第 9 章 d: 隐函数的求导公式

数学系 梁卓滨

2016-2017 **学年** II



Outline

1. 一个方程的情形

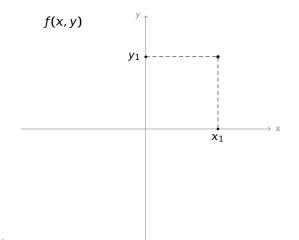
2. 方程组的情形

We are here now...

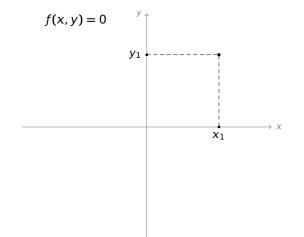
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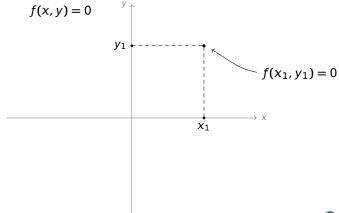
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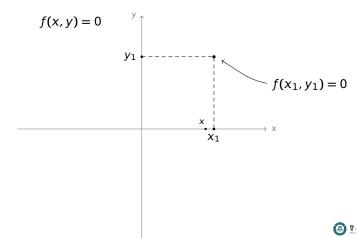
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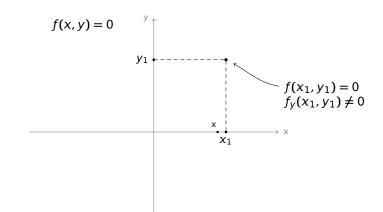
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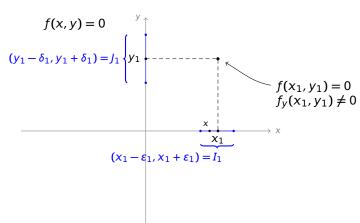


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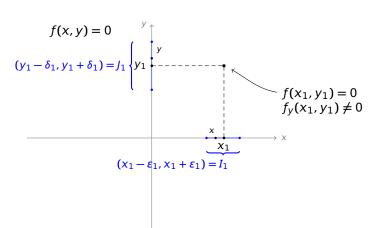


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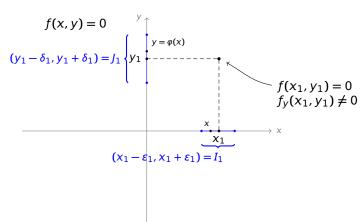




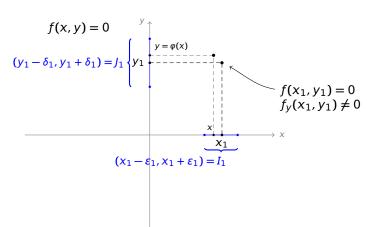
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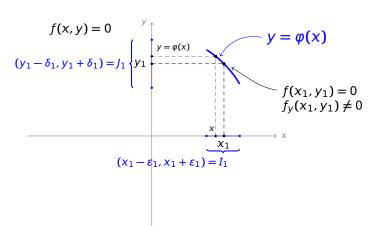
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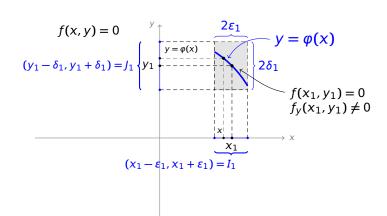
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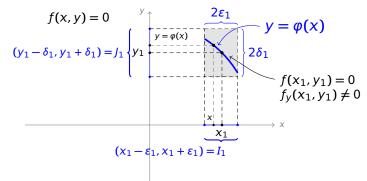
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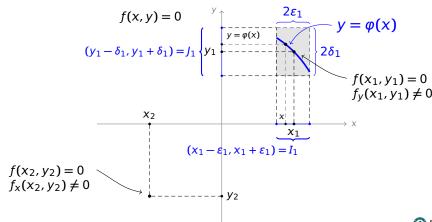
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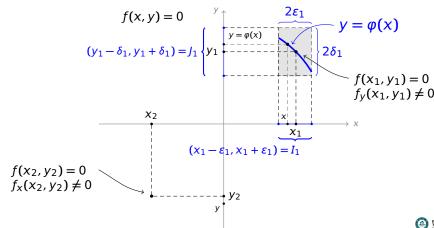
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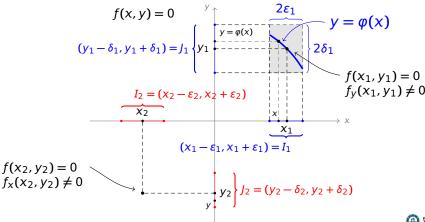
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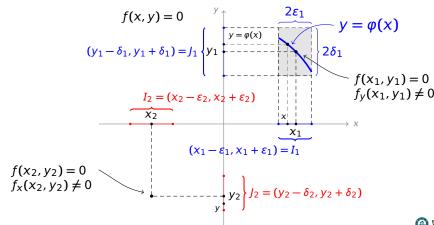


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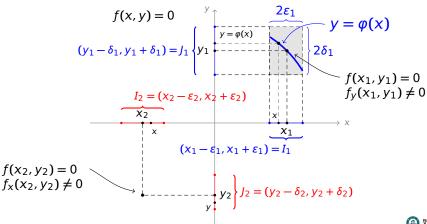
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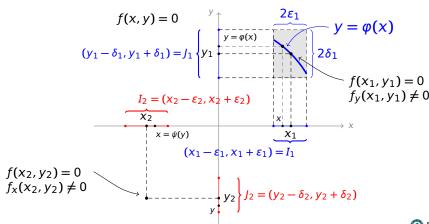
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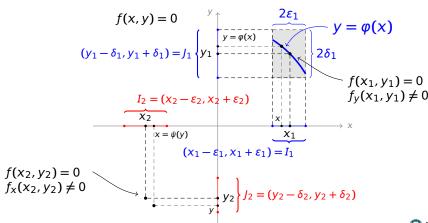


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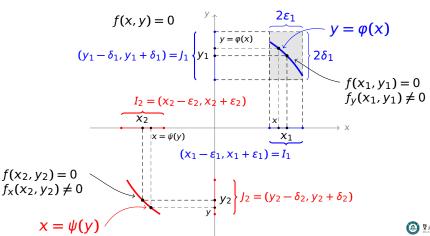


