Product Reallocation and Market Concentration

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Abstract

This paper connects firm-level concentration to product reallocation. To do so, we link an endogenous growth model with product creation, maturity, and exchange to a novel matched product-firm dataset. The dataset informs key ingredients in the model by delivering the universe of registered brands with their prices, sales, and movement across firms. We find (i) consumer markets are dominated by large firms and mature products, (ii) products with greater sales and maturity are more likely to be exchanged, (iii) product-ownership transactions exhibit evidence of being driven by both efficient and strategic considerations. The model reconciles these flows through the lens of firm dynamics, product dynamics, and product reallocation across firms. The framework incorporates three key margins. Motivated by the facts, the model incorporates heterogeneity at the firm level, product level and firm $\times$ product level. Due to *efficiency gains*, product reallocation exhibits evidence of efficiency, increasing sales with a minimal or negative effect on prices. Due to *strategic* motive concerns, product reallocation may induce higher concentration and markups. We estimate the model to ask how these margins interact and to study counterfactual policies. We find that in aggregate, transactions are on net efficient, but significant heterogeneity exists across product categories. Coarse anti-trust policy may backfire, but optimal antitrust policies exhibit significant heterogeneity across different product categories.

Key Words: Concentration, Product Innovation, Firm Dynamics, Reallocation, Mergers & Acquisitions

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1 Introduction

Large firms are multi-product by nature. The product scope of firms explains a large share of the concentration of sales (Hottman et al., 2016). Further, product ownership is a dynamic process; products are very heterogeneous, exhibit rich life cycles (e.g., Argente et al., 2020a), and ownership moves across firms. These dynamics have important implications for market concentration and economic efficiency. Despite a rising interest in the product life cycle, the mechanics of product ownership and firm dynamics have received less attention from the literature. Empirical and theoretical frameworks to understand the macroeconomic implications are needed.

This paper links together the empirical patterns of product-ownership allocation to aggregate concentration and efficiency. We start by documenting empirical facts that link product reallocation to concentration. These facts connect the following (i) firm dynamics, as firms persist and grow through innovation, product maturity, and acquisition; (ii) product dynamics; as products exhibit rich heterogeneity and life cycles; and (iii) firm x product dynamics, as product ownership shifts across firms, more so from small to large than large to small firms. We leverage USPTO Trademark data to track the history of brands and the movement of product ownership to Nielsen Scanner data, which provide the evolution of a products prices and sales.

Although we find product ownership is exchanged from both small to large and large to small firms, net product ownership tends to move from small to large firms. This general pattern has drawn the attention of policymakers and economists (David, 2020; Cunningham et al., 2021). Product ownership may move across firms for multiple reasons. We discuss in these paper two transactions in particular, transactions that are *efficient* and *strategic*. Efficient product exchange may occur because the buying firm is better equipped to sell and distribute the product and hold ownership. Here, the interests of the firms and the overall economy align. On the other hand, strategic product exchange may occur because the buying firm can exert market power through pricing and limited consumer substitution across products. Here, the firm's interest in consolidation generates costs for the economy. Many product exchanges may exhibit evidence of both efficient and strategic incentives driving behavior.

Whether a product-ownership transaction is strategic or efficient has different predictions on prices and sales. In efficient transactions, we expect sales to increase and prices to weakly decrease. In strategic transactions, we expect prices to increase and sales to stay flat or decline. If transactions are a mix, we may see some features of both of these types of exchange. The patterns of gross and net flows and the outcomes of these events provide a criterion for evaluating the efficiency implications of product reallocation. To understand the aggregate implications, fully characterizing the product dynamics, firm dynamics, and their interaction is essential.

Given various gross and net flows of product ownership across firms, we build a model that can aggregate these flows into a measurement of efficiency. The model incorporates three main margins, in line with our study in the data. The firm margin focuses on firm interaction and firm growth, driven by product entry, maturity, and transaction. The product margin focuses on product heterogeneity and the product life cycle of sales and transactions. The firm x product margin focuses on the interaction of products and firms through ownership transfer. The model builds in benign and strategic reasons for product-ownership flows.

