



# 教学要求

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◇ 熟练掌握分部积分法



# Outline of §5.4

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# 分部积分法，能干啥？

能够计算如下的不定积分：

$$\int x e^x dx, \quad \int x^2 \ln x dx, \quad \int x \arctan x dx, \quad \int x \cos x dx$$

$$\int \ln x dx, \quad \int \arcsin x dx, \quad \int \arctan x dx, \quad \int \ln(1+x^2) dx$$

$$\int x^2 e^x dx, \quad \int e^x \cos x dx \quad \dots\dots$$

（可能要结合前面学的换元积分法）

# 分部积分引入

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- 微分公式

$$d(uv) = u dv + v du$$

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- 微分公式

$$d(uv) = u dv + v du \quad \Rightarrow \quad u dv = d(uv) - v du$$

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- 微分公式

$$d(uv) = u dv + v du \Rightarrow u dv = d(uv) - v du$$

练习

$$x \cos x dx =$$

$$x e^x dx =$$

$$\ln x dx =$$

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- 微分公式

$$d(uv) = u dv + v du \Rightarrow u dv = d(uv) - v du$$

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$$x \cos x dx = x d \sin x =$$

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$$d(uv) = u dv + v du \Rightarrow u dv = d(uv) - v du$$

## 练习

$$x \cos x dx = x d \sin x = d(x \sin x) - \sin x dx$$

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## 应用

$$x \cos x dx = \quad x d \sin x = \quad d(x \sin x) - \quad \sin x dx$$

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应用

$$\int x \cos x dx = \int x d \sin x = \int d(x \sin x) - \int \sin x dx$$

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## 应用

$$\begin{aligned} \int x \cos x dx &= \int x d \sin x = \int d(x \sin x) - \int \sin x dx \\ &= x \sin x + \cos x + C \end{aligned}$$



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$$\begin{aligned} \int x \cos x dx &= \int x d \sin x = \int d(x \sin x) - \int \sin x dx \\ &= x \sin x + \cos x + C \end{aligned}$$

$$\int \ln x dx = \int d(x \ln x) - \int x d \ln x$$

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$$\begin{aligned} \int x \cos x dx &= \int x d \sin x = \int d(x \sin x) - \int \sin x dx \\ &= x \sin x + \cos x + C \end{aligned}$$

$$\int \ln x dx = \int d(x \ln x) - \int x d \ln x = \int d(x \ln x) - \int dx$$

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$$\ln x dx = d(x \ln x) - x d \ln x = d(x \ln x) - dx$$

## 应用

$$\begin{aligned} \int x \cos x dx &= \int x d \sin x = \int d(x \sin x) - \int \sin x dx \\ &= x \sin x + \cos x + C \end{aligned}$$

$$\begin{aligned} \int \ln x dx &= \int d(x \ln x) - \int x d \ln x = \int d(x \ln x) - \int dx \\ &= x \ln x - x + C \end{aligned}$$

# 分部积分公式

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- 微分公式

$$d(uv) = u dv + v du \quad \Rightarrow \quad u dv = d(uv) - v du$$

- 两边积分得：

# 分部积分公式

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- 微分公式

$$d(uv) = u dv + v du \quad \Rightarrow \quad u dv = d(uv) - v du$$

- 两边积分得：

$$\int u dv = uv - \int v du$$

# 分部积分公式

- 微分公式

$$d(uv) = u dv + v du \Rightarrow u dv = d(uv) - v du$$

- 两边积分得：

$$\int u dv = uv - \int v du$$

- 实际应用时的步骤：

$$\int \text{"İıżğÆć"} dx =$$



# 分部积分公式

- 微分公式

$$d(uv) = u dv + v du \Rightarrow u dv = d(uv) - v du$$

- 两边积分得：

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$$\int u' v dx = \int u v' dx$$

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$$d(uv) = u dv + v du \Rightarrow u dv = d(uv) - v du$$

