Inflation, Skill Loss During Unemployment, and TFP in the Long Run*

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February 4, 2025

Abstract

We develop a search model with frictional goods and labor markets to study the long run relationship between inflation, unemployment, and TFP when workers lose skills during unemployment. As inflation increases, fewer jobs are created, workers experience longer unemployment durations and lose human capital, causing TFP to decline. Our calibrated results show that transitioning from the Friedman rule to 10% annual inflation lowers TFP by 4.3%. A stochastic version of the model demonstrates that prolonged periods of inflation can have lasting negative effects on productivity.

JEL Classification: E24, J24, E40, D24

Keywords: Inflation, TFP, Skill Loss, Unemployment, Human Capital

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*We thank Mohammed Aït Lahcen, Lukas Altermatt, Aleksander Berentsen, Mei Dong, Ioannis Kospentaris, Lien Laureys, Florian Madison, Sephorah Mangin, Victor Ortego-Marti, Guillaume Rocheteau, Serene Tan, Hugo van Buggenum, Liang Wang, and seminar participants at the National University of Singapore, UC Irvine, UC Riverside, and the University of Basel for their helpful comments and suggestions.

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1 Introduction

The relationship between inflation and labor market performance has been widely studied in macroeconomics, where labor market performance is typically measured by the unemployment rate. While this literature is vast and dates back to Phillips (1958), the connection between inflation and unemployment has featured prominently in policy debates following the worldwide inflationary episode following the COVID-19 pandemic. Alongside this literature and policy discussions, a large body of evidence has been documented that workers lose skills during unemployment.[1] When workers lose skills during unemployment, the economy's productivity becomes endogenous, as the aggregate stock of human capital is a function of labor market flows (Ortego-Marti, 2017b, 2020). From a macroeconomic perspective, skill loss during unemployment provides a channel through which inflation can impact the economy's aggregate human capital and TFP. The goal of this paper is to develop a framework which captures and quantifies the long run relationship between inflation, unemployment, and an additional measure of labor market performance, TFP, when workers lose skills during unemployment.

We build a microfounded model of money with frictional goods and labor markets as in Berentsen et al. (2011). The frictional labor market follows the Diamond (1982) and Mortensen and Pissarides (1994) model of equilibrium unemployment, whereas the monetary side of the model is built on Lagos and Wright (2005) and Rocheteau and Wright (2005). Firms post vacancies in order to hire workers. Those firms who match with a worker can sell their output at a markup in a subsequent retail market where there is anonymity and lack of commitment, which makes a means of payment (fiat money) essential. As households use real balances to consume in the retail market, there is a direct linkage between the value of money and the expected revenue of filling a vacancy. The key ingredient in our baseline framework is that workers are heterogeneous in the skill level and that highly-skilled workers who do not find a job are subject to skill loss shocks. TFP, as measured by average labor productivity, becomes a function of both the skill composition of unemployed workers and net output produced in the retail market.

An escalation in the inflation tax impacts TFP through two channels. First, as is standard in monetary search models, higher inflation causes households to hold less real balances, thereby reducing their consumption in the retail market. This reduction in demand decreases the expected productivity of a match, as the net output generated by selling output in the retail market falls. Second, as the expected productivity of a vacancy decreases, firms create less vacancies and workers' job finding probability decreases. When workers face longer unemployment durations, they are more exposed to skill loss shocks and the skill composition of the labor force deteriorates. As the skill composition decays, the average production in the labor market also decreases. Ultimately, TFP falls due to declines in both the net output generated by the retail market and the average skill level of the workforce.

Next, motivated by evidence showing that occupations which require more skills experience

[1] Section 1.1 reviews the empirical evidence documenting skill loss during unemployment.

higher rates of skill decay (Ortego-Marti, 2017a), we extend our model to include heterogeneous firms. Firms can create a simple or complex vacancy as in Albrecht and Vroman (2002), where there are two distinctions between simple and complex jobs. First, highly-skilled workers produce more output in complex jobs. Second, the productivity of highly-skilled workers who are hit with a skill loss shock declines more in complex than simple jobs. In this version of the model, the aggregate stock of vacancies and the composition of job complexity (i.e., the fraction of jobs that are complex) are endogenous and jointly determined with the value of money and skill composition of unemployed workers. Therefore, TFP becomes a function of the composition of job complexity in addition to the skill composition of the unemployed and value of money. A hike in anticipated inflation now has an adverse impact on the creation of complex vacancies, as skill loss has a larger impact on productivity in complex jobs. Thus, the total effect of increasing inflation on TFP can be decomposed into three components: (i) a worsening of the skill distribution, (ii) a reduction in the net output generated by the retail market, and (iii) a shift in the composition of job complexity from complex to simple jobs.

We calibrate the model to the US economy between 1955-2017 to quantify the effects of anticipated inflation on unemployment and TFP. We also use the calibrated model to decompose changes in TFP into the aforementioned three channels. A key target in our calibration comes from the empirical literature which has estimated the effect of an additional month of unemployment duration on reemployment wages. We choose the skill loss parameters so that when we simulate employment histories, the effects of unemployment history on wages are consistent with the empirical evidence. The model can match this evidence and other targets commonly used in the literature well.

In our main quantitative exercise, we estimate the productivity costs of inflation. To begin, we vary the nominal interest rate from its minimum value observed between 1955-2017 (a level that is consistent with 1% annual inflation) to its maximum (consistent with 11% annual inflation). The model implies that such an escalation in inflation reduces TFP by nearly 4%.[2] Our results show that the effect is non-linear, as TFP is most sensitive to changes in inflation at low rates of inflation. This is also illustrated by comparing an economy at the Friedman Rule to one with 10% annual inflation, where our results show that such an increase in inflation causes the unemployment rate to rise from 5.1% to 6.2% and TFP to decrease by 4.3%. When we decompose the productivity costs of inflation into the three channels, we find that shifts in the skill composition of workers accounts for nearly 75% of TFP losses, whereas the shift from complex to simple jobs accounts for 15% and the remaining 10% is attributable to the decline in retail market sales.

While our main quantitative exercises highlight the contribution of changes in the skill composition of unemployed workers to the productivity costs of inflation, these exercises also compare different steady-states. As a final exercise, we solve the version of the model where anticipated inflation (as measured by the nominal interest rate) follows a stochastic process and present the resulting simulations after feeding nominal interest rate data between 1955-2017 through the model.[3]

[2] We provide four alternative calibrations in Supplemental Appendix E and show that the productivity costs of inflation are largely unchanged.

[3] A primary motivation for this exercise is that the economy may not quickly transition between steady-states

The stochastic version of the model generates an increase in trend unemployment through the 1970s and 1980s as anticipated inflation increased in the US. At the same time, the skill composition of the unemployed decayed and composition of job complexity shifted from complex to simple jobs. We find that TFP reached its lowest level in 1985, with a 0.82% decline relative to its level in 1955. At this level, the contribution of the three channels are relatively equal showing that changes in the skill composition of the unemployed cause larger productivity losses and account for a higher share of productivity losses following a permanent increase in anticipated inflation. The simulations also show there are scarring effects of prolonged periods of high inflation, as TFP remains depressed even after the unemployment rate has begun to recover.

The rest of this paper is organized as follows. Section 1.1 discusses our contributions relative to the related literature. Section 2 introduces the baseline environment and Section 3 discusses the equilibrium. Section 4 provides an extension with heterogeneous firms. Section 5 performs the quantitative analysis. Finally, Section 6 concludes. All theoretical proofs, empirical details, and quantitative technicalities are delegated to the Supplemental Appendix and referenced through the main text.

1.1 Related Literature

This paper is most closely related to Berentsen et al. (2011), who find a positive correlation between anticipated inflation and unemployment in the US between 1955-2005.[4] Berentsen et al. (2011) explain this relationship by integrating the Diamond (1982) and Mortensen and Pissarides (1994) model of unemployment with the monetary search framework of Lagos and Wright (2005).[5] The inflation tax causes households to carry less real balances, which generates a decrease in demand and therefore a decline in the expected revenue of a job. Following the decline in revenue, firms post less vacancies and unemployment increases.[6] Our contributions relative to Berentsen et al. (2011) are two-fold. First, on the theory, we incorporate skill loss during unemployment and heterogeneous firms, which opens two channels through which inflation impacts TFP. On the quantitative side, we show that these two channels amplify the transmission of higher expected inflation to unemployment and TFP.

More broadly, our paper contributes to the literature which studies the relationship between anticipated inflation and labor market performance. A majority of this literature measures labor

following a change in the nominal interest rate, as it takes time for the skill composition of unemployed workers to evolve.

[4]Further evidence supporting an upward sloping long run Phillips curve is provided by Haug and King (2014), who used a strategy centered around the band-pass filter, and Ait Lahcen et al. (2022) who use panel data from OECD countries.

[5]See Rocheteau and Nosal (2017) and Lagos et al. (2017) and references therein for a survey of this New Monetarist literature.

[6]Additional, but closely frameworks developed to study the relationship between inflation and unemployment include Rocheteau et al. (2007) and Dong (2011), whose model of unemployment follows Rogerson (1988). Rocheteau et al. (2021) study equilibria in which it takes multiple periods for unemployed workers to reach their optimal holdings of real balances. Ait Lahcen et al. (2022) show that higher inflation increases both the level and volatility of unemployment.

market performance via the unemployment rate and recent contributions emphasize how additional financial frictions interact with the effect of inflation on unemployment (Gu et al., 2023; Bethune et al., 2015) or optimal policy (Laureys, 2014; Gomis-Porqueras et al., 2013).[7] A recent exception is Gomis-Porqueras et al. (2020), who studied the long run relationship between inflation, unemployment, and capital. Our contribution to this literature is to focus on unemployment and TFP, as measured by average labor productivity, and to propose skill loss during unemployment as a channel which links anticipated inflation to TFP. We find that even when it appears that expected inflation does have a large impact on the labor market, it can be misleading to focus solely on the unemployment rate as small shifts in unemployment can lead to sizeable changes in TFP through a change in the skill composition of workers. A complementary study to ours Berentsen et al. (2012), who find that countries with higher inflation rates tend to have lower growth rates. While our focus is not on growth, our findings are related in that we study a channel (skill loss during unemployment) through which inflation lowers productivity.

Our paper also builds on previous work on skill loss during unemployment. Studies which provide direct evidence on skill loss during unemployment include Edin and Gustavsson (2008) and Dinerstein et al. (2022), who exploit quasi-random variation in the assignment of teachers in Greece and estimate an annual skill depreciation rate of 4.2%.[8] Further, there is additional literature which studies the macroeconomic consequences of skill loss during unemployment. Pissarides (1992) showed that unemployment is more persistent when unemployed workers lose skills during unemployment, whereas Ljungqvist and Sargent (1998) argued that generous UI benefits with skill loss provides an explanation for high unemployment in Europe relative to the US. Ortego-Marti (2017b, 2020) illustrated how loss of skill during unemployment impacts TFP while Doppelt (2019) proposed skill loss as a mechanism to explain the long-run relationship between growth and unemployment.[9] Motivated by these insights, our contribution is to incorporate skill loss during unemployment into a microfounded model of money and study the quantitative implications of this channel for the effects of anticipated inflation on the labor market.

2 Model Environment

Time is discrete and goes on forever. There are two types of agents indexed by $j \in \{h, f\}$: a measure 1 of households, h, and a large measure of firms, f, where the measure of active firms is endogenous.[10] Each period is divided into three stages. In stage 1, households and firms trade labor

[7] There is an interesting and closely literature which has studied the relationship the provision of public liquidity on unemployment. See, for example, Rocheteau and Rodriguez-Lopez (2014) and Dong and Xiao (2019).

[8] Additional studies documenting skill loss due to breaks in production include David and Brachet (2011); Hockenberry et al. (2008); Hockenberry and Helmchen (2014), Globerson et al. (1989); Bailey (1989) and Shafer et al. (2001). These findings are consistent with studies which find large effects of job displacement on wages and employment (Jacobson et al., 1993; Davis and von Wachter, 2011; Jarosch, 2023).

[9] Further evidence on the relationship between unemployment and productivity is provided by Kuhn et al. (2024) who document a negative relationship between labor productivity and unemployment across local labor markets. Our model rationalizes this correlation through skill loss during unemployment.

[10] Throughout the analysis, we use 'household' and 'worker' interchangeably.

services and produce a general good in a decentralized labor market. In stage 2, households and firms trade specialized goods in a retail market. In stage 3, agents trade fiat money and the general good in a frictionless centralized market. The general good is taken as the numéraire. All goods are non-storable across time periods.[11] The sequence of markets within a representative time period is summarized in Figure 1.

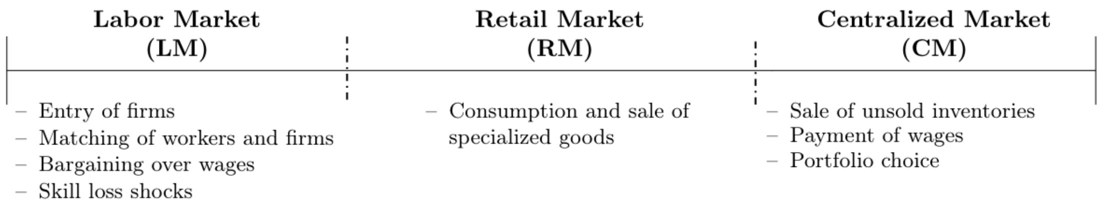


Figure 1: Timing of a Representative Period

Households are heterogeneous in their skill, which is indexed by $\varepsilon \in \{L, H\}$: low (L) and high (H). Employed high skill workers produce y units of output per period. Low skill workers produce δy with $\delta \in (0,1)$. Skill loss occurs as follows. At the beginning of each period, firms post vacancies and hire workers. After hiring takes place, high skill households who entered the period unemployed and did not find a job become permanently low-skilled with probability σ.[12]

The household's lifetime discounted utility is given by

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t[\epsilon_t \upsilon(q_t) + x_t], \tag{1}$$

where $\beta = (1+\rho)^{-1} \in (0,1)$ is a discount factor, $q_t \in \mathbb{R}_+$ is consumption of the specialized good, $\epsilon_t \in \{0,1\}$ is a preference shock for the specialized good, and $x_t \in \mathbb{R}_+$ is consumption of the numéraire. We assume $\upsilon' > 0 > \upsilon''$, $\upsilon'(0) = \infty$, $\upsilon'(\infty) = 0$, and $\upsilon(0) = 0$. The preference shocks, $\{\epsilon_t\}_{t=0}^{\infty}$, are i.i.d. across agents and time with $\Pr[\epsilon_t = 1] = \alpha$ and $\Pr[\epsilon_t = 0] = 1 - \alpha$.

Households and firms meet each other in stage 1 according to a constant returns to scale meeting technology $\mathcal{N}(u, v)$ where u is the stock of unemployed workers and v is the stock of vacancies. The probability that an unemployed household meets a firm is given by $\xi_h = \mathcal{N}(u,v)/u = \mathcal{N}(1,\theta)$ where $\theta = v/u$ is labor market tightness. We assume that $\mathcal{N}(1,\theta)$ is strictly increasing in θ with $\lim_{\theta\to 0}\xi_h = 0$, and $\lim_{\theta\to\infty}\xi_h = 1$. Similarly, the probability that a firm meets a worker is given by $\xi_f = \mathcal{N}(u,v)/v = \mathcal{N}(1,\theta)/\theta$. We assume that $\mathcal{N}(1,\theta)/\theta$ is strictly decreasing in θ with $\lim_{\theta\to 0}\xi_f = 1$ and $\lim_{\theta\to\infty}\xi_f = 0$. Existing matches in the labor market are destroyed at the beginning of stage

[11]This assumption is crucial in motivating money's role as a medium of exchange. If goods were storable, households could carry them into the next retail market and use them in quid pro quo exchanges. Hence, to focus on money's role as a medium of exchange, we assume all goods are non-storable.

[12]This assumption is supported by evidence which shows skill losses are extremely persistent. Ortego-Marti (2016) shows that a month of unemployment accumulated in the previous 5 years lowers wages by 1.61% whereas a month of unemployment experienced more than 5 years ago still decreases wages by 1.04%. Additionally, Davis and von Wachter (2011) and Jarosch (2023) find that wage losses follow workers for more than 20 years.

1 with probability $\lambda \in [0, 1]$. Those whose match is destroyed in period t can not be matched again until period $t + 1$.

There is anonymity and lack of commitment in the retail market, making a means of payment essential (Kocherlakota, 1998). We further assume that fiat money is always recognizable and cannot be counterfeited, whereas counterfeit claims to real assets (e.g., claims on firms' profits) cannot be recognized, leaving money as the medium of exchange in the retail market.[13] Firms who produced in stage 1 can sell q units of their inventory in stage 2 at cost $c(q)$ where $c' > 0$ and $c'' > 0$.

In stage 3, unemployed households are paid an unemployment benefit b, where $b < \delta y$, while employed households are paid their wage.[14] All households pay lump-sum taxes and firm's profits are paid out as dividends to households. Vacant firms can pay k units of the numéraire to enter the next labor market with a vacancy and agents have the opportunity to accumulate real balances. To keep the distribution of skills stationary, we assume that between periods a fraction $\mu \in (0, 1)$ of workers leave the labor force and that a measure μ of workers enter the labor force as unemployed who are highly-skilled. The real balances among those who leave the labor force are equally redistributed among the new entrants.

There is a government who finances government expenditures, G, and unemployment benefits b, by levying lump-sum taxes on households in stage 3 and by printing fiat money at rate π.

3 Equilibrium

Sections 3.1 and 3.2 describe the value functions and optimization problems agent faces throughout the three stages. Section 3.3 solves for the equilibrium in the retail market. Section 3.4 solves the wage bargaining problem in the labor market and describes the entry of firms. Section 3.5 characterizes the set of stationary equilibria. Finally, Section 3.6 discusses the relationship between inflation and TFP.

Throughout the analysis, we focus on stationary equilibria where aggregate real balances are constant across time. Letting ϕ_t denote the price of money in terms of the numéraire, it follows that $\phi_t = (1 + \pi)\phi_{t+1}$ in a stationary equilibrium and the real gross rate of return of money is $1 + r = \phi_{t+1}/\phi_t = 1/(1 + \pi)$.

3.1 Households

We solve the model through backwards induction, beginning with stage 3.

[13]See Rocheteau and Rodriguez-Lopez (2014) for an environment where agents trade claims on firm's profits. In addition, Lester et al. (2012) provide a formal analysis of how the recognizability of an asset determines the acceptability in exchange.

[14]As in Berentsen et al. (2011), we have wages paid in the centralized market to abstract from specifying whether the wage is paid in money or goods, as it does not make a difference how they are paid in the centralized market. See Gu et al. (2023) for an analysis where firms must pay wages in stage 1 using cash.

Stage 3: Centralized Market

Consider a type $\Omega \in \{L, H\} \times \{0, 1\}$ household where $\Omega = (\varepsilon, 0)$ denotes an unemployed household of skill level ε and $\Omega = (\varepsilon, 1)$ denotes a worker of skill level ε who is employed. The value of household with real balances z in the centralized market is given by

$$W_\Omega(z) = \max_{x,z'} \big\{x + \bar{\beta} U_\Omega(z')\big\}, \tag{2}$$

$$\text{s.t.} \;\; x + \frac{z'}{1+r} = w_\Omega + \Delta + z + T, \tag{3}$$

$$x \geq 0, \tag{4}$$

where x is consumption of the numéraire, z is current real balances, z' is real balances brought into the next period, w_Ω is labor market income, Δ is dividends, T is transfers net of taxes, $U_\Omega(z')$ is the continuation value of entering stage 1 with labor market status Ω and holding z' real balances, and $\bar{\beta} \equiv \beta(1-\mu)$ is the effective discount factor. Note that the non-negativity constraint, (4), will not bind if the unemployment benefits, b, are large enough. Assuming that (4) does not bind and substituting for x using the budget constraint gives

$$W_\Omega(z) = I_\Omega + z + \max_{z'} \left\{ -\frac{z'}{1+r} + \bar{\beta} U_\Omega(z') \right\}, \tag{5}$$

where $I_\Omega = w_\Omega + \Delta + T$ is net income. From (5), the household's value function is linear in z and thus their choice of real balances, z', is independent of their current holdings of real balances. It will also be shown later that the household's choice of z' is independent of their labor market status, Ω. As a result, the distribution of real balances is degenerate.

Stage 2: Retail Market

We consider a competitive retail market, where all agents take prices, p, as given. The value of a household with real balances z and labor market status Ω in the retail market, $V_\Omega(z)$, satisfies

$$V_\Omega(z) = \alpha \max_{pq \leq z} \big\{\upsilon(q) + W_\Omega(z - pq)\big\} + (1-\alpha) W_\Omega(z). \tag{6}$$

From equation (6), households are hit with a preference shock with probability α, subsequently choose their consumption, q, to maximize their lifetime discounted utility, and enter the centralized market with $z - pq$ units of real balances. With probability $1 - \alpha$ the household does not consume and enters stage 3 with z units of real balances.

Stage 1: Labor Market

Households enter the labor market with labor market status Ω and real balances z. The value of an unemployed household with high skills, $U_{H,0}(z)$, satisfies

$$U_{H,0}(z) = \xi_h V_{H,1}(z) + (1-\xi_h)\big\{\sigma V_{L,0}(z) + (1-\sigma)V_{H,0}(z)\big\}. \tag{7}$$

From (7), a high skill household meets a firm and becomes employed with probability ξ_h. However, With probability $1-\xi_h$, the worker does not meet a firm and is susceptible to skill loss. If they do not find a job, they become low-skilled with probability σ and remain highly skilled with probability $1-\sigma$.

The value of an unemployed household with low skills, $U_{L,0}$, satisfies

$$U_{L,0}(z) = \xi_h V_{L,1} + (1-\xi_h)V_{L,0}(z), \tag{8}$$

which has a similar interpretation as equation (7), except that low skill households are not susceptible to skill loss shocks.

The value function of a household of skill level ε who enters the labor market employed is given by

$$U_{\varepsilon,1}(z) = \lambda V_{\varepsilon,0}(z) + (1-\lambda)V_{\varepsilon,1}(z). \tag{9}$$

Substituting (6) into (7)-(9) and using the linearity of the $W_\Omega(z)$ gives the following value functions which summarizes all three markets:

$$\begin{aligned} U_{H,0}(z) &= \alpha\big[\upsilon(q) + z - pq\big] + (1-\alpha)z + \xi_h W_{H,1}(0) + \\ &\qquad (1-\xi_h)\big\{\sigma W_{L,0}(0) + (1-\sigma)W_{H,0}(0)\big\}, \qquad (10)\\ U_{L,0}(z) &= \alpha\big[\upsilon(q) + z - pq\big] + (1-\alpha)z + \xi_h W_{L,1}(0) + (1-\xi_h)W_{L,0}(0), \qquad (11)\\ U_{\varepsilon,1}(z) &= \alpha\big[\upsilon(q) + z - pq\big] + (1-\alpha)z + \lambda W_{\varepsilon,0}(0) + (1-\lambda)W_{\varepsilon,1}(0), \qquad (12)\end{aligned}$$

which can in turn be substituted into (5) to give:

$$W_\Omega(z) = I_\Omega + z + \max_{z'\geq 0}\left\{-\frac{z'}{1+r} + \bar{\beta}\Big[\alpha\big[\upsilon(q) + z' - pq\big] + (1-\alpha)z' + \mathbb{E}W_{\Omega'}(0)\Big]\right\}, \tag{13}$$

where the expectation is taken with respect to next period's type, Ω'. From (13), the choice of real balances is independent of Ω and z. Therefore, all households enter next period holding the same amount of real balances, z'.

3.2 Firms

Stage 3: Centralized Market

Firms who are matched enter the centralized market with unsold inventory x and real balances z. As carrying money is costly, firms do not need to carry real balances into the next time period. It follows that the value function for a firm who is currently matched with a type ε household is given by

$$\Pi_{\varepsilon,1}(x,z) = x + z - w_\varepsilon + \bar{\beta} J_{\varepsilon,1}, \tag{14}$$

where $J_{\varepsilon,1}$ is the continuation value of entering next period's labor market in a match.

Firms who enter the centralized market unmatched have no inventory as they did not produce in the labor market. They have a choice of whether to pay k units of the numéraire to enter the next labor market with a vacancy. Therefore, the problem of a vacant firm is given by

$$\Pi_0(z) = z + \max\big\{0, -k + \bar{\beta} J_0\big\}. \tag{15}$$

Stage 2: Retail Market

Let $K_{\varepsilon,1}$ denote the value of a firm who enters the retail market having produced y_ε units of output in the labor market. It follows that

$$K_{\varepsilon,1} = \max_{q \leq y_\varepsilon} \Pi_{\varepsilon,1}(y_\varepsilon - c(q), pq). \tag{16}$$

From (16), firms sell q units of specialized goods in the retail market and enter the centralized market with $y_\varepsilon - c(q)$ units of unsold inventory and pq units of real balances. Finally, for unmatched firms, $K_0 = \Pi_0$ as they have no inventory to sell in the retail market.

Stage 1: Labor Market

Let φ denote the share of job seekers with low skills. The value function for a firm who enters the labor market with a vacancy, J_0, is given by

$$J_0 = \xi_f \big\{\varphi K_{L,1} + (1-\varphi) K_{H,1}\big\} + (1-\xi_f) K_0, \tag{17}$$

whereas the value function of an employed firm entering the labor market is given by

$$J_{\varepsilon,1} = \lambda K_0 + (1-\lambda) K_{\varepsilon,1}. \tag{18}$$

The firm's value functions can be simplified by substituting (14) and (18) into (16) to obtain

$$K_{\varepsilon,1} = R_\varepsilon - w_\varepsilon + \bar{\beta}\big[\lambda K_0 + (1-\lambda) K_{\varepsilon,1}\big], \tag{19}$$

where $R_\varepsilon = y_\varepsilon + pq - c(q)$ is the total revenue of a filled job.