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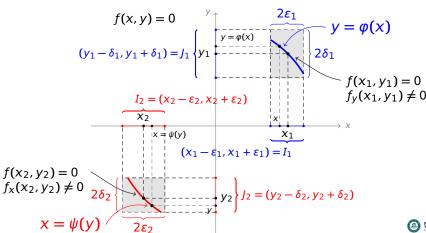
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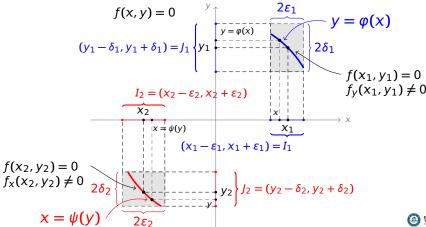
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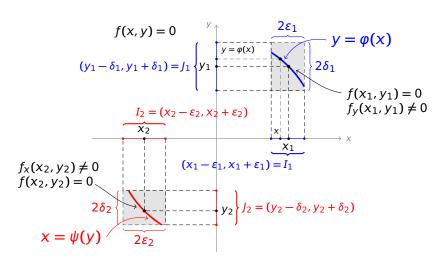
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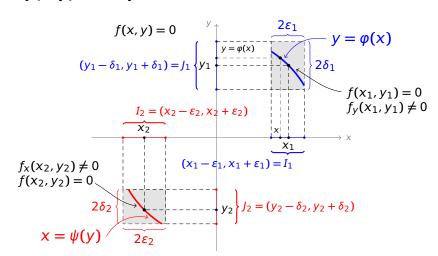


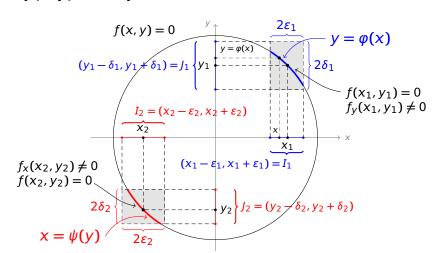
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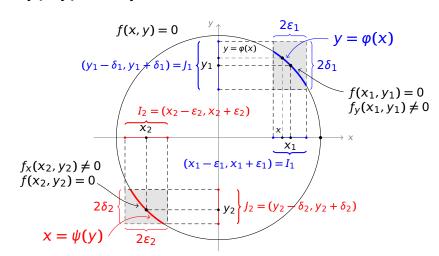
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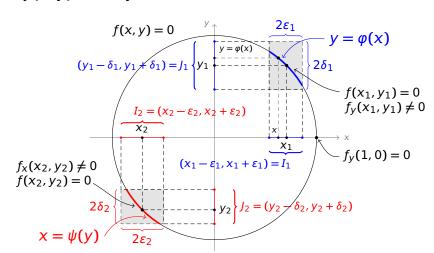


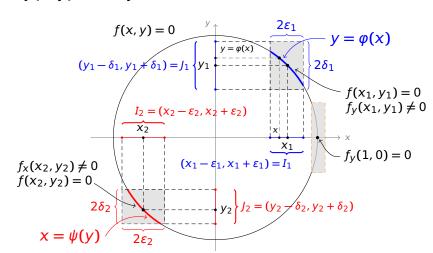


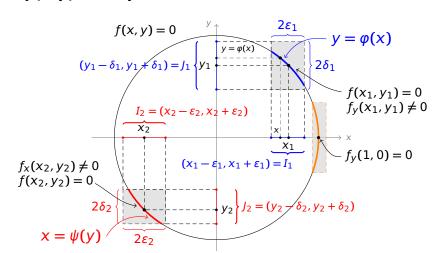


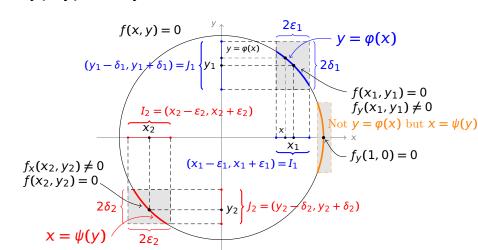


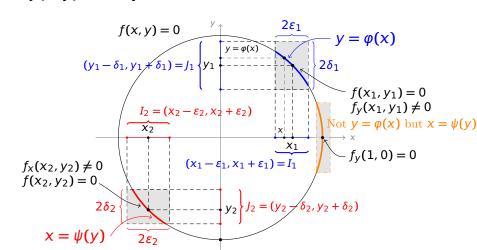


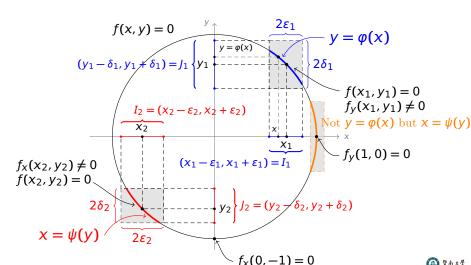




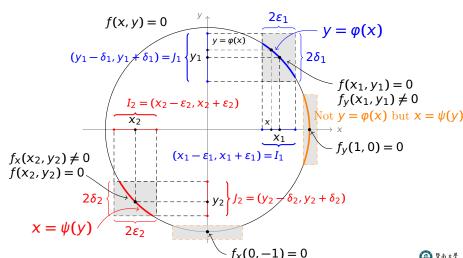






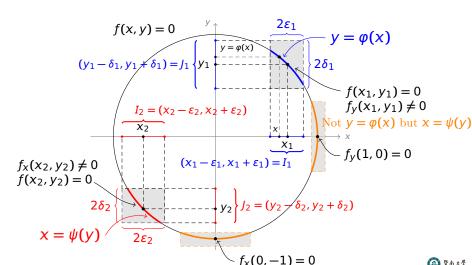


注 隐函数定理中条件 $f_{v}(x_{1},y_{1}) \neq 0$ 和 $f_{x}(x_{2},y_{2}) \neq 0$ 不能去掉,否 则结论可能不成立



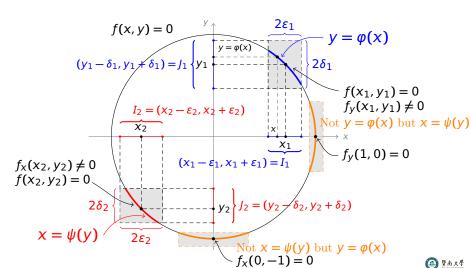
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以 $f(x, y) = x^2 - y^2 - 1$ 为例说明:



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隐函数定理的一个理论应用

性质 设 f(x, y) 具有连续偏导,且 f_x , f_y 处处不同时为零,若 $E = \{(x, y) | f(x, y) = 0\}$

非空,则 E 是平面上"一条曲线"。



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$$f(x, y, z) = 0$$
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$$I_1 = (x_0 - \varepsilon, x_0 + \varepsilon), \quad I_2 = (y_0 - \varepsilon, y_0 + \varepsilon), \quad I = (z_0 - \delta, z_0 + \delta)$$

使得:

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使得:

1. 对任意
$$(x, y) \in I_1 \times I_2$$
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使得:

- 1. 对任意 $(x,y) \in I_1 \times I_2$,方程 f(x,y,z) = 0 有唯一的解 $z = \varphi(x,y) \in J$
- 2. 函数 $z = \varphi(x, y)$ 在区域 $I_1 \times I_2$ 上具有连续导数。

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隐函数的求导法Ⅰ

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隐函数的求导法Ⅰ

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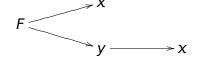
$$\therefore \quad 0 = \frac{d}{dx} F(x, y(x)) =$$

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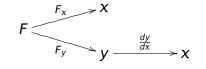


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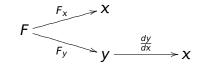


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证明
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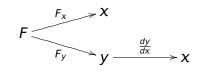


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$$\therefore \quad 0 = \frac{d}{dx} F(x, y(x)) = F_x + F_y \cdot \frac{dy}{dx}$$



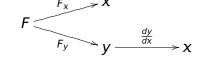
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例 设
$$y = f(x)$$
 满足 $\sin y + e^x = xy^2$,求 $\frac{dy}{dx}$

方法一

$$F(x, y) = 0$$

$$\frac{dy}{dx} = -\frac{r_x}{F_y}$$

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方法一 注意
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$$= -\frac{\frac{2x}{x^2 + y^2} + 3y}{\frac{2y}{x^2 + y^2} + 3x}$$

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公式 设 z = f(x, y) 满足 F(x, y, z) = 0,

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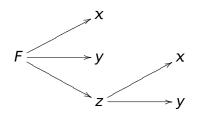
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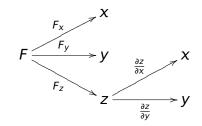


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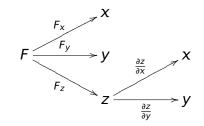


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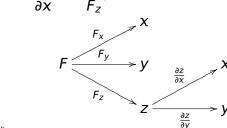
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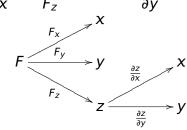
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例 设 z = f(x, y) 满足 $x + y + xz = e^z - 1$, 求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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$$\mathbf{H} \Leftrightarrow F(x, y, z) = x + y + xz - e^z + 1, \quad F(x, y, z) = 0$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

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$$F(x, y, z) = x + y + xz - e^z + 1$$
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$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(x+y+xz-e^z+1)_x'}{(x+y+xz-e^z+1)_z'}$$

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$$= -\frac{0}{0}$$

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$$= -\frac{1}{0+0+x-e^z+0}$$

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$$= -\frac{0+1+0-0+0}{0+0+x-e^z+0} = -\frac{1}{x-e^z}$$



例设 z = f(x, y) 满足 $2\sin(x + 2y - 3z) = x + 2y - 3z$, 求 $\frac{\partial z}{\partial x}$ 和 $\frac{\partial z}{\partial y}$

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F(x, y, z) = 0

$$\frac{\partial Z}{\partial X} = -\frac{F_X}{F_Z} =$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F} =$$

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$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

解
$$\Leftrightarrow F(x, y, z) = 2 \sin(x + 2y - 3z) - x - 2y + 3z$$
, 则 $F(x, y, z) = 0$,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_x'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$
$$= -\frac{(2\sin(x+2y-3z)-x-2y+3z)_z'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

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= ------



$$F(x, y, z) = 0$$
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$$= -\frac{-6\cos(x+2y-3z)}{-6\cos(x+2y-3z)}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$



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$$= -\frac{-6\cos(x+2y-3z)+3}{-6\cos(x+2y-3z)+3}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(2\sin(x+2y-3z)-x-2y+3z)_y'}{(2\sin(x+2y-3z)-x-2y+3z)_z'}$$

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$$F(x, y, z) = 0$$
,所以

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$$= -\frac{2\cos(x+2y-3z)-1}{-6\cos(x+2y-3z)+3}$$

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$$F(x, y, z) = 2 \sin(x + 2y - 3z) - x - 2y + 3z$$
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例 设 z = f(x, y) 满足 $2 \sin(x + 2y - 3z) = x + 2y - 3z$,求 $\frac{\partial z}{\partial y}$ 和 $\frac{\partial z}{\partial y}$

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$$= -\frac{4\cos(x+2y-3z)}{-6\cos(x+2y-3z)+3}$$



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$$= -\frac{4\cos(x+2y-3z)-2}{-6\cos(x+2y-3z)+3}$$



例 设
$$z = f(x, y)$$
 满足 $z - y - x + xe^{z-y-x} = 0$, 求 dz

$$\frac{\partial Z}{\partial X} =$$

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$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} =$$

$$\frac{\partial z}{\partial v} = -\frac{F_y}{F_z} =$$

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例 设
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= -

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{1}{(z - y - x + xe^{z - y - x})_z'}$$

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$$= -\frac{1 + xe^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{1}{2}$$

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$$= -\frac{1}{1 + xe^{z - y - x}}$$

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$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则 $F(x, y, z) = 0$,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$
$$= -\frac{-1 + e^{z - y - x}}{1 + xe^{z - y - x}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_y'}{(z - y - x + xe^{z - y - x})_z'} = -\frac{1}{(z - y - x + xe^{z - y - x})_z'}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



解令
$$F(x, y, z) = z - y - x + xe^{z-y-x}$$
,则 $F(x, y, z) = 0$,所以

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(z - y - x + xe^{z - y - x})_x'}{(z - y - x + xe^{z - y - x})_z'}$$
$$= -\frac{-1 + e^{z - y - x} - xe^{z - y - x}}{1 + xe^{z - y - x}}$$

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$$= -\frac{-1 + e^{z - y - x} - xe^{z - y - x}}{1 + xe^{z - y - x}} = -\frac{1 + (x - 1)e^{z - y - x}}{1 + xe^{z - y - x}}$$

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例 设 z = f(x, y) 满足 $z - y - x + xe^{z-y-x} = 0$,求 dz

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$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$



例 设 z = f(x, y) 满足 $z - y - x + xe^{z-y-x} = 0$,求 dz

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$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy = -\frac{1 + (x - 1)e^{z - y - x}}{1 + xe^{z - y - x}}dx + dy$$



$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

例 设 $\Phi(u, v)$ 具有连续偏导数,函数 z = z(x, y) 满足

$$\Phi(cx - az, cy - bz) = 0$$
, 证明:

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\mathbf{F}(x, y, z) = \Phi(cx - az, cy - bz),$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{\partial z}{\partial y} = \frac{F_y}{F_z} = \frac{F_y}{F_z}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\frac{dz}{dx} = -\frac{F_x}{F_z} = \frac{dz}{dy} = -\frac{F_y}{F_z} = \frac{dz}{dy} = -\frac{F_y}{F_z} = \frac{dz}{dy}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\partial X = \partial Y$$

d: 隐函数的求导公式

$$F_{x} =$$
 $F_{y} =$

$$F_z =$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial y} = -\frac{F_y}{F} =$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

