Adverse Selection and Small Business Finances

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Abstract

I propose a motive for firms to hold liquid assets, the *signaling motive*: firms carry liquid assets to signal their investment quality. The signaling motive complements the well-studied precautionary motive: liquidity holdings are useful outside of credit markets when the external credit is impossible or too costly to obtain. Using a directed search model with asymmetric information, this paper rationalizes both motives for small businesses that have a simple capital structure consisting of liquid assets and bank loans. The novelty of this model is the endogenous outside option of borrowing, self-finance, that creates firms' precautionary demand for liquidity, which further allows firms to use it to signal. Thus, in equilibrium, banks use firms' liquidity holdings as well as loan approval rate to screen out risky firms. Without the self-finance option, liquid assets become redundant as both motives disappear. While the equilibrium is distorted, I also show that constrained efficiency can be restored via an appropriately designed tax scheme: taxing the safe borrowers and subsidize the risky ones.

Keywords: Adverse Selection, Competitive Search, Liquidity Holdings, Small Business Finances

JEL codes: D82, D83, G30, E41

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1 Introduction

I study liquid assets held by small businesses who have a simple capital structure consisting of liquid assets and commercial bank loans.[1] One primary goal of this paper is to rationalize the usefulness of liquid assets both inside and outside credit markets. In the traditional literature on corporate liquidity management, many papers have focused on the precautionary motive for holding liquid assets (e.g. T. Opler et al. 1999, Almeida et al. 2004, Bates et al. 2009, and V. Acharya et al. 2012): liquidity acts as a buffer stock when the credit market is not accessible or external credit is too costly to obtain. However, even when the credit market is accessible, liquidity holdings help firms obtain credit: about 7% of commercial business loans are secured by liquid collateral, which is business deposits or securities (such as stocks and bonds) pledged against the loan borrowed, according to the Survey of Small Business Finances (SSBF) 2003.[2] One plausible explanation is that liquidity holdings serve as a screening device in the credit market when lenders cannot distinguish borrowers' ability to repay. Thus, I propose a motive called the *signaling motive*, which causes firms to use liquidity holdings as collateral to signal their ability to repay the loan.

To rationalize the precautionary and signaling motives for holding liquid assets, I introduce an endogenous self-finance channel in a classic screening model by allowing firms to self-support investment using liquidity holdings. I further investigate why the self-finance channel is essential to explain both motives. I also study if the competitive equilibrium can be constrained efficient. If not, I describe what the government intervention can be to mitigate the effects of adverse selection.

I use the Kauffman Firm Survey (KFS) data to provide empirical support for my theo-

[1]Cash and cash equivalents as liquid assets are essential to small businesses. In the U.S., about 28% of total small business assets are cash and cash equivalent, according to the 2011 wave of the Kauffman Firm Survey (KFS). The average cash-to-assets ratio was around 23% among U.S. large and mostly public trading firms in 2006 (Bates et al. 2009). I focus on commercial business loans since they are small firms' most used financial resource; 43% of small firms sought one in 2020, as documented by the Small Business Credit Survey (SBCS).

[2]According to the SSBF 2003, about 55% of commercial business loans are secured by collateral. Among collateralized loans, 13% are secured by liquid collateral, and the rest are secured by illiquid collateral, which can be real estate, equipment, inventory, etc.

retical work. I find that after controlling for the standard liquidity motives in the literature – namely the precautionary, transaction, agency, and tax motives[3] – liquid collateral pledged remains significant in explaining liquidity holdings.[4] Considering the liquidity holdings make up about 28% of an average firm's total assets, firms that plan to pledge liquid collateral hold additional liquidity, equivalent to 20% of total assets, before the initiation of debt. Signaling motive can be a plausible explanation of this phenomenon. Furthermore, my estimations confirm the existence of the precautionary and transaction motives for holding liquid assets.

To rationalize the precautionary and signaling motives for holding liquid assets,[5] I use a general equilibrium approach to liquidity that builds on Lagos & Wright (2005) and Rocheteau & Wright (2005)[6] with directed search[7] and asymmetric information. In the model, there are two types of entrepreneurs: risky and safe, who differ in their investment quality. It is assumed that entrepreneurs know their type, but bankers do not.[8] Entrepreneurs receive random opportunities to invest but cannot issue debt.[9] Thus, in order to finance investment projects, they either use their own liquidity holdings (i.e., the self-finance channel) or borrow from a bank (i.e., the external finance channel). I adapt the mechanism of Guerrieri,

[3]The transaction motive states that firms benefit from holding assets so that they can save on transaction costs, mainly the cost of converting illiquid assets to liquid assets. The agency motive, first proposed by Jensen (1986), suggests that managers accumulate cash even when they do not have good investment opportunities rather than returning cash to firm owners. Another motive is connected to the tax system. The income of C-corporations is taxed at corporate level, and owners have to pay taxes on dividends and capital gains at individual level. In contrast, the income of S-corporations is taxed only once as personal returns. Multinational firms tend to collect cash in foreign accounts to avoid the repatriation tax, as discussed in Fritz Foley et al. (2007).

[4]For empirical papers studying corporate liquidity holdings, see Graham & Leary (2018) using Compustat and Moddy's Industrial Manuals 1920–2014, Bates et al. (2009) using Compustat 1980 – 2006, and Faulkender (2002) using SSBF 1993.

[5]In a simple environment with only one kind of liquid asset, the transaction motive comes with the precautionary and signaling motives. My empirical work shows that the agency and tax motives are not significant for the small firms I study. Thus, in the rest of the paper I restrict attention to the precautionary and signaling motives.

[6]My paper studies entrepreneurs financing investment using liquid assets rather than households financing consumption as in Lagos & Wright (2005), Rocheteau & Wright (2005), and most of their extensions.

[7]One can build a similar model with random search, where entrepreneurs and bankers bargain over loan terms. However, bankers in a directed search model can use loan approval rate to screen, while bankers in a random search model cannot.

[8]Borrowers often have more information about their profitability than lenders do. Crawford et al. (2018) find evidence for adverse selection in credit markets by estimating a structural model of credit demand and default.

[9]This can be rationalized as entrepreneurs' lack of commitment.

Shimer, & Wright (2010), hereafter GSW, to model the external finance channel: bankers post contracts on one side of a competitive and frictional loan market and entrepreneurs search directly on the other side.[10] The ex ante homogenous bankers incur a fixed cost of opening a branch. Bank branches can be located in a place that is characterized by the contract, which is called a submarket. Thus, a banker decides which submarket to enter and open his branch. After observing bankers' entry decisions and contracts, each entrepreneur optimally chooses in which submarket to apply for a loan, where matching is bilateral. A loan contract specifies the loan amount,[11] the down payment (inside collateral),[12] and the repayment. In this framework, search frictions naturally imply that some entrepreneurs will not be successful in acquiring a loan.[13] The loan approval rate (i.e., matching probability) endogenously depends on the entry of bankers to submarkets (i.e., number of loans supplied).

If the entrepreneur foresees the possibility of not obtaining a loan, she will hold some liquidity up front and turn to the self-finance channel when needed; this is the precautionary motive for holding liquid assets. Entrepreneurs use liquidity holdings to purchase capital and finance investment in a frictionless capital market. Thus, entrepreneurs hold liquid assets to better cope with the risk of being financially constrained, as argued by Almeida et al. (2004), etc.

With asymmetric information, liquidity is also essential in the loan market because it signals entrepreneur's investment quality; this is the signaling motive. Risky types have an incentive to pose as the safe type to receive more favorable loan conditions, e.g. lower repayment. As a result, bankers employ two devices to screen out risky types: down pay-

[10]See Wright et al. (2021) for a survey of directed search and competitive search models.

[11]The amount of intraperiod liabilities issued is the amount of loans, which are represented by banknotes. One can assume that banknotes are counterfeitable one period after their issuance, so banknotes will not circulate across periods. Banks commit to redeem banknotes later, so banknotes circulate as payment instruments.

[12]In the real world, there are two types of collateral: inside collateral and outside collateral. Inside collateral, like the down payment in this paper, is pledged assets that are used in the financed project. Outside collateral, on the other hand, is assets not used in the project; for example, inventory pledged against an equipment loan.

[13]Obtaining a loan is a time-consuming process that is not always successful. See Agarwal et al. (2020) for the study of search behavior and loan approval in mortgage markets.

ment and loan approval rate.[14] In equilibrium, there is a unique separating equilibrium where two types of contracts are offered (i.e., two submarkets): (1) a low-down-payment, high-repayment small loan catering to the risky types and (2) a high-down-payment, low-repayment large loan catering to the safe types. Applying for a safe type contract comes with two costs: it requires investing more in costly liquidity beforehand, and it lowers the probability of acquiring a loan. Bankers ask for a large down payment that is paid by liquidity holdings because they know that the safe types have a stronger precautionary motive. As the safe types have higher marginal benefit of holding liquid assets in the self-finance channel, they hold more liquid assets to to insure themselves against their potential failure in the loan market. In addition to asking for a large down payment, bankers endogenously make the safe type contracts scarce, so the probability of obtaining such a loan becomes lower. Because the safe types receive higher surplus when they borrow and carry large amount of liquidity as buffer stock, they are more willing to accept this lower probability in return for better loan terms when they do get to borrow. In contrast, the risky types are not willing to accept this lower probability for their inferior projects. This logic explains why down payment and loan approval rate are used to prevent the risky types from seeking a safe type contract. To sum up, liquidity serves two roles: to self-finance investment as an outside option to bank loans and to signal the borrower's ability to repay.

Key to these results is the endogenous self-finance channel. Without self-finance, the model reduces to GSW with an application to asset markets. Safe types have no underlying incentive to hold liquid assets, so it is optimal to screen using only loan approval rate, rendering the liquid asset redundant. In addition to the self-finance channel, directed search is also crucial to my results. If the decentralized search is replaced with centralized contract posting, then my model boils down to Rothschild & Stiglitz (1976). With a centralized loan market, entrepreneurs obtain loans with certainty, and the self-finance channel becomes inactive. In turn, the precautionary motive disappears, and liquidity is nothing more than

[14] See Coco (2000) for a survey of the use of collateral as a screening or an incentive device in credit markets with asymmetric information.

a costly signal to obtain loans. To screen entrepreneurs, banks use only down payment.

As is common in models with adverse selection, the competitive equilibrium is in general not efficient. Under complete information where entrepreneurs' type is observable, submarkets are independent from each other; the change in the entrepreneurs' payoff in one submarket does not affect the entrepreneurs' payoff in the other submarket. Under private information, however, bankers face an additional incentive compatibility constraint to ensure that entrepreneurs are willing to reveal their type and apply for loans designed for them. Submarkets are no longer independent from each other since the change in one type's payoff affects the other type's payoff through the incentive compatibility constraint that bankers face in the other submarket. The change in this constraint alters the set of feasible contracts that bankers can offer to attract the other type of entrepreneurs and thus affects their payoff. Bankers in one submarket of the market economy take the entrepreneurs' payoff in the other submarket as given; bankers cannot internalize this externality, but a planner can.

To think about government intervention that can improve total welfare, it is useful to analyze a planner's problem. I consider a constrained efficiency problem akin to the direct mechanism from Davoodalhosseini (2019): a social planner who faces the same information and search frictions chooses loan contracts and liquidity holdings, or equivalently transfers contingent on contracts, so that an entrepreneur will reveal her type and receive a contract that depends on her revealed type. I show that under some conditions, a utilitarian planner can always achieve higher welfare than in the market economy by subsidizing the risky types and taxing the safe types. Intuitively, when the risky types are subsidized, they have a higher payoff and lower incentive to misreport their type. This leads to a less tight incentive compatibility constraint that bankers face in the safe submarket. Bankers then screen with lower intensity, and allocations are less distorted: down payments become smaller and more loans are offered. The safe types benefit from lower screening intensity because the cost of carrying liquidity is reduced and the expected investment is increased. Moreover, when this benefit is greater than the cost of being taxed, the safe types are also better off with cross-subsidization. Such allocation Pareto dominates the competitive equilibrium allocation. This result is reminiscent of the finding of Greenwald & Stiglitz (1986); the competitive equilibria with adverse selection may not be constrained Pareto optimal.

The optimal intervention depends on how a planner values the payoff of agents when considering the weighted total welfare. I find a sufficient condition such that a utilitarian planner, who cares about all entrepreneurs equally, can completely undo the effect of adverse selection and recover the complete information allocations and welfare using cross-subsidization: taxing the safe types and subsidizing the risky types. This happens when the opportunity cost of holding liquid assets is not too low, there are very few risky types, or the risky types' net benefit of applying for a safe type contract is small. At the other extreme, for a planner who may not care about all entrepreneurs equally, I provide a sufficient condition for the distorted competitive equilibrium to be constrained efficient, in which case no transfers are needed. This happens when the population or the failure probability of the risky types is large so that subsidizing them is costly. This result corresponds with findings in Rothschild & Stiglitz (1976).

In the baseline, it is assumed that banknotes can only be used to purchase capital for investment, which is equivalent to observable investment. In the extension, by relaxing this assumption and allowing banknotes to be used to buy consumption goods, I further incorporate moral hazard into the model and study dual deviation: entrepreneurs may not only misreport their type but also deviate from the investment level that is expected by the banker. As a result, in addition to liquidity holdings and loan approval rate, bankers employ size of the loan as an extra screening tool, which means allocations are distorted in an additional dimension. I also find that equilibrium allocations are not only more distorted but also more likely to be distorted, in the sense that distortions occur in a large area of the parameter space.

The rest of my paper is organized as follows: Section 2 overviews the related literature, Section 3 gives empirical support, Section 4 outlines the theoretical model, Section 5 describes the equilibrium, Section 6 examines the constrained efficiency problem, Section 7 incorporates moral hazard, and Section 8 concludes the paper.

2 Literature Review

My paper ties into the literature on adverse selection (e.g., Akerlof 1970, Spence 1973, Levin 2001). To illustrate, I review Rothschild & Stiglitz (1976), one of the pioneer models. They study competitive and centralized insurance markets where only the insurants know the probability of their house burning down. The authors show the presence of insurants with different risks can lead to the nonexistence of insurance in equilibrium. In my paper, if directed search is replaced with centralized contracts posting, then my model boils down to the Rothschild and Stiglitz model, and equilibrium may not exist when there are very few risky types or the risky types' probability of failure is too small relative to the safe types. GSW solve this nonexistence problem by replacing the competitive market with a frictional market with directed search and capacity limits, thereby providing a tractable general framework to analyze adverse selection in competitive search markets.[15] In my paper, the competitive bank loan market builds on the model of GSW. However, the drawback of this version of GSW, and of most standard screening models with collateral (e.g., Besanko & Thakor 1987 and Bester 1985, 1987) is that the outside option of borrowing is set exogenously, preventing the precautionary motive to be operative. The possibility for entrepreneurs to self-support investment endogenizes the outside option and makes the model consistent with the empirical importance of the precautionary motive, in additional to making liquidity essential in the world of GSW.

Stiglitz & Weiss (1981) develop a canonical model of credit rationing, in which among loan applicants who have different risks but seem to be identical some of them are fully funded but some are denied for a loan, although the rejected applicants are willing to pay a higher interest rate. The phenomenon that some borrowers obtain a loan and some do

[15] In the context of GSW environment, the capacity limit is that one banker can serve only one borrower.

not is similar to my model, but the failure of obtaining a loan is caused by search frictions rather than credit rationing. In the circumstance of credit rationing, equilibrium interest rate fails to clear excess demand of credit because raising interest rate will lower bank's profit; however, this cannot happen in a competitive bank loan market.

Another paper related to mine is Leland & Pyle (1977), which study signaling through self-finance. In their setup, entrepreneurs have private information about the quality of their projects and are in need of external funds. When the level of projects' self-financing is observable, there exists a continuum of signaling equilibrium, where entrepreneurs with good projects choose to self-finance a fraction of their projects. This is likewise to my finding that safe types provide larger down payment. However, my model has a unique separating equilibrium due to the incorporation of directed search rather than a continuum of equilibrium.

My paper also contributes to the literature on corporate liquidity management. Rocheteau et al. (2018) use the dynamic general equilibrium approach to liquidity to study internal and external finance in a complete information environment with random search. When an entrepreneur and banker meet bilaterally and bargain over loan terms, liquidity holdings improve the entrepreneur's bargaining position because an entrepreneur with more liquidity has a better outside option. I incorporate asymmetric information by replacing random search and bargaining with directed search and loan contracts posting. In my model, liquidity is essential for signaling. Specifically, for signaling reasons, entrepreneurs hold liquid assets to satisfy down payment requirements and obtain better loan terms.[16]

When studying the constrained efficiency problem, I borrow the direct mechanism of GSW developed in Davoodalhosseini (2019). He shows that a planner who maximizes the weighted average of the payoff to agents can attain a first best allocation by conducting cross-

[16] There are also many papers focusing on the precautionary motive for holding liquid assets. For example, Almeida et al. (2004) model precautionary demand for liquidity and find that financially constrained firms invest in liquidity, while unconstrained firms do not. However, they fail to consider how liquidity holdings help entrepreneurs to obtain external finance. Kim et al. (1998) show firms demand more liquidity when external finance costs or future investment returns increase; my model confirms their results.

subsidization and relaxing incentive constraints (e.g., Miyazaki 1977 and Spence 1978). I show that a similar result holds in my model. Moreover, I also consider a generalized constrained efficiency problem with a planner who weights agents arbitrarily. I prove that under some conditions, the competitive equilibrium can be constrained efficient. Similar result is discussed in Rothschild & Stiglitz (1976). Crocker & Snow (1985) consider efficiencies of the equilibrium notion such as Miyazaki-Wilson (Miyazaki 1977 and Wilson 1977) in the setup of Rothschild & Stiglitz (1976) and show competitive equilibrium can be constrained efficient.

In addition, my paper is related to empirical studies of liquid collateral usage. Berger et al. (2016) use Bolivian business loan data to show that loans with liquid collateral are associated with lower interest rates and lower default or delinquency rates than similar loans with illiquid or no collateral. These findings match with the equilibrium loan terms in my model; loans designed for the safe type entrepreneurs require liquid collateral and smaller repayment. To my knowledge, similar studies have not been done using U.S. data. Because KFS does not record ex post outcomes or interest rates of loans, I am not able to address weather loans with liquid collateral perform better empirically in this paper.

There are empirical papers verifying findings from the planner's problem: constrained efficiency can be restored through cross-subsidization. Cowan et al. (2015) use Chilean loan data to study the impact of a loan guarantee program, which is one of the most common ways to subsidize small firms.[17] They find that firms that borrow with guarantees are more likely to default than similar firms that borrow without guarantees. This higher default rate is caused by adverse selection: firms enrolled in the guarantee program are generally weaker or riskier. They also show that loan guarantees increase the amount of credit supplied both at the intensive margin (i.e., loan size) and the extensive margin (i.e., number of new loans

[17] A business loan guarantee program is widely used in most OECD countries. For example, in 2019, the U.S. Small Business Administration (SBA) guaranteed over 28 billion dollars to entrepreneurs. A business loan guarantee program (e.g., the SBA 7(a) Loan Program) is designed to help small businesses obtain financing when they might not be eligible for a reasonably priced business loan. The SBA does not make loans but rather guarantees a portion of loans made by commercial lending institutions. In case of default, the SBA will compensate a portion of the loan to the issued institution.

issued). The supply of guaranteed loans increases because they are the ones being subsidized, but, surprisingly, the supply of non-guaranteed loans also increases. These findings match with my results from the constrained efficiency problem that is equivalent to the problem faced in the loan guarantee program: using cross-subsidization, a planner can improve loans supplied both in the intensive margin (i.e., lower down payment) and the extensive margin (i.e., higher probability of getting a loan), thereby increasing overall welfare. Bachas et al. (2021) study the U.S. loan guarantee program and its credit supply, but they do not address the impact of the program on non-guaranteed loans or the overall credit supply.

