ESE 303 – Homework 12

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November 29, 2018

Problem 1

Assume otherwise, if an arbitrage opportunity exists, it means that no matter the outcome of the game, the bidder always walks home with positive amounts of money.

We employ the famously announced equation by Prof. Ribeiro in class, that if there exists an arbitrage, then the odds of all possible outcomes (in this example say $\{a, b, c\}$) must satisfies:

$$1/a + 1/b + 1/c < 1$$

We verify that the arbitrage doesn't exist:

$$1/6.6 + 1/8.1 + 1/1.2 = 1.10830527497 > 1$$

Therefore the arbitrage opportunity does not exist.

Problem 2

There just cannot be an arbitrage when the return of one of the outcomes is less than 1. Specifically, if **Other** happens, then we get back at most 0.3 of what we bid. Therefore no arbitrage is possible. However, we can sort of do the best we can by playing the best odds from each booker:

Specifically, bidding on Brazil with booker 1, on Spain with booker 3 and on Other with booker 2 yields the best return rate. Suppose we know the true probability of which team winning, which is p of Brazil, q of Spain, and $1 - p - q$ of Other. Then we invest x on Brazil, y on Spain, and $1 - x - y$ on Other.

Then the expected payoff is $P = p*(5.6x-1)+q*(7.4y-1)+(1-p-q)*(0.3*(1-x-y)-1)$ Then we maximize with respect to x, y. The x derivative is $5.9p + 0.3q - 0.3$; the y derivative is $0.3p + 7.7q - 0.3$. Suppose $p = 0.2, q = 0.3$. Then the max is $(x, y) = (0, 1)$. A result of this optimization is shown in Figure 1.

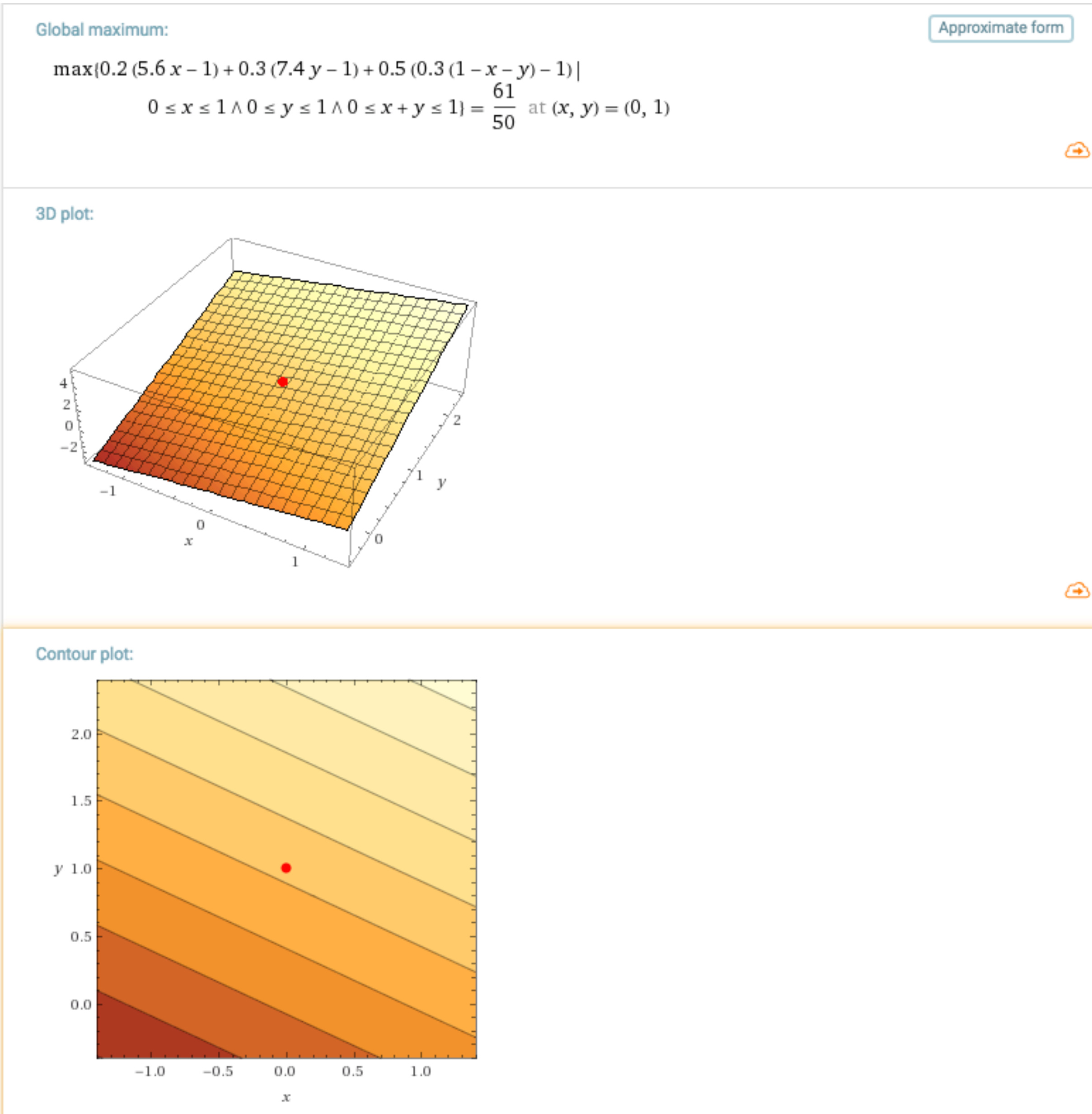

Figure 1: Wolfram Alpha Maximizer when $p = 0.2, q = 0.3$.