解令
$$F(x, y, z) = \Phi(cx - \alpha z, cy - bz)$$
,则

$$F_X = \Phi_u \cdot u_X + \Phi_v \cdot v_X$$
$$F_y =$$

$$F_y =$$
 $F_z =$

$$\frac{\partial Z}{\partial x} = -\frac{F_X}{F_Z} =$$

$$\frac{F_y}{F_z} =$$



$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\partial x = \partial y$$

解令 $F(x, y, z) = \Phi(cx - \alpha z, cy - bz)$,则

$$F_{X} = \Phi_{u} \cdot u_{X} + \Phi_{v} \cdot v_{X} = c\Phi_{u}$$

$$F_{y} =$$

$$F_y =$$
 $F_z =$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{F_y}{$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\partial x \partial y \partial z$$

解令 $F(x, y, z) = \Phi(cx - az, cy - bz)$,则

$$F_X = \Phi_u \cdot u_X + \Phi_V \cdot V_X = c\Phi_u$$

$$F_V = \Phi_U \cdot u_V + \Phi_V \cdot V_V$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot v_y$$

$$F_{z} = \frac{\partial z}{\partial x} = -\frac{F_{x}}{F_{z}} = \frac{\partial z}{\partial y} = -\frac{F_{y}}{F_{z}} = -\frac{F_{y}}{F_{z}} = \frac{\partial z}{\partial y} = -\frac{F_{y}}{F_{z}} = \frac{\partial z}{\partial y} = -\frac{F_{y}}{F_{z}} = -\frac{F_{y}}{F_{$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\partial x \partial y$$
 解令 $F(x, y, z) = \Phi(cx - az, cy - bz)$,则

$$F_{X} = \Phi_{U} \cdot u_{X} + \Phi_{V} \cdot V_{X} = c\Phi_{U}$$

$$F_{V} = \Phi_{U} \cdot u_{V} + \Phi_{V} \cdot V_{V} = c\Phi_{V}$$

$$F_{y} = \Phi_{u} \cdot u_{y} + \Phi_{v} \cdot v_{y} = c\Phi_{v}$$

$$F_{z} =$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{F_y}{$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$\partial x \partial y$$
 解令 $F(x, y, z) = \Phi(cx - az, cy - bz)$,则

$$F_X = \Phi_u \cdot u_X + \Phi_V \cdot V_X = c\Phi_u$$

$$F_{x} = \Phi_{u} \cdot u_{x} + \Phi_{v} \cdot v_{x} = c\Phi_{u}$$

$$F_{v} = \Phi_{u} \cdot u_{v} + \Phi_{v} \cdot v_{v} = c\Phi_{v}$$

$$\Phi_u \cdot u_y + \Phi_v \cdot v_y = c\Phi_v$$

$$F_z = \Phi_u \cdot u_z + \Phi_V \cdot \nu_z$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{F_$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$F_X = \Phi_u \cdot u_X + \Phi_V \cdot V_X = c\Phi_u$$

$$= \Psi_{\mathcal{U}} \cdot \mathcal{U}_{\mathcal{X}} + \Psi_{\mathcal{V}} \cdot \mathcal{V}_{\mathcal{X}} = \mathcal{C}\Psi_{\mathcal{U}}$$

$$F_y = \Phi_u \cdot u_y + \Phi_v \cdot \nu_y = c\Phi_v$$

$$F_{y} = \Phi_{u} \cdot u_{y} + \Phi_{v} \cdot V_{y} = C\Phi_{v}$$

$$F_{z} = \Phi_{u} \cdot u_{z} + \Phi_{v} \cdot V_{z} = -a\Phi_{u} - b\Phi_{v}$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{\partial z}{\partial x} = -\frac{F_y}{F_z} = -\frac{F_$$



$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

解令 $F(x, y, z) = \Phi(cx - az, cy - bz)$,则 $F_x = \Phi_{U} \cdot u_x + \Phi_{V} \cdot V_x = c\Phi_{U}$

$$F_{y} = \Phi_{u} \cdot u_{y} + \Phi_{v} \cdot v_{y} = c\Phi_{v}$$

$$F_{z} = \Phi_{u} \cdot u_{z} + \Phi_{v} \cdot v_{z} = -\alpha\Phi_{u} - b\Phi_{v}$$

$$\frac{\partial z}{\partial x} = -\frac{F_{x}}{F_{z}} = \frac{c\Phi_{u}}{\alpha\Phi_{u} + b\Phi_{v}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_{y}}{F_{z}} = \frac{c\Phi_{u}}{\sigma\Phi_{v} + \sigma_{v}}$$

$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

$$a \frac{\partial}{\partial x} + b \frac{\partial}{\partial y} = C.$$
解令 $F(x, y, z) = \Phi(cx - az, cy - bz)$,则

$$F_{\mathsf{x}} = \Phi_{\mathsf{u}} \cdot u_{\mathsf{x}} + \Phi_{\mathsf{v}} \cdot \mathsf{v}_{\mathsf{x}} = c\Phi_{\mathsf{u}}$$

$$F_{x} = \Phi_{u} \cdot u_{x} + \Phi_{v} \cdot v_{x} = C\Phi_{u}$$

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$$\frac{\partial z}{\partial x} = -\frac{F_{x}}{F_{z}} = \frac{c\Phi_{u}}{a\Phi_{u} + b\Phi_{v}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_{y}}{F_{z}} = \frac{c\Phi_{v}}{a\Phi_{u} + b\Phi_{v}}$$

$$\frac{\partial Z}{\partial y} =$$



$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

 \mathbf{H} 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$,则 $F_x = \Phi_{II} \cdot u_x + \Phi_{V} \cdot V_x = c\Phi_{II}$

$$F_{y} = \Phi_{u} \cdot u_{y} + \Phi_{v} \cdot v_{y} = c\Phi_{v}$$

$$F_{z} = \Phi_{u} \cdot u_{z} + \Phi_{v} \cdot v_{z} = -\alpha\Phi_{u} - b\Phi_{v}$$

$$\frac{\partial z}{\partial x} = -\frac{F_{x}}{F_{z}} = \frac{c\Phi_{u}}{\alpha\Phi_{u} + b\Phi_{v}}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{c\Phi_v}{a\Phi_u + b\Phi_v}$$
$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = \frac{ac\Phi_u}{a\Phi_u + b\Phi_v} + \frac{bc\Phi_v}{a\Phi_u + b\Phi_v}$$



$$a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = c.$$

 $F_{\mathbf{Y}} = \Phi_{II} \cdot u_{\mathbf{X}} + \Phi_{\mathbf{V}} \cdot \mathbf{V}_{\mathbf{X}} = c\Phi_{II}$

