第 12 章 e: 傅里叶级数

数学系 梁卓滨

2016-2017 **学年** II

Outline

1. 傅里叶级数的概念

2. 周期为 2π 的周期函数的傅里叶级数

3. 一般周期函数的傅里叶级数



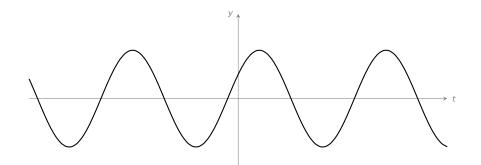
We are here now...

1. 傅里叶级数的概念

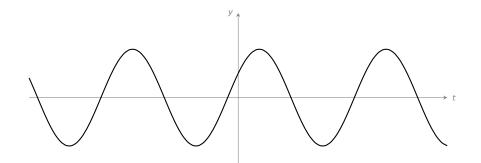
2. 周期为 2π 的周期函数的傅里叶级数

3. 一般周期函数的傅里叶级数

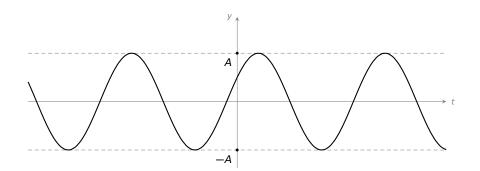
正弦函数 $y = A \sin(\omega t + \varphi)$



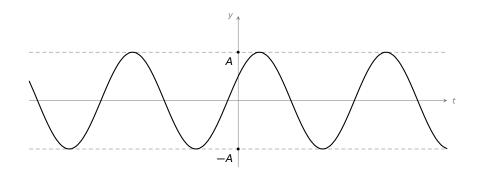
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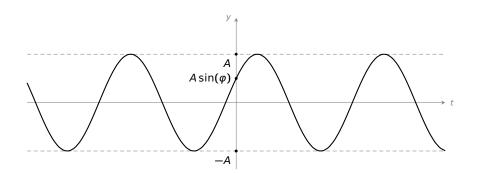
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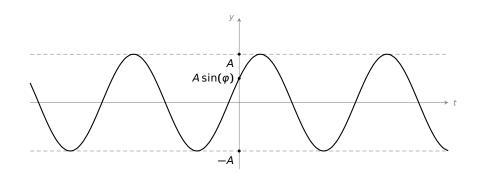
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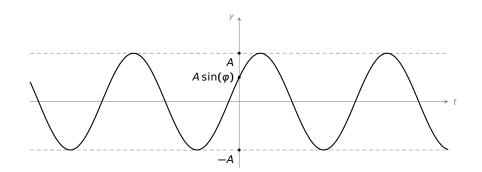


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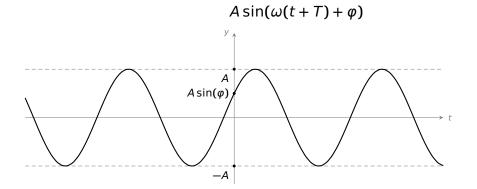
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具有周期 $T = \frac{2\pi}{\omega}$



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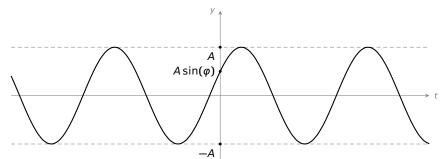




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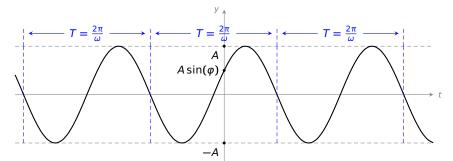




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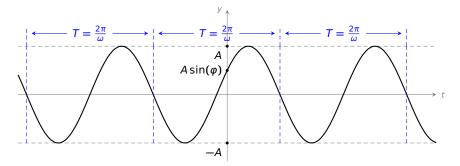




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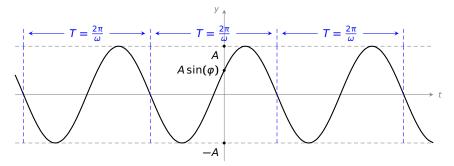
设 n 为正整数,正弦函数 $y = A_n \sin(n\omega t + \varphi_n)$



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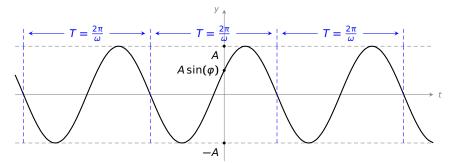
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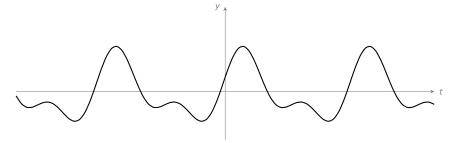
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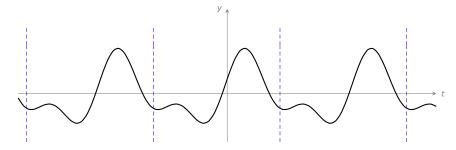


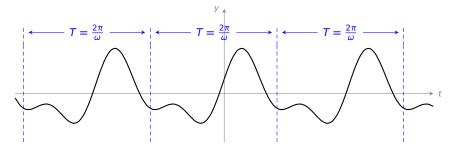
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然 $T = \frac{2\pi}{\alpha}$ 也是周期

