

Outline

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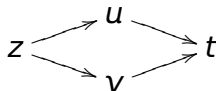
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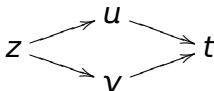


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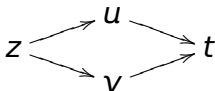
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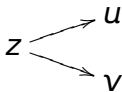
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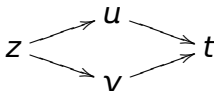


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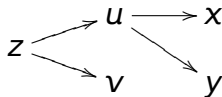
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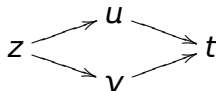


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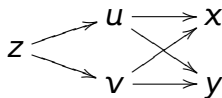
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二元复合函数求导公式——中间变量是一元函数

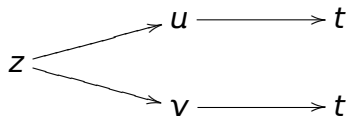
公式 设 $z = f(u, v)$, $u = \varphi(t)$, $v = \psi(t)$, 则 $z = f(\varphi(t), \psi(t))$ 的**全导数**

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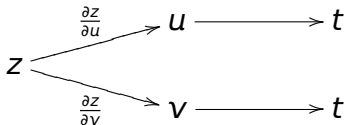
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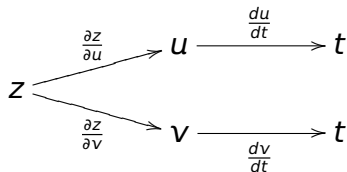
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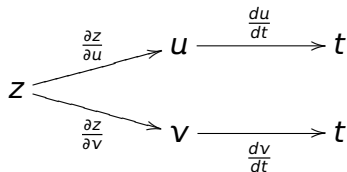
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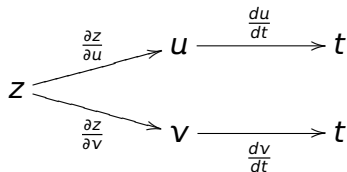
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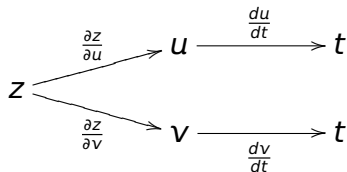
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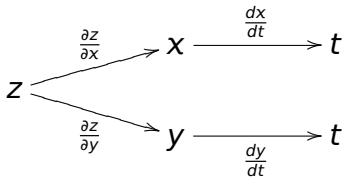
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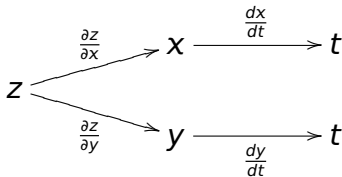
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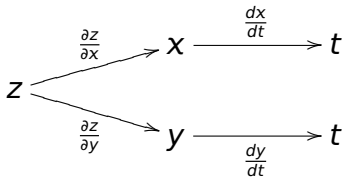
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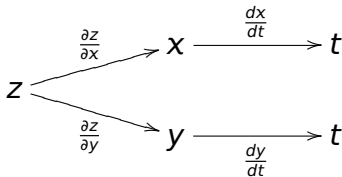
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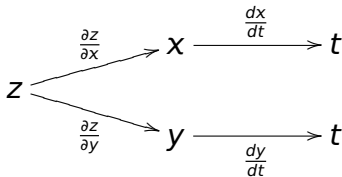
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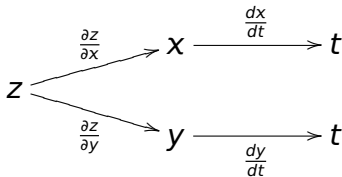
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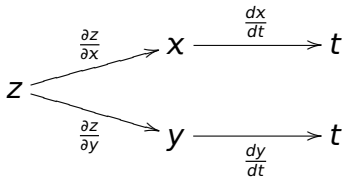
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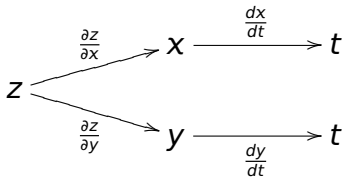
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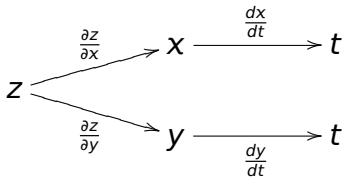
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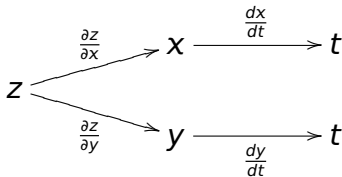
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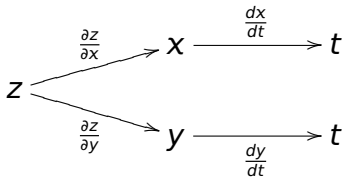
$$\begin{aligned}\frac{dz}{dt} &= \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt} = \left(\frac{y}{x}\right)'_x \cdot (e^t)'_t + \left(\frac{y}{x}\right)'_y \cdot (1 - e^{2t})'_t \\ &= -\frac{y}{x^2} \cdot e^t + \frac{1}{x} \cdot (-2e^{2t}) =\end{aligned}$$