 \mathbf{H} 令 $F(x, y, z) = \Phi(cx - az, cy - bz)$,则

$$F_{y} = \Phi_{u} \cdot u_{y} + \Phi_{v} \cdot v_{y} = c\Phi_{v}$$

$$F_{z} = \Phi_{u} \cdot u_{z} + \Phi_{v} \cdot v_{z} = -a\Phi_{u} - b\Phi_{v}$$

$$\frac{\partial z}{\partial x} = -\frac{F_{x}}{F_{z}} = \frac{c\Phi_{u}}{a\Phi_{u} + b\Phi_{v}}$$

 $\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{c\Phi_V}{a\Phi_U + b\Phi_V}$ $a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = \frac{ac\Phi_u}{a\Phi_u + b\Phi_y} + \frac{bc\Phi_y}{a\Phi_u + b\Phi_y} = c$ 例 设 z = f(x, y) 满足 $z = x + ye^z$, 求 $\frac{\partial^2 z}{\partial x \partial y}$

例 设
$$z = f(x, y)$$
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解
$$F(x, y, z) = x + ye^z - z$$
,则 $F(x, y, z) = 0$

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$$z = f(x, y)$$
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$$F(x, y, z) = x + ye^z - z$$
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$$= \frac{-e^z}{(ye^z - 1)^3} = \frac{e^z}{(1 + x - z)^3}$$

We are here now...

1. 一个方程的情形

2. 方程组的情形

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{22} \\ a_{21}x + a_{22}y = b_2 & (2) \times a_{12} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去 y , 得:

二元线性方程组

$$\begin{cases} a_{11} a_{22} x + a_{12} a_{22} y = a_{22} b_1 & (1) \times a_{22} \\ a_{21} x + a_{22} y = b_2 & (2) \times a_{12} \end{cases}$$

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$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

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, 消去 y , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去 x , 得:

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \times a_{21} \\ a_{21}a_{11}x + a_{22}a_{11}y = a_{11}b_2 & (2) \times a_{11} \end{cases}$$

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去 y , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去 x , 得:

二元线性方程组

$$\begin{cases} a_{11} a_{21} x + a_{12} a_{21} y = a_{21} b_1 & (1) \times a_{21} \\ a_{21} a_{11} x + a_{22} a_{11} y = a_{11} b_2 & (2) \times a_{11} \end{cases}$$

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$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去 x , 得:

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用消元法解:

$$(1) \times a_{22} - (2) \times a_{12}$$
, 消去 y , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{a_{11} a_{12}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

(2) × a_{11} – (1) × a_{21} , 消去 x, 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{a_{11} a_{12}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$



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 $(2) \times a_{11} - (1) \times a_{21}$, 消去 x, 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{a_{11}a_{12}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

二元线性方程组

$$\begin{cases} a_{11}x + a_{12}y = b_1 & (1) \\ a_{21}x + a_{22}y = b_2 & (2) \end{cases}$$

(1)×
$$a_{22}$$
-(2)× a_{12} , 消去 y , 得:

$$x = \frac{b_1 a_{22} - a_{12} b_2}{a_{11} a_{22} - a_{12} a_{21}} = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$(2) \times a_{11} - (1) \times a_{21}$$
, 消去 x , 得:

$$y = \frac{a_{11}b_2 - b_1a_{21}}{a_{11}a_{22} - a_{12}a_{21}} = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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1.
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \qquad , \quad y =$$

2.
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1.
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = -- \qquad , \quad y = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}} = --$$

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$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x =$$

y =

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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2.
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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,
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$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

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2.
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = , y =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1.
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$

2.
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x =$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1.
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$
2.
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - , \quad y = \frac{1}{1} = \frac{$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1.
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$
2.
$$\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - , \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = - \end{cases}$$

第 9 章 d: 隐函数的求导公式

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

1.
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \begin{vmatrix} 0 & 5 \\ 4 & 8 \\ 2 & 5 \\ 3 & 8 \end{vmatrix} = \frac{-20}{1} = -20, \quad y = \begin{vmatrix} 2 & 0 \\ 3 & 4 \\ 2 & 5 \\ 3 & 8 \end{vmatrix} = \frac{8}{1} = 8$$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{}{3} , y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{}{3}$





第 9 章 d: 隐函数的求导公式

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{1}{3} \quad y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{1}{3}$

3.
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases}$$
 $x = \begin{vmatrix} 0 & 5 \\ 4 & 8 \\ \hline{2 & 5} \\ 3 & 8 \end{vmatrix} = \frac{-20}{1} = -20, \quad y = \begin{vmatrix} 2 & 0 \\ 3 & 4 \\ \hline{2 & 5} \\ 3 & 8 \end{vmatrix} = \frac{8}{1} = 8$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}$$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} \quad , y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{3}{3}$

1.
$$\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

练习 利用二阶行列式求解下面二元线性方程组

$$\begin{array}{c}
1. & \begin{cases}
2x + 5 \\
2x + 6
\end{cases}
\end{array}$$

$$\begin{cases}
2x + 5y = 0 \\
3x + 8y = 4
\end{cases}$$



1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$ 2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} , y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3}$

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

1.
$$\begin{cases} 2x + 3y = 0 \\ 3x + 8y = 4 \end{cases} x = \frac{\begin{vmatrix} 4 & 6 \\ 2 & 5 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = \frac{2}{1} = -20, \quad y = \frac{\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 2 & 5 \end{vmatrix}} = \frac{8}{1} = \frac{2}{1} = -20, \quad y = \frac{\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 2 & 5 \end{vmatrix}} = \frac{8}{1} = \frac{2}{1} = -20, \quad y = \frac{\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 2 & 5 \end{vmatrix}} = \frac{8}{1} = \frac{2}{1} = -20, \quad y = \frac{\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & 8 \end{vmatrix}} = \frac{8}{1} = \frac{2}{1} = -20, \quad y = \frac{\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & 8 \end{vmatrix}} = \frac{8}{1} = \frac{8}{1} = \frac{2}{1} = -20, \quad y = \frac{\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & 8 \end{vmatrix}} = \frac{8}{1} = \frac{8}{1} = \frac{1}{1} = -20, \quad y = \frac{\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & 8 \end{vmatrix}} = \frac{8}{1} = \frac{1}{1} = -20, \quad y = \frac{\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & 8 \end{vmatrix}} = \frac{8}{1} = \frac{1}{1} = -20, \quad y = \frac{\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & 8 \end{vmatrix}} = \frac{8}{1} = \frac{1}{1} = -20, \quad y = \frac{\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & 8 \end{vmatrix}} = \frac{8}{1} = \frac{1}{1} = -20, \quad y = \frac{3}{1} = \frac{3}{1} = \frac{1}{1} = -20, \quad y = \frac{3}{1} = \frac{3}{1} = \frac{3}{1} = -20, \quad y = \frac{3}{1} = \frac{3}{1} = \frac{3}{1} = \frac{3}{1} = -20, \quad y = \frac{3}{1} = \frac{3}{$$