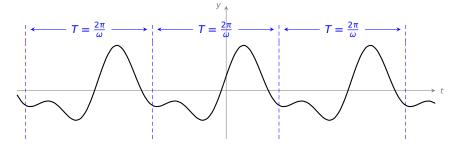








假设 f(t) 是定义域为 \mathbb{R} 的周期函数,周期也是 $T = \frac{2\pi}{\omega}$ 。

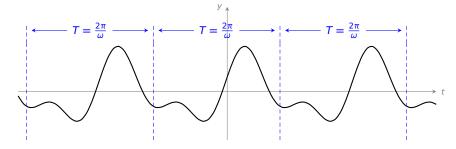


问题 是否有如下展开

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin(n\omega t + \varphi_n)$$



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$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin(n\omega t + \varphi_n)$$

注 在电工学中,上述展开称为谐波分析; A_0 称为直流分量;

 $A_n \sin(n\omega t + \varphi_n)$ 称为 n 次谐波



设 $T = \frac{2\pi}{\omega} = 2l$,

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注意到
$$\omega = \frac{\pi}{l}$$
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$$\sin \varphi_n \cos \frac{n\pi t}{l} + \cos \varphi_n \sin \frac{n\pi t}{l}$$

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注意到
$$\omega = \frac{\pi}{7}$$
,所以

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$$=: a_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l}$$

这时
$$f(t) = A_0 + \sum_{n=0}^{\infty} A_n \sin(n\omega t + \varphi_n)$$



设 $T = \frac{2\pi}{\Omega} = 2l$,故区间 [-l, l] 是 f(t) 的一个完整周期。

注意到
$$\omega = \frac{\pi}{7}$$
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注息到
$$\omega = \overline{l}$$
,则

$$A_n \sin(n\omega t + \varphi_n) = A_n \sin(\frac{n\pi t}{l} + \varphi_n)$$

$$A_n \sin(n\omega t +$$

- $f(t) = A_0 + \sum_{n=0}^{\infty} A_n \sin(n\omega t + \varphi_n) \qquad \sum_{n=0}^{\infty} \left(a_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l} \right)$

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$$=: \alpha_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l}$$

$$=: a_n \cos \frac{mt}{l} + b_n \sin \frac{mt}{l}$$

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$$\omega = \frac{1}{l}$$
, H^{-1}





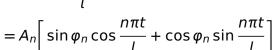
- 以下不妨先设周期 $T = 2\pi (l = \pi)$ 。 f(x) 的周期区间为 $[-\pi, \pi]$,
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- $= A_n \left[\sin \varphi_n \cos \frac{n\pi t}{\iota} + \cos \varphi_n \sin \frac{n\pi t}{\iota} \right]$ $=: a_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l}$

设 $T = \frac{2\pi}{4} = 2l$,故区间 [-l, l] 是 f(t) 的一个完整周期。

注意到 $\omega = \frac{\pi}{7}$,所以

江思到
$$\omega = \overline{l}$$
, H

 $A_n \sin(n\omega t + \varphi_n) = A_n \sin(\frac{n\pi t}{t} + \varphi_n)$



 $f(t) = A_0 + \sum_{n=0}^{\infty} A_n \sin(n\omega t + \varphi_n) = \frac{a_0}{2} + \sum_{n=0}^{\infty} \left(a_n \cos \frac{n\pi t}{l} + b_n \sin \frac{n\pi t}{l} \right)$

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应的展开为

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1. 傅里叶级数的概念

2. 周期为 2π 的周期函数的傅里叶级数

3. 一般周期函数的傅里叶级数

性质 三角函数系

1, $\cos x$, $\sin x$, $\cos 2x$, $\sin 2x$, ..., $\cos nx$, $\sin nx$, ...

在区间 $[-\pi, \pi]$ 上正交。

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$$\int_{-\pi}^{\pi} \cos nx dx = 0, \qquad \int_{-\pi}^{\pi} \sin nx dx = 0 \qquad (n = 1, 2, 3, \dots)$$

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$$\int_{-\pi}^{\pi} \cos nx \, dx = 0, \qquad \int_{-\pi}^{\pi} \sin nx \, dx = 0 \qquad (n = 1, 2, 3, \dots)$$
$$\int_{-\pi}^{\pi} \sin kx \cdot \cos nx \, dx = 0 \qquad (k, n = 1, 2, 3, \dots, k \neq n)$$

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在区间 $[-\pi, \pi]$ 上正交。即上述任意相异两个函数的乘积,在 $[-\pi, \pi]$ 上的积分为零:

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另外

$$\int_{-\pi}^{\pi} \sin^2 nx dx = \int_{-\pi}^{\pi} \cos^2 nx dx = \pi \qquad (n = 1, 2, 3, \dots)$$



$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

则

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \qquad (n = 0, 1, 2, 3, \dots)$$

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"形式推导" (1) 当 $n = 1, 2, 3, \cdots$ 时,

$$\int_{-\pi}^{\pi} f(x) \cos nx dx$$

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$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

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$$= \int_{-\pi}^{\pi} a_n \cos nx \cdot \cos nx dx = \pi a_n$$



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$$n = 1, 2, 3, \cdots$$
 时,

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$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

