Adverse Selection and Small Business Finances

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This version: June 11, 2022

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Abstract

This paper builds a directed search model with asymmetric information and argues that small firms hold liquid assets not only to self-finance investment but also to signal their investment quality to obtain better loan terms. Because self-finance is an outside option of borrowing, it affects bank loans and the credit market structure. Monetary policy affects the cost of self-finance and hence affects the market structure and the screening regimes in the credit market. An increase in the policy rate can trigger banks to use screening contracts, which distort allocations and reduce welfare. I find that a negative pass-through from the policy rate to the real lending rate is possible. I also show that when the equilibrium is distorted, constrained efficiency can be restored via an appropriately designed tax scheme that involves taxing the safe borrowers and subsidizing the risky ones.

Keywords: Adverse Selection, Competitive Search, Liquidity Holdings, Small Business Finances

JEL codes: D82, D83, G30, E41

*I am grateful to Martin Gervais for his extensive feedback and support throughout the writing of this paper. I also thank Roozbeh Hosseini, Michael Choi, Scott Spitze, Jie (Jack) He, as well as all participants in the Economics Seminar at UGA, the Macro Brownbag at UC-Irvine, and I-85 Macro at UNC Charlotte for their valuable comments and suggestions. Special thanks to the Kauffman Foundation for sponsorship of the Kauffman Firm Survey. Email: fan.liang@uga.edu

1 Introduction

This paper studies small firms' incentives of holding liquid assets, i.e., cash and cash equivalents that have low or even negative returns, and credit market structure in an environment with information and search frictions.[1] In the traditional literature on corporate liquidity management, many papers have focused on the precautionary motive for holding liquid assets (e.g., T. Opler et al. 1999, Almeida et al. 2004, Bates et al. 2009, and Acharya et al. 2012): liquidity acts as a buffer stock when the credit market is not accessible or external credit is too costly to obtain. However, even when the credit market is accessible, liquidity holdings help firms obtain credit: about 7% of commercial business loans are secured by liquid collateral, which is business deposits or securities (such as stocks and bonds) pledged against the loan borrowed, according to the Survey of Small Business Finances (SSBF) 2003.[2] One plausible explanation is that liquidity holdings serve as a screening device in the credit market when lenders cannot distinguish borrowers' ability to repay. Thus, I propose a motive, called the *signaling motive*, suggesting that firms use liquidity holdings as collateral to signal their ability to repay the loan in exchange for better loan terms.

To rationalize the precautionary and signaling motives for holding liquid assets, I introduce an endogenous self-finance channel in a classic screening model by allowing firms to self-support investment using liquidity holdings. I then investigate how monetary policy affects credit market structure and screening regimes. In addition, I also study if the competitive equilibrium can be constrained efficient; if it cannot be so, I describe what fiscal policy can be to mitigate the effects of adverse selection.

[1] This paper restricts attention to small businesses with simple balance sheet consisting of liquid assets and commercial bank loans. In the U.S., about 28% of total small business assets are cash and cash equivalent, according to the 2011 wave of the Kauffman Firm Survey (KFS). The average cash-to-assets ratio was around 23% among U.S. large and mostly public trading firms in 2006 (Bates et al. 2009). I focus on commercial business loans because they are small firms' most used financial resource; 43% of small firms sought one in 2020, as documented by the Small Business Credit Survey (SBCS).

[2] According to the SSBF 2003, about 55% of commercial business loans are secured by collateral. Among collateralized loans, 13% are secured by liquid collateral, and the rest are secured by illiquid collateral, which can be real estate, equipment, inventory, etc.

To rationalize the precautionary and signaling motives for holding liquid assets,[3] I use a general equilibrium approach to liquidity that builds on Lagos & Wright (2005) and Rocheteau & Wright (2005)[4] with directed search[5] and asymmetric information. In the model, there are two types of entrepreneurs: risky and safe, and they differ in their investment quality. It is assumed that the entrepreneurs know their type, but the bankers do not.[6] The entrepreneurs receive random opportunities to invest but cannot issue debt. Thus, in order to finance investment projects, they either use their own liquidity holdings (i.e., the self-finance channel) or borrow from a bank (i.e., the external finance channel). I adapt the mechanism of Guerrieri, Shimer, & Wright (2010), hereafter GSW, to model the external finance channel: the bankers post contracts on one side of a competitive and frictional loan market and the entrepreneurs search directly on the other side.[7] The ex ante homogenous bankers incur a fixed cost of opening a branch. Bank branches can be located in a submarket, a place that is characterized by the contract; thus, a banker decides which submarket to enter and open his branch. After observing bankers' entry decisions and contracts, each entrepreneur optimally chooses in which submarket to apply for a loan, where matching is bilateral. A loan contract specifies the loan amount,[8] the down payment (inside collateral),[9] and the repayment. In

[3]In Appendix A, I document the existence of precautionary, signaling and transaction motives using the Kauffman Firm Survey (KFS) data to support my model. My result shows that the agency and tax motives, which are suggested by the existing literature, are not significant for the small firms. In a simple environment with only one kind of liquid asset, the transaction motive comes with the precautionary and signaling motives. Thus, in the rest of the paper I restrict attention to the precautionary and signaling motives.

[4]My paper studies entrepreneurs financing investment using liquid assets rather than households financing consumption as in Lagos & Wright (2005), Rocheteau & Wright (2005), and most of their extensions.

[5]One can build a similar model with random search, where the entrepreneurs and bankers bargain over loan terms. However, the bankers in a directed search model can use loan approval rate to screen out the risky entrepreneurs, while the bankers in a random search model cannot.

[6]Borrowers often have more information about their profitability than lenders do. Crawford et al. (2018) find evidence for adverse selection in credit markets by estimating a structural model of credit demand and default.

[7]See Wright et al. (2021) for a survey of directed search and competitive search models.

[8]The amount of intraperiod liabilities issued is the amount of loans, represented by banknotes. One can assume that banknotes are counterfeitable one period after their issuance, so they will not circulate across periods. The banks commit to redeem banknotes later, so they circulate as payment instruments.

[9]In reality, there are two types of collateral: inside collateral and outside collateral. Inside collateral, like the down payment in this paper, is pledged assets that are used in the financed project. Outside collateral, on the other hand, is assets not used in the project; for example, inventory pledged against an equipment loan. Assuming inside collateral will not change the results of the model.

this framework, search frictions naturally imply that some entrepreneurs will not be successful in acquiring a loan.[10] The loan approval rate (i.e., matching probability) endogenously depends on the entry of bankers to submarkets (i.e., number of loans supplied).

If the entrepreneur foresees the possibility of not obtaining a loan, she will hold some liquidity up front and turn to the self-finance channel when needed; this is the precautionary motive for holding liquid assets. The entrepreneurs use liquidity holdings to purchase capital and finance investment in a frictionless capital market. Thus, the entrepreneurs hold liquid assets to better cope with the risk of being financially constrained, as argued by Almeida et al. (2004), for example.

Even when credit is accessible, liquidity is essential because it helps the entrepreneur to obtain better loan terms by signaling her investment quality; this is the signaling motive. In this environment, there is a unique separating equilibrium where two types of contracts are offered (i.e., two submarkets): (1) a low-down-payment, high-repayment small loan catering to the risky types and (2) a high-down-payment, low-repayment large loan catering to the safe types. Because the risky types have an incentive to pose as the safe type to save on the repayment, the bankers in the safe submarket may adopt two devices to screen out the risky types: down payment and loan approval rate.[11] As a result, applying for a safe type contract comes with two costs: it requires investing more in costly liquidity beforehand, and it lowers the probability of acquiring a loan. Bankers ask for a large down payment that is paid by liquidity holdings because they know that the safe types have a stronger precautionary motive, as the safe types have a higher marginal benefit of holding liquid assets in the self-finance channel. The risky types, on the other hand, have lower marginal benefit of holding liquid assets in the self-finance channel and thus are less willing to do so. In addition to asking for a large down payment, the bankers endogenously make the safe type contracts scarce, so the probability of obtaining such a loan becomes lower and the risky types are

[10] Obtaining a loan is a time-consuming process that is not always successful. See Agarwal et al. (2020) for the study of search behavior and loan approval in mortgage markets.

[11] See Coco (2000) for a survey of the use of collateral as a screening or an incentive device in credit markets with asymmetric information.

further discouraged from applying for a safe type loan. Because the safe types receive a higher surplus when they borrow and carry a large amount of liquidity as buffer stock, they are more willing to accept this lower probability in return for better loan terms when they do get to borrow. In contrast, the risky types are not willing to accept this lower probability for their inferior projects. This logic explains why down payment and loan approval rate are used to prevent the risky types from seeking a safe type contract.

The signaling motive complements the precautionary motive in this environment. Without self-finance, the model reduces to GSW with an application to asset markets. In this classic screening model, entrepreneurs have no precautionary demand for liquid assets. Thus, it becomes optimal for bankers in the safe type submarket to use only loan approval rate to screen, which saves entry costs and creates less distortion than using a combination of down payment and loan approval rate. As a result, the removal of the self-finance channel renders the liquid assets redundant.

The opportunity cost of holding liquid assets, which is equivalent to the policy interest rate in this setup, affects screening behaviors in the loan market. There are four possible types of equilibrium allocations, which one occurs depends on policy rate. As the policy rate increases, the type of equilibria transitions from no participation in the credit market, no screening, to screening with one screening tool, and then screening with two screening tools. Note that when the policy rate is high, the economy enters the fourth type of equilibrium, where both down payment and loan approval rate are used to screen, resulting in distorted equilibrium allocations and lower payoffs of entrepreneurs. The model also suggests that the interest rate pass-through can be different across submarkets, depending on the matching technology. An increase in the policy rate can reduce the real lending rate for the safe entrepreneurs while rise the real lending rate for the risky ones.

As is common in models with adverse selection, the competitive equilibrium is not efficient when two screening tools are used. Under complete information where entrepreneurs' type is observable, submarkets are independent from each other; the change in the entrepreneurs' payoff in one submarket does not affect the entrepreneurs' payoff in the other submarket. Under private information, however, the bankers face an additional incentive compatibility constraint to ensure that entrepreneurs are willing to reveal their type and apply for loans designed for them. Submarkets are no longer independent from each other since the change in one type's payoff affects the other type's payoff through the incentive compatibility constraint that the bankers face in the other submarket. The change in this constraint alters the set of feasible contracts that the bankers can offer to attract the other type of entrepreneurs and thus affects their payoff. The bankers in one submarket of the market economy take the entrepreneurs' payoff in the other submarket as given; the bankers cannot internalize this externality, but a planner can.

When thinking about government interventions that can improve total welfare, it is useful to analyze a planner's problem. I consider a constrained efficiency problem akin to the direct mechanism from Davoodalhosseini (2019): a social planner who faces the same information and search frictions chooses loan contracts and liquidity holdings, or equivalently transfers contingent on contracts, so that an entrepreneur will reveal her type and receive a contract that depends on her revealed type. I show that under some conditions, a utilitarian planner can always achieve higher welfare than in the market economy by subsidizing the risky types and taxing the safe types. Intuitively, when the risky types are subsidized, they have a higher payoff and lower incentive to misreport their type. This leads to a less tight incentive compatibility constraint that the bankers face in the safe submarket. The bankers then screen out the risky entrepreneurs with a lower intensity and allocations are less distorted: down payments become smaller and more loans are offered. The safe types benefit from lower screening intensity because the cost of carrying liquidity is reduced and the expected investment is increased. Moreover, when this benefit is greater than the cost of being taxed, the safe types are also better off with cross-subsidization. Such allocation Pareto dominates the competitive equilibrium allocation. This result is reminiscent of the finding of Greenwald & Stiglitz (1986); the competitive equilibria with adverse selection may not be constrained

Pareto optimal.

There are two major findings concerning the constrained efficiency. First, I provide a sufficient condition for the competitive equilibrium to be constrained efficient, in which case no transfers are needed. This condition is fulfilled when the population or the failure probability of the risky types is so large that subsidizing them is costly. This result corresponds with findings in Rothschild & Stiglitz (1976). Second, I find a sufficient condition such that, when the equilibrium is distorted, a utilitarian planner who cares about all entrepreneurs equally can completely undo the effect of adverse selection and recover the complete information allocations and welfare using cross-subsidization: taxing the safe types and subsidizing the risky types. This condition is fulfilled when the opportunity cost of holding liquid assets is not too low, there are very few risky types, or the risky types' net benefit of applying for a safe type contract is small.

In the baseline, it is assumed that banknotes can be used only to purchase capital for investment, which is equivalent to observable investment. In the extension, by relaxing this assumption and allowing banknotes to be used to buy consumption goods, I further incorporate moral hazard into the model and study dual deviation: the entrepreneurs may not only misreport their type but also deviate from the investment level that is expected by the banker. As a result, in addition to liquidity holdings and loan approval rate, the bankers employ loan size as an extra screening tool, which means the allocations are distorted in an additional dimension. I also find that the equilibrium allocations are not only more distorted but also more likely to be distorted, in the sense that distortions occur in a large area of the parameter space.

The rest of my paper is organized as follows: Section 2 overviews the related literature, Section 3 outlines the theoretical model, Section 4 describes the equilibrium, Section 5 examines the constrained efficiency problem, Section 6 incorporates moral hazard into the model, and Section 7 concludes the paper.

2 Literature Review

My paper ties into the literature on adverse selection (e.g., Akerlof 1970, Spence 1973, Levin 2001). To illustrate, I review Rothschild & Stiglitz (1976), one of the pioneer models. They study competitive and centralized insurance markets where only the insurants know the probability of their house burning down. The authors show the presence of insurants with different risks can lead to the nonexistence of insurance in equilibrium. In my paper, if directed search is replaced with centralized contracts posting, then my model boils down to the Rothschild and Stiglitz model, and equilibrium may not exist when there are very few risky types or the risky types' probability of failure is too small relative to that of the safe types. GSW solve this nonexistence problem by replacing the competitive market with a frictional market with directed search and capacity limits, thereby providing a tractable general framework to analyze adverse selection in competitive search markets.[12] In my paper, the competitive bank loan market builds on the model of GSW. However, the drawback of this version of GSW, and of most standard screening models with collateral (e.g., Besanko & Thakor 1987 and Bester 1985, 1987) is that the outside option of borrowing is set exogenously, preventing the precautionary motive from being operative. The possibility for entrepreneurs to self-support investment endogenizes the outside option and makes the model consistent with the empirical importance of the precautionary motive, in addition to making liquidity essential in the world of GSW.

Stiglitz & Weiss (1981) develop a canonical model of credit rationing in which among loan applicants who have different risks but seem to be identical, some of them are fully funded but some are denied for a loan, although the rejected applicants are willing to pay a higher interest rate. The phenomenon that some borrowers obtain a loan and some do not is similar to my model, but the failure of obtaining a loan is caused by search frictions rather than credit rationing. In the circumstance of credit rationing, the equilibrium interest rate fails to clear excess demand of credit because raising the interest rate will lower the bank's

[12] In the context of a GSW environment, the capacity limit is that one banker can serve only one borrower.

profit; however, this cannot happen in a competitive bank loan market.

Another paper related to mine is Leland & Pyle (1977), who study signaling through self-finance. In their setup, entrepreneurs have private information about the quality of their projects and are in need of external funds. When the level of projects' self-financing is observable, there exists a continuum of signaling equilibrium, where entrepreneurs with good projects choose to self-finance a fraction of their projects. This is likewise to my finding that the safe types provide a larger down payment than do the risky types. However, my model has a unique separating equilibrium because of the incorporation of directed search rather than a continuum of equilibrium.

My paper also contributes to the literature on corporate liquidity management. Rocheteau et al. (2018) use the dynamic general equilibrium approach to liquidity to study internal and external finance in a complete information environment with random search. When an entrepreneur and banker meet bilaterally and bargain over loan terms, liquidity holdings improve the entrepreneur's bargaining position because an entrepreneur with more liquidity has a better outside option. I incorporate asymmetric information by replacing random search and bargaining with directed search and loan contracts posting. In my model, liquidity is essential for signaling an entrepreneur's ability to repay a loan. Specifically, for signaling reasons, the entrepreneur holds liquid assets to satisfy down payment requirements and obtain better loan terms.[13]

When studying the constrained efficiency problem, I borrow the direct mechanism of GSW developed in Davoodalhosseini (2019). He shows that a planner who maximizes the weighted average of the payoff to agents can attain a first best allocation by conducting cross-subsidization and relaxing incentive constraints (e.g., Miyazaki 1977 and Spence 1978). I show that a similar result holds in my model. Moreover, I also consider a generalized con-

[13]There are also many papers focusing on the precautionary motive for holding liquid assets. For example, Almeida et al. (2004) model precautionary demand for liquidity and find that financially constrained firms invest in liquidity, while unconstrained firms do not. However, they fail to consider how liquidity holdings help entrepreneurs to obtain external finance. Kim et al. (1998) show firms demand more liquidity when external finance costs or future investment returns increase; my model confirms their results.

strained efficiency problem with a planner who weights agents arbitrarily and prove that under some conditions, the competitive equilibrium can be constrained efficient. A similar result is discussed in Rothschild & Stiglitz (1976). Crocker & Snow (1985) consider efficiencies of the equilibrium notion such as Miyazaki-Wilson (Miyazaki 1977 and Wilson 1977) in the setup of Rothschild & Stiglitz (1976) and show competitive equilibrium can be constrained efficient.

In addition, my paper is related to empirical studies of liquid collateral usage. Berger et al. (2016) use Bolivian business loan data to show that loans with liquid collateral are associated with lower interest rates and lower default or delinquency rates than similar loans with illiquid or no collateral. These findings match with the equilibrium loan terms in my model; loans designed for the safe type entrepreneurs require liquid collateral and smaller repayment. To my knowledge, similar studies have not been done using U.S. data. Because KFS does not record ex post outcomes or loan interest rates, I am not able to empirically study whether loans with liquid collateral perform better.

There are empirical papers verifying the findings from the planner's problem, specifically, that constrained efficiency can be restored through cross-subsidization. Cowan et al. (2015) use Chilean loan data to study the impact of a loan guarantee program, which is one of the most common ways to subsidize small firms.[14] They find that firms that borrow with guarantees are more likely to default than similar firms that borrow without guarantees. This higher default rate is caused by adverse selection: firms enrolled in the guarantee program are generally weaker or riskier. They also show that loan guarantees increase the amount of credit supplied both at the intensive margin (i.e., loan size) and the extensive margin (i.e., number of new loans issued). The supply of guaranteed loans increases because they are the ones being subsidized, but, surprisingly, the supply of non-guaranteed loans also increases.

[14] A business loan guarantee program is widely used in most OECD countries. For example, in 2019, the U.S. Small Business Administration (SBA) guaranteed over 28 billion dollars to entrepreneurs. A business loan guarantee program (e.g., the SBA 7(a) Loan Program) is designed to help small businesses obtain financing when they might not be eligible for a reasonably priced business loan. The SBA does not make loans but rather guarantees a portion of loans made by commercial lending institutions. In case of default, the SBA will compensate a portion of the loan to the institution that issued the loan.

These findings match with my results from the constrained efficiency problem, which is equivalent to the problem faced in the loan guarantee program: using cross-subsidization, a planner can improve loans supplied both in the intensive margin (i.e., lower down payment) and the extensive margin (i.e., higher probability of getting a loan), thereby increasing overall welfare. Bachas et al. (2021) study the U.S. loan guarantee program and its credit supply, but they do not address the impact of the program on non-guaranteed loans or the overall credit supply.

3 Model

3.1 Agents and Assets

Time is discrete and infinite, $t = 1, 2, ...$ Each period is divided into two subperiods: in the first subperiod, there is a decentralized frictional banking market (DM) together with a competitive capital market; in the second, there is a centralized Walrasian market (CM) where agents settle debts and trade production goods and assets. All agents discount between the CM and DM at $\beta \in (0, 1)$. This alternating CM and DM structure helps to keep track of the distribution of assets. There are three groups of agents: a unit measure of entrepreneurs (e) who want to invest and need capital, a large number of ex ante homogenous bankers (b) who issue loans, and a large number of capital producers (p) who provide capital. There are two types of entrepreneurs: the risky types (r) with measure ν_r and the safe types (s) with measure ν_s. They differ in the probability of success; with probability δ_j, $j = r, s$, type-j entrepreneurs will have a profitable investment project. The safe types are assumed to be more likely to be successful than the risky types, $\delta_r < \delta_s$. Type is the entrepreneurs' private information, and the bankers know only the distribution of entrepreneurs. All agents are risk neutral and have linear utility over numeraire goods c, where $c > 0$ is consumption and $c < 0$ is production.

There is one type of liquid asset; for example, fiat money. The supply of liquid assets evolves according to $M_{t+1} = (1+\pi)M_t$, where π is the growth rate of liquid asset supply implemented by lump-sum transfers/taxes, T. The price of liquid assets in terms of numeraire is q_t^m, and in a stationary world $q_t^m = (1+\pi)q_{t+1}^m$, as suggested by the quantity theory of money. The real rate of return on liquid assets is $1 + r^z = q_{t+1}^m / q_t^m = 1/(1+\pi)$. As it is usually done, I impose $\pi \geq \beta - 1$ or, equivalently, $r^z \leq 1/\beta - 1$.

The timeline is displayed in Figure 1. In the CM, the idiosyncratic shock on investment is revealed. If the investment turns out to be successful, an entrepreneur with previously acquired capital k produces $f(k)$ units of numeraire goods, where $f(0) = 0$, $f'(0) = \infty$, $f'(\infty) = 0$, and $f'(k) > 0 > f''(k), \forall k > 0$. For simplicity, k fully depreciates after production, which is equivalent to renting. If the investment turns out to be a failure, an entrepreneur produces nothing and exits the market; meanwhile, a new entrepreneur is born to replace her.[15] The entrepreneurs then choose the consumption and production and the real balances of liquidity z. At the same time, the bankers decide whether to enter the bank loan market. If a banker enters this market, he pays a fixed cost $\tilde{\kappa}$ and posts a loan contract $\Omega = (d, R, \ell)$ that specifies the down payment d, the repayment R, and the loan amount ℓ measured in numeraire.[16] Also, $\Omega \in \mathbf{\Omega}$, where $\mathbf{\Omega} \subset \mathbb{R}_+^3$ is the set of contracts. The down payment is paid with the liquidity holdings, and the loan amount is the amount of banknotes (deposit claims) issued that are valid only for capital purchases.[17] As in GSW, bankers face capacity limits; one banker can serve at most one entrepreneur.[18]

In the DM, the entrepreneurs' opportunity to invest arrives randomly with probability α, as in Kiyotaki & Moore (1997), in which case they can have production technology f. The entrepreneurs who have an investment opportunity can participate in the bank loan market;

[15] The entrepreneurs are assumed to consume all remaining assets and leave the market. However, in equilibrium the entrepreneurs would not have any assets at this point.