There is free entry of firms so that the expected profits of creating a vacancy are equal to zero, i.e. $K_0 = \Pi_0 = 0$. It follows that the firm's entry decision solves

$$\Pi_0 = \max\{0, -k + \bar{\beta}\xi_f\left[\varphi K_{L,1} + (1-\varphi)K_{H,1}\right]\}, \tag{20}$$

where, from equation (19), $K_{\varepsilon,1} = (R_\varepsilon - w_\varepsilon)/(1-\bar{\beta}(1-\lambda))$. It follows that firms post vacancies until the cost of creating a vacancy is equated to the expected profits of filling a vacancy:

$$k = \frac{\bar{\beta}\xi_f}{1-\bar{\beta}(1-\lambda)}\left\{\varphi(R_L - w_L) + (1-\varphi)(R_H - w_H)\right\}. \tag{21}$$

3.3 Retail Market Equilibrium

Recall that all households enter the retail market holding the same amount of real balances. Their problem is given by

$$\max_{q^D} \; \upsilon(q^D) - pq^D \text{ s.t. } pq^D \le z. \tag{22}$$

If cash is costly to hold, households will not accumulate more than they need to consume in the retail market and their budget constraint will bind. It follows that $q^D = z/p$.

The problem of a firm matched with a household of skill level ε is

$$\max_{q_\varepsilon^S} \; pq_\varepsilon^S - c(q_\varepsilon^S) \text{ s.t. } c(q_\varepsilon^S) \le y_\varepsilon. \tag{23}$$

According to (23), the firm maximizes revenue net of costs incurred to sell their inventory in the retail market. Assuming $\lim_{q_\varepsilon \to \delta y} c'(q_\varepsilon)$ is large enough, the inventory constraint will not bind for any firm. It follows that $c'(q_\varepsilon^S) = p$, which equates price with marginal costs and implies that the firm's supply can be expressed as

$$q_\varepsilon^S = c'^{-1}(p). \tag{24}$$

From (24), it is straightforward to see that a firm's quantity supplied in the retail market, q_ε^S, is independent of the worker's skill level, ε, i.e. $q_L^S = q_H^S = q^S$ where $q^S = c'^{-1}(p)$.

Prices are determined through market clearing. There is a measure one of households and a fraction α receive a preference shock, so aggregate demand is given by $Q^D = \alpha q^D$. Denoting u as the unemployment rate, the measure of firms with inventory to sell is $1-u$. Hence, market clearing is given by

$$\alpha q^D = (1-u)c'^{-1}(p). \tag{25}$$

With the outcome of the retail market in hand, we revisit the household's choice of real balances.

From equation (5), we have

$$\max_{z' \geq 0} \left\{ -\frac{z'}{1+r} + \bar{\beta}\big[\alpha \upsilon(q) + (1-\alpha)z'\big] \right\}, \tag{26}$$

where q is a function of z'. By the Fisher equation, $(1+i) = (1+\rho)(1+\pi)$, the household's portfolio choice is equivalent to

$$\max_{z' \geq 0} \big\{ -(1+i)z' + (1-\mu)\big[\alpha \upsilon(q) + (1-\alpha)z'\big] \big\}, \tag{27}$$

and the first-order condition is

$$i = (1-\mu)\left[\alpha \upsilon'(q)\frac{\partial q}{\partial z'} + (1-\alpha)\right] - 1. \tag{28}$$

It is clear from (28) that real balances and retail market consumptions are independent of the worker's labor market status, Ω.[15] As households are price takers in the retail market, we have $\partial q/\partial z' = 1/p$. Combining $c'(q^S) = p$ and market clearing conditions with (28) gives

$$i = (1-\mu)\left[\alpha \frac{\upsilon'(q)}{c'\big(\frac{\alpha q}{1-u}\big)} + (1-\alpha)\right] - 1. \tag{29}$$

We refer to equation (29) as the Retail Market (RM) curve, which, given the unemployment rate u, determines q.[16] As in Berentsen et al. (2011), the RM curve is downward sloping in the (u, q) space, as long as firms make positive profits in the retail market. The intuition is straightforward: as u increases, there are less firms with inventory to sell in the retail market, which increases the price, p. Therefore, households carry less real balances across periods, causing q to decrease. Proposition 1 summarizes.[17]

Proposition 1. *Suppose that $c'' > 0$. For all $i > 0$, the RM curve slopes downward in (u, q) space with $u = 0$ implying $q = \hat{q} \in (0, q^*)$, where $q^* \equiv \arg\max\{\upsilon(q) - c(q)\}$, and $u = 1$ implying $q = 0$.*

Next, we show how parameter changes affect money holding decisions and hence shift RM curve.

Lemma 1. *The RM curve shifts down in μ and i and up in α.*

A higher α leads to greater household consumption and higher real balance holdings, increasing q for each u. When μ rises, households become effectively less patient and hold less real balances,

[15] In Supplemental Appendix A, we extend the model to incorporate both money and credit. By introducing an endogenous credit limit linked to labor income, high-skilled workers receive a higher credit limit than low-skilled workers. As a result, high-skilled workers borrow more and hold lower real balances compared to low-skilled workers. Consequently, they also consume more in the retail market.

[16] Unlike most money search papers, implementing the Friedman rule ($i = 0$) alone does not restore efficiency in money holdings or first-best consumption q. This is because although the Friedman rule eliminates the intertemporal distortion caused by the inflation tax, it does not eliminate the hold-up problem caused by the exogenous exit shock μ. See Supplemental Appendix B for a detailed discussion on efficiency.

[17] All proofs are delegated to Supplemental Appendix C.

reducing q for each u. Finally, an increase in i increases the cost of holding real balances, causing households to hold less real balances, which also decreases q for each u.

3.4 Labor Market Equilibrium

Upon meeting in the labor market, the household's wage is determined through Nash bargaining. A type ε household's surplus of forming a match in the labor market is given by $S^h_\varepsilon = V_{\varepsilon,1}(z) - V_{\varepsilon,0}(z)$ whereas the firm's surplus is given by $S^f_{\varepsilon,1} = K_{\varepsilon,1}$. It follows that wages solve

$$w_\varepsilon \in \arg\max \left[V_{\varepsilon,1}(z) - V_{\varepsilon,0}(z)\right]^\gamma \left[K_{\varepsilon,1}\right]^{1-\gamma}, \tag{30}$$

where $\gamma \in [0,1]$ is the worker's bargaining power. Letting $S_\varepsilon = S^h_\varepsilon + S^f_\varepsilon$ denote the total surplus of a match between a type ε worker and firm, the solution to (30) gives the surplus sharing rules $S^h_\varepsilon = \gamma S_\varepsilon$; $S^f_\varepsilon = (1-\gamma)S_\varepsilon$. Using the Bellman equations, surplus sharing rules, and letting $\Delta_\sigma \equiv V_{H,0}(z) - V_{L,0}(z)$ denote the cost of skill loss, we have:[18]

$$S_L = \frac{R_L - b}{\bar{\beta}(\mu + \rho(1+\mu) + \lambda + \gamma\xi_h)};\ S_H = \frac{R_H - b + \bar{\beta}(1-\xi_h)\sigma\Delta_\sigma}{\bar{\beta}(\mu + \rho(1+\mu) + \lambda + \gamma\xi_h)}. \tag{31}$$

Substituting the surpluses into equation (21) gives the job creation condition:

$$\frac{k}{\xi_f} = (1-\gamma)\left[\frac{y[1-\varphi(1-\delta)] + (1-\varphi)\bar{\beta}(1-\xi_h)\sigma\Delta_\sigma + c'\left(\frac{\alpha q}{1-u}\right)\frac{\alpha q}{1-u} - c\left(\frac{\alpha q}{1-u}\right) - b}{\mu + \rho(1+\mu) + \lambda + \gamma\xi_h}\right]. \tag{32}$$

Equation (32) is the Labor Market (LM) curve, as it determines entry of firms and thus, u, for a given q. It is straightforward to show the LM curve is also downward sloping in the (u,q) space, as a reduction in q decreases the expected revenue from a filled job, leading to less vacancies being created and for the unemployment rate to increase. The novel aspect of the LM curve in our framework is that the entry of firms also depends on the skill composition of unemployed workers, φ, and the cost of skill loss, Δ_σ. As we show below, the skill composition, φ, is itself a function of market tightness. When more firms post vacancies, workers face shorter unemployment durations, and the skill composition of the unemployed improves, increasing the expected benefit of filling a vacancy. This complementarity is present in many search models with endogenous entry and skill loss (Pissarides, 1992) and can lead to multiplicity of equilibria. This happens only under extreme parameter values. Thus, throughout the analysis we assume parameter values are such that, for a given q, there is a unique θ which solves (32). Proposition 2 summarizes.

Proposition 2. *Suppose that parameter values are such that there is a unique θ which solves (32). Further assume $k(\mu + \rho(1+\mu) + \lambda) < (1-\gamma)(\delta y - b)$. The LM curve slopes downward in (u,q) space. It passes through $(\underline{u}, q^*)$ with $\underline{u} \in (0,1)$ and $(\bar{u}, 0)$ where $\bar{u} < 1$.*

In the following lemma, we show how parameter changes affect job creation and thus the LM

[18] See Supplemental Appendix C.3 for a derivation of the cost of skill loss, Δ_σ.

curve.

Lemma 2. *Assume that $\gamma = 0$. The LM curve shifts to the right following an increase in σ, k, b, and λ and shifts to the left as δ, y, and α increase.*

A higher σ increases the likelihood of matching with low-skilled workers, leading to lower output and vacancies, which increases u for each q. A higher vacancy creation cost k discourages job postings, further rising u for each q. Additionally, when the household's outside option b improves, firms must pay higher wages, increasing hiring costs and reducing job creation, leading to higher u for each q. A higher λ lowers the expected return on job creation, also resulting in fewer jobs and higher u for each q. In contrast, a higher δ or y increases expected output, encouraging job creation and reducing u for each q. Likewise, when α increases, stronger retail demand boosts the expected revenues of firms, leading to more vacancies being posted and lower u for each q.

3.5 General Equilibrium

To close the model, all that is left to do is to derive the stationary distribution of workers. Let u_ε and n_ε denote the measure of unemployed and employed households of skill level ε, respectively. It follows that

$$u_{L,t+1} = (1-\mu)\big[(1-\xi_h)[u_{L,t} + \sigma u_{H,t}] + \lambda n_{L,t}\big];\ \ n_{L,t+1} = (1-\mu)\big[(1-\lambda)n_{L,t} + \xi_h u_{L,t}\big], \tag{33}$$

$$u_{H,t+1} = \mu + (1-\mu)\big[(1-\xi_h)(1-\sigma)u_{H,t} + \lambda n_{H,t}\big];\ \ n_{H,t+1} = (1-\mu)\big[(1-\lambda)n_{H,t} + \xi_h u_{H,t}\big]. \tag{34}$$

From equations (33)-(34), it is straightforward to derive the stationary unemployment rate, $u = u_L + u_H$:

$$u = \frac{\mu + (1-\mu)\lambda}{\mu + (1-\mu)(\lambda + \xi_h)}, \tag{35}$$

and the fraction of unemployed workers who are less-skilled, φ:

$$\varphi = \frac{\sigma(1-\mu)(1-\xi_h)[1-(1-\mu)(1-\lambda)]}{\mu(1-\mu)\xi_h + [\mu + (1-\mu)(1-\xi_h)\sigma][1-(1-\mu)(1-\lambda)]}. \tag{36}$$

Definition 1. A stationary equilibrium is a vector $\{q, \theta, u, \varphi\}$ such that: retail market allocations, q, satisfies (29), market tightness, θ, satisfies (32), the unemployment rate, u, is given by equation (35), and the skill composition of unemployed workers, φ, satisfies (36).

Figure 2 maps the RM and LM curves into the $\mathcal{B} = [0,1] \times [0, q^*]$ space where the intersection of the curves determines the equilibrium value of money, q, and unemployment rate, u. As discussed previously, the RM curve enters $\mathcal{B}$ at $(0, \hat{q})$ where $\hat{q} \le q^*$ and exits at $(1, 0)$. From Proposition 2, the LM curve enters at $(q^*, \underline{u})$ and exits at $(0, \bar{u})$ as long as k is small enough. There is always the possibility of a non-monetary equilibria and monetary equilibria. Additionally, the non-monetary and monetary equilibria need not be unique, as discussed previously due to the effect posting a vacancy has on the skill composition of unemployed workers. Proceeding under the assumption

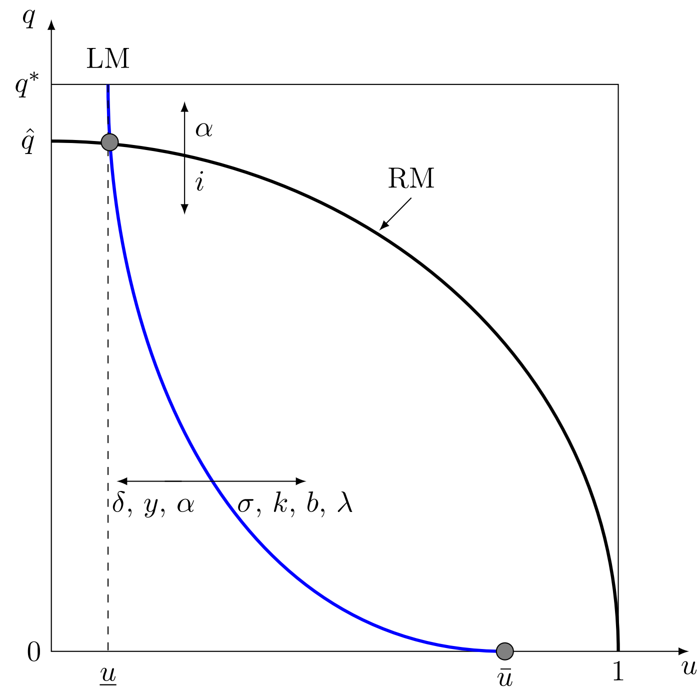


Figure 2: Steady-State Equilibrium. *Notes:* The RM curve represents equation (29). The LM represents equation (32). Comparative statics are signified by the arrows next to the listed parameters.

that parameters values are such that there is a unique θ and hence, u, for each value of q (which we verify in our quantitative analysis), and assuming workers' bargaining power is low, we have the following results regarding the skill loss parameters. An increase in the frequency of skill loss shocks, σ, or a decrease in δ, shifts the LM curve to the right, causing it to intersect the RM curve at a lower value of q and higher value of u. The intuition is straightforward: an increase in σ makes it more likely firms meet low skill workers, which causes less firms to enter and unemployment to increase. As less firms enter, the price of the retail market good increases, causing households to hold less real balances and q to decrease. Similarly, a decrease in δ means that skill loss reduces the productivity of a worker by a larger amount, which decreases the expected value of creating a vacancy.

Now consider a monetary equilibrium. If i increases, the opportunity cost of holding real balances increases, causing households to hold fewer real balances and the RM curve to shift down. As the RM curve shifts downward, demand for the specialized good in the retail market decreases, causing less firms to enter and for unemployment to increase. There is also a multiplier effect due to workers losing skills during unemployment: as less firms enter following an increase in i, the skill composition among the unemployed deteriorates, which further decreases the entry of firms and increases unemployment.[19] Proposition 3 summarizes.

Proposition 3. *Assume that $k(\mu+\rho(1+\mu)+\lambda) < (1-\gamma)(\delta y-b)$ and $i > 0$. At least one monetary equilibrium with $q \in (0, q^*)$ and $u \in (0, 1)$ exists. There also exists at least one non-monetary*

[19] We quantify this multiplier effect in Section 5.6.

equilibrium with $q = 0$ and $u = \bar{u} \in (0, 1)$.

Following Lemma 1 and 2, it is straightforward to analyze how parameter changes affect equilibrium retail consumption q and unemployment rate u. Proposition 4 summarizes these results.

Proposition 4. *If the monetary equilibrium is unique and $\gamma = 0$, an increase in σ, i, k, b, and λ, or decrease in α, δ, and y reduces q and increases u.*

3.6 Inflation and TFP

We now focus on the relationship between monetary policy and TFP, where TFP is simply given by average labor productivity:

$$\text{TFP} = \overbrace{\varphi(\theta)\delta y + (1 - \varphi(\theta))y}^{\text{Average production of numéraire}} + \underbrace{c'\left(\frac{\alpha q}{1 - u(\theta)}\right)\frac{\alpha q}{1 - u(\theta)} - c\left(\frac{\alpha q}{1 - u(\theta)}\right)}_{\text{Net production in retail market}}. \tag{37}$$

The first two terms on the right side of (37) are the average production of the numéraire in the labor market, while the remaining terms are the net production of each firm in the retail market. Thus, there are two channels through which a change in monetary policy impacts TFP. An increase to the nominal interest rate directly affects the entry of firms, the skill distribution of workers, and therefore the average production in the labor market. Secondly, as both q and the unemployment rate, u, react to the change in the nominal interest rate, so does the net output produced by each firm in the retail market. More formally,

$$\frac{d\text{TFP}}{di} = -y(1 - \delta)\varphi'(\theta)\frac{\partial \theta}{\partial i} + g_1(q, \theta)\frac{\partial q}{\partial i} + g_2(q, \theta)\frac{\partial \theta}{\partial i}, \tag{38}$$

where $g(q, \theta) \equiv c'\left(\frac{\alpha q}{1-u(\theta)}\right)\frac{\alpha q}{1-u(\theta)} - c\left(\frac{\alpha q}{1-u(\theta)}\right)$. The effect of the nominal interest rate on TFP is three-fold. First, there is the decline in average production in the labor market following a shift in the skill composition of workers towards less-skilled workers. Second, the net output of each firm in the retail market declines as households hold less real balances. The third effect follows from the fact that as less firms post vacancies, each remaining firm will, conditional on q, supply a larger amount to the retail market. The first two effects decrease TFP, whereas the third effect increases TFP as each active firm supplies a larger amount of output to the retail market.[20]

4 Heterogeneous Firms

Motivated by evidence that jobs which require more skills also experience higher rates of skill loss, we now extend the model to have heterogeneous firms. We do so by allowing firms to create one

[20]In the extension where credit is available (see Supplemental Appendix A), we also analyze TFP as defined in equation (A.6) and examine how monetary policy influences TFP as shown in equation (A.7). Although the first two effects of the baseline model remain in the extension, the third effect is absent due to changes in the structure of the retail market.

of two types of jobs indexed by $\chi \in \{s, c\}$: simple (s) and complex (c), which determines the specialized good the firm produces in the retail market. Employed high skill workers now produce y_χ units of output per period, where $y_s < y_c$. Low skill workers produce $\delta_\chi y_\chi$ with $\delta_\chi \in (0,1)$ and $\delta_c < \delta_s$. Thus, there are two distinguishing characteristics of complex jobs. The first is that high skill workers produce more output in complex jobs. Second, skill loss has a larger impact on a worker's productivity in complex jobs than in simple jobs, which is consistent with evidence that an additional month of unemployment history has a larger impact on wages in highly-skilled occupations (Ortego-Marti, 2017a).

Utility from consumption of specialized goods is given by $v(q_s, q_c)$, where $q_\chi \in \mathbb{R}_+$ is consumption of good χ, $v'(0, q_c) = v'(q_s, 0) = \infty$, $v'(\infty, q_c) = v'(q_s, \infty) = 0$, and $v(0,0) = 0$. The only other differences relative to the baseline model are that we assume the utility while unemployed is bounded by output in the lowest productivity match: $b < \min\{\delta_s y_s, \delta_c y_c\}$. We also allow for different vacancy posting costs across the two jobs, i.e. vacant firms can pay k_χ units of the numéraire to enter the labor market with a type χ vacancy.

A household's type is now given by $\Omega \in \{L, H\} \times \{0, s, c\}$ where $\Omega = (\varepsilon, 0)$ denotes an unemployed household of skill level ε and $\Omega = (\varepsilon, \chi)$ denotes a worker of skill level ε who is matched with a type χ job. Let p_χ denote the price of good χ in terms of the numéraire. The value function of household with real balances z and labor market status Ω in the retail market, $V_\Omega(z)$, is now given by

$$V_\Omega(z) = \alpha \max_{\sum_\chi p_\chi q_\chi \leq z} \big\{ v(q_s, q_c) + W_\Omega(z - p_s q_s - p_c q_c) \big\} + (1-\alpha) W_\Omega(z). \tag{39}$$

From equation (39), households choose their consumption of q_s and q_c to maximize their lifetime discounted utility and enter the centralized market with $z - \sum_\chi p_\chi q_\chi$ units of real balances.

Letting $\zeta = v_s/[v_s + v_c]$ denote the fraction of vacancies that are simple, the value of an highly-skilled and unemployed household is given by

$$U_{H,0}(z) = \xi_h \big\{ \zeta V_{H,s}(z) + (1-\zeta) V_{H,c}(z) \big\} + (1-\xi_h) \big\{ \sigma V_{L,0}(z) + (1-\sigma) V_{H,0}(z) \big\}. \tag{40}$$

Relative to the baseline model and equation (7), a high skill household can now meet a simple or complex job in the labor market. The value of an unemployed household with low skills now satisfies

$$U_{L,0}(z) = \xi_h \big\{ \zeta V_{L,s}(z) + (1-\zeta) V_{L,c}(z) \big\} + (1-\xi_h) V_{L,0}(z). \tag{41}$$

Through the same process as the baseline model, we can use the linearity of the $W_\Omega(z)$ to obtain value functions which summarizes all three markets, which can in turn be substituted into (5) to give

$$W_\Omega(z) = I_\Omega + z + \max_{z' \geq 0} \left\{ -\frac{z'}{1+r} + \bar{\beta} \left[\alpha \big[v(q_s, q_c) + z' - \sum_\chi p_\chi q_\chi \big] + (1-\alpha) z' + \mathbb{E} W_{\Omega'}(0) \right] \right\}. \tag{42}$$

From (42), the choice of real balances is still independent of current type, Ω, and real balances, z.

Firms who enter the centralized market unmatched have a choice of whether to pay k_χ units of the numéraire to enter the next labor market with a type χ vacancy. A vacant firm's problem is now given by

$$\Pi_0(z) = z + \max\left\{0, -k_s + \bar{\beta}J_{0,s}, -k_c + \bar{\beta}J_{0,c}\right\}. \tag{43}$$

With free entry of firms, the expected profits of creating either type of vacancy are equal to zero. It follows that firms post type χ vacancies until

$$k_\chi = \frac{\bar{\beta}\xi_f}{1-\bar{\beta}(1-\lambda)}\left\{\varphi(R_{L,\chi} - w_{L,\chi}) + (1-\varphi)(R_{H,\chi} - w_{H,\chi})\right\}, \tag{44}$$

where $R_{\varepsilon,\chi} = y_{\varepsilon,\chi} + p_\chi q_{\varepsilon,\chi} - c(q_{\varepsilon,\chi})$ for $\chi \in \{s, c\}$. Relative to the baseline model, there are two entry conditions, one for each type of job.

The household's problem in the retail market is now given by

$$\max_{q_s^D, q_c^D} \upsilon(q_s^D, q_c^D) - p_s q_s^D - p_c q_c^D; \text{ s.t. } p_s q_s^D + p_c q_c^D \leq z. \tag{45}$$

If cash is costly to hold, households will not accumulate more than they need to consume in the retail market and their budget constraint will bind. It follows that q_s^D and q_c^D satisfy

$$\frac{\upsilon_2\left(q_s^D, q_c^D\right)}{\upsilon_1\left(q_s^D, q_c^D\right)} = \frac{p_c}{p_s}, \tag{46}$$

$$\frac{z - p_c q_c^D}{p_s} = q_s^D. \tag{47}$$

The problem of type χ firm matched with a household of skill level ε is given by

$$\max_{q_{\varepsilon,\chi}^S} p_\chi q_{\varepsilon,\chi}^S - c(q_{\varepsilon,\chi}^S); \text{ s.t. } c(q_{\varepsilon,\chi}^S) \leq y_{\varepsilon,\chi}. \tag{48}$$

Assuming again that the inventory constraint does not bind for any firm, the solution is given by $c'(q_{\varepsilon,\chi}^S) = p_\chi$ and the firm's supply can be expressed as $q_{\varepsilon,\chi}^S = c'^{-1}(p_\chi)$. It follows that a type χ firm's quantity supplied in the retail market, $q_{\varepsilon,\chi}^S$, is independent of the worker's skill level, i.e. $q_{L,\chi}^S = q_{H,\chi}^S = q_\chi^S$ where $q_\chi^S = c'^{-1}(p_\chi)$.

The total measure of firms with simple goods is given by $(1-u)\zeta$ whereas the measure of firms who supply complex goods is $(1-u)(1-\zeta)$. Market clearing conditions are given by

$$\alpha q_s^D = (1-u)\zeta c'^{-1}(p_s), \tag{49}$$

$$\alpha q_c^D = (1-u)(1-\zeta)c'^{-1}(p_c). \tag{50}$$

Equations (49)-(50) define a system of equations to determine retail market prices. Given the equilibrium prices, the quantities of consumption by each household and production by each firm satisfy the respective firm order conditions above.

The household's first-order condition for real balances is given by

$$i = (1-\mu)\left[\alpha\left(\upsilon_1(q_s, q_c)\frac{\partial q_s}{\partial z'} + \upsilon_2(q_s, q_c)\frac{\partial q_c}{\partial z'}\right) + (1-\alpha)\right] - 1, \tag{51}$$

where $\partial q_s/\partial z' = \omega(z')/p_s$, $\partial q_c/\partial z' = (1-\omega(z'))/p_c$, and $\omega(z')$ is the fraction of an additional unit of real balances spent on simple goods. Combining $c'(q_\chi^S) = p_\chi$ and market clearing conditions with (46) and (51) gives

$$\frac{\upsilon_2(q_s, q_c)}{\upsilon_1(q_s, q_c)} = \frac{c'\left(\frac{\alpha q_c}{(1-u)(1-\zeta)}\right)}{c'\left(\frac{\alpha q_s}{(1-u)\zeta}\right)}, \tag{52}$$

$$i = (1-\mu)\left[\alpha\left(\frac{\upsilon_1(q_s, q_c)\omega(z')}{c'\left(\frac{q_s}{(1-u)\zeta}\right)} + \frac{\upsilon_2(q_s, q_c)(1-\omega(z'))}{c'\left(\frac{q_c}{(1-u)(1-\zeta)}\right)}\right) + (1-\alpha)\right] - 1, \tag{53}$$

which, given the unemployment rate, u, and composition of jobs, ζ, jointly determines q_s and q_c.

