§5.2 基本积分表

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教学要求

- ♢ 熟练运用基本积分公式
- ♣ 直接积分法
- ♡
- ♠

Outline of §5.2

1. 基本积分表

2. 直接积分法

We are here now...

1. 基本积分表

2. 直接积分法

基本积分表

基本初等函数的导数公式 $\longrightarrow$ 基本的积分公式

基本积分表

基本初等函数的导数公式 $\longrightarrow$ 基本的积分公式

- $\int 0dx = C$
- $\int x^{\alpha}dx = \frac{1}{\alpha+1}x^{\alpha+1} + C,\ (\alpha \neq -1)$
- $\int \frac{1}{x}dx = \ln|x| + C$

基本积分表

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- ♣ $\int a^{x}dx = \frac{1}{\ln a}a^{x} + C$
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- ♡ $\int \sin x dx = -\cos x + C$
- ♡ $\int \cos x dx = \sin x + C$
- ♡ $\int \frac{1}{\cos^2 x}dx = \frac{\sin x}{\cos x} + C = \tan x + C$
- ♡ $\int \frac{1}{\sin^2 x}dx = -\frac{\cos x}{\sin x} + C = -\cot x + C$

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- ♣ $\int a^x dx = \frac{1}{\ln a}a^x + C$
- ♣ $\int e^x dx = e^x + C$
- ♠ $\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin x + C$
- ♠ $\int \frac{1}{1+x^2}dx = \arctan x + C$

We are here now...

1. 基本积分表

2. 直接积分法

求不定积分 I——“直接”利用基本积分公式

例子 求不定积分 $\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1-t^2}} - 5^t \right) dt$

解 $\csc t = \frac{1}{\sin t}$

求不定积分 I——“直接”利用基本积分公式

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$=$

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$$\int \left(\frac{2}{t} - 3\cos t - \csc^2 t + \frac{2}{\sqrt{1-t^2}} - 5^t \right) dt$$

$$= \int \frac{2}{t} dt - \int 3\cos t dt - \int \csc^2 t dt + \int \frac{2}{\sqrt{1-t^2}} dt - \int 5^t dt$$

$$=$$

求不定积分 I——“直接”利用基本积分公式

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$$\int\left(\frac{2}{t}-3\cos t-\csc^2 t+\frac{2}{\sqrt{1-t^2}}-5^t\right)dt$$

$$=\int\frac{2}{t}dt-\int 3\cos t dt-\int\csc^2 t dt+\int\frac{2}{\sqrt{1-t^2}}dt-\int 5^t dt$$

$$=2\int\frac{1}{t}dt-3\int\cos t dt-\int\frac{1}{\sin^2 t}dt+2\int\frac{1}{\sqrt{1-t^2}}dt-\int 5^t dt$$

$=$

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$$=2\int\frac{1}{t}dt-3\int\cos t dt-\int\frac{1}{\sin^2 t}dt+2\int\frac{1}{\sqrt{1-t^2}}dt-\int 5^t dt$$

$$=2\ln|t|$$

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$$=2\int\frac{1}{t}dt-3\int\cos t dt-\int\frac{1}{\sin^2 t}dt+2\int\frac{1}{\sqrt{1-t^2}}dt-\int 5^t dt$$

$$=2\ln|t|-3\sin t$$

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$$=2\ln|t|-3\sin t+\cot t$$

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$$=2\int\frac{1}{t}dt-3\int\cos t dt-\int\frac{1}{\sin^2 t}dt+2\int\frac{1}{\sqrt{1-t^2}}dt-\int 5^t dt$$

$$=2\ln|t|-3\sin t+\cot t+2\arcsin t-\frac{1}{\ln 5}5^t+C$$

求不定积分 I——“直接”利用基本积分公式 (Cont.)

熟练计算 $\int x^{\alpha} dx = \begin{cases} \frac{1}{\alpha+1} x^{\alpha+1} + C, & \alpha \neq -1 \\ \ln|x| + C, & \alpha = -1 \end{cases}$

求不定积分 I——“直接”利用基本积分公式 (Cont.)

