§5.3 换元积分法

数学系 梁卓滨

2017-2018 学年 II



教学要求

◇ 熟练掌握换元积分法: "凑微分", "变量代换"





4

Outline of §5.3

1. 第一类换元积分法: 凑微分

第二类换元积分法: 变量代换



We are here now...

1. 第一类换元积分法: 凑微分

第二类换元积分法: 变量代换



能够计算如下的不定积分:

$$\int \frac{dx}{2x+1}, \quad \int \cos(\frac{5}{2}x)dx$$

$$\int \frac{x}{\sqrt{3-x^2}}dx, \quad \int x\sin(x^2)dx$$

$$\int \frac{(\ln x)^2}{x}dx, \quad \int e^{\sin x}\cos xdx$$

$$\int \frac{1}{\cos x}dx$$

$$\int "l\% »§Æ¢È" dx$$

$$\int \text{"i½»§Æ¢È"} dx = \int f(\varphi(x))\varphi'(x)dx$$

$$\int \text{"i}\% \text{ §\mathcal{E}c\`{E}"} dx = \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\int \text{"i}\% \text{ §Acce" } dx = \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$
$$= \frac{\varphi(x) = u}{u} \int f(u)du$$

$$\int \text{"i}\% \text{sAcce"} dx = \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$
$$= \frac{\varphi(x)=u}{m} \int f(u)du$$
$$= F(u) + C$$

$$\int \text{"i} \frac{1}{2} \text{sg.} \mathcal{E} \text{ce'} dx = \int f(\varphi(x)) \varphi'(x) dx = \int f(\varphi(x)) d\varphi(x)$$

$$= \frac{\varphi(x) = u}{2} \int f(u) du$$

$$= F(u) + C = \frac{u = \varphi(x)}{2} F(\varphi(x)) + C$$

$$\int \text{"i½»§Æ¢È"} dx = \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$
$$= \frac{\varphi(x)=u}{m} \int f(u)du$$
$$= F(u) + C = \frac{u=\varphi(x)}{m} F(\varphi(x)) + C$$

• 计算步骤:

$$\int \text{"l½»§Æ¢È"} dx = \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$
$$= \frac{\varphi(x)=u}{\int} f(u)du$$
$$= F(u) + C = \frac{u=\varphi(x)}{\int} F(\varphi(x)) + C$$

验证: F(φ(x)) 确是 "l½»§Æ¢è" 的原函数!

• 计算步骤:

$$\int \text{"i½»§ÆcÈ"} dx = \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$
$$= \frac{\varphi(x) = u}{\int} f(u)du$$
$$= F(u) + C = \frac{u = \varphi(x)}{\int} F(\varphi(x)) + C$$

验证: F(φ(x)) 确是 "l½»§Æ¢è" 的原函数!

$$\frac{d}{dx}F(\varphi(x)) =$$

• 计算步骤:

验证: F(φ(x)) 确是 "l½»§Æ¢È" 的原函数!

$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) =$$

• 计算步骤:

$$\int \text{"i½»§ÆcÈ"} dx = \frac{\text{養微分}}{\text{=}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$
$$= \frac{\varphi(x) = u}{\text{=}} \int f(u)du$$
$$= F(u) + C = \frac{u = \varphi(x)}{\text{=}} F(\varphi(x)) + C$$

验证: F(φ(x)) 确是 "l½»§Æ¢È" 的原函数!

$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \cdot \varphi'(x) =$$

• 计算步骤:

验证: F(φ(x)) 确是 "l½»§Æ¢È" 的原函数!

$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \cdot \varphi'(x) = \text{"i}\% \text{ s.e.}$$

• 计算步骤:

$$\int \text{"l½»§ÆcÈ"} dx \xrightarrow{\frac{}{}} \int f(\varphi(x))\varphi'(x)dx = \int f(\varphi(x))d\varphi(x)$$

$$\frac{\varphi(x)=u}{} \int f(u)du$$

$$= F(u) + C \frac{u=\varphi(x)}{} F(\varphi(x)) + C$$

验证: F(φ(x)) 确是 "ì½»§Æ¢È" 的原函数!

$$\frac{d}{dx}F(\varphi(x)) = F'(\varphi(x)) \cdot \varphi'(x) = f(\varphi(x)) \cdot \varphi'(x) = \text{"i}_{2} \text{ sacc}$$

总之
$$\int$$
 "ì½»§Æ¢È" dx $=$ $\int f(\varphi(x))d\varphi(x)$
$$= \int f(u)du = F(u) + C = F(\varphi(x)) + C$$

例子 求不定积分

$$\int \frac{1}{1+2x} dx, \qquad \int \frac{1}{2-3x} dx, \qquad \int \sqrt{3x-1} dx,$$

$$\int \frac{1}{\sqrt{1-5x}} dx, \qquad \int \cos(\frac{3}{2}x) dx, \qquad \int e^{-\frac{1}{2}x+4} dx.$$

凑微分 " $\int "ì½»§Æc\`e" dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ "例 1: 热身

例子 求不定积分

$$\int \frac{1}{1+2x} dx, \qquad \int \frac{1}{2-3x} dx, \qquad \int \sqrt{3x-1} dx,$$

$$\int \frac{1}{\sqrt{1-5x}} dx, \qquad \int \cos(\frac{3}{2}x) dx, \qquad \int e^{-\frac{1}{2}x+4} dx.$$

$$dx = d(2x + 1), dx = d(2 - 3x),$$

$$dx = d(\frac{3}{2}x),$$
 $dx = \underline{\qquad} d(-\frac{1}{2}x + 4).$

凑微分 "ʃ"쉾Æcè" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$ " 例 I: 热身

例子 求不定积分

$$\int \frac{1}{1+2x} dx, \qquad \int \frac{1}{2-3x} dx, \qquad \int \sqrt{3x-1} dx,$$

$$\int \frac{1}{\sqrt{1-5x}} dx, \qquad \int \cos(\frac{3}{2}x) dx, \qquad \int e^{-\frac{1}{2}x+4} dx.$$

$$dx = \frac{1}{2} d(2x+1), \qquad dx = d(2-3x),$$

$$dx = d(\frac{3}{2}x),$$
 $dx = \underline{\qquad} d(-\frac{1}{2}x + 4).$

凑微分 "ʃ"쉾Æcè" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$ " 例 I: 热身

例子 求不定积分

$$\int \frac{1}{1+2x} dx, \qquad \int \frac{1}{2-3x} dx, \qquad \int \sqrt{3x-1} dx,$$

$$\int \frac{1}{\sqrt{1-5x}} dx, \qquad \int \cos(\frac{3}{2}x) dx, \qquad \int e^{-\frac{1}{2}x+4} dx.$$

$$dx = \frac{1}{2} d(2x+1),$$
 $dx = -\frac{1}{3} d(2-3x),$

$$dx = d(\frac{3}{2}x),$$
 $dx = \underline{\qquad} d(-\frac{1}{2}x + 4).$

凑微分 "ʃ"쉾Æcè" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$ " 例 I: 热身

$$\int \frac{1}{1+2x} dx, \qquad \int \frac{1}{2-3x} dx, \qquad \int \sqrt{3x-1} dx,$$

$$\int \frac{1}{\sqrt{1-5x}} dx, \qquad \int \cos(\frac{3}{2}x) dx, \qquad \int e^{-\frac{1}{2}x+4} dx.$$

$$dx = \frac{1}{2} d(2x+1),$$
 $dx = -\frac{1}{3} d(2-3x),$

$$dx = \frac{2}{3} d(\frac{3}{2}x),$$
 $dx = \underline{\qquad} d(-\frac{1}{2}x + 4).$



$$\int \frac{1}{1+2x} dx, \qquad \int \frac{1}{2-3x} dx, \qquad \int \sqrt{3x-1} dx,$$

$$\int \frac{1}{\sqrt{1-5x}} dx, \qquad \int \cos(\frac{3}{2}x) dx, \qquad \int e^{-\frac{1}{2}x+4} dx.$$

$$dx = \frac{1}{2} d(2x+1),$$
 $dx = -\frac{1}{3} d(2-3x),$

$$dx = \frac{2}{3} d(\frac{3}{2}x),$$
 $dx = \underline{-2} d(-\frac{1}{2}x + 4).$

