第3章c:向量组的线性相关性

数学系 梁卓滨

2019-2020 学年 I

 $\alpha_1 \qquad \alpha_2 \quad \cdots \quad \alpha_n$

定义 如果存在不全为零的一组数 k_1, k_2, \ldots, k_n 使得:

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_n\alpha_n = 0$$

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则称向量组 $\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性相关性

线性相关性 1/14 < ▷ △ ▽

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$$"k_1\alpha_1+k_2\alpha_2+\cdots+k_n\alpha_n=0\quad \Rightarrow\quad k_1=k_2=\cdots=k_n=0\,".$$

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例

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$$\alpha_1 = \begin{pmatrix} 3 \\ -6 \end{pmatrix} \ni \alpha_2 = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$$
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 α_1 , α_2 , . . . , α_n 线性无关

$$lpha_1, lpha_2, \ldots, lpha_n$$
 线性无关 $lpha_1$ $lpha_2$ $lpha_n$ $lpha_n$ $lpha_n$ $lpha_{n}$ $$

$$\alpha_1, \alpha_2, \dots, \alpha_n$$
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$$k_1 \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_2 \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \dots + k_n \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = 0$$

线性相关性 2/14 < ▷ △ ▽

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线性相关性 2/14 < ▷ △ ▽

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$$\begin{pmatrix} a_{1} & \alpha_{2} & \alpha_{n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \begin{pmatrix} k_{1} \\ k_{2} \\ \vdots \\ k_{n} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$\alpha_{1}, \alpha_{2}, \dots, \alpha_{n}$$
 线性无关
$$\alpha_{2} \qquad \alpha_{n}$$

$$\Leftrightarrow k_{1} \begin{pmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{m1} \end{pmatrix} + k_{2} \begin{pmatrix} \alpha_{12} \\ \alpha_{22} \\ \vdots \\ \alpha_{m2} \end{pmatrix} + \dots + k_{n} \begin{pmatrix} \alpha_{1n} \\ \alpha_{2n} \\ \vdots \\ \alpha_{mn} \end{pmatrix} = 0$$
 只有解 $k_{1} = \dots = k_{n} = 0$
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2/14 ✓ ▷ △ ▽

⇔ 方程
$$Ax = 0$$
 只有零解

线性相关性 2/14 ⊲ ▷ △ ▽

$$\alpha_{1}, \alpha_{2}, \dots, \alpha_{n}$$
 线性无关
$$\alpha_{1} \qquad \alpha_{2} \qquad \alpha_{n}$$

$$\Leftrightarrow k_{1} \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} + k_{2} \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} + \dots + k_{n} \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = 0$$
 只有解 $k_{1} = \dots = k_{n} = 0$

$$\Leftrightarrow \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} k_{1} \\ k_{2} \\ \vdots \\ k_{n} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$
 只有解 $k_{1} = \dots = k_{n} = 0$

$$\Leftrightarrow$$
 方程 $Ax = 0$ 只有零解

$$\Leftrightarrow r(A) = n$$

线性相关性 2/14 ⊲ ▷ △ ▽

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 只有解 $k_{1} = \dots = k_{n} = 0$

$$⇔$$
 方程 $Ax = 0$ 只有零解

$$\Leftrightarrow r(A) = n$$

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线性相关性 2/14 < ▶ △ ▼

$$\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}$$
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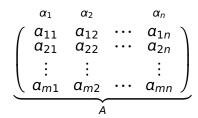
线性相关性

 $\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性相关

$$\alpha_{1}, \alpha_{2}, \ldots, \alpha_{n}$$
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线性相关性

定理 设



则

$$\alpha_1, \alpha_2, \ldots, \alpha_n$$
 线性相关 \Leftrightarrow $r(A) < n$

$$\alpha_1, \alpha_2, \ldots, \alpha_n$$
 线性无关 \Leftrightarrow $r(A) = n$

定理 设

$$\begin{pmatrix}
\alpha_{1} & \alpha_{2} & \alpha_{n} \\
\alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\
\alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\
\vdots & \vdots & & \vdots \\
\alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn}
\end{pmatrix}$$

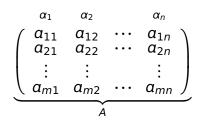
则

$$\alpha_1, \alpha_2, \ldots, \alpha_n$$
 线性相关 \Leftrightarrow $r(A) < n$

$$\alpha_1, \alpha_2, \ldots, \alpha_n$$
 线性无关 \Leftrightarrow $r(A) = n$

推论 1 如果 m = n(向量维数 = 向量个数),则

定理 设



则

$$\alpha_1, \alpha_2, \ldots, \alpha_n$$
 线性相关 \iff $r(A) < n$

$$\alpha_1, \alpha_2, \ldots, \alpha_n$$
 线性无关 \iff $r(A) = n$

推论 1 如果
$$m = n$$
(向量维数 = 向量个数),则
线性相关 \Leftrightarrow $|A| = 0$, 线性无关 \Leftrightarrow $|A| \neq 0$

线性相关性

定理 设

$$\underbrace{\begin{pmatrix}
\alpha_{11} & \alpha_{12} & \cdots & \alpha_{n} \\
\alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\
\alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\
\vdots & \vdots & & \vdots \\
\alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn}
\end{pmatrix}}_{A}$$

则

$$\alpha_1, \alpha_2, \ldots, \alpha_n$$
 线性相关 $\Leftrightarrow r(A) < n$
 $\alpha_1, \alpha_2, \ldots, \alpha_n$ 线性无关 $\Leftrightarrow r(A) = n$

推论 1 如果 m = n(向量维数 = 向量个数),则 线性相关 $\iff |A| = 0$, 线性无关 $\iff |A| \neq 0$

推论 2 如果 m < n (向量维数 < 向量个数),则一定线性相关。

定理 设

$$\underbrace{\begin{pmatrix}
\alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} \\
\alpha_{11} & \alpha_{12} & \cdots & \alpha_{2n} \\
\alpha_{21} & \alpha_{22} & \cdots & \alpha_{2n} \\
\vdots & \vdots & & \vdots \\
\alpha_{m1} & \alpha_{m2} & \cdots & \alpha_{mn}
\end{pmatrix}}_{A}$$

则

$$\alpha_1, \alpha_2, \ldots, \alpha_n$$
 线性相关 \iff $r(A) < n$

$$\alpha_1, \alpha_2, \ldots, \alpha_n$$
 线性无关 \iff $r(A) = n$

推论 1 如果 m = n (向量维数 = 向量个数),则

线性相关
$$\Leftrightarrow$$
 $|A| = 0$, 线性无关 \Leftrightarrow $|A| \neq 0$

推论 2 如果 m < n(向量维数 < 向量个数),则一定线性相关。这是:

$$r(A) \le m < n$$
.

