

§5.4 分部积分法

数学系 梁卓滨

2017-2018 学年 II

教学要求

◇ 熟练掌握分部积分法



Outline of §5.4

分部积分法，能干啥？

能够计算如下的不定积分：

$$\int x e^x dx, \quad \int x^2 \ln x dx, \quad \int x \arctan x dx, \quad \int x \cos x dx$$

$$\int \ln x dx, \quad \int \arcsin x dx, \quad \int \arctan x dx, \quad \int \ln(1+x^2) dx$$

$$\int x^2 e^x dx, \quad \int e^x \cos x dx \quad \dots\dots$$

（可能要结合前面学的换元积分法）

分部积分引入

- 微分公式

$$d(uv) = u dv + v du$$

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$$d(uv) = u dv + v du \quad \Rightarrow \quad u dv = d(uv) - v du$$

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- 微分公式

$$d(uv) = u dv + v du \quad \Rightarrow \quad u dv = d(uv) - v du$$

练习

$$x \cos x dx =$$

$$x e^x dx =$$

$$\ln x dx =$$

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应用

$$x \cos x dx = \quad x d \sin x = \quad d(x \sin x) - \quad \sin x dx$$

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应用

$$\begin{aligned} \int x \cos x dx &= \int x d \sin x = \int d(x \sin x) - \int \sin x dx \\ &= x \sin x + \cos x + C \end{aligned}$$

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$$\begin{aligned} \int \ln x dx &= \int d(x \ln x) - \int x d \ln x = \int d(x \ln x) - \int dx \\ &= x \ln x - x + C \end{aligned}$$

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- 两边积分得：

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$$d(uv) = u dv + v du \Rightarrow u dv = d(uv) - v du$$

- 两边积分得：

$$\int u dv = uv - \int v du$$

- 实际应用时的步骤：

$$\int \ln x dx =$$

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- 实际应用时的步骤：

$$\int u v' dx = uv - \int v u' dx$$

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- 两边积分得：

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- 实际应用时的步骤：

$$\int u v' dx = \int u v' dx$$

$$\underline{\underline{\text{凑微分}}} \int u dv =$$

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$$d(uv) = u dv + v du \Rightarrow u dv = d(uv) - v du$$

- 两边积分得：

$$\int u dv = uv - \int v du$$

- 实际应用时的步骤：

$$\int u' v dx = \int u v' dx$$

$$\underline{\underline{\text{凑微分}}} \int u dv = uv - \int v du$$

分部积分公式

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$$d(uv) = u dv + v du \Rightarrow u dv = d(uv) - v du$$

- 两边积分得：

$$\int u dv = uv - \int v du$$

- 实际应用时的步骤：

$$\int f(x) g'(x) dx = \int u v' dx$$

$$\underline{\underline{\text{凑微分}}} \int u dv = uv - \underbrace{\int v du}_{\text{简单、易算}}$$

例 1 求 $\int x \cos x dx$, $\int x e^x dx$, $\int x \sin x dx$

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$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx$$

例 1 求 $\int x \cos x dx$, $\int x e^x dx$, $\int x \sin x dx$

解

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

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$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

$$\int x \cos x dx = \int \cos x \cdot d\left(\frac{1}{2}x^2\right)$$

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$$\int x \cos x dx = \int \cos x \cdot d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cos x - \int \frac{1}{2}x^2 d \cos x$$

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$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

行不通的做法

$$\begin{aligned} \int x \cos x dx &= \int \cos x \cdot d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cos x - \int \frac{1}{2}x^2 d \cos x \\ &= \frac{1}{2}x^2 \cos x + \underbrace{\int \frac{1}{2}x^2 \sin x dx}_{\text{更加复杂!}} = \dots \end{aligned}$$

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$$\int x \sin x dx =$$

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例 1 求 $\int x \cos x dx$, $\int x e^x dx$, $\int x \sin x dx$

解

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

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$$\int x \sin x dx = \int x d(-\cos x) =$$

行不通的做法

$$\begin{aligned} \int x \cos x dx &= \int \cos x \cdot d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cos x - \int \frac{1}{2}x^2 d \cos x \\ &= \frac{1}{2}x^2 \cos x + \underbrace{\int \frac{1}{2}x^2 \sin x dx}_{\text{更加复杂!}} = \dots \end{aligned}$$

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$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