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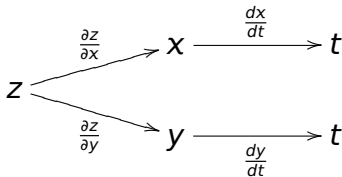
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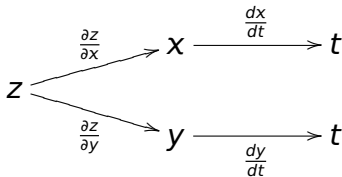
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三元复合函数求导公式——中间变量是一元函数

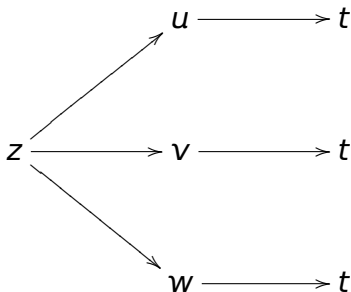
公式 设 $z = f(u, v, w)$, $u = \varphi(t)$, $v = \psi(t)$, $w = \omega(t)$, 则 $z = f(\varphi(t), \psi(t), \omega(t))$ 的全导数

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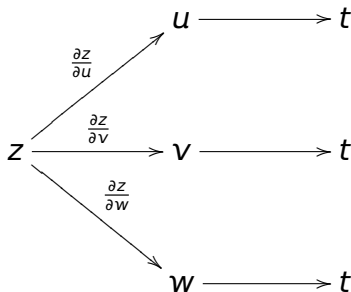
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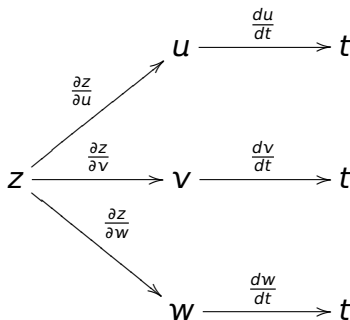
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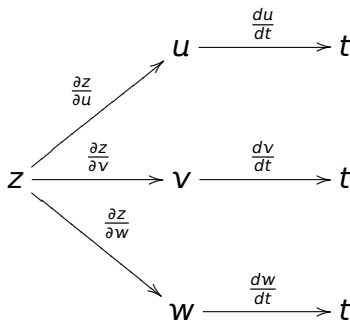
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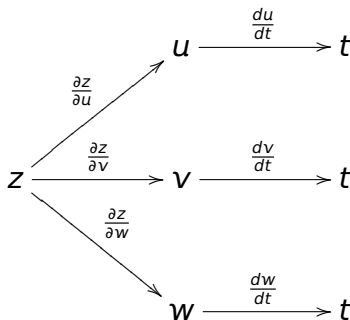
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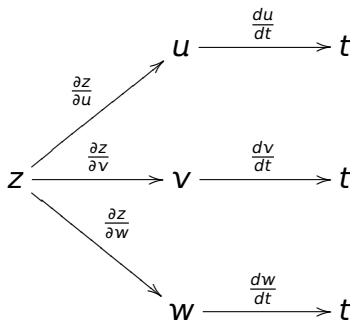
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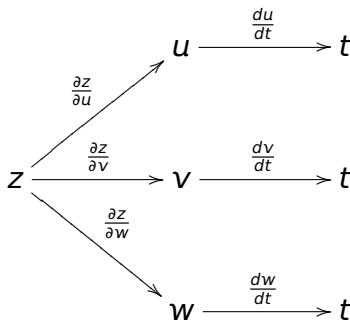
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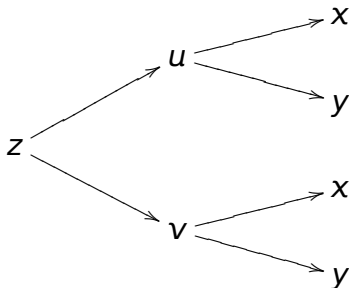
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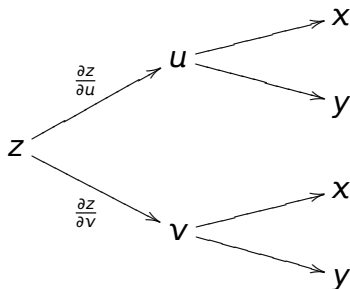