例1
$$\alpha_1 = \begin{pmatrix} 1\\2\\-1\\5 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 2\\-1\\1\\1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4\\3\\-1\\11 \end{pmatrix}$ 是否线性相关性?如果

是,求出一个"线性相关性表达式"

例 1
$$\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 5 \end{pmatrix}$$
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是, 求出一个"线性相关性表达式"

$$\begin{array}{cccc}
\mathbf{f} & \alpha_1 & \alpha_2 & \alpha_3 \\
1 & 2 & 4 \\
2 & -1 & 3 \\
-1 & 1 & -1 \\
5 & 1 & 11 \\
\end{array}$$

$$\frac{r_2 - 2r_1}{r_3 + r_1} + r_4 - 5r_1$$

例 1
$$\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 5 \end{pmatrix}$$
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$$\begin{pmatrix} 1 & 2 & 4 \\ 2 & -1 & 3 \\ -1 & 1 & -1 \end{pmatrix} \xrightarrow[r_4-5r_1]{r_2-2r_1} \begin{pmatrix} 1 & 2 & 4 \\ 0 & -5 & -5 \end{pmatrix}$$

线性相关性 4/14 ⊲ ▷ △ ▽

例 1
$$\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 5 \end{pmatrix}$$
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$$\begin{pmatrix} 1 & 2 & 4 \\ 2 & -1 & 3 \\ -1 & 1 & -1 \end{pmatrix} \xrightarrow[r_3+r_1]{r_2-2r_1} \begin{pmatrix} 1 & 2 & 4 \\ 0 & -5 & -5 \\ 0 & 3 & 3 \end{pmatrix}$$

例 1
$$\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 5 \end{pmatrix}$$
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$$\begin{pmatrix}
1 & 2 & 4 \\
2 & -1 & 3 \\
-1 & 1 & -1 \\
5 & 1 & 11
\end{pmatrix}
\xrightarrow[r_4-5r_1]{r_2-2r_1}
\begin{pmatrix}
1 & 2 & 4 \\
0 & -5 & -5 \\
0 & 3 & 3 \\
0 & -9 & -9
\end{pmatrix}$$

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例 1
$$\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 5 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ -1 \\ 11 \end{pmatrix}$ 是否线性相关性?如果

$$\begin{pmatrix}
1 & 2 & 4 \\
2 & -1 & 3 \\
-1 & 1 & -1 \\
5 & 1 & 11
\end{pmatrix}
\xrightarrow[r_4-5r_1]{r_2-2r_1}
\begin{pmatrix}
1 & 2 & 4 \\
0 & -5 & -5 \\
0 & 3 & 3 \\
0 & -9 & -9
\end{pmatrix}
\xrightarrow[-\frac{1}{3}\times r_4]{\frac{-\frac{1}{5}\times r_2}{\frac{1}{3}\times r_3}}
\xrightarrow[-\frac{1}{3}\times r_4]{\frac{1}{3}\times r_4}$$

线性相关性 4/14 riangleright 5/14 riangleright 4/14 riangleright 5/14 riangleright 5/14 riangleright

例 1
$$\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 5 \end{pmatrix}$$
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线性相关性 4/14 ◁ ▷ △ ヾ

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$$r_3-r_2$$

线性相关性 4/14 riangleright 6/14 riangleright 1/14 riangleright 1/14 riangleright 1/14 riangleright 1/14 riangleright 1/14 riangleright

例1
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, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ -1 \\ 11 \end{pmatrix}$ 是否线性相关性?如果

$$\begin{array}{c|cccc}
r_3 - r_2 \\
\hline
r_4 - r_2 \\
\hline
\end{array}
\begin{pmatrix}
1 & 2 & 4 \\
0 & 1 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}$$

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例 1
$$\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 5 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ -1 \\ 11 \end{pmatrix}$ 是否线性相关性?如果

$$\frac{r_3 - r_2}{r_4 - r_2} \begin{pmatrix}
1 & 2 & 4 \\
0 & 1 & 1 \\
0 & 0 & 0
\end{pmatrix} \xrightarrow{r_1 - 2r_2}$$

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例 1
$$\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 5 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 4 \\ 3 \\ -1 \\ 11 \end{pmatrix}$ 是否线性相关性?如果

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1 & 2 & 4 \\
2 & -1 & 3 \\
-1 & 1 & -1 \\
5 & 1 & 11
\end{pmatrix}
\xrightarrow[r_4-5r_1]{r_2-2r_1}
\begin{pmatrix}
1 & 2 & 4 \\
0 & -5 & -5 \\
0 & 3 & 3 \\
0 & -9 & -9
\end{pmatrix}
\xrightarrow[\frac{1}{3}\times r_4]{-\frac{1}{5}\times r_2}
\begin{pmatrix}
1 & 2 & 4 \\
0 & 1 & 1 \\
0 & 1 & 1
\end{pmatrix}$$

$$\begin{array}{c|cccc}
r_3 - r_2 \\
\hline
r_4 - r_2 \\
\hline
\end{array}
\begin{pmatrix}
1 & 2 & 4 \\
0 & 1 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\xrightarrow{r_1 - 2r_2}
\begin{pmatrix}
1 & 0 & 2 \\
0 & 1 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}$$

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$$\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 5 \end{pmatrix}$$
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1 & 2 & 4 \\
2 & -1 & 3 \\
-1 & 1 & -1 \\
5 & 1 & 11
\end{pmatrix}
\xrightarrow[r_{4}-5r_{1}]{r_{2}-2r_{1}}
\begin{pmatrix}
1 & 2 & 4 \\
0 & -5 & -5 \\
0 & 3 & 3 \\
0 & -9 & -9
\end{pmatrix}
\xrightarrow[-\frac{1}{3}\times r_{4}]{-\frac{1}{5}\times r_{2}}
\begin{pmatrix}
1 & 2 & 4 \\
0 & 1 & 1 \\
0 & 1 & 1
\end{pmatrix}$$

$$\frac{r_3 - r_2}{r_4 - r_2} \begin{pmatrix}
1 & 2 & 4 \\
0 & 1 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\xrightarrow{r_1 - 2r_2} \begin{pmatrix}
1 & 0 & 2 \\
0 & 1 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}$$

例 1
$$\alpha_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 5 \end{pmatrix}$$
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$$\begin{pmatrix}
1 & 2 & 4 \\
2 & -1 & 3 \\
-1 & 1 & -1 \\
5 & 1 & 11
\end{pmatrix}
\xrightarrow[r_4-5r_1]{r_2-2r_1}
\begin{pmatrix}
1 & 2 & 4 \\
0 & -5 & -5 \\
0 & 3 & 3 \\
0 & -9 & -9
\end{pmatrix}
\xrightarrow[\frac{1}{9}\times r_4]{\frac{1}{5}\times r_2}
\begin{pmatrix}
1 & 2 & 4 \\
0 & 1 & 1 \\
0 & 1 & 1 \\
0 & 1 & 1
\end{pmatrix}$$

$$\begin{array}{c|cccc}
r_{3}-r_{2} \\
\hline
r_{4}-r_{2}
\end{array}
\begin{pmatrix}
1 & 2 & 4 \\
0 & 1 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\xrightarrow{r_{1}-2r_{2}}
\begin{pmatrix}
1 & 0 & 2 \\
0 & 1 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}$$

可见 $r(\alpha_1\alpha_2\alpha_3) = 2 < 3$,线性相关性;

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5 & 1 & 11
\end{pmatrix}
\xrightarrow[r_4-5r_1]{r_2-2r_1}
\begin{pmatrix}
1 & 2 & 4 \\
0 & -5 & -5 \\
0 & 3 & 3 \\
0 & -9 & -9
\end{pmatrix}
\xrightarrow[-\frac{1}{9}\times r_4]{-\frac{1}{5}\times r_2}
\begin{pmatrix}
1 & 2 & 4 \\
0 & 1 & 1 \\
0 & 1 & 1 \\
0 & 1 & 1
\end{pmatrix}$$

$$\begin{array}{c|cccc}
r_3 - r_2 \\
\hline
r_4 - r_2
\end{array}
\begin{pmatrix}
1 & 2 & 4 \\
0 & 1 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\xrightarrow{r_1 - 2r_2}
\begin{pmatrix}
1 & 0 & 2 \\
0 & 1 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}$$

