

## §8.4 偏导数与全微分

2017-2018 学年 II

# Outline of §8.4

---

1. 二元函数偏导数定义

3. 全微分的定义与计算

# We are here now...

---

## 1. 二元函数偏导数定义

## 3. 全微分的定义与计算

# 偏导数引入

---

- 对一元函数  $y = f(x)$ : 导数  $y' = f'(x) \longleftrightarrow$  变化率

# 偏导数引入

---

- 对一元函数  $y = f(x)$ : 导数  $y' = f'(x) \longleftrightarrow$  变化率
- 对二元函数  $z = f(x, y)$ : 导数?

# 偏导数引入

- 对一元函数  $y = f(x)$ : 导数  $y' = f'(x) \longleftrightarrow$  变化率
- 对二元函数  $z = f(x, y)$ : 导数?
  1. 固定  $y$ , 对  $x$  求导
  2. 固定  $x$ , 对  $y$  求导

# 偏导数引入

- 对一元函数  $y = f(x)$ : 导数  $y' = f'(x) \longleftrightarrow$  变化率
- 对二元函数  $z = f(x, y)$ : 导数?

1. 固定  $y$ , 对  $x$  求导

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \quad \text{对 } x \text{ 偏导数}$$

2. 固定  $x$ , 对  $y$  求导

# 偏导数引入

- 对一元函数  $y = f(x)$ : 导数  $y' = f'(x) \longleftrightarrow$  变化率
- 对二元函数  $z = f(x, y)$ : 导数?

1. 固定  $y$ , 对  $x$  求导

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \quad \text{对 } x \text{ 偏导数}$$

2. 固定  $x$ , 对  $y$  求导

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \quad \text{对 } y \text{ 偏导数}$$



# 偏导数引入

- 对一元函数  $y = f(x)$ : 导数  $y' = f'(x) \longleftrightarrow$  变化率
- 对二元函数  $z = f(x, y)$ : 导数?
  1. 固定  $y$ , 对  $x$  求导:  $z = f(x, y)$  关于  $x$  的变化率

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \quad \text{对} x \text{偏导数}$$

2. 固定  $x$ , 对  $y$  求导

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \quad \text{对} y \text{偏导数}$$

# 偏导数引入

- 对一元函数  $y = f(x)$ : 导数  $y' = f'(x) \longleftrightarrow$  变化率
- 对二元函数  $z = f(x, y)$ : 导数?
  1. 固定  $y$ , 对  $x$  求导:  $z = f(x, y)$  关于  $x$  的变化率

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \quad \text{对} x \text{偏导数}$$

2. 固定  $x$ , 对  $y$  求导:  $z = f(x, y)$  关于  $y$  的变化率

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \quad \text{对} y \text{偏导数}$$

# 偏导数引入

- 对一元函数  $y = f(x)$ : 导数  $y' = f'(x) \longleftrightarrow$  变化率
- 对二元函数  $z = f(x, y)$ : 导数?

1. 固定  $y$ , 对  $x$  求导:  $z = f(x, y)$  关于  $x$  的变化率

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \quad \text{对 } x \text{ 偏导数}$$

2. 固定  $x$ , 对  $y$  求导:  $z = f(x, y)$  关于  $y$  的变化率

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \quad \text{对 } y \text{ 偏导数}$$

例 1 设  $z = f(x, y) = x^2y + 2x + y + 1$ , 则

# 偏导数引入

- 对一元函数  $y = f(x)$ : 导数  $y' = f'(x) \longleftrightarrow$  变化率
- 对二元函数  $z = f(x, y)$ : 导数?
  1. 固定  $y$ , 对  $x$  求导:  $z = f(x, y)$  关于  $x$  的变化率

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \quad \text{对} x \text{偏导数}$$

2. 固定  $x$ , 对  $y$  求导:  $z = f(x, y)$  关于  $y$  的变化率

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \quad \text{对} y \text{偏导数}$$

例 1 设  $z = f(x, y) = x^2y + 2x + y + 1$ , 则

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

# 偏导数引入

- 对一元函数  $y = f(x)$ : 导数  $y' = f'(x) \longleftrightarrow$  变化率
- 对二元函数  $z = f(x, y)$ : 导数?

1. 固定  $y$ , 对  $x$  求导:  $z = f(x, y)$  关于  $x$  的变化率

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \quad \text{对 } x \text{ 偏导数}$$

2. 固定  $x$ , 对  $y$  求导:  $z = f(x, y)$  关于  $y$  的变化率

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \quad \text{对 } y \text{ 偏导数}$$

例 1 设  $z = f(x, y) = x^2y + 2x + y + 1$ , 则

$$\frac{\partial z}{\partial x} = (x^2y + 2x + y + 1)'_x =$$

$$\frac{\partial z}{\partial y} =$$

# 偏导数引入

- 对一元函数  $y = f(x)$ : 导数  $y' = f'(x) \longleftrightarrow$  变化率
- 对二元函数  $z = f(x, y)$ : 导数?

1. 固定  $y$ , 对  $x$  求导:  $z = f(x, y)$  关于  $x$  的变化率

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \quad \text{对} x \text{偏导数}$$

2. 固定  $x$ , 对  $y$  求导:  $z = f(x, y)$  关于  $y$  的变化率

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \quad \text{对} y \text{偏导数}$$

例 1 设  $z = f(x, y) = x^2y + 2x + y + 1$ , 则

$$\frac{\partial z}{\partial x} = (x^2y + 2x + y + 1)'_x = 2xy +$$

$$\frac{\partial z}{\partial y} =$$

# 偏导数引入

- 对一元函数  $y = f(x)$ : 导数  $y' = f'(x) \longleftrightarrow$  变化率
- 对二元函数  $z = f(x, y)$ : 导数?
  1. 固定  $y$ , 对  $x$  求导:  $z = f(x, y)$  关于  $x$  的变化率

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \quad \text{对} x \text{偏导数}$$

2. 固定  $x$ , 对  $y$  求导:  $z = f(x, y)$  关于  $y$  的变化率

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \quad \text{对} y \text{偏导数}$$

例 1 设  $z = f(x, y) = x^2y + 2x + y + 1$ , 则

$$\frac{\partial z}{\partial x} = (x^2y + 2x + y + 1)'_x = 2xy + 2$$

$$\frac{\partial z}{\partial y} =$$

# 偏导数引入

- 对一元函数  $y = f(x)$ : 导数  $y' = f'(x) \longleftrightarrow$  变化率
- 对二元函数  $z = f(x, y)$ : 导数?

1. 固定  $y$ , 对  $x$  求导:  $z = f(x, y)$  关于  $x$  的变化率

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \quad \text{对} x \text{偏导数}$$

2. 固定  $x$ , 对  $y$  求导:  $z = f(x, y)$  关于  $y$  的变化率

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \quad \text{对} y \text{偏导数}$$

例 1 设  $z = f(x, y) = x^2y + 2x + y + 1$ , 则

$$\frac{\partial z}{\partial x} = (x^2y + 2x + y + 1)'_x = 2xy + 2$$

$$\frac{\partial z}{\partial y} = (x^2y + 2x + y + 1)'_y =$$



# 偏导数引入

- 对一元函数  $y = f(x)$ : 导数  $y' = f'(x) \longleftrightarrow$  变化率
- 对二元函数  $z = f(x, y)$ : 导数?

1. 固定  $y$ , 对  $x$  求导:  $z = f(x, y)$  关于  $x$  的变化率

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \quad \text{对} x \text{偏导数}$$

2. 固定  $x$ , 对  $y$  求导:  $z = f(x, y)$  关于  $y$  的变化率

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \quad \text{对} y \text{偏导数}$$

例 1 设  $z = f(x, y) = x^2y + 2x + y + 1$ , 则

$$\frac{\partial z}{\partial x} = (x^2y + 2x + y + 1)'_x = 2xy + 2$$

$$\frac{\partial z}{\partial y} = (x^2y + 2x + y + 1)'_y = x^2 +$$

# 偏导数引入

- 对一元函数  $y = f(x)$ : 导数  $y' = f'(x) \longleftrightarrow$  变化率
- 对二元函数  $z = f(x, y)$ : 导数?

1. 固定  $y$ , 对  $x$  求导:  $z = f(x, y)$  关于  $x$  的变化率

$$\frac{\partial z}{\partial x} \quad \text{或} \quad z'_x \quad \text{或} \quad z_x \quad \text{或} \quad f_x \quad \text{对} x \text{偏导数}$$

2. 固定  $x$ , 对  $y$  求导:  $z = f(x, y)$  关于  $y$  的变化率

$$\frac{\partial z}{\partial y} \quad \text{或} \quad z'_y \quad \text{或} \quad z_y \quad \text{或} \quad f_y \quad \text{对} y \text{偏导数}$$

例 1 设  $z = f(x, y) = x^2y + 2x + y + 1$ , 则

$$\frac{\partial z}{\partial x} = (x^2y + 2x + y + 1)'_x = 2xy + 2$$

$$\frac{\partial z}{\partial y} = (x^2y + 2x + y + 1)'_y = x^2 + 1$$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x =$$

$$\frac{\partial z}{\partial y} =$$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x =$$
$$\frac{\partial z}{\partial y} =$$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} +$$

$$\frac{\partial z}{\partial y} =$$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} =$$



例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y =$$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y =$$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} +$$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

---

例 3 设  $z = f(x, y) = 2y \sin(3x)$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

---

例 3 设  $z = f(x, y) = 2y \sin(3x)$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial y}$$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

---

例 3 设  $z = f(x, y) = 2y \sin(3x)$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y \sin(3x))'_x =$$

$$\frac{\partial z}{\partial y}$$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设  $z = f(x, y) = 2y \sin(3x)$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y \sin(3x))'_x = 2y(\sin(3x))'_x =$$

$$\frac{\partial z}{\partial y}$$



例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设  $z = f(x, y) = 2y \sin(3x)$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y \sin(3x))'_x = 2y(\sin(3x))'_x = 2y \cdot 3 \cos(3x) =$$

$$\frac{\partial z}{\partial y}$$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

---

例 3 设  $z = f(x, y) = 2y \sin(3x)$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y \sin(3x))'_x = 2y(\sin(3x))'_x = 2y \cdot 3 \cos(3x) = 6y \cos(3x)$$

$$\frac{\partial z}{\partial y}$$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设  $z = f(x, y) = 2y \sin(3x)$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y \sin(3x))'_x = 2y(\sin(3x))'_x = 2y \cdot 3 \cos(3x) = 6y \cos(3x)$$

$$\frac{\partial z}{\partial y} = (2y \sin(3x))'_y =$$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设  $z = f(x, y) = 2y \sin(3x)$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y \sin(3x))'_x = 2y(\sin(3x))'_x = 2y \cdot 3 \cos(3x) = 6y \cos(3x)$$

$$\frac{\partial z}{\partial y} = (2y \sin(3x))'_y = (2y)'_y \cdot \sin(3x) =$$

例 2 设  $z = f(x, y) = e^{xy} + 2xy^2$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$\frac{\partial z}{\partial y} = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

例 3 设  $z = f(x, y) = 2y \sin(3x)$ , 求  $\frac{\partial z}{\partial x}$  和  $\frac{\partial z}{\partial y}$

解

$$\frac{\partial z}{\partial x} = (2y \sin(3x))'_x = 2y(\sin(3x))'_x = 2y \cdot 3 \cos(3x) = 6y \cos(3x)$$

$$\frac{\partial z}{\partial y} = (2y \sin(3x))'_y = (2y)'_y \cdot \sin(3x) = 2 \sin(3x)$$

例 4 求三元函数  $u = xyz + \frac{z}{x}$  的全部一阶偏导数

例 4 求三元函数  $u = xyz + \frac{z}{x}$  的全部一阶偏导数

解

$$u_x =$$

$$u_y =$$

$$u_z =$$

例 4 求三元函数  $u = xyz + \frac{z}{x}$  的全部一阶偏导数

解 
$$u_x = (xyz + \frac{z}{x})'_x =$$

$$u_y =$$

$$u_z =$$



例 4 求三元函数  $u = xyz + \frac{z}{x}$  的全部一阶偏导数

解 
$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x =$$

$$u_y =$$

$$u_z =$$

例 4 求三元函数  $u = xyz + \frac{z}{x}$  的全部一阶偏导数

解 
$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz$$

$$u_y =$$

$$u_z =$$

例 4 求三元函数  $u = xyz + \frac{z}{x}$  的全部一阶偏导数

解 
$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y =$$

$$u_z =$$

例 4 求三元函数  $u = xyz + \frac{z}{x}$  的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y =$$

$$u_z =$$

例 4 求三元函数  $u = xyz + \frac{z}{x}$  的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y =$$

$$u_z =$$

例 4 求三元函数  $u = xyz + \frac{z}{x}$  的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z =$$

例 4 求三元函数  $u = xyz + \frac{z}{x}$  的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z =$$

例 4 求三元函数  $u = xyz + \frac{z}{x}$  的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z =$$



例 4 求三元函数  $u = xyz + \frac{z}{x}$  的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z = xy$$

例 4 求三元函数  $u = xyz + \frac{z}{x}$  的全部一阶偏导数

解

$$u_x = (xyz + \frac{z}{x})'_x = (xyz)'_x + (\frac{z}{x})'_x = yz - \frac{z}{x^2}$$

$$u_y = (xyz + \frac{z}{x})'_y = (xyz)'_y + (\frac{z}{x})'_y = xz$$

$$u_z = (xyz + \frac{z}{x})'_z = (xyz)'_z + (\frac{z}{x})'_z = xy + \frac{1}{x}$$