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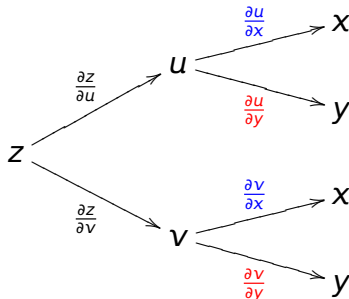
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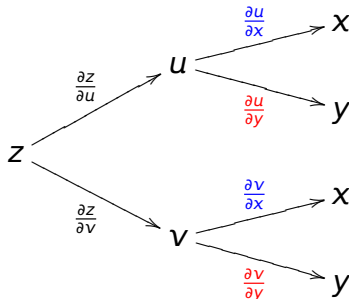
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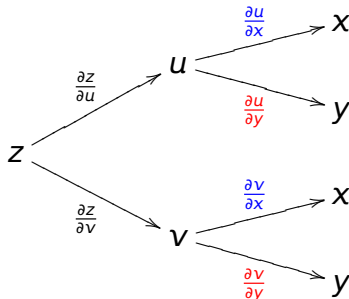
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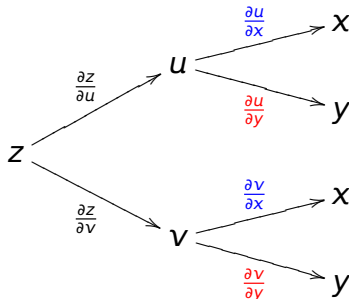
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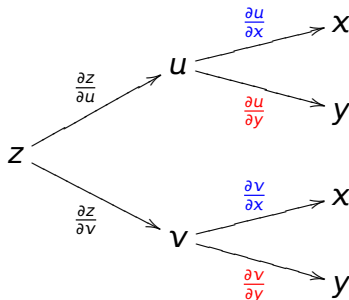
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例 设 $z = e^{2u} \sin v$, $u = x^3 y$, $v = x^2 + y^2$, 求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$\begin{aligned}\frac{\partial z}{\partial y} &= \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y} \\&= (e^{2u} \sin v)'_u \cdot (x^3 y)'_y +\end{aligned}$$

例 设 $z = e^{2u} \sin v$, $u = x^3 y$, $v = x^2 + y^2$, 求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$\begin{aligned}\frac{\partial z}{\partial x} &= \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x} \\&= (e^{2u} \sin v)'_u \cdot (x^3 y)'_x + (e^{2u} \sin v)'_v \cdot (x^2 + y^2)'_x \\&= 2e^{2u} \sin v \cdot 3x^2 y + e^{2u} \cos v \cdot 2x \\&= 2e^{2x^3 y} \sin(x^2 + y^2) \cdot 3x^2 y + e^{2x^3 y} \cos(x^2 + y^2) \cdot 2x\end{aligned}$$