例子 求不定积分 $\int \frac{1}{1+2x} dx$, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\mathbf{H} \int \frac{1}{1+2x} dx = 0$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\mathbf{H} \int \frac{1}{1+2x} dx = \int \frac{1}{1+2x}$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\iint \frac{1}{1+2x} dx = \int \frac{1}{1+2$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\mathbf{H} \int \frac{1}{1+2x} dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x)$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\Re \int \frac{1}{1+2x} dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x) = \frac{1}{2} \int \frac{1}{u} du$$

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\int \sqrt{3x - 1} dx =$$



凑微分 " \int "ì%»§Æcè" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\int \sqrt{3x-1}dx =$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\int \sqrt{3x-1}dx =$$



$$\int \sqrt{3x - 1} dx =$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$ 解 $\int 1 \int 1 \int 1 dx$

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$

$$\int \sqrt{3x-1}dx =$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\int \sqrt{3x - 1} dx =$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\int \sqrt{3x-1}dx =$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\int \sqrt{3x-1} dx = \int \sqrt{3x-1}$$

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\int \sqrt{3x-1} dx = \int \sqrt{3x-1} \qquad d(3x-1)$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\int \sqrt{3x-1} dx = \int \sqrt{3x-1} \cdot \frac{1}{3} d(3x-1)$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\frac{1}{1+2x}dx = \int \frac{1}{1+2x} \cdot \frac{1}{2}d(1+2x) = \frac{1}{2} \int \frac{1}{u}du$$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1+2x| + C$$

$$\int \frac{1}{2-3x}dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3})d(2-3x) = -\frac{1}{3} \int \frac{1}{u}du$$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

$$\int \sqrt{3x - 1} \, dx = \int \sqrt{3x - 1} \cdot \frac{1}{3} \, d(3x - 1) = \frac{1}{3} \int \sqrt{u} \, du$$

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2 - 3x| + C$$

$$\int \sqrt{3x - 1} dx = \int \sqrt{3x - 1} \cdot \frac{1}{3} d(3x - 1) = \frac{1}{3} \int \sqrt{u} du = \frac{1}{3} \int u^{1/2} du$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1 + 2x| + C$$

$$\int \frac{1}{2 - 3x} dx = \int \frac{1}{2 - 3x} \cdot (-\frac{1}{3}) d(2 - 3x) = -\frac{1}{3} \int \frac{1}{u} du$$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2 - 3x| + C$$

$$\int \sqrt{3x - 1} dx = \int \sqrt{3x - 1} \frac{1}{3} d(3x - 1) = \frac{1}{3} \int \sqrt{u} du =$$

$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2 - 3x| + C$$

$$\int \sqrt{3x - 1} dx = \int \sqrt{3x - 1} \cdot \frac{1}{3} d(3x - 1) = \frac{1}{3} \int \sqrt{u} du = \frac{1}{3} \int u^{1/2} du$$



例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1 + 2x| + C$$

$$\int \frac{1}{2 - 3x} dx = \int \frac{1}{2 - 3x} \cdot (-\frac{1}{3}) d(2 - 3x) = -\frac{1}{3} \int \frac{1}{u} du$$

$$= -\frac{1}{2} \ln|u| + C = -\frac{1}{2} \ln|2 - 3x| + C$$

 $\frac{2}{5}u^{3/2}$

 $= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2 - 3x| + C$ $\int \sqrt{3x - 1} dx = \int \sqrt{3x - 1} \cdot \frac{1}{3} d(3x - 1) = \frac{1}{3} \int \sqrt{u} du = \frac{1}{3} \int u^{1/2} du$

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

例于 永不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{2}{2-3x} dx$, $\int \sqrt{3x-1} dx$ 解 $\int 1$ 1 1 1 $\int 1$

$$\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$$
$$= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2-3x| + C$$

 $\int \sqrt{3x-1} dx = \int \sqrt{3x-1} \cdot \frac{1}{3} d(3x-1) = \frac{1}{3} \int \sqrt{u} du = \frac{1}{3} \int u^{1/2} du$

$$\int \int 3 \cdot 3$$

$$= \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C$$

凑微分 " $\lceil r$ ì½»§Æ¢è" $dx = \lceil f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{1+2x} dx$$
, $\int \frac{1}{2-3x} dx$, $\int \sqrt{3x-1} dx$

$$\mathbf{H} \int \frac{1}{1+2x} dx = \int \frac{1}{1+2x} \cdot \frac{1}{2} d(1+2x) = \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1 + 2x| + C$$

$$\int \frac{1}{2 - 3x} dx = \int \frac{1}{2 - 3x} \cdot (-\frac{1}{3}) d(2 - 3x) = -\frac{1}{3} \int \frac{1}{u} du$$

 $\int \frac{1}{2-3x} dx = \int \frac{1}{2-3x} \cdot (-\frac{1}{3}) d(2-3x) = -\frac{1}{3} \int \frac{1}{u} du$ $= -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|2 - 3x| + C$

$$= -\frac{1}{3}\ln|u| + C = -\frac{1}{3}\ln|2 - 3x| + C$$

$$\int \sqrt{3x - 1} dx = \int \sqrt{3x - 1} \cdot \frac{1}{3}d(3x - 1) = \frac{1}{3}\int \sqrt{u}du = \frac{1}{3}\int u^{1/2}du$$

 $= \frac{1}{3} \cdot \frac{2}{3} u^{3/2} + C = \frac{2}{9} (3x - 1)^{3/2} + C$

例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\int_{0}^{\infty} \frac{1}{\sqrt{1-5x}} dx =$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4} dx =$$



例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\int_{1}^{\infty} \frac{1}{\sqrt{1-5x}} dx = \int_{1}^{\infty} (1-5x)^{-1/2}$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4} dx =$$



例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\iint_{\sqrt{1-5x}} \frac{1}{dx} = \int (1-5x)^{-1/2} \qquad d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4} dx =$$



例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\int_{0}^{\infty} \frac{1}{\sqrt{1-5x}} dx = \int_{0}^{\infty} (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x)$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



凑微分 " $\lceil r$ ì½»§Æ¢è" $dx = \lceil f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\int_{0}^{\frac{1}{2}} \frac{1}{\sqrt{1-5x}} dx = \int_{0}^{2} (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int_{0}^{2} u^{-1/2} du$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



凑微分 " $\lceil r$ ì½»§Æ¢è" $dx = \lceil f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\int_{0}^{1} \frac{1}{\sqrt{1-5x}} dx = \int_{0}^{1} (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int_{0}^{1} u^{-1/2} du$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$

$$= 2u^{1/2}$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



$$\iint_{\infty} \frac{1}{\sqrt{1-5x}} dx = \int_{\infty}^{\infty} (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int_{\infty}^{\infty} u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \sqrt{1-5x} \qquad \int \qquad 5 \qquad 5 \int$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x) dx =$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$= -\frac{1}{5} \cdot 2u^{1/3}$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \quad d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x)$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



$$\frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$\int e^{-\frac{1}{2}x+4}dx =$$



例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}} dx$, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\frac{1}{\sqrt{1-5x}}dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5})d(1-5x) = -\frac{1}{5} \int u^{-1/2}du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3} \int \cos u du$$

$$= \int \cos \frac{3}{2} x \cdot \frac{2}{3} d(\frac{3}{2}x) = \frac{2}{3} \int \cos u du$$
$$= \frac{2}{3} \sin(u) + C$$

 $\int e^{-\frac{1}{2}x+4} dx =$



凑微分 " $\lceil r$ ì½»§Æ¢è" $dx = \lceil f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 I

$$\Re \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(-x) dx = \int \cos(-x) dx =$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$
$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4} dx =$$



凑微分 " $\lceil r$ ì½»§Æ¢è" $dx = \lceil f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 I

$$\iint_{-\infty} \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int_{-\infty}^{\infty} \frac{3}{5} dx = \int_{-\infty}^{\infty} \frac{3}{5} \frac{2}{5} dx = \frac{3}{5} \frac{2}{5} \int_{-\infty}^{\infty} \frac{3}{5} \frac{3}{5} \frac{3}{5} \frac{2}{5} \int_{-\infty}^{\infty} \frac{3}{5} \frac{3}$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$
$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4} dx = \int e^{-\frac{1}{2}x+4}$$



