## 第 9 章 d: 隐函数的求导公式

数学系 梁卓滨

2016-2017 **学年** II



## Outline

1. 一个方程的情形

2. 方程组的情形

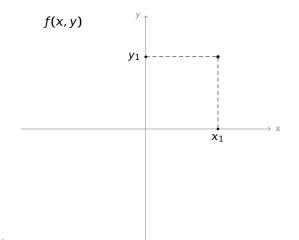


We are here now...

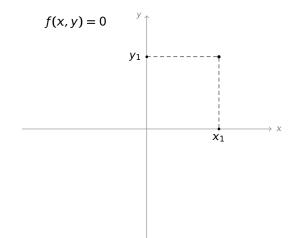
1. 一个方程的情形

2. 方程组的情形

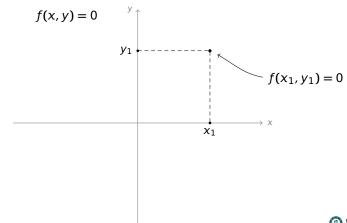
隐函数定理 设 f(x,y) 在点  $p_1(x_1,y_1)$  附近有定义,具有连续偏导;



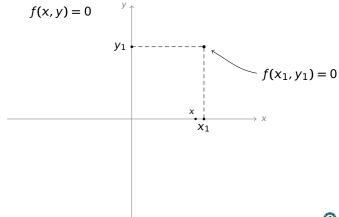
隐函数定理 设 f(x,y) 在点  $p_1(x_1,y_1)$  附近有定义,具有连续偏导;



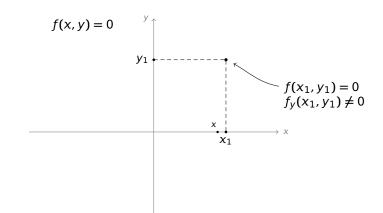
隐函数定理 设 f(x, y) 在点  $p_1(x_1, y_1)$  附近有定义,具有连续偏导;  $f(x_1, y_1) = 0$ ;

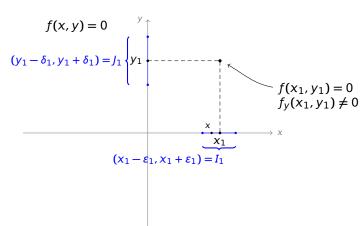


隐函数定理 设 f(x, y) 在点  $p_1(x_1, y_1)$  附近有定义,具有连续偏导;  $f(x_1, y_1) = 0$ ;

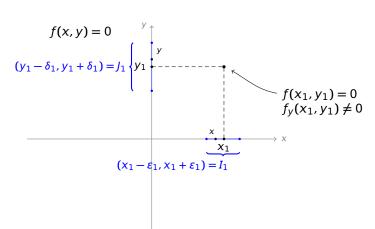


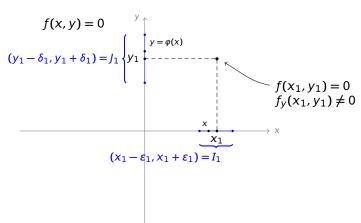
隐函数定理 设 f(x,y) 在点  $p_1(x_1,y_1)$  附近有定义,具有连续偏导;  $f(x_1,y_1)=0$ ;  $f_V(x_1,y_1)\neq 0$ 。

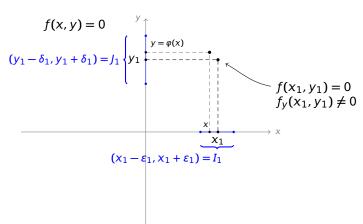


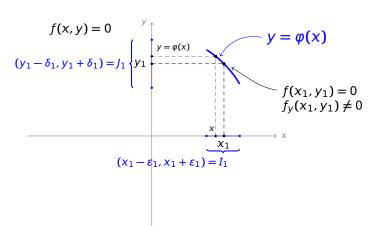


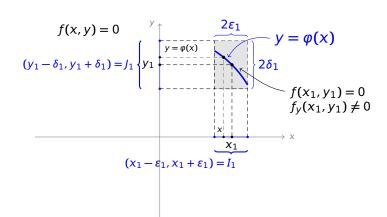
1. 对任意  $x \in I_1$ , 方程 f(x, y) = 0 有唯一的解  $y \in J_1$ 。



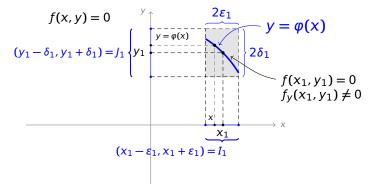




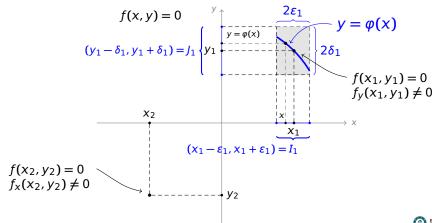




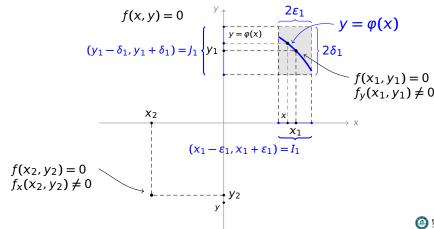
- 1. 对任意  $x \in I_1$ , 方程 f(x,y) = 0 有唯一的解  $y = \varphi(x) \in J_1$ 。
- 2. 函数  $y = \varphi(x)$  在区间 I 上具有连续导数。

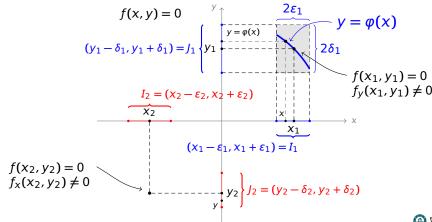


隐函数定理 设 f(x, y) 在点  $p_2(x_2, y_2)$  附近有定义,具有连续偏导;  $f(x_2, y_2) = 0$ ;  $f_V(x_2, y_2) \neq 0$ 。

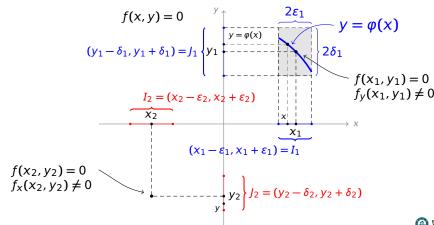


隐函数定理 设 f(x,y) 在点  $p_2(x_2,y_2)$  附近有定义,具有连续偏导;  $f(x_2, y_2) = 0$ ;  $f_v(x_2, y_2) \neq 0$ .

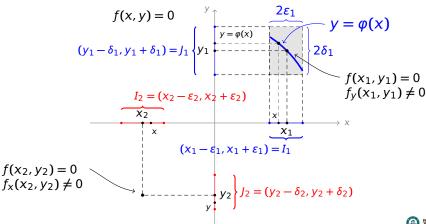




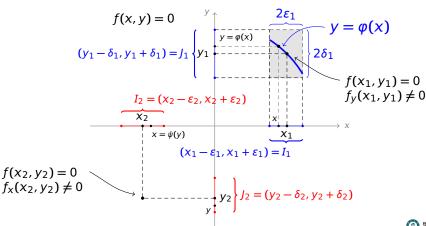
1. 对任意 *y* ∈ *J*<sub>2</sub>



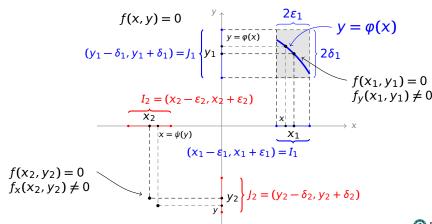
1. 对任意  $y \in J_2$ , 方程 f(x,y) = 0 有唯一的解  $x \in I_2$ 。



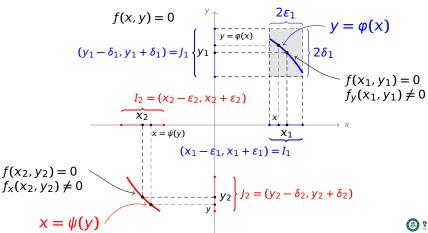
1. 对任意  $y \in J_2$ ,方程 f(x,y) = 0 有唯一的解  $x = \psi(y) \in I_2$ 。



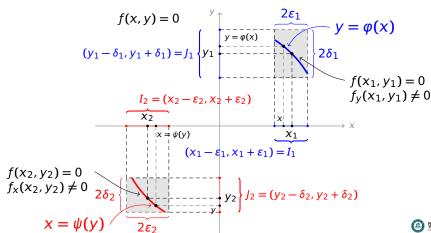
1. 对任意  $y \in J_2$ ,方程 f(x,y) = 0 有唯一的解  $x = \psi(y) \in I_2$ 。