熟练计算 $\int x^{\alpha}dx = \begin{cases} \frac{1}{\alpha+1}x^{\alpha+1}+C, & \alpha \neq -1 \\ \ln|x|+C, & \alpha = -1 \end{cases}$

例子 求不定积分 $\int \sqrt{x^5}dx$, $\int \frac{1}{\sqrt{x^3}}dx$

求不定积分 I——“直接”利用基本积分公式 (Cont.)

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例子 求不定积分 $\int \sqrt{x^5} dx$, $\int \frac{1}{\sqrt{x^3}} dx$

解

$$\int \sqrt{x^5} dx$$

$$\int \frac{1}{\sqrt{x^3}} dx$$

求不定积分 I——“直接”利用基本积分公式 (Cont.)

熟练计算 $\int x^{\alpha}dx = \begin{cases} \frac{1}{\alpha+1}x^{\alpha+1}+C, & \alpha \neq -1 \\ \ln|x|+C, & \alpha = -1 \end{cases}$

例子 求不定积分 $\int \sqrt{x^5}dx$, $\int \frac{1}{\sqrt{x^3}}dx$

解

$$\int \sqrt{x^5}dx = \int (x^5)^{\frac{1}{2}}dx$$

$$\int \frac{1}{\sqrt{x^3}}dx$$

求不定积分 I——“直接”利用基本积分公式 (Cont.)

熟练计算 $\int x^{\alpha}dx = \begin{cases} \frac{1}{\alpha+1}x^{\alpha+1}+C, & \alpha \neq -1 \\ \ln|x|+C, & \alpha = -1 \end{cases}$

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例子 求不定积分 $\int \sqrt{x^5}dx$, $\int \frac{1}{\sqrt{x^3}}dx$

解

$$\int \sqrt{x^5}dx = \int (x^5)^{\frac{1}{2}}dx = \int x^{5/2}dx = \qquad x^{\frac{5}{2}+1}$$

$$\int \frac{1}{\sqrt{x^3}}dx$$

求不定积分 I——“直接”利用基本积分公式 (Cont.)

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求不定积分 I——“直接”利用基本积分公式 (Cont.)

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例子 求不定积分 $\int \sqrt{x^5}dx$, $\int \frac{1}{\sqrt{x^3}}dx$

解

$$\int \sqrt{x^5}dx = \int (x^5)^{\frac{1}{2}}dx = \int x^{5/2}dx = \frac{1}{\frac{5}{2}+1}x^{\frac{5}{2}+1}+C = \frac{2}{7}x^{\frac{7}{2}}+C$$

$$\int \frac{1}{\sqrt{x^3}}dx$$

求不定积分 I——“直接”利用基本积分公式 (Cont.)

熟练计算 $\int x^{\alpha}dx = \begin{cases} \frac{1}{\alpha+1}x^{\alpha+1}+C, & \alpha \neq -1 \\ \ln|x|+C, & \alpha = -1 \end{cases}$

例子 求不定积分 $\int \sqrt{x^5}dx$, $\int \frac{1}{\sqrt{x^3}}dx$

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$$\int \frac{1}{\sqrt{x^3}}dx = \int \frac{1}{x^{3/2}}dx$$

求不定积分 I——“直接”利用基本积分公式 (Cont.)

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$$\int \sqrt{x^5}dx = \int (x^5)^{\frac{1}{2}}dx = \int x^{5/2}dx = \frac{1}{\frac{5}{2}+1}x^{\frac{5}{2}+1}+C = \frac{2}{7}x^{\frac{7}{2}}+C$$

$$\int \frac{1}{\sqrt{x^3}}dx = \int \frac{1}{x^{3/2}}dx = \int x^{-3/2}dx$$

$$=$$

求不定积分 I——“直接”利用基本积分公式 (Cont.)