$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$
$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4} dx = \int e^{-\frac{1}{2}x+4} \qquad d(-\frac{1}{2}x+4)$$



$$\iint_{0}^{\mathbb{R}} \frac{1}{\sqrt{1-5x}} dx = \int_{0}^{\mathbb{R}} (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int_{0}^{\mathbb{R}} u^{-1/2} du$$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$
$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

$$\int e^{-\frac{1}{2}x+4} dx = \int e^{-\frac{1}{2}x+4} \cdot (-2) d(-\frac{1}{2}x+4)$$



例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}} dx$, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$
$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

 $= \frac{1}{3}\sin(u) + C = \frac{1}{3}\sin(\frac{1}{2}x) + C$ $\int e^{-\frac{1}{2}x+4} dx = \int e^{-\frac{1}{2}x+4} \cdot (-2)d(-\frac{1}{2}x+4) = -2\int e^{u} du$



凑微分 " $\lceil r$ ì½»§Æ¢è" $dx = \lceil f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 I

例子 求不定积分 $\int \frac{1}{\sqrt{1-5x}} dx$, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

$$\iint \frac{1}{\sqrt{1-5x}} dx = \int (1-5x)^{-1/2} \cdot (-\frac{1}{5}) d(1-5x) = -\frac{1}{5} \int u^{-1/2} du$$
$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5} (1-5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$
$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

 $\int e^{-\frac{1}{2}x+4} dx = \int e^{-\frac{1}{2}x+4} \cdot (-2) d(-\frac{1}{2}x+4) = -2 \int e^{u} du$

例子 求不定积分
$$\int \frac{1}{\sqrt{1-5x}} dx$$
, $\int \cos(\frac{3}{2}x) dx$, $\int e^{-\frac{1}{2}x+4} dx$

解
$$\frac{1}{\sqrt{1-5x}}dx$$
, $\int \cos(\frac{\pi}{2}x)dx$, $\int e^{-2\pi x}dx$

$$= -\frac{1}{5} \cdot 2u^{1/2} + C = -\frac{2}{5}(1 - 5x)^{1/2} + C$$

$$\int \cos(\frac{3}{2}x)dx = \int \cos\frac{3}{2}x \cdot \frac{2}{3}d(\frac{3}{2}x) = \frac{2}{3}\int \cos u du$$

$$= \frac{2}{3}\sin(u) + C = \frac{2}{3}\sin(\frac{3}{2}x) + C$$

 $\int e^{-\frac{1}{2}x+4} dx = \int e^{-\frac{1}{2}x+4} \cdot (-2) d(-\frac{1}{2}x+4) = -2 \int e^{u} du$

 $= -2e^{u} + C = -2e^{-\frac{1}{2}x+4} + C$

(1)
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$,

(2)
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

例子 求不定积分

(1)
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$,

(2)
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

热身

$$xdx = \underline{\qquad} d(x^2) \qquad \qquad xdx = \underline{\qquad} d(1-x^2)$$

$$xdx = d(3-x^2) \quad xdx = d(1+3x^2)$$

例子 求不定积分

(1)
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$,

(2)
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$xdx = \frac{1}{2} d(x^2) \qquad xdx = \underline{\qquad} d(1-x^2)$$

$$xdx = d(3-x^2) \quad xdx = d(1+3x^2)$$

例子 求不定积分

(1)
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$,

(2)
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$xdx = \frac{1}{2} d(x^2)$$
 $xdx = \frac{1}{2} d(1-x^2)$

$$xdx = d(3-x^2) \quad xdx = d(1+3x^2)$$

例子 求不定积分

(1)
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$,

(2)
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$xdx = \frac{1}{2} d(x^2) \qquad xdx = \frac{1}{2} d(1-x^2)$$

$$xdx = -\frac{1}{2} d(3-x^2) \quad xdx = d(1+3x^2)$$



例子 求不定积分

(1)
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$,

(2)
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$xdx = \frac{1}{2} d(x^2)$$
 $xdx = \frac{1}{2} d(1-x^2)$

$$xdx = -\frac{1}{2} d(3-x^2)$$
 $xdx = \frac{1}{6} d(1+3x^2)$



凑微分 "ʃ"ì½»§ÆcÈ" $dx=\int f(\varphi(x))d\varphi(x)=F(\varphi(x))+C$ " 例 III

例子 求 $\int xe^{x^2}dx$, $\int x\sin(x^2)dx$

例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int x e^{x^2} dx =$$

例子求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int x e^{x^2} dx = \int e^{x^2}$$

例子求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int x e^{x^2} dx = \int e^{x^2} d(x^2)$$

例子求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2)$$

例子求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^{2}}dx = \int e^{x^{2}} \frac{1}{2}d(x^{2}) = \frac{1}{2} \int e^{u}du$$

例子求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^udu = \frac{1}{2}e^u + C$$

例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^udu = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$



例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^udu = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x \sin(x^2) dx =$$



例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2}dx = \int e^{x^2}\frac{1}{2}d(x^2) = \frac{1}{2}\int e^udu = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x \sin(x^2) dx = \int \sin(x^2)$$



例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2}dx = \int e^{x^2} \frac{1}{2}d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2) d(x^2)$$

例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2}dx = \int e^{x^2} \frac{1}{2}d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2) \cdot \frac{1}{2}d(x^2)$$

例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2}dx = \int e^{x^2} \frac{1}{2}d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$$

$$\int x\sin(x^2)dx = \int \sin(x^2) \cdot \frac{1}{2}d(x^2) = \frac{1}{2} \int \sin u du$$



例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

$$\int x \sin(x^2) dx = \int \sin(x^2) \cdot \frac{1}{2} d(x^2) = \frac{1}{2} \int \sin u du$$

$$= -\frac{1}{2} \cos u + C$$



例子 求
$$\int xe^{x^2}dx$$
, $\int x\sin(x^2)dx$

$$\int xe^{x^2} dx = \int e^{x^2} \frac{1}{2} d(x^2) = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2} + C$$

$$\int x \sin(x^2) dx = \int \sin(x^2) \cdot \frac{1}{2} d(x^2) = \frac{1}{2} \int \sin u du$$

$$= -\frac{1}{2} \cos u + C = -\frac{1}{2} \cos(x^2) + C$$

$$\cos u + C = -\frac{1}{2}\cos(x^2) + C$$

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$ 解 $\int x\sqrt{1-x^2}dx =$



例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$ 解
$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}$$



例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$
解 $\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}}$ $d(1-x^2)$

例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$ 解
$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2)$$



例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2} \int u^{\frac{1}{2}}du$$



凑微分 "ʃ"ì½»§Æcè" dx=ʃƒ(φ(x))dφ(x)=F(φ(x))+C" 例 IV

例子求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

凑微分 "ʃ"ì½»§Æ¢È" dx=ʃƒ(φ(x))dφ(x)=F(φ(x))+C" 例 Ⅳ

例子求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$ 解
$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$\frac{2}{3}u^{3/2}$$

凑微分 "ʃ"ì½»§Æ¢È" dx=ʃƒ(φ(x))dφ(x)=F(φ(x))+C" 例 Ⅳ

例子求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$
$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$
$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C$$

例子求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

例子求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx =$$

$$\int \frac{x}{1+3x^2} dx =$$



例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}}$$

$$\int \frac{x}{1+3x^2} dx =$$



例子求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$
解
 $\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$
 $= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$
 $\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} dx$

$$\int \frac{x}{1+3x^2} dx =$$



例子求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$\int \frac{x}{1+3x^2} dx =$$



例子求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$
解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du$$

$$\int \frac{x}{1+3x^2} dx =$$



例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$
解

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du \qquad 2u^{1/2}$$

$$\int \frac{x}{1+3x^2} dx =$$



例子求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}}dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$

$$= -\frac{1}{2}\int u^{-1/2}du = -\frac{1}{2} \cdot 2u^{1/2} + C$$

$$\int \frac{x}{1+3x^2} dx =$$



例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

例子求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$
$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C = -\frac{1}{3} (1 - x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3 - x^2}} dx = \int (3 - x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3 - x^2)$$

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$
$$= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2} dx =$$



例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

例子求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{\hat{A}}{\sqrt{3-x^2}}dx$, $\int \frac{\hat{A}}{1+3x^2}dx$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

$$= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3 - x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1 + 3x^2} dx = \int \frac{1}{1 + 3x^2}$$



例子求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

例子求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2})d(3-x^2)$$
$$= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1+3x^2} dx = \int \frac{1}{1+3x^2} d(1+3x^2)$$