可见
$$r(\alpha_1\alpha_2\alpha_3) = 2 < 3$$
,线性相关性;且 $\alpha_3 = 2\alpha_1 + \alpha_2$

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\xrightarrow[r_4-5r_1]{r_2-2r_1}
\begin{pmatrix}
1 & 2 & 4 \\
0 & -5 & -5 \\
0 & 3 & 3 \\
0 & -9 & -9
\end{pmatrix}
\xrightarrow[\frac{1}{9}\times r_4]{-\frac{1}{5}\times r_2}
\begin{pmatrix}
1 & 2 & 4 \\
0 & 1 & 1 \\
0 & 1 & 1 \\
0 & 1 & 1
\end{pmatrix}$$

$$\frac{r_3 - r_2}{r_4 - r_2} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{r_1 - 2r_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

可见
$$r(\alpha_1\alpha_2\alpha_3) = 2 < 3$$
,线性相关性;且
$$\alpha_3 = 2\alpha_1 + \alpha_2 \quad \Rightarrow \quad 2\alpha_1 + \alpha_2 - \alpha_3 = 0$$

例 2
$$\alpha_1 = \begin{pmatrix} 0 \\ 4 \\ 0 \\ 2 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 6 \\ 0 \\ 4 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ -1 \\ 2 \\ 0 \end{pmatrix}$ 是否线性相关性?如果是,

例 2
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$$\begin{pmatrix}
\alpha_1 & \alpha_2 & \alpha_3 \\
0 & 6 & 3 \\
4 & 0 & -1 \\
0 & 4 & 2 \\
2 & 1 & 0
\end{pmatrix}
\xrightarrow{r_1 \leftrightarrow r_4}$$

例 2
$$\alpha_1 = \begin{pmatrix} 0 \\ 4 \\ 0 \\ 2 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 6 \\ 0 \\ 4 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ -1 \\ 2 \\ 0 \end{pmatrix}$ 是否线性相关性?如果是,

$$\begin{pmatrix}
\alpha_1 & \alpha_2 & \alpha_3 \\
0 & 6 & 3 \\
4 & 0 & -1 \\
0 & 4 & 2 \\
2 & 1 & 0
\end{pmatrix}
\xrightarrow{r_1 \leftrightarrow r_4}
\begin{pmatrix}
2 & 1 & 0 \\
4 & 0 & -1 \\
0 & 4 & 2 \\
0 & 6 & 2
\end{pmatrix}$$

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$$\begin{pmatrix}
0 & 6 & 3 \\
4 & 0 & -1 \\
0 & 4 & 2
\end{pmatrix}
\xrightarrow{r_1 \leftrightarrow r_4}
\begin{pmatrix}
2 & 1 & 0 \\
4 & 0 & -1 \\
0 & 4 & 2
\end{pmatrix}
\xrightarrow{r_2 - 2r_1}$$

例 2
$$\alpha_1 = \begin{pmatrix} 0 \\ 4 \\ 0 \\ 2 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 6 \\ 0 \\ 4 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ -1 \\ 2 \\ 0 \end{pmatrix}$ 是否线性相关性?如果是,

$$\begin{array}{ccccc}
\mathbf{f} & \alpha_1 & \alpha_2 & \alpha_3 \\
 & \alpha_1 & \alpha_2 & \alpha_3
\end{array}$$

$$\begin{pmatrix} 0 & 6 & 3 \\ 4 & 0 & -1 \\ 0 & 4 & 2 \\ 2 & 1 & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \\ 0 & 4 & 2 \\ 0 & 6 & 3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 2 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 4 & 2 \\ 0 & 6 & 3 \end{pmatrix}$$

线性相关性 5/14 < ▷ △ ▽

例 2
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$$\begin{array}{ccccc}
\mathbf{m} & \alpha_1 & \alpha_2 & \alpha_3 \\
 & 0 & 6 & 3
\end{array}$$

$$\begin{pmatrix} 0 & 6 & 3 \\ 4 & 0 & -1 \\ 0 & 4 & 2 \\ 2 & 1 & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \\ 0 & 4 & 2 \\ 0 & 6 & 3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 2 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 4 & 2 \\ 0 & 6 & 3 \end{pmatrix}$$

$$\frac{r_3 + 2r_2}{r_4 + 3r_2}$$

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例 2
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, $\alpha_2 = \begin{pmatrix} 6 \\ 0 \\ 4 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ -1 \\ 2 \\ 0 \end{pmatrix}$ 是否线性相关性?如果是,

$$\mathbf{M}$$
 α_1 α_2 α_3

$$\begin{pmatrix} 0 & 6 & 3 \\ 4 & 0 & -1 \\ 0 & 4 & 2 \\ 2 & 1 & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \\ 0 & 4 & 2 \\ 0 & 6 & 3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 2 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 4 & 2 \\ 0 & 6 & 3 \end{pmatrix}$$

$$\begin{array}{c|cccc}
r_{3}+2r_{2} \\
\hline
r_{4}+3r_{2}
\end{array}
\begin{pmatrix}
2 & 1 & 0 \\
0 & -2 & -1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}$$

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$$\mathbf{R}$$
 α_1 α_2 α_3

$$\begin{pmatrix} 0 & 6 & 3 \\ 4 & 0 & -1 \\ 0 & 4 & 2 \\ 2 & 1 & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \\ 0 & 4 & 2 \\ 0 & 6 & 3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 2 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 4 & 2 \\ 0 & 6 & 3 \end{pmatrix}$$

$$\xrightarrow[r_4+3r_2]{r_3+2r_2} \begin{pmatrix} 2 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{\frac{1}{2} \times r_1} \xrightarrow{-\frac{1}{2} \times r_2}$$

例 2
$$\alpha_1 = \begin{pmatrix} 0 \\ 4 \\ 0 \\ 2 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 6 \\ 0 \\ 4 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ -1 \\ 2 \\ 0 \end{pmatrix}$ 是否线性相关性?如果是,

$$\mathbf{H}$$
 α_1 α_2 α_3

$$\begin{pmatrix} 0 & 6 & 3 \\ 4 & 0 & -1 \\ 0 & 4 & 2 \\ 2 & 1 & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \\ 0 & 4 & 2 \\ 0 & 6 & 3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 2 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 4 & 2 \\ 0 & 6 & 3 \end{pmatrix}$$

$$\frac{r_{3}+2r_{2}}{r_{4}+3r_{2}} \begin{pmatrix} 2 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{\frac{1}{2} \times r_{1}} \begin{pmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

线性相关性 5/14 ⊲ ⊳ Δ ⊽

例 2
$$\alpha_1 = \begin{pmatrix} 0 \\ 4 \\ 0 \\ 2 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 6 \\ 0 \\ 4 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ -1 \\ 2 \\ 0 \end{pmatrix}$ 是否线性相关性?如果是,

$$\mathbf{m}$$
 α_1 α_2 α_3

$$\begin{pmatrix} 0 & 6 & 3 \\ 4 & 0 & -1 \\ 0 & 4 & 2 \\ 2 & 1 & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \\ 0 & 4 & 2 \\ 0 & 6 & 3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 2 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 4 & 2 \\ 0 & 6 & 3 \end{pmatrix}$$

$$\frac{r_{3}+2r_{2}}{r_{4}+3r_{2}}\begin{pmatrix}2&1&0\\0&-2&-1\\0&0&0\\0&0&0\end{pmatrix}\frac{\frac{1}{2}\times r_{1}}{-\frac{1}{2}\times r_{2}}\begin{pmatrix}1&\frac{1}{2}&0\\0&1&\frac{1}{2}\\0&0&0\\0&0&0\end{pmatrix}\xrightarrow{r_{1}-\frac{1}{2}r_{2}}$$