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三元复合函数求导公式：举例

公式 设 $z = f(x, y, u)$, $u = u(x, y)$,

三元复合函数求导公式：举例

公式 设 $z = f(x, y, u)$, $u = u(x, y)$, 则复合函数

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的偏导数是：

$$\frac{\partial z}{\partial x} = \quad , \quad \frac{\partial z}{\partial y} =$$

三元复合函数求导公式：举例

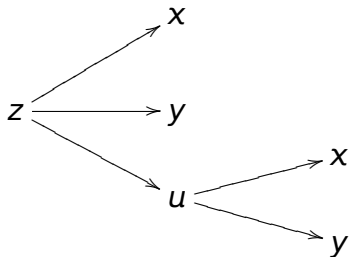
公式 设 $z = f(x, y, u)$, $u = u(x, y)$, 则复合函数

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图示



三元复合函数求导公式：举例

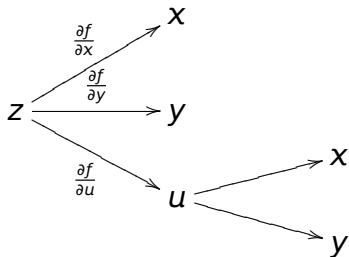
公式 设 $z = f(x, y, u)$, $u = u(x, y)$, 则复合函数

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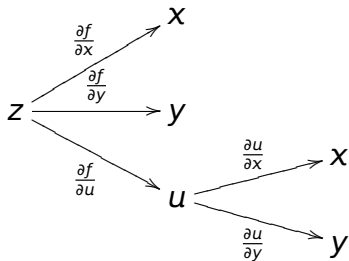
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的偏导数是：

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图示



三元复合函数求导公式：举例

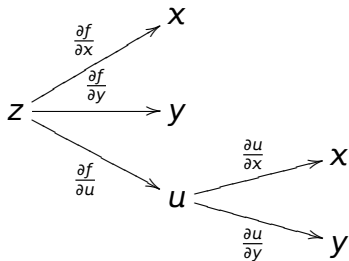
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图示



三元复合函数求导公式：举例

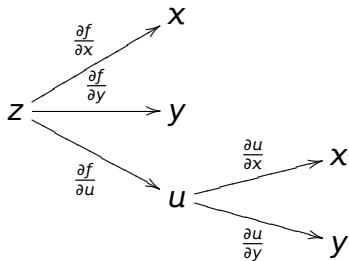
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$$z = f(x, y, u(x, y))$$

的偏导数是：

$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x}, \quad \frac{\partial z}{\partial y} =$$

图示



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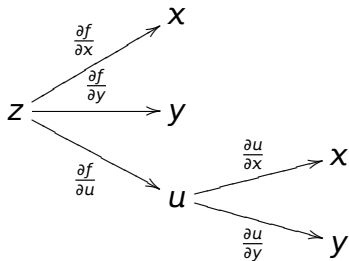
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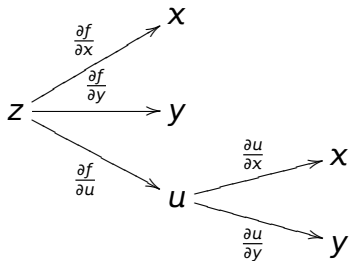
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图示



复合函数的高阶导数

公式 设 $z = f(u, v)$, $u = u(x, y)$, $v = v(x, y)$, 则复合函数

$$z = f(u(x, y), v(x, y))$$

的偏导数是:

$$z_x = z_u \cdot u_x + z_v \cdot v_x,$$

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$$z_{xy} =$$

$$z_{yy} =$$

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$$z_{xy} =$$

$$z_{yy} =$$

复合函数的高阶导数

公式 设 $z = f(u, v)$, $u = u(x, y)$, $v = v(x, y)$, 则复合函数

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例 设 $z = f(xy^2, x^2y)$, 求 $\frac{\partial^2 z}{\partial x \partial y}$

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