```
from google.colab import drive
drive.mount('/content/drive')
```

```
Go to this URL in a browser: https://accounts.google.com/o/oauth2/auth?client_id=947318989803-6bn6qk8qdgf4n4g3pfee6491hc0brc4

Enter your authorization code:
··········
Mounted at /content/drive
```

1. Unsupervised Learning

```
%matplotlib inline
import scipy
import numpy as np
import itertools
import matplotlib.pyplot as plt
import time
```

1. Generating the data

First, we will generate some data for this problem. Set the number of points $N = 400$, their dimension $D = 2$, and the number of clusters $K = 2$, and generate data from the distribution $p(x|z = k) = \mathcal{N}(\mu_k, \Sigma_k)$. Sample 200 data points for $k = 1$ and 200 for $k = 2$, with

$$\mu_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \mu_2 = \begin{bmatrix} 6.0 \\ 0.1 \end{bmatrix} \text{ and } \Sigma_1 = \Sigma_2 = \begin{bmatrix} 10 & 7 \\ 7 & 10 \end{bmatrix}$$

Here, $N = 400$. Since you generated the data, you already know which sample comes from which class. Run the cell in the IPython notebook to generate the data.

```
# TODO: Run this cell to generate the data
num_samples = 400
cov = np.array([[1., .7], [.7, 1.]]) * 10
mean_1 = [.1, .1]
mean_2 = [6., .1]

x_class1 = np.random.multivariate_normal(mean_1, cov, num_samples // 2)
x_class2 = np.random.multivariate_normal(mean_2, cov, num_samples // 2)
xy_class1 = np.column_stack((x_class1, np.zeros(num_samples // 2)))
xy_class2 = np.column_stack((x_class2, np.ones(num_samples // 2)))
data_full = np.row_stack([xy_class1, xy_class2])
np.random.shuffle(data_full)
data = data_full[:, :2]
labels = data_full[:, 2]
```

Make a scatter plot of the data points showing the true cluster assignment of each point using different color codes and shape (x for first class and circles for second class):

```
# TODO: Make a scatterplot for the data points showing the true cluster assignments of each point
# plt.plot(...) # first class, x shape
# plt.plot(...) # second class, circle shape
plt.plot(x_class1[:, 0], x_class1[:, 1], 'x', c='red')
plt.plot(x_class2[:, 0], x_class2[:, 1], 'o', c='green')
plt.show()
```

2. Implement and Run K-Means algorithm

Now, we assume that the true class labels are not known. Implement the k-means algorithm for this problem. Write two functions: `km_assignment_step`, and `km_refitting_step` as given in the lecture (Here, `km_` means k-means). Identify the correct arguments, and the order to run them. Initialize the algorithm with

$$\hat{\mu}_1 = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}, \hat{\mu}_2 = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$

and run it until convergence. Show the resulting cluster assignments on a scatter plot either using different color codes or shape or both. Also plot the cost vs. the number of iterations. Report your misclassification error.

```
def cost(data, R, Mu):
    N, D = data.shape
    K = Mu.shape[1]
    J = 0
    for k in range(K):
        J += np.sum(np.dot(np.linalg.norm(data - np.array([Mu[:, k], ] * N), axis=1)**2, R))
    return J
```

```
# TODO: K-Means Assignment Step
def km_assignment_step(data, Mu):
    """ Compute K-Means assignment step

    Args:
        data: a NxD matrix for the data points
        Mu: a DxK matrix for the cluster means locations

    Returns:
        R_new: a NxK matrix of responsibilities
    """
    N, D = data.shape  # Number of datapoints and dimension of datapoint
    K = Mu.shape[1]  # number of clusters
    r = np.zeros([N, K])
    #a matrix of NxK dimension for the distances of the the N datapoints to each of the K cluster centers.
    for k in range(K):
         r[:, k] = np.linalg.norm(data - np.array([Mu[:, k], ] * N), axis=1)**2
    arg_min = np.argmin(r, axis=1) # argmax/argmin along dimension 1
    R_new = np.zeros([N, K]) # Set to zeros/ones with shape (N, K)
    R_new[np.arange(N),arg_min] = 1 # Assign to 1
    return R_new
```

```
# TODO: K-means Refitting Step
def km_refitting_step(data, R, Mu):
    """ Compute K-Means refitting step.

    Args:
        data: a NxD matrix for the data points
        R: a NxK matrix of responsibilities
        Mu: a DxK matrix for the cluster means locations

    Returns:
        Mu_new: a DxK matrix for the new cluster means locations
    """
    N, D = data.shape
    K = Mu.shape[1]
    Mu_new = np.zeros((D, K))
    for k in range(K):
        index = np.where(R[:, k] == 1)
        Mu_new[:, k] = np.mean(data[index, :], axis = 1)
    return Mu_new
```

```
N, D = data.shape
K = 2
max_iter = 100
class_init = np.random.binomial(1., .5, size=N)
R = np.vstack([class_init, 1 - class_init]).T

Mu = np.zeros([D, K])
Mu[:, 1] = 1.
R.T.dot(data), np.sum(R, axis=0)

iteration = []
clst = []
for it in range(max_iter):
    R = km_assignment_step(data, Mu)
    Mu = km_refitting_step(data, R, Mu)
    c = cost(data, R, Mu)
    iteration.append(it)
    clst.append(c)
    print(it, c)

class_1 = np.where(R[:, 0])
class_2 = np.where(R[:, 1])
```

```
0 37292.58165797178
1 37250.88887891708
2 37167.849589052785
3 37036.73687303193
4 36928.79657380079
5 36844.57439989758
6 36773.98217006367
7 36739.05559204959
8 36726.52159971727
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```

```
96 36726.52159971727
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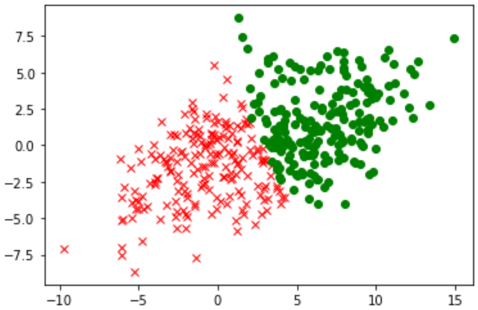
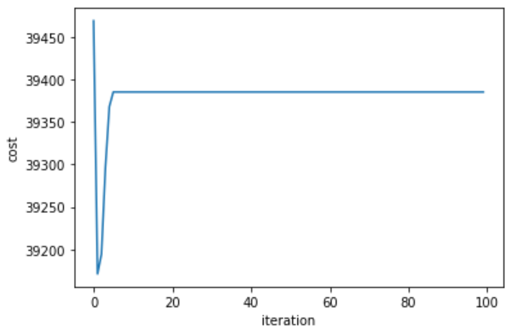
99 36726.52159971727
```