1. 对任意  $y \in J_2$ ,方程 f(x,y) = 0 有唯一的解  $x = \psi(y) \in I_2$ 。



1. 对任意  $y \in I_2$ ,方程 f(x, y) = 0 有唯一的解  $x = \psi(y) \in I_2$ 。

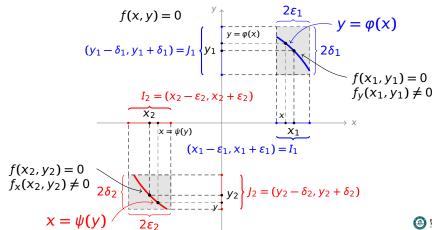


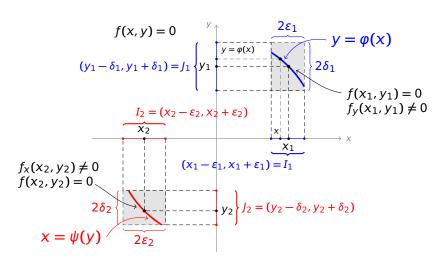
隐函数定理 设 f(x,y) 在点  $p_2(x_2,y_2)$  附近有定义,具有连续偏导;  $f(x_2, y_2) = 0$ ;  $f_{V}(x_2, y_2) \neq 0$ 。则存在开区间  $I_2 = (x_2 - \varepsilon, x_2 + \varepsilon)$ 

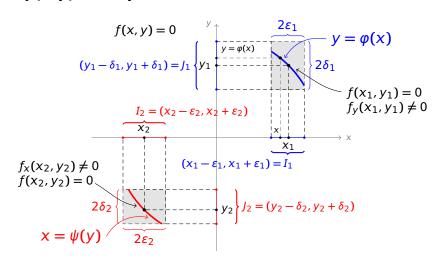
1. 对任意  $y \in I_2$ ,方程 f(x,y) = 0 有唯一的解  $x = \psi(y) \in I_2$ 。

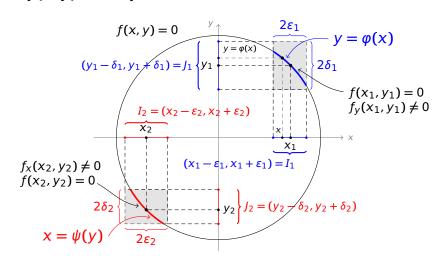
和  $I_2 = (y_2 - \delta, y_2 + \delta)$ , 使得

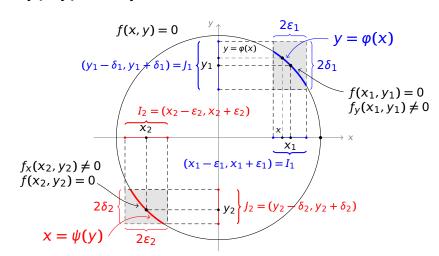
2. 函数  $x = \psi(y)$  在区间 I 上具有连续导数。

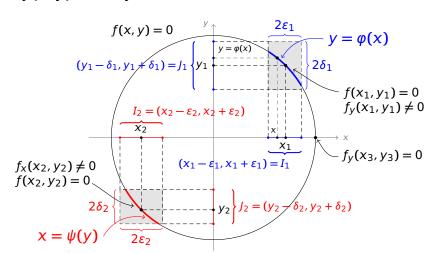


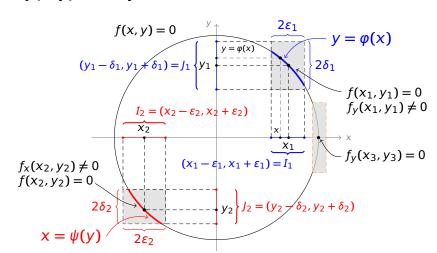


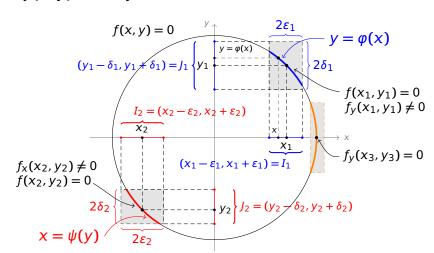


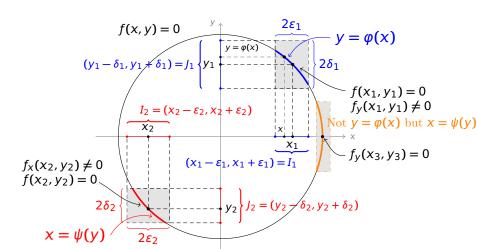


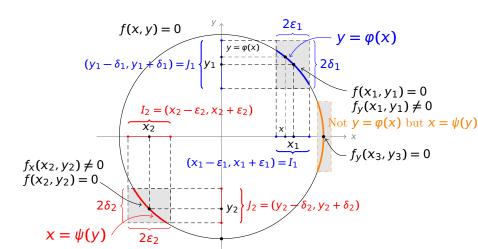


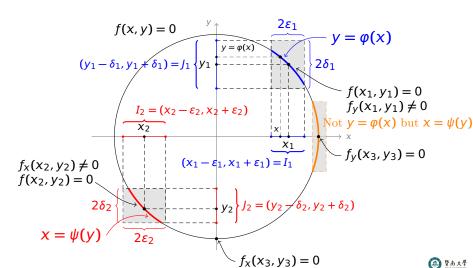


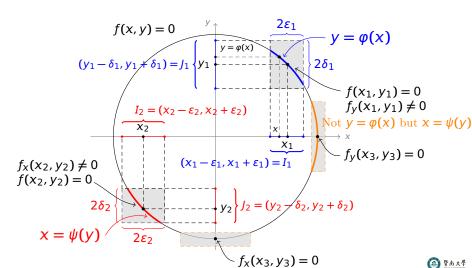






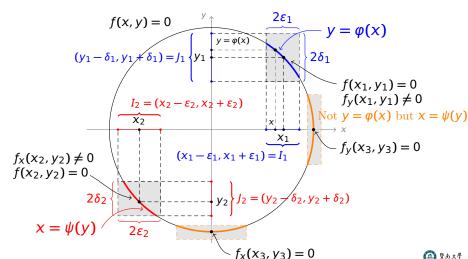






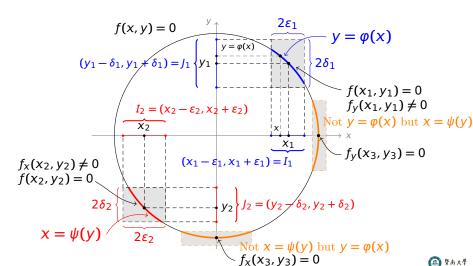
注 隐函数定理中条件  $f_y(x_1, y_1) \neq 0$  和  $f_y(x_2, y_2) \neq 0$  不能去掉,否则结论可能不成立

以  $f(x, y) = x^2 - y^2 - 1$  为例说明:



注 隐函数定理中条件  $f_y(x_1, y_1) \neq 0$  和  $f_y(x_2, y_2) \neq 0$  不能去掉,否则结论可能不成立

以  $f(x, y) = x^2 - y^2 - 1$  为例说明:



导;  $f(x_0, y_0, z_0) = 0$ ;

导; 
$$f(x_0, y_0, z_0) = 0$$
;

方程 
$$f(x, y, z) = 0$$
 有唯一的解

$$z = \varphi(x, y)$$



导; 
$$f(x_0, y_0, z_0) = 0$$
;  $f_z(x_0, y_0, z_0) \neq 0$ 。

方程 
$$f(x, y, z) = 0$$
 有唯一的解

$$z = \varphi(x, y)$$



导; 
$$f(x_0, y_0, z_0) = 0$$
;  $f_z(x_0, y_0, z_0) \neq 0$ 。则存在开区间
$$I_1 = (x_0 - \varepsilon, x_0 + \varepsilon), \quad I_2 = (y_0 - \varepsilon, y_0 + \varepsilon), \quad I = (z_0 - \delta, z_0 + \delta)$$

使得:

方程 
$$f(x, y, z) = 0$$
 有唯一的解

$$z = \varphi(x, y)$$



导; 
$$f(x_0, y_0, z_0) = 0$$
;  $f_z(x_0, y_0, z_0) \neq 0$ 。则存在开区间
$$I_1 = (x_0 - \varepsilon, x_0 + \varepsilon), \quad I_2 = (y_0 - \varepsilon, y_0 + \varepsilon), \quad I = (z_0 - \delta, z_0 + \delta)$$

