§5.2 基本积分表

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教学要求

- ◇ 熟练运用基本积分公式
- ♣ 直接积分法



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Outline of §5.2

1. 基本积分表

2. 直接积分法



We are here now...

1. 基本积分表

2. 直接积分法

$$\Diamond$$
 $\int 0 dx = C$

$$\oint \int x^{\alpha} dx = \frac{1}{\alpha + 1} x^{\alpha + 1} + C, (\alpha \neq -1)$$

$$\oint \int \frac{1}{x} dx = \ln|x| + C$$

$$\oint \int 0 dx = C$$

$$\oint \int a^{x} dx = \frac{1}{\ln a} a^{x} + C$$

$$\oint \int x^{\alpha} dx = \frac{1}{\alpha + 1} x^{\alpha + 1} + C, (\alpha \neq -1)$$

$$\oint \int e^{x} dx = e^{x} + C$$

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$$\oint \int e^{x} dx = e^{x} + C$$

$$\oint \int \frac{1}{x} dx = \ln|x| + C$$

$$\bigvee \int \sin x dx = -\cos x + C$$

$$\bigvee \int \frac{1}{\cos^2 x} dx = \frac{\sin x}{\cos x} + C = \tan x + C$$



$$\Diamond$$
 $\int 0 dx = C$

$$\oint \int x^{\alpha} dx = \frac{1}{\alpha + 1} x^{\alpha + 1} + C, (\alpha \neq -1) \quad \bullet \quad \int e^{x} dx = e^{x} + C$$

$$\oint \int \frac{1}{x} dx = \ln|x| + C$$

$$\bigvee \int \frac{1}{\cos^2 x} dx = \frac{\sin x}{\cos x} + C = \tan x + C$$

$$\bullet \int a^x dx = \frac{1}{\ln a} a^x + C$$

$$\oint \int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$$

$$\oint \int \frac{1}{1+x^2} dx = \arctan x + C$$



We are here now...

1. 基本积分表

2. 直接积分法

例子 求不定积分
$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1-t^2}} - 5^t\right) dt$$

解 csc
$$t = \frac{1}{\sin t}$$



例子 求不定积分
$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1 - t^2}} - 5^t\right) dt$$

解 $\csc t = \frac{1}{\sin t}$

$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1 - t^2}} - 5^t\right) dt$$

$$= \int \frac{2}{t} dt - \int 3\cos t dt - \int \csc^2 t dt + \int \frac{2}{\sqrt{1 - t^2}} dt - \int 5^t dt$$



例子 求不定积分
$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1-t^2}} - 5^t\right) dt$$

解
$$\csc t = \frac{1}{\sin t}$$

$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1 - t^2}} - 5^t\right) dt$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty$$

$$= \int \frac{2}{t} dt - \int 3\cos t dt - \int \csc^2 t dt + \int \frac{2}{\sqrt{1-t^2}} dt - \int 5^t dt$$

$$= 2 \int \frac{1}{t} dt - 3 \int \cos t dt - \int \frac{1}{\sin^2 t} dt + 2 \int \frac{1}{\sqrt{1 - t^2}} dt - \int 5^t dt$$

例子 求不定积分
$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1-t^2}} - 5^t\right) dt$$

解 csc
$$t = \frac{1}{\sin t}$$

$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1 - t^2}} - 5^t\right) dt$$

$$\int \left(\frac{1}{t} - 3\cos t - \csc t + \frac{1}{\sqrt{1 - t^2}} - 3\right) dt$$

$$= \int \frac{2}{t} dt - \int 3\cos t dt - \int \csc^2 t dt + \int \frac{2}{\sqrt{1-t^2}} dt - \int 5^t dt$$

$$= 2 \int \frac{1}{t} dt - 3 \int \cos t dt - \int \frac{1}{\sin^2 t} dt + 2 \int \frac{1}{\sqrt{1 - t^2}} dt - \int 5^t dt$$

 $= 2 \ln |t|$



例子 求不定积分
$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1-t^2}} - 5^t\right) dt$$

$$\mathbf{m} \csc t = \frac{1}{\sin t}$$

$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1 - t^2}} - 5^t\right) dt$$

$$\int \left(\frac{1}{t} - 3\cos t - \csc t\right) + \frac{1}{\sqrt{1 - t^2}} - 3 \int dt$$

$$= \int \frac{2}{t}dt - \int 3\cos t dt - \int \csc^2 t dt + \int \frac{2}{\sqrt{1-t^2}}dt - \int 5^t dt$$

$$= 2 \int \frac{1}{t} dt - 3 \int \cos t dt - \int \frac{1}{\sin^2 t} dt + 2 \int \frac{1}{\sqrt{1 - t^2}} dt - \int 5^t dt$$

 $= 2 \ln |t| - 3 \sin t$



例子 求不定积分
$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1-t^2}} - 5^t\right) dt$$

解
$$\csc t = \frac{1}{\sin t}$$

$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1 - t^2}} - 5^t\right) dt$$

$$= \int \frac{2}{t} dt - \int 3\cos t dt - \int \csc^2 t dt + \int \frac{2}{\sqrt{1-t^2}} dt - \int 5^t dt$$

$$= \int \frac{dt}{t} - \int 3\cos t dt - \int \csc^2 t dt + \int \frac{1}{\sqrt{1 - t^2}} dt - \int 5^t dt$$

$$= 2 \int \frac{1}{t} dt - 3 \int \cos t dt - \int \frac{1}{\sin^2 t} dt + 2 \int \frac{1}{\sqrt{1 - t^2}} dt - \int 5^t dt$$

 $= 2 \ln |t| - 3 \sin t + \cot t$



例子 求不定积分
$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1-t^2}} - 5^t\right) dt$$

$$\mathbf{m} \csc t = \frac{1}{\sin t}$$

$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1 - t^2}} - 5^t\right) dt$$

$$\int \int t \sqrt{1-t^2} dt$$

$$= \int \frac{2}{3} dt - \int 3 \cos t dt - \int \csc^2 t dt + \int \frac{2}{\sqrt{1-t^2}} dt$$

$$= \int \frac{2}{t} dt - \int 3 \cos t dt - \int \csc^2 t dt + \int \frac{2}{\sqrt{1 - t^2}} dt - \int 5^t dt$$

$$= 2 \int \frac{1}{t} dt - 3 \int \cos t dt - \int \frac{1}{\sin^2 t} dt + 2 \int \frac{1}{\sqrt{1 - t^2}} dt - \int 5^t dt$$

 $= 2 \ln |t| - 3 \sin t + \cot t + 2 \arcsin t$



例子 求不定积分
$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1-t^2}} - 5^t\right) dt$$

解 csc
$$t = \frac{1}{\sin t}$$

$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1 - t^2}} - 5^t\right) dt$$

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$$= 2 \int \frac{1}{t} dt - 3 \int \cos t dt - \int \frac{1}{\sin^2 t} dt + 2 \int \frac{1}{\sqrt{1 - t^2}} dt - \int 5^t dt$$