- 两边积分得：

$$\int u dv = uv - \int v du$$

- 实际应用时的步骤：

$$\int u v' dx = \int u dv$$

凑微分

$$\int u dv =$$

# 分部积分公式

- 微分公式

$$d(uv) = u dv + v du \Rightarrow u dv = d(uv) - v du$$

- 两边积分得：

$$\int u dv = uv - \int v du$$

- 实际应用时的步骤：

$$\int u' v dx = \int u v' dx$$

$$\underline{\underline{\text{凑微分}}} \int u dv = uv - \int v du$$

# 分部积分公式

- 微分公式

$$d(uv) = u dv + v du \Rightarrow u dv = d(uv) - v du$$

- 两边积分得：

$$\int u dv = uv - \int v du$$

- 实际应用时的步骤：

$$\int f(x) g'(x) dx = \int u v' dx$$

$$\underline{\underline{\text{凑微分}}} \int u dv = uv - \underbrace{\int v du}_{\text{简单、易算}}$$

例子 求不定积分  $\int x \cos x dx$ ,  $\int x e^x dx$ ,  $\int x \sin x dx$

例子 求不定积分  $\int x \cos x dx$ ,  $\int x e^x dx$ ,  $\int x \sin x dx$

解

$$\int x \cos x dx =$$

例子 求不定积分  $\int x \cos x dx$ ,  $\int x e^x dx$ ,  $\int x \sin x dx$

解

$$\int x \cos x dx = \int x d \sin x =$$

例子 求不定积分  $\int x \cos x dx$ ,  $\int x e^x dx$ ,  $\int x \sin x dx$

解

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx$$



例子 求不定积分  $\int x \cos x dx$ ,  $\int x e^x dx$ ,  $\int x \sin x dx$

解

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

例子 求不定积分  $\int x \cos x dx$ ,  $\int x e^x dx$ ,  $\int x \sin x dx$

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$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

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行不通的做法

$$\int x \cos x dx = \int \cos x \cdot d\left(\frac{1}{2}x^2\right)$$

例子 求不定积分  $\int x \cos x dx$ ,  $\int x e^x dx$ ,  $\int x \sin x dx$

解

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

---

行不通的做法

$$\int x \cos x dx = \int \cos x \cdot d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cos x - \int \frac{1}{2}x^2 d \cos x$$

例子 求不定积分  $\int x \cos x dx$ ,  $\int x e^x dx$ ,  $\int x \sin x dx$

解

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

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行不通的做法

$$\begin{aligned} \int x \cos x dx &= \int \cos x \cdot d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cos x - \int \frac{1}{2}x^2 d \cos x \\ &= \frac{1}{2}x^2 \cos x + \int \frac{1}{2}x^2 \sin x dx \end{aligned}$$

例子 求不定积分  $\int x \cos x dx$ ,  $\int x e^x dx$ ,  $\int x \sin x dx$

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$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

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$$\begin{aligned} \int x \cos x dx &= \int \cos x \cdot d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cos x - \int \frac{1}{2}x^2 d \cos x \\ &= \frac{1}{2}x^2 \cos x + \underbrace{\int \frac{1}{2}x^2 \sin x dx}_{\text{更加复杂!}} = \dots \end{aligned}$$

例子 求不定积分  $\int x \cos x dx$ ,  $\int x e^x dx$ ,  $\int x \sin x dx$

解

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

$$\int x e^x dx =$$

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$$\begin{aligned} \int x \cos x dx &= \int \cos x \cdot d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cos x - \int \frac{1}{2}x^2 d \cos x \\ &= \frac{1}{2}x^2 \cos x + \underbrace{\int \frac{1}{2}x^2 \sin x dx}_{\text{更加复杂!}} = \dots \end{aligned}$$

例子 求不定积分  $\int x \cos x dx$ ,  $\int x e^x dx$ ,  $\int x \sin x dx$

解

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

$$\int x e^x dx = \int x d e^x =$$

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行不通的做法

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例子 求不定积分  $\int x \cos x dx$ ,  $\int x e^x dx$ ,  $\int x \sin x dx$