3 Empirical Support

In this section, I provide empirical support for the existence of signaling and precautionary motives for holding liquid assets. For this purpose, I use the Kauffman Firm Survey (KFS), which is a longitudinal survey of new businesses in the U.S. The KFS collected information on 4928 new firms and surveyed them every year from 2004 to 2011. After removing the firms that have missing information or permanently closed before 2011, there are 660 firms left in my balanced panel data. Information on these firms includes industry, capital structure (equity and debt), employment, and firm owner characteristics. One drawback of the data that significantly limits the scope of my study is that the type of debt collateral is recorded only in the last three years of the survey. In my sample, an average firm has total assets amounting to \$284,526 and holds 27.9% of them as liquid assets. About 2.5% of all firms have pledged liquid collateral to obtain loans, and about 13.8% have pledged illiquid collateral. Many of the firms surveyed are financially constrained; specifically, 3.4% of them have difficulties in obtaining external credit and 13.1% are in need of credit but choose to not apply for it due to possible denial.[18]

I adopt a dynamic approach which allows me to study firms' adjustment in liquidity holdings over time. It is common that the targeted level of liquidity holdings cannot be

[18] Column 1 in Table A1 presents summary details for the sample of the KFS data.

instantaneously achieved due to transaction and other adjustment costs, so a lag term of liquidity is included in the regression as an explanatory variable. To verify the existence of precautionary, transaction, tax and agency motives, I follow the traditional methods on studying corporate cash holdings (e.g., Ozkan & Ozkan 2004, Bates et al. 2009, Faulkender 2002, and Bigelli & Sánchez-Vidal 2012). I also add liquid collateral pledged to capture the intention of using liquid assets as collateral to receive favorable loan terms, which serves as a proxy of the signaling motive. The empirical counterpart of the liquidity demand function can be written as follows:

$$\begin{aligned} Lqd\_Ass_{i,t} = \ & \beta_{lla} Lqd\_Ass_{i,t-1} + \beta_{flc} LqdC_{i,t} \\ & + \beta_{rd} RD\_Ass_{i,t} + \beta_{cp} C\_prob_{i,t} + \beta_{a} ln\_Ass_{i,t} + \beta_{mo} MO_{i,t} + \beta_{cc} C\_corp_{i,t} \\ & + X_{i,t} \cdot \beta_X + Firm_i + \epsilon_{i,t}, \end{aligned} \tag{1}$$

where $Lqd\_Ass_{i,t}$ is the liquidity-to-assets ratio of firm i at the end of year t, $Lqd\_Ass_{i,t-1}$ is the lagged liquidity-to-assets ratio (Ozkan & Ozkan 2004), $LqdC_{i,t}$ equals 1 if liquid collateral is pledged to obtain any of the debt financing options that are used in year $t+1$ and 0 if otherwise, $RD\_Ass_{i,t}$ is the ratio of research and development (R&D) expenditure to total assets in year t (Bates et al. 2009), $C\_prob_{i,t}$ equals 1 if the firm considers itself as having difficulties in obtaining external credit in year t and 0 if otherwise (Faulkender 2002), $ln\_Ass_{i,t}$ is the natural log of total assets (Bates et al. 2009), $MO_{i,t}$ equals 1 if the primary owner is the manger and 0 if otherwise (Faulkender 2002), $C\_corp_{i,t}$ equals 1 if the firm is a C-corporation and 0 if otherwise (Faulkender 2002), $X_{i,t}$ is a vector of firm characteristics (Bigelli & Sánchez-Vidal 2012), $Firm_i$ is the firm fixed effect that is used to capture time-invariant liquidity preference, and $\epsilon_{i,t}$ is the error term.[19]

[19]This liquid collateral can be pledged against any type of debt, including business loans, credit cards, and lines of credit. However, I am unable to further narrow down the type of debt financing due to the limitation of the data. Firm financial characteristics controls are natural log of sales, total-expenditure-to-assets ratio, profit margin, total-liability-to-assets ratio, total-loan-to-assets ratio, dividends-to-assets ratio, natural log of remaining available credit, credit score, if the firm has multiple locations, if the firm provides services, if the firm has comparative advantages (patent, trademark, or copyright), if the firm considers itself as having

	Theoretical prediction	(1) OLS	(2) FE	(3) FDT	(4) FDT-IV	(5) FDT-IV
lagged liquidity-to-assets	+	0.4920***	-0.1833***	-0.3445***	0.2781*	0.2766*
		(0.0259)	(0.0383)	(0.0285)	(0.1515)	(0.1516)
future liquid collateral	+	-0.0009	-0.0075	0.0238	0.1023**	0.1983**
		(0.0381)	(0.0668)	(0.0387)	(0.0513)	(0.0954)
R&D-to-assets	+	0.0221*	0.0116	0.0142	0.0910***	0.0905***
		(0.0128)	(0.0157)	(0.0136)	(0.0174)	(0.0174)
credit prob.	+	0.0061	0.0461	0.0636*	0.0935*	0.0993*
		(0.0252)	(0.0445)	(0.0343)	(0.0525)	(0.0527)
ln(assets)	−	-0.0426***	-0.0879***	-0.0741***	-0.0893***	-0.0895***
		(0.0062)	(0.0142)	(0.0111)	(0.0168)	(0.0168)
C-corp	+	0.0021	-0.0729	-0.0319	-0.4090	-0.4419
		(0.0301)	(0.1514)	(0.1393)	(0.3010)	(0.2943)
primary-owner-manager	−	0.0135	0.0127	0.0109	-0.0013	0.0002
		(0.0143)	(0.0288)	(0.0220)	(0.0338)	(0.0339)
firm financial char.		Yes	Yes	Yes	Yes	Yes
firm FE		No	Yes	No	No	No
adj R-sqrd		0.3386	0.5701	0.2006	.	.
N		1980	1980	1320	660	660
F-stat on lagged liq.-to-assets					65.34	32.77
F-stat on future liq. col.						14.87
Cragg-Donald Wald F-stat						62.056

Note: *** p<0.01, ** p<0.05, * p<0.1. I further restrict firms to have positive revenues and non-negative liquidity holdings (cash and cash equivalent), and to primarily conduct business in the U.S. Standard errors are clustered at the industry level. Year 2008–10 observations are included in (1) and (2), year 2009–10 observations are included in (3) and (4), and only year 2010 observations are included in (5).

Table 1: **Dynamic Panel Data Estimation**

Regression results are presented in Table 1. The ordinary least square (OLS) in column 1 estimates Equation (1) without the firm fixed effect, yielding an estimated *future liquid collateral* coefficient of -0.0009. In this case, the *lagged liquidity-to-assets* ratio is positively correlated with the error term, and the correlation will not vanish with a large number of firms. By standard results for omitted variable bias, the OLS estimator of the *lagged liquidity-to-assets* ratio, 0.4920, is biased upwards. Although the *lagged liquidity-to-assets* is not of direct interest, resolving its endogeneity issues helps recover a consistent estimate of *future liquid collateral*. The one-way fixed effects (FE) model in column 2 estimates Equation (1), yielding an estimated *future liquid collateral* coefficient of -0.0075. As Nickell (1981) shows,

cash flow problems, and if the firm needs credit but has not applied for it because of possible denial.

the FE estimator of the *lagged liquidity-to-assets* ratio, -0.1833, is biased downwards because the demeaning process which subtracts the individual's mean value from the respective variable creates a correlation between the *lagged liquidity-to-assets* ratio and the error term.[20] One solution to this problem involves taking the first difference transformation (FDT) that removes both the constant and the firm fixed effect:

$$\begin{aligned}\Delta Lqd\_Ass_{i,t} = \; & \beta_{lla}\Delta Lqd\_Ass_{i,t-1} + \beta_{flc}\Delta LqdC_{i,t+1} \\ & + \beta_{rd}\Delta RD\_Ass_{i,t} + \beta_{cp}\Delta C\_prob_{i,t} + \beta_{a}\Delta ln\_Ass_{i,t} + \beta_{mo}\Delta MO_{i,t} + \beta_{cc}\Delta C\_corp_{i,t} \\ & + \Delta X_{i,t} \cdot \beta_X + \Delta\epsilon_{i,t},\end{aligned} \tag{2}$$

where $\Delta Y_{i,t} = Y_{i,t} - Y_{i,t-1}$ for a variable Y. In column 3, OLS is then used to estimate the FDT of Equation (2), resulting in an estimated *future liquid collateral* coefficient of 0.0238. Nevertheless, the FDT introduces correlation between the *lagged differenced liquidity-to-assets* ($\Delta Lqd\_Ass_{i,t-1}$) and the differenced errors ($\Delta\epsilon_{i,t}$) through the *lagged liquidity-to-assets* ($Lqd\_Ass_{i,t-1}$) and the lagged error ($\epsilon_{i,t-1}$). This issue advocates the Anderson-Hsiao estimator (Anderson & Hsiao 1981) in column 4, which uses the *second lag of liquidity-to-assets* in the form of differences ($\Delta Lqd\_Ass_{i,t-2}$) as an instrument for the *lagged differenced liquidity-to-assets* ratio, resulting a *second lag of liquidity-to-assets* coefficient of 0.2781 and a *future liquid collateral* coefficient of 0.1023. A positive coefficient of *future liquid collateral* can imply two plausible explanations: firms collect more liquidity and then pay for liquid collateral in the future, or firms choose to pay for liquid collateral because they have collected large amount of liquidity in the past. Another estimation problem is that the decision of pledging liquid collateral in the next year is endogenous and may depend on the omitted variables. To further address these issues, in column 5, I instrument the *future differenced liquid collateral* using the *current liquid collateral* in the form of differences ($\Delta LqdC_{i,t}$).[21]

[20] Nickell (1981) demonstrates that the bias can be quite large when the total number of periods (T) is small. In a large sample, the bias becomes $\text{plim}_{N\to\infty}(\hat{\beta}_{lla} - \beta_{lla}) = -(2+\beta_{lla})(1+\beta_{lla})/2$ for $T = 3$, where $\hat{\beta}_{lla}$ is the FE estimator. Suppose the true β_{lla} is 0.28, then the bias can be -1.46.

[21] The validity of instruments requires the absence of higher-order serial correlation in the idiosyncratic

This additional instrument resolves the omitted variable issue and rules out the possibility that firms pay for liquid collateral since they have large amount of liquidity holdings. The Anderson-Hsiao strategy yields an estimated signaling motive of 0.1983, indicating that a firm that chooses to obtain debts and pledge liquid collateral would hold more liquid assets, equivalent to 19.83% of its total assets, before the debt application than similar firms that choose to pledge illiquid or no collateral.[22] In other words, firms hold more liquidity ex ante to help themselves obtain external credit with favorable terms by pledging liquid collateral.[23] There are many ways that liquid collateral can help firms obtain loans. One plausible explanation is that firms use liquid collateral to signal their ability to repay, which is the focus of this paper. Another is that firms pledge liquid collateral to improve their bargaining position during the negotiation with banks, as hypothesized by Rocheteau et al. (2018). My results verify both scenarios; however, I am not able to separate them due to the limitation of the data. Also note that the estimates might understate the signaling motive since the desire to pledge liquid collateral is not directly observed in the data. It is possible that some firms want to obtain a loan and pledge liquid collateral but fail to get one due to various reasons, leaving excess amount of liquid assets in their accounts. So those firms who have the desire are failed to be identified by the regression. This increases the liquidity holdings of firms whom are assumed of not having a desire. Hence, the regression might underestimate the true effect of signaling motive.

The results of column 5 pass this consistency test. The Cragg-Donald Wald F-statistic is 62.056, which is larger than the critical value of the Stock-Yogo weak identification test of 7.03, rejecting weak instruments. Bond (2002) suggests that the candidate consistent estimator of *lagged liquidity-to-assets* (0.2766) should live between the OLS and FE estimator

component of the error term ϵ. However, with only one year of observations included in the regression in column (5), it is no longer a concern in this case.

[22] Arellano & Bond (1991) find that the Anderson-Hsiao estimator fails to take all potential orthogonality conditions into account. However, due to the data limitation that the KFS records the type of collateral only in the last three years, the FDT with generalized method of moments (GMM), where the instruments applicable to each period of equation differ, cannot be feasibly applied here.

[23] Similar results using the SSBF are shown in Appendix B.

(0.4920 and −0.1833) because these two estimators are biased in opposite directions.

In addition to showing the existence of a signaling motive, my estimations also show the existence of a precautionary motive, as found in Faulkender (2002) using SSBF 1993. R&D measures growth opportunities and is usually financed with cash. A firm that considers itself as having growth opportunities invests more in R&D and holds more liquidity so that it would not miss any investment opportunities, for the precautionary motive.[24] A firm that considers itself as having difficulties in accessing external credit should hold more liquid assets since the firm's marginal benefit from liquidity is higher. Hence, by the precautionary motive, the coefficient of *R&D-to-assets* and *credit problem* should be positive. The estimates in column 5 confirm the existence of a precautionary motive. Spending one percentage additional assets on R&D leads to higher liquidity holdings, equivalent to 0.09% of the firm's total assets. Also, a firm that has difficulties in raising external credit carries additional liquid assets, equivalent to 9.93% of its total assets.

My estimations also verify the transaction motive, as larger firms hold a lesser amount of liquidity proportionally, as shown in Mulligan (1997) using Compustat data. According to the theory, a larger firm holds relatively less liquidity than a smaller firm because of the increasing economies of scale in liquidity holdings. If firm size is measured by total assets, then the estimates in column 5 suggest that a 1 percentage point increase in total assets leads to lower amount of liquidity holdings, equivalent to 0.09% of the firm's total assets.

However, my estimations fails to show the existence of tax and agency motives. The agency motive, first proposed by Jensen (1986), suggests that managers accumulate cash even when they do not have good investment opportunities rather than returning cash to firm owners. If the firm's primary owner is the manager, then liquidity holdings should be lower because the agency motive is lower. So the coefficient of *primary-owner-manager* should

[24] T. C. Opler & Titman (1994) use R&D expenditure as a proxy for financial distress costs. In time of distress, firms would cut R&D expenditure, which means giving up investment opportunities. So firms that spend the most in R&D are those firms that would lose the most in case of distress. However, a reduction in R&D expenditure may not be caused only by indebtedness, but also by limited access to external credit. Thus, firms have an incentive to hold more liquid assets as R&D expenditure increases, regardless of indebtedness.

have a negative sign. The tax motive is tied to the tax system. Because a C-corporation is taxed at both corporate income and dividend payout, a C-corporation has a larger incentive to hold liquid assets, resulting in a positive coefficient of *C-corp*. The coefficients of tax and agency motives across my estimations are against the theoretical prediction; however, they are not statistically significant.

4 Model

4.1 Agents and Assets

Time is discrete and infinite, $t = 1, 2, ...$ Each period is divided into two subperiods: in the first subperiod, there is a decentralized frictional banking market (DM) together with a competitive capital market; in the second, there is a centralized Walrasian market (CM) where agents settle debts and trade production goods and assets. All agents discount between the CM and DM at $\beta \in (0, 1)$. This alternating CM and DM structure helps to keep track of the distribution of assets. In this setting, there are three groups of agents: a unit measure of entrepreneurs (e) who want to invest and need capital, a large number of ex ante homogenous bankers (b) who issue loans, and a large number of capital producers (p) who provide capital. There are two types of entrepreneurs: the risky types (r) with measure ν_r and the safe types (s) with measure ν_s. They differ in the probability of success; with probability δ_j, $j = r, s$, type-j entrepreneurs will have a profitable investment project. With probability $1 - \delta_j$, the entrepreneur's investment will be unprofitable; then she will leave the market, and a new entrepreneur with the same type will be born to replace her. The safe types are more likely to be successful than the risky types, $\delta_r < \delta_s$. Type is entrepreneurs' private information, and bankers know only the distribution of entrepreneurs. All agents are risk neutral and have linear utility over numeraire goods c, where $c > 0$ is consumption and $c < 0$ is production.

There is one type of liquid asset; for example, fiat money or T-bills. The supply of liquid assets evolves according to $M_{t+1} = (1 + \pi)M_t$, where π is the growth rate of liquid asset supply implemented by some unproductive government consumption. The price of liquid assets in terms of numeraire is q_t^m, and in a stationary world $q_t^m = (1+\pi)q_{t+1}^m$, as suggested by the quantity theory of money.[25] The real rate of return on liquid assets is $1 + r^z = q_{t+1}^m/q_t^m = 1/(1+\pi)$. As it is usually done, I impose $\pi \geq \beta - 1$ or equivalently $r^z \leq 1/\beta - 1$.

The timeline of my model is displayed in Figure 1. In the CM, the idiosyncratic shock on investment is revealed. If the investment turns out to be successful, an entrepreneur with previously acquired capital k produces $f(k)$ units of numeraire goods, where $f(0) = 0, f'(0) = \infty, f'(\infty) = 0$, and $f'(k) > 0 > f''(k), \forall k > 0$. For simplicity, k fully depreciates after production.[26] If the investment turns out to be a failure, an entrepreneur produces nothing and exits the market; meanwhile, a new entrepreneur is born to replace her.[27] Entrepreneurs then choose the consumption and production and the real balances of liquidity z. At the same time, bankers decide whether to enter the bank loan market. If a banker enters this market, he pays a fixed cost $\tilde{\kappa}$ and posts a loan contract $\Omega = (d, R, \ell)$ that specifies the down payment d, the repayment R, and the loan amount ℓ measured in numeraire.[28] Also, $\Omega \in \mathbf{\Omega}$, where $\mathbf{\Omega} \subset \mathbb{R}_+^3$ is the set of contracts. The down payment is paid with the liquidity holdings, and the loan amount is the amount of banknotes (deposit claims) issued that are valid only for capital purchases.[29] As in GSW, bankers face capacity limits; one banker can serve only one entrepreneur.[30]

In the DM, entrepreneurs' opportunity to invest arrives randomly with probability α, as in Kiyotaki & Moore (1997), in which case they can have production technology f. En-

[25]Suppose government consumption at period t is G_t. The policy maker's budget constraint is $G_t + q_t^m M_t = q_{t+1}^m M_{t+1}$. If $\pi > 0$ (resp. $\pi < 0$), the policy maker consumes positive (resp. negative) amount.

[26]One can interpret it as renting capital every period.

[27]I assume entrepreneurs consume all remaining assets and leave the market. However, in equilibrium entrepreneurs would not have any assets at this point.

[28]Posting one contract is equivalent to posting a menu of contracts in this environment, skin to GSW. I also formally prove it in Appendix C.

[29]I relax this assumption in Section 7 by allowing the entrepreneurs to use the banknotes for purchasing consumption goods.

[30]Another interpretation is that each bank has a large collection of bankers. In that sense, a bank can serve multiple entrepreneurs, and banker in this paper is similar to a vacancy in the Diamond-Mortensen-Pissarides models.

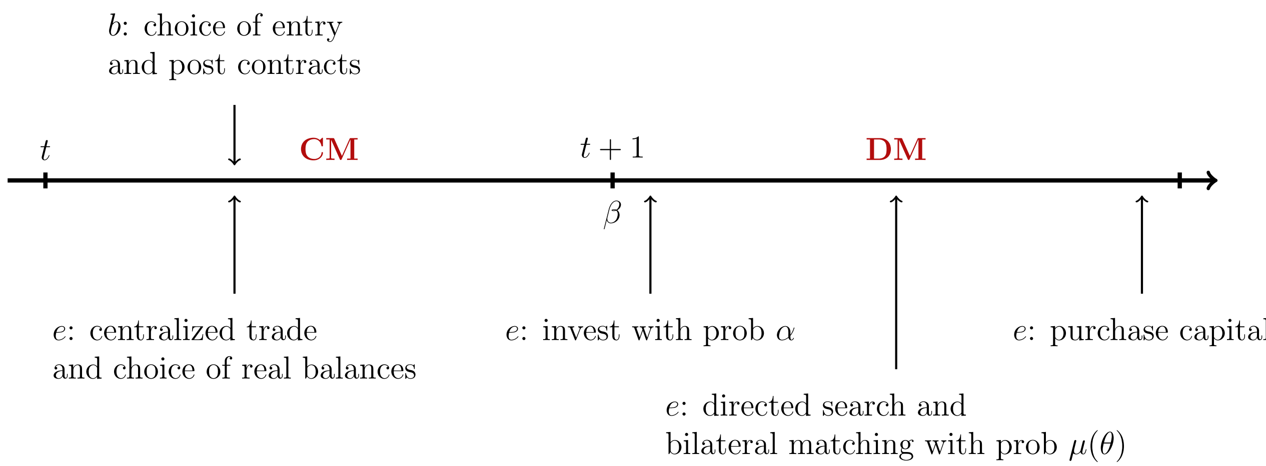
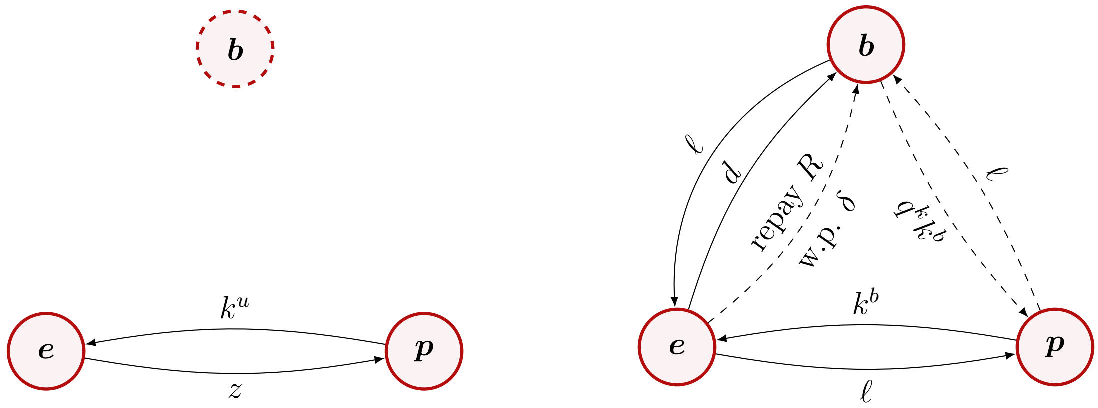


Figure 1: **Timeline**

trepreneurs who have an investment opportunity can participate in the bank loan market; then, they meet with a banker and obtain a loan with some endogenous probability, which is discussed in detail in the next subsection. Meanwhile, in the competitive capital market, capital is produced by capital producers at unit cost and is sold at price q^k.

There are two channels to finance investment: self-finance and external finance. The self-finance channel is illustrated in Figure 2a. If an entrepreneur fails to meet a banker, she turns to the capital market and uses z to buy k^u, which is the amount of capital bought when she is unbanked. Because entrepreneurs have limited commitment, trade credits are ruled out; therefore, liquidity holdings are essential in the self-finance channel.

The external finance channel is illustrated in Figure 2b, where the entrepreneur is matched with a banker. Bankers have an advantage in enforcing repayments from borrowers, meaning that entrepreneurs cannot renege on loan repayments unless they have an unprofitable project and exit the market. If an entrepreneur obtains a loan, she passes a d amount of liquid assets to the banker for an ℓ amount of banknotes, and promises to pay back an R amount of numeraire. Then she goes to the capital market and uses ℓ in exchange for k^b, which is the amount of capital purchased when she is banked. In the next period, the entrepreneur produces $f(k^b)$ and pays back R with probability δ. The capital producer then goes to



(a) Self-finance w/ liquidity holdings

(b) External finance w/ banknotes

Figure 2: **Self-finance vs External Finance:** The left panel shows the self-finance channel, in which bankers b are inactive. The right panel shows the external finance channel, in which bankers b participate and banknotes circulate. Solid arrows indicate transactions that happen before production, and dashed arrows indicate transactions that happen after production.

the bank and uses ℓ to claim a $q^k k^b$ amount of numeraire after production. Bankers are assumed to have deep pockets to back up their liabilities and thus can commit. So even if the entrepreneur fails to pay back R, the capital producer can still get $q^k k^b$ back from the banker. Hence, banknotes circulate as inside liquidity and facilitate trade in the external finance channel.[31]

4.2 Matching in the Loan Market

A submarket of the loan market consists of a set of bankers posting loan contracts and a set of entrepreneurs searching for those loans. It is assumed that bankers and entrepreneurs meet pairwise (i.e., there is a bilateral matching technology). Each banker in a given submarket posts a single contract Ω_j, $j = r, s$, that is designed for a type-j entrepreneur. Entrepreneurs observe all posted contracts and then choose the contract that is in their best interest. Any contract Ω_j is associated with a market tightness $\theta(\Omega_j) : \mathbb{R}^3_+ \to [0, \infty]$, which is the bankers-to-entrepreneurs ratio in submarket j. A share of entrepreneurs applying for Ω_j that are

[31]The case that banknotes circulate is equivalent to the case that k^b is passed to bankers first and then to entrepreneurs. But in either case bankers issue liabilities and have full commitment on their liabilities.

type-j is $\lambda_j(\Omega_j) \geq 0$. Entrepreneurs' probability of getting a loan (i.e., the matching probability) depends on the market tightness in that submarket. An entrepreneur searching in submarket-j obtains a type-j contract with probability $\mu(\theta_j)$, independent of entrepreneur's type, where the matching function $\mu : [0, \infty] \to [0, 1]$ is strictly increasing and continuous. With probability $1 - \mu(\theta_j)$, the entrepreneur is unmatched and needs to self-finance the investment project. A banker offering contract-j matches with an entrepreneur with probability $\eta(\theta_j)$, where $\eta : [0, \infty] \to [0, 1]$ is strictly decreasing and continuous, and otherwise is unmatched. Let $\mu(\theta) = \theta\eta(\theta)$ for all θ and $\eta(\infty) = \mu(0) = 0$. The market tightness and probability of matching are endogenously determined in equilibrium.