=
$$2 \ln |t| - 3 \sin t + \cot t + 2 \arcsin t - \frac{1}{\ln 5} 5^t$$



例子 求不定积分
$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1-t^2}} - 5^t\right) dt$$

解
$$\csc t = \frac{1}{\sin t}$$

$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1 - t^2}} - 5^t\right) dt$$

$$\int \left(t - \frac{2}{\sqrt{1 - t^2}} \right)^{dt}$$

$$= \int \frac{2}{-dt} - \int 3\cos t dt - \int \csc^2 t dt + \int \frac{2}{-dt}$$

$$= \int \frac{2}{t} dt - \int 3\cos t dt - \int \csc^2 t dt + \int \frac{2}{\sqrt{1 - t^2}} dt - \int 5^t dt$$

$$= 2 \int \frac{1}{t} dt - 3 \int \cos t dt - \int \frac{1}{\sin^2 t} dt + 2 \int \frac{1}{\sqrt{1 - t^2}} dt - \int 5^t dt$$

=
$$2 \ln |t| - 3 \sin t + \cot t + 2 \arcsin t - \frac{1}{\ln 5} 5^t + C$$



熟练计算
$$\int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$$

求不定积分Ⅰ── "直接"利用基本积分公式 (Cont.)

熟练计算
$$\int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$$

例子 求不定积分
$$\int \sqrt{x^5} dx$$
, $\int \frac{1}{\sqrt{x^3}} dx$

求不定积分Ⅰ── "直接"利用基本积分公式 (Cont.)

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$$\int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$$

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例子 求不定积分
$$\int \sqrt{x^5} dx$$
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$$\int \sqrt{x^5} dx = \int (x^5)^{\frac{1}{2}} dx$$
$$\int \frac{1}{\sqrt{x^3}} dx$$

熟练计算
$$\int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$$

例子 求不定积分
$$\int \sqrt{x^5} dx$$
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$$\int \sqrt{x^5} dx = \int (x^5)^{\frac{1}{2}} dx = \int x^{5/2} dx =$$

$$\int \frac{1}{\sqrt{x^3}} dx$$

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例子 求不定积分
$$\int \sqrt{x^5} dx$$
, $\int \frac{1}{\sqrt{x^3}} dx$

$$\int \sqrt{x^5} dx = \int (x^5)^{\frac{1}{2}} dx = \int x^{5/2} dx = x^{\frac{5}{2} + 1}$$

$$\int \frac{1}{\sqrt{x^3}} dx$$



熟练计算
$$\int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$$

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$$\int \sqrt{x^5} dx$$
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$$\int \frac{1}{\sqrt{x^3}} dx$$

求不定积分Ⅰ── "直接"利用基本积分公式 (Cont.)

熟练计算
$$\int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$$

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求不定积分Ⅰ── "直接"利用基本积分公式 (Cont.)

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$$\int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$$

例子 求不定积分
$$\int \sqrt{x^5} dx$$
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$$\int \sqrt{x^5} dx = \int (x^5)^{\frac{1}{2}} dx = \int x^{5/2} dx = \frac{1}{\frac{5}{2} + 1} x^{\frac{5}{2} + 1} + C = \frac{2}{7} x^{\frac{7}{2}} + C$$

$$\int \frac{1}{\sqrt{x^3}} dx$$



熟练计算
$$\int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$$

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$$\int \frac{1}{\sqrt{x^3}} dx = \int \frac{1}{x^{3/2}} dx$$

求不定积分Ⅰ── "直接"利用基本积分公式 (Cont.)

熟练计算
$$\int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$$

例子 求不定积分
$$\int \sqrt{x^5} dx$$
, $\int \frac{1}{\sqrt{x^3}} dx$

$$\int \sqrt{x^5} dx = \int (x^5)^{\frac{1}{2}} dx = \int x^{5/2} dx = \frac{1}{\frac{5}{2} + 1} x^{\frac{5}{2} + 1} + C = \frac{2}{7} x^{\frac{7}{2}} + C$$

$$\int \frac{1}{\sqrt{x^3}} dx = \int \frac{1}{x^{3/2}} dx = \int x^{-3/2} dx$$

熟练计算
$$\int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$$

例子 求不定积分
$$\int \sqrt{x^5} dx$$
, $\int \frac{1}{\sqrt{x^3}} dx$

解

$$\int \sqrt{x^5} dx = \int (x^5)^{\frac{1}{2}} dx = \int x^{5/2} dx = \frac{1}{\frac{5}{2} + 1} x^{\frac{5}{2} + 1} + C = \frac{2}{7} x^{\frac{7}{2}} + C$$

$$\int \frac{1}{\sqrt{x^3}} dx = \int \frac{1}{x^{3/2}} dx = \int x^{-3/2} dx$$

 $x^{-\frac{3}{2}+1}$

熟练计算
$$\int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$$

例子 求不定积分
$$\int \sqrt{x^5} dx$$
, $\int \frac{1}{\sqrt{x^3}} dx$

$$\int \sqrt{x^5} dx = \int (x^5)^{\frac{1}{2}} dx = \int x^{5/2} dx = \frac{1}{\frac{5}{2} + 1} x^{\frac{5}{2} + 1} + C = \frac{2}{7} x^{\frac{7}{2}} + C$$

$$\int \frac{1}{\sqrt{x^3}} dx = \int \frac{1}{x^{3/2}} dx = \int x^{-3/2} dx$$

$$= \frac{1}{-\frac{3}{2} + 1} x^{-\frac{3}{2} + 1}$$



熟练计算
$$\int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$$

例子 求不定积分
$$\int \sqrt{x^5} dx$$
, $\int \frac{1}{\sqrt{x^3}} dx$

解

$$\int \sqrt{x^5} dx = \int (x^5)^{\frac{1}{2}} dx = \int x^{5/2} dx = \frac{1}{\frac{5}{2} + 1} x^{\frac{5}{2} + 1} + C = \frac{2}{7} x^{\frac{7}{2}} + C$$

$$\int \frac{1}{\sqrt{x^3}} dx = \int \frac{1}{x^{3/2}} dx = \int x^{-3/2} dx$$

$$= \frac{1}{-\frac{3}{2} + 1} x^{-\frac{3}{2} + 1} + C$$

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熟练计算
$$\int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$$

例子 求不定积分
$$\int \sqrt{x^5} dx$$
, $\int \frac{1}{\sqrt{x^3}} dx$

$$\int \sqrt{x^5} dx = \int (x^5)^{\frac{1}{2}} dx = \int x^{5/2} dx = \frac{1}{\frac{5}{2} + 1} x^{\frac{5}{2} + 1} + C = \frac{2}{7} x^{\frac{7}{2}} + C$$

$$\int \frac{1}{\sqrt{x^3}} dx = \int \frac{1}{x^{3/2}} dx = \int x^{-3/2} dx$$

$$\int \sqrt{x^3} \int \sqrt{x^3} dx = \int \sqrt{1 + 1} x^{-\frac{3}{2} + 1} + C = -2x^{-\frac{1}{2}} + C$$