熟练计算 $\int x^{\alpha}dx = \begin{cases} \frac{1}{\alpha+1}x^{\alpha+1}+C, & \alpha \neq -1 \\ \ln|x|+C, & \alpha = -1 \end{cases}$

例子 求不定积分 $\int \sqrt{x^5}dx$, $\int \frac{1}{\sqrt{x^3}}dx$

解

$$\int \sqrt{x^5}dx = \int (x^5)^{\frac{1}{2}}dx = \int x^{5/2}dx = \frac{1}{\frac{5}{2}+1}x^{\frac{5}{2}+1}+C = \frac{2}{7}x^{\frac{7}{2}}+C$$

$$\int \frac{1}{\sqrt{x^3}}dx = \int \frac{1}{x^{3/2}}dx = \int x^{-3/2}dx$$

$$= x^{-\frac{3}{2}+1}$$

求不定积分 I——“直接”利用基本积分公式 (Cont.)

熟练计算 $\int x^\alpha dx = \begin{cases} \frac{1}{\alpha+1}x^{\alpha+1}+C, & \alpha \neq -1 \\ \ln|x|+C, & \alpha = -1 \end{cases}$

例子 求不定积分 $\int \sqrt{x^5}dx$, $\int \frac{1}{\sqrt{x^3}}dx$

解

$$\int \sqrt{x^5}dx = \int (x^5)^{\frac{1}{2}}dx = \int x^{5/2}dx = \frac{1}{\frac{5}{2}+1}x^{\frac{5}{2}+1}+C = \frac{2}{7}x^{\frac{7}{2}}+C$$

$$\int \frac{1}{\sqrt{x^3}}dx = \int \frac{1}{x^{3/2}}dx = \int x^{-3/2}dx$$

$$= \frac{1}{-\frac{3}{2}+1}x^{-\frac{3}{2}+1}$$

求不定积分 I——“直接”利用基本积分公式 (Cont.)

熟练计算 $\int x^{\alpha}dx = \begin{cases} \frac{1}{\alpha+1}x^{\alpha+1}+C, & \alpha \neq -1 \\ \ln|x|+C, & \alpha = -1 \end{cases}$

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$$\int \sqrt{x^5}dx = \int (x^5)^{\frac{1}{2}}dx = \int x^{5/2}dx = \frac{1}{\frac{5}{2}+1}x^{\frac{5}{2}+1}+C = \frac{2}{7}x^{\frac{7}{2}}+C$$

$$\int \frac{1}{\sqrt{x^3}}dx = \int \frac{1}{x^{3/2}}dx = \int x^{-3/2}dx$$

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求不定积分 I——“直接”利用基本积分公式 (Cont.)

熟练计算 $\int x^{\alpha}dx = \begin{cases} \frac{1}{\alpha+1}x^{\alpha+1}+C, & \alpha \neq -1 \\ \ln|x|+C, & \alpha = -1 \end{cases}$

例子 求不定积分 $\int \sqrt{x^5}dx$, $\int \frac{1}{\sqrt{x^3}}dx$

解

$$\int \sqrt{x^5}dx = \int (x^5)^{\frac{1}{2}}dx = \int x^{5/2}dx = \frac{1}{\frac{5}{2}+1}x^{\frac{5}{2}+1}+C = \frac{2}{7}x^{\frac{7}{2}}+C$$

$$\int \frac{1}{\sqrt{x^3}}dx = \int \frac{1}{x^{3/2}}dx = \int x^{-3/2}dx$$

$$= \frac{1}{-\frac{3}{2}+1}x^{-\frac{3}{2}+1}+C = -2x^{-1/2}+C$$

求不定积分 I——“直接”利用基本积分公式 (Cont.)