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$$\begin{aligned} \int x \sin x dx &= \int x d(-\cos x) = x(-\cos x) - \int (-\cos x) dx \\ &= -x \cos x + \sin x + C \end{aligned}$$

行不通的做法

$$\begin{aligned} \int x \cos x dx &= \int \cos x \cdot d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cos x - \int \frac{1}{2}x^2 d \cos x \\ &= \frac{1}{2}x^2 \cos x + \underbrace{\int \frac{1}{2}x^2 \sin x dx}_{\text{更加复杂!}} = \dots \end{aligned}$$

例 2 求 $\int x \ln x dx$, $\int x^2 \ln x dx$

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$$\int x \ln x dx = \int \ln x d\left(\frac{1}{2}x^2\right) =$$

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例 2 求 $\int x \ln x dx$, $\int x^2 \ln x dx$

解
$$\int x \ln x dx = \int \ln x d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 d \ln x$$

$$\int x^2 \ln x dx =$$

例 2 求 $\int x \ln x dx$, $\int x^2 \ln x dx$

解
$$\begin{aligned}\int x \ln x dx &= \int \ln x d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 d \ln x \\ &= \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx\end{aligned}$$

$$\int x^2 \ln x dx =$$

例 2 求 $\int x \ln x dx$, $\int x^2 \ln x dx$

解
$$\begin{aligned}\int x \ln x dx &= \int \ln x d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 d \ln x \\ &= \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx \\ &= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C\end{aligned}$$

$$\int x^2 \ln x dx =$$

例 2 求 $\int x \ln x dx$, $\int x^2 \ln x dx$

解
$$\begin{aligned}\int x \ln x dx &= \int \ln x d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 d \ln x \\ &= \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx \\ &= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C \\ \int x^2 \ln x dx &= \int \ln x d\left(\frac{1}{3}x^3\right) =\end{aligned}$$

例 2 求 $\int x \ln x dx$, $\int x^2 \ln x dx$

解
$$\begin{aligned}\int x \ln x dx &= \int \ln x d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 d \ln x \\ &= \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx \\ &= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C\end{aligned}$$
$$\int x^2 \ln x dx = \int \ln x d\left(\frac{1}{3}x^3\right) = \frac{1}{3}x^3 \cdot \ln x - \int \frac{1}{3}x^3 d \ln x$$

例2 求 $\int x \ln x dx$, $\int x^2 \ln x dx$

解

$$\begin{aligned}\int x \ln x dx &= \int \ln x d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 d \ln x \\&= \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx \\&= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C\end{aligned}$$
$$\begin{aligned}\int x^2 \ln x dx &= \int \ln x d\left(\frac{1}{3}x^3\right) = \frac{1}{3}x^3 \cdot \ln x - \int \frac{1}{3}x^3 d \ln x \\&= \frac{1}{3}x^3 \cdot \ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x} dx\end{aligned}$$

例2 求 $\int x \ln x dx$, $\int x^2 \ln x dx$

解
$$\begin{aligned}\int x \ln x dx &= \int \ln x d\left(\frac{1}{2}x^2\right) = \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 d \ln x \\ &= \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx \\ &= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C\end{aligned}$$

$$\begin{aligned}\int x^2 \ln x dx &= \int \ln x d\left(\frac{1}{3}x^3\right) = \frac{1}{3}x^3 \cdot \ln x - \int \frac{1}{3}x^3 d \ln x \\ &= \frac{1}{3}x^3 \cdot \ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x} dx \\ &= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C\end{aligned}$$

例 3 求 $\int x \arctan x dx$

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解 $\int x \arctan x dx =$

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解
$$\int x \arctan x dx = \int \arctan x d\left(\frac{1}{2}x^2\right)$$

例 3 求 $\int x \arctan x dx$

解
$$\begin{aligned}\int x \arctan x dx &= \int \arctan x d\left(\frac{1}{2}x^2\right) \\ &= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 d \arctan x\end{aligned}$$

例 3 求 $\int x \arctan x dx$

解
$$\begin{aligned}\int x \arctan x dx &= \int \arctan x d\left(\frac{1}{2}x^2\right) \\&= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 d \arctan x \\&= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2} dx\end{aligned}$$

例3 求 $\int x \arctan x dx$

解

$$\begin{aligned}\int x \arctan x dx &= \int \arctan x d\left(\frac{1}{2}x^2\right) \\&= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 d \arctan x \\&= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2} dx \\&= \frac{1}{2}x^2 \arctan x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx\end{aligned}$$

