

### §3.2 向量与向量组的线性组合

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- 零向量  $O = (0, 0, \dots, 0)$

# 向量的线性运算

• 设  $\alpha = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$ ,  $\beta = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$ ,  $k \in \mathbb{R}$ , 则

$$\alpha + \beta = \quad , \quad \alpha - \beta = \quad , \quad k\alpha =$$

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- 行向量类似

# 线性组合问题

给定向量组

$$\alpha_1 = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}, \alpha_2 = \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix}, \dots, \alpha_n = \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix}$$

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如果能, 则称  $\beta$  是向量组  $\alpha_1, \alpha_2, \dots, \alpha_n$  的线性组合。

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例 判断  $\beta$  能否由  $\alpha_1, \alpha_2, \alpha_3$  线性表示, 若能, 写出线性表示等式

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• (1) 问

$$\begin{array}{cccc} \beta & \alpha_1 & \alpha_2 & \alpha_3 \\ \begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} & \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \end{array}$$

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所以  $\beta$  不能由  $\alpha_1, \alpha_2, \alpha_3$  线性表出!

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例 设  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ ; 及  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$ , 问

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### 问题

- 一般地, 如何判断  $\beta$  能否由  $\alpha_1, \alpha_2, \dots, \alpha_n$  线性表出?
- 如果能线性表出, 如何求出  $k_1, k_2, \dots, k_n$  使

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_n\alpha_n = \beta?$$



# 线性表示问题与线性方程组求解问题的联系

问题是否存在数  $k_1, k_2, \dots, k_n$  使

$$\begin{matrix} \beta \\ \left( \begin{array}{c} b_1 \\ b_2 \\ \vdots \\ b_m \end{array} \right) \end{matrix} = k_1 \begin{matrix} \alpha_1 \\ \left( \begin{array}{c} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{array} \right) \end{matrix} + k_2 \begin{matrix} \alpha_2 \\ \left( \begin{array}{c} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{array} \right) \end{matrix} + \cdots + k_n \begin{matrix} \alpha_n \\ \left( \begin{array}{c} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{array} \right) \end{matrix}$$

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$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix}$$

## 线性表示问题与线性方程组求解问题的联系

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$$\begin{matrix} \beta \\ \left( \begin{array}{c} b_1 \\ b_2 \\ \vdots \\ b_m \end{array} \right) \end{matrix} = k_1 \begin{matrix} \alpha_1 \\ \left( \begin{array}{c} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{array} \right) \end{matrix} + k_2 \begin{matrix} \alpha_2 \\ \left( \begin{array}{c} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{array} \right) \end{matrix} + \cdots + k_n \begin{matrix} \alpha_n \\ \left( \begin{array}{c} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{array} \right) \end{matrix}$$

$$\begin{matrix} \alpha_1 \\ \left( \begin{array}{cccc} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{array} \right) \end{matrix} \left( \begin{array}{c} k_1 \\ k_2 \\ \vdots \\ k_n \end{array} \right)$$

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$$\begin{pmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix}$$

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$$r(A) = r(A:\beta)$$

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方程有解等价于

$$r(A) = r(A:\beta) \Leftrightarrow r(\alpha_1 \alpha_2 \cdots \alpha_n) = r(\alpha_1 \alpha_2 \cdots \alpha_n \beta)$$

# 初等行变换求线性表示问题——例 1

**例** 判断  $\beta$  是否能由  $\alpha_1, \alpha_2, \alpha_3$  线性表示, 若能, 写出线性表示等式。

(1)

$$(\alpha_1 \ \alpha_2 \ \alpha_3 | \beta) = \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right)$$

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- 所以  $r(\alpha_1 \alpha_2 \alpha_3) = 3$ ,  $r(\alpha_1 \alpha_2 \alpha_3 \beta) = 3$ , 成立

$$r(\alpha_1 \alpha_2 \alpha_3) = r(\alpha_1 \alpha_2 \alpha_3 \beta)$$

$\beta$  可由  $\alpha_1, \alpha_2, \alpha_3$  线性表示。

- 显然  $\beta' = \alpha'_1 - \alpha'_2 + \alpha'_3$ ,

## 初等行变换求线性表示问题——例 1

**例** 判断  $\beta$  是否能由  $\alpha_1, \alpha_2, \alpha_3$  线性表示, 若能, 写出线性表示等式。

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**注** 可证明: 作初等行变换不改变列与列之间的“线性关系”。