[16] Posting one contract is equivalent to posting a menu of contracts in this environment, akin to GSW. I also formally prove it in Appendix C.

[17] I relax this assumption in Section 6 by allowing the entrepreneurs to use the banknotes for purchasing consumption goods.

[18] Another interpretation is that each bank has a large collection of bankers. In that sense, a bank can serve multiple entrepreneurs, and a banker in this paper is similar to a vacancy in the Diamond-Mortensen-Pissarides model.

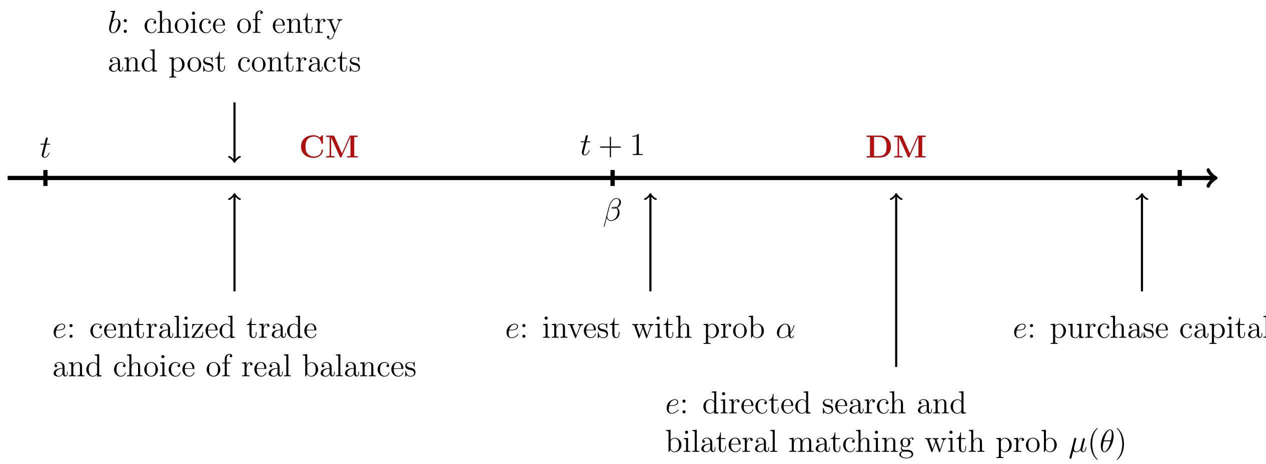
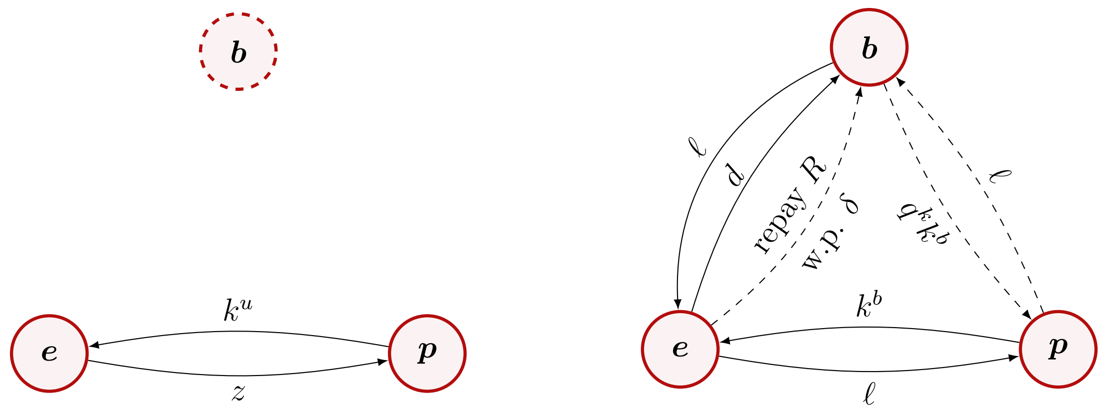


Figure 1: **Timeline**

then, they meet with a banker and obtain a loan with some endogenous probability, which is discussed in detail in the next subsection. Meanwhile, in the competitive capital market, capital is produced by capital producers at unit cost and is sold at price q^k.

There are two channels to finance investment: self-finance and external finance. The self-finance channel is illustrated in Figure 2a. If an entrepreneur fails to meet a banker, she turns to the capital market and uses z to buy k^u, which is the amount of capital bought when she is unbanked. Because the entrepreneurs have limited commitment, trade credits are ruled out; therefore, liquidity holdings are essential in the self-finance channel.

The external finance channel is illustrated in Figure 2b, where the entrepreneur is matched with a banker. The bankers have an advantage in enforcing repayments from their borrowers, meaning that the entrepreneurs cannot renege on loan repayments unless they have an unprofitable project and exit the market. If an entrepreneur obtains a loan, she passes a d amount of liquid assets to the banker for an ℓ amount of banknotes and promises to pay back an R amount of numeraire. Then she goes to the capital market and uses ℓ in exchange for k^b, which is the amount of capital purchased when she is banked. In the next period, the entrepreneur produces $f(k^b)$ and pays back R with probability δ. The capital producer then goes to the bank and uses ℓ to claim a $q^k k^b$ amount of numeraire after production. The bankers are assumed to have deep pockets to back up their liabilities and thus can commit.



(a) Self-finance w/ liquidity holdings

(b) External finance w/ banknotes

Figure 2: **Self-finance vs External Finance:** The left panel shows the self-finance channel, in which bankers b are inactive. The right panel shows the external finance channel, in which bankers b participate and banknotes circulate. Solid arrows indicate transactions that happen before production, and dashed arrows indicate transactions that happen after production.

So even if the entrepreneur fails to pay back R, the capital producer can still get $q^k k^b$ back from the banker. Hence, banknotes circulate as inside liquidity and facilitate trade in the external finance channel.[19]

3.2 Matching in the Loan Market

A submarket of the loan market consists of a set of bankers posting loan contracts and a set of entrepreneurs searching for those loans. It is assumed that bankers and entrepreneurs meet pairwise (i.e., there is a bilateral matching technology). Each banker in a given submarket posts a single contract Ω_j, $j = r, s$, that is designed for a type-j entrepreneur. The entrepreneurs observe all posted contracts and then choose the contract that is in their best interest. Any contract Ω_j is associated with a market tightness $\theta(\Omega_j) : \mathbb{R}^3_+ \to [0, \infty]$, which is the bankers-to-entrepreneurs ratio in submarket j. A share of entrepreneurs applying for Ω_j that are type-j is $\lambda_j(\Omega_j) \geq 0$. The entrepreneurs' probability of getting a loan (i.e., the matching probability) depends on the market tightness in that submarket. An entrepreneur

[19]The case that banknotes circulate is equivalent to the case that k^b is passed to bankers first and then to entrepreneurs. But in either case bankers issue liabilities and have full commitment on their liabilities.

searching in submarket-j obtains a type-j contract with probability $\mu(\theta_j)$, independent of entrepreneur's type, where the matching function $\mu : [0, \infty] \to [0, 1]$ is strictly increasing and continuous. With probability $1 - \mu(\theta_j)$, the entrepreneur is unmatched and needs to self-finance the investment project. A banker offering contract-j matches with an entrepreneur with probability $\eta(\theta_j)$, where $\eta : [0, \infty] \to [0, 1]$ is strictly decreasing and continuous, and otherwise is unmatched. Let $\mu(\theta) = \theta\eta(\theta)$ for all θ and $\eta(\infty) = \mu(0) = 0$. The market tightness and probability of matching are endogenously determined in equilibrium.

3.3 Optimization Problem

At the beginning of the CM, a type-j entrepreneur with capital k and financial wealth w, including liquidity holdings minus debts denominated in numeraire goods, has value $W_j(k, w)$ as follows:

$$W_j(k, w) = \max_{c_j, z'_j} \; c_j + \beta V_j(z'_j) \;\text{ s.t. }\; c_j + \frac{z'_j}{1 + r^z} \leq f(k) + w + T,$$

where $V_j(z'_j)$ is the expected continuation value of the type-j entrepreneur in the DM of the next period with a new liquidity holding z'_j. The constraint is the budget constraint indicating that the change in financial wealth, $w - z'_j/(1+r^z)$, is covered by retained earnings, $f(k) + T - c_j$. Eliminating c_j using the binding budget constraint, the objective function becomes

$$W_j(k, w) = w + f(k) + T + \max_{z'_j} \; \Big\{ -\frac{z'_j}{1 + r^z} + \beta V_j(z'_j) \Big\}.$$

As W_j is linear in financial wealth and output, the choice of z'_j is independent of (k, w).

In the DM, suppose a type-j entrepreneur applies for a loan contract while observing the set of contracts posted in the market, $\mathbf{\Omega}^{pt}$, and then purchases capital for production.

$$\begin{aligned} V_j(z_j) = \max_{\Omega_j \in \mathbf{\Omega}^{pt}, k_j^b, k_j^u} \quad & \alpha\mu(\theta(\Omega_j))\Big[\delta_j[f(k_j^b) + z_j - d_j - R_j + W_j^0] + (1-\delta_j)(z_j - d_j)\Big] \\ & + \alpha(1-\mu(\theta(\Omega_j)))\Big[\delta_j[f(k_j^u) + z_j - q^k k_j^u + W_j^0] + (1-\delta_j)(z_j - q^k k_j^u)\Big] \\ & + (1-\alpha)\Big[z_j + W_j^0\Big] \\ \text{s.t.} \;\; & z_j \geq d_j, \;\; q^k k_j^b \leq \ell_j, \;\; q^k k_j^u \leq z_j, \end{aligned}$$

where $W_j^0 \equiv W_j(0,0)$ is a constant representing type-j entrepreneur's continuation value with zero financial wealth and capital. The first component describes the expected value of using the external finance channel. The entrepreneur obtains a loan Ω_j with probability $\alpha\mu(\theta(\Omega_j))$ that depends on the chosen submarket Ω_j and purchases capital k^b. With probability δ_j, the investment is successful, and the entrepreneur produces $f(k_j^b)$ and has wealth $z_j - d_j - R_j$. With probability $1-\delta_j$, the entrepreneur consumes his remaining wealth $z_j - d_j$ and leaves the market afterwards. The second component describes the expected value of using the self-finance channel: the entrepreneur fails to obtain a loan with probability $\alpha(1-\mu(\theta(\Omega_j)))$ and purchases capital k^u. With probability δ_j, the investment is successful and the entrepreneur produces $f(k_j^u)$ and has wealth $z_j - q^k k_j^u$. With probability $1-\delta_j$, the entrepreneur consumes his remaining wealth $z_j - q^k k_j^u$ and leaves the market afterwards. The third component describes the case of not having an investment opportunity with probability $1-\alpha$. The first constraint is the feasibility constraint (FC), meaning that an entrepreneur applying for Ω_j must hold at least a d_j amount of liquidity. The type-j entrepreneur purchases k_j^b at cost $q^k k_j^b$ using banknotes ℓ_j when she is banked (i.e., matched with a banker). She purchases k_j^u at cost $q^k k_j^u$ using liquidity holding z_j when she is unbanked. Because banknotes can be used only to purchase capital, the entrepreneurs will use up all ℓ_j, $\ell_j = q^k k_j^b$. When liquidity is costly to hold, the entrepreneurs have no incentive to hold more liquidity than $q^k k_j^*$, which is the cost to purchase an efficient amount of capital, characterized by $\delta_j f'(k_j^*) = 1$, and then spend all liquidity holdings to purchase capital, $z_j = q^k k_j^u$. It is easy to show that if the competitive capital market is active, then the price of capital $q^k = 1$.[20] If a liquid asset is costly to hold, i.e., $r^z \leq 1/\beta - 1$, the producers have no incentive to hold liquid assets.

Substituting V_j into W_j, a type-j entrepreneur who anticipates contracts posted, $\boldsymbol{\Omega}^{pt}$, makes a portfolio choice and investment decision according to

$$U_j = \max_{z_j, \Omega_j \in \boldsymbol{\Omega}^{pt}} \quad -iz_j + \alpha\mu(\theta(\Omega_j))[\delta_j f(\ell_j) - d_j - \delta_j R_j] + \alpha(1 - \mu(\theta(\Omega_j)))[\delta_j f(z_j) - z_j] \quad (1)$$

$$\text{s.t.} \;\; z_j \geq d_j,$$

where i is the opportunity cost of holding liquidity, $i = 1/(\beta(1 + r^z)) - 1$, and U_j is the instantaneous payoff of type-j when she actively searches for a contract that is designed for her type in the loan market.

Meanwhile, consider the value function of a banker with financial wealth w in the CM as follows:

$$W_b(w) = w + \max_{\text{enter,not enter}} \Big\{ -\tilde{\kappa} + \max_{z_b', \Omega_j'} \big\{ -\frac{z_b'}{1 + r^z} + \beta V_b(z_b', \Omega_j') \big\}, \; \max_{z_b'} \big\{ -\frac{z_b'}{1 + r^z} + \beta V_b(z_b', \mathbf{0}) \big\} \Big\},$$

where $V_b(z_b', \Omega_b')$ is the expected continuation value of the banker in the DM of the next period with a new liquidity holding z_b' and contract Ω_j' posted for the type-j entrepreneurs, and $V_b(z_b', \mathbf{0})$ is the expected value if the banker carries z_b' but chooses to not enter the loan market.

[20]The value function of a capital producer with financial wealth w in the CM is

$$W_p(w) = w + \max_{z'} \Big\{ -\frac{z'}{1 + r^z} + \beta V_p(z') \Big\}.$$

In the DM,

$$V_p(z) = \max_{k \geq 0} \Big\{ -k + W_p(z + q^k k) \Big\}.$$

Hence, the producer produces k at unit cost and sells at price q^k so that his financial wealth increases by $q^k k$. Using the linearity of W_p, $q^k = 1$ and $V_p(z) = W_p(z)$.

In the DM,

$$V_b(z_b,\Omega_j) = \eta(\theta(\Omega_j))\Big[z_b + \sum_{i=r,s}\lambda_i(\Omega_j)(d_j - \ell_j + \delta_i R_j) + W_b^0\Big] + (1-\eta(\theta(\Omega_j)))\Big[z_b + W_b^0\Big],$$

where $W_b^0 \equiv W_b(0)$ represents the banker's continuation value with zero financial wealth. If the banker posted Ω_j, he will be matched with probability $\eta(\theta(\Omega_j))$, which is endogenously determined in the submarket Ω_j. Here $\lambda_i(\Omega_j)$ is the share of type-i entrepreneurs applying for Ω_j, and $d_j - \ell_j + \delta_i R_j$ is the expected profit collected from type-i. So, $\sum_{i=r,s}\lambda_i(\Omega_j)(d_j - \ell_j + \delta_i R_j)$ is the banker's total profit collected by posting Ω_j. With probability $1-\eta(\theta(\Omega_j))$, the banker will be unmatched and have zero profit in the DM. Likewise to the producers, the bankers have no incentive to hold liquid assets, so $z_b' = 0$. Thus, the banker's one-period net profit earned from posting Ω_j is

$$\Pi(\Omega_j) = -\kappa + \eta(\theta(\Omega_j))\sum_{i=r,s}\lambda_i(\Omega_j)(d_j - \ell_j + \delta_i R_j), \tag{2}$$

where $\kappa = \tilde{\kappa}/\beta$ is the fixed cost measured in numeraire of period $t+1$. If the banker chooses to not post any contract, the banker has value $V_b(z_b,\mathbf{0}) = z_b + W_b^0$.

An entrepreneur may prefer not participating in the loan market. Let $\hat{U}_j$ be the entrepreneur's payoff of using the self-finance channel only:

$$\hat{U}_j = \max_{z_j} \ -iz_j + \alpha[\delta_j f(z_j) - z_j]. \tag{3}$$

The entrepreneur's decision of entry is thus made according to

$$\mathcal{U}_j = \max_{\text{enter, not enter}} \Big\{U_j, \hat{U}_j\Big\},$$

where $\mathcal{U}_j$ is type-j entrepreneur's payoff.

4 Equilibrium

In this section, I first define and characterize the equilibrium under complete information. Then I characterize the equilibrium under incomplete information and compare the allocations with those of the complete information case. Moreover, I discuss how classic models are nested into mine and why the endogenous self-finance channel is critical to firm capital structure and loan contracts posted.

4.1 Symmetric Information

I first determine the equilibrium allocations in the case of perfect information, where both the entrepreneur and the banker know the entrepreneur's type. This part serves as a benchmark for the case of asymmetric information, where the banker does not observe the entrepreneur's type. Under symmetric information, the bankers simply offer loan contracts that are conditional on the entrepreneur's type, $\Omega_j^* = (d_j^*, R_j^*, \ell_j^*)$, and $\theta_j^* = \theta(\Omega_j^*)$. The payoff of a type-j entrepreneur while she is participating in the loan market is $U_j^* = U(\Omega_j^*)$, and the net profit of the banker is $\Pi_j^* = \Pi(\Omega_j^*)$.

Now define an equilibrium under symmetric information.

Definition 1 *In the case of complete information, there exists a stationary equilibrium that consists of*

(i). payoffs U_j^* *and* $\hat{U}_j$, $j = r, s$, *such that if* $U_j^* > \hat{U}_j$, Ω_j^*, θ_j^*, *and* z_j^* *solve* (1) *with* $\mathbf{\Omega}^{pt} = \{\Omega_j^*\}$ *and the free-entry condition* $\Pi_j^* \geq 0$ *as in* (2) *with* $\lambda_i(\Omega_j) = 0$, $i \neq j$; *if* $U_j^* \leq \hat{U}_j$, $\hat{z}_j$ *solves* (3);

(ii). price q^m *solves* $\sum_{j=r,s} \nu_j \bar{z}_j = q^m M$, *where* $\bar{z}_j = z_j^*$ *if* $U_j^* > \hat{U}_j$ *and* $\bar{z}_j = \hat{z}_j$ *otherwise.*

Suppose the type-j entrepreneurs participate in the loan market; the market designer solves the following simplified problem:[21]

$$U_j^* = \max_{z_j, \Omega_j, \theta_j} \; -iz_j + \alpha\mu(\theta_j)[\delta_j f(\ell_j) - d_j - \delta_j R_j] + \alpha(1-\mu(\theta_j))[\delta_j f(z_j) - z_j] \tag{P*-j}$$

$$\text{s.t. } \; z_j \geq d_j, \;\; \eta(\theta_j)(d_j - \ell_j + \delta_j R_j) \geq \kappa.$$

If there exists a competitive search equilibrium as first defined by Moen (1997), the bankers make zero expected profit and the free-entry condition must bind. Because there is no need to screen out the risky entrepreneurs, $d_j^* = 0$. The loan amount is issued at the optimal value,

$$\delta_j f'(\ell_j^*) = 1, \tag{4}$$

such that the marginal gain of having one more unit of capital when the type-j entrepreneur is banked equals the marginal cost. Repayment R_j^* is such that the free-entry condition binds. Given θ_j^*, liquidity holding is characterized by

$$\alpha(1-\mu(\theta_j^*))[\delta_j f(z_j^*) - 1] = i, \tag{5}$$

where the left hand side is the marginal gain of carrying one more unit of liquidity into the self-finance channel, and the right hand side is the marginal cost of holding liquidity. Given z_j^*, market tightness is implicitly given by

$$\mu'(\theta_j^*)[\delta_j f(\ell_j^*) - \ell_j^* - \delta_j f(z_j^*) + z_j^*] = \kappa, \tag{6}$$

where the left hand side is the marginal increase of matching probability in market tightness times the surplus of a match in the submarket-j, and the right hand side is the cost of posting

[21] In this setup, the market designer's problem is the same as the entrepreneur's problem and the banker's problem. Since the bankers make zero expected profit, the total welfare in a submarket equals the payoff of the entrepreneurs. Thus in each submarket the market designer maximizes the payoff of the entrepreneurs, which is also the objective of the entrepreneurs. Likewise, the bankers maximize the payoff of the entrepreneurs in order to attract more borrowers.

a contract. In other words, there is an efficient number of contracts supplied in the loan market.[22]

Suppose the type-j entrepreneurs do not participate in the bank loan market; then liquidity holding $\hat{z}_j$ is characterized by

$$\alpha[\delta_j f(\hat{z}_j) - 1] = i, \tag{7}$$

where the marginal benefit of bringing one more unit of liquidity when the entrepreneur needs to make an investment equals the marginal cost of holding liquidity.

Lemma 1 *Under complete information, for $j = r, s$,*

1. *if $U_j^* > \hat{U}_j$, liquidity holdings z_j^* and market tightness θ_j^* satisfy Equation (5) and (6), and contract $\Omega_j^* = (d_j^*, \ell_j^*, R_j^*)$ satisfies $d_j^* = 0$, Equation (4), and the binding free-entry condition holds;*

2. *otherwise, submarket-j is inactive and liquidity holdings $\hat{z}_j$ satisfy Equation (7).*

As shown in Equation (4)-(6), the loans issued to the safe types are larger, more loans are supplied to them, and they carry more liquidity for the precautionary reason.

