第3章b:向量与向量组的线性组合

数学系 梁卓滨

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● n 维行向量

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• 行向量、列向量统称向量。

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- 零向量 O = (0, 0, ···, 0)

•
$$\mathfrak{P} \alpha = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}, \ \beta = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}, \ k \in \mathbb{R}, \ \mathbb{M}$$

$$\alpha + \beta =$$
 , $\alpha - \beta =$, $k\alpha =$

• 设
$$\alpha = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$
, $\beta = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$, $k \in \mathbb{R}$, 则

$$\alpha + \beta = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{pmatrix}, \quad \alpha - \beta = \qquad , \quad k\alpha =$$

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$$\alpha = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}, \beta = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}, k \in \mathbb{R}, 则$$

$$\alpha + \beta = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{pmatrix}, \quad \alpha - \beta = \begin{pmatrix} a_1 - b_1 \\ a_2 - b_2 \\ \vdots \\ a_n - b_n \end{pmatrix}, \quad k\alpha = \begin{pmatrix} a_1 - b_1 \\ \vdots \\ a_n - b_n \end{pmatrix}$$

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, $\beta = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$, $k \in \mathbb{R}$, 则

$$\alpha+\beta=\begin{pmatrix} a_1+b_1\\ a_2+b_2\\ \vdots\\ a_n+b_n \end{pmatrix},\quad \alpha-\beta=\begin{pmatrix} a_1-b_1\\ a_2-b_2\\ \vdots\\ a_n-b_n \end{pmatrix},\quad k\alpha=\begin{pmatrix} ka_1\\ ka_2\\ \vdots\\ ka_n \end{pmatrix}$$

• 行向量类似

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• 给定向量组

$$\alpha_1 = \begin{pmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{m1} \end{pmatrix}, \ \alpha_2 = \begin{pmatrix} \alpha_{12} \\ \alpha_{22} \\ \vdots \\ \alpha_{m2} \end{pmatrix}, \dots, \ \alpha_n = \begin{pmatrix} \alpha_{1n} \\ \alpha_{2n} \\ \vdots \\ \alpha_{mn} \end{pmatrix}$$

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设 k_1 , k_2 , · · · , k_n 为任意数,则称

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_n\alpha_n$$

为向量组 α_1 , α_2 , ..., α_n 的 **线性组合** 。

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为向量组 α_1 , α_2 , ..., α_n 的**线性组合** 。

• 问题 给定向量 $\beta = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ c \end{pmatrix}$,问 β 能否由 $\alpha_1, \alpha_2, \cdots, \alpha_n$ 线性表示?

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• 问题 给定向量 $\beta = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$,问 β 能否由 $\alpha_1, \alpha_2, \cdots, \alpha_n$ 线性表示?

即:是否存在数 k_1, k_2, \ldots, k_n 使得:

$$\beta = k_1 \alpha_1 + k_2 \alpha_2 + \dots + k_n \alpha_n?$$

例 判断 β 能否由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式 $\beta = k_1\alpha_1 + k_2\alpha_2 + k_3\alpha_3$ 。

$$\begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} \qquad \begin{pmatrix} \alpha_1 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \alpha_2 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} \alpha_3 \\ 0 \\ 0 \\ 2 \end{pmatrix}$$

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$$\begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} = -\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + --\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + -\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

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$$\begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} = \mathbf{2} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \mathbf{2} \begin{pmatrix} \alpha_1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \mathbf{3} \begin{pmatrix} \alpha_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

线性组合

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$$\begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} = \mathbf{2} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \mathbf{-7} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \mathbf{\frac{5}{2}} \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

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(1)
$$\overrightarrow{P}$$

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所以 $\beta = 2\alpha_1 - 7\alpha_2 + \frac{5}{2}\alpha_3$;

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所以 $\beta = 2\alpha_1 - 7\alpha_2 + \frac{5}{2}\alpha_3$; β 能由 α_1 , α_2 , α_3 线性表示

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(1)
$$\Box$$

$$\begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} = \frac{2}{2} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \frac{-7}{2} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \frac{5}{2} \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

所以 $\beta = \frac{2\alpha_1 - 7\alpha_2}{3} + \frac{5}{2}\alpha_3$; β 能由 α_1 , α_2 , α_3 线性表示

(2)
$$\[\bigcap$$
 β α_1 α_2 α_3 α_3 α_4 α_5 α_5 α_5 α_6 α_7 α_8 α_9 α_9

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(1)
$$\Box$$

$$\begin{pmatrix}
2 \\
-7 \\
5
\end{pmatrix} = \frac{2}{2} \begin{pmatrix}
1 \\
0 \\
0
\end{pmatrix} + \frac{-7}{2} \begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix} + \frac{5}{2} \begin{pmatrix}
0 \\
0 \\
2
\end{pmatrix}$$

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(2)
$$\Box$$
 β α_1 α_2 α_3 α_4 α_5 α_5

例 判断 β 能否由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式 $\beta = k_1\alpha_1 + k_2\alpha_2 + k_3\alpha_3$ 。

(1)
$$\Box$$

$$\begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} = \frac{2}{2} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \frac{-7}{2} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \frac{5}{2} \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

所以 $\beta = \frac{2\alpha_1 - 7\alpha_2}{3} + \frac{5}{2}\alpha_3$; β 能由 α_1 , α_2 , α_3 线性表示

(2) 问
$$\begin{pmatrix}
2 \\
-7 \\
5
\end{pmatrix}
\times
\begin{pmatrix}
\alpha_1 \\
0 \\
0
\end{pmatrix}
+
\begin{pmatrix}
2 \\
3 \\
0
\end{pmatrix}
+
\begin{pmatrix}
0 \\
2 \\
0
\end{pmatrix}$$

例 判断 β 能否由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式 $\beta = k_1\alpha_1 + k_2\alpha_2 + k_3\alpha_3$ 。

(1)
$$\Box$$

$$\begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} = \frac{2}{2} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \frac{-7}{2} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \frac{5}{2} \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

所以 $\beta = \frac{2\alpha_1 - 7\alpha_2}{3} + \frac{5}{2}\alpha_3$; β 能由 α_1 , α_2 , α_3 线性表示

(2)
$$\triangleright$$
 β

$$\begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

所以 β 不能由 $\alpha_1, \alpha_2, \alpha_3$ 线性表示!

例 问
$$\begin{bmatrix} \beta & \alpha_1 & \alpha_2 & \alpha_3 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} + - \begin{bmatrix} 2 & \alpha_3 & \alpha_3 \\ -1 & 1 & 1 \\ 1 & -2 \end{bmatrix} + - \begin{bmatrix} 3 & 2 & \alpha_3 & \alpha$$

即: β 能否由 α_1 , α_2 , α_3 线性表示? 如果能,线性表达式是什么?

例 问
$$\begin{bmatrix} \beta & \alpha_1 & \alpha_2 & \alpha_3 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} + - \begin{bmatrix} 2 & \alpha_3 & \alpha_4 & \alpha_5 \\ -1 & 1 & 1 \\ 1 & -2 \end{bmatrix} + - \begin{bmatrix} 3 & 2 & \alpha_5 & \alpha_5 & \alpha_5 & \alpha_5 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

即: β 能否由 α_1 , α_2 , α_3 线性表示? 如果能,线性表达式是什么?

问题

• 一般地,如何判断 β 能否由 $\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性表示?

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$$\begin{bmatrix} \beta & \alpha_1 & \alpha_2 & \alpha_3 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} + - \begin{pmatrix} 2 & \alpha_3 & \alpha_3 \\ -1 & 1 & 1 \\ 1 & -2 \end{pmatrix} + - \begin{pmatrix} 3 & 2 & \alpha_3 & \alpha_3 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

即: β 能否由 α_1 , α_2 , α_3 线性表示? 如果能,线性表达式是什么?

问题

- 一般地,如何判断 β 能否由 $\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性表示?
- 如果能线性表出,如何求出 *k*₁, *k*₂, . . . , *k*_n 使

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_n\alpha_n = \beta$$
?

例 问
$$\begin{bmatrix} \beta & \alpha_1 & \alpha_2 & \alpha_3 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} + - \begin{pmatrix} 2 & \alpha_3 & \alpha_3 \\ -1 & 1 & 1 \\ 1 & -2 \end{pmatrix} + - \begin{pmatrix} 3 & 2 & \alpha_3 & \alpha_3 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

即: β 能否由 α_1 , α_2 , α_3 线性表示?如果能,线性表达式是什么?

问题

- 一般地,如何判断 β 能否由 $\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性表示?
- 如果能线性表出,如何求出 k_1 , k_2 , ..., k_n 使

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_n\alpha_n = \beta$$
?