例子求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\Re \int x \sqrt{1-x^2} dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2}) d(1-x^2) = -\frac{1}{2} \int u^{\frac{1}{2}} du$$

$$= -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C = -\frac{1}{3} (1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

$$= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3-x^2)^{\frac{1}{2}} + C$$

$$\int x du = \int 1 du + 2u^{2} du$$

$$\int \frac{x}{1+3x^2} dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6} d(1+3x^2)$$



凑微分 " $\int n^2 dx = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ "例 IV

例子求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

例子求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\int x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2} \int u^{\frac{1}{2}}du$$

$$= -\frac{1}{2} \cdot \frac{2}{3}u^{3/2} + C = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

$$\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

$$\int \frac{\lambda}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

$$\int \frac{\lambda}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$$

$$= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3 - x^2)^{\frac{1}{2}} + C$$

$$\int \frac{x}{1 + 3x^2} dx = \int \frac{1}{1 + 3x^2} \cdot \frac{1}{6} d(1 + 3x^2) = \frac{1}{6} \int \frac{1}{u} du$$



凑微分 " $\int n^2 dx = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ "例 IV

例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\int_{0}^{\infty} x\sqrt{1-x^{2}} dx = \int_{0}^{\infty} (1-x^{2})^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^{2}) = 0$$

 $\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$

 $= \frac{1}{6} \ln |u| + C$

 $= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3 - x^2)^{\frac{1}{2}} + C$

 $\int_{0}^{\infty} x\sqrt{1-x^{2}}dx = \int_{0}^{\infty} (1-x^{2})^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^{2}) = -\frac{1}{2} \int_{0}^{\infty} u^{\frac{1}{2}}du$

 $= -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C = -\frac{1}{3} (1 - x^2)^{\frac{3}{2}} + C$

凑微分 " $\int n^2 dx = \int f(\varphi(x)) d\varphi(x) = F(\varphi(x)) + C$ "例 IV

例子 求
$$\int x\sqrt{1-x^2}dx$$
, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

$$\int_{0}^{\infty} x\sqrt{1-x^2} dx = \int_{0}^{\infty} (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}$$

$$x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -$$

$$c\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2} \int u^{\frac{1}{2}}du$$
1 2 1

$$x\sqrt{1-x^2}dx = \int (1-x^2)^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^2) = -\frac{1}{2}\int u^{\frac{1}{2}}du$$

 $= -\frac{1}{2} \cdot \frac{2}{3} u^{3/2} + C = -\frac{1}{3} (1 - x^2)^{\frac{3}{2}} + C$

 $\int_{0}^{\infty} x\sqrt{1-x^{2}}dx = \int_{0}^{\infty} (1-x^{2})^{\frac{1}{2}} \cdot (-\frac{1}{2})d(1-x^{2}) = -\frac{1}{2} \int_{0}^{\infty} u^{\frac{1}{2}}du$

 $= -\frac{1}{2} \int u^{-1/2} du = -\frac{1}{2} \cdot 2u^{1/2} + C = -(3 - x^2)^{\frac{1}{2}} + C$

 $\int \frac{x}{\sqrt{3-x^2}} dx = \int (3-x^2)^{-\frac{1}{2}} \cdot (-\frac{1}{2}) d(3-x^2)$

例子 求 $\int x\sqrt{1-x^2}dx$, $\int \frac{x}{\sqrt{3-x^2}}dx$, $\int \frac{x}{1+3x^2}dx$

 $\int \frac{x}{1+3x^2} dx = \int \frac{1}{1+3x^2} \cdot \frac{1}{6} d(1+3x^2) = \frac{1}{6} \int \frac{1}{u} du$

 $= \frac{1}{6} \ln|u| + C = \frac{1}{6} \ln|1 + 3x^2| + C$

例子求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^{x}dx = de^{x}$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^{x}dx = de^{x}$$

$$\int e^{x} \sin(e^{x}) dx =$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^{x}dx = de^{x}$$

$$\int e^x \sin(e^x) dx = \int \sin(e^x)$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^x dx = de^x$$

$$\iint e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x}$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^x dx = de^x$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^x dx = de^x$$

$$\Re \int e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x}$$

$$= \int \sin u du = -\cos u + C$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^x dx = de^x$$

$$\iint e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x}$$

$$= \int \sin u du = -\cos u + C = -\cos(e^{x}) + C$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^x dx = de^x$$

$$\iint e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x}$$

$$= \int \sin u du = -\cos u + C = -\cos(e^{x}) + C$$

$$\int \frac{e^{x}}{1 + e^{x}} dx$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^{x}dx = de^{x}$$

$$\begin{aligned}
\mathbf{f} & = \int e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x} \\
& = \int \sin u du = -\cos u + C = -\cos(e^{x}) + C \\
& = \int \frac{e^{x}}{1 + e^{x}} dx = \int \frac{1}{1 + e^{x}} de^{x}
\end{aligned}$$



例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^x dx = de^x$$

$$\begin{aligned}
\mathbf{f} & = \int e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x} \\
& = \int \sin u du = -\cos u + C = -\cos(e^{x}) + C
\end{aligned}$$

$$\begin{aligned}
\mathbf{f} & = \int e^{x} dx &$$

$$\int \frac{e^{x}}{1 + e^{x}} dx = \int \frac{1}{1 + e^{x}} de^{x} = \int \frac{1}{1 + e^{x}} d(e^{x} + 1)$$

凑微分 "「"ì½»§Æcè" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 V

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^{x}dx = de^{x}$$

$$\iint e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x}$$

$$= \int \sin u du = -\cos u + C = -\cos(e^{x}) + C$$

$$\int \frac{e^x}{1+e^x} dx = \int \frac{1}{1+e^x} de^x = \int \frac{1}{1+e^x} d(e^x + 1)$$
$$= \int \frac{1}{u} du$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^{x}dx = de^{x}$$

$$\begin{aligned}
\mathbf{R} & \int e^{x} \sin(e^{x}) dx = \int \sin(e^{x}) de^{x} \\
&= \int \sin u du = -\cos u + C = -\cos(e^{x}) + C \\
& \int \frac{e^{x}}{1 + e^{x}} dx = \int \frac{1}{1 + e^{x}} de^{x} = \int \frac{1}{1 + e^{x}} d(e^{x} + 1) \\
&= \int \frac{1}{u} du = \ln|u| + C
\end{aligned}$$

例子 求
$$\int e^x \sin(e^x) dx$$
, $\int \frac{e^x}{1+e^x} dx$

$$e^{x}dx = de^{x}$$

$$= \int \sin u du = -\cos u + C = -\cos(e^{x}) + C$$

$$\int \frac{e^{x}}{1 + e^{x}} dx = \int \frac{1}{1 + e^{x}} de^{x} = \int \frac{1}{1 + e^{x}} d(e^{x} + 1)$$

$$= \int \frac{1}{u} du = \ln|u| + C = \ln(e^{x} + 1) + C$$



例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int_{-\infty}^{\infty} \ln x dx =$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$
$$= \frac{1}{2} u^2 + C$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$
$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$
$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$
$$\int \frac{1}{x^2} dx = \frac{1}{2} (\ln x)^2 + C$$

$$\int \frac{1}{x \ln x} dx =$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$
$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$
$$\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d \ln x$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$
$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$
$$\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d \ln x = \int \frac{1}{u} du$$

例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$
$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$
$$\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d \ln x = \int \frac{1}{u} du$$
$$= \ln |u| + C$$



例子 求
$$\int \frac{1}{x} \ln x dx$$
, $\int \frac{1}{x \ln x} dx$

$$\frac{1}{x}dx = d\ln x$$

$$\int \frac{1}{x} \ln x dx = \int \ln x d \ln x = \int u du$$

$$= \frac{1}{2} u^2 + C = \frac{1}{2} (\ln x)^2 + C$$

$$\int \frac{1}{x \ln x} dx = \int \frac{1}{\ln x} d \ln x = \int \frac{1}{u} du$$

$$= \ln |u| + C = \ln |\ln x| + C$$



凑微分 " \int "ì½»§Æ¢È" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 VII

例子求 $\int e^{\cos x} \sin x dx$, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

凑微分 " \int "ì½»§Æcè" $dx = \int f(\varphi(x))d\varphi(x) = F(\varphi(x)) + C$ " 例 **VII**