Proposition 1 *With symmetric information, the loan amount, market tightness, and liquidity holdings of the safe types are greater than those of the risky types, i.e., $\ell_s^* > \ell_r^*$, $\theta_s^* > \theta_r^*$, $z_s^* > z_r^*$.*

The bankers provide larger loans to the safe types simply because the safe types are more likely to succeed than the risky types. Also, more loans are supplied in the safe type submarket because investments made by the safe types generate a larger surplus. Using Equation (5), it is also easy to see that with perfect information, liquidity is held to cope

[22] One can rewrite condition (6) and get $\epsilon(\theta_j^*) = [\kappa/\eta(\theta_j^*)]/[\delta_j f(\ell_j^*) - \ell_j^* - \delta_j f(z_j^*) + z_j^*]$, where $\epsilon(\theta) = \theta\mu'(\theta)/\mu(\theta)$ is the elasticity of the matching probability of the entrepreneur with respect to θ. This condition implies that there is an efficient number of banks entering the loan market given liquidity holdings.

with the risk of not getting a loan, so only for the precautionary motive. The safe types in fact have a stronger precautionary motive than do the risky types, as $z_s^* > z_r^*$. This is important to the decision of screening tools that the bankers use under asymmetric information, which is discussed in the following subsection.

4.2 Asymmetric Information

Under asymmetric information, the bankers do not directly observe the entrepreneurs' type, but the bankers can screen the entrepreneurs by offering different loan contracts. In equilibrium, the bankers offer profit-maximizing loan contracts subject to the free entry condition, and the entrepreneurs direct their search to the most preferred contract, conditional on the contracts offered and the entrepreneurs' beliefs. A stationary equilibrium with contract posting is defined as follows:

Definition 2 *A competitive search equilibrium is a set of entrepreneurs' payoff* $\mathcal{U}_j$, $j = r, s$, *liquidity holdings* $z(\Omega)$ *and* $\hat{z}_j$, *market tightness* $\theta(\Omega)$, *and market composition* $\lambda_j(\Omega)$ *defined over* $\mathbf{\Omega}$, *a cumulative distribution function* $\Gamma(\Omega)$, *a set of posted contracts* $\mathbf{\Omega}^{pt} \subset \mathbf{\Omega}$, *and price of the liquid asset* q^m *that satisfy the following conditions:*

(i). the bankers' profit maximization and free entry: for any $\Omega \in \mathbf{\Omega}$,

$$-\kappa + \eta(\theta(\Omega)) \sum_{j=r,s} \lambda_j(\Omega)[d(\Omega) - \ell(\Omega) + \delta_j R(\Omega)] \leq 0,$$

with equality if $\Omega \in \mathbf{\Omega}^{pt}$;

(ii). the entrepreneurs' optimal search: let

$$\mathcal{U}_j = \max\{U_j, \hat{U}_j\},$$

where $\mathcal{U}_j = \hat{U}_j$ *if* $\boldsymbol{\Omega}^{pt} = \emptyset$*; then for any* $\Omega \in \boldsymbol{\Omega}$ *and* j*,*

$$U_j \geq U_j(\Omega),$$

with equality if $\theta(\Omega) < \infty$ *and* $\lambda_j(\Omega) > 0$*, where*

$$\begin{aligned} U_j &= \max_{\Omega \in \boldsymbol{\Omega}^{pt}} \; U_j(\Omega) \\ &= \max_{\Omega \in \boldsymbol{\Omega}^{pt}} \; \{-iz(\Omega) + \alpha\mu(\theta(\Omega))[\delta_j f(\ell(\Omega)) - d(\Omega) - \delta_j R(\Omega)] \\ &\qquad + \alpha(1 - \mu(\theta(\Omega)))[\delta_j f(z(\Omega)) - z(\Omega)]\}; \end{aligned}$$

moreover, if $U_j < \hat{U}_j$*,* $z(\Omega) = 0$*,* $\hat{z}_j = \arg\max \; \hat{U}_j$ *and either* $\theta(\Omega) = \infty$ *or* $\lambda_j(\Omega) = 0$*;*

(iii). feasibility:

$$\int_{\boldsymbol{\Omega}^{pt}} \frac{\lambda_j(\Omega)}{\theta(\Omega)} d\Gamma(\Omega) \leq \nu_j \;\; \textit{for any} \;\; j,$$

with equality if $U_j > \hat{U}_j$*;*

(iv). and the liquid asset market clears:

$$\sum_{j=r,s} \nu_j \bar{z}_j = q^m M,$$

where $\bar{z}_j = \int_{\boldsymbol{\Omega}^{pt}} \frac{\lambda_j(\Omega)}{\theta(\Omega)} z(\Omega) d\Gamma(\Omega)$ *if* $U_j > \hat{U}_j$ *and* $\bar{z}_j = \hat{z}_j$ *if otherwise.*

The first set of conditions (i) determines the loan terms in each submarket; (ii) given the loan terms, pins down the corresponding market tightness θ and liquidity holding z; (iii) ensures all type-j entrepreneurs apply for the same type of contract when the payoff from participating in the loan market is larger than that of not participating in it; and (iv) determines the price of the liquid asset.

In Appendix C, I show the results of Guerrieri, Shimer, & Wright (2010) also apply in my setting: there exists a unique competitive search equilibrium with contract posting that is payoff-equivalent to a competitive search equilibrium with revelation mechanisms, in which entrepreneurs prefer to reveal their type. As in GSW, pooling equilibrium never exists in this model because of search frictions and the capacity limit. Suppose a banker posts a pooling contract designed to attract all types of entrepreneurs. The more entrepreneurs that search for this pooling contract, the less likely it is for any entrepreneur to be matched with a banker. This lower matching probability corresponds to a longer queue in front of the bank that offers a pooling contract. This longer queue discourages entrepreneurs from seeking this kind of contract. In particular, because the safe types have a higher surplus of obtaining a separating contract than the risky types, they will leave the queue first. As a result, the risky types are the ones remaining in the queue, leaving the banker unprofitable. Therefore, pooling contracts are never offered.

Without loss of generality, for the bankers, posting a menu of contracts is the same as posting one contract. In separating equilibrium, any contract that attracts the type-j entrepreneurs should solve the following optimization problem P-j: for any type $j = r, s$,

$$U_j = \max_{z_j,(d_j,\ell_j,R_j),\theta_j} \quad -iz_j + \alpha\mu(\theta_j)[\delta_j f(\ell_j) - d_j - \delta_j R_j] + \alpha(1-\mu(\theta_j))[\delta_j f(z_j) - z_j] \tag{P-j}$$

$$\text{s.t. } z_j \geq d_j, \;\; \eta(\theta_j)(d_j - \ell_j + \delta_j R_j) \geq \kappa,$$

$$-iz_j + \alpha\mu(\theta_j)[\delta_{\tilde{j}} f(\ell_j) - d_j - \delta_j R_j] + \alpha(1-\mu(\theta_j))[\delta_{\tilde{j}} f(z_j) - z_j] \leq U_{\tilde{j}},$$

$$\text{for any participating } \tilde{j} \neq j. \tag{IC-$\tilde{\text{j}}$j}$$

Relative to the problems under symmetric information (P*-j), this problem adds an incentive compatibility constraint (IC-$\tilde{\text{j}}$j), which makes sure that any entrepreneur who is not type-j should be better off by applying for a contract that is designed for her, when both types of entrepreneurs participate in the loan market. The left (resp., right) hand side of the constraint is the value of a type-$\tilde{j}$ entrepreneur planning to search in the submarket featuring Ω_j (resp., $\Omega_{\tilde{j}}$) with tightness θ_j (resp., $\theta_{\tilde{j}}$) and thus holding z_j (resp., $z_{\tilde{j}}$) before entering the bank loan market tomorrow.

Because the opportunity cost of holding liquid assets, i, affect the cost of self-finance, it affects the bank loan market structure and hence screening behaviors. The following Proposition describes how the economy switches regimes as i changes.

Proposition 2 *For any δ_r, there exist cutoffs $\underline{i}$, $\bar{i}$, and $\bar{\bar{i}}$ that are ranked, $\underline{i} < \bar{i} < \bar{\bar{i}}$. Types of equilibrium allocations can be classified as follows:*

(1) No participation: if $i \leq \underline{i}$, both the risky and safe types do not enter the loan market;

(2) No screening: if $i \in (\underline{i}, \bar{i}]$, the risky types do not enter the loan market, but the safe types do;

(3) Screening with z: if $i \in (\bar{i}, \bar{\bar{i}}]$, both types enter the loan market, and the bankers use down payment to screen out the risky types in the safe type submarket;

(4) Screening with z and θ: if $i > \bar{\bar{i}}$, both types enter the loan market, and the bankers use both down payment and market tightness to screen out the risky types in the safe type submarket.

The risky types' submarket transitions from *no participation* to *no screening* when i increases, as shown in Figure 3a. (1) *No participation*: when liquid assets are relatively cheap to hold, i.e., when i is small or κ is large, $\hat{U}_r > U_r$ and the risky type entrepreneurs are better off by self-financing investment, because liquid assets are relatively cheap to hold. (2) *No screening*: when liquid assets become more expensive to hold, i.e., i becomes larger, the risky types choose to participate in the loan market. As might be expected with adverse selection (e.g., Mirrlees 1971), the incentive compatibility constraint for the risky types (IC-sr) is slack; in other words, the safe type entrepreneurs have no incentive to mimic the risky types. The risky types' problem under asymmetric information (P-r) thus becomes identical to the one under symmetric information (P*-r). As a result, $U_r = U_r^*$ and the allocations of the risky types are not distorted by adverse selection, i.e., $z_r = z_r^*$, $d_r = d_r^* = 0$, $\ell_r = \ell_r^*$, $R_r = R_r^*$, and $\theta_r = \theta_r^*$.

The safe types' submarket transitions between *no participation*, *no screening*, *screening with* z, and *screening with* z *and* θ when i changes, as illustrated in Figure 3b. (1) *No participation*: when i is very small, $\hat{U}_s > U_s$ and the safe types prefer self-finance to bank loans. (2) *No screening*: when i becomes larger, the safe types choose to participate in the loan market, but the risky types do not. This is because safe types have a larger surplus of obtaining a loan than risky types do. Then the safe types are the only type of borrowers in the loan market and their problem does not face an incentive compatibility constraint; thus, the allocations and payoffs are not distorted, i.e., $z_s = z_s^*$, $d_s = d_s^* = 0$, $\ell_s = \ell_s^*$, $R_s = R_s^*$, $\theta_s = \theta_s^*$, and $U_s = U_s^*$. (3) *Screening with* z: when i becomes even larger, both types of entrepreneurs participate in the bank loan market. The risky types have a small incentive to pose themselves as safe types, so a small amount of down payment ($d_s = z_s^*$) is enough to make safe type contracts incentive compatible. There is no distortion in market tightness even when there is screening via asset holdings. Thus, the allocations are $z_s = d_s = z_s^*$, $\ell_s = \ell_s^*$, and $\theta_s = \theta_s^*$, R_s solves the binding free entry condition, and the payoffs are not distorted, $U_s = U_s^*$.[23] (4) *Screening with* z *and* θ: when i is large, the risky types have a large incentive to pose themselves as safe types, so the upward incentive compatibility constraint (IC-rs) binds. As a result, the safe types' allocations are distorted and the safe types have lower payoffs than they do in the complete information case, $U_s < U_s^*$. Using the first-order conditions (FOC) of problem P-s, ℓ_s is characterized by

$$\delta_s f'(\ell_s) = 1, \tag{8}$$

meaning that the loan amount ℓ_s is optimal. It is also easy to show that the entrepreneurs

[23] The difference between the second and third scenarios *(no screening* and *screening with* z*)* is down payment. In the second scenario, $d_s = 0$ because there is no need to screen entrepreneurs. In the third scenario, $d_s = z_s^*$ and the safe types do not need to bring more liquidity than do z_s^*. So, in either scenario, the safe type entrepreneurs have payoff U_s^* and undistorted allocations.

pledge all liquidity holdings as down payment, i.e., $d_s = z_s$. Here θ_s and d_s solve

$$\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s - \delta_s f(z_s) + z_s] = \kappa \tag{9}$$

and the binding (IC-rs) constraint. Given θ_s, ℓ_s, and d_s, the binding free entry condition characterizes R_s. The equilibrium allocations when both types of entrepreneurs participate in the loan market can be summarized as follows.

Lemma 2 *In the separating equilibrium when all entrepreneurs participate in the loan market with asymmetric information,*

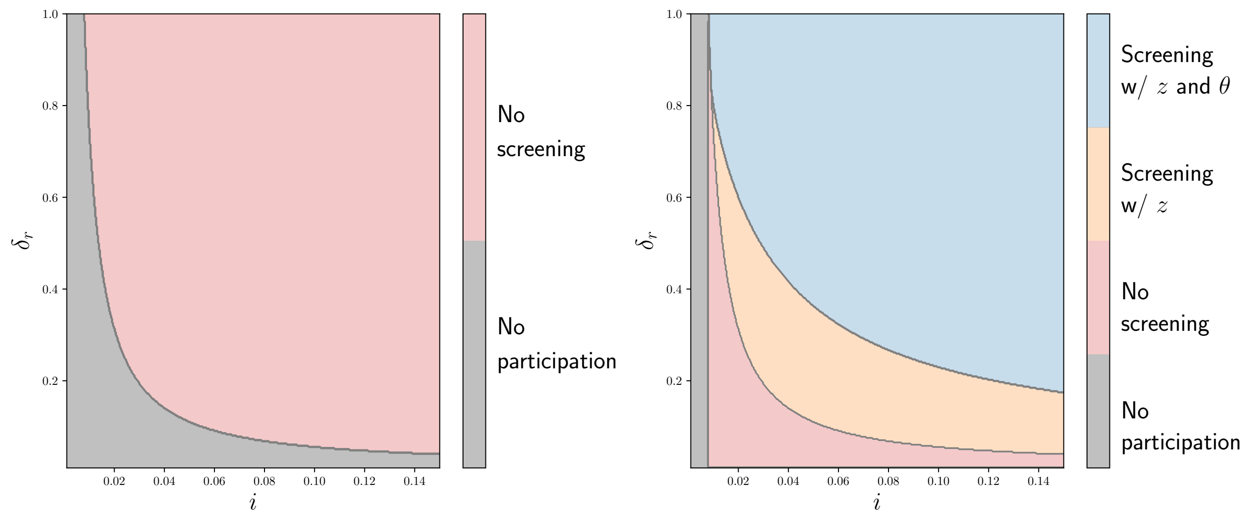
1. *for the risky types, the allocations are the same as the ones under symmetric information,* $(z_r, d_r, R_r, \ell_r, \theta_r) = (z_r^*, d_r^*, R_r^*, \ell_r^*, \theta_r^*)$ *;*

2. *for the safe types, market tightness* θ_s *and liquidity holdings* z_s *solve Equation* (9) *and the binding IC-rs constraint, and the loan contract* $\Omega_s = (d_s, \ell_s, R_s)$ *satisfies* $d_s = z_s$*, Equation* (8)*, and the binding free-entry condition.*

The next proposition compares the allocations under asymmetric information with the ones under complete information in the parameter space of scenario (4) described above *(screening with* z *and* θ*)*.

Proposition 3 *The allocations under asymmetric information are distorted in the extensive margin; in particular,* $d_s = z_s > z_s^*$ *and* $\theta_s < \theta_s^*$*. However, in the intensive margin, the loan size is not distorted;* $\ell_s = \ell_s^*$*.*

In this case, the loan contracts posted under symmetric information are no longer incentive compatible; the risky types have an incentive to misreport their type and choose a contract designed for the safe types so that the risky types can repay less. As a result, the bankers use two devices to screen out the risky entrepreneurs in the submarket of safe types: down payment (i.e., liquidity holdings) and market tightness (i.e., probability of matching).



(a) The risky types

(b) The safe types

Figure 3: **Types of Equilibrium Allocations:** $\delta_s = 1$. Appendix D describes other parameters and functions used in the numerical examples.

Knowing that the safe type entrepreneurs have a higher marginal benefit of holding liquidity when they are credit constrained (i.e., they have a stronger precautionary motive), the bankers ask for a large down payment from anyone applying for a safe type contract. The safe types in turn hold more liquidity to satisfy this requirement. The bankers also make the safe type contract hard to obtain by making the supply scarce. Because the safe types have a higher surplus of getting a loan, they are more willing to accept a lower probability of getting a loan in return for better loan terms when they do get to borrow.[24] However, once the bankers are able to use down payment and market tightness to screen out the risky types, they have no incentive to distort the amount of banknotes provided to the borrowers once the loan is issued. Hence, the loan amount is always at the optimal level.

The precautionary and signaling motives of holding liquid assets can be shown using the following equation. Taking the FOC with respect to z_s, the marginal benefit (MB) of holding

[24] Another way to interpret the probability of matching is the waiting period or the queue length. Because the risky types have an inferior investment project, their surplus of getting matched is low and therefore they are less willing to take on the cost of waiting.

one more unit of z_s must equal the marginal cost (MC) of doing so:

$$\underbrace{\alpha(1-\mu(\theta_s))[\delta_s f'(z_s)-1]}_{\text{MB of self-finance}} + \underbrace{\Big[-\Delta\frac{\partial U_r(\Omega_s)}{\partial z_s}\Big]}_{\text{MB of signaling}} = \underbrace{i}_{\text{MC}}, \tag{10}$$

where Δ is the Lagrange multiplier of the IC-rs constraint and $U_r(\Omega_s)$ is type-r's payoff of choosing Ω_s. Because liquidity z_s is too big for the risky types, it is costly for them to bring in z_s for the down payment requirement d_s, i.e., $\partial U_r(\Omega_s)/\partial z_s = -i - \alpha\mu(\theta_s)\delta_r/\delta_s + \alpha(1-\mu(\theta_s))[\delta_r f'(z_s)-1] < 0$. The first component of the left hand side represents the marginal gain of using one more unit of liquidity to self-finance investment when the entrepreneur wants to invest but fails to obtain a loan. The second component represents the marginal gain of signaling, that is the impact of an increase in z_s on safe types' payoff U_s through the channel of IC-rs. Intuitively, the safe type entrepreneurs would like to hold a large amount of z_s and pledge all z_s as down payment until the amount of z_s is so large that the risky type entrepreneurs have no incentive to do so. Therefore, as shown in Equation (10), under asymmetric information, there are two motives for holding liquidity: precautionary and signaling.

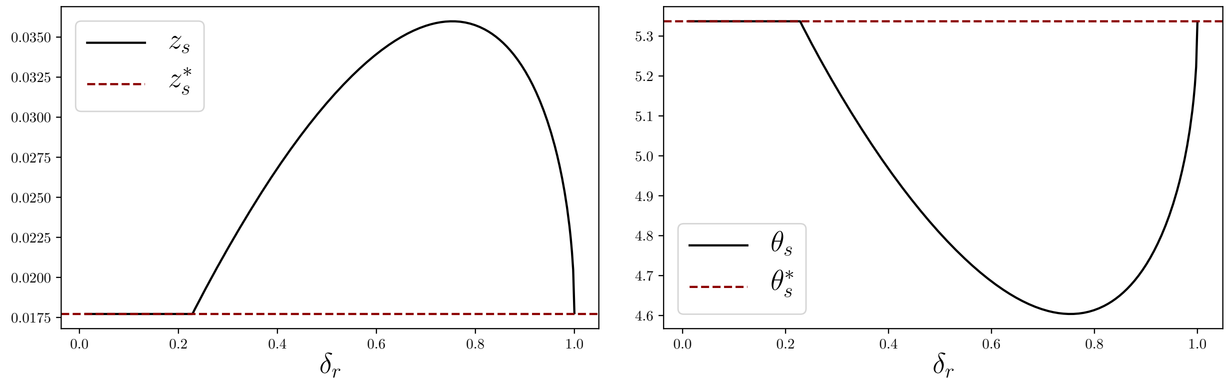
Another finding is that screening intensity, i.e., the level of z and θ, is non-monotone in the relative riskiness. This is caused by two competing forces: repayment saved by misreporting (i.e., the benefit of misreporting) and the usefulness of liquid assets in the self-finance channel (i.e., the cost of misreporting) for the risky types.

Proposition 4 *The safe types' liquidity holdings z_s and market tightness θ_s are non-monotone in δ_r: there exists a cutoff $\bar{\delta}_r$ such that*

(i). z_s increases (resp., decreases) in δ_r when $\delta_r < \bar{\delta}_r$ (resp., $\delta_r > \bar{\delta}_r$) and

(ii). θ_s decreases (resp., increases) in δ_r when $\delta_r > \bar{\delta}_r$ (resp., $\delta_r < \bar{\delta}_r$).

How z_s and θ_s change in δ_r is illustrated in Figure 4. When δ_r is very small, the risky types prefer not participating in the loan market (*no participation* in Figure 3b) or a small down



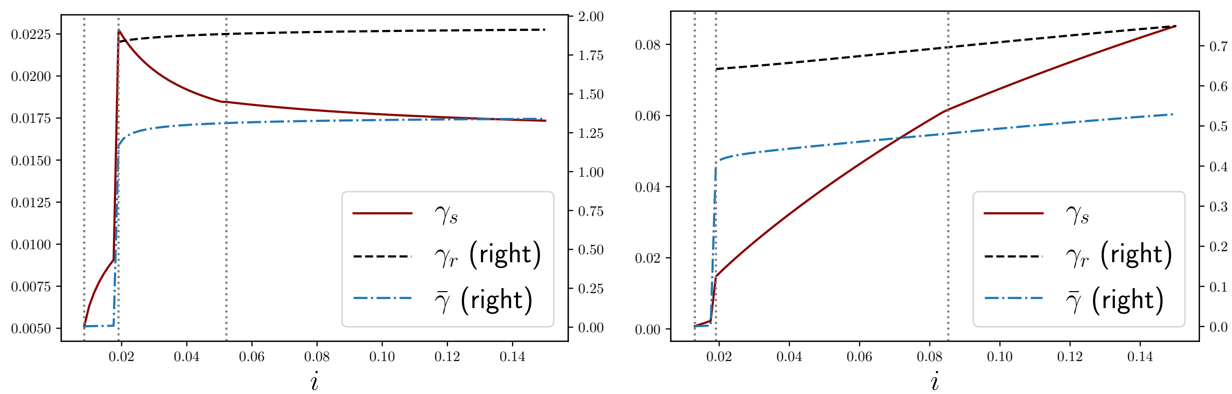
(a) Liquidity holdings (b) Market tightness

Figure 4: **Screening Intensity**: $i = 0.1$, $\delta_s = 1$.

payment $d_s = z_s^*$ is enough to prevent them from misreporting their type (*no screening* in Figure 3b), so z_s and θ_s are the same as the ones under complete information. Then, consider the case when two screening devices are used (*screening with z and θ* in Figure 3b). When δ_r increases but remains very low compared with δ_s, even though the benefit of misreporting their type is large for the risky types, the cost of mimicking the safe types is large, because the marginal benefit of bringing liquid assets is very low outside the loan market. When δ_s is large, the cost of mimicking the safe types is small, because the risky types are not very different from the safe types, but the benefit of misreporting their type is small. So, when δ_r is small or large, a small d_s and a lenient θ_s are enough to screen out the risky types. However, when δ_r is not too small or too big, the bankers have to use a large d_s and a tight θ_s to screen out the risky types; thus, allocations are the most distorted, and payoff U_s is the lowest.