使得:

1. 对任意  $(x, y) \in I_1 \times I_2$ ,方程 f(x, y, z) = 0 有唯一的解  $z = \varphi(x, y)$ 



导; 
$$f(x_0, y_0, z_0) = 0$$
;  $f_z(x_0, y_0, z_0) \neq 0$ 。则存在开区间
$$I_1 = (x_0 - \varepsilon, x_0 + \varepsilon), \quad I_2 = (y_0 - \varepsilon, y_0 + \varepsilon), \quad I = (z_0 - \delta, z_0 + \delta)$$

使得:

1. 对任意  $(x, y) \in I_1 \times I_2$ ,方程 f(x, y, z) = 0 有唯一的解  $z = \varphi(x, y) \in I$ 



导; 
$$f(x_0, y_0, z_0) = 0$$
;  $f_z(x_0, y_0, z_0) \neq 0$ 。则存在开区间
$$I_1 = (x_0 - \varepsilon, x_0 + \varepsilon), \quad I_2 = (y_0 - \varepsilon, y_0 + \varepsilon), \quad I = (z_0 - \delta, z_0 + \delta)$$

使得:

- 1. 对任意  $(x,y) \in I_1 \times I_2$ ,方程 f(x,y,z) = 0 有唯一的解  $z = \varphi(x,y) \in I$
- 2. 函数  $z = \varphi(x, y)$  在区域  $I_1 \times I_2$  上具有连续导数。



#### 隐函数的求导法Ⅰ

公式 设 y = f(x) 满足 F(x, y) = 0,

公式 设 y = f(x) 满足 F(x, y) = 0, 即 F(x, y(x)) = 0,

#### 隐函数的求导法Ⅰ

公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则 
$$\frac{dy}{dx} =$$

公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则 
$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则 
$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则 
$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

证明 
$$: F(x, y(x)) = 0$$

公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则 
$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

$$:: F(x, y(x)) = 0$$

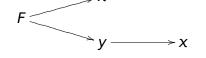
$$0 = \frac{d}{dx}F(x, y(x)) =$$

公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则

$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

证明 
$$: F(x, y(x)) = 0$$

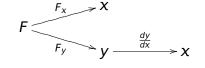
$$\therefore \quad 0 = \frac{d}{dx} F(x, y(x)) =$$



公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则 
$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

$$: F(x, y(x)) = 0$$

$$\therefore \quad 0 = \frac{d}{dx} F(x, y(x)) =$$

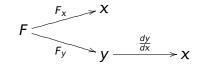


公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则

$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

证明 
$$: F(x, y(x)) = 0$$

$$\therefore 0 = \frac{d}{dx} F(x, y(x)) = F_x +$$

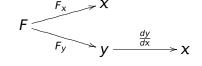


公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则

$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

证明 
$$: F(x, y(x)) = 0$$

$$\therefore \quad 0 = \frac{d}{dx} F(x, y(x)) = F_x + F_y \cdot \frac{dy}{dx}$$

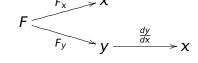


公式 设 
$$y = f(x)$$
 满足  $F(x, y) = 0$ ,即  $F(x, y(x)) = 0$ ,则 
$$\frac{dy}{dx} = -\frac{F_x}{F_y} \qquad (F_y \neq 0)$$

$$: F(x, y(x)) = 0$$

$$\therefore \quad 0 = \frac{d}{dx} F(x, y(x)) = F_x + F_y \cdot \frac{dy}{dx}$$

$$\therefore \quad \frac{dy}{dx} = -\frac{F_x}{F_y}$$



例 设 
$$y = f(x)$$
 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

#### 方法一

$$F(x,\,y)=0$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} =$$

例 设 
$$y = f(x)$$
 满足  $\sin y + e^x = xy^2$ , 求  $\frac{dy}{dx}$ 

方法一 注意 
$$\sin y + e^x - xy^2 = 0$$

$$F(x, y) = 0$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} =$$

例设 
$$y = f(x)$$
 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

方法一 注意 
$$\sin y + e^x - xy^2 = 0$$
, 令  $F(x, y) = \sin y + e^x - xy^2$ ,  $F(x, y) = 0$ 

$$\frac{dy}{dx} = -\frac{F_x}{F_y} =$$

方法一 注意 
$$\sin y + e^x - xy^2 = 0$$
, 令  $F(x, y) = \sin y + e^x - xy^2$ , 则

$$F(x, y) = 0$$
,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} =$$

方法一 注意 
$$\sin y + e^x - xy^2 = 0$$
, 令  $F(x, y) = \sin y + e^x - xy^2$ , 则

$$F(x, y) = 0$$
,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -$$

方法一 注意  $\sin y + e^x - xy^2 = 0$ ,令  $F(x, y) = \sin y + e^x - xy^2$ ,则

$$F(x,y)=0$$
,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{(\sin y + e^x - xy^2)_y'}$$

方法一 注意 
$$\sin y + e^x - xy^2 = 0$$
, 令  $F(x, y) = \sin y + e^x - xy^2$ , 则

$$F(x,y)=0$$
,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法一 注意 
$$\sin y + e^x - xy^2 = 0$$
, 令  $F(x, y) = \sin y + e^x - xy^2$ , 则

$$F(x, y) = 0$$
,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

#### 方法二



方法一 注意 
$$\sin y + e^x - xy^2 = 0$$
,令  $F(x, y) = \sin y + e^x - xy^2$ ,则

$$F(x,y)=0$$
,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 
$$\sin y(x) + e^x - xy(x)^2 = 0$$
,



方法一 注意  $\sin y + e^x - xy^2 = 0$ ,令  $F(x, y) = \sin y + e^x - xy^2$ ,则

F(x,y)=0,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 
$$\sin y(x) + e^x - xy(x)^2 = 0$$
,所以 
$$0 = (\sin y(x) + e^x - xy(x)^2)_x'$$

例设 
$$y = f(x)$$
 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

方法一 注意 
$$\sin y + e^x - xy^2 = 0$$
, 令  $F(x, y) = \sin y + e^x - xy^2$ , 则

$$F(x, y) = 0$$
,所以 
$$dy F_x (\sin y + e^x - xy^2)_x'$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 
$$\sin y(x) + e^x - xy(x)^2 = 0$$
,所以
$$0 = (\sin y(x) + e^x - xy(x)^2)_x'$$

$$= (\sin y(x))_y' + (e^x)_y' - (xy(x)^2)_y'$$



例设 
$$y = f(x)$$
 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

方法一 注意  $\sin y + e^x - xy^2 = 0$ , 令  $F(x, y) = \sin y + e^x - xy^2$ , 则

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 
$$\sin y(x) + e^x - xy(x)^2 = 0$$
,所以
$$0 = (\sin y(x) + e^x - xy(x)^2)_x'$$

$$= (\sin y(x))_x' + (e^x)_x' - (xy(x)^2)_x'$$

$$= \cos y \cdot y'$$



F(x, y) = 0,所以

例 设 
$$y = f(x)$$
 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

方法一 注意  $\sin y + e^x - xy^2 = 0$ ,令  $F(x, y) = \sin y + e^x - xy^2$ ,则 F(x, y) = 0,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 
$$\sin y(x) + e^x - xy(x)^2 = 0$$
,所以
$$0 = (\sin y(x) + e^x - xy(x)^2)_x'$$

$$= (\sin y(x))_x' + (e^x)_x' - (xy(x)^2)_x'$$

$$= \cos y \cdot y' + e^x$$



例设 
$$y = f(x)$$
 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

方法一 注意  $\sin y + e^x - xy^2 = 0$ ,令  $F(x, y) = \sin y + e^x - xy^2$ ,则 F(x, y) = 0,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 
$$\sin y(x) + e^x - xy(x)^2 = 0$$
,所以
$$0 = (\sin y(x) + e^x - xy(x)^2)_x'$$

$$= (\sin y(x))_x' + (e^x)_x' - (xy(x)^2)_x'$$

$$= \cos y \cdot y' + e^x - y^2 - 2xy \cdot y'$$



例设 y = f(x) 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

方法一 注意  $\sin y + e^x - xy^2 = 0$ ,令  $F(x, y) = \sin y + e^x - xy^2$ ,则

$$F(x,y)=0$$
,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 
$$\sin y(x) + e^x - xy(x)^2 = 0$$
,所以
$$0 = (\sin y(x) + e^x - xy(x)^2)_x'$$

$$= (\sin y(x))_x' + (e^x)_x' - (xy(x)^2)_x'$$

$$= \cos y \cdot y' + e^x - y^2 - 2xy \cdot y'$$

$$= e^x - y^2 + (\cos y - 2xy)y'$$

例 设 y = f(x) 满足  $\sin y + e^x = xy^2$ ,求  $\frac{dy}{dx}$ 

方法一 注意  $\sin y + e^x - xy^2 = 0$ ,令  $F(x, y) = \sin y + e^x - xy^2$ ,则

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\sin y + e^x - xy^2)_x'}{(\sin y + e^x - xy^2)_y'} = -\frac{e^x - y^2}{\cos y - 2xy}$$