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$$\int_{-\pi}^{\pi} f(x) \sin nx dx \qquad \left[\frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_k \cos kx + b_k \sin kx \right) \right] \sin nx dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

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$$= \int_{-\pi}^{\pi} \frac{a_0}{2} dx = \pi a_0$$

定义 f(x) 的傅里叶级数定义为

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问题 何时成立
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$
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定理(收敛定理, 狄利克雷充分条件)



- 1. 在一个周期内连续或只有有限个第一类间断点;
- 2. 在一个周期内至多只有有限个极值点,

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当 x 是 f(x) 的连续点时,

• 当 x 是 f(x) 的间断点时,

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• 当 $x \in f(x)$ 的间断点时,

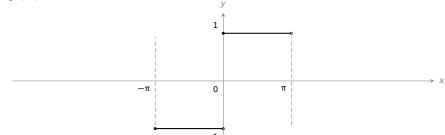
$$\frac{1}{2} \Big[f(x^{-}) + f(x^{+}) \Big] = \frac{a_0}{2} + \sum_{n=1}^{\infty} \Big(a_n \cos nx + b_n \sin nx \Big)$$

$$f(x) = \begin{cases} -1, & -\pi \le x < 0, \\ 1, & 0 \le x < \pi. \end{cases}$$

将 f(x) 展出傅里叶级数。

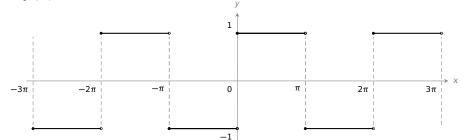
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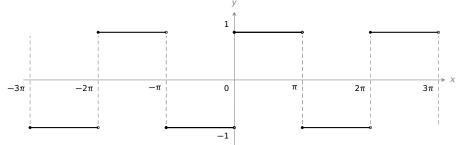
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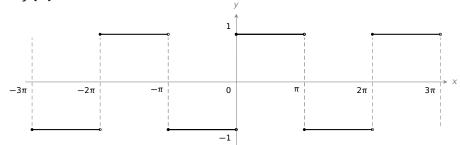
解 计算傅里叶系数如下:

 a_n



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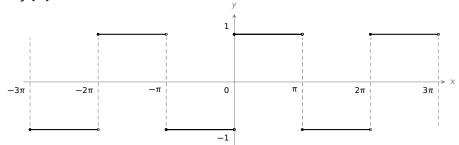
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解 计算傅里叶系数如下:

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{3}} 0$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{3}} 0,$$

 b_n

$$\alpha_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{\pi} \text{ (model}} 0,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{4}} 0,$$

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$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \stackrel{\text{fight}}{=} 0,$$

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$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{6}} 0,$$

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$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \stackrel{\text{fight}}{===} 0,$$

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$$= \left\{ \begin{array}{c} n = 1, 3, 5, \cdots \\ n = 2, 4, 6, \cdots . \end{array} \right.$$



$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\frac{6}{6}} 0,$$

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所以傅里叶级数为

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right) = \sum_{n=1}^{\infty} b_n \sin nx$$



第 12 草 e: 傳里叶级数

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所以傅里叶级数为

 $\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right) = \sum_{n=1}^{\infty} b_n \sin nx$

$$= \frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$$

注 1 f(x) 的傅里叶级数是 $\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$

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$$f(x)$$
 的傅里叶级数是 $\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$, 利用

当 x ≠ nπ 时,

• 当 $x = n\pi$ 是,

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注
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 的傅里叶级数是 $\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$, 利用 收敛定理分析可知:

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(显然,可直接看出当 $x = n\pi$ 时傅里叶级数的值为 0)

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$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right] = \frac{1}{2} \left[f(x^{-}) + f(x^{+}) \right] = 0$$
(显然,可直接看出当 $x = n\pi$ 时傅里叶级数的值为 0)

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$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots$$



注
$$1 f(x)$$
 的傅里叶级数是 $\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$, 利用 收敛定理分析可知:

• 当 $x \neq n\pi$ 时,是 f 的连续点,此时

$$f(x) = \frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$$

• 当 $x = n\pi$ 是,是 f 的间断点,此时

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(显然,可直接看出当 $x = n\pi$ 时傅里叶级数的值为 0)

$$1 - \frac{1}{2} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots = \frac{\pi}{4}$$



• 当 $x \neq n\pi$ 时,是 f 的连续点,此时

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注 1 f(x) 的傅里叶级数是 $\frac{4}{\pi} | \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots |$,利用

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \dots = \frac{\pi}{4}$$

注 3 奇函数 f(x) 的傅里叶级数是 $\sum_{n=1}^{\infty} b_n \sin nx$



$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \cdots \right]$$

$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

$$\sum_{i=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$



$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

考虑部分和

$$\sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$



 3π

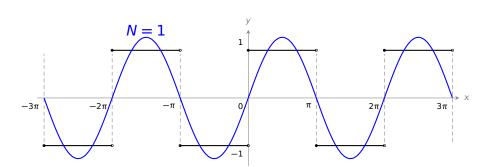
 -3π

 -2π

 2π

$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

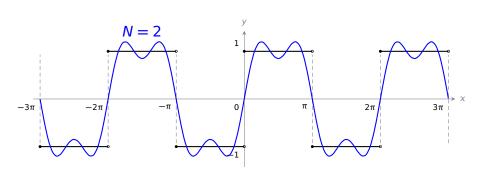
$$\sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$