4.3 Optimization Problem

At the beginning of the CM, a type-j entrepreneur with capital k and financial wealth w, including liquidity holdings minus debts denominated in numeraire goods, has value $W_j(k, w)$ as follows:

$$\begin{aligned} W_j(k, w) = &\max_{c_j, z'_j} \quad c_j + \beta V_j(z'_j) \\ &\text{s.t. } \; c_j + \frac{z'_j}{1 + r^z} \leq f(k) + w, \end{aligned}$$

where $V_j(z'_j)$ is the expected continuation value of the type-j entrepreneur in the DM of next period with a new liquidity holding z'_j. The constraint is the budget constraint indicating that the change in financial wealth, $w - \frac{z'_j}{1+r^z}$, is covered by retained earnings, $f(k) - c_j$. Eliminating c_j using the binding budget constraint, the objective function becomes

$$W_j(k, w) = w + f(k) + \max_{z'_j} \; \Big\{ -\frac{z'_j}{1 + r^z} + \beta V_j(z'_j) \Big\}.$$

W_j is linear in financial wealth and output, so the choice of z'_j is independent of (k, w).

In the DM, a type-j entrepreneur applies for a loan contract while observing the set of contracts posted in the market, $\mathbf{\Omega}^{pt}$, and then purchases capital for production.

$$
\begin{aligned}
V_j(z_j) = \max_{\Omega_j \in \mathbf{\Omega}^{pt}, k_j^b, k_j^u} \quad & \alpha\mu(\theta(\Omega_j))\Big[\delta_j[f(k_j^b) + z_j - d_j - R_j + W_j^0] + (1-\delta_j)(z_j - d_j)\Big] \\
& + \alpha(1-\mu(\theta(\Omega_j)))\Big[\delta_j[f(k_j^u) + z_j - q^k k_j^u + W_j^0] + (1-\delta_j)(z_j - q^k k_j^u)\Big] \\
& + (1-\alpha)\Big[z_j + W_j^0\Big] \\
\text{s.t.} \;\; & z_j \geq d_j, \;\; q^k k_j^b \leq \ell_j, \;\; q^k k_j^u \leq z_j,
\end{aligned}
$$

where $W_j^0 \equiv W_j(0,0)$ is a constant representing type-j entrepreneur's continuation value with zero financial wealth and capital. The first component describes the expected value of using the external finance channel. The entrepreneur obtains a loan Ω_j with probability $\alpha\mu(\theta(\Omega_j))$ that depends on the chosen submarket Ω_j and purchases capital k^b. With probability δ_j, the investment is successful, and the entrepreneur produces $f(k_j^b)$ and has wealth $z_j - d_j - R_j$. With probability $1-\delta_j$, the entrepreneur consumes his remaining wealth $z_j - d_j$ and leaves the market afterwards. The second component describes the expected value of using the self-finance channel: the entrepreneur fails to obtain a loan with probability $\alpha(1-\mu(\theta(\Omega_j)))$ and purchases capital k^u. With probability δ_j, the investment is successful and the entrepreneur produces $f(k_j^u)$ and has wealth $z_j - q^k k_j^u$. With probability $1-\delta_j$, the entrepreneur consumes his remaining wealth $z_j - q^k k_j^u$ and leaves the market afterwards. The third component describes the case of not having an investment opportunity with probability $1-\alpha$. The first constraint is the feasibility constraint (FC), meaning that an entrepreneur applying for Ω_j must hold at least a d_j amount of liquidity. The type-j entrepreneur purchases k_j^b at cost $q^k k_j^b$ using banknotes ℓ_j when she is banked (i.e., matched with a banker). She purchases k_j^u at cost $q^k k_j^u$ using liquidity holding z_j when she is unbanked. Because banknotes can be used only to purchase capital, entrepreneurs will use up all ℓ_j, $\ell_j = q^k k_j^b$. When liquidity is costly to hold, entrepreneurs have no incentive to hold more liquidity than $q^k k_j^*$, which is the cost to purchase an efficient amount of capital characterized by $\delta_j f'(k_j^*) = 1$, and then spend all liquidity holdings to purchase capital, $z_j = q^k k_j^u$. It is easy to show that if the competitive capital market is active, then the price of capital $q^k = 1$.[32] If a liquid asset is costly to hold, i.e., $r^z \leq 1/\beta - 1$, producers have no incentive to hold liquid assets.

Substituting V_j into W_j, a type-j entrepreneur who anticipates contracts posted, $\mathbf{\Omega}^{pt}$, makes a portfolio choice and investment decision according to

$$U_j = \max_{z_j, \Omega_j \in \mathbf{\Omega}^{pt}} \quad -iz_j + \alpha\mu(\theta(\Omega_j))[\delta_j f(\ell_j) - d_j - \delta_j R_j] + \alpha(1-\mu(\theta(\Omega_j)))[\delta_j f(z_j) - z_j] \tag{3}$$

$$\text{s.t. } z_j \geq d_j,$$

where i is the opportunity cost of holding liquidity, $i = 1/(\beta(1+r^z)) - 1$, and U_j is the instantaneous payoff of type-j when she actively searches for a contract that is designed for her type in the loan market.

Meanwhile, consider the value function of a banker with financial wealth w in the CM as follows:

$$W_b(w) = w + \max_{\text{enter,not enter}} \Big\{ -\tilde{\kappa} + \max_{z_b', \Omega_j'} \big\{ -\frac{z_b'}{1+r^z} + \beta V_b(z_b', \Omega_j') \big\},\ \max_{z_b'} \big\{ -\frac{z_b'}{1+r^z} + \beta V_b(z_b', \mathbf{0}) \big\} \Big\},$$

where $V_b(z_b', \Omega_b')$ is the expected continuation value of the banker in the DM of the next period with a new liquidity holding z_b' and contract Ω_j' posted for type-j entrepreneurs, and $V_b(z_b', \mathbf{0})$ is the expected value if the banker carries z_b' but chooses to not enter the loan market.

[32] The value function of a capital producer with financial wealth w in the CM is

$$W_p(w) = w + \max_{z'} \Big\{ -\frac{z'}{1+r^z} + \beta V_p(z') \Big\}.$$

In the DM,

$$V_p(z) = \max_{k \geq 0} \Big\{ -k + W_p(z + q^k k) \Big\}.$$

Hence, the producer produces k at unit cost and sells at price q^k so that his financial wealth increases by $q^k k$. Using the linearity of W_p, $q^k = 1$ and $V_p(z) = W_p(z)$.

In the DM,

$$V_b(z_b,\Omega_j) = \eta(\theta(\Omega_j))\Big[z_b + \sum_{i=r,s}\lambda_i(\Omega_j)(d_j - \ell_j + \delta_i R_j) + W_b^0\Big] + (1-\eta(\theta(\Omega_j)))\Big[z_b + W_b^0\Big],$$

where $W_b^0 \equiv W_b(0)$ representing banker's continuation value with zero financial wealth. If the banker posted Ω_j, he will be matched with probability $\eta(\theta(\Omega_j))$, which is endogenously determined in the submarket Ω_j. Here $\lambda_i(\Omega_j)$ is the share of type-i entrepreneurs applying for Ω_j, and $d_j - \ell_j + \delta_i R_j$ is the expected profit collected from type-i. So, $\sum_{i=r,s}\lambda_i(\Omega_j)(d_j - \ell_j + \delta_i R_j)$ is the banker's total profit collected by posting Ω_j. With probability $1-\eta(\theta(\Omega_j))$, the banker will be unmatched and have zero profit in the DM. Likewise to producers, bankers have no incentive to hold liquid assets, so $z_b' = 0$. Thus, bankers' one-period net profit earned from posting Ω_j is

$$\Pi(\Omega_j) = -\kappa + \eta(\theta(\Omega_j))\sum_{i=r,s}\lambda_i(\Omega_j)(d_j - \ell_j + \delta_i R_j), \tag{4}$$

where $\kappa = \tilde{\kappa}/\beta$ is the fixed cost measured in numeraire of period $t+1$. If the bank chooses to not post any contract, the bank has value $V_b(z_b,\mathbf{0}) = z_b + W_b^0$.

An entrepreneur may prefer not participating in the loan market. Let $\hat{U}_j$ be the entrepreneur's payoff of not entering the loan market (i.e., of using the self-finance channel only):

$$\hat{U}_j = \max_{z_j} \;\; -iz_j + \alpha[\delta_j f(z_j) - z_j]. \tag{5}$$

Entrepreneurs' decision of entry is thus made according to

$$\mathcal{U}_j = \max_{\text{enter, not enter}} \Big\{U_j, \hat{U}_j\Big\},$$

where $\mathcal{U}_j$ is type-j entrepreneur's payoff.

5 Equilibrium

In this section, I first define and characterize the equilibrium under complete information. Then I characterize the equilibrium under incomplete information and compare the allocations with those of the complete information case. Moreover, I discuss how classic models are nested into mine and why the endogenous self-finance channel is critical to firm capital structure and loan contracts posted.

5.1 Symmetric Information

I first determine equilibrium allocations in the case of perfect information, where both the entrepreneur and the banker know the entrepreneur's type. This part serves as a benchmark for the case of asymmetric information, where the banker does not observe the entrepreneur's type. Under symmetric information, bankers simply offer loan contracts that are conditional on entrepreneur's type, $\Omega_j^* = (d_j^*, R_j^*, \ell_j^*)$, and $\theta_j^* = \theta(\Omega_j^*)$. The payoff of a type-j entrepreneur while participating in the loan market is $U_j^* = U(\Omega_j^*)$, and the net profit of the banker is $\Pi_j^* = \Pi(\Omega_j^*)$.

Now define an equilibrium under symmetric information.

Definition 1 *In the case of complete information, there exists a stationary equilibrium that consists of*

i). payoffs U_j^* and $\hat{U}_j$, $j = r, s$, such that if $U_j^* > \hat{U}_j$, Ω_j^*, θ_j^*, and z_j^* solve (3) with $\mathbf{\Omega}^{pt} = \{\Omega_j^*\}$ and the free-entry condition $\Pi_j^* \geq 0$ as in (4) with $\lambda_i(\Omega_j) = 0$, $i \neq j$; if $U_j^* \leq \hat{U}_j$, $\hat{z}_j$ solves (5);

ii). price q^m solves $\sum_{j=r,s} \nu_j \bar{z}_j = q^m M$, where $\bar{z}_j = z_j^*$ if $U_j^* > \hat{U}_j$ and $\bar{z}_j = \hat{z}_j$ otherwise.

Suppose type-j entrepreneurs participate in the loan market; the market designer solves the following simplified problem:[33]

$$U_j^* = \max_{z_j, \Omega_j, \theta_j} \quad -iz_j + \alpha\mu(\theta_j)[\delta_j f(\ell_j) - d_j - \delta_j R_j] + \alpha(1-\mu(\theta_j))[\delta_j f(z_j) - z_j] \qquad \text{(P}^*\text{-j)}$$

$$\text{s.t.} \;\; z_j \geq d_j, \;\; \eta(\theta_j)(d_j - \ell_j + \delta_j R_j) \geq \kappa.$$

If there exists a competitive search equilibrium as first defined by Moen (1997), bankers make zero expected profit and the free-entry condition must bind. Because there is no need to screen entrepreneurs, $d_j^* = 0$. The loan amount is issued at the optimal value:

$$\delta_j f'(\ell_j^*) = 1, \tag{6}$$

such that the marginal gain of having one more unit of capital when the type-j entrepreneur is banked equals the marginal cost. Repayment R_j^* is such that the free-entry condition binds. Given θ_j^*, liquidity holding is characterized by

$$\alpha(1-\mu(\theta_j^*))[\delta_j f(z_j^*) - 1] = i, \tag{7}$$

where the left hand side is the marginal gain of carrying one more unit of liquidity into the self-finance channel and the right hand side is the marginal cost of holding liquidity. Given z_j^*, market tightness is implicitly given by

$$\mu'(\theta_j^*)[\delta_j f(\ell_j^*) - \ell_j^* - \delta_j f(z_j^*) + z_j^*] = \kappa, \tag{8}$$

where the left hand side is the marginal increase of matching probability in market tightness times the surplus of a match in the submarket-j, and the right hand side is the cost of posting

[33] In this setup, the market designer's problem is the same as the entrepreneur's problem and the banker's problem. Since bankers make zero expected profit, the total welfare in a submarket equals the payoff of entrepreneurs. Thus in each submarket the market designer maximizes the payoff of entrepreneurs that is also the objective of entrepreneurs. Likewise, bankers maximize the payoff of entrepreneurs in order to attract more borrowers.

a contract. In other words, there is an efficient number of contracts supplied in the loan market.[34]

Suppose type-j entrepreneurs do not participate in the bank loan market; then liquidity holding $\hat{z}_j$ is characterized by

$$\alpha[\delta_j f(\hat{z}_j) - 1] = i, \tag{9}$$

where the marginal benefit of bringing one more unit of liquidity when the entrepreneur needs to make an investment equals the marginal cost of holding liquidity.

Lemma 1 *Under complete information, for $j = r, s$,*

1. *if $U_j^* > \hat{U}_j$, liquidity holdings z_j^* and market tightness θ_j^* satisfy Equation (7) and (8), and contract $\Omega_j^* = (d_j^*, \ell_j^*, R_j^*)$ satisfies $d_j^* = 0$, Equation (6), and the binding free-entry condition holds;*

2. *otherwise, submarket-j is inactive and liquidity holdings $\hat{z}_j$ satisfies Equation (9).*

As shown in Equation (6)-(8), the loans issued to the safe types are larger, more loans are supplied to them, and they carry more liquidity for the precautionary reason.

Proposition 1 *With symmetric information, the loan amount, market tightness, and liquidity holdings of the safe types are greater than those of the risky types, i.e., $\ell_s^* > \ell_r^*$, $\theta_s^* > \theta_r^*$, $z_s^* > z_r^*$.*

Proof. See Appendix E. ■

Bankers provide larger loans to the safe types simply because the safe types are more likely to succeed than the risky types. More loans are supplied in the safe type submarket

[34] One can rewrite condition (8) and get $\eta(\theta_j^*)E(\theta_j^*)[\delta_j f(\ell_j^*) - \ell_j^* - \delta_j f(z_j^*) + z_j^*] = \kappa$, where $E(\theta_j^*) = \frac{\partial \theta_j^* \eta(\theta_j^*)}{\partial \theta_j^*} \frac{\theta_j^*}{\theta_j^* \eta(\theta_j^*}$ is the elasticity of the matching probability of the entrepreneur with respect to θ_j^*. This condition implies that there is an efficient number of banks entering the loan market.

since investments made by the safe types generate a larger surplus. Using Equation (7), it is also easy to see that with perfect information, liquidity is held to cope with the risk of not getting a loan. So liquidity is held only for the precautionary motive. The safe types in fact have a stronger precautionary motive, as $z_s^* > z_r^*$. This is important to the decision of screening tools that bankers use under asymmetric information, which is discussed in the following subsection.

5.2 Asymmetric Information

Under asymmetric information, bankers do not directly observe entrepreneurs' type, but they can screen entrepreneurs by offering different loan contracts. In equilibrium, bankers offer profit-maximizing loan contracts subject to the free entry condition, and entrepreneurs direct their search to the most preferred contract, conditional on the contracts offered and entrepreneurs' beliefs. A stationary equilibrium with contract posting is defined as follows:

Definition 2 *A competitive search equilibrium is a set of entrepreneurs' payoff $\mathcal{U}_j$, $j = r, s$, liquidity holdings $z(\Omega)$ and $\hat{z}_j$, market tightness $\theta(\Omega)$, and market composition $\lambda_j(\Omega)$ defined over $\mathbf{\Omega}$, a cumulative distribution function $\Gamma(\Omega)$, a set of posted contracts $\mathbf{\Omega}^{pt} \subset \mathbf{\Omega}$, and price of the liquid asset q^m that satisfy the following conditions:*

(i). bankers' profit maximization and free entry: for any $\Omega \in \mathbf{\Omega}$,

$$-\kappa + \eta(\theta(\Omega)) \sum_{j=r,s} \lambda_j(\Omega)[d(\Omega) - \ell(\Omega) + \delta_j R(\Omega)] \leq 0,$$

with equality if $\Omega \in \mathbf{\Omega}^{pt}$;

(ii). entrepreneurs' optimal search: let

$$\mathcal{U}_j = \max\{U_j, \hat{U}_j\},$$

where $\mathcal{U}_j = \hat{U}_j$ *if* $\boldsymbol{\Omega}^{pt} = \emptyset$*; then for any* $\Omega \in \boldsymbol{\Omega}$ *and* j*,*

$$U_j \geq U_j(\Omega),$$

with equality if $\theta(\Omega) < \infty$ *and* $\lambda_j(\Omega) > 0$*, where*

$$\begin{aligned} U_j &= \max_{\Omega \in \boldsymbol{\Omega}^{pt}} \; U_j(\Omega) \\ &= \max_{\Omega \in \boldsymbol{\Omega}^{pt}} \; \{-iz(\Omega) + \alpha\mu(\theta(\Omega))[\delta_j f(\ell(\Omega)) - d(\Omega) - \delta_j R(\Omega)] \\ &\qquad + \alpha(1-\mu(\theta(\Omega)))[\delta_j f(z(\Omega)) - z(\Omega)]\}; \end{aligned}$$

moreover, if $U_j < \hat{U}_j$*,* $z(\Omega) = 0$*,* $\hat{z}_j = \arg\max \; \hat{U}_j$ *and either* $\theta(\Omega) = \infty$ *or* $\lambda_j(\Omega) = 0$*;*

(iii). feasibility:

$$\int_{\boldsymbol{\Omega}^{pt}} \frac{\lambda_j(\Omega)}{\theta(\Omega)} d\Gamma(\Omega) \leq \nu_j \;\; \textit{for any} \;\; j,$$

with equality if $U_j > \hat{U}_j$*;*

(iv). and liquid asset market clears:

$$\sum_{j=r,s} \nu_j \bar{z}_j = q^m M,$$

where $\bar{z}_j = \int_{\boldsymbol{\Omega}^{pt}} \frac{\lambda_j(\Omega)}{\theta(\Omega)} z(\Omega) d\Gamma(\Omega)$ *if* $U_j > \hat{U}_j$ *and* $\bar{z}_j = \hat{z}_j$ *if otherwise.*

The first set of conditions (i) determines the loan terms in each submarket; (ii) given the loan terms, pins down the corresponding market tightness θ and liquidity holding z; (iii) ensures all type-j entrepreneurs apply for the same type of contract when the payoff from participating in the loan market is larger than that of not participating; and (iv) determines the price of the liquid asset.

In Appendix C, I show the results of Guerrieri, Shimer, & Wright (2010) also apply in my setting: there exists a unique competitive search equilibrium with contract posting that is payoff-equivalent to a competitive search equilibrium with revelation mechanisms, in which entrepreneurs prefer to reveal their type. Without loss of generality, posting a menu of contracts is the same as posting one contract for bankers. In separating equilibrium, any contract that attracts type-j entrepreneurs should solve the following optimization problem P-j: for any type $j = r, s$,

$$
\begin{aligned}
U_j = \max_{z_j,(d_j,\ell_j,R_j),\theta_j} \quad & -iz_j + \alpha\mu(\theta_j)[\delta_j f(\ell_j) - d_j - \delta_j R_j] + \alpha(1-\mu(\theta_j))[\delta_j f(z_j) - z_j] \qquad \text{(P-j)} \\
\text{s.t.} \quad & z_j \geq d_j, \quad \eta(\theta_j)(d_j - \ell_j + \delta_j R_j) \geq \kappa, \\
& -iz_j + \alpha\mu(\theta_j)[\delta_{\tilde{j}} f(\ell_j) - d_j - \delta_j R_j] + \alpha(1-\mu(\theta_j))[\delta_{\tilde{j}} f(z_j) - z_j] \leq U_{\tilde{j}}, \\
& \text{for any participating } \tilde{j} \neq j. \quad \text{(IC-}\tilde{\text{j}}\text{j)}
\end{aligned}
$$

Relative to the problems under symmetric information (P*-j), this problem adds a incentive compatibility constraint (IC-$\tilde{\text{j}}$j), which makes sure that any entrepreneur who is not type-j should be better off by applying for a contract that is designed for her, when both types of entrepreneurs participate in the loan market. The left (resp., right) hand side of the constraint is the value of a type-$\tilde{j}$ entrepreneur planning to search in the submarket featuring Ω_j (resp., $\Omega_{\tilde{j}}$) with tightness θ_j (resp., $\theta_{\tilde{j}}$) and thus holding z_j (resp., $z_{\tilde{j}}$) before entering the bank loan market tomorrow.