例子求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

热身
$$\sin x dx = d$$
_____ , $\cos x dx = d$ _____

例子求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

热身
$$\sin x dx = d - \cos x$$
 , $\cos x dx = d$ _____

,
$$\cos x dx = d$$

例子求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

热身
$$\sin x dx = d - \cos x = -d \cos x$$
, $\cos x dx = d$

例子求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

热身
$$\sin x dx = d - \cos x = -d \cos x$$
, $\cos x dx = d \sin x$

例子求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$
热身 $\sin x dx = d - \cos x = -d \cos x$, $\cos x dx = d \sin x$
解 $\int e^{\cos x} \sin x dx =$

例子求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$
热身 $\sin x dx = d - \cos x = -d \cos x$, $\cos x dx = d \sin x$
解
$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x$$

例子求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$
热身 $\sin x dx = d - \cos x = -d \cos x$, $\cos x dx = d \sin x$
解
$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^u du$$

例子求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$
热身 $\sin x dx = d - \cos x = -d \cos x$, $\cos x dx = d \sin x$
解
$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^u du$$
$$= -e^u + C$$



例子求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$
热身 $\sin x dx = d - \cos x = -d \cos x$, $\cos x dx = d \sin x$
解
$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^u du$$
$$= -e^u + C = -e^{\cos x} + C$$

例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^{u} du$$
$$= -e^{u} + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1 + \cos^2 x} dx =$$

$$\int \frac{\cos x}{\sin x} dx =$$

例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^{u} du$$
$$= -e^{u} + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d\cos x$$

$$\int \frac{\cos x}{\sin x} dx =$$



例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

热身
$$\sin x dx = d - \cos x = -d \cos x$$
, $\cos x dx = d \sin x$

$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^{u} du$$
$$= -e^{u} + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d\cos x = -\int \frac{1}{1 + u^2} du$$

$$\int \frac{\cos x}{\sin x} dx =$$



例子求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

热身
$$\sin x dx = d - \cos x = -d \cos x$$
, $\cos x dx = d \sin x$

$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^{u} du$$
$$= -e^{u} + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d\cos x = -\int \frac{1}{1 + u^2} du$$
$$= -\arctan u + C$$

$$\int \frac{\cos x}{\sin x} dx =$$



例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^{u} du$$
$$= -e^{u} + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d\cos x = -\int \frac{1}{1 + u^2} du$$
$$= -\arctan u + C = -\arctan(\cos x) + C$$

$$\int \frac{\cos x}{\sin x} dx =$$



例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^{u} du$$
$$= -e^{u} + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d\cos x = -\int \frac{1}{1 + u^2} du$$
$$= -\arctan u + C = -\arctan(\cos x) + C$$

$$\int \frac{\cos x}{\sin x} dx = \int \frac{1}{\sin x} d\sin x$$

例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^{u} du$$
$$= -e^{u} + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d\cos x = -\int \frac{1}{1 + u^2} du$$
$$= -\arctan u + C = -\arctan(\cos x) + C$$

$$\int \frac{\cos x}{\sin x} dx = \int \frac{1}{\sin x} d\sin x = \int \frac{1}{u} du$$

例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1+\cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

热身
$$\sin x dx = d - \cos x = -d \cos x$$
, $\cos x dx = d \sin x$

$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^{u} du$$
$$= -e^{u} + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d\cos x = -\int \frac{1}{1 + u^2} du$$
$$= -\arctan u + C = -\arctan(\cos x) + C$$

$$\int \frac{\cos x}{\sin x} dx = \int \frac{1}{\sin x} d\sin x = \int \frac{1}{u} du = \ln|u| + C$$



例子 求
$$\int e^{\cos x} \sin x dx$$
, $\int \frac{\sin x}{1 + \cos^2 x} dx$, $\int \frac{\cos x}{\sin x} dx$

$$\int e^{\cos x} \sin x dx = \int e^{\cos x} (-1) d \cos x = -\int e^{u} du$$
$$= -e^{u} + C = -e^{\cos x} + C$$

$$\int \frac{\sin x}{1 + \cos^2 x} dx = \int \frac{1}{1 + \cos^2 x} (-1) d\cos x = -\int \frac{1}{1 + u^2} du$$
$$= -\arctan u + C = -\arctan(\cos x) + C$$

$$\int \frac{\cos x}{\sin x} dx = \int \frac{1}{\sin x} d\sin x = \int \frac{1}{u} du = \ln|u| + C$$
$$= \ln|\sin x| + C$$

凑微分法 " \int "ì½»§ÆcÈ" $dx = \int f(\varphi(x))d\varphi(x)$ ": 例子总结

$$\int \frac{1}{1 - 3x} dx =$$

$$\int \sqrt{3x - 1} dx =$$

$$\int xe^{x^2} dx =$$

$$\int x\sqrt{1 - x^2} dx =$$

$$\int \frac{\ln x}{x} dx =$$

$$\int e^{\cos x} \sin x dx =$$

凑微分法 "「"ì½»§ÆcÈ" $dx= \int f(\varphi(x))d\varphi(x)$ ": 例子总结

$$\int \frac{1}{1-3x} dx = -\frac{1}{3} \int \frac{1}{1-3x} d(1-3x) = -\frac{1}{3} \int \frac{1}{u} du = \cdots$$

$$\int \sqrt{3x-1} dx = \frac{1}{3} \int \sqrt{3x-1} d(3x-1) = \frac{1}{3} \int u^{1/2} du = \cdots$$

$$\int x e^{x^2} dx =$$

$$\int x \sqrt{1-x^2} dx =$$

$$\int \frac{\ln x}{x} dx =$$



 $\int e^{\cos x} \sin x dx =$

凑微分法 " $\lceil r$ ì½»§Æcè" $dx = \lceil f(\varphi(x))d\varphi(x)$ ": 例子总结

$$\int \frac{1}{1-3x} dx = -\frac{1}{3} \int \frac{1}{1-3x} d(1-3x) = -\frac{1}{3} \int \frac{1}{u} du = \cdots$$

$$\int \sqrt{3x-1} dx = \frac{1}{3} \int \sqrt{3x-1} d(3x-1) = \frac{1}{3} \int u^{1/2} du = \cdots$$

$$\int x e^{x^2} dx = \frac{1}{2} \int e^{x^2} dx^2 = \frac{1}{2} \int e^u du = \cdots$$

$$\int x \sqrt{1-x^2} dx = -\frac{1}{2} \int \sqrt{1-x^2} d(1-x^2) = -\frac{1}{2} \int u^{1/2} du = \cdots$$

$$\int \frac{\ln x}{x} dx =$$

$$\int e^{\cos x} \sin x dx =$$



凑微分法 "["ì½»§Æcè" $dx=[f(\varphi(x))d\varphi(x)$ ": 例子总结

$$\int \frac{1}{1 - 3x} dx = -\frac{1}{3} \int \frac{1}{1 - 3x} d(1 - 3x) = -\frac{1}{3} \int \frac{1}{u} du = \cdots$$

$$\int \sqrt{3x - 1} dx = \frac{1}{3} \int \sqrt{3x - 1} d(3x - 1) = \frac{1}{3} \int u^{1/2} du = \cdots$$

$$\int x e^{x^2} dx = \frac{1}{2} \int e^{x^2} dx^2 = \frac{1}{2} \int e^u du = \cdots$$

$$\int x \sqrt{1 - x^2} dx = -\frac{1}{2} \int \sqrt{1 - x^2} d(1 - x^2) = -\frac{1}{2} \int u^{1/2} du = \cdots$$

$$\int \frac{\ln x}{x} dx = \int \ln x d(\ln x) = \int u du = \cdots$$

$$\int e^{\cos x} \sin x dx = -\int e^{\cos x} d\cos x = -\int e^u du = \cdots$$

We are here now...

1. 第一类换元积分法: 凑微分

第二类换元积分法: 变量代换



第二类换元积分法——"变量代换"法,能干啥?