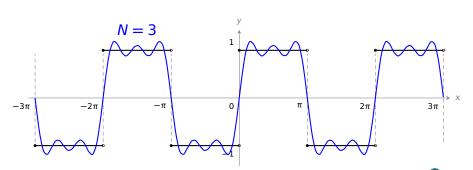
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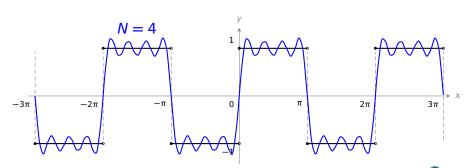
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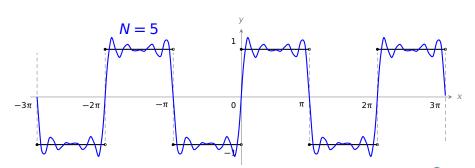
$$\sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$





$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

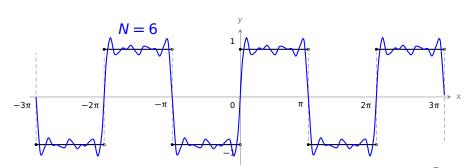
$$\sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$





$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

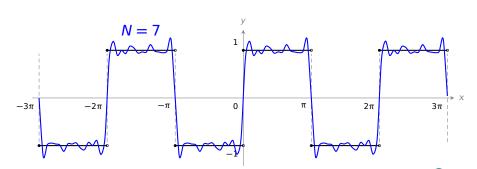
$$\sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$





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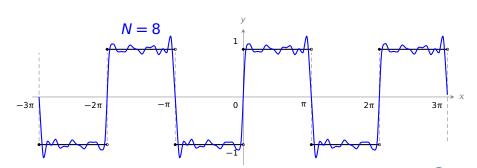
$$\sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$





$$\frac{4}{\pi} \left[\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right] = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin[(2n-1)x]$$

$$\sum_{n=1}^{N} \frac{1}{2n-1} \sin[(2n-1)x]$$

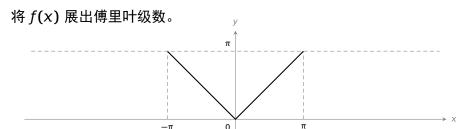




例 设 f(x) 是周期为 2π 的周期函数,在 $[-\pi, \pi)$ 上的表达式为 f(x) = |x|

将 f(x) 展出傅里叶级数。

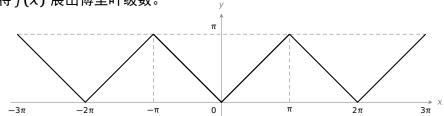
$$f(x) = |x|$$



 $-\pi$

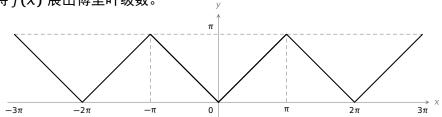
$$f(x) = |x|$$

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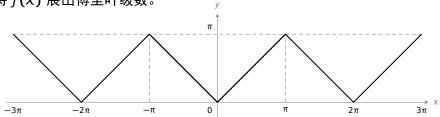


解 计算傅里叶系数如下:

 b_n

$$f(x) = |x|$$

将 f(x) 展出傅里叶级数。



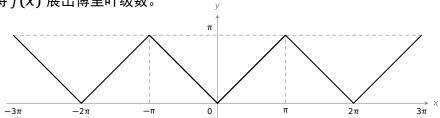
解 计算傅里叶系数如下:

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx$$



$$f(x) = |x|$$

将 f(x) 展出傅里叶级数。



解 计算傅里叶系数如下:

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{§fight}} 0,$$

$$a_n =$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fille } 0},$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{§fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{§fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$= \frac{2}{n\pi} \int_{0}^{\pi} x d\sin nx = \frac{2}{n\pi} \left[x \sin nx \right]_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$= \frac{2}{n\pi} \int_0^{\pi} x d\sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$
$$= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \Big|_0^{\pi} \right]$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$\frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \sin$$

$$= \frac{2}{n\pi} \int_0^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$
$$= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \Big|_0^{\pi} \right] = \frac{2}{n^2 \pi} \left[(-1)^n - 1 \right]$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = -\int_{0}^{\pi} f(x) \cos nx dx = -\int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{\pi} \left[x \sin nx \right]^{\pi} - \left[x \sin nx dx \right]^{\pi}$$