## 初等行变换求线性表示问题——例 2

**例** 判断  $\beta$  是否能由  $\alpha_1, \alpha_2, \alpha_3$  线性表示, 若能, 写出线性表示等式。

(2)

$$(\alpha_1 \ \alpha_2 \ \alpha_3 | \beta) = \left( \begin{array}{ccc|c} 1 & 2 & -1 & 4 \\ 2 & -1 & 3 & 3 \\ -1 & 1 & -2 & 0 \\ 5 & 1 & 4 & 11 \end{array} \right)$$

## 初等行变换求线性表示问题——例 2

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## 初等行变换求线性表示问题——例 2

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$\beta$  不能由  $\alpha_1, \alpha_2, \alpha_3$  线性表示。

# 初等行变换求线性表示问题——总结

**问题**  $\beta$  能否由  $\alpha_1, \alpha_2, \dots, \alpha_n$  线性表示？若能，写出线性表示等式。

**步骤**

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$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n \mid \beta)$$

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$$\beta \text{ 由 } \alpha_1, \alpha_2, \dots, \alpha_n \text{ 线性表示} \iff r(\alpha_1 \cdots \alpha_n) = r(\alpha_1 \cdots \alpha_n \ \beta)$$

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2. 行变换前后列与列的线性关系不变, 即:

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例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解

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例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4 - 2r_1]{r_3 - r_1} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4 + 6r_2]{r_3 + r_2}$$

例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 1 & 1 & 0 & | & 0 \\ 2 & -2 & 1 & | & 5 \end{pmatrix} \xrightarrow[r_4 - 2r_1]{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix}$$
$$\xrightarrow[r_4 + 6r_2]{r_3 + r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & -1 & | & -5 \\ 0 & 0 & 7 & | & -17 \end{pmatrix}$$

例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 1 & 1 & 0 & | & 0 \\ 2 & -2 & 1 & | & 5 \end{pmatrix} \xrightarrow[r_4 - 2r_1]{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix}$$
$$\xrightarrow[r_4 + 6r_2]{r_3 + r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & -5 & | & -5 \\ 0 & 0 & -5 & | & -5 \end{pmatrix}$$



例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 1 & 1 & 0 & | & 0 \\ 2 & -2 & 1 & | & 5 \end{pmatrix} \xrightarrow[r_4 - 2r_1]{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix}$$
$$\xrightarrow[r_4 + 6r_2]{r_3 + r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & -5 & | & -5 \\ 0 & 0 & -17 & | & -17 \end{pmatrix}$$

例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 1 & 1 & 0 & | & 0 \\ 2 & -2 & 1 & | & 5 \end{pmatrix} \xrightarrow[r_4 - 2r_1]{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix}$$
$$\xrightarrow[r_4 + 6r_2]{r_3 + r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & -5 & | & -5 \\ 0 & 0 & -17 & | & -17 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & 1 & | & 1 \\ 0 & 0 & 1 & | & 1 \end{pmatrix}$$

例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 1 & 1 & 0 & | & 0 \\ 2 & -2 & 1 & | & 5 \end{pmatrix} \xrightarrow[r_4 - 2r_1]{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix}$$

$$\xrightarrow[r_4 + 6r_2]{r_3 + r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & -5 & | & -5 \\ 0 & 0 & -17 & | & -17 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & 1 & | & 1 \\ 0 & 0 & 1 & | & 1 \end{pmatrix} \xrightarrow{r_4 - r_3}$$

例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 1 & 1 & 0 & | & 0 \\ 2 & -2 & 1 & | & 5 \end{pmatrix} \xrightarrow[r_4 - 2r_1]{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix}$$

$$\xrightarrow[r_4 + 6r_2]{r_3 + r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & -5 & | & -5 \\ 0 & 0 & -17 & | & -17 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & 1 & | & 1 \\ 0 & 0 & 1 & | & 1 \end{pmatrix} \xrightarrow{r_4 - r_3} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & 0 & 1 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3}$$

例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left( \begin{array}{ccc|c} & & & \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left( \begin{array}{ccc|c} 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left( \begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$



例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left( \begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{r_1-2r_2}$$

例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left( \begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{r_1-2r_2} \left( \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left( \begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{r_1-2r_2} \left( \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left( \begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{r_1-2r_2} \left( \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

所以  $r(\alpha_1 \alpha_2 \alpha_3) = r(\alpha_1 \alpha_2 \alpha_3 \beta)$ ,

例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left( \begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{r_1-2r_2} \left( \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

所以  $r(\alpha_1 \alpha_2 \alpha_3) = r(\alpha_1 \alpha_2 \alpha_3 \beta)$ , 能线性表示

例  $\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$  线性表示?