解

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

$$\int x e^x dx = \int x d e^x = x e^x - \int e^x dx =$$

$$\int x \sin x dx =$$

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$$\begin{aligned} \int x \cos x dx &= \int \cos x \cdot d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cos x - \int \frac{1}{2}x^2 d \cos x \\ &= \frac{1}{2}x^2 \cos x + \underbrace{\int \frac{1}{2}x^2 \sin x dx}_{\text{更加复杂!}} = \dots \end{aligned}$$



例子 求不定积分  $\int x \cos x dx$ ,  $\int x e^x dx$ ,  $\int x \sin x dx$

解

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

$$\int x e^x dx = \int x d e^x = x e^x - \int e^x dx = x e^x - e^x + C$$

$$\int x \sin x dx =$$

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例子 求不定积分  $\int x \cos x dx$ ,  $\int x e^x dx$ ,  $\int x \sin x dx$

解

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

$$\int x e^x dx = \int x d e^x = x e^x - \int e^x dx = x e^x - e^x + C$$

$$\int x \sin x dx = \int x d(-\cos x) =$$

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行不通的做法

$$\begin{aligned} \int x \cos x dx &= \int \cos x \cdot d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cos x - \int \frac{1}{2}x^2 d \cos x \\ &= \frac{1}{2}x^2 \cos x + \underbrace{\int \frac{1}{2}x^2 \sin x dx}_{\text{更加复杂!}} = \dots \end{aligned}$$

例子 求不定积分  $\int x \cos x dx$ ,  $\int x e^x dx$ ,  $\int x \sin x dx$

解

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

$$\int x e^x dx = \int x d e^x = x e^x - \int e^x dx = x e^x - e^x + C$$

$$\int x \sin x dx = \int x d(-\cos x) = x(-\cos x) - \int (-\cos x) dx$$

---

行不通的做法

$$\begin{aligned} \int x \cos x dx &= \int \cos x \cdot d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cos x - \int \frac{1}{2}x^2 d \cos x \\ &= \frac{1}{2}x^2 \cos x + \underbrace{\int \frac{1}{2}x^2 \sin x dx}_{\text{更加复杂!}} = \dots \end{aligned}$$

例子 求不定积分  $\int x \cos x dx$ ,  $\int x e^x dx$ ,  $\int x \sin x dx$

解

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

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$$\begin{aligned} \int x \sin x dx &= \int x d(-\cos x) = x(-\cos x) - \int (-\cos x) dx \\ &= -x \cos x + \sin x + C \end{aligned}$$

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例子 求不定积分  $\int x \ln x dx$ ,  $\int x^2 \ln x dx$

例子 求不定积分  $\int x \ln x dx$ ,  $\int x^2 \ln x dx$

解 
$$\int x \ln x dx =$$

$$\int x^2 \ln x dx =$$

例子 求不定积分  $\int x \ln x dx$ ,  $\int x^2 \ln x dx$

解 
$$\int x \ln x dx = \int \ln x d\left(\frac{1}{2}x^2\right) =$$

$$\int x^2 \ln x dx =$$

例子 求不定积分  $\int x \ln x dx$ ,  $\int x^2 \ln x dx$

解 
$$\int x \ln x dx = \int \ln x d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 d \ln x$$

$$\int x^2 \ln x dx =$$



例子 求不定积分  $\int x \ln x dx$ ,  $\int x^2 \ln x dx$

解 
$$\begin{aligned}\int x \ln x dx &= \int \ln x d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 d \ln x \\ &= \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx\end{aligned}$$

$$\int x^2 \ln x dx =$$

例子 求不定积分  $\int x \ln x dx$ ,  $\int x^2 \ln x dx$

解

$$\begin{aligned}\int x \ln x dx &= \int \ln x d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 d \ln x \\ &= \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx \\ &= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C\end{aligned}$$

$$\int x^2 \ln x dx =$$

例子 求不定积分  $\int x \ln x dx$ ,  $\int x^2 \ln x dx$

解

$$\begin{aligned}\int x \ln x dx &= \int \ln x d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 d \ln x \\&= \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx \\&= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C\end{aligned}$$
$$\int x^2 \ln x dx = \int \ln x d\left(\frac{1}{3}x^3\right) =$$