Empirically, we integrate two datasets central to firm- and product-level analysis: USPTO Trademark data, to study the reallocation of product ownership among firms and product age; and Nielsen retail scanner data, to study product-level sales and prices. We document the following facts regarding product reallocation and market-share concentration:

Fact 1 *Trademark and consumer product markets are dominated by large firms that persistently lead their market group. Existing product growth and product-ownership acquisitions play a significant role in firm concentration.*

Fact 2 *Products are (a) heterogeneous and (b) dynamic; products start out different and over time become transacted and make up a large share of sales in the market.*

Fact 3 *Firms frequently exchange product ownership, we find evidence for strategic (increasing markups) and efficiency (increasing sales) effects of ownership exchange.*

To focus specifically on the strategic and efficient tension, we zoom into product reallocation flows among small and large firms and focus on the differences in prices and sales. First, we find sizable product reallocation among firms, both from small to large firms and from large to small firms. In net, products tend to flow from small to large firms. Second, we find the reallocation of products among firms typically involves an increase in sales, which implies *reallocation via efficient motives*. However, when products reallocate from small to large firms, their prices increase, which implies *reallocation via strategic motives*. Through the lens of a Cobb-Douglas nested CES demand system as used in Hottman et al. (2016), we empirically decompose the sales of products into a firm component, product component, and product-firm match component.

The model quantifies the efficiency implications of the observed product reallocation flows among firms. In the model, each product group is populated with one multi-product leader and endogenous measure of single-product fringe firms. Leaders internalize their impact on the group-level price index and charge a markup that is above the monopolistically competitive level (Atkeson and Burstein, 2008), which generates variable markups. Firms can build market share either by innovation or ownership transfers.

To model the product reallocation in a tractable way, we assume firms engage in directed search: buyers post terms of trade and sellers direct search to postings. We incorporate the three components of product heterogeneity into our model: firm productivity, product quality, and firm-product fit. These three components, together with variable markups, generate both gross and net flows of product reallocation. In the limit where markup is constant, all reallocation is due to an *efficiency motive*. In the limit where firms are identical in productivity, all reallocation is due to a *strategic motive*. Our model is able to span between these limits, and to rationalize the observed reallocation flows and their impacts on sales and prices. We quantify the model with the microdata on products to understand the role of reallocation across firms and optimal antitrust and innovation policies. We explore a rich set of out-of-sample moments before turning to policy counterfactuals, comparing optimal policy with current antitrust frameworks. As a result, this paper addresses a question central to a growing debate on concentration and reallocation: What is the impact of firm- and product-level reallocation on concentration and efficiency, and what are the policy implications?

We link our model to the empirical results to identify the parameters that underlie these three forces. Through the lens of the estimated model, we address the welfare incidence of product reallocation. Along the entry margin, a lower fixed cost leads to more entry, less concentration, and higher welfare in the steady state. Along the acquisition margin, a higher search cost (e.g., more regulatory compliance or acquisition taxes) can increase or decrease welfare. Along the maturing margin, a faster growth of entering products leads to higher entry yet higher concentration. The welfare cost of concentration interacts with the life cycle of products. Concentration becomes less costly and can even lead to higher welfare when products mature quickly. We find that encouraging acquisition of products can increase welfare, but too high a transaction subsidy eventually decreases efficiency.

Our findings inform an important and growing debate in macroeconomics and industrial organization on the interconnection of concentration, markups, and innovation. We begin by building an empirical framework that links the life cycle of products, and their distribution across firms, to aggregate concentration. We motivate our analysis with three facts about firms and products from USPTO Trademark data and Nielsen Scanner data. We merge these two datasets to explore the intersection of product dynamics and firm dynamics and find the following three key facts.

Our model also makes theoretical contribution to the literature. To our knowledge, this model is the first to incorporate competitive search equilibrium into an endogenous growth model with variable markups. Due to the free entry of fringe buyers of product ownership, the equilibrium is block recursive (as in Menzio and Shi, 2011). We can calculate the group-level equilibria and aggregate to the general equilibrium without iteration.

With this model in hand, we return to our earlier question: *What is the impact of product-level reallocation on concentration and efficiency, and what are the policy implications?* We answer this question both analytically and quantitatively. Analytically, we characterize the underlying conditions that drive efficient and strategic transactions, which are shaped by the entire market. Quantitatively, we find that there is significant heterogeneity across product groups, and on while transactions exhibit strategic costs and efficient benefits, they are on average efficient, even in markets with high concentration.

The paper is structured as follows. The remainder this section reviews the literature. Section 2 introduces the USPTO Trademark Dataset and Nielsen Scanner Data. Section 3 documents the key empirical facts that frame our investigation. Section 4 introduces the model of product creation and acquisition with variable firm productivity and variable markups. Section 5 estimates the model. Section 6 uses the quantified model to understand the contribution of specific margins and perform policy counterfactuals. Section 7 concludes.

Related Literature

This paper builds on and contributes to several literatures. With product life cycles at the core of our analysis, we build on a recent body of work on product market dynamics and their life cycles. Turning to the dynamics of shifts in market shares, we link this literature to work on firm dynamics and endogenous growth. As a result, we speak to an ongoing literature on concentration, mergers and acquisitions, markups, and welfare.