# 偏导数的极限定义

---

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为:

$$f'(x_0) =$$

# 偏导数的极限定义

---

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为：

$$f'(x_0) = \lim \text{—————}$$

# 偏导数的极限定义

---

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为:

$$f'(x_0) = \lim \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

# 偏导数的极限定义

---

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为:

$$f'(x_0) = \lim_{\Delta x} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为：

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为：

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $x$  的偏导数：

$$\frac{\partial f}{\partial x}(x_0, y_0) =$$



# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为：

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $x$  的偏导数：

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim \frac{\quad}{\quad}$$

# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为:

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $x$  的偏导数:

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$

# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为：

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $x$  的偏导数：

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$

# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为：

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $x$  的偏导数：

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x}$$

# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为:

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $x$  的偏导数:

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} = f'(x, y_0)$$

# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为：

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $x$  的偏导数：

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} = \frac{d}{dx} [f(x, y_0)]$$

# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为:

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $x$  的偏导数:

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} = \frac{d}{dx} \left[ f(x, y_0) \right] \Big|_{x=x_0}$$

# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为：

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $x$  的偏导数：

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} = \frac{d}{dx} \left[ f(x, y_0) \right] \Big|_{x=x_0}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $y$  的偏导数：

$$\frac{\partial f}{\partial y}(x_0, y_0) =$$



# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为：

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $x$  的偏导数：

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} = \frac{d}{dx} \left[ f(x, y_0) \right] \Big|_{x=x_0}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $y$  的偏导数：

$$\frac{\partial f}{\partial y}(x_0, y_0) = \lim \frac{\quad}{\quad}$$

# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为：

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $x$  的偏导数：

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} = \frac{d}{dx} \left[ f(x, y_0) \right] \Big|_{x=x_0}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $y$  的偏导数：

$$\frac{\partial f}{\partial y}(x_0, y_0) = \lim_{\Delta y \rightarrow 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$$

# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为:

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $x$  的偏导数:

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} = \frac{d}{dx} \left[ f(x, y_0) \right] \Big|_{x=x_0}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $y$  的偏导数:

$$\frac{\partial f}{\partial y}(x_0, y_0) = \lim_{\Delta y \rightarrow 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$$

# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为：

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $x$  的偏导数：

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} = \frac{d}{dx} \left[ f(x, y_0) \right] \Big|_{x=x_0}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $y$  的偏导数：

$$\frac{\partial f}{\partial y}(x_0, y_0) = \lim_{\Delta y \rightarrow 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}$$

# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为:

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $x$  的偏导数:

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} = \frac{d}{dx} \left[ f(x, y_0) \right] \Big|_{x=x_0}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $y$  的偏导数:

$$\frac{\partial f}{\partial y}(x_0, y_0) = \lim_{\Delta y \rightarrow 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y} = \frac{d}{dy} f(x_0, y)$$

# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为:

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $x$  的偏导数:

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} = \frac{d}{dx} \left[ f(x, y_0) \right] \Big|_{x=x_0}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $y$  的偏导数:

$$\frac{\partial f}{\partial y}(x_0, y_0) = \lim_{\Delta y \rightarrow 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y} = \frac{d}{dy} \left[ f(x_0, y) \right]$$

# 偏导数的极限定义

- 一元函数  $y = f(x)$  在  $x = x_0$  处的导数定义为:

$$f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $x$  的偏导数:

$$\frac{\partial f}{\partial x}(x_0, y_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x, y_0) - f(x_0, y_0)}{\Delta x} = \frac{d}{dx} \left[ f(x, y_0) \right] \Big|_{x=x_0}$$

- $z = f(x, y)$  在点  $(x_0, y_0)$  处关于  $y$  的偏导数:

$$\frac{\partial f}{\partial y}(x_0, y_0) = \lim_{\Delta y \rightarrow 0} \frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y} = \frac{d}{dy} \left[ f(x_0, y) \right] \Big|_{y=y_0}$$

注 求偏导数的值  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$  有两种方式:



注 求偏导数的值  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$  有两种方式:

- 先求出  $f(x, y)$  的偏导数  $f_x(x, y)$  和  $f_y(x, y)$  的一般形式,

注 求偏导数的值  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$  有两种方式:

- 先求出  $f(x, y)$  的偏导数  $f_x(x, y)$  和  $f_y(x, y)$  的一般形式, 然后赋值求出  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$ 。

注 求偏导数的值  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$  有两种方式:

- 先求出  $f(x, y)$  的偏导数  $f_x(x, y)$  和  $f_y(x, y)$  的一般形式, 然后赋值求出  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$ 。

- $$\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}$$

$$\frac{\partial f}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

注 求偏导数的值  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$  有两种方式:

- 先求出  $f(x, y)$  的偏导数  $f_x(x, y)$  和  $f_y(x, y)$  的一般形式, 然后赋值求出  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$ 。

- $$\frac{\partial f}{\partial x}(x_0, y_0) = f_x(x_0, y_0)$$

$$\frac{\partial f}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

**注** 求偏导数的值  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$  有两种方式:

- 先求出  $f(x, y)$  的偏导数  $f_x(x, y)$  和  $f_y(x, y)$  的一般形式, 然后赋值求出  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$ 。

- $$\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)]$$

$$\frac{\partial f}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

注 求偏导数的值  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$  有两种方式:

- 先求出  $f(x, y)$  的偏导数  $f_x(x, y)$  和  $f_y(x, y)$  的一般形式, 然后赋值求出  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$ 。

- $$\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}$$

$$\frac{\partial f}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

注 求偏导数的值  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$  有两种方式:

- 先求出  $f(x, y)$  的偏导数  $f_x(x, y)$  和  $f_y(x, y)$  的一般形式, 然后赋值求出  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$ 。

- $$\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}$$

$$\frac{\partial f}{\partial y}(x_0, y_0) = f_y(x_0, y_0)$$

注 求偏导数的值  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$  有两种方式:

- 先求出  $f(x, y)$  的偏导数  $f_x(x, y)$  和  $f_y(x, y)$  的一般形式, 然后赋值求出  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$ 。

- $$\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}$$

$$\frac{\partial f}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)]$$



注 求偏导数的值  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$  有两种方式:

- 先求出  $f(x, y)$  的偏导数  $f_x(x, y)$  和  $f_y(x, y)$  的一般形式, 然后赋值求出  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$ 。

- $$\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}$$

$$\frac{\partial f}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

**注** 求偏导数的值  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$  有两种方式:

- 先求出  $f(x, y)$  的偏导数  $f_x(x, y)$  和  $f_y(x, y)$  的一般形式, 然后赋值求出  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$ 。
- $$\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}$$
$$\frac{\partial f}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

(先对无关的变量赋值, 然后求导, 最后对求导的变量赋值)

**注** 求偏导数的值  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$  有两种方式:

- 先求出  $f(x, y)$  的偏导数  $f_x(x, y)$  和  $f_y(x, y)$  的一般形式, 然后赋值求出  $\frac{\partial f}{\partial x}(x_0, y_0)$  和  $\frac{\partial f}{\partial y}(x_0, y_0)$ 。

- $$\frac{\partial f}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}$$

$$\frac{\partial f}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

(先对无关的变量赋值, 然后求导, 最后对求导的变量赋值)

两种方式各有优点, 要灵活运用

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法一

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法一

$$\frac{\partial z}{\partial x} =$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left. \frac{\partial z}{\partial x} \right|_{\substack{x=2 \\ y=1}} =$$

$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=2 \\ y=1}} =$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点 (2, 1) 处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x =$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left. \frac{\partial z}{\partial x} \right|_{\substack{x=2 \\ y=1}} =$$

$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=2 \\ y=1}} =$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x =$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left. \frac{\partial z}{\partial x} \right|_{\substack{x=2 \\ y=1}} =$$

$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=2 \\ y=1}} =$$



例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y +$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left. \frac{\partial z}{\partial x} \right|_{\substack{x=2 \\ y=1}} =$$

$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=2 \\ y=1}} =$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left. \frac{\partial z}{\partial x} \right|_{\substack{x=2 \\ y=1}} =$$

$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=2 \\ y=1}} =$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left. \frac{\partial z}{\partial x} \right|_{\substack{x=2 \\ y=1}} = \left( y + \frac{1}{y} \right) \bigg|_{\substack{x=2 \\ y=1}} =$$

$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=2 \\ y=1}} =$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\frac{\partial z}{\partial x} \Big|_{\substack{x=2 \\ y=1}} = (y + \frac{1}{y}) \Big|_{\substack{x=2 \\ y=1}} = 1 + \frac{1}{1} =$$

$$\frac{\partial z}{\partial y} \Big|_{\substack{x=2 \\ y=1}} =$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} =$$

所以

$$\left. \frac{\partial z}{\partial x} \right|_{\substack{x=2 \\ y=1}} = \left( y + \frac{1}{y} \right) \bigg|_{\substack{x=2 \\ y=1}} = 1 + \frac{1}{1} = 2$$

$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=2 \\ y=1}} =$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y =$$

所以

$$\left. \frac{\partial z}{\partial x} \right|_{\substack{x=2 \\ y=1}} = \left( y + \frac{1}{y} \right) \bigg|_{\substack{x=2 \\ y=1}} = 1 + \frac{1}{1} = 2$$

$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=2 \\ y=1}} =$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y =$$

所以

$$\left. \frac{\partial z}{\partial x} \right|_{\substack{x=2 \\ y=1}} = \left( y + \frac{1}{y} \right) \bigg|_{\substack{x=2 \\ y=1}} = 1 + \frac{1}{1} = 2$$

$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=2 \\ y=1}} =$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x$$

所以

$$\left. \frac{\partial z}{\partial x} \right|_{\substack{x=2 \\ y=1}} = \left( y + \frac{1}{y} \right) \bigg|_{\substack{x=2 \\ y=1}} = 1 + \frac{1}{1} = 2$$

$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=2 \\ y=1}} =$$



例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x - \frac{x}{y^2}$$

所以

$$\left. \frac{\partial z}{\partial x} \right|_{\substack{x=2 \\ y=1}} = \left( y + \frac{1}{y} \right) \bigg|_{\substack{x=2 \\ y=1}} = 1 + \frac{1}{1} = 2$$

$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=2 \\ y=1}} =$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x - \frac{x}{y^2}$$

所以

$$\left. \frac{\partial z}{\partial x} \right|_{\substack{x=2 \\ y=1}} = \left( y + \frac{1}{y} \right) \Big|_{\substack{x=2 \\ y=1}} = 1 + \frac{1}{1} = 2$$

$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=2 \\ y=1}} = \left( x - \frac{x}{y^2} \right) \Big|_{\substack{x=2 \\ y=1}} =$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x - \frac{x}{y^2}$$

所以

$$\left. \frac{\partial z}{\partial x} \right|_{\substack{x=2 \\ y=1}} = \left( y + \frac{1}{y} \right) \bigg|_{\substack{x=2 \\ y=1}} = 1 + \frac{1}{1} = 2$$

$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=2 \\ y=1}} = \left( x - \frac{x}{y^2} \right) \bigg|_{\substack{x=2 \\ y=1}} = 2 - \frac{2}{1} =$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法一

$$\frac{\partial z}{\partial x} = (xy + \frac{x}{y})'_x = (xy)'_x + (\frac{x}{y})'_x = y + \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = (xy + \frac{x}{y})'_y = (xy)'_y + (\frac{x}{y})'_y = x - \frac{x}{y^2}$$

所以

$$\left. \frac{\partial z}{\partial x} \right|_{\substack{x=2 \\ y=1}} = \left( y + \frac{1}{y} \right) \bigg|_{\substack{x=2 \\ y=1}} = 1 + \frac{1}{1} = 2$$

$$\left. \frac{\partial z}{\partial y} \right|_{\substack{x=2 \\ y=1}} = \left( x - \frac{x}{y^2} \right) \bigg|_{\substack{x=2 \\ y=1}} = 2 - \frac{2}{1} = 0$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

所以  $f(x, 1)$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

所以  $f(x, 1) = 2x$



例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

所以  $f(x, 1) = 2x \Rightarrow \frac{d}{dx}[f(x, 1)] =$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

所以  $f(x, 1) = 2x \Rightarrow \frac{d}{dx}[f(x, 1)] = 2$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

所以  $f(x, 1) = 2x \Rightarrow \frac{d}{dx}[f(x, 1)] = 2$

$$\Rightarrow \frac{d}{dx}[f(x, 1)] \Big|_{x=2} = 2,$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

所以  $f(x, 1) = 2x \Rightarrow \frac{d}{dx}[f(x, 1)] = 2$

$$\Rightarrow \frac{\partial z}{\partial x} \Big|_{\substack{x=2 \\ y=1}} = \frac{d}{dx}[f(x, 1)] \Big|_{x=2} = 2,$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

所以  $f(x, 1) = 2x \Rightarrow \frac{d}{dx}[f(x, 1)] = 2$

$$\Rightarrow \frac{\partial z}{\partial x} \Big|_{\substack{x=2 \\ y=1}} = \frac{d}{dx}[f(x, 1)] \Big|_{x=2} = 2,$$

$$f(2, y)$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

所以  $f(x, 1) = 2x \Rightarrow \frac{d}{dx}[f(x, 1)] = 2$

$$\Rightarrow \frac{\partial z}{\partial x} \Big|_{\substack{x=2 \\ y=1}} = \frac{d}{dx}[f(x, 1)] \Big|_{x=2} = 2,$$

$$f(2, y) = 2y + \frac{2}{y}$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

所以  $f(x, 1) = 2x \Rightarrow \frac{d}{dx}[f(x, 1)] = 2$

$$\Rightarrow \frac{\partial z}{\partial x} \Big|_{\substack{x=2 \\ y=1}} = \frac{d}{dx}[f(x, 1)] \Big|_{x=2} = 2,$$

$$f(2, y) = 2y + \frac{2}{y} \Rightarrow \frac{d}{dy}[f(2, y)] =$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