能够计算如下的不定积分:

$$\int x\sqrt{3x-1}dx, \quad \int \frac{x}{\sqrt{x-2}}dx$$

$$\int \frac{1}{1+\sqrt{x}}dx, \quad \int \frac{1}{1+\sqrt[3]{x+1}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

• 计算步骤: $\int f(x)dx$

$$\int f(x)dx \stackrel{x=\varphi(t)}{=}$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t)$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int f(\varphi(t))\varphi'(t)dt$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\nabla m \otimes H} dt$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单, 容易求!}} dt$$
$$= G(t) + C$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{\overline{D}}} dt$$
$$= G(t) + C \xrightarrow{t=\varphi^{-1}(x)}$$

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{one of }, \text{ as } \text{sp. } t} dt$$
$$= G(t) + C \xrightarrow{t=\varphi^{-1}(x)} G(\varphi^{-1}(x)) + C$$

• 计算步骤:

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{∇m fill β, α}, α} dt$$
$$= G(t) + C \xrightarrow{t=\varphi^{-1}(x)} G(\varphi^{-1}(x)) + C$$

• 关键是:如何选取函数 $x = \varphi(t)$?



• 计算步骤:

$$\int f(x)dx \xrightarrow{x=\varphi(t)} \int f(\varphi(t))d\varphi(t) = \int \underbrace{f(\varphi(t))\varphi'(t)}_{\text{反而简单, }\text{容易求!}} dt$$
$$= G(t) + C \xrightarrow{t=\varphi^{-1}(x)} G(\varphi^{-1}(x)) + C$$

• 关键是:如何选取函数 $x = \varphi(t)$? 在后面的例子中,选取函数 $x = \varphi(t)$ 的方法:

把被积函数 f(x) 中复杂的部分整个设为 t , 从而得到 x 与 t 的函数关系!



变量代换 " $\int f(x)dx = \frac{x=\varphi(t)}{\int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t)) + C}$ " 例 I

例子 求不定积分 $\int \sqrt{1-x^2} dx$

变量代换 " $\int f(x)dx = \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t)) + C$ " **例 I**

例子 求不定积分
$$\int \sqrt{1-x^2} dx$$

$$\mathbb{H}$$
 :: $-1 \le x \le 1$, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$,

变量代换 " $\int f(x)dx = \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t)) + C$ " 例 I

例子 求不定积分
$$\int \sqrt{1-x^2} dx$$

$$\mathbf{F}$$
 $\because -1 \le x \le 1$, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$,

$$\therefore \int \sqrt{1-x^2} dx = \int \sqrt{1-\sin^2 t} d\sin t$$

变量代换 " $\int f(x)dx = \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t)) + C$ " 例 I

例子 求不定积分
$$\int \sqrt{1-x^2} dx$$

$$\mathbb{H}$$
 $\because -1 \le x \le 1$, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\cos t \ge 0$

$$\therefore \int \sqrt{1-x^2} dx = \int \sqrt{1-\sin^2 t} d\sin t$$

例子 求不定积分
$$\int \sqrt{1-x^2} dx$$

$$\mathbb{H}$$
 :: $-1 \le x \le 1$, $0 = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\cos t \ge 0$

$$\therefore \int \sqrt{1-x^2} dx = \int \sqrt{1-\sin^2 t} d\sin t = \int \cos^2 t dt$$

例子 求不定积分
$$\int \sqrt{1-x^2} dx$$

$$\mathbb{H}$$
 :: $-1 \le x \le 1$, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\cos t \ge 0$

$$\therefore \int \sqrt{1 - x^2} dx = \int \sqrt{1 - \sin^2 t} d\sin t = \int \cos^2 t dt$$
$$= \frac{1}{2} \int \cos 2t + 1 dt$$

例子 求不定积分
$$\int \sqrt{1-x^2} dx$$

解
$$: -1 \le x \le 1$$
, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\cos t \ge 0$

$$: \int \sqrt{1 - x^2} dx = \int \sqrt{1 - \sin^2 t} d\sin t = \int \cos^2 t dt$$

$$= \frac{1}{2} \int \cos 2t + 1 dt \qquad \frac{1}{2} \sin 2t$$

例子 求不定积分
$$\int \sqrt{1-x^2} dx$$

$$\mathbb{H}$$
 :: $-1 \le x \le 1$, $\Im x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\cos t \ge 0$

$$\therefore \int \sqrt{1-x^2} dx = \int \sqrt{1-\sin^2 t} d\sin t = \int \cos^2 t dt$$
$$= \frac{1}{2} \int \cos 2t + 1 dt = \frac{1}{2} \cdot \frac{1}{2} \sin 2t + \frac{1}{2} t + C$$

例子 求不定积分
$$\int \sqrt{1-x^2} dx$$

$$\mathbb{H}$$
 :: $-1 \le x \le 1$, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\cos t \ge 0$

$$\therefore \int \sqrt{1-x^2} dx = \int \sqrt{1-\sin^2 t} d\sin t = \int \cos^2 t dt$$

$$= \frac{1}{2} \int \cos 2t + 1 dt = \frac{1}{2} \cdot \frac{1}{2} \sin 2t + \frac{1}{2} t + C$$

$$= \frac{1}{2} \sin t \cos t + \frac{1}{2} t + C$$

例子 求不定积分
$$\int \sqrt{1-x^2} dx$$

$$\mathbb{H}$$
 :: $-1 \le x \le 1$, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\cos t \ge 0$

$$\therefore \int \sqrt{1 - x^2} dx = \int \sqrt{1 - \sin^2 t} d \sin t = \int \cos^2 t dt$$

$$= \frac{1}{2} \int \cos 2t + 1 dt = \frac{1}{2} \cdot \frac{1}{2} \sin 2t + \frac{1}{2} t + C$$

$$= \frac{1}{2} \sin t \cos t + \frac{1}{2} t + C$$

$$= \frac{1}{2} x \sqrt{1 - x^2} + \frac{1}{2} \arcsin x + C$$



变量代换 " $\int f(x)dx = \frac{x=\varphi(t)}{\int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t)) + C$ " 例 I

例子 求不定积分 $\int \sqrt{1-x^2} dx$

$$\mathbb{H}$$
 : $-1 \le x \le 1$, 设 $x = \sin t$, $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$, $\cos t \ge 0$

$$\therefore \int \sqrt{1 - x^2} dx = \int \sqrt{1 - \sin^2 t} d\sin t = \int \cos^2 t dt$$

$$= \frac{1}{2} \int \cos 2t + 1 dt = \frac{1}{2} \cdot \frac{1}{2} \sin 2t + \frac{1}{2} t + C$$

$$= \frac{1}{2} \sin t \cos t + \frac{1}{2} t + C$$

$$= \frac{1}{2} x \sqrt{1 - x^2} + \frac{1}{2} \arcsin x + C$$

 $\dot{\mathbf{E}}$ 可见选取合适 $\mathbf{x} = \boldsymbol{\varphi}(t)$ 很关键!



例子 求不定积分 $\int x\sqrt{3x-1}dx$, $\int \frac{x}{\sqrt{x-2}}dx$



例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$ 解 (1) 设 $t = (3x-1)^{\frac{1}{2}}$,

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

$$\therefore \int x\sqrt{3x-1}dx =$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

(1) 设
$$t = (3x-1)^{\frac{1}{2}}$$
, $\therefore x = \frac{1}{3}(t^2+1)$,

$$\therefore \int x\sqrt{3x-1}dx =$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

(1) 设
$$t = (3x-1)^{\frac{1}{2}}$$
, $\therefore x = \frac{1}{3}(t^2+1)$, $dx = \frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx =$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t.$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$
$$= \frac{2}{45}t^5 + \frac{2}{27}t^3 + C$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

(1) 设
$$t = (3x-1)^{\frac{1}{2}}$$
, $x = \frac{1}{3}(t^2+1)$, $dx = \frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$
$$= \frac{2}{45}t^5 + \frac{2}{27}t^3 + C = \frac{2}{45}(3x-1)^{\frac{5}{2}} + \frac{2}{27}(3x-1)^{\frac{3}{2}} + C$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

(1)
$$\partial_t t = (3x-1)^{\frac{1}{2}}, \quad \therefore x = \frac{1}{3}(t^2+1), \quad dx = \frac{2}{3}tdt$$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$
$$= \frac{2}{45}t^5 + \frac{2}{27}t^3 + C = \frac{2}{45}(3x-1)^{\frac{5}{2}} + \frac{2}{27}(3x-1)^{\frac{3}{2}} + C$$

(2)
$$\[\psi \] t = (x-2)^{\frac{1}{2}}, \]$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