To study the determinants of pass-through from the policy rate i to the real lending rate $\gamma_j = R_j/(\ell_j - d_j) - 1$. When *no screening*, $d_j^* = 0$ and R_j^* increases in i, so γ_j rises in i, suggesting a positive pass-through. When screening is needed, the real lending rate for the safe types can be rewritten as

$$\gamma_s = \frac{\kappa/(\delta_s \eta(\theta_s))}{\ell_s - z_s},$$



(a) Urn-ball matching function (b) Cobb-Douglas matching function

Figure 5: **Interest Rate Pass-through**: $\delta_s = 1$, $\delta_r = 0.36$. Dotted gray lines separate different screening regimes in the safe types' submarket: *no participation, no screening, screening with z, and screening with z and θ.*

where the numerator is the cost of posting a contract scaled by probability of receiving a repayment and the denominator is the net amount of lending. Define an elasticity ratio

$$\frac{\epsilon(\eta(\theta_s))}{\epsilon(\mu'(\theta_s))} = \frac{[\theta_s\mu'(\theta_s) - \mu(\theta_s)]\mu'(\theta_s)}{\theta_s\mu(\theta_s)\mu''(\theta_s)}.$$

When i and $\epsilon(\eta(\theta_s))/(\epsilon(\mu'(\theta_s)))$ are small, $\ell_s - z_s$ rises in i, but $\kappa/(\delta_s\eta(\theta_s))$ is not responsive to the changes in i, so γ_s can fall in i, suggesting a negative pass-through for the safe type loans. When $i \to 0$, $z_s \to 0$, so γ_s can still increase in i even when $\epsilon(\eta(\theta_s))/(\epsilon(\mu'(\theta_s)))$ is small. In Figure 5, I show numerical examples of lending rates. In the left panel where an Urn-ball matching function is used[25], γ_s rises in i when bankers do not screen and falls when they do, while the aggregate lending rate $\bar{\gamma} = \nu_s\gamma_s + \nu_r\gamma_r$ can still monotonically increase in i. In the right panel where a Cobb-Douglas matching function is used, lending rates always increase in i.

[25] Assume the borrower is chosen randomly from a group of arriving borrowers.

4.3 Nested Models

In this subsection, I aim to show how classic models are nested into mine, as summarized in Table 1. If the decentralized direct search is replaced with centralized contracts posting, my model coincides with that of Rothschild & Stiglitz (1976).[26] Liquidity holdings and loan contracts solve the following problem: for $j = r, s$,

$$U_j^{RS} = \max_{z_j,(d_j,\ell_j,R_j)} -iz_j + \alpha[\delta_j f(\ell_j) - d_j - \delta_j R_j] \tag{P$^{\mathrm{RS}}$-j}$$

$$\text{s.t. } z_j \geq d_j, \;\; d_j - \ell_j + \delta_j R_j \geq \kappa,$$

$$-iz_j + \alpha[\delta_{\tilde{j}} f(\ell_j) - d_j - \delta_{\tilde{j}} R_j] \leq U_{\tilde{j}}^{RS} \;\text{ for }\; \tilde{j} \neq j. \tag{IC$^{\mathrm{RS}}$-$\tilde{\mathrm{j}}$j}$$

Because of the absence of search friction, the entrepreneurs will always get matched with their preferred contract once they participate in the loan market. The self-finance channel is thus inactive, and the precautionary motive for holding liquidity is missing. By the standard single-crossing property,[27]the IC$^{\mathrm{RS}}$-rs constraint binds but the IC$^{\mathrm{RS}}$-sr constraint does not. Then for the risky types, $z_r = d_r = 0$, ℓ_r solves $\delta_r f'(\ell_r) = 1$, and R_r solves the binding free entry condition. For the safe types, $z_s = d_s > 0$ solves the binding IC$^{\mathrm{RS}}$-rs condition, ℓ_s solves $\delta_s f'(\ell_s) = 1$, and R_s solves the binding free entry condition. Liquidity holdings therefore serve as a pure costly signal, and down payment is the only screening device used by the bankers.

In this Rothschild & Stiglitz environment, a competitive equilibrium with bank loans need not exist. First, as shown in Rothschild & Stiglitz (1976), a pooling equilibrium can never exist. Suppose there is a pooling equilibrium; a banker can deviate from the pooling contract by offering the safe type a contract with better loan terms. Then this banker attracts all safe types in the market and makes a strictly positive profit, which is in contradiction with the

[26] One can relabel things and my model becomes identical to that of Rothschild & Stiglitz once search friction is removed.

[27] If the risky types prefer the safe type contract, then the safe types must as well. See Milgrom & Shannon (1994).

Table 1: **Nested Models**

		Rothschild & Stiglitz No search friction	Guerrieri, Shimer & Wright No self-finance	This paper
Liquidity	Precautionary	✗	✗	✓
	Signaling	✓	✗	✓
Screening	Down payment	✓	✗	✓
	Market tightness	✗	✓	✓

competitive loan market. So, if an equilibrium exists, it must be a separating equilibrium. Second, suppose there is a separating equilibrium; a banker can make a strictly positive profit by offering a pooling contract when the cost to pool is small, e.g., the risky and safe types are very alike in terms of success probability, or the measure of the risky type entrepreneurs is small. By selecting the pooling contract, the safe types can be better off because they do not need to make a down payment, although they need to subsidize the risky types. The risky types also benefit from the pooling contract simply because they are being subsidized. Therefore, when the cost to pool is small (or the cost of separating is high), an equilibrium with bank loans does not exist. If the bank loan equilibrium does not exist or loans are too expensive ($U_j^{RS} \leq \hat{U}_j$), the type-j entrepreneurs will switch to the self-finance channel.

Without the endogenous self-finance channel, my model turns into an application of GSW in the spirit of Akerlof (1970), which solves the non-existence problem of Rothschild & Stiglitz (1976) by replacing centralized contract posting with directed search. Liquidity holdings, loan contracts, and market tightness, respectively, solve the following problem: for $j = r, s$

$$U_j^{GSW} = \max_{z_j,(d_j,\ell_j,R_j),\theta_j} -iz_j + \alpha\mu(\theta_j)[\delta_j f(\ell_j) - d_j - \delta_j R_j] \tag{P$^{\text{GSW}}$-j}$$

$$\text{s.t. } z_j \geq d_j, \ \ \eta(\theta_j)[d_j - \ell_j + \delta_j R_j] \geq \kappa,$$

$$-iz_j + \alpha\mu(\theta_j)[\delta_{\tilde{j}} f(\ell_j) - d_j - \delta_{\tilde{j}} R_j] \leq U_{\tilde{j}}^{GSW} \ \text{ for } \ \tilde{j} \neq j. \tag{IC$^{\text{GSW}}$-$\tilde{\text{j}}$j}$$

As shown in GSW, the IC$^{\text{GSW}}$-rs constraint binds but the IC$^{\text{GSW}}$-sr constraint does not. Then for the risky types, $z_r = d_r = 0$, ℓ_r solves $\delta_r f'(\ell_r) = 1$, R_r solves the binding free entry condition, and θ_r solves $\mu'(\theta_r)[\delta_r f(\ell_r) - \ell_r] = \kappa$. For the safe types, $z_s = d_s = 0$, ℓ_s solves $\delta_s f'(\ell_s) = 1$, R_s solves the binding free entry condition, and θ_r solves the binding IC^{GSW}-rs constraint.

Proposition 5 *Without the self-finance option, neither a precautionary nor a signaling motive exists; $z_j = 0$, for $j = r, s$.*

Notice that in this setup liquidity becomes redundant. If the outside option is removed, the precautionary motive for holding liquid assets no longer exists. Then there is only one potential reason to hold liquid assets: to show the borrower's ability to repay by providing down payments. Down payments require obtaining liquidity beforehand, which is costly, because later the borrower may or may not be matched with a banker. A tighter loan market, on the other hand, leads to less bank entry, which saves on fixed entry costs. When liquidity has no use outside the credit market, the loan approval rate becomes the dominant screening device that provides higher payoffs to the entrepreneurs who honestly apply for contracts that are designed for them. Hence, when the self-finance channel is shut down, neither the precautionary nor the signaling motive exists and liquidity plays no role in the economy.

5 Constrained Efficiency Problem

In this section, I first discuss the constrained efficient problem in the language of GSW. Then I assume the planner uses the direct mechanism, in which entrepreneurs directly report their type to the planner and then the planner allocates them loan contracts, and specify the planner's optimization problem. I also describe constrained efficient allocations and discuss the origin of inefficiency in the baseline market economy.

5.1 Planner's Problem Using Taxation

In the market economy as described in the baseline, it is not possible to transfer funds from one submarket to another. However, a planner who has the power of taxation can tax the agents in some submarkets and subsidize the agents in other submarkets, thus changing the payoffs that the entrepreneurs may receive. This cross-subsidization is central to the constrained efficiency problem. Here the planner is assumed to have the power to implement taxation contingent on bank loans. First, the planner collects a lump sum tax, $\tau_0 \in \mathbb{R}_+$ measured in the numeraire good from all bankers. Second, the planner gives subsidies to submarkets. The subsidy will be made contingent on loan contracts, $\tau(\Omega) : \mathbb{R}^3_+ \to \mathbb{R}$.[28] Let $\{\tau_0, \tau\}$ be the planner's policy. Because the planner faces the same information and search frictions as agents do, the implementable allocations are defined as follows:

Definition 3 *An allocation $\{z_j, \mathbf{\Omega}^{pt}, \theta, \lambda_j, \Gamma, \hat{z}_j, q^m\}$ is implementable through policy $\{\tau_0, \tau\}$ if such allocation satisfies the following conditions:*

(i). the bankers' profit maximization and free entry: for any $\Omega \in \mathbf{\Omega}$,

$$-\kappa - \tau_0 + \eta(\theta(\Omega)) \sum_{j=r,s} \lambda_j(\Omega)[d(\Omega) - \ell(\Omega) + \delta_j R(\Omega) + \tau(\Omega)] \leq 0,$$

with equality if $\Omega \in \mathbf{\Omega}^{pt}$;

(ii). the entrepreneurs' optimal search: let

$$\mathcal{U}_j = \max\{U_j, \hat{U}_j\},$$

where $\mathcal{U}_j = \hat{U}_j$ if $\mathbf{\Omega}^{pt} = \emptyset$; then for any $\Omega \in \mathbf{\Omega}$ and j,

$$U_j \geq U_j(\Omega),$$

[28] It is equivalent if taxes are levied on and subsidies given to borrowers.

with equality if $\theta(\Omega) < \infty$ *and* $\lambda_j(\Omega) > 0$*, where*

$$\begin{aligned} U_j &= \max_{\Omega \in \mathbf{\Omega}^{pt}} \; U_j(\Omega) \\ &= \max_{\Omega \in \mathbf{\Omega}^{pt}} \; \{-iz(\Omega) + \alpha\mu(\theta(\Omega))[\delta_j f(\ell(\Omega)) - d(\Omega) - \delta_j R(\Omega)] \\ &\qquad\qquad + \alpha(1 - \mu(\theta(\Omega)))[\delta_j f(z(\Omega)) - z(\Omega)]\}; \end{aligned}$$

moreover, if $U_j < \hat{U}_j$*,* $z(\Omega) = 0$*,* $\hat{z}_j = \arg\max \; \hat{U}_j$*, and either* $\theta(\Omega) = \infty$ *or* $\lambda_j(\Omega) = 0$*;*

(iii). feasibility:

$$\int_{\mathbf{\Omega}^{pt}} \frac{\lambda_j(\Omega)}{\theta(\Omega)} d\Gamma(\Omega) \leq \nu_j \;\; \textit{for any} \;\; j,$$

with equality if $U_j > \hat{U}_j$*;*

(iv). the liquid asset market clears:

$$\sum_{j=r,s} \nu_j \bar{z}_j = q^m M,$$

where $\bar{z}_j = \int_{\mathbf{\Omega}^{pt}} \frac{\lambda_j(\Omega)}{\theta(\Omega)} z(\Omega) d\Gamma(\Omega)$ *if* $U_j > \hat{U}_j$ *and* $\bar{z}_j = \hat{z}_j$ *if otherwise;*

(v). and the planner's budget balances:

$$\int_{\mathbf{\Omega}^{pt}} \eta(\theta(\Omega))\tau(\Omega) d\Gamma(\Omega) \leq \int_{\mathbf{\Omega}^{pt}} \tau_0 d\Gamma(\Omega).$$

Note that when a zero taxation policy is implemented, $\tau_0 = 0$ and $\tau(\Omega) = 0$ for all Ω. This definition boils down to Definition 2, so the competitive equilibrium is implementable through zero taxation in the planner's problem. The bankers need to take into account the taxation policy and then decide what contract to post or in which submarket to participate. The entrepreneurs in turn choose the submarket to enter among all active submarkets. Also note that Condition (i) is different from the one in Definition 2 because the bankers have to take taxation policy into account when posting contracts in the planner's problem. For all submarkets that the planner wants to be active in, the bankers must not receive any positive profits. Condition (v) states that the planner does not have any external resources to finance the transfers. All other conditions are the same as in Definition 2.

Let $\bar{\sigma}_j$ be the Pareto weight of each type-j entrepreneur with $\sum_j \bar{\sigma}_j = 1$. Also define $\sigma \equiv \bar{\sigma}_j \nu_j$. A constrained efficient allocation is described as follows:

Definition 4 *A constrained efficient allocation solves the problem:*

$$\max_{\{z_i, \boldsymbol{\Omega}^{pt}, \theta, \lambda_i, \Gamma, \hat{z}_i, q^m\}, \{\tau_0, \tau\}} \sum_{i=r,s} \sigma_i \, \mathcal{U}_i$$

$$s.t. \quad \{z_i, \boldsymbol{\Omega}^{pt}, \theta, \lambda_i, \Gamma, \hat{z}_i, q^m\} \textit{ is implementable via policy } \{\tau, \tau_0\}.$$

A constrained efficient allocation is an implementable allocation as described in Definition 3 that maximizes total welfare weighted by σ among all implementable allocations. In the next subsection, the planner's problem is defined using a direct mechanism that yields the same results as in this definition and is more convenient to work with.

5.2 Direct Mechanism

In this subsection, the planner is assumed to use a direct mechanism. In it, entrepreneurs report their types to the planner, who acts as a middleman, and then the planner allocates them to a submarket Ω with θ. Intuitively, the planner sets up two shops, a certain number of entrepreneurs are allocated to match with bankers at each shop, and then the bankers matched with entrepreneurs are asked to issue loans with certain loan terms specified. The planner also implements transfers among bankers. Davoodalhosseini (2019) shows that a planner who uses a direct mechanism obtains the same amount of welfare as the planner in a dynamic version of the equilibrium as in Definition 4,[29] and all results from the direct

[29]Unrestricted power of taxation is the key here. Because there is no limit on the taxation power of the planner, the planner can effectively shut down any submarket by imposing a large amount of taxes on that submarket so that the same result as in the direct mechanism is obtained.

mechanism can be found in the setting if the planner has the power of taxation. Without loss of generality, I use the direct mechanism to study the constrained efficiency problem. Conditional on both types of entrepreneurs participating in the loan market,[30] the planner who weights the type-j entrepreneur with σ_j solves the following problem to maximize total welfare $\mathbb{W}$, which is the weighted average payoffs of entrepreneurs:[31]

$$\mathbb{W} = \max_{\{z_j,(d_j,\ell_j,R_j),\theta_j\}_{j=r,s}} \sum_{j=r,s} \sigma_j U_j(z_j,\Omega_j,\theta_j) \tag{PP}$$

$$\text{s.t. } z_j \geq d_j,$$

$$\sum_{j=r,s} \nu_j[\mu(\theta_j)(d_j - \ell_j + \delta_j R_j) - \theta_j \kappa] \geq 0, \tag{BB}$$

$$U_j(z_j,\Omega_j,\theta_j) \geq \hat{U}_j, \tag{PC-j}$$

$$U_{\tilde{j}}(z_j,\Omega_j,\theta_j) \leq U_{\tilde{j}}(z_{\tilde{j}},\Omega_{\tilde{j}},\theta_{\tilde{j}}), \quad \text{for any } \tilde{j} \neq j,, \tag{PIC-$\tilde{\text{j}}$j}$$

where $U_{\tilde{j}}(z_j,\Omega_j,\theta_j) \equiv -iz_j + \alpha\mu(\theta_j)[\delta_{\tilde{j}} f(\ell_j) - d_j - \delta_j R_j] + \alpha(1-\mu(\theta_j))[\delta_{\tilde{j}} f(z_j) - z_j]$ is the expected payoff of the type-$\tilde{j}$ entrepreneur who plans to choose z_j and Ω_j and anticipates θ_j. The first condition guarantees that type-j entrepreneurs hold at least a d_j amount of z_j. The second condition is the government budget balance (BB) condition. There are two shops set up by the planner, and each location has an $\alpha\nu_j\theta_j$ number of bankers (contracts) as each banker makes an expected net profit $\eta(\theta_j)(d_j - \ell_j + \delta_j R_j) - \kappa$. The planner makes transfers (i.e., collects taxes and gives subsidies) among bankers; some bankers might make positive profits before taxation, and some might make negative profits before subsidization. Then repayments are implicit functions of transfers. So, the left hand side of the BB condition is the sum of the expected net profits of all bankers over the two shops and must be non-negative. The third condition, the participation constraint (PC), ensures that entrepreneurs would not be worse off by participating in the loan market. The fourth condition, the planner's

[30]If any type does not participate in the loan market, the planner cannot use cross-subsidization to interfere with the market effectively.

[31]Because the bankers and capital producers make zero expected profit, the total welfare in the economy is the weighted average payoffs of the entrepreneurs.

incentive compatibility (PIC) constraint, guarantees that all entrepreneurs would truthfully report their types to the planner. The difference between the PIC and IC constraints is that in the baseline model, when bankers design contracts in one submarket, they take the expected payoff of the entrepreneurs in the other submarket as given (i.e., the right hand side of the IC constraint); in contrast, the planner can manipulate the payoff of the entrepreneurs in the other submarket (i.e., the right hand side of the PIC constraint) by choosing contracts posted in each submarket and making transfers across submarkets.

5.3 Constrained Efficient Allocations

This subsection focuses on the circumstances where the competitive equilibrium is distorted, i.e., the bankers use liquidity holdings and market tightness to screen out the risky types. When some entrepreneurs are inclined to misreport their type, the bankers face an additional IC constraint to ensure that the entrepreneurs truthfully report their types. Then the change in one type's payoff affects the entrepreneurs in the other submarket through the IC constraint. Furthermore, it has an impact on the set of contracts that the bankers can offer to attract the entrepreneurs in the other submarket and thus affects their payoffs. An agent in the market economy cannot take this externality into account, but a planner can internalize it by implementing transfers. When the risky types receive subsidies, they have higher payoffs and thus are less inclined to apply for a safe type loan. In turn, the bankers in the safe type submarket screen out the risky types with a smaller down payment and higher market tightness. Therefore, if the benefit of lower screening intensity outweighs the cost of paying taxes, the safe types can be better off, resulting in higher total welfare of all entrepreneurs, i.e., a Pareto superior allocation.

As shown in Figure 6, PIC-$\tilde{\text{j}}$j, PC-j, and BB, respectively, represent the following conditions that are plotted as functions of repayment pair (R_r, R_s) given full information allocations z^*, ℓ^*, θ^*, and d^*:

$$
\begin{aligned}
&U_{\tilde{j}}(z_j^*, d_j^*, R_j, \ell_j^*, \theta_j^*) = U_{\tilde{j}}(z_{\tilde{j}}^*, d_{\tilde{j}}^*, R_{\tilde{j}}, \ell_{\tilde{j}}^*, \theta_{\tilde{j}}^*),\\
&U_j(z_j^*, d_j^*, R_j, \ell_j^*, \theta_j^*) = \hat{U}_j,\\
&\sum_{j=r,s} \nu_j[\mu(\theta_j^*)(d_j^* - \ell_j^* + \delta_j R_j) - \theta_j^* \kappa] = 0.
\end{aligned}
$$

It is easy to see that given full information allocations, any area below PIC-sr, above PIC-rs, below PC-s, and to the right of PC-r (highlighted in pink in Figure 6) can support a repayment pair (R_r, R_s) that satisfies the PC and PIC constraints. The BB constraint is a straight line in (R_r, R_s), and any point on it represents a feasible repayment pair. Thus, any point on the thick dashed red line, which demonstrates the set of repayment pairs that are incentive compatible, individual rational, and feasible, can support the complete information allocation z^*, ℓ^*, θ^*, and d^*. Also note that with $z^*, \ell^*, \theta^*, d^*$ and $\sigma_j = \nu_j$, the utilitarian planner's indifference curve has the same slope as that of the BB constraint. So, any repayment pair on the thick dashed red line recovers the complete information total welfare. This is because z^*, ℓ^*, θ^*, and d^* are supported and the sum of repayments $(\sum_j \nu_j \mu(\theta_j^*) \delta_j R_j)$ equals the sum of complete information repayments $(\sum_j \nu_j \mu(\theta_j^*) \delta_j R_j^*)$. As a result, total welfare $\mathbb{W} = \mathbb{W}^* \equiv \sum_j \sigma_j U_j^*$, although type-j entrepreneur's payoff may be different from U_j^* because of transfers made contingent on the entrepreneur's type.