练习 利用二阶行列式求解下面二元线性方程组 1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$



$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases} \Rightarrow x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}, y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

2. $\begin{cases} 7x + 16y = 1 \\ 2x + 5y = -1 \end{cases} x = \frac{\begin{vmatrix} 1 & 16 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{21}{3} = 7, \ y = \frac{\begin{vmatrix} 7 & 1 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 7 & 16 \\ 2 & 5 \end{vmatrix}} = \frac{-9}{3} = -3$

1. $\begin{cases} 2x + 5y = 0 \\ 3x + 8y = 4 \end{cases} \quad x = \frac{\begin{vmatrix} 0 & 5 \\ 4 & 8 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{-20}{1} = -20, \quad y = \frac{\begin{vmatrix} 2 & 0 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 2 & 5 \\ 3 & 8 \end{vmatrix}} = \frac{8}{1} = 8$



$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

假设函数
$$u = u(x, y), v = v(x, y)$$
 满足方程组
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

假设函数
$$u = u(x, y), v = v(x, y)$$
 满足方程组
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题:如何计算 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$?

假设函数
$$u = u(x, y), v = v(x, y)$$
 满足方程组
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$?

求解如下:

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \\ G(x, y, u, v) = 0 & \Longrightarrow \end{cases}$$

假设函数
$$u = u(x, y), v = v(x, y)$$
 满足方程组
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

假设函数
$$u = u(x, y), v = v(x, y)$$
 满足方程组
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$?

$$\begin{cases} F(x,y,u,v) = 0 \\ G(x,y,u,v) = 0 \end{cases} \stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

假设函数
$$u = u(x, y), v = v(x, y)$$
 满足方程组
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \\ G(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \end{cases} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

假设函数
$$u = u(x, y), v = v(x, y)$$
 满足方程组
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$?

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xrightarrow{\frac{\partial}{\partial x}} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$



假设函数
$$u = u(x, y), v = v(x, y)$$
 满足方程组
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \\ G(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \end{cases} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

假设函数
$$u = u(x, y), v = v(x, y)$$
 满足方程组
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial X}} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

$$\Rightarrow u_x = \begin{vmatrix} -F_x & F_v \\ -G_x & G_v \end{vmatrix}, \quad v_x = \begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}$$



假设函数
$$u = u(x, y), v = v(x, y)$$
 满足方程组
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题:如何计算 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial x}$?

求解如下:
$$\begin{cases}
F(x, y, u, v) = 0 \\
G(x, y, u, v) = 0
\end{cases} \Rightarrow \begin{cases}
F_u \cdot u_x + F_v \cdot v_x = -F_x \\
G_u \cdot u_x + G_v \cdot v_x = -G_x
\end{cases}$$

$$\Rightarrow u_x = \frac{\begin{vmatrix} -F_x & F_v \\ -G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_x = \frac{\begin{vmatrix} -F_u & F_x \\ -G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

假设函数
$$u = u(x, y), v = v(x, y)$$
 满足方程组
$$\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$$

问题: 如何计算 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$?

$$\begin{cases} F(x, y, u, v) = 0 & \stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

$$\Rightarrow u_x = -\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_x = -\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$



假设函数 u = u(x, y), v = v(x, y) 满足方程组 $\begin{cases} F(x, y, u, v) = 0, \\ G(x, v, u, v) = 0. \end{cases}$

问题:如何计算 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial x}$?

$$\begin{cases} F(x, y, u, v) = 0 & \stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G(x, y, u, v) = 0 \end{cases} \end{cases}$$

$$\Rightarrow u_x = -\frac{\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_x = -\frac{\begin{vmatrix} F_u & F_x \\ G_u & G_x \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$= -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}$$



假设函数 u = u(x, y), v = v(x, y) 满足方程组 $\begin{cases} F(x, y, u, v) = 0, \\ G(x, y, u, v) = 0. \end{cases}$

问题: 如何计算 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$?

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \\ G(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial x}} \end{cases} \begin{cases} F_u \cdot u_x + F_v \cdot v_x = -F_x \\ G_u \cdot u_x + G_v \cdot v_x = -G_x \end{cases}$$

$$\Rightarrow u_{X} = -\frac{\begin{vmatrix} F_{X} & F_{V} \\ G_{X} & G_{V} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{V} \\ G_{u} & G_{V} \end{vmatrix}}, \quad v_{X} = -\frac{\begin{vmatrix} F_{u} & F_{X} \\ G_{u} & G_{X} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{V} \\ G_{u} & G_{V} \end{vmatrix}}$$
$$= -\frac{1}{J} \frac{\partial(F, G)}{\partial(X, V)} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(U, X)} \underbrace{\bigcirc \underbrace{\bullet \bullet \land \bullet}_{22/25 \, \triangleleft P \land A}}_{22/25 \, \triangleleft P \land A} \underbrace{\bigcirc \underbrace{\bullet \bullet \land \bullet}_{22/25 \, \triangleleft P \land A}}_{22/25 \, \triangleleft P \land A}$$

$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \\ G(x, y, u, v) = 0 & \Longrightarrow \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \\ G(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \end{cases} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y =$$
 ——, $v_y =$ ——

$$\begin{cases} F(x, y, u, v) = 0 & \stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = \frac{ }{ \left| \begin{array}{cc} F_u & F_v \\ G_u & G_v \end{array} \right| }, \quad V_y = \frac{ }{ \left| \begin{array}{cc} F_u & F_v \\ G_u & G_v \end{array} \right| }$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = \frac{\begin{vmatrix} -F_y & F_v \\ -G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = \frac{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$



$$\begin{cases} F(x, y, u, v) = 0 & \stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = \begin{vmatrix} -F_y & F_v \\ -G_y & G_v \end{vmatrix}, \quad v_y = \begin{vmatrix} -F_u & F_y \\ -G_u & G_y \end{vmatrix}$$

$$\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$= -\frac{1}{J} \frac{\partial (F, G)}{\partial (y, v)}$$



$$\begin{cases} F(x, y, u, v) = 0 & \xrightarrow{\frac{\partial}{\partial y}} \begin{cases} F_u \cdot u_y + F_v \cdot v_y = -F_y \\ G_u \cdot u_y + G_v \cdot v_y = -G_y \end{cases}$$

$$\Rightarrow u_y = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}, \quad v_y = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}}$$

$$= -\frac{1}{J} \frac{\partial (F, G)}{\partial (y, v)} \qquad = -\frac{1}{J} \frac{\partial (F, G)}{\partial (u, y)}$$