The equilibrium is featured multiple types of equilibrium allocations that depend on the relative riskiness between entrepreneurs, δ_r/δ_s, and the opportunity cost of holding liquid assets, i. Figure 3a illustrates the risky types' different types of equilibrium allocations in the parameter space of δ_r and i, holding other parameters (e.g., δ_s) constant. (1) *No participation*: when liquid assets are relatively cheap to hold, i.e., when i is small or κ is large, $\hat{U}_r > U_r$ and the risky type entrepreneurs are better off by self-finance investment, since liquid assets are relatively cheap to hold. (2) *No screening*: when liquid assets become more expensive to hold, i.e., i becomes larger, the risky types choose to participate in the loan market. As might be expected with adverse selection (e.g., Mirrlees 1971), the incentive compatibility constraint for the risky types (IC-sr) is slack; in other words, the safe type entrepreneurs have no incentive to mimic the risky types. The risky types' problem under asymmetric information (P-r) thus becomes identical to the one under symmetric information (P*-r). As a result, $U_r = U_r^*$ and the allocations of the risky types are not distorted by adverse selection, i.e., $z_r = z_r^*$, $d_r = d_r^* = 0$, $\ell_r = \ell_r^*$, $R_r = R_r^*$, and $\theta_r = \theta_r^*$.

Figure 3b presents the safe types' different types of equilibrium allocations. (1) *No participation*: when i is very small, $\hat{U}_s > U_s$ and the safe types prefer self-finance to bank loans. (2) *No screening*: when i becomes larger, the safe types choose to participate in the loan market, but the risky types do not. Then the safe types are the only type of borrowers in the loan market, and their problem does not face an incentive compatibility constraint. So allocations and payoffs are not distorted, i.e., $z_s = z_s^*$, $d_s = d_s^* = 0$, $\ell_s = \ell_s^*$, $R_s = R_s^*$, $\theta_s = \theta_s^*$, and $U_s = U_s^*$. (3) *Screening with z*: if the risky types enter the loan market but are very unlikely to succeed compared to the safe types (i.e., $\delta_r << \delta_s$), then a small amount of down payment ($d_s = z_s^*$) is enough to make contracts incentive compatible. This is because the risky types have a very small need for investment, so a small down payment requirement is enough to turn the risky types away. There is no distortion in market tightness even when there is screening via asset holding. Thus, allocations are $z_s = d_s = z_s^*$, $\ell_s = \ell_s^*$, $\theta_s = \theta_s^*$, R_s solves the binding free entry condition, and payoffs are not distorted, $U_s = U_s^*$.[35] (4) *Screening with z and θ*: if the risky types enter the loan market and are not too different from the safe types, safe types' allocations are distorted and safe types have lower payoffs than the complete information case, $U_s < U_s^*$, due to the binding upward incentive compatibility constraint (IC-rs). Using the first-order conditions (FOC) of problem P-s, ℓ_s is characterized by

$$\delta_s f'(\ell_s) = 1, \tag{10}$$

[35] The difference between the scenario *No screening* and *Screening with z* is down payment. In the scenario *No screening*, $d_s = 0$ because there is no need to screen entrepreneurs. In the scenario *Screening with z*, $d_s = z_s^*$ and the safe types do not need to bring more liquidity than z_s^*. So in either scenario, the safe type entrepreneurs have payoff U_s^* and undistorted allocations.

meaning that the loan amount ℓ_s is optimal. It is also easy to show that entrepreneurs pledge all liquidity holdings as down payment, i.e., $d_s = z_s$. Here θ_s and d_s solve

$$\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s - \delta_s f(z_s) + z_s] = \kappa \tag{11}$$

and the binding (IC-rs) constraint. Given θ_s, ℓ_s, and d_s, the binding free entry condition characterizes R_s. The equilibrium allocations when both types of entrepreneurs participate in the loan market can be summarized as follows.

Lemma 2 *In the separating equilibrium when all entrepreneurs participate in the loan market with asymmetric information,*

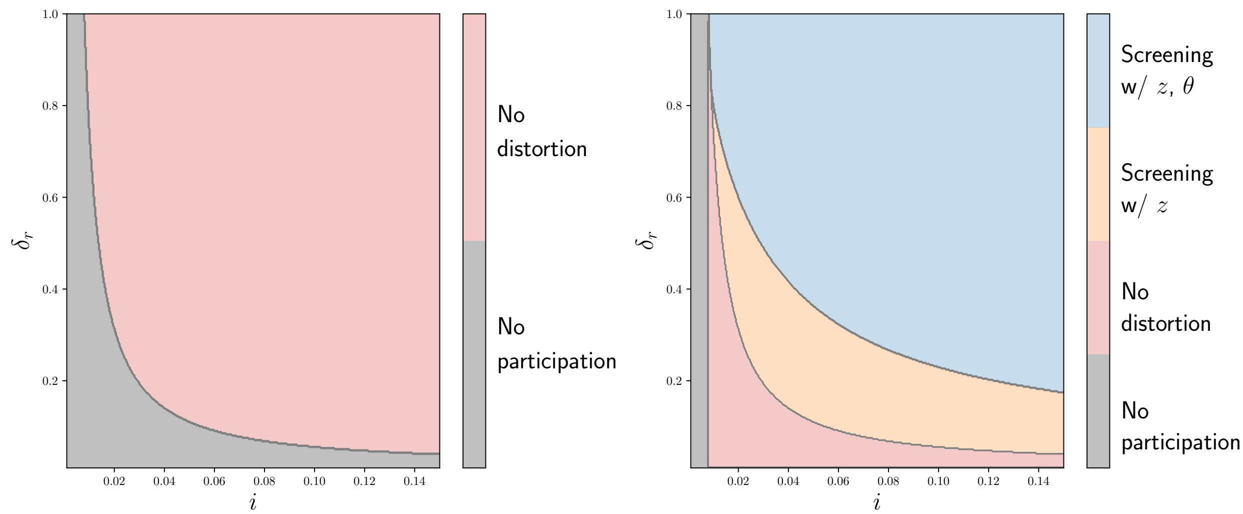
1. *for the risky types, the allocations are the same as the ones under symmetric information, $(z_r, d_r, R_r, \ell_r, \theta_r) = (z_r^*, d_r^*, R_r^*, \ell_r^*, \theta_r^*)$;*
2. *for the safe types, market tightness θ_s and liquidity holdings z_s solve Equation (11) and the binding IC-rs constraint, and the loan contract $\Omega_s = (d_s, \ell_s, R_s)$ satisfies $d_s = z_s$, Equation (10), and the binding free-entry condition.*

The next Proposition compares the allocations under asymmetric information with the ones under complete information in the parameter space *screening with z and θ*.

Proposition 2 *The allocations under asymmetric information are distorted in the extensive margin; in particular, $d_s = z_s > z_s^*$ and $\theta_s < \theta_s^*$. However, in the intensive margin, loan amount is not distorted; $\ell_s = \ell_s^*$.*

Proof. See Appendix F. ■

In this case, the loan contracts posted under symmetric information are no longer incentive compatible; the risky types have an incentive to misreport and choose a contract designed for the safe types so that the risky types can repay less. As a result, bankers use



(a) The risky types

(b) The safe types

Figure 3: **Types of Equilibrium Allocations:** $\delta_s = 1$. Appendix D describes other parameters and functions used in the numerical examples.

two devices to screen out the risky entrepreneurs in the submarket of safe types: down payment (i.e., liquidity holdings) and market tightness (i.e., probability of matching). Knowing that the safe type entrepreneurs have a higher marginal benefit of holding liquidity when they are credit constrained (i.e., they have a stronger precautionary motive), bankers ask for a large down payment from anyone applying for a safe type contract. The safe types in turn hold more liquidity to satisfy this requirement. Bankers also make the safe type contract hard to obtain by making the supply scarce. Because the safe types have higher surplus of getting a loan, they are more willing to accept a lower probability of getting a loan in return for better loan terms when they do get to borrow.[36] However, once bankers are able to use down payment and market tightness to screen out the risky types, they have no incentive to distort the amount of banknotes provided to the borrowers once the loan is issued. Hence, loan amount is always at the optimal level.

The precautionary and signaling motives of holding liquid assets can be shown using the

[36] Another way to interpret the probability of matching is the waiting period or the queue length. Because the risky types have an inferior investment project, their surplus of getting matched is low and therefore they are less willing to take on the cost of waiting.

following equation. Taking the FOC with respect to z_s, the marginal benefit (MB) of holding one more unit of z_s must equal the marginal cost (MC) of doing so:

$$\underbrace{\alpha(1-\mu(\theta_s))[\delta_s f'(z_s)-1]}_{\text{MB of self-finance}} + \underbrace{\left[-\Delta\frac{\partial U_r(\Omega_s)}{\partial z_s}\right]}_{\text{MB of signaling}} = \underbrace{i}_{\text{MC}}, \tag{12}$$

where Δ is the Lagrange multiplier of the IC-rs constraint that solves

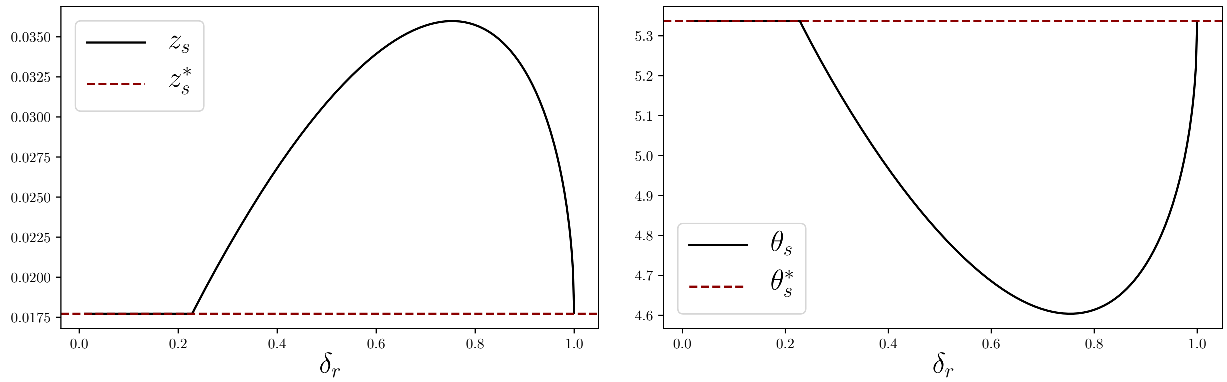
$$\Delta = \frac{-i+\alpha(1-\mu(\theta_s))[\delta_s f'(z_s)-1]}{-i-\alpha\mu(\theta_s)\delta_r/\delta_s+\alpha(1-\mu(\theta_s))[\delta_r f'(z_s)-1]}, \tag{13}$$

and $U_r(\Omega_s)$ is type-r's payoff of choosing Ω_s. Because liquidity z_s is too big for the risky types, it is costly for them to bring in z_s for the down payment requirement d_s, i.e., $\frac{\partial U_r(\Omega_s)}{\partial z_s} = -i-\alpha\mu(\theta_s)\delta_r/\delta_s+\alpha(1-\mu(\theta_s))[\delta_r f'(z_s)-1] < 0$. The first component of the left hand side represents the marginal gain of using one more unit of liquidity to self-finance when the entrepreneur wants to invest but fails to obtain a loan. The second component represents the marginal gain of holding one more unit of liquidity to signal the entrepreneur's type. Intuitively, the safe type entrepreneurs would like to hold a large amount of z_s and pledge all z_s as down payment until the amount of z_s is too large so that the risky type entrepreneurs have no incentive to do so. Therefore, as shown in Equation (12), under asymmetric information, there are two motives for holding liquidity: precautionary and signaling.

Another finding is that screening intensity, i.e., the level of z and θ, is non-monotone in the relative riskiness. This is caused by two competing forces: repayment saved by misreporting (the benefit of misreporting) and the usefulness of liquid assets in the self-finance channel (the cost of misreporting) for the risky types.

Proposition 3 *The safe types' liquidity holdings z_s and market tightness θ_s are non-monotone in δ_r: there exists a cutoff $\bar{\delta}_r$ such that*

i). z_s increases (resp., decreases) in δ_r when $\delta_r < \bar{\delta}_r$ (resp., $\delta_r > \bar{\delta}_r$).



(a) Liquidity holdings

(b) Market tightness

Figure 4: **Screening Intensity**: $i = 0.1$, $\delta_s = 1$.

ii). θ_s decreases (resp., increases) in δ_r when $\delta_r > \bar{\bar{\delta}}_r$ (resp., $\delta_r < \bar{\bar{\delta}}_r$).

Proof. See Appendix G. ■

How z_s and θ_s change in δ_r is illustrated in Figure 4. When δ_r is very small, the risky types prefer not participating in the loan market (*No participation* in Figure 3b) or a small down payment $d_s = z_s^*$ is enough to prevent them from misreporting (*No screening* in Figure 3b), so z_s and θ_s are the same as the ones under complete information. Then, consider the case when two screening devices are used (*Screening with z and θ* in Figure 3b). When δ_r increases but remains very low comparing with δ_s, even though the benefit of misreporting is large for the risky types, the cost of mimicking the safe types is large since the marginal benefit of bringing liquid assets is very low outside the loan market. When δ_s is large, the cost of mimicking the safe types is small since the risky types are not very different from the safe types, but the benefit of misreporting is small. So when δ_r is small or large, a small d_s and a lenient θ_s are enough to screen out the risky types. However, when δ_r is not too small or too big, bankers have to use a large d_s and a tight θ_s to screen out the risky types; thus, allocations are the most distorted, and payoff U_s is the lowest.

Then, I classify types of equilibrium allocations as presented in Figure 3.

Proposition 4 *For any δ_r, there exists cutoffs $\underline{i}$, $\bar{i}$, and $\bar{\bar{i}}$ that are ranked, $\underline{i} < \bar{i} < \bar{\bar{i}}$, and* *(i). if $i \leq \underline{i}$, both risky and safe types do not enter the loan market;*

(ii). if $i \in (\underline{i}, \bar{i}]$, risky types do not enter the loan market, but safe types do;

(iii). if $i \in (\bar{i}, \bar{\bar{i}}]$, both types enter, and bankers use down payment to screen;

(iv). if $i > \bar{\bar{i}}$, both types enter, and bankers use both down payment and market tightness to screen.

Proof. See Appendix H. ∎

When $i > \bar{\bar{i}}$, multiple screening tools are used by banks. Allocations of safe type entrepreneurs are distorted, so do their payoffs, $U_s < U_s^*$. In Section 6, I restrict attention to the case of $i > \bar{\bar{i}}$ to consider whether government interventions can improve total welfare.

Note that, as in GSW, pooling equilibrium never exists in this model because of search friction and capacity limit. Suppose a banker posts a pooling contract designed to attract all types of entrepreneurs. The more entrepreneurs that search for this pooling contract, the less likely it is for anyone to be matched. This lower matching probability corresponds with longer queue in front of the bank that offers a pooling contract. This longer queue discourages entrepreneurs from seeking it. In particular, because the safe types have a higher surplus of obtain a separating contract than the risky types, they will leave the queue first. As a result, the risky types are ones remain in the queue, leaving the banker unprofitable. Therefore, pooling contracts are never offered.

5.3 Nested Models

In this subsection, I aim to show how classic models are nested into mine, as summarized in Table 2. If the decentralized direct search is replaced with centralized contracts posting, my model coincides with that of Rothschild & Stiglitz (1976).[37] Liquidity holdings and loan

[37] One can relabel things, and my model becomes identical to that of Rothschild & Stiglitz once search friction is removed.

contracts solve the following problem: for $j = r, s$,

$$U_j^{RS} = \max_{z_j,(d_j,\ell_j,R_j)} -iz_j + \alpha[\delta_j f(\ell_j) - d_j - \delta_j R_j] \tag{P$^{\text{RS}}$-j}$$

$$\text{s.t. } z_j \geq d_j, \;\; d_j - \ell_j + \delta_j R_j \geq \kappa,$$

$$-iz_j + \alpha[\delta_{\tilde{j}} f(\ell_j) - d_j - \delta_{\tilde{j}} R_j] \leq U_{\tilde{j}}^{RS} \text{ for } \tilde{j} \neq j. \tag{IC$^{\text{RS}}$-$\tilde{\text{j}}$j}$$

Due to the absence of search friction, entrepreneurs will always get matched with their preferred contract once they participate in the loan market. The self-finance channel is thus inactive, and the precautionary motive for holding liquidity is missing. By the standard single-crossing property,[38] IC$^{\text{RS}}$-rs binds but IC$^{\text{RS}}$-sr does not. Then for the risky types, $z_r = d_r = 0$, ℓ_r solves $\delta_r f'(\ell_r) = 1$, and R_r solves the binding free entry condition. For the safe types, $z_s = d_s > 0$ solves the binding IC$^{\text{RS}}$-rs condition, ℓ_s solves $\delta_s f'(\ell_s) = 1$, and R_s solves the binding free entry condition. Liquidity holdings therefore serve as a pure costly signal, and down payment is the only screening device used by bankers.

In this Rothschild & Stiglitz environment, a competitive equilibrium with bank loans need not exist. First, as shown in Rothschild & Stiglitz (1976), a pooling equilibrium can never exist. Suppose there is a pooling equilibrium; a banker can deviate from the pooling contract by offering the safe type a contract with better loan terms. Then this banker attracts all safe types in the market and makes a strictly positive profit, which is in contradiction with the competitive loan market. So if an equilibrium exists, it must be a separating equilibrium. Second, suppose there is a separating equilibrium; a banker can make a strictly positive profit by offering a pooling contract when the cost to pool is small, e.g., the risky and safe types are very alike in terms of success probability, or the measure of the risky type entrepreneurs is small. By selecting the pooling contract, the safe types can be better off because they do not need to make a down payment, although they need to subsidize the risky types. The risky types also benefit from the pooling contract simply because they are being subsidized.

[38] If the risky types prefer the safe type contract, then the safe types must as well. See Milgrom & Shannon (1994).

		Rothschild & Stiglitz No search friction	Guerrieri, Shimer & Wright No self-finance	This paper
Liquidity	Precautionary	✗	✗	✓
	Signaling	✓	✗	✓
Screening	Down payment	✓	✗	✓
	Market tightness	✗	✓	✓

Table 2: **Nested Models**

Therefore, when the cost to pool is small (or the cost of separating is high), an equilibrium with bank loans does not exist. If the bank loan equilibrium does not exist or loans are too expensive ($U_j^{RS} \leq \hat{U}_j$), the type-j entrepreneurs will switch to the self-finance channel.

Without the endogenous self-finance channel, my model turns into an application of GSW in the spirit of Akerlof (1970), which solves the non-existence problem of Rothschild & Stiglitz (1976) by replacing centralized contract posting with directed search. Liquidity holdings, loan contracts, and market tightness solve the following problem: for $j = r, s$

$$U_j^{GSW} = \max_{z_j,(d_j,\ell_j,R_j),\theta_j} -iz_j + \alpha\mu(\theta_j)[\delta_j f(\ell_j) - d_j - \delta_j R_j] \tag{P$^{\text{GSW}}$-j}$$

$$\text{s.t. } \; z_j \geq d_j, \;\; \eta(\theta_j)[d_j - \ell_j + \delta_j R_j] \geq \kappa,$$

$$-iz_j + \alpha\mu(\theta_j)[\delta_{\tilde{j}} f(\ell_j) - d_j - \delta_{\tilde{j}} R_j] \leq U_{\tilde{j}}^{GSW} \;\text{ for }\; \tilde{j} \neq j. \tag{IC$^{\text{GSW}}$-$\tilde{\text{j}}$j}$$

As shown in GSW, IC$^{\text{GSW}}$-rs binds but IC$^{\text{GSW}}$-sr does not. Then for the risky types, $z_r = d_r = 0$, ℓ_r solves $\delta_r f'(\ell_r) = 1$, R_r solves the binding free entry condition, and θ_r solves $\mu'(\theta_r)[\delta_r f(\ell_r) - \ell_r] = \kappa$. For the safe types, $z_s = d_s = 0$, ℓ_s solves $\delta_s f'(\ell_s) = 1$, R_s solves the binding free entry condition, and θ_r solves the binding IC$^{\text{GSW}}$-rs constraint.

Proposition 5 *Without the self-finance option, neither a precautionary nor a signaling motive exists; $z_j = 0$, for $j = r, s$.*

Proof. See Appendix I. ■

Notice that in this setup liquidity becomes redundant. If the outside option is removed, the precautionary motive for holding liquid assets no longer exists. Then there is only one potential reason to hold liquid assets: to show the borrower's ability to repay by providing down payments. Down payments require obtaining liquidity beforehand, which is costly since later the borrower may or may not be matched with a banker. A tighter loan market, on the other hand, leads to less bank entry, which saves on fixed entry costs. When liquidity has no use outside the credit market, the loan approval rate becomes the dominant screening device that provides higher payoffs to entrepreneurs who honestly apply for contracts that are designed for them. Hence, when the self-finance channel is shut down, neither the precautionary nor the signaling motive exists, and liquidity plays no role in the economy.

6 Constrained Efficiency Problem

In this section, I first discuss the constrained efficient problem in the language of GSW. Then I assume the planner uses the direct mechanism, in which entrepreneurs directly report their type to the planner and then the planner allocates them loan contracts, and specify the planner's optimization problem. I also describe constrained efficient allocations and discuss the origin of inefficiency in the baseline market economy.