熟练计算 $\int x^{\alpha}dx = \begin{cases} \frac{1}{\alpha+1}x^{\alpha+1}+C, & \alpha \neq -1 \\ \ln|x|+C, & \alpha = -1 \end{cases}$

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$$\int \frac{1}{\sqrt{x^3}}dx = \int \frac{1}{x^{3/2}}dx = \int x^{-3/2}dx$$

$$= \frac{1}{-\frac{3}{2}+1}x^{-\frac{3}{2}+1}+C = -2x^{-1/2}+C = -\frac{2}{\sqrt{x}}+C$$

求不定积分 II：化为 $\int x^\alpha dx$

例子 求不定积分 $\int \frac{x}{\sqrt{x^5}}dx$, $\int \frac{(3-x\sqrt{x})^2}{x}dx$, $\int \frac{(1-\sqrt{x})^2}{x}dx$

求不定积分 II：化为 $\int x^\alpha dx$

例子 求不定积分 $\int \frac{x}{\sqrt{x^5}}dx$, $\int \frac{(3-x\sqrt{x})^2}{x}dx$, $\int \frac{(1-\sqrt{x})^2}{x}dx$

提示 整理被积函数，化不定积分为 $\int x^\alpha dx$ 形式

求不定积分 II：化为 $\int x^\alpha dx$

例子 求不定积分 $\int \frac{x}{\sqrt{x^5}}dx$, $\int \frac{(3-x\sqrt{x})^2}{x}dx$, $\int \frac{(1-\sqrt{x})^2}{x}dx$

提示 整理被积函数，化不定积分为 $\int x^\alpha dx$ 形式

解 $\int \frac{x}{\sqrt{x^5}}dx =$

求不定积分 II：化为 $\int x^\alpha dx$

例子 求不定积分 $\int \frac{x}{\sqrt{x^5}}dx$, $\int \frac{(3-x\sqrt{x})^2}{x}dx$, $\int \frac{(1-\sqrt{x})^2}{x}dx$

提示 整理被积函数，化不定积分为 $\int x^\alpha dx$ 形式

解 $$\int \frac{x}{\sqrt{x^5}}dx = \int \frac{x}{x^{5/2}}dx =$$

求不定积分 II：化为 $\int x^\alpha dx$

例子 求不定积分 $\int \frac{x}{\sqrt{x^5}}dx$, $\int \frac{(3-x\sqrt{x})^2}{x}dx$, $\int \frac{(1-\sqrt{x})^2}{x}dx$

提示 整理被积函数，化不定积分为 $\int x^\alpha dx$ 形式

解 $$\int \frac{x}{\sqrt{x^5}}dx = \int \frac{x}{x^{5/2}}dx = \int x^{1-5/2}dx =$$

求不定积分 II：化为 $\int x^\alpha dx$

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求不定积分 II：化为 $\int x^\alpha dx$

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求不定积分 II：化为 $\int x^\alpha dx$

例子 求不定积分 $\int \frac{x}{\sqrt{x^5}}dx$, $\int \frac{(3-x\sqrt{x})^2}{x}dx$, $\int \frac{(1-\sqrt{x})^2}{x}dx$

提示 整理被积函数，化不定积分为 $\int x^\alpha dx$ 形式

解 $$\int \frac{x}{\sqrt{x^5}}dx = \int \frac{x}{x^{5/2}}dx = \int x^{1-5/2}dx = \int x^{-3/2}dx = -2x^{-\frac{1}{2}}+C$$

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求不定积分 III: 形如 $\int \frac{\text{Ìá½»½ó¸¢È}}{\text{×ª · ¢: Ñ§ÆÚÌÕ}}dx$

提示 将分式 “$\frac{\text{Ìá½»½ó¸¢È}}{\text{×ª · ¢: Ñ§ÆÚÌÕ}}$” 拆成两个（或多个）简单的式子/分式

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$$\int \frac{3+x^2}{1+x^2}dx = \int 1 + \frac{2}{1+x^2}dx =$$

求不定积分 III: 形如 $\int \frac{\text{Ìá½»½ó¸ç È}}{\text{x² · ç: Ñ§ÆÚÌÕ}}dx$

提示 将分式 “$\frac{\text{Ìá½»½ó¸ç È}}{\text{x² · ç: Ñ§ÆÚÌÕ}}$” 拆成两个（或多个）简单的式子/分式

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求不定积分 III: 形如 $\int \frac{\text{乱码}}{\text{乱码}}dx$

提示 将分式 “$\frac{\text{乱码}}{\text{乱码}}$” 拆成两个（或多个）简单的式子/分式

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例子 求不定积分 $\int \frac{1}{x^2(1+x^2)}dx$, $\int \frac{e^{2x}-1}{e^x+1}dx$

