

Outline

1. 一个方程的情形

2. 方程组的情形

We are here now...

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隐函数的求导法 I

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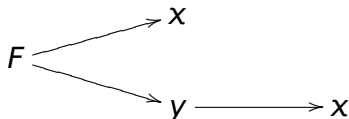
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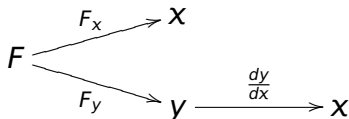
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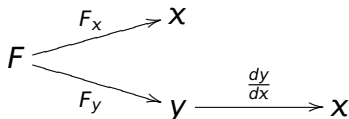
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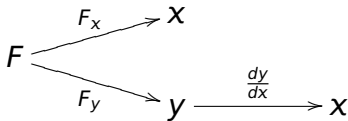
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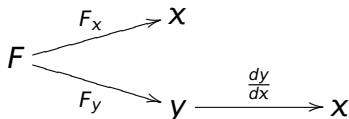
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$$\text{所以 } y' = -\frac{e^x - y^2}{\cos y - 2xy}$$

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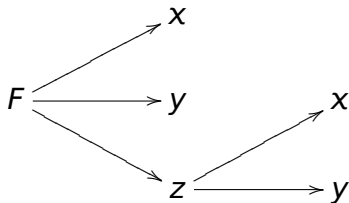
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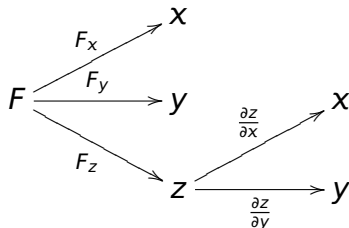
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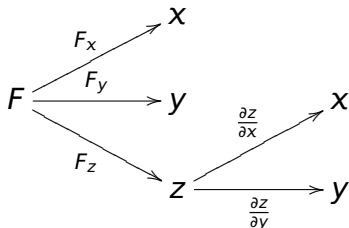
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$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \quad (F_z \neq 0)$$

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$$\because F(x, y, z(x, y)) = 0$$

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隐函数的求导法 II

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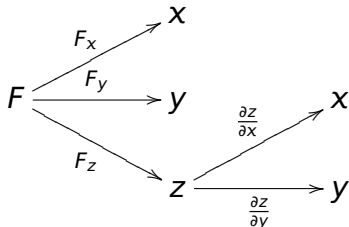
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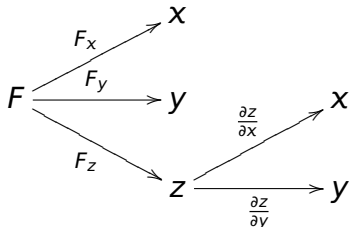
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例 设 $z = f(x, y)$ 满足 $2 \sin(x + 2y - 3z) = x + 2y - 3z$, 求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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解 令 $F(x, y, z) = z - y - x + xe^{z-y-x}$, 则 $F(x, y, z) = 0$, 所以

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$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy = \frac{1 + (x-1)e^{z-y-x}}{1 + xe^{z-y-x}} dx + dy$$

例 设 $\Phi(u, v)$ 具有连续偏导数, 函数 $z = z(x, y)$ 满足 $\Phi(cx - az, cy - bz) = 0$, 证明:

$$a \frac{\partial z}{\partial x} + b \frac{\partial z}{\partial y} = c.$$

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$$F_y =$$

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解 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$, 则

$$F_x = \Phi_u \cdot u_x + \Phi_v \cdot v_x = c \Phi_u$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y$$

$$F_z =$$

$$\frac{\partial z}{\partial x} = - \frac{F_x}{F_z} =$$

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We are here now...

1. 一个方程的情形

2. 方程组的情形

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

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回顾：二元线性方程组的求解

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(1) $\times a_{22}$ - (2) $\times a_{12}$ ，消去 y ，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

(2) $\times a_{11}$ - (1) $\times a_{21}$ ，消去 x ，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

(1) $\times a_{22}$ - (2) $\times a_{12}$ ，消去 y ，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

(2) $\times a_{11}$ - (1) $\times a_{21}$ ，消去 x ，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

(1) $\times a_{22}$ - (2) $\times a_{12}$ ，消去 y ，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{b_1 a_{22} - a_{12} b_2}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

(2) $\times a_{11}$ - (1) $\times a_{21}$ ，消去 x ，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{a_{11} b_2 - b_1 a_{21}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

(1) $\times a_{22}$ - (2) $\times a_{12}$ ，消去 y ，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

(2) $\times a_{11}$ - (1) $\times a_{21}$ ，消去 x ，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

回顾：二元线性方程组的求解

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解：

(1) $\times a_{22}$ - (2) $\times a_{12}$ ，消去 y ，得：

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

(2) $\times a_{11}$ - (1) $\times a_{21}$ ，消去 x ，得：

$$y = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

公式：

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

公式:

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练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \quad , \quad y =$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad , \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \text{---} \quad , \quad y =$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad , \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \text{---} \quad , \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \text{---}$$

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公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -\frac{1}{1}, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1}, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \bar{1}$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad , \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

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$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \quad, \quad y =$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-17}{-17} = 1, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-7-2}{-17} = \frac{9}{17}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-17}{35}, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-10}{35} = -\frac{2}{7}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-17}{3}, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-10}{3}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{1}{3}, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-5}{3}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3}, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-3}{3}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

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公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$2. \begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} \quad x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3}$$

公式:

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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方程组的隐函数求导公式

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

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总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