熟练计算
$$\int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$$

例子 求不定积分
$$\int \sqrt{x^5} dx$$
, $\int \frac{1}{\sqrt{x^3}} dx$

$$\int \sqrt{x^5} dx = \int (x^5)^{\frac{1}{2}} dx = \int x^{5/2} dx = \frac{1}{\frac{5}{2} + 1} x^{\frac{5}{2} + 1} + C = \frac{2}{7} x^{\frac{7}{2}} + C$$

 $= \frac{1}{-\frac{3}{2}+1}x^{-\frac{3}{2}+1} + C = -2x^{-1/2} + C = -\frac{2}{\sqrt{x}} + C$

$$\int \frac{1}{\sqrt{x^3}} dx = \int \frac{1}{x^{3/2}} dx = \int x^{-3/2} dx$$



例子 求不定积分 $\int \frac{x}{\sqrt{x^5}} dx$, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\iint \frac{x}{\sqrt{x^5}} dx =$$

例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\iiint \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx =$$

例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = 0$$

例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\iiint \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = \int x^{-3/2} dx$$

例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\Re$$
 $\int \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = \int x^{-3/2} dx = \int x^{-\frac{1}{2}}$

例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\iint \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = \int x^{-3/2} dx = -2x^{-\frac{1}{2}}$$

例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\iiint \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = \int x^{-3/2} dx = -2x^{-\frac{1}{2}} + C$$

例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\Re \int \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = \int x^{-3/2} dx = -2x^{-\frac{1}{2}} + C$$

$$\int \frac{(3-x\sqrt{x})^2}{x} dx =$$

例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\Re \int \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = \int x^{-3/2} dx = -2x^{-\frac{1}{2}} + C$$

$$\int \frac{(3 - x\sqrt{x})^2}{x} dx = \int \frac{(3 - x^{\frac{3}{2}})^2}{x} dx =$$

例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\int \frac{(3 - x\sqrt{x})^2}{x} dx = \int \frac{(3 - x^{\frac{3}{2}})^2}{x} dx = \int \frac{9 - 6x^{\frac{3}{2}} + x^3}{x} dx$$



例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\Re \int \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = \int x^{-3/2} dx = -2x^{-\frac{1}{2}} + C$$

$$C (3 - x\sqrt{x})^2 \qquad C (3 - x^{\frac{3}{2}})^2 \qquad C 9 - 6x^{\frac{3}{2}} + x^3$$

$$\int \frac{(3-x\sqrt{x})^2}{x} dx = \int \frac{(3-x^{\frac{3}{2}})^2}{x} dx = \int \frac{9-6x^{\frac{3}{2}}+x^3}{x} dx$$
$$= \int \frac{9}{x} - 6x^{\frac{1}{2}} + x^2 dx =$$

例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\Re \int \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = \int x^{-3/2} dx = -2x^{-\frac{1}{2}} + C$$

$$(3 - x\sqrt{x})^2 \qquad (3 - x\sqrt{x})^2 \qquad (9 - 6x^{\frac{3}{2}} + x^3)$$

$$\int \frac{(3-x\sqrt{x})^2}{x} dx = \int \frac{(3-x^{\frac{3}{2}})^2}{x} dx = \int \frac{9-6x^{\frac{3}{2}}+x^3}{x} dx$$
$$= \int \frac{9}{x} - 6x^{\frac{1}{2}} + x^2 dx = 9 \ln|x|$$



例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\iint \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = \int x^{-3/2} dx = -2x^{-\frac{1}{2}} + C$$

$$\int \frac{(3-x\sqrt{x})^2}{x} dx = \int \frac{(3-x^{\frac{3}{2}})^2}{x} dx = \int \frac{9-6x^{\frac{3}{2}}+x^3}{x} dx$$
$$= \int \frac{9}{x} - 6x^{\frac{1}{2}} + x^2 dx = 9 \ln|x| \qquad x^{\frac{3}{2}}$$



例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\Re \int \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = \int x^{-3/2} dx = -2x^{-\frac{1}{2}} + C$$

$$\int \frac{(3-x\sqrt{x})^2}{x} dx = \int \frac{(3-x^{\frac{3}{2}})^2}{x} dx = \int \frac{9-6x^{\frac{3}{2}}+x^3}{x} dx$$
$$= \int \frac{9}{x} - 6x^{\frac{1}{2}} + x^2 dx = 9 \ln|x| - 4x^{\frac{3}{2}}$$



例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\Re \int \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = \int x^{-3/2} dx = -2x^{-\frac{1}{2}} + C$$

$$\int \frac{(3-x\sqrt{x})^2}{x} dx = \int \frac{(3-x^{\frac{3}{2}})^2}{x} dx = \int \frac{9-6x^{\frac{3}{2}}+x^3}{x} dx$$
$$= \int \frac{9}{x} - 6x^{\frac{1}{2}} + x^2 dx = 9 \ln|x| - 4x^{\frac{3}{2}} + \frac{1}{3}x^3$$



例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\int \frac{(3-x\sqrt{x})^2}{x} dx = \int \frac{(3-x^{\frac{3}{2}})^2}{x} dx = \int \frac{9-6x^{\frac{3}{2}}+x^3}{x} dx$$

$$= \int \frac{9}{x} - 6x^{\frac{1}{2}} + x^2 dx = 9 \ln|x| - 4x^{\frac{3}{2}} + \frac{1}{3}x^3 + C$$

例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\iiint \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = \int x^{-3/2} dx = -2x^{-\frac{1}{2}} + C$$

$$\int \frac{(3 - x\sqrt{x})^2}{x} dx = \int \frac{(3 - x^{\frac{3}{2}})^2}{x} dx = \int \frac{9 - 6x^{\frac{3}{2}} + x^3}{x} dx$$

$$\int \frac{(3-x\sqrt{x})}{x} dx = \int \frac{(3-x\sqrt{x})}{x} dx = \int \frac{3-3x\sqrt{x}}{x} dx$$
$$= \int \frac{9}{x} - 6x^{\frac{1}{2}} + x^2 dx = 9 \ln|x| - 4x^{\frac{3}{2}} + \frac{1}{3}x^3 + C$$

$$\int \frac{(1-\sqrt{x})^2}{x} dx =$$

例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\iiint \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = \int x^{-3/2} dx = -2x^{-\frac{1}{2}} + C$$

$$\int \frac{(3 - x\sqrt{x})^2}{x} dx = \int \frac{(3 - x^{\frac{3}{2}})^2}{x} dx = \int \frac{9 - 6x^{\frac{3}{2}} + x^3}{x} dx$$

$$\int \frac{1}{x} dx = \int \frac{1}{x} dx = \int \frac{1}{x} dx$$

$$= \int \frac{9}{x} - 6x^{\frac{1}{2}} + x^2 dx = 9 \ln|x| - 4x^{\frac{3}{2}} + \frac{1}{3}x^3 + C$$

$$\int \frac{(1-\sqrt{x})^2}{x} dx = \int \frac{(1-x^{\frac{1}{2}})^2}{x} dx =$$



例子 求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

$$\iiint \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = \int x^{-3/2} dx = -2x^{-\frac{1}{2}} + C$$

$$\int \frac{(3-x\sqrt{x})^2}{x} dx = \int \frac{(3-x^{\frac{3}{2}})^2}{x} dx = \int \frac{9-6x^{\frac{3}{2}}+x^3}{x} dx$$

$$= \int \frac{9}{x} - 6x^{\frac{1}{2}} + x^2 dx = 9 \ln|x| - 4x^{\frac{3}{2}} + \frac{1}{3}x^3 + C$$

$$\int \frac{(1 - \sqrt{x})^2}{x} dx = \int \frac{(1 - x^{\frac{1}{2}})^2}{x} dx = \int \frac{1 - 2x^{\frac{1}{2}} + x}{x} dx$$