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例 2
$$\alpha_1 = \begin{pmatrix} 0 \\ 4 \\ 0 \\ 2 \end{pmatrix}$$
, $\alpha_2 = \begin{pmatrix} 6 \\ 0 \\ 4 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ -1 \\ 2 \\ 0 \end{pmatrix}$ 是否线性相关性?如果是,

$$\begin{pmatrix} 0 & 6 & 3 \\ 4 & 0 & -1 \\ 0 & 4 & 2 \\ 2 & 1 & 0 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \\ 0 & 4 & 2 \\ 0 & 6 & 3 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 2 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 4 & 2 \\ 0 & 6 & 3 \end{pmatrix}$$

$$\xrightarrow[r_4+3r_2]{r_4+3r_2}
\begin{pmatrix}
2 & 1 & 0 \\
0 & -2 & -1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\xrightarrow[-\frac{1}{2}\times r_2]{\frac{\frac{1}{2}\times r_1}{-\frac{1}{2}\times r_2}}
\begin{pmatrix}
1 & \frac{1}{2} & 0 \\
0 & 1 & \frac{1}{2} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\xrightarrow[r_1-\frac{1}{2}r_2]{r_1+\frac{1}{2}r_2}
\begin{pmatrix}
1 & 0 & -\frac{1}{4} \\
0 & 1 & \frac{1}{2} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}$$

线性相关性 5/14 < ▷ △ ▽

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1 & \frac{1}{2} & 0 \\
0 & 1 & \frac{1}{2} \\
0 & 0 & 0 \\
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\end{pmatrix}
\xrightarrow[r_1-\frac{1}{2}r_2]{r_1-\frac{1}{2}r_2}
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线性相关性 5/14 < ▷ △ ▽

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\end{pmatrix}
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\begin{pmatrix}
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0 & -2 & -1 \\
0 & 4 & 2 \\
0 & 6 & 3
\end{pmatrix}$$

$$\xrightarrow{r_3 + 2r_2}
\xrightarrow{r_4 + 3r_2}
\begin{pmatrix}
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0 & -2 & -1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\xrightarrow{\frac{1}{2} \times r_1}
\xrightarrow{-\frac{1}{2} \times r_2}
\begin{pmatrix}
1 & \frac{1}{2} & 0 \\
0 & 1 & \frac{1}{2} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\xrightarrow{r_1 - \frac{1}{2}r_2}
\begin{pmatrix}
1 & 0 & -\frac{1}{4} \\
0 & 1 & \frac{1}{2} \\
0 & 0 & 0
\end{pmatrix}$$

可见 $r(\alpha_1\alpha_2\alpha_3) = 2 < 3$,线性相关性;

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\begin{pmatrix}
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线性相关性 5/14 < ▷ △ ▽

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\end{pmatrix}
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\xrightarrow{-\frac{1}{2} \times r_2}
\begin{pmatrix}
1 & \frac{1}{2} & 0 \\
0 & 1 & \frac{1}{2} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\xrightarrow{r_1 - \frac{1}{2}r_2}
\begin{pmatrix}
1 & 0 & -\frac{1}{4} \\
0 & 1 & \frac{1}{2} \\
0 & 0 & 0 \\
0 & 0 & 0
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$$r(\alpha_1\alpha_2\alpha_3)=2<3$$
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 $\alpha_3 = -\frac{1}{4}\alpha_1 + \frac{1}{2}\alpha_2$

$$\frac{r_{3}+2r_{2}}{r_{4}+3r_{2}} \xrightarrow{\begin{pmatrix} 2 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}} \xrightarrow{\frac{1}{2} \times r_{1}} \xrightarrow{\begin{pmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}} \xrightarrow{r_{1}-\frac{1}{2}r_{2}} \xrightarrow{\begin{pmatrix} 1 & 0 & -\frac{1}{4} \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

 $\begin{pmatrix} 0 & 6 & 3 \\ 4 & 0 & -1 \\ 0 & 4 & 2 \end{pmatrix} \xrightarrow{r_1 \leftrightarrow r_4} \begin{pmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \\ 0 & 4 & 2 \end{pmatrix} \xrightarrow{r_2 - 2r_1} \begin{pmatrix} 2 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 4 & 2 \end{pmatrix}$

例 2 $\alpha_1 = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ 是否线性相关性? 如果是,

可见 $r(\alpha_1 \alpha_2 \alpha_3) = 2 < 3$,线性相关性;且 $\alpha_3 = -\frac{1}{4}\alpha_1 + \frac{1}{2}\alpha_2 \quad \Rightarrow \quad -\frac{1}{4}\alpha_1 + \frac{1}{2}\alpha_2 - \alpha_3 = 0$

求出一个"线性相关性表达式"

线性相关性 6/14 ◁ ▷ △ ▽

$$0 = k_1(\alpha + \beta) + k_2(\beta + \gamma) + k_3(\gamma + \alpha)$$

$$0 = k_1(\alpha + \beta) + k_2(\beta + \gamma) + k_3(\gamma + \alpha)$$

= ()\alpha + ()\beta + ()\gamma

$$0 = k_1(\alpha + \beta) + k_2(\beta + \gamma) + k_3(\gamma + \alpha)$$

= $(k_1 + k_3)\alpha + (\beta + \gamma) + (k_3(\gamma + \alpha))$

$$0 = k_1(\alpha + \beta) + k_2(\beta + \gamma) + k_3(\gamma + \alpha)$$

= $(k_1 + k_3)\alpha + (k_1 + k_2)\beta + ($)

$$0 = k_1(\alpha + \beta) + k_2(\beta + \gamma) + k_3(\gamma + \alpha)$$

= $(k_1 + k_3)\alpha + (k_1 + k_2)\beta + (k_2 + k_3)\gamma$

证明设

$$0 = k_1(\alpha + \beta) + k_2(\beta + \gamma) + k_3(\gamma + \alpha)$$

= $(k_1 + k_3)\alpha + (k_1 + k_2)\beta + (k_2 + k_3)\gamma$

$$\begin{cases} k_1 + k_3 = 0 \\ k_1 + k_2 = 0 \\ k_2 + k_3 = 0 \end{cases}$$

证明设

$$0 = k_1(\alpha + \beta) + k_2(\beta + \gamma) + k_3(\gamma + \alpha)$$

= $(k_1 + k_3)\alpha + (k_1 + k_2)\beta + (k_2 + k_3)\gamma$

所以

$$\begin{cases} k_1 + k_3 = 0 \\ k_1 + k_2 = 0 \\ k_2 + k_3 = 0 \end{cases} \qquad \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

线性相关性 6/14 ◁ ▷ △ ヾ

证明设

$$0 = k_1(\alpha + \beta) + k_2(\beta + \gamma) + k_3(\gamma + \alpha)$$

= $(k_1 + k_3)\alpha + (k_1 + k_2)\beta + (k_2 + k_3)\gamma$

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线性相关性 6/14 ◁ ▷ △ ▼

证明设

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所以

$$\begin{cases} k_1 & + k_3 = 0 \\ k_1 + k_2 & = 0 \\ k_2 + k_3 = 0 \end{cases} \Rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow Ax = 0$$