In the following proposition, I provide a sufficient condition such that a utilitarian planner can choose a repayment pair (R_r, R_s), which is implementable through policy (τ_0, τ), and completely undo the effect of adverse selection, recovering the allocations $(z^*, \ell^*, \theta^*, d^*)$ and the complete information total welfare $(\mathbb{W}^*)$.

Proposition 6 *A utilitarian planner ($\sigma_j = \nu_j$) can choose a repayment pair (R_r, R_s) to completely nullify the effect of adverse selection and achieve the complete information allo-*

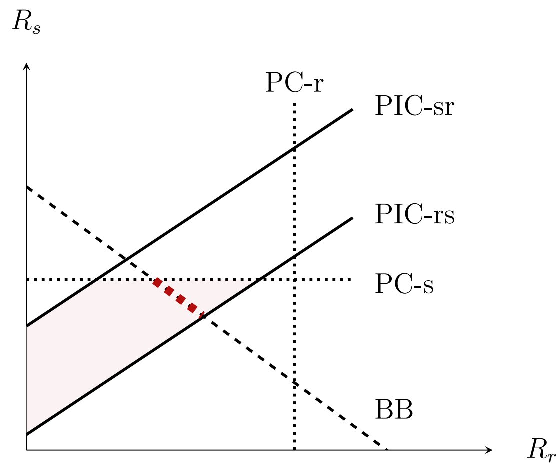


Figure 6: **Utilitarian Planner:** All PIC, PC, and BB conditions are plotted as functions of repayment pair (R_r, R_s) such that z^*, ℓ^*, θ^*, and d^* are chosen. Then all repayment pairs on the thick dashed red line (▪▪▪) can support the complete information allocation z^*, ℓ^*, θ^*, and d^*.

cations $z^*, \ell^*, \theta^*, d^*$ if

$$U_s^* - \hat{U}_s \geq \frac{U_r(z_s^*, \Omega_s^*, \theta_s^*) - U_r^*}{\frac{\nu_s}{\nu_r} + \frac{\delta_r}{\delta_s}}. \tag{11}$$

In Condition (11), $U_s^* - \hat{U}_s$ represents the safe types' net surplus of obtaining a safe type loan, as in the complete information case, rather than not borrowing, and $U_r(z_s^*, \Omega_s^*, \theta_s^*) - U_r^*$ represents the risky types' net surplus of obtaining a safe type loan, as in the complete information case, rather than a risky type loan. This condition is more likely to be satisfied in the following situations. When liquid assets are very costly to hold, i.e., i is large, the safe types' gain of participating in the loan market, $U_s^* - \hat{U}_s$, is large, so the safe types are willing to pay taxes and still apply for loans. When the population of risky types is small, i.e., ν_r is small, the right hand side of the equation is small. Intuitively, the safe types need to pay a small amount of tax since there are few risky types to subsidize. When the risky types' net benefit of misreporting their type, $U_r(z_s^*, \Omega_s^*, \theta_s^*) - U_r^*$, is small, the right hand side of the equation is small. This happens when δ_r is very different from or close to δ_s, as discussed in Proposition 4. Figure 7a shows where the full information allocations can be achieved in the parameter space of δ_r and i. When i and δ_r are not too big or too small (area *Not achievable*), the complete information allocations can never be recovered.

Then consider a general planner with weight σ_j, which can be different from ν_j. In the following proposition, I show a sufficient condition for the distorted competitive equilibrium to be constrained efficient, suggesting that the first welfare theorem holds and no transfers should be made.

Proposition 7 *If there exist δ_j and ν_j, $j = r, s$, such that*

$$\frac{\delta_r}{\delta_s} + \frac{\nu_s}{\nu_r} < 1, \tag{12}$$

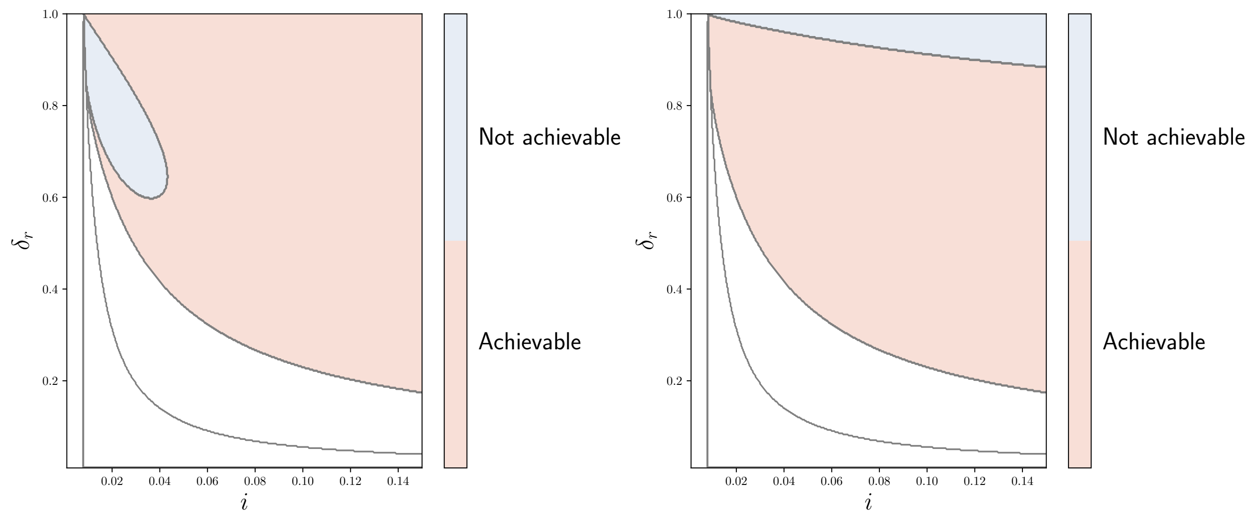
then there exists a Pareto weight σ_j such that the competitive equilibrium solves the planner's problem.

Equation (12) is satisfied when the risky types' investment quality is poor (low δ_r) or their population is large (high ν_r). In this case, the cost of subsidization is large, so there exists a σ_j, $j = r, s$, such that the competitive equilibrium is constrained efficient. In particular, such σ_j must satisfy

$$\frac{1 - (\nu_s \sigma_r)/(\nu_r \sigma_s)}{\delta_r/\delta_s + \nu_s/\nu_r} = \Delta^{ce}, \tag{13}$$

where Δ^{ce} is the Lagrange multiplier of the IC-rs constraint in the competitive equilibrium that is characterized in Equation (10). The multiplier $\Delta^{ce} \to 1$ as $\delta_r/\delta_s \to 1$, and $\Delta^{ce} = 0$ if δ_r is very small, because the risky types will not borrow when $\delta_r = 0$. Thus, the distorted competitive equilibrium must have $0 < \Delta^{ce} < 1$. By Equation (13), the planner must care more about the safe types ($\sigma_s > \nu_s$) for the distorted competitive equilibrium to be constrained efficient. In this case, the planner has to value the safe types' payoff enough such that she does not want to help the risky types at the safe types' expense but not too much such that she wants to tax the risky types instead.

As illustrated in Figure 7b, given ν_r, when δ_r is very close to δ_s (area *Not achievable*), the competitive equilibrium can never be constrained efficient. Suppose there is cross-



(a) Complete information allocations

(b) First Welfare Theorem

Figure 7: **The Planner's Problem:** $\delta_s = 1$

subsidization and δ_r is large; the cost of subsidization (or of taxation) is smaller than the benefit (i.e., less liquidity holdings and a larger loan supply) to the safe type entrepreneurs; in turn, cross-subsidization improves U_s. The closer δ_r is to δ_s, the easier it is to increase U_s. Thus, when δ_r is very close to δ_s, there does not exist a σ_s such that the planner implements zero taxation.

6 Moral Hazard

The baseline model assumes that banknotes (ℓ) can be used only to purchase capital (k); in this section, I relax this assumption and allow the entrepreneurs to use banknotes to buy both consumption goods and capital. This is equivalent to unobservable investment. In this setup, an entrepreneur can potentially deviate twice: she can misreport her type and apply for a loan that is not designed for her; moreover, she can use the banknotes that are intended to support investment to purchase consumption goods.[32] As a result,

[32] Cole & Kocherlakota (2001) provide a characterization of the efficient consumption in an environment in which individuals have hidden income and storage. However, the entrepreneurs in my model have no incentive to save across periods because (1) banknotes cannot circulate across periods, (2) the opportunity cost of holding liquid assets is non-negative, $i \geq 0$, and (3) the utility function of the CM is quasi-linear.

moral hazard is introduced in addition to adverse selection. Using banknotes to purchase consumption goods incurs unit cost $C(\tilde{\chi})$, where $C : [0, \infty] \to [0, 1]$ and $\tilde{\chi}$ is the exogenous difficulty level of doing so. Note that when the cost of conducting moral hazard behavior is very high, $\tilde{\chi} \to \infty$, $C \to 1$, and the problem coincides with the baseline. Let $k_{\tilde{j}j}$ and $x_{\tilde{j}j}$ denote type-$\tilde{\text{j}}$ entrepreneurs' investment and consumption when they apply for type-j contract, respectively. Also use U_j^{mh} to denote the equilibrium payoff of type-j in the setup with moral hazard. Then, the market designer faces a new MIC constraint rather than the IC constraint in the baseline:[33]

$$
\begin{aligned}
&-iz_j + \alpha\mu(\theta_j)\Big[\max_{\substack{k_{\tilde{j}j}, x_{\tilde{j}j} \text{ s.t.} \\ k_{\tilde{j}j} \le \ell_j + z_j - d_j \\ x_{\tilde{j}j} \le \ell_j - k_{\tilde{j}j} + z_j - d_j}} \delta_{\tilde{j}} f(k_{\tilde{j}j}) + (1 - C(\tilde{\chi}))x_{\tilde{j}j} - d_j - \delta_{\tilde{j}} R_j \Big] \\
&\quad + \alpha(1 - \mu(\theta_j))[\delta_{\tilde{j}} f(z_j) - z_j] \le U_{\tilde{j}}^{mh} \qquad \text{(MIC-}\tilde{\text{j}}\text{j)}
\end{aligned}
$$

Conditional on obtaining a type-j loan, the type-$\tilde{\text{j}}$ entrepreneur can use ℓ_j and any remaining liquidity $z_j - d_j$ to purchase capital goods $k_{\tilde{j}j}$ from the producers in the DM, and produce $\delta_{\tilde{j}} f(k_{\tilde{j}j})$ in expectation. If there are any remaining banknotes and liquidity, in the next CM she can use them to buy consumption goods at most $\ell_j - k_{\tilde{j}j} + z_j - d_j$ before the debt is repaid. For simplicity, let $\chi \equiv 1 - C(\tilde{\chi})$ denote the net amount of numeraire per unit of banknotes spent on purchasing consumption goods. Let $k_{\tilde{j}j}^{mh}$ denote type-$\tilde{\text{j}}$ entrepreneurs' amount of capital invested such that $\delta_{\tilde{j}} f'(k_{\tilde{j}j}^{mh}) = \chi$. To simplify the MIC constraint, it is worth looking into an entrepreneur's investment and consumption when she misreports her type.

Lemma 3 *Consider a type-$\tilde{j}$ entrepreneur's investment $k_{\tilde{j}j}$ and consumption $x_{\tilde{j}j}$ in the problem of type-j, $\tilde{j} \neq j$.*

1. *In the safe type entrepreneurs' problem, $j = s$:*

[33] I show the complete problem and derive the MIC constraint step by step in Appendix E.

(i). if $\chi > \delta_r/\delta_s$, $k_{rs} = k_{rs}^{mh} < \ell_s$ and $x_{rs} = \ell_s - k_{rs}^{mh}$;

(ii). otherwise, $k_{rs} = \ell_s$ and $x_{rs} = 0$; the MIC-rs constraint vanishes into the IC-rs constraint in the baseline.

2. In the risky type entrepreneurs' problem, $j = r$: $k_{sr} = \ell_r$ and $x_{sr} = 0$; the MIC-sr constraint vanishes into the IC-sr constraint in the baseline.

When $\chi > \delta_r/\delta_s$, the cost of purchasing consumption goods using banknotes is small. The amount of banknotes issued for the safe type loan is too big for the risky types to invest in their projects. Because the marginal gain from investing an additional unit of capital on top of k_{rs}^{mh} is less than the marginal gain from purchasing a unit of consumption good, the risky type of entrepreneurs who applied for a safe type loan would choose to invest k_{rs}^{mh} and consume a net amount of $\chi(\ell_s - k_{rs}^{mh})$. Then, the MIC-rs constraint can be simplified to

$$-iz_s + \alpha\mu(\theta_s)[\delta_r f(k_{rs}^{mh}) + \chi(\ell_s - k_{rs}^{mh}) - d_s - \delta_r R_s] + \alpha(1 - \mu(\theta_s))[\delta_r f(z_s) - z_s] \leq U_r^{mh}.$$

In contrast, if banknotes can be used only to buy capital, the risky types have to use up ℓ_s and produce $\delta_r f(\ell_s)$ in expectation, which generates less payoff than that from producing $\delta_r f(k_{rs}^{mh})$ and consuming $\chi(\ell_s - k_{rs}^{mh})$. So, the risky types have higher incentives to apply for a safe type loan, because their payoffs of misreporting their type is higher than in the baseline.

When the cost of purchasing consumption goods using banknotes is high, $\chi \leq \delta_r/\delta_s$, the risky types exhaust the banknotes for investment, $k_{rs}^{mh} = \ell_s$, because using ℓ_s for investment generates a higher payoff than using ℓ_s for consumption does. The MIC-rs constraint coincides with the IC-rs constraint; as a result, the equilibrium allocations in this case are identical to the ones in the baseline.

When $\{z_s, \ell_s, d_s, R_s, \theta_s\} = \{z_s^*, \ell_s^*, z_s^*, (\kappa/\eta(\theta_s^*)+\ell_s^*-z_s^*)/\delta_s, \theta_s^*\}$ is not incentive compatible,

$$
\begin{aligned}
&-iz_s^* + \alpha\mu(\theta_s^*)[\delta_r f(k_r) + \chi(\ell_s^* - k_r) - (1 - \delta_r/\delta_s)z_s^* - \ell_s^*\delta_r/\delta_s] - \alpha\theta_s^*(\delta_r/\delta_s)\kappa \\
&\qquad + \alpha(1 - \mu(\theta_s^*))[\delta_r f(z_s^*) - z_s^*] > U_r^{mh}, \text{ where } \begin{cases} k_r = k_{rs}^{mh} & \text{if } \chi > \delta_r/\delta_s, \\ k_r = \ell_s^* & \text{if else,} \end{cases}
\end{aligned}
$$

more screening is needed, and the allocations are distorted in another dimension in this case. Let mh be the superscript denoting the equilibrium allocations of the setting with moral hazard. The following proposition compares the moral hazard allocations with the complete information allocations.

Proposition 8 *In the environment with adverse selection and moral hazard,*

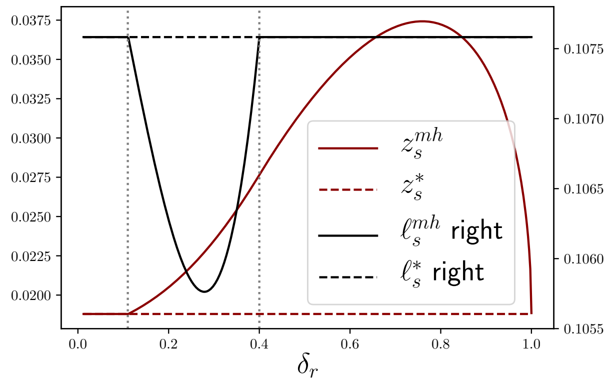
1. *if $\chi > \delta_r/\delta_s$,*

   (i). the allocations of the safe types are distorted both in the intensive margin, $\ell_s^{mh} < \ell_s^*$, and the extensive margin, $z_s^{mh} > z_s^*$, and $\theta_s^{mh} < \theta_s^*$;

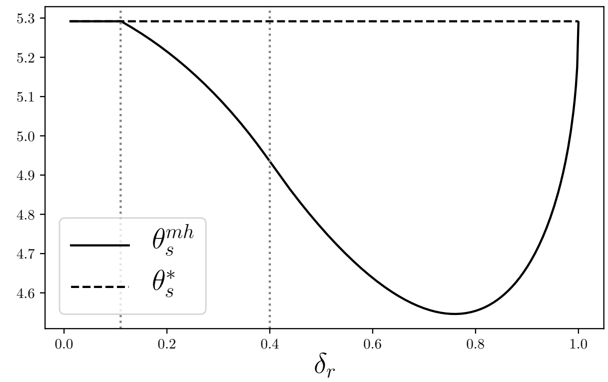
   (ii). while the allocations of the risky types are not distorted, $\ell_r^{mh} = \ell_r^*$, $z_r^{mh} = z_r^*$, and $\theta_r^{mh} = \theta_r^*$.

2. *if $\chi \leq \delta_r/\delta_s$, the allocations coincide with the ones in the baseline.*

With the possibility of dual deviation, the allocations are distorted in an additional dimension – loan size ℓ. Bankers issue a smaller loan amount in order to screen out the entrepreneurs who have an incentive to deviate ex post. In this case, three screening tools are used by the bankers: loan amount, liquidity holdings, and market tightness. Figure 8 shows a numerical exercise of equilibrium allocation z_s, ℓ_s, and θ_s under this extension relative to the complete information case. When δ_r is very small and below the first dotted line, the risky types either choose to not enter the loan market or do not have an incentive to misreport their type even if the bankers use only a small down payment $d_s^{mh} = z_s^*$ to screen



(a) Liquidity holdings and loan amount



(b) Market tightness

Figure 8: **Screening Intensity Under Moral Hazard**: $i = 0.1$, $\delta_s = 1$, $\chi = 0.4$.

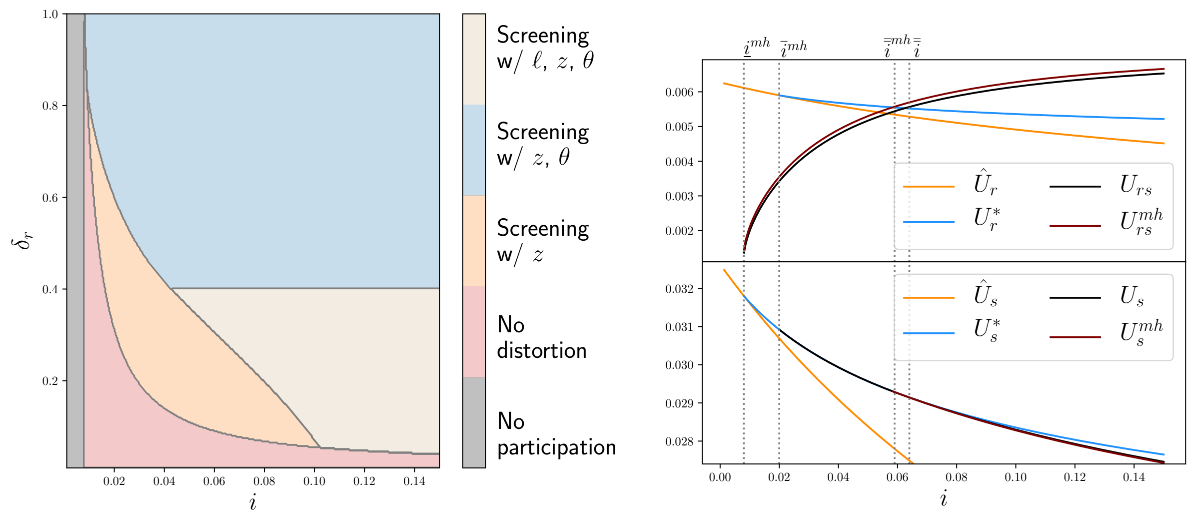
out the risky types. When δ_r is large and above the second dotted line, which is χ in this exercise, the problem becomes identical to the baseline model, and so do the allocations. When δ_r falls between the two dotted lines, $\ell_s^{mh} < \ell_s^*$, $z_s^{mh} > z_s^*$, $\theta_s^{mh} < \theta_s^*$, and $U_s^{mh} < U_s^*$.

Consider other types of equilibrium allocations for the safe type entrepreneurs in a parameter space of δ_r and i as in Figure 9a. Comparing with the baseline as in Figure 3b, the market is distorted in both the extensive and the intensive margin, as the bankers may *screen with ℓ, z, and θ*. The market is also distorted in a larger parameter space, as the bankers are less likely to *screen with z* only. Furthermore, because the safe types do not have an incentive to misreport their type and mimic the risky types, the risky types' equilibrium allocations are identical to the ones under the complete information and the baseline case. In the following proposition, I classify the types of equilibrium allocations using the opportunity cost of holding liquid assets i.

Proposition 9 *In the environment with moral hazard, there exist cutoffs $\underline{i}^{mh}$, $\bar{i}^{mh}$, and $\bar{\bar{i}}^{mh}$ that are ranked, $\underline{i}^{mh} < \bar{i}^{mh} \leq \bar{\bar{i}}^{mh}$.*

1. *When $\chi > \delta_r/\delta_s$,*

   (i). if $i \leq \underline{i}^{mh}$, both the risky and safe types do not enter the loan market;



(a) Safe type entrepreneurs

(b) Payoffs: $\delta_r = 0.3$

Figure 9: **Types of Equilibrium Allocations Under Moral Hazard**: $i = 0.1$, $\delta_s = 1$, $\chi = 0.4$.