$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \Big|_{0}^{\pi} \right] = \frac{2}{n^{2}\pi} \left[(-1)^{n} - 1 \right] = \begin{cases} n = 1, 3, 5, \dots \\ n = 2, 4, 6, \dots \end{cases}$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$f(x)\cos nx dx = -\int_{-\pi}^{\pi} \int_{0}^{\pi} f(x)\cos nx dx = -\int_{0}^{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= \frac{2}{\pi} \int_{0}^{\pi} x d\sin nx = \frac{2}{\pi} \left[x \sin nx \right]^{\pi} - \int_{0}^{\pi} \sin nx dx$$

$$\pi \int_{-\pi}^{\pi} \pi \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \Big|_{0}^{\pi} \right] = \frac{2}{n^{2}\pi} \left[(-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \dots \\ n = 2, 4, 6, \dots \end{cases}$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$f(x)\cos nx dx = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} f(x)\cos nx dx = \frac{1}{\pi} \int_{0}^{\pi} x\cos nx dx$$
$$= \frac{2}{n\pi} \int_{0}^{\pi} xd\sin nx = \frac{2}{n\pi} \left[x\sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

$$\pi \int_{-\pi}^{\pi} (x) \cos nx dx \qquad \pi \int_{0}^{\pi} (x) \cos nx dx \qquad \pi \int_{0}^{\pi} (x) \cos nx dx$$

$$= \frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

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$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$= \frac{2}{n\pi} \int_0^{\pi} x d\sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$

$$n\pi \int_{0}^{\pi} n\pi \left[\frac{1}{n} \cos nx \right]_{0}^{\pi} = \frac{2}{n^{2}\pi} \left[(-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

 a_0

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$\pi \int_{-\pi}^{\pi} \pi \int_{0}^{\pi} \pi \int$$

$$= \frac{1}{n\pi} \int_0^{\pi} x ds \sin nx = \frac{1}{n\pi} \left[x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$

$$= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \Big|_0^{\pi} \right] = \frac{2}{n^2 \pi} \left[(-1)^n - 1 \right] = \begin{cases} -\frac{4}{n^2 \pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

$$= \frac{1}{n\pi} \int_0^{\pi} f(x) dx$$

$$a_0 = \frac{1}{\pi} \int_0^{\pi} f(x) dx$$

 $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$\int_{-\pi}^{\pi} \frac{1}{\pi} \int_{-\pi}^{\pi} \int_{0}^{\pi} x d\sin nx = \frac{2}{n\pi} \int_{0}^{\pi} \int_{0}^{\pi} x d\sin nx = \frac{2}{n\pi} \left[x \sin nx \right]_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx$$

$$= \frac{2}{n\pi} \left[\frac{1}{n} \cos nx \Big|_{0}^{\pi} \right] = \frac{2}{n^{2}\pi} \left[(-1)^{n} - 1 \right] = \begin{cases} -\frac{4}{n^{2}\pi}, & n = 1, 3, 5, \dots \\ 0, & n = 2, 4, 6, \dots \end{cases}$$

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$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

$$d_n = -\frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = -\frac{1}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = -\frac{1}{\pi} \int_{0}^{\pi} x \cos nx dx$$
$$= -\frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = -\frac{2}{n\pi} \left[x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

$$= \frac{1}{n\pi} \int_{0}^{\pi} x d \sin nx = \frac{1}{n\pi} \left[x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

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$$a_{0} = \frac{1}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} x dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} x dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{§fight}} 0,$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

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$$a_{0} = \frac{1}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} x dx = \frac{2}{\pi} \cdot \frac{1}{2} x^{2} \Big|_{0}^{\pi}$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) dx = \frac{2}{\pi} \int_{0}^{\pi} x dx = \frac{2}{\pi} \cdot \frac{1}{2} x^2 \Big|_{0}^{\pi}$$

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$$\pi J_{-\pi} \qquad \pi J_0 \qquad \pi J_0$$

$$= \frac{2}{n\pi} \int_0^{\pi} x d\sin nx = \frac{2}{n\pi} \left[x \sin nx \Big|_0^{\pi} - \int_0^{\pi} \sin nx dx \right]$$

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$$= -\frac{2}{n\pi} \int_{0}^{\pi} x d \sin nx = -\frac{2}{n\pi} \left[x \sin nx \Big|_{0}^{\pi} - \int_{0}^{\pi} \sin nx dx \right]$$

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所以傅里叶级数为
$$\frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos nx$$



$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \xrightarrow{\text{fight}} 0,$$

$$dx = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx = \frac{2}{\pi} \int_{0}^{\pi} x \cos nx dx$$

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$$\frac{1}{n} \int_{0}^{\pi} f(x) dx = \frac{2}{n^{2}\pi} \int_{0}^{\pi} f(x) dx$$

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 $\frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos nx = \frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$

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$$\dot{x}$$
 2 取 $x = 0$. 可得到

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$$

$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

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 ≥ 2 取 x = 0,可得到

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$$

注 3 偶函数 f(x) 的傅里叶级数是 $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$



$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right]$$

$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$



$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

考虑部分和

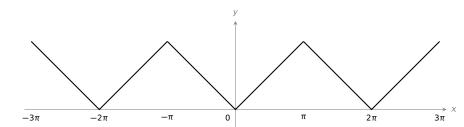
$$\sum_{1}^{N} \frac{\pi}{2} - \frac{4}{\pi} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$



$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

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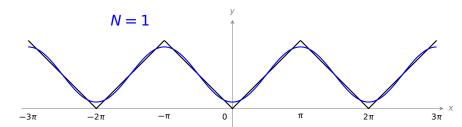




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考虑部分和

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 $\sum_{n=1}^{\infty} \frac{\pi}{2} - \frac{4}{\pi} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$

考虑部分和

$$N=2$$

π



3π

第 12 草 e: 傅里叶级数

 -3π

 -2π

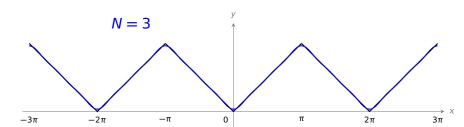
 $-\pi$

2π

$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

考虑部分和

$$\sum_{n=1}^{N} \frac{\pi}{2} - \frac{4}{\pi} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$