线性相关性 6/14 < ▶ △ ▽

证明设

$$0 = k_1(\alpha + \beta) + k_2(\beta + \gamma) + k_3(\gamma + \alpha)$$

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$$\overline{m} |A| = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$$

证明设

$$0 = k_1(\alpha + \beta) + k_2(\beta + \gamma) + k_3(\gamma + \alpha)$$

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$$m|A| =$$

$$\begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$$

$$\begin{vmatrix} r_2 - r_1 \\ 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix}$$

证明设

$$0 = k_1(\alpha + \beta) + k_2(\beta + \gamma) + k_3(\gamma + \alpha)$$

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$$\overline{m}|A| = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} \xrightarrow{r_2 - r_1} \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 2 \neq 0$$

证明设

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所以只有零解: $k_1 = k_2 = k_3 = 0$,

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所以只有零解: $k_1 = k_2 = k_3 = 0$,所以线性无关

$$(\alpha + \beta \quad \beta + \gamma \quad \gamma + \alpha) = (\alpha \quad \beta \quad \gamma) \left(\qquad \qquad \right)$$

$$(\alpha + \beta \quad \beta + \gamma \quad \gamma + \alpha) = (\alpha \quad \beta \quad \gamma) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$(\alpha + \beta \quad \beta + \gamma \quad \gamma + \alpha) = (\alpha \quad \beta \quad \gamma) \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$(\alpha + \beta \quad \beta + \gamma \quad \gamma + \alpha) = (\alpha \quad \beta \quad \gamma) \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

另证 注意到

$$\underbrace{\left(\alpha + \beta \quad \beta + \gamma \quad \gamma + \alpha\right)}_{Q} = \underbrace{\left(\alpha \quad \beta \quad \gamma\right)}_{P} \underbrace{\left(\begin{matrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{matrix}\right)}_{A}$$

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另证 注意到

$$\underbrace{\left(\alpha + \beta \quad \beta + \gamma \quad \gamma + \alpha\right)}_{Q} = \underbrace{\left(\alpha \quad \beta \quad \gamma\right)}_{P} \underbrace{\left(\begin{matrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{matrix}\right)}_{Q} \quad \Rightarrow \quad Q = PA$$

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另证 注意到

$$\underbrace{\left(\alpha + \beta \quad \beta + \gamma \quad \gamma + \alpha\right)}_{Q} = \underbrace{\left(\alpha \quad \beta \quad \gamma\right)}_{P} \underbrace{\left(\begin{array}{ccc} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{array}\right)}_{Q} \quad \Rightarrow \quad Q = PA$$

$$r(Q) = r(PA)$$

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另证 注意到

$$\underbrace{(\alpha + \beta \quad \beta + \gamma \quad \gamma + \alpha)}_{Q} = \underbrace{(\alpha \quad \beta \quad \gamma)}_{P} \underbrace{\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}}_{A} \Rightarrow Q = PA$$

而
$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} r_2 - r_1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 2 \neq 0$$
,所以 A 可逆,
$$r(O) = r(PA)$$

r(Q) = r(PA)

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另证 注意到

$$\underbrace{(\alpha + \beta \quad \beta + \gamma \quad \gamma + \alpha)}_{Q} = \underbrace{(\alpha \quad \beta \quad \gamma)}_{P} \underbrace{\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}}_{A} \quad \Rightarrow \quad Q = PA$$

而
$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} \frac{r_2 - r_1}{2} \\ 0 & 1 & 1 \end{vmatrix} = 2 \neq 0$$
,所以 A 可逆,从而

$$r(Q) = r(PA) = r(P)$$

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另证 注意到

$$\underbrace{\left(\alpha + \beta \quad \beta + \gamma \quad \gamma + \alpha\right)}_{Q} = \underbrace{\left(\alpha \quad \beta \quad \gamma\right)}_{P} \underbrace{\left(\begin{matrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{matrix}\right)}_{A} \quad \Rightarrow \quad Q = PA$$

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,所以 A 可逆,从而

$$r(Q) = r(PA) = r(P) = 3$$

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另证 注意到

$$\underbrace{\left(\alpha + \beta \quad \beta + \gamma \quad \gamma + \alpha\right)}_{Q} = \underbrace{\left(\alpha \quad \beta \quad \gamma\right)}_{P} \underbrace{\left(\begin{matrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{matrix}\right)}_{A} \quad \Rightarrow \quad Q = PA$$

$$r(O) = r(PA) = r(P) = 3$$

所以 $\alpha + \beta$, $\beta + \gamma$, $\gamma + \alpha$ 线性无关。

线性相关 \Leftrightarrow $\exists k \neq 0$ 使得 $k\alpha = 0$

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例 2 两个向量 α , β 线性相关当且仅当它们成比例。

线性相关 \Leftrightarrow $\exists k \neq 0$ 使得 $k\alpha = 0$ \Leftrightarrow $\alpha = 0$

例 2 两个向量 α , β 线性相关当且仅当它们成比例。

证明

1. 设 α, β 线性相关:

线性相关 \Leftrightarrow $\exists k \neq 0$ 使得 $k\alpha = 0$ \Leftrightarrow $\alpha = 0$

例 2 两个向量 α , β 线性相关当且仅当它们成比例。

证明

1. 设 α , β 线性相关:存在不全为零的 k_1 , k_2 使 $k_1\alpha + k_2\beta = 0$ 。

线性相关
$$\Leftrightarrow$$
 $\exists k \neq 0$ 使得 $k\alpha = 0$ \Leftrightarrow $\alpha = 0$

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$$\alpha = -\frac{k_2}{k_1}\beta$$

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所以 α , β 成比例

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线性相关 \Leftrightarrow $\exists k \neq 0$ 使得 $k\alpha = 0$ \Leftrightarrow $\alpha = 0$

例 2 两个向量 α , β 线性相关当且仅当它们成比例。

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$$\alpha = -\frac{k_2}{k_1}\beta$$

所以 α , β 成比例

2. 设 α , β 成比例: 不妨设 $\alpha = k\beta$, 则

$$1 \cdot \alpha - k\beta = 0$$

线性相关
$$\Leftrightarrow$$
 $\exists k \neq 0$ 使得 $k\alpha = 0$ \Leftrightarrow $\alpha = 0$

例 2 两个向量 α , β 线性相关当且仅当它们成比例。

证明

1. 设 α , β 线性相关:存在不全为零的 k_1 , k_2 使 $k_1\alpha + k_2\beta = 0$ 。不 妨设 $k_1 \neq 0$,则

$$\alpha = -\frac{k_2}{k_1}\beta$$

所以 α , β 成比例

2. 设 α , β 成比例: 不妨设 $\alpha = k\beta$, 则

$$1 \cdot \alpha - k\beta = 0$$

所以 α , β 线性相关

证明 不妨设

$$\alpha = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \beta = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \gamma = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

证明 不妨设

$$\alpha = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \beta = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \gamma = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

则

$$\alpha, \beta, \gamma$$
线性相关 \iff $\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$ 秩小于3

证明 不妨设

$$\alpha = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \beta = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \gamma = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