例子 求不定积分  $\int x \ln x dx$ ,  $\int x^2 \ln x dx$

解

$$\begin{aligned}\int x \ln x dx &= \int \ln x d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 d \ln x \\&= \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx \\&= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C\end{aligned}$$
$$\int x^2 \ln x dx = \int \ln x d\left(\frac{1}{3}x^3\right) = \frac{1}{3}x^3 \cdot \ln x - \int \frac{1}{3}x^3 d \ln x$$

例子 求不定积分  $\int x \ln x dx$ ,  $\int x^2 \ln x dx$

解

$$\begin{aligned}\int x \ln x dx &= \int \ln x d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 d \ln x \\&= \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx \\&= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C\end{aligned}$$
$$\begin{aligned}\int x^2 \ln x dx &= \int \ln x d\left(\frac{1}{3}x^3\right) = \frac{1}{3}x^3 \cdot \ln x - \int \frac{1}{3}x^3 d \ln x \\&= \frac{1}{3}x^3 \cdot \ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x} dx\end{aligned}$$

例子 求不定积分  $\int x \ln x dx$ ,  $\int x^2 \ln x dx$

解

$$\begin{aligned}\int x \ln x dx &= \int \ln x d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 d \ln x \\&= \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx \\&= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C\end{aligned}$$
$$\begin{aligned}\int x^2 \ln x dx &= \int \ln x d\left(\frac{1}{3}x^3\right) = \frac{1}{3}x^3 \cdot \ln x - \int \frac{1}{3}x^3 d \ln x \\&= \frac{1}{3}x^3 \cdot \ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x} dx \\&= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C\end{aligned}$$

例子 求不定积分  $\int x \arctan x dx$

例子 求不定积分  $\int x \arctan x dx$

解  $\int x \arctan x dx =$



例子 求不定积分  $\int x \arctan x dx$

解 
$$\int x \arctan x dx = \int \arctan x d\left(\frac{1}{2}x^2\right)$$

例子 求不定积分  $\int x \arctan x dx$

解 
$$\begin{aligned}\int x \arctan x dx &= \int \arctan x d\left(\frac{1}{2}x^2\right) \\ &= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 d \arctan x\end{aligned}$$

例子 求不定积分  $\int x \arctan x dx$

解

$$\begin{aligned}\int x \arctan x dx &= \int \arctan x d\left(\frac{1}{2}x^2\right) \\&= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 d \arctan x \\&= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2} dx\end{aligned}$$

例子 求不定积分  $\int x \arctan x dx$

解

$$\begin{aligned}\int x \arctan x dx &= \int \arctan x d\left(\frac{1}{2}x^2\right) \\&= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 d \arctan x \\&= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2} dx \\&= \frac{1}{2}x^2 \arctan x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx\end{aligned}$$

例子 求不定积分  $\int x \arctan x dx$

解

$$\begin{aligned}\int x \arctan x dx &= \int \arctan x d\left(\frac{1}{2}x^2\right) \\&= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 d \arctan x \\&= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2} dx \\&= \frac{1}{2}x^2 \arctan x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx \\&= \frac{1}{2}x^2 \arctan x - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2}\right) dx\end{aligned}$$

例子 求不定积分  $\int x \arctan x dx$

解

$$\begin{aligned}\int x \arctan x dx &= \int \arctan x d\left(\frac{1}{2}x^2\right) \\&= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 d \arctan x \\&= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2} dx \\&= \frac{1}{2}x^2 \arctan x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx \\&= \frac{1}{2}x^2 \arctan x - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2}\right) dx \\&= \frac{1}{2}x^2 \arctan x - \frac{1}{2}x + \frac{1}{2} \arctan x + C\end{aligned}$$