```
# TODO: Make a scatterplot for the data points showing the K-Means cluster assignments of each point
# plt.plot(...) # first class, x shape
# plt.plot(...) # second class, circle shape
plt.plot(data[class_1, 0], data[class_1,1], 'x', color='red')
plt.plot(data[class_2, 0], data[class_2,1], 'o', color='green')
plt.show()
```




```
plt.ylabel('cost')
plt.xlabel('iteration')
plt.plot(iterations, costs)
```

```
[<matplotlib.lines.Line2D at 0x7f5141a4f898>]
```




```
labels_predict = np.argwhere(R == 1)[:, 1]
print("Error rate: ", np.mean(labels != labels_predict))
```

3. Implement EM algorithm for Gaussian mixtures

Next, implement the EM algorithm for Gaussian mixtures. Write three functions: `log_likelihood`, `gm_e_step`, and `gm_m_step` as given in the lecture. Identify the correct arguments, and the order to run them. Initialize the algorithm with means as in Qs 2.1 k-means initialization, covariances with $\hat{\Sigma}_1 = \hat{\Sigma}_2 = I$, and $\hat{\pi}_1 = \hat{\pi}_2$.

In addition to the update equations in the lecture, for the M (Maximization) step, you also need to use this following equation to update the covariance Σ_k:

$$\hat{\mathbf{\Sigma}}_k = \frac{1}{N_k} \sum_{n=1}^{N} r_k^{(n)} (\mathbf{x}^{(n)} - \hat{\mu}_k)(\mathbf{x}^{(n)} - \hat{\mu}_k)^\top$$

Run the algorithm until convergence and show the resulting cluster assignments on a scatter plot either using different color codes or shape or both. Also plot the log-likelihood vs. the number of iterations. Report your misclassification error.

```
def normal_density(x, mu, Sigma):
    return np.exp(-.5 * np.dot(x - mu, np.linalg.solve(Sigma, x - mu))) \
        / np.sqrt(np.linalg.det(2 * np.pi * Sigma))
```

```
def log_likelihood(data, Mu, Sigma, Pi):
    """ Compute log likelihood on the data given the Gaussian Mixture Parameters.

    Args:
        data: a NxD matrix for the data points
        Mu: a DxK matrix for the means of the K Gaussian Mixtures
        Sigma: a list of size K with each element being DxD covariance matrix
        Pi: a vector of size K for the mixing coefficients

    Returns:
        L: a scalar denoting the log likelihood of the data given the Gaussian Mixture
    """
```

```
    """
    # Fill this in:
    # N, D = ...  # Number of datapoints and dimension of datapoint
    # K = ... # number of mixtures
    # L, T = 0., 0.
    # for n in range(N):
    #     for k in range(K):
    #         # T += ... # Compute the likelihood from the k-th Gaussian weighted by the mixing coefficients
    #     L += np.log(T)
    # return L

    N, D = data.shape  # Number of datapoints and dimension of datapoint
    K = Mu.shape[1] # number of mixtures
    L, T = 0., 0.
    for n in range(N):
        T = 0.
        for k in range(K):
            T += Pi[k] * normal_density(data[n], Mu[:, k], Sigma[k]) # Compute the likelihood from the k-th Gaussian weighted
        L += np.log(T)
    return L
```

```
# TODO: Gaussian Mixture Expectation Step
def gm_e_step(data, Mu, Sigma, Pi):
    """ Gaussian Mixture Expectation Step.

    Args:
        data: a NxD matrix for the data points
        Mu: a DxK matrix for the means of the K Gaussian Mixtures
        Sigma: a list of size K with each element being DxD covariance matrix
        Pi: a vector of size K for the mixing coefficients

    Returns:
        Gamma: a NxK matrix of responsibilities
    """
    # Fill this in:
    # N, D = ... # Number of datapoints and dimension of datapoint
    # K = ... # number of mixtures
    # Gamma = ... # zeros of shape (N,K), matrix of responsibilities
    # for n in range(N):
    #     for k in range(K):
    #         # Gamma[n, k] = ....
    #     # Gamma[n, :] /= ... # Normalize by sum across second dimension (mixtures)
    # return Gamma
    N, D = data.shape # Number of datapoints and dimension of datapoint
    K = Mu.shape[1] # number of mixtures
    Gamma = np.zeros((N,K)) # zeros of shape (N,K), matrix of responsibilities
    for n in range(N):
        for k in range(K):
            Gamma[n, k] = Pi[k] * normal_density(data[n], Mu[:,k], Sigma[k])
        Gamma[n, :] /= np.sum(Gamma[n, :]) # Normalize by sum across second dimension (mixtures)
    return Gamma
```

```
# TODO: Gaussian Mixture Maximization Step
def gm_m_step(data, Gamma):
    """ Gaussian Mixture Maximization Step.

    Args:
        data: a NxD matrix for the data points
        Gamma: a NxK matrix of responsibilities

    Returns:
        Mu: a DxK matrix for the means of the K Gaussian Mixtures
        Sigma: a list of size K with each element being DxD covariance matrix
        Pi: a vector of size K for the mixing coefficients
    """
    N, D = data.shape # Number of datapoints and dimension of datapoint
    K = Gamma.shape[1]  # number of mixtures
    Nk = np.sum(Gamma, axis=0) # Sum along first axis
    Mu = np.zeros([D, K])
    Sigma = np.zeros((K,D,D))

    for k in range(K):
        for n in range(N):
            Mu[:,k] += Gamma[n,k] * data[n,:]/Nk[k]
    for k in range(K):
        for n in range(N):
            temp = (data[n,:] - Mu[:,k]).reshape((D,1))
            Sigma[k] += Gamma[n,k] * np.dot(temp, temp.T)/Nk[k]
    Pi = Nk / N
    return Mu, Sigma, Pi
```

```
# TODO: Run this cell to call the Gaussian Mixture EM algorithm
N, D = data.shape
K = 2
Mu = np.zeros([D, K])
```