解  $\alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta$

$$\left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{array} \right) \xrightarrow[r_4-2r_1]{r_3-r_1} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right) \xrightarrow{(-1) \times r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{array} \right)$$

$$\xrightarrow[r_4+6r_2]{r_3+r_2} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array} \right) \rightarrow \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{r_4-r_3} \left( \begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow[r_1-3r_3]{r_2-2r_3} \left( \begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{r_1-2r_2} \left( \begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

所以  $r(\alpha_1 \alpha_2 \alpha_3) = r(\alpha_1 \alpha_2 \alpha_3 \beta)$ , 能线性表示, 且  $\beta = \alpha_1 - \alpha_2 + \alpha_3$

例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \end{array}$$



例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) & \xrightarrow{r_1 \leftrightarrow r_3} & & \end{array}$$

例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) & \xrightarrow{r_1 \leftrightarrow r_3} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \end{array}$$

例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) & \xrightarrow{r_1 \leftrightarrow r_3} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \end{array}$$

$$\begin{array}{l} \xrightarrow{r_2 + r_1} \\ r_3 - 2r_1 \\ r_4 - r_1 \end{array}$$

例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) & \xrightarrow{r_1 \leftrightarrow r_3} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\ & & \xrightarrow{\substack{r_2+r_1 \\ r_3-2r_1 \\ r_4-r_1}} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ & & & \\ & & & \\ & & & \end{array} \right) \end{array}$$

例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) & \xrightarrow{r_1 \leftrightarrow r_3} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\ & & \xrightarrow{\substack{r_2+r_1 \\ r_3-2r_1 \\ r_4-r_1}} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ & & & \\ & & & \end{array} \right) \end{array}$$

例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) & \xrightarrow{r_1 \leftrightarrow r_3} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\ & & \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \end{array} \right) \end{array}$$

例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) & \xrightarrow{r_1 \leftrightarrow r_3} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\ & & \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \end{array}$$

例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2}
 \end{array}$$



例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) & \xrightarrow{r_1 \leftrightarrow r_3} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\ & \xrightarrow{\substack{r_2+r_1 \\ r_3-2r_1 \\ r_4-r_1}} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) & \xrightarrow{\substack{r_3-3r_2 \\ r_4-6r_2}} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & -2 \\ 0 & 0 & -5 & -5 \end{array} \right) \end{array}$$

例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) & \xrightarrow{r_1 \leftrightarrow r_3} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\ & \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) & \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 1 \\ 0 & 0 & -5 & 0 \end{array} \right) \end{array}$$

例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) & \xrightarrow{r_1 \leftrightarrow r_3} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\ & \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) & \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2} & \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \end{array}$$

例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \\
 \xrightarrow{-\frac{1}{5} \times r_3} \rightarrow
 \end{array}$$

例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \\
 \xrightarrow{-\frac{1}{5} \times r_3} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right)
 \end{array}$$

例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \\
 \xrightarrow{-\frac{1}{5} \times r_3} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \xrightarrow{r_4 + 7r_3}
 \end{array}$$

例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \\
 \xrightarrow{-\frac{1}{5} \times r_3} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \xrightarrow{r_4 + 7r_3} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right)
 \end{array}$$

例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
 \xrightarrow[r_4 - r_1]{\begin{array}{l} r_2 + r_1 \\ r_3 - 2r_1 \end{array}} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array} \right) \xrightarrow[r_4 - 6r_2]{r_3 - 3r_2} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \\
 \xrightarrow{-\frac{1}{5} \times r_3} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \xrightarrow{r_4 + 7r_3} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right)
 \end{array}$$



例  $\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$  能否由  $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$ ,  $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$ ,  $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$  线性表示?