例子 求不定积分  $\int \ln x dx$ ,  $\int \ln(1 + x^2) dx$

例子 求不定积分  $\int \ln x dx$ ,  $\int \ln(1+x^2)dx$

解 
$$\int \ln x dx =$$



例子 求不定积分  $\int \ln x dx$ ,  $\int \ln(1+x^2)dx$

解 
$$\int \ln x dx = x \ln x - \int x d \ln x =$$

例子 求不定积分  $\int \ln x dx$ ,  $\int \ln(1+x^2)dx$

解 
$$\int \ln x dx = x \ln x - \int x d \ln x = x \ln x - \int x \cdot \frac{1}{x} dx$$

例子 求不定积分  $\int \ln x dx$ ,  $\int \ln(1+x^2)dx$

解

$$\begin{aligned}\int \ln x dx &= x \ln x - \int x d \ln x = x \ln x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - x + C\end{aligned}$$

例子 求不定积分  $\int \ln x dx$ ,  $\int \ln(1+x^2)dx$

解

$$\begin{aligned}\int \ln x dx &= x \ln x - \int x d \ln x = x \ln x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - x + C\end{aligned}$$

$$\int \ln(1+x^2)dx =$$

例子 求不定积分  $\int \ln x dx$ ,  $\int \ln(1+x^2)dx$

解

$$\begin{aligned}\int \ln x dx &= x \ln x - \int x d \ln x = x \ln x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - x + C\end{aligned}$$

$$\int \ln(1+x^2)dx = x \ln(1+x^2) - \int x d \ln(1+x^2)$$

例子 求不定积分  $\int \ln x dx$ ,  $\int \ln(1+x^2) dx$

解

$$\begin{aligned}\int \ln x dx &= x \ln x - \int x d \ln x = x \ln x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - x + C\end{aligned}$$

$$\begin{aligned}\int \ln(1+x^2) dx &= x \ln(1+x^2) - \int x d \ln(1+x^2) \\ &= x \ln(1+x^2) - \int x \cdot \frac{2x}{1+x^2} dx\end{aligned}$$

例子 求不定积分  $\int \ln x dx$ ,  $\int \ln(1+x^2) dx$

解

$$\begin{aligned}\int \ln x dx &= x \ln x - \int x d \ln x = x \ln x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - x + C\end{aligned}$$

$$\begin{aligned}\int \ln(1+x^2) dx &= x \ln(1+x^2) - \int x d \ln(1+x^2) \\ &= x \ln(1+x^2) - \int x \cdot \frac{2x}{1+x^2} dx \\ &= x \ln(1+x^2) - 2 \int \frac{x^2}{1+x^2} dx\end{aligned}$$

例子 求不定积分  $\int \ln x dx$ ,  $\int \ln(1+x^2) dx$

解

$$\begin{aligned}\int \ln x dx &= x \ln x - \int x d \ln x = x \ln x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - x + C\end{aligned}$$

$$\begin{aligned}\int \ln(1+x^2) dx &= x \ln(1+x^2) - \int x d \ln(1+x^2) \\ &= x \ln(1+x^2) - \int x \cdot \frac{2x}{1+x^2} dx \\ &= x \ln(1+x^2) - 2 \int \frac{x^2}{1+x^2} dx \\ &= x \ln(1+x^2) - 2 \int \left(1 - \frac{1}{1+x^2}\right) dx\end{aligned}$$



例子 求不定积分  $\int \ln x dx$ ,  $\int \ln(1+x^2) dx$

解

$$\begin{aligned}\int \ln x dx &= x \ln x - \int x d \ln x = x \ln x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - x + C\end{aligned}$$

$$\begin{aligned}\int \ln(1+x^2) dx &= x \ln(1+x^2) - \int x d \ln(1+x^2) \\ &= x \ln(1+x^2) - \int x \cdot \frac{2x}{1+x^2} dx \\ &= x \ln(1+x^2) - 2 \int \frac{x^2}{1+x^2} dx \\ &= x \ln(1+x^2) - 2 \int \left(1 - \frac{1}{1+x^2}\right) dx \\ &= x \ln(1+x^2) - 2x + 2 \arctan x + C\end{aligned}$$