```
Mu[:, 1] = 1.
Sigma = [np.eye(2), np.eye(2)]
Pi = np.ones(K) / K
Gamma = np.zeros([N, K]) # Gamma is the matrix of responsibilities

max_iter  = 200

iterations = []
costs = []
for it in range(max_iter):
    Gamma = gm_e_step(data, Mu, Sigma, Pi)
    Mu, Sigma, Pi = gm_m_step(data, Gamma)
    loglike = log_likelihood(data, Mu, Sigma, Pi)
    iterations.append(it)
    costs.append(loglike)
    print(it, loglike) # This function makes the computation longer, but good for debugging

class_1 = np.where(Gamma[:, 0] >= .5)
class_2 = np.where(Gamma[:, 1] >= .5)
```

```
0 -2108.569030648646
1 -2103.523067787013
2 -2101.578628940565
3 -2100.39571336378
4 -2099.4236632574143
5 -2098.464055686756
6 -2097.408395697052
7 -2096.1536982077464
8 -2094.5612603236423
9 -2092.519545762133
10 -2090.247433322861
11 -2088.2949298098933
12 -2086.9395731095738
13 -2086.1168033743156
14 -2085.660531138905
15 -2085.421344453549
16 -2085.298761138278
17 -2085.235414098758
18 -2085.2015105004816
19 -2085.1822768473003
20 -2085.1704904611533
21 -2085.1626047237182
22 -2085.1568538546535
23 -2085.1523420208
24 -2085.148604067255
25 -2085.1453911308645
26 -2085.1425645465742
27 -2085.140042670383
28 -2085.137773932699
29 -2085.13572305107
30 -2085.1338639070595
31 -2085.1321758226945
32 -2085.130641581603
33 -2085.1292463507466
34 -2085.1279770703873
35 -2085.1268220897928
36 -2085.1257709341003
37 -2085.1248141428205
38 -2085.1239431490208
39 -2085.1231501829784
40 -2085.122428191501
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43 -2085.1206268735164
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46 -2085.1192660133675
47 -2085.1188907041155
48 -2085.1185487919975
49 -2085.1182372862168
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51 -2085.117694856276
52 -2085.117459204643
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65 -2085.115760621329
66 -2085.115696192309
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74 -2085.115348285397
75 -2085.115320197835
76 -2085.115294582412
77 -2085.1152712210865
78 -2085.1152499150935
79 -2085.1152304832535
80 -2085.1152127604037
81 -2085.115196595972
82 -2085.1151818526905
83 -2085.115168405407
84 -2085.115156140023
85 -2085.1151449524973
86 -2085.1151347479804
87 -2085.115125439978
88 -2085.1151169496216
89 -2085.115109205
90 -2085.115102140534
91 -2085.115095696413
92 -2085.1150898181154
93 -2085.1150844558997
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```

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96 -2085.1150710318134
97 -2085.1150673185743
98 -2085.1150639311895
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101 -2085.1150554503643
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118 -2085.115034319264
119 -2085.115033826356
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130 -2085.115030566197
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132 -2085.11503025266
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134 -2085.1150299916653
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148 -2085.1150290542655
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153 -2085.115028922195
154 -2085.1150289023008
155 -2085.1150288841523
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162 -2085.115028794612
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168 -2085.1150287525174
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170 -2085.1150287429027
171 -2085.115028738715
172 -2085.1150287348955
173 -2085.115028731412
174 -2085.1150287282317
175 -2085.115028725332
176 -2085.1150287226865
177 -2085.115028720271
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180 -2085.115028714223
181 -2085.115028712549
182 -2085.115028711023
183 -2085.1150287096307
184 -2085.115028708359
185 -2085.115028707199
186 -2085.115028706141
187 -2085.1150287051746
188 -2085.1150287042956
189 -2085.1150287034893
190 -2085.1150287027563
191 -2085.1150287020887
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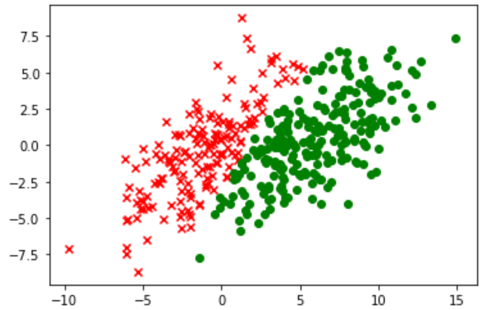
192 -2085.115028701477
193 -2085.115028700922
194 -2085.115028700415
195 -2085.115028699949
196 -2085.1150286995266
197 -2085.1150286991387
198 -2085.1150286987895
199 -2085.115028698466
```

```
# TODO: Make a scatterplot for the data points showing the Gaussian Mixture cluster assignments of each point

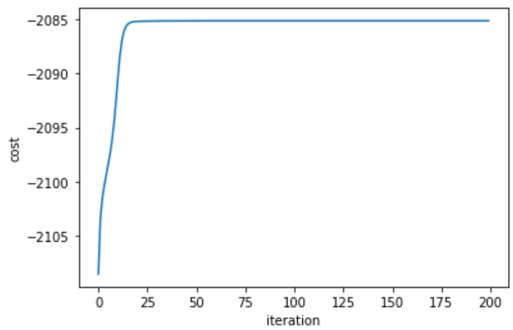
plt.scatter(data[class_1][:, 0], data[class_1][:, 1], marker='x', color='red')
plt.scatter(data[class_2][:, 0], data[class_2][:, 1], marker='o', color = 'green')
```

```
<matplotlib.collections.PathCollection at 0x7f513ea6bc50>
```




```
plt.ylabel('cost')
plt.xlabel('iteration')
plt.plot(iterations, costs)
```

```
[<matplotlib.lines.Line2D at 0x7f5141c02a90>]
```




```
labels_predict_EM = np.argwhere(Gamma >= 0.5)[:, 1]
print("Error rate: ", np.mean(labels != labels_predict_EM))
```

```
Error rate:  0.11
```

```
k_means_error_list = []
EM_error_list = []
k_means_time_list = []
EM_time_list = []

for i in range(5):
    num_samples = 400
    cov = np.array([[1., .7], [.7, 1.]]) * 10
    mean_1 = [.1, .1]
    mean_2 = [6., .1]

    x_class1 = np.random.multivariate_normal(mean_1, cov, num_samples // 2)
    x_class2 = np.random.multivariate_normal(mean_2, cov, num_samples // 2)
    xy_class1 = np.column_stack((x_class1, np.zeros(num_samples // 2)))
    xy_class2 = np.column_stack((x_class2, np.ones(num_samples // 2)))
    data_full = np.row_stack([xy_class1, xy_class2])
    np.random.shuffle(data_full)
    data = data_full[:, :2]
    labels = data_full[:, 2]

    N, D = data.shape
    K = 2
    max_iter = 200
    class_init = np.random.binomial(1., .5, size=N)
    R = np.vstack([class_init, 1 - class_init]).T

    start_time=time.clock()
    Mu = np.zeros([D, K])
    Mu[:, 1] = 1.
    R.T.dot(data), np.sum(R, axis=0)
    for it in range(max_iter):
```