In the labor market, wages continued to be determined through Nash bargaining. Letting $S_{\varepsilon,\chi} = S^h_{\varepsilon,\chi} + S^f_{\varepsilon,\chi}$ denote the total surplus of a match between a type ε worker and type χ firm and combining the resulting surplus sharing rules with the Bellmans, we have the total match surpluses with low skill households correspond to

$$S_{L,s} = \frac{R_{L,s} - b}{\bar{\beta}\rho_2} - \frac{\gamma\xi_h(1-\zeta)(R_{L,c} - R_{L,s})}{\bar{\beta}\rho_1\rho_2}, \tag{54}$$

$$S_{L,c} = \frac{R_{L,c} - b}{\bar{\beta}\rho_2} - \frac{\gamma\xi_h\zeta(R_{L,s} - R_{L,c})}{\bar{\beta}\rho_1\rho_2}, \tag{55}$$

where $\rho_1 \equiv \mu + \rho(1+\mu) + \lambda$ and $\rho_2 \equiv \rho_1 + \gamma\xi_h$, and the total match surpluses with high skill households are given by:[21]

$$S_{H,s} = \frac{(R_{H,s} - b + \bar{\beta}(1-\xi_h)\sigma\Delta_\sigma)}{\bar{\beta}\rho_2} - \frac{\gamma\xi_h(1-\zeta)(R_{H,c} - R_{H,s})}{\bar{\beta}\rho_1\rho_2}, \tag{56}$$

$$S_{H,c} = \frac{(R_{H,c} - b + \bar{\beta}(1-\xi_h)\sigma\Delta_\sigma)}{\bar{\beta}\rho_2} - \frac{\gamma\xi_h\zeta(R_{H,s} - R_{H,c})}{\bar{\beta}\rho_1\rho_2}. \tag{57}$$

Note that the surpluses need not be positive. For example, consider $S_{H,s}$ from equation (56). If workers have high bargaining power, there are many complex jobs, and the difference in revenue a high-skilled worker generates in a complex job relative to a simple job is large, then the highly-skilled worker's reservation wage may be high enough to the point where they are better off forgoing a match with a simple job with the chance of matching with a complex job the next period. The fact that the differences in revenue, $R_{H,c} - R_{H,s}$, is an important determinant of the match surplus provides a channel for monetary policy to impact which types of jobs are created, as the relative profits of selling output in the retail market are tied to the value of money, and hence monetary

[21] See Supplemental Appendix C.7 for a derivation of the cost of skill loss, Δ_σ, with heterogeneous firms.

policy. Proposition 5 provides a sufficient condition on fundamentals to ensure all matches generate a positive surplus, which is the class of equilibria we restrict our attention to.

Proposition 5. *Define $\bar{q} \equiv \arg\max\{c'(q)q - c(q)\}$. All matches generate a positive surplus if*

$$\frac{\delta_\chi y_\chi + [c'(\bar{q})\bar{q} - c(\bar{q})] - b}{\delta_{\chi'} y_{\chi'} - b} < \frac{\rho_1 + \gamma}{\gamma} \text{ for } \chi \in \{s, c\}, \tag{58}$$

where $\chi' = c$ if $\chi = s$ and $\chi' = s$ if $\chi = c$.

With the surpluses in hand, we then have the job creation conditions

$$\frac{k_s}{\xi_f} = \bar{\beta}(1-\gamma)\big[\varphi S_{L,s} + (1-\varphi)S_{H,s}\big], \tag{59}$$

$$\frac{k_c}{\xi_f} = \bar{\beta}(1-\gamma)\big[\varphi S_{L,c} + (1-\varphi)S_{H,c}\big]. \tag{60}$$

Equation (59) is the entry condition for simple jobs while equation (60) is the entry condition for complex jobs. Assuming that (58) is satisfied, the flow equations of workers across their states are the same as in the baseline model.

Definition 2. A stationary equilibrium is a vector $\{q_s, q_c, \theta, \zeta, u, \varphi\}$ such that: retail market allocations, q_χ for $\chi \in \{s, c\}$, satisfy (52)-(53), market tightness and composition of vacancies, θ and ζ, satisfy (59)-(60), the unemployment rate, u, is given by equation (35), and the skill composition of unemployed workers, φ, is given by (36).

Proposition 6. *Assume that $i > 0$ and*

$$k_\chi < \frac{(1-\gamma)\sigma(1-\mu)(\delta_\chi y_\chi - b)}{\rho_1(\mu + (1-\mu)\sigma)} \text{ for } \chi \in \{s, c\}. \tag{61}$$

At least one monetary equilibrium with $q_\chi > 0$ for $\chi \in \{s, c\}$, $u \in (0,1)$, and $\zeta \in (0,1)$ exists. There also exists at least one non-monetary equilibrium with $q_\chi = 0$ for $\chi \in \{s, c\}$, $u \in (0,1)$, and $\zeta \in (0,1)$.

As in the case with homogeneous firms, existence of an active equilibrium in which firms create a positive measure of vacancies is not guaranteed. Proposition 6 establishes that if each respective vacancy posting cost is low enough, then a positive measure of both simple and complex vacancies will be created, and hence $\theta > 0$ and $\zeta \in (0,1)$. Also, the complementarity between vacancy posting decisions made by firms and the skill composition of unemployed workers is still present, giving rise to the possibility of multiplicity. The equilibrium can no longer be represented graphically, as the equilibrium is now comprised of four endogenous variables $(q_s, q_c, \theta, \zeta)$, as opposed to two, (q, θ), in the case of homogeneous firms. While the model is somewhat more complicated with heterogeneous firms, there is now an additional channel through which inflation can impact TFP, which we discuss in the next section.

4.1 Inflation and TFP Revisited

With heterogeneous firms, TFP is now given by

$$\text{TFP} = \overbrace{\zeta\left[\varphi\delta_s y_s + (1-\varphi)y_s + c'(\hat{q}_s)\hat{q}_s - c(\hat{q}_s)\right]}^{\text{Average production in simple jobs}} + \underbrace{(1-\zeta)\left[\varphi\delta_c y_c + (1-\varphi)y_c + c'(\hat{q}_c)\hat{q}_c - c(\hat{q}_c)\right]}_{\text{Average production in complex jobs}}, \tag{62}$$

where $\hat{q}_s \equiv \alpha q_s/(1-u)\zeta$ and $\hat{q}_c \equiv \alpha q_c/(1-u)(1-\zeta)$. There are several differences relative to TFP in the case of homogeneous firms (equation (37)). First, and most notably, is TFP now captures production from simple and complex jobs and is a function of the composition of job complexity, as measured by ζ. Second, the average production within each type of job now depends on several job specific characteristics. Production of the numéraire by low skill workers is determined by δ_χ, the job-specific production of less-skilled workers. Also, net production in the retail market varies across the two types of jobs and are determined by demand for the specialized good, q_χ, and the composition of job complexity.

There are now three channels through which inflation impacts TFP. The first two are the same as in the case with homogeneous firms: an increase in the nominal interest rate or money growth rate increases the opportunity cost of real balances and leads to a decline in demand, q_s and q_c. Second, as firms post less vacancies, workers experience prolonged unemployment durations and the skill composition of unemployed workers deteriorates, lowering average production in the labor market. It is at this point where a third channel is active, and that is a change in the composition of job complexity. While the model is now too complicated for analytical results, the idea is the following: a deterioration in the skill composition of the unemployed has an adverse impact on complex vacancies as the impact of skill loss on production in complex jobs is larger than in simple jobs (recall $\delta_c < \delta_s$). Thus, in addition to the skill composition of the unemployed worsening, the composition of vacancies shifts away from complex jobs and towards simple jobs (i.e., ζ increases), which can cause a further decline in TFP as simple jobs are less productive than complex jobs. We revisit this in Section 5 when we quantitatively evaluate the contribution of these three channels to the productivity costs of inflation.

5 Quantitative Analysis

In this Section, we quantitatively evaluate the effect of inflation on TFP. Section 5.1 introduces our measure of job complexity while Section 5.2 details the calibration strategy. Section 5.3 introduces our main findings while Section 5.4 evaluates the contribution of the three channels to the effect of anticipated inflation on TFP. Section 5.5 presents the version of our model and resulting simulations when the nominal interest rate follows a stochastic process. Finally, Section 5.6 studies the quantitative implications of shutting down the skill loss channel. Supplemental Appendix D provides details on data used to calibrate the model.

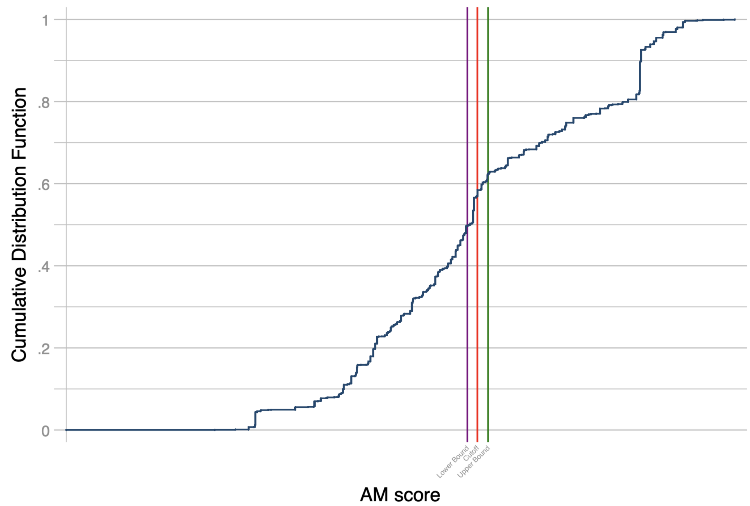


Figure 3: CDF of Job Complexity Among Workers Between 1968-2017. *Notes:* The middle vertical line labelled "Cutoff" represents the AM score used to distinguish between simple and complex jobs in our baseline calibration. The left (right) vertical line labelled "Lower Bound" ("Upper Bound") represents the lowest (highest) threshold AM score used to perform alternative calibrations.

5.1 Measuring Job Complexity

We measure job complexity by using abstract and manual task input measures constructed by Autor and Dorn (2013). The task requirements of each occupation are based on the US department of Labor's Dictionary of Occupational Titles, and hence merged with the census occupation classification. We then construct a normalized measure of job complexity for each occupation k, AM_k:

$$AM_k = \frac{(T^A_{k,1980} - T^M_{k,1980}) - \underline{AM}}{\overline{AM} - \underline{AM}}, \tag{63}$$

where $T^A_{k,1980}$ and $T^M_{k,1980}$ are the abstract and manual task input in each occupation k in 1980, and $\overline{AM}$ ($\underline{AM}$) is the maximum (minimum) AM score across all occupations.[22]

To map our measure of job complexity to the theory, we choose a cutoff value of AM where occupations below the cutoff are labelled as "simple" and occupations above the cutoff are "complex". To identify the cutoff, Figure 3 plots the CDF of the average AM score between 1968-2017. Our baseline cutoff value of AM is shown by the red line in Figure 3, where there appears to be a discrete jump in the CDF. This corresponds to an AM score of 0.615. Therefore, as a baseline, we label complex occupations as those with an AM score above or equal to 0.615, and simple occupations as those with an AM score below 0.615. Under this cutoff, we find that an average of 52.5% of employed workers are in simple jobs, which establishes our target for the composition of job complexity. Clearly, the choice of the cutoff is arbitrary. We think it does well in separating what we think of as "simple" and "complex" occupations. For example, occupations such as salespersons and secretaries are just below the baseline cutoff, whereas bookkeepers and accounting clerks are

[22]See Supplemental Appendix D.2 for a list of occupations with the highest and lowest complexity measures and more details on abstract and manual task measurements.

Table 1: Parameter Values

Parameter	Definition	Value
	Panel A: Assigned parameters	
ρ	Discount rate	1.68×10^{-3}
μ	Probability of exiting the labor force	2×10^{-3}
λ	Separation probability	0.035
η	Elasticity of matching function	0.50
γ	Worker's bargaining power	0.50
y_c	Productivity of high skill workers in complex jobs	1.00
σ	Probability of skill loss	1/3
i	Annual nominal interest rate	6.89×10^{-2}
a	Elasticity of cost function	1.30
	Panel B: Calibrated parameters	
A	Matching efficiency	0.590
b	Value of unemployment	0.554
k_s	Vacancy posting cost: simple jobs	0.245
k_c	Vacancy posting cost: complex jobs	0.589
δ_s	Human capital decay in simple jobs	0.825
δ_c	Human capital decay in complex jobs	0.650
y_s	Productivity of high skill workers in simple jobs	0.787
α	Pr. of consuming in RM	0.050
ϱ	RM utility weight	1.693

Notes: Parameters in Panel A are pre-assigned and not calibrated. Parameters listed in Panel B are chosen to minimize the distance between the model and empirical targets described in the main text.

right above it.[23] For robustness, we perform our quantitative exercises under alternative definitions of a simple and complex job where we instead use a lower cutoff (labelled "lower bound" in Figure 3) and a higher cutoff (labelled "upper bound" in Figure 3). Our quantitative findings are similar across all three cutoff values. See Supplemental Appendix E for additional details and results.

5.2 Calibration Strategy

A unit of time is one month and we calibrate to US data covering 1955-2017. We set the rate of time preference to 1.68×10^{-3} to target a discount factor of $\beta = 0.98^{1/12}$. The probability of leaving the labor force is set to $\mu = 1/480$, which corresponds to being in the labor force on average for 40 years. The separation probability is set to $\lambda = 0.035$ following Shimer (2005). We assume a Cobb-Douglas matching technology: $\mathcal{N}(u, v) = Au^{\eta}v^{1-\eta}$. The elasticity of the matching function

[23] Figure D1 in Supplemental Appendix D.2 shows the occupations around the cutoff which distinguishes simple and complex jobs.

is set to $\eta = 0.5$, which is in line with the empirical evidence (Petrongolo and Pissarides, 2001). The worker's bargaining power is then set to $\gamma = 0.5$ to implement the Hosios (1990) condition. The matching efficiency, A, is set to target a steady-state unemployment rate of 5.9%. Combined with normalizing steady-state market tightness to one, we have $A = 0.5902$. We then normalize the output produced in matches between high-skill workers and complex jobs to $y_c = 1$ and choose the value of unemployment, b, so that the ratio of b to average labor productivity is equal to 0.79, which is in-between the common targets proposed by Hall and Milgrom (2008) and Hagedorn and Manovskii (2008). We find $b = 0.5542$. We then choose the entry costs, k_s and k_c, to target a market tightness of $\theta = 1$ as in Shimer (2005) and a composition of vacancies where $\zeta = 0.525$, which follows from our definition of simple and complex occupations. We find $k_s = 0.2454$ and $k_c = 0.5896$, indicating that complex jobs have significantly larger entry costs.

The remaining labor market parameters are the skill loss parameters $\{\delta_s, \delta_c, \sigma\}$. We calibrate these parameters to match the empirical evidence on the effect of unemployment duration on wages. As discussed by Laureys (2020), the empirical evidence on the effect of unemployment duration on wages can not be used to choose a unique value of σ, δ_s, and δ_c. Thus, we set $\sigma = 1/3$, which corresponds to skill loss taking 3 months on average and is well supported by the empirical evidence on how quickly skill loss occurs (Ortego-Marti, 2016). We then choose δ_s and δ_c to match the estimated effects of unemployment duration on wages. That is, we choose a value of δ_s and δ_c, and given the wages and transition probabilities between employment and unemployment, we simulate 10,000 employment histories and estimate the following regression:

$$\log(wage_\chi) = \beta_0 + \beta_1 \times Unhis + \epsilon,$$

where $Unhis$ is the length of the unemployment spell in months and $\ln(wage_\chi)$ are log wages in type χ jobs. For each simulation of employment histories, we compute β_1 for simple and complex jobs and repeat this process 100 times where we then have an average estimate of β_1 for each type of job. We vary δ_s and δ_c and repeat this exercise until our average estimate of β_1 is -0.0093 for simple jobs and -0.0193 for complex jobs.[24] Through this procedure, we find $\delta_s = 0.825$ and $\delta_c = 0.65$.

Proceeding to the monetary side of the model, we target an average annual nominal interest rate of $i = 0.0689$, which corresponds to the average Aaa nominal corporate bond yield between 1955-2017. The cost to sell inventory in the retail market is $c(q) = q^{1.3}$, which generates mark-ups over average total costs of 30% (Faig and Jerez, 2005), and the retail market utility function is given by $\upsilon(q_s, q_c) = \varrho\sqrt{q_s q_c}$. The remaining parameters are y_s, the production of high-skilled workers in simple jobs, α, the frequency of preference shocks, and ϱ, the RM utility weight. We choose these parameters to minimize the distance between the empirical and model generated money demand curves. Through this process, we find $y_s = 0.7875$, $\alpha = 0.0501$, and $\varrho = 1.6932$.[25]

[24] Our target of -0.0093 for simple occupations follows Ortego-Marti (2017a) and is the average effect of unemployment duration on log wages in clerical, sales, craftsmen, foreman, and operator occupations. The target for complex jobs of -0.0193 also follows from Ortego-Marti (2017a) and is the average effect of unemployment duration on log wages in professional, technical, managers, and officials occupations.

[25] Our empirical definition of money demand is M/pY where M is the sum of $M1$ and money market deposit

Table 2: Targeted Moments

Moment	Data	Model
Unemployment rate	0.0590	0.0590
$b/\mathbb{E}$[labor productivity]	0.7900	0.7900
Fraction of jobs that are simple	0.5250	0.5250
Unemployment duration on wages in simple jobs (negative)	0.0093	0.0094
Unemployment duration on wages in complex jobs (negative)	0.0193	0.0191

Notes: Model moments listed in the first three rows are steady-state outcomes, where $\mathbb{E}$[labor productivity] is the average labor productivity among employed workers. Moments in the last two rows are computed by simulating the model and regressing an individual's wage on their unemployment history.

Table 1 summarizes the parameter values while Table 2 and Figure 4(a) show that the model closely matches the targeted moments. For robustness, we perform four alternative calibrations. One is where markups are 20%, rather than 30%. The second is where it takes on average six months of unemployment to become low-skilled. The third and fourth target alternative values for the composition of job complexity. Parameter values and quantitative results under these alternative calibrations are very similar to our baseline results and are presented in Supplemental Appendix E.

5.3 Results

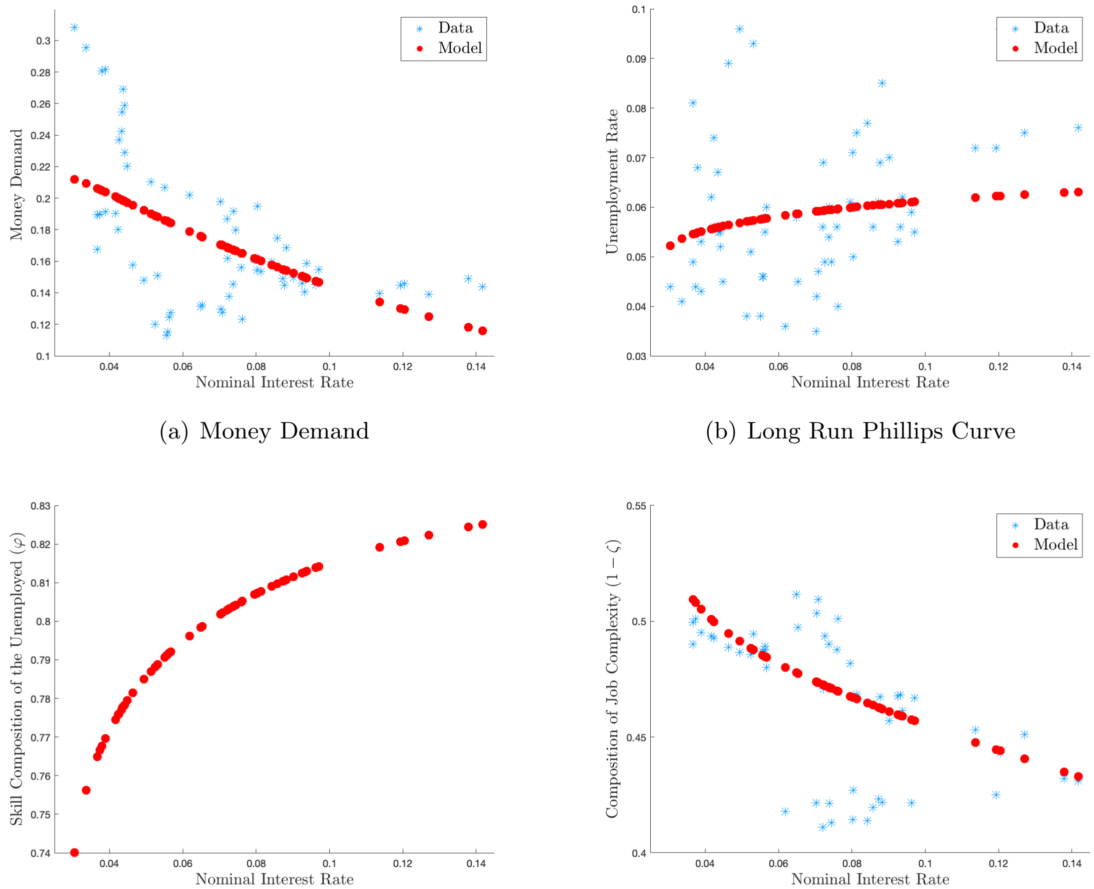
To begin, we present the effect of changes to anticipated inflation on equilibrium outcomes, where anticipated inflation is measured by the nominal interest rate. Figure 4 contains the results, where we compute the equilibrium outcomes at each value of the nominal interest rate observed in the data between 1955-2017.

Starting with Figure 4(a), we see the standard result that money demand is decreasing in the nominal interest rate, as an increase in the nominal interest rate increases the opportunity cost of holding real balances. Figure 4(b) shows the model generates the positive correlation between anticipated inflation and unemployment that is observed in US data over this period.[26]. The intuition follows from Berentsen et al. (2011): as money demand decreases (as shown in Figure 4(a)), so do the profits a firm can make by selling their inventory in the retail market, which thereby reduces the expected revenue of a filled job. This leads to less firms posting vacancies and for the unemployment rate to increase.

Figure 4(c) demonstrates the effect of a change in anticipated inflation on the skill composition of the unemployed. As the nominal rate increases, money demand decreases, firms post less vacancies, and unemployed workers face longer unemployment durations. As a result, unemployed workers are more exposed to skill loss shocks and the skill distribution among the unemployed deteriorates.

accounts and pY is nominal GDP. The model produces an average money demand of 0.174, which exactly matches the average ratio of M/pY between 1955-2017. Moreover, the model produces an elasticity of money demand with respect to the Aaa nominal corporate bond yield of -0.383, which matches the empirical elasticity over 1955-2017.

[26]See Supplemental Appendix Figures D2-D3 for a closer examination of the correlation between anticipated inflation and unemployment in the US between 1955-2017.



(a) Money Demand

(b) Long Run Phillips Curve

(c) Skill Composition of the Unemployed

(d) Composition of Job Complexity

Figure 4: Effects of a Change in Anticipated Inflation. *Notes:* The red circles represent the model generated outcome at each respective value of the nominal interest rate, i. The blue stars represent the scatter plot between each outcome and the Aaa nominal corporate bond yield from the US data between 1955-2017.

Lastly, Figure 4(d) shows the effect of anticipated inflation on the composition of job complexity, as measured by the fraction of vacancies which are complex, $(1-\zeta)$. As discussed in Section 4.1, a deterioration of the skill distribution has an adverse impact on complex jobs, as skill loss causes a larger decline in productivity in complex than simple jobs. Figure 4(d) shows that as the nominal interest increases and more workers become less-skilled, the composition of vacancies shifts from complex jobs towards simple jobs (i.e., ζ increases).

5.4 Productivity Costs of Inflation

Turning to the the productivity costs of inflation, Figure 5 shows the TFP generated by the model at values of the nominal interest rate observed in the data.[27] The first set of results we wish to discuss are the red circles, which represents TFP when all three channels described in Section 4.1

[27] We normalize the highest level of TFP to 1 so that any respective differences are interpreted as percentage deviations.

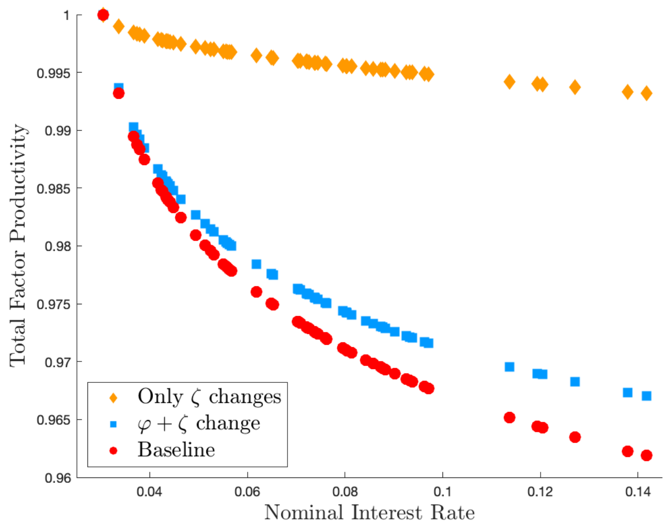


Figure 5: The Productivity Costs of Inflation. *Notes:* Baseline is the TFP produced by the baseline model with all channels active. The "$\varphi + \zeta$" change series holds the net retail market output fixed at its value when TFP is equal to one. The "Only ζ changes" series holds the net retail market output and skill composition of unemployed workers fixed at their respective values when TFP is equal to one. Each series is normalized so that TFP is equal to 1 at the lowest value of the nominal interest rate.

are active. The model shows a negative relationship between TFP and the nominal interest rate. Moreover, the effects are significant: an economy with a nominal interest rate of 14%, or an annual inflation rate of nearly 11%, is 3.9% less productive than an economy where the nominal interest rate is 3% and the annual inflation rate is 1%.