方法二 注意 
$$\sin y(x) + e^x - xy(x)^2 = 0$$
,所以
$$0 = (\sin y(x) + e^x - xy(x)^2)_x'$$

$$= (\sin y(x))_x' + (e^x)_x' - (xy(x)^2)_x'$$

$$= \cos y \cdot y' + e^x - y^2 - 2xy \cdot y'$$

$$= e^x - y^2 + (\cos y - 2xy)y'$$

所以  $y' = -\frac{e^{x}-y^{2}}{\cos y-2xy}$ 

F(x, y) = 0,所以

例 设 y = f(x) 满足  $\ln(x^2 + y^2) + 3xy = 4$ , 求  $\frac{dy}{dx}$ 

例 设 y = f(x) 满足  $\ln(x^2 + y^2) + 3xy = 4$ ,求  $\frac{dy}{dx}$ 

解

$$F(x, y) = 0$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = 0$$

例 设 
$$y = f(x)$$
 满足  $\ln(x^2 + y^2) + 3xy = 4$ ,求  $\frac{dy}{dx}$ 解 注意  $\ln(x^2 + y^2) + 3xy - 4 = 0$ 

$$F(x, y) = 0$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = 0$$

例 设 
$$y = f(x)$$
 满足  $\ln(x^2 + y^2) + 3xy = 4$ ,求  $\frac{dy}{dx}$ 

解注意 
$$ln(x^2 + y^2) + 3xy - 4 = 0$$
,令

$$F(x, y) = \ln(x^2 + y^2) + 3xy - 4$$

$$F(x, y) = 0$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = 0$$

例 设 
$$y = f(x)$$
 满足  $\ln(x^2 + y^2) + 3xy = 4$ , 求  $\frac{dy}{dx}$ 

解注意 
$$ln(x^2 + y^2) + 3xy - 4 = 0$$
, 令

$$F(x, y) = \ln(x^2 + y^2) + 3xy - 4$$

则 
$$F(x, y) = 0$$
,所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\ln(x^2 + y^2) + 3xy - 4)_x'}{(\ln(x^2 + y^2) + 3xy - 4)_y'}$$

例 设 
$$y = f(x)$$
 满足  $\ln(x^2 + y^2) + 3xy = 4$ , 求  $\frac{dy}{dx}$ 

解注意 
$$ln(x^2 + y^2) + 3xy - 4 = 0$$
, 令

$$F(x, y) = \ln(x^2 + y^2) + 3xy - 4$$

则 
$$F(x, y) = 0$$
, 所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\ln(x^2 + y^2) + 3xy - 4)_x'}{(\ln(x^2 + y^2) + 3xy - 4)_y'}$$

例 设 
$$y = f(x)$$
 满足  $\ln(x^2 + y^2) + 3xy = 4$ ,求  $\frac{dy}{dx}$ 

解注意 
$$ln(x^2 + y^2) + 3xy - 4 = 0$$
, 令

$$F(x, y) = \ln(x^2 + y^2) + 3xy - 4$$

则 
$$F(x, y) = 0$$
, 所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\ln(x^2 + y^2) + 3xy - 4)_x'}{(\ln(x^2 + y^2) + 3xy - 4)_y'}$$
$$\frac{2x}{x^2 + y^2} + 3y$$

例 设 
$$y = f(x)$$
 满足  $\ln(x^2 + y^2) + 3xy = 4$ ,求  $\frac{dy}{dx}$ 

解注意 
$$ln(x^2 + y^2) + 3xy - 4 = 0$$
, 令

$$F(x, y) = \ln(x^2 + y^2) + 3xy - 4$$

则 
$$F(x, y) = 0$$
, 所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\ln(x^2 + y^2) + 3xy - 4)_x'}{(\ln(x^2 + y^2) + 3xy - 4)_y'}$$

$$\frac{2x}{x^2 + y^2} + 3y$$

$$= -\frac{\frac{2x}{x^2 + y^2} + 3y}{\frac{2y}{x^2 + y^2} + 3x}$$

例 设 
$$y = f(x)$$
 满足  $\ln(x^2 + y^2) + 3xy = 4$ ,求  $\frac{dy}{dx}$ 

解注意 
$$ln(x^2 + y^2) + 3xy - 4 = 0$$
, 令

$$F(x, y) = \ln(x^2 + y^2) + 3xy - 4$$

则 
$$F(x, y) = 0$$
, 所以

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{(\ln(x^2 + y^2) + 3xy - 4)_x'}{(\ln(x^2 + y^2) + 3xy - 4)_y'}$$

$$= -\frac{\frac{2x}{x^2 + y^2} + 3y}{\frac{2y}{x^2 + y^2} + 3x}$$

$$=\frac{2x+3x^2y+3y^3}{2y+3xy^2+3x^3}$$

公式 设 z = f(x, y) 满足 F(x, y, z) = 0,

公式 设 z = f(x, y) 满足 F(x, y, z) = 0, 即 F(x, y, z(x, y)) = 0,

公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial Z}{\partial X} = \qquad , \qquad \frac{\partial Z}{\partial y} =$$

公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} =$$

公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ ,即  $F(x, y, z(x, y)) = 0$ ,则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ ,即  $F(x, y, z(x, y)) = 0$ ,则

$$\frac{\partial z}{\partial x} = -\frac{F_X}{F_Z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_Z} \qquad (F_Z \neq 0)$$

公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

$$F(x, y, z(x, y)) = 0$$

公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

$$F(x, y, z(x, y)) = 0$$

$$\therefore \quad 0 = \frac{\partial}{\partial x} F(x, y, z(x, y)) =$$



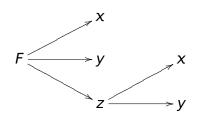
公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

证明

$$F(x, y, z(x, y)) = 0$$

$$\therefore \quad 0 = \frac{\partial}{\partial x} F(x, y, z(x, y)) =$$

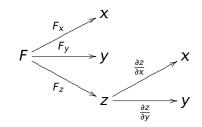


公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

证明 
$$: F(x, y, z(x, y)) = 0$$

$$\therefore \quad 0 = \frac{\partial}{\partial x} F(x, y, z(x, y)) =$$

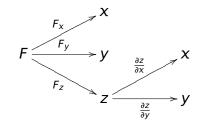


公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

证明 
$$:: F(x, y, z(x, y)) = 0$$

$$\therefore \quad 0 = \frac{\partial}{\partial x} F(x, y, z(x, y)) = F_x + F_z \cdot \frac{\partial z}{\partial x}$$



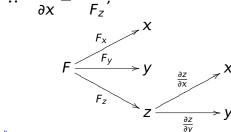
公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

证明 
$$:: F(x, y, z(x, y)) = 0$$

$$\therefore \quad 0 = \frac{\partial}{\partial x} F(x, y, z(x, y)) = F_X + F_Z \cdot \frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{a}$$



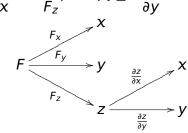
公式 设 
$$z = f(x, y)$$
 满足  $F(x, y, z) = 0$ , 即  $F(x, y, z(x, y)) = 0$ , 则

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}, \qquad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} \qquad (F_z \neq 0)$$

证明 
$$:: F(x, y, z(x, y)) = 0$$

$$\therefore \quad 0 = \frac{\partial}{\partial x} F(x, y, z(x, y)) = F_X + F_Z \cdot \frac{\partial z}{\partial x}$$

∴ 
$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$
,  $= \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$ 



例 设 z = f(x, y) 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ ,求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$  解

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

$$\frac{\partial Z}{\partial y} = -\frac{F_y}{F_z} =$$

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ ,求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

$$\mathbb{R} \diamondsuit F(x, y, z) = x + y + xz - e^z + 1, \qquad F(x, y, z) = 0$$

$$\frac{\partial Z}{\partial X} = -\frac{F_X}{F_Z} =$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} =$$

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + y + xz - e^z + 1)_y'}{(x + y + xz - e^z + 1)_z'}$$

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
= -

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1}{0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
= -

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1}{0+0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
= -