(ii). if $i \in (\underline{i}^{mh}, \bar{i}^{mh}]$, the risky types do not enter the loan market, but the safe types do;

(iii). if $i \in (\bar{i}^{mh}, \bar{\bar{i}}^{mh}]$, both types enter the loan market and the bankers use down payment to screen out the risky types;

(iv). if $i > \bar{\bar{i}}^{mh}$, both types enter the loan market and the bankers use loan amount, down payment, and market tightness to screen out the risky types.

2. When $\chi \leq \delta_r/\delta_s$, $\{\underline{i}^{mh}, \bar{i}^{mh}, \bar{\bar{i}}^{mh}\} = \{\underline{i}, \bar{i}, \bar{\bar{i}}\}$.

Comparing with the baseline, screening is not only more intense but also more likely to happen.

Proposition 10 *In the environment with moral hazard, the cutoff of using multiple screening tools is lower than in the baseline, $\bar{\bar{i}}^{mh} \leq \bar{\bar{i}}$.*

In Figure 9b, U_{rs} and U_{rs}^{mh} are the payoff of the risky types when they mimic the safe types, given $\{z_s, \ell_s, d_s, R_s, \theta_s\} = \{z_s^*, \ell_s^*, z_s^*, (\kappa/\eta(\theta_s^*) + \ell_s^* - z_s^*)/\delta_s, \theta_s^*\}$, under the baseline and the moral hazard extension, respectively. (i) When i is very small and below $\underline{i}^{mh}$, none of the entrepreneurs enters the loan market. (ii) When i is between $\underline{i}^{mh}$ and $\bar{i}^{mh}$, the safe types prefer entering the loan market while the risky types do not, because the safe types have a higher surplus of getting a loan. (iii) When i is between $\bar{i}^{mh}$ and $\bar{\bar{i}}^{mh}$, both types enter the loan market and the bankers use $d_s = z_s^*$ to screen out the risky types. Because the safe types are indifferent between making some down payment up front and making a larger repayment afterwards, the safe types have the payoff U_s^* in this case. (iv) When i is above $\bar{\bar{i}}^{mh}$, which happens when $U_{rs}^{mh} > U_r^*$, a small down payment is not enough to keep the risky types away from the safe type contracts. As a result, the bankers use multiple screening devices, and the payoff of the safe types is lower than their payoff in the complete information case. The risky types, on the other hand, always have the complete information payoff once they enter the loan market. In Figure 9b, it is also easy to see how the third cutoff, $\bar{\bar{i}}^{mh}$, differs from the one in the baseline, $\bar{\bar{i}}$. When the risky types can spend some ℓ_s on consumption, U_{rs}^{mh} is higher than U_{rs}, and the magnitude depends on the cost of conducting moral hazard behavior. The lower the cost is, the higher U_{rs}^{mh} is, and the lower $\bar{\bar{i}}^{mh}$ is. In other words, with potential moral hazard behavior, allocations are not only more distorted but also more likely to be distorted.

7 Conclusion

In this paper, I propose firms have a signaling motive for holding liquid assets in addition to the precautionary motive, which is well-studied in the literature. I build a directed search model with asymmetric information to rationalize liquidity holdings both inside and outside the credit market. First, liquid assets are useful inside the credit market as entrepreneurs use them to signal their ability to repay, so they help entrepreneurs obtain external credit. Second, liquid assets are useful outside the credit market by acting as a buffer stock as entrepreneurs fail to secure a loan. By introducing a self-finance channel to a classic screening model with costly collateral, both liquidity holdings and loan approval rate are used to screen out risky entrepreneurs. The self-finance channel acts as an endogenous outside option which is crucial to credit contracts and screening devices. I show that without the self-finance option, liquid assets become redundant, as both the precautionary and signaling motives disappear.

While the bankers in the market economy use liquidity holdings and loan approval rate to screen out the risky borrowers, the safe borrowers are worse off, because they need to bring more liquid assets and are less likely to obtain a loan. The risky borrowers, thus, cause an externality, resulting in lower payoffs for the safe borrowers. This is because the bankers in one submarket do not take into account the effects of their entry on the set of feasible contracts that the bankers in the other submarket can offer to attract entrepreneurs. However, unlike the agents in the market economy, a planner can internalize this externality by levying taxation. I show that under some conditions, a planner can always achieve higher welfare by subsidizing the risky borrowers and taxing the safe borrowers. In particular, a utilitarian planner can completely undo the effect of adverse selection and recover the complete information allocations. I also find that the competitive market equilibrium can be constrained efficient, in which case no transfers are needed.

There are many related research questions that can be addressed in the future. For example, long-term banking relationships (e.g., Bethune et al. Forthcoming), such as business credit cards, significantly reduce firms' demand for liquidity, as illustrated by the fact that credit cards were the second most used financial resource in 2020. One can study this issue by allowing entrepreneurs to build long-term relationships with certain bankers in this model; however, it requires extensive work with dynamic contracting. Another issue is that some firms might be more financially constrained than others, meaning that some entrepreneurs are not able to raise as much liquidity as they need to. One can study this issue by introducing multiple dimensions of private information (e.g., Chang 2018, Guerrieri & Shimer 2018, and Williams 2021) into the model, i.e., the entrepreneurs differ in their investment quality and ability of raising liquidity in the centralized Walrasian market, and the bankers observe either of it.

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Appendices

A Empirical Support

In this section, I provide empirical support for the existence of the signaling and precautionary motives for holding liquid assets. For this purpose, I use the Kauffman Firm Survey (KFS), which is a longitudinal survey of new businesses in the U.S. The KFS collected information on 4,928 new firms and surveyed them every year from 2004 to 2011. After I removed the firms that have missing information or permanently closed before 2011, there are 660 firms left in my balanced panel data. Information on these firms includes industry, capital structure (equity and debt), employment, and firm owner characteristics. One drawback of the data that significantly limits the scope of my study is that the type of debt collateral is recorded only in the last three years of the survey. In my sample, an average firm has total assets amounting to $284,526 and holds 27.9% of them as liquid assets. About 2.5% of all firms have pledged liquid collateral to obtain loans, and about 13.8% have pledged illiquid collateral. Many of the firms surveyed are financially constrained; specifically, 3.4% of them have difficulties in obtaining external credit and 13.1% are in need of credit but choose to not apply for it because of possible denial.[34]

I adopt a dynamic approach which allows me to study firms' adjustment in liquidity holdings over time. It is common that the targeted level of liquidity holdings cannot be instantaneously achieved because of transaction and other adjustment costs, so a lag term of liquidity is included in the regression as an explanatory variable. To verify the existence of the precautionary, transaction, tax and agency motives, I follow the traditional methods on studying corporate cash holdings (e.g., Ozkan & Ozkan 2004, Bates et al. 2009, Faulkender 2002, and Bigelli & Sánchez-Vidal 2012). I also add liquid collateral pledged to capture the intention of using liquid assets as collateral to receive favorable loan terms, which serves as a proxy of the signaling motive. The empirical counterpart of the liquidity demand function

[34] Column 1 in Table A2 presents summary details for the sample of the KFS data.

can be written as follows:

$$
\begin{aligned}
Lqd\_Ass_{i,t} = \; & \beta_{lla} Lqd\_Ass_{i,t-1} + \beta_{flc} LqdC_{i,t} \\
& + \beta_{rd} RD\_Ass_{i,t} + \beta_{cp} C\_prob_{i,t} + \beta_{a} ln\_Ass_{i,t} + \beta_{mo} MO_{i,t} + \beta_{cc} C\_corp_{i,t} \\
& + X_{i,t} \cdot \beta_X + Firm_i + \epsilon_{i,t},
\end{aligned} \tag{A.1}
$$

where $Lqd\_Ass_{i,t}$ is the liquidity-to-assets ratio of firm i at the end of year t, $Lqd\_Ass_{i,t-1}$ is the lagged liquidity-to-assets ratio (Ozkan & Ozkan 2004), $LqdC_{i,t}$ equals 1 if liquid collateral is pledged to obtain any of the debt financing options that are used in year t and 0 if otherwise, $RD\_Ass_{i,t}$ is the ratio of research and development (R&D) expenditure to total assets in year t (Bates et al. 2009), $C\_prob_{i,t}$ equals 1 if the firm considers itself as having difficulties in obtaining external credit in year t and 0 if otherwise (Faulkender 2002), $ln\_Ass_{i,t}$ is the natural log of total assets (Bates et al. 2009), $MO_{i,t}$ equals 1 if the primary owner is the manager and 0 if otherwise (Faulkender 2002), $C\_corp_{i,t}$ equals 1 if the firm is a C-corporation and 0 if otherwise (Faulkender 2002), $X_{i,t}$ is a vector of firm characteristics (Bigelli & Sánchez-Vidal 2012), $Firm_i$ is the firm fixed effect that is used to capture time-invariant liquidity preference, and $\epsilon_{i,t}$ is the error term.[35]

Regression results are presented in Table A1. The ordinary least squares method (OLS) in column 1 estimates Equation (A.1) without the firm fixed effect, yielding an estimated *liquid collateral* coefficient of -0.0009. In this case, the *lagged liquidity-to-assets* ratio is positively correlated with the error term, and the correlation will not vanish with a large number of firms. By standard results for omitted variable bias, the OLS estimator of the *lagged liquidity-to-assets* ratio, 0.4920, is biased upwards. Although the *lagged liquidity-to-assets* is

[35]This liquid collateral can be pledged against any type of debt, including business loans, credit cards, and lines of credit. However, I am unable to further narrow down the type of debt financing because of data limitations. Firm financial characteristics controls are natural log of sales, total-expenditure-to-assets ratio, profit margin, total-liability-to-assets ratio, total-loan-to-assets ratio, dividends-to-assets ratio, natural log of remaining available credit, credit score, if the firm has multiple locations, if the firm provides services, if the firm has comparative advantages (patent, trademark, or copyright), if the firm considers itself as having cash flow problems, and if the firm needs credit but has not applied for it because of possible denial.

Table A1: **Dynamic Panel Data Estimation**

	Theoretical prediction	(1) OLS	(2) FE	(3) FDT	(4) FDT-IV	(5) FDT-IV
lagged liquidity-to-assets	+	0.4920***	-0.1833***	-0.3445***	0.2781*	0.2766*
		(0.0259)	(0.0383)	(0.0285)	(0.1515)	(0.1516)
liquid collateral	+	-0.0009	-0.0075	0.0238	0.1023**	0.1983**
		(0.0381)	(0.0668)	(0.0387)	(0.0513)	(0.0954)
R&D-to-assets	+	0.0221*	0.0116	0.0142	0.0910***	0.0905***
		(0.0128)	(0.0157)	(0.0136)	(0.0174)	(0.0174)
credit prob.	+	0.0061	0.0461	0.0636*	0.0935*	0.0993*
		(0.0252)	(0.0445)	(0.0343)	(0.0525)	(0.0527)
ln(assets)	−	-0.0426***	-0.0879***	-0.0741***	-0.0893***	-0.0895***
		(0.0062)	(0.0142)	(0.0111)	(0.0168)	(0.0168)
C-corp	+	0.0021	-0.0729	-0.0319	-0.4090	-0.4419
		(0.0301)	(0.1514)	(0.1393)	(0.3010)	(0.2943)
primary-owner-manager	−	0.0135	0.0127	0.0109	-0.0013	0.0002
		(0.0143)	(0.0288)	(0.0220)	(0.0338)	(0.0339)
firm financial char.		Yes	Yes	Yes	Yes	Yes
firm FE		No	Yes	No	No	No
adj R-sqrd		0.3386	0.5701	0.2006	.	.
N		1980	1980	1320	660	660
F-stat on lagged liq.-to-assets					65.34	32.77
F-stat on future liq. col.						14.87
Cragg-Donald Wald F-stat						62.056

Note: *** p<0.01, ** p<0.05, * p<0.1. I further restrict firms to have positive revenues and non-negative liquidity holdings (cash and cash equivalent) and to primarily conduct business in the U.S. Standard errors are clustered at the industry level. Year 2008–10 observations are included in (1) and (2), year 2009–10 observations are included in (3) and (4), and only year 2010 observations are included in (5).

not of direct interest, resolving its endogeneity issues helps recover a consistent estimate of *liquid collateral*. The one-way fixed effects (FE) model in column 2 estimates Equation (A.1), yielding an estimated *liquid collateral* coefficient of -0.0075. As Nickell (1981) shows, the FE estimator of the *lagged liquidity-to-assets* ratio, -0.1833, is biased downwards because the demeaning process which subtracts the individual's mean value from the respective variable creates a correlation between the *lagged liquidity-to-assets* ratio and the error term.[36] One solution to this problem involves taking the first difference transformation (FDT), which

[36] Nickell (1981) demonstrates that the bias can be quite large when the total number of periods (T) is small. In a large sample, the bias becomes $\text{plim}_{N\to\infty}(\hat{\beta}_{lla} - \beta_{lla}) = -(2+\beta_{lla})(1+\beta_{lla})/2$ for $T = 3$, where $\hat{\beta}_{lla}$ is the FE estimator. Suppose the true β_{lla} is 0.28, then the bias can be -1.46.

removes both the constant and the firm fixed effect:

$$\begin{aligned}\Delta Lqd\_Ass_{i,t} = \ & \beta_{lla}\Delta Lqd\_Ass_{i,t-1} + \beta_{flc}\Delta LqdC_{i,t+1} \\ & + \beta_{rd}\Delta RD\_Ass_{i,t} + \beta_{cp}\Delta C\_prob_{i,t} + \beta_{a}\Delta ln\_Ass_{i,t} + \beta_{mo}\Delta MO_{i,t} + \beta_{cc}\Delta C\_corp_{i,t} \\ & + \Delta X_{i,t} \cdot \beta_X + \Delta\epsilon_{i,t},\end{aligned} \tag{A.2}$$

where $\Delta Y_{i,t} = Y_{i,t} - Y_{i,t-1}$ for a variable Y. In column 3, the OLS is then used to estimate the FDT of Equation (A.2), resulting in an estimated *liquid collateral* coefficient of 0.0238. Nevertheless, the FDT introduces correlation between the *lagged differenced liquidity-to-assets* ($\Delta Lqd\_Ass_{i,t-1}$) and the differenced errors ($\Delta\epsilon_{i,t}$) through the *lagged liquidity-to-assets* ($Lqd\_Ass_{i,t-1}$) and the lagged error ($\epsilon_{i,t-1}$). This issue advocates the Anderson-Hsiao estimator (Anderson & Hsiao 1981) in column 4, which uses the *second lag of liquidity-to-assets* in the form of differences ($\Delta Lqd\_Ass_{i,t-2}$) as an instrument for the *lagged differenced liquidity-to-assets* ratio, resulting in a *second lag of liquidity-to-assets* coefficient of 0.2781 and a *liquid collateral* coefficient of 0.1023. A positive coefficient of *liquid collateral* can imply two plausible explanations: either firms collect more liquidity and then pay for liquid collateral in the future or firms choose to pay for liquid collateral because they have collected a large amount of liquidity in the past. Another estimation problem is that the decision of pledging liquid collateral in the next year is endogenous and may depend on the omitted variables. To further address these issues, in column 5, I instrument the *differenced liquid collateral* using the *lagged liquid collateral* in the form of differences ($\Delta LqdC_{i,t}$).[37] This additional instrument resolves the omitted variable issue and rules out the possibility that firms pay for liquid collateral because they have a large amount of liquidity holdings. The Anderson-Hsiao strategy yields an estimated signaling motive of 0.1983, indicating that a firm that chooses to obtain loans and pledge liquid collateral would hold more liquid assets,

[37] The validity of instruments requires the absence of higher-order serial correlation in the idiosyncratic component of the error term ϵ. However, with only one year of observations included in the regression in column 5, it is no longer a concern in this case.

equivalent to 19.83% of its total assets, before the loan application than would similar firms that choose to pledge illiquid or no collateral.[38] In other words, firms hold more liquidity ex ante to help themselves obtain external credit with favorable terms by pledging liquid collateral. There are many ways that liquid collateral can help firms obtain loans. One plausible explanation is that firms use liquid collateral to signal their ability to repay, which is the focus of this paper. Another is that firms pledge liquid collateral to improve their bargaining position during the loan negotiation with banks, as hypothesized by Rocheteau et al. (2018). My results verify both scenarios; however, I am not able to separate them because of data limitations.

The results in column 5 pass this consistency test. The Cragg-Donald Wald F-statistic is 62.056, which is larger than the critical value of the Stock-Yogo weak identification test (7.03), rejecting weak instruments. Bond (2002) suggests that the candidate consistent estimator of *lagged liquidity-to-assets* (0.2766) should live between the OLS and FE estimators (0.4920 and -0.1833) because they are biased in opposite directions.

The estimation of the signaling motive can be biased, and the direction of bias is ambiguous. On the one hand, the regression might underestimate the true effect of the signaling motive. Since the desire to pledge liquid collateral is not directly observed in the data, it is possible that some firms are eligible and plan to obtain a loan and pledge liquid collateral but fail to get one because of various reasons, leaving an excessive amount of liquid assets in their accounts. The fact that these firms cannot be identified by the regression increases the liquidity holdings of the firms assumed to not want a loan. On the other hand, the regression might overestimate the true effect of the signaling motive, as it is possible that some firms are not eligible, e.g., not pledging enough liquid assets, and fail to obtain a loan. These noneligible firms lower liquidity holdings of the firms assumed to not want a loan. This bias can be corrected if the regression is conditional on suceesful loan applications. However, the

[38] Arellano & Bond (1991) find that the Anderson-Hsiao estimator fails to take all potential orthogonality conditions into account. However, because of the data limitation that the KFS records the type of collateral only in the last three years, the FDT with generalized method of moments (GMM), where the instruments applicable to each period of equation differ, cannot be feasibly applied here.

loan application outcomes are not available in the KFS.

In addition to showing the existence of the signaling motive, my estimations also show the existence of the precautionary motive, as found in Faulkender (2002) using SSBF 1993. R&D measures growth opportunities and is usually financed with cash. A firm that considers itself as having growth opportunities invests more in R&D and holds more liquidity so that it would not miss any investment opportunities, for the precautionary motive.[39] A firm that considers itself as having difficulties in accessing external credit should hold more liquid assets, because the firm's marginal benefit from liquidity is higher. Hence, by the precautionary motive, the coefficient of *R&D-to-assets* and *credit problem* should be positive. The estimates in column 5 confirm the existence of the precautionary motive. Spending one percentage additional assets on R&D leads to higher liquidity holdings, equivalent to 0.09% of the firm's total assets. Also, a firm that has difficulties in raising external credit carries additional liquid assets, equivalent to 9.93% of its total assets.

My estimations also verify the transaction motive, as larger firms hold a lesser amount of liquidity proportionally, as shown in Mulligan (1997) using Compustat data. According to the theory, a larger firm holds relatively less liquidity than a smaller firm because of the increasing economies of scale in liquidity holdings. If firm size is measured by total assets, then the estimates in column 5 suggest that a one percentage point increase in total assets leads to a lower amount of liquidity holdings, equivalent to 0.09% of the firm's total assets.

However, my estimations fail to show the existence of the tax and agency motives. The agency motive, first proposed by Jensen (1986), suggests that managers accumulate cash even when they do not have good investment opportunities rather than returning cash to firm owners. If the firm's primary owner is the manager, then liquidity holdings should be lower because the agency motive is lower. So, the coefficient of *primary-owner-manager* should

[39] T. C. Opler & Titman (1994) use R&D expenditure as a proxy for financial distress costs. In time of distress, firms would cut R&D expenditure, which means giving up investment opportunities. So firms that spend the most in R&D are those firms that would lose the most in case of distress. However, a reduction in R&D expenditure may not be caused only by indebtedness, but also by limited access to external credit. Thus, firms have an incentive to hold more liquid assets as R&D expenditure increases, regardless of indebtedness.

have a negative sign. The tax motive is tied to the tax system. Because a C-corporation is taxed at both corporate income and dividend payout, a C-corporation has a larger incentive to hold liquid assets, resulting in a positive coefficient of *C-corp*. The coefficients of the tax and agency motives across my estimations are against the theoretical prediction; however, they are not statistically significant.

B Summary Statistics of the the Kauffman Firm Survey

Table A2 illustrates the summary statistics of the the Kauffman Firm Survey (KFS) and Survey of Small Business Finances 2003 (SSBF 2003) used in my paper. In the KFS sample, I restrict firms to not file for bankruptcy or merge with other firms, to have positive revenues and non-negative liquidity holdings (i.e., cash and cash equivalent), to primarily conduct business in the U.S., and to have non-missing key information. In the SSBF 2003 sample, I restrict firms to be private, to have no bad credit mark (i.e., no personal or firm bankruptcy, no personal or firm debt delinquency), to be unbanked (i.e., no business credit card, no line of credit), to conduct business primarily in the U.S., to have non-negative cash and cash equivalent, and to have applied for a business loan (e.g., mortgage, equipment loan, vehicle loan) in the last three years.

Table A2: **Summary Statistics of KFS and SSBF 2003**

	(1) KFS		(2) SSBF 2003	
	Mean	SD	Mean	SD
liquidity-to-assets	0.279	0.356	0.213	0.268
liquid collateral	0.025	0.155	0.045	0.207
illiquid collateral	0.138	0.345	0.612	0.488
assets	284,526	318,754	617,493	1,003,017
revenue	421,460	350,759	1,188,200	1,887,146
R&D-to-assets	0.103	1.175		
credit prob.	0.034	0.180	0.045	0.207
C-corp	0.062	0.241	0.179	0.384
primary-owner-manager	0.672	0.470	0.896	0.306
need credit, didn't apl	0.131	0.338	0.119	0.325
N	660		335	

C Verifying the Assumptions of the Guerrieri, Shimer and Wright model

In this subsection, I show that the assumptions of Guerrieri, Shimer, & Wright (2010) (GSW) are satisfied in this environment; thus, the results of GSW can be applied directly to my model. Let $u_j(\Omega)$ be the net surplus of a type-j entrepreneur if she is successfully matched with a banker in submarket Ω, such that $u_j(\Omega) = \delta_j f(\ell) - \delta_j R - d - \delta_j f(z) + z$, where $z \geq d$. Let $v_j(\Omega)$ be the payoff of a banker in submarket Ω who matches with a type-j entrepreneur and gets payoff $v_j(\Omega) = \delta_j R + d - \ell$. Also, let $\bar{\Omega}_j$ be the set of contracts that deliver a non-negative net surplus upon matching to a type-j entrepreneur while permitting the banker to make non-negative profits if the market tightness is 0, such that

$$\bar{\Omega}_j = \{\Omega \in \mathbf{\Omega} \mid \bar{\eta} v_j(\Omega) \geq \kappa \text{ and } u_j(\Omega) \geq 0\},$$

where $\bar{\eta} = \eta(0)$, and

$$\bar{\Omega} = \bigcup_j \bar{\Omega}_j.$$

In equilibrium, contracts that are not in $\bar{\Omega}$ are not traded, because bankers cannot make non-negative profit while attracting entrepreneurs.