Our next goal is to quantify the contributions of the three respective channels through which a change to anticipated inflation affects TFP. As a first step, we compute the implied TFP while holding the net output produced by firms constant at the value corresponding to TFP $= 1$ while the composition of unemployed workers and job complexity change. This is represented by the blue squares in Figure 5, so that any differences between the blue squares and red circles are due to the change in retail market net production. As seen by comparing the two series, there is little difference between them, suggesting that the change in net production induced by a change in monetary policy does not contribute much to the aggregate effects on TFP. In a second step, we hold the skill composition of the unemployed fixed at its value where TFP $= 1$ (in addition to holding net production in the retail market fixed). This is represented by the orange diamonds in Figure 5, where differences between the orange diamonds and blue squares are due to changes in the skill composition of unemployed workers. The large gap between the two series indicates that the shift in the composition of the unemployed makes a large contribution to the aggregate impact of a change in anticipated inflation on TFP. We will revisit this shortly. Finally, any difference between a TFP level of 1 and the orange diamonds is due to a change in the composition of job complexity.

Next, we compute the fraction of the total decline in TFP that is accounted for by a change in the net production in the retail market, skill composition of the unemployed, and composition

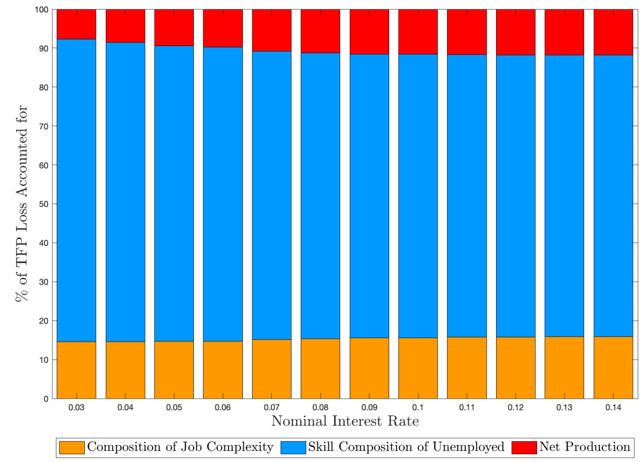


Figure 6: Decomposition of the Productivity Costs of Inflation. *Notes:* Each bar represents the decomposition of the decrease in TFP relative to the TFP at the lowest nominal interest rate. The orange region represents the fraction of the TFP loss that is attributed to a change in the fraction of jobs that are simple, ζ. The blue region captures the portion of the TFP loss driven by changes in the fraction of unemployed workers who are less-skilled, φ. The red region is the portion of TFP losses which are due to a reduction in net retail market production.

of job complexity. First, we compute the total decline in TFP at a given nominal interest rate. We then compute the fraction of the total decline that is due to the change in net production by computing the ratio of the difference between the "Baseline" and "$\varphi + \zeta$" change series in Figure 5 and the total decline in TFP. The fraction of the total decline which is due to the change in the skill composition of the unemployed is simply the ratio of the difference between the "Only ζ changes" and "$\varphi + \zeta$ change" series in Figure 5 to the total decline in TFP. Finally, the fraction of the decline due to the composition of job complexity changing is the ratio of the total decline in TFP which is not yet accounted for and the total decline in TFP.

As seen in Figure 6, the fraction of the total decline in TFP due to a change in anticipated inflation which is due to a change in net output is around 10%, whereas the contribution of a change in the composition of job complexity is approximately 15%. This means that nearly 75% of the decline in TFP following an increase in anticipated inflation is due to the change in the skill composition of the unemployed. Moreover, these shares are relatively constant across different levels of the nominal interest rate.

Next, we quantitatively evaluate the effect of increasing the nominal interest rate from $i = 0$ (the Friedman rule) to a level which is consistent with an annual inflation rate of 10%. From Table 3, increasing the nominal rate from 0 to a level which is consistent with an annual inflation rate of 10% increases the unemployment rate from 5.1% to 6.2%. As the unemployment rate increases and skills among the unemployed deteriorates, the percentage of jobs which are complex decreases from 53.6% to 44.0%. Finally, the last row shows that TFP decreases by 4.3%.

To provide some context for the productivity costs of inflation through the skill loss channel, we compare productivity costs generated by the model to TFP differences across low- and high-

Table 3: From the Friedman Rule to 10% Annual Inflation

	Friedman rule	10% annual inflation
Unemployment rate	0.051	0.062
Fraction of jobs that are complex	0.536	0.440
TFP	1.000	0.957

Notes: Friedman rule corresponds to the model outcomes when $i = 0$. The right column represents the model's outcome when the nominal interest rate is set so that the implied annual inflation rate is 10%.

Table 4: Comparison with TFP Differences Across OECD Countries

Country type	Inflation	TFP (data, normalized)	TFP (model, normalized)	% Explained
Low Inflation	5.45	1.000	1.000	–
High Inflation	8.20	0.938	0.994	9.67

Notes: The second and third columns represent the average inflation and (normalized) TFP over low- and high-inflation countries, respectively. The fourth column is the normalized TFP generated by the model at a nominal interest rate commensurate with the annual inflation rates listed in the second column. The final column represents the fraction of the difference in TFPs listed in the third column which are accounted for by the model.

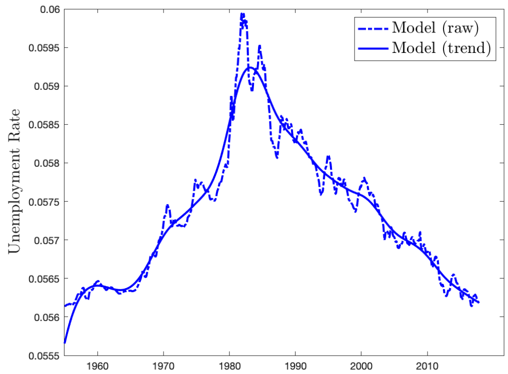
inflation OECD countries.[28] From Table 4, low inflation OECD countries have an average annual inflation rate of 5.45%, whereas high inflation countries have an average annual inflation rate of 8.20% and 6.2% lower TFP. When we compute our model at two values of the nominal interest rate which correspond to 5.45% and 8.20% annual inflation rates, we find that TFP is 0.6% lower in the high inflation economies than the low inflation economies. Therefore, the model accounts for approximately 9.67% of the productivity differences observed in the data between low- and high-inflation OECD economies.

5.5 Stochastic Nominal Interest Rate

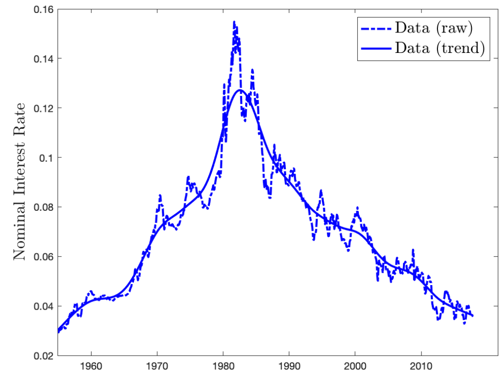
We now proceed to study the stochastic version of our model. In particular, we assume the nominal interest rate follows an AR(1) process, solve the stochastic version of the model, and feed the nominal Aaa corporate bond yield series between 1955-2017 through the model.[29] Our motivation for doing so is twofold. First, Section 5.4 illustrated that the productivity costs of inflation can be sizeable and that the skill composition of unemployed workers accounts for nearly 75% of the productivity costs. However, these exercises compared different steady-states. If it takes time for the skill composition of unemployed workers to reach the new steady-state value following a change in the nominal interest rate, the steady-state experiments may overstate the contribution of the skill composition of unemployed workers changing to the productivity costs of inflation in an economy where the nominal interest rate evolves and is subject to shocks. Second, we can use the stochastic

[28]We follow Berentsen et al. (2012)'s categorization of OECD countries as low- or high-inflation economies. Further, TFP data across OECD countries is from the Penn World Table 10.0 (Feenstra et al., 2015), downloadable at `https://www.rug.nl/ggdc/productivity/pwt/?lang=en`.

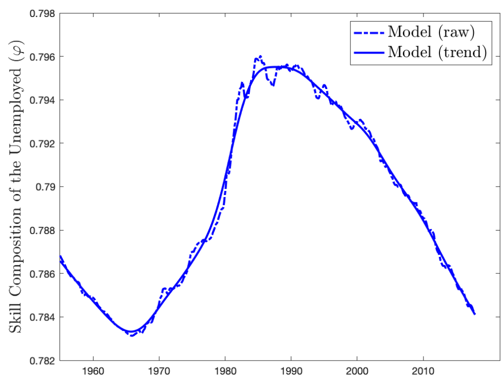
[29]See Supplemental Appendix F for a definition of the stochastic recursive equilibrium.



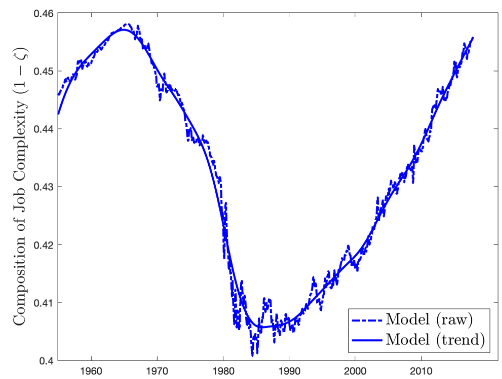
(a) Counterfactual Unemployment



(b) Nominal Interest Rate



(c) Skill Composition of the Unemployed



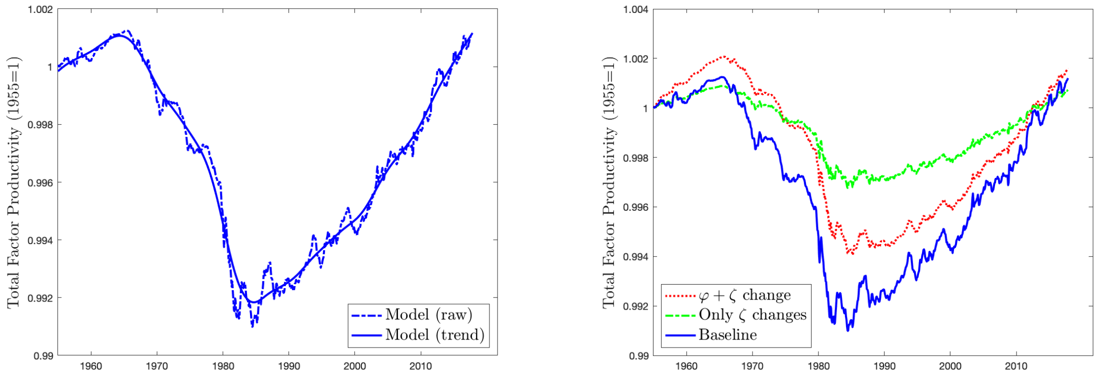
(d) Composition of Job Complexity

Figure 7: Counterfactual Series from the Stochastic Model. *Notes:* Each series is first generated by solving the stochastic version of the model. The "Model (raw)" series is obtained by feeding the Aaa nominal corporate bond yield series through the policy functions obtained through solving the stochastic recursive equilibrium. The "Model (trend)" series are obtained by applying the Hodrick-Prescott filter with smoothing parameter 129600 to the raw monthly series.

version of the model to evaluate the effect of low-frequency changes to anticipated inflation in the US on the unemployment rate, skill composition of the unemployed, composition of job complexity, and TFP. Figures 7-8 show the results.

Figure 7(a) demonstrates the counterfactual unemployment rate, which is generated only by changes in the nominal interest rate. The model generates an increase in trend unemployment through the 1970s and 1980s following the increase in the nominal interest rate during this period. As inflation decreased through the 1980s, the model generates a decrease in trend unemployment.

Turning to the compositions, Figures 7(c) shows the evolution of the skill composition of the unemployed. As the unemployment rate increases through the 1970s and 1980s, the skill composition of the unemployed degrades, as φ increased from 0.783 to 0.796 between 1966-1988. There are two additional features of φ to emphasize. First, the deterioration of the skill composition lags behind the unemployment rate. From Figure 7(a), the unemployment rate began to decline in 1983, whereas



(a) TFP

(b) Decomposition

Figure 8: Simulated TFP. *Notes:* The raw and trend series of TFP are obtained following the same method used to generate Figure 7. The baseline series represents TFP when all channels in the model are active. The "$\varphi + \zeta$" series holds the net retail market output fixed at its level in 1955. The "Only ζ changes" series additionally holds the skill composition of unemployed workers, φ, fixed at its 1955 value. Each TFP series is normalized to 1 in 1955.

the skill composition only begins to recover after 1988. Second, the skill composition recovers at a slower rate than it takes to decay. It took 22 years for the skill composition to deteriorate between 1966-1988, whereas it required 29 years to recover and reach the levels observed in the late 1960s. Figure 7(d) presents the composition of the job complexity. As in the steady-state experiments, the fraction of jobs which are complex, $1 - \zeta$, and fraction of unemployed workers who are less-skilled, φ, are inversely related. As the skill composition of unemployed workers deteriorates through the 1970s and 1980s, so does the composition of job complexity. Following the decline in unemployment and improvement of the skill composition of unemployed, the fraction of jobs which are complex increases through the 1990s and 2000s.

Next, Figure 8(a) displays the TFP series, which is normalized to 1 in 1955. As expected, TFP declines throughout the 1970s and 1980s as higher trend inflation causes the unemployment rate to increase, the skill composition of unemployed to worsen, and composition of job complexity to deteriorate. Our results show that trend TFP reaches its lowest value of 0.9918 in 1985, indicating a 0.82% decline in TFP relative to the 1955 level. Clearly this decline in TFP is smaller than the results in Section 5.4 which compared different steady-states and follows from the fact that it takes time for the skill composition of the unemployed to deteriorate following an increase in anticipated inflation. However, as discussed above, it also takes time for the skill composition of the unemployed to recover. For this reason, the recovery in TFP lags behind the improvement in unemployment: the unemployment rate begins to recover in 1983, whereas TFP continues to decline until 1985.

Finally, Figure 8(b) carries out the same decomposition exercise as in Figure 5. We find that when TFP reaches its lowest value in the beginning of 1985, 35.7% of the decline in TFP relative to its 1955 level is due to the shift in the composition of job complexity, 33.24% is attributable to the change in the skill composition of unemployed workers, and the remaining 31.06% is a result of the decline in net production. Therefore, the contributions of the three channels are quantitatively

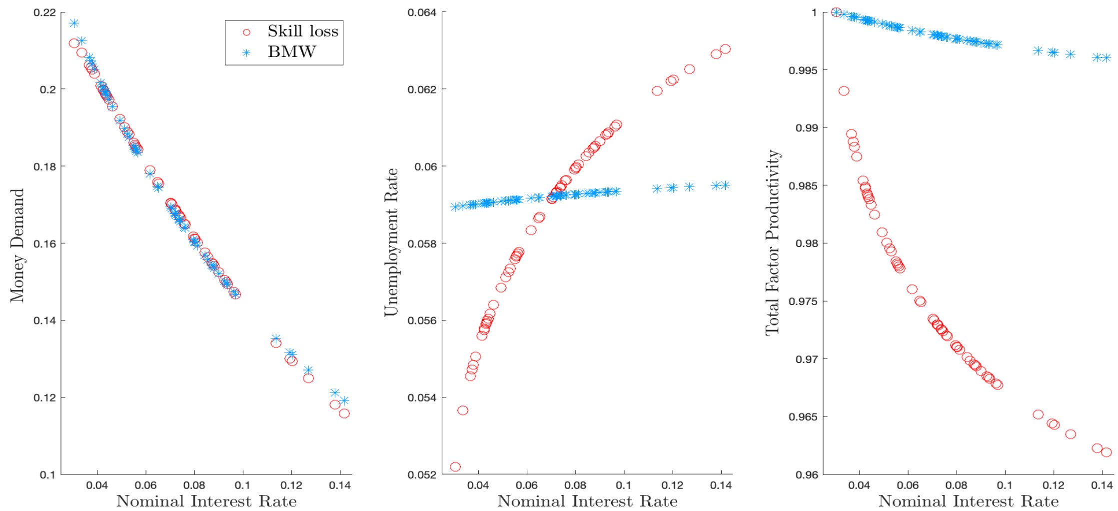


Figure 9: Steady-state Comparison. *Notes:* Series labelled "Skill loss" correspond to the baseline model with $\sigma > 0$ and heterogeneous firms. The "BMW" series correspond to the calibrated version of the model with $\sigma = 0$ and homogenous firms.

similar in the stochastic version of the model. We attribute this result to the fact that it takes time for the skill composition of the unemployed to reach its new steady-state value following a change to anticipated inflation.

5.6 Shutting Down the Skill Loss Channel

As a final quantitative exercise, we shut down the skill loss channel in order to further quantify its contribution to the productivity costs of inflation. This is done by setting the arrival probability of a skill loss shock, σ, to zero. In doing so, the model essentially collapses to that of Berentsen et al. (2011) with homogeneous workers and firms and a competitive retail market.[30] To that end, we re-calibrate our model without the skill loss channel and perform the same quantitative experiments found above. Supplemental Appendix G contains a comparison of the calibrated parameters and targeted moments in the model with and without skill loss. Both match the targeted moments well.

To begin, Figure 9 compares steady-state outcomes with and without skill loss. The series labelled "Skill loss" are the same as in Sections 5.3-5.4 whereas the series labelled BMW are the outcomes generated by the model without skill loss. Proceeding from left to right, we first see that both models produce essentially the same money demand curve. Thus, any differences in labor market outcomes in response to a change in anticipated inflation is not driven by differences in money demand. The middle panel presents the long run Phillips curve (LRPC). The LRPC is essentially flat in the version of the model without skill loss, whereas unemployment is much

[30]Heterogeneity among firms may still emerge from the household's preferences over the goods produced in simple and complex jobs and differences in vacancy posting costs. However, to facilitate a comparison between our quantitative version of the model and Berentsen et al. (2011), we abstract from these differences and compare our quantitative findings to a version of our model with homogeneous firms, as initially presented in Section 2, and with $\sigma = 0$.

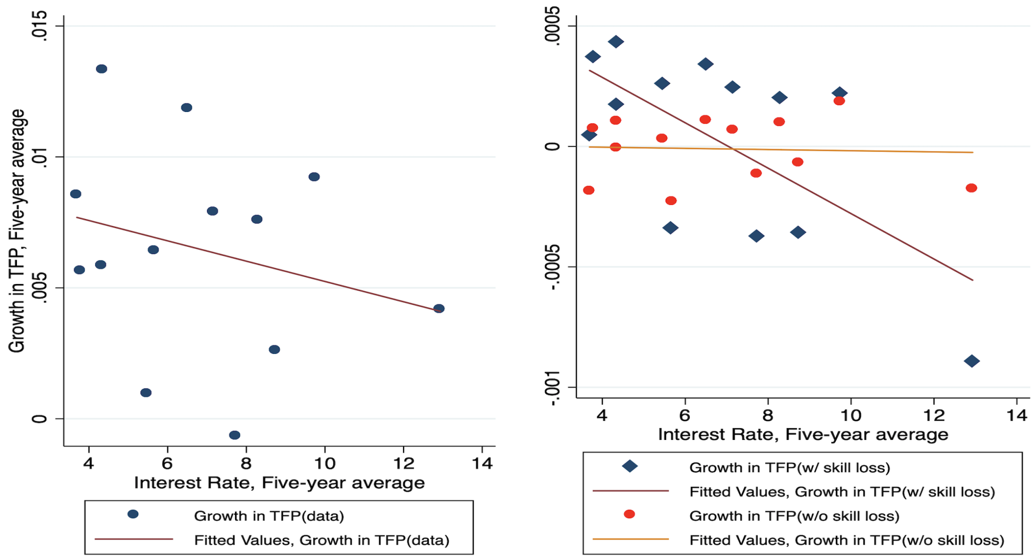


Figure 10: TFP Growth and the Interest Rate. *Notes:* The left panel represents five year averages of TFP growth and the Aaa nominal corporate bond yield between 1955-2017. In the right panel, model generated data is obtained by solving the stochastic version of the model and feeding through the Aaa nominal corporate bond yield between 1955-2017. Blue diamonds (red circles) in the right panel represent data generated by the model with (without) an active skill loss channel.

more responsive to changes in the nominal interest rate with an active skill loss channel. This is because as firms post less vacancies, the skill distribution of workers deteriorates, which further reduces the expected benefits of posting a vacancy, hence a larger response in vacancy posting and unemployment following an increase in the nominal rate that is generated by the skill loss channel.

The right panel of Figure 9 shows that the productivity costs of inflation are much larger with an active skill loss channel. There are several reasons for this. First, the model with skill loss has two additional channels through which inflation can impact TFP: the skill distribution of workers and the composition of jobs created. The only active channel in the BMW version of the model is a change in net retail market output, which as we showed in Figure 6, did not contribute much to the aggregate productivity costs of inflation. Further, as shown in the middle panel, unemployment is much more responsive to a change in the nominal interest rate with an active skill loss channel, causing a shift in the skill distribution of workers, thereby lowering productivity by even more.

Finally, we compare the results from simulating the stochastic version of the models with the data. To do so, we first simulate each model (with and without skill loss), and compute five-year averages of annual TFP growth using the counterfactual TFP series generated by the simulation. The results are shown in the right panel of Figure 10, whereas the left panel is the five-year averages from the data. First, comparing the two sets of results in the right panel, we see that the version of the model with skill loss generates a much stronger negative correlation between the interest rate and TFP growth; as the relationship between the two is essentially flat when the skill loss channel is shut down. This results from the fact that the unemployment rate is much more responsive to a change in the nominal interest rate with an active skill loss channel, as demonstrated in Figure 9 and holds true in the stochastic version of the model. Second, we see that the negative correlation generated by the model is much more in line with the data. While the magnitudes of TFP growth generated by the model do not perfectly match the data, our model suggests that the skill loss channel is an important component of understanding the negative correlation observed in the data.[31]

6 Conclusion

Economists have long been interested in the relationship between inflation and unemployment. Recently, models with both payment and labor frictions have been developed to analyze their relationship. Alongside this literature, a large body of evidence has shown workers' human capital decays when they are unemployed, generating a linkage between labor market flows and aggregate productivity. This paper has developed a framework to link these two large literatures and quantified the relationship between inflation, unemployment, and TFP when workers lose skills during unemployment. The model shows that the economy's labor market flows, skill composition of unemployed workers, and TFP are linked to the value of money and anticipated inflation. The quantitative exercises demonstrate that the productivity costs of inflation can be sizeable and that shifts in the skill composition of unemployed workers are a primary driver of these productivity costs. The stochastic version of the model illustrates that extended periods of high inflation, such as the 1970s in the US, can scar the economy as productivity lags behind the recovery in unemployment. We argue that this result is relevant for policy discussions about the potential trade-offs of allowing the economy to run above the inflation target for an extended period and for many economies who are still experiencing elevated inflation following the COVID-19 pandemic.

[31] It should not be surprising that the magnitudes of TFP growth generated by the model are not in line with the data, as our model is not one of endogenous growth.

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Supplemental Appendix

A Extension with Money and Credit

In this extension, we introduce credit by incorporating a competitive credit market with the following features: there is a large number of lenders with free entry and no cost of issuing debt, ensuring that lenders make zero profit. Borrowers can take out a loan ℓ with an endogenous credit limit and limited commitment in the retail market to finance their consumption by pledging their future labor income w, which is earned in the centralized market (CM). Post consumption, borrowers can renege on their debt, but lenders could garnish their labor income w in the CM and recover $\chi \in [0,1]$ of w, meaning that lenders extend credit up to χw. We refer to χ the pledgeability, which is an exogenous technology that depends on, for example, the accounting system and reputable auditors and accountants. In addition to credit, we impose the following modifications to the baseline model to ease the analysis: (1) random search in the labor market is replaced with directed search, (2) matching and separation in the labor market occur at the end of the labor market as in Gu et al. (2023), and (3) it is assumed that the competitive retail market is segmented. We discuss how we approach these changes and the reasoning later.

A.1 Households

Following the baseline model, we solve the model through backward induction, beginning with stage 3.

A.1.1 Stage 3: Centralized Market

The value of a household with real balances z, labor market income receivable w and debt ℓ in the CM is given by

$$W_\Omega(z,w,\ell) = \max_{x,z'} \ \{x + \bar{\beta}U_\Omega(z')\}$$
$$\text{s.t. } x + z'/(1+r) + (1+i_\ell)\ell = \Delta + z + w + T \ \text{ and } x \geq 0,$$

where x is consumption, i_ℓ is the interest rate of debt, z is current real balances, z' is the real balances for next period, w is the labor market income of type Ω in the last labor market, Δ is dividends, T is lump-sum transfers, $U_\Omega(z')$ is the continuation value of entering stage 1 with labor market status Ω and holding real balances z', and $\bar{\beta} \equiv \beta(1-\mu)$ is the effective discount factor. Also note that unemployed workers receive unemployment benefits b, that is, $w_{\epsilon,0} = b$. We assume that b is large enough so that the non-negativity constraint does not bind. Using the budget constraint and substituting x, the problem is simplified to

$$W_\Omega(z,w,\ell) = z + w - (1+i_\ell)\ell + \Delta + T + \max_{z'}\{-z'/(1+r) + \bar{\beta}U_\Omega(z')\}.$$

The first-order condition corresponds to

$$-1/(1+r) + \bar{\beta}U'_{\Omega}(z') \leq 0$$

with equality if and only if $z' > 0$. Later we show that, unlike the baseline, the household's choice of z' depends on their labor market status Ω as credit limit varies with Ω. Hence, there is a non-degenerate distribution of real balances.

