第 04 周作业解答

练习 1. 求不定积分
(1) $\int(3x-1)e^x dx$, (2) $\int xe^{2x}dx$.

解：(1)

$$\begin{aligned}\int(3x-1)e^x dx &= \int(3x-1)de^x \\ &= (3x-1)e^x - \int e^x d(3x-1) \\ &= (3x-1)e^x - 3\int e^x dx \\ &= (3x-1)e^x - 3e^x + C \\ &= (3x-4)e^x + C.\end{aligned}$$

(2)

$$\begin{aligned}\int xe^{2x}dx &= \int xd(\frac{1}{2}e^{2x}) \\ &= x\cdot\frac{1}{2}e^{2x} - \int\frac{1}{2}e^{2x}dx \\ &= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C.\end{aligned}$$

练习 2. 求不定积分
(1) $\int(3\theta-1)\sin\theta d\theta$, (2) $\int\theta\sin(2\theta)d\theta$.

解：(1)

$$\begin{aligned}\int(3\theta-1)\sin\theta d\theta &= -\int(3\theta-1)d\cos\theta \\ &= -\left[(3\theta-1)\cos\theta - \int\cos\theta d(3\theta-1)\right] \\ &= -(3\theta-1)\cos\theta + 3\int\cos\theta d\theta \\ &= -(3\theta-1)\cos\theta + 3\sin\theta + C.\end{aligned}$$

(2)

$$\begin{aligned}\int\theta\sin(2\theta)d\theta &= -\frac{1}{2}\int\theta d\cos 2\theta \\ &= -\frac{1}{2}\left(\theta\cos 2\theta - \int\cos 2\theta d\theta\right) \\ &= -\frac{1}{2}\theta\cos 2\theta + \frac{1}{4}\int\cos 2\theta d(2\theta) \\ &= -\frac{1}{2}\theta\cos 2\theta + \frac{1}{4}\sin 2\theta + C.\end{aligned}$$

练习 3. 求不定积分 $\int \frac{\ln x}{x^2}dx$.

解：利用 $\frac{1}{x^2}dx = -d\frac{1}{x}$：

$$\begin{aligned}\int \frac{\ln x}{x^2}dx &= -\int \ln x d\frac{1}{x} \\ &= -\left(\ln x \cdot \frac{1}{x} - \int \frac{1}{x} d\ln x\right) \\ &= -\frac{1}{x}\ln x + \int \frac{1}{x^2}dx \\ &= -\frac{1}{x}\ln x - \frac{1}{x} + C.\end{aligned}$$

练习 4. 求不定积分 $\int \ln(5x-1)dx$.

解：

$$\begin{aligned}\int \ln(5x-1)dx &= x\ln(5x-1) - \int x d\ln(5x-1) \\ &= x\ln(5x-1) - \int \frac{5x}{5x-1}dx \\ &= x\ln(5x-1) - \int 1 + \frac{1}{5x-1}dx \\ &= x\ln(5x-1) - x - \int \frac{1}{5x-1}dx \\ &= x\ln(5x-1) - x - \frac{1}{5}\int \frac{1}{5x-1}d(5x-1) \\ &= x\ln(5x-1) - x - \frac{1}{5}\ln(5x-1) + C\end{aligned}$$

练习 5. 求不定积分 $\int \sin(\frac{1}{x})\frac{1}{x^2}dx$.

解：

$$\int \sin(\frac{1}{x})\frac{1}{x^2}dx = \int \sin(\frac{1}{x})\cdot(-1)d(\frac{1}{x}) \xlongequal{u=\frac{1}{x}} -\int \sin u du = \cos u + C = \cos\frac{1}{x} + C$$

练习 6. 求不定积分 $\int \cos\sqrt{x}dx$.

解：令 $t = \sqrt{x}$，则 $x = t^2$，$dx = dt^2 = 2tdt$。所以

$$\begin{aligned}\int \cos\sqrt{x}dx &= \int \cos t \cdot 2tdt = 2\int t\cos t dt \\ &= 2\int t d\sin t \\ &= 2\left(t\sin t - \int \sin t dt\right) \\ &= 2\left(t\sin t + \cos t\right) + C \\ &= 2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} + C\end{aligned}$$

练习 7. 设生产某产品的固定成本为 20 元，而生产 x 个单位时，边际成本函数为 $C'(x) = 0.4x + 2$。试求出总成本函数 $C(x)$。

预计当售价定为 18 元/单位时产品可售罄，试求出此时的利润函数 $L(x)$，并以此确定生产多少单位产品可获取最大利润？最大利润是多少？

解：(1)

$$C(x) = \int C'(x)dx = \int 0.4x + 2dx = 0.2x^2 + 2x + c$$

因为 $C(0) = 20$，所以 $c = 20$，$C(x) = 0.2x^2 + 2x + 20$。

(2)

$$L(x) = 18x - C(x) = -0.2x^2 + 16x - 20$$

这是开口向下的抛物线，有唯一最大值点。解 $L'(x) = -0.4x + 16 = 0$ 得 $x = 40$。所以生产 40 单位产品可获取最大利润，最大利润是

$$L(40) = -0.2 \cdot 40^2 + 16 \cdot 40 - 20 = 300\,(\text{元})$$