例子 求不定积分  $\int \arctan x dx$

例子 求不定积分  $\int \arctan x dx$

解

$$\int \arctan x dx =$$

例子 求不定积分  $\int \arctan x dx$

解

$$\int \arctan x dx = x \arctan x - \int x d \arctan x$$

例子 求不定积分  $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x \arctan x - \int x d \arctan x \\ &= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx\end{aligned}$$

例子 求不定积分  $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x \arctan x - \int x d \arctan x \\&= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx \\&= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} dx^2\end{aligned}$$

例子 求不定积分  $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x \arctan x - \int x d \arctan x \\&= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx \\&= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} dx^2 \\&= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} d(1+x^2)\end{aligned}$$

例子 求不定积分  $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x \arctan x - \int x d \arctan x \\&= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx \\&= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} dx^2 \\&= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} d(1+x^2)\end{aligned}$$

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$$\int \frac{1}{1+x^2} d(1+x^2) =$$



例子 求不定积分  $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x \arctan x - \int x d \arctan x \\&= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx \\&= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} dx^2 \\&= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} d(1+x^2)\end{aligned}$$

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$$\int \frac{1}{1+x^2} d(1+x^2) = \int \frac{1}{u} du =$$

例子 求不定积分  $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x \arctan x - \int x d \arctan x \\&= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx \\&= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} dx^2 \\&= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} d(1+x^2)\end{aligned}$$

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$$\int \frac{1}{1+x^2} d(1+x^2) = \int \frac{1}{u} du = \ln u + C =$$

例子 求不定积分  $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x \arctan x - \int x d \arctan x \\&= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx \\&= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} dx^2 \\&= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} d(1+x^2)\end{aligned}$$

---

$$\int \frac{1}{1+x^2} d(1+x^2) = \int \frac{1}{u} du = \ln u + C = \ln(1+x^2) + C$$

例子 求不定积分  $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x \arctan x - \int x d \arctan x \\&= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx \\&= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} dx^2 \\&= x \arctan x - \frac{1}{2} \int \frac{1}{1+x^2} d(1+x^2) \\&= x \arctan x - \frac{1}{2} \ln(1+x^2) + C.\end{aligned}$$

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$$\int \frac{1}{1+x^2} d(1+x^2) = \int \frac{1}{u} du = \ln u + C = \ln(1+x^2) + C$$

例子 求不定积分  $\int \arcsin x dx$

例子 求不定积分  $\int \arcsin x dx$

解  $\int \arcsin x dx =$

例子 求不定积分  $\int \arcsin x dx$

解 
$$\int \arcsin x dx = x \arcsin x - \int x d \arcsin x$$

例子 求不定积分  $\int \arcsin x dx$

解 
$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\ &= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx\end{aligned}$$



例子 求不定积分  $\int \arcsin x dx$

解

$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\&= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\&= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx^2\end{aligned}$$

例子 求不定积分  $\int \arcsin x dx$

解 
$$\int \arcsin x dx = x \arcsin x - \int x d \arcsin x$$

$$= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$$

$$= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx^2$$

$$= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} \cdot (-1) d(1-x^2)$$

例子 求不定积分  $\int \arcsin x dx$

解

$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\&= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\&= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx^2 \\&= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} \cdot (-1) d(1-x^2)\end{aligned}$$

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$$\int \frac{1}{\sqrt{1-x^2}} d(1-x^2) =$$

例子 求不定积分  $\int \arcsin x dx$

解

$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\&= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\&= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx^2 \\&= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} \cdot (-1) d(1-x^2)\end{aligned}$$