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1}{0+0+x}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)'_y}{(x+y+xz-e^z+1)'_z}$$
= -

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{(x+y+xz-e^z+1)_z'}{(x+y+xz-e^z+1)_z'}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
= -

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1}{0+0+x-e^z+0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)'_y}{(x+y+xz-e^z+1)'_z}$$
= -

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1}{0+0+x-e^z+0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{(x+y+xz-e^z+1)_z'}{(x+y+xz-e^z+0)}$$



例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1}{0+0+x-e^z+0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{(x+y+xz-e^z+1)_z'}{(x+y+xz-e^z+0)}$$



例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0}{0+0+x-e^z+0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{(x+y+xz-e^z+1)_z'}{(x+y+xz-e^z+0)}$$



例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0+z}{0+0+x-e^z+0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{(x+y+xz-e^z+1)_z'}{(x+y+xz-e^z+0)}$$



例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0+z-0}{0+0+x-e^z+0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{(x+y+xz-e^z+1)_z'}{(x+y+xz-e^z+0)}$$

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0+z-0+0}{0+0+x-e^z+0}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{(x+y+xz-e^z+1)_z'}{(x+y+xz-e^z+0)}$$



例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0+z-0+0}{0+0+x-e^z+0} = -\frac{1+z}{x-e^z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{(x+y+xz-e^z+1)_z'}{(x+y+xz-e^z+0)}$$

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0+z-0+0}{0+0+x-e^z+0} = -\frac{1+z}{x-e^z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{0}{0+0+x-e^z+0}$$



例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0+z-0+0}{0+0+x-e^z+0} = -\frac{1+z}{x-e^z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{0+1}{0+0+x-e^z+0}$$

例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0+z-0+0}{0+0+x-e^z+0} = -\frac{1+z}{x-e^z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{0+1+0}{0+0+x-e^z+0}$$



例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0+z-0+0}{0+0+x-e^z+0} = -\frac{1+z}{x-e^z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{0+1+0-0}{0+0+x-e^z+0}$$



例 设 
$$z = f(x, y)$$
 满足  $x + y + xz = e^z - 1$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

解令
$$F(x, y, z) = x + y + xz - e^z + 1$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{1+0+z-0+0}{0+0+x-e^z+0} = -\frac{1+z}{x-e^z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x+y+xz-e^z+1)_y'}{(x+y+xz-e^z+1)_z'}$$
$$= -\frac{0+1+0-0+0}{0+0+x-e^z+0} = -\frac{1}{x-e^z}$$



例设 z = f(x, y) 满足  $2\sin(x + 2y - 3z) = x + 2y - 3z$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

例设 z = f(x, y) 满足  $2\sin(x + 2y - 3z) = x + 2y - 3z$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$ 

F(x, y, z) = 0

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} =$$

$$\mathbb{R} \Leftrightarrow F(x, y, z) = 2\sin(x + 2y - 3z) - x - 2y + 3z,$$
  
 $F(x, y, z) = 0$ 

$$\frac{\partial Z}{\partial X} = -\frac{F_X}{F_Z} =$$

$$\frac{\partial Z}{\partial V} = -\frac{F_y}{F_z} =$$

$$F(x, y, z) = 0$$
,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

$$F(x, y, z) = 0$$
,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{(2\sin(x+2y-3z)-x-2y+3z)_z'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

= -

$$F(x, y, z) = 0$$
,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{-6\cos(x+2y-3z)}{-6\cos(x+2y-3z)}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

$$F(x, y, z) = 0$$
,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{-6\cos(x+2y-3z)+3}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$



$$F(x, y, z) = 0$$
,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{2\cos(x+2y-3z)}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

= -

$$F(x, y, z) = 0$$
,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

鱼蟹鱼

解 令 
$$F(x, y, z) = 2 \sin(x + 2y - 3z) - x - 2y + 3z$$
, 则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)'_y}{(2\sin(x+2y-3z)-x-2y+3z)'_z}$$

 $-6\cos(x+2y-3z)+3$ 



解 令 
$$F(x, y, z) = 2 \sin(x + 2y - 3z) - x - 2y + 3z$$
,则  $F(x, y, z) = 0$ ,所以

例 设 z = f(x, y) 满足  $2 \sin(x + 2y - 3z) = x + 2y - 3z$ ,求  $\frac{\partial z}{\partial y}$  和  $\frac{\partial z}{\partial y}$ 

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{4\cos(x+2y-3z)}{-6\cos(x+2y-3z)+3}$$



解 令 
$$F(x, y, z) = 2 \sin(x + 2y - 3z) - x - 2y + 3z$$
, 则  $F(x, y, z) = 0$ , 所以

例 设 z = f(x, y) 满足  $2 \sin(x + 2y - 3z) = x + 2y - 3z$ ,求  $\frac{\partial z}{\partial y}$  和  $\frac{\partial z}{\partial y}$ 

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{4\cos(x+2y-3z)-2}{-6\cos(x+2y-3z)+3}$$



例 设 
$$z = f(x, y)$$
 满足  $z - y - x + xe^{z-y-x} = 0$ , 求  $dz$ 

解

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$

例 设 
$$z = f(x, y)$$
 满足  $z - y - x + xe^{z-y-x} = 0$ , 求  $dz$ 

解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ 

$$\frac{\partial Z}{\partial X} =$$

$$\frac{\partial Z}{\partial V} =$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$

例 设 
$$z = f(x, y)$$
 满足  $z - y - x + xe^{z-y-x} = 0$ , 求  $dz$ 

解 令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

$$\frac{\partial z}{\partial v} = -\frac{F_y}{F_z} =$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



例 设 
$$z = f(x, y)$$
 满足  $z - y - x + xe^{z-y-x} = 0$ , 求  $dz$ 

解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})'_y}{(z - y - x + xe^{z - y - x})'_z}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$
$$= -$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{1}{(z - y - x + xe^{z - y - x})_z'}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$
$$= -\frac{1 + xe^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{1}{(z - y - x + xe^{z - y - x})_z'}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$
$$= -\frac{1}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$
$$= -\frac{-1 + e^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{1}{(z - y - x + xe^{z - y - x})_z'}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$
$$= -\frac{-1 + e^{z - y - x} - xe^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{1}{(z - y - x + xe^{z - y - x})_z'}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则 $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$

$$= -\frac{-1 + e^{z - y - x} - xe^{z - y - x}}{1 + xe^{z - y - x}} = -\frac{1 + (x - 1)e^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{(z - y - x + xe^{z - y - x})_z'}{(z - y - x + xe^{z - y - x})_z'}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解 令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$

$$= -\frac{-1 + e^{z - y - x} - xe^{z - y - x}}{1 + xe^{z - y - x}} = -\frac{1 + (x - 1)e^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{1 + xe^{z - y - x}}{1 + xe^{z - y - x}}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解 令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$

$$= -\frac{-1 + e^{z - y - x} - xe^{z - y - x}}{1 + xe^{z - y - x}} = -\frac{1 + (x - 1)e^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{1}{1 + xe^{z - y - x}}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解 令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$

$$= -\frac{-1 + e^{z - y - x} - xe^{z - y - x}}{1 + xe^{z - y - x}} = -\frac{1 + (x - 1)e^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{-1 - xe^{z - y - x}}{1 + xe^{z - y - x}}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



例 设 z = f(x, y) 满足  $z - y - x + xe^{z-y-x} = 0$ ,求 dz

解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$
$$= -\frac{-1 + e^{z - y - x} - xe^{z - y - x}}{1 + xe^{z - y - x}} = -\frac{1 + (x - 1)e^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{-1 - xe^{z - y - x}}{1 + xe^{z - y - x}} = 1$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



例 设 z = f(x, y) 满足  $z - y - x + xe^{z-y-x} = 0$ ,求 dz

解令 
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$
$$= -\frac{-1 + e^{z - y - x} - xe^{z - y - x}}{1 + xe^{z - y - x}} = -\frac{1 + (x - 1)e^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{-1 - xe^{z - y - x}}{1 + xe^{z - y - x}} = 1$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy = -\frac{1 + (x - 1)e^{z - y - x}}{1 + xe^{z - y - x}}dx + dy$$



$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

例 设  $\Phi(u, v)$  具有连续偏导数,函数 z = z(x, y) 满足

$$Φ(cx - αz, cy - bz) = 0$$
, 证明:

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\mathbf{F}(x, y, z) = \Phi(cx - az, cy - bz),$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{\partial z}{\partial y} = \frac{F_y}{F_z} = \frac{F_y}{F_z}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{\partial z}{\partial y} = \frac{1}{2} = \frac{$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

