§6.6 定积分的分部积分法

2016-2017 学年 II

教学要求

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Outline of §6.6

- 求定积分 $\int_a^b f(x)dx$ 可分成两步：
  1. 求出不定积分 $\int f(x)dx = F(x) + C$

     （方法：直接积分法、换元积分法、分部积分法（第五章））
  2. $\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$

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  2. $\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$
- 在实际操作中，两步可合成一步：
  - 以分部积分法为例说明

- 不定积分的分部积分：

$$\int u dv = uv - \int v du$$

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$$\int_a^b u dv = uv\Big|_a^b - \int_a^b v du$$

分部积分法——例

例 计算定积分 $\int_0^{\frac{1}{2}} \arcsin x dx$

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$$\frac{1}{\sqrt{1-x^2}} dx$$

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练习 计算定积分 $\int_0^1 xe^{-x}dx$

解

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$$\int_0^1 xe^{-x}dx = -\int_0^1 xde^{-x} = -\left(xe^{-x} \quad - \int \quad e^{-x}dx\right)$$

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$$\int_0^1 xe^{-x}dx = -\int_0^1 x de^{-x} = -\left(xe^{-x}\Big|_0^1 - \int_0^1 e^{-x}dx \right)$$

练习 计算定积分 $\int_0^1 xe^{-x}dx$

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$$\int_0^1 xe^{-x}dx = -\int_0^1 xde^{-x} = -\left(xe^{-x}\Big|_0^1 - \int_0^1 e^{-x}dx\right)$$

$$= -\Big([e^{-1} - 0] - \qquad\qquad\qquad\Big)$$

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$$= -\left([e^{-1} - 0] - (-e^{-x})\Big|_0^1\right)$$

$$= -\left(e^{-1} + e^{-x}\Big|_0^1\right)$$

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$$= -(e^{-1} + e^{-1} - 1)$$

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$$= -\left(e^{-1} + e^{-x}\Big|_0^1\right)$$

$$= -\left(e^{-1} + e^{-1} - 1\right) = 1 - \frac{2}{e}$$

练习 计算定积分 $\int_0^{\frac{\pi}{2}} x\sin x dx$

解

$$\int_0^{\frac{\pi}{2}} x\sin x dx =$$

练习 计算定积分 $\int_0^{\frac{\pi}{2}} x \sin x dx$

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$$= \sin x\Big|_0^{\frac{\pi}{2}}$$

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$$= -\left([0-0] - \sin x\Big|_0^{\frac{\pi}{2}} \right)$$

$$= \sin x\Big|_0^{\frac{\pi}{2}} = 1 - 0 = 1$$