例子 求不定积分 $\int \frac{x}{\sqrt{x^5}} dx$, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

提示 整理被积函数,化不定积分为
$$\int x^{\alpha} dx$$
 形式

$$\int \frac{dx}{\sqrt{x^5}} dx = \int \frac{x^{1-3/2}}{x^{5/2}} dx = \int x^{1-3/2} dx = \int x^{-3/2} dx = -2x^{-\frac{7}{2}} dx$$

$$\int (3-x\sqrt{x})^2 dx = \int (3-x^{\frac{3}{2}})^2 dx = \int 9-6x^{\frac{3}{2}} + x^3 dx$$

 $\int \frac{(3 - x\sqrt{x})^2}{x} dx = \int \frac{(3 - x^{\frac{3}{2}})^2}{x} dx = \int \frac{9 - 6x^{\frac{3}{2}} + x^3}{x} dx$

 $= \int \frac{9}{x} - 6x^{\frac{1}{2}} + x^2 dx = 9 \ln|x| - 4x^{\frac{3}{2}} + \frac{1}{3}x^3 + C$

 $\int \frac{(1-\sqrt{x})^2}{x} dx = \int \frac{(1-x^{\frac{1}{2}})^2}{x} dx = \int \frac{1-2x^{\frac{1}{2}}+x}{x} dx$ $=\int_{X}^{1}\frac{1}{x}-2x^{-\frac{1}{2}}+1dx=$



例子 求不定积分 $\int \frac{x}{\sqrt{x^5}} dx$, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

提示 整理被积函数,化不定积分为
$$\int x^{\alpha} dx$$
 形式

$$\int \frac{x}{\sqrt{x}} dx = \int \frac{x}{\sqrt{x}} dx = \int x^{1-5/2} dx = \int x^{1-5/2} dx$$

提示 整理被积函数,化不定积分为 $\int x^{\alpha}dx$ 形式 $\iiint \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = \int x^{-3/2} dx = -2x^{-\frac{1}{2}} + C$

 $\int \frac{(3 - x\sqrt{x})^2}{x} dx = \int \frac{(3 - x^{\frac{3}{2}})^2}{x} dx = \int \frac{9 - 6x^{\frac{3}{2}} + x^3}{x} dx$

 $\int \frac{(1-\sqrt{x})^2}{x} dx = \int \frac{(1-x^{\frac{1}{2}})^2}{x} dx = \int \frac{1-2x^{\frac{1}{2}}+x}{x} dx$

 $= \int_{-\infty}^{\infty} \frac{1}{x} - 2x^{-\frac{1}{2}} + 1dx = \ln|x|$

 $= \int \frac{9}{x} - 6x^{\frac{1}{2}} + x^2 dx = 9 \ln|x| - 4x^{\frac{3}{2}} + \frac{1}{3}x^3 + C$

例子 求不定积分 $\int \frac{x}{\sqrt{x^5}} dx$, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

提示 整理被积函数,化不定积分为
$$\int x^{\alpha} dx$$
 形式

 $\int \frac{(1-\sqrt{x})^2}{x} dx = \int \frac{(1-x^{\frac{1}{2}})^2}{x} dx = \int \frac{1-2x^{\frac{1}{2}}+x}{x} dx$

$$\int \frac{x}{\sqrt{x}} dx = \int \frac{x}{\sqrt{x}} dx = \int x^{1-5/2} dx = \int x^{1-5/2} dx$$

求不定积分
$$\int \frac{x}{\sqrt{x^5}} dx$$
, $\int \frac{(3-x\sqrt{x})^2}{x}$

 $= \int \frac{9}{x} - 6x^{\frac{1}{2}} + x^2 dx = 9 \ln|x| - 4x^{\frac{3}{2}} + \frac{1}{3}x^3 + C$

 $= \int \frac{1}{x} - 2x^{-\frac{1}{2}} + 1dx = \ln|x| \qquad x^{\frac{1}{2}}$

 $\int \frac{(3 - x\sqrt{x})^2}{x} dx = \int \frac{(3 - x^{\frac{3}{2}})^2}{x} dx = \int \frac{9 - 6x^{\frac{3}{2}} + x^3}{x} dx$

$$= \int x^{1-5/2} dx = \int$$

例子 求不定积分 $\int \frac{x}{\sqrt{x^5}} dx$, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

提示 整理被积函数,化不定积分为
$$\int x^{\alpha} dx$$
 形式

$$\int \frac{\sqrt{x^5}}{\sqrt{x^5}} dx = \int \frac{x^{5/2}}{x^{5/2}} dx = \int x^{-1} dx = -2x^{-25}$$

$$\int \frac{(3 - x\sqrt{x})^2}{x^5} dx = \int \frac{(3 - x^{\frac{3}{2}})^2}{x^5} dx = \int \frac{9 - 6x^{\frac{3}{2}} + x^3}{x^5} dx$$

$$\int \frac{(3-x\sqrt{x})^2}{x} dx = \int \frac{(3-x^{\frac{3}{2}})^2}{x} dx = \int \frac{9-6x^{\frac{3}{2}}+x^3}{x} dx$$

$$\int \frac{(3-x^2x)}{x} dx = \int \frac{(3-x^2)}{x} dx = \int \frac{9-6x^2+x}{x} dx$$

$$= \int \frac{9}{x^2} + x^2 dx = 0 \ln|x| + 4x^{\frac{3}{2}} + \frac{1}{x^3} + C$$

$$\int \frac{1}{x} dx = \int \frac{1}{x} dx = \int \frac{1}{x} dx$$
$$= \int \frac{9}{x} - 6x^{\frac{1}{2}} + x^2 dx = 9 \ln|x| - 4x^{\frac{3}{2}} + \frac{1}{3}x^3 + C$$

$$= \int \frac{9}{x} - 6x^{\frac{1}{2}} + x^2 dx = 9 \ln|x| - 4x^{\frac{3}{2}} + \frac{1}{3}x^3 + C$$

$$\int \frac{(1 - \sqrt{x})^2}{x} dx = \int \frac{(1 - x^{\frac{1}{2}})^2}{x} dx = \int \frac{1 - 2x^{\frac{1}{2}} + x}{x} dx$$