例3 求 $\int x \arctan x dx$

解

$$\begin{aligned}\int x \arctan x dx &= \int \arctan x d\left(\frac{1}{2}x^2\right) \\&= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 d \arctan x \\&= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2} dx \\&= \frac{1}{2}x^2 \arctan x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx \\&= \frac{1}{2}x^2 \arctan x - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2}\right) dx\end{aligned}$$

例3 求 $\int x \arctan x dx$

解

$$\begin{aligned}\int x \arctan x dx &= \int \arctan x d\left(\frac{1}{2}x^2\right) \\&= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 d \arctan x \\&= \frac{1}{2}x^2 \arctan x - \int \frac{1}{2}x^2 \cdot \frac{1}{1+x^2} dx \\&= \frac{1}{2}x^2 \arctan x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx \\&= \frac{1}{2}x^2 \arctan x - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2}\right) dx \\&= \frac{1}{2}x^2 \arctan x - \frac{1}{2}x + \frac{1}{2} \arctan x + C\end{aligned}$$

例 4 求 $\int \ln x dx$, $\int \ln(1 + x^2) dx$

例 4 求 $\int \ln x dx$, $\int \ln(1+x^2) dx$

解

$$\int \ln x dx =$$

例 4 求 $\int \ln x dx$, $\int \ln(1+x^2) dx$

解
$$\int \ln x dx = x \ln x - \int x d \ln x =$$

例 4 求 $\int \ln x dx$, $\int \ln(1+x^2)dx$

解
$$\int \ln x dx = x \ln x - \int x d \ln x = x \ln x - \int x \cdot \frac{1}{x} dx$$

例 4 求 $\int \ln x dx$, $\int \ln(1+x^2) dx$

解

$$\begin{aligned}\int \ln x dx &= x \ln x - \int x d \ln x = x \ln x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - x + C\end{aligned}$$

例 4 求 $\int \ln x dx$, $\int \ln(1+x^2)dx$

解

$$\begin{aligned}\int \ln x dx &= x \ln x - \int x d \ln x = x \ln x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - x + C\end{aligned}$$

$$\int \ln(1+x^2)dx =$$

例 4 求 $\int \ln x dx$, $\int \ln(1+x^2)dx$

解

$$\begin{aligned}\int \ln x dx &= x \ln x - \int x d \ln x = x \ln x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - x + C\end{aligned}$$

$$\int \ln(1+x^2)dx = x \ln(1+x^2) - \int x d \ln(1+x^2)$$

例 4 求 $\int \ln x dx$, $\int \ln(1+x^2)dx$

解

$$\begin{aligned}\int \ln x dx &= x \ln x - \int x d \ln x = x \ln x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - x + C\end{aligned}$$

$$\begin{aligned}\int \ln(1+x^2)dx &= x \ln(1+x^2) - \int x d \ln(1+x^2) \\ &= x \ln(1+x^2) - \int x \cdot \frac{2x}{1+x^2} dx\end{aligned}$$

例 4 求 $\int \ln x dx$, $\int \ln(1+x^2) dx$

解

$$\begin{aligned}\int \ln x dx &= x \ln x - \int x d \ln x = x \ln x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - x + C\end{aligned}$$

$$\begin{aligned}\int \ln(1+x^2) dx &= x \ln(1+x^2) - \int x d \ln(1+x^2) \\ &= x \ln(1+x^2) - \int x \cdot \frac{2x}{1+x^2} dx \\ &= x \ln(1+x^2) - 2 \int \frac{x^2}{1+x^2} dx\end{aligned}$$

例 4 求 $\int \ln x dx$, $\int \ln(1+x^2)dx$

解

$$\begin{aligned}\int \ln x dx &= x \ln x - \int x d \ln x = x \ln x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - x + C\end{aligned}$$

$$\begin{aligned}\int \ln(1+x^2)dx &= x \ln(1+x^2) - \int x d \ln(1+x^2) \\ &= x \ln(1+x^2) - \int x \cdot \frac{2x}{1+x^2} dx \\ &= x \ln(1+x^2) - 2 \int \frac{x^2}{1+x^2} dx \\ &= x \ln(1+x^2) - 2 \int \left(1 - \frac{1}{1+x^2}\right) dx\end{aligned}$$

