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分块矩阵的运算：乘积

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则

$$AI$$

分块矩阵的运算：乘积

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$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I=\left(\begin{array}{c:c:c:c} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{array}\right)=\begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix}$$

则

$$AI=A\begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix}=$$

分块矩阵的运算：乘积

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则

$$AI = A\begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix} = \begin{pmatrix} A\varepsilon_1 & A\varepsilon_2 & \cdots & A\varepsilon_n \end{pmatrix}$$

分块矩阵的运算：乘积

例 5 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I=\left(\begin{array}{c|c|c|c} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{array}\right)=\begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix}$$

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$$=\left(\begin{array}{c|c|c|c} \quad & \quad & \quad & \quad \end{array}\right)$$

分块矩阵的运算：乘积

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$$AI=A\begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix}=\begin{pmatrix} A\varepsilon_1 & A\varepsilon_2 & \cdots & A\varepsilon_n \end{pmatrix}$$

$$=\left(\begin{array}{c:c:c:c} a_{11} & & & \\ a_{21} & & & \\ \vdots & & & \\ a_{m1} & & & \end{array}\right)$$

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分块矩阵的运算：乘积

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分块矩阵的运算：乘积

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分块矩阵的运算：乘积

例 5 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad I=\left(\begin{array}{c:c:c:c} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{array}\right)=\begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix}$$

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$$= \left(\begin{array}{c:c:c:c} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{array}\right) = \begin{pmatrix} A_1 & A_2 & \cdots & A_n \end{pmatrix} = A$$

分块矩阵的运算：乘积

例 6 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B=\begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{pmatrix}$$

分块矩阵的运算：乘积

例 6 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B=\left(\begin{array}{c:c:c:c} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{array}\right)$$

分块矩阵的运算：乘积

例 6 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B=\left(\begin{array}{c:c:c:c} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{array}\right)=\begin{pmatrix}\beta_1 & \beta_2 & \cdots & \beta_l\end{pmatrix}$$

分块矩阵的运算：乘积

例 6 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B=\left(\begin{array}{c:c:c:c} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{array}\right)=\begin{pmatrix} \beta_1 & \beta_2 & \cdots & \beta_l \end{pmatrix}$$

则

$$AB=A(\beta_1, \beta_2, \cdots, \beta_l)$$

分块矩阵的运算：乘积

例 6 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B=\left(\begin{array}{c:c:c:c} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{array}\right)=\begin{pmatrix} \beta_1 & \beta_2 & \cdots & \beta_l \end{pmatrix}$$

则

$$AB=A(\beta_1,\beta_2,\cdots,\beta_l)=(A\beta_1,A\beta_2,\cdots,A\beta_l)$$

分块矩阵的运算：乘积

例 6 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B=\left(\begin{array}{c:c:c:c} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{array}\right)=(\beta_1 \ \ \beta_2 \ \ \cdots \ \ \beta_l)$$

则

$$AB=A(\beta_1,\beta_2,\cdots,\beta_l)=(A\beta_1,A\beta_2,\cdots,A\beta_l)$$

而

$A\beta_i=$

分块矩阵的运算：乘积

例 6 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B=\left(\begin{array}{c:c:c:c} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{array}\right)=(\beta_1 \ \ \beta_2 \ \ \cdots \ \ \beta_l)$$

则

$$AB=A(\beta_1, \beta_2, \cdots, \beta_l)=(A\beta_1, A\beta_2, \cdots, A\beta_l)$$

而

$$A\beta_i=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}\begin{pmatrix} b_{1i} \\ b_{2i} \\ \vdots \\ b_{ni} \end{pmatrix}$$

分块矩阵的运算：乘积

例 6 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B=\left(\begin{array}{c:c:c:c} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{array}\right)=(\beta_1 \ \ \beta_2 \ \ \cdots \ \ \beta_l)$$

则

$$AB=A(\beta_1,\beta_2,\cdots,\beta_l)=(A\beta_1,A\beta_2,\cdots,A\beta_l)$$

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分块矩阵的运算：乘积

例 6 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B=\left(\begin{array}{c:c:c:c} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{array}\right)=\begin{pmatrix} \beta_1 & \beta_2 & \cdots & \beta_l \end{pmatrix}$$

则

$$AB=A(\beta_1,\beta_2,\cdots,\beta_l)=(A\beta_1,A\beta_2,\cdots,A\beta_l)$$

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分块矩阵的运算：乘积

例 6 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B=\left(\begin{array}{c|c|c|c} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{array}\right)=\begin{pmatrix} \beta_1 & \beta_2 & \cdots & \beta_l \end{pmatrix}$$

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$$AB=A(\beta_1,\beta_2,\cdots,\beta_l)=(A\beta_1,A\beta_2,\cdots,A\beta_l)$$

而

$$A\beta_i=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}\begin{pmatrix} b_{1i} \\ b_{2i} \\ \vdots \\ b_{ni} \end{pmatrix}=\begin{pmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_n \end{pmatrix}\begin{pmatrix} b_{1i} \\ b_{2i} \\ \vdots \\ b_{ni} \end{pmatrix}$$

$$=b_{1i}\alpha_1+b_{2i}\alpha_2+\cdots+b_{ni}\alpha_n$$

分块矩阵的运算：乘积

例 6 设

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad B=\left(\begin{array}{c:c:c:c} b_{11} & b_{12} & \cdots & b_{1l} \\ b_{21} & b_{22} & \cdots & b_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nl} \end{array}\right)=(\beta_1 \ \beta_2 \ \cdots \ \beta_l)$$

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$$AB=A(\beta_1, \beta_2, \cdots, \beta_l)=(A\beta_1, A\beta_2, \cdots, A\beta_l)$$

而

$$A\beta_i=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}\begin{pmatrix} b_{1i} \\ b_{2i} \\ \vdots \\ b_{ni} \end{pmatrix}=(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n)\begin{pmatrix} b_{1i} \\ b_{2i} \\ \vdots \\ b_{ni} \end{pmatrix}$$

$$=b_{1i}\alpha_1+b_{2i}\alpha_2+\cdots+b_{ni}\alpha_n=b_{1i}\begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}+b_{2i}\begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix}+\cdots+b_{ni}\begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix}$$