EPSIC: Electromagnetic Polarization Simulation in C++

Willem van Straten

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Abstract

EPSIC can be used to simulate the effects of integration over finite samples when more than one source is present. The sources can be disjoint (mutually exclusive), such that only one source emits at a given instant, or superposed, such that the electric fields are summed. It is also possible to simulate integration over a composite sample of unresolved disjoint modes.

1 Introduction

EPSIC performs the following Monte Carlo simulation.

1. Generate a sequence of M random electric field vector instances $\boldsymbol{e}$, each with statistically independent and identically distributed (iid) circular complex normal components; such a sequence is described by the population mean Stokes parameters [1,0,0,0].

2. To yield the desired population mean Stokes parameters, S_μ, transform each electric field vector instance by the Hermitian square root of $2\boldsymbol{\rho} = S_\mu \boldsymbol{\sigma}_\mu$.

3. Optionally perform amplitude modulation by multiplying each instance of $\boldsymbol{e}$ by an iid random variate u that is drawn from a log-normal distribution. The log-normally distributed variate is generated from a normally distributed iid variate with zero mean and standard deviation ς and is normalized by the mean of the distribution, $\langle u \rangle = \exp(\varsigma^2/2)$, such that the mean of the amplitude modulating function is unity.

   - To simulate rectangular subpulses defined by the sub-sample size n, a single value of u is applied to n consecutive instances of $\boldsymbol{e}$.
   - The modulation function can also be smoothed using a box-car of specified width.

4. If simulating *superposed samples*, repeat all of the previous steps to produce M instances of the electric field vector in the other mode then, for each instance of the electric field vectors from modes A and B, produce M new instances $\boldsymbol{e} = \boldsymbol{e}_A + \boldsymbol{e}_B$.

5. Compute the instantaneous Stokes parameters, $s_\mu = \boldsymbol{e}^\dagger \boldsymbol{\sigma}_\mu \boldsymbol{e}$.

6. Optionally divide the sequence of M instantaneous Stokes vectors into mutually exclusive Stokes samples of n instances, yielding a sequence of $N = M/n$ Stokes samples. This step is not optional when simulating composite samples.
7. If simulating *composite samples*, replace $(1 - f)n$ instances in each Stokes sample with instantaneous Stokes vectors in the other mode.
8. If simulating *disjoint samples*, replace $(1 - F)N$ Stokes samples with Stokes samples that contain only instantaneous Stokes vectors in the other mode.
9. For each Stokes sample, compute the sample mean Stokes parameters $\bar{S}_\mu$.
10. Compute the 4×4 covariances between the Stokes parameters using either the M instantaneous Stokes parameters or the N sample mean Stokes parameters.

Following the simulation, EPSIC reports

- the expected population mean Stokes parameters and 4×4 matrix of covariances between the instantaneous or sample mean Stokes parameters;
- the values of the above quantities as derived from the simulated data;
- the mean value of the degree of polarization, derived from the simulated instantaneous or sample mean Stokes parameters; and
- the modulation index derived from simulated data.

EPSIC currently does not verify that the computed covariance matrix matches the theoretical prediction within the uncertainty due to noise.

2 Simulation Model Parameters

The simulation can be configured using the following parameters, each of which corresponds to a command line option. For a brief list of options and their arguments, run `epsic -h`.

- By default, EPSIC simulates the polarized electromagnetic radiation of a single source. To simulate a combination of two sources, use one of the following three options:
  - Enable **superposed modes** using the `-S` option, which has no arguments.
  - Enable **disjont modes** using the `-D` option; the argument to this option is the fraction of Stokes samples that occur in mode A in the simulated population.
  - Enable **composite samples of disjoint modes** using the `-C` option; the argument to this option is the fraction of electric field instances that occur in mode A in each Stokes sample.

- The **Stokes sample size** can be set using the `-n` option. The argument to this option is the integer number of instances of the electric field in each Stokes sample. By default, EPSIC calculates the statistics of the instantaneous Stokes parameters (i.e. $n = 1$).

- The **population size** can be set using the `-N` option. The argument to this option is the floating point number of mega Stokes samples to simulate, where a mega Stokes sample is 2^{20} Stokes samples. The total number of instances of electric field vectors that will be simulated is the population size times the Stokes sample size.

- The **population mean Stokes parameters** can be set using the `-s`, `-A`, and `-B` options. If simulating only a single source, use either `-s` or `-A`. If simulating a combination of two sources, use `-A` and `-B` to specify the population mean Stokes parameters of the first and second sources, respectively. The argument to this option is a comma separated list of the four population mean Stokes parameters, I, Q, U, V; e.g. `epsic -A 5.3,0,0,-1.4` will simulate polarized noise with a total intensity (Stokes I) of 5.3 units and circularly polarized intensity (Stokes V) of -1.4 units. By default, EPSIC will simulate unpolarized noise from a single source with population mean intensity equal to unity.

- Optional **amplitude modulation** is enabled using the `-l` option. The argument to this option is the standard deviation ς of the normally distributed variate that is used to generate a lognormal distribution. For example, `epsic -l 0.2` will cause `epsic` to multiply each electric field instance by $u = \exp(v/2)$, where v is normally distributed with mean equal to zero and standard deviation equal to 0.2. When amplitude modulation is enabled using the `-l` option, the following options apply.

  - The amplitude modulating function can be set to a contiguous sequence of **rectangular pulses** using the `-r` option. The argument to this option is the integer number of instances of the electric field to be multiplied by a single value of u.
  - The amplitude modulating function can be **boxcar smoothed** using the `-b` option. The argument to this option is the width of the boxcar expressed as the integer number of instances of the electric field that it spans.

  If simulating a combination of two sources, modulation properties of the second source can be specified by preceding the argument to any of `-l`, `-r` and/or `-b` with the letter ‘`B`’; e.g. `epsic -S -l B0.5 -b B4`.