 $= \int_{x}^{1} \frac{1}{x} - 2x^{-\frac{1}{2}} + 1dx = \ln|x| - 4x^{\frac{1}{2}}$

$$\int \frac{(3-x\sqrt{x})^2}{x} dx = \int \frac{(3-x^{\frac{3}{2}})^2}{x} dx = \int \frac{9-6x^{\frac{3}{2}}+x^3}{x} dx$$

例子 求不定积分 $\int \frac{x}{\sqrt{x^5}} dx$, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

 $\int \frac{(1-\sqrt{x})^2}{x} dx = \int \frac{(1-x^{\frac{1}{2}})^2}{x} dx = \int \frac{1-2x^{\frac{1}{2}}+x}{x} dx$

提示 整理被积函数,化不定积分为
$$\int x^{\alpha} dx$$
 形式 $\int x^{\alpha} = \int x^{\alpha} \int x^{\alpha} dx$

$$\mathbf{H} \qquad \int \frac{x}{\sqrt{x^5}} dx = \int \frac{x}{x^{5/2}} dx = \int x^{1-5/2} dx = \int x^{-3/2} dx = -2x^{-\frac{1}{2}} + C$$

提示 整理被积函数,化不定积分为
$$\int x^{\alpha} dx$$
 形式 $\int x^{\alpha} dx$ 形式

$$x = \int x^{1-5/2} dx = \int$$

 $= \int \frac{1}{x} - 2x^{-\frac{1}{2}} + 1dx = \ln|x| - 4x^{\frac{1}{2}} + x$

$$x = \int \frac{9 - 6x^{\frac{3}{2}} + x^3}{x} dx$$

$$\int \frac{(3-x\sqrt{x})^2}{x} dx = \int \frac{(3-x^{\frac{3}{2}})^2}{x} dx = \int \frac{9-6x^{\frac{3}{2}}+x^3}{x} dx$$

 $= \int \frac{9}{x} - 6x^{\frac{1}{2}} + x^2 dx = 9 \ln|x| - 4x^{\frac{3}{2}} + \frac{1}{3}x^3 + C$

例子 求不定积分 $\int \frac{x}{\sqrt{x^5}} dx$, $\int \frac{(3-x\sqrt{x})^2}{x} dx$, $\int \frac{(1-\sqrt{x})^2}{x} dx$

 $\int \frac{(3 - x\sqrt{x})^2}{x} dx = \int \frac{(3 - x^{\frac{3}{2}})^2}{x} dx = \int \frac{9 - 6x^{\frac{3}{2}} + x^3}{x} dx$

 $\int \frac{(1-\sqrt{x})^2}{x} dx = \int \frac{(1-x^{\frac{1}{2}})^2}{x} dx = \int \frac{1-2x^{\frac{1}{2}}+x}{x} dx$

提示 整理被积函数,化不定积分为
$$\int x^{\alpha} dx$$
 形式

$$\begin{cases} X & \text{for } X \\ X & \text{for } X \end{cases}$$

 $= \int \frac{9}{x} - 6x^{\frac{1}{2}} + x^2 dx = 9 \ln|x| - 4x^{\frac{3}{2}} + \frac{1}{3}x^3 + C$

 $= \int \frac{1}{x} - 2x^{-\frac{1}{2}} + 1 dx = \ln|x| - 4x^{\frac{1}{2}} + x + C$

求不定积分 III: 形如∫ lá½»½ó,cè xª·c: ѧÆÚÌÕ dx

提示 将分式 " $\frac{la^{\prime}_{x}$ " $\frac{\lambda^{\prime}_{x}}{x^{2} \cdot c}$ " 拆成两个(或多个)简单的式子/分式

求不定积分 III: 形如 ∫ lá½»½ó,cÈ x² · c: ÑsÆúìō dx

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$

解

求不定积分 III: 形如 $\int \frac{\hat{l} \hat{a}/x y x \hat{c}_{i} \hat{c}_{k} \hat{c}_{k}}{x^{2} \cdot \hat{c}_{i} \hat{N} \delta \mathcal{F}_{k} \hat{U} \hat{I} \hat{O}} dx$

提示 将分式 " $\frac{ldV.»V.o.,c.}{x^2 \cdot c. \tilde{N}SE(\tilde{I})\tilde{O}}$ " 拆成两个(或多个)简单的式子/分式

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$ 解
$$\int \frac{x^2}{1+x^2} dx =$$

求不定积分 III: 形如∫ lá½»½ó,cÈ x² · c: ÑsÆúìō dx

提示 将分式 " $\frac{lá½ » ½ ó, cè}{x^2 + c: NSEÚIÕ}$ " 拆成两个(或多个)简单的式子/分式

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$ 解
$$\int \frac{x^2}{1+x^2} dx = \int \frac{1+x^2-1}{1+x^2} dx =$$

求不定积分 III: 形如 $\int \frac{\hat{l} \hat{a}/x y x \hat{c}_{i} \hat{c}_{k} \hat{c}_{k}}{x^{2} \cdot \hat{c}_{i} \hat{N} \delta \mathcal{F}_{k} \hat{U} \hat{I} \hat{O}} dx$

提示 将分式 " $\frac{ldV.»V.o.,c.}{x^2 \cdot c. \tilde{N}SE(\tilde{I})\tilde{O}}$ " 拆成两个(或多个)简单的式子/分式

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$
解 $\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx =$

求不定积分 III: 形如 $\int \frac{|\dot{a}/x|^3 \times \dot{c}_i \tilde{N} \delta_f(\dot{c})}{x^4 + c_i \tilde{N} \delta_f(\dot{c}) \tilde{D}} dx$

提示 将分式 " $\frac{ldV.»V.o.ci}{x^2 + c: NSE(II)O}$ " 拆成两个(或多个)简单的式子/分式

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$
解 $\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \frac{1}{1+x^2} dx$

求不定积分 III: 形如 ∫ lá½»½ó,cÈ / x² · c: ÑsÆúìõ dx

提示 将分式 " $\frac{ldV.»V.o.,c.}{x^2 \cdot c. \tilde{N}SE(\tilde{I})\tilde{O}}$ " 拆成两个(或多个)简单的式子/分式

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$
解 $\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x$

求不定积分 III: 形如 ∫ lá½»½ó,cÈ / x² · c: ÑsÆúìõ dx

提示 将分式 " $\frac{ldV.»V.o.,ct}{x^2 \cdot c. \tilde{N} \delta \mathcal{E}(\tilde{D})}$ " 拆成两个(或多个)简单的式子/分式

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$
解 $\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$

求不定积分 III: 形如 ∫ lá½»½ó,cÈ / x² · c: ÑsÆúìõ dx

提示 将分式 " $\frac{ldV.»V.o.,CE}{x^2 \cdot C: \tilde{N}SE(\tilde{U})\tilde{O}}$ " 拆成两个(或多个)简单的式子/分式

