§6.5 定积分的换元积分法

2016-2017 学年 II

教学要求

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Outline of §6.5

- 求定积分 $\int_a^b f(x)dx$ 可分成两步：
  1. 求出不定积分 $\int f(x)dx = F(x) + C$

     （方法：直接积分法、换元积分法、分部积分法（第五章））
  2. $\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$

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  2. $\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$
- 在实际操作中，两步可合成一步：
  - 以换元积分法、分部积分法为例说明

凑微分——例

例 计算定积分 $\int_0^{\frac{\pi}{2}} \sin^2 x \cos x dx$

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凑微分——练习

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解法二 令 $t=\sqrt{x}$，则 $x=t^2$，$dx=2tdt$，$t=1\ldots2$

$$\int_1^4 \frac{1}{x+\sqrt{x}}dx = \int_1^2 \frac{1}{t^2+t}\cdot 2tdt = \int_1^2 \frac{2}{t+1}dt = 2\ln|t+1|\Big|_1^2 = 2\ln\frac{3}{2}$$

变量代换——例

例 计算定积分 $\int_1^4 \frac{1}{x+\sqrt{x}}dx$

解法一 先求出 $\int \frac{1}{x+\sqrt{x}}dx$，令 $t=\sqrt{x}$，则 $x=t^2$，$dx=2tdt$

$$\because \int \frac{1}{x+\sqrt{x}}dx = \int \frac{1}{t^2+t}\cdot 2tdt = \int \frac{2}{t+1}dt$$

$$= 2\ln|t+1| + C = 2\ln(\sqrt{x}+1) + C$$

$$\therefore \int_1^4 \frac{x}{1+x^2}dx = 2\ln(\sqrt{x}+1)\Big|_1^4 = 2(\ln 3 - \ln 2) = 2\ln\frac{3}{2}$$

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练习 计算定积分 $\int_1^4 \frac{1}{\sqrt{x}+1}dx$

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解一 先求 $\int \frac{1}{\sqrt{x}+1}dx$,

练习 计算定积分 $\int_1^4 \frac{1}{\sqrt{x}+1}dx$

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练习 计算定积分 $\int_1^4 \frac{1}{\sqrt{x}+1}dx$

解一 先求 $\int \frac{1}{\sqrt{x}+1}dx$, 令 $t=\sqrt{x}+1$,

$$\because \int \frac{1}{\sqrt{x}+1}dx = \int \frac{1}{t} \cdot$$

练习 计算定积分 $\int_1^4 \frac{1}{\sqrt{x}+1} dx$

解一 先求 $\int \frac{1}{\sqrt{x}+1} dx$, 令 $t = \sqrt{x}+1$, 则 $x = (t-1)^2$,

$$\because \int \frac{1}{\sqrt{x}+1} dx = \int \frac{1}{t} \cdot$$

练习 计算定积分 $\int_1^4 \frac{1}{\sqrt{x}+1}dx$

解一 先求 $\int \frac{1}{\sqrt{x}+1}dx$, 令 $t=\sqrt{x}+1$, 则 $x=(t-1)^2$, $dx=2(t-1)dt$

$$\because \int \frac{1}{\sqrt{x}+1}dx = \int \frac{1}{t} \cdot$$

练习 计算定积分 $\int_1^4 \frac{1}{\sqrt{x}+1}dx$

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$$\because \int \frac{1}{\sqrt{x}+1}dx = \int \frac{1}{t}\cdot 2(t-1)dt$$

练习 计算定积分 $\int_1^4 \frac{1}{\sqrt{x}+1}dx$

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$$\because \int \frac{1}{\sqrt{x}+1}dx = \int \frac{1}{t}\cdot 2(t-1)dt = 2\int 1-\frac{1}{t}dt =$$

练习 计算定积分 $\int_1^4 \frac{1}{\sqrt{x}+1}dx$

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$$= 2(t-\ln|t|)+C$$

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$$\because \int \frac{1}{\sqrt{x}+1}dx = \int \frac{1}{t}\cdot 2(t-1)dt = 2\int 1-\frac{1}{t}dt =$$

$$= 2(t-\ln|t|)+C = 2(\sqrt{x}+1-\ln(\sqrt{x}+1))+C$$

$$\therefore \int_1^4 \frac{1}{\sqrt{x}+1}dx = 2(\sqrt{x}-\ln|\sqrt{x}+1|)\Big|_1^4$$