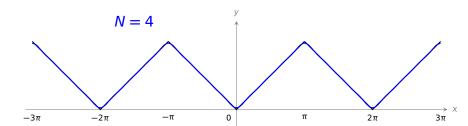




第 12 草 e: 傳里叶级数

$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

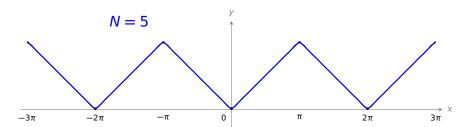
$$\sum_{n=1}^{N} \frac{\pi}{2} - \frac{4}{\pi} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$





$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

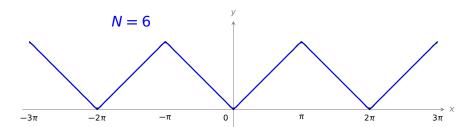
$$\sum_{n=1}^{N} \frac{\pi}{2} - \frac{4}{\pi} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$





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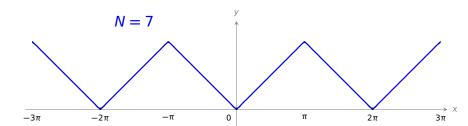




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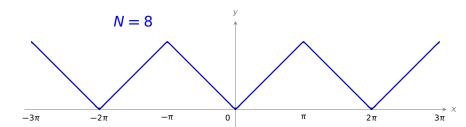




第 12 草 e: 傅里叶级数

$$\frac{\pi}{2} - \frac{4}{\pi} \left[\cos x + \frac{1}{3^2} \cos 3x + \frac{1}{5^2} \cos 5x + \cdots \right] = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$$

$$\sum_{n=1}^{N} \frac{\pi}{2} - \frac{4}{\pi} \frac{1}{(2n-1)^2} \cos[(2n-1)x]$$





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证明(1)假设f为奇函数,则

$$a_n =$$

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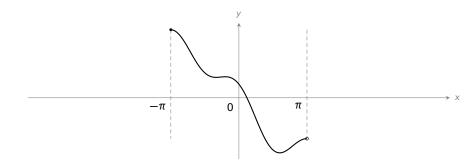
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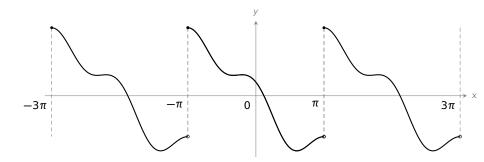
$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \xrightarrow{\text{fight}} \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx$$

设 f(x) 是定义在区间 $[-\pi, \pi]$ (或 $(-\pi, \pi]$)上的函数,可以对其进行周期延拓,从而得到定义在 \mathbb{R} 上的周期函数

设 f(x) 是定义在区间 $[-\pi, \pi)$ (或 $(-\pi, \pi]$)上的函数,可以对其进行周期延拓,从而得到定义在 \mathbb{R} 上的周期函数,如图:

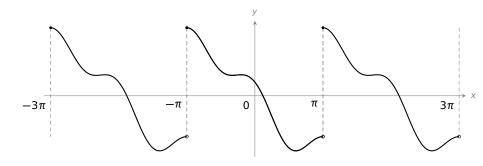


设 f(x) 是定义在区间 $[-\pi, \pi)$ (或 $(-\pi, \pi]$)上的函数,可以对其进行周期延拓,从而得到定义在 \mathbb{R} 上的周期函数,如图:





设 f(x) 是定义在区间 $[-\pi, \pi)$ (或 $(-\pi, \pi]$)上的函数,可以对其进行周期延拓,从而得到定义在 \mathbb{R} 上的周期函数,如图:



延拓后的周期函数任然记为 f(x), 此时可以进行傅里叶展开。



设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得到定义在 \mathbb{R} 上的周期奇函数。

设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得

到定义在 ℝ 上的周期奇函数。

奇延拓步骤:

设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得到定义在 \mathbb{R} 上的周期奇函数。

奇延拓步骤:

定义 f(0) = 0

设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得到定义在 \mathbb{R} 上的周期奇函数。

奇延拓步骤:

• 定义 f(0) = 0; 当 $x \in (-\pi, 0)$ 时, 定义 f(x) = -f(-x);

设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得到定义在 \mathbb{R} 上的周期奇函数。

奇延拓步骤:

• 定义 f(0) = 0; 当 $x \in (-\pi, 0)$ 时,定义 f(x) = -f(-x); (此时 f 在 $(-\pi, \pi]$ 上有定义,且在 $(-\pi, \pi)$ 上为奇函数)

设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得到定义在 \mathbb{R} 上的周期奇函数。

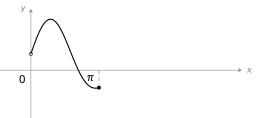
奇延拓步骤:

- 定义 f(0) = 0; 当 $x \in (-\pi, 0)$ 时,定义 f(x) = -f(-x); (此时 f 在 $(-\pi, \pi]$ 上有定义,且在 $(-\pi, \pi)$ 上为奇函数)
- 周期延拓 f 在 $(-\pi, \pi]$ 上的取值。