则

$$\alpha$$
, β , γ 线性相关 \Leftrightarrow $\begin{pmatrix} \alpha_1 & b_1 & c_1 \\ \alpha_2 & b_2 & c_2 \\ \alpha_3 & b_3 & c_3 \end{pmatrix}$ 秩小于3 \Leftrightarrow $\begin{vmatrix} \alpha_1 & b_1 & c_1 \\ \alpha_2 & b_2 & c_2 \\ \alpha_3 & b_3 & c_3 \end{vmatrix} = 0$

例 3 \mathbb{R}^3 中三个向量 α , β , γ 线性相关当且仅当它们共面。

证明 不妨设

$$\alpha = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \beta = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \gamma = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

则

$$\alpha$$
, β , γ 线性相关 \Leftrightarrow $\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$ 秩小于3 \Leftrightarrow $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0 \Leftrightarrow \alpha$, β , γ 共面

线性相关性

线性相关性 10/14 ◁ ▷ △ ▽

证明 设

$$\underline{\alpha_1, \alpha_2, \ldots, \alpha_r}, \alpha_{r+1}, \ldots \alpha_s$$

线性相关

线性相关性 10/14 ◁ ▷ △ ▽

证明 设

$$\underline{\alpha_1, \alpha_2, \ldots, \alpha_r}, \alpha_{r+1}, \ldots \alpha_s$$

线性相关

则存在不全为零的数 k_1, k_2, \ldots, k_r 使

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_r\alpha_r = 0$$

线性相关性 10/14 < ▶ △ ▼

证明 设

$$\underline{\alpha_1, \alpha_2, \ldots, \alpha_r}$$
, $\alpha_{r+1}, \ldots \alpha_s$ 线性相关

则存在不全为零的数 k_1, k_2, \ldots, k_r 使

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_r\alpha_r = 0$$

所以

$$k_1\alpha_1 + k_2\alpha_2 + \dots + k_r\alpha_r + 0\alpha_{r+1} + \dots + 0\alpha_s = 0$$

线性相关性 10/14 < ▷ △ ▽

证明设

$$\underline{\alpha_1, \alpha_2, \ldots, \alpha_r}, \alpha_{r+1}, \ldots \alpha_s$$

线性相关

则存在不全为零的数 k_1, k_2, \ldots, k_r 使

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_r\alpha_r = 0$$

所以

$$k_1\alpha_1+k_2\alpha_2+\cdots+k_r\alpha_r+0\alpha_{r+1}+\cdots+0\alpha_s=0$$

其中系数不全为零,

证明设

$$\underline{\alpha_1, \alpha_2, \ldots, \alpha_r}, \alpha_{r+1}, \ldots \alpha_s$$

线性相关

则存在不全为零的数 k_1, k_2, \ldots, k_r 使

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_r\alpha_r = 0$$

所以

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_r\alpha_r + 0\alpha_{r+1} + \cdots + 0\alpha_s = 0$$

其中系数不全为零,所以 α_1 , α_2 , ..., α_s 线性相关。

证明

1. "⇒"

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证明

1. "⇒"设 $\alpha_1, \alpha_2, \ldots, \alpha_s$ 线性相关,

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证明

1. " \Rightarrow "设 $\alpha_1, \alpha_2, \ldots, \alpha_s$ 线性相关,存在不全为零 k_1, k_2, \ldots, k_s 使

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_s\alpha_s = 0$$

证明

1. "⇒"设 α_1 , α_2 , ..., α_s 线性相关,存在不全为零 k_1 , k_2 , ..., k_s 使

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_s\alpha_s = 0$$

不妨设 $k_1 \neq 0$,

证明

1. "⇒"设 α_1 , α_2 , ..., α_s 线性相关,存在不全为零 k_1 , k_2 , ..., k_s 使

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_s\alpha_s = 0$$

不妨设
$$k_1 \neq 0$$
,则 $\alpha_1 = -\frac{k_2}{k_1}\alpha_2 - \cdots - \frac{k_s}{k_1}\alpha_s$

证明

1. "⇒"设 α_1 , α_2 , ..., α_s 线性相关,存在不全为零 k_1 , k_2 , ..., k_s 使

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所以 α_1 为 α_2 , ..., α_s 的线性组合

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所以 α_1 为 $\alpha_2, \ldots, \alpha_s$ 的线性组合

2. "←"假设 α_1 为 α_2 ,..., α_s 的线性组合,

证明

1. "⇒"设 α_1 , α_2 , ..., α_s 线性相关,存在不全为零 k_1 , k_2 , ..., k_s 使

$$k_1\alpha_1 + k_2\alpha_2 + \cdots + k_s\alpha_s = 0$$

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所以 α_1 为 α_2 , ..., α_s 的线性组合

2. "←"假设 α_1 为 α_2 , . . . , α_s 的线性组合,

$$\alpha_1 = k_2 \alpha_2 + \dots + k_s \alpha_s$$

证明

1. "⇒"设 α_1 , α_2 , ..., α_s 线性相关,存在不全为零 k_1 , k_2 , ..., k_s 使

$$k_1\alpha_1 + k_2\alpha_2 + \dots + k_s\alpha_s = 0$$

不妨设
$$k_1 \neq 0$$
,则 $\alpha_1 = -\frac{k_2}{k_1}\alpha_2 - \dots - \frac{k_s}{k_1}\alpha_s$

所以 α_1 为 α_2 , ..., α_s 的线性组合

2. "←"假设 α_1 为 α_2 ,..., α_s 的线性组合,

$$\alpha_1 = k_2 \alpha_2 + \dots + k_s \alpha_s$$

$$-\alpha_1 + k_2\alpha_2 + \dots + k_s\alpha_s = 0$$

证明

1. "⇒"设 α_1 , α_2 , ..., α_s 线性相关,存在不全为零 k_1 , k_2 , ..., k_s 使

$$k_1\alpha_1 + k_2\alpha_2 + \dots + k_s\alpha_s = 0$$

不妨设
$$k_1 \neq 0$$
,则 $\alpha_1 = -\frac{k_2}{k_1}\alpha_2 - \dots - \frac{k_s}{k_1}\alpha_s$

所以 α_1 为 $\alpha_2, \ldots, \alpha_s$ 的线性组合

2. "←"假设 α_1 为 α_2 ,..., α_s 的线性组合,

$$\alpha_1 = k_2 \alpha_2 + \dots + k_s \alpha_s$$

$$-\alpha_1 + k_2\alpha_2 + \cdots + k_s\alpha_s = 0$$

且系数不全为零,所以 α_1 , α_2 , ..., α_s 线性相关。

定理 3 设 α_1 , α_2 , . . . , α_s 线性无关,但 α_1 , α_2 , . . . , α_s , β 线性相关,

则 β 可由 α_1 , α_2 , ..., α_s 线性表示,且表示法唯一。

证明

1. 存在不全为零的 k_1 , k_2 , ..., k_s , k 使

$$k_1\alpha_1 + \cdots + k_s\alpha_s + k\beta = 0$$

证明

1. 存在不全为零的 k_1 , k_2 , ..., k_s , k 使

$$k_1\alpha_1 + \cdots + k_s\alpha_s + k\beta = 0 \xrightarrow{\overline{\eta} \exists k \neq 0}$$

证明

1. 存在不全为零的 $k_1, k_2, ..., k_s, k$ 使

$$k_1\alpha_1 + \dots + k_s\alpha_s + k\beta = 0 \xrightarrow{\text{pii} k \neq 0} \beta = -\frac{k_1}{k}\alpha_1 - \dots - \frac{k_s}{k}\alpha_s$$