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$
解 $\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$
 $\int \frac{3+x^2}{1+x^2} dx =$

求不定积分 III: 形如∫ lá½»½ó,cÈ / x²·c: ÑsÆúìỗ dx

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$
解 $\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$

$$\int \frac{3+x^2}{1+x^2} dx = \int \frac{1+x^2+2}{1+x^2} dx =$$

求不定积分 III: 形如∫ lá½»½ó,cÈ /x² · c: Ñ\$ÆÚÌÕ dx

提示 将分式 " lá½ »½ó,cĒ " 拆成两个(或多个)简单的式子/分式 表 c: Ñsē Úlò

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$
解 $\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$
 $\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx =$

求不定积分 III: 形如∫ lá½»½ó,cÈ / x²·c: ÑsÆúìỗ dx

提示 将分式 " lá½ »½ó,cĒ " 拆成两个(或多个)简单的式子/分式 表 c: Ñsē Úlò

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$
解 $\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$
 $\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x +$

求不定积分 III: 形如 ∫ lá½»½ó,cÈ / x² · c: ÑsÆúìõ dx

提示 将分式 " $\frac{lá½ »½ó,ct}{x^2 \cdot c: \tilde{N} \delta \mathcal{E} U \tilde{I} \tilde{O}}$ " 拆成两个(或多个)简单的式子/分式

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$

$$\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$

$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x$$

求不定积分 III: 形如∫ lá½»½ó,cÈ / x²·c: ÑsÆúìỗ dx

提示 将分式 " $\frac{l\acute{a}\acute{x}.»\acute{x}\acute{c},c\acute{t}}{x^a\cdot c: \, N\acute{s}\cancel{E}())\ddot{0}}$ " 拆成两个(或多个)简单的式子/分式

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$
解 $\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$
 $\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$

求不定积分 III: 形如∫ lá½»½ó,cÈ /x²·c: ÑsÆúìỗ dx

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$$\int \frac{x^2}{1+x^2} dx$$
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 $\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$

例子 求不定积分
$$\int \frac{1}{x^2(1+x^2)} dx$$
, $\int \frac{e^{2x}-1}{e^x+1} dx$

求不定积分 III: 形如∫ lá½»½ó,cÈ / x²·c: ÑsÆúìỗ dx

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$
解 $\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$
 $\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$

例子 求不定积分
$$\int \frac{1}{x^2(1+x^2)} dx$$
, $\int \frac{e^{2x}-1}{e^x+1} dx$ 解 $\int \frac{1}{x^2(1+x^2)} dx =$



求不定积分 III: 形如∫ lá½»½ó,cÈ / x²·c: ÑsÆúìÕ dx

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$
解 $\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$
 $\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2\arctan x + C$

例子 求不定积分
$$\int \frac{1}{x^2(1+x^2)} dx$$
, $\int \frac{e^{2x}-1}{e^x+1} dx$ 解 $\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx =$



求不定积分 III: 形如∫ lá½»½ó,cÈ / x²·c: ÑsÆúìÕ dx

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$
解 $\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$
 $\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$

例子 求不定积分
$$\int \frac{1}{x^2(1+x^2)} dx$$
, $\int \frac{e^{2x}-1}{e^x+1} dx$ 解 $\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx = -\frac{1}{x}$



求不定积分 III: 形如∫ lá½»½ó,cÈ /x²·c: ÑsÆúìỗ dx

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$
解 $\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$
 $\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$

例子 求不定积分
$$\int \frac{1}{x^2(1+x^2)} dx$$
, $\int \frac{e^{2x}-1}{e^x+1} dx$
解 $\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx = -\frac{1}{x} - \arctan x$



求不定积分 III: 形如∫ lá½»½ó,cè /x²·c: ѧÆÚÌÕ dx

提示 将分式 " $\frac{|\dot{a}\dot{x}\rangle \sqrt{\lambda}\dot{b},\dot{c}\dot{c}}{x^2+c:\,\dot{N}\delta E(\dot{D}\dot{d})}$ " 拆成两个(或多个)简单的式子/分式

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$
解 $\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$
 $\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2\arctan x + C$



求不定积分 III: 形如 $\int \frac{\hat{a}_{x}^{2} \times \hat{c}_{x} \hat{v}_{x}^{2} \hat{c}_{x} \hat{c}_{x}}{x^{2} \cdot \hat{c}_{x} \hat{v}_{x}^{2} \hat{c}_{x} \hat{v}_{x}^{2}} dx$

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$

$$\iint \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$

$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$$

$$\int \frac{e^{2x} - 1}{e^x + 1} dx =$$

求不定积分 III: 形如∫ lá½»½ó,¢È / x² · c: ÑsÆúìõ dx

提示 将分式 " $\frac{|\hat{a}\%\rangle\%,c\hat{c}}{x^2+c:\hat{N}\delta\mathcal{A}(\hat{l})\hat{l}}$ " 拆成两个(或多个)简单的式子/分式

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$

$$\iint \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$

$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$$

例子 求不定积分 $\int \frac{1}{x^2(1+x^2)} dx$, $\int \frac{e^{2x}-1}{e^x+1} dx$ 解 $\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx = -\frac{1}{x} - \arctan x + C$

$$\int \frac{e^{2x} - 1}{e^x + 1} dx = \int \frac{(e^x + 1)(e^x - 1)}{e^x + 1} dx =$$

求不定积分 III: 形如 $\int \frac{\hat{a}_{x}^{2} \times \hat{c}_{x} \hat{v}_{x}^{2} \cdot \hat{c}_{x}}{\hat{v}_{x}^{2} \cdot \hat{c}_{x} \hat{v}_{x}^{2} \cdot \hat{c}_{x} \hat{v}_{x}^{2} \cdot \hat{c}_{x}^{2}} dx$

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$

$$\iint \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$

$$\int \frac{3+x^2}{1+x^2} dx = \int 1 + \frac{2}{1+x^2} dx = x + 2 \arctan x + C$$

例子 求不定积分 $\int \frac{1}{x^2(1+x^2)} dx$, $\int \frac{e^{2x}-1}{e^x+1} dx$ $\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx = -\frac{1}{x} - \arctan x + C$

$$\int \frac{e^{2x}-1}{e^x+1} dx = \int e^x - 1 dx =$$



求不定积分 III: 形如 $\int \frac{|\hat{a}/x|^3 \times \hat{c}_i \hat{N} \delta_f(\hat{c})}{x^3 + \hat{c}_i \hat{N} \delta_f(\hat{c}) \hat{l}_0} dx$

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$

$$\iint \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$

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例子 求不定积分 $\int \frac{1}{x^2(1+x^2)} dx$, $\int \frac{e^{2x}-1}{e^x+1} dx$