所以  $f(x, 1) = 2x \Rightarrow \frac{d}{dx}[f(x, 1)] = 2$

$$\Rightarrow \frac{\partial z}{\partial x} \Big|_{\substack{x=2 \\ y=1}} = \frac{d}{dx}[f(x, 1)] \Big|_{x=2} = 2,$$

$$f(2, y) = 2y + \frac{2}{y} \Rightarrow \frac{d}{dy}[f(2, y)] = 2 - \frac{2}{y^2}$$



例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

所以

$$\begin{aligned} f(x, 1) = 2x &\Rightarrow \frac{d}{dx}[f(x, 1)] = 2 \\ &\Rightarrow \frac{\partial z}{\partial x} \Big|_{\substack{x=2 \\ y=1}} = \frac{d}{dx}[f(x, 1)] \Big|_{x=2} = 2, \\ f(2, y) = 2y + \frac{2}{y} &\Rightarrow \frac{d}{dy}[f(2, y)] = 2 - \frac{2}{y^2} \\ &\Rightarrow \frac{d}{dy}[f(2, y)] \Big|_{y=1} = 0. \end{aligned}$$

例 设  $z = xy + \frac{x}{y}$ , 求  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  和在点  $(2, 1)$  处的偏导数值

解法二 利用

$$\frac{\partial z}{\partial x}(x_0, y_0) = \frac{d}{dx}[f(x, y_0)] \Big|_{x=x_0}, \quad \frac{\partial z}{\partial y}(x_0, y_0) = \frac{d}{dy}[f(x_0, y)] \Big|_{y=y_0}$$

所以

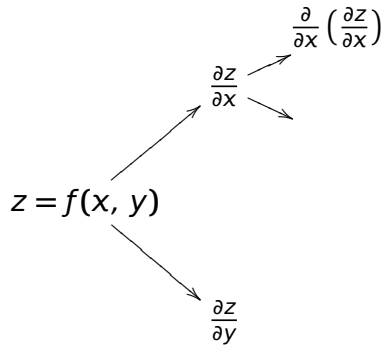
$$\begin{aligned} f(x, 1) = 2x &\Rightarrow \frac{d}{dx}[f(x, 1)] = 2 \\ &\Rightarrow \frac{\partial z}{\partial x} \Big|_{\substack{x=2 \\ y=1}} = \frac{d}{dx}[f(x, 1)] \Big|_{x=2} = 2, \\ f(2, y) = 2y + \frac{2}{y} &\Rightarrow \frac{d}{dy}[f(2, y)] = 2 - \frac{2}{y^2} \\ &\Rightarrow \frac{\partial z}{\partial y} \Big|_{\substack{x=2 \\ y=1}} = \frac{d}{dy}[f(2, y)] \Big|_{y=1} = 0. \end{aligned}$$

# 二阶偏导数

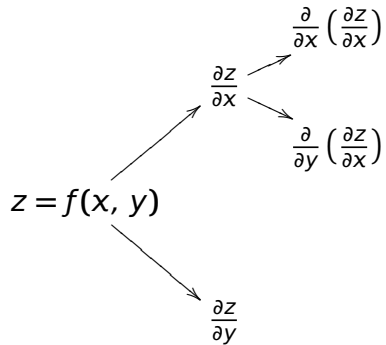
---

$$\begin{array}{c} \nearrow \frac{\partial z}{\partial x} \\ z = f(x, y) \\ \searrow \frac{\partial z}{\partial y} \end{array}$$