6.1 Planner's Problem Using Taxation

In the market economy as described in the baseline, it is not possible to transfer funds from one submarket to another. However, a planner who has the power of taxation can tax agents in some submarkets and subsidize agents in other submarkets, thus changing the payoffs that entrepreneurs may receive. This cross-subsidization is central to the constrained efficiency problem. Here the planner is assumed to have the power to implement taxation contingent on bank loans. First, the planner collects a lump sum tax, $\tau_0 \in \mathbb{R}_+$ measured in the numeraire good from all bankers. Second, the planner gives subsidies to submarkets. The subsidy will be made contingent on loan contracts, $\tau(\Omega) : \mathbb{R}^3_+ \rightarrow \mathbb{R}$.[39] Let $\{\tau_0, \tau\}$ be

[39] It is equivalent if taxes are levied on and subsidies given to borrowers.

the planner's policy. Because the planner faces the same information and search frictions as agents do, the implementable allocations are defined as follows:

Definition 3 *An allocation* $\{z_j, \mathbf{\Omega}^{pt}, \theta, \lambda_j, \Gamma, \hat{z}_j, q^m\}$ *is implementable through policy* $\{\tau_0, \tau\}$ *if such allocation satisfies the following conditions:*

(i). bankers' profit maximization and free entry: for any $\Omega \in \mathbf{\Omega}$*,*

$$-\kappa - \tau_0 + \eta(\theta(\Omega)) \sum_{j=r,s} \lambda_j(\Omega)[d(\Omega) - \ell(\Omega) + \delta_j R(\Omega) + \tau(\Omega)] \leq 0,$$

with equality if $\Omega \in \mathbf{\Omega}^{pt}$*;*

(ii). entrepreneurs' optimal search: let

$$\mathcal{U}_j = \max\{U_j, \hat{U}_j\},$$

where $\mathcal{U}_j = \hat{U}_j$ *if* $\mathbf{\Omega}^{pt} = \emptyset$*; then for any* $\Omega \in \mathbf{\Omega}$ *and* j*,*

$$U_j \geq U_j(\Omega),$$

with equality if $\theta(\Omega) < \infty$ *and* $\lambda_j(\Omega) > 0$*, where*

$$\begin{aligned} U_j &= \max_{\Omega \in \mathbf{\Omega}^{pt}} \; U_j(\Omega) \\ &= \max_{\Omega \in \mathbf{\Omega}^{pt}} \; \{-iz(\Omega) + \alpha\mu(\theta(\Omega))[\delta_j f(\ell(\Omega)) - d(\Omega) - \delta_j R(\Omega)] \\ &\quad + \alpha(1 - \mu(\theta(\Omega)))[\delta_j f(z(\Omega)) - z(\Omega)]\}; \end{aligned}$$

moreover, if $U_j < \hat{U}_j$*,* $z(\Omega) = 0$*,* $\hat{z}_j = \arg\max \; \hat{U}_j$*, and either* $\theta(\Omega) = \infty$ *or* $\lambda_j(\Omega) = 0$*;*

(iii). feasibility:

$$\int_{\mathbf{\Omega}^{pt}} \frac{\lambda_j(\Omega)}{\theta(\Omega)} d\Gamma(\Omega) \leq \nu_j \;\; \text{for any} \;\; j,$$

with equality if $U_j > \hat{U}_j$*;*

(iv). liquid asset market clears:

$$\sum_{j=r,s} \nu_j \bar{z}_j = q^m M,$$

where $\bar{z}_j = \int_{\mathbf{\Omega}^{pt}} \frac{\lambda_j(\Omega)}{\theta(\Omega)} z(\Omega) d\Gamma(\Omega)$ *if* $U_j > \hat{U}_j$ *and* $\bar{z}_j = \hat{z}_j$ *if otherwise.*

(v). and the planner's budget balances:

$$\int_{\mathbf{\Omega}^{pt}} \eta(\theta(\Omega))\tau(\Omega) d\Gamma(\Omega) \leq \int_{\mathbf{\Omega}^{pt}} \tau_0 d\Gamma(\Omega).$$

Note that when a zero taxation policy is implemented, $\tau_0 = 0$ and $\tau(\Omega) = 0$ for all Ω. This definition boils down to Definition 2, so the competitive equilibrium is implementable through zero taxation in the planner's problem. Bankers need to take into account the taxation policy and then decide what contract to post or in which submarket to participate. Entrepreneurs in turn choose the submarket to enter among all active submarkets. Also note that Condition (i) is different from the one in Definition 2 because bankers have to take taxation policy into account when posting contracts in the planner's problem. For all submarkets that the planner wants to be active in, bankers must not receive any positive profits. Condition (v) states that the planner does not have any external resources to finance the transfers. All other conditions are the same as in Definition 2.

Let $\bar{\sigma}_j$ be the Pareto weight of each type-j entrepreneur with $\sum_j \bar{\sigma}_j = 1$. Also define $\sigma \equiv \bar{\sigma}_j \nu_j$. A constrained efficient allocation is described as follows:

Definition 4 *A constrained efficient allocation solves the problem:*

$$\max_{\{z_i, \mathbf{\Omega}^{pt}, \theta, \lambda_i, \Gamma, \hat{z}_i, q^m\}, \{\tau_0, \tau\}} \sum_{i=r,s} \sigma_i \, \mathcal{U}_i$$

$$s.t. \quad \{z_i, \mathbf{\Omega}^{pt}, \theta, \lambda_i, \Gamma, \hat{z}_i, q^m\} \text{ is implementable via policy } \{\tau, \tau_0\}.$$

A constrained efficient allocation is an implementable allocation as described in Definition 3 that maximizes total welfare weighted by σ among all implementable allocations. In the next subsection, the planner's problem is defined using a direct mechanism that yields the same results as in this definition and is more convenient to work with.

6.2 Direct Mechanism

In this subsection, the planner is assumed to use a direct mechanism. In this mechanism, entrepreneurs report their types to the planner who acts as a middleman, and then the planner allocates them to a submarket Ω with θ. Intuitively, the planner sets up two shops, a certain number of entrepreneurs are allocated to match with bankers at each shop, and then bankers are asked to issue loans with certain loan terms specified if the bankers are matched with entrepreneurs. The planner also implements transfers among bankers. Davoodalhosseini (2019) shows that a planner who uses a direct mechanism obtains the same amount of welfare as the planner in a dynamic version of the equilibrium as in Definition 4,[40] and all results from the direct mechanism can be found in the setting if the planner has the power of taxation. Without loss of generality, I use the direct mechanism to study the constrained efficiency problem. Conditional on both types of entrepreneurs participating in the loan market,[41] the planner who weights the type-j entrepreneur with σ_j solves the following problem to

[40]Unrestricted power of taxation is the key here. Because there is no limit on the taxation power of the planner, the planner can effectively shut down any submarket by imposing a large amount of taxes on that submarket so that the same result as in the direct mechanism is obtained.

[41]If any type does not participate in the loan market, the planner cannot use cross-subsidization to interfere with the market effectively.

maximize total welfare $\mathbb{W}$, which is the weighted average payoffs of entrepreneurs:[42]

$$\mathbb{W} = \max_{\{z_j,(d_j,\ell_j,R_j),\theta_j\}_{j=r,s}} \sum_{j=r,s} \sigma_j U_j(z_j, \Omega_j, \theta_j) \tag{PP}$$

$$\text{s.t.} \quad z_j \geq d_j,$$

$$\sum_{j=r,s} \nu_j [\mu(\theta_j)(d_j - \ell_j + \delta_j R_j) - \theta_j \kappa] \geq 0, \tag{BB}$$

$$U_j(z_j, \Omega_j, \theta_j) \geq \hat{U}_j, \tag{PC-j}$$

$$U_{\tilde{j}}(z_j, \Omega_j, \theta_j) \leq U_{\tilde{j}}(z_{\tilde{j}}, \Omega_{\tilde{j}}, \theta_{\tilde{j}}), \quad \text{for any } \; \tilde{j} \neq j,, \tag{PIC-$\tilde{\text{j}}$j}$$

where $U_{\tilde{j}}(z_j, \Omega_j, \theta_j) \equiv -iz_j + \alpha\mu(\theta_j)[\delta_{\tilde{j}} f(\ell_j) - d_j - \delta_j R_j] + \alpha(1 - \mu(\theta_j))[\delta_{\tilde{j}} f(z_j) - z_j]$ is the expected payoff of the type-$\tilde{j}$ entrepreneur who plans to choose z_j and Ω_j and anticipates θ_j. The first condition guarantees that type-j entrepreneurs hold at least a d_j amount of z_j. The second condition is the government budget balance (BB) condition. There are two shops set up by the planner, and each location has an $\alpha\nu_j\theta_j$ number of bankers (contracts) as each banker makes an expected net profit $\eta(\theta_j)(d_j - \ell_j + \delta_j R_j) - \kappa$. The planner makes transfers (i.e., collects taxes and gives subsidies) among bankers; some bankers might make positive profits before taxation, and some might make negative profits before subsidization. Then repayments are implicit functions of transfers. So the left hand side of the BB condition is the sum of the expected net profits of all bankers over the two shops and must be non-negative. The third condition, the participation constraint (PC), ensures that entrepreneurs would not be worse off by participating in the loan market. The fourth condition, the planner's incentive compatibility (PIC) constraint, guarantees that all entrepreneurs would truthfully report their types to the planner. The difference between the PIC and IC constraints is that in the baseline model when bankers design contracts in one submarket, they take the expected payoff of entrepreneurs in the other submarket as given (i.e., the right hand side of the IC constraint); in contrast, the planner can manipulate the payoff of entrepreneurs in

[42]Because bankers and capital producers make zero expected profit, the total welfare in the economy is the weighted average payoffs of entrepreneurs.

the other submarket (i.e., the right hand side of the PIC constraint) by choosing contracts posted in each submarket and making transfers across submarkets.

6.3 Constrained Efficient Allocations

This subsection focuses on the circumstances where the competitive equilibrium is distorted, i.e., bankers use liquidity holdings and market tightness to screen. When some entrepreneurs are inclined to misreport their type, bankers face an additional IC constraint to ensure that they truthfully report their types. Then the change in one type's payoff affects the entrepreneurs in the other submarket through the IC constraint. Furthermore, it has an impact on the set of contracts that bankers can offer to attract the entrepreneurs in the other submarket and thus affects their payoffs. An agent in the market economy cannot take this externality into account, but a planner can internalize it by implementing transfers. When the risky types receive subsidies, they have higher payoffs and thus are less inclined to apply for a safe type loan. In turn, bankers in the safe type submarket screen with a smaller down payment and higher market tightness. Therefore, if the benefit of lower screening intensity outweighs the cost of paying taxes, the safe types can be better off, resulting in higher total welfare of all entrepreneurs, i.e., a Pareto superior allocation.

As shown in Figure 5, PIC-$\tilde{\text{j}}$j, PC-j, and BB represent the following conditions that are plotted as functions of repayment pair (R_r, R_s) given full information allocations z^*, ℓ^*, θ^*, and d^*:

$$
\begin{aligned}
&U_{\tilde{j}}(z_j^*, d_j^*, R_j, \ell_j^*, \theta_j^*) = U_{\tilde{j}}(z_{\tilde{j}}^*, d_{\tilde{j}}^*, R_{\tilde{j}}, \ell_{\tilde{j}}^*, \theta_{\tilde{j}}^*),\\
&U_j(z_j^*, d_j^*, R_j, \ell_j^*, \theta_j^*) = \hat{U}_j,\\
&\sum_{j=r,s} \nu_j[\mu(\theta_j^*)(d_j^* - \ell_j^* + \delta_j R_j) - \theta_j^* \kappa] = 0.
\end{aligned}
$$

It is easy to see that given full information allocations, any area below PIC-sr, above PIC-rs, below PC-s, and to the right of PC-r, which is demonstrated by the pink area, can support a repayment pair (R_r, R_s) that satisfies the PC and PIC constraints. The BB constraint is a straight line in (R_r, R_s), and any point on it represents a feasible repayment pair. Thus, any point on the thick dashed red line, which demonstrates the set of repayment pairs that are incentive compatible, individual rational, and feasible, can support the complete information allocation z^*, ℓ^*, θ^*, and d^*. Also note that with $z^*, \ell^*, \theta^*, d^*$ and $\sigma_j = \nu_j$, the utilitarian planner's indifference curve has the same slope as BB. So any repayment pair on the thick dashed red line recovers the complete information total welfare. This is because z^*, ℓ^*, θ^*, and d^* are supported and the sum of repayments $(\sum_j \nu_j \mu(\theta_j^*) \delta_j R_j)$ equals the sum of complete information repayments $(\sum_j \nu_j \mu(\theta_j^*) \delta_j R_j^*)$. As a result, total welfare $\mathbb{W} = \mathbb{W}^* \equiv \sum_j \sigma_j U_j^*$, although type-j entrepreneur's payoff may be different from U_j^* because of transfers made contingent on the entrepreneur's type.

In the following proposition, I provide a sufficient condition such that a utilitarian planner can choose a repayment pair (R_r, R_s), which is implementable through policy (τ_0, τ), and completely undo the effect of adverse selection, recovering the allocations $(z^*, \ell^*, \theta^*, d^*)$ and the complete information total welfare $(\mathbb{W}^*)$.

Proposition 6 *A utilitarian planner ($\sigma_j = \nu_j$) can choose a repayment pair (R_r, R_s) to completely nullifies the effect of adverse selection and achieve the complete information allocations $z^*, \ell^*, \theta^*, d^*$ if*

$$U_s^* - \hat{U}_s \geq \frac{U_r(z_s^*, \Omega_s^*, \theta_s^*) - U_r^*}{\frac{\nu_s}{\nu_r} + \frac{\delta_r}{\delta_s}}. \tag{14}$$

Proof. See Appendix J for the detailed proof. ■

In Condition (14), $U_s^* - \hat{U}_s$ represents the safe types' net surplus of obtaining a safe type loan as in complete information rather than not borrowing, and $U_r(z_s^*, \Omega_s^*, \theta_s^*) - U_r^*$ represents the risky types' net surplus of obtaining a safe type loan as in complete information rather than a risky type loan. This condition is more likely to be satisfied in the following situations. When liquid assets are very costly to hold, i.e., i is large, safe types' gain of participating in

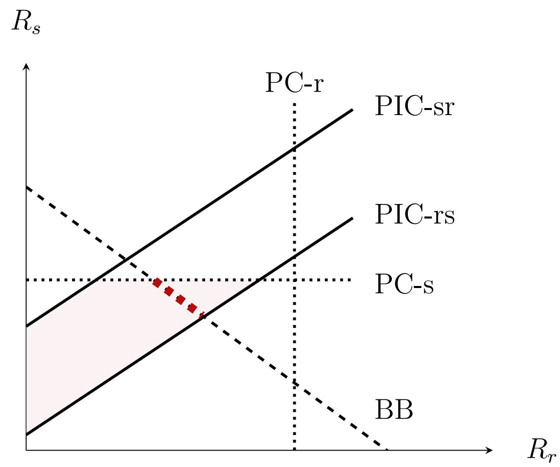


Figure 5: **Utilitarian Planner:** All PIC, PC, and BB conditions are plotted as functions of repayment pair (R_r, R_s) such that z^*, ℓ^*, θ^*, and d^* are chosen. Then all repayment pairs on the thick dashed red line (▪▪▪) can support the complete information allocation z^*, ℓ^*, θ^*, and d^*.

the loan market, $U_s^* - \hat{U}_s$, is large, so safe types are willing to pay taxes and still apply for loans. When the population of risky types is small, i.e., ν_r is small, the right hand side of the equation is small. Intuitively, safe types need to pay a small amount of tax since there are few risky types to subsidize. When risky types' net benefit of misreporting their type, $U_r(z_s^*, \Omega_s^*, \theta_s^*) - U_r^*$, is small, the right hand side of the equation is small. This happens when δ_r is very different or close to δ_s, as discussed in Proposition 3. Figure 6a shows where the full information allocations can be achieved in the parameter space of δ_r and i. When i and δ_r are not too big or too small (area *not achievable*), complete information allocations can never be recovered.

Then consider a general planner with weight σ_j that can be different from ν_j. In the following proposition, I show a sufficient condition for the distorted competitive equilibrium to be constrained efficient, suggesting that the first welfare theorem holds and no transfers should be made.

Proposition 7 *If there exist δ_j and ν_j, $j = r, s$, such that*

$$\frac{\delta_r}{\delta_s} + \frac{\nu_s}{\nu_r} < 1, \tag{15}$$

then there exist Pareto weight σ_j such that the competitive equilibrium solves the planner's problem.

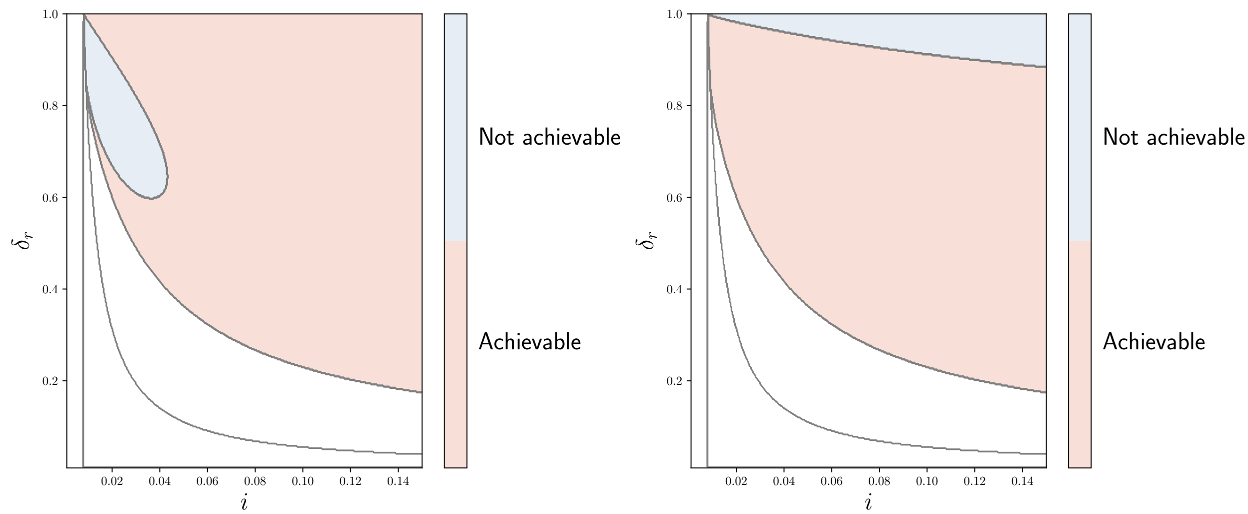
Proof. See Appendix K. ■

Equation (15) is satisfied when the risky types' investment quality is poor (low δ_r) or population is large (high ν_r). In this case, the cost of subsidization is large, so there exist σ_j, $j = r, s$, such that the competitive equilibrium is constrained efficient. In particular, such σ_j must satisfy

$$\frac{1 - (\nu_s \sigma_r)/(\nu_r \sigma_s)}{\delta_r/\delta_s + \nu_s/\nu_r} = \Delta^{ce}, \tag{16}$$

where Δ^{ce} is the Lagrange multiplier of the IC-rs constraint in the competitive equilibrium that is characterized in Equation (13). The multiplier $\Delta^{ce} \to 1$ as $\delta_r/\delta_s \to 1$, and $\Delta^{ce} = 0$ if δ_r is very small since the risky types will not borrow when $\delta_r = 0$. Thus, the distorted competitive equilibrium must have $0 < \Delta^{ce} < 1$. By Equation (16), the planner must care more about the safe types ($\sigma_s > \nu_s$) for the distorted competitive equilibrium to be constrained efficient. In this case, the planner has to value the safe types' payoff enough such that she does not want to help the risky types at the safe types' expense but not too much such that she wants to tax the risky types instead.

As illustrated in Figure 6b, given ν_r, when δ_r is very close to δ_s (area *Not achievable*), the competitive equilibrium can never be constrained efficient. Suppose there is cross-subsidization and δ_r is large; the cost of subsidization (or of taxation) is smaller than the benefit (i.e., less liquidity holdings and a larger loan supply) to the safe type entrepreneurs; in turn, cross-subsidization improves U_s. The closer δ_r is to δ_s, the easier it is to increase U_s. Thus, when δ_r is very close to δ_s, there does not exist a σ_s such that the planner implements zero taxation.



(a) Complete information allocations

(b) First Welfare Theorem

Figure 6: **The Planner's Problem:** $\delta_s = 1$

7 Extension: Moral Hazard

The baseline model assums that banknotes ℓ can be used only to purchase capital k; in this section, I relax this assumption and allow entrepreneurs to use banknotes to buy both consumption goods and capital. This is equivalent to unobservable investment. In this setup, an entrepreneur can potentially deviate twice: she can misreport her type and apply for a loan that is not designed for her; moreover, she can use the banknotes that are intended to support investment to purchase consumption goods.[43] As a result, moral hazard is introduced in addition to adverse selection. Using banknotes to purchase consumption goods incurs unit cost $C(\tilde{\chi})$, where $C : [0, \infty] \to [0, 1]$ and $\tilde{\chi}$ is the exogenous difficulty level of doing so. Note that when the cost of conducting moral hazard behavior is very high $\tilde{\chi} \to \infty$, $C \to 1$, and the problem coincides with the baseline. Let $k_{\tilde{j}j}$ and $x_{\tilde{j}j}$ to denote type-$\tilde{\text{j}}$ entrepreneurs' investment and consumption when they apply for type-j contract. Also use U_j^{mh} to denote equilibrium payoff of type-j in the setup with moral hazard. Then, the market designer faces

[43] Cole & Kocherlakota (2001) provide a characterization of the efficient consumption in an environment in which individuals have hidden income and storage. However, entrepreneurs in my model have no incentive to save across periods because 1) banknotes cannot circulate across periods, 2) opportunity cost of holding liquid assets is non-negative, $i \geq 0$, and 3) the utility function of CM is quasi-linear.

a new MIC constraint rather than the IC constraint in the baseline:[44]

$$-iz_j + \alpha\mu(\theta_j)\Big[\max_{\substack{k_{\tilde{j}j}, x_{\tilde{j}j} \text{ s.t.} \\ k_{\tilde{j}j} \le \ell_j + z_j - d_j \\ x_{\tilde{j}j} \le \ell_j - k_{\tilde{j}j} + z_j - d_j}} \delta_{\tilde{j}} f(k_{\tilde{j}j}) + (1 - C(\tilde{\chi}))x_{\tilde{j}j} - d_j - \delta_{\tilde{j}} R_j \Big]$$
$$+ \alpha(1-\mu(\theta_j))[\delta_{\tilde{j}} f(z_j) - z_j] \le U^{mh}_{\tilde{j}} \tag{MIC-$\tilde{\text{j}}$j}$$

Conditional on obtaining a type-j loan, the type-$\tilde{\text{j}}$ entrepreneur can use ℓ_j and any remaining liquidity $z_j - d_j$ to purchase capital goods $k_{\tilde{j}j}$ from the producers in the DM, and produce $\delta_{\tilde{j}} f(k_{\tilde{j}j})$ in expectation. If there are any remaining banknotes and liquidity, in the next CM she can use them to buy consumption goods at most $\ell_j - k_{\tilde{j}j} + z_j - d_j$ before the debt is repaid. For simplicity, let $\chi \equiv 1 - C(\tilde{\chi})$ denote the net amount of numeraires per unit of banknotes spent on purchasing consumption goods. Let $k^{mh}_{\tilde{j}j}$ denote type-$\tilde{\text{j}}$ entrepreneurs' amount of capital invested such that $\delta_{\tilde{j}} f'(k^{mh}_{\tilde{j}j}) = \chi$. To simplify the MIC constraint, it is worth to look into an entrepreneur's investment and consumption when misreport.