不难看出, k_1, \dots, k_n 的求解可归结为线性方程组的求解。

 $\begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} \qquad \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} \qquad \cdots \qquad \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} \qquad \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{mn} \end{pmatrix}$

 β 可由 α_1 , α_2 , \cdots , α_n 线性表示

$lpha_1$	α_2	α_n	β
$\begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix}$	$\begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix}$	$\begin{pmatrix} a_{1n} \\ a_{2n} \end{pmatrix}$	$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$
$\left(\begin{array}{c} \vdots \\ a_{m1} \end{array}\right)$	$\left(\begin{array}{c} \vdots \\ a_{m2} \end{array}\right)$	$\begin{pmatrix} \vdots \\ a_{mn} \end{pmatrix}$	$\left(\begin{array}{c} \vdots \\ b_m \end{array}\right)$

$$\Leftrightarrow k_1 \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_2 \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \dots + k_n \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

$$\Leftrightarrow k_{1} \begin{pmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{m1} \end{pmatrix} + k_{2} \begin{pmatrix} \alpha_{12} \\ \alpha_{22} \\ \vdots \\ \alpha_{m2} \end{pmatrix} + \cdots + k_{n} \begin{pmatrix} \alpha_{1n} \\ \alpha_{2n} \\ \vdots \\ \alpha_{mn} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\begin{pmatrix} \alpha_{1} & \alpha_{2} & \alpha_{n} \\ \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} \end{pmatrix} \begin{pmatrix} k_{1} \\ k_{2} \\ \vdots \\ k_{n} \end{pmatrix}$$

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$$\Leftrightarrow k_{1} \begin{pmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{m1} \end{pmatrix} + k_{2} \begin{pmatrix} \alpha_{12} \\ \alpha_{22} \\ \vdots \\ \alpha_{m2} \end{pmatrix} + \cdots + k_{n} \begin{pmatrix} \alpha_{1n} \\ \alpha_{2n} \\ \vdots \\ \alpha_{mn} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

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$$\Leftrightarrow k_{1} \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_{2} \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \cdots + k_{n} \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} k_{1} \\ k_{2} \\ \vdots \\ k_{n} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

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$$\Leftrightarrow k_{1}\begin{pmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{m1} \end{pmatrix} + k_{2}\begin{pmatrix} \alpha_{12} \\ \alpha_{22} \\ \vdots \\ \alpha_{m2} \end{pmatrix} + \cdots + k_{n}\begin{pmatrix} \alpha_{1n} \\ \alpha_{2n} \\ \vdots \\ \alpha_{mn} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\ \vdots & \vdots & & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn} \end{pmatrix} \begin{pmatrix} k_{1} \\ k_{2} \\ \vdots \\ k_{n} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

 \Leftrightarrow $Ax = \beta$ 有解

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$$\Leftrightarrow k_1 \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_2 \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \cdots + k_n \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

$$\Leftrightarrow Ax = \beta f M \qquad (k_1, \dots, k_n \Xi f \Xi h M M)$$

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$$\Leftrightarrow k_{1} \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_{2} \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \cdots + k_{n} \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} k_{1} \\ k_{2} \\ \vdots \\ k_{n} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow$$
 $Ax = β$ 有解 $(k_1, \dots, k_n$ 是方程的解)

$$\Leftrightarrow$$
 $r(A) = r(A:\beta)$

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$$\Leftrightarrow$$
 $Ax = \beta$ 有解 $(k_1, \dots, k_n$ 是方程的解)

$$\Leftrightarrow r(A) = r(A:\beta) \iff (\alpha_1 \alpha_2 \cdots \alpha_n) \quad (\alpha_1 \alpha_2 \cdots \alpha_n \beta)$$

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$$\Leftrightarrow k_{1} \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_{2} \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \cdots + k_{n} \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} k_{1} \\ k_{2} \\ \vdots \\ k_{n} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$⇔$$
 $Ax = β$ 有解 $(k_1, \dots, k_n$ 是方程的解)

$$\Leftrightarrow$$
 $r(A) = r(A:\beta)$ \Leftrightarrow $r(\alpha_1 \alpha_2 \cdots \alpha_n) = r(\alpha_1 \alpha_2 \cdots \alpha_n \beta)$

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$$\beta \text{可由}\alpha_{1}, \alpha_{2}, \cdots, \alpha_{n}$$
线性表示
$$\Leftrightarrow k_{1} \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_{2} \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \cdots + k_{n} \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} k_{1} \\ k_{2} \\ \vdots \\ k_{n} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow Ax = \beta \text{ fig } (k_{1}, \cdots, k_{n}) \text{ Epsilon}$$

$$\Leftrightarrow$$
 $r(A) = r(A:\beta)$ \Leftrightarrow $r(\alpha_1 \alpha_2 \cdots \alpha_n) = r(\alpha_1 \alpha_2 \cdots \alpha_n \beta)$

定理 β 可由 $\alpha_1, \alpha_2, \cdots, \alpha_n$ 线性表示 $\Leftrightarrow r(\alpha_1 \alpha_2 \cdots \alpha_n) = r(\alpha_1 \alpha_2 \cdots \alpha_n \beta)$

$$\Leftrightarrow k_{1}\begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_{2}\begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \cdots + k_{n}\begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} k_{1} \\ k_{2} \\ \vdots \\ k_{n} \end{pmatrix} = \begin{pmatrix} b_{1} \\ b_{2} \\ \vdots \\ b_{m} \end{pmatrix}$$

$$\Leftrightarrow Ax = \beta \overline{\beta} \beta \beta \qquad (k_{1}, \cdots, k_{n}) \mathbb{E} \overline{\beta} \beta \beta \beta \beta \beta$$

定理 β 可由 $\alpha_1, \alpha_2, \dots, \alpha_n$ 线性表示 $\Leftrightarrow r(\alpha_1 \alpha_2 \dots \alpha_n) = r(\alpha_1 \alpha_2 \dots \alpha_n \beta)$

 \Leftrightarrow $r(A) = r(A:\beta)$ \Leftrightarrow $r(\alpha_1 \alpha_2 \cdots \alpha_n) = r(\alpha_1 \alpha_2 \cdots \alpha_n \beta)$

 $\mathbf{\dot{z}}$ 实际中, k_1, \dots, k_n 的求解不需要特意解方程 $Ax = \beta$,方法见下例

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (1)

$$(\alpha_1 \ \alpha_2 \ \alpha_3 | \beta) = \begin{pmatrix} 1 & 2 & 3 | 2 \\ 0 & -1 & 2 | 3 \\ 1 & 1 & 0 | 0 \\ 2 & -2 & 1 | 5 \end{pmatrix}$$

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (1)

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$$r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta) = 3,$$

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (1)

$$(\alpha_1 \ \alpha_2 \ \alpha_3 \ | \) = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 2 \\ 1 & 1 & 0 \\ 2 & -2 & 1 \end{pmatrix} \xrightarrow{\text{初等行变换}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

•
$$r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta) = 3$$

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (1)

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• 所以 $r(\alpha_1\alpha_2\alpha_3) = 3$, $r(\alpha_1\alpha_2\alpha_3\beta) = 3$,

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (1)

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• 所以
$$r(\alpha_1\alpha_2\alpha_3) = 3$$
, $r(\alpha_1\alpha_2\alpha_3\beta) = 3$,成立
$$r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta)$$

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 **(1)**

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• 所以
$$r(\alpha_1\alpha_2\alpha_3) = 3$$
, $r(\alpha_1\alpha_2\alpha_3\beta) = 3$, 成立

$$r(\alpha_1\alpha_2\alpha_3)=r(\alpha_1\alpha_2\alpha_3\beta)$$

 β 可由 α_1 , α_2 , α_3 线性表示。

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。

(1)
$$(\alpha_1 \ \alpha_2 \ \alpha_3 \ | \ \beta \) = \begin{pmatrix} 1 & 2 & 3 \ | & 2 \ 0 & -1 & 2 \ | & 3 \ 1 & 1 & 0 \ | & 0 \ 2 & -2 & 1 \ | & 5 \end{pmatrix} \xrightarrow{\overline{ay}} \begin{pmatrix} \alpha_1' \ \alpha_2' \ \alpha_3' \ \beta' \ 0 & 1 & 0 \ | & 1 \ 0 & 0 & 1 \ | & 1 \ 0 & 0 & 0 \ | & 0 \end{pmatrix}$$

• 所以
$$r(\alpha_1\alpha_2\alpha_3) = 3$$
, $r(\alpha_1\alpha_2\alpha_3\beta) = 3$, 成立

$$r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta)$$

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• 所以
$$r(\alpha_1\alpha_2\alpha_3) = 3$$
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$$r(\alpha_1\alpha_2\alpha_3)=r(\alpha_1\alpha_2\alpha_3\beta)$$

 β 可由 α_1 , α_2 , α_3 线性表示。

• 显然 $\beta' = \alpha'_1 - \alpha'_2 + \alpha'_3$,

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。

(1)
$$(\alpha_1 \ \alpha_2 \ \alpha_3 \ | \ \beta \) = \begin{pmatrix} 1 & 2 & 3 \ | & 2 \ 0 & -1 & 2 \ | & 3 \ 1 & 1 & 0 \ | & 0 \ 2 & -2 & 1 \ | & 5 \end{pmatrix} \xrightarrow{\overline{\text{初等行变换}}} \begin{pmatrix} \alpha_1' & \alpha_2' & \alpha_3' & \beta' \\ 1 & 0 & 0 \ | & 1 \ 0 \\ 0 & 0 & 1 \ | & 1 \ 0 \\ 0 & 0 & 0 \ | & 0 \end{pmatrix}$$

• 所以 $r(\alpha_1\alpha_2\alpha_3) = 3$, $r(\alpha_1\alpha_2\alpha_3\beta) = 3$, 成立

$$r(\alpha_1\alpha_2\alpha_3)=r(\alpha_1\alpha_2\alpha_3\beta)$$

 β 可由 α_1 , α_2 , α_3 线性表示。

• 显然 $\beta' = \alpha'_1 - \alpha'_2 + \alpha'_3$,是否也有 $\beta = \alpha_1 - \alpha_2 + \alpha_3$?