Assumption 1 *Monotonicity: for all* $\Omega \in \bar{\Omega}$, $v_r(\Omega) \leq v_s(\Omega)$.

This assumption is satisfied, since $\delta_r R + d - \ell < \delta_s R + d - \ell$ is true for all $R > 0$, which is guaranteed by $\Omega \in \bar{\Omega}$.

For the next assumption, let $B_\epsilon(\Omega) \equiv \{\Omega' \in \mathbf{\Omega} \mid d(\Omega', \Omega) < \epsilon\}$ be a ball of radius ϵ around Ω.

Assumption 2 *Local nonsatiation: for* $\Omega \in \bar{\Omega}_s$ *and* $\epsilon > 0$, *there exists a* $\Omega' \in B_\epsilon(\Omega)$ *such that* $v_s(\Omega') > v_s(\Omega)$ *and* $u_r(\Omega') \leq u_r(\Omega)$.

Consider a contract with $\ell' = \ell$, $d' = d$, and $R' = R + \epsilon$. For bankers, $\delta_s(R+\epsilon) + d - \ell > \delta_s R +$

$d-\ell$, so $v_s(\Omega') > v_s(\Omega)$ is satisfied. For entrepreneurs, $\delta_r f(\ell)-\delta_r(R+\epsilon)-d < \delta_r f(\ell)-\delta_r R-d$, so $u_r(\Omega') \le u_r(\Omega)$ is also satisfied.

The last assumption ensures that it is possible to make the contract attractive to some entrepreneurs while making it not attractive to some other entrepreneurs.

Assumption 3 *Sorting: for $\Omega \in \bar{\Omega}_s$, and $\epsilon > 0$, there exists a contract $\Omega' \in B_\epsilon(\Omega)$ such that $u_s(\Omega') > u_s(\Omega)$ and $u_r(\Omega') < u_r(\Omega)$.*

For fixed $\Omega \in \bar{\Omega}$, $\tilde{\delta} \in (\delta_r, \delta_s)$, and an arbitrary $\tilde{\epsilon} > 0$, consider $z = d + \tilde{\epsilon}$ and a contract with $\ell' = \ell$, $R' = R - \tilde{\epsilon}/\tilde{\delta}$, and $d' = d + \tilde{\epsilon}$. This is feasible for small $\tilde{\epsilon}$, since $\Omega \in \bar{\Omega}_s$ makes sure that $R > 0$. Then, $\delta_s f(\ell) - \delta_s(R - \tilde{\epsilon}/\tilde{\delta}) - (d + \tilde{\epsilon}) > \delta_s f(\ell) - \delta_s R - d$, so $u_s(\Omega') > u_s(\Omega)$. Also, $\delta_r f(\ell) - \delta_r(R - \tilde{\epsilon}/\tilde{\delta}) - (d + \tilde{\epsilon}) < \delta_r f(\ell) - \delta_r R - d$, so $u_r(\Omega') < u_r(\Omega)$. Now for a given $\epsilon > 0$, choose a $\tilde{\epsilon} \le \epsilon/\sqrt{1 + 1/\tilde{\delta}^2}$ that guarantees $\Omega' \in B_\epsilon(\Omega)$. Hence, this assumption is satisfied.

Under Assumption 1, 2, and 3, without loss of generality one can assume that each banker posts a single contract instead of posting a menu of contracts (revelation mechanism), as established by Proposition 5 of GSW.

D Parameters for Numerical Examples

For all numerical examples, I use the matching function $\mu(\theta) = 0.7(1 - e^{-\theta})$ and production function $f(k) = 0.7k^{0.3}$. The other parameters are $\beta = 0.99, \kappa = 0.0002, \alpha = 0.13, \delta_s = 1, \nu_s = 0.3$. In Figure 5, the Cobb-Douglas matching function is $\mu(\theta) = \theta^{0.7}$. In the moral hazard extension, I use $C(\tilde{\chi}) = e^{-\tilde{\chi}}$. The other parameters are $\tilde{\chi} = 0.51$ and $\chi = 0.4$.

E Moral Hazard Problem

A type-j entrepreneur has the following value functions. In the CM,

$$W_j^{mh}(k, w) = w + f(k) + \max_{z'_j, \Omega'_j \in \mathbf{\Omega}^{pt}, \theta'_j} \Big\{ - \frac{z'_j}{1 + r^z} + \beta V_j^{mh}(z'_j, \Omega'_j, \theta'_j) \Big\}.$$

In the DM,

$$
\begin{aligned}
V_j^{mh}(z_j,\Omega_j,\theta_j) = \max_{k_j^b,k_j^u}\ & \alpha\mu(\theta_j)\Big[\delta_j[f(k_j^b)+z_j-d_j-R_j+(1-C(\tilde{\chi}))(\ell_j-q^k k_j^b)+W_j^{mh,0}] \\
& + (1-\delta_j)(z_j-d_j+(1-C(\tilde{\chi}))(\ell_j-q^k k_j^b))\Big] \\
& + \alpha(1-\mu(\theta_j))\Big[\delta_j[f(k_j^u)+z_j-q^k k_j^u+W_j^{mh,0}]+(1-\delta_j)(z_j-q^k k_j^u)\Big] \\
& + (1-\alpha)\Big[z_j+W_j^{mh,0}\Big] \\
\text{s.t. }\ & q^k k_j^b \le \ell_j,\ \ q^k k_j^u \le z_j,
\end{aligned}
$$

where $W_j^{mh,0}=W_j^{mh}(0,0)$. The banks and capital producers have similar value functions as in the baseline. Then, the market designer solves the optimization problem below:

$$
\begin{aligned}
U_j^{mh} = \max_{z_j,(d_j,\ell_j,R_j),\theta_j}\ & -iz_j+\alpha\mu(\theta_j)[\delta_j f(\ell_j)-d_j-\delta_j R_j]+\alpha(1-\mu(\theta_j))[\delta_j f(z_j)-z_j] \\
\text{s.t. }\ & z_j \ge d_j,\ \ \eta(\theta_j)(d_j-\ell_j+\delta_j R_j)\ge\kappa, \\
& -iz_j+\alpha\mu(\theta_j)\Big[\max_{\substack{k_{\tilde{j}j},x_{\tilde{j}j}\ \text{s.t.}\\ k_{\tilde{j}j}\le \ell_j+z_j-d_j\\ x_{\tilde{j}j}\le \ell_j-k_{\tilde{j}j}+z_j-d_j}} \delta_{\tilde{j}}f(k_{\tilde{j}j})+(1-C(\tilde{\chi}))x_{\tilde{j}j}-d_j-\delta_{\tilde{j}}R_j\Big] \\
& + \alpha(1-\mu(\theta_j))[\delta_{\tilde{j}}f(z_j)-z_j] \le U_{\tilde{j}}^{mh}
\end{aligned}
$$

F Omitted Proofs

Proof of Proposition 1.

Under the case of symmetric information, we can rewrite the utility function as (omit subscript j and superscript $*$, for simplicity)

$$U = -iz+\alpha\mu(\theta)[\delta f(\ell)-\ell]-\alpha\theta\kappa+\alpha(1-\mu(\theta))[\delta f(z)-z].$$

By monotone comparative statics as in Theorem 2.3 in Vives (2001),

$$
\begin{aligned}
\frac{\partial U}{\partial z} &= -i + \alpha(1-\mu(\theta))[\delta f'(z) - 1] \\
\frac{\partial U}{\partial \theta} &= \alpha\mu'(\theta)[\delta f(\ell) - \ell - \delta f(z) + z] - \alpha\kappa \\
\frac{\partial^2 U}{\partial z \partial \theta} &= -\alpha\mu'(\theta)[\delta f'(z) - 1] < 0 \\
\frac{\partial^2 U}{\partial z \partial \delta} &= \alpha(1-\mu(\theta))f'(z) > 0 \\
\frac{\partial^2 U}{\partial \theta \partial \delta} &= \alpha\mu'(\theta)[f(\ell) - f(z)] > 0 \\
\frac{\partial^2 U}{\partial z \partial i} &= -1 < 0 \\
\frac{\partial^2 U}{\partial \theta \partial i} &= 0 \\
\frac{\partial^2 U}{\partial z \partial \kappa} &= 0 \\
\frac{\partial^2 U}{\partial \theta \partial \kappa} &= -\alpha < 0.
\end{aligned}
$$

Because $\frac{\partial^2 U}{\partial z \partial \theta} < 0$, $\frac{\partial^2 U}{\partial z \partial i} < 0$, and $\frac{\partial^2 U}{\partial \theta \partial i} = 0$, as i increases either (i) $\frac{\partial z}{\partial i} \leq 0$ and $\frac{\partial \theta}{\partial i} \geq 0$ or (ii) $U(z_1, \theta_1, i_1) - U(z_2, \theta_2, i_1) = U(z_1, \theta_1, i_2) - U(z_2, \theta_2, i_2) = 0$, which is true if $\theta_1 = \theta_2$ and $z_1 = z_2$. Thus, in both cases $\frac{\partial z}{\partial i} \leq 0$ and $\frac{\partial \theta}{\partial i} \geq 0$.

Because $\frac{\partial^2 U}{\partial z \partial \theta} < 0$, $\frac{\partial^2 U}{\partial z \partial \kappa} = 0$, and $\frac{\partial^2 U}{\partial \theta \partial \kappa} < 0$, as κ increases either (i) $\frac{\partial z}{\partial \kappa} \geq 0$ and $\frac{\partial \theta}{\partial \kappa} \leq 0$ or (ii) $U(z_1, \theta_1, \kappa_1) - U(z_2, \theta_2, \kappa_1) = U(z_1, \theta_1, \kappa_2) - U(z_2, \theta_2, \kappa_2) = 0$, which is true if $\theta_1 = \theta_2$ and $z_1 = z_2$. Thus, in both cases $\frac{\partial z}{\partial \kappa} \geq 0$ and $\frac{\partial \theta}{\partial \kappa} \leq 0$.

Because $\frac{\partial^2 U}{\partial z \partial \theta} < 0$, $\frac{\partial^2 U}{\partial z \partial \delta} > 0$, and $\frac{\partial^2 U}{\partial \theta \partial \delta} > 0$, we cannot apply monotone comparative statics.

Define the following function:

$$
\begin{aligned}
G &\equiv -i + \alpha[1-\mu(\theta(z,\delta))][\delta f'(z) - 1] = 0 \\
\frac{\partial G}{\partial \delta} &= -\alpha\mu'(\theta)\frac{\partial \theta(z,\delta)}{\partial \delta}[\delta f'(z) - 1] - \alpha[1-\mu(\theta)]f'(z) < 0 \\
\frac{\partial G}{\partial z} &= -\alpha\mu'(\theta)\frac{\partial \theta(z,\delta)}{\partial z}[\delta f'(z) - 1] - \alpha[1-\mu(\theta)]\delta f''(z) > 0.
\end{aligned}
$$

where

$$
\begin{aligned}
\theta(z,\delta) &= \mu'^{-1}\Big(\frac{\kappa}{\delta f(\ell) - \ell - \delta f(z) + z}\Big)\\
\frac{\partial \theta(z,\delta)}{\partial \delta} &= \frac{\kappa}{(\delta f(\ell) - \ell - \delta f(z) + z)^2}\frac{f(\ell) - f(z)}{-\mu''(\theta)} > 0\\
\frac{\partial \theta(z,\delta)}{\partial z} &= \frac{\kappa}{(\delta f(\ell) - \ell - \delta f(z) + z)^2}\frac{\delta f'(z) - 1}{\mu''(\theta)} < 0.
\end{aligned}
$$

By fundamental theorem,

$$
\frac{\partial z}{\delta \delta} = -\frac{\frac{\partial G}{\partial \delta}}{\frac{\partial G}{\partial z}} = \frac{a[f(\ell) - f(z)] + bf'(z)}{a[\delta f'(z) - 1] - b\delta f''(z)} > 0, \tag{A.3}
$$

where $a = -\alpha\mu'(\theta)[\delta f'(z)-1][\delta f(\ell)-\ell-\delta f(z)+z)]^{-2}(\mu''(\theta))^{-1}\kappa > 0$ and $b = \alpha(1-\mu(\theta)) > 0$.

Equation (A.3) can be rewritten as

$$
a\Big[f(\ell) - f(z) - (\delta f'(z) - 1)\frac{\partial z}{\partial \delta}\Big] + b\Big[f'(z) + \delta f''(z)\frac{\partial z}{\partial \delta}\Big] = 0.
$$

We know $f'(z) + \delta f''(z)\frac{\partial z}{\partial \delta} < 0$ since $\frac{\partial \delta f'(z(\delta))}{\partial \delta} < 0$. So, it must be true that $f(\ell) - f(z) - (\delta f'(z) - 1)\frac{\partial z}{\partial \delta} > 0$.

Then, consider the total surplus of obtaining a loan $\Lambda(\delta)$ defined as follows:

$$
\Lambda(\delta) = \delta f(\ell(\delta)) - \ell(\delta) - \delta f(z(\delta)) + z(\delta).
$$

We know

$$
\frac{\partial \Lambda(\delta)}{\partial \delta} = f(\ell) - f(z) - (\delta f'(z) - 1)\frac{\partial z}{\partial \delta} > 0.
$$

Finally, by $\mu'(\theta)\Lambda(\delta) = \kappa$, we know $\frac{\partial \theta}{\partial \delta} > 0$.

■

Proof of Proposition 2.

First, consider $\hat{U}_j = -i\hat{z}_j + \alpha(\delta_j f(\hat{z}_j) - \hat{z}_j$, where $\hat{z}_j$ is pinned down by $i = \alpha(\delta_j f'(\hat{z}_j) - 1)$.

Then,

$$\frac{\partial \hat{U}_j}{\partial i} = -\hat{z}_j < 0,$$
$$\frac{\partial^2 \hat{U}_j}{\partial i^2} = -\frac{1}{\alpha \delta_j f''(\hat{z}_j)} > 0,$$

So, $\hat{U}_j$ is decreasing and convex in i. Second, consider U_j^*, and

$$\frac{\partial U_j^*}{\partial i} = -z_j^* < 0,$$
$$\frac{\partial^2 U_j^*}{\partial i^2} = -\frac{\partial z_j^*}{\partial i} > 0.$$

So, U_j^* is decreasing and convex in i, too. But $\hat{U}_j$ has a steeper slope than U_j^* does, since $\hat{z}_j > z_j^*$. Also recall $\hat{z}_s > \hat{z}_r$ and $z_s^* > z_r^*$. So, $\hat{U}_s$ is steeper than $\hat{U}_r$, and U_s^* is steeper than U_r^*. Since $\delta_s > \delta_r$, $\hat{U}_s$ is higher than $\hat{U}_r$ at $i = 0$, where entrepreneurs use self-finance and enjoy the highest payoff. Thus, $\underline{i}$, the intersection of $\hat{U}_s$ and U_s^*, is lower than $\bar{i}$, the intersection of $\hat{U}_r$ and U_r^*, as shown in Figure 9b. When both types enter the loan market, $\bar{\bar{i}}$ is characterized by

$$-iz_s^* + \alpha\mu(\theta_s^*)[\delta_r f(\ell_s^*) - (1-\delta_r/\delta_s)z_s^* - \ell_s^*\delta_r/\delta_s] - \alpha\theta_s^*(\delta_r/\delta_s)\kappa + \alpha(1-\mu(\theta_s^*))[\delta_r f(z_s^*) - z_s^*] = U_r^*.$$

If $i > \bar{\bar{i}}$, the safe type contract becomes more attractive to the risky types and $\{z_s, \ell_s, d_s, R_s, \theta_s\} = \{z_s^*, \ell_s^*, z_s^*, (\kappa/\eta(\theta_s^*) + \ell_s^* - z_s^*)/\delta_s, \theta_s^*\}$ is no longer incentive compatible. Hence, the bankers need to ask for a large d_s and lower θ_s to reduce the risky types' incentive of misreporting their type.

■

Proof of Proposition 3.

Let $U_r(\Omega)$ be type-r's payoff of applying for a contract Ω. Suppose $U_r(\Omega_s^*) > U_r^*$ such that Ω_s^* is not incentive compatible when information is asymmetric. Using the binding free-entry condition, $U_r(\Omega_s)$ can be written as

$$U_r(\Omega_s) = -iz_s + \alpha\mu(\theta_s)[\delta_r f(\ell_s) - \frac{\delta_r}{\delta_s}\ell_s + \frac{\delta_r}{\delta_s}d_s - d_s] - \frac{\delta_r}{\delta_s}\kappa\theta_s + \alpha(1-\mu(\theta_s))[\delta_r f(z_s) - z_s].$$

With asymmetric information, down payment is used to screen entrepreneurs, so $d_s = z_s$. Then,

$$\frac{\partial U_r(\Omega_s)}{\partial \theta_s} = \alpha\frac{\delta_r}{\delta_s}\Big\{\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s - \delta_s f(z_s) + z_s] - \kappa Big\},$$

where $\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s - \delta_s f(z_s) + z_s]$ is the expected marginal surplus of an additional match in the safe type submarket, and κ is the marginal cost. The net surplus is greater than or equal to zero, $\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s - \delta_s f(z_s) + z_s] - \kappa \geq 0$; otherwise, bankers would not enter the loan market. So $\frac{\partial U_r(\Omega_s)}{\partial \theta_s} \geq 0$, indicating that the risky types' incentive to misreport increases in market tightness in the safe type submarket. Meanwhile,

$$\frac{\partial U_r(\Omega_s)}{\partial z_s} = -i - \alpha\mu(\theta_s)\frac{\delta_r}{\delta_s} + \alpha(1-\mu(\theta_s))[\delta_r f'(z_s) - 1].$$

Since $-i + \alpha(1-\mu(\theta_s^*))[\delta_s f'(z_s^*) - 1] = 0$, $\frac{\partial U_r(\Omega_s)}{\partial z_s}|_{z_s=z_s^*,\theta_s=\theta_s^*} < 0$, meaning that the risky types' incentive to misreport their type decreases in down payment required in the safe type submarket. In equilibrium, $\mu'(\theta_s) = \frac{\kappa}{\delta_s f(\ell_s) - \ell_s - \delta_s f(z_s) + z_s}$, θ_s and z_s move in opposite directions. Therefore, if Ω_s is incentive compatible, it must be the case that $z_s > z_s^*$ and $\theta_s < \theta_s^*$.

■

Proof of Proposition 4.