A.1.2 Stage 2: Retail Market

Suppose the retail market is competitive and segmented. Households with different labor status or liquidity may choose to shop in different submarkets that are characterized by different prices. In equilibrium, high-skilled employed households with high credit limits purchase goods from sellers with large amounts of inventory and higher prices, and low-skilled employed households with low credit limits purchase goods from sellers with little inventory and lower prices. Equivalently, we could assume that employed households buy goods from their employer. Another way to interpret this is that workers get paid partially in the retail market with the firm's output, and the rest of the wage is paid in the next CM. For unemployed households, we assume they do not participate in the retail market. Additionally, suppose there is a competitive credit market where buyers pledge their wages w and receive a line of credit with a limit of χw, where $\chi > 0$, to finance their consumption in the retail market. The value of a household with status Ω, real balances z, and receivable wage w entering a segmented market with price p_ϵ is given by

$$\begin{aligned} V_{\Omega}(z,w) &= \alpha \max_{q,\hat{z},\ell}[v(q) + W_{\Omega}(z-\hat{z},w,\ell)] + (1-\alpha)W_{\Omega}(z,w,0) \\ &\text{s.t. } p_\epsilon q \leq \hat{z} + \ell,\ \hat{z} \leq z,\ \ell \leq \chi w. \end{aligned}$$

The households are hit with a preference shock and consume with probability α, subsequently choose a quantity of trade q, cash transfer $\hat{z}$, and debt ℓ to optimize their lifetime utility and enter the next CM with real balances $z - \hat{z}$, receivable wage w, and debt ℓ. The households face three constraints: 1) the total cost of obtaining the consumption pq cannot be larger than the cash transfer plus the debt, 2) the cash transfer cannot be greater than the real balances holdings, 3) debt taken out cannot be higher than the credit limit. Let q^* denote the first-best retail consumption that solves $v'(q^*) = p$. The first-order conditions result in $\ell = \chi w$, $\hat{z} = z$ and $q_\epsilon = (z+\ell)/p_\epsilon < q^*$ if $z + \chi w < p_\epsilon q^*$; and $\ell = p_\epsilon q^* - \hat{z} < \chi w$, $\hat{z} < z$ and $q_\epsilon = q^*$ if $z + \chi w \geq p_\epsilon q^*$.

A.1.3 Stage 1: Labor Market

To ease the analytical difficulty, we assume a labor market with directed search, where workers observe wages and then choose which jobs to apply for. As a result, high-skilled workers search for a high-skilled submarket with high wages, and low-skilled workers search for a low-skilled submarket with low wages. Without loss of generality, we also modify the timing of the model. In this extension, matching and job separation occur at the end of the labor market. Households enter the labor market with labor market status Ω and real balances z. The value of an unemployed household with high skills satisfies:

$$U_{H,0}(z) = \xi_H^h V_{H,1}(z,b) + (1-\xi_H^h)[\sigma V_{L,0}(z,b) + (1-\sigma)V_{H,0}(z,b)],$$

where ξ_H^h is the matching probability in the high-skilled workers' submarket with endogenous tightness θ_H. With probability ξ_H^h, the unemployed household with high skills becomes matched. With probability $1-\xi_H^h$, the unemployed household with high skills remains unmatched, but with probability σ the high-skilled household becomes low-skilled. Because the matching occurs at the end of the labor market, the household would collect b in the next CM and start working in the next labor market. Similarly, the value of an unemployed household with low skills satisfies:

$$U_{L,0}(z) = \xi_L^h V_{L,1}(z,b) + (1-\xi_L^h)V_{L,0}(z,b),$$

where ξ_L^h is the matching probability in the low-skilled workers' submarket with endogenous tightness θ_L. The value of employed households with ϵ-skills satisfies:

$$U_{\epsilon,1} = \lambda V_{\epsilon,0}(z,w_\epsilon) + (1-\lambda)V_{\epsilon,1}(z,w_\epsilon).$$

After working, the separation shock occurs. With probability λ, the employed household becomes unemployed in the next period; and with probability $1-\lambda$, the unemployed household becomes employed in the next period. However, in any case, the worker with the initial status $(\epsilon, 1)$ will collect w_ϵ in the next CM.

A.2 Firms

A.2.1 Stage 3: Centralized Market

A firm matched with an ϵ-skilled worker enters the CM with y units of unsold inventory, w payable to the worker, and z amount of real balances has value as follows:

$$\Pi_{\epsilon,1}(y,w,z) = \max_{x \text{ s.t. } x\le y+z-w} \{x + \bar{\beta} J_{\epsilon,1}\},$$

where the firm chooses consumption x that cannot be greater than the net wealth, $y+x-w$ to maximize the firm's continuation value. Furthermore, an unmatched firm enters the CM with y units of unsold inventory, w payable to the worker, and z amount of real balances has value as follows:

$$\Pi_0(y,w,z) = \max\left\{ \max_{x \text{ s.t. } x\le y+z-w} \{x+\bar{\beta}\Pi_0\}, \quad -k + \max_{x \text{ s.t. } x\le y+z-w} \{x+\bar{\beta}J_0\} \right\},$$

where the firm chooses whether to post a new vacancy. If the firm does not post a new vacancy, the firm chooses consumption x and skips the labor and retail market. If the firm chooses to post a vacancy, the firm pays k in numeraire, chooses consumption x in the CM, and engages in the directed search process in the labor market.

A.2.2 Stage 2: Retail market

The value of a firm with a next period worker labor status Ω, y units of output produced in the last labor market, and payable wage w in the retail market, $K_\Omega(y, w)$, solves

$$K_\Omega(y, w) = \max\Big\{ \max_{q_L \leq y} \Pi_\Omega(y - c(q_L), w, p_L q_L), \quad \max_{q_H \leq y} \Pi_\Omega(y - c(q_H), w, p_H q_H) \Big\},$$

where the firm chooses one segmented market to sell the inventory. In the ϵ-type submarket, the firm takes price p_ϵ as given, sells q_ϵ units of specialized goods, and enters the next CM with $y - c(q_\epsilon)$, payable wage w, and $p_\epsilon q_\epsilon$ real balances. The problem of a firm with y units inventory is:

$$\max_{q_\epsilon} \quad p_\epsilon q_\epsilon - c(q_\epsilon) \quad \text{s.t.} \quad c(q_\epsilon) \leq y.$$

Assuming $\lim_{q_\epsilon \to \delta y} c'(q_\epsilon)$ is large enough, the constraint doesn't bind, and hence the price solves

$$p_\epsilon = c'(q_\epsilon). \tag{A.1}$$

Together with the first-order conditions of the households' problem, we show equilibrium allocations in the retail market:

$$\begin{cases} \ell = \chi w, \hat{z} = z, q_\epsilon = (\hat{z} + \ell)/c'^{-1}(q_\epsilon) < q^* & \text{if } z + \chi w < c'^{-1}(q^*)q^*; \\ \ell + \hat{z} = c'^{-1}(q^*)q^*, q_\epsilon = q^* & \text{if otherwise.} \end{cases} \tag{A.2}$$

If $z + \chi w < c'^{-1}(q^*)q^*$, liquidity is scarce, so households exhaust their real balances and borrow up to the credit limit but still consume a suboptimal amount; otherwise, households consume the first-best by using any combination of ℓ and $\hat{z}$, which means that households may leave the retail market with idle real balances and/or a slack borrowing constraint.

A.2.3 Stage 1: Labor market

Firms with a newly created vacancy in the labor market have value:

$$J_0 = \max\Big\{ \xi_L^f K_{L,1}(0,0) + (1 - \xi_L^f) K_0(0,0), \quad \xi_H^f K_{H,1}(0,0) + (1 - \xi_H^f) K_0(0,0) \Big\},$$

where the firm chooses to enter a submarket that maximizes their continuation value. If the firm enters the low-skilled (high-skilled) submarket, the firm matches with probability ξ_L^f (ξ_H^f) and continues as a matched firm with a low-skilled (high-skilled) worker; however, with probability

$1-\xi^f_L\ (1-\xi^f_H)$ the firm fails to match and continues as an unmatched firm. Furthermore, matched firms with an ϵ-skilled worker in the labor market has value:

$$J_{\epsilon,1} = \lambda K_0(y_\epsilon, w_\epsilon) + (1-\lambda)K_{\epsilon,1}(y_\epsilon, w_\epsilon),$$

where the match produces y_ϵ during the labor market and the firm is liable to pay the ϵ-skilled worker w_ϵ, and then the match is destroyed exogenously at the end of the labor market with probability λ.

A.3 Equilibrium

Using the outcome of the retail market, we first revisit the household's choice of real balances in the CM. Also using the linearity of W, an ϵ-skilled employed worker solves

$$\max_{z'\geq 0} -\frac{z'}{1+r} + \bar{\beta}\{z' + \alpha[v(q(z'+\ell'_{\epsilon,1})) - p_\epsilon q(z'+\ell'_{\epsilon,1}) - i_\ell \ell'_{\epsilon,1}]\},$$

where the interest rate on debt $i_\ell = 0$ as the credit market is assumed to be competitive. The first-order condition solves

$$-\frac{1}{1+r} + \bar{\beta}\{\alpha[v'(q(z'_{\epsilon,1}+\ell'_{\epsilon,1}) - p_\epsilon)q'(z'_{\epsilon,1}+\ell'_{\epsilon,1})] + 1\} \leq 0,$$

with strict equality if and only if $z'_{\epsilon,1} > 0$. Using $\bar{\beta} = \beta(1-\mu)$, $1+r = 1/(1+\pi)$, the first order condition can be simplified into

$$-(1+i) + (1-\mu)\{\alpha[v'(q(z'_{\epsilon,1}+\ell'_{\epsilon,1}) - p_\epsilon)q'(z'_{\epsilon,1}+\ell'_{\epsilon,1})] + 1\} \leq 0,$$

where $1+i = (1+\pi)(1+\rho)$. Let $\tilde{i}_\epsilon$ denote the cutoff policy rate that solves $1+\tilde{i}_\epsilon = (1-\mu)\Big\{\alpha\big[\frac{v'(\tilde{q}_{\epsilon,1})}{c'(\tilde{q}_{\epsilon,1})} - 1\big] + 1\Big\}$, where $\tilde{q}_{\epsilon,1}$ is characterized by $c'(\tilde{q}_{\epsilon,1})\tilde{q}_{\epsilon,1} = \chi w_\epsilon$. Households' portfolio choices are shown in the following lemma.

Lemma 3. *There exists a cutoff $\tilde{i}_\epsilon$ such that workers with labor skill ϵ choose*

$$z_{\epsilon,1}\begin{cases} > 0 \text{ and solves } 1+i = (1-\mu)\Big\{\alpha\big[\frac{v'(q_{\epsilon,1})}{c'(q_{\epsilon,1})} - 1\big] + 1\Big\} & \text{if} \quad i < \tilde{i}_\epsilon; \\ = 0 & \text{otherwise.} \end{cases} \tag{A.3}$$

When the cost of holding real balances over a period is low such that $i < \tilde{i}_\epsilon$, a ϵ-skilled worker chooses $z_{\epsilon,1} > 0$ such that the marginal cost of holding real balances equals the marginal benefit given the credit limit that depends on labor skill ϵ. If matching and separation take place at the start of the labor market, as assumed in the baseline model, the marginal benefit of an ϵ-skilled worker carrying a unit of real balance into the retail market is a weighted average of the marginal benefits in two scenarios: when the worker continues as employed and when the worker becomes separated. So, making the change in timing helps us reduce the complexity of the analysis. On the other hand, when

$i \geq \tilde{i}$, an ϵ-skilled worker does not hold any real balances, $z_{\epsilon,1} = 0$, as the marginal cost of holding real balances is greater than the marginal benefit of doing so, $1 + i \geq (1-\mu)\Big\{\alpha\big[\frac{v'(q_{\epsilon,1})}{c'(q_{\epsilon,1})} - 1\big] + 1\Big\}$, where $q_{\epsilon,1} = \chi w_\epsilon / p_\epsilon$. Next, we summarize the real balance holdings and loans of different types of households as follows:

Lemma 4. *If $w_H > w_L > b$, then the loans and real balance holdings are ranked: $\ell_{H,1} \geq \ell_{L,1} \geq \ell_{\epsilon,0} = 0$ and $z_{L,1} \geq z_{H,1} \geq z_{\epsilon,0} = 0$. As a result, the retail market consumptions are ranked: $q_{H,1} \geq q_{L,1} \geq q_{\epsilon,0} = 0$.*

If high-skilled workers earn higher wages than low-skilled workers $w_H > w_L > b$, which we show later, high-skilled workers have higher credit limits than low-skilled workers, and hence high-skilled workers borrow weakly more than low-skilled workers do, $\ell_{H,1} \geq \ell_{L,1}$. Also note that the policy rate cutoffs are ranked, $\tilde{i}_H < \tilde{i}_L$, and thus high-skilled workers hold fewer real balances than low-skilled workers do, $z_{L,1} \geq z_{H,1}$ as money and credit are substitutes. Because unemployed workers do not participate in the retail market by assumption, they do not borrow loans or hold real balances, $\ell_{\epsilon,0} = z_{\epsilon,0} = 0$. Also note that when holding real balances is costly, it is optimal for firms not to hold any.

Second, we characterize the equilibrium wages. Let $\bar{W}_\Omega \equiv W_\Omega(0,0,0)$. To find the equilibrium wages, we first show the matching surplus of the low- and high-skilled households as follows:

$$
\begin{aligned}
\bar{W}_{L,1} - \bar{W}_{L,0} &= \frac{\bar{\beta}\nu_{L,1}}{1 - \bar{\beta}(1 - \lambda - \xi_L^h)} \\
\bar{W}_{H,1} - \bar{W}_{H,0} &= \frac{\bar{\beta}\nu_{H,1}}{1 - \bar{\beta}(1 - \lambda - \xi_H^h)} + \sigma \frac{\bar{\beta}(1 - \xi_H^h)[\bar{W}_{H,0} - \bar{W}_{L,0}]}{1 - \bar{\beta}(1 - \lambda - \xi_H^h)},
\end{aligned}
$$

where $\nu_{\epsilon,1}$ is used to denote the per-period surplus of a ϵ-skilled employed worker, $\nu_{\epsilon,1} = -(\frac{1}{\beta(1+r)} - \bar{\beta})z'_{\epsilon,1} + \alpha[v(q_{\epsilon,1}) - p_\epsilon q_{\epsilon,1}] + w_\epsilon$, and $\bar{W}_{H,0}$ and $\bar{W}_{L,0}$ are

$$
\begin{aligned}
\bar{W}_{L,0} &= \frac{\Delta + T}{1 - \bar{\beta}} + \frac{\bar{\beta}\xi_L^h \nu_{L,1}}{(1 - \bar{\beta})[1 - \bar{\beta}(1 - \lambda - \xi_L^h)]}, \\
\bar{W}_{H,0} &= \frac{\Delta + T}{1 - \bar{\beta}} + \frac{\bar{\beta}^2 \xi_H^h \nu_{H,1}}{[1 - \bar{\beta}(1 - \xi_H^h)(1 - \sigma)][1 - \bar{\beta}(1 - \lambda)] - \bar{\beta}^2 \xi_H^h \lambda} \\
&\quad + \frac{\bar{\beta}^2(1 - \xi_H^h)\sigma[1 - \bar{\beta}(1 - \lambda)]\xi_L^h \nu_{L,1}}{(1 - \bar{\beta})[1 - \bar{\beta}(1 - \lambda - \xi_L^h)]\{[1 - \bar{\beta}(1 - \xi_H^h)(1 - \sigma)][1 - \bar{\beta}(1 - \lambda)] - \bar{\beta}^2 \xi_H^h \lambda\}},
\end{aligned}
$$

respectively. Additionally, the matching surplus of the firm with an ϵ-skilled worker is:

$$
\bar{\Pi}_\epsilon = \frac{\bar{\beta}[R_\epsilon + k - w_\epsilon]}{1 - \bar{\beta}(1 - \xi_\epsilon^f - \lambda)},
$$

where the firm revenue $R_\epsilon \equiv y_\epsilon + p_\epsilon q_\epsilon - c(q_\epsilon)$ and the firm's matching probability for a type-ϵ worker, ξ_ϵ^f, depends on the tightness of the ϵ-type submarket. According to the equilibrium condition of the retail market, $p_\epsilon = c'(q_\epsilon)$.

Firms' free entry condition to an ϵ-skilled labor submarket and ϵ-typed retail submarket becomes

$$k = \frac{\bar{\beta}^2 \xi_\epsilon^f}{1 - \bar{\beta}(1-\lambda)} (R_\epsilon - w_\epsilon). \tag{A.4}$$

It is easy to see that $\partial \xi_\epsilon^f / \partial w_\epsilon > 0$. A higher wage decreases firms' per-period net profit as $0 \leq \partial R_\epsilon / \partial w_\epsilon < 1$. As a result, it discourages firms' entry, tightness decreases, and the matching probability for the firm increases (the matching probability for the worker decreases). Upon meeting in the labor market, the households' wages are determined through Nash bargaining, as shown in the baseline model, wages of ϵ-skilled households, w_ϵ, are determined by

$$\gamma \bar{\Pi}_\epsilon = (1-\gamma)(\bar{W}_{\epsilon,1} - \bar{W}_{\epsilon,0}), \tag{A.5}$$

where γ is the households' bargaining power.

Lastly, we close the model by deriving the stationary distribution of workers. Let u_ϵ and n_ϵ denote the measure of unemployed and employed households of skill level ϵ, respectively. It follows that

$$\begin{aligned}
u_L' &= (1-\mu)[(1-\xi_L^h)(u_L + \sigma u_H) + \lambda n_L], \\
u_H' &= (1-\mu)[(1-\xi_H^h)(1-\sigma) u_H + \lambda n_H] + \mu, \\
n_L' &= (1-\mu)[(1-\lambda) n_L + \xi_L^h u_L], \\
n_H' &= (1-\mu)[(1-\lambda) n_H + \xi_H^h u_H].
\end{aligned}$$

In a steady state, the stationary n_H solves

$$n_H = (1-\mu)\mu \xi_H^h / \bar{\xi}_H^h$$

where $\bar{\xi}_H^h = [1-(1-\mu)(1-\lambda)][1-(1-\mu)(1-\xi_H^h)(1-\sigma)] - (1-\mu)^2 \xi_H^h \lambda$; the stationary u_H solves

$$u_H = \frac{(1-\mu)\lambda n_H + \mu}{1-(1-\mu)(1-\xi_H^h)(1-\sigma)};$$

the stationary n_L solves

$$n_L = (1-\mu)^2 \xi_L^h (1-\xi_L^h) \sigma u_H / \bar{\xi}_L^h,$$

where $\bar{\xi}_L^h = [1-(1-\mu)(1-\lambda)][1-(1-\mu)(1-\xi_L^h)] - (1-\mu)^2 \xi_L^h \lambda$; and the stationary u_L solves

$$u_L = \frac{(1-\mu)(1-\xi_L^h)\sigma u_H + (1-\mu)\lambda n_L}{1-(1-\mu)(1-\xi_L^h)}.$$

Note that n_H and u_H depend on ξ_H^h, but n_L and u_L depend on both ξ_L^h and ξ_H^h.

We finally define the monetary equilibrium with credit as follows:

Definition 3. When loan repayment can be enforced, a monetary equilibrium with a competitive credit market is $\{z_{\epsilon,1}, \ell_{\epsilon,1}, q_{\epsilon,1}, p_\epsilon, \theta_\epsilon, w_\epsilon\}_{\epsilon=H,L}$ that solves (A.1), (A.2), (A.3), (A.4), and (A.5) for $\epsilon = H, L$.

The wage of low-skilled workers is determined within the low-skilled labor and retail submarket due to the assumption of directed search in the labor market and segmented retail markets. Without this assumption, the wage of low-skilled workers would also depend on the wage of high-skilled workers, significantly complicating the analytical solutions. This consideration motivates the use of directed search and segmented retail markets in place of random search and a unified retail market in the baseline model. However, regardless of these assumptions, the wage of high-skilled workers always depends on the wage of low-skilled workers due to the possibility of skill loss.

Next, we consider how the monetary policy (i) affects total factor productivity (TFP). TFP, defined as average labor productivity, is given by:

$$\text{TFP} = \sum_{\epsilon=H,L} n_\epsilon \left[\overbrace{y_\epsilon}^{\text{Numéraire}} + \underbrace{c'(q_{\epsilon,1})q_{\epsilon,1} - c(q_{\epsilon,1})}_{\text{Specialized good}} \right]. \tag{A.6}$$

In the submarket of ϵ-skilled workers, there are n_ϵ matches, and each of them produces y_ϵ units of numéraire in the labor market and $c'(q_{\epsilon,1})q_{\epsilon,1} - c(q_{\epsilon,1})$ net units of the specialized good in the retail market. In this extension, monetary policy affects TFP through the extensive and intensive when money is valued ($i < \tilde{i}_L$). An increase in i reduces the consumption and production in the retail market for each match, and therefore affects entry and the distribution of workers across skill levels. Formally,

$$\frac{d\text{TFP}}{di} = \sum_{\epsilon=H,L} \frac{\partial n_\epsilon}{\partial i} \left[y_\epsilon + c'(q_{\epsilon,1})q_{\epsilon,1} - c(q_{\epsilon,1}) \right] + \frac{\partial c'(q_{\epsilon,1})q_{\epsilon,1} - c(q_{\epsilon,1})}{\partial q_{\epsilon,1}} \frac{\partial q_{\epsilon,1}}{\partial i}. \tag{A.7}$$

The effect of monetary policy on TFP has two components. First, the extensive margin is that the monetary policy changes the number of matches in each labor submarket (n_ϵ), though the direction of this effect ($\partial n_\epsilon / \partial i$) is unclear. Second, the intensive margin is that the net output of each match in the retail market declines as workers hold less real balance given their debt limit and therefore consume less amount, as shown in (A.3), resulting in a negative effect on TFP. When credit is available, consumption in the retail market is higher than the baseline given the prices. Consequently, a higher i has a smaller impact on $q_{\epsilon,1}$ in the extension compared to the baseline, i.e., $\partial q_{\epsilon,1}/\partial i < \partial q^{BL}/\partial i$, where q^{BL} is the baseline retail market consumption solving (29) at given retail market prices. In the baseline model, as shown in (38), there is an additional effect: as i increases, fewer firms produce in the retail market, and those producing firms increase their production. However, in this extension, the retail market is segmented, so this effect does not occur. On the other hand, if money is not valued ($i \geq \tilde{i}_L$), a change in i does not influence retail market consumption $q_{\epsilon,1}$ as $q_{\epsilon,1}$ is purely financed by credit that depends on wages, and thus it does not affect firms' entry decision or the distribution of workers across skill levels. As a result, monetary policy has no impact on TFP when i is too large.

Furthermore, we analyze how pledgeability χ as an exogenous technology affects wages.

Lemma 5. *The effect of pledgeability, χ, is summarized by the following cases.*

1. *When $i < \tilde{i}_L$, the wage of low-skilled workers decreases in pledgeability, $\partial w_L/\partial\chi < 0$;*

2. *When $i \geq \tilde{i}_L$, the wage of low-skilled workers could increase in pledgeability; in particular, if $q_{L,1} \to q^*$ and $\gamma \to 1$, $\partial w_L/\partial\chi > 0$;*

3. *However, the wage of high-skilled workers could increase or decrease in pledgeability,i.e., the sign of $\partial w_H/\partial\chi$ is ambiguous.*

Proof. See Section A.4. ■

Surprisingly, low-skilled workers' wages always decrease in χ when the policy rate is low, $i < \tilde{i}_L$. This is because as χ increases, for a high enough γ, credit limit χw_L increases in χ, and workers are willing to give up a little bit wage income to have a higher retail market consumption that is supported by higher credit limit. However, when $i \geq \tilde{i}_L$, there is a greater retail market demand and thus greater matching surplus, causing w_L rises in χ under some conditions.

A.4 Proof of Lemma 5

Proof. We analyze $\partial w_L/\partial\chi$ in two cases, $z_{L,1} > 0$ and $= 0$. Rewrite (A.5) for $\epsilon = L$ and define

$$G_L \equiv -\gamma[y_L + p_L q_{L,1} - c(q_{L,1}) + k - w_L] + (1-\gamma)\frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}\nu_{L,1}.$$

- First, when $z_{L,1} > 0$, we know $\partial q_{L,1}/\partial w_L = 0$, $\partial z_{L,1}/\partial w_L = -\chi$, $\partial\xi_L^f/\partial w_L > 0$, and $\partial\xi_L^h/\partial w_L < 0$. So

$$\frac{\partial G_L}{\partial w_L} = \gamma + (1-\gamma)\frac{\partial\frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}}{\partial w_L}\nu_{L,1} + (1-\gamma)\frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}[(\frac{1}{\bar{\beta}(1+r)}-1)\chi+1] > 0,$$

$$\frac{\partial G_L}{\partial\chi} = (1-\gamma)\frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}(\frac{1}{\bar{\beta}(1+r)}-1)w_L > 0.$$

  Then, using the fundamental theorem, we find $\partial w_L/\partial\chi < 0$ when $z_L > 0$.