练习 计算定积分 $\int_1^4 \frac{1}{\sqrt{x}+1}dx$

解一 先求 $\int \frac{1}{\sqrt{x}+1}dx$，令 $t=\sqrt{x}+1$，则 $x=(t-1)^2$，$dx=2(t-1)dt$

$$\because \int \frac{1}{\sqrt{x}+1}dx = \int \frac{1}{t}\cdot 2(t-1)dt = 2\int 1-\frac{1}{t}dt =$$

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$$\therefore \int_1^4 \frac{1}{\sqrt{x}+1}dx = 2(\sqrt{x}-\ln|\sqrt{x}+1|)\Big|_1^4 = 2+2\ln\frac{2}{3}$$

练习 计算定积分 $\int_1^4 \frac{1}{\sqrt{x}+1}dx$

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解二 令 $t=\sqrt{x}+1$，则 $x=(t-1)^2$，$dx=2(t-1)dt$，

练习 计算定积分 $\int_1^4 \frac{1}{\sqrt{x}+1}dx$

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解二 令 $t=\sqrt{x}+1$，则 $x=(t-1)^2$，$dx=2(t-1)dt$，

$$\int_1^4 \frac{1}{\sqrt{x}+1}dx = \int \quad \frac{1}{t}\cdot 2(t-1)dt$$

练习 计算定积分 $\int_1^4 \frac{1}{\sqrt{x}+1}dx$

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解二 令 $t=\sqrt{x}+1$, 则 $x=(t-1)^2$, $dx=2(t-1)dt$, $t=2\ldots 3$

$$\int_1^4 \frac{1}{\sqrt{x}+1}dx = \int \frac{1}{t}\cdot 2(t-1)dt$$

练习 计算定积分 $\int_1^4 \frac{1}{\sqrt{x}+1}dx$

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解二 令 $t=\sqrt{x}+1$，则 $x=(t-1)^2$，$dx=2(t-1)dt$，$t=2\ldots3$

$$\int_1^4 \frac{1}{\sqrt{x}+1}dx = \int_2^3 \frac{1}{t}\cdot 2(t-1)dt$$

练习 计算定积分 $\int_1^4 \frac{1}{\sqrt{x}+1}dx$

解一 先求 $\int \frac{1}{\sqrt{x}+1}dx$，令 $t=\sqrt{x}+1$，则 $x=(t-1)^2$，$dx=2(t-1)dt$

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$$= 2(t-\ln|t|)+C = 2(\sqrt{x}+1-\ln(\sqrt{x}+1))+C$$

$$\therefore \int_1^4 \frac{1}{\sqrt{x}+1}dx = 2(\sqrt{x}-\ln|\sqrt{x}+1|)\Big|_1^4 = 2+2\ln\frac{2}{3}$$

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练习 计算定积分 $\int_1^4 \frac{1}{\sqrt{x}+1}dx$

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练习 计算定积分 $\int_0^{\ln 2} \sqrt{e^x - 1} dx$

解

练习 计算定积分 $\int_0^{\ln 2} \sqrt{e^x - 1} dx$

解 令 $t = \sqrt{e^x - 1}$,

练习 计算定积分 $\int_0^{\ln 2} \sqrt{e^x - 1} dx$

解 令 $t = \sqrt{e^x - 1}$,

$$\int_0^{\ln 2} \sqrt{e^x - 1} dx = \int \quad t \cdot$$

练习 计算定积分 $\int_0^{\ln 2} \sqrt{e^x - 1} dx$

解 令 $t = \sqrt{e^x - 1}$，则 $x = \ln(1 + t^2)$，

$$\int_0^{\ln 2} \sqrt{e^x - 1} dx = \int \quad t \cdot$$

练习 计算定积分 $\int_0^{\ln 2}\sqrt{e^x-1}dx$

解 令 $t=\sqrt{e^x-1}$，则 $x=\ln(1+t^2)$，$dx=\frac{2t}{1+t^2}dt$，

$$\int_0^{\ln 2}\sqrt{e^x-1}dx=\int\ t\cdot$$

变量代换——练习

练习 计算定积分 $\int_0^{\ln 2} \sqrt{e^x - 1} dx$

解 令 $t = \sqrt{e^x - 1}$，则 $x = \ln(1 + t^2)$，$dx = \frac{2t}{1+t^2} dt$，

$$\int_0^{\ln 2} \sqrt{e^x - 1} dx = \int \quad t \cdot \frac{2t}{1 + t^2} dt$$

练习 计算定积分 $\int_0^{\ln 2} \sqrt{e^x - 1} dx$

解 令 $t = \sqrt{e^x - 1}$，则 $x = \ln(1 + t^2)$，$dx = \frac{2t}{1+t^2} dt$，$t = 0 \ldots 1$