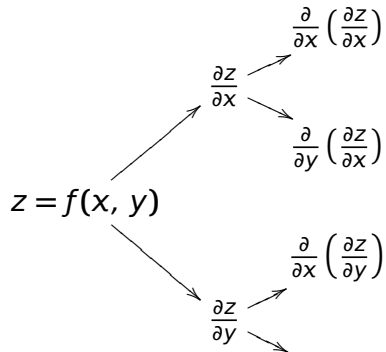
## 二阶偏导数



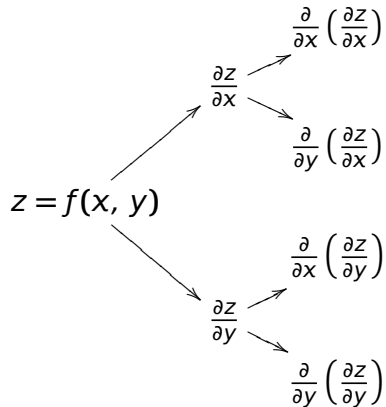
## 二阶偏导数



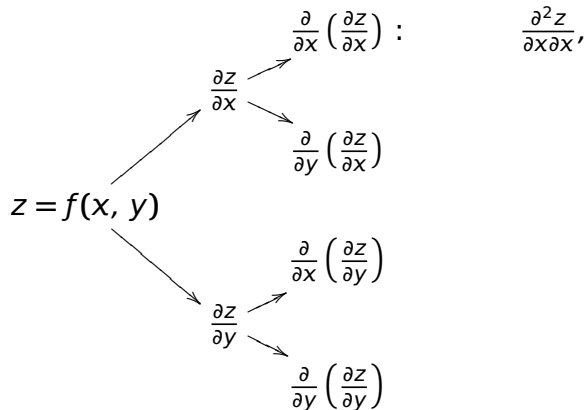
## 二阶偏导数



## 二阶偏导数

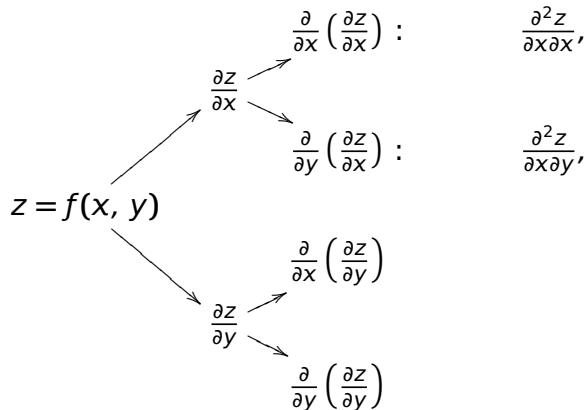


## 二阶偏导数

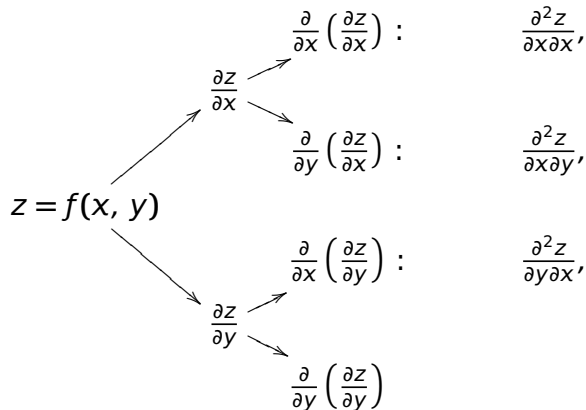




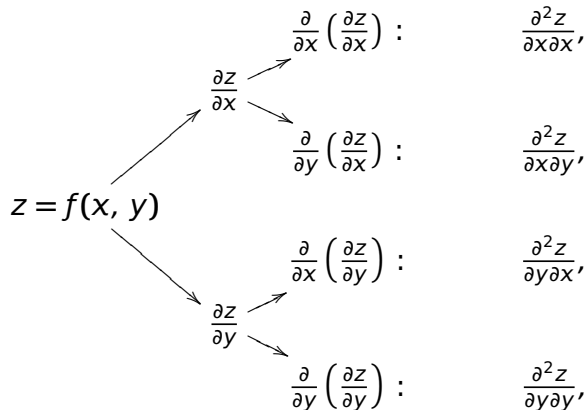
## 二阶偏导数



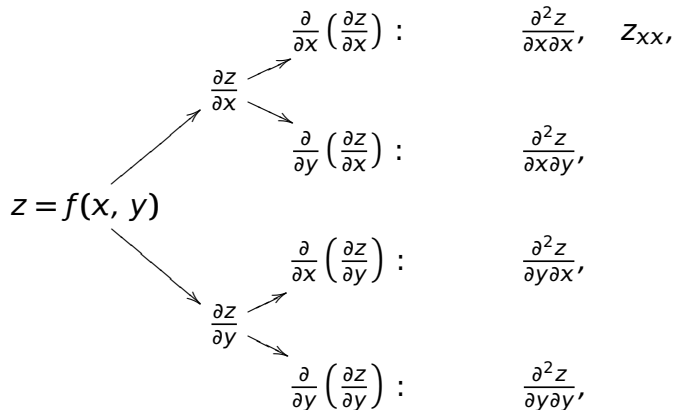
## 二阶偏导数



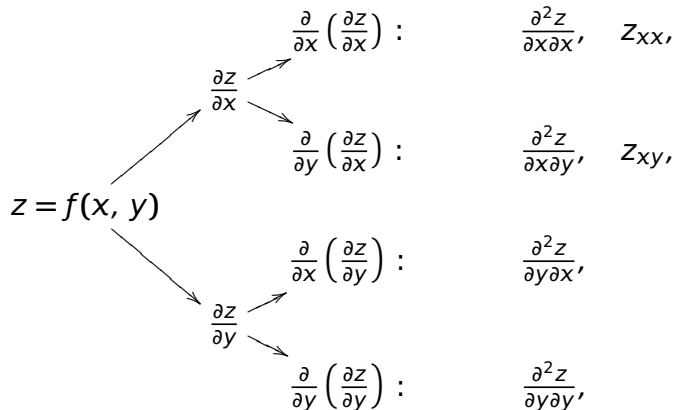
## 二阶偏导数



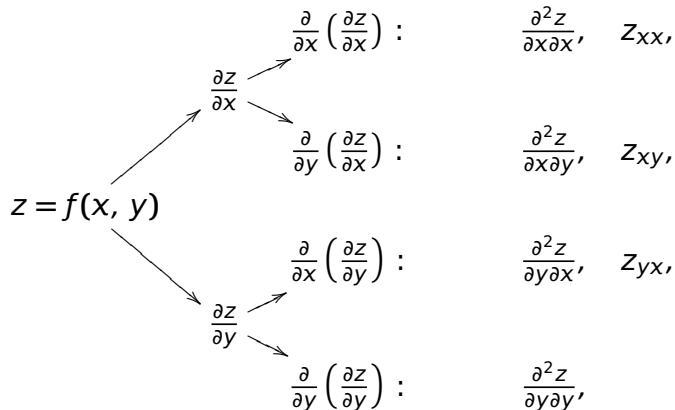
## 二阶偏导数



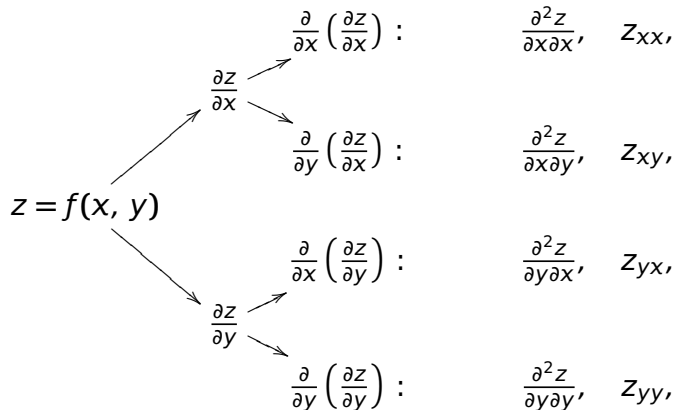
## 二阶偏导数



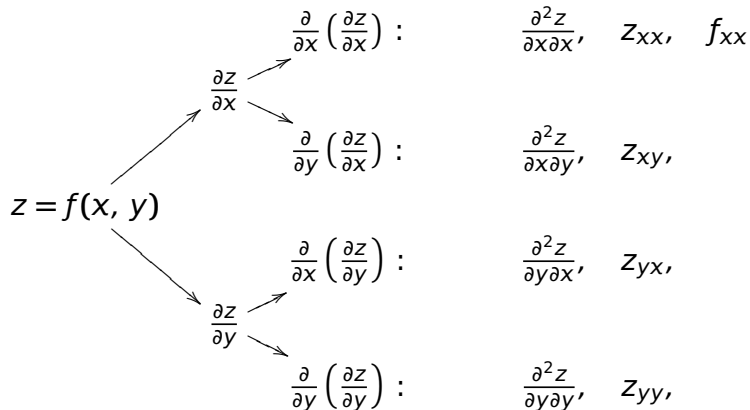
## 二阶偏导数



## 二阶偏导数

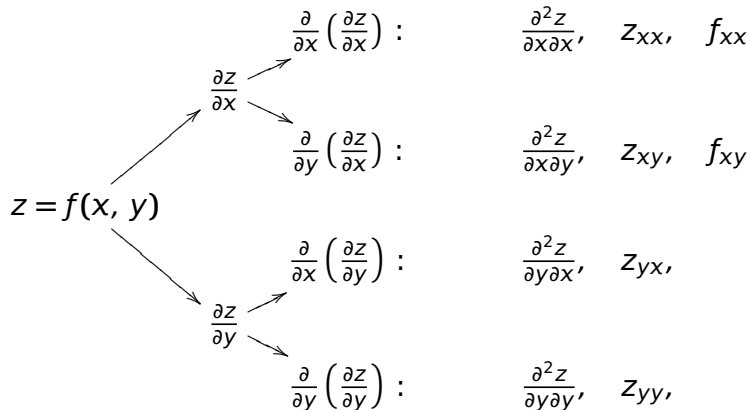


## 二阶偏导数

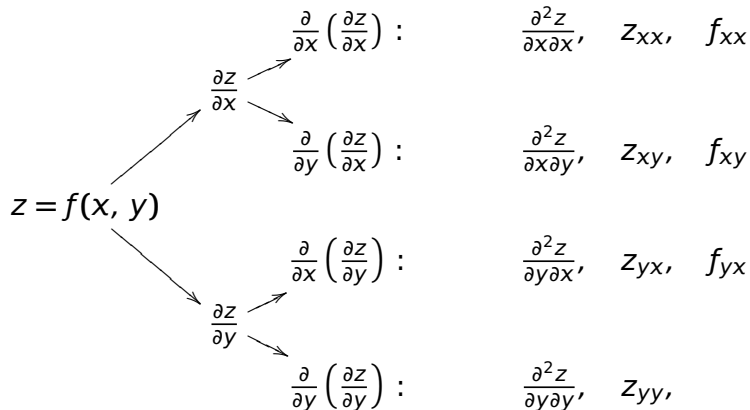




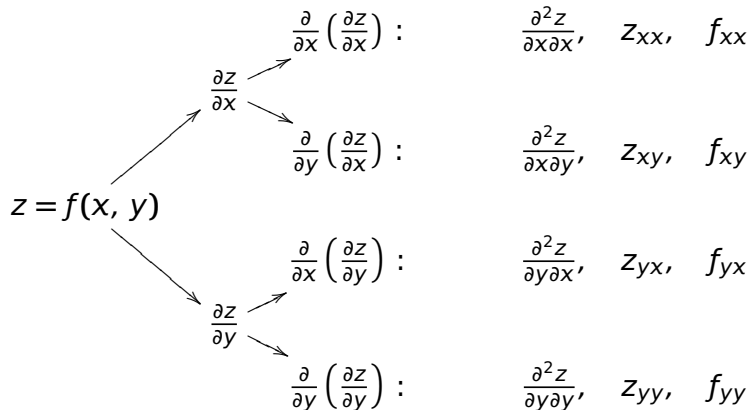
## 二阶偏导数



## 二阶偏导数



## 二阶偏导数



例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x =$$

$$z_y =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$



例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} +$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} +$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$



例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} +$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x =$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} +$$

$$z_{yy} =$$



例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y =$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2 e^{xy} +$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2 e^{xy} + 4x$$

例 求  $z = e^{xy} + 2xy^2$  全部二阶偏导数

解

$$z_x = (e^{xy} + 2xy^2)'_x = (e^{xy})'_x + (2xy^2)'_x = ye^{xy} + 2y^2$$

$$z_y = (e^{xy} + 2xy^2)'_y = (e^{xy})'_y + (2xy^2)'_y = xe^{xy} + 4xy$$

$$z_{xx} = (ye^{xy} + 2y^2)'_x = (ye^{xy})'_x + (2y^2)'_x = y^2 e^{xy}$$

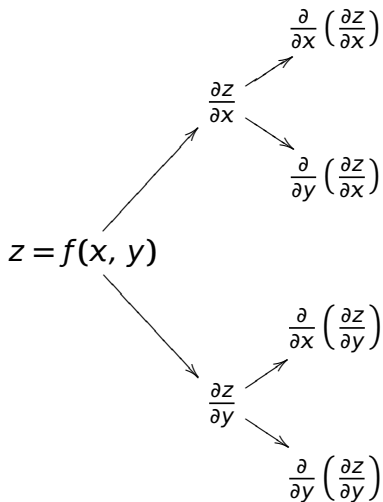
$$z_{xy} = (ye^{xy} + 2y^2)'_y = (ye^{xy})'_y + (2y^2)'_y = e^{xy} + xye^{xy} + 4y$$

$$z_{yx} = (xe^{xy} + 4xy)'_x = (xe^{xy})'_x + (4xy)'_x = e^{xy} + xye^{xy} + 4y$$

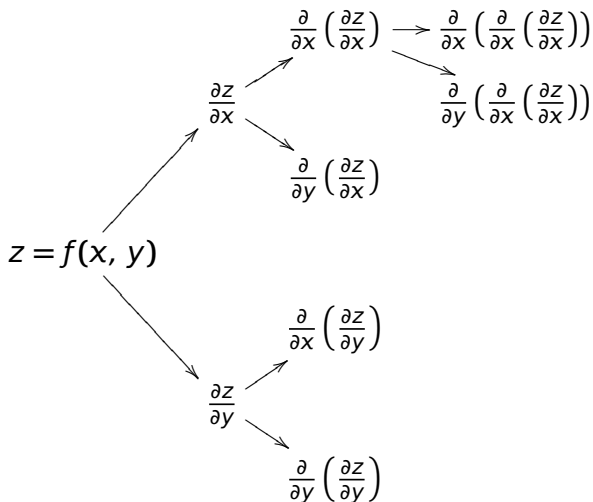
$$z_{yy} = (xe^{xy} + 4xy)'_y = (xe^{xy})'_y + (4xy)'_y = x^2 e^{xy} + 4x$$