```
        R = km_assignment_step(data, Mu)
        Mu = km_refitting_step(data, R, Mu)
        c = cost(data, R, Mu)
    labels_predict = np.argwhere(R == 1)[:, 1]
    k_means_error_list.append(np.mean(labels != labels_predict))
    time_use=time.clock()-start_time
    k_means_time_list.append(time_use)

    start_time = time.clock()
    Sigma = [np.eye(2), np.eye(2)]
    Pi = np.ones(K) / K
    Gamma = np.zeros([N, K]) # Gamma is the matrix of responsibilities
    for it in range(max_iter):
        Gamma = gm_e_step(data, Mu, Sigma, Pi)
        Mu, Sigma, Pi = gm_m_step(data, Gamma)
        loglike = log_likelihood(data, Mu, Sigma, Pi)
    labels_predict = np.argwhere(Gamma >= 0.5)[:, 1]
    EM_error_list.append(np.mean(labels != labels_predict))
    time_use=time.clock()-start_time
    EM_time_list.append(time_use)
```

```
print("BELOW IS THE RESULT FOR FIVE DIFFERENT DATA REALIZATION")
print("=================================================================")
print("THE K_MEANS REUSLT IS:", k_means_error_list)
print("=================================================================")
print("THE K_MEANS TIME IS:", k_means_time_list)
print("=================================================================")
print("THE EM_ERROR REUSLT IS:", EM_error_list)
print("=================================================================")
print("THE EM_ERROR REUSLT IS:", EM_time_list)
```

```
BELOW IS THE RESULT FOR FIVE DIFFERENT DATA REALIZATION
=================================================================
THE K_MEANS REUSLT IS: [0.2325, 0.2675, 0.2675, 0.2675, 0.2375]
=================================================================
THE K_MEANS TIME IS: [0.1449140000000284, 0.1250660000000039, 0.12583499999999503, 0.12067200000001321, 0.12221499999998287]
=================================================================
THE EM_ERROR REUSLT IS: [0.065, 0.0875, 0.1025, 0.1075, 0.0975]
=================================================================
THE EM_ERROR REUSLT IS: [11.571901000000025, 11.817532999999969, 11.66621299999997, 11.625350999999966, 11.75290799999999]
```

4. Comment on findings + additional experiments

Comment on the results:

- Compare the performance of k-Means and EM based on the resulting cluster assignments. **Answer:** Compare the performance of k-Means and EM besed on resulting cluster assignments, we could see that they are different. Also, notice the misclassification rate for k-Means is significantly larger than the misclassification rate for EM, we could draw the conclusion that EM has a better clusterring result than k-Means.
- Compare the performance of k-Means and EM based on their convergence rate. What is the bottleneck for which method? **Answer:** From the result above we could see that for k-Means the convergenece time is between 0.1 point to 0.2 point. For EM method, the convergence time is roughly over 10 seconds. Therefore, k-Means converges way faster then EM, and the bottleneck for EM method is EM is more accurate, but takes longer computataion time, and for k-Means the bottleneck is it indeed takes relatively shorter time but the prediction is not as accurate as EM's.
- Experiment with 5 different data realizations (generate new data), run your algorithms, and summarize your findings. Does the algorithm performance depend on different realizations of data? **Answer:** From

2. Reinforcement Learning

There are 3 files:

1. `maze.py` : defines the `MazeEnv` class, the simulation environment which the Q-learning agent will interact in.
2. `qlearning.py` : defines the `qlearn` function which you will implement, along with several helper functions. Follow the instructions in the file.
3. `plotting_utils.py` : defines several plotting and visualization utilities. In particular, you will use `plot_steps_vs_iters`, `plot_several_steps_vs_iters`, `plot_policy_from_q`

```
# from qlearning import qlearn
# from maze import MazeEnv, ProbabilisticMazeEnv
# from plotting_utils import plot_steps_vs_iters, plot_several_steps_vs_iters, plot_policy_from_q
```

```
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
# from qlearning import *
# from maze import *

#  UTILITY FUNCTIONS
```

```
color_cycle = ['#377eb8', '#ff7f00', '#a65628',
               '#f781bf','#4daf4a',  '#984ea3',
               '#999999', '#e41a1c', '#dede00']

def plot_steps_vs_iters(steps_vs_iters, block_size=10):
    num_iters = len(steps_vs_iters)
    block_size = 10
    num_blocks = num_iters // block_size
    smooted_data = np.zeros(shape=(num_blocks, 1))
    for i in range(num_blocks):
        lower = i * block_size
        upper = lower + 9
        smooted_data[i] = np.mean(steps_vs_iters[lower:upper])

    plt.figure()
    plt.title("Steps to goal vs episodes")
    plt.ylabel("Steps to goal")
    plt.xlabel("Episodes")
    plt.plot(np.arange(1,num_iters,block_size), smooted_data, color=color_cycle[0])

    return

def plot_several_steps_vs_iters(steps_vs_iters_list, label_list, block_size=10):
    smooted_data_list = []
    for steps_vs_iters in steps_vs_iters_list:
        num_iters = len(steps_vs_iters)
        block_size = 10
        num_blocks = num_iters // block_size
        smooted_data = np.zeros(shape=(num_blocks, 1))
        for i in range(num_blocks):
            lower = i * block_size
            upper = lower + 9
            smooted_data[i] = np.mean(steps_vs_iters[lower:upper])
        smooted_data_list.append(smooted_data)

    plt.figure()
    plt.title("Steps to goal vs episodes")
    plt.ylabel("Steps to goal")
    plt.xlabel("Episodes")
    index = 0
    for label, smooted_data in zip(label_list, smooted_data_list):
        plt.plot(np.arange(1,num_iters,block_size), smooted_data, label=label, color=color_cycle[index])
        index += 1
    plt.legend()

    return


# this function sets color values for
# Q table cells depending on expected reward value
def get_color(value, min_val, max_val):

    switcher={
                0:'gray',
                1:'indigo',
                2:'darkmagenta',
                3:'orchid',
                4:'lightpink',
             }

    step = (max_val-min_val)/5
    i = 0
    color='lightpink'

    for limit in np.arange(min_val, max_val, step):
        if limit <= value < limit+step:
            color = switcher.get(i)
        i+=1
    return color



# get first cell out of the start state
def get_next_cell(x1,x2,heatmap,policy_table,xlim=9,ylim=9):
    up_reward=-10000
    down_reward=-10000
    left_reward=-10000
    right_reward=-10000

    if (x1<ylim):
        if (policy_table[x1-1][x2]!=3):
            up_reward = heatmap[x1-1][x2]
    else:
        up_reward = -1000
```

```
    if (x1>0):
        if (policy_table[x1+1][x2]!=0):
            down_reward = heatmap[x1+1][x2]
    else:
        down_reward = -1000

    if (x2>0):
        if (policy_table[x1][x2-1]!=1):
            left_reward = heatmap[x1][x2-1]

    else:
        left_reward = -1000

    if (x2<xlim):
        if (policy_table[x1][x2+1]!=2):
            right_reward = heatmap[x1][x2+1]

    else:
        right_reward = -1000

    rewards = np.array([up_reward, down_reward, left_reward, right_reward])
    idx = np.argmax(rewards)
    next_cell = [(x1-1,x2), (x1+1,x2), (x1,x2-1), (x1,x2+1)][idx]
    choice = ['up', 'down', 'left', 'right']
    #print ('picking ',choice[idx])
    return next_cell




