第7章 c: 可降阶微分方程

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提要

假设 y = y(x) 为未知函数,探讨如何求解以下三种类型的:

- $y^{(n)} = f(x)$
- y'' = f(x, y')
- y'' = f(y, y')



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上述方程都是所谓 可降阶微分方程



Outline

♦
$$y^{(n)} = f(x)$$
 型的微分方程

♣
$$y'' = f(x, y')$$
 型的微分方程



We are here now...

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两边积分 ------

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 型的微分方程

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$$\longrightarrow$$
 $y^{(n-1)} = \int f(x)dx + C_1$

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 $\Rightarrow \qquad y^{(n-1)} = \int f(x)dx + C_1$
 $\Rightarrow \qquad y^{(n-2)} = \int \left[\int f(x)dx + C_1\right]dx + C_2$

.....

两边积分
$$y = \int \left\{ \cdots \int \left[\int f(x) dx + C_1 \right] dx + C_2 \cdots \right\} dx + C_n$$





$$y''' = e^{2x} - \cos x \Rightarrow$$

$$y''' = e^{2x} - \cos x \implies y'' =$$

$$y''' = e^{2x} - \cos x \implies y'' = \frac{1}{2}e^{2x} - \frac{1}{2}e^{2x}$$

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例 2 求
$$y''' = \frac{1}{\sqrt{x}}$$
 的通解.

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解 连续两边积分

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$$y''' = x^{-\frac{1}{2}} \implies y'' = 2x^{\frac{1}{2}} + C_1 \implies y' = 2 \cdot \frac{2}{3}x^{\frac{3}{2}}$$



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$$\Rightarrow$$

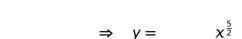
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$$\Rightarrow y = 2 \cdot \frac{2}{3} \cdot \frac{2}{5} x^{\frac{5}{2}}$$

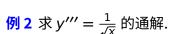
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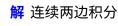
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$$C + C_2$$



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这两类特殊类型的二阶微分方程,都可以通过变量代换,降阶成一阶微 分方程。



We are here now...

♣
$$y'' = f(x, y')$$
 型的微分方程

将 y'' = f(x, y') 看成关于 y' 的一阶微分方程.

计算通解的方法:

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$$p=\varphi(x,\,C_1)$$

3. 代回变量 p = y' 得:

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3. 代回变量 p = y' 得:

$$y'=\varphi(x,\,C_1)$$

所以

$$y = \int \varphi(x, C_1) dx + C_2$$

例 1 求 $(1 + x^2)y'' = 2xy'$ 的通解.





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$$\Rightarrow \quad \ln|p| = \ln(1+x^2) + C_1'$$



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$$\Rightarrow \quad \ln|p| = \ln(1+x^2) + C_1'$$
$$\Rightarrow \quad p = C_1(1+x^2)$$

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2. 这是可分离变量微分方程

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3. 还原变量



例 1 求 $(1 + x^2)y'' = 2xy'$ 的通解.

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 $\Rightarrow p = C_1(1+x^2)$ 3. 还原变量 , 并两边积分

$$p = 1 + x^{2}$$

$$\Rightarrow \ln |p| = \ln(1 + x^{2}) + C'_{1}$$

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 $y' = C_1(1+x^2) \implies y = C_1 \left[(1+x^2)dx = C_1\left(\frac{1}{3}x^3 + x + C_2\right) \right]$ 思考 求在初始条件 $y|_{x=0} = 1$, $y'|_{x=0} = 3$ 的特解. 7c 可降阶微分方程 7/11 ⊲ ⊳ ∆ ⊽

解



$$p' = p + x$$

$$p' = p + x \Rightarrow p' - p = x$$

 \mathbf{H} 1. 作变量代换 p = y',得

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2. 这是一阶线性微分方程

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- 2.1 齐次部分
- 2.2 常数变易

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$$y' = -(1+x) + C_1 e^x \Rightarrow y = -x - \frac{1}{2}x^2 + C_1 e^x + C_2$$

We are here now...

计算通解的方法:



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2. 改写: $\frac{dp}{dx} = \frac{dp}{dy} \cdot \frac{dy}{dx} = p\frac{dp}{dy}$. 代入上式得:

$$\rho \frac{dp}{dy} = f(y, p)$$

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$$p=\varphi(y,\,C_1)$$

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$$y' = p = \varphi(y, C_1) \Rightarrow \frac{dy}{dx} = \varphi(y, C_1)$$

计算通解的方法:

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$$\frac{dp}{dx} = f(y, p)$$

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3. 假设可解得:

$$y' = p = \varphi(y, C_1) \Rightarrow \frac{dy}{dx} = \varphi(y, C_1)$$

$$x = \int \frac{1}{\varphi(v, C_1)} dy + C_2$$

$$y\frac{dp}{dx} - p^2 = 0$$

 \mathbf{H} 1. 作变量代换 p = y':

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4. 还原变量:

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$$\frac{dp}{dx} = \frac{dp}{dx} \cdot \frac{dy}{dx} = p\frac{dp}{dx}$$
. 代入上式得:

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 ay
 p
 y

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$$\frac{dy}{dx} = C_1 y$$

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 $\int \frac{1}{p} dp = \int \frac{1}{y} dy \quad \Rightarrow \quad \ln|p| = \ln|y| + C_1' \quad \Rightarrow \quad p = C_1 y$

$$\frac{dy}{dx} = C_1 y \quad \Rightarrow \quad y = C_2 e^{C_1 x}.$$



解 作变量代换 p = y':

 \mathbf{M} 作变量代换 p = y':

$$y^3 \frac{dp}{dx} + 1 = 0$$

$$\mathbf{m}$$
 作变量代换 $p = y'$:

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$$y^{3}p\frac{dp}{dy} + 1 = y^{3}\frac{dp}{dx} + 1 = 0$$

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$$\Rightarrow pdp = -y^{-3}dy$$

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$$y^{3}p\frac{dp}{dy} + 1 = 0$$

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解 作变量代换 *p* = *y'*:

$$y^{3}p\frac{dp}{dy} + 1 = 0$$

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 \mathbf{p} 作变量代换 p = y':

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$$\Rightarrow \frac{1}{2}p^{2} = \frac{1}{2}y^{-2} + C_{1} \xrightarrow{x=1} \frac{x=1}{y=1} \stackrel{\text{poly}}{p=0} C_{1} = -\frac{1}{2},$$

 \mathbf{K} 作变量代换 p = y':

$$y^{3} \rho \frac{d\rho}{dy} + 1 = 0$$

$$\Rightarrow \rho d\rho = -y^{-3} dy \Rightarrow \int \rho d\rho = -\int y^{-3} dy$$

$$\Rightarrow \frac{1}{2} \rho^{2} = \frac{1}{2} y^{-2} + C_{1} \xrightarrow{\underset{v=1, \rho=0}{x=1} \text{ p}} C_{1} = -\frac{1}{2}, \ \rho^{2} = y^{-2} - 1$$

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$$\Rightarrow \frac{1}{2}p^{2} = \frac{1}{2}y^{-2} + C_{1} \xrightarrow{\frac{x=1}{y}} C_{1} = -\frac{1}{2}, p^{2} = y^{-2} - 1$$

$$\Rightarrow \qquad p = \pm \sqrt{y^{-2} - 1}$$

解 作变量代换 p = v':

$$y^{3}p\frac{dp}{dy} + 1 = 0$$

$$\Rightarrow pdp = -y^{-3}dy \Rightarrow \int pdp = -\int y^{-3}dy$$

$$\Rightarrow \frac{1}{2}p^{2} = \frac{1}{2}y^{-2} + C_{1} \xrightarrow{\frac{x=1}{y=1}, p=0} C_{1} = -\frac{1}{2}, p^{2} = y^{-2} - 1$$

$$\Rightarrow \frac{dy}{dy} = p = \pm \sqrt{y^{-2} - 1}$$

$$\Rightarrow \frac{dy}{dx} = p = \pm \sqrt{y^{-2} - 1}$$



解 作变量代换 p = y':

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解 作变量代换
$$p = y'$$
:

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$$\Rightarrow \quad \frac{dy}{dx} = p = \pm \sqrt{y^{-2} - 1}$$

$$\Rightarrow \frac{ydy}{\sqrt{1-y^2}} = \pm dx \Rightarrow -\sqrt{1-y^2} = \pm x + C_2$$

$$\Rightarrow \frac{x=1}{y-1}$$

$$y^{3}p\frac{dp}{dy} + 1 = 0$$

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$$\Rightarrow \frac{1}{2}p^{2} = \frac{1}{2}y^{-2} + C_{1} \xrightarrow{\frac{x=1}{y=1}, p=0} C_{1} = -\frac{1}{2}, p^{2} = y^{-2} - 1$$

$$\Rightarrow \quad \frac{dy}{dx} = p = \pm \sqrt{y^{-2} - 1}$$

$$\Rightarrow \frac{ydy}{\sqrt{1-y^2}} = \pm dx \Rightarrow -\sqrt{1-y^2} = \pm x + C_2$$

$$\Rightarrow \frac{x=1 \text{ Pd}}{y=1} \quad C_2 = \mp 1,$$

解 作变量代换
$$p = y'$$
:
$$y^3 p \frac{dp}{dv} + 1 = 0$$

$$\Rightarrow pdp = -y^{-3}dy \Rightarrow \int pdp = -\int y^{-3}dy$$

$$\Rightarrow \frac{1}{2}p^{2} = \frac{1}{2}y^{-2} + C_{1} \xrightarrow{\frac{x=1}{9}} C_{1} = -\frac{1}{2}, p^{2} = y^{-2} - 1$$

$$\Rightarrow \frac{dy}{dx} = p = \pm \sqrt{y^{-2} - 1}$$

$$\Rightarrow \frac{ydy}{\sqrt{1 - y^2}} = \pm dx \Rightarrow -\sqrt{1 - y^2} = \pm x + C_2$$

$$\sqrt{1-y^2}$$

$$\Rightarrow \xrightarrow{x=1 \text{ by}} C_2 = \mp 1, -\sqrt{1-y^2} = \pm x \mp 1$$