$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

总结 设
$$u = u(x, y), v = v(x, y)$$
 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

$$u_x =$$

$$v_x =$$

$$u_v =$$

$$\nu_{\scriptscriptstyle Y} =$$

总结 设
$$u = u(x, y), v = v(x, y)$$
 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

$$u_x = v_x = v_x$$

$$u_{V} = v_{V} = v_{V}$$

总结 设
$$u = u(x, y), v = v(x, y)$$
 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\xrightarrow{\frac{\partial}{\partial x}} \begin{cases} G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$u_x =$$

$$v_x =$$

$$u_v =$$

$$y' = 1$$

总结 设
$$u = u(x, y), v = v(x, y)$$
 满足方程组

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$
$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

$$u_{\mathsf{x}} =$$
 v

$$u_{v} = v_{v} = v_{v}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

$$\Rightarrow \frac{\frac{\partial}{\partial x}}{\frac{\partial}{\partial y}}$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \Rightarrow \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$
$$\stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$u_v =$$

$$v_y =$$

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

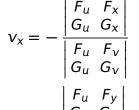
$$\stackrel{\frac{\partial}{\partial X}}{\Longrightarrow} \begin{cases} F_X + F_U \cdot u_X + F_V \cdot V_X = 0 \\ G_X + G_U \cdot u_X + G_V \cdot V_X = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial Y}}{\Longrightarrow} \begin{cases} F_Y + F_U \cdot u_Y + F_V \cdot V_Y = 0 \\ G_Y + G_U \cdot u_Y + G_V \cdot V_Y = 0 \end{cases}$$

所以

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$
$$u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{v} \\ G_{y} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

$$F_{\nu}$$
 G_{ν}
 F_{ν}
 G_{ν}



$$v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$





$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \Rightarrow \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$
$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{y} \\ G_{x} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, y)}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}$$

$$u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{y} \\ G_{y} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}$$

$$v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}$$





$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{y} \\ G_{x} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, y)}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(u, x)}$$

$$u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{y} \\ G_{y} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}$$

$$v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}$$

第 9 章 d: 隐函数的求导公司

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{\longleftrightarrow} \begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)},$$

$$u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{v} \\ G_{y} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)}, \quad v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}}$$

第 9 章 d: 隐函数的求导公



$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases} \begin{cases} F_x + F_u \cdot u_x + F_v \cdot v_x = 0 \\ G_x + G_u \cdot u_x + G_v \cdot v_x = 0 \end{cases}$$

$$\begin{cases} F_y + F_u \cdot u_y + F_v \cdot v_y = 0 \\ G_y + G_u \cdot u_y + G_v \cdot v_y = 0 \end{cases}$$

所以

$$u_{x} = -\frac{\begin{vmatrix} F_{x} & F_{v} \\ G_{x} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(x, v)}, \quad v_{x} = -\frac{\begin{vmatrix} F_{u} & F_{x} \\ G_{u} & G_{x} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)},$$

$$u_{y} = -\frac{\begin{vmatrix} F_{y} & F_{v} \\ G_{y} & G_{v} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(y, v)}, \quad v_{y} = -\frac{\begin{vmatrix} F_{u} & F_{y} \\ G_{u} & G_{y} \end{vmatrix}}{\begin{vmatrix} F_{u} & F_{v} \\ G_{u} & G_{v} \end{vmatrix}} = -\frac{1}{J} \frac{\partial(F, G)}{\partial(u, y)}$$

第 9 章 d: 隐函数的求导公

例设 $\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}, \ \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$

例设
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}$$

$$u_x = v_x = v_x$$

$$u_y = v_y = v_y$$

例设
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}$$

$$u_x = v_x = v_x$$

$$u_y = v_y = v_y$$

例设
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \left\{ (e^u + \sin v) u_x + u \cos v \cdot v_x = 1 \right.$$

$$u_x =$$

$$\nu_{\chi} =$$

$$u_v =$$

$$\nu_{\scriptscriptstyle V} =$$

例设
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\stackrel{\frac{d}{dx}}{\Longrightarrow} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1\\ (e^u - \cos v)u_x + u\sin v \cdot v_x = 0 \end{cases}$$

$$u_x =$$

$$\nu_{x} =$$

$$u_v =$$

$$v_v =$$

例设
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{x} + u\cos v \cdot v_{x} = 1\\ (e^{u} - \cos v)u_{x} + u\sin v \cdot v_{x} = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0 \end{cases}$$

$$u_x =$$

$$\nu_{x} =$$

$$u_v =$$

$$v_v =$$

例设
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{x} + u\cos v \cdot v_{x} = 1\\ (e^{u} - \cos v)u_{x} + u\sin v \cdot v_{x} = 0 \end{cases}$$

$$\stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0\\ (e^{u} - \cos v)u_{y} + u\sin v \cdot v_{y} = 1 \end{cases}$$

$$u_x =$$

$$\nu_{x} =$$

$$u_v =$$

$$v_v =$$

例设
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}$$
, 求 $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$, $\frac{\partial v}{\partial y}$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{=} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

所以
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$u_x = v_x = v_x$$

$$u_y = v_y = v_y$$



例设
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} (e^u + \sin v)u_x + u\cos v \cdot v_x = 1\\ (e^u - \cos v)u_x + u\sin v \cdot v_x = 0 \end{cases}$$

$$\stackrel{\frac{\sigma}{\partial y}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u\sin v \cdot v_{y} = 1 \end{cases}$$

$$u\cos v$$

所以
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$u_{x} = \frac{1}{J}$$

$$u_{y} = \frac{1}{J}$$





例设
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

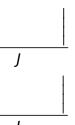
$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{x} + u\cos v \cdot v_{x} = 1\\ (e^{u} - \cos v)u_{x} + u\sin v \cdot v_{x} = 0 \end{cases}$$

所以
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix}$$

$$u_{x} = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$v_x = \frac{1}{v_y}$$





例设
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

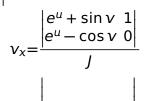
$$= x \qquad \stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{x} + u\cos v \cdot v_{x} = 1\\ (e^{u} - \cos v)u_{x} + u\sin v \cdot v_{x} = 0 \end{cases}$$

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$$u_{x} = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$u_{y} = \frac{\begin{vmatrix} u & u & u \\ 0 & u & u \end{vmatrix}}{J}$$





例设
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

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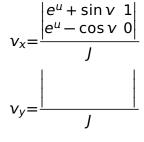
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$$|1 \ u \cos v|$$

$$u_{x} = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$u_{y} = \frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}$$