证明

1. 存在不全为零的 k_1 , k_2 , ..., k_s , k 使

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(证明 $k \neq 0$:

证明

1. 存在不全为零的 $k_1, k_2, ..., k_s, k$ 使

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(证明 $k \neq 0$: 否则 $(k = 0)$

$$k_1\alpha_1 + k_2\alpha_2 + \dots + k_s\alpha_s = 0$$

证明

1. 存在不全为零的 $k_1, k_2, ..., k_s, k$ 使

$$k_1\alpha_1 + \dots + k_s\alpha_s + k\beta = 0$$
 $\xrightarrow{\exists \exists k \neq 0}$ $\beta = -\frac{k_1}{k}\alpha_1 - \dots - \frac{k_s}{k}\alpha_s$
(证明 $k \neq 0$: 否则($k = 0$), k_1, k_2, \dots, k_s 不全为零,且
$$k_1\alpha_1 + k_2\alpha_2 + \dots + k_s\alpha_s = 0$$

证明

1. 存在不全为零的 $k_1, k_2, ..., k_s, k$ 使

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$$k_1\alpha_1 + k_2\alpha_2 + \dots + k_s\alpha_s = 0$$

推出 α_1 , α_2 , ..., α_s 线性相关,矛盾。)

证明

1. 存在不全为零的 k_1 , k_2 , ..., k_s , k 使

$$k_1\alpha_1 + \dots + k_s\alpha_s + k\beta = 0$$
 $\xrightarrow{\overline{\text{qik}} \neq 0} \beta = -\frac{k_1}{k}\alpha_1 - \dots - \frac{k_s}{k}\alpha_s$
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推出 α_1 , α_2 , ..., α_s 线性相关,矛盾。)

2. 设

$$\beta = h_1 \alpha_1 + \dots + h_s \alpha_s$$

$$\beta = l_1 \alpha_1 + \dots + l_s \alpha_s$$

证明

1. 存在不全为零的 $k_1, k_2, ..., k_s, k$ 使

$$k_1\alpha_1 + \dots + k_s\alpha_s + k\beta = 0$$
 $\xrightarrow{\text{可证}k\neq 0}$ $\beta = -\frac{k_1}{k}\alpha_1 - \dots - \frac{k_s}{k}\alpha_s$
(证明 $k \neq 0$:否则($k = 0$), k_1, k_2, \dots, k_s 不全为零,且
 $k_1\alpha_1 + k_2\alpha_2 + \dots + k_s\alpha_s = 0$

推出 α_1 , α_2 , ..., α_s 线性相关,矛盾。)

2. 设

$$\beta = h_1 \alpha_1 + \dots + h_s \alpha_s$$

$$\beta = l_1 \alpha_1 + \dots + l_s \alpha_s$$

$$\beta = l_1 \alpha_1 + \dots + l_s \alpha_s$$

$$(h_1 - l_1) \alpha_1 + \dots + (h_s - l_s) \alpha_s = 0$$

证明

1. 存在不全为零的 k₁, k₂, . . . , k_s, k 使

$$k_1\alpha_1 + \dots + k_s\alpha_s + k\beta = 0$$
 $\xrightarrow{\overline{\text{olik}} \neq 0} \beta = -\frac{k_1}{k}\alpha_1 - \dots - \frac{k_s}{k}\alpha_s$
(证明 $k \neq 0$:否则($k = 0$), k_1, k_2, \dots, k_s 不全为零,且

 $k_1\alpha_1 + k_2\alpha_2 + \cdots + k_s\alpha_s = 0$

推出 $lpha_1$, $lpha_2$, . . . , $lpha_s$ 线性相关,矛盾。)

2. 设

$$\beta = h_1 \alpha_1 + \dots + h_s \alpha_s$$

$$\beta = l_1 \alpha_1 + \dots + l_s \alpha_s$$

$$(h_1 - l_1) \alpha_1 + \dots + (h_s - l_s) \alpha_s = 0$$

由线性无关性, $h_1 = l_1, \ldots, h_s = l_s$ 。

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

若 (B) 可由 (A) 线性表示,且 t > s,

定理 4 两个向量组 (A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

若(B)可由(A)线性表示,且t > s,则向量组(B)线性相关。

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

若 (B) 可由 (A) 线性表示,且 t > s,则向量组 (B) 线性相关。

证明 要找不全为零的 k_1, \dots, k_t 使下式为零:

$$k_1\beta_1 + k_2\beta_2 + \cdots + k_t\beta_t$$

(A):
$$\alpha_1, \alpha_2, \ldots, \alpha_s$$

(B):
$$\beta_1, \beta_2, \ldots, \beta_t$$

若 (B) 可由 (A) 线性表示,且 t > s,则向量组 (B) 线性相关。

证明 要找不全为零的
$$k_1, \dots, k_t$$
 使下式为零:
$$k_1\beta_1 + k_2\beta_2 + \dots + k_t\beta_t = (\beta_1\beta_2 \dots \beta_t) \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_t \end{pmatrix}$$

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

 $(B): \beta_1, \beta_2, \ldots, \beta_t$

若 (B) 可由 (A) 线性表示,且 t > s,则向量组 (B) 线性相关。

证明 要找不全为零的
$$k_1, \dots, k_t$$
 使下式为零:
$$k_1\beta_1 + k_2\beta_2 + \dots + k_t\beta_t = (\beta_1\beta_2 \dots \beta_t) \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_t \end{pmatrix}$$

$$= (\alpha_1 \, \alpha_2 \, \cdots \, \alpha_s) \left(\begin{array}{c} k_1 \\ k_2 \\ \vdots \\ k_t \end{array} \right)$$

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

 $(B): \beta_1, \beta_2, \ldots, \beta_t$

若 (B) 可由 (A) 线性表示,且 t > s,则向量组 (B) 线性相关。

证明 要找不全为零的
$$k_1, \dots, k_t$$
 使下式为零:
$$k_1\beta_1 + k_2\beta_2 + \dots + k_t\beta_t = (\beta_1\beta_2 \dots \beta_t) \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_t \end{pmatrix}$$

$$= (\alpha_1 \, \alpha_2 \, \cdots \, \alpha_s) \begin{pmatrix} \alpha_{11} \\ \alpha_{21} \\ \vdots \\ \alpha_{s1} \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_t \end{pmatrix}$$

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

 $(B): \beta_1, \beta_2, \ldots, \beta_t$

证明 要找不全为零的
$$k_1, \dots, k_t$$
 使下式为零:
$$k_1\beta_1 + k_2\beta_2 + \dots + k_t\beta_t = (\beta_1\beta_2 \dots \beta_t) \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_t \end{pmatrix}$$

$$= (\alpha_1 \alpha_2 \cdots \alpha_s) \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ \vdots & \vdots \\ a_{s1} & a_{s2} \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_t \end{pmatrix}$$

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

 $(B): \beta_1, \beta_2, \ldots, \beta_t$

证明 要找不全为零的
$$k_1, \dots, k_t$$
 使下式为零:
$$k_1\beta_1 + k_2\beta_2 + \dots + k_t\beta_t = (\beta_1\beta_2 \dots \beta_t) \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_t \end{pmatrix}$$

$$= (\alpha_1 \alpha_2 \cdots \alpha_s) \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1t} \\ a_{21} & a_{22} & \cdots & a_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ a_{s1} & a_{s2} & \cdots & a_{st} \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_t \end{pmatrix}$$