(1) 设
$$t = (3x-1)^{\frac{1}{2}}$$
, $\therefore x = \frac{1}{3}(t^2+1)$, $dx = \frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$
$$= \frac{2}{45}t^5 + \frac{2}{27}t^3 + C = \frac{2}{45}(3x-1)^{\frac{5}{2}} + \frac{2}{27}(3x-1)^{\frac{3}{2}} + C$$

(2)
$$\[\mathcal{U} \] t = (x-2)^{\frac{1}{2}}, \]$$

$$\therefore \int \frac{x}{\sqrt{x-2}} dx =$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

(1) 设
$$t = (3x-1)^{\frac{1}{2}}$$
, $\therefore x = \frac{1}{3}(t^2+1)$, $dx = \frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$
$$= \frac{2}{45}t^5 + \frac{2}{27}t^3 + C = \frac{2}{45}(3x-1)^{\frac{5}{2}} + \frac{2}{27}(3x-1)^{\frac{3}{2}} + C$$

(2)
$$\mbox{if } t = (x-2)^{\frac{1}{2}}, \quad \therefore x = t^2 + 2,$$

$$\therefore \int \frac{x}{\sqrt{x-2}} dx =$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

(1) 设
$$t = (3x-1)^{\frac{1}{2}}$$
, $\therefore x = \frac{1}{3}(t^2+1)$, $dx = \frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$
$$= \frac{2}{45}t^5 + \frac{2}{27}t^3 + C = \frac{2}{45}(3x-1)^{\frac{5}{2}} + \frac{2}{27}(3x-1)^{\frac{3}{2}} + C$$

(2)
$$\partial t = (x-2)^{\frac{1}{2}}$$
, ∴ $x = t^2 + 2$, $dx = 2tdt$

$$\therefore \int \frac{x}{\sqrt{x-2}} dx =$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$
$$= \frac{2}{45}t^5 + \frac{2}{27}t^3 + C = \frac{2}{45}(3x-1)^{\frac{5}{2}} + \frac{2}{27}(3x-1)^{\frac{3}{2}} + C$$

$$\therefore \int \frac{x}{\sqrt{x-2}} dx = \int \frac{t^2+2}{t} \cdot 2t dt$$

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$
$$= \frac{2}{45}t^5 + \frac{2}{27}t^3 + C = \frac{2}{45}(3x-1)^{\frac{5}{2}} + \frac{2}{27}(3x-1)^{\frac{3}{2}} + C$$

(2)
$$\[\] t = (x-2)^{\frac{1}{2}}, \quad \therefore x = t^2 + 2, \quad dx = 2tdt \]$$

$$\therefore \int \frac{x}{\sqrt{x-2}} dx = \int \frac{t^2+2}{t} \cdot 2t dt = 2 \int t^2 + 2 dt$$



例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

解 (1) 设
$$t = (3x-1)^{\frac{1}{2}}$$
, $\therefore x = \frac{1}{3}(t^2+1)$, $dx = \frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$

$$= \frac{2}{45}t^5 + \frac{2}{27}t^3 + C = \frac{2}{45}(3x-1)^{\frac{5}{2}} + \frac{2}{27}(3x-1)^{\frac{3}{2}} + C$$

(2)
$$\mbox{if } t = (x-2)^{\frac{1}{2}}, \quad \therefore x = t^2 + 2, \quad dx = 2tdt$$

$$\therefore \int \frac{x}{\sqrt{x-2}} dx = \int \frac{t^2+2}{t} \cdot 2t dt = 2 \int t^2 + 2 dt = \frac{2}{3} t^3 + 4t + C$$



变量代换 " $\int f(x)dx = \frac{x=\varphi(t)}{\int f(\varphi(t))\varphi'(t)dt} = G(\varphi^{-1}(t)) + C$ " **例** I

例子 求不定积分
$$\int x\sqrt{3x-1}dx$$
, $\int \frac{x}{\sqrt{x-2}}dx$

解
(1) 设
$$t = (3x-1)^{\frac{1}{2}}$$
, $\therefore x = \frac{1}{3}(t^2+1)$, $dx = \frac{2}{3}tdt$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{2}{3}tdt = \frac{2}{9}\int t^4+t^2dt$$

$$\therefore \int x\sqrt{3x-1}dx = \int \frac{1}{3}(t^2+1)t \cdot \frac{1}{3}tdt = \frac{1}{9}\int t^4+t^2dt$$
$$= \frac{2}{45}t^5 + \frac{2}{27}t^3 + C = \frac{2}{45}(3x-1)^{\frac{5}{2}} + \frac{2}{27}(3x-1)^{\frac{3}{2}} + C$$

 $\therefore \int \frac{x}{\sqrt{x-2}} dx = \int \frac{t^2+2}{t} \cdot 2t dt = 2 \int t^2 + 2 dt = \frac{2}{3}t^3 + 4t + C$

$$= \frac{2}{3}(x-2)^{\frac{3}{2}} + 4(x-2)^{\frac{1}{2}} + C$$

例子 求不定积分 $\int \frac{1}{1+\sqrt{\chi}} dx$, $\int \frac{1}{1+\sqrt[3]{1+\chi}} dx$

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

$$\mathbf{m}$$
 (1)设 $t = 1 + x^{\frac{1}{2}}$,

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

$$\mu$$
 (1)设 $t = 1 + x^{\frac{1}{2}}$,

$$\therefore \int \frac{1}{1+\sqrt{x}} dx =$$

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设
$$t = 1 + x^{\frac{1}{2}}$$
, $\therefore x = (t-1)^2$,

$$\therefore \int \frac{1}{1+\sqrt{x}} dx =$$

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

$$\mathbf{H}$$
 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx =$$

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

$$\mathbf{H}$$
 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt$$



例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

$$\mathbf{H}$$
 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt = 2 \int 1 - \frac{1}{t} dt$$

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

$$\mathbf{H}$$
 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt = 2 \int 1 - \frac{1}{t} dt$$

$$= 2t - 2 \ln t + C$$



例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

$$\mathbb{H}$$
 (1) 0 $t = 1 + x^{\frac{1}{2}}$, $x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt = 2 \int 1 - \frac{1}{t} dt$$
$$= 2t - 2 \ln t + C = 2(1+x^{\frac{1}{2}}) - 2 \ln(1+x^{\frac{1}{2}}) + C$$

$$= 2t - 2\ln t + C = 2(1 + x^{\frac{1}{2}}) - 2\ln(1 + x^{\frac{1}{2}}) +$$



例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

$$\mathbf{H}$$
 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt = 2 \int 1 - \frac{1}{t} dt$$
$$= 2t - 2 \ln t + C = 2(1+x^{\frac{1}{2}}) - 2 \ln(1+x^{\frac{1}{2}}) + C$$

(2)
$$\[\mathcal{C} \] t = 1 + (1+x)^{\frac{1}{3}}, \]$$

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

$$\mathbf{H}$$
 (1) 设 $t = 1 + x^{\frac{1}{2}}$, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt = 2 \int 1 - \frac{1}{t} dt$$
$$= 2t - 2 \ln t + C = 2(1+x^{\frac{1}{2}}) - 2 \ln(1+x^{\frac{1}{2}}) + C$$

(2) 设
$$t = 1 + (1 + x)^{\frac{1}{3}}$$
,
∴ $\int \frac{1}{1 + \sqrt[3]{1 + x}} dx =$



例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设
$$t = 1 + x^{\frac{1}{2}}$$
, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2 \int 1 - \frac{1}{t} dt$$
$$= 2t - 2\ln t + C = 2(1+x^{\frac{1}{2}}) - 2\ln(1+x^{\frac{1}{2}}) + C$$

(2) 设
$$t = 1 + (1 + x)^{\frac{1}{3}}$$
, ∴ $x = (t - 1)^3 - 1$,
∴ $\int \frac{1}{1 + \sqrt[3]{1 + x}} dx =$

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设
$$t = 1 + x^{\frac{1}{2}}$$
, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1) dt = 2 \int 1 - \frac{1}{t} dt$$
$$= 2t - 2 \ln t + C = 2(1+x^{\frac{1}{2}}) - 2 \ln(1+x^{\frac{1}{2}}) + C$$

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}} dx =$$

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设
$$t = 1 + x^{\frac{1}{2}}$$
, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2 \int 1 - \frac{1}{t} dt$$
$$= 2t - 2\ln t + C = 2(1+x^{\frac{1}{2}}) - 2\ln(1+x^{\frac{1}{2}}) + C$$