设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得到定义在 \mathbb{R} 上的周期奇函数。

奇延拓步骤:

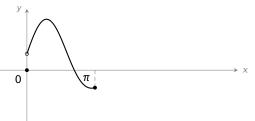
- 定义 f(0) = 0; 当 $x \in (-\pi, 0)$ 时,定义 f(x) = -f(-x); (此时 f 在 $(-\pi, \pi]$ 上有定义,且在 $(-\pi, \pi)$ 上为奇函数)
- 周期延拓 f 在 $(-\pi, \pi]$ 上的取值。



设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得到定义在 \mathbb{R} 上的周期奇函数。

奇延拓步骤:

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- 周期延拓 f 在 $(-\pi, \pi]$ 上的取值。

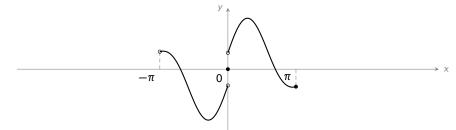


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设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得到定义在 \mathbb{R} 上的周期奇函数。

奇延拓步骤:

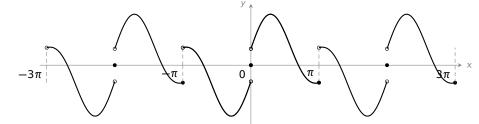
- 定义 f(0) = 0; 当 $x \in (-\pi, 0)$ 时,定义 f(x) = -f(-x); (此时 f 在 $(-\pi, \pi]$ 上有定义,且在 $(-\pi, \pi)$ 上为奇函数)
- 周期延拓 f 在 (-π, π] 上的取值。



设 f(x) 是定义在区间 $(0, \pi]$ 上的函数,可以对其进行奇延拓,从而得到定义在 \mathbb{R} 上的周期奇函数。

奇延拓步骤:

- 定义 f(0) = 0; 当 $x \in (-\pi, 0)$ 时,定义 f(x) = -f(-x); (此时 f 在 $(-\pi, \pi]$ 上有定义,且在 $(-\pi, \pi)$ 上为奇函数)
- 周期延拓 f 在 (-π, π] 上的取值。



设 f(x) 是定义在区间 $[0, \pi]$ 上的函数,可以对其进行偶延拓,从而得到定义在 \mathbb{R} 上的周期偶函数。

偶延拓步骤:

设 f(x) 是定义在区间 $[0, \pi]$ 上的函数,可以对其进行偶延拓,从而得

到定义在 ℝ 上的周期偶函数。

设 f(x) 是定义在区间 $[0, \pi]$ 上的函数,可以对其进行偶延拓,从而得到定义在 \mathbb{R} 上的周期偶函数。

偶延拓步骤:

• 当 $x \in [-\pi, 0]$ 时,定义 f(x) = f(-x);

设 f(x) 是定义在区间 $[0, \pi]$ 上的函数,可以对其进行偶延拓,从而得到定义在 \mathbb{R} 上的周期偶函数。

偶延拓步骤:

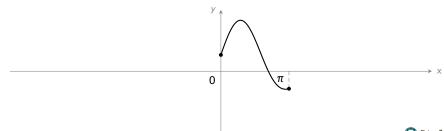
• 当 $x \in [-\pi, 0]$ 时,定义 f(x) = f(-x); (此时 f 成为定义在 $[-\pi, \pi]$ 上为偶函数)

设 f(x) 是定义在区间 $[0, \pi]$ 上的函数,可以对其进行偶延拓,从而得到定义在 \mathbb{R} 上的周期偶函数。

- 当 $x \in [-\pi, 0]$ 时,定义 f(x) = f(-x); (此时 f 成为定义在 $[-\pi, \pi]$ 上为偶函数)
- 周期延拓 f 在 $[-\pi, \pi]$ 上的取值。

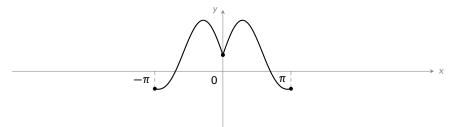
设 f(x) 是定义在区间 $[0, \pi]$ 上的函数,可以对其进行偶延拓,从而得到定义在 \mathbb{R} 上的周期偶函数。

- 当 $x \in [-\pi, 0]$ 时,定义 f(x) = f(-x); (此时 f 成为定义在 $[-\pi, \pi]$ 上为偶函数)
- 周期延拓 f 在 [-π, π] 上的取值。



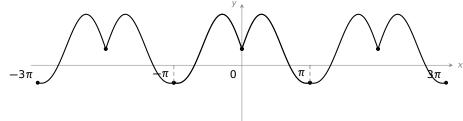
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- 周期延拓 f 在 $[-\pi, \pi]$ 上的取值。



设 f(x) 是定义在区间 $[0, \pi]$ 上的函数,可以对其进行偶延拓,从而得到定义在 \mathbb{R} 上的周期偶函数。

- 当 $x \in [-\pi, 0]$ 时,定义 f(x) = f(-x); (此时 f 成为定义在 $[-\pi, \pi]$ 上为偶函数)
- 周期延拓 f 在 $[-\pi, \pi]$ 上的取值。



We are here now...