解

求不定积分 III: 形如 $\int \frac{\text{Ìá½»½ó¸ç È}}{\text{x² · ç: Ñ§ÆÚÌÕ}}dx$

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例子 求不定积分 $\int \frac{1}{x^2(1+x^2)}dx$, $\int \frac{e^{2x}-1}{e^x+1}dx$

解
$$\int \frac{1}{x^2(1+x^2)}dx = \int \frac{1}{x^2}-\frac{1}{1+x^2}dx = -\frac{1}{x}$$

求不定积分 III: 形如 $\int \frac{\text{Ìá½»½ó¸ç È}}{\text{x² · ç: Ñ§ÆÚÌÕ}}dx$

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$$\int \frac{1}{x^2(1+x^2)}dx = \int \frac{1}{x^2} - \frac{1}{1+x^2}dx = -\frac{1}{x} - \arctan x$$

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求不定积分 IV: 化为 $\int a^x dx$

掌握 $\int a^x dx = \frac{1}{\ln a} a^x + C, (a > 0)$

例子 求不定积分 $\int 3^x e^x dx$, $\int 5^{-x} e^x dx$

求不定积分 IV: 化为 $\int a^x dx$

掌握 $\int a^x dx = \frac{1}{\ln a} a^x + C, (a > 0)$

例子 求不定积分 $\int 3^x e^x dx$, $\int 5^{-x} e^x dx$

解

$$\int 3^x e^x dx =$$

求不定积分 IV: 化为 $\int a^x dx$

掌握 $\int a^x dx = \frac{1}{\ln a} a^x + C, (a > 0)$

例子 求不定积分 $\int 3^x e^x dx$, $\int 5^{-x} e^x dx$

解

$$\int 3^x e^x dx = \int (3e)^x dx =$$

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解

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解

$$\int 3^x e^x dx = \int (3e)^x dx = \frac{1}{\ln(3e)} (3e)^x + C = \frac{3^x e^x}{1 + \ln 3} + C$$

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$$= \frac{1}{\ln\left(\frac{1}{5} e\right)} \left(\frac{1}{5} e\right)^x + C$$

求不定积分 IV: 化为 $\int a^x dx$

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$$\int 3^x e^x dx = \int (3e)^x dx = \frac{1}{\ln(3e)} (3e)^x + C = \frac{3^x e^x}{1 + \ln 3} + C$$

$$\int 5^{-x} e^x dx = \int \left(\frac{1}{5}\right)^x e^x dx = \int \left(\frac{1}{5} e\right)^x dx$$

$$= \frac{1}{\ln\left(\frac{1}{5}e\right)} \left(\frac{1}{5} e\right)^x + C = \frac{e^x}{(1 - \ln 5) 5^x} + C$$

求不定积分 V: $\int$ "triangle functions" dx

求不定积分

$$(1)\int \tan^2 x dx, \quad \int \cot^2 x dx$$

$$(2)\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx, \quad \int \frac{1}{\cos^2 x \sin^2 x} dx$$

$$(3)\int \sin^2 \frac{x}{2} dx, \quad \int \cos^2 \frac{x}{2} dx$$

求不定积分 V: $\int$ "triangle functions" dx

求不定积分

$$(1)\int \tan^2 x dx, \quad \int \cot^2 x dx$$

$$(2)\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx, \quad \int \frac{1}{\cos^2 x \sin^2 x} dx$$

$$(3)\int \sin^2 \frac{x}{2} dx, \quad \int \cos^2 \frac{x}{2} dx$$

提示 利用“三角恒等式”