 $(A): \alpha_1, \alpha_2, \ldots, \alpha_s$

 $(B): \beta_1, \beta_2, \ldots, \beta_t$

证明 要找不全为零的
$$k_1, \dots, k_t$$
 使下式为零:
$$k_1\beta_1 + k_2\beta_2 + \dots + k_t\beta_t = (\beta_1\beta_2 \dots \beta_t) \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_t \end{pmatrix}$$

$$= (\alpha_1 \alpha_2 \cdots \alpha_s) \underbrace{\begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1t} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{s1} & \alpha_{s2} & \cdots & \alpha_{st} \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_t \end{pmatrix}}_{k}$$

 $(A): \alpha_1, \alpha_2, \ldots, \alpha_s$

 $(B): \beta_1, \beta_2, \ldots, \beta_t$

证明 要找不全为零的
$$k_1, \dots, k_t$$
 使下式为零:
$$k_1\beta_1 + k_2\beta_2 + \dots + k_t\beta_t = (\beta_1\beta_2 \dots \beta_t) \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_t \end{pmatrix}$$

$$= (\alpha_1 \, \alpha_2 \, \cdots \, \alpha_s) \underbrace{\begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1t} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{s1} & \alpha_{s2} & \cdots & \alpha_{st} \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_t \end{pmatrix}}_{k} \quad \therefore r(A) \leq s < t$$

 $(A): \alpha_1, \alpha_2, \ldots, \alpha_s$

 $(B): \beta_1, \beta_2, \ldots, \beta_t$

若 (B) 可由 (A) 线性表示,且 t > s,则向量组 (B) 线性相关。

证明 要找不全为零的
$$k_1, \dots, k_t$$
 使下式为零:
$$k_1\beta_1 + k_2\beta_2 + \dots + k_t\beta_t = (\beta_1\beta_2 \dots \beta_t) \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_t \end{pmatrix}$$

$$= (\alpha_1 \alpha_2 \cdots \alpha_s) \underbrace{\begin{pmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1t} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2t} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{s1} & \alpha_{s2} & \cdots & \alpha_{st} \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_t \end{pmatrix}}_{k} \quad \begin{array}{c} \vdots \\ \text{$r(A) \leq s < t$} \\ \text{c $? c $? c $? $$} \\ \text{$\rlap{$c$ $? c $? $$} $} \\ \text{$\rlap{c $? c $? $$} } \\ \text{$\rlap{$c$ $? c $? $} } \\ \text{$\rlap{c $? c $? $$} } \\ \text{$\rlap{$c$ $$ $\> $$} } \\ \text{$\rlap{$c$ $$ $$} } \\ \text{$\rlap{$c$ $$ $$} } \\ \text{$\rlap{$c$ $$ $$} } \\ \text{$\rlap{$c$ $$} } \\ \text{$\rlap{$c$ $$} } \\ \text{$\rlap{$c$ $$ $$} } \\ \text{$\rlap{$c$ $$} } \\ \text{$\rlap{$$} } \\ \text{$\rlap{$$c$ $$} } \\ \text{$\rlap{$c$ $$} } \\ \text{$\rlap{$$} } \\ \text{$\rlap{$$}$$

线性相关性

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$ 定理 4 两个向量组

 $(B): \beta_1, \beta_2, \ldots, \beta_t$

若 (B) 可由 (A) 线性表示,且 t > s,则向量组 (B) 线性相关。

证明 要找不全为零的
$$k_1, \dots, k_t$$
 使下式为零:
$$k_1\beta_1 + k_2\beta_2 + \dots + k_t\beta_t = (\beta_1\beta_2 \dots \beta_t) \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_t \end{pmatrix}$$

所以向量组 (B) 线性相关。

定理 4" 两个向量组 (A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

假设向量组 (B) 可由 (A) 线性表示,结论:

1. 若 t > s,则向量组 (B) 线性相关。

定理 4" 两个向量组 (A):

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

假设向量组 (B) 可由 (A) 线性表示,结论:

- 1. 若 t > s,则向量组 (B) 线性相关。
- 2. 若向量组 (B) 线性无关,则 $t \le s$ 。

$$\alpha_1, \alpha_2, \ldots, \alpha_s$$

 $\beta_1, \beta_2, \ldots, \beta_t$

定理 4"两个向量组

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

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推论 两个向量组

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

定理 4"两个向量组

 $(A): \alpha_1, \alpha_2, \ldots, \alpha_s$

 $(B): \beta_1, \beta_2, \ldots, \beta_t$

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推论 两个向量组

 $(A): \alpha_1, \alpha_2, \ldots, \alpha_s$

 $(B): \beta_1, \beta_2, \ldots, \beta_t$

如果向量组 (A) 与 (B) 等价,且均线性无关,

定理 4"两个向量组

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

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假设向量组(B)可由(A)线性表示,结论:

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线性相关性 14/14 ▽ △ ▽

定理 4"两个向量组 (A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

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推论 两个向量组 (A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

如果向量组 (A) 与 (B) 等价,且均线性无关,则 s=t。

证明

● (B) 由 (A) 线性表示,且 (B) 线性无关

定理 4"两个向量组 (A): C

(A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

假设向量组(B)可由(A)线性表示,结论:

- 1. 若 t > s,则向量组 (B) 线性相关。
- 2. 若向量组 (B) 线性无关,则 $t \le s$ 。

推论 两个向量组 $(A): \alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

如果向量组 (A) 与 (B) 等价,且均线性无关,则 s=t。

证明

• (B) 由 (A) 线性表示,且(B) 线性无关 ⇒ t ≤ s

定理 4" 两个向量组 (A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

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推论 两个向量组 $(A): \alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

如果向量组 (A) 与 (B) 等价,且均线性无关,则 s=t。

证明

- (B) 由 (A) 线性表示,且 (B) 线性无关 ⇒ t ≤ s
- (A) 由 (B) 线性表示,且(A) 线性无关

定理 4"两个向量组 (A): $\alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

假设向量组 (B) 可由 (A) 线性表示,结论:

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推论 两个向量组 $(A): \alpha_1, \alpha_2, \ldots, \alpha_s$

(B): $\beta_1, \beta_2, \ldots, \beta_t$

如果向量组 (A) 与 (B) 等价,且均线性无关,则 s = t。

证明

- (B) 由 (A) 线性表示,且 (B) 线性无关 ⇒ t ≤ s
- (A)由(B)线性表示,且(A)线性无关⇒s≤t

定理 4" 两个向量组 (A):
$$\alpha_1, \alpha_2, \ldots, \alpha_s$$
 (B): $\beta_1, \beta_2, \ldots, \beta_t$

假设向量组 (B) 可由 (A) 线性表示,结论:

- 1. 若 t > s,则向量组 (B) 线性相关。
- 2. 若向量组 (B) 线性无关,则 $t \leq s$ 。

推论 两个向量组
$$(A)$$
: $\alpha_1, \alpha_2, \ldots, \alpha_s$ (B) : $\beta_1, \beta_2, \ldots, \beta_t$

如果向量组 (A) 与 (B) 等价,且均线性无关,则 s = t。

证明

- (B) 由 (A) 线性表示,且 (B) 线性无关 ⇒ t ≤ s
- (A)由(B)线性表示,且(A)线性无关⇒s≤t

所以s=t