注 此例成立  $z_{xy} = z_{yx}$

# 三阶偏导数

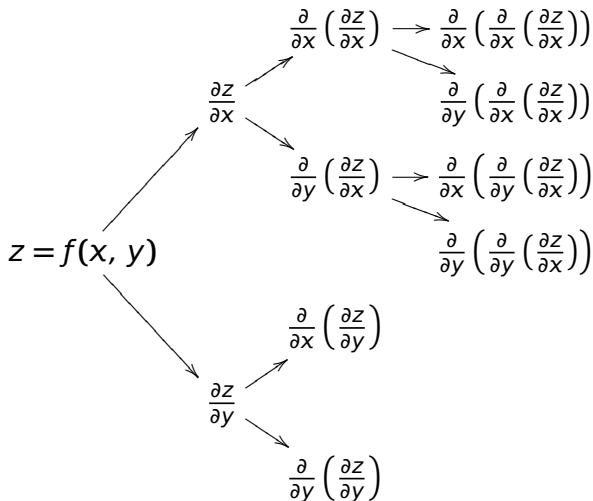


# 三阶偏导数

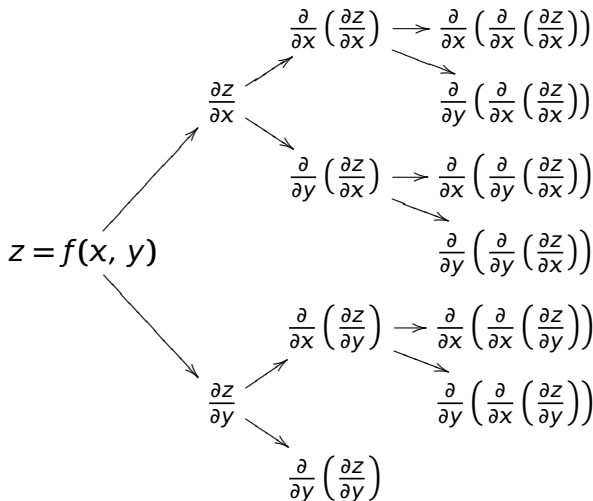




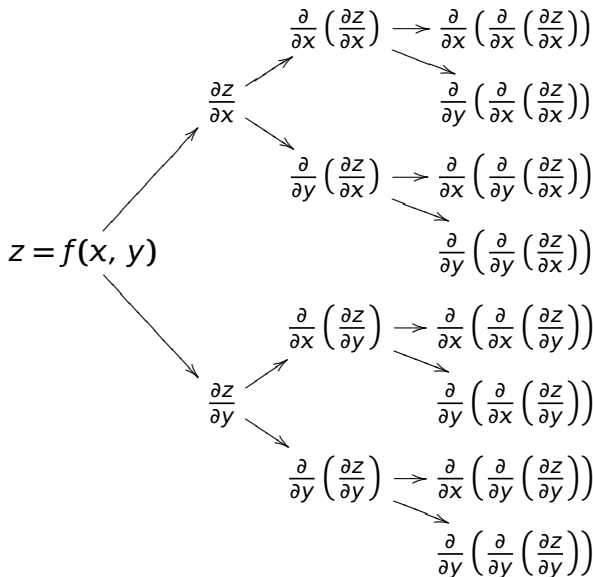
# 三阶偏导数



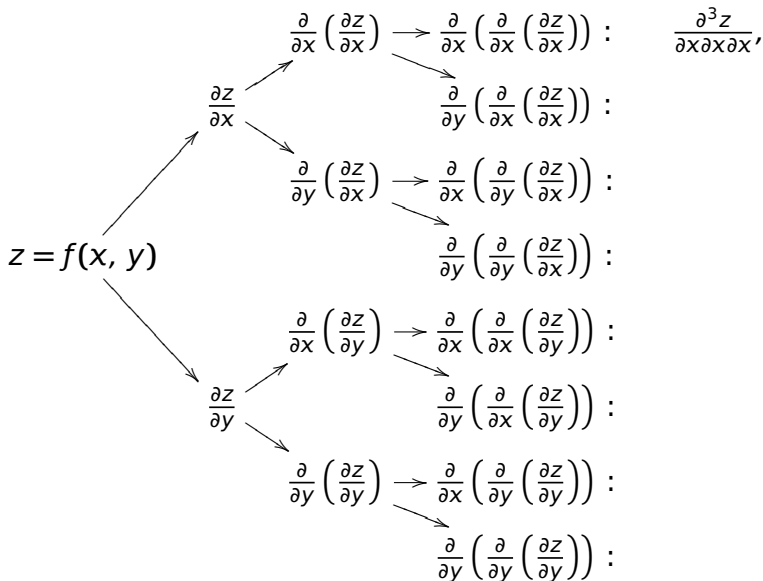
# 三阶偏导数



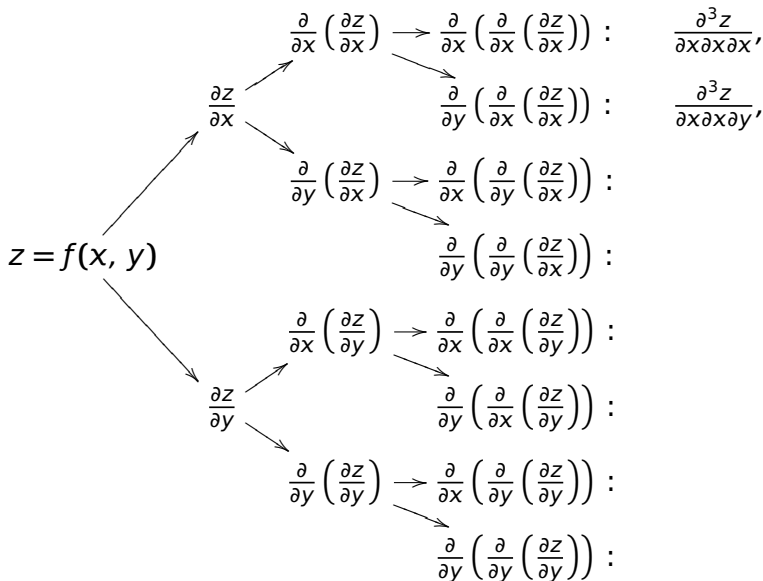
# 三阶偏导数



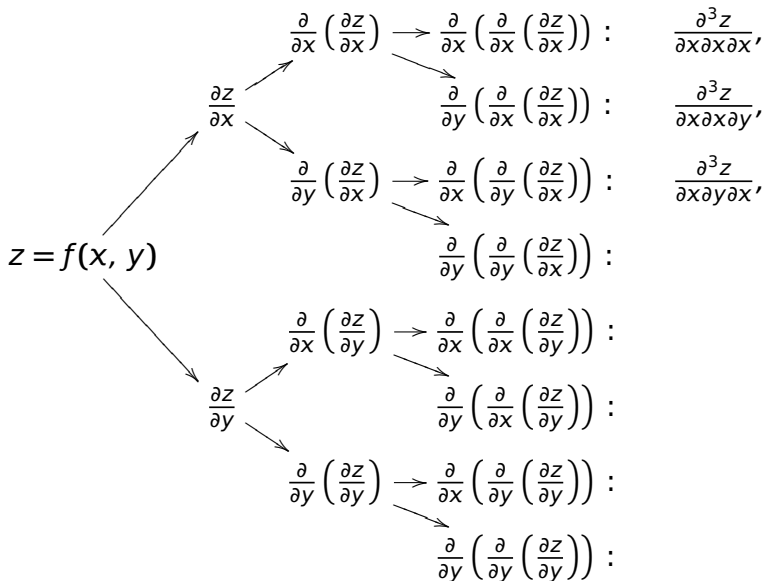
# 三阶偏导数



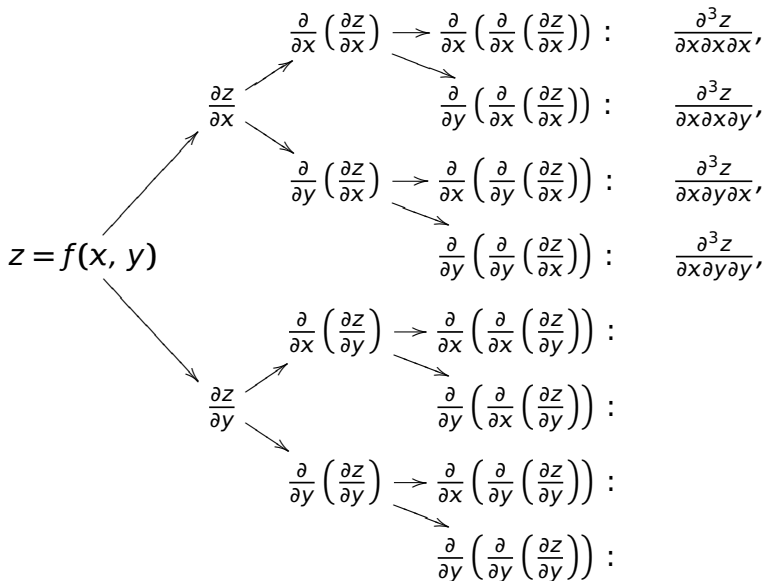
# 三阶偏导数



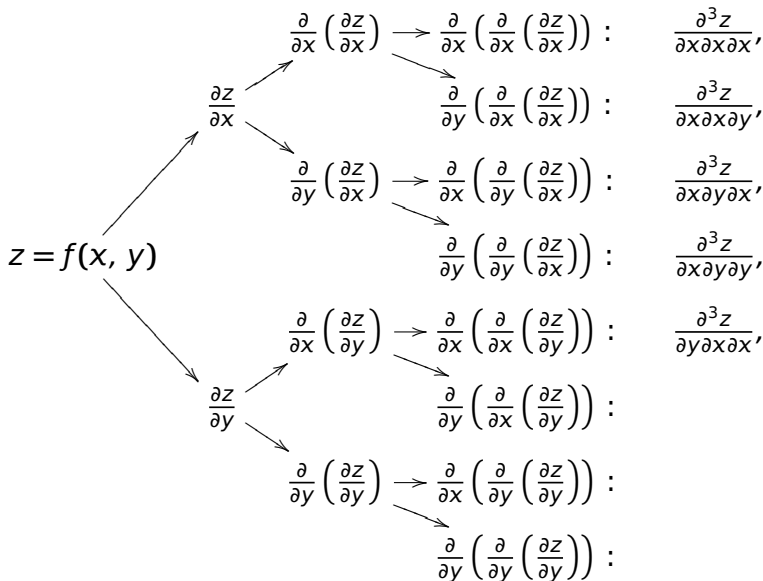
# 三阶偏导数



# 三阶偏导数

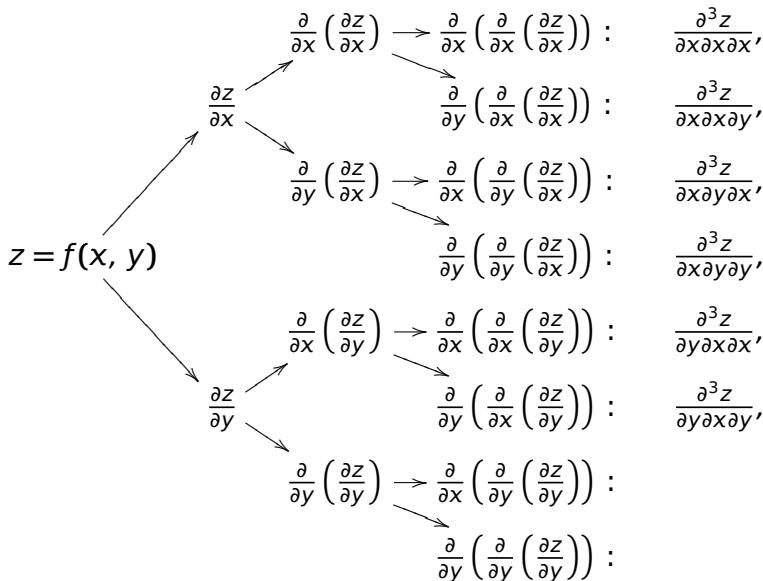


# 三阶偏导数

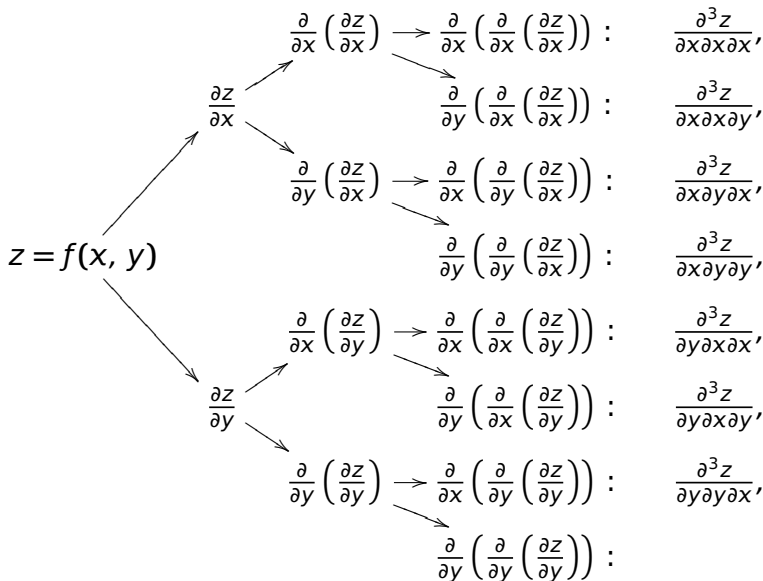




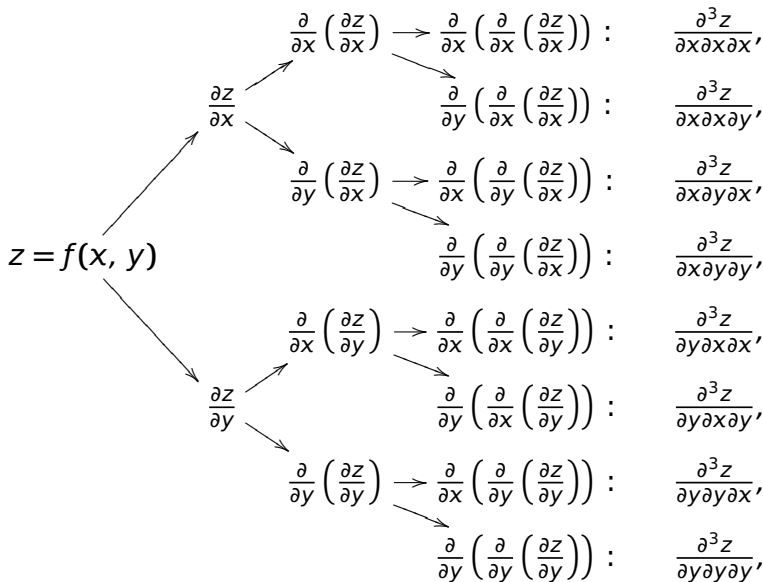
# 三阶偏导数



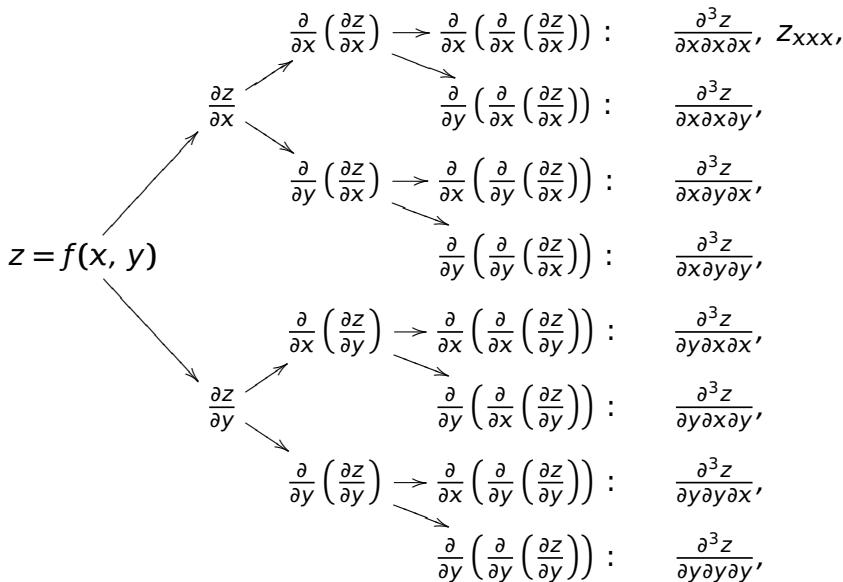
# 三阶偏导数



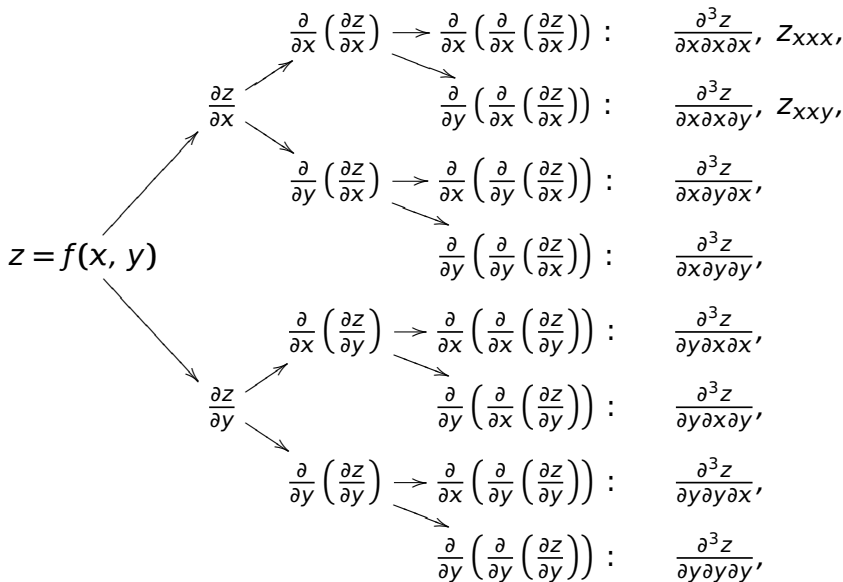
# 三阶偏导数



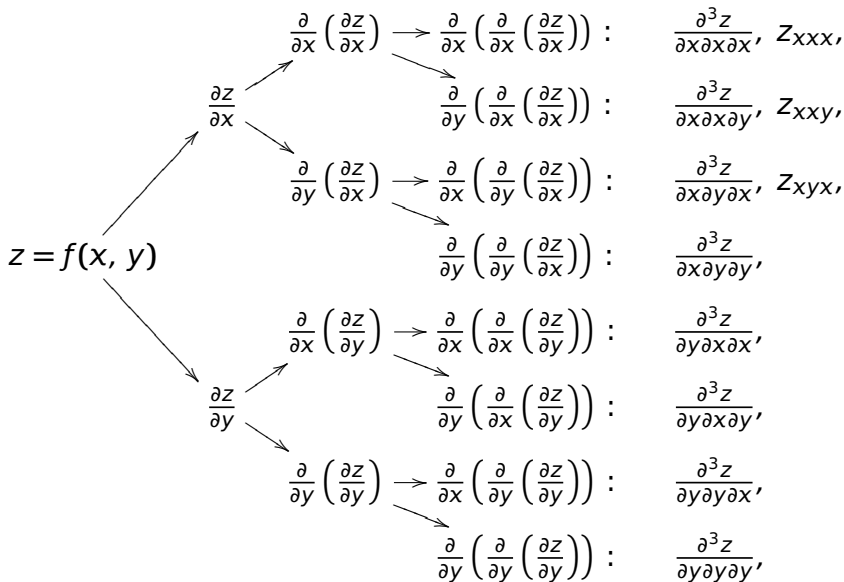
# 三阶偏导数



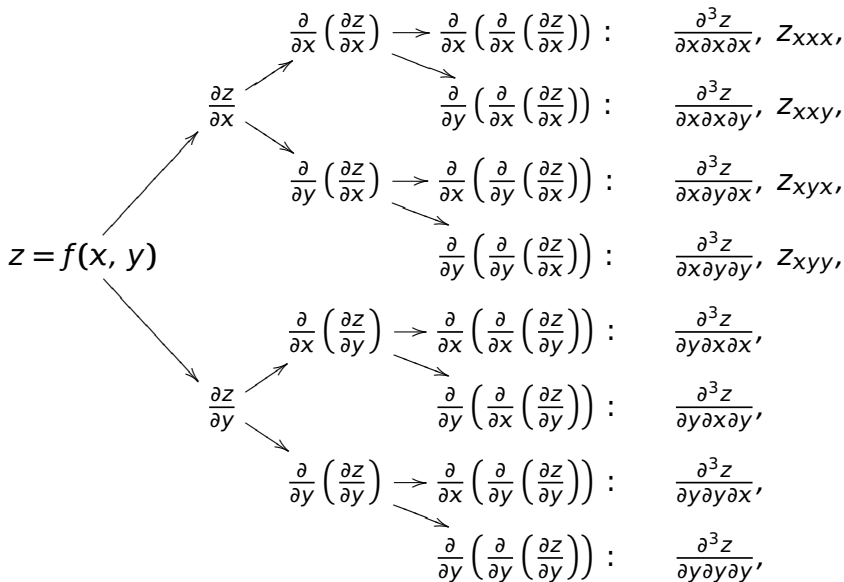
# 三阶偏导数



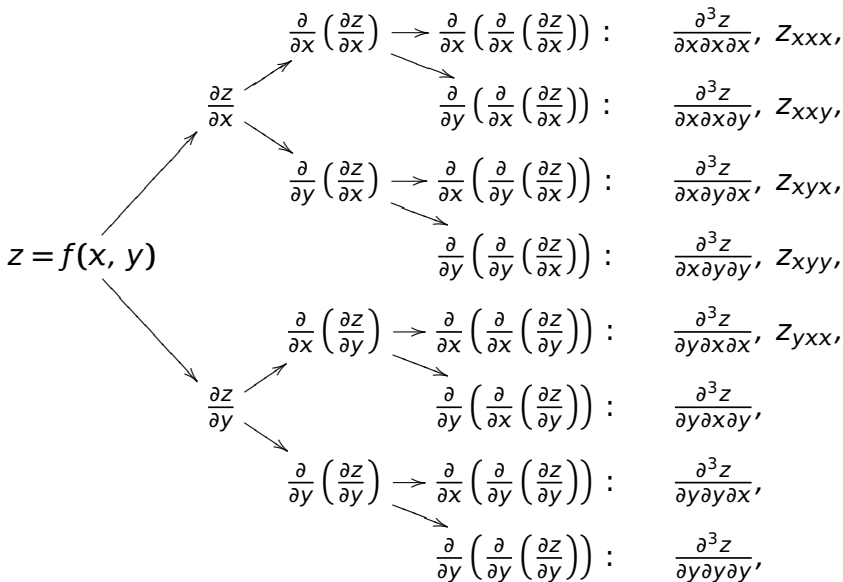
# 三阶偏导数



# 三阶偏导数

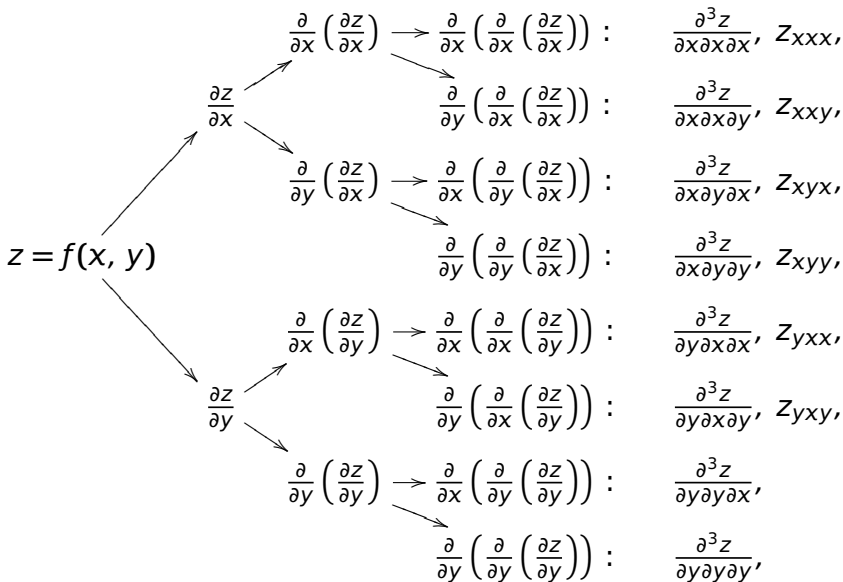


# 三阶偏导数

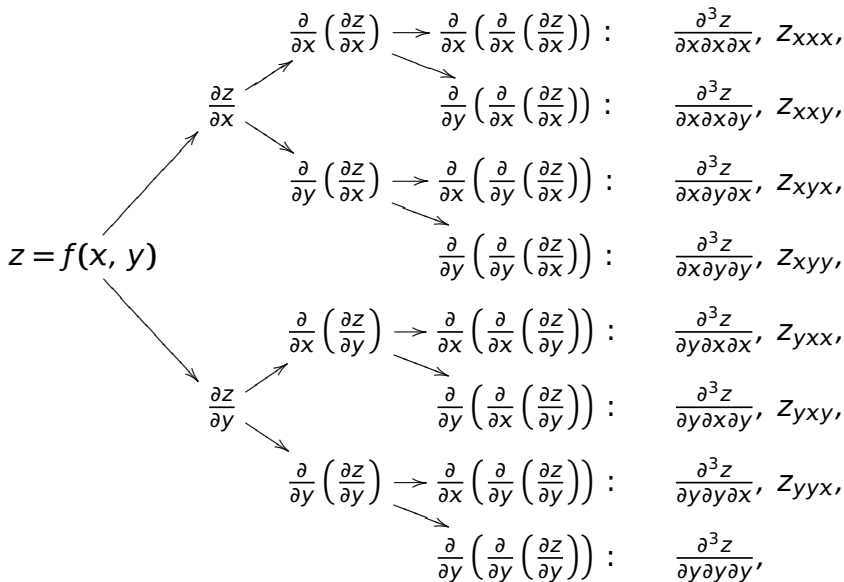




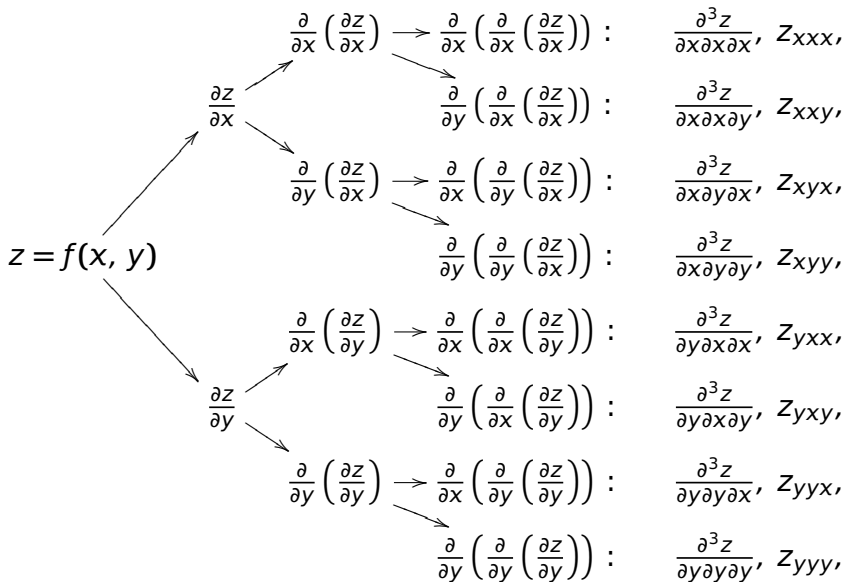
# 三阶偏导数



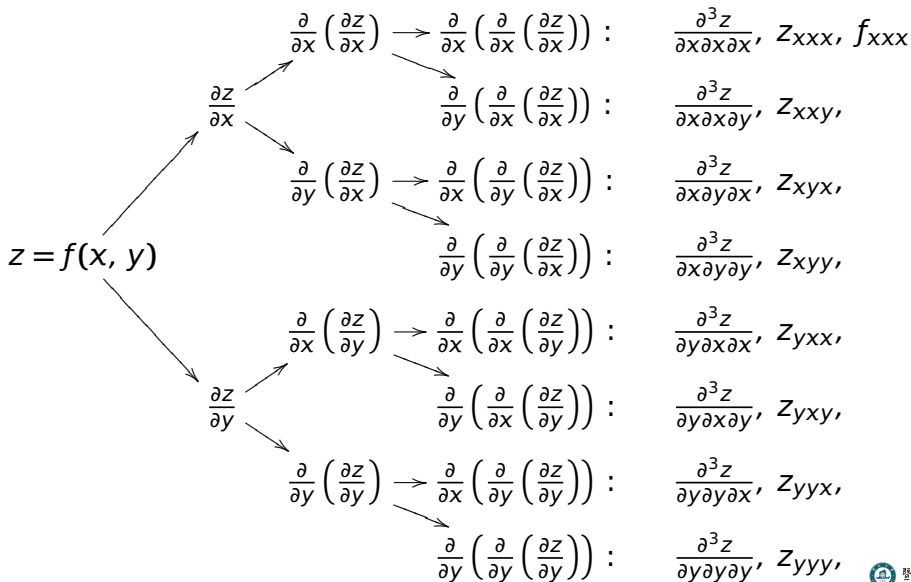
# 三阶偏导数



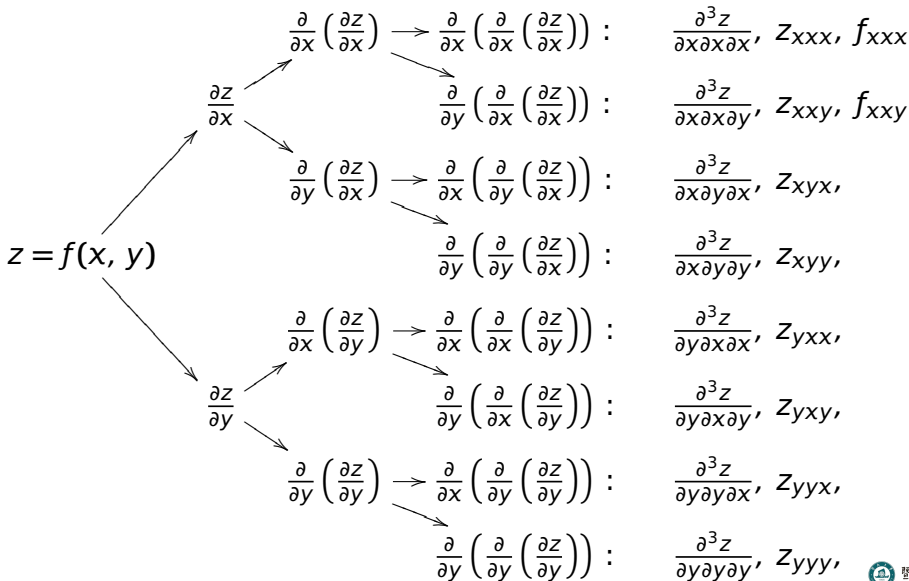
# 三阶偏导数



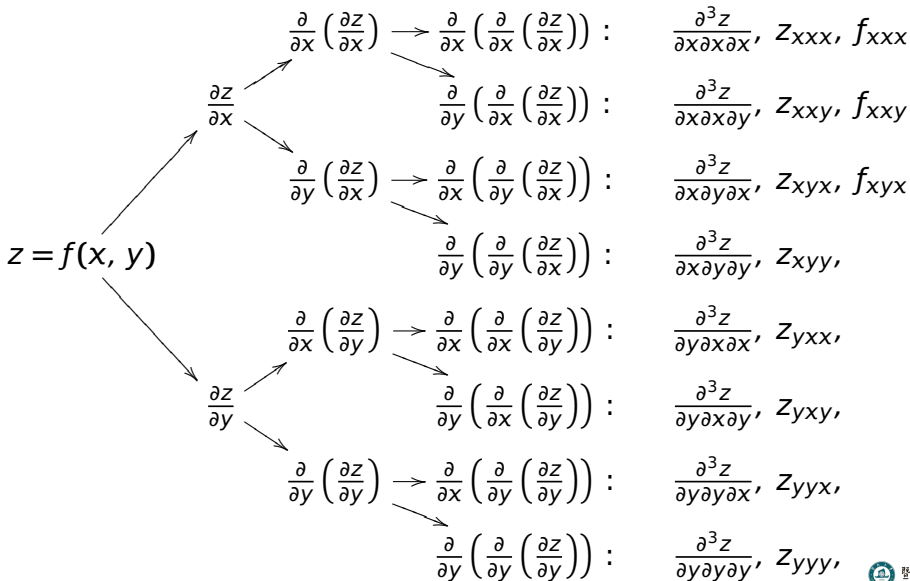
# 三阶偏导数



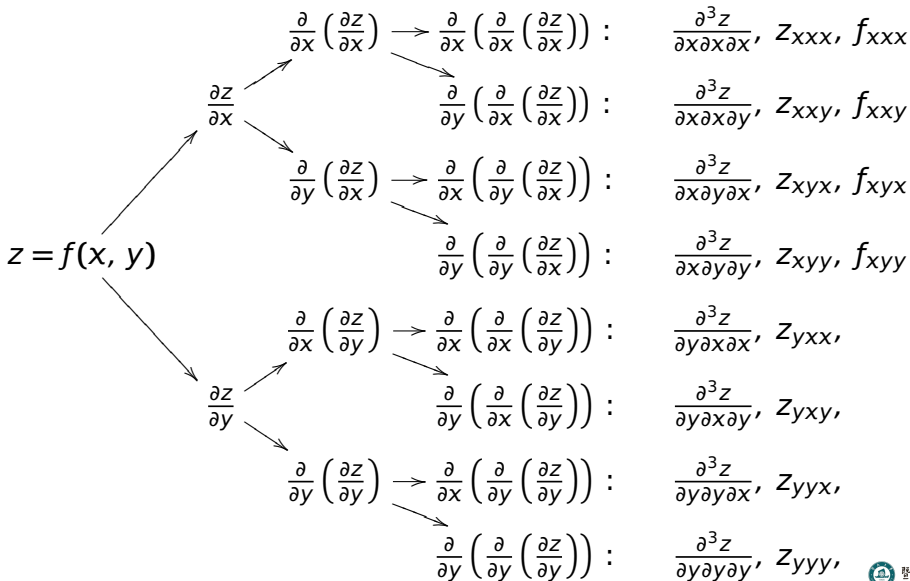
# 三阶偏导数



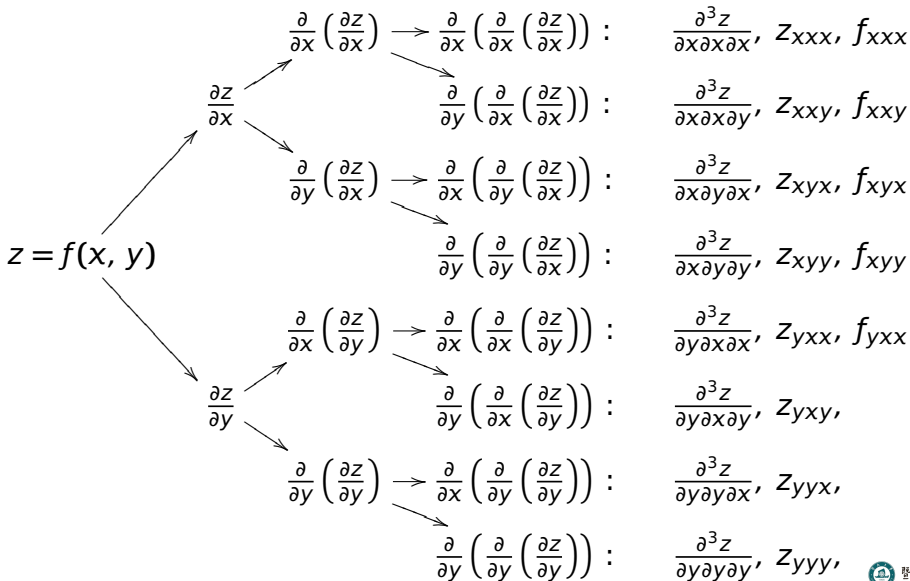
# 三阶偏导数



# 三阶偏导数

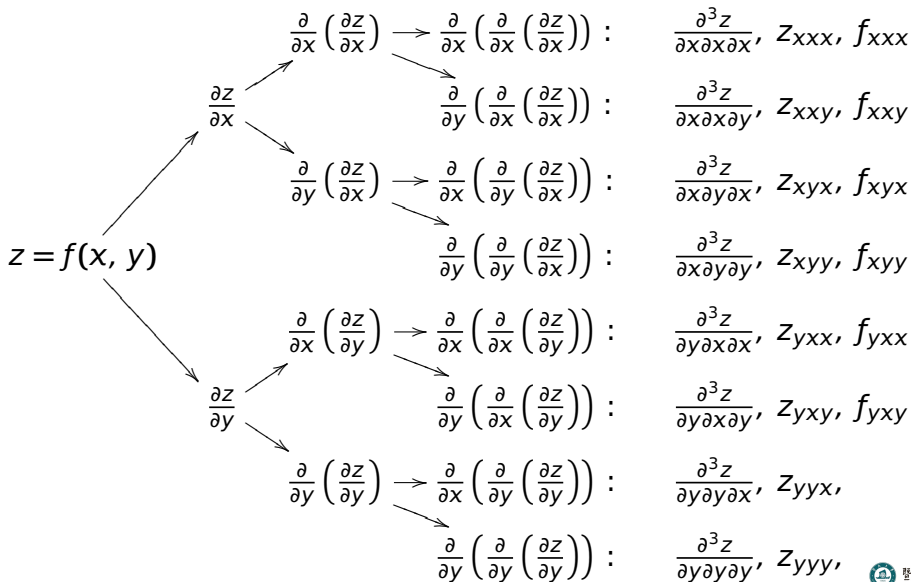


# 三阶偏导数

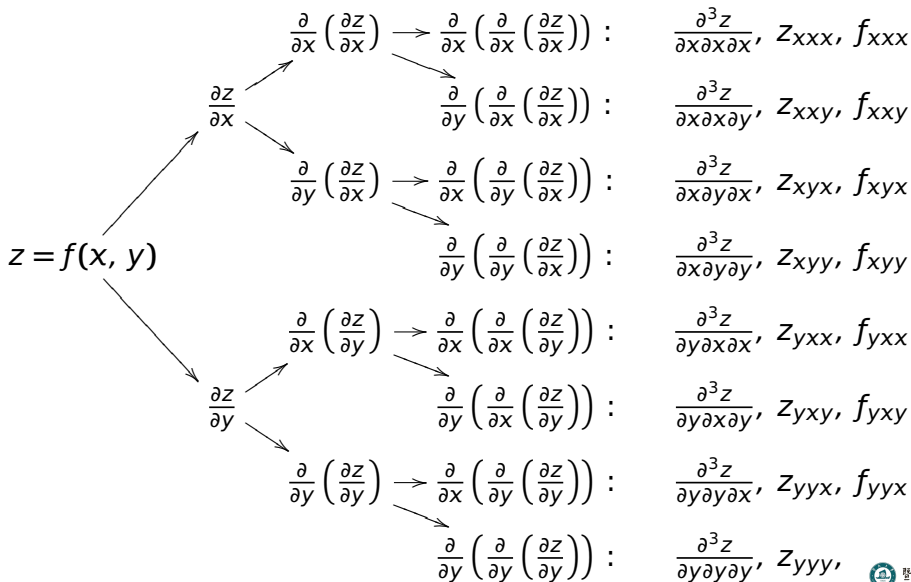




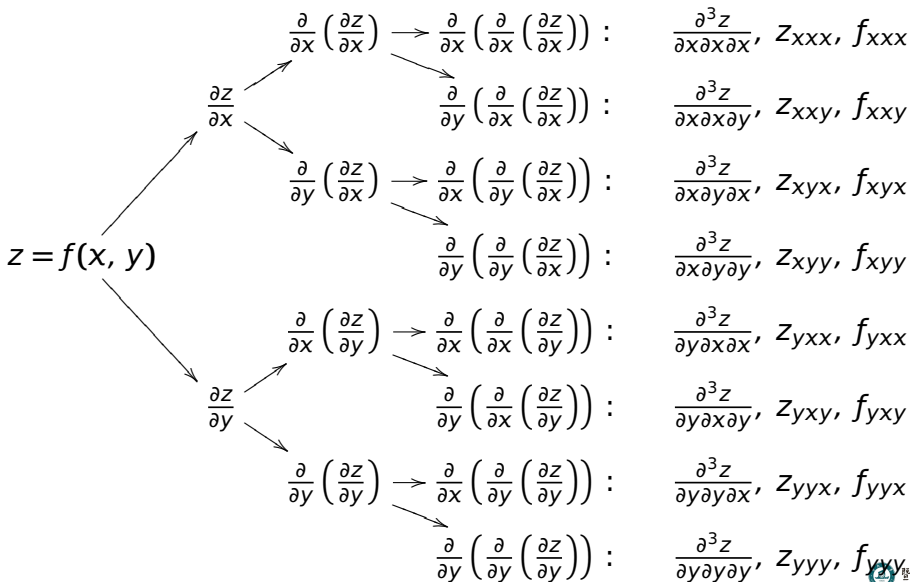
# 三阶偏导数



# 三阶偏导数



# 三阶偏导数



例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解  $z_x =$

$z_y =$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解  $z_x =$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解  $z_x =$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解 
$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$



例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解 
$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解 
$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解 
$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$
$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$
$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9x^2y^2 - x + 1$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$
$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$



例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x =$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2$$

$$z_{yy} =$$

$$z_{xxx} =$$



例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y =$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} = (6xy^2)'_x =$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} = (6xy^2)'_x = 6y^2$$

例 求  $z = x^3y^2 - 3xy^3 - xy + 1$  全部二阶偏导数及  $\frac{\partial^3 z}{\partial x^3}$

解