Now I show how δ_r affects z_s and θ_s. We can write θ_s as a function of z_s. Then the binding IC constraint can be rewritten as

$$I \equiv -iz_s + \alpha\mu(\theta_s(z_s))[\delta_r f(\ell_s) - (1 - \frac{\delta_r}{\delta_s})z_s - \frac{\delta_r}{\delta_s}\ell_s] - \frac{\alpha\theta_s(z_s)\kappa\delta_r}{\delta_s}$$
$$+ \alpha(1-\mu(\theta_s(z_s)))[\delta_r f(z_s) - z_s] - U_r = 0$$

$$
\begin{aligned}
\frac{\partial I}{\partial z_s} &= -i - \alpha\mu(\theta_s(z_s))(1-\frac{\delta_r}{\delta_s}) + \alpha(1-\mu(\theta_s(z_s)))[\delta_r f'(z_s) - 1] \\
&\quad + \alpha\mu'(\theta_s)\theta_s'(z_s)[\delta_r f(\ell_s) - \delta_r f(z_s) + \frac{\delta_r}{\delta_s} z_s - \frac{\delta_r}{\delta_s}\ell_s] - \alpha\frac{\delta_r\kappa\theta_s'(z_s)}{\delta_s} \\
&= -i - \alpha\mu(\theta_s(z_s))(1-\frac{\delta_r}{\delta_s}) + \alpha(1-\mu(\theta_s(z_s)))[\delta_r f'(z_s) - 1] < 0 \\
\frac{\partial I}{\partial \delta_r} &= \alpha\mu(\theta_s(z_s))[f(\ell_s) + z_s/\delta_s - \ell_s/\delta_s] - \frac{\alpha\kappa\theta_s(z_s)}{\delta_s} + \alpha(1-\mu(\theta_s(z_s)))f(z_s) - \frac{\partial U_r}{\partial \delta_r} \\
&= \frac{\alpha}{\delta_s}[\mu(\theta_s(z_s))[\delta_s f(\ell_s) + z_s - \ell_s] - \kappa\theta_s(z_s) + (1-\mu(\theta_s(z_s)))\delta_s f(z_s)] - \frac{\partial U_r}{\partial \delta_r} \\
&= \frac{\alpha}{\delta_s}[\mu(\theta_s(z_s))[\delta_s f(\ell_s) + z_s - \ell_s] - \kappa\theta_s(z_s) + (1-\mu(\theta_s(z_s)))\delta_s f(z_s)] \\
&\quad - \alpha[\mu(\theta_r)f(\ell_r) + (1-\mu(\theta_r))f(z_r)] \\
\frac{\partial I}{\partial i} &= -z_s + z_r < 0.
\end{aligned}
$$

Note that when $\delta_r \to \delta_s$, $\frac{\partial I}{\partial \delta_r} < 0$; when δ_r is very small, $\frac{\partial I}{\partial \delta_r} > 0$. By fundamental theorem,

$$
\frac{\partial z_s}{\partial i} = -\frac{\frac{\partial I}{\partial i}}{\frac{\partial I}{\partial z_s}} < 0 \quad \text{and} \quad \frac{\partial z_s}{\partial \delta_r} = -\frac{\frac{\partial I}{\partial \delta_r}}{\frac{\partial I}{\partial z_s}} \begin{cases} > 0 \text{ when } \delta_r \text{ is small;} \\ < 0 \text{ when } \delta_r \text{ is big.} \end{cases}
$$

We can write z_s as a function of θ_s. Then the binding IC constraint can be rewritten as

$$
\begin{aligned}
I \equiv -iz_s(\theta_s) + \alpha\mu(\theta_s)[\delta_r f(\ell_s) - (1-\frac{\delta_r}{\delta_s})z_s(\theta_s) - \frac{\delta_r}{\delta_s}\ell_s] - \frac{\alpha\theta_s\kappa\delta_r}{\delta_s} \\
+ \alpha(1-\mu(\theta_s))[\delta_r f(z_s(\theta_s)) - z_s(\theta_s)] - U_r = 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial I}{\partial \theta_s} &= -iz_s'(\theta_s) + \alpha\mu'(\theta_s)[\delta_r f(\ell_s) - (1-\frac{\delta_r}{\delta_s})z_s(\theta_s) - \frac{\delta_r}{\delta_s}\ell_s] - \alpha\mu(\theta_s)(1-\frac{\delta_r}{\delta_s})z_s'(\theta_s) \\
&- \alpha\mu'(\theta_s)[\delta_r f(z_s(\theta_s)) - z_s(\theta_s)] + \alpha(1-\mu(\theta_s))[\delta_r f'(z_s(\theta_s)) - 1]z_s'(\theta_s) - \alpha\frac{\delta_r\kappa}{\delta_s} \\
&= z_s'(\theta_s)[-i - \alpha\mu(\theta_s)(1-\frac{\delta_r}{\delta_s}) + \alpha(1-\mu(\theta_s))[\delta_r f'(z_s) - 1]] > 0 \\
\frac{\partial I}{\partial \delta_r} &= \frac{\alpha}{\delta_s}[\mu(\theta_s)[\delta_s f(\ell_s) + z_s(\theta_s) - \ell_s] - \kappa\theta_s + (1-\mu(\theta_s))\delta_s f(z_s(\theta_s))] - \frac{\partial U_r}{\partial \delta_r} \\
&= \frac{\alpha}{\delta_s}[\mu(\theta_s)[\delta_s f(\ell_s) + z_s(\theta_s) - \ell_s] - \kappa\theta_s + (1-\mu(\theta_s))\delta_s f(z_s(\theta_s))] \\
&- \alpha[\mu(\theta_r)f(\ell_r) + (1-\mu(\theta_r))f(z_r)] \\
\frac{\partial I}{\partial i} &= -z_s(\theta_s) < 0
\end{aligned}
$$

By fundamental theorem,

$$
\frac{\partial \theta_s}{\partial i} = -\frac{\frac{\partial I}{\partial i}}{\frac{\partial I}{\partial \theta_s}} > 0 \text{ and } \frac{\partial \theta_s}{\partial \delta_r} = -\frac{\frac{\partial I}{\partial \delta_r}}{\frac{\partial I}{\partial \theta_s}} \begin{cases} < 0 \text{ when } \delta_r \text{ is small;} \\ > 0 \text{ when } \delta_r \text{ is big.} \end{cases}
$$

■

Proof of Proposition 5.

First solve the symmetric information case. With complete information, down payment is not needed, $d_j = 0$. Using the binding BB constraint to eliminate R_j, the optimization problem becomes

$$
\max_{z_j,\ell_j,\theta_j} -iz_j + \alpha\mu(\theta_j)[\delta_j f(\ell_j) - \ell_j] - \theta_j\kappa.
$$

It is obvious that $\delta_j f'(\ell_j^*) = 1$, $\mu'(\theta_j^*) = \frac{\kappa}{\delta_j f(\ell_j^*) - \ell_j^*}$, and $z_j^* = 0$.

Then consider the asymmetric information case. Let Δ^{IC} be the Lagrangian multiplier of the IC^{GSW}-rs constraint. In this setup, z_s has no use but to pay d_s, so $d_s = z_s$. By taking first order conditions, $\delta_s f'(\ell_s) = 1$. Suppose $\theta_s > 0$,

$$
\Delta^{IC}\frac{\delta_r}{\delta_s} = \frac{\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s] - \kappa}{\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s - \frac{\delta_s}{\delta_r}z_s + z_s] - \kappa},
$$

which can be rewritten as

$$(1 - \Delta^{IC}\frac{\delta_r}{\delta_s})[\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s] - \kappa] = \Delta^{IC}\frac{\delta_r}{\delta_s}\mu'(\theta_s)(1 - \frac{\delta_s}{\delta_r})z_s. \tag{A.4}$$

Suppose $z_s > 0$,

$$\Delta^{IC} = \frac{i}{i + \alpha\mu(\theta_s)(1 - \delta_r/\delta_s)} < 1.$$

Then, $1 - \Delta^{IC}\frac{\delta_r}{\delta_s} > 0$ and the expected marginal net surplus of posting one more contract in the safe type submarket is greater than or equal to zero, $\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s] - \kappa \geq 0$, so the left hand side of Equation (A.4) is non-negative. The right hand side of Equation (A.4), however, is strictly negative if $z_s > 0$, but this is a contradiction; thus, $z_s = 0$. Equation (A.4) becomes

$$(1 - \Delta^{IC}\frac{\delta_r}{\delta_s})[\mu'(\theta_s)[\delta_s f(\ell_s) - \ell_s] - \kappa] = 0.$$

So, $\Delta^{IC} = \delta_s/\delta_r$ since $\theta_s \neq \theta_s^*$ under asymmetric information; θ_s solves the binding IC$^{\text{GSW}}$-rs constraint, and R_s solves the binding free-entry condition.

■ **Proof of Proposition 6.**

Let $\bar{\tau}_j = \mu(\theta_j^*)(R_j - R_j^*)$ be the net transfer to the type-j entrepreneurs upon matching to restore the complete information contract Ω_j^*. Then, the BB, PIC-rs, PIC-sr, PC-r, and PC-s constraints become, respectively,

$$\begin{aligned}
&\nu_s\bar{\tau}_s + \nu_r\bar{\tau}_s \geq 0,\\
&U_r(z_s^*, \Omega_s^*, \theta_s^*) + \alpha\frac{\delta_r}{\delta_s}\bar{\tau}_s \leq U_r^* + \alpha\bar{\tau}_r,\\
&U_s(z_r^*, \Omega_r^*, \theta_r^*) + \alpha\frac{\delta_s}{\delta_r}\bar{\tau}_r \leq U_s^* + \alpha\bar{\tau}_s,\\
&U_r^* + \alpha\bar{\tau}_r \geq \hat{U}_r,\\
&U_s^* + \alpha\bar{\tau}_s \geq \hat{U}_s.
\end{aligned}$$

Since the risky types have an incentive to misreport their type and $U_r(z_s^*, \Omega_s^*, \theta_s^*) - U_r^* > 0$, it must be true that $\bar{\tau}_s < 0$ and $\bar{\tau}_r > 0$ for the PIC-rs constraint to be satisfied. If $\bar{\tau}_r > 0$, the PC-r constraint must be slack. Suppose there exists a net transfer $\bar{\bar{\tau}}_s$ such that the BB and PIC-rs constraints bind, $\bar{\bar{\tau}}_s = -\frac{U_r(z_s^*,\Omega_s^*,\theta_s^*)-U_r^*}{\alpha(\nu_s/\nu_r+\delta_r/\delta_s)}$. Since $U_s^* - U_s(z_r^*, \Omega_r^*, \theta_r^*) > [U_r(z_s^*, \Omega_s^*, \theta_s^*) - U_r^*]\delta_s/\delta_r$, such $\bar{\bar{\tau}}_s$ satisfies the PIC-sr constraint. Note that it must also satisfy the PC-s constraint such that $\bar{\bar{\tau}}_s \geq -\frac{U_s^*-\hat{U}_s}{\alpha}$. Thus, a sufficient condition to restore z_j^*, d_j^*, ℓ_j^*, and θ_j^* is obtained:

$$U_s^* - \hat{U}_s \geq \frac{U_r(z_s^*, \Omega_s^*, \theta_s^*) - U_r^*}{\frac{\nu_s}{\nu_r} + \frac{\delta_r}{\delta_s}}.$$

■

Proof of Proposition 7.

Let Δ^x be the Lagrangian multiplier of condition x. Suppose the PC-r, PC-s, and PIC-sr constraints are slack; the FOCs become

$$\begin{aligned}
&\delta_r f'(\ell_r) = 1, \\
&-i + \alpha(1-\mu(\theta_r))[\delta f'(z_r) - 1] = 0, \\
&\mu'(\theta_r) = \frac{\kappa}{\delta_r f(\ell_r) - \ell_r - \delta_r f(z_r) + z_r}, \\
&\Delta^{BB}\nu_r = \alpha(\sigma_r + \Delta^{PIC-rs}).
\end{aligned} \tag{A.5}$$

Together with condition $d_r \leq z_r$, it is obvious that the risky types' allocations are not distorted. The safe types' allocations are characterized by the following conditions:

$$d_s = z_s,$$

$$\delta_s f'(\ell_s) = 1,$$

$$\Delta^{BB}\nu_s = \alpha(\sigma_s - \frac{\delta_r}{\delta_s}\Delta^{PIC-rs}), \tag{A.6}$$

$$\mu'(\theta_s) = \frac{\kappa}{\delta_s f(\ell_s) - \ell_s - \delta_s f(z_s) + z_s}, \tag{A.7}$$

$$\frac{\Delta^{PIC-rs}}{\sigma_s} = \frac{-i + \alpha(1-\mu(\theta_s))[\delta_s f'(z_s) - 1]}{-i - (1-\delta_r/\delta_s)\alpha\mu(\theta_s) + \alpha(1-\mu(\theta_s))[\delta_r f'(z_s) - 1]}. \tag{A.8}$$

Using Equation (A.5) and (A.6), Δ^{BB} and Δ^{PIC-rs} are obtained. Then $\frac{\Delta^{PIC-rs}}{\sigma_s} = \frac{1-\frac{\nu_s\sigma_r}{\nu_r\sigma_s}}{\frac{\delta_r}{\delta_s}+\frac{\nu_s}{\nu_r}}$. Equation (A.8) can be rewritten as

$$\frac{1-\frac{\nu_s\sigma_r}{\nu_r\sigma_s}}{\frac{\delta_r}{\delta_s}+\frac{\nu_s}{\nu_r}} = \Delta^{IC} \equiv \frac{-i+\alpha(1-\mu(\theta_s))[\delta_s f'(z_s)-1]}{-i-(1-\delta_r/\delta_s)\alpha\mu(\theta_s)+\alpha(1-\mu(\theta_s))[\delta_r f'(z_s)-1]}.$$

Suppose all types choose to enter the loan market. Recall the market equilibrium allocation z_s^{ce} and θ_s^{ce} solve the same condition as in (A.7) and the binding IC-rs constraint, given the R_s that is pinned down by the binding free entry condition. Also note that $-i-(1-\delta_r/\delta_s)\alpha\mu(\theta_s^{ce})+\alpha(1-\mu(\theta_s^{ce}))[\delta_r f'(z_s^{ce})-1] < -i+\alpha(1-\mu(\theta_s^{ce}))[\delta_s f'(z_s^{ce})-1] < 0$. Therefore, if there exists a σ_s such that

$$\frac{1-\frac{\nu_s(1-\sigma_s)}{\nu_r\sigma_s}}{\frac{\delta_r}{\delta_s}+\frac{\nu_s}{\nu_r}} = \Delta^{IC} = \Delta^{ce},$$

where Δ^{ce} is the multiplier in the competitive equilibrium and $0 < \Delta^{ce} < 1$; then the allocations are identical to the ones in the competitive equilibrium, $\{\theta_j, z_j, \ell_j, d_j, R_j\}_{j=r,s} = \{\theta_j^{ce}, z_j^{ce}, \ell_j^{ce}, d_j^{ce}, R_j^{ce}\}_{j=r,s}$.

The above equation can be rewritten as

$$1 = \Delta^{ce}\left(\frac{\delta_r}{\delta_s}+\frac{\nu_s}{\nu_r}\right)+\frac{\nu_s}{\nu_r}\frac{\sigma_r}{\sigma_s}, \tag{A.9}$$

where $\frac{\sigma_r}{\sigma_s} \to 0$ as $\sigma_s \to 1$ and $\frac{\sigma_r}{\sigma_s} \to \infty$ as $\sigma_s \to 0$. If

$$\frac{\delta_r}{\delta_s}+\frac{\nu_s}{\nu_r} < 1,$$

there always exists a σ_s such that Equation (A.9) holds. Therefore, I have found a sufficient condition for the competitive equilibrium to be constrained efficient.

■

Proof of Lemma 3.

The risky types' allocations are identical to the ones under complete information, as expected. For the problem of safe types, first solve the risky types' choice of investment when they apply for a safe type contract. The risky types choose either $k_{rs} = k_{rs}^{mh} < \ell_s$ or $k_{rs} = \ell_s$. Suppose $k_{rs} = k_{rs}^{mh} < \ell_s$, ℓ_s, θ_s, z_s and the multiplier Δ of the constraint MIC-rs are characterized by

$$\delta_s f'(\ell_s) - 1 = (\chi - \delta_r/\delta_s)\Delta, \tag{A.10}$$

$$\Delta = \frac{-i + \alpha(1-\mu(\theta_s))[\delta_s f'(z_s) - 1]}{-i - \alpha\mu(\theta_s)(1-\delta_r/\delta_s) + \alpha(1-\mu(\theta_s))[\delta_r f'(z_s) - 1]}, \tag{A.11}$$

$$\mu'(\theta_s) = \frac{\kappa}{\frac{[\delta_s f(\ell_s) - \ell_s] - \Delta\delta_r/\delta_s[\delta_s f(k_{rs}^{mh}) + (\chi\delta_s/\delta_r - 1)\ell_s - \chi k_{rs}^{mh}\delta_s/\delta_r]}{1-\Delta\delta_r/\delta_s} - \delta_s f(z_s) + z_s}, \tag{A.12}$$

and the binding IC constraint

$$\begin{aligned} -iz_s + &\alpha\mu(\theta_s)[\delta_r f(k_{rs}^{mh}) - z_s + \chi(\ell_s - k_{rs}^{mh}) + z_s\delta_r/\delta_s - \ell_s\delta_r/\delta_s] - \alpha\theta_s(\kappa)(\delta_r/\delta_s) \\ &+ \alpha(1-\mu(\theta_s))[\delta_r f(z_s) - z_s] = U_r^*, \end{aligned} \tag{A.13}$$

which is simplified using the binding BB constraint and $d_s = z_s$.

By Equation (A.10), it is either (1) $\chi > \delta_r/\delta_s$ and $\ell_s < \ell_s^*$ or (2) $\chi \leq \delta_r/\delta_s$ and $\ell_s \geq \ell_s^*$. Since $\delta_r f'(k_{rs}^{mh}) = \chi$ and $1 > \Delta > 0$, Equation (A.10) can be rewritten as $1 > (\delta_s f'(\ell_s) - 1)/(\delta_s f'(k_{rs}^{mh}) - 1) = \Delta\delta_r/\delta_s > 0$. If (1) is true, $\ell_s > k_{rs}^{mh}$. If (2) is true, $k_{rs}^{mh} > \ell_s$, which is not feasible knowing that the risky types have no other assets, $d_s = z_s$. So, if $\chi > \delta_r/\delta_s$, $k_{rs} = k_{rs}^{mh}$ and the equilibrium allocations are characterized by Equation (A.10) - (A.13).

Suppose $k_{rs} = \ell_s$; then the MIC-rs becomes IC-rs and the problem coincides with the one in the baseline.

■

Proof of Proposition 8.

By Condition (A.10), it is easy to see that $\delta_s f'(\ell_s) - 1 > 0$, so $\ell_s < \ell_s^*$. By Condition (A.11), we know $0 < \Delta < 1$ since $-i - \alpha\mu(\theta_s)(1-\delta_r/\delta_s) + \alpha(1-\mu(\theta_s))[\delta_r f'(z_s) - 1] < -i +$

$\alpha(1-\mu(\theta_s))[\delta_s f'(z_s)-1] < 0$. Also, consider Condition (A.12). Since $\delta_s f(\ell_s)-\ell_s < \delta_s f(k_r^*)+(\delta_s/\delta_r-1)\ell_s-k_r^*\delta_s/\delta_r$ and $1-\Delta\delta_r/\delta_s > 0$, we know $\frac{[\delta_s f(\ell_s)-\ell_s]-\Delta\delta_r/\delta_s[\delta_s f(k_r^*)+(\delta_s/\delta_r-1)\ell_s-k_r^*\delta_s/\delta_r]}{1-\Delta\delta_r/\delta_s} < \delta_s f(\ell_s)-\ell_s$. Then, for a given z_s, $\mu'(\theta_s) = \frac{\kappa}{\frac{[\delta_s f(\ell_s)-\ell_s]-\Delta\delta_r/\delta_s[\delta_s f(k_r^*)+(\delta_s/\delta_r-1)\ell_s-k_r^*\delta_s/\delta_r]}{1-\Delta\delta_r/\delta_s}-\delta_s f(z_s)+z_s} > \frac{\kappa}{\delta_s f(\ell_s)-\ell_s-\delta_s f(z_s)+z_s} > \frac{\kappa}{\delta_s f(\ell_s^*)-\ell_s^*-\delta_s f(z_s)+z_s} = \mu'(\theta_s^{bl})$, so $\theta_s < \theta_s^{bl}$ given z_s. Finally, consider Condition (A.13). We know $\delta_r f(k_r^*)-z_s+\ell_s-k_r^*+z_s\delta_r/\delta_s-\ell_s\delta_r/\delta_s > \delta_r f(\ell_s^*)-z_s+z_s\delta_r/\delta_s-\ell_s^*\delta_r/\delta_s$. Given a θ_s, for the IC-rs constraint to bind in both the moral hazard and the baseline model, $-iz_s+\alpha\mu(\theta_s)[\delta_r f(k_r^*)-z_s+\ell_s-k_r^*+z_s\delta_r/\delta_s-\ell_s\delta_r/\delta_s]-\alpha\theta_s(\kappa)(\delta_r/\delta_s)+\alpha(1-\mu(\theta_s))[\delta_r f(z_s)-z_s] = -iz_s^{bl}+\alpha\mu(\theta_s)[\delta_r f(\ell_s^*)-z_s^{bl}+z_s^{bl}\delta_r/\delta_s-\ell_s^*\delta_r/\delta_s]-\alpha\theta_s(\kappa)(\delta_r/\delta_s)+\alpha(1-\mu(\theta_s))[\delta_r f(z_s^{bl})-z_s^{bl}] = U_r^*$, it must be the case that $z_s > z_s^{bl}$, given θ_s.

■

Proof of Proposition 9.

Since the dual deviation does not affect $\hat{U}_j$ or U_j^*, $\underline{i}^{mh} = \underline{i}$ and $\bar{i}^{mh} = \bar{i}$. However, when both types enter the loan market and $\chi > \delta_r/\delta_s$, $\bar{\bar{i}}^{mh}$ is characterized by

$$-iz_s^* + \alpha\mu(\theta_s^*)[\delta_r f(k_{rs}^{mh}) + \chi(\ell_s^* - k_{rs}^{mh}) - (1-\delta_r/\delta_s)z_s^* - \ell_s^*\delta_r/\delta_s] - \alpha\theta_s^*(\delta_r/\delta_s)(\kappa)$$
$$+ \alpha(1-\mu(\theta_s^*))[\delta_r f(z_s^*) - z_s^*] = U_r^*.$$

If $i > \bar{\bar{i}}^{mh}$, the safe type contract becomes more attractive to the risky types and $\{z_s, \ell_s, d_s, R_s, \theta_s\} = \{z_s^*, \ell_s^*, z_s^*, (\kappa/\eta(\theta_s^*) + \ell_s^* - z_s^*)/\delta_s, \theta_s^*\}$ is no longer incentive compatible. Hence, the bankers need to ask for a large d_s and lower ℓ_s and θ_s to reduce the risky types' incentive of misreporting their type.

■ **Proof of Proposition 10.**

When $\chi > \delta_r/\delta_s$ and $\{z_s, \ell_s, d_s, R_s, \theta_s\} = \{z_s^*, \ell_s^*, z_s^*, (\kappa/\eta(\theta_s^*) + \ell_s^* - z_s^*)/\delta_s, \theta_s^*\}$, the risky types' payoff of misreporting their type in the moral hazard extension is

$$\begin{aligned} U_{rs}^{mh} =& -iz_s^* + \alpha\mu(\theta_s^*)[\delta_r f(k_{rs}^{mh}) + \chi(\ell_s^* - k_{rs}^{mh}) - (1-\delta_r/\delta_s)z_s^* - \ell_s^*\delta_r/\delta_s] - \alpha\theta_s^*(\delta_r/\delta_s)\kappa \\ &+ \alpha(1-\mu(\theta_s^*))[\delta_r f(z_s^*) - z_s^*]. \end{aligned}$$

The risky types' payoff of misreporting their type in the baseline is

$$\begin{aligned} U_{rs} =& -iz_s^* + \alpha\mu(\theta_s^*)[\delta_r f(\ell_s^*) - (1-\delta_r/\delta_s)z_s^* - \ell_s^*\delta_r/\delta_s] - \alpha\theta_s^*(\delta_r/\delta_s)\kappa \\ &+ \alpha(1-\mu(\theta_s^*))[\delta_r f(z_s^*) - z_s^*]. \end{aligned}$$

It is obvious that $U_{rs}^{mh} \geq U_{rs}$ since $k_r^* \leq k_{rs}^{mh} \leq \ell_s^*$. So, U_{rs}^{mh} intersects with U_r^* at a lower i than U_{rs} does, as Figure 9b illustrates. ■