解 令  $F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则

 $F_x =$ 

$$F_{y} = F_{z} = \frac{\partial z}{\partial x} = -\frac{F_{x}}{F_{z}} = \frac{\partial z}{\partial y} = -\frac{F_{y}}{F_{z}} = -\frac{F_{y}}{F_{z}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

解令
$$F(x, y, z) = \Phi(cx - \alpha z, cy - bz)$$
,则

$$F_X = \Phi_u \cdot u_X + \Phi_V \cdot V_X$$

$$F_y =$$
 $F_z =$ 

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial x} = \frac{F_y}{F_y} = \frac{\partial z}{\partial y} = \frac{\partial z}{\partial y} = \frac{F_y}{F_y} = \frac{\partial z}{\partial y} =$$

$$\frac{\partial Z}{\partial V} =$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\mathbf{F}$$
 会  $\mathbf{F}(x, y, z) = \Phi(cx - \alpha z, cy - bz)$ ,则

$$F_X = \Phi_u \cdot u_X + \Phi_v \cdot \nu_X = c\Phi_u$$

$$F_X = \Phi_u \cdot U_X + \Phi_V \cdot V_X = C\Phi_U$$
$$F_Y =$$

$$F_z = \frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{1}{2}$$

$$= -\frac{F_z}{F_z} = -\frac{F_y}{F_z} =$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\partial x = \partial y$$
  
解令  $F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则

$$F_X = \Phi_u \cdot u_X + \Phi_V \cdot V_X = c\Phi_u$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y$$

$$F_{z} = \frac{\partial z}{\partial x} = -\frac{F_{x}}{F_{z}} = \frac{\partial z}{\partial x} = -\frac{F_{y}}{F_{z}} = \frac{\partial z}{\partial x} = \frac{\partial z}{\partial x} = -\frac{F_{y}}{F_{z}} = -\frac{F_{y}}{$$

$$\frac{\partial z}{\partial v} =$$

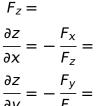
$$\partial \frac{\partial z}{\partial v} =$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$
解令  $F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则

$$F_X = \Phi_u \cdot u_X + \Phi_v \cdot v_X = c\Phi_u$$

$$F_V = \Phi_u \cdot u_V + \Phi_v \cdot v_V = c\Phi_v$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot \nu_y = c\Phi_v$$



$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\partial x \partial y \partial y$$
  
解令  $F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则

$$F_X = \Phi_u \cdot u_X + \Phi_V \cdot V_X = c\Phi_u$$

$$F_{y} = \Phi_{u} \cdot u_{y} + \Phi_{v} \cdot v_{y} = c\Phi_{v}$$

$$F_z = \Phi_u \cdot u_z + \Phi_v \cdot v_z$$
$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

$$\frac{F_z}{F_z} =$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\mathbf{F}(x, y, z) = \Phi(cx - az, cy - bz)$$
, 则

$$F_X = \Phi_u \cdot u_X + \Phi_v \cdot v_X = c\Phi_u$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y = c\Phi_v$$

$$= \Phi_u \cdot u_y + \Phi_v \cdot v_y = c\Phi_v$$

$$= \Phi_u \cdot u_z + \Phi_v \cdot v_z = -a\Phi$$

$$F_z = \Phi_u \cdot u_z + \Phi_v \cdot v_z = -a\Phi_u - b\Phi_v$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial x} = -\frac{F_y}{F_z} = -\frac{F_$$



$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$F_{X} = \Phi_{u} \cdot u_{X} + \Phi_{v} \cdot v_{X} = c\Phi_{u}$$

$$F_{Y} = \Phi_{u} \cdot u_{Y} + \Phi_{v} \cdot v_{Y} = c\Phi_{v}$$

$$F_{Z} = \Phi_{u} \cdot u_{Z} + \Phi_{v} \cdot v_{Z} = -\alpha\Phi_{u} - b\Phi_{v}$$

$$\frac{\partial z}{\partial x} = -\frac{F_{x}}{F_{z}} = \frac{c\Phi_{u}}{\alpha\Phi_{u} + b\Phi_{v}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_{y}}{F_{z}} = \frac{c\Phi_{v}}{\sigma\Phi_{v} + \sigma_{v}}$$



$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

解令
$$F(x, y, z) = \Phi(cx - az, cy - bz)$$
,则

$$F_X = \Phi_u \cdot u_X + \Phi_V \cdot \nu_X = c\Phi_u$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot \nu_y = c\Phi_v$$

$$F_{y} = \Psi_{u} \cdot u_{y} + \Psi_{v} \cdot v_{y} = c\Psi_{v}$$

$$F_{z} = \Phi_{u} \cdot u_{z} + \Phi_{v} \cdot v_{z} = -a\Phi_{u} - b\Phi_{v}$$

$$\frac{\partial Z}{\partial x} = -\frac{F_X}{F_Z} = \frac{c\Phi_u}{\alpha\Phi_u + b\Phi_v}$$

$$\frac{\partial Z}{\partial y} = -\frac{F_y}{F_z} = \frac{c\Phi_v}{a\Phi_u + b\Phi_v}$$

$$\frac{\partial Z}{\partial V} =$$



$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

 $\mathbf{H}$  令  $F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则  $F_x = \Phi_{II} \cdot u_x + \Phi_{V} \cdot V_x = c\Phi_{II}$ 

$$F_{y} = \Phi_{u} \cdot u_{y} + \Phi_{v} \cdot v_{y} = c\Phi_{v}$$

$$F_{z} = \Phi_{u} \cdot u_{z} + \Phi_{v} \cdot v_{z} = -\alpha\Phi_{u} - b\Phi_{v}$$

$$\partial z \qquad F_{x} \qquad c\Phi_{u}$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{c\Phi_u}{a\Phi_u + b\Phi_v}$$
$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{c\Phi_v}{a\Phi_u + b\Phi_v}$$





$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

 $\mathbf{H}$  令  $F(x, y, z) = \Phi(cx - az, cy - bz)$ ,则  $F_x = \Phi_{II} \cdot u_x + \Phi_{V} \cdot V_x = c\Phi_{II}$ 

$$F_{y} = \Phi_{u} \cdot u_{y} + \Phi_{v} \cdot v_{y} = c\Phi_{v}$$

$$F_{z} = \Phi_{u} \cdot u_{z} + \Phi_{v} \cdot v_{z} = -a\Phi_{u} - b\Phi_{v}$$

| ∂Z         | $F_{x}$               | $c\Phi_u$                      |
|------------|-----------------------|--------------------------------|
| 9x         | $\frac{1}{F_z}$       | $\overline{a\Phi_u + b\Phi_v}$ |
| ∂Z         | $_{-}$ $F_{y}$ $_{-}$ | $c\Phi_{ m V}$                 |
| ∂ <i>y</i> | $=-\frac{1}{F_z}=$    | $\overline{a\Phi_u + b\Phi_v}$ |

例 设 z = f(x, y) 满足  $z = x + ye^z$ , 求  $\frac{\partial^2 z}{\partial x \partial y}$ 

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ , 求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$ 

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ , 求  $\frac{\partial^2 z}{\partial x \partial y}$ 

$$F(x, y, z) = x + ye^z - z, \ 则 \ F(x, y, z) = 0, \ 所以$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{\partial z}{\partial y} = \frac{F_y}{F_z} = \frac{F_y}{F_z}$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ , 求  $\frac{\partial^2 z}{\partial x \partial y}$ 

$$F(x, y, z) = x + ye^z - z, \ 则 \ F(x, y, z) = 0, \ 所以$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

$$F(x, y, z) = x + ye^z - z, \ 则 \ F(x, y, z) = 0, \ 所以$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z}$$
$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} =$$
$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) =$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{e^z}{e^z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{e^z}{e^z}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \frac{e^z}{e^z}$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} =$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) =$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) =$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) =$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)_y'$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)_y' = \frac{(ye^z - 1)_y'}{(ye^z - 1)^2}$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)_y' = \frac{(ye^z - 1)_y'}{(ye^z - 1)^2}$$

$$= \frac{e^z + y(e^z)_y'}{(ye^z - 1)^2}$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$  ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)_y' = \frac{(ye^z - 1)_y'}{(ye^z - 1)^2}$$

$$= \frac{e^z + y(e^z)_y'}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \frac{\partial z}{\partial y}}{(ye^z - 1)^2}$$

例 设 
$$z = f(x, y)$$
 满足  $z = x + ye^z$ , 求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)'_y = \frac{(ye^z - 1)'_y}{(ye^z - 1)^2}$$

$$= \frac{e^z + y(e^z)'_y}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \frac{\partial z}{\partial y}}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \left(-\frac{e^z}{ye^z - 1}\right)}{(ye^z - 1)^2}$$