$$z_x = (x^3y^2 - 3xy^3 - xy + 1)'_x = 3x^2y^2 - 3y^3 - y$$

$$z_y = (x^3y^2 - 3xy^3 - xy + 1)'_y = 2x^3y - 9xy^2 - x$$

$$z_{xx} = (3x^2y^2 - 3y^3 - y)'_x = 6xy^2$$

$$z_{xy} = (3x^2y^2 - 3y^3 - y)'_y = 6x^2y - 9y^2 - 1$$

$$z_{yx} = (2x^3y - 9xy^2 - x)'_x = 6x^2y - 9y^2 - 1$$

$$z_{yy} = (2x^3y - 9xy^2 - x)'_y = 2x^3 - 18xy$$

$$z_{xxx} = (6xy^2)'_x = 6y^2$$

注 此例成立  $z_{xy} = z_{yx}$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解



例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解  $z_x =$

$z_y =$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解

$$z_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解 
$$z_x = (x \sin(3y))'_x =$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解 
$$z_x = (x \sin(3y))'_x = \sin(3y)$$

$$z_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解 
$$z_x = (x \sin(3y))'_x = \sin(3y)$$

$$z_y = (x \sin(3y))'_y =$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解 
$$z_x = (x \sin(3y))'_x = \sin(3y)$$
$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$

$$z_{xx} =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解 
$$z_x = (x \sin(3y))'_x = \sin(3y)$$
$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$

$$z_{xx} = (\sin(3y))'_x =$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$



例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解 
$$z_x = (x \sin(3y))'_x = \sin(3y)$$
$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解 
$$z_x = (x \sin(3y))'_x = \sin(3y)$$
$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y =$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解 
$$z_x = (x \sin(3y))'_x = \sin(3y)$$
$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3 \cos(3y)$$

$$z_{yx} =$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解

$$z_x = (x \sin(3y))'_x = \sin(3y)$$
$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$
$$z_{xx} = (\sin(3y))'_x = 0$$
$$z_{xy} = (\sin(3y))'_y = 3 \cos(3y)$$
$$z_{yx} = (3x \cos(3y))'_x =$$
$$z_{yy} =$$
$$z_{xyy} =$$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解 
$$z_x = (x \sin(3y))'_x = \sin(3y)$$

$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3 \cos(3y)$$

$$z_{yx} = (3x \cos(3y))'_x = 3 \cos(3y)$$

$$z_{yy} =$$

$$z_{xyy} =$$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解 
$$z_x = (x \sin(3y))'_x = \sin(3y)$$

$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3 \cos(3y)$$

$$z_{yx} = (3x \cos(3y))'_x = 3 \cos(3y)$$

$$z_{yy} = (3x \cos(3y))'_y =$$

$$z_{xyy} =$$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解 
$$z_x = (x \sin(3y))'_x = \sin(3y)$$

$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3 \cos(3y)$$

$$z_{yx} = (3x \cos(3y))'_x = 3 \cos(3y)$$

$$z_{yy} = (3x \cos(3y))'_y = -9x \sin(3y)$$