解

$$\begin{array}{c}
 \alpha_1 \quad \alpha_2 \quad \alpha_3 \quad \beta \\
 \left( \begin{array}{ccc|c} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{array} \right) \xrightarrow{r_1 \leftrightarrow r_3} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{array} \right) \\
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 \xrightarrow{-\frac{1}{5} \times r_3} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -7 & 1 \end{array} \right) \xrightarrow{r_4 + 7r_3} \left( \begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right)
 \end{array}$$

可见  $r(\alpha_1 \alpha_2 \alpha_3 \beta) = 4 > 3 = r(\alpha_1 \alpha_2 \alpha_3)$ ,

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 \end{array}$$

可见  $r(\alpha_1 \alpha_2 \alpha_3 \beta) = 4 > 3 = r(\alpha_1 \alpha_2 \alpha_3)$ , 所以不能线性表示。

# 向量组的线性组合

定义 设有两个向量组

$$(A): \alpha_1, \alpha_2, \dots, \alpha_s$$

$$(B): \beta_1, \beta_2, \dots, \beta_t$$

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$$\Longrightarrow \begin{cases} \alpha_1 = & \beta_1 + & \beta_2 + & \beta_3 \\ \alpha_2 = & \beta_1 + & \beta_2 + & \beta_3 \end{cases}$$

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$$\Longrightarrow \begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 + a_{31}\beta_3 \\ \alpha_2 = \beta_1 + \beta_2 + \beta_3 \end{cases}$$

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**注 1** 一般地, 向量组  $\alpha_1, \alpha_2, \dots, \alpha_s$  可由向量组  $\beta_1, \beta_2, \dots, \beta_t$  线性表示, 当且仅当存在矩阵  $A_{t \times s}$  满足:

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这时  $A$  的每一列表示线性组合的系数。



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这时  $A$  的每一列表示线性组合的系数。例如,

$$\alpha_j = a_{1j}\beta_1 + a_{2j}\beta_2 + \cdots + a_{tj}\beta_t$$

其中的系数就是  $A$  的第  $j$  列  $\begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{sj} \end{pmatrix}$ 。

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$$= (\beta_1, \beta_2, \dots, \beta_t) \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1s} \\ a_{21} & a_{22} & \cdots & a_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ a_{t1} & a_{t2} & \cdots & a_{ts} \end{pmatrix}$$

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**定理（向量组线性表示的传递性）** 假设向量组  $(A)$ ,  $(B)$ ,  $(C)$  满足： $(A)$  可由  $(B)$  线性表示， $(B)$  可由  $(C)$  线性表示，则  $(A)$  可由  $(C)$  线性表示。

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$$(\beta_1, \beta_2, \dots, \beta_t) = (\gamma_1, \gamma_2, \dots, \gamma_k)B_{k \times t}.$$

**定理（向量组线性表示的传递性）** 假设向量组  $(A)$ ,  $(B)$ ,  $(C)$  满足： $(A)$  可由  $(B)$  线性表示， $(B)$  可由  $(C)$  线性表示，则  $(A)$  可由  $(C)$  线性表示。

**证明** 设向量组  $\alpha_1, \alpha_2, \dots, \alpha_s$  可由向量组  $\beta_1, \beta_2, \dots, \beta_t$  线性表示：

$$(\alpha_1, \alpha_2, \dots, \alpha_s) = (\beta_1, \beta_2, \dots, \beta_t)A_{t \times s}.$$

向量组  $\beta_1, \beta_2, \dots, \beta_t$  可由向量组  $\gamma_1, \gamma_2, \dots, \gamma_k$  线性表示：

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将第 2 式代入第 1 式，可得

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$$(\beta_1, \beta_2, \dots, \beta_t) = (\gamma_1, \gamma_2, \dots, \gamma_k)B_{k \times t}.$$

将第 2 式代入第 1 式，可得

$$(\alpha_1, \alpha_2, \dots, \alpha_s) = (\gamma_1, \gamma_2, \dots, \gamma_k)B_{k \times t}A_{t \times s}$$

**定理（向量组线性表示的传递性）** 假设向量组  $(A)$ ,  $(B)$ ,  $(C)$  满足： $(A)$  可由  $(B)$  线性表示， $(B)$  可由  $(C)$  线性表示，则  $(A)$  可由  $(C)$  线性表示。

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将第 2 式代入第 1 式，可得

$$(\alpha_1, \alpha_2, \dots, \alpha_s) = (\gamma_1, \gamma_2, \dots, \gamma_k) \underbrace{B_{k \times t} A_{t \times s}}_{C_{k \times s}}$$

**定理（向量组线性表示的传递性）** 假设向量组  $(A)$ ,  $(B)$ ,  $(C)$  满足： $(A)$  可由  $(B)$  线性表示， $(B)$  可由  $(C)$  线性表示，则  $(A)$  可由  $(C)$  线性表示。