 $\int \frac{e^{2x}-1}{e^x+1} dx = \int e^x - 1 dx = e^x$



求不定积分 III: 形如 $\int \frac{\hat{a}_{x}^{2} \times \hat{c}_{x} \hat{v}_{x}^{2} \cdot \hat{c}_{x}}{\hat{v}_{x}^{2} \cdot \hat{c}_{x} \hat{v}_{x}^{2} \cdot \hat{c}_{x} \hat{v}_{x}^{2} \cdot \hat{c}_{x}^{2}} dx$

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
, $\int \frac{3+x^2}{1+x^2} dx$

$$\iint \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$

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例子 求不定积分 $\int \frac{1}{x^2(1+x^2)} dx$, $\int \frac{e^{2x}-1}{e^x+1} dx$ 解 $\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx = -\frac{1}{x} - \arctan x + C$

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求不定积分 III: 形如 $\int \frac{\hat{a}_{x}^{2} \times \hat{c}_{x} \hat{v}_{x}^{2} \cdot \hat{c}_{x}}{\hat{v}_{x}^{2} \cdot \hat{c}_{x} \hat{v}_{x}^{2} \cdot \hat{c}_{x} \hat{v}_{x}^{2} \cdot \hat{c}_{x}^{2}} dx$

例子 求不定积分
$$\int \frac{x^2}{1+x^2} dx$$
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$$\iint \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx = x - \arctan x + C$$

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例子 求不定积分 $\int \frac{1}{x^2(1+x^2)} dx$, $\int \frac{e^{2x}-1}{e^x+1} dx$ 解 $\int \frac{1}{x^2(1+x^2)} dx = \int \frac{1}{x^2} - \frac{1}{1+x^2} dx = -\frac{1}{x} - \arctan x + C$

$$\int \frac{e^{2x} - 1}{e^x + 1} dx = \int e^x - 1 dx = e^x - x + C$$

掌握
$$\int a^x dx = \frac{1}{\ln a} a^x + C$$
, $(a > 0)$

例子 求不定积分 $\int 3^x e^x dx$, $\int 5^{-x} e^x dx$

掌握
$$\int a^x dx = \frac{1}{\ln a} a^x + C$$
, $(a > 0)$

例子 求不定积分
$$\int 3^x e^x dx$$
, $\int 5^{-x} e^x dx$

$$\int 3^x e^x dx =$$

掌握
$$\int a^x dx = \frac{1}{\ln a} a^x + C$$
, $(a > 0)$

例子 求不定积分
$$\int 3^x e^x dx$$
, $\int 5^{-x} e^x dx$

$$\int 3^{x}e^{x}dx = \int (3e)^{x}dx =$$



掌握
$$\int a^x dx = \frac{1}{\ln a} a^x + C$$
, $(a > 0)$

例子 求不定积分
$$\int 3^x e^x dx$$
, $\int 5^{-x} e^x dx$

$$\int 3^{x}e^{x}dx = \int (3e)^{x}dx = \frac{1}{\ln(3e)}(3e)^{x} + C$$

掌握
$$\int a^x dx = \frac{1}{\ln a} a^x + C$$
, $(a > 0)$

例子 求不定积分
$$\int 3^x e^x dx$$
, $\int 5^{-x} e^x dx$

$$\int 3^{x}e^{x}dx = \int (3e)^{x}dx = \frac{1}{\ln(3e)}(3e)^{x} + C = \frac{3^{x}e^{x}}{1 + \ln 3} + C$$



掌握
$$\int a^x dx = \frac{1}{\ln a} a^x + C$$
, $(a > 0)$

例子 求不定积分
$$\int 3^x e^x dx$$
, $\int 5^{-x} e^x dx$

$$\int 3^{x} e^{x} dx = \int (3e)^{x} dx = \frac{1}{\ln(3e)} (3e)^{x} + C = \frac{3^{x} e^{x}}{1 + \ln 3} + C$$
$$\int 5^{-x} e^{x} dx =$$

掌握
$$\int a^x dx = \frac{1}{\ln a} a^x + C$$
, $(a > 0)$

例子 求不定积分
$$\int 3^x e^x dx$$
, $\int 5^{-x} e^x dx$

$$\int 3^{x} e^{x} dx = \int (3e)^{x} dx = \frac{1}{\ln(3e)} (3e)^{x} + C = \frac{3^{x} e^{x}}{1 + \ln 3} + C$$
$$\int 5^{-x} e^{x} dx = \int \left(\frac{1}{5}\right)^{x} e^{x} dx =$$

掌握
$$\int a^x dx = \frac{1}{\ln a} a^x + C$$
, $(a > 0)$

例子 求不定积分
$$\int 3^x e^x dx$$
, $\int 5^{-x} e^x dx$

$$\int 3^{x} e^{x} dx = \int (3e)^{x} dx = \frac{1}{\ln(3e)} (3e)^{x} + C = \frac{3^{x} e^{x}}{1 + \ln 3} + C$$
$$\int 5^{-x} e^{x} dx = \int \left(\frac{1}{5}\right)^{x} e^{x} dx = \int \left(\frac{1}{5}e\right)^{x} dx$$

掌握
$$\int a^x dx = \frac{1}{\ln a} a^x + C$$
, $(a > 0)$

例子 求不定积分
$$\int 3^x e^x dx$$
, $\int 5^{-x} e^x dx$

$$\int 3^{x} e^{x} dx = \int (3e)^{x} dx = \frac{1}{\ln(3e)} (3e)^{x} + C = \frac{3^{x} e^{x}}{1 + \ln 3} + C$$

$$\int 5^{-x} e^{x} dx = \int \left(\frac{1}{5}\right)^{x} e^{x} dx = \int \left(\frac{1}{5}e\right)^{x} dx$$

$$= \frac{1}{\ln\left(\frac{1}{5}e\right)} \left(\frac{1}{5}e\right)^{x} + C$$



掌握
$$\int a^x dx = \frac{1}{\ln a} a^x + C$$
, $(a > 0)$

例子 求不定积分
$$\int 3^x e^x dx$$
, $\int 5^{-x} e^x dx$

$$\int 3^{x} e^{x} dx = \int (3e)^{x} dx = \frac{1}{\ln(3e)} (3e)^{x} + C = \frac{3^{x} e^{x}}{1 + \ln 3} + C$$

$$\int 5^{-x} e^{x} dx = \int \left(\frac{1}{5}\right)^{x} e^{x} dx = \int \left(\frac{1}{5}e\right)^{x} dx$$

$$= \frac{1}{\ln\left(\frac{1}{5}e\right)} \left(\frac{1}{5}e\right)^{x} + C = \frac{e^{x}}{(1 - \ln 5)5^{x}} + C$$