- Second, when $z_{L,1} = 0$ and $q_{L,1} < q^*$, where q^* solves $u'(q^*) = c'(q^*)$, $p_L q_{L,1} = \chi w_L$, and $p_L = c'(q_{L,1})$.

$$\frac{\partial G_L}{\partial w_L} = (1-\gamma)\frac{\partial\frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}}{\partial w_L}\nu_{L,1} + (1-\gamma)\frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}\{\alpha\chi[\frac{v'(q_{L,1})}{c''(q_{L,1})q_{L,1}+c'(q_{L,1})}-1]+1\} + \gamma.$$

If $q_{L,1}$ is large, the elasticity of the cost function is large, then $\frac{v'(q_{L,1})}{c''(q_{L,1})q_{L,1}+c'(q_{L,1})} - 1 < 0$. However, for $\gamma \to 1$, $\partial G_L/\partial w_L > 0$, and

$$\begin{aligned}\frac{\partial G_L}{\partial \chi} =&(1-\gamma)\Bigg\{\frac{\partial \frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}}{\partial \chi}[\alpha(v(q_{L,1})-p_L q_{L,1})+w_L] \\ &+\frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}\alpha[v'(q_{L,1})-c''(q_{L,1})q_{L,1}-c'(q_{L,1})]\frac{\partial q_{L,1}}{\partial \chi}\Bigg\} - \gamma\frac{\partial R_L}{\partial \chi}.\end{aligned}$$

When χ increases, $q_{L,1}$ increases as credit increases given w_L ($\partial q_{L,1}/\partial w_L > 0$), and thus R_L goes higher. Holding w_L constant, $\partial \xi_L^f/\partial \chi < 0$ since higher profits encourage firm entry and thus lower the matching probability for firms. Then, $\partial\left[\frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}\right]/\partial \chi < 0$, holding w_L constant. When $q_{L,1} \to q^*$, $v'(q_{L,1}) - c''(q_{L,1})q_{L,1} - c'(q_{L,1}) < 0$. When χ increases, firms earn higher revenues due to higher sales, so $\partial R_L/\partial \chi = c''(q_{L,1})q_{L,1} > 0$. Hence, for large $q_{L,1}$, the elasticity of the cost function is large, $\partial G_L/\partial \chi < 0$. Using the fundamental theorem, we find $\partial w_L/\partial \chi > 0$ when $z_L = 0$, $q_{L,1} \to q^*$, and $\gamma \to 1$. For $q_{L,1} = q^*$, $\partial w_L/\partial \chi = 0$.

Wages w_H are determined by

$$\gamma\bar{\Pi}_H = (1-\gamma)(\bar{W}_{H,1} - \bar{W}_{H,0}).$$

We study $\partial w_H/\partial \chi$ in two cases again, $z_{H,1} > 0$ and $z_{H,1} = 0$. Rewrite the above equation and define $G_H \equiv -\gamma[y_H + p_H q_{H,1} - c(q_{H,1}) + k - w_H] + (1-\gamma)\frac{1-\bar{\beta}(1-\lambda-\xi_H^f)}{1-\bar{\beta}(1-\lambda-\xi_H^h)}[\nu_{H,1} + \sigma(1-\xi_H^h)(\bar{W}_{H,1} - \bar{W}_{L,1})]$. When there is no skills loss $\sigma = 0$, $\partial w_H/\partial \chi$ would have the same sign as $\partial w_L/\partial \chi$. When $\sigma > 0$, we study $\partial w_H/\partial \chi$ under different parameter spaces.

- First, when $z_{H,1} > 0$ and $z_{L,1} > 0$, we know $\partial \xi_H^h/\partial w_H < 0$, $\nu_{H,1}/\partial w_H > 0$, then

$$\begin{aligned}\frac{\partial G_H}{\partial w_H} = (1-\gamma)&\Bigg\{\frac{\partial \frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}}{\partial w_H}[\nu_{H,1} + \sigma(1-\xi_H^h)(\bar{W}_{H,1} - \bar{W}_{L,1})] \\ &+\frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}\Big[\frac{\partial \nu_{H,1}}{\partial w_H} - \sigma\frac{\partial \xi_H^h}{\partial w_H}(\bar{W}_{H,0} - \bar{W}_{L,0}) + \sigma(1-\xi_H^h)\frac{\partial \bar{W}_{H,0}}{\partial w_H}\Big]\Bigg\} + \gamma.\end{aligned}$$

We know $\partial\left[\frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}\right]/\partial w_H > 0$, but the sign of $\partial \bar{W}_{H,0}/\partial w_H$ is ambiguous. However, if

$\gamma \to 1$, then $\partial G_H/\partial w_H > 0$, and

$$\begin{aligned}
\frac{\partial G_H}{\partial \chi} &= (1-\gamma)\frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}\Big\{\frac{\partial \nu_{H,1}}{\partial \chi} + \sigma(1-\xi_H^h)[\frac{\partial \bar{W}_{H,0}}{\partial \chi} - \frac{\partial \bar{W}_{L,0}}{\partial \chi}]\Big\} \\
&= (1-\gamma)\frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}\Bigg\{\Big[1+\frac{\bar{\beta}^2\xi_H^h(1-\xi_H^h)\sigma}{[1-\bar{\beta}(1-\xi_H^h)(1-\sigma)][1-\bar{\beta}(1-\lambda)]-\bar{\beta}^2\xi_H^h\lambda}\Big]\frac{\partial \nu_{H,1}}{\partial \chi} \\
&\quad - \frac{\sigma(1-\xi_H^h)(1-\bar{\beta}(1-\lambda-\xi_H^h))}{[1-\bar{\beta}(1-\xi_H^h)(1-\sigma)][1-\bar{\beta}(1-\lambda)]-\bar{\beta}^2\xi_H^h\lambda} \\
&\quad \times \Big[\frac{\frac{\partial \xi_L^h}{\partial \chi}\bar{\beta}(1-\bar{\beta}(1-\lambda))(\nu_{L,1}-\bar{\beta}\nu_{L,0})}{[1-\bar{\beta}(1-\lambda-\xi_L^h)]^2} + \frac{\bar{\beta}\xi_L^h\frac{\partial \nu_{L,1}}{\partial \chi}}{1-\bar{\beta}(1-\lambda-\xi_L^h)}\Big]\Bigg\}.
\end{aligned}$$

Again, the sign of $\partial G_H/\partial \chi$ is ambiguous as both components in the curly bracket are positive. Thus, when $z_L > 0$ and $z_H > 0$, the sign of $\partial w_H/\partial \chi$ is ambiguous.

- Second, suppose $z_H = 0$, $q_{H,1} = q^*$, $z_L > 0$ and $q_{L,1} < q^*$. The partial derivatives are similar to the above case. When $\gamma \to 1$, $\partial G_H/\partial w_H > 0$. Because $\partial \nu_{H,1}/\partial \chi = 0$, $\partial \nu_{L,1}/\partial \chi > 0$, $\partial \xi_L^h/\partial \chi > 0$ and $\partial \xi_H^h/\partial \chi = 0$, when $\nu_{L,1} > \bar{\beta}\nu_{L,0}$, $\partial G_H/\partial \chi < 0$. Therefore, when $z_H = 0$, $q_{H,1} = q^*$, $z_L > 0$, $q_{L,1} < q^*$, $\gamma \to 1$ and $\nu_{L,1} > \bar{\beta}\nu_{L,0}$, we find $\partial w_H/\partial \chi > 0$.

- Third, suppose $z_H = 0$, $q_{H,1} < q^*$, $z_L > 0$ and $q_{L,1} < q^*$. Then we have

$$\begin{aligned}
\frac{\partial G_H}{\partial w_H} = (1-\gamma)\Bigg\{ & \frac{\partial \frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}}{\partial w_H}[\nu_{H,1}+\sigma(1-\xi_H^h)(\bar{W}_{H,1}-\bar{W}_{L,1})] \\
& + \frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}\Big[\frac{\partial \nu_{H,1}}{\partial w_H} - \sigma\frac{\partial \xi_H^h}{\partial w_H}(\bar{W}_{H,0}-\bar{W}_{L,0}) + \sigma(1-\xi_H^h)\frac{\partial \bar{W}_{H,0}}{\partial w_H}\Big]\Bigg\} + \gamma,
\end{aligned}$$

where $\partial\Big[\frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}\Big]/\partial w_H > 0$, $\partial \nu_{H,1}/\partial w_H < 0$ when $q_{H,1}$ is large, but the sign of $\partial \bar{W}_{H,0}/\partial w_H$ is ambiguous. However, if $\gamma \to 1$, then $\partial G_H/\partial w_H > 0$, and

$$\begin{aligned}
\frac{\partial G_H}{\partial \chi} =& (1-\gamma)\frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}\Bigg\{\frac{\partial \nu_{H,1}}{\partial \chi} + \sigma(1-\xi_H^h)[\frac{\partial \bar{W}_{H,0}}{\partial \chi} - \frac{\partial \bar{W}_{L,0}}{\partial \chi}] - \sigma\frac{\partial \xi_H^h}{\partial \chi}[\bar{W}_{H,1}-\bar{W}_{L,0}]\Bigg\} \\
& + (1-\gamma)\frac{\partial \frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}}{\partial \chi}\Big\{\nu_{H,1}+\sigma(1-\xi_H^h)[\bar{W}_{H,1}-\bar{W}_{L,0}]\Big\} \\
& - \gamma\frac{\partial R_H}{\partial \chi},
\end{aligned}$$

where $\partial \nu_{H,1}/\partial \chi < 0$ when $q_{H,1}$ is large, $\partial \xi_H^h/\partial \chi > 0$, $\partial R_H/\partial \chi > 0$, and $\partial \frac{1-\bar{\beta}(1-\lambda-\xi_L^f)}{1-\bar{\beta}(1-\lambda-\xi_L^h)}/\partial \chi < 0$. However, the sign of $\partial[\bar{W}_{H,0}-\bar{W}_{L,0}]/\partial \chi$ is uncertain. So, the sign of $\partial w_H/\partial \chi$ is ambiguous. But because of continuity, $\partial w_H/\partial \chi$ should be positive when $q_{H,1}$ is close to q^*.

■

B Welfare

We examine efficiency by analyzing the social planner's problem:

$$J(u_H, u_L, v_H, v_L) = \max_{q,v} \Bigg\{ \alpha[\upsilon(q) - pq] + y_H n_H + y_L n_L + [pq^s - c(q^s)](1-u) - kv \qquad \text{(B.1)}$$

$$+ \beta J(\hat{u}_H, \hat{u}_L, \hat{v}_H, \hat{v}_L) \Bigg\}$$

$$\text{s.t. (33) and (34),} \qquad \text{(B.2)}$$

where the planners maximizes the total welfare J by choosing the retail market consumption, q, and vacancies, v. The first-order condition with respect to q provides

$$\upsilon'(q^*) = p. \qquad \text{(B.3)}$$

It is easy to see that the retail market consumption q^{CE} as in (29) coincides with the efficient retail market consumption q^* characterized in (B.3), if and only if $(1+i)/(1-\mu) = 1$. Under the Friedman rule ($i = 0$), holding real balances across periods becomes costless, eliminating the intertemporal distortion caused by the inflation tax. However, this alone does not restore efficiency in real balance holdings and thus q because households exit the economy exogenously with probability μ. The planner is unaffected by this, as the exiting households' balances are redistributed among the newborns. In contrast, in a competitive equilibrium, households hold fewer real balances, as they do not internalize the surplus of newborns. Obviously, the condition $(1+i)/(1-\mu) = 1$ requires i to be negative—contradicting our assumption—monetary policy alone cannot achieve efficient retail market consumption.

When Hosios' rule holds, the worker's bargaining power equals the elasticity of the matching function with respect to workers, eliminating the conventional distortion caused by matching frictions.[32] However, firms' entry remains discouraged due to lower revenue in the retail market, as $pq^s - c(q^s)$ increases in q and $q^{CE} < q^*$. Consequently, labor market efficiency cannot be achieved by Hosios' rule alone. One possible approach is to follow Gomis-Porqueras et al. (2013), which explores policy tools designed to restore efficiency within frameworks with frictional goods and labor markets as in Berentsen et al. (2011).

[32] Suppose the matching function takes a functional form $\mathcal{N}(u, v) = u^a v^{1-a}$. Hosios' rule requires $a = \gamma$.

C Proofs and Derivations

C.1 Proof of Proposition 1

We first establish that the RM curve is downward sloping in the (u, q) space. Recall that the RM curve:

$$i = (1-\mu)\left[\alpha \frac{\upsilon'(q)}{c'\left(\frac{q}{1-u}\right)} + (1-\alpha)\right] - 1. \tag{C.1}$$

Suppose that u increases. It follows that the right hand side of (C.1) increases as $c'' > 0$. It follows that q must decrease to ensure (C.1) is satisfied, as $\upsilon'' < 0$.

We now establish that for $i > 0$ and $u = 0$ that $q = \hat{q} \in (0, q^*)$. If $i > 0$ and $u = 0$, it is straightforward to show $\hat{q}$ solves

$$\frac{i+\mu}{1-\mu} = \alpha\left[\frac{\upsilon'(\hat{q})}{c'(\hat{q})} - 1\right]. \tag{C.2}$$

As the left hand side of (C.2) is greater than zero, we require that $\upsilon'(q)/c'(q) > 1$. It follows that $\hat{q} < q^*$ as $\upsilon'(q^*)/c'(q^*) = 1$ and $\upsilon'(q)/c'(q)$ is decreasing in q.

Next we show that $q \to 0$ as $u \to 1$. We can see that as $u \to 1$, that $c'(q/(1-u)) \to \infty$ as $q/(1-u) \to \infty$. It follows that $q \to 0$ to ensure (C.1) is satisfied when $u \to 1$ as $\upsilon'(0) = \infty$. ■

C.2 Proof of Lemma 1

Suppose that i increases. It follows that the left hand side of (C.1) increases, meaning that the right hand side must also increase. Holding fixed the unemployment rate, u, it is straightforward to see that q must decrease so that (C.1) is satisfied, as $\upsilon'(q)/c'(q/(1-u))$ is decreasing in q. Thus, following an increase in i, the equilibrium value of q at each unemployment rate as determined through the RM curve decreases and the curve shifts downward. Similarly, we show that when μ increases, q must decrease given an u so that (C.1) is satisfied. However, with a higher α, q must increase given an u as $\upsilon'(q)/c'(q/(1-u)) > 1$. ■

C.3 The Cost of Skill Loss: Homogeneous firms

The cost of skill loss is defined by $\Delta_\sigma = V_{H,0}(z) - V_{L,0}(z)$, which after substituting the relevant value functions into (6) is given by

$$\Delta_\sigma = \frac{\bar{\beta}\gamma\xi_h(S_H - S_L)}{1 - \bar{\beta}(1-(1-\xi_h)\sigma)}. \tag{C.3}$$

From (C.3), the cost of skill loss is the discounted sum of the additional surplus a highly-skilled worker obtains in the labor market. Combining equation (C.3) with S_L and S_H from (31) and solving for Δ_σ gives

$$\Delta_\sigma = \frac{\gamma\xi_h y(1-\delta)}{(\mu+\rho(1+\mu)+\lambda+\gamma\xi_h)[1-\bar{\beta}(1-(1-\xi_h)\sigma)] - \gamma\xi_h\bar{\beta}(1-\xi_h)\sigma}. \tag{C.4}$$

C.4 Proof of Proposition 2

Our first objective is to show that there exists at least one θ (and hence u) that satisfies the LM curve for all $q \in [0, q^*]$. Recall the job creation condition:

$$\frac{k}{\xi_f} = (1-\gamma)\left[\frac{y[1-\varphi(1-\delta)] + (1-\varphi)\bar{\beta}(1-\xi_h)\sigma\Delta_\sigma + c'\left(\frac{\alpha q}{1-u}\right)\frac{\alpha q}{1-u} - c\left(\frac{\alpha q}{1-u}\right) - b}{\mu + \rho(1+\mu) + \lambda + \gamma\xi_h}\right]. \tag{C.5}$$

We know that if $\theta \to \infty$, then $\xi_f \to 0$ and the left hand side of (C.5) approaches ∞. The right hand side, however goes to

$$(1-\gamma)\left[\frac{y[1-\varphi(1-\delta)] + c'(\alpha q)\alpha q - c(\alpha q) - b}{\mu + \rho(1+\mu) + \lambda + \gamma}\right] < \infty. \tag{C.6}$$

Therefore, there exists at least one $\theta > 0$ which satisfies the job creation condition as long as the left hand side of (C.5) is less than the right hand side when $\theta \to 0$. As $\theta \to 0$, the left hand side of (C.5) approaches k. It is then straightforward to show that there is at least one $\theta > 0$ which satisfies (C.5) if

$$k < (1-\gamma)\left[\frac{y[1-\varphi(1-\delta)] + (1-\varphi)\bar{\beta}\sigma\Delta_\sigma - b}{\mu + \rho(1+\mu) + \lambda}\right]. \tag{C.7}$$

The right hand side of (C.7) is clearly decreasing in φ. To establish a sufficient condition for the existence of $\theta > 0$, we suppose that $\varphi \to 1$. It follows that the left hand side of (C.5) is less than the right hand side as $\theta \to 0$ and there exists at least one $\theta > 0$ which satisfies (C.5) if

$$k(\mu + \rho(1+\mu) + \lambda) < (1-\gamma)(\delta y - b). \tag{C.8}$$

For the rest of the proof, we assume that (C.8) holds and further assume that parameters are such that there is a unique θ which satisfies (C.5). This need not always be the case as an increase in θ improves the skill distribution among the unemployed (φ decreases), which means the right hand side of (C.5) can be upward sloping. This typically only occurs under extreme parameters quantitatively.

We proceed to show that the LM curve slopes downward in the (u, q) space. Suppose that q increases within the interval $[0, q^*]$. It follows that the profits a firm obtains from selling inventory in the retail market, $c'\left(\frac{\alpha q}{1-u}\right)\frac{\alpha q}{1-u} - c\left(\frac{\alpha q}{1-u}\right)$, increase, causing the right hand side of the job creation condition to increase. Therefore, market tightness, θ, must increase so that the left hand side of (C.5) also increases. As market tightness increases, the unemployment rate decreases, and hence, the LM curve is downward sloping in the (u, q) space. The lowest level of the unemployment rate, $\underline{u} = (\mu + (1-\mu)\lambda)/(\mu + (1-\mu)(\lambda + \xi_h(\bar{\theta})))$, is reached when $q = q^*$ and $\bar{\theta}$ solves (C.5) with $q = q^*$. Moreover, the highest level of the unemployment rate, $\bar{u} = (\mu + (1-\mu)\lambda)/(\mu + (1-\mu)(\lambda + \xi_h(\underline{\theta}))$ where $\underline{\theta}$ solves (C.5) when $q = 0$. Clearly, $\underline{\theta} > 0$ and therefore $\bar{u} < 1$. ■

C.5 Proof of Lemma 2

Now suppose that $\gamma = 0$. The job creation condition is given by

$$\frac{k}{\xi_f} = \frac{y[1-\varphi(1-\delta)] + c'\left(\frac{\alpha q}{1-u}\right)\frac{\alpha q}{1-u} - c\left(\frac{\alpha q}{1-u}\right) - b}{\mu + \rho(1+\mu) + \lambda}, \tag{C.9}$$

where the skill distribution among the unemployed is given by

$$\varphi = \frac{\sigma(1-\mu)(1-\xi_h)[1-(1-\mu)(1-\lambda)]}{\mu(1-\mu)\xi_h + [\mu + (1-\mu)(1-\xi_h)\sigma][1-(1-\mu)(1-\lambda)]}. \tag{C.10}$$

Suppose that δ or y increases. The right hand side of (C.9) increases. Therefore, market tightness must increase to satisfy the job creation condition. It then follows that for each q, market tightness increases and the unemployment rate decreases, i.e., the LM curve shifts to the left. Also, suppose that α increases, given q, u must decrease.

Now consider an increase to σ. From (C.10), φ increases. As φ increases, the right side of (C.9) decreases. Therefore, market tightness decreases and the economy exhibits a higher unemployment rate at each value of q. This establishes that the LM curve shifts to the right following an increase in σ. Furthermore, suppose that k increases, the left hand side of (C.9) increases, given q, u must increase for (C.9) to hold. Also, as b rises, the right hand side of (C.9) decreases, so for a given q, u must increase. Lastly, when λ increases, the right hand side of (C.9) decreases as $\partial\varphi/\partial\lambda > 0$. As a result, for a given q, u increases. ■

C.6 Proof of Proposition 3

We first establish the existence of at least one monetary equilibrium with $q \in (0, q^*)$ and $u \in (0, 1)$. Following the Proofs of Propositions 1 and 2, both the RM and LM curves are downward sloping in the (u, q) space. However, we know that at $u = 0$, the RM curve is below the LM curve and as $u \to 1$, the RM curve is above the LM curve as shown in Figure 2, indicating that the curves must cross at least once at a point where $q > 0$, $q \in (0, q^*)$, and $u \in (0, 1)$. In general, the monetary equilibrium may not be unique as there may be multiple θ which satisfy the job creation condition, as discussed in the Proof of Proposition 2.

Second, there also exists at least one non-monetary equilibrium as $q = 0$ is always an equilibrium outcome to the household's portfolio choice, irrespective of the unemployment rate. Additionally, as established in the Proof of Proposition 2, $u = \bar{u}$ where $\bar{u} < 1$ when $q = 0$ and (C.8) is satisfied.

Turning to the comparative statics and focusing on a monetary equilibrium. If $\gamma = 0$, then by the Proof of Proposition 2, an increase in σ or decrease in δ both cause the LM curve to shift to the right. It follows that the new LM curve intersects the RM curve at a lower value of q and higher unemployment rate, u. Finally, if i increases, then the RM curve shifts outward (following the Proof of Proposition 1) and the RM curve intersects the LM curve at a lower q and higher level of the unemployment rate, u. ■

C.7 The Cost of Skill Loss: Heterogeneous firms

As in the case with homogeneous firms, the cost of skill loss is given by $\Delta_\sigma = V_{H,0} - V_{L,0}$ and after substituting stage 1 and 3 value functions into (39) and solving for Δ_σ, we have

$$\Delta_\sigma = \frac{\bar{\beta}\gamma\xi_h\{\zeta(S_{H,s} - S_{L,S}) + (1-\zeta)(S_{H,c} - S_{L,c})\}}{1 - \bar{\beta}(1 - (1-\xi_h)\sigma)}. \tag{C.11}$$

Equation (C.11) has the same interpretation as equation (C.3): the cost of skill loss is the discounted sum of the additional surplus a highly-skilled worker earns in the labor market. In the version of the heterogeneous firms, the additional surplus obtained by a highly-skilled worker is weighted by the composition of job complexity. Combining equation (C.11) with equations (54)-(57) and solving for Δ_σ gives

$$\Delta_\sigma = \frac{\gamma\xi_h\{\zeta[y_s(1-\delta_s)\rho_1 - \gamma\xi_h(1-\zeta)(y_c(1-\delta_c) - y_s(1-\delta_s))] + (1-\zeta)[y_c(1-\delta_c)\rho_1 - \gamma\xi_h\zeta(y_s(1-\delta_s) - y_c(1-\delta_c))]\}}{\rho_1\rho_2[1 - \bar{\beta}(1-(1-\xi_h)\sigma)] - \rho_1\gamma\xi_h\bar{\beta}(1-\xi_h)\sigma}, \tag{C.12}$$

where $\rho_1 = \mu + \rho(1+\mu) + \lambda$ and $\rho_2 = \mu + \rho(1+\mu) + \lambda + \gamma\xi_h$.

C.8 Proof of Proposition 5

To establish sufficient conditions for all match surpluses to be positive, we only need to establish sufficient conditions for match surpluses with low skill workers to be positive. This is because match surpluses with high skill workers are always higher due to the fact that highly-skilled workers are more productive and forming a match saves them the risk of skill loss, captured by the term $\bar{\beta}(1-\xi_h)\sigma\Delta_\sigma$ in equations (56)-(57). From equation (54), $S_{L,s} > 0$ if

$$\frac{R_{L,s} - b}{R_{L,c} - b} > \frac{\gamma\xi_h(1-\zeta)}{\mu + \rho(1+\mu) + \lambda + \gamma\xi_h(1-\zeta)}. \tag{C.13}$$

The right hand side of (C.13) is decreasing in ζ and increasing in ξ_h. Thus, we set $\xi_h = 1$ and $\zeta = 0$ to find a sufficient condition on only fundamentals for $S_{L,s} > 0$:

$$\frac{R_{L,s} - b}{R_{L,c} - b} > \frac{\gamma}{\mu + \rho(1+\mu) + \lambda + \gamma}. \tag{C.14}$$

Further suppose that $q_s = 0$ and $q_c = \bar{q}$ where $\bar{q} \equiv \arg\max\{c(q)q - c(q)\}$. It follows that (C.14) and, therefore, (C.13), are satisfied if

$$\frac{\delta_s y_s - b}{\delta_c y_c + [c'(\bar{q})\bar{q} - c(\bar{q})] - b} > \frac{\gamma}{\mu + \rho(1+\mu) + \lambda + \gamma}, \tag{C.15}$$

which establishes the sufficient condition for $S_{L,s} > 0$.

Turning to matches between less-skilled workers and complex jobs, from (55), $S_{L,c} > 0$ if

$$\frac{R_{L,c} - b}{R_{L,s} - b} > \frac{\gamma \xi_h \zeta}{\mu + \rho(1+\mu) + \lambda + \gamma \xi_h \zeta}. \tag{C.16}$$

Following the same logic as above, it is straightforward to show that $S_{L,c} > 0$ if

$$\frac{\delta_c y_c - b}{\delta_s y_s + [c'(\bar{q})\bar{q} - c(\bar{q})] - b} > \frac{\gamma}{\mu + \rho(1+\mu) + \lambda + \gamma}. \tag{C.17}$$

Equations (C.15) and (C.17) establish sufficient conditions for all matches to generate a positive surplus. ■

C.9 Proof of Proposition 6

To show the existence of at least one monetary equilibrium, we first derive sufficient conditions to ensure firms post vacancies. Following the same steps as in the Proof of Proposition 2 to derive equation (C.8), it is straightforward to show that firms will always post a positive measure of type χ vacancies if

$$k_\chi < \frac{(1-\gamma)\sigma(1-\mu)(\delta_\chi y_\chi - b)}{(\mu + \rho(1+\mu) + \lambda)(\mu + (1-\mu)\sigma)}. \tag{C.18}$$

Therefore, if equation (C.18) holds for both $\chi = \{s, c\}$, then a positive measure of both types of vacancies are always created. It follows that $\theta > 0$, $u < 1$, and $\zeta \in (0,1)$.