例 设 z = f(x, y) 满足  $z = x + ye^z$ , 求  $\frac{\partial^2 z}{\partial x \partial y}$ 

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)_y' = \frac{(ye^z - 1)_y'}{(ye^z - 1)^2}$$

$$= \frac{e^z + y(e^z)_y'}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \frac{\partial z}{\partial y}}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \left(-\frac{e^z}{ye^z - 1}\right)}{(ye^z - 1)^2}$$

$$= \frac{-e^z}{(ye^z - 1)^2}$$

例 设 z = f(x, y) 满足  $z = x + ye^z$ ,求  $\frac{\partial^2 z}{\partial x \partial y}$ 

解 
$$F(x, y, z) = x + ye^z - z$$
,则  $F(x, y, z) = 0$ ,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x + ye^z - z)_x}{(x + ye^z - z)_z} = -\frac{1}{ye^z - 1}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(x + ye^z - z)_y}{(x + ye^z - z)_z} = -\frac{e^z}{ye^z - 1}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x}\right) = \left(-\frac{1}{ye^z - 1}\right)_y' = \frac{(ye^z - 1)_y'}{(ye^z - 1)^2}$$

$$= \frac{e^z + y(e^z)_y'}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \frac{\partial z}{\partial y}}{(ye^z - 1)^2} = \frac{e^z + ye^z \cdot \left(-\frac{e^z}{ye^z - 1}\right)}{(ye^z - 1)^2}$$

$$= \frac{-e^z}{(ye^z - 1)^3} = \frac{e^z}{(1 + x - z)^3}$$

We are here now...

1. 一个方程的情形

2. 方程组的情形

## 回顾: 二元线性方程组的求解

## 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

## 用消元法解:

#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11} a_{22} x + a_{12} a_{22} y = a_{22} b_1 & (1) \times a_{22} \\ a_{21} x + a_{22} y = b_2 & (2) \times a_{12} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11} a_{22} x + a_{12} a_{22} y = a_{22} b_1 & (1) \times a_{22} \\ a_{21} a_{12} x + a_{22} a_{12} y = a_{12} b_2 & (2) \times a_{12} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11} a_{22} x + a_{12} a_{22} y = a_{22} b_1 & (1) \times a_{22} \\ a_{21} a_{12} x + a_{22} a_{12} y = a_{12} b_2 & (2) \times a_{12} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{11} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}a_{11}x + a_{22}a_{11}y = a_{11}b_2 & (2) \times a_{11} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11} a_{21} x + a_{12} a_{21} y = a_{21} b_1 & (1) \times a_{21} \\ a_{21} a_{11} x + a_{22} a_{11} y = a_{11} b_2 & (2) \times a_{11} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

#### 二元线性方程组

$$\begin{cases} a_{11} a_{21} x + a_{12} a_{21} y = a_{21} b_1 & (1) \times a_{21} \\ a_{21} a_{11} x + a_{22} a_{11} y = a_{11} b_2 & (2) \times a_{11} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$



#### 二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$



二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

用消元法解:

(1) × 
$$a_{22}$$
 – (2) ×  $a_{12}$ , 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

(2)× $a_{11}$ -(1)× $a_{21}$ , 消去x, 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{a_{11} a_{12}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$



二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

(2) × 
$$a_{11}$$
 – (1) ×  $a_{21}$ , 消去  $x$ , 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{a_{11}a_{12}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$



二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去  $y$ , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去  $x$ , 得:

$$y = rac{a_{11} - (1) \times a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = rac{\begin{vmatrix} a_{11} & b_1 \ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \ a_{21} & a_{22} \end{vmatrix}}$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \qquad , \quad y =$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -- \qquad , \quad y = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}} = --$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = - - , \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = - -$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{1}{1} \qquad , \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -\frac{1}{1}$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{1}{1}$$
, 
$$y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{1}{1}$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} \qquad , \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{1}{1}$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} \qquad , \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1}$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1}$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \begin{vmatrix} 0 & 5 \\ 4 & 8 \\ 2 & 5 \\ 3 & 8 \end{vmatrix} = \frac{-20}{1} = -20, \quad y = \begin{vmatrix} 2 & 0 \\ 3 & 4 \\ 2 & 5 \\ 3 & 8 \end{vmatrix} = \frac{8}{1} = 8$$

2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x =$$





$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$
2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - , \quad y = \frac{3}{1} = 8$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$
2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - , \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - \end{cases}$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$
2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-3}{3}, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-3}{3}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1.  $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$ 

2.  $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} \quad , y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{3}{3}$ 

d: 隐函数的求导公式

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

练习 利用二阶行列式求解下面二元线性方程组
$$1 \quad \begin{cases} 2x + 5y = 0 \\ x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 4 & 8 \end{vmatrix}} = \frac{-20}{3} = -20 \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \end{vmatrix}} = \frac{8}{3} = \frac{8}{3}$$

d: 隐函数的求异公式

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

2.  $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \ y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3}$ 

练习 利用二阶行列式求解下面二元线性方程组
$$1. \begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \begin{vmatrix} 0 & 5 \\ 4 & 8 \\ \hline \begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix} = \frac{-20}{1} = -20, \quad y = \begin{vmatrix} 2 & 0 \\ 3 & 4 \\ \hline \begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix} = \frac{8}{1} = 8$$

d: 隐函数的求异公式

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1. 
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$
2. 
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3} = -3$$

练习 利用二阶行列式求解下面二元线性方程组  $1. \begin{cases} 2x + 5y = 0 \\ x = \begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix} = \frac{-20}{120} = -20, \quad y = \begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix} = \frac{8}{120} = \frac$ 

$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题:如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

#### 求解如下:

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \\ G(x, y, u, v) = 0 & \Longrightarrow \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题:如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

#### 求解如下:

$$\begin{cases} F(x, y, u, v) = 0 & \stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x,y,u,v) = 0 \\ G(x,y,u,v) = 0 \end{cases} \stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \\ G(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \end{cases} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$



假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \\ G(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \end{cases} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题:如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial x}$ ?

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \Rightarrow \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

$$\Rightarrow u_x = \begin{vmatrix} -F_x & F_v \\ -G_x & G_v \end{vmatrix}, \quad v_x = \begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}$$



假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

求解如下:
$$\begin{cases}
F(x, y, u, v) = 0 \\
G(x, y, u, v) = 0
\end{cases} \Rightarrow \begin{cases}
F_u \cdot u_x + F_v \cdot v_x = -F_x \\
G_u \cdot u_x + G_v \cdot v_x = -G_x
\end{cases}$$

$$\Rightarrow u_x = \frac{\begin{vmatrix} -F_x & F_v \\ -G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_x = \frac{\begin{vmatrix} -F_u & F_x \\ -G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$



假设函数 
$$u = u(x, y), v = v(x, y)$$
 满足方程组 
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ ?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

$$\Rightarrow u_x = -\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_x = -\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$



假设函数 u = u(x, y), v = v(x, y) 满足方程组  $\begin{cases} F(x, y, u, v) = 0, \\ G(x, v, u, v) = 0. \end{cases}$ 

问题:如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial x}$ ?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

$$\Rightarrow u_x = -\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_x = -\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$\frac{\partial(F,G)}{\partial(x,v)}$$



假设函数 u = u(x, y), v = v(x, y) 满足方程组  $\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$ 

问题:如何计算  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial x}$ ?