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$$\int \frac{1}{\sqrt{1-x^2}} d(1-x^2) = \int u^{-\frac{1}{2}} du =$$

例子 求不定积分  $\int \arcsin x dx$

解 
$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\&= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\&= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx^2 \\&= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} \cdot (-1) d(1-x^2)\end{aligned}$$

---

$$\int \frac{1}{\sqrt{1-x^2}} d(1-x^2) = \int u^{-\frac{1}{2}} du = 2u^{\frac{1}{2}} + C =$$

例子 求不定积分  $\int \arcsin x dx$

解 
$$\int \arcsin x dx = x \arcsin x - \int x d \arcsin x$$

$$= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$$

$$= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx^2$$

$$= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} \cdot (-1) d(1-x^2)$$

---

$$\int \frac{1}{\sqrt{1-x^2}} d(1-x^2) = \int u^{-\frac{1}{2}} du = 2u^{\frac{1}{2}} + C = 2(1-x^2)^{\frac{1}{2}} + C$$

例子 求不定积分  $\int \arcsin x dx$

解 
$$\int \arcsin x dx = x \arcsin x - \int x d \arcsin x$$

$$= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$$

$$= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx^2$$

$$= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} \cdot (-1) d(1-x^2)$$

$$= x \arcsin x + \sqrt{1-x^2} + C.$$

---

$$\int \frac{1}{\sqrt{1-x^2}} d(1-x^2) = \int u^{-\frac{1}{2}} du = 2u^{\frac{1}{2}} + C = 2(1-x^2)^{\frac{1}{2}} + C$$

例子 求不定积分  $\int x^2 e^x dx$ ,  $\int x^2 \sin x dx$



例子 求不定积分  $\int x^2 e^x dx$ ,  $\int x^2 \sin x dx$  (提示 两次分部积分)

例子 求不定积分  $\int x^2 e^x dx$ ,  $\int x^2 \sin x dx$  (提示 两次分部积分)

解  $\int x^2 e^x dx =$

例子 求不定积分  $\int x^2 e^x dx$ ,  $\int x^2 \sin x dx$  (提示 两次分部积分)

解 
$$\int x^2 e^x dx = \int x^2 de^x =$$

例子 求不定积分  $\int x^2 e^x dx$ ,  $\int x^2 \sin x dx$  (提示 两次分部积分)

解 
$$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 =$$

例子 求不定积分  $\int x^2 e^x dx$ ,  $\int x^2 \sin x dx$  (提示 两次分部积分)

解 
$$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx$$

例子 求不定积分  $\int x^2 e^x dx$ ,  $\int x^2 \sin x dx$  (提示 两次分部积分)

解 
$$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx$$
$$= x^2 e^x - 2 \left( \int x de^x \right) =$$

例子 求不定积分  $\int x^2 e^x dx$ ,  $\int x^2 \sin x dx$  (提示 两次分部积分)

解 
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left( \int x de^x \right) = x^2 e^x - 2 \left( x e^x - \int e^x dx \right)\end{aligned}$$

例子 求不定积分  $\int x^2 e^x dx$ ,  $\int x^2 \sin x dx$  (提示 两次分部积分)

解 
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left( \int x de^x \right) = x^2 e^x - 2 \left( x e^x - \int e^x dx \right) \\ &= x^2 e^x - 2 x e^x + 2 e^x + C\end{aligned}$$



例子 求不定积分  $\int x^2 e^x dx$ ,  $\int x^2 \sin x dx$  (提示 两次分部积分)

解 
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left( \int x de^x \right) = x^2 e^x - 2 \left( x e^x - \int e^x dx \right) \\ &= x^2 e^x - 2 x e^x + 2 e^x + C\end{aligned}$$

$$\int x^2 \sin x dx =$$

例子 求不定积分  $\int x^2 e^x dx$ ,  $\int x^2 \sin x dx$  (提示 两次分部积分)