$$z_{xyy} =$$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解 
$$z_x = (x \sin(3y))'_x = \sin(3y)$$

$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3 \cos(3y)$$

$$z_{yx} = (3x \cos(3y))'_x = 3 \cos(3y)$$

$$z_{yy} = (3x \cos(3y))'_y = -9x \sin(3y)$$

$$z_{xyy} = (3 \cos(3y))'_y =$$



例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解 
$$z_x = (x \sin(3y))'_x = \sin(3y)$$

$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3 \cos(3y)$$

$$z_{yx} = (3x \cos(3y))'_x = 3 \cos(3y)$$

$$z_{yy} = (3x \cos(3y))'_y = -9x \sin(3y)$$

$$z_{xyy} = (3 \cos(3y))'_y = -9 \sin(3y)$$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解 
$$z_x = (x \sin(3y))'_x = \sin(3y)$$

$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3 \cos(3y)$$

$$z_{yx} = (3x \cos(3y))'_x = 3 \cos(3y)$$

$$z_{yy} = (3x \cos(3y))'_y = -9x \sin(3y)$$

$$z_{xyy} = (3 \cos(3y))'_y = -9 \sin(3y)$$

注 此例成立  $z_{xy} = z_{yx}$

例 求  $z = x \sin(3y)$  全部二阶偏导数及  $z_{xyy}$

解 
$$z_x = (x \sin(3y))'_x = \sin(3y)$$

$$z_y = (x \sin(3y))'_y = 3x \cos(3y)$$

$$z_{xx} = (\sin(3y))'_x = 0$$

$$z_{xy} = (\sin(3y))'_y = 3 \cos(3y)$$

$$z_{yx} = (3x \cos(3y))'_x = 3 \cos(3y)$$

$$z_{yy} = (3x \cos(3y))'_y = -9x \sin(3y)$$

$$z_{xyy} = (3 \cos(3y))'_y = -9 \sin(3y)$$

注 此例成立  $z_{xy} = z_{yx}$

性质 设有二元函数  $z = f(x, y)$ 。若  $\frac{\partial^2 z}{\partial y \partial x}$  和  $\frac{\partial^2 z}{\partial x \partial y}$  均连续, 则

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial^2 z}{\partial x \partial y}$$

# We are here now...

---

1. 二元函数偏导数定义

3. 全微分的定义与计算

# 回顾一元函数的微分

---

- 函数  $y = f(x)$  的增量

$$\Delta y = f(x + \Delta x) - f(x)$$

# 回顾一元函数的微分

---

- 函数  $y = f(x)$  的增量

$$\Delta y = f(x + \Delta x) - f(x)$$

- 若  $y = f(x)$  可微, 则

$$\Delta y = f(x + \Delta x) - f(x) = A\Delta x + o(\Delta x)$$

# 回顾一元函数的微分

---

- 函数  $y = f(x)$  的增量

$$\Delta y = f(x + \Delta x) - f(x)$$

- 若  $y = f(x)$  可微, 则

$$\Delta y = f(x + \Delta x) - f(x) = A\Delta x + o(\Delta x) = f'(x)\Delta x + o(\Delta x)$$

# 回顾一元函数的微分

- 函数  $y = f(x)$  的增量

$$\Delta y = f(x + \Delta x) - f(x)$$

- 若  $y = f(x)$  可微, 则

$$\Delta y = f(x + \Delta x) - f(x) = A\Delta x + o(\Delta x) = f'(x)\Delta x + o(\Delta x)$$

此时可用  $f'(x)\Delta x$  近似代替  $\Delta y$ ,



# 回顾一元函数的微分

- 函数  $y = f(x)$  的增量

$$\Delta y = f(x + \Delta x) - f(x)$$

- 若  $y = f(x)$  可微, 则

$$\Delta y = f(x + \Delta x) - f(x) = A\Delta x + o(\Delta x) = f'(x)\Delta x + o(\Delta x)$$

此时可用  $f'(x)\Delta x$  近似代替  $\Delta y$ , 称为函数  $y = f(x)$  的微分,

# 回顾一元函数的微分

- 函数  $y = f(x)$  的增量

$$\Delta y = f(x + \Delta x) - f(x)$$

- 若  $y = f(x)$  可微, 则

$$\Delta y = f(x + \Delta x) - f(x) = A\Delta x + o(\Delta x) = f'(x)\Delta x + o(\Delta x)$$