1. 傅里叶级数的概念

2. 周期为 2π 的周期函数的傅里叶级数

3. 一般周期函数的傅里叶级数



假设 f(x) 是定义在 \mathbb{R} 上周期函数,周期为 T=2l,

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_{n} = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$1 \int_{-l}^{l} n\pi x$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

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$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导"
$$\Leftrightarrow q(x) = f(\frac{l}{\pi}x),$$



$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导" 令 $g(x) = f(\frac{l}{\pi}x)$, 则 g 是周期为 2π 的周期函数:



$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

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"推导" \Leftrightarrow $g(x) = f(\frac{l}{\pi}x)$, 则 g 是周期为 2π 的周期函数:

$$q(x+2\pi)$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导" $\Leftrightarrow g(x) = f(\frac{l}{\pi}x)$,则 g 是周期为 2π 的周期函数:

$$g(x + 2\pi) = f(\frac{l}{\pi}(x + 2\pi))$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

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"推导" $\Leftrightarrow g(x) = f(\frac{l}{\pi}x)$,则 g 是周期为 2π 的周期函数:

$$g(x+2\pi) = f(\frac{l}{\pi}(x+2\pi)) = f(\frac{l}{\pi}x+2l)$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

"推导" 令 $g(x) = f(\frac{l}{\pi}x)$, 则 g 是周期为 2π 的周期函数:

$$g(x+2\pi) = f(\frac{l}{\pi}(x+2\pi)) = f(\frac{l}{\pi}x+2l) = f(\frac{l}{\pi}x)$$



$$\frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

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"推导" 令 $g(x) = f(\frac{l}{\pi}x)$, 则 g 是周期为 2π 的周期函数: $g(x+2\pi) = f(\frac{l}{\pi}(x+2\pi)) = f(\frac{l}{\pi}x+2l) = f(\frac{l}{\pi}x) = g(x)$

 $g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$

$$b_n = \frac{1}{l} \int_{-l}^{l} f(x)$$

 $b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx$ $(n = 0, 1, 2, 3, \dots)$

$$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx \qquad (n = 0, 1, 2, 3, \dots)$$

其中

 $\frac{a_0}{2} + \sum_{i=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$

假设 f(x) 是定义在 \mathbb{R} 上周期函数,周期为 T=2l,其傅里叶级数应为: $\frac{a_0}{2} + \sum_{i=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$

 $a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx$ $(n = 0, 1, 2, 3, \cdots)$

其中

 $b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx$ $(n = 0, 1, 2, 3, \dots)$ "推导" 令 $g(x) = f(\frac{l}{\pi}x)$, 则 g 是周期为 2π 的周期函数:

 $g(x+2\pi) = f(\frac{l}{\pi}(x+2\pi)) = f(\frac{l}{\pi}x+2l) = f(\frac{l}{\pi}x) = g(x)$ 所以 $f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$



既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$



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其中

 a_n

 b_n



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其中

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz$$

 b_n



既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

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$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

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既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

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$$\underline{x = \frac{l}{\pi}z}$$

$$x = \frac{l}{\pi}z$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz$$



既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

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$$\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos \frac{n\pi x}{l}$$

$$1 \int_{-\pi}^{\pi} f(x) \cos \frac{n\pi x}{l}$$

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$$= \frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz$$



既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$= \frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz$$



既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

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$$\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz$$



既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \sin nz dz$$



既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

$$b_{n} = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \sin nz dz$$



既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$\xrightarrow{x = \frac{l}{\pi}z} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

 $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(-z) \sin nz dz$

$$\stackrel{x=\frac{l}{\pi}z}{=} \frac{1}{\pi} \int f(x) \sin \frac{n\pi x}{l}$$



既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

 $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(-z) \sin nz dz$

$$\frac{x=\frac{l}{\pi}z}{\pi}\frac{1}{\pi}\int f(x)\sin\frac{n\pi x}{l}d(\frac{\pi}{l}x)$$



既然
$$f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)$$

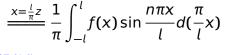
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

其中

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$x = \frac{l}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

 $\frac{x=\frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L} d(\frac{\pi}{L}x) = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L} dx,$ $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(-z) \sin nz dz$





既然 $f(\frac{l}{\pi}x) = g(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$

所以
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \cos nz dz = \frac{1}{\pi} \int_{-\pi}^{\pi} f(\frac{l}{\pi}z) \cos nz dz$$

$$\frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-\pi}^{l} f(x) \cos \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-\pi}^{l} f(x) \cos \frac{n\pi x}{l} dx,$$

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$b = \frac{1}{\pi} \int_{0}^{\pi}$	a(z) sin na	1	\int_{0}^{π}	f(7) cir	n nada

$$b_n = -\frac{1}{\pi} \int_{-\pi}^{\pi} g(z) \sin nz dz = -\frac{1}{\pi} \int_{-\pi}^{\pi} f(-z) \sin nz dz$$

$$= \frac{x = \frac{l}{\pi}z}{\pi} \frac{1}{\pi} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} d(\frac{\pi}{l}x) = \frac{1}{l} \int_{-l}^{l} f(x) \sin \frac{n\pi x}{l} dx.$$

更多内容

傅里叶变换在工程中有许多应用,更多内容可以浏览在"Stanford Engineering Everywhere"中的课程"The Fourier Transform and Its Applications",讲义在 这里。