解 
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left( \int x de^x \right) = x^2 e^x - 2 \left( x e^x - \int e^x dx \right) \\ &= x^2 e^x - 2 x e^x + 2 e^x + C\end{aligned}$$

$$\int x^2 \sin x dx = - \int x^2 d \cos x =$$

例子 求不定积分  $\int x^2 e^x dx$ ,  $\int x^2 \sin x dx$  (提示 两次分部积分)

解 
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left( \int x de^x \right) = x^2 e^x - 2 \left( x e^x - \int e^x dx \right) \\ &= x^2 e^x - 2 x e^x + 2 e^x + C\end{aligned}$$

$$\int x^2 \sin x dx = - \int x^2 d \cos x = -x^2 \cos x + \int \cos x dx^2$$

例子 求不定积分  $\int x^2 e^x dx$ ,  $\int x^2 \sin x dx$  (提示 两次分部积分)

解 
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left( \int x de^x \right) = x^2 e^x - 2 \left( x e^x - \int e^x dx \right) \\ &= x^2 e^x - 2 x e^x + 2 e^x + C\end{aligned}$$

$$\begin{aligned}\int x^2 \sin x dx &= - \int x^2 d \cos x = -x^2 \cos x + \int \cos x dx^2 \\ &= -x^2 \cos x + 2 \int x \cos x dx\end{aligned}$$

例子 求不定积分  $\int x^2 e^x dx$ ,  $\int x^2 \sin x dx$  (提示 两次分部积分)

解 
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left( \int x de^x \right) = x^2 e^x - 2 \left( x e^x - \int e^x dx \right) \\ &= x^2 e^x - 2 x e^x + 2 e^x + C\end{aligned}$$

$$\begin{aligned}\int x^2 \sin x dx &= - \int x^2 d \cos x = -x^2 \cos x + \int \cos x dx^2 \\ &= -x^2 \cos x + 2 \int x \cos x dx \\ &= -x^2 \cos x + 2 \int x d \sin x\end{aligned}$$

例子 求不定积分  $\int x^2 e^x dx$ ,  $\int x^2 \sin x dx$  (提示 两次分部积分)

解 
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left( \int x de^x \right) = x^2 e^x - 2 \left( x e^x - \int e^x dx \right) \\ &= x^2 e^x - 2 x e^x + 2 e^x + C\end{aligned}$$

$$\begin{aligned}\int x^2 \sin x dx &= - \int x^2 d \cos x = -x^2 \cos x + \int \cos x dx^2 \\ &= -x^2 \cos x + 2 \int x \cos x dx \\ &= -x^2 \cos x + 2 \int x d \sin x \\ &= -x^2 \cos x + 2 \left( x \sin x - \int \sin x dx \right)\end{aligned}$$

例子 求不定积分  $\int x^2 e^x dx$ ,  $\int x^2 \sin x dx$  (提示 两次分部积分)

解 
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left( \int x de^x \right) = x^2 e^x - 2 \left( x e^x - \int e^x dx \right) \\ &= x^2 e^x - 2 x e^x + 2 e^x + C\end{aligned}$$

$$\begin{aligned}\int x^2 \sin x dx &= - \int x^2 d \cos x = -x^2 \cos x + \int \cos x dx^2 \\ &= -x^2 \cos x + 2 \int x \cos x dx \\ &= -x^2 \cos x + 2 \int x d \sin x \\ &= -x^2 \cos x + 2 \left( x \sin x - \int \sin x dx \right) \\ &= -x^2 \cos x + 2 x \sin x + 2 \cos x + C\end{aligned}$$

## 分部积分：例子总结

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$$\int x e^x dx =$$

$$\int x \cos x dx =$$

$$\int x^2 \ln x dx =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$



## 分部积分：例子总结

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$$\int x e^x dx = \int x de^x =$$

$$\int x \cos x dx =$$

$$\int x^2 \ln x dx =$$

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**例子** 假设某产品的边际成本为  $C'(x) = 33 + 38x - 12x^2$ ，固定成本  $C(0) = 68$ 。求：

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