例 4 求 $\int \ln x dx$, $\int \ln(1+x^2)dx$

解

$$\begin{aligned}\int \ln x dx &= x \ln x - \int x d \ln x = x \ln x - \int x \cdot \frac{1}{x} dx \\ &= x \ln x - x + C\end{aligned}$$

$$\begin{aligned}\int \ln(1+x^2)dx &= x \ln(1+x^2) - \int x d \ln(1+x^2) \\ &= x \ln(1+x^2) - \int x \cdot \frac{2x}{1+x^2} dx \\ &= x \ln(1+x^2) - 2 \int \frac{x^2}{1+x^2} dx \\ &= x \ln(1+x^2) - 2 \int \left(1 - \frac{1}{1+x^2}\right) dx \\ &= x \ln(1+x^2) - 2x + 2 \arctan x + C\end{aligned}$$

例 5 求 $\int \arctan x dx$

例 5 求 $\int \arctan x dx$

解 $\int \arctan x dx =$

例 5 求 $\int \arctan x dx$

解
$$\int \arctan x dx = x \arctan x - \int x d \arctan x$$

例 5 求 $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x \arctan x - \int x d \arctan x \\ &= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx\end{aligned}$$

例 5 求 $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x \arctan x - \int x d \arctan x \\ &= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx\end{aligned}$$

$$d(1+x^2)$$

例 5 求 $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x \arctan x - \int x d \arctan x \\ &= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx \\ &\quad \frac{1}{2} d(1+x^2)\end{aligned}$$

例 5 求 $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x \arctan x - \int x d \arctan x \\&= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx \\&= x \arctan x - \int \frac{1}{1+x^2} \cdot \frac{1}{2} d(1+x^2)\end{aligned}$$

例 5 求 $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x \arctan x - \int x d \arctan x \\&= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx \\&= x \arctan x - \int \frac{1}{1+x^2} \cdot \frac{1}{2} d(1+x^2) \\&= x \arctan x - \int \frac{1}{u} du\end{aligned}$$

例 5 求 $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x \arctan x - \int x d \arctan x \\&= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx \\&= x \arctan x - \int \frac{1}{1+x^2} \cdot \frac{1}{2} d(1+x^2) \\&= x \arctan x - \int \frac{1}{u} du \\&= x \arctan x - \ln |u| + C\end{aligned}$$

例 5 求 $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x \arctan x - \int x d \arctan x \\&= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx \\&= x \arctan x - \int \frac{1}{1+x^2} \cdot \frac{1}{2} d(1+x^2) \\&= x \arctan x - \int \frac{1}{u} du \\&= x \arctan x - \ln |u| + C \\&= x \arctan x - \frac{1}{2} \ln(1+x^2) + C.\end{aligned}$$

例 5 求 $\int \arctan x dx$

解

$$\begin{aligned}\int \arctan x dx &= x \arctan x - \int x d \arctan x \\&= x \arctan x - \int x \cdot \frac{1}{1+x^2} dx \\&= x \arctan x - \int \frac{1}{1+x^2} \cdot \frac{1}{2} d(1+x^2) \\&= x \arctan x - \int \frac{1}{u} du \\&= x \arctan x - \ln |u| + C \\&= x \arctan x - \frac{1}{2} \ln(1+x^2) + C.\end{aligned}$$

例 6 求 $\int \arcsin x dx$

例 6 求不定积分 $\int \arcsin x dx$

例 6 求不定积分 $\int \arcsin x dx$

解 $\int \arcsin x dx =$

例 6 求不定积分 $\int \arcsin x dx$

解
$$\int \arcsin x dx = x \arcsin x - \int x d \arcsin x$$

例 6 求不定积分 $\int \arcsin x dx$

解
$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\ &= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx\end{aligned}$$

例 6 求不定积分 $\int \arcsin x dx$

解
$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\ &= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx\end{aligned}$$

$$d(1-x^2)$$

例 6 求不定积分 $\int \arcsin x dx$

解

$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\ &= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\ &\quad - \frac{1}{2} d(1-x^2)\end{aligned}$$

例 6 求不定积分 $\int \arcsin x dx$

解

$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\&= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\&= x \arcsin x - \int \frac{1}{\sqrt{1-x^2}} \cdot -\frac{1}{2} d(1-x^2)\end{aligned}$$