# get coordinates of the cells
# on the way from the start to goal state
def get_path(x1,x2, policy_table):
    x_coords = [x1]
    y_coords = [x2]
    x1_new = x1
    x2_new = x2

    i=0
    num_steps = 0
    total_cells = len(policy_table)*len(policy_table[0])
    while (policy_table[x1][x2]!='G') and num_steps < total_cells:
        if (policy_table[x1][x2]==1): # right
            x2_new=x2+1
            #print(i, ' - moving right')

        elif (policy_table[x1][x2]==0):
            x1_new=x1-1
            #print(i, ' - moving up')

        elif (policy_table[x1][x2]==3):
            x1_new=x1+1
            #print(i, ' - moving down')

        elif (policy_table[x1][x2]==2):
            x2_new=x2-1
            #print(i, ' - moving left')

        x1 = x1_new
        x2 = x2_new
        x_coords.append(x1)
        y_coords.append(x2)
        num_steps += 1
    return x_coords, y_coords

# plot Q table
# optimal path is highlighted and cells colored by their values
def plot_table(env, table_data, heatmap, goal_states, start_state, max_val, min_val, x_coords, y_coords):
    fig = plt.figure(dpi=80)
    ax = fig.add_subplot(1,1,1)
    plt.figure(figsize=(10,10))

    width = len(table_data[0])
    height = len(table_data)

    new_table = []

    for i in range(height):
        new_row = []

        for j in range(width):
            if env.map[i][j] == 0:
                new_row.append('')
            else:
                digit = table_data[i][j]
```

```
                if (digit==0):
                    new_row.append('\u2191') # up
                elif (digit==1):
                    new_row.append('\u2192') # right
                elif (digit==2):
                    new_row.append('\u2190') # left
                elif (digit==3):
                    new_row.append('\u2193') # down
                elif (digit=='G'):
                    new_row.append('G') # goal state
                elif (digit=='S'):
                    new_row.append('S') # goal state
                elif (digit==-1):
                    new_row.append('+') # All four directions
                else:
                    new_row.append('x') # unknown

        new_table.append(new_row)

    table = ax.table(cellText=new_table, loc='center',cellLoc='center')

    table.scale(1,2)

    for i in range(height):
        new_row = []

        for j in range(width):
            if new_table[i][j] == '':
                table[i, j].set_facecolor('black')
            else:
                table[i, j].set_facecolor(get_color(heatmap[i][j],min_val,max_val))

    for goal_state in goal_states:
        table[(goal_state[0], goal_state[1])].set_facecolor("limegreen")
    table[(start_state[0], start_state[1])].set_facecolor("yellow")
    ax.axis('off')
    table.set_fontsize(16)

    for i in range(len(x_coords)):
        table[(x_coords[i], y_coords[i])].get_text().set_color('red')
    plt.show()


# this function takes 3D Q table as an input
# and outputs optimal trajectory table (policy table)
# and corresponding excpected reward values of different cells (heatmap)
def get_policy_table(q_hat_3D, start_state, goal_states):
    policy_table = []
    heatmap = []

    for i in range(q_hat_3D.shape[0]):
        row = []
        heatmap_row = []
        for j in range(q_hat_3D.shape[1]):

            heatmap_row.append(np.max(q_hat_3D[i,j,:]))

            for goal_state in goal_states:
                if (goal_state[0]==i) and (goal_state[1]==j):
                    row.append('G')

            if (start_state[0]==i) and (start_state[1]==j):
                row.append('S')
            else:
                if np.max(q_hat_3D[i,j,:]) == 0:
                    row.append(-1) # All zeros
                else:
                    row.append(np.argmax(q_hat_3D[i,j,:]))
        policy_table.append(row)
        heatmap.append(heatmap_row)

    return policy_table, heatmap

def plot_policy_from_q(q_hat, env):
    q_hat_3D = np.reshape(q_hat, (env.m_size, env.m_size, env.num_actions))
    max_val = q_hat_3D.max()
    min_val = q_hat_3D.min()
    start_state = env.get_coords_from_state(env._get_start_state)
    goal_states = env._get_goal_state
    goal_states = [env.get_coords_from_state(goal_state) for goal_state in goal_states]
    policy_table, heatmap = get_policy_table(q_hat_3D, start_state, goal_states)
    x,y = get_next_cell(start_state[0],start_state[1],heatmap,policy_table)
    x_coords, y_coords = get_path(x,y,policy_table)
    plot_table(env, policy_table, heatmap, goal_states, start_state,max_val,min_val, x_coords, y_coords)

    return
```

```python
import numpy as np
import copy
import math
import random

ACTION_MEANING = {
    0: "UP",
    1: "RIGHT",
    2: "LEFT",
    3: "DOWN",
}

SPACE_MEANING = {
    1: "ROAD",
    0: "BARRIER",
    -1: "GOAL",
}


class MazeEnv:

    def __init__(self, start=[6,3], goals=[[1, 8]]):
        """Deterministic Maze Environment"""

        self.m_size = 10
        self.reward = 10
        self.num_actions = 4
        self.num_states = self.m_size * self.m_size

        self.map = np.ones((self.m_size, self.m_size))
        self.map[3, 4:9] = 0
        self.map[4:8, 4] = 0
        self.map[5, 2:4] = 0

        for goal in goals:
            self.map[goal[0], goal[1]] = -1

        self.start = start
        self.goals = goals
        self.obs = self.start

    def step(self, a):
        """ Perform a action on the environment

            Args:
                a (int): action integer

            Returns:
                obs (list): observation list
                reward (int): reward for such action
                done (int): whether the goal is reached
        """
        done, reward = False, 0.0
        next_obs = copy.copy(self.obs)

        if a == 0:
            next_obs[0] = next_obs[0] - 1
        elif a == 1:
            next_obs[1] = next_obs[1] + 1
        elif a == 2:
            next_obs[1] = next_obs[1] - 1
        elif a == 3:
            next_obs[0] = next_obs[0] + 1
        else:
            raise Exception("Action is Not Valid")

        if self.is_valid_obs(next_obs):
            self.obs = next_obs

        if self.map[self.obs[0], self.obs[1]] == -1:
            reward = self.reward
            done = True

        state = self.get_state_from_coords(self.obs[0], self.obs[1])

        return state, reward, done

    def is_valid_obs(self, obs):
        """ Check whether the observation is valid

            Args:
                obs (list): observation [x, y]