Lemma 3 *Consider a type-$\tilde{j}$ entrepreneur's investment $k_{\tilde{j}j}$ and consumption $x_{\tilde{j}j}$ in the problem of type-j, $\tilde{j} \neq j$.*

1. *In the safe type entrepreneurs' problem, $j = s$:*

   (i). if $\chi > \delta_r/\delta_s$, $k_{rs} = k^{mh}_{rs} < \ell_s$ and $x_{rs} = \ell_s - k^{mh}_{rs}$.

   (ii). otherwise, $k_{rs} = \ell_s$ and $x_{rs} = 0$; MIC-rs vanishes into IC-rs in the baseline.

2. *In the risky type entrepreneurs' problem, $j = r$: $k_{sr} = \ell_r$ and $x_{sr} = 0$; MIC-sr vanishes into IC-sr in the baseline.*

Proof. See Appendix M. ∎

When $\chi > \delta_r/\delta_s$, the cost of purchasing consumption goods using banknotes is small. The amount of banknotes issued by the safe type loan are too big for the risky types to invest

[44] I show the complete problem and derive the MIC constraint step by step in Appendix L.

in their projects. Because the marginal gain from investing an additional unit of capital on top of k_{rs}^{mh} is less than the marginal gain from purchasing a unit of consumption good, the risky type of entrepreneurs who applied for a safe type loan would choose to invest k_{rs}^{mh} and consume a net amount of $\chi(\ell_s - k_{rs}^{mh})$. Then, the MIC-rs constraint can be simplified to

$$-iz_s + \alpha\mu(\theta_s)[\delta_r f(k_{rs}^{mh}) + \chi(\ell_s - k_{rs}^{mh}) - d_s - \delta_r R_s] + \alpha(1-\mu(\theta_s))[\delta_r f(z_s) - z_s] \leq U_r^{mh}.$$

In contrast, if banknotes can be used only to buy capital, the risky types have to use up ℓ_s and produce $\delta_r f(\ell_s)$ in expectation, which generates less payoff than producing $\delta_r f(k_{rs}^{mh})$ and consuming $\chi(\ell_s - k_{rs}^{mh})$. So, risky types have higher incentives to apply for a safe type loan as their payoffs of misreporting is higher than the baseline.

When the cost of purchasing consumption goods using banknotes is high, $\chi \leq \delta_r/\delta_s$, the risky types exhaust the banknotes for investment, $k_{rs}^{mh} = \ell_s$, because using ℓ_s for investment generates higher payoff than for consumption. MIC-rs constraint coincides with IC-rs; as a result, equilibrium allocations are identical as the ones in the baseline.

When $\{z_s, \ell_s, d_s, R_s, \theta_s\} = \{z_s^*, \ell_s^*, z_s^*, (\kappa/\eta(\theta_s^*)+\ell_s^*-z_s^*)/\delta_s, \theta_s^*\}$ is not incentive compatible,

$$\begin{aligned} &-iz_s^* + \alpha\mu(\theta_s^*)[\delta_r f(k_r) + \chi(\ell_s^* - k_r) - (1-\delta_r/\delta_s)z_s^* - \ell_s^*\delta_r/\delta_s] - \alpha\theta_s^*(\delta_r/\delta_s)\kappa \\ &+ \alpha(1-\mu(\theta_s^*))[\delta_r f(z_s^*) - z_s^*] > U_r^{mh}, \text{ where } \begin{cases} k_r = k_{rs}^{mh} & \text{if } \chi > \delta_r/\delta_s, \\ k_r = \ell_s^* & \text{if else,} \end{cases} \end{aligned}$$

more screening is needed, and allocations are distorted in another dimension in this case. Let mh be the superscript to denote the equilibrium allocations of the setting with moral hazard. The following Proposition compares the moral hazard allocations with the complete information allocations.

Proposition 8 *In the environment with adverse selection and moral hazard,*

1. *if* $\chi > \delta_r/\delta_s$,

(i). the allocations of the safe types are distorted both in the intensive margin, $\ell_s^{mh} < \ell_s^*$, and the extensive margin, $z_s^{mh} > z_s^*$, and $\theta_s^{mh} < \theta_s^*$;

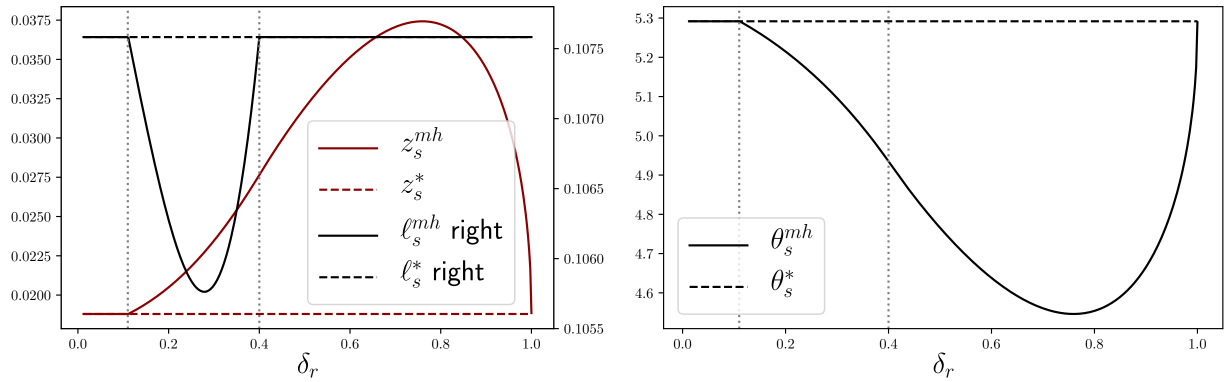
(ii). while allocations of the risky types are not distorted, $\ell_r^{mh} = \ell_r^*$, $z_r^{mh} = z_r^*$, and $\theta_r^{mh} = \theta_r^*$.

2. if $\chi \leq \delta_r/\delta_s$, allocations coincide with the ones in the baseline.

Proof. See Appendix N. ■

With the possibility of dual deviation, allocations are distorted in an additional dimension – loan size ℓ. Bankers issue a smaller loan amount in order to screen out entrepreneurs who have an incentive to deviate ex post. In this case, three screening tools are used by bankers: loan amount, liquidity holdings, and market tightness. Figure 7 shows a numerical exercise of equilibrium allocation z_s, ℓ_s, and θ_s under this extension relative to the complete information case. When δ_r is very small and below the first dotted line, risky types either choose to not enter the loan market or do not have incentive to misreport even if bankers use only a small down payment $d_s^{mh} = z_s^*$ to screen. When δ_r is large and above the second dotted line, which is χ in this exercise, the problem becomes identical as the baseline model, so do the allocations. When δ_r falls between the two dotted lines, $\ell_s^{mh} < \ell_s^*$, $z_s^{mh} > z_s^*$, $\theta_s^{mh} < \theta_s^*$, and $U_s^{mh} < U_s^*$.

Consider other types of equilibrium allocations for the safe type entrepreneurs in a parameter space of δ_r and i as in Figure 8a. Comparing with the baseline as in Figure 3b, the market is distorted in both extensive and intensive margin, as bankers may *screen with ℓ, z, and θ*. The market is also distorted in a larger parameter space, as bankers are less likely to *screen with z* only. Furthermore, because the safe types do not have incentive to misreport and mimic the risky types, risky types' equilibrium allocations are identical to the ones under the complete information and the baseline case. In the following proposition, I classify types of equilibrium allocations using the opportunity cost of holding liquid assets i.



(a) Liquidity holdings and loan amount (b) Market tightness

Figure 7: **Screening Intensity Under Moral Hazard**: $i = 0.1$, $\delta_s = 1$, $\chi = 0.4$.

Proposition 9 *In the environment with moral hazard, there exists cutoffs $\underline{i}^{mh}$, $\bar{i}^{mh}$, and $\bar{\bar{i}}^{mh}$ that are ranked, $\underline{i}^{mh} < \bar{i}^{mh} \leq \bar{\bar{i}}^{mh}$.*

1. *When $\chi > \delta_r/\delta_s$,*

   (i). if $i \leq \underline{i}^{mh}$, both risky and safe types do not enter the loan market;

   (ii). if $i \in (\underline{i}^{mh}, \bar{i}^{mh}]$, risky types do not enter the loan market, but safe types do;

   (iii). if $i \in (\bar{i}^{mh}, \bar{\bar{i}}^{mh}]$, both types enter, and bankers use down payment to screen;

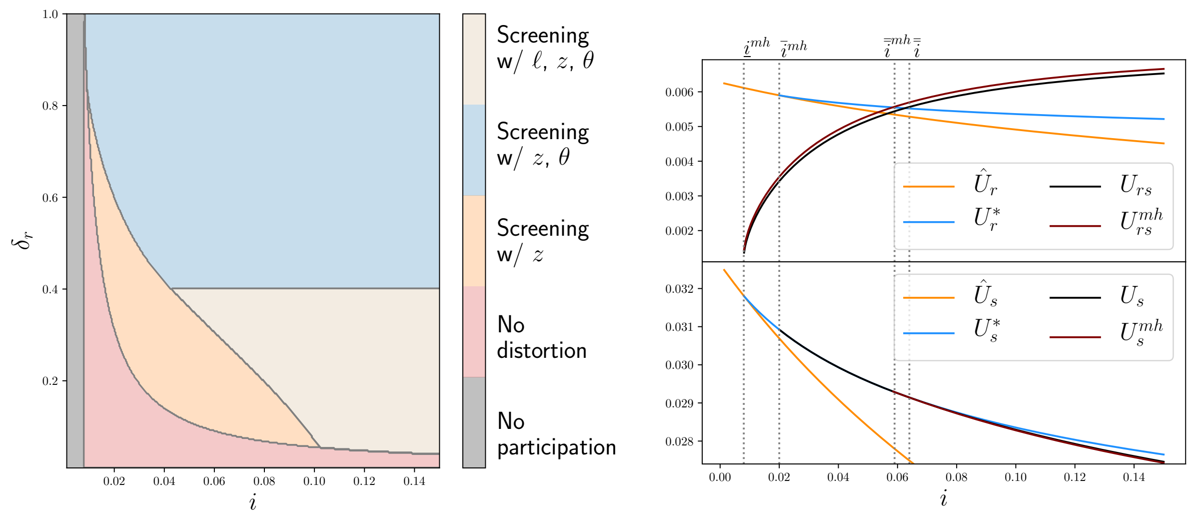
   (iv). if $i > \bar{\bar{i}}^{mh}$, both types enter, and bankers use loan amount, down payment, and market tightness to screen.

2. *When $\chi \leq \delta_r/\delta_s$, $\{\underline{i}^{mh}, \bar{i}^{mh}, \bar{\bar{i}}^{mh}\} = \{\underline{i}, \bar{i}, \bar{\bar{i}}\}$.*

Proof. See Appendix O. ∎

Comparing with the baseline, screening is not only more intense but also more likely to happen.

Proposition 10 *In the environment with moral hazard, the cutoff of using multiple screening tools is lower than the baseline, $\bar{\bar{i}}^{mh} \leq \bar{\bar{i}}$.*



(a) Safe type entrepreneurs

(b) Payoffs: $\delta_r = 0.3$

Figure 8: **Types of Equilibrium Allocations Under Moral Hazard**: $i = 0.1$, $\delta_s = 1$, $\chi = 0.4$.

Proof. See Appendix P. ■

In Figure 8b, U_{rs} and U_{rs}^{mh} are risky types' payoff when they mimic safe types, given $\{z_s, \ell_s, d_s, R_s, \theta_s\} = \{z_s^*, \ell_s^*, z_s^*, (\kappa/\eta(\theta_s^*) + \ell_s^* - z_s^*)/\delta_s, \theta_s^*\}$, under the baseline and the moral hazard extension respectively. (i) When i is very small and below $\underline{i}^{mh}$, none of the entrepreneurs enters the loan market. (ii) When i is between $\underline{i}^{mh}$ and $\bar{i}^{mh}$, safe types prefer entering while risky types do not since safe types have higher surplus of getting a loan. (iii) When i is between $\bar{i}^{mh}$ and $\bar{\bar{i}}^{mh}$, both types enter, and bankers use $d_s = z_s^*$ to screen out risky types. Because safe types are indifferent between paying some down payment up front and giving larger repayment afterwards, safe types have the payoff U_s^* in this case. (iv) When i is above $\bar{\bar{i}}^{mh}$, which happens when $U_{rs}^{mh} > U_r^*$, a small down payment is not enough to keep risky types away from the safe type contracts. As a result, bankers use multiple screening devices, and the payoff of safe types is lower than the complete information payoff. Risky types, on the other hand, always have the complete information payoff once they enter. From this figure, it is also easy to see how the third cutoff $\bar{\bar{i}}^{mh}$ differs from the one in baseline $\bar{\bar{i}}$. When the risky types can spend some ℓ_s on consumption, U_{rs}^{mh} is higher than U_{rs}, where the magnitude depends on the cost of conducting moral hazard behavior. The lower the cost is, the higher U_{rs}^{mh} is, and the lower $\bar{\bar{i}}^{mh}$ is. In other words, with potential moral hazard behavior, allocations are not only more distorted but also more likely to be distorted.

8 Conclusion

In this paper, I propose a signaling motive for holding liquid assets in addition to the well-studied precautionary motive in the literature. I build a directed search model with asymmetric information to rationalize liquidity holdings both inside and outside the credit market. First, liquid assets are useful inside the credit market as entrepreneurs use them to signal their ability to repay, so they help entrepreneurs obtain external credit. Second, liquid assets are useful outside the credit market by acting as a buffer stock as entrepreneurs fail to secure a loan. By introducing a self-finance channel to a classic screening model with costly collateral, both liquidity holdings and loan approval rate are used to screen out risky entrepreneurs. The self-finance channel acts as an endogenous outside option which is crucial to credit contracts and screening devices. I show that without a self-finance option, liquid assets become redundant, as both the precautionary and signaling motives disappear.

While bankers in the market economy use liquidity holdings and loan approval rate to screen out risky borrowers, the safe borrowers are worse off since they need to bringing more liquid assets and are less likely to obtain a loan. The risky borrowers, thus, cause an externality, resulting in lower payoffs for the safe borrowers. This is because the bankers in one submarket do not take into account the effects of their entry on the set of feasible contracts that the bankers in the other submarket can offer to attract entrepreneurs. However, unlike the agents in the market economy, a planner can internalize this externality by levying taxation. I show that under some conditions, a planner can always achieve higher welfare by subsidizing risky borrowers and taxing safe borrowers. In particular, a utilitarian planner can completely undo the effect of adverse selection and recover the allocations under complete information. I also find that the competitive market equilibrium can be constrained efficient, in which case no transfers are needed.

There are many related research questions that can be addressed in the future. For example, long-term banking relationships (e.g., Bethune et al. 2019), such as business credit cards, significantly reduce firms' demand for liquidity, as credit cards were the second most used financial resource in 2020. One can study this issue by allowing entrepreneurs to build long-term relationships with certain bankers in this model; however, it requires extensive work with dynamic contracting. Another issue is that in reality some firms are more financially constrained than others, meaning that some entrepreneurs are not able to raise as much liquidity as they need to. One can study this issue by introducing multiple dimensions of private information (e.g., Chang 2017, Guerrieri & Shimer 2018, and Williams 2021) into the model, i.e., entrepreneurs differ in investment quality and ability of raising liquidity in the centralized Walrasian market, and bankers observe either of it.

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Appendices

A Summary Statistics of KFS

Table A1 illustrates summary statistics of KFS and SSBF 2003 used in my paper. In the sample of KFS, I restrict firms to not file bankruptcy or merge with other firms, to have positive revenues and non-negative liquidity holdings (cash and cash equivalent), to primarily conduct business in the U.S., and to have non-missing key information. In the sample of SSBF 2003, I restrict firms to be private, to have no bad credit mark (i.e., no personal or firm bankruptcy, no personal or firm debt delinquency), to be unbanked (i.e., no business credit card, no line of credit), to conduct business primarily in the U.S., to have non-negative cash and cash equivalent, and to have applied for a business loan (e.g., mortgage, equipment loan, vehicle loan) in the last three years.

	(1) KFS		(2) SSBF 2003	
	Mean	SD	Mean	SD
liquidity-to-assets	0.279	0.356	0.213	0.268
liquid collateral	0.025	0.155	0.045	0.207
illiquid collateral	0.138	0.345	0.612	0.488
assets	284,526	318,754	617,493	1,003,017
revenue	421,460	350,759	1,188,200	1,887,146
R&D-to-assets	0.103	1.175		
credit prob.	0.034	0.180	0.045	0.207
C-corp	0.062	0.241	0.179	0.384
primary-owner-manager	0.672	0.470	0.896	0.306
need credit, didn't apl	0.131	0.338	0.119	0.325
N	660		335	

Table A1: **Summary Statistics of KFS and SSBF 2003**

B Estimations Using SSBF

First, I use the Survey of Small Business Finances 2003 (SSBF). I restrict the dataset to firms that are private, have no bad credit mark, are unbanked, conduct business primarily in the U.S., have non-negative cash and cash equivalent, and have applied for a loan (excluding capital lease or line of credit) in the last three years. Column 2 in Table A1 shows summary statistics of my sample. I then estimate the following regression:

$$
\begin{aligned}
Lqd\_Ass_i = &\beta_{lc} LqdC_i + \beta_{cp} CProb_i + \beta_a ln\_Ass_i + \beta_{mo} MO_i + \beta_{cc} C\_corp_i \\
&+ X_i \cdot \beta_X + Time + Ind + \epsilon_i,
\end{aligned}
$$

where Lqd_Ass_i is the liquidity-to-total-assets ratio of firm i, $LqdC_i$ equals 1 if the firm has pledged a liquid collateral for the most recent loan and 0 if otherwise, $CProb_i$ equals 1 if the firm considers itself as having difficulties getting credit and 0 if otherwise, ln_Ass_i is the natural log of total assets, MO_i equals 1 if the owner is the manager and 0 if otherwise, C_corp_i equals 1 if the firm is a C-corporation and 0 if otherwise, X_i is a vector of firm and owner characteristics, $Time$ and Ind are the time and industry fixed effects, respectively, and ϵ_i is the error term.[45]

A firm that faces credit problems has a precautionary motive to hold more liquidity; then β_{cp} should have a positive sign. According to the transaction motive, a larger firm holds a relatively lower amount of liquidity because of the increasing economy of scale in liquidity holdings; then β_a should have a negative sign. If the firm owner is the manager, then liquidity holdings should be lower because the agency motive is lower; then β_{mo} should have a negative sign. Because a C-corporation is taxed at both corporate income and dividend payout, a C-corporation has a larger incentive to hold liquid assets; then β_{cc} should have a positive sign. The time fixed effect is included to capture the impact of changes in nominal risk free

[45] In the SSBF, there is the question "What is the single most important problem facing your business today?" Given a list of possible issues, if the firm picked "financing and interest rate", then I identified the firm as having a credit problem. Firm and owner characteristics controlled are total-loan-to-assets ratio, total-expenditure-to-assets ratio, profit margin, credit score, Herfindahl-Hirschman Index (HHI) banking market concentration index of MSA or country where firm's headquarter is located, if located in MSA, Census division, how firm is established, number of employees, firm age, number of loan applications in the last three years, if the firm needs a loan but has decided not to apply because of possible rejection in the last three years, if the firm is family business, and owner's education level, race, age, and gender.

interest rates from June 2004 to January 2005.[46] The industry fixed effect is used to capture systematic differences across industries in liquidity holdings.

In an ordinary least squares (OLS) regression, the coefficient of liquid collateral, β_{lc}, represents the correlation between the liquid collateral and the liquidity-to-total-assets ratio. A positive correlation has two possible explanations: the firm holds more liquidity in order to pledge a liquid collateral (i.e., the signaling motive) or the firm pledges liquid collateral because the firm has a large amount of liquidity. Then I instrument the liquid collateral using the three-month T-bill rate at the time of loan application, which can affect only the liquidity-to-total-assets ratio at the survey time through the decision of pledging liquid collateral.

Results from the OLS and the instrumental variable (IV) estimation using the SSBF are shown in column (1) and (2) in Table A2. Conditional on applying for a loan recently, the liquidity ratio of a firm who has recently pledged liquid collateral is 31.01 percentage points higher than the liquidity ratio of a similar firm who has pledged illiquid or no collateral. Using IV, the liquidity ratio of a firm who has recently pledged liquid collateral is 38.31 percentage points higher, suggesting that firms who have pledged a liquid collateral hold extra liquid assets that amount to 38.31% of total assets.The F-stats in the first stage regression is 94, which is larger than the rule of thumb 10 suggested by Staiger & Stock (1997), so the 3-month T-bill rate at time of loan application is not a weak instrument for liquid collateral.[47] Coefficient of credit problem, the proxy of precautionary motive, is positive and consistent across OLS and IV, suggesting financially constrained firms hold more liquid assets as buffer stock. The coefficients of log of assets, the proxy of transaction motive, is negative and consistent across OLS and IV, suggesting that larger firms hold less liquidity proportionally. The coefficients of manager is owner, the proxy of agency motive, is against the prediction, so agency motive is not found using SSBF. The coefficients of c-corp, the proxy of tax motive,

[46]The nominal risk free interest rates from June 2004 to January 2005 vary from 1.3% to 2.3%.

[47]I find that the coefficient of liquid collateral using IV estimate is much larger than OLS; however, this is not unique, as researchers find that the 2SLS estimates are larger than the OLS estimates by approximately 25%-50%, e.g., Card (1999, 2001).

	(1) OLS	(2) IV
liquid collateral	0.3101***	0.3831***
	(7.4859)	(5.0769)
credit prob	0.3237***	0.3279***
	(6.3702)	(7.0208)
ln(asset)	-0.1284***	-0.1281***
	(-14.2032)	(-15.4572)
owner is manager	0.1388***	0.1475***
	(5.0345)	(5.5814)
c-corp	0.0634*	0.0745*
	(2.0535)	(2.4864)
firm & owner char	Yes	Yes
adj R-squared	0.8844	.
N	335	335
F-stats of liquid collateral		94.084

Note: *** p<0.01, ** p<0.05, * p<0.1.