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例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。

(1)
$$(\alpha_1 \ \alpha_2 \ \alpha_3 \ | \ \beta \) = \begin{pmatrix} 1 & 2 & 3 \ | & 2 \ 0 & -1 & 2 \ | & 3 \ 1 & 1 & 0 \ | & 0 \ 2 & -2 & 1 \ | & 5 \end{pmatrix} \xrightarrow{\overline{\text{初等行变换}}} \begin{pmatrix} \alpha_1' & \alpha_2' & \alpha_3' & \beta' \\ 1 & 0 & 0 & | & 1 \ 0 & 0 & 1 \\ 0 & 0 & 1 & | & 1 \ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

• 所以
$$r(\alpha_1\alpha_2\alpha_3) = 3$$
, $r(\alpha_1\alpha_2\alpha_3\beta) = 3$, 成立

$$r(\alpha_1\alpha_2\alpha_3)=r(\alpha_1\alpha_2\alpha_3\beta)$$

 β 可由 α_1 , α_2 , α_3 线性表示。

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是的

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。

(1)
$$(\alpha_1 \ \alpha_2 \ \alpha_3 \ | \ \beta \) = \begin{pmatrix} 1 & 2 & 3 \ | & 2 \ 0 & -1 & 2 \ | & 3 \ 1 & 1 & 0 \ | & 0 \ 2 & -2 & 1 \ | & 5 \end{pmatrix} \xrightarrow{\overline{ay} \oplus 7\overline{y}} \begin{pmatrix} 1 & 0 & 0 \ | & 1 \ 0 & 1 & 0 \ | & -1 \ 0 & 0 & 1 \ | & 1 \ 0 & 0 & 0 \ | & 0 \end{pmatrix}$$

• 所以
$$r(\alpha_1\alpha_2\alpha_3) = 3$$
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$$\beta' = \alpha'_1 - \alpha'_2 + \alpha'_3$$
,是否也有 $\beta = \alpha_1 - \alpha_2 + \alpha_3$?

是的

注 可证明:作初等行变换不改变列与列之间的"线性关系"。

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (2)

$$(\alpha_1 \ \alpha_2 \ \alpha_3 | \beta) = \begin{pmatrix} 1 & 2 & -1 & | & 4 \\ 2 & -1 & 3 & | & 3 \\ -1 & 1 & -2 & | & 0 \\ 5 & 1 & 4 & | & 11 \end{pmatrix}$$

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (2)

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例 判断 $oldsymbol{eta}$ 是否能由 $lpha_1$, $lpha_2$, $lpha_3$ 线性表示,若能,写出线性表示等式。

(2)

$$(\alpha_1 \ \alpha_2 \ \alpha_3 \ | \ \beta \) = \begin{pmatrix} 1 & 2 & -1 & 4 \\ 2 & -1 & 3 & 3 \\ -1 & 1 & -2 & 0 \\ 5 & 1 & 4 \ | \ 11 \end{pmatrix} \xrightarrow{\text{disfree}} \begin{pmatrix} 1 & 2 & -1 & | \ 0 & 1 & -1 & | \ 1 \\ 0 & 0 & 0 & | \ 1 \\ 0 & 0 & 0 & | \ 0 \end{pmatrix}$$

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (2)

$$(\alpha_1 \ \alpha_2 \ \alpha_3 \ | \ \beta \) = \begin{pmatrix} 1 & 2 & -1 & | & 4 \\ 2 & -1 & 3 & | & 3 \\ -1 & 1 & -2 & | & 0 \\ 5 & 1 & 4 & | & 11 \end{pmatrix} \xrightarrow{\text{初等行变换}} \begin{pmatrix} 1 & 2 & -1 & | & 4 \\ 0 & 1 & -1 & | & 1 \\ 0 & 0 & 0 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (2)

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•
$$r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta) = 3$$

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (2)

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•
$$r(\alpha_1\alpha_2\alpha_3) = 2$$
, $r(\alpha_1\alpha_2\alpha_3\beta) = 3$,

例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (2)

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• 所以
$$r(\alpha_1\alpha_2\alpha_3) = 2$$
, $r(\alpha_1\alpha_2\alpha_3\beta) = 3$,成立
$$r(\alpha_1\alpha_2\alpha_3) \neq r(\alpha_1\alpha_2\alpha_3\beta)$$

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例 判断 β 是否能由 α_1 , α_2 , α_3 线性表示,若能,写出线性表示等式。 (2)

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• 所以
$$r(\alpha_1\alpha_2\alpha_3) = 2$$
, $r(\alpha_1\alpha_2\alpha_3\beta) = 3$,成立
$$r(\alpha_1\alpha_2\alpha_3) \neq r(\alpha_1\alpha_2\alpha_3\beta)$$

 β 不能由 α_1 , α_2 , α_3 线性表示。

<mark>问题</mark> β 能否由 $α_1, α_2, ..., α_n$ 线性表示? 若能,写出线性表示等式。

步骤

<mark>问题</mark> β 能否由 $α_1$, $α_2$, ..., $α_n$ 线性表示? 若能,写出线性表示等式。

步骤 作初等 行 变换:

$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n | \beta)$$

<mark>问题</mark> β 能否由 $α_1, α_2, ..., α_n$ 线性表示? 若能,写出线性表示等式。

步骤 作初等 行 变换:

$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n | \beta) \xrightarrow{\overline{\eta} + \overline{\eta} + \overline{\eta}}$$

问题 β 能否由 $α_1, α_2, ..., α_n$ 线性表示? 若能,写出线性表示等式。

步骤 作初等 行 变换:

$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n | eta) \xrightarrow{ ext{ inftition}} (\alpha_1' \ \alpha_2' \ \cdots \ \alpha_n' | eta')$$
 (简化)

问题 β 能否由 $α_1$, $α_2$, ..., $α_n$ 线性表示? 若能,写出线性表示等式。

步骤 作初等 行 变换:

$$(lpha_1 \ lpha_2 \ \cdots \ lpha_n \, | \, eta \,) \stackrel{ ext{instance}}{\longrightarrow} \ (lpha_1' \ lpha_2' \ \cdots \ lpha_n' \, | \, eta' \,)$$
 (简化) 所梯型矩阵

1.

$$\beta$$
由 $\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性表示 $\Leftrightarrow r(\alpha_1 \cdots \alpha_n) = r(\alpha_1 \cdots \alpha_n \beta)$

线性组合

问题 β 能否由 $\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性表示? 若能,写出线性表示等式。 **步骤** 作初等 \uparrow 变换:

$$(lpha_1 \ lpha_2 \ \cdots \ lpha_n \, | \, eta \,) \stackrel{ar{n}$$
等行变换 $(lpha_1' \ lpha_2' \ \cdots \ lpha_n' \, | \, eta' \,)$ (简化) 所梯型矩阵

1.

$$\beta$$
由 $\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性表示 $\iff r(\alpha_1 \cdots \alpha_n) = r(\alpha_1 \cdots \alpha_n \beta)$

$$\updownarrow$$

$$r(\alpha'_1 \cdots \alpha'_n) = r(\alpha'_1 \cdots \alpha'_n \beta')$$

线性组合

问题 β 能否由 α_1 , α_2 , ..., α_n 线性表示? 若能,写出线性表示等式。 **步骤** 作初等 行 变换:

$$(lpha_1 \ lpha_2 \ \cdots \ lpha_n \ | eta \) \stackrel{ ext{nsffeph}}{\longrightarrow} \ (lpha_1' \ lpha_2' \ \cdots \ lpha_n' \ | eta' \)$$
 (简化)

1.

2. 行变换前后列与列的线性关系不变,即:

问题 β 能否由 α_1 , α_2 , ..., α_n 线性表示? 若能,写出线性表示等式。 **步骤** 作初等 行 变换:

$$(\alpha_1 \ \alpha_2 \ \cdots \ \alpha_n \ | \ eta \) \xrightarrow{ ext{初等行变换}} \ (\alpha_1' \ \alpha_2' \ \cdots \ \alpha_n' \ | \ eta' \)$$
 (简化)

1.

$$\beta$$
由 $\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性表示 $\iff r(\alpha_1 \cdots \alpha_n) = r(\alpha_1 \cdots \alpha_n \beta)$
 \uparrow

$$r(\alpha'_1 \cdots \alpha'_n) = r(\alpha'_1 \cdots \alpha'_n \beta')$$

2. 行变换前后列与列的线性关系不变,即:

$$\beta' = k_1 \alpha_1' + \dots + k_n \alpha_n' \Rightarrow$$

<mark>问题</mark> β 能否由 $α_1$, $α_2$, ..., $α_n$ 线性表示? 若能,写出线性表示等式。 步骤 作初等 $\frac{1}{1}$ 变换:

$$(lpha_1 \ lpha_2 \ \cdots \ lpha_n | eta) \xrightarrow{rac{\partial (lpha_1 + lpha_2 + lpha_n + eta)}{n}} (lpha_1' \ lpha_2' \ \cdots \ lpha_n' | eta')$$
 (简化)

1.

$$\beta$$
由 $\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性表示 $\iff r(\alpha_1 \cdots \alpha_n) = r(\alpha_1 \cdots \alpha_n \beta)$

$$\updownarrow$$

$$r(\alpha'_1 \cdots \alpha'_n) = r(\alpha'_1 \cdots \alpha'_n \beta')$$

2. 行变换前后列与列的线性关系不变,即:

$$\beta' = k_1 \alpha_1' + \dots + k_n \alpha_n' \Rightarrow \beta = k_1 \alpha_1 + \dots + k_n \alpha_n$$

例 1
$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

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$$\mathbf{H}$$
 α_1 α_2 α_3 β

$$\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & -1 & 2 & 3 \\
1 & 1 & 0 & 0 \\
2 & -2 & 1 & 5
\end{pmatrix}$$

例 1
$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
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 α_1 α_2 α_3 β

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \xrightarrow{r_4 - 2r_1}$$

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能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