平方关系 $\sin^2 x + \cos^2 x = 1$

倍角公式 $\cos 2x = \cos^2 x - \sin^2 x$；$\sin 2x = 2 \sin x \cos x$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx$, $\int \cot^2 x dx$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x}$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1-\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - 1$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx =$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1-\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - 1$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1-\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - 1$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - 1$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x} = \frac{1 - \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - 1$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1,$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样，

$$\int \cot^2 x dx =$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \qquad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样，

$$\int \cot^2 x dx =$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \qquad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样，

$$\int \cot^2 x dx =$$

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x}$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \qquad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样，

$$\int \cot^2 x dx =$$

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x}$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \qquad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样，

$$\int \cot^2 x dx =$$

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \qquad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样，

$$\int \cot^2 x dx = \int \frac{1}{\sin^2 x} - 1 dx$$

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \qquad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样,

$$\int \cot^2 x dx = \int \frac{1}{\sin^2 x} - 1 dx = -\cot x$$

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \qquad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样,

$$\int \cot^2 x dx = \int \frac{1}{\sin^2 x} - 1 dx = -\cot x - x$$

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \tan^2 x dx, \quad \int \cot^2 x dx$

解 因为

$$\tan^2 x = \frac{1}{\cos^2 x} - 1, \qquad \cot^2 x = \frac{1}{\sin^2 x} - 1$$

所以

$$\int \tan^2 x dx = \int \frac{1}{\cos^2 x} - 1 dx = \tan x - x + C$$

同样，

$$\int \cot^2 x dx = \int \frac{1}{\sin^2 x} - 1 dx = -\cot x - x + C.$$

$$\cot^2 x = \frac{\cos^2 x}{\sin^2 x} = \frac{1 - \sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - 1$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \frac{1}{\cos^2 x \sin^2 x} dx$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \frac{1}{\cos^2 x \sin^2 x} dx$

解

$$\int \frac{1}{\cos^2 x \sin^2 x} dx$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \frac{1}{\cos^2 x \sin^2 x} dx$

解

$$\int \frac{1}{\cos^2 x \sin^2 x} dx = \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} dx$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \frac{1}{\cos^2 x \sin^2 x} dx$

解

$$\int \frac{1}{\cos^2 x \sin^2 x} dx = \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} dx$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \frac{1}{\cos^2 x \sin^2 x} dx$

解

$$\begin{aligned}\int \frac{1}{\cos^2 x \sin^2 x} dx &= \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} dx \\ &= \int \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} dx \\ &= \tan x\end{aligned}$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \frac{1}{\cos^2 x \sin^2 x} dx$

解

$$\begin{aligned}\int \frac{1}{\cos^2 x \sin^2 x} dx &= \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} dx \\ &= \int \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} dx \\ &= \tan x - \cot x\end{aligned}$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用：$\sin^2 x + \cos^2 x = 1$

例子 求不定积分 $\int \frac{1}{\cos^2 x \sin^2 x} dx$

解

$$\begin{aligned}\int \frac{1}{\cos^2 x \sin^2 x} dx &= \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x} dx \\ &= \int \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} dx \\ &= \tan x - \cot x + C\end{aligned}$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分 $\int \cos^2 \frac{x}{2} dx, \quad \int \sin^2 \frac{x}{2} dx$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x)$$

例子 求不定积分 $\int \cos^2 \frac{x}{2} dx, \quad \int \sin^2 \frac{x}{2} dx$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

例子 求不定积分 $\int \cos^2 \frac{x}{2} dx, \quad \int \sin^2 \frac{x}{2} dx$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

$$\therefore \quad \cos^2 x = \frac{1 + \cos 2x}{2},$$

例子 求不定积分 $\int \cos^2 \frac{x}{2} dx, \quad \int \sin^2 \frac{x}{2} dx$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

$$\therefore \quad \cos^2 x = \frac{1 + \cos 2x}{2}, \qquad \sin^2 x = \frac{1 - \cos 2x}{2}$$

例子 求不定积分 $\int \cos^2 \frac{x}{2} dx, \quad \int \sin^2 \frac{x}{2} dx$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

$$\therefore \quad \cos^2 x = \frac{1 + \cos 2x}{2}, \qquad \sin^2 x = \frac{1 - \cos 2x}{2}$$