            Returns:
                is_valid (bool)
```

```
        """

        if obs[0] >= self.m_size or obs[0] < 0:
            return False

        if obs[1] >= self.m_size or obs[1] < 0:
            return False

        if self.map[obs[0], obs[1]] == 0:
            return False

        return True

    @property
    def _get_obs(self):
        """ Get current observation
        """
        return self.obs

    @property
    def _get_state(self):
        """ Get current observation
        """
        return self.get_state_from_coords(self.obs[0], self.obs[1])

    @property
    def _get_start_state(self):
        """ Get the start state
        """
        return self.get_state_from_coords(self.start[0], self.start[1])

    @property
    def _get_goal_state(self):
        """ Get the start state
        """
        goals = []
        for goal in self.goals:
            goals.append(self.get_state_from_coords(goal[0], goal[1]))
        return goals

    def reset(self):
        """ Reset the observation into starting point
        """
        self.obs = self.start
        state = self.get_state_from_coords(self.obs[0], self.obs[1])
        return state

    def get_state_from_coords(self, row, col):
        state = row * self.m_size + col
        return state

    def get_coords_from_state(self, state):
        row = int(math.floor(state/self.m_size))
        col = int(state % self.m_size)
        return row, col


class ProbabilisticMazeEnv(MazeEnv):
    """ (Q2.3) Hints: you can refer the implementation in MazeEnv
    """

    def __init__(self, goals=[[2, 8]], p_random=0.05):
        """ Probabilistic Maze Environment

            Args:
                goals (list): list of goals coordinates
                p_random (float): random action rate
        """
        self.m_size = 10
        self.reward = 10
        self.num_actions = 4
        self.num_states = self.m_size * self.m_size
        self.map = np.ones((self.m_size, self.m_size))
        self.map[3, 4:9] = 0
        self.map[4:8, 4] = 0
        self.map[5, 2:4] = 0
        for goal in goals:
            self.map[goal[0], goal[1]] = -1
        self.goals = goals
        self.start = [6,3]
        self.obs = self.start
        self.p_random = p_random

    def step(self, a):
        done, reward = False, 0.0
        next_obs = copy.copy(self.obs)
```

```
        rand  = random.uniform(0, 1)
        if rand <= self.p_random:
            a = np.random.choice(4)

        if a == 0:
            next_obs[0] = next_obs[0] - 1
        elif a == 1:
            next_obs[1] = next_obs[1] + 1
        elif a == 2:
            next_obs[1] = next_obs[1] - 1
        elif a == 3:
            next_obs[0] = next_obs[0] + 1
        else:
            raise Exception("Action is Not Valid")

        if self.is_valid_obs(next_obs):
            self.obs = next_obs

        if self.map[self.obs[0], self.obs[1]] == -1:
            reward = self.reward
            done = True

        state = self.get_state_from_coords(self.obs[0], self.obs[1])

        return state, reward, done


```

```
import numpy as np
import math
import copy

def qlearn(env, num_iters, alpha, gamma, epsilon, max_steps, use_softmax_policy, init_beta=None, k_exp_sched=None):
    """ Runs tabular Q learning algorithm for stochastic environment.

    Args:
        env: instance of environment object
        num_iters (int): Number of episodes to run Q-learning algorithm
        alpha (float): The learning rate between [0,1]
        gamma (float): Discount factor, between [0,1)
        epsilon (float): Probability in [0,1] that the agent selects a random move instead of
                selecting greedily from Q value
        max_steps (int): Maximum number of steps in the environment per episode
        use_softmax_policy (bool): Whether to use softmax policy (True) or Epsilon-Greedy (False)
        init_beta (float): If using stochastic policy, sets the initial beta as the parameter for the softmax
        k_exp_sched (float): If using stochastic policy, sets hyperparameter for exponential schedule
            on beta

    Returns:
        q_hat: A Q-value table shaped [num_states, num_actions] for environment with with num_states
            number of states (e.g. num rows * num columns for grid) and num_actions number of possible
            actions (e.g. 4 actions up/down/left/right)
        steps_vs_iters: An array of size num_iters. Each element denotes the number
            of steps in the environment that the agent took to get to the goal
            (capped to max_steps)
    """
    action_space_size = env.num_actions
    state_space_size = env.num_states
    q_hat = np.zeros(shape=(state_space_size, action_space_size))
    steps_vs_iters = np.zeros(num_iters)

    for i in range(num_iters):
        # TODO: Initialize current state by resetting the environment
        # curr_state = ...
        curr_state = env.reset()
        num_steps = 0
        done = False

        # TODO: Keep looping while environment isn't done and less than maximum steps
        # while ...:
        while (done != True) and (num_steps < max_steps):
            num_steps += 1

            # Choose an action using policy derived from either softmax Q-value
            # or epsilon greedy
            if use_softmax_policy:
                assert(init_beta is not None)
                assert(k_exp_sched is not None)

                beta = beta_exp_schedule(init_beta, i, k_exp_sched)
                action = softmax_policy(q_hat, beta, curr_state, action_space_size)
            else:
                # TODO: Epsilon-greedy
                action = epsilon_greedy(q_hat, epsilon, curr_state, action_space_size)
            next_state, reward, done = env.step(action)
```

```

            # TODO: Update Q_value
            if next_state != curr_state:
                new_value = alpha * (reward + gamma * np.max(q_hat[next_state, :]) - q_hat[curr_state, action])
                q_hat[curr_state, action] = q_hat[curr_state, action] + new_value
                curr_state = next_state
        steps_vs_iters[i] = num_steps

    return q_hat, steps_vs_iters


def epsilon_greedy(q_hat, epsilon, state, action_space_size):
    """ Chooses a random action with p_rand_move probability,
    otherwise choose the action with highest Q value for
    current observation

    Args:
        q_hat_3D: A Q-value table shaped [num_rows, num_col, num_actions] for
            grid environment with num_rows rows and num_col columns and num_actions
            number of possible actions
        epsilon (float): Probability in [0,1] that the agent selects a random
            move instead of selecting greedily from Q value
        obs: A 2-element array with integer element denoting the row and column
            that the agent is in
        action_space_size (int): number of possible actions

    Returns:
        action (int): A number in the range [0, action_space_size-1]
            denoting the action the agent will take
    """
    # TODO: Implement your code here
    # Hint: Sample from a uniform distribution and check if the sample is below
    # a certain threshold
    prob = np.random.uniform()
    if np.all(q_hat[state, :] == 0) == True:
        action = np.random.choice(action_space_size)
    elif (prob <= epsilon):
        action = np.random.choice(action_space_size)
    else:
        action = q_hat[state,:].argmax()
    return action


def softmax_policy(q_hat, beta, state, action_space_size):
    """ Choose action using policy derived from Q, using
    softmax of the Q values divided by the temperature.