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 α_1 α_2 α_3 β

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \end{pmatrix}$$

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$$\mathbf{H}$$
 α_1 α_2 α_3 β

$$\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & -1 & 2 & 3 \\
1 & 1 & 0 & 0 \\
2 & -2 & 1 & 5
\end{pmatrix}
\xrightarrow{r_3 - r_1}
\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & -1 & 2 & 3 \\
0 & -1 & -3 & -2 \\
0 & -6 & -5 & 1
\end{pmatrix}$$

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$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\mathbf{H}$$
 α_1 α_2 α_3 β

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2}$$

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$$\mathbf{H}$$
 α_1 α_2 α_3 μ

$$\begin{pmatrix}
1 & 2 & 3 & | & 2 \\
0 & -1 & 2 & | & 3 \\
1 & 1 & 0 & | & 0 \\
2 & -2 & 1 & | & 5
\end{pmatrix}
\xrightarrow{r_3 - r_1}
\begin{pmatrix}
1 & 2 & 3 & | & 2 \\
0 & -1 & 2 & | & 3 \\
0 & -1 & -3 & | & -2 \\
0 & -6 & -5 & | & 1
\end{pmatrix}
\xrightarrow{(-1) \times r_2}
\begin{pmatrix}
1 & 2 & 3 & | & 2 \\
0 & 1 & -2 & | & -3 \\
0 & -1 & -3 & | & -2 \\
0 & -6 & -5 & | & 1
\end{pmatrix}$$

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$$\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & -1 & 2 & 3 \\
1 & 1 & 0 & 0 \\
2 & -2 & 1 & 5
\end{pmatrix}
\xrightarrow[r_4-2r_1]{r_3-r_1}
\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & -1 & 2 & 3 \\
0 & -1 & -3 & -2 \\
0 & -6 & -5 & 1
\end{pmatrix}
\xrightarrow[r_4-2r_1]{(-1)\times r_2}
\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & 1 & -2 & -3 \\
0 & -1 & -3 & -2 \\
0 & -6 & -5 & 1
\end{pmatrix}$$

$$r_3+r_2$$

 r_4+6r_2

线性组合

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$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\frac{r_3 + r_2}{r_4 + 6r_2} \begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & 1 & -2 & -3
\end{pmatrix}$$

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$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\begin{array}{c|cccc} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \end{array}}$$

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$$\begin{array}{c|cccc}
r_3+r_2 \\
\hline
r_4+6r_2
\end{array}
\begin{pmatrix}
1 & 2 & 3 & 2 \\
0 & 1 & -2 & -3 \\
0 & 0 & -5 & -5 \\
0 & 0 & -17 & -17
\end{pmatrix}$$

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$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\begin{array}{c|cccc} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array}} \rightarrow \left(\begin{array}{ccccc} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

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例1
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$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow{r_3+r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{r_4-r_3}$$

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$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\begin{array}{cccc}1 & 2 & 3 & 2\\0 & 1 & -2 & -3\\0 & 0 & -5 & -5\\0 & 0 & -17 & -17\end{array}} \longrightarrow \begin{pmatrix}1 & 2 & 3 & 2\\0 & 1 & -2 & -3\\0 & 0 & 1 & 1\\0 & 0 & 1 & 1\end{pmatrix}\xrightarrow[r_4-r_3]{\left(\begin{array}{ccccc}1 & 2 & 3 & 2\\0 & 1 & -2 & -3\\0 & 0 & 1 & 1\\0 & 0 & 0 & 0\end{array}\right)}$$

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$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\begin{array}{c|cccc}1&2&3&2\\0&1&-2&-3\\0&0&-5&-5\\0&0&-17\end{array}} \longrightarrow \begin{pmatrix}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&1&1\end{pmatrix} \xrightarrow[r_4-r_3]{\begin{array}{c|cccc}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&0&0\end{array}}$$

$$r_2-2r_3$$
 r_1-3r_3

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$$\mathbf{H}$$
 α_1 α_2 α_3 β

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\begin{array}{c}1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17\end{array}} \longrightarrow \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1\end{pmatrix} \xrightarrow[r_4-r_3]{\begin{array}{c}1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0\end{array}}$$

$$\frac{r_2-2r_3}{r_1-3r_3} \left(\begin{array}{ccc|c} & & & \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

线性组合

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$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\begin{array}{c}1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17\end{array}} \longrightarrow \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1\end{pmatrix} \xrightarrow[r_4-r_3]{\begin{array}{c}1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0\end{array}}$$

$$\frac{r_2 - 2r_3}{r_1 - 3r_3} \left(\begin{array}{ccc} 0 & 1 & 0 & | & -1 \\ 0 & 0 & 1 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{array} \right)$$

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$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\left(\begin{array}{ccc|c}1&2&3&2\\0&1&-2&-3\\0&0&-5&-5\\0&0&-17&-17\end{array}\right)} \rightarrow \left(\begin{array}{ccc|c}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&1&1\end{array}\right)\xrightarrow[r_4-r_3]{\left(\begin{array}{ccc|c}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&0&0\end{array}\right)}$$

$$\frac{r_2 - 2r_3}{r_1 - 3r_3} \begin{pmatrix} 1 & 2 & 0 & | & -1 \\ 0 & 1 & 0 & | & -1 \\ 0 & 0 & 1 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

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$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\left(\begin{array}{ccc|c}1&2&3&2\\0&1&-2&-3\\0&0&-5&-5\\0&0&-17&-17\end{array}\right)} \rightarrow \left(\begin{array}{ccc|c}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&1&1\end{array}\right)\xrightarrow[r_4-r_3]{\left(\begin{array}{ccc|c}1&2&3&2\\0&1&-2&-3\\0&0&1&1\\0&0&0&0\end{array}\right)}$$

$$\frac{r_2 - 2r_3}{r_1 - 3r_3} \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xrightarrow{r_1 - 2r_2}$$

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例 1
$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\begin{array}{c|cccc}
1 & 2 & 3 & 2 \\
0 & 1 & -2 & -3 \\
0 & 0 & -5 & -5 \\
0 & 0 & -17 & -17
\end{array}} \longrightarrow \left(\begin{array}{ccccc}
1 & 2 & 3 & 2 \\
0 & 1 & -2 & -3 \\
0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1
\end{array}\right) \xrightarrow[r_4-r_3]{\left(\begin{array}{cccccc}
1 & 2 & 3 & 2 \\
0 & 1 & -2 & -3 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0
\end{array}\right)}$$

$$\xrightarrow[r_1-3r_3]{\begin{array}{c}1&2&0&-1\\0&1&0&-1\\0&0&1&1\\0&0&0&0\end{array}}\xrightarrow[r_1-2r_2]{\begin{array}{c}1&0&0&1\\0&1&0&-1\\0&0&1&1\\0&0&0&0\end{array}}$$

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例1
$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\xrightarrow[r_4+6r_2]{\begin{array}{c|cccc}
1 & 2 & 3 & 2 \\
0 & 1 & -2 & -3 \\
0 & 0 & -5 & -5 \\
0 & 0 & -17 & -17
\end{array}} \longrightarrow \left(\begin{array}{ccccc}
1 & 2 & 3 & 2 \\
0 & 1 & -2 & -3 \\
0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1
\end{array}\right) \xrightarrow[r_4-r_3]{\left(\begin{array}{cccccc}
1 & 2 & 3 & 2 \\
0 & 1 & -2 & -3 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0
\end{array}\right)}$$

$$\xrightarrow[r_1-3r_3]{\begin{array}{c}1&2&0&-1\\0&1&0&-1\\0&0&1&1\\0&0&0&0\end{array}}\xrightarrow[r_1-2r_2]{\begin{array}{c}1&0&0&1\\0&1&0&-1\\0&0&1&1\\0&0&0&0\end{array}}$$

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例 1
$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\frac{r_{3}+r_{2}}{r_{4}+6r_{2}} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{r_{4}-r_{3}} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{r_{4}-r_{3}} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow{r_{2}-2r_{3}} \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{r_{1}-2r_{2}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

所以
$$r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta)$$
,

例 1
$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 1 & 1 & 0 & | & 0 \\ 2 & -2 & 1 & | & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & -1 & 2 & | & 3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & | & 2 \\ 0 & 1 & -2 & | & -3 \\ 0 & -1 & -3 & | & -2 \\ 0 & -6 & -5 & | & 1 \end{pmatrix}$$

$$\xrightarrow{r_3+r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{r_4-r_3} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow[r_1-3r_3]{\left(\begin{array}{cc|c}1&2&0&-1\\0&1&0&-1\\0&0&1&1\\0&0&0&0\end{array}\right)}\xrightarrow[r_1-2r_2]{\left(\begin{array}{cc|c}1&0&0&1\\0&1&0&-1\\0&0&1&1\\0&0&0&0\end{array}\right)}$$

所以 $r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta)$,能线性表示

例 1
$$\beta = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 5 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -2 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 2 \\ 0 \\ 1 \end{pmatrix}$ 线性表示?