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}} dx = \int \frac{1}{t} \cdot 3(t-1)^2 dt =$$

例子 求不定积分
$$\int \frac{1}{1+\sqrt{x}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+x}} dx$

解 (1) 设
$$t = 1 + x^{\frac{1}{2}}$$
, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2 \int 1 - \frac{1}{t} dt$$
$$= 2t - 2\ln t + C = 2(1+x^{\frac{1}{2}}) - 2\ln(1+x^{\frac{1}{2}}) + C$$



例子 求不定积分
$$\int \frac{1}{1+\sqrt{\chi}} dx$$
, $\int \frac{1}{1+\sqrt[3]{1+\chi}} dx$

解 (1) 设
$$t = 1 + x^{\frac{1}{2}}$$
, $\therefore x = (t-1)^2$, $dx = 2(t-1)dt$

$$\therefore \int \frac{1}{1 + \sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2 \int 1 - \frac{1}{t} dt$$

$$= 2t - 2\ln t + C = 2(1 + x^{\frac{1}{2}}) - 2\ln(1 + x^{\frac{1}{2}}) + C$$



变量代换 " $\int f(x)dx = \int f(\varphi(t))\varphi'(t)dt = G(\varphi^{-1}(t)) + C$ " 例 II 例子 求不定积分 $\int \frac{1}{1+\sqrt{x}} dx$, $\int \frac{1}{1+\sqrt{3}/1+x} dx$

$$\mathbf{H} \qquad (1) \ \partial_t t = 1 + x^{\frac{1}{2}}, \quad \therefore x = (t-1)^2, \quad dx = 2(t-1)dt$$

$$\therefore \int \frac{1}{1+\sqrt{x}} dx = \int \frac{1}{t} \cdot 2(t-1)dt = 2 \int 1 - \frac{1}{t} dt$$

$$= 2t - 2\ln t + C = 2(1 + x^{\frac{1}{2}}) - 2\ln(1 + x^{\frac{1}{2}}) + C$$
(2) $\mathfrak{L} t = 1 + (1 + x)^{\frac{1}{3}}, \quad \therefore x = (t - 1)^3 - 1, \quad dx = 3(t - 1)^2 dt$

$$\therefore \int \frac{1}{1+\sqrt[3]{1+x}} dx = \int \frac{1}{t} \cdot 3(t-1)^2 dt = 3 \int t - 2 + \frac{1}{t} dt$$
$$= \frac{3}{2}t^2 - 6t + 3\ln|t| + C$$

 $= \frac{3}{2}(1 + (1+x)^{\frac{1}{3}})^2 - 6(1 + (1+x)^{\frac{1}{3}}) + 3\ln|1 + (1+x)^{\frac{1}{3}}| + C$

例子 求不定积分 $\int \frac{1}{\sqrt{1+e^x}} dx$

例子 求不定积分
$$\int \frac{1}{\sqrt{1+e^x}} dx$$

设
$$t = \sqrt{1 + e^x}$$
,

例子 求不定积分
$$\int \frac{1}{\sqrt{1+e^x}} dx$$

设
$$t = \sqrt{1 + e^x}$$
, $\therefore x = \ln(t^2 - 1)$,

例子 求不定积分
$$\int \frac{1}{\sqrt{1+a^x}} dx$$

设
$$t = \sqrt{1 + e^x}$$
, $\therefore x = \ln(t^2 - 1)$, $dx = \frac{2t}{t^2 - 1}dt$

例子 求不定积分
$$\int \frac{1}{\sqrt{1+a^x}} dx$$

设
$$t = \sqrt{1 + e^x}$$
, $\therefore x = \ln(t^2 - 1)$, $dx = \frac{2t}{t^2 - 1}dt$

$$\therefore \int \frac{1}{\sqrt{1+e^x}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1} dt$$

例子 求不定积分
$$\int \frac{1}{\sqrt{1+a^x}} dx$$

设
$$t = \sqrt{1 + e^x}$$
, $\therefore x = \ln(t^2 - 1)$, $dx = \frac{2t}{t^2 - 1}dt$

$$\therefore \int \frac{1}{\sqrt{1 + e^x}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1} dt = \int \frac{1}{t - 1} - \frac{1}{t + 1} dt$$

例子 求不定积分
$$\int \frac{1}{\sqrt{1+\alpha^2}} dx$$

设
$$t = \sqrt{1 + e^x}$$
, $\therefore x = \ln(t^2 - 1)$, $dx = \frac{2t}{t^2 - 1}dt$

$$\therefore \int \frac{1}{\sqrt{1 + e^x}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1} dt = \int \frac{1}{t - 1} - \frac{1}{t + 1} dt$$

$$= \ln|t - 1| - \ln|t + 1| + C$$

例子 求不定积分 $\int \frac{1}{\sqrt{1+a^x}} dx$

设
$$t = \sqrt{1 + e^x}$$
, $\therefore x = \ln(t^2 - 1)$, $dx = \frac{2t}{t^2 - 1}dt$

$$\therefore \int \frac{1}{\sqrt{1 + e^x}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1} dt = \int \frac{1}{t - 1} - \frac{1}{t + 1} dt$$

$$= \ln|t - 1| - \ln|t + 1| + C = \ln|\frac{t - 1}{t + 1}| + C$$

例子 求不定积分 $\int \frac{1}{\sqrt{1+\alpha^{X}}} dx$

设
$$t = \sqrt{1 + e^x}$$
, $\therefore x = \ln(t^2 - 1)$, $dx = \frac{2t}{t^2 - 1}dt$

$$\therefore \int \frac{1}{\sqrt{1 + e^x}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1} dt = \int \frac{1}{t - 1} - \frac{1}{t + 1} dt$$

$$= \ln|t - 1| - \ln|t + 1| + C = \ln|\frac{t - 1}{t + 1}| + C$$

$$= \ln\left(\frac{\sqrt{1 + e^x} - 1}{\sqrt{1 + e^x} + 1}\right) + C$$

例子 求不定积分 $\int \frac{1}{\sqrt{1+a^{\chi}}} dx$

解

设
$$t = \sqrt{1 + e^x}$$
, $\therefore x = \ln(t^2 - 1)$, $dx = \frac{2t}{t^2 - 1}dt$

$$\therefore \int \frac{1}{\sqrt{1 + e^x}} dx = \int \frac{1}{t} \cdot \frac{2t}{t^2 - 1} dt = \int \frac{1}{t - 1} - \frac{1}{t + 1} dt$$

$$= \ln|t - 1| - \ln|t + 1| + C = \ln|\frac{t - 1}{t + 1}| + C$$

$$= \ln\left(\frac{\sqrt{1 + e^x} - 1}{\sqrt{1 + e^x} + 1}\right) + C$$

 $= 2 \ln(\sqrt{1 + e^x} - 1) - x + C$

a

变量代换法 " $\int f(x)dx = \int f(\varphi(t))\varphi'(t)dt$ ": 例子总结

$$\int x\sqrt{3x-1}dx$$

$$\int \frac{1}{1+\sqrt{x}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

变量代换法 " $\int f(x)dx = \int f(\varphi(t))\varphi'(t)dt$ ": 例子总结

$$\int x\sqrt{3x-1}dx \xrightarrow{t=\sqrt{3x-1}} \cdots$$

$$\int \frac{1}{1+\sqrt{x}}dx$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

变量代换法 " $\int f(x)dx = \int f(\varphi(t))\varphi'(t)dt$ ": 例子总结

$$\int x\sqrt{3x-1}dx \xrightarrow{t=\sqrt{3x-1}} \cdots$$

$$\int \frac{1}{1+\sqrt{x}}dx \xrightarrow{t=1+\sqrt{x}} \cdots$$

$$\int \frac{1}{\sqrt{1+e^x}}dx$$

变量代换法 " $\int f(x)dx = \frac{x=\varphi(t)}{\int f(\varphi(t))\varphi'(t)dt}$ ": 例子总结

$$\int x\sqrt{3x-1}dx \stackrel{t=\sqrt{3x-1}}{====}\cdots$$

$$\int \frac{1}{1+\sqrt{x}} dx \, \frac{t=1+\sqrt{x}}{2} \cdots$$

$$\int \frac{1}{\sqrt{1+e^x}} dx \stackrel{t=\sqrt{1+e^x}}{=} \cdots$$