    Args:
        q_hat: A Q-value table shaped [num_rows, num_col, num_actions] for
            grid environment with num_rows rows and num_col columns
        beta (float): Parameter for controlling the stochasticity of the action
        obs: A 2-element array with integer element denoting the row and column
            that the agent is in

    Returns:
        action (int): A number in the range [0, action_space_size-1]
            denoting the action the agent will take
    """
    prob = stable_softmax(beta * q_hat)
    if all(ele == 0 for ele in q_hat[state]):
        action = np.random.choice(action_space_size)
    else:
        action = np.random.choice(action_space_size, p=prob[state,:])
    return action


def beta_exp_schedule(init_beta, iteration, k=0.1):
   beta = init_beta * np.exp(k * iteration)
   return beta

def stable_softmax(x, axis=1):
    """ Numerically stable softmax:
    softmax(x) = e^x /(sum(e^x))
               = e^x / (e^max(x) * sum(e^x/e^max(x)))

    Args:
        x: An N-dimensional array of floats
        axis: The axis for normalizing over.

    Returns:
        output: softmax(x) along the specified dimension
    """
    max_x = np.max(x, axis, keepdims=True)
    z = np.exp(x - max_x)
    output = z / np.sum(z, axis, keepdims=True)

```

```
    return output
```

1. Basic Q Learning experiments

(a) Run your algorithm several times on the given environment. Use the following hyperparameters:

1. Number of episodes = 200
2. Alpha (α) learning rate = 1.0
3. Maximum number of steps per episode = 100. An episode ends when the agent reaches a goal state, or uses the maximum number of steps per episode
4. Gamma (γ) discount factor = 0.9
5. Epsilon (ϵ) for ϵ-greedy = 0.1 (10% of the time)

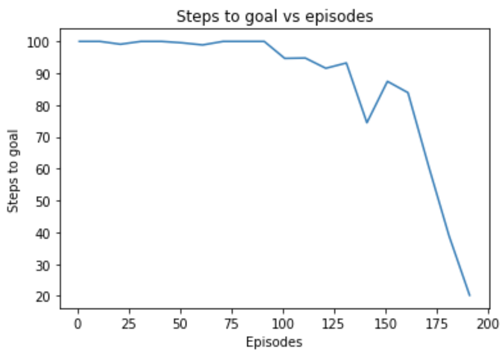
```
# TODO: Fill this in
num_iters = 200
alpha = 1.0
gamma = 0.9
epsilon = 0.1
max_steps = 100
use_softmax_policy = False

# TODO: Instantiate the MazeEnv environment with default arguments
env = MazeEnv()

# TODO: Run Q-learning:
q_hat, steps_vs_iters = qlearn(env, num_iters, alpha, gamma, epsilon, max_steps, use_softmax_policy)
```

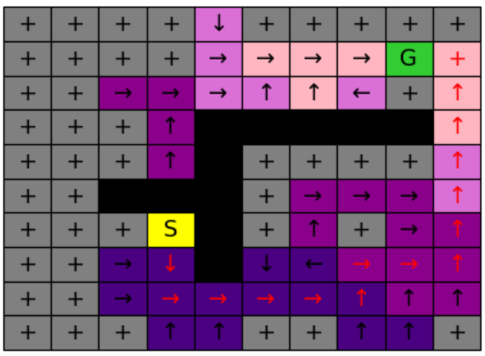
Plot the steps to goal vs training iterations (episodes):

```
# TODO: Plot the steps vs iterations
plot_steps_vs_iters(steps_vs_iters)
```



Visualize the learned greedy policy from the Q values:

```
# TODO: plot the policy from the Q value
plot_policy_from_q(q_hat, env)
```




```
<Figure size 720x720 with 0 Axes>
```

(b) Run your algorithm by passing in a list of 2 goal locations: (1,8) and (5,6). Note: we are using 0-indexing, where (0,0) is top left corner. Report on the results.

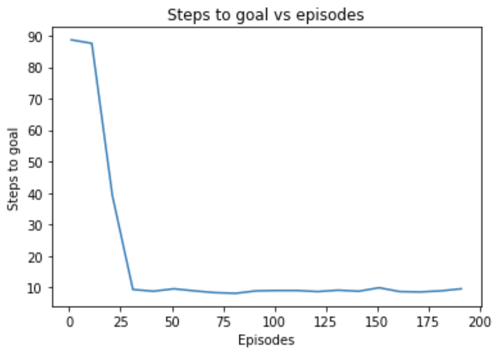
```
# TODO: Fill this in (same as before)
num_iters = 200
alpha = 1.0
gamma = 0.9
epsilon = 0.1
max_steps = 100
use_softmax_policy = False
```

```
# TODO: Set the goal
goal_locs = [[1, 8], [5, 6]]
env = MazeEnv(goals= goal_locs)

# TODO: Run Q-learning:
q_hat, steps_vs_iters = qlearn(env, num_iters, alpha, gamma, epsilon, max_steps, use_softmax_policy)
```

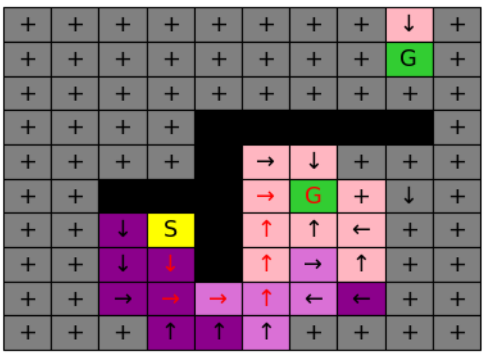
Plot the steps to goal vs training iterations (episodes):

```
# TODO: Plot the steps vs iterations
plot_steps_vs_iters(steps_vs_iters)
```



Plot the steps to goal vs training iterations (episodes):

```
# TODO: plot the policy from the Q values
plot_policy_from_q(q_hat, env)
```




```
<Figure size 720x720 with 0 Axes>
```

2. Experiment with the exploration strategy, in the original environment

(a) Try different ϵ values in ϵ-greedy exploration: We asked you to use a rate of ϵ=10%, but try also 50% and 1%. Graph the results (for 3 epsilon values) and discuss the costs and benefits of higher and lower exploration rates.

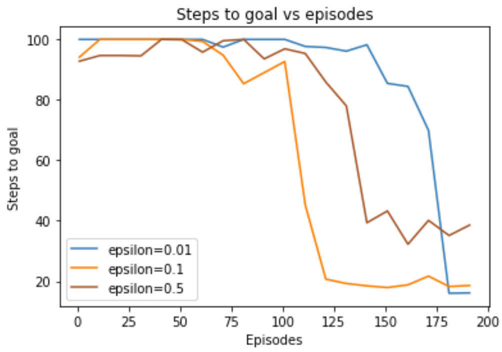
```
# TODO: Fill this in (same as before)
num_iters = 200
alpha = 1.0
gamma = 0.9
max_steps = 100
use_softmax_policy = False