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

$$\Rightarrow u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$
$$= -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)} \qquad = -\frac{1}{J} \frac{\partial(F, G)}{\partial(u, x)} \underbrace{\bigcirc \underbrace{\Diamond F, A^{+}}_{20/23 \, \triangleleft \, \triangleright \, A^{-}}_{20/23 \, \triangleleft \, \triangleright \, A^{-}}}$$

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \\ G(x, y, u, v) = 0 & \Longrightarrow \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ \end{cases}$$



$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \\ G(x, y, u, v) = 0 & \Longrightarrow \end{cases} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 & \stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \\ G(x, y, u, v) = 0 & \stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \end{cases} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y =$$
 ———,  $v_y =$  ———



$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = \frac{ }{ \left| \begin{array}{cc} F_u & F_v \\ G_u & G_v \end{array} \right| }, \quad V_y = \frac{ }{ \left| \begin{array}{cc} F_u & F_v \\ G_u & G_v \end{array} \right| }$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = \frac{\begin{vmatrix} -F_y & F_v \\ -G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = \frac{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = \begin{vmatrix} -F_y & F_v \\ -G_y & G_v \end{vmatrix}, \quad v_y = \begin{vmatrix} -F_u & F_y \\ -G_u & G_y \end{vmatrix}$$

$$\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$= -\frac{1}{J} \frac{\partial (F, G)}{\partial (y, v)}$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$= -\frac{1}{J} \frac{\partial (F, G)}{\partial (y, v)} \qquad = -\frac{1}{J} \frac{\partial (F, G)}{\partial (u, y)}$$



$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

总结 设 
$$u = u(x, y), v = v(x, y)$$
 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

$$u_{x} = v_{x} = v_{x} = v_{x}$$

$$u_{V} = v_{V} = v_{V} = v_{V}$$

总结 设 
$$u = u(x, y), v = v(x, y)$$
 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

$$u_x = v_x = v_x$$

$$u_{V} = v_{V} = v_{V}$$

总结 设 
$$u = u(x, y), v = v(x, y)$$
 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$u_x =$$

$$v_x =$$

$$u_v =$$

$$y =$$

总结 设 
$$u = u(x, y), v = v(x, y)$$
 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

$$u_x = v_x = v_x$$

$$u_{V} = v_{V} = v_{V}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$
$$\xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

$$\begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$u_v =$$

$$v_y =$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \end{cases} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$
$$u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{v} \\ G_{y} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{y} \end{vmatrix}}$$





$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \Rightarrow \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$
$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{v} \\ G_{y} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{y} \end{vmatrix}}$$

$$v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{y} \end{vmatrix}}$$





$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以
$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(u, x)}$$

$$u_{y} = -\frac{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{y} & F_{v} \\ G_{y} & G_{v} \end{vmatrix}}$$

$$G_{u} = -\frac{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

 $v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{y} \end{vmatrix}}$ 



$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \end{cases}$$

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(u, x)}$$

 $u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{v} \\ G_{y} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)}, \quad v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$ 



$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G_y + G_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)},$$

$$u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{v} \\ G_{y} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{y} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)}, \quad v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(u, y)}$$

9 章 *d*: 隐函数的求导公式

② 暨南大學 22/23 ◁ ▷ A ▽ 例设  $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}, \ \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$ 

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}$$

$$u_x = v_x = v_x$$

 $\nu_{\nu} =$ 

 $u_v =$ 

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}$$

$$u_x = v_x = v_x$$

 $\nu_{\nu} =$ 

 $u_v =$ 

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

 $\stackrel{\frac{\sigma}{\partial x}}{\Longrightarrow} \left\{ (e^u + \sin v) u_x + u \cos v \cdot v_x = 1 \right.$ 

23/23 ▷ ▷ ▷ ▽

 $\nu_x =$ 

 $\nu_{\nu} =$ 

 $\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$ 







 $u_x =$ 

 $u_v =$ 

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow}$$

$$\stackrel{\frac{\sigma}{\partial x}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{x} + u\cos v \cdot v_{x} = 1\\ (e^{u} - \cos v)u_{x} + u\sin v \cdot v_{x} = 0 \end{cases}$$

$$u_x =$$

$$v_{\chi}$$
=

$$u_v =$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\Rightarrow \begin{cases} (e^{u} + \sin v)u_{x} + u\cos v \cdot v_{x} = 1 \\ (e^{u} - \cos v)u_{x} + u\sin v \cdot v_{x} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0 \end{cases}$$

$$u_x =$$

$$\nu_{x} =$$

$$u_v =$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$x \qquad \stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{x} + u\cos v \cdot v_{x} = 1\\ (e^{u} - \cos v)u_{x} + u\sin v \cdot v_{x} = 0 \end{cases}$$

 $\stackrel{\frac{\sigma}{\partial y}}{\Longrightarrow} \begin{cases} (e^u + \sin v)u_y + u\cos v \cdot v_y = 0 \\ (e^u - \cos v)u_v + u\sin v \cdot v_v = 1 \end{cases}$ 

$$u_x =$$

$$=$$
  $v_{\chi}=$ 

$$u_v =$$

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u\sin v \cdot v_{y} = 1 \end{cases}$$

$$\begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u\sin v \cdot v_{y} = 1 \end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$u_x = v_x = v_x$$

$$u_y = v_y = v_y$$

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\begin{cases} e^{u} - u\cos v = y \\ \Longrightarrow \end{cases} \begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u\sin v \cdot v_{y} = 1 \end{cases}$$

$$\text{所以 } J = \begin{vmatrix} e^{u} + \sin v & u\cos v \\ e^{u} - \cos v & u\sin v \end{vmatrix}$$

所以 
$$J = \begin{vmatrix} e^{u} + \sin v & u \cos v \\ e^{u} - \cos v & u \sin v \end{vmatrix}$$

$$v_{x} = -\frac{1}{J}$$

$$v_{y} = -\frac{1}{J}$$

$$v_{y} = -\frac{1}{J}$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases} \begin{cases} (e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\ (e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0 \end{cases}$$
$$\xrightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1 \end{cases}$$

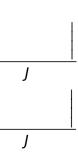
所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

所以
$$J = \begin{vmatrix} e^{u} + \sin v & u \cos v \\ e^{u} - \cos v & u \sin v \end{vmatrix}$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_{x} = -\frac{\begin{vmatrix} u & u & u \\ 0 & u & v \end{vmatrix}}{J}$$

$$v_{y} = -\frac{\begin{vmatrix} u & u & u \\ 0 & u & v \end{vmatrix}}{J}$$





例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{=} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以  $J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$ 

$$u_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_{x} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{J}$$

$$u_{y} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{J}$$

例设 
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$$
, 求  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ 

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{=} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_{x} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{J}$$

$$u_{y} = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}$$

$$v_{y} = -\frac{\begin{vmatrix} 1 & u \cos v & 0 \\ 1 & u \sin v & 0 \end{vmatrix}}{J}$$

例设 
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$$
, 求  $\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$ 

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{=} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_{x} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{J}$$

$$u_{y} = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}$$

$$v_{y} = -\frac{\begin{vmatrix} e^{u} + \sin v & 0 \\ e^{u} - \cos v & 1 \end{vmatrix}}{J}$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\int e^{u} + u \sin v =$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases} \begin{cases} (e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\ (e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0 \end{cases}$$

$$v = x$$
 $v = y$ 

$$\stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u\sin v \cdot v_{y} = 1 \end{cases}$$

 $v_y = -\frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{r}$ 

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$u_x = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_x = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J}$$

 $u_y = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{\cdot}$ 

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{=} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$|1 \ u \cos v| \qquad |e^u + \sin v \ 1|$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J} = \frac{-\sin v}{e^{u(\sin v - \cos v) + 1}}, v_{x} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{J}$$
$$u_{y} = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}$$
$$v_{y} = -\frac{\begin{vmatrix} e^{u} + \sin v & 0 \\ e^{u} - \cos v & 1 \end{vmatrix}}{J}$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

 $u_{x} = -\frac{\left|0 \ u \sin v\right|}{J} = \frac{-\sin v}{e^{u(\sin v - \cos v) + 1}}, v_{x} = -\frac{\left|e^{u} - \cos v \ 0\right|}{J}$   $u_{y} = -\frac{\left|0 \ u \cos v\right|}{I}$   $v_{y} = -\frac{\left|e^{u} + \sin v \ 0\right|}{I}$   $v_{y} = -\frac{\left|e^{u} - \cos v \ 1\right|}{I}$ 

例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{=} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$u_{x} = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{\int} = \frac{-\sin v}{e^{u(\sin v - \cos v) + 1}}, v_{x} = -\frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{\int} = \frac{e^{u - \cos v}}{ue^{u(\sin v - \cos v)}}$$
$$u_{y} = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{\int} = \frac{\cos v}{e^{u(\sin v - \cos v) + 1}}, v_{y} = -\frac{\begin{vmatrix} e^{u} + \sin v & 0 \\ e^{u} - \cos v & 1 \end{vmatrix}}{\int}$$



例设 
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\frac{\partial}{\partial y} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

所以 
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u (\sin v - \cos v) + u$$

$$u_X = -\frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{\int} = \frac{-\sin v}{e^u (\sin v - \cos v) + 1}, v_X = -\frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{\int} = \frac{e^u - \cos v}{u \sin v \cos v}$$

$$u_{y} = -\frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J} = \frac{\cos v}{e^{u(\sin v - \cos v) + 1}}, v_{y} = -\frac{\begin{vmatrix} e^{u} + \sin v & 0 \\ e^{u} - \cos v & 1 \end{vmatrix}}{J} = -\frac{e^{u + \sin v}}{u e^{u(\sin v - \cos v) + u}}$$