例子 求不定积分 $\int \cos^2 \frac{x}{2} dx, \quad \int \sin^2 \frac{x}{2} dx$

解

$$\int \cos^2 \frac{x}{2} dx =$$

$$\int \sin^2 \frac{x}{2} dx =$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

$$\therefore \quad \cos^2 x = \frac{1+\cos 2x}{2}, \qquad \sin^2 x = \frac{1-\cos 2x}{2}$$

例子 求不定积分 $\int \cos^2 \frac{x}{2} dx, \quad \int \sin^2 \frac{x}{2} dx$

解

$$\int \cos^2 \frac{x}{2} dx = \int \frac{1+\cos x}{2} dx =$$

$$\int \sin^2 \frac{x}{2} dx =$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

$$\therefore \quad \cos^2 x = \frac{1+\cos 2x}{2}, \qquad \sin^2 x = \frac{1-\cos 2x}{2}$$

例子 求不定积分 $\int \cos^2 \frac{x}{2} dx, \quad \int \sin^2 \frac{x}{2} dx$

解

$$\int \cos^2 \frac{x}{2} dx = \int \frac{1+\cos x}{2} dx = \frac{1}{2}x + \frac{1}{2}\sin x + C$$

$$\int \sin^2 \frac{x}{2} dx =$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

$$\therefore \quad \cos^2 x = \frac{1+\cos 2x}{2}, \qquad \sin^2 x = \frac{1-\cos 2x}{2}$$

例子 求不定积分 $\int \cos^2 \frac{x}{2} dx, \quad \int \sin^2 \frac{x}{2} dx$

解

$$\int \cos^2 \frac{x}{2} dx = \int \frac{1+\cos x}{2} dx = \frac{1}{2}x + \frac{1}{2}\sin x + C$$

$$\int \sin^2 \frac{x}{2} dx = \int \frac{1-\cos x}{2} dx =$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = 2\cos^2 x - 1$$

$$\therefore \quad \cos^2 x = \frac{1+\cos 2x}{2}, \qquad \sin^2 x = \frac{1-\cos 2x}{2}$$

例子 求不定积分 $\int \cos^2 \frac{x}{2} dx, \quad \int \sin^2 \frac{x}{2} dx$

解

$$\int \cos^2 \frac{x}{2} dx = \int \frac{1+\cos x}{2} dx = \frac{1}{2}x + \frac{1}{2}\sin x + C$$

$$\int \sin^2 \frac{x}{2} dx = \int \frac{1-\cos x}{2} dx = \frac{1}{2}x - \frac{1}{2}\sin x + C$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分 $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分 $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

解

$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} dx$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分 $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

解

$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} dx$$

$$= \int \frac{\cos^2 x}{\cos^2 x \sin^2 x} - \frac{\sin^2 x}{\cos^2 x \sin^2 x} dx$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分 $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

解

$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} dx$$

$$= \int \frac{\cos^2 x}{\cos^2 x \sin^2 x} - \frac{\sin^2 x}{\cos^2 x \sin^2 x} dx$$

$$= \int \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} dx$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分 $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

解

$$\begin{aligned}
\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx &= \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} dx \\
&= \int \frac{\cos^2 x}{\cos^2 x \sin^2 x} - \frac{\sin^2 x}{\cos^2 x \sin^2 x} dx \\
&= \int \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} dx \\
&= -\cot x
\end{aligned}$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分 $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

解

$$\begin{aligned}
\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx &= \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} dx \\
&= \int \frac{\cos^2 x}{\cos^2 x \sin^2 x} - \frac{\sin^2 x}{\cos^2 x \sin^2 x} dx \\
&= \int \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} dx \\
&= -\cot x - \tan x
\end{aligned}$$

求不定积分 V: $\int$ "triangle functions" dx (Cont.)

利用“倍角公式”

$$\cos 2x = \cos^2 x - \sin^2 x$$

例子 求不定积分 $\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx$

解

$$\begin{aligned}
\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx &= \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} dx \\
&= \int \frac{\cos^2 x}{\cos^2 x \sin^2 x} - \frac{\sin^2 x}{\cos^2 x \sin^2 x} dx \\
&= \int \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} dx \\
&= -\cot x - \tan x + C
\end{aligned}$$