# TODO: set the epsilon lists in increasing order:
epsilon_list = [0.01, 0.1, 0.5]

env = MazeEnv()

steps_vs_iters_list = []
for epsilon in epsilon_list:
   q_hat, steps_vs_iters = qlearn(env, num_iters, alpha, gamma, epsilon, max_steps, use_softmax_policy)
   steps_vs_iters_list.append(steps_vs_iters)
```

```
# TODO: Plot the results
label_list = ["epsilon={}".format(eps) for eps in epsilon_list]
plot_several_steps_vs_iters(steps_vs_iters_list, label_list)
```

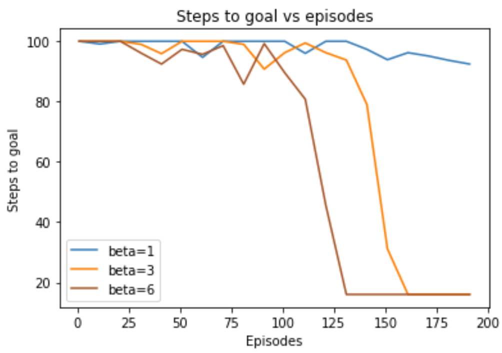


(b) Try exploring with policy derived from **softmax of Q-values** described in the Q learning lecture. Use the values of $\beta \in \{1, 3, 6\}$ for your experiment, keeping β fixed throughout the training.

```
# TODO: Fill this in for Static Beta with softmax of Q-values
num_iters = 200
alpha = 1.0
gamma = 0.9
epsilon = 0.1
max_steps = 100

# TODO: Set the beta
beta_list = [1, 3, 6]
use_softmax_policy = True
k_exp_schedule = 0.0
env = MazeEnv()
steps_vs_iters_list = []
for beta in beta_list:
    q_hat, steps_vs_iters = qlearn(env, num_iters, alpha, gamma, epsilon, max_steps, use_softmax_policy, beta, k_exp_schedule
    steps_vs_iters_list.append(steps_vs_iters)
```

```
label_list = ["beta={}".format(beta) for beta in beta_list]
# TODO:
plot_several_steps_vs_iters(steps_vs_iters_list, label_list)
```



(c) Instead of fixing the $\beta = \beta_0$ to the initial value, we will increase the value of β as the number of episodes t increase:

$$\beta(t) = \beta_0 e^{kt}$$

That is, the β value is fixed for a particular episode. Run the training again for different values of $k \in \{0.05, 0.1, 0.25, 0.5\}$, keeping $\beta_0 = 1.0$. Compare the results obtained with this approach to those obtained with a static β value.

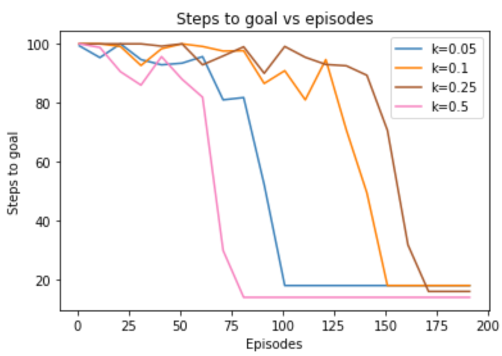
```
# TODO: Fill this in for Dynamic Beta
num_iters = 200
alpha = 1.0
gamma = 0.9
epsilon = 0.1
max_steps = 100

# TODO: Set the beta
beta = 1.0
use_softmax_policy = True
k_exp_schedule_list = [0.05, 0.1, 0.25, 0.5]
env = MazeEnv()

steps_vs_iters_list = []
for k_exp_schedule in k_exp_schedule_list:
    q_hat, steps_vs_iters = qlearn(env, num_iters, alpha, gamma, epsilon, max_steps, use_softmax_policy, beta, k_exp_schedule
    steps_vs_iters_list.append(steps_vs_iters)
```

```
# TODO: Plot the steps vs iterations
```

```
label_list = ["k={}".format(k_exp_schedule) for k_exp_schedule in k_exp_schedule_list]
plot_several_steps_vs_iters(steps_vs_iters_list, label_list)
```



3. Stochastic Environments

(a) Make the environment stochastic (uncertain), such that the agent only has a 95% chance of moving in the chosen direction, and has a 5% chance of moving in some random direction.

```
# TODO: Implement ProbabilisticMazeEnv in maze.py
```

(b) Change the learning rule to handle the non-determinism, and experiment with different probability of environment performing random action $p_{rand} \in \{0.05, 0.1, 0.25, 0.5\}$ in this new rule. How does performance vary as the environment becomes more stochastic?

Use the same parameters as in first part, except change the alpha (α) value to be **less than 1**, e.g. 0.5.

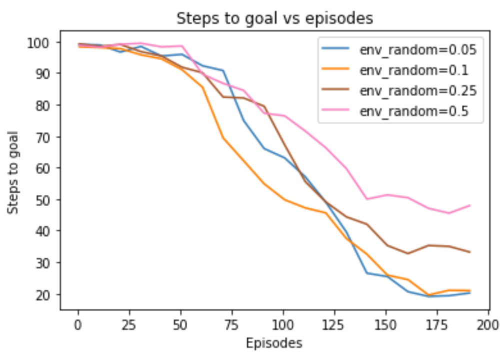
```
# TODO: Fill this in for Dynamic Beta
num_iters = 200
alpha = 0.5
gamma = 0.9
epsilon = 0.1
max_steps = 100
use_softmax_policy = False
beta = 1.0

# Set the environment probability of random
env_p_rand_list = [0.05, 0.1, 0.25, 0.5]

steps_vs_iters_list = []
for env_p_rand in env_p_rand_list:
    # Instantiate with ProbabilisticMazeEnv
    env = ProbabilisticMazeEnv(p_random=env_p_rand)

    # Note: We will repeat for several runs of the algorithm to make the result less noisy
    avg_steps_vs_iters = np.zeros(num_iters)
    for i in range(10):
        # q_hat, steps_vs_iters = qlearn(env, num_iters, alpha, gamma, epsilon, max_steps, use_softmax_policy, init_beta=6, k
        q_hat, steps_vs_iters = qlearn(env, num_iters, alpha, gamma, epsilon, max_steps, use_softmax_policy)
        avg_steps_vs_iters += steps_vs_iters
    avg_steps_vs_iters /= 10
    steps_vs_iters_list.append(avg_steps_vs_iters)
```

```
label_list = ["env_random={}".format(env_p_rand) for env_p_rand in env_p_rand_list]
plot_several_steps_vs_iters(steps_vs_iters_list, label_list)
```



3. Did you complete the course evaluation?

ABSOLUTELY YES Thank you so much my dearest professors and TAs.