例设
$$\begin{cases} x = e^{u} + u \sin v \\ y = e^{u} - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

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$$u_{x} = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{J}$$

$$u_{y} = \frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{J}$$

$$v_{x} = \frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{J}$$

$$\begin{vmatrix} e^{u} + \sin v & 0 \end{vmatrix}$$

$$v_x = \frac{\int_{a^u + \sin v \ 0} \left| e^u + \sin v \ 0 \right|}{\int_{a^u + \cos v \ 1} \left| e^u + \cos v \ 1 \right|}$$



例设
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}, \ \ \vec{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases}$$

$$\stackrel{\frac{\partial}{\partial x}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{x} + u\cos v \cdot v_{x} = 1\\ (e^{u} - \cos v)u_{x} + u\sin v \cdot v_{x} = 0 \end{cases}$$

$$e^{u} - u \cos v = y$$

$$\stackrel{\frac{\partial}{\partial y}}{\Longrightarrow} \begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u\sin v \cdot v_{y} = 1 \end{cases}$$

$$u_{x} = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{\int}$$

所以
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$u_x = \begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}$$

$$v_x = \frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{J}$$

$$u_{x} = \frac{\int_{J} u \cos v}{\int u \sin v}$$

$$u_{y} = \frac{\left| u \cos v \right|}{\int u \sin v}$$

$$v_{y} = \frac{\begin{vmatrix} e^{u} + \sin v & 0 \\ e^{u} - \cos v & 1 \end{vmatrix}}{J}$$



例设
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}, \ \ \dot{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases} \begin{cases} (e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\ (e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0 \end{cases}$$
$$\xrightarrow{\frac{\partial}{\partial x}} \begin{cases} (e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1 \end{cases}$$

所以
$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

$$u_{x} = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{\int} = \frac{\begin{vmatrix} u \cos v \\ 0 & u \sin v \end{vmatrix}}{\int} = \frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u(\sin v - \cos v) + 1} \end{vmatrix}}{\int}$$

$$u_{y} = \frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{\int}$$

$$v_{y} = \frac{\begin{vmatrix} e^{u} + \sin v & 0 \\ e^{u} - \cos v & 1 \end{vmatrix}}{\int}$$



例设
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}, \ \ \dot{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases} e^{u} + u \sin v = x \\ e^{u} - u \cos v = y \end{cases} \begin{cases} (e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\ (e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0 \end{cases}$$
$$\xrightarrow{\frac{\partial}{\partial y}} \begin{cases} (e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1 \end{cases}$$

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$$J = \begin{vmatrix} e^u + \sin v & u \cos v \\ e^u - \cos v & u \sin v \end{vmatrix} = ue^u(\sin v - \cos v) + u$$

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$$u_x = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{\int_{-e^u(\sin v - \cos v) + 1}^{e^u(\sin v - \cos v) + 1}}, v_x = \frac{\begin{vmatrix} e^u + \sin v & 1 \\ e^u - \cos v & 0 \end{vmatrix}}{\int_{-e^u(\sin v - \cos v) + 1}^{e^u(\sin v - \cos v) + 1}}$$

$$u_{x} = \frac{\int_{0}^{\infty} u \cos v}{\int_{0}^{\infty} u \sin v}$$

$$v_y = \frac{\begin{vmatrix} e^u + \sin v & 0 \\ e^u - \cos v & 1 \end{vmatrix}}{J}$$



例设
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}, \ \ \dot{x} \ \frac{\partial u}{\partial x}, \ \frac{\partial u}{\partial y}, \ \frac{\partial v}{\partial x}, \ \frac{\partial v}{\partial y} \end{cases}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$\frac{\partial}{\partial y} \begin{cases}
(e^{u} + \sin v)u_{y} + u \cos v \cdot v_{y} = 0 \\
(e^{u} - \cos v)u_{y} + u \sin v \cdot v_{y} = 1
\end{cases}$$

$$\Longrightarrow \begin{cases} (e^{u} + \sin v)u_{y} + u\cos v \cdot v_{y} = 0 \\ (e^{u} - \cos v)u_{y} + u\sin v \cdot v_{y} = 1 \end{cases}$$

$$\iiint J = \begin{vmatrix} e^{u} + \sin v & u\cos v \\ e^{u} - \cos v & u\sin v \end{vmatrix} = ue^{u}(\sin v - \cos v) + u$$

$$u_{x} = \frac{\begin{vmatrix} 1 & u\cos v \\ 0 & u\sin v \end{vmatrix}}{\int} = \frac{\sin v}{e^{u}(\sin v - \cos v) + 1}, v_{x} = \frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{\int} = \frac{-e^{u} + \cos v}{ue^{u}(\sin v - \cos v) + u}$$

$$u_{x} = \frac{|v + \sin v|}{\int} = \frac{\sin v}{e^{u(\sin v - \cos v) + 1}}, v_{x} = \frac{|v + \sin v|}{\int}$$

$$u_{y} = \frac{\left| 0 + \sin v \right|}{\int} = \frac{|e^{u} + \sin v|}{e^{u(\sin v - \cos v) + 1}}, v_{y} = \frac{|e^{u} + \sin v|}{\int}$$



例设
$$\begin{cases} x = e^u + u \sin v \\ y = e^u - u \cos v \end{cases}, \ \vec{x} \ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$$

$$\begin{cases}
e^{u} + u \sin v = x \\
e^{u} - u \cos v = y
\end{cases}
\begin{cases}
(e^{u} + \sin v)u_{x} + u \cos v \cdot v_{x} = 1 \\
(e^{u} - \cos v)u_{x} + u \sin v \cdot v_{x} = 0
\end{cases}$$

$$u_{x} = \frac{\begin{vmatrix} 1 & u \cos v \\ 0 & u \sin v \end{vmatrix}}{\int} = \frac{\begin{vmatrix} \sin v \\ \sin v - \cos v \end{vmatrix}}{e^{u(\sin v - \cos v) + 1}}, v_{x} = \frac{\begin{vmatrix} e^{u} + \sin v & 1 \\ e^{u} - \cos v & 0 \end{vmatrix}}{\int} = \frac{-e^{u + \cos v}}{ue^{u(\sin v - \cos v) + u}}$$

$$u_{y} = \frac{\begin{vmatrix} 0 & u \cos v \\ 1 & u \sin v \end{vmatrix}}{\int} = \frac{-\cos v}{e^{u(\sin v - \cos v) + 1}}, v_{y} = \frac{\begin{vmatrix} e^{u} + \sin v & 0 \\ e^{u} - \cos v & 1 \end{vmatrix}}{\int} = \frac{e^{u + \sin v}}{ue^{u(\sin v - \cos v) + u}}$$