 \mathbf{m} α_1 α_2 α_3

$$\begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 5 \end{pmatrix} \xrightarrow{r_3 - r_1} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & 2 & 3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix} \xrightarrow{(-1) \times r_2} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & -1 & -3 & -2 \\ 0 & -6 & -5 & 1 \end{pmatrix}$$

$$\frac{r_{3}+r_{2}}{r_{4}+6r_{2}} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -5 & -5 \\ 0 & 0 & -17 & -17 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{r_{4}-r_{3}} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right) \xrightarrow{r_{4}-r_{3}} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

$$\frac{r_{2}-2r_{3}}{r_{1}-3r_{3}} \left(\begin{array}{ccc|c} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow{r_{1}-2r_{2}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

所以
$$r(\alpha_1\alpha_2\alpha_3) = r(\alpha_1\alpha_2\alpha_3\beta)$$
,能线性表示,且 $\beta = \alpha_1 - \alpha_2 + \alpha_3$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 2 & -1 & 4 & | & 1 \\ -1 & 3 & 3 & | & 2 \\ 1 & -2 & 0 & | & -1 \\ 1 & 4 & 11 & | & 6 \end{pmatrix}$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \beta \\ 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3}$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & -2 & 0 & | -1 \\ -1 & 3 & 3 & | & 2 \\ 2 & -1 & 4 & | & 1 \\ 1 & 4 & 11 & | & 6 \end{pmatrix}$$

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例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & -2 & 0 & | -1 \\ -1 & 3 & 3 & | & 2 \\ 2 & -1 & 4 & | & 1 \\ 1 & 4 & 11 & | & 6 \end{pmatrix}$$

$$r_2+r_1$$
 r_3-2r_1
 r_4-r_1

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} \alpha_{1} & \alpha_{2} & \alpha_{3} & \beta \\ 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{pmatrix} \xrightarrow{r_{1} \longleftrightarrow r_{3}} \begin{pmatrix} 1 & -2 & 0 & | -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 1 -2 & 0 & | -1 \end{pmatrix}$$

$$\xrightarrow[r_{3}-2r_{1}\\r_{4}-r_{1}]{r_{2}-2r_{1}}\left(\begin{array}{cccc} 1-2 & 0 & |-1\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & & |\\ & |\\ & & |\\ & & |\\ & |\\ & & |\\ & |\\ & |\\ & |\\ & |\\ & |\\$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} a_1 & a_2 & a_3 & \beta \\ 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & -2 & 0 & | -1 \\ -1 & 3 & 3 & | & 2 \\ 2 & -1 & 4 & | & 1 \\ 1 & 4 & 11 & | & 6 \end{pmatrix}$$

$$\xrightarrow[r_{4}]{r_{2}+r_{1}} \begin{pmatrix} 1-2 & 0 & | -1 \\ 0 & 1 & 3 & | 1 \\ & & & | 1 \end{pmatrix}$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & -2 & 0 & | -1 \\ -1 & 3 & 3 & | & 2 \\ 2 & -1 & 4 & | & 1 \\ 1 & 4 & 11 & | & 6 \end{pmatrix}$$

$$\xrightarrow[r_{4}-r_{1}]{r_{2}-r_{1}} \begin{pmatrix} 1-2 & 0 & | -1 \\ 0 & 1 & 3 & | 1 \\ 0 & 3 & 4 & | 3 \end{pmatrix}$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & -2 & 0 & | -1 \\ -1 & 3 & 3 & 2 \\ 2 & -1 & 4 & 1 \\ 1 & 4 & 11 & 6 \end{pmatrix}$$

$$\xrightarrow[r_{4}-r_{1}]{r_{2}-r_{1}}
\xrightarrow[r_{4}-r_{1}]{r_{2}-r_{1}}
\begin{pmatrix}
1-2 & 0 & | -1 \\
0 & 1 & 3 & | 1 \\
0 & 3 & 4 & | 3 \\
0 & 6 & 11 & 7
\end{pmatrix}$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix} 2 & -1 & 4 & 1 \\ -1 & 3 & 3 & 2 \\ 1 & -2 & 0 & -1 \\ 1 & 4 & 11 & 6 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_3} \begin{pmatrix} 1 & -2 & 0 & | -1 \\ -1 & 3 & 3 & | & 2 \\ 2 & -1 & 4 & | & 1 \\ 1 & 4 & 11 & | & 6 \end{pmatrix}$$

$$\frac{r_2+r_1}{r_3-2r_1} \begin{pmatrix}
1-2 & 0 & | & -1 \\
0 & 1 & 3 & | & 1 \\
0 & 3 & 4 & | & 3 \\
0 & 6 & 11 & | & 7
\end{pmatrix} \xrightarrow{r_3-3r_2} \xrightarrow{r_4-6r_2}$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix}
\alpha_{1} & \alpha_{2} & \alpha_{3} & \beta \\
2 & -1 & 4 & 1 \\
-1 & 3 & 3 & 2 \\
1 & -2 & 0 & -1 \\
1 & 4 & 11 & 6
\end{pmatrix}
\xrightarrow{r_{1} \leftrightarrow r_{3}}
\begin{pmatrix}
1 & -2 & 0 & | -1 \\
-1 & 3 & 3 & 2 \\
2 & -1 & 4 & 1 \\
1 & 4 & 11 & 6
\end{pmatrix}$$

$$\xrightarrow{r_{2} + r_{1}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 3 & 4 & 3 \\
0 & 6 & 11 & 7
\end{pmatrix}
\xrightarrow{r_{3} - 3r_{2}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 6 & 11 & 7
\end{pmatrix}
\xrightarrow{r_{3} - 3r_{2}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 6 & 11 & 7
\end{pmatrix}$$

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例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix}
\alpha_{1} & \alpha_{2} & \alpha_{3} & \beta \\
2 & -1 & 4 & 1 \\
-1 & 3 & 3 & 2 \\
1 & -2 & 0 & -1 \\
1 & 4 & 11 & 6
\end{pmatrix}
\xrightarrow{r_{1} \leftrightarrow r_{3}}
\begin{pmatrix}
1 & -2 & 0 & | -1 \\
-1 & 3 & 3 & | 2 \\
2 & -1 & 4 & | 1 \\
1 & 4 & 11 & 6
\end{pmatrix}$$

$$\xrightarrow{r_{2} + r_{1}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & | 1 \\
0 & 1 & 3 & | 1
\end{pmatrix}
\xrightarrow{r_{3} - 3r_{2}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & | 1
\end{pmatrix}$$

$$\frac{r_{2}+r_{1}}{r_{3}-2r_{1}} \left(\begin{array}{ccc|c} 1-2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{array}\right) \xrightarrow[r_{4}-6r_{2}]{r_{3}-3r_{2}} \left(\begin{array}{ccc|c} 1-2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \end{array}\right)$$

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
能否由 $\alpha_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ 0 \\ 11 \end{pmatrix}$ 线性表示?

$$\begin{pmatrix}
\alpha_{1} & \alpha_{2} & \alpha_{3} & \beta \\
2 & -1 & 4 & 1 \\
-1 & 3 & 3 & 2 \\
1 & -2 & 0 & -1 \\
1 & 4 & 11 & 6
\end{pmatrix}
\xrightarrow{r_{1} \leftrightarrow r_{3}}
\begin{pmatrix}
1 & -2 & 0 & | -1 \\
-1 & 3 & 3 & | & 2 \\
2 & -1 & 4 & | & 1 \\
1 & 4 & 11 & | & 6
\end{pmatrix}$$

$$\xrightarrow{r_{2} + r_{1}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & | & 1 \\
0 & 3 & 4 & | & 3
\end{pmatrix}
\xrightarrow{r_{3} - 3r_{2}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & | & 1 \\
0 & 0 & -5 & | & 0
\end{pmatrix}$$

$$\frac{r_2+r_1}{r_3-2r_1} \xrightarrow{r_3-2r_1} \begin{pmatrix} 1-2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 3 & 4 & 3 \\ 0 & 6 & 11 & 7 \end{pmatrix} \xrightarrow{r_3-3r_2} \begin{pmatrix} 1-2 & 0 & -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & -7 & 1 \end{pmatrix}$$

线性组合

例 2
$$\beta = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 6 \end{pmatrix}$$
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$$-\frac{1}{5} \times r_3$$

线性组合

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$$\xrightarrow{-\frac{1}{5} \times r_3} \begin{pmatrix} 1 - 2 & 0 & | -1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -7 & 1 \end{pmatrix}$$

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\alpha_{1} & \alpha_{2} & \alpha_{3} & \beta \\
-1 & 3 & 3 & 2 \\
1 & -2 & 0 & -1 \\
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\end{pmatrix}
\xrightarrow{r_{1} \leftrightarrow r_{3}}
\begin{pmatrix}
1 & -2 & 0 & | -1 \\
-1 & 3 & 3 & 2 \\
2 & -1 & 4 & 1 \\
1 & 4 & 11 & 6
\end{pmatrix}$$

$$\xrightarrow{r_{2}+r_{1}}
\begin{pmatrix}
1-2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 3 & 4 & 3 \\
0 & 6 & 11 & 7
\end{pmatrix}
\xrightarrow{r_{3}-3r_{2}}
\begin{pmatrix}
1-2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 0 & -5 & 0 \\
0 & 0 & -7 & 1
\end{pmatrix}$$

$$\xrightarrow{-\frac{1}{5} \times r_{3}}
\begin{pmatrix}
1-2 & 0 & | -1 \\
0 & 1 & 3 & 1 \\
0 & 0 & 1 & 3 \\
0 & 0 & 0 & 7 & 1
\end{pmatrix}
\xrightarrow{r_{4}+7r_{3}}$$

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$$\xrightarrow{r_{2} + r_{1}}
\begin{pmatrix}
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0 & 3 & 4 & 3 \\
0 & 6 & 11 & 7
\end{pmatrix}
\xrightarrow{r_{3} - 3r_{2}}
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1 - 2 & 0 & | -1 \\
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0 & 0 & 1 & 0 \\
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\end{pmatrix}$$

线性组合

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可见
$$r(\alpha_1\alpha_2\alpha_3\beta) = 4 > 3 = r(\alpha_1\alpha_2\alpha_3)$$
,

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\begin{pmatrix}
1 - 2 & 0 & | -1 \\
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0 & 0 & 1 & | 0 \\
0 & 0 & 1 & | 0
\end{pmatrix}
\xrightarrow{r_{4} + 7r_{3}}
\begin{pmatrix}
1 - 2 & 0 & | -1 \\
0 & 1 & 3 & | 1 \\
0 & 0 & 1 & | 0
\end{pmatrix}$$