Table A2: **Regressions using SSBF**

is positive and consistent across OLS and IV, suggesting that c-corp firms tend to hold more liquid assets to avoid tax expenditure.

C Verify the Assumptions of the Guerrieri, Shimer and Wright model

In this subsection, I show that the assumptions of GSW are satisfied in this environment; thus, the results of GSW can be applied directly to my model. Let $u_j(\Omega)$ be the net surplus of type-j entrepreneur if she is successfully matched with a banker in submarket Ω, such that $u_j(\Omega) = \delta_j f(\ell) - \delta_j R - d - \delta_j f(z) + z,$ where $z \geq d$. Let $v_j(\Omega)$ be the payoff of banker in submarket Ω who matches with a type-j entrepreneur gets payoff $v_j(\Omega) = \delta_j R + d - \ell$. Also, let $\bar{\Omega}_j$ be the set of contracts that deliver non-negative net surplus upon matching to a type-j entrepreneur while permitting the banker to make non-negative profits if the market tightness is 0, such that

$$\bar{\Omega}_j = \{\Omega \in \mathbf{\Omega} \mid \bar{\eta} v_j(\Omega) \geq \kappa \text{ and } u_j(\Omega) \geq 0\}$$

where $\bar{\eta} = \eta(0)$, and

$$\bar{\Omega} = \bigcup_j \bar{\Omega}_j.$$

In equilibrium, contracts that are not in $\bar{\Omega}$ are not traded since bankers cannot make non-negative profit while attracting entrepreneurs.

Assumption 1 *Monotonicity: for all* $\Omega \in \bar{\Omega}$, $v_r(\Omega) \leq v_s(\Omega)$.

This assumption is satisfied since $\delta_r R + d - \ell < \delta_s R + d - \ell$ is true for all $R > 0$ which is guaranteed by $\Omega \in \bar{\Omega}$.

For the next assumption, let $B_\epsilon(\Omega) \equiv \{\Omega' \in \mathbf{\Omega} \mid d(\Omega', \Omega) < \epsilon\}$ be a ball of radius ϵ around Ω.

Assumption 2 *Local non-satiation: for* $\Omega \in \bar{\Omega}_s$, *and* $\epsilon > 0$, *there exists a* $\Omega' \in B_\epsilon(\Omega)$ *such that* $v_s(\Omega') > v_s(\Omega)$ *and* $u_r(\Omega') \leq u_r(\Omega)$.

Consider a contract with $\ell' = \ell$, $d' = d$, and $R' = R+\epsilon$. For bankers, $\delta_s(R+\epsilon)+d-\ell > \delta_s R + d-\ell$, so $v_s(\Omega') > v_s(\Omega)$ is satisfied. For entrepreneurs, $\delta_r f(\ell)-\delta_r(R+\epsilon)-d < \delta_r f(\ell)-\delta_r R-d$, so $u_r(\Omega') \leq u_r(\Omega)$ is also satisfied.

The last assumption ensures that it is possible to make the contract attractive to some entrepreneurs while not attractive to some other entrepreneurs.

Assumption 3 *Sorting: for* $\Omega \in \bar{\Omega}_s$, *and* $\epsilon > 0$, *there exists a contract* $\Omega' \in B_\epsilon(\Omega)$ *such that* $u_s(\Omega') > u_s(\Omega)$ *and* $u_r(\Omega') < u_r(\Omega)$.

For fixed $\Omega \in \bar{\Omega}$, $\tilde{\delta} \in (\delta_r, \delta_s)$ and an arbitrary $\tilde{\epsilon} > 0$, consider $z = d + \tilde{\epsilon}$ and a contract with $\ell' = \ell$, $R' = R - \tilde{\epsilon}/\tilde{\delta}$, and $d' = d + \tilde{\epsilon}$. This is feasible for small $\tilde{\epsilon}$ since $\Omega \in \bar{\Omega}_s$ makes sure that $R > 0$. Then, $\delta_s f(\ell) - \delta_s(R - \tilde{\epsilon}/\tilde{\delta}) - (d + \tilde{\epsilon}) > \delta_s f(\ell) - \delta_s R - d$, so $u_s(\Omega') > u_s(\Omega)$. Also, $\delta_r f(\ell) - \delta_r(R - \tilde{\epsilon}/\tilde{\delta}) - (d + \tilde{\epsilon}) < \delta_r f(\ell) - \delta_r R - d$, so $u_r(\Omega') < u_r(\Omega)$. Now for a given $\epsilon > 0$, choose a $\tilde{\epsilon} \leq \epsilon/\sqrt{1 + 1/\tilde{\delta}^2}$ that guarantees $\Omega' \in B_\epsilon(\Omega)$. Hence, this assumption is satisfied.

Under Assumption 1, 2, and 3, without loss of generality one can assume that each banker post a single contract instead of posting a menu of contracts (revelation mechanism), as established by Proposition 5 of GSW.

D Parameters for Numerical Examples

For all numerical examples, I use the matching function $\mu(\theta) = 0.7(1 - e^{-\theta})$ and production function $f(k) = 0.7k^{0.3}$. Other parameters are $\beta = 0.99, \kappa = 0.0002, \alpha = 0.13, \delta_s = 1, \nu_s = 0.3$. In the moral hazard extension, I use $C(\tilde{\chi}) = e^{-\tilde{\chi}}$. Other parameters are $\tilde{\chi} = 0.51$ and $\chi = 0.4$.

E Comparative Statics

Under the case of symmetric information, we can rewrite the utility function as (omit subscript j and superscript $*$, for simplicity)

$$U = -iz + \alpha\mu(\theta)[\delta f(\ell) - \ell] - \alpha\theta\kappa + \alpha(1 - \mu(\theta))[\delta f(z) - z].$$

By monotone comparative statics as in Theorem 2.3 in Vives (2001),

$$
\begin{aligned}
\frac{\partial U}{\partial z} &= -i + \alpha(1-\mu(\theta))[\delta f'(z) - 1] \\
\frac{\partial U}{\partial \theta} &= \alpha\mu'(\theta)[\delta f(\ell) - \ell - \delta f(z) + z] - \alpha\kappa \\
\frac{\partial^2 U}{\partial z \partial \theta} &= -\alpha\mu'(\theta)[\delta f'(z) - 1] < 0 \\
\frac{\partial^2 U}{\partial z \partial \delta} &= \alpha(1-\mu(\theta))f'(z) > 0 \\
\frac{\partial^2 U}{\partial \theta \partial \delta} &= \alpha\mu'(\theta)[f(\ell) - f(z)] > 0 \\
\frac{\partial^2 U}{\partial z \partial i} &= -1 < 0 \\
\frac{\partial^2 U}{\partial \theta \partial i} &= 0 \\
\frac{\partial^2 U}{\partial z \partial \kappa} &= 0 \\
\frac{\partial^2 U}{\partial \theta \partial \kappa} &= -\alpha < 0.
\end{aligned}
$$

Because $\frac{\partial^2 U}{\partial z \partial \theta} < 0$, $\frac{\partial^2 U}{\partial z \partial i} < 0$, and $\frac{\partial^2 U}{\partial \theta \partial i} = 0$, as i increases either (i) $\frac{\partial z}{\partial i} \leq 0$ and $\frac{\partial \theta}{\partial i} \geq 0$ or (ii) $U(z_1, \theta_1, i_1) - U(z_2, \theta_2, i_1) = U(z_1, \theta_1, i_2) - U(z_2, \theta_2, i_2) = 0$, which is true if $\theta_1 = \theta_2$ and $z_1 = z_2$. Thus, in both cases $\frac{\partial z}{\partial i} \leq 0$ and $\frac{\partial \theta}{\partial i} \geq 0$.

Because $\frac{\partial^2 U}{\partial z \partial \theta} < 0$, $\frac{\partial^2 U}{\partial z \partial \kappa} = 0$, and $\frac{\partial^2 U}{\partial \theta \partial \kappa} < 0$, as κ increases either (i) $\frac{\partial z}{\partial \kappa} \geq 0$ and $\frac{\partial \theta}{\partial \kappa} \leq 0$ or (ii) $U(z_1, \theta_1, \kappa_1) - U(z_2, \theta_2, \kappa_1) = U(z_1, \theta_1, \kappa_2) - U(z_2, \theta_2, \kappa_2) = 0$, which is true if $\theta_1 = \theta_2$ and $z_1 = z_2$. Thus, in both cases $\frac{\partial z}{\partial \kappa} \geq 0$ and $\frac{\partial \theta}{\partial \kappa} \leq 0$.

Because $\frac{\partial^2 U}{\partial z \partial \theta} < 0$, $\frac{\partial^2 U}{\partial z \partial \delta} > 0$, and $\frac{\partial^2 U}{\partial \theta \partial \delta} > 0$, we cannot apply monotone comparative statics.

Define the following function:

$$
\begin{aligned}
G &\equiv -i + \alpha[1 - \mu(\theta(z, \delta))][\delta f'(z) - 1] = 0 \\
\frac{\partial G}{\partial \delta} &= -\alpha\mu'(\theta)\frac{\partial \theta(z, \delta)}{\partial \delta}[\delta f'(z) - 1] - \alpha[1 - \mu(\theta)]f'(z) < 0 \\
\frac{\partial G}{\partial z} &= -\alpha\mu'(\theta)\frac{\partial \theta(z, \delta)}{\partial z}[\delta f'(z) - 1] - \alpha[1 - \mu(\theta)]\delta f''(z) > 0.
\end{aligned}
$$

where

$$\theta(z,\delta) = \mu'^{-1}\Big(\frac{\kappa}{\delta f(\ell) - \ell - \delta f(z) + z}\Big)$$
$$\frac{\partial\theta(z,\delta)}{\partial\delta} = \frac{\kappa}{(\delta f(\ell) - \ell - \delta f(z) + z)^2}\frac{f(\ell) - f(z)}{-\mu''(\theta)} > 0$$
$$\frac{\partial\theta(z,\delta)}{\partial z} = \frac{\kappa}{(\delta f(\ell) - \ell - \delta f(z) + z)^2}\frac{\delta f'(z) - 1}{\mu''(\theta)} < 0$$

By fundamental theorem,

$$\frac{\partial z}{\delta\delta} = -\frac{\frac{\partial G}{\partial\delta}}{\frac{\partial G}{\partial z}} = \frac{a[f(\ell) - f(z)] + bf'(z)}{a[\delta f'(z) - 1] - b\delta f''(z)} > 0 \tag{A.1}$$

where $a = -\alpha\mu'(\theta)[\delta f'(z) - 1][\delta f(\ell) - \ell - \delta f(z) + z)]^{-2}(\mu''(\theta))^{-1}\kappa > 0$ and $b = \alpha(1 - \mu(\theta)) > 0$.

Equation (A.1) can be rewritten as

$$a\Big[f(\ell) - f(z) - (\delta f'(z) - 1)\frac{\partial z}{\partial\delta}\Big] + b\Big[f'(z) + \delta f''(z)\frac{\partial z}{\partial\delta}\Big] = 0.$$

We know $f'(z) + \delta f''(z)\frac{\partial z}{\partial\delta} < 0$ since $\frac{\partial\delta f'(z(\delta))}{\partial\delta} < 0$. So it must be true that $f(\ell) - f(z) - (\delta f'(z) - 1)\frac{\partial z}{\partial\delta} > 0$.

Then, consider the total surplus of obtaining a loan $\Lambda(\delta)$ defined as follows

$$\Lambda(\delta) = \delta f(\ell(\delta)) - \ell(\delta) - \delta f(z(\delta)) + z(\delta).$$

We know

$$\frac{\partial\Lambda(\delta)}{\partial\delta} = f(\ell) - f(z) - (\delta f'(z) - 1)\frac{\partial z}{\partial\delta} > 0$$

Finally, by $\mu'(\theta)\Lambda(\delta) = \kappa$, we know $\frac{\partial\theta}{\partial\delta} > 0$.

F Proof of Proposition 2

Let $U_r(\Omega)$ be type-r's payoff of applying for a contract Ω. Suppose $U_r(\Omega_s^*) > U_r^*$ such that Ω_s^* is not incentive compatible when information is asymmetric. Using the binding free-entry condition, $U_r(\Omega_s)$ can be written as

$$U_r(\Omega_s) = -iz_s + \alpha\mu(\theta_s)[\delta_r f(\ell_s) - \frac{\delta_r}{\delta_s}\ell_s + \frac{\delta_r}{\delta_s}d_s - d_s] - \frac{\delta_r}{\delta_s}\kappa\theta_s + \alpha(1-\mu(\theta_s))[\delta_r f(z_s) - z_s]$$

With asymmetric information, down payment is used to screen entrepreneurs, so $d_s = z_s$. Then,

$$\frac{\partial U_r(\Omega_s)}{\partial \theta_s} = \alpha\frac{\delta_r}{\delta_s}\Big\{\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s - \delta_s f(z_s) + z_s] - \kappa Big\},$$

where $\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s - \delta_s f(z_s) + z_s]$ is the expected marginal surplus of an additional match in the safe type submarket, and κ is the marginal cost. The net surplus is greater than or equal to zero, $\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s - \delta_s f(z_s) + z_s] - \kappa \geq 0$, otherwise bankers would not enter the loan market. So $\frac{\partial U_r(\Omega_s)}{\partial \theta_s} \geq 0$, indicating that risky types' incentive to misreport increases in market tightness in the safe type submarket. Meanwhile,

$$\frac{\partial U_r(\Omega_s)}{\partial z_s} = -i - \alpha\mu(\theta_s)\frac{\delta_r}{\delta_s} + \alpha(1-\mu(\theta_s))[\delta_r f'(z_s) - 1].$$

Since $-i + \alpha(1-\mu(\theta_s^*))[\delta_s f'(z_s^*) - 1] = 0$, $\frac{\partial U_r(\Omega_s)}{\partial z_s}|_{z_s=z_s^*,\theta_s=\theta_s^*} < 0$, meaning that risky types' incentive to misreport decreases in down payment required in the safe type submarket. In equilibrium, $\mu'(\theta_s) = \frac{\kappa}{\delta_s f(\ell_s) - \ell_s - \delta_s f(z_s) + z_s}$, θ_s and z_s move in opposite directions. Therefore, if Ω_s is incentive compatible, it must be the case that $z_s > z_s^*$ and $\theta_s < \theta_s^*$.

G Proof of Proposition 3

Now I show how δ_r affects z_s and θ_s. We can write θ_s as a function of z_s. Then the binding IC constraint can be rewritten as

$$I \equiv -iz_s + \alpha\mu(\theta_s(z_s))[\delta_r f(\ell_s) - (1 - \frac{\delta_r}{\delta_s})z_s - \frac{\delta_r}{\delta_s}\ell_s] - \frac{\alpha\theta_s(z_s)\kappa\delta_r}{\delta_s}$$
$$+ \alpha(1 - \mu(\theta_s(z_s)))[\delta_r f(z_s) - z_s] - U_r = 0$$

$$\begin{aligned}
\frac{\partial I}{\partial z_s} &= -i - \alpha\mu(\theta_s(z_s))(1 - \frac{\delta_r}{\delta_s}) + \alpha(1 - \mu(\theta_s(z_s)))[\delta_r f'(z_s) - 1] \\
&+ \alpha\mu'(\theta_s)\theta_s'(z_s)[\delta_r f(\ell_s) - \delta_r f(z_s) + \frac{\delta_r}{\delta_s}z_s - \frac{\delta_r}{\delta_s}\ell_s] - \alpha\frac{\delta_r\kappa\theta_s'(z_s)}{\delta_s} \\
&= -i - \alpha\mu(\theta_s(z_s))(1 - \frac{\delta_r}{\delta_s}) + \alpha(1 - \mu(\theta_s(z_s)))[\delta_r f'(z_s) - 1] < 0 \\
\frac{\partial I}{\partial \delta_r} &= \alpha\mu(\theta_s(z_s))[f(\ell_s) + z_s/\delta_s - \ell_s/\delta_s] - \frac{\alpha\kappa\theta_s(z_s)}{\delta_s} + \alpha(1 - \mu(\theta_s(z_s)))f(z_s) - \frac{\partial U_r}{\partial \delta_r} \\
&= \frac{\alpha}{\delta_s}[\mu(\theta_s(z_s))[\delta_s f(\ell_s) + z_s - \ell_s] - \kappa\theta_s(z_s) + (1 - \mu(\theta_s(z_s)))\delta_s f(z_s)] - \frac{\partial U_r}{\partial \delta_r} \\
&= \frac{\alpha}{\delta_s}[\mu(\theta_s(z_s))[\delta_s f(\ell_s) + z_s - \ell_s] - \kappa\theta_s(z_s) + (1 - \mu(\theta_s(z_s)))\delta_s f(z_s)] \\
&- \alpha[\mu(\theta_r)f(\ell_r) + (1 - \mu(\theta_r))f(z_r)] \\
\frac{\partial I}{\partial i} &= -z_s < 0.
\end{aligned}$$

Note that when $\delta_r \to \delta_s$, $\frac{\partial I}{\partial \delta_r} < 0$; when δ_r is very small, $\frac{\partial I}{\partial \delta_r} > 0$. By fundamental theorem,

$$\frac{\partial z_s}{\partial i} = -\frac{\frac{\partial I}{\partial i}}{\frac{\partial I}{\partial z_s}} < 0 \text{ and } \frac{\partial z_s}{\partial \delta_r} = -\frac{\frac{\partial I}{\partial \delta_r}}{\frac{\partial I}{\partial z_s}} \begin{cases} > 0 \text{ when } \delta_r \text{ is small;} \\ < 0 \text{ when } \delta_r \text{ is big.} \end{cases}$$

We can write z_s as a function of θ_s. Then the binding IC constraint can be rewritten as

$$I \equiv -iz_s(\theta_s) + \alpha\mu(\theta_s)[\delta_r f(\ell_s) - (1-\frac{\delta_r}{\delta_s})z_s(\theta_s) - \frac{\delta_r}{\delta_s}\ell_s] - \frac{\alpha\theta_s\kappa\delta_r}{\delta_s}$$
$$+ \alpha(1-\mu(\theta_s))[\delta_r f(z_s(\theta_s)) - z_s(\theta_s)] - U_r = 0$$

$$\begin{aligned}
\frac{\partial I}{\partial \theta_s} &= -iz_s'(\theta_s) + \alpha\mu'(\theta_s)[\delta_r f(\ell_s) - (1-\frac{\delta_r}{\delta_s})z_s(\theta_s) - \frac{\delta_r}{\delta_s}\ell_s] - \alpha\mu(\theta_s)(1-\frac{\delta_r}{\delta_s})z_s'(\theta_s) \\
&- \alpha\mu'(\theta_s)[\delta_r f(z_s(\theta_s)) - z_s(\theta_s)] + \alpha(1-\mu(\theta_s))[\delta_r f'(z_s(\theta_s)) - 1]z_s'(\theta_s) - \alpha\frac{\delta_r\kappa}{\delta_s} \\
&= z_s'(\theta_s)[-i - \alpha\mu(\theta_s)(1-\frac{\delta_r}{\delta_s}) + \alpha(1-\mu(\theta_s))[\delta_r f'(z_s) - 1]] > 0 \\
\frac{\partial I}{\partial \delta_r} &= \frac{\alpha}{\delta_s}[\mu(\theta_s)[\delta_s f(\ell_s) + z_s(\theta_s) - \ell_s] - \kappa\theta_s + (1-\mu(\theta_s))\delta_s f(z_s(\theta_s))] - \frac{\partial U_r}{\partial \delta_r} \\
&= \frac{\alpha}{\delta_s}[\mu(\theta_s)[\delta_s f(\ell_s) + z_s(\theta_s) - \ell_s] - \kappa\theta_s + (1-\mu(\theta_s))\delta_s f(z_s(\theta_s))] \\
&- \alpha[\mu(\theta_r)f(\ell_r) + (1-\mu(\theta_r))f(z_r)] \\
\frac{\partial I}{\partial i} &= -z_s(\theta_s) < 0
\end{aligned}$$

By fundamental theorem,

$$\frac{\partial \theta_s}{\partial i} = -\frac{\frac{\partial I}{\partial i}}{\frac{\partial I}{\partial \theta_s}} > 0 \ \text{ and } \ \frac{\partial \theta_s}{\partial \delta_r} = -\frac{\frac{\partial I}{\partial \delta_r}}{\frac{\partial I}{\partial \theta_s}} \begin{cases} < 0 \text{ when } \delta_r \text{ is small;} \\ > 0 \text{ when } \delta_r \text{ is big.} \end{cases}$$

H Proof of Proposition 4

First, consider $\hat{U}_j = -i\hat{z}_j + \alpha(\delta_j f(\hat{z}_j) - \hat{z}_j$, where $\hat{z}_j$ is pinned down by $i = \alpha(\delta_j f'(\hat{z}_j) - 1)$. Then,

$$\begin{aligned}
\frac{\partial \hat{U}_j}{\partial i} &= -\hat{z}_j < 0, \\
\frac{\partial^2 \hat{U}_j}{\partial i^2} &= -\frac{1}{\alpha\delta_j f''(\hat{z}_j)} > 0,
\end{aligned}$$

So, $\hat{U}_j$ is decreasing and convex in i. Second, consider U_j^*, and

$$\frac{\partial U_j^*}{\partial i} = -z_j^* < 0,$$
$$\frac{\partial^2 U_j^*}{\partial i^2} = -\frac{\partial z_j^*}{\partial i} > 0.$$

So, U_j^* is decreasing and convex in i too. But $\hat{U}_j$ has a steeper slope than U_j^* since $\hat{z}_j > z_j^*$. Also recall $\hat{z}_s > \hat{z}_r$ and $z_s^* > z_r^*$. So $\hat{U}_s$ is steeper than $\hat{U}_r$, and U_s^* is steeper than U_r^*. Since $\delta_s > \delta_r$, $\hat{U}_s$ is higher than $\hat{U}_r$ at $i = 0$, where entrepreneurs use self-finance and enjoy the highest payoff. Thus, $\underline{i}$, the intersection of $\hat{U}_s$ and U_s^*, is lower than $\bar{i}$, the intersection of $\hat{U}_r$ and U_r^*, as shown in Figure 8b. When both types enter the loan market, $\bar{\bar{i}}$ is characterized by

$$-iz_s^* + \alpha\mu(\theta_s^*)[\delta_r f(\ell_s^*) - (1-\delta_r/\delta_s)z_s^* - \ell_s^*\delta_r/\delta_s] - \alpha\theta_s^*(\delta_r/\delta_s)\kappa + \alpha(1-\mu(\theta_s^*))[\delta_r f(z_s^*) - z_s^*] = U_r^*.$$

If $i > \bar{\bar{i}}$, the safe type contract becomes more attractive to the risky types, and $\{z_s, \ell_s, d_s, R_s, \theta_s\} = \{z_s^*, \ell_s^*, z_s^*, (\kappa/\eta(\theta_s^*) + \ell_s^* - z_s^*)/\delta_s, \theta_s^*\}$ is no longer incentive compatible. Hence, bankers need to ask for a large d_s and lower θ_s to reduce the risky types' incentive of misreporting.