例 6 求不定积分 $\int \arcsin x dx$

解

$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\&= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\&= x \arcsin x - \int \frac{1}{\sqrt{1-x^2}} \cdot -\frac{1}{2} d(1-x^2) \\&= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{u}} \cdot (-1) du\end{aligned}$$

例 6 求不定积分 $\int \arcsin x dx$

解

$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\&= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\&= x \arcsin x - \int \frac{1}{\sqrt{1-x^2}} \cdot -\frac{1}{2} d(1-x^2) \\&= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{u}} \cdot (-1) du \\&\quad 2u^{\frac{1}{2}}\end{aligned}$$

例 6 求不定积分 $\int \arcsin x dx$

解

$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\&= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\&= x \arcsin x - \int \frac{1}{\sqrt{1-x^2}} \cdot -\frac{1}{2} d(1-x^2) \\&= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{u}} \cdot (-1) du \\&= x \arcsin x + \frac{1}{2} \cdot 2u^{\frac{1}{2}}\end{aligned}$$

例 6 求不定积分 $\int \arcsin x dx$

解

$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\&= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\&= x \arcsin x - \int \frac{1}{\sqrt{1-x^2}} \cdot -\frac{1}{2} d(1-x^2) \\&= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{u}} \cdot (-1) du \\&= x \arcsin x + \frac{1}{2} \cdot 2u^{\frac{1}{2}} \\&= x \arcsin x + \sqrt{1-x^2} + C.\end{aligned}$$

例 6 求不定积分 $\int \arcsin x dx$

解

$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\&= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\&= x \arcsin x - \int \frac{1}{\sqrt{1-x^2}} \cdot -\frac{1}{2} d(1-x^2) \\&= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{u}} \cdot (-1) du \\&= x \arcsin x + \frac{1}{2} \cdot 2u^{\frac{1}{2}} \\&= x \arcsin x + \sqrt{1-x^2} + C.\end{aligned}$$

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$

例 6 求不定积分 $\int \arcsin x dx$

解

$$\begin{aligned}\int \arcsin x dx &= x \arcsin x - \int x d \arcsin x \\&= x \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx \\&= x \arcsin x - \int \frac{1}{\sqrt{1-x^2}} \cdot -\frac{1}{2} d(1-x^2) \\&= x \arcsin x - \frac{1}{2} \int \frac{1}{\sqrt{u}} \cdot (-1) du \\&= x \arcsin x + \frac{1}{2} \cdot 2u^{\frac{1}{2}} \\&= x \arcsin x + \sqrt{1-x^2} + C.\end{aligned}$$

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$

(提示 两次分部积分)

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$

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(提示 两次分部积分)

解 $\int x^2 e^x dx =$

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$

(提示 两次分部积分)

解
$$\int x^2 e^x dx = \int x^2 de^x =$$

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$ (提示 两次分部积分)

解
$$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 =$$

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$ (提示 两次分部积分)

解
$$\int x^2 e^x dx = \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx$$

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$ (提示 两次分部积分)

解
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left(\int x de^x \right) =\end{aligned}$$

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$ (提示 两次分部积分)

解
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx x^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left(\int x de^x \right) = x^2 e^x - 2 \left(x e^x - \int e^x dx \right)\end{aligned}$$

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$ (提示 两次分部积分)

解
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx x^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left(\int x de^x \right) = x^2 e^x - 2 \left(x e^x - \int e^x dx \right) \\ &= x^2 e^x - 2 x e^x + 2 e^x + C\end{aligned}$$

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$ (提示 两次分部积分)

解
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left(\int x de^x \right) = x^2 e^x - 2 \left(x e^x - \int e^x dx \right) \\ &= x^2 e^x - 2 x e^x + 2 e^x + C\end{aligned}$$

$$\int x^2 \sin x dx =$$

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$ (提示 两次分部积分)

解
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left(\int x de^x \right) = x^2 e^x - 2 \left(x e^x - \int e^x dx \right) \\ &= x^2 e^x - 2 x e^x + 2 e^x + C\end{aligned}$$

$$\int x^2 \sin x dx = - \int x^2 d \cos x =$$

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$ (提示 两次分部积分)

解
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left(\int x de^x \right) = x^2 e^x - 2 \left(x e^x - \int e^x dx \right) \\ &= x^2 e^x - 2 x e^x + 2 e^x + C\end{aligned}$$

$$\int x^2 \sin x dx = - \int x^2 d \cos x = -x^2 \cos x + \int \cos x dx^2$$

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$ (提示 两次分部积分)