此时可用  $f'(x)\Delta x$  近似代替  $\Delta y$ , 称为函数  $y = f(x)$  的微分, 记为:

$$dy = f'(x)dx \quad \text{或} \quad df = f'(x)dx$$

# 多元函数的全微分

---

- 二元函数  $z = f(x, y)$

$$f(x + \Delta x, y + \Delta y) - f(x, y)$$

# 多元函数的全微分

---

- 二元函数  $z = f(x, y)$

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

# 多元函数的全微分

- 二元函数  $z = f(x, y)$  的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

# 多元函数的全微分

- 二元函数  $z = f(x, y)$  的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称  $z = f(x, y)$  可微是指：

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

# 多元函数的全微分

- 二元函数  $z = f(x, y)$  的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称  $z = f(x, y)$  可微是指：

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y +\end{aligned}$$

# 多元函数的全微分

- 二元函数  $z = f(x, y)$  的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称  $z = f(x, y)$  可微是指:

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right)\end{aligned}$$



# 多元函数的全微分

- 二元函数  $z = f(x, y)$  的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称  $z = f(x, y)$  可微是指:

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \approx A\Delta x + B\Delta y\end{aligned}$$

# 多元函数的全微分

- 二元函数  $z = f(x, y)$  的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称  $z = f(x, y)$  可微是指:

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \approx A\Delta x + B\Delta y\end{aligned}$$

例 设  $z = f(x, y) = x^2 + y^2$ ,

# 多元函数的全微分

- 二元函数  $z = f(x, y)$  的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称  $z = f(x, y)$  可微是指:

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \approx A\Delta x + B\Delta y\end{aligned}$$

例 设  $z = f(x, y) = x^2 + y^2$ , 则

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

# 多元函数的全微分

- 二元函数  $z = f(x, y)$  的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称  $z = f(x, y)$  可微是指:

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \approx A\Delta x + B\Delta y\end{aligned}$$

例 设  $z = f(x, y) = x^2 + y^2$ , 则

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= [(x + \Delta x)^2 + (y + \Delta y)^2] - [x^2 + y^2]\end{aligned}$$

# 多元函数的全微分

- 二元函数  $z = f(x, y)$  的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称  $z = f(x, y)$  可微是指:

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \approx A\Delta x + B\Delta y\end{aligned}$$

例 设  $z = f(x, y) = x^2 + y^2$ , 则

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= [(x + \Delta x)^2 + (y + \Delta y)^2] - [x^2 + y^2]\end{aligned}$$

# 多元函数的全微分

- 二元函数  $z = f(x, y)$  的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称  $z = f(x, y)$  可微是指:

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \approx A\Delta x + B\Delta y\end{aligned}$$

例 设  $z = f(x, y) = x^2 + y^2$ , 则

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= [(x + \Delta x)^2 + (y + \Delta y)^2] - [x^2 + y^2] \\ &= 2x\Delta x + 2y\Delta y + [(\Delta x)^2 + (\Delta y)^2]\end{aligned}$$

# 多元函数的全微分

- 二元函数  $z = f(x, y)$  的全增量

$$\Delta z = f(x + \Delta x, y + \Delta y) - f(x, y)$$

- 称  $z = f(x, y)$  可微是指:

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= A\Delta x + B\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right) \approx A\Delta x + B\Delta y\end{aligned}$$

例 设  $z = f(x, y) = x^2 + y^2$ , 则

$$\begin{aligned}\Delta z &= f(x + \Delta x, y + \Delta y) - f(x, y) \\ &= [(x + \Delta x)^2 + (y + \Delta y)^2] - [x^2 + y^2] \\ &= 2x\Delta x + 2y\Delta y + [(\Delta x)^2 + (\Delta y)^2]\end{aligned}$$

所以  $z = x^2 + y^2$  可微。

## 多元函数的全微分 (Cont.)

---

- 若  $z = f(x, y)$  可微, 则连续, 且存在偏导数  $z_x, z_y$ , 还有
$$\Delta z = f(x + \Delta x) - f(x)$$



## 多元函数的全微分 (Cont.)

- 若  $z = f(x, y)$  可微, 则连续, 且存在偏导数  $z_x, z_y$ , 还有

$$\Delta z = f(x + \Delta x) - f(x)$$

$$= z_x(x, y)\Delta x + z_y(x, y)\Delta y +$$

## 多元函数的全微分 (Cont.)

- 若  $z = f(x, y)$  可微, 则连续, 且存在偏导数  $z_x, z_y$ , 还有

$$\Delta z = f(x + \Delta x) - f(x)$$

$$= z_x(x, y)\Delta x + z_y(x, y)\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right)$$

## 多元函数的全微分 (Cont.)

- 若  $z = f(x, y)$  可微, 则连续, 且存在偏导数  $z_x, z_y$ , 还有

$$\Delta z = f(x + \Delta x) - f(x)$$

$$= z_x(x, y)\Delta x + z_y(x, y)\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right)$$

$$\approx z_x(x, y)\Delta x + z_y(x, y)\Delta y$$

## 多元函数的全微分 (Cont.)

- 若  $z = f(x, y)$  可微, 则连续, 且存在偏导数  $z_x, z_y$ , 还有

$$\Delta z = f(x + \Delta x) - f(x)$$

$$= z_x(x, y)\Delta x + z_y(x, y)\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right)$$

$$\approx z_x(x, y)\Delta x + z_y(x, y)\Delta y$$

$z = f(x, y)$  的全微分:  $dz = z_x(x, y)dx + z_y(x, y)dy$

## 多元函数的全微分 (Cont.)

- 若  $z = f(x, y)$  可微, 则连续, 且存在偏导数  $z_x, z_y$ , 还有

$$\Delta z = f(x + \Delta x) - f(x)$$

$$= z_x(x, y)\Delta x + z_y(x, y)\Delta y + o\left(\sqrt{(\Delta x)^2 + (\Delta y)^2}\right)$$

$$\approx z_x(x, y)\Delta x + z_y(x, y)\Delta y$$

$z = f(x, y)$  的全微分:  $dz = z_x(x, y)dx + z_y(x, y)dy$

- 若  $z = f(x, y)$  可微, 则  $\Delta z \approx dz$

## 多元函数的全微分 (Cont.)

---

- 对三元函数  $u = \varphi(x, y, z)$ , 其全微分

$$du = u_x dx + u_y dy + u_z dz$$

## 多元函数的全微分 (Cont.)

- 对三元函数  $u = \varphi(x, y, z)$ , 其全微分

$$du = u_x dx + u_y dy + u_z dz$$

此时

$$\Delta u = \varphi(x + \Delta x, y + \Delta y, z + \Delta z) - \varphi(x, y, z)$$

## 多元函数的全微分 (Cont.)

- 对三元函数  $u = \varphi(x, y, z)$ , 其全微分

$$du = u_x dx + u_y dy + u_z dz$$

此时

$$\Delta u = \varphi(x + \Delta x, y + \Delta y, z + \Delta z) - \varphi(x, y, z) \approx du$$



例 计算函数  $z = \frac{y}{x}$  的全微分

解

例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x =$$

$$z_y =$$

例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x =$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x =$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy =$$

例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$



例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$

---

例 计算函数  $z = x^2y + y^2$  的全微分

例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$

---

例 计算函数  $z = x^2y + y^2$  的全微分

解

$$z_x =$$

$$z_y =$$

例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$

---

例 计算函数  $z = x^2y + y^2$  的全微分

解

$$z_x =$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$

---

例 计算函数  $z = x^2y + y^2$  的全微分

解

$$z_x = (x^2y + y^2)'_x =$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$

---

例 计算函数  $z = x^2y + y^2$  的全微分

解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x =$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$

---

例 计算函数  $z = x^2y + y^2$  的全微分

解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x = 2xy$$

$$z_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$

---

例 计算函数  $z = x^2y + y^2$  的全微分

解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x = 2xy$$

$$z_y = (x^2y + y^2)'_y =$$

$$dz = z_x dx + z_y dy =$$

例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$

---

例 计算函数  $z = x^2y + y^2$  的全微分

解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x = 2xy$$

$$z_y = (x^2y + y^2)'_y = (x^2y)'_y + (y^2)'_y =$$

$$dz = z_x dx + z_y dy =$$



例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$

---

例 计算函数  $z = x^2y + y^2$  的全微分

解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x = 2xy$$

$$z_y = (x^2y + y^2)'_y = (x^2y)'_y + (y^2)'_y = x^2 + 2y$$

$$dz = z_x dx + z_y dy =$$

例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$

---

例 计算函数  $z = x^2y + y^2$  的全微分

解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x = 2xy$$

$$z_y = (x^2y + y^2)'_y = (x^2y)'_y + (y^2)'_y = x^2 + 2y$$

$$dz = z_x dx + z_y dy =$$

例 计算函数  $z = \frac{y}{x}$  的全微分

解

$$z_x = \left(\frac{y}{x}\right)'_x = -\frac{y}{x^2}$$

$$z_y = \left(\frac{y}{x}\right)'_y = \frac{1}{x}$$

$$dz = z_x dx + z_y dy = -\frac{y}{x^2} dx + \frac{1}{x} dy$$

---

例 计算函数  $z = x^2y + y^2$  的全微分

解

$$z_x = (x^2y + y^2)'_x = (x^2y)'_x + (y^2)'_x = 2xy$$

$$z_y = (x^2y + y^2)'_y = (x^2y)'_y + (y^2)'_y = x^2 + 2y$$

$$dz = z_x dx + z_y dy = 2xy dx + (x^2 + 2y) dy$$

例 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

解

例 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

解  $z_x =$  ,  $z_y =$

例 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

解

$$z_x = \quad , \quad z_y =$$
$$dz = z_x dx + z_y dy =$$

例 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

解

$$z_x = (xy)'_x = \quad , \quad z_y =$$
$$dz = z_x dx + z_y dy =$$

例 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

解

$$z_x = (xy)'_x = y, \quad z_y =$$
$$dz = z_x dx + z_y dy =$$



例 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y =$$
$$dz = z_x dx + z_y dy =$$

例 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy =$$

例 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = ydx + xdy$$

例 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = ydx + xdy$$

将  $(x, y) = (2, 3)$  及  $\Delta x = 0.1$ 、 $\Delta y = 0.2$  代入得:

$$dz =$$

例 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = ydx + xdy$$

将  $(x, y) = (2, 3)$  及  $\Delta x = 0.1$ 、 $\Delta y = 0.2$  代入得:

$$dz = 3 \times 0.1 +$$

例 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = ydx + xdy$$

将  $(x, y) = (2, 3)$  及  $\Delta x = 0.1$ 、 $\Delta y = 0.2$  代入得:

$$dz = 3 \times 0.1 + 2 \times 0.2 =$$

例 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = ydx + xdy$$

将  $(x, y) = (2, 3)$  及  $\Delta x = 0.1$ 、 $\Delta y = 0.2$  代入得:

$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

例 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = ydx + xdy$$

将  $(x, y) = (2, 3)$  及  $\Delta x = 0.1$ 、 $\Delta y = 0.2$  代入得:

$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

而全增量为  $\Delta z =$



**例** 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

**解**

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = ydx + xdy$$

将  $(x, y) = (2, 3)$  及  $\Delta x = 0.1$ 、 $\Delta y = 0.2$  代入得:

$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

而全增量为

$$\Delta z = z(2 + 0.1, 3 + 0.2) - z(2, 3)$$

**例** 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

**解**

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = y dx + x dy$$

将  $(x, y) = (2, 3)$  及  $\Delta x = 0.1$ 、 $\Delta y = 0.2$  代入得:

$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

而全增量为

$$\begin{aligned} \Delta z &= z(2 + 0.1, 3 + 0.2) - z(2, 3) \\ &= (2 + 0.1) \times (3 + 0.2) - \end{aligned}$$

**例** 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

**解**

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = y dx + x dy$$

将  $(x, y) = (2, 3)$  及  $\Delta x = 0.1$ 、 $\Delta y = 0.2$  代入得:

$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

而全增量为

$$\begin{aligned}\Delta z &= z(2 + 0.1, 3 + 0.2) - z(2, 3) \\ &= (2 + 0.1) \times (3 + 0.2) - 2 \times 3\end{aligned}$$

例 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = ydx + xdy$$

将  $(x, y) = (2, 3)$  及  $\Delta x = 0.1$ 、 $\Delta y = 0.2$  代入得:

$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

而全增量为

$$\begin{aligned}\Delta z &= z(2 + 0.1, 3 + 0.2) - z(2, 3) \\ &= (2 + 0.1) \times (3 + 0.2) - 2 \times 3 \\ &= 0.72\end{aligned}$$

例 求  $z = xy$  在点  $(2, 3)$  处, 关于  $\Delta x = 0.1$ ,  $\Delta y = 0.2$  的全增量  $\Delta z$  及全微分  $dz$ 。

解

$$z_x = (xy)'_x = y, \quad z_y = (xy)'_y = x$$
$$dz = z_x dx + z_y dy = ydx + xdy$$

将  $(x, y) = (2, 3)$  及  $\Delta x = 0.1$ 、 $\Delta y = 0.2$  代入得:

$$dz = 3 \times 0.1 + 2 \times 0.2 = 0.7$$

而全增量为

$$\begin{aligned}\Delta z &= z(2 + 0.1, 3 + 0.2) - z(2, 3) \\ &= (2 + 0.1) \times (3 + 0.2) - 2 \times 3 \\ &= 0.72 \\ &\approx dz\end{aligned}$$