可见
$$r(\alpha_1\alpha_2\alpha_3\beta) = 4 > 3 = r(\alpha_1\alpha_2\alpha_3)$$
,所以不能线性表示。

定义 设有两个向量组

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

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如果中(A)中每一向量均可由(B)线性表示,则称向量组(A)可由向量组(B)线性表示。

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$$\stackrel{\text{<}}{\Longrightarrow}$$
 $\left\{ \begin{array}{ll} \alpha_1 = & \beta_1 + & \beta_2 + & \beta_3 \\ \alpha_2 = & \beta_1 + & \beta_2 + & \beta_3 \end{array} \right.$

定义 设有两个向量组

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$$\stackrel{\text{ᡯ妨设}}{\Longrightarrow} \left\{ \begin{array}{l} \alpha_1 = \alpha_{11}\beta_1 + \alpha_{21}\beta_2 + \alpha_{31}\beta_3 \\ \alpha_2 = \beta_1 + \beta_2 + \beta_3 \end{array} \right.$$

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本妨设

$$\begin{cases} \alpha_1 = \alpha_{11}\beta_1 + \alpha_{21}\beta_2 + \alpha_{31}\beta_3 \\ \alpha_2 = \alpha_{12}\beta_1 + \alpha_{22}\beta_2 + \alpha_{32}\beta_3 \end{cases}$$

$$ightharpoonup$$
 $(lpha_1, \, lpha_2) = (eta_1, \, eta_2, \, eta_3)$

定义 设有两个向量组

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

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改写为

$$(\alpha_1, \alpha_2) = (\beta_1, \beta_2, \beta_3)$$
 $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$

定义 设有两个向量组

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

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如果中(A)中每一向量均可由(B)线性表示,则称向量组(A)可由向量组(B)线性表示。

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$$Arr$$
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$$(\alpha_1, \alpha_2, \ldots, \alpha_s) = (\beta_1, \beta_2, \ldots, \beta_t)A$$

$$(\alpha_1, \alpha_2, \ldots, \alpha_s) = (\beta_1, \beta_2, \ldots, \beta_t)A$$

$$= (\beta_1, \beta_2, \dots, \beta_t) \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1s} \\ a_{21} & a_{22} & \cdots & a_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ a_{t1} & a_{t2} & \cdots & a_{ts} \end{pmatrix}$$

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这时 A 的每一列表示线性组合的系数。

$$(\alpha_1, \alpha_2, \ldots, \alpha_s) = (\beta_1, \beta_2, \ldots, \beta_t)A$$

$$= (\beta_1, \beta_2, \ldots, \beta_t) \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1s} \\ a_{21} & a_{22} & \cdots & a_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ a_{t1} & a_{t2} & \cdots & a_{ts} \end{pmatrix}$$

这时A的每一列表示线性组合的系数。例如,

$$\alpha_j = \beta_1 + \beta_2 + \cdots + \beta_t$$

线性组合

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这时A的每一列表示线性组合的系数。例如,

$$\alpha_j = \alpha_{1j}\beta_1 + \alpha_{2j}\beta_2 + \cdots + \alpha_{tj}\beta_t$$

其中的系数就是
$$A$$
 的第 j 列 $\begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{si} \end{pmatrix}$ 。

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注2 设上述向量均为列向量,

$$\underbrace{(\alpha_1, \, \alpha_2, \, \ldots, \, \alpha_s)}_{P} = (\beta_1, \, \beta_2, \, \ldots, \, \beta_t) A$$

$$= (\beta_1, \beta_2, \dots, \beta_t) \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1s} \\ a_{21} & a_{22} & \cdots & a_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ a_{t1} & a_{t2} & \cdots & a_{ts} \end{pmatrix}$$

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注2 设上述向量均为列向量,

$$\underbrace{(\alpha_1, \alpha_2, \dots, \alpha_s)}_{P} = \underbrace{(\beta_1, \beta_2, \dots, \beta_t)}_{Q} A$$

$$= (\beta_1, \beta_2, \dots, \beta_t) \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1s} \\ a_{21} & a_{22} & \cdots & a_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ a_{t1} & a_{t2} & \cdots & a_{ts} \end{pmatrix}$$

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$$= (\beta_1, \beta_2, \dots, \beta_t) \begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1s} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{t1} & \alpha_{t2} & \cdots & \alpha_{ts} \end{pmatrix}$$

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 ≥ 2 设上述向量均为列向量,则上式正好表示矩阵乘积: P = QA

 $\mathbf{\dot{L}1}$ 一般地,向量组 $\alpha_1, \alpha_2, \ldots, \alpha_s$ 可由向量组 $\beta_1, \beta_2, \ldots, \beta_t$ 线性表示,当且仅当存在矩阵 $A_{t \times s}$ 满足:

$$\underbrace{(\alpha_1, \alpha_2, \dots, \alpha_s)}_{P} = \underbrace{(\beta_1, \beta_2, \dots, \beta_t)}_{Q} A$$

$$= (\beta_1, \beta_2, \dots, \beta_t) \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1s} \\ a_{21} & a_{22} & \cdots & a_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ a_{t1} & a_{t2} & \cdots & a_{ts} \end{pmatrix}$$

这时A的每一列表示线性组合的系数。例如,

$$\alpha_j = \alpha_{1j}\beta_1 + \alpha_{2j}\beta_2 + \cdots + \alpha_{tj}\beta_t$$

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 的第 j 列 $\begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{si} \end{pmatrix}$ 。

 $\mathbf{\dot{z}}$ 2 设上述向量均为列向量,则上式正好表示矩阵乘积: P = QA

注 3 反之,若 P = QA,则说明 P 的列向量组可由的 Q 列向量组线性表示

证明 设向量组 $\alpha_1, \alpha_2, \ldots, \alpha_s$ 可由向量组 $\beta_1, \beta_2, \ldots, \beta_t$ 线性表示:

向量组 β_1 , β_2 , ..., β_t 可由向量组 γ_1 , γ_2 , ..., γ_k 线性表示:

证明 设向量组 $\alpha_1, \alpha_2, \ldots, \alpha_s$ 可由向量组 $\beta_1, \beta_2, \ldots, \beta_t$ 线性表示:

$$(\alpha_1, \alpha_2, \ldots, \alpha_s) = (\beta_1, \beta_2, \ldots, \beta_t) A_{t \times s}.$$

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证明 设向量组 α_1 , α_2 , ..., α_s 可由向量组 β_1 , β_2 , ..., β_t 线性表示:

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向量组 β_1 , β_2 , ..., β_t 可由向量组 γ_1 , γ_2 , ..., γ_k 线性表示:

$$(\beta_1, \beta_2, \ldots, \beta_t) = (\gamma_1, \gamma_2, \ldots, \gamma_k) B_{k \times t}.$$

 $\overline{\mathbf{u}}$ 明 设向量组 α_1 , α_2 , ..., α_s 可由向量组 β_1 , β_2 , ..., β_t 线性表示:

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将第2式代入第1式,可得

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$$(\alpha_1, \alpha_2, \ldots, \alpha_s) = (\gamma_1, \gamma_2, \ldots, \gamma_k) \underbrace{B_{k \times t} A_{t \times s}}_{C_{k \times s}}$$

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所以向量组 α_1 , α_2 , ..., α_s 可由向量组 γ_1 , γ_2 , ..., γ_k 线性表示。(并且,线性组合的系数就是矩阵 C 的列。)

 $\left\{egin{align*} & \alpha_1, \ \alpha_2 \pm \beta_1, \ \beta_2$ 线性表示 $\beta_1, \ \beta_2 \pm \gamma_1, \ \gamma_2, \ \gamma_3$ 线性表示 $\beta_1, \ \beta_2 \pm \gamma_1, \ \gamma_2, \ \gamma_3$ 线性表示

例
$$\alpha_1, \alpha_2$$
由 β_1, β_2 线性表示 β_1, β_2 由 $\gamma_1, \gamma_2, \gamma_3$ 线性表示 $\beta \Rightarrow \alpha_1, \alpha_2$ 由 $\gamma_1, \gamma_2, \gamma_3$ 线性表示

具体地,设

$$\left\{ \begin{array}{l} \alpha_1 = \alpha_{11}\beta_1 + \alpha_{21}\beta_2 \\ \alpha_2 = \alpha_{12}\beta_1 + \alpha_{22}\beta_2 \end{array} \right.$$

$$egin{array}{ll} & lpha_1, \, lpha_2 ext{由}eta_1, \, eta_2 ext{线性表示} \ eta_1, \, eta_2 ext{e}eta_1, \, \gamma_2, \, \gamma_3 ext{线性表示} \end{array}
ight\} \Rightarrow lpha_1, \, lpha_2 ext{e}eta_1, \, \gamma_2, \, \gamma_3 ext{线性表示}$$

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例
$$\alpha_1$$
, α_2 由 β_1 , β_2 线性表示 β_1 , β_2 由 γ_1 , γ_2 , γ_3 线性表示 β_1 , β_2 由 γ_1 , γ_2 , γ_3 线性表示
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$$\alpha_1 =$$

$$\alpha_2 =$$

例
$$\alpha_1, \alpha_2 \oplus \beta_1, \beta_2$$
线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示
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 则
$$\alpha_1 = \alpha_{11} () + \alpha_{21} ()$$