解 作变量代换
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$$1 \quad 1 \quad x = 1$$

$$\Rightarrow \frac{1}{2}p^{2} = \frac{1}{2}y^{-2} + C_{1} \xrightarrow{\frac{x=1}{3}} C_{1} = -\frac{1}{2}, \ p^{2} = y^{-2} - 1$$

$$\Rightarrow \frac{dy}{dx} = p = \pm \sqrt{y^{-2} - 1}$$

$$\Rightarrow \frac{ydy}{\sqrt{1-y^2}} = \pm dx \Rightarrow -\sqrt{1-y^2} = \pm x + C_2$$

$$\Rightarrow \frac{x=1}{y=1} \qquad C_2 = \mp 1, -\sqrt{1-y^2} = \pm x \mp 1 = \pm (x-1)$$





例 2 求 $y^3y'' + 1 = 0$ 在初始条件 $y|_{x=1} = 1$, $y'|_{x=1} = 0$ 下的特解. \mathbf{M} 作变量代换 p = y':

$$y^{3}p\frac{dp}{dy} + 1 = 0$$

$$\Rightarrow pdp = -y^{-3}dy \Rightarrow \int pdp = -\int y^{-3}dy$$

$$\Rightarrow pdp = -y^{-3}dy \Rightarrow \int pdp = -\int y^{-3}dy$$

$$\Rightarrow \frac{1}{2}p^2 = \frac{1}{2}y^{-2} + C_1 \xrightarrow{x=1} C_1 = -\frac{1}{2}, p^2 = y^{-2} - 1$$

$$\Rightarrow \frac{1}{2}p^2 = \frac{1}{2}y^{-2} + C_1 \xrightarrow{\frac{\chi = 1py}{y=1, p=0}} C_1 = -\frac{1}{2}, p^2 = y^{-2} - 1$$

$$dy$$

$$\Rightarrow \frac{dy}{dx} = p = \pm \sqrt{y^{-2} - 1}$$

$$\Rightarrow \frac{dy}{dx} = p = \pm \sqrt{y^{-2} - 1}$$

$$y dy$$

$$\frac{dx}{dx} = \frac{ydy}{dx} = +dx \Rightarrow -\sqrt{1-y^2} = +x + Cx$$

$$\Rightarrow \frac{ydy}{\sqrt{1-y^2}} = \pm dx \Rightarrow -\sqrt{1-y^2} = \pm x + C_2$$

$$\Rightarrow \frac{ydy}{\sqrt{1-y^2}} = \pm dx \Rightarrow -\sqrt{1-y^2} = \pm x + C_2$$

$$\Rightarrow \frac{y}{\sqrt{1-y^2}} = \pm dx \quad \Rightarrow \quad -\sqrt{1-y^2} = \pm x + C_2$$

$$\Rightarrow \frac{1}{\sqrt{1-y^2}} = \pm ax \Rightarrow -\sqrt{1-y^2} = \pm x + C_2$$

$$\sqrt{1-y^2}$$

$$y = 1 \text{ Hd}$$

 $\Rightarrow \xrightarrow[y=1]{x=1}^{x=1}$ $C_2 = \mp 1, -\sqrt{1-y^2} = \pm x \mp 1 = \pm (x-1)$

例 2 求 $y^3y'' + 1 = 0$ 在初始条件 $y|_{x=1} = 1$, $y'|_{x=1} = 0$ 下的特解. \mathbf{M} 作变量代换 p = y': $y^3p\frac{dp}{dv} + 1 =$

$$\Rightarrow pdp = -y^{-3}dy \Rightarrow \int pdp = -\int y^{-3}dy$$

$$\Rightarrow \frac{1}{2}p^{2} = \frac{1}{2}y^{-2} + C_{1} \xrightarrow{\underset{y=1, p=0}{\times}} C_{1} = -\frac{1}{2}, p^{2} = y^{-2} - 1$$

$$dy$$

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$$\Rightarrow \frac{y \, dy}{\sqrt{1 - y^2}} = \pm dx \quad \Rightarrow \quad -\sqrt{1 - y^2} = \pm x + C_2$$

$$\Rightarrow \frac{3}{\sqrt{1-y^2}} = \pm dx \quad \Rightarrow \quad -\sqrt{1-y^2} = \pm x + C_2$$

$$\Rightarrow \frac{1}{\sqrt{1-y^2}} = \pm dx \Rightarrow -\sqrt{1-y^2} = \pm x + C_2$$

$$\sqrt{1-y^2}$$
 $x=1$ 时 $\sqrt{1-x^2}$ $\sqrt{1-x^2}$

$$\xrightarrow{x=1} C_2 - x_1 - \sqrt{1-y^2} - + x + 1 - + (x-1)$$

$$\Rightarrow \xrightarrow[y=1]{x=1} C_2 = \mp 1, -\sqrt{1-y^2} = \pm x \mp 1 = \pm (x-1)$$