The value and development of brands relates to a discussion of rising interest amongst economists in the life cycle of products. Argente et al. (2018, 2020a) explore how product creation and destruction are pervasive in product markets. Argente et al. (2021) and Einav et al. (2021) document the expansion of product sales is largely due to expansion of the customer base. Foster et al. (2016) discuss how plants grow also through building a consumer base, as entrants start out well behind incumbents and converge to them. We find this life-cycle pattern to be true in the product space as well.

Given that firms hold many products, the product life cycle has important implications for firm concentration. Hottman et al. (2016) study multi-product firms and find the scope of products explains a large share of sales variations across firms. In this paper, the sources of market power come from oligopolistic competition across firms, following the theoretical framework in Atkeson and Burstein (2008). As a result, the framework captures how the nature of product substitution interacts with market power and productivity (as noted by Syverson, 2004a,b; Melitz and Ottaviano, 2008). These sources of monopoly power interact with the size of firms, thus connecting to concerns about dominant superstar firms that potentially hold significant market power both in product and labor markets (Berger et al., 2019; Autor et al., 2020).

Market power interacts with product-ownership transfers across firms. Recent work has focused on the aggregate implications of these ownership transfers (David, 2020). Intellectual property transfer plays an important role in the distribution of technologies and products across firms. Our paper is thus related to Akcigit et al. (2016), who study the effect of patent transfers on productivity growth, where the gains from trade in patent transfers come from matching firms to technologies. This current paper differs from Akcigit et al. (2016) in focusing on product markets, firm concentration, and the strategic effect of ownership transfer. Related to the ability to capture rents from innovation, Shi and Hopenhayn (2017) study how the appropriability of innovation, the ability to license or sell intellectual property, induces upstream incentives. Relatedly, Abrams et al. (2019) study how intermediaries in intellectual property transfers can have competing negative and positive effects. We focus on the demand side by documenting facts of product-ownership transfers.

The introduction of new products is a bedrock component of much of modern endogenous growth theory (Romer, 1990; Grossman and Helpman, 1991a), as well as consumer welfare (Jaravel, 2018). Product creation has also been noted as a key empirical component of both economic growth and gains from trade as in Bils and Klenow (2001) and Broda and Weinstein (2006). Further, the ability of individuals to exchange products allows for products to expand into new markets and may spur upstream innovation (Eaton and Kortum, 1996). We contribute to this literature by documenting how products continue to grow post-introduction and how their transaction shapes new product innovation.

The quantitative model in this paper is based on the endogenous product-creation model developed by Grossman and Helpman (1991b) and oligopolistic-competition model by Atkeson and Burstein (2010). Markups are a central incentive for innovation in most models of endogenous growth. Models that incorporate tend to assume limit pricing to gain tractability, for example, Peters (2020). A recent paper by Liu et al. (2019) differs by considering a model with duopolistic competition. This current paper, like Liu et al. (2019), features endogenous markup dispersion, whereas we additionally feature endogenous acquisition and product heterogeneity.

Monopoly power is not an exogenous force. Market structure interacts with monopoly power (noted by Sutton, 1991 ; Berry, 1992). The industrial organization literature has paid significant attention to this interaction (Bresnahan, 1989; Bresnahan and Reiss, 1991). Our goal is to speak to these papers through focusing on the sources of reallocation driving concentration. Our attempt is informed by recent work on the core assumptions that can be taken for granted in measuring the current status of concentration and markups (Berry et al., 2019).

A long literature investigates how brands play a significant role in markets and economies (e.g., Brown, 1953; Nelson, 1970, 1974). Firms spend capital to invest in building their brands to connect with consumers (Sutton, 1991). Industrial organization economists have noted brands are potentially the most powerful force in generating monopoly rents (Bain, 1956). Although countervailing views of advertising and branding as good and bad exist (discussed in Becker and Murphy, 1993), economists have shown brands are persistent across space and time (Bronnenberg et al., 2009, 2012), often due to persistent consumer loyalty (Dubé et al., 2010) rather than more short-run considerations such as limited options. Bronnenberg et al. (2019) reviews this literature, discussing the economics of brands and branding.

One source of monopoly power is the stickiness of consumer preferences. The stickiness of brand preferences (e.g., as noted by Bronnenberg et al., 2012, and reviewed by Bronnenberg et al., 2019) naturally lead to a product life cycle. Products are born, build a consumer base, and then achieve profits from consumer appeal. As a result, a firm must incorporate its set of products into how it optimizes (Dhingra, 2013). Gourio and Rudanko (2014) note how, as a result, consumer goodwill is a relevant state variable for firms and products. Our current paper points out that when brand ownership is transacted, this life-cycle element becomes essential for understanding concentration.

This paper applies insights from search theory to study the market