解
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left(\int x de^x \right) = x^2 e^x - 2 \left(x e^x - \int e^x dx \right) \\ &= x^2 e^x - 2 x e^x + 2 e^x + C\end{aligned}$$

$$\begin{aligned}\int x^2 \sin x dx &= - \int x^2 d \cos x = -x^2 \cos x + \int \cos x dx^2 \\ &= -x^2 \cos x + 2 \int x \cos x dx\end{aligned}$$

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$ (提示 两次分部积分)

解
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\ &= x^2 e^x - 2 \left(\int x de^x \right) = x^2 e^x - 2 \left(x e^x - \int e^x dx \right) \\ &= x^2 e^x - 2 x e^x + 2 e^x + C\end{aligned}$$

$$\begin{aligned}\int x^2 \sin x dx &= - \int x^2 d \cos x = -x^2 \cos x + \int \cos x dx^2 \\ &= -x^2 \cos x + 2 \int x \cos x dx \\ &= -x^2 \cos x + 2 \int x d \sin x\end{aligned}$$

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$ (提示 两次分部积分)

解
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\&= x^2 e^x - 2 \left(\int x de^x \right) = x^2 e^x - 2 \left(x e^x - \int e^x dx \right) \\&= x^2 e^x - 2 x e^x + 2 e^x + C\end{aligned}$$

$$\begin{aligned}\int x^2 \sin x dx &= - \int x^2 d \cos x = -x^2 \cos x + \int \cos x dx^2 \\&= -x^2 \cos x + 2 \int x \cos x dx \\&= -x^2 \cos x + 2 \int x d \sin x \\&= -x^2 \cos x + 2 \left(x \sin x - \int \sin x dx \right)\end{aligned}$$

例 7 求不定积分 $\int x^2 e^x dx$, $\int x^2 \sin x dx$ (提示 两次分部积分)

解
$$\begin{aligned}\int x^2 e^x dx &= \int x^2 de^x = x^2 e^x - \int e^x dx^2 = x^2 e^x - 2 \int e^x x dx \\&= x^2 e^x - 2 \left(\int x de^x \right) = x^2 e^x - 2 \left(x e^x - \int e^x dx \right) \\&= x^2 e^x - 2 x e^x + 2 e^x + C\end{aligned}$$

$$\begin{aligned}\int x^2 \sin x dx &= - \int x^2 d \cos x = -x^2 \cos x + \int \cos x dx^2 \\&= -x^2 \cos x + 2 \int x \cos x dx \\&= -x^2 \cos x + 2 \int x d \sin x \\&= -x^2 \cos x + 2 \left(x \sin x - \int \sin x dx \right) \\&= -x^2 \cos x + 2 x \sin x + 2 \cos x + C\end{aligned}$$

分部积分：例子总结

$$\int x e^x dx =$$

$$\int x \cos x dx =$$

$$\int x^2 \ln x dx =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int x e^x dx = \int x de^x =$$

$$\int x \cos x dx =$$

$$\int x^2 \ln x dx =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int x e^x dx = \int x d e^x = x e^x - \int e^x dx = \dots$$

$$\int x \cos x dx =$$

$$\int x^2 \ln x dx =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int x e^x dx = \int x d e^x = x e^x - \int e^x dx = \dots$$

$$\int x \cos x dx = \int x d \sin x =$$

$$\int x^2 \ln x dx =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int x e^x dx = \int x d e^x = x e^x - \int e^x dx = \dots$$

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = \dots$$

$$\int x^2 \ln x dx =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int x e^x dx = \int x d e^x = x e^x - \int e^x dx = \dots$$

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = \dots$$

$$\int x^2 \ln x dx = \int \ln x d\left(\frac{1}{3}x^3\right) =$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

$$\int x e^x dx = \int x d e^x = x e^x - \int e^x dx = \dots$$

$$\int x \cos x dx = \int x d \sin x = x \sin x - \int \sin x dx = \dots$$

$$\int x^2 \ln x dx = \int \ln x d\left(\frac{1}{3}x^3\right) = \frac{1}{3}x^3 \ln x - \frac{1}{3} \int x^3 d \ln x = \dots$$

$$\int \ln x dx =$$

$$\int \arctan x dx =$$

分部积分：例子总结

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