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所以

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$$v_y = - \frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{matrix} \xRightarrow{\frac{\partial}{\partial x}} \\ \xRightarrow{\frac{\partial}{\partial y}} \end{matrix} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \\ F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$\begin{aligned} u_x &= - \frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, & v_x &= - \frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \\ u_y &= - \frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, & v_y &= - \frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \end{aligned}$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_x = - \frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_x = - \frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(u, x)}$$

$$u_y = - \frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = - \frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_x = - \frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_x = - \frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(u, x)}$$

$$u_y = - \frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)}, \quad v_y = - \frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

总结 设 $u = u(x, y)$, $v = v(x, y)$ 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_x = - \frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_x = - \frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(u, x)}$$

$$u_y = - \frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)}, \quad v_y = - \frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = - \frac{1}{J} \frac{\partial(F, G)}{\partial(u, y)}$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \begin{matrix} \xRightarrow{\frac{\partial}{\partial x}} \\ \xRightarrow{\frac{\partial}{\partial y}} \end{matrix}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

$$v_y =$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \begin{matrix} \xRightarrow{\frac{\partial}{\partial x}} \\ \xRightarrow{\frac{\partial}{\partial y}} \end{matrix}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

$$v_y =$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \end{cases}$$
$$\xRightarrow{\frac{\partial}{\partial y}}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

$$v_y =$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u \sin v \cdot v_x = 0 \end{cases}$$
$$\xRightarrow{\frac{\partial}{\partial y}}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

$$v_y =$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u \sin v \cdot v_x = 0 \end{cases}$$
$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \end{cases}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

$$v_y =$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u \sin v \cdot v_x = 0 \end{cases}$$
$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

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例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u \sin v \cdot v_x = 0 \end{cases}$$
$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

$$\text{所以 } J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$u_x =$$

$$v_x =$$

$$u_y =$$

$$v_y =$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u \sin v \cdot v_x = 0 \end{cases}$$
$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

所以 $J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$

$$u_x = \frac{\begin{vmatrix} \quad & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}}{J}$$

$$v_x = \frac{\begin{vmatrix} e^u + \sin v & \quad \\ e^u - \cos v & \quad \end{vmatrix}}{J}$$

$$u_y = \frac{\begin{vmatrix} \quad & u \cos v \\ e^u + \sin v & u \sin v \end{vmatrix}}{J}$$

$$v_y = \frac{\begin{vmatrix} e^u + \sin v & \quad \\ e^u - \cos v & \quad \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u \sin v \cdot v_x = 0 \end{cases}$$
$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

$$\text{所以 } J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$u_x = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_x = \frac{\begin{vmatrix} & \\ & \end{vmatrix}}{J}$$

$$u_y = \frac{\begin{vmatrix} & \\ & \end{vmatrix}}{J}$$

$$v_y = \frac{\begin{vmatrix} & \\ & \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

所以 $J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$

$$u_x = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_x = \frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J}$$

$$u_y = \frac{\begin{vmatrix} & u \cos v \\ & u \sin v \end{vmatrix}}{J}$$

$$v_y = \frac{\begin{vmatrix} & 1 \\ & 0 \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u \sin v \cdot v_x = 0 \end{cases}$$
$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

$$\text{所以 } J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$u_x = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_x = \frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J}$$

$$u_y = \frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}$$

$$v_y = \frac{\begin{vmatrix} & \\ & \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

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$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u \sin v \cdot v_x = 0 \end{cases}$$
$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

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$$u_x = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

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$$u_y = \frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}$$

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例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

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$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

$$\text{所以 } J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$u_x = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_x = \frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J}$$

$$u_y = \frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}$$

$$v_y = \frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u \sin v \cdot v_x = 0 \end{cases}$$
$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

$$\text{所以 } J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$u_x = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J} = \frac{\sin v}{e^u(\sin v - \cos v) + 1}, \quad v_x = \frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J}$$

$$u_y = \frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}, \quad v_y = \frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u \sin v \cdot v_x = 0 \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

$$\text{所以 } J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$u_x = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J} = \frac{\sin v}{e^u(\sin v - \cos v) + 1}, \quad v_x = \frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J} = \frac{-e^u + \cos v}{ue^u(\sin v - \cos v) + u}$$

$$u_y = \frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J} \quad v_y = \frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u \sin v \cdot v_x = 0 \end{cases}$$
$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

$$\text{所以 } J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$u_x = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J} = \frac{\sin v}{e^u(\sin v - \cos v) + 1}, \quad v_x = \frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J} = \frac{-e^u + \cos v}{ue^u(\sin v - \cos v) + u}$$

$$u_y = \frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J} = \frac{-\cos v}{e^u(\sin v - \cos v) + 1}, \quad v_y = \frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J}$$

例 设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$, 求 $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

解

$$\begin{cases} e^u + u \sin v = x \\ e^u - u \cos v = y \end{cases} \xRightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^u + \sin v)u_x + u \cos v \cdot v_x = 1 \\ (e^u - \cos v)u_x + u \sin v \cdot v_x = 0 \end{cases}$$

$$\xRightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^u + \sin v)u_y + u \cos v \cdot v_y = 0 \\ (e^u - \cos v)u_y + u \sin v \cdot v_y = 1 \end{cases}$$

$$\text{所以 } J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$u_x = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J} = \frac{\sin v}{e^u(\sin v - \cos v) + 1}, \quad v_x = \frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J} = \frac{-e^u + \cos v}{ue^u(\sin v - \cos v) + u}$$

$$u_y = \frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J} = \frac{-\cos v}{e^u(\sin v - \cos v) + 1}, \quad v_y = \frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J} = \frac{e^u + \sin v}{ue^u(\sin v - \cos v) + u}$$