We proceed by assuming (C.18) holds for all $\chi \in \{s, c\}$. It is straightforward to show the existence of a monetary equilibrium. First, from equations (46)-(47), there is a unique $q_\chi^D > 0$ for $\chi \in \{s, c\}$ which solves the household's optimization problem in the retail market for a given level of real balances, z, and prices p_s and p_c. Moreover, from equation (51), the marginal benefit to an additional unit of real balances is strictly decreasing in z'. Therefore, for a given $u \in (0,1)$ and $\zeta \in (0,1)$, there exists $q_\chi > 0$ for $\chi \in \{s, c\}$ which solves the household's portfolio choice and utility maximization problem within the retail market.

Finally, as in the case of homogeneous firms, there can be both a non-monetary equilibrium as $q_\chi = 0$ for $\chi \in \{s, c\}$ is always a solution to the household's portfolio choice. ■

D Empirical Appendix

D.1 Data Sources and Construction

This section describes the data used in the quantitative exercises. All data were downloaded directly from the Federal Reserve Bank of St. Louis' FRED database.[33] The following are the data series we download from the FRED database, covering 1955-2017.

- Unemployment rate.

[33] Data from the FRED database can be directly downloaded from https://fred.stlouisfed.org/.

- Series title: Civilian unemployment rate.
- Series ID: UNRATE.

- Interest rate.
  - Series title: Moody's Seasoned Aaa Corporate Bond Yield.
  - Series ID: AAA.
- TFP.
  - Series title: Total Factor Productivity at Constant National Prices for United States (as constructed by Feenstra et al. (2015)).
  - Series ID: RTFPNAUSA632NRUG.
- CPI.
  - Series title: Consumer Price Index for All Urban Consumers: All Items.
  - Series ID: CPIAUCSL.
- Labor Productivity.
  - Series title: Nonfarm Business Sector: Real Output Per Hour of All Persons.
  - Series ID: PRS85006091.

We then use the data to construct the following series.

- TFP growth is the percentage growth in TFP between year $t-1$ and year t.
- Inflation is the percentage growth in CPI between year $t-1$ and year t.

The calibration uses several additional series downloaded from the FRED database:

- M1 (1955-1958).
  - Series title: M1 Money Stock.
  - Series ID: M1SL.
- M1 (1959-2017).
  - Series title: M1 Money Stock.
  - Series ID: M1SA.
- Money market deposit accounts.

  - Series title: Money Market Funds; Total Time and Savings Deposits.
  - Series ID: BOGZ1FL633030000Q.

- Nominal GDP.
  - Series title: Gross Domestic Product.
  - Series ID: GDP.

We then construct the series for money demand as follows. First, we construct the total money supply by adding together the M1 and money market deposit accounts. Second, we calculate money demand by taking the ratio of the total money supply and nominal GDP.

D.2 Task Scores and Job Complexity

Measures of tasks are created by Autor and Dorn (2013), where they derived the abstract, routine and manual tasks using the US Department of Labor's *Dictionary of Occupational Titles* (DOT). The original five task measures of Autor et al. (2003) are collapsed into three categories, following Autor et al. (2006). The manual task measurement, $T^{M}_{k,1980}$, is the DOT variable for an occupation's demand for "eye-hand-foot coordination" in 1980. The abstract task measure, $T^{A}_{k,1980}$, is the average of the DOT's variables for "direction control and planning" which measures managerial and interactive tasks and "GED Math", measuring mathematical and formal reasoning requirement in 1980. More details could be found in Autor et al. (2003) Supplemental Appendix Table 1. Table D1 shows the occupations with the twenty highest and lowest AM measures.

After constructing the AM measures for each occupation, we then match the AM scores by occupation to the Annual Social and Economic Supplement (ASEC) of the Current Population Survey (CPS) in order to calculate the fraction of total employment above and below our cutoff that distinguishes simple and complex jobs.[34] To merge the AM scores into the ASEC data, we used the 1990 Census Bureau occupational classification. That is, we use the "occ1990" variable in the ASEC data and merge with AM occupation scores using the crosswalk developed by Autor and Dorn (2013).[35] We then restrict our sample to individuals between ages 25-65 (inclusive).

Figure D1 shows several occupations around the cutoff between simple and complex occupations. The vertical axis is the occupation's average level of employment between 1968-2017. While the red line represents our baseline cutoff between simple and complex jobs, we also consider alternative cutoffs as shown by the purple and green vertical lines. See Supplemental Appendix E for quantitative results under these alternative thresholds.

[34] We download the ASEC data directly from IPUMS CPS (Flood et al., 2023), which is available at https://cps.ipums.org/cps/.

[35] The "occ1990" variable reports the respondent's primary occupation.

Table D1: Occupation with the Highest and Lowest *AM* Score

Highest 20	*AM*	Lowest 20	*AM*
Physical Scientist	1	Dancers	0
Chemical Engineers	0.983	Parking Lot Attendant	0.222
Chemists	0.952	Paving and surfacing equipment operators	0.253
Actuaries	0.944	Operating Engineers of construction equipment	0.273
Dietitians and Nutritionists	0.942	Fire Fighting	0.273
Metallurgical and Materials Engineers	0.926	Excavating and Loading Machine Operators	0.281
Mechanical Engineers	0.926	Bus Driver	0.283
Funeral Directors	0.924	Truck, Delivery, and Tractor Drivers	0.283
Accountants and Auditors	0.922	Taxi Cab Driver	0.285
Petroleum, Mining and Geological Engineers	0.921	Roofer and Slaters	0.291
Managers of Medicine	0.914	Crane, derrick, winch, and hoist operators	0.291
Financial Managers	0.911	Structural Metal Workers	0.302
Aerospace Engineer	0.897	Plasterers	0.306
Atmospheric and Space Scientists	0.895	Textile and Sewing Machine Operator	0.343
Other Financial Specialist	0.893	Garbage and Recyclable Material Collector	0.343
Subject Instructor (HS/College)	0.892	Driller of Earth	0.361
Managers and Specialists in Marketing, Advertising, and Public relations	0.883	Railroad brake, coupler, and switch operators	0.362
Biological Scientists	0.882	Millwrights	0.370
Computer Software Developer	0.879	Carpenter	0.371

Notes: The *AM* score is computed for each occupation observed in the ASEC data among individuals aged 25-65 years old.

D.3 Supplementary Figures

To begin, Figure D2 uses US data from 1955-2017 and presents scatter plots containing the relationship between average annual unemployment and anticipated inflation, as measured by nominal Aaa corporate bond interest rates. Following Berentsen et al. (2011), we remove higher frequency fluctuations by applying an HP filter with a larger smoothing parameter while the last panel presents five-year averages. Figure D2 shows a positive relationship between the lower-frequency movements in unemployment and anticipated inflation. Figure D3 presents the scatter plots for the relationship between low frequency trends in CPI inflation and the unemployment rate.

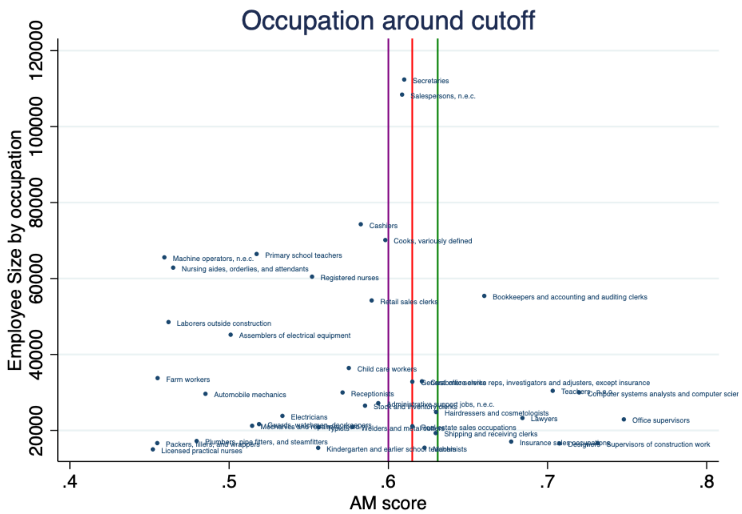


Figure D1: Occupations Close to the Cutoff Between Simple and Complex Jobs. *Notes:* The middle vertical line represents the AM score used to distinguish between simple and complex jobs in our baseline calibration. The left (right) vertical line represents the lowest (highest) threshold AM score used to perform alternative calibrations.

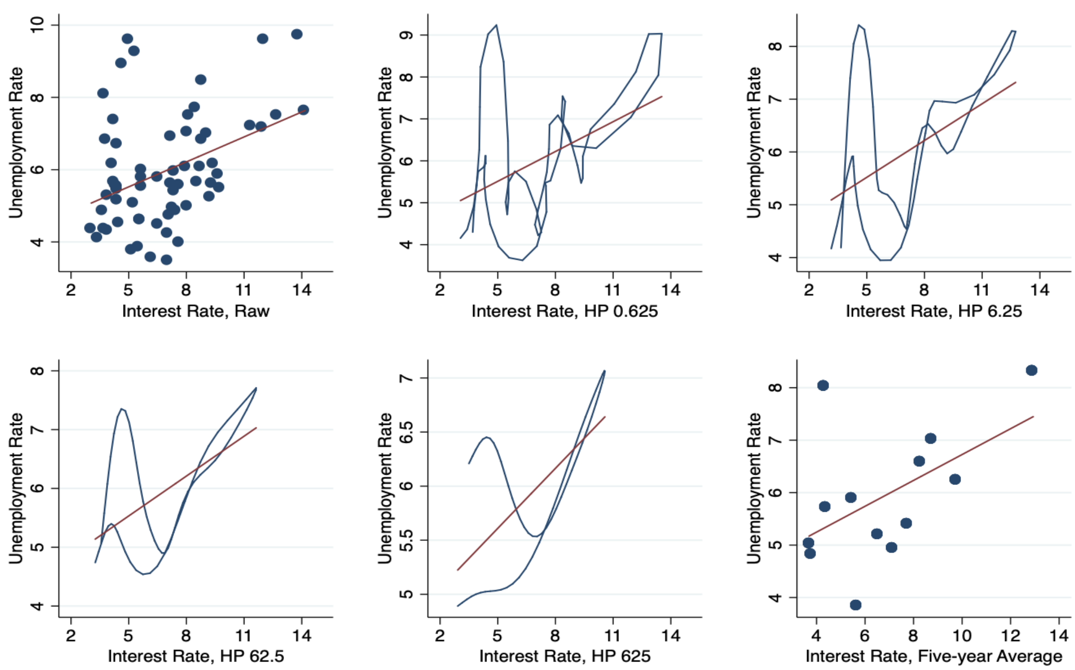


Figure D2: Interest Rate and Unemployment. *Notes:* Interest rate is the nominal Aaa corporate bond rates. Series denoted "HP" are detrended using the Hodrick-Prescott filter with respective smoothing parameters.

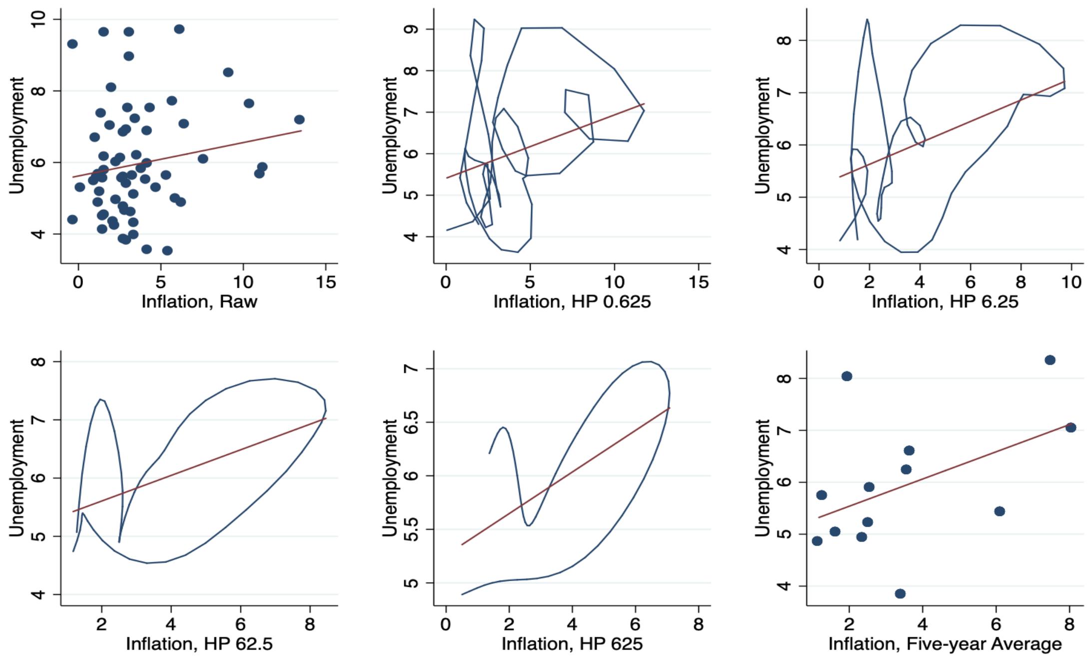


Figure D3: Inflation and Unemployment. *Notes:* Inflation is the annual inflation rate derived from the Consumer Price Index (CPI). Series denoted “HP” are detrended using the Hodrick-Prescott filter with respective smoothing parameters.

E Additional Quantitative Results

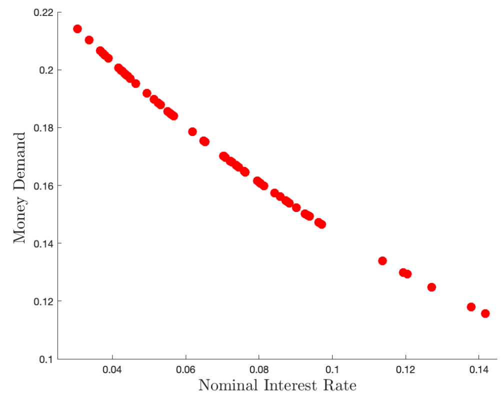
In this section, we present four alternative calibration strategies. The first one, “Markup”, targets a lower level of markups in the retail sector (20%), as opposed to 30% in the baseline calibration. The second strategy, “Skill loss”, assumes that skill loss takes (on average) six months to occur ($\sigma = 1/6$) rather than three months ($\sigma = 1/3$). The third strategy targets $\zeta = 0.50$ instead of $\zeta = 0.52$, where $\zeta = 0.50$ is the composition of job complexity when we use an AM score of 0.60 as the lower bound for a complex occupation. This is the AM score labelled as “Lower Bound” in Figure 3. The last strategy targets $\zeta = 0.62$, which corresponds to an AM score of 0.631 as being the lower bound for a complex occupation. In Figure 3, this is the cutoff labelled “Upper Bound”. The intuitive mapping from parameters to moments is the same as in our baseline calibration strategy outlined in Section 5.2. Table E1 presents parameter values under our baseline and four alternative strategies.

In the rest of this section, we present the quantitative results under the alternative strategies. Figures E1-E3 contain the results for the markup calibration, Figures E4-E6 for the skill loss calibration, Figures E7-E9 for the strategy which targets $\zeta = 0.50$, and Figures E10-E12 for the strategy which targets $\zeta = 0.62$.

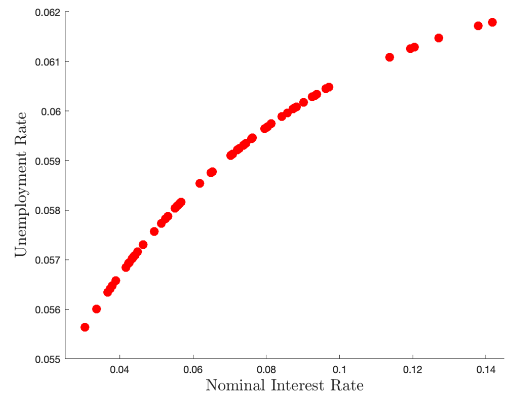
Table E1: Parameter Values Under Alternative Calibrations

Parameter	Baseline	Markup	Skill loss	$\zeta = 0.50$	$\zeta = 0.62$
Panel A: Assigned parameters					
ρ	1.68×10^{-3}	1.68×10^{-3}	1.68×10^{-3}	1.68×10^{-3}	1.68×10^{-3}
μ	2×10^{-3}	2×10^{-3}	2×10^{-3}	2×10^{-3}	2×10^{-3}
λ	0.035	0.035	0.035	0.035	0.035
η	0.50	0.50	0.50	0.50	0.50
γ	0.50	0.50	0.50	0.50	0.50
y_c	1.00	1.00	1.00	1.00	1.00
σ	1/3	1/3	1/6	1/3	1/3
i	6.89×10^{-2}	6.89×10^{-2}	6.89×10^{-2}	6.89×10^{-2}	6.89×10^{-2}
a	1.30	1.20	1.30	1.30	1.30
Panel B: Calibrated parameters					
A	0.590	0.590	0.590	0.590	0.590
b	0.554	0.547	0.570	0.550	0.554
k_s	0.245	0.195	0.202	0.224	0.273
k_c	0.589	0.635	0.853	0.602	0.591
δ_s	0.825	0.826	0.839	0.825	0.825
δ_c	0.650	0.648	0.642	0.648	0.641
y_s	0.787	0.770	0.760	0.775	0.797
α	0.050	0.073	0.051	0.049	0.057
ϱ	1.693	1.847	1.705	1.686	1.731

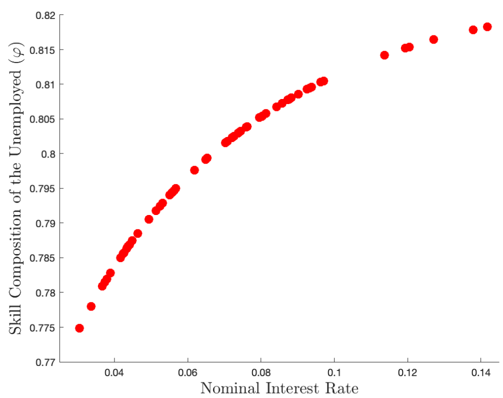
Notes: Parameters in Panel A are pre-assigned and not calibrated. Parameters listed in Panel B are chosen to minimize the distance between the model and empirical targets described in the text. Each respective column indicates the calibration strategy used to obtain the parameters listed in Panel B.



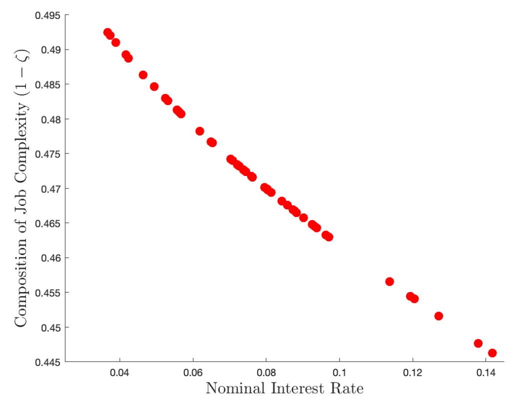
(a) Money Demand



(b) Long Run Phillips Curve



(c) Skill Composition of the Unemployed



(d) Composition of Job Complexity

Figure E1: Effects of a Change in Anticipated Inflation (markup calibration). *Notes:* The red circles represent the model generated outcome at each respective value of the nominal interest rate, i.

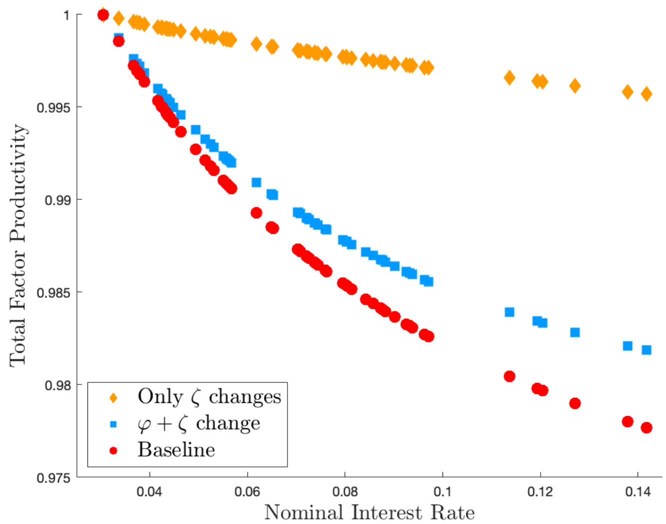


Figure E2: The Productivity Costs of Inflation (markup calibration). *Notes:* Baseline is the TFP produced by the baseline model with all channels active. The "$\varphi + \zeta$" change series holds the net retail market output fixed at its value when TFP is equal to one. The "Only ζ changes" series holds the net retail market output and skill composition of unemployed workers fixed at their respective values when TFP is equal to one. Each series is normalized so that TFP is equal to 1 at the lowest value of the nominal interest rate.

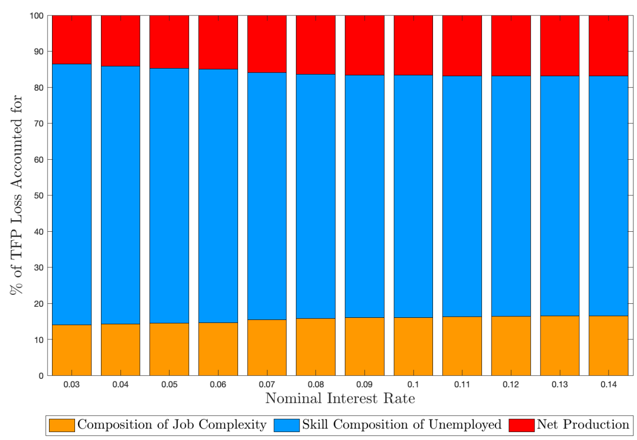
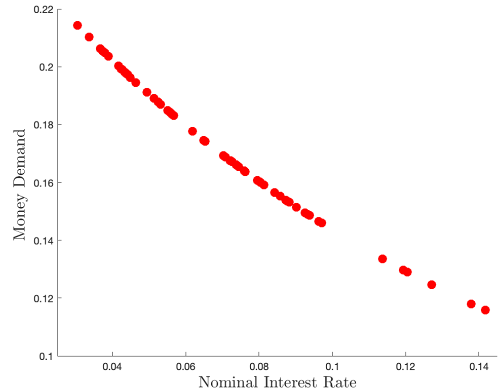
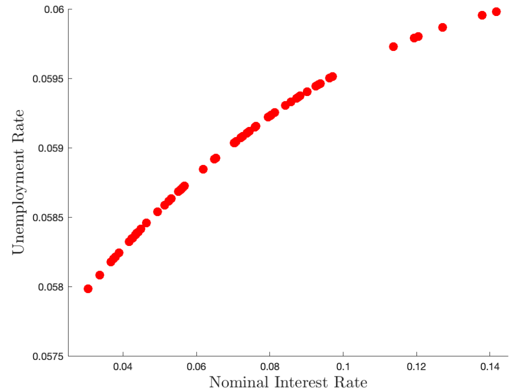


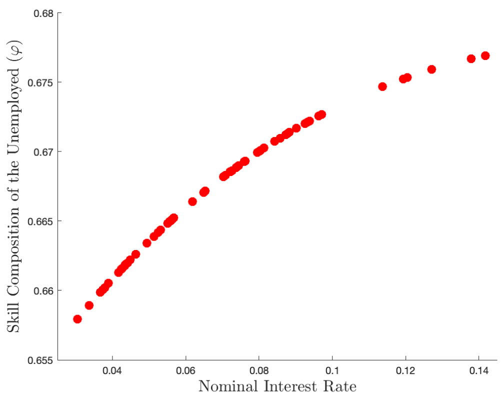
Figure E3: Decomposition of the Productivity Costs of Inflation (markup calibration). *Notes:* Each bar represents the decomposition of the decrease in TFP relative to the TFP at the lowest nominal interest rate. The orange region represents the fraction of the TFP loss that is attributed to a change in the fraction of jobs that are simple, ζ. The blue region captures the portion of the TFP loss driven by changes in the fraction of unemployed workers who are less-skilled, φ. The red region is the portion of TFP losses which are due to a reduction in net retail market production.



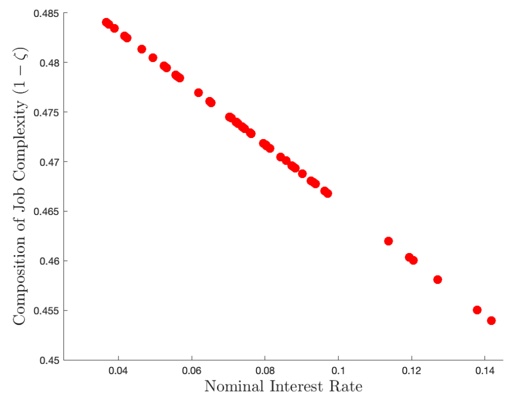
(a) Money Demand



(b) Long Run Phillips Curve



(c) Skill Composition of the Unemployed



(d) Composition of Job Complexity

Figure E4: Effects of a Change in Anticipated Inflation (skill loss calibration). *Notes:* The red circles represent the model generated outcome at each respective value of the nominal interest rate, i.

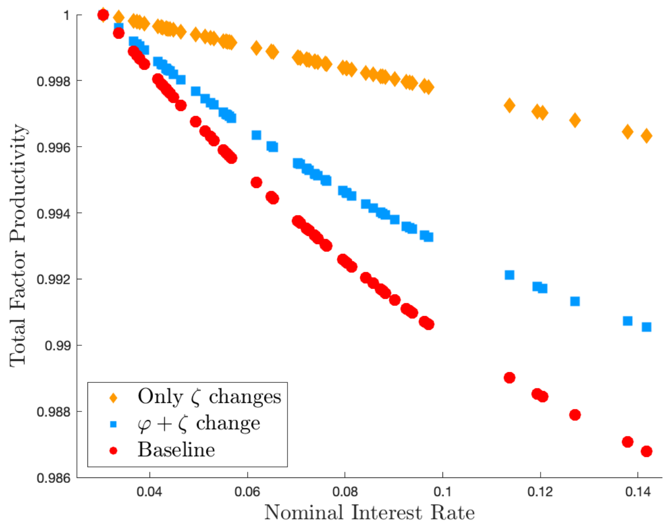


Figure E5: The Productivity Costs of Inflation (skill loss calibration). *Notes:* Baseline is the TFP produced by the baseline model with all channels active. The "$\varphi + \zeta$" change series holds the net retail market output fixed at its value when TFP is equal to one. The "Only ζ changes" series holds the net retail market output and skill composition of unemployed workers fixed at their respective values when TFP is equal to one. Each series is normalized so that TFP is equal to 1 at the lowest value of the nominal interest rate.