求不定积分 \mathbf{V} : \int "triangle functions" dx

求不定积分

$$(1) \int \tan^2 x dx, \quad \int \cot^2 x dx$$

$$(2) \int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx, \quad \int \frac{1}{\cos^2 x \sin^2 x} dx$$

$$(3) \int \sin^2 \frac{x}{2} dx, \quad \int \cos^2 \frac{x}{2} dx$$



求不定积分 V: \int "triangle functions" dx

求不定积分

$$(1) \int \tan^2 x dx, \quad \int \cot^2 x dx$$

$$(2) \int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx, \quad \int \frac{1}{\cos^2 x \sin^2 x} dx$$

$$(3) \int \sin^2 \frac{x}{2} dx, \quad \int \cos^2 \frac{x}{2} dx$$

提示 利用"三角恒等式"

平方关系 $\sin^2 x + \cos^2 x = 1$

倍角公式 $\cos 2x = \cos^2 x - \sin^2 x$; $\sin 2x = 2 \sin x \cos x$

利用: $\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx$, $\int \cot^2 x dx$

利用:
$$\sin^2 x + \cos^2 x = 1$$

例子 求不定积分
$$\int tan^2 x dx$$
, $\int cot^2 x dx$

解因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

利用:
$$\sin^2 x + \cos^2 x = 1$$

例子 求不定积分
$$\int \tan^2 x dx$$
, $\int \cot^2 x dx$

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$$

利用:
$$\sin^2 x + \cos^2 x = 1$$

例子 求不定积分
$$\int tan^2 x dx$$
, $\int cot^2 x dx$

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x}$$

利用:
$$\sin^2 x + \cos^2 x = 1$$

例子 求不定积分
$$\int tan^2 x dx$$
, $\int cot^2 x dx$

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - 1$$



利用:
$$\sin^2 x + \cos^2 x = 1$$

例子 求不定积分
$$\int tan^2 x dx$$
, $\int cot^2 x dx$

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx =$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - 1$$



利用:
$$\sin^2 x + \cos^2 x = 1$$

例子 求不定积分
$$\int tan^2 x dx$$
, $\int cot^2 x dx$

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - 1$$



利用:
$$\sin^2 x + \cos^2 x = 1$$

例子 求不定积分
$$\int tan^2 x dx$$
, $\int cot^2 x dx$

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - 1$$



利用:
$$\sin^2 x + \cos^2 x = 1$$

例子 求不定积分
$$\int tan^2 x dx$$
, $\int cot^2 x dx$

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - 1$$

利用:
$$\sin^2 x + \cos^2 x = 1$$

例子 求不定积分
$$\int tan^2 x dx$$
, $\int cot^2 x dx$

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

$$\int \cot^2 x dx =$$

利用:
$$\sin^2 x + \cos^2 x = 1$$

例子 求不定积分
$$\int tan^2 x dx$$
, $\int cot^2 x dx$

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \qquad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

$$\int \cot^2 x dx =$$



利用:
$$\sin^2 x + \cos^2 x = 1$$

例子 求不定积分
$$\int tan^2 x dx$$
, $\int cot^2 x dx$

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \qquad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样, $\int \cot^2 x dx =$

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x}$$



利用:
$$\sin^2 x + \cos^2 x = 1$$

例子 求不定积分
$$\int tan^2 x dx$$
, $\int cot^2 x dx$

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \qquad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样, $\int \cot^2 x dx =$

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x}$$



利用:
$$\sin^2 x + \cos^2 x = 1$$

例子 求不定积分
$$\int tan^2 x dx$$
, $\int cot^2 x dx$

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \qquad \cot^2 x = \frac{1}{\sin^2 x} - 1$$
所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

$$\Box \vec{H}, \qquad c$$

$$\int \cot^2 x dx =$$

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$$



利用:
$$\sin^2 x + \cos^2 x = 1$$

例子 求不定积分
$$\int tan^2 x dx$$
, $\int cot^2 x dx$

解因为
$$\tan^2 x = \frac{1}{\cos^2 x} - 1$$
, $\cot^2 x = \frac{1}{\sin^2 x} - 1$

所以
$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$
同样,

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$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$$



利用:
$$\sin^2 x + \cos^2 x = 1$$

例子 求不定积分
$$\int tan^2 x dx$$
, $\int cot^2 x dx$

解因为
$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \qquad \cot^2 x = \frac{1}{\sin^2 x} - 1$$
 所以

$$\int \cot^2 x dx = \int \frac{1}{\sin^2 x} - 1 dx = -\cot x$$

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$$



利用:
$$\sin^2 x + \cos^2 x = 1$$

例子 求不定积分
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利用:
$$\sin^2 x + \cos^2 x = 1$$

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利用: $\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \frac{1}{\cos^2 x \sin^2 x} dx$

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$$\int \frac{1}{\cos^2 x \sin^2 x} dx$$

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$$= \tan x$$

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$$= \int \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} dx$$
$$= \tan x - \cot x$$

利用:
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$$\int \frac{1}{\cos^2 x \sin^2 x} dx$$

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$$= \int \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} dx$$
$$= \tan x - \cot x + C$$

利用 "倍角公式" $\cos 2x = \cos^2 x - \sin^2 x$

例子 求不定积分 $\int \cos^2 \frac{x}{2} dx$, $\int \sin^2 \frac{x}{2} dx$

利用"倍角公式"

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x)$$

例子 求不定积分 $\int \cos^2 \frac{x}{2} dx$, $\int \sin^2 \frac{x}{2} dx$

利用"倍角公式"

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

例子 求不定积分
$$\int \cos^2 \frac{x}{2} dx$$
, $\int \sin^2 \frac{x}{2} dx$

利用"倍角公式"

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

$$\therefore \cos^2 x = \frac{1 + \cos 2x}{2},$$

例子 求不定积分
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例子 求不定积分
$$\int \cos^2 \frac{x}{2} dx$$
, $\int \sin^2 \frac{x}{2} dx$

$$\int \cos^2 \frac{x}{2} dx = \int \frac{1 + \cos x}{2} dx = \frac{1}{2}x + \frac{1}{2}\sin x + C$$
$$\int \sin^2 \frac{x}{2} dx =$$

利用"倍角公式"

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

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$$\int \cos^2 \frac{x}{2} dx$$
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$$\int \sin^2 \frac{x}{2} dx = \int \frac{1 - \cos x}{2} dx =$$

利用"倍角公式"

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

$$\therefore \cos^2 x = \frac{1 + \cos 2x}{2}, \qquad \sin^2 x = \frac{1 - \cos 2x}{2}$$

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$$\int \cos^2 \frac{x}{2} dx$$
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$$\int \sin^2 \frac{x}{2} dx = \int \frac{1 - \cos x}{2} dx = \frac{1}{2} x - \frac{1}{2} \sin x + C$$

利用"倍角公式"

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分 $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

利用"倍角公式"

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分
$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$$

$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} dx$$

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$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$$

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$$= \int \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} dx$$

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例子 求不定积分
$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$$

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$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} dx$$

$$= \int \frac{\cos^2 x}{\cos^2 x \sin^2 x} - \frac{\sin^2 x}{\cos^2 x \sin^2 x} dx$$

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$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$$

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= $-\cot x - \tan x$

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$$= \int \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} dx$$

$$= -\cot x - \tan x + C$$