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**定理（向量组线性表示的传递性）** 假设向量组  $(A)$ ,  $(B)$ ,  $(C)$  满足： $(A)$  可由  $(B)$  线性表示， $(B)$  可由  $(C)$  线性表示，则  $(A)$  可由  $(C)$  线性表示。

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所以向量组  $\alpha_1, \alpha_2, \dots, \alpha_s$  可由向量组  $\gamma_1, \gamma_2, \dots, \gamma_k$  线性表示。

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将第 2 式代入第 1 式，可得

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所以向量组  $\alpha_1, \alpha_2, \dots, \alpha_s$  可由向量组  $\gamma_1, \gamma_2, \dots, \gamma_k$  线性表示。

（并且，线性组合的系数就是矩阵  $C$  的列。）



例 
$$\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

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则

$$\alpha_1 =$$

$$\alpha_2 =$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

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则

$$\alpha_1 = a_{11}( \quad ) + a_{21}( \quad )$$

$$\alpha_2 =$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

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则

$$\alpha_1 = a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}( \quad )$$

$$\alpha_2 =$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

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$$\alpha_1 = a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

$$\alpha_2 =$$

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$$\alpha_2 =$$



例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

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例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

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$$\alpha_2 = a_{12}(\quad) + a_{22}(\quad)$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

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$$\alpha_2 = a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}( \quad )$$

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$$\alpha_2 = a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

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则

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$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= ( \quad )\gamma_1 + ( \quad )\gamma_2 + ( \quad )\gamma_3 \end{aligned}$$

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$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + ( \quad )\gamma_2 + ( \quad )\gamma_3 \end{aligned}$$



例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{aligned}$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{aligned}$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{aligned}$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{aligned}$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{aligned}$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + \end{aligned}$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + \end{aligned}$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$



例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$

其中

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{pmatrix}$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$

其中

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{11}b_{21} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{12} & a_{21}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{12}b_{32} & a_{21}b_{31} + a_{22}b_{32} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases} \Rightarrow (\alpha_1, \alpha_2) = (\beta_1, \beta_2) \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$

其中

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{11}b_{21} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{12} & a_{21}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{12}b_{32} & a_{21}b_{31} + a_{22}b_{32} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases} \Rightarrow (\alpha_1, \alpha_2) = (\beta_1, \beta_2) \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases} \Rightarrow (\beta_1, \beta_2) = (\gamma_1, \gamma_2, \gamma_3) \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$

其中

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{11}b_{21} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{12} & a_{21}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{12}b_{32} & a_{21}b_{31} + a_{22}b_{32} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases} \Rightarrow (\alpha_1, \alpha_2) = (\beta_1, \beta_2) \underbrace{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}}_A$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases} \Rightarrow (\beta_1, \beta_2) = (\gamma_1, \gamma_2, \gamma_3) \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$

其中

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{11}b_{21} + a_{12}b_{22} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases} \Rightarrow (\alpha_1, \alpha_2) = (\beta_1, \beta_2) \underbrace{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}}_A$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases} \Rightarrow (\beta_1, \beta_2) = (\gamma_1, \gamma_2, \gamma_3) \underbrace{\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}}_B$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$

其中

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{11}b_{21} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{12} & a_{21}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

例  $\left. \begin{array}{l} \alpha_1, \alpha_2 \text{ 由 } \beta_1, \beta_2 \text{ 线性表示} \\ \beta_1, \beta_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示} \end{array} \right\} \Rightarrow \alpha_1, \alpha_2 \text{ 由 } \gamma_1, \gamma_2, \gamma_3 \text{ 线性表示}$

具体地, 设

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases} \Rightarrow (\alpha_1, \alpha_2) = (\beta_1, \beta_2) \underbrace{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}}_A$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases} \Rightarrow (\beta_1, \beta_2) = (\gamma_1, \gamma_2, \gamma_3) \underbrace{\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}}_B$$

则

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \end{aligned}$$

$$\begin{aligned} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{aligned}$$

其中

$$(c_{ij}) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{11}b_{21} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{12} & a_{21}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{12}b_{32} & a_{21}b_{31} + a_{22}b_{32} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = BA$$

定义 设有两个向量组

$$(A): \alpha_1, \alpha_2, \dots, \alpha_s$$

$$(B): \beta_1, \beta_2, \dots, \beta_t$$

如果  $(A)$  与  $(B)$  可相互线性表示, 则称向量组  $(A)$  与  $(B)$  等价。