I Proof of Proposition 5

First solve the symmetric information case. With complete information, down payment is not needed, $d_j = 0$. Using the binding BB constraint to eliminate R_j, the optimization problem becomes

$$\max_{z_j, \ell_j, \theta_j} -iz_j + \alpha\mu(\theta_j)[\delta_j f(\ell_j) - \ell_j] - \theta_j\kappa.$$

It is obvious that $\delta_j f'(\ell_j^*) = 1$, $\mu'(\theta_j^*) = \frac{\kappa}{\delta_j f(\ell_j^*) - \ell_j^*}$, and $z_j^* = 0$.

Then consider the asymmetric information case. Let Δ^{IC} be the Lagrangian multiplier of the IC^{GSW}-rs constraint. In this setup, z_s has no use but to pay d_s, so $d_s = z_s$. By taking first order conditions, $\delta_s f'(\ell_s) = 1$. Suppose $\theta_s > 0$,

$$\Delta^{IC}\frac{\delta_r}{\delta_s} = \frac{\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s] - \kappa}{\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s - \frac{\delta_s}{\delta_r} z_s + z_s] - \kappa},$$

which can be rewritten as

$$(1 - \Delta^{IC}\frac{\delta_r}{\delta_s})[\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s] - \kappa] = \Delta^{IC}\frac{\delta_r}{\delta_s}\mu'(\theta_s)(1 - \frac{\delta_s}{\delta_r})z_s. \tag{A.2}$$

Suppose $z_s > 0$,

$$\Delta^{IC} = \frac{i}{i + \alpha\mu(\theta_s)(1 - \delta_r/\delta_s)} < 1.$$

Then, $1 - \Delta^{IC}\frac{\delta_r}{\delta_s} > 0$ and the expected marginal net surplus of posting one more contract in the safe type submarket is greater than or equal to zero, $\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s] - \kappa \geq 0$, so the left hand side of Equation (A.2) is non-negative. The right hand side of Equation (A.2), however, is strictly negative if $z_s > 0$, but this is a contradiction; thus, $z_s = 0$. Equation (A.2) becomes

$$(1 - \Delta^{IC}\frac{\delta_r}{\delta_s})[\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s] - \kappa] = 0.$$

So $\Delta^{IC} = \delta_s/\delta_r$ since $\theta_s \neq \theta_s^*$ under asymmetric information; θ_s solves the binding IC^{GSW}-rs constraint, and R_s solves the binding free-entry condition.

J Proof of Proposition 6

Let $\bar{\tau}_j = \mu(\theta_j^*)(R_j - R_j^*)$ be the net transfer to type-j entrepreneurs upon matching to restore the complete information contract Ω_j^*. Then, the BB, PIC-rs, PIC-sr, PC-r, and

PC-s constraints become

$$
\begin{aligned}
&\nu_s \bar{\tau}_s + \nu_r \bar{\tau}_s \geq 0, \\
&U_r(z_s^*, \Omega_s^*, \theta_s^*) + \alpha \frac{\delta_r}{\delta_s} \bar{\tau}_s \leq U_r^* + \alpha \bar{\tau}_r, \\
&U_s(z_r^*, \Omega_r^*, \theta_r^*) + \alpha \frac{\delta_s}{\delta_r} \bar{\tau}_r \leq U_s^* + \alpha \bar{\tau}_s, \\
&U_r^* + \alpha \bar{\tau}_r \geq \hat{U}_r, \\
&U_s^* + \alpha \bar{\tau}_s \geq \hat{U}_s.
\end{aligned}
$$

Since risky types have an incentive to misreport and $U_r(z_s^*, \Omega_s^*, \theta_s^*) - U_r^* > 0$, it must be true that $\bar{\tau}_s < 0$ and $\bar{\tau}_r > 0$ for the PIC-rs constraint to be satisfied. If $\bar{\tau}_r > 0$, the PC-r constraint must be slack. Suppose there exists a net transfer $\bar{\bar{\tau}}_s$ such that the BB and PIC-rs constraints bind, $\bar{\bar{\tau}}_s = -\frac{U_r(z_s^*, \Omega_s^*, \theta_s^*) - U_r^*}{\alpha(\nu_s/\nu_r + \delta_r/\delta_s)}$. Since $U_s^* - U_s(z_r^*, \Omega_r^*, \theta_r^*) > [U_r(z_s^*, \Omega_s^*, \theta_s^*) - U_r^*]\delta_s/\delta_r$, such $\bar{\bar{\tau}}_s$ satisfies the PIC-sr constraint. Note that it must also satisfy the PC-s constraint such that $\bar{\bar{\tau}}_s \geq -\frac{U_s^* - \hat{U}_s}{\alpha}$. Thus, a sufficient condition to restore z_j^*, d_j^*, ℓ_j^*, and θ_j^* is obtained:

$$
U_s^* - \hat{U}_s \geq \frac{U_r(z_s^*, \Omega_s^*, \theta_s^*) - U_r^*}{\frac{\nu_s}{\nu_r} + \frac{\delta_r}{\delta_s}}.
$$

K Proof of Proposition 7

Let Δ^x be the Lagrangian multiplier of condition x. Suppose the PC-r, PC-s and PIC-sr constraints are slack; the FOCs become

$$
\begin{aligned}
&\delta_r f'(\ell_r) = 1, \\
&-i + \alpha(1 - \mu(\theta_r))[\delta f'(z_r) - 1] = 0, \\
&\mu'(\theta_r) = \frac{\kappa}{\delta_r f(\ell_r) - \ell_r - \delta_r f(z_r) + z_r}, \\
&\Delta^{BB} \nu_r = \alpha(\sigma_r + \Delta^{PIC-rs}).
\end{aligned}
\tag{A.3}
$$

Together with condition $d_r \leq z_r$, it is obvious that the allocations of risky types are not distorted. The allocations of safe types are characterized by the following conditions:

$$
\begin{aligned}
& d_s = z_s, \\
& \delta_s f'(\ell_s) = 1, \\
& \Delta^{BB}\nu_s = \alpha(\sigma_s - \frac{\delta_r}{\delta_s}\Delta^{PIC-rs}), && \text{(A.4)} \\
& \mu'(\theta_s) = \frac{\kappa}{\delta_s f(\ell_s) - \ell_s - \delta_s f(z_s) + z_s}, && \text{(A.5)} \\
& \frac{\Delta^{PIC-rs}}{\sigma_s} = \frac{-i + \alpha(1-\mu(\theta_s))[\delta_s f'(z_s) - 1]}{-i - (1-\delta_r/\delta_s)\alpha\mu(\theta_s) + \alpha(1-\mu(\theta_s))[\delta_r f'(z_s) - 1]}. && \text{(A.6)}
\end{aligned}
$$

Using Equation (A.3) and (A.4), Δ^{BB} and Δ^{PIC-rs} are obtained. Then $\frac{\Delta^{PIC-rs}}{\sigma_s} = \frac{1-\frac{\nu_s\sigma_r}{\nu_r\sigma_s}}{\frac{\delta_r}{\delta_s}+\frac{\nu_s}{\nu_r}}$. Equation (A.6) can be rewritten as

$$
\frac{1 - \frac{\nu_s\sigma_r}{\nu_r\sigma_s}}{\frac{\delta_r}{\delta_s} + \frac{\nu_s}{\nu_r}} = \Delta^{IC} \equiv \frac{-i + \alpha(1-\mu(\theta_s))[\delta_s f'(z_s) - 1]}{-i - (1-\delta_r/\delta_s)\alpha\mu(\theta_s) + \alpha(1-\mu(\theta_s))[\delta_r f'(z_s) - 1]}.
$$

Suppose all types choose to enter the loan market. Recall the market equilibrium allocation z_s^{ce} and θ_s^{ce} solve the same condition as in (A.5) and the binding IC-rs constraint, given the R_s that is pinned down by the binding free entry condition. Also note that $-i - (1 - \delta_r/\delta_s)\alpha\mu(\theta_s^{ce}) + \alpha(1-\mu(\theta_s^{ce}))[\delta_r f'(z_s^{ce}) - 1] < -i + \alpha(1-\mu(\theta_s^{ce}))[\delta_s f'(z_s^{ce}) - 1] < 0$. Therefore, if there exists a σ_s such that

$$
\frac{1 - \frac{\nu_s(1-\sigma_s)}{\nu_r\sigma_s}}{\frac{\delta_r}{\delta_s} + \frac{\nu_s}{\nu_r}} = \Delta^{IC} = \Delta^{ce},
$$

where Δ^{ce} is the multiplier in the competitive equilibrium and $0 < \Delta^{ce} < 1$, then allocations are identical as ones in the competitive equilibrium, $\{\theta_j, z_j, \ell_j, d_j, R_j\}_{j=r,s} = \{\theta_j^{ce}, z_j^{ce}, \ell_j^{ce}, d_j^{ce}, R_j^{ce}\}_{j=r,s}$.

The above equation can be rewritten as

$$
1 = \Delta^{ce}\Big(\frac{\delta_r}{\delta_s} + \frac{\nu_s}{\nu_r}\Big) + \frac{\nu_s}{\nu_r}\frac{\sigma_r}{\sigma_s}, \tag{A.7}
$$

where $\frac{\sigma_r}{\sigma_s} \to 0$ as $\sigma_s \to 1$ and $\frac{\sigma_r}{\sigma_s} \to \infty$ as $\sigma_s \to 0$. If

$$\frac{\delta_r}{\delta_s} + \frac{\nu_s}{\nu_r} < 1,$$

there always exists a σ_s such that the Equation (A.7) holds. Therefore, I have found a sufficient condition for the competitive equilibrium to be constrained efficient.

L Moral Hazard Problem

A type-j entrepreneur has the following value functions. In the CM,

$$W_j^{mh}(k,w) = w + f(k) + \max_{z_j', \Omega_j' \in \boldsymbol{\Omega}^{pt}, \theta_j'} \Big\{ -\frac{z_j'}{1+r^z} + \beta V_j^{mh}(z_j', \Omega_j', \theta_j') \Big\}.$$

In the DM,

$$\begin{aligned} V_j^{mh}(z_j, \Omega_j, \theta_j) = \max_{k_j^b, k_j^u} \;& \alpha\mu(\theta_j)\Big[\delta_j[f(k_j^b) + z_j - d_j - R_j + (1 - C(\tilde{\chi}))(\ell_j - q^k k_j^b) + W_j^{mh,0}] \\ & + (1-\delta_j)(z_j - d_j + (1 - C(\tilde{\chi}))(\ell_j - q^k k_j^b))\Big] \\ & + \alpha(1-\mu(\theta_j))\Big[\delta_j[f(k_j^u) + z_j - q^k k_j^u + W_j^{mh,0}] + (1-\delta_j)(z_j - q^k k_j^u)\Big] \\ & + (1-\alpha)\Big[z_j + W_j^{mh,0}\Big] \\ \text{s.t.} \;\; & q^k k_j^b \le \ell_j, \;\; q^k k_j^u \le z_j, \end{aligned}$$

where $W_j^{mh,0} = W_j^{mh}(0,0)$. Banks and capital producers have similar value functions as in the baseline. Then, the market designer solves the optimization problem below:

$$
\begin{aligned}
U_j^{mh} = \max_{z_j,(d_j,\ell_j,R_j),\theta_j} \quad & -iz_j + \alpha\mu(\theta_j)[\delta_j f(\ell_j) - d_j - \delta_j R_j] + \alpha(1-\mu(\theta_j))[\delta_j f(z_j) - z_j] \\
\text{s.t.} \quad & z_j \geq d_j, \quad \eta(\theta_j)(d_j - \ell_j + \delta_j R_j) \geq \kappa, \\
& -iz_j + \alpha\mu(\theta_j)\Big[\max_{\substack{k_{\tilde{j}j}, x_{\tilde{j}j} \text{ s.t.} \\ k_{\tilde{j}j} \leq \ell_j + z_j - d_j \\ x_{\tilde{j}j} \leq \ell_j - k_{\tilde{j}j} + z_j - d_j}} \delta_{\tilde{j}} f(k_{\tilde{j}j}) + (1 - C(\tilde{\chi}))x_{\tilde{j}j} - d_j - \delta_{\tilde{j}} R_j \Big] \\
& + \alpha(1-\mu(\theta_j))[\delta_{\tilde{j}} f(z_j) - z_j] \leq U_{\tilde{j}}^{mh}
\end{aligned}
$$

M Proof of Lemma 3

The allocations of the risky type entrepreneurs are identical to the ones under complete information, as expected. For the problem of safe types, first solve the risky types' choice of investment when they apply for a safe type contract. The risky types either choose $k_{rs} = k_{rs}^{mh} < \ell_s$ or $k_{rs} = \ell_s$. Suppose $k_{rs} = k_{rs}^{mh} < \ell_s$, ℓ_s, θ_s, z_s and multiplier Δ of the constraint MIC-rs are characterized by

$$
\delta_s f'(\ell_s) - 1 = (\chi - \delta_r/\delta_s)\Delta, \tag{A.8}
$$

$$
\Delta = \frac{-i + \alpha(1-\mu(\theta_s))[\delta_s f'(z_s) - 1]}{-i - \alpha\mu(\theta_s)(1 - \delta_r/\delta_s) + \alpha(1-\mu(\theta_s))[\delta_r f'(z_s) - 1]}, \tag{A.9}
$$

$$
\mu'(\theta_s) = \frac{\kappa}{\frac{[\delta_s f(\ell_s) - \ell_s] - \Delta\delta_r/\delta_s[\delta_s f(k_{rs}^{mh}) + (\chi\delta_s/\delta_r - 1)\ell_s - \chi k_{rs}^{mh}\delta_s/\delta_r]}{1 - \Delta\delta_r/\delta_s} - \delta_s f(z_s) + z_s}, \tag{A.10}
$$

and the binding IC constraint

$$
\begin{aligned}
-iz_s + \alpha\mu(\theta_s)[\delta_r f(k_{rs}^{mh}) - z_s + \chi(\ell_s - k_{rs}^{mh}) + z_s\delta_r/\delta_s - \ell_s\delta_r/\delta_s] - \alpha\theta_s(\kappa)(\delta_r/\delta_s) \\
+ \alpha(1-\mu(\theta_s))[\delta_r f(z_s) - z_s] = U_r^*,
\end{aligned} \tag{A.11}
$$

that is simplified using binding BB constraint and $d_s = z_s$.

By Equation (A.8), it is either (1) $\chi > \delta_r/\delta_s$ and $\ell_s < \ell_s^*$, or (2) $\chi \leq \delta_r/\delta_s$ and $\ell_s \geq \ell_s^*$. Since $\delta_r f'(k_{rs}^{mh}) = \chi$ and $1 > \Delta > 0$, Equation (A.8) can be rewritten as $1 > (\delta_s f'(\ell_s) - 1)/(\delta_s f'(k_{rs}^{mh}) - 1) = \Delta\delta_r/\delta_s > 0$. If (1) is true, $\ell_s > k_{rs}^{mh}$. If (2) is true, $k_{rs}^{mh} > \ell_s$, which is not feasible knowing that the risky types have no other assets, $d_s = z_s$. So, if $\chi > \delta_r/\delta_s$, $k_{rs} = k_{rs}^{mh}$ and equilibrium allocations are characterized by Equation (A.8) - (A.11).

Suppose $k_{rs} = \ell_s$, then the MIC-rs becomes IC-rs, and the problem coincides with the one in the baseline.

N Proof of Proposition 8

By Condition (A.8), it is easy to see that $\delta_s f'(\ell_s) - 1 > 0$, so $\ell_s < \ell_s^*$. By Condition (A.9), we know $0 < \Delta < 1$ since $-i - \alpha\mu(\theta_s)(1 - \delta_r/\delta_s) + \alpha(1 - \mu(\theta_s))[\delta_r f'(z_s) - 1] < -i + \alpha(1 - \mu(\theta_s))[\delta_s f'(z_s) - 1] < 0$. Also, consider Condition (A.10). Since $\delta_s f(\ell_s) - \ell_s < \delta_s f(k_r^*) + (\delta_s/\delta_r - 1)\ell_s - k_r^*\delta_s/\delta_r$ and $1 - \Delta\delta_r/\delta_s > 0$, we know $\frac{[\delta_s f(\ell_s) - \ell_s] - \Delta\delta_r/\delta_s[\delta_s f(k_r^*) + (\delta_s/\delta_r - 1)\ell_s - k_r^*\delta_s/\delta_r]}{1 - \Delta\delta_r/\delta_s} < \delta_s f(\ell_s) - \ell_s$. Then, for a given z_s, $\mu'(\theta_s) = \frac{\kappa}{\frac{[\delta_s f(\ell_s) - \ell_s] - \Delta\delta_r/\delta_s[\delta_s f(k_r^*) + (\delta_s/\delta_r - 1)\ell_s - k_r^*\delta_s/\delta_r]}{1 - \Delta\delta_r/\delta_s} - \delta_s f(z_s) + z_s} > \frac{\kappa}{\delta_s f(\ell_s) - \ell_s - \delta_s f(z_s) + z_s} > \frac{\kappa}{\delta_s f(\ell_s^*) - \ell_s^* - \delta_s f(z_s) + z_s} = \mu'(\theta_s^{bl})$, so $\theta_s < \theta_s^{bl}$ given z_s. Finally, consider Condition (A.11). We know $\delta_r f(k_r^*) - z_s + \ell_s - k_r^* + z_s\delta_r/\delta_s - \ell_s\delta_r/\delta_s > \delta_r f(\ell_s^*) - z_s + z_s\delta_r/\delta_s - \ell_s^*\delta_r/\delta_s$. Given a θ_s, for IC-rs constraint to bind in both the moral hazard and the baseline model, $-iz_s + \alpha\mu(\theta_s)[\delta_r f(k_r^*) - z_s + \ell_s - k_r^* + z_s\delta_r/\delta_s - \ell_s\delta_r/\delta_s] - \alpha\theta_s(\kappa)(\delta_r/\delta_s) + \alpha(1 - \mu(\theta_s))[\delta_r f(z_s) - z_s] = -iz_s^{bl} + \alpha\mu(\theta_s)[\delta_r f(\ell_s^*) - z_s^{bl} + z_s^{bl}\delta_r/\delta_s - \ell_s^*\delta_r/\delta_s] - \alpha\theta_s(\kappa)(\delta_r/\delta_s) + \alpha(1 - \mu(\theta_s))[\delta_r f(z_s^{bl}) - z_s^{bl}] = U_r^*$, it must be the case that $z_s > z_s^{bl}$, given θ_s.

O Proof of Proposition 9

Since the dual deviation does not affect $\hat{U}_j$ or U_j^*, $\underline{i}^{mh} = \underline{i}$ and $\bar{i}^{mh} = \bar{i}$. However, when both types enter the loan market and $\chi > \delta_r/\delta_s$, $\bar{\bar{i}}^{mh}$ is characterized by

$$-iz_s^* + \alpha\mu(\theta_s^*)[\delta_r f(k_{rs}^{mh}) + \chi(\ell_s^* - k_{rs}^{mh}) - (1-\delta_r/\delta_s)z_s^* - \ell_s^*\delta_r/\delta_s] - \alpha\theta_s^*(\delta_r/\delta_s)(\kappa)$$
$$+ \alpha(1-\mu(\theta_s^*))[\delta_r f(z_s^*) - z_s^*] = U_r^*.$$

If $i > \bar{\bar{i}}^{mh}$, the safe type contract becomes more attractive to the risky types, and $\{z_s, \ell_s, d_s, R_s, \theta_s\} = \{z_s^*, \ell_s^*, z_s^*, (\kappa/\eta(\theta_s^*) + \ell_s^* - z_s^*)/\delta_s, \theta_s^*\}$ is no longer incentive compatible. Hence, bankers need to ask for a large d_s and lower ℓ_s and θ_s to reduce the risky types' incentive of misreporting.

P Proof of Proposition 10

When $\chi > \delta_r/\delta_s$ and $\{z_s, \ell_s, d_s, R_s, \theta_s\} = \{z_s^*, \ell_s^*, z_s^*, (\kappa/\eta(\theta_s^*) + \ell_s^* - z_s^*)/\delta_s, \theta_s^*\}$, risky types' payoff of misreport in the moral hazard extension is

$$U_{rs}^{mh} = -iz_s^* + \alpha\mu(\theta_s^*)[\delta_r f(k_{rs}^{mh}) + \chi(\ell_s^* - k_{rs}^{mh}) - (1-\delta_r/\delta_s)z_s^* - \ell_s^*\delta_r/\delta_s] - \alpha\theta_s^*(\delta_r/\delta_s)\kappa$$
$$+ \alpha(1-\mu(\theta_s^*))[\delta_r f(z_s^*) - z_s^*]$$

Risky types' payoff of misreport in the baseline is

$$U_{rs} = -iz_s^* + \alpha\mu(\theta_s^*)[\delta_r f(\ell_s^*) - (1-\delta_r/\delta_s)z_s^* - \ell_s^*\delta_r/\delta_s] - \alpha\theta_s^*(\delta_r/\delta_s)\kappa$$
$$+ \alpha(1-\mu(\theta_s^*))[\delta_r f(z_s^*) - z_s^*]$$

It is obvious that $U_{rs}^{mh} \geq U_{rs}$ since $k_r^* \leq k_{rs}^{mh} \leq \ell_s^*$. So U_{rs}^{mh} intersects with U_r^* at a lower i than U_{rs} does, as Figure 8b illustrates.