线性组合

 $\alpha_2 =$

例
$$\alpha_1, \alpha_2 \oplus \beta_1, \beta_2$$
线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示
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 则

$$\alpha_1 = a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}($$

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$$= ()\gamma_1 + ()\gamma_2 + ()\gamma_2 + ()\gamma_2 + ()\gamma_3 + ()\gamma_4 + ()\gamma_4 + ()\gamma_4 + ()\gamma_5 + ()\gamma$$

线性组合

 $\alpha_2 =$

)γ3

例
$$\alpha_1, \alpha_2 \oplus \beta_1, \beta_2$$
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 则
$$\alpha_1 = a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ = (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + ()\gamma_2 + ()\gamma_3 + ()\gamma_4 +$$

线性组合

 $\alpha_2 =$

)γ3

例
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$$= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + ($$

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$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \end{aligned}$$

$$\alpha_2 =$$

例
$$\alpha_1, \alpha_2$$
由 β_1, β_2 线性表示 β_1, β_2 的性表示 β_1, β_2 的 β_1, β_2 的 β_1 β_2 β_1 β_2 β_2 β_3 β_4 β_2 β_3 β_4 β_4 β_4 β_5 β_5 β_6 $\beta_$

例
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$$= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3$$

$$\alpha_2 = a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}($$

$$\left.egin{align*} & \alpha_1, \ \alpha_2 ext{由} eta_1, \ eta_2 ext{dt} eta_1, \ eta_2 ext{h} eta_2, \ eta_3 ext{dt} \ eta_3, \ eta_3 ext{dt} \ eta_3, \ eta_4 ext{h} eta_2 ext{h} eta_3, \ eta_4 ext{h} eta_2, \ eta_3 ext{dt} \ eta_3, \ eta_4 ext{h} eta_3, \ eta_4 ext{h} eta_4, \ eta_4 ext{h} eta_4, \ eta_5 ext{h} eta_4, \ eta_5 ext{h} eta_4, \ eta_5 ext{h} eta_5, \ eta_5 ext{h} eta_5$$

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= $(a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3$

$$\alpha_2 = a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

例
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$$\alpha_1 = a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

$$= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3$$

$$\alpha_2 = a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

 $)\gamma_2 + ($

 $)\gamma_3$

 $)\gamma_{1} + ($

= (

例
$$\alpha_1, \alpha_2 \oplus \beta_1, \beta_2$$
线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示
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$$= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3$$

$$\alpha_2 = a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

 $=(a_{12}b_{11}+a_{22}b_{12})\gamma_1+($

线性组合 15/16 < ▷ △ ▽

 $)\gamma_2 + ($

 $)\gamma_3$

例
$$\alpha_1, \alpha_2 \oplus \beta_1, \beta_2$$
线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示
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$$\alpha_2 = a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

 $=(a_{12}b_{11}+a_{22}b_{12})\gamma_1+(a_{12}b_{21}+a_{22}b_{22})\gamma_2+(a_{12}b_{12}+a_{12}b_{12})\gamma_2$

 $)\gamma_3$

例
$$\alpha_1, \alpha_2$$
由 β_1, β_2 线性表示 β_1, β_2 由 $\gamma_1, \gamma_2, \gamma_3$ 线性表示 β_1, β_2 由 $\gamma_1, \gamma_2, \gamma_3$ 线性表示
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$$\begin{split} \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{split}$$

例
$$\alpha_1, \alpha_2 \oplus \beta_1, \beta_2$$
线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 具体地,设
$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\alpha_{1} = a_{11}(b_{11}\gamma_{1} + b_{21}\gamma_{2} + b_{31}\gamma_{3}) + a_{21}(b_{12}\gamma_{1} + b_{22}\gamma_{2} + b_{32}\gamma_{3})$$

$$= (a_{11}b_{11} + a_{21}b_{12})\gamma_{1} + (a_{11}b_{21} + a_{21}b_{22})\gamma_{2} + (a_{11}b_{31} + a_{21}b_{32})\gamma_{3}$$

$$= c_{11}\gamma_{1} +$$

$$\alpha_{2} = a_{12}(b_{11}\gamma_{1} + b_{21}\gamma_{2} + b_{31}\gamma_{3}) + a_{22}(b_{12}\gamma_{1} + b_{22}\gamma_{2} + b_{32}\gamma_{3})$$

$$= (a_{12}b_{11} + a_{22}b_{12})\gamma_{1} + (a_{12}b_{21} + a_{22}b_{22})\gamma_{2} + (a_{12}b_{31} + a_{22}b_{32})\gamma_{3}$$

例
$$\alpha_1, \alpha_2 \oplus \beta_1, \beta_2$$
线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 具体地,设
$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\alpha_{1} = a_{11}(b_{11}\gamma_{1} + b_{21}\gamma_{2} + b_{31}\gamma_{3}) + a_{21}(b_{12}\gamma_{1} + b_{22}\gamma_{2} + b_{32}\gamma_{3})$$

$$= (a_{11}b_{11} + a_{21}b_{12})\gamma_{1} + (a_{11}b_{21} + a_{21}b_{22})\gamma_{2} + (a_{11}b_{31} + a_{21}b_{32})\gamma_{3}$$

$$= c_{11}\gamma_{1} + c_{21}\gamma_{2} +$$

$$\alpha_{2} = a_{12}(b_{11}\gamma_{1} + b_{21}\gamma_{2} + b_{31}\gamma_{3}) + a_{22}(b_{12}\gamma_{1} + b_{22}\gamma_{2} + b_{32}\gamma_{3})$$

$$= (a_{12}b_{11} + a_{22}b_{12})\gamma_{1} + (a_{12}b_{21} + a_{22}b_{22})\gamma_{2} + (a_{12}b_{31} + a_{22}b_{32})\gamma_{3}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

 $\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$

$$\begin{aligned} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \\ \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \end{aligned}$$

$$\begin{cases} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{cases}$$

$$\left\{ \begin{array}{l} \beta_1 = b_{11} \gamma_1 + b_{21} \gamma_2 + b_{31} \gamma_3 \\ \beta_2 = b_{12} \gamma_1 + b_{22} \gamma_2 + b_{32} \gamma_3 \end{array} \right.$$

$$\begin{split} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \\ \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + \end{split}$$

例
$$\alpha_1, \alpha_2 \oplus \beta_1, \beta_2$$
线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 具体地,设
$$\begin{cases} \alpha_1 = \alpha_{11}\beta_1 + \alpha_{21}\beta_2 \\ \alpha_2 = \alpha_{12}\beta_1 + \alpha_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\begin{split} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \\ \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + \end{split}$$

例
$$\alpha_1, \alpha_2 \oplus \beta_1, \beta_2$$
线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 具体地,设
$$\begin{cases} \alpha_1 = \alpha_{11}\beta_1 + \alpha_{21}\beta_2 \\ \alpha_2 = \alpha_{12}\beta_1 + \alpha_{22}\beta_2 \end{cases}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases}$$

$$\begin{split} \alpha_1 &= a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ &= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \\ \alpha_2 &= a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ &= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ &= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{split}$$

例
$$\alpha_1, \alpha_2$$
由 β_1, β_2 线性表示 β_1, β_2 由 $\gamma_1, \gamma_2, \gamma_3$ 线性表示 β_1, β_2 由 $\gamma_1, \gamma_2, \gamma_3$ 线性表示 β_1, β_2 由 $\gamma_1, \gamma_2, \gamma_3$ 线性表示
具体地,设
$$\left\{ \begin{array}{l} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{array} \right.$$

$$\left\{ \begin{array}{l} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{array} \right.$$
则
$$\alpha_1 = a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ = (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3 \\ = c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3 \\ \alpha_2 = a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3) \\ = (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3 \\ = c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3 \end{array}$$
其中
$$\left(c_{ij} \right) = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{pmatrix}$$

$$(c_{ij}) = \left(\begin{array}{ccc} a_{11}b_{11} + a_{12}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{array} \right) = \left(\begin{array}{ccc} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{array} \right) \left(\begin{array}{ccc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)$$

例
$$\alpha_1, \alpha_2 \oplus \beta_1, \beta_2 \otimes detenty$$
 $\Rightarrow \alpha_1, \alpha_2 \oplus \gamma_1, \gamma_2, \gamma_3 \otimes detenty$ $\Rightarrow \alpha_1, \alpha_2 \oplus \gamma_1, \gamma_2, \gamma_3 \otimes detenty$ $\Rightarrow \alpha_1, \alpha_2 \oplus \gamma_1, \gamma_2, \gamma_3 \otimes detenty$ $\Rightarrow \alpha_1, \alpha_2 \oplus \gamma_1, \gamma_2, \gamma_3 \otimes detenty$ $\Rightarrow \alpha_1, \alpha_2 \oplus \gamma_1, \gamma_2, \gamma_3 \otimes detenty$ $\Rightarrow \alpha_1, \alpha_2 \oplus \gamma_1, \gamma_2, \gamma_3 \otimes detenty$ $\Rightarrow \alpha_1, \alpha_2 \oplus \alpha_2, \alpha_2 \oplus \alpha_3, \alpha_2 \oplus \alpha_4, \alpha_4 \oplus \alpha_$

其中

 $= C_{12}\gamma_1 + C_{22}\gamma_2 + C_{32}\gamma_3$

$$(c_{ij}) = \left(\begin{array}{ccc} a_{11}b_{11} + a_{12}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{array} \right) = \left(\begin{array}{ccc} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{array} \right) \left(\begin{array}{ccc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)$$

例
$$\alpha_1, \alpha_2 \oplus \beta_1, \beta_2$$
线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示 $\beta_1, \beta_2 \oplus \gamma_1, \gamma_2, \gamma_3$ 线性表示