$$\int_0^{\ln 2} \sqrt{e^x - 1} dx = \int \quad t \cdot \frac{2t}{1 + t^2} dt$$

练习 计算定积分 $\int_0^{\ln 2}\sqrt{e^x-1}dx$

解 令 $t=\sqrt{e^x-1}$，则 $x=\ln(1+t^2)$，$dx=\frac{2t}{1+t^2}dt$，$t=0\ldots1$

$$\int_0^{\ln 2}\sqrt{e^x-1}dx=\int_0^1 t\cdot\frac{2t}{1+t^2}dt$$

练习 计算定积分 $\int_0^{\ln 2} \sqrt{e^x - 1}dx$

解 令 $t = \sqrt{e^x - 1}$，则 $x = \ln(1 + t^2)$，$dx = \frac{2t}{1+t^2}dt$，$t = 0 \ldots 1$

$$\int_0^{\ln 2} \sqrt{e^x - 1}dx = \int_0^1 t \cdot \frac{2t}{1 + t^2}dt = 2\int_0^1 \frac{t^2}{1 + t^2}dt$$

变量代换——练习

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解 令 $t=\sqrt{e^x-1}$，则 $x=\ln(1+t^2)$，$dx=\frac{2t}{1+t^2}dt$，$t=0\ldots 1$

$$\int_0^{\ln 2}\sqrt{e^x-1}dx=\int_0^1 t\cdot\frac{2t}{1+t^2}dt=2\int_0^1\frac{t^2}{1+t^2}dt$$

$$=2\int_0^1\left(1-\frac{1}{1+t^2}\right)dt$$

练习 计算定积分 $\int_0^{\ln 2} \sqrt{e^x - 1} dx$

解 令 $t = \sqrt{e^x - 1}$，则 $x = \ln(1 + t^2)$，$dx = \frac{2t}{1+t^2} dt$，$t = 0 \dots 1$

$$\int_0^{\ln 2} \sqrt{e^x - 1} dx = \int_0^1 t \cdot \frac{2t}{1 + t^2} dt = 2 \int_0^1 \frac{t^2}{1 + t^2} dt$$

$$= 2 \int_0^1 \left(1 - \frac{1}{1 + t^2}\right) dt$$

$$= 2(t - \arctan t)$$

变量代换——练习

练习 计算定积分 $\int_0^{\ln 2} \sqrt{e^x - 1}dx$

解 令 $t = \sqrt{e^x - 1}$，则 $x = \ln(1+t^2)$，$dx = \frac{2t}{1+t^2}dt$，$t = 0 \ldots 1$

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$$= 2(t - \arctan t)\Big|_0^1$$

练习 计算定积分 $\int_0^{\ln 2} \sqrt{e^x - 1}dx$

解 令 $t = \sqrt{e^x - 1}$，则 $x = \ln(1+t^2)$，$dx = \frac{2t}{1+t^2}dt$，$t = 0 \dots 1$

$$\int_0^{\ln 2} \sqrt{e^x - 1}dx = \int_0^1 t \cdot \frac{2t}{1+t^2}dt = 2\int_0^1 \frac{t^2}{1+t^2}dt$$

$$= 2\int_0^1 \left(1 - \frac{1}{1+t^2}\right)dt$$

$$= 2(t - \arctan t)\Big|_0^1 = 2[(1 - \frac{\pi}{4}) - 0] =$$

练习 计算定积分 $\int_0^{\ln 2}\sqrt{e^x-1}dx$

解 令 $t=\sqrt{e^x-1}$，则 $x=\ln(1+t^2)$，$dx=\frac{2t}{1+t^2}dt$，$t=0\ldots1$

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$$=2(t-\arctan t)\Big|_0^1=2[(1-\frac{\pi}{4})-0]=2-\frac{\pi}{2}$$