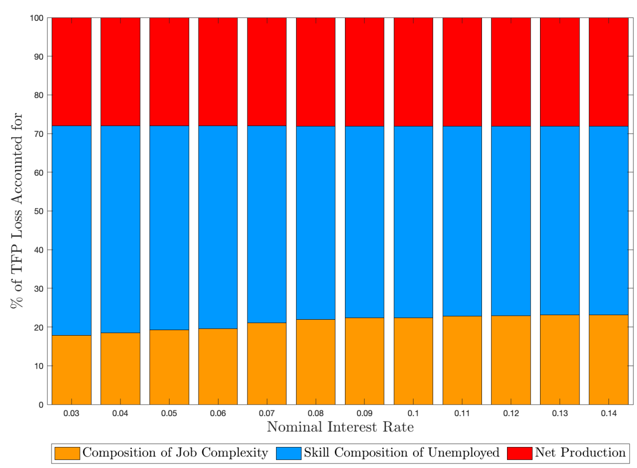
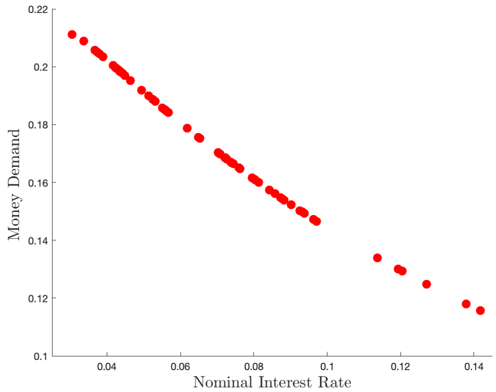
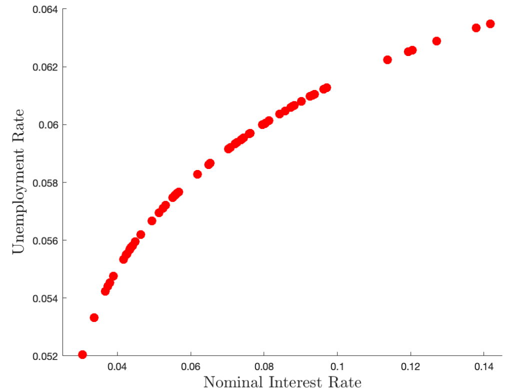


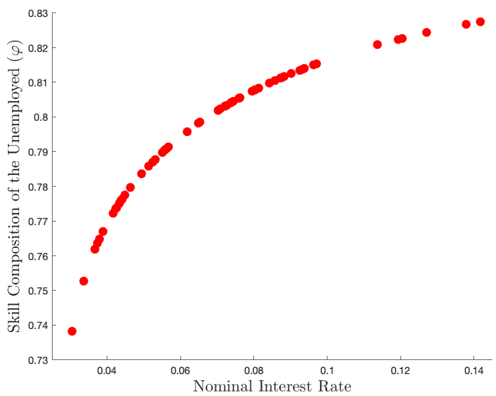
Figure E6: Decomposition of the Productivity Costs of Inflation (skill loss calibration). *Notes:* Each bar represents the decomposition of the decrease in TFP relative to the TFP at the lowest nominal interest rate. The orange region represents the fraction of the TFP loss that is attributed to a change in the fraction of jobs that are simple, ζ. The blue region captures the portion of the TFP loss driven by changes in the fraction of unemployed workers who are less-skilled, φ. The red region is the portion of TFP losses which are due to a reduction in net retail market production.



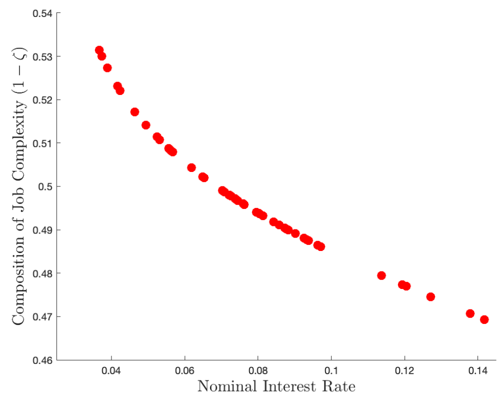
(a) Money Demand



(b) Long Run Phillips Curve



(c) Skill Composition of the Unemployed



(d) Composition of Job Complexity

Figure E7: Effects of a Change in Anticipated Inflation (low ζ calibration). *Notes:* The red circles represent the model generated outcome at each respective value of the nominal interest rate, i.

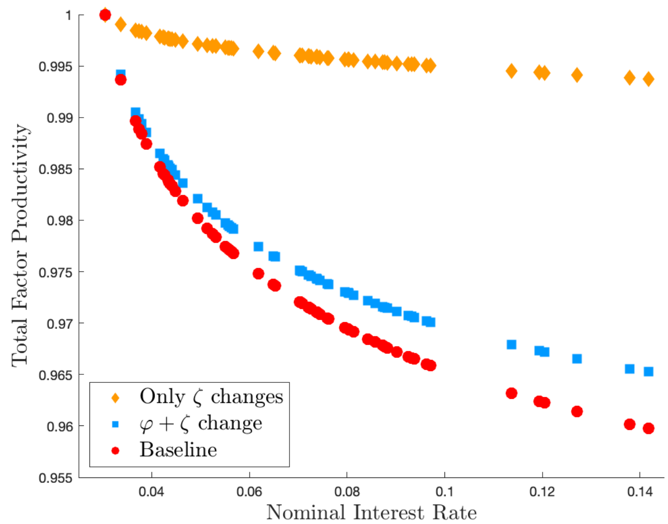


Figure E8: The Productivity Costs of Inflation (low ζ calibration). *Notes:* Baseline is the TFP produced by the baseline model with all channels active. The "$\varphi + \zeta$" change series holds the net retail market output fixed at its value when TFP is equal to one. The "Only ζ changes" series holds the net retail market output and skill composition of unemployed workers fixed at their respective values when TFP is equal to one. Each series is normalized so that TFP is equal to 1 at the lowest value of the nominal interest rate.

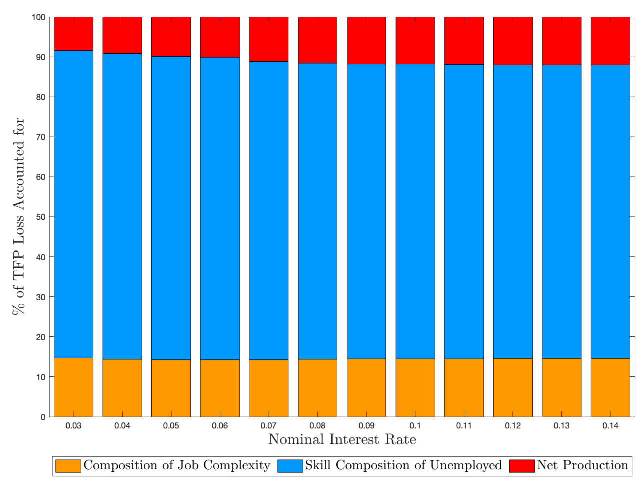
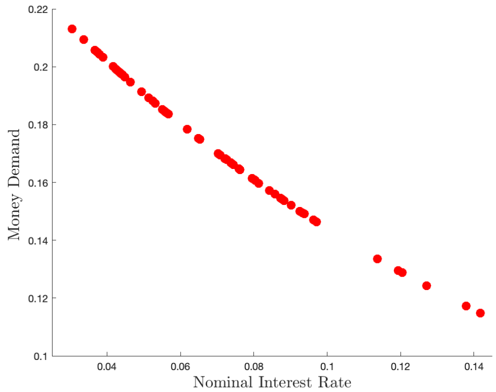
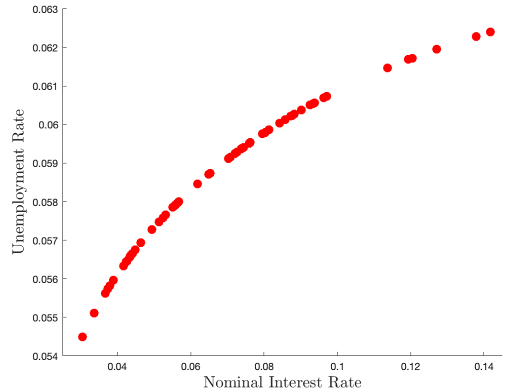


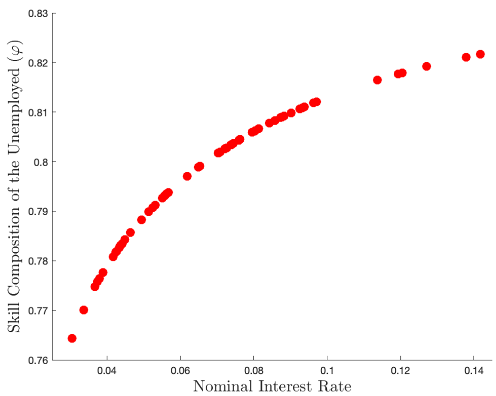
Figure E9: Decomposition of the Productivity Costs of Inflation (low ζ calibration). *Notes:* Each bar represents the decomposition of the decrease in TFP relative to the TFP at the lowest nominal interest rate. The orange region represents the fraction of the TFP loss that is attributed to a change in the fraction of jobs that are simple, ζ. The blue region captures the portion of the TFP loss driven by changes in the fraction of unemployed workers who are less-skilled, φ. The red region is the portion of TFP losses which are due to a reduction in net retail market production.



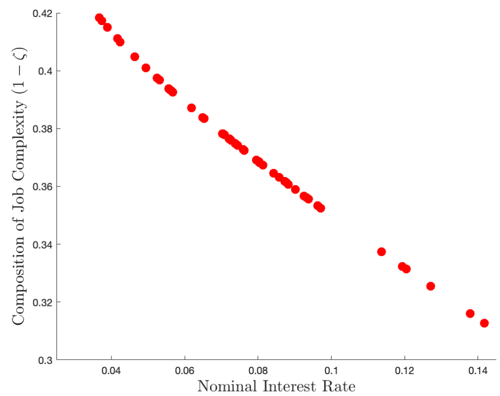
(a) Money Demand



(b) Long Run Phillips Curve



(c) Skill Composition of the Unemployed



(d) Composition of Job Complexity

Figure E10: Effects of a Change in Anticipated Inflation (high ζ calibration). *Notes:* The red circles represent the model generated outcome at each respective value of the nominal interest rate, i.

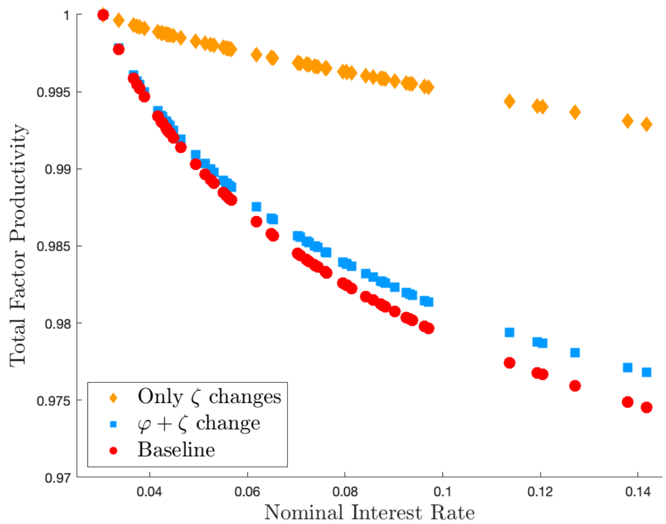


Figure E11: The Productivity Costs of Inflation (high ζ calibration). *Notes:* Baseline is the TFP produced by the baseline model with all channels active. The "$\varphi + \zeta$" change series holds the net retail market output fixed at its value when TFP is equal to one. The "Only ζ changes" series holds the net retail market output and skill composition of unemployed workers fixed at their respective values when TFP is equal to one. Each series is normalized so that TFP is equal to 1 at the lowest value of the nominal interest rate.

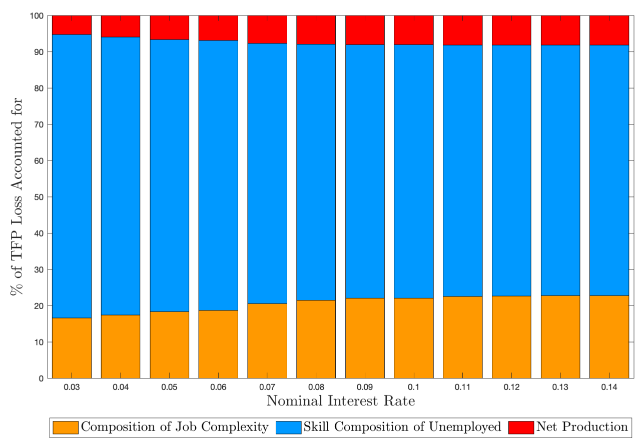


Figure E12: Decomposition of the Productivity Costs of Inflation (high ζ calibration). *Notes:* Each bar represents the decomposition of the decrease in TFP relative to the TFP at the lowest nominal interest rate. The orange region represents the fraction of the TFP loss that is attributed to a change in the fraction of jobs that are simple, ζ. The blue region captures the portion of the TFP loss driven by changes in the fraction of unemployed workers who are less-skilled, φ. The red region is the portion of TFP losses which are due to a reduction in net retail market production.

F Stochastic Version of the Model

In the stochastic version of the model, the state vector is given by $\psi = (u_L, u_H, n_L, i)$, where the nominal interest rate follows a stochastic process that is given by:

$$\hat{i} = \bar{i} + \rho_i(i - \bar{i}) + \varepsilon_i,\ \varepsilon_i \sim N(0, \sigma_i), \tag{F.1}$$

and the laws of motion for u_L, u_H, and n_L are given by

$$\hat{u}_L(\psi) = (1-\mu)\big[\big(1 - \xi_h(\theta(\psi))\big)[u_L + \sigma u_H]\big], \tag{F.2}$$

$$\hat{u}_H(\psi) = \mu + (1-\mu)\big[\big(1 - \xi_h(\theta(\psi))\big)(1-\sigma)u_H + \lambda(1 - u_L - u_H - n_L)\big], \tag{F.3}$$

$$\hat{n}_L(\psi) = \mu + (1-\mu)\big[(1-\lambda)n_L + \xi_h(\theta(\psi))u_L\big]. \tag{F.4}$$

We assume that the shock to the nominal interest rate is realized in stage 3 before households make their portfolio choice and firms make their vacancy posting decision.

Denoting $P(\hat{\psi};\psi)$ as the transition probability function between ψ and ψ', a recursive equilibrium is a list of functions $\{\theta(\psi), \zeta(\psi), R_{\varepsilon,\chi}(\psi), \Delta_\sigma(\psi), S_{\varepsilon,\chi}(\psi), q_\chi(\psi), P(\hat{\psi};\psi)\}$ for $\varepsilon \in \{L, H\}$ and $\chi \in$

$\{s, c\}$ such that:

$$\frac{k_s}{\xi_f(\theta(\psi))} = \bar{\beta}(1-\gamma)\Bigg[\varphi(\psi)R_{L,s}(\psi) + (1-\varphi(\psi))R_{H,s}(\psi) - b + \bar{\beta}\mathbb{E}\Bigg\{\frac{(1-\lambda-\gamma\xi_h(\theta(\hat{\psi}))\zeta(\hat{\psi}))k_s}{\xi_f(\theta(\hat{\psi}))\bar{\beta}(1-\gamma)} - \frac{\xi_h(\theta(\hat{\psi}))(1-\zeta(\hat{\psi}))\gamma k_c}{\xi_f(\theta(\hat{\psi}))\bar{\beta}(1-\gamma)} + (1-\varphi(\psi))(1-\xi_h(\theta(\hat{\psi})))\sigma\Delta_\sigma(\hat{\psi})\Bigg\}\Bigg], \tag{F.5}$$

$$\frac{k_c}{\xi_f(\theta(\psi))} = \bar{\beta}(1-\gamma)\Bigg[\varphi(\psi)R_{L,c}(\psi) + (1-\varphi(\psi))R_{H,c}(\psi) - b + \bar{\beta}\mathbb{E}\Bigg\{\frac{(1-\lambda-\gamma\xi_h(\theta(\hat{\psi}))(1-\zeta(\hat{\psi})))k_c}{\xi_f(\theta(\hat{\psi}))\bar{\beta}(1-\gamma)} - \frac{\xi_h(\theta(\hat{\psi}))\zeta(\hat{\psi})\gamma k_s}{\xi_f(\theta(\hat{\psi}))\bar{\beta}(1-\gamma)} + (1-\varphi(\psi))(1-\xi_h(\theta(\hat{\psi})))\sigma\Delta_\sigma(\hat{\psi})\Bigg\}\Bigg], \tag{F.6}$$

$$\Delta_\sigma(\psi) = \bar{\beta}\mathbb{E}\Bigg[\xi_h(\theta(\hat{\psi}))\gamma\big\{\zeta(\hat{\psi})[S_{H,s}(\hat{\psi}) - S_{L,s}(\hat{\psi})] + (1-\zeta(\hat{\psi}))\big[S_{H,c}(\hat{\psi}) - S_{L,c}(\hat{\psi})]\big\} + \big(1-(1-\xi_h(\theta(\hat{\psi})))\sigma\big)\Delta_\sigma(\hat{\psi})\Bigg], \tag{F.7}$$

$$S_{L,s}(\psi) = R_{L,s}(\psi) - b + \bar{\beta}\mathbb{E}\big[(1-\lambda-\gamma\xi_h(\theta(\hat{\psi}))\zeta(\hat{\psi}))S_{L,s}(\hat{\psi}) - \xi_h(\theta(\hat{\psi}))(1-\zeta(\hat{\psi}))\gamma S_{L,c}(\hat{\psi})\big], \tag{F.8}$$

$$S_{L,c}(\psi) = R_{L,c}(\psi) - b + \bar{\beta}\mathbb{E}\big[(1-\lambda-\gamma\xi_h(\theta(\hat{\psi}))(1-\zeta(\hat{\psi})))S_{L,c}(\hat{\psi}) - \xi_h(\theta(\hat{\psi}))\zeta(\hat{\psi})\gamma S_{L,s}(\hat{\psi})\big], \tag{F.9}$$

$$S_{H,s}(\psi) = R_{H,s}(\psi) - b + \bar{\beta}\mathbb{E}\big[(1-\lambda-\gamma\xi_h(\theta(\hat{\psi}))\zeta(\hat{\psi}))S_{H,s}(\hat{\psi}) - \xi_h(\theta(\hat{\psi}))(1-\zeta(\hat{\psi}))\gamma S_{H,c}(\hat{\psi}) + (1-\xi_h(\theta(\hat{\psi})))\sigma\Delta_\sigma(\hat{\psi})\big], \tag{F.10}$$

$$S_{H,c}(\psi) = R_{H,c}(\psi) - b + \bar{\beta}\mathbb{E}\big[(1-\lambda-\gamma\xi_h(\theta(\hat{\psi}))(1-\zeta(\hat{\psi})))S_{H,c}(\hat{\psi}) - \xi_h(\theta(\hat{\psi}))\zeta(\hat{\psi})\gamma S_{H,s}(\hat{\psi}) + (1-\xi_h(\theta(\hat{\psi})))\sigma\Delta_\sigma(\hat{\psi})\big], \tag{F.11}$$

where $\varphi(\psi) = u_L(\psi)/(u_L(\psi) + u_H(\psi))$ and the revenues of a filled job, $R_{\varepsilon,\chi}(\psi)$, are given by

$$R_{L,S}(\psi) = \delta_s y_s + c'\left(\frac{\alpha q_s(\psi)}{\zeta(\psi)(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))}\right)\frac{\alpha q_s(\psi)}{\zeta(\psi)(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))} - c\left(\frac{\alpha q_s(\psi)}{\zeta(\psi)(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))}\right), \tag{F.12}$$

$$R_{H,S}(\psi) = y_s + c'\left(\frac{\alpha q_s(\psi)}{\zeta(\psi)(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))}\right)\frac{\alpha q_s(\psi)}{\zeta(\psi)(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))} - c\left(\frac{\alpha q_s(\psi)}{\zeta(\psi)(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))}\right), \tag{F.13}$$

$$R_{L,C}(\psi) = \delta_c y_c + c'\left(\frac{\alpha q_c(\psi)}{(1-\zeta(\psi))(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))}\right)\frac{\alpha q_c(\psi)}{(1-\zeta(\psi))(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))} - c\left(\frac{\alpha q_c(\psi)}{(1-\zeta(\psi))(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))}\right), \tag{F.14}$$

$$R_{H,C}(\psi) = y_c + c'\left(\frac{\alpha q_c(\psi)}{(1-\zeta(\psi))(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))}\right)\frac{\alpha q_c(\psi)}{(1-\zeta(\psi))(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))} - c\left(\frac{\alpha q_c(\psi)}{(1-\zeta(\psi))(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))}\right), \tag{F.15}$$

$(q_s(\psi), q_c(\psi))$ solve

$$\frac{\upsilon_2(q_s(\psi), q_c(\psi))}{\upsilon_1(q_s(\psi), q_c(\psi))} = \frac{c'\left(\frac{\alpha q_c(\psi)}{(1-\zeta(\psi))(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))}\right)}{c'\left(\frac{\alpha q_s(\psi)}{\zeta(\psi)(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))}\right)}, \tag{F.16}$$

$$i = (1-\mu)\left[\alpha\left(\frac{\upsilon_1(q_s(\psi), q_c(\psi))\omega(\psi)}{c'\left(\frac{\alpha q_s(\psi)}{\zeta(\psi)(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))}\right)} + \frac{\upsilon_2(q_s(\psi), q_c(\psi))(1-\omega(\psi))}{c'\left(\frac{\alpha q_c(\psi)}{(1-\zeta(\psi))(1-\hat{u}_L(\psi)-\hat{u}_H(\psi))}\right)}\right) + (1-\alpha)\right] - 1, \tag{F.17}$$

and, finally, $P(\hat{\psi}; \psi)$ is consistent with the laws of motion of (u_L, u_H, n_L, i) as defined by (F.1)-(F.4). The model is solved in several steps. First, we approximate the stochastic process for the nominal interest rate using the Rouwenhorst (1995) method, where we estimate the first-order autocorrelation in the monthly nominal Aaa corporate bond rate to be 0.9965 and a standard deviation of 0.0021. Second, we estimate the policy functions $\big(\theta(\psi), \zeta(\psi)\big)$ through a projection algorithm as in Petrosky-Nadeau and Zhang (2017).

Table G1: Comparison of Parameter Values

Parameter	Definition	Skill loss	BMW
	Panel A: Assigned parameters		
ρ	Discount rate	1.68×10^{-3}	1.68×10^{-3}
μ	Probability of exiting the labor force	2×10^{-3}	2×10^{-3}
λ	Separation probability	0.035	0.035
η	Elasticity of matching function	0.50	0.50
γ	Worker's bargaining power	0.50	0.50
y_c	Productivity of high skill workers in complex jobs	1.00	–
y	Productivity of a match	–	1.00
σ	Probability of skill loss	1/3	–
i	Annual nominal interest rate	6.89×10^{-2}	6.89×10^{-2}
a	Elasticity of cost function	1.30	1.30
	Panel B: Calibrated parameters		
A	Matching efficiency	0.590	0.590
b	Value of unemployment	0.554	0.797
k_s	Vacancy posting cost: simple jobs	0.245	–
k_c	Vacancy posting cost: complex jobs	0.589	–
k	Vacancy posting cost	–	0.1865
δ_s	Human capital decay in simple jobs	0.825	–
δ_c	Human capital decay in complex jobs	0.650	–
y_s	Productivity of high skill workers in simple jobs	0.787	–
α	Pr. of consuming in RM	0.050	0.014
ϱ	RM utility weight	1.693	3.127

Notes: Parameters in Panel A are pre-assigned and not calibrated. Parameters listed in Panel B are chosen to minimize the distance between the model and empirical targets described in the text. Each respective column indicates the calibration strategy used to obtain the parameters listed in Panel B.

G Calibration without Skill Loss

In this section, we detail the calibration of the version of the model without skill loss presented in Section 5.6. The model without skill loss essentially boils down to the version of the model presented in Section 2 with $\delta = 1$ or $\sigma = 0$. That is, firms are homogeneous and incur a vacancy posting cost k to open a vacancy. All matches in the labor market produce y units of output. Further, there is only one specialized good, q, produced in the retail market and we assume household's preferences from consuming q units of output in the retail market are given by $\upsilon(q) = \varrho\sqrt{q}$.

We follow the same calibration strategy as outlined in Section 5.2. The only differences are that, in calibrating the BMW version of the model, we do not target a composition of job complexity as the firms are homogeneous and we do not target the effect of unemployment duration of wages as

Table G2: Comparison of Targeted Moments

Moment	Data	Skill loss	BMW
Unemployment rate	0.0590	0.0590	0.0590
$b/\mathbb{E}$[labor productivity]	0.7900	0.7900	0.7900
Fraction of jobs that are simple	0.5250	0.5250	–
Unemployment duration on wages in simple jobs (negative)	0.0093	0.0094	–
Unemployment duration on wages in complex jobs (negative)	0.0193	0.0191	–

Notes: Model moments listed in the first three rows are steady-state outcomes, where $\mathbb{E}$[labor productivity] is the average labor productivity among employed workers. Moments in the last two rows are computed by simulating the model and regressing an individual's wage on their unemployment history in the simulated data. Each respective column indicates whether the moments correspond to the data or a specified version of the model (Skill loss or BMW).

there is no skill loss. Table G1 compares the parameter values between the model with (labelled "Skill loss") and without the skill loss channel (labeled "BMW"). Table G2 compares the targeted moments across the calibrations.