具体地,设

$$\left\{ \begin{array}{l} \alpha_1 = \alpha_{11}\beta_1 + \alpha_{21}\beta_2 \\ \alpha_2 = \alpha_{12}\beta_1 + \alpha_{22}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \; \alpha_2) = (\beta_1, \; \beta_2) \left(\begin{array}{ll} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{array} \right)$$

$$\begin{cases}
\beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\
\beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3
\end{cases} \Rightarrow (\beta_1, \beta_2) = (\gamma_1, \gamma_2, \gamma_3) \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

则

$$\alpha_{1} = a_{11}(b_{11}\gamma_{1} + b_{21}\gamma_{2} + b_{31}\gamma_{3}) + a_{21}(b_{12}\gamma_{1} + b_{22}\gamma_{2} + b_{32}\gamma_{3})$$

$$= (a_{11}b_{11} + a_{21}b_{12})\gamma_{1} + (a_{11}b_{21} + a_{21}b_{22})\gamma_{2} + (a_{11}b_{31} + a_{21}b_{32})\gamma_{3}$$

$$= c_{11}\gamma_{1} + c_{21}\gamma_{2} + c_{31}\gamma_{3}$$

$$\alpha_{2} = a_{12}(b_{11}\gamma_{1} + b_{21}\gamma_{2} + b_{31}\gamma_{3}) + a_{22}(b_{12}\gamma_{1} + b_{22}\gamma_{2} + b_{32}\gamma_{3})$$

$$= (a_{12}b_{11} + a_{22}b_{12})\gamma_{1} + (a_{12}b_{21} + a_{22}b_{22})\gamma_{2} + (a_{12}b_{31} + a_{22}b_{32})\gamma_{3}$$

$$= c_{12}\gamma_{1} + c_{22}\gamma_{2} + c_{32}\gamma_{3}$$

$$(c_{ij}) = \left(\begin{array}{ccc} a_{11}b_{11} + a_{12}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{array}\right) = \left(\begin{array}{ccc} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{array}\right) \left(\begin{array}{ccc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array}\right)$$

$$\alpha_1$$
, α_2 由 β_1 , β_2 线性表示
 β_1 , β_2 由 γ_1 , γ_2 , γ_3 线性表示
 β_1 , β_2 由 γ_1 , γ_2 , γ_3 线性表示
 β_1 β_2 β_2 β_3 β_4 β_2 β_3 β_4 β_5 β_4 β_5 β_5 β_6 β_7 β_8 β_7 β_8 β_9 β_9

具体地,设

$$\left\{ \begin{array}{l} \alpha_1 = a_{11}\beta_1 + a_{21}\beta_2 \\ \alpha_2 = a_{12}\beta_1 + a_{22}\beta_2 \end{array} \right. \Rightarrow (\alpha_1, \, \alpha_2) = (\beta_1, \, \beta_2) \underbrace{ \left(\begin{array}{ll} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)}_{A}$$

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases} \Rightarrow (\beta_1, \beta_2) = (\gamma_1, \gamma_2, \gamma_3) \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

则

$$\alpha_{1} = a_{11}(b_{11}\gamma_{1} + b_{21}\gamma_{2} + b_{31}\gamma_{3}) + a_{21}(b_{12}\gamma_{1} + b_{22}\gamma_{2} + b_{32}\gamma_{3})$$

$$= (a_{11}b_{11} + a_{21}b_{12})\gamma_{1} + (a_{11}b_{21} + a_{21}b_{22})\gamma_{2} + (a_{11}b_{31} + a_{21}b_{32})\gamma_{3}$$

$$= c_{11}\gamma_{1} + c_{21}\gamma_{2} + c_{31}\gamma_{3}$$

$$\alpha_{2} = a_{12}(b_{11}\gamma_{1} + b_{21}\gamma_{2} + b_{31}\gamma_{3}) + a_{22}(b_{12}\gamma_{1} + b_{22}\gamma_{2} + b_{32}\gamma_{3})$$

$$= (a_{12}b_{11} + a_{22}b_{12})\gamma_{1} + (a_{12}b_{21} + a_{22}b_{22})\gamma_{2} + (a_{12}b_{31} + a_{22}b_{32})\gamma_{3}$$

$$= c_{12}\gamma_{1} + c_{22}\gamma_{2} + c_{32}\gamma_{3}$$

$$(c_{ij}) = \left(\begin{array}{ccc} a_{11}b_{11} + a_{12}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{array} \right) = \left(\begin{array}{ccc} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{array} \right) \left(\begin{array}{ccc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)$$

例
$$\alpha_1, \alpha_2$$
由 β_1, β_2 线性表示 β_1, β_2 会性表示 β_1, β_2 由 β_2, β_2 由 β_1, β_2 由 β_1, β_2 由 β_2, β_2 由 β_1, β_2 由 β_1, β_2 由 β_2 由 β_1, β_2 由 β_1, β_2 由 β_2 由 β_1, β_2 由 β_1, β_2 由 β_2 由 β_1, β_2 由 β_2, β_2 和 β_1, β_2 和 β_2 和 β_1, β_2 和 β_1, β_2 和 β_2 和 β_1, β_2 和 β_1, β_2 和 β_2 和 β_1, β_2 和 β_2 和 β_1, β_2 和 β_2 和 β_1, β_2 和 β_1, β_2 和 β_2 和 β_1, β_2 和 β_2 和 β_1, β_2 和 β_1, β_2 和 β_2 和 β_1, β_2 和 β_2 和 β_1, β_2 和 β_2 和 β_1, β_2 和 β_1, β_2 和 β_2 和 β_1, β_2 和 β_2 和 β_1, β_2 和 β_1, β_2 和 β_2 和 β_1, β_2 和 β_2 和 β_1, β_2 和 β_1, β_2 和 β_2 和 β_1, β_2 和 β_1, β_2 和 β_1, β_2 和 β_2 和 β_1, β_2

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases} \Rightarrow (\beta_1, \beta_2) = (\gamma_1, \gamma_2, \gamma_3) \underbrace{\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}}_{B}$$

$$\alpha_{1} = a_{11}(b_{11}\gamma_{1} + b_{21}\gamma_{2} + b_{31}\gamma_{3}) + a_{21}(b_{12}\gamma_{1} + b_{22}\gamma_{2} + b_{32}\gamma_{3})$$

$$= (a_{11}b_{11} + a_{21}b_{12})\gamma_{1} + (a_{11}b_{21} + a_{21}b_{22})\gamma_{2} + (a_{11}b_{31} + a_{21}b_{32})\gamma_{3}$$

$$= c_{11}\gamma_{1} + c_{21}\gamma_{2} + c_{31}\gamma_{3}$$

$$\alpha_{2} = a_{12}(b_{11}\gamma_{1} + b_{21}\gamma_{2} + b_{31}\gamma_{3}) + a_{22}(b_{12}\gamma_{1} + b_{22}\gamma_{2} + b_{32}\gamma_{3})$$

$$= (a_{12}b_{11} + a_{22}b_{12})\gamma_{1} + (a_{12}b_{21} + a_{22}b_{22})\gamma_{2} + (a_{12}b_{31} + a_{22}b_{32})\gamma_{3}$$

$$= c_{12}\gamma_{1} + c_{22}\gamma_{2} + c_{32}\gamma_{3}$$

$$(c_{ij}) = \left(\begin{array}{ccc} a_{11}b_{11} + a_{12}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{array} \right) = \left(\begin{array}{ccc} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{array} \right) \left(\begin{array}{ccc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right)$$

例
$$\alpha_1$$
, α_2 由 β_1 , β_2 线性表示 β_1 , β_2 由 β_2 由 β_1 中 β_2 中 β_1 中 β_1 中 β_2 中 β_1 中 β

$$\begin{cases} \beta_1 = b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3 \\ \beta_2 = b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3 \end{cases} \Rightarrow (\beta_1, \beta_2) = (\gamma_1, \gamma_2, \gamma_3) \underbrace{\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}}_{B}$$

 $\alpha_1 = a_{11}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{21}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$

则

$$= (a_{11}b_{11} + a_{21}b_{12})\gamma_1 + (a_{11}b_{21} + a_{21}b_{22})\gamma_2 + (a_{11}b_{31} + a_{21}b_{32})\gamma_3$$

$$= c_{11}\gamma_1 + c_{21}\gamma_2 + c_{31}\gamma_3$$

$$\alpha_2 = a_{12}(b_{11}\gamma_1 + b_{21}\gamma_2 + b_{31}\gamma_3) + a_{22}(b_{12}\gamma_1 + b_{22}\gamma_2 + b_{32}\gamma_3)$$

$$= (a_{12}b_{11} + a_{22}b_{12})\gamma_1 + (a_{12}b_{21} + a_{22}b_{22})\gamma_2 + (a_{12}b_{31} + a_{22}b_{32})\gamma_3$$

$$= c_{12}\gamma_1 + c_{22}\gamma_2 + c_{32}\gamma_3$$

$$(c_{ij}) = \left(\begin{array}{ccc} a_{11}b_{11} + a_{12}b_{12} & a_{12}b_{11} + a_{22}b_{12} \\ a_{11}b_{21} + a_{21}b_{22} & a_{12}b_{21} + a_{22}b_{22} \\ a_{11}b_{31} + a_{21}b_{32} & a_{12}b_{31} + a_{22}b_{32} \end{array} \right) = \left(\begin{array}{ccc} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{array} \right) \left(\begin{array}{ccc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right) = BA$$

定义 设有两个向量组

(A):
$$\alpha_1, \alpha_2, \ldots, \alpha_s$$

(B):
$$\beta_1, \beta_2, \ldots, \beta_t$$

如果 (A) 与 (B) 可相互线性表示,则称向量组 (A) 与 (B) 等价。

线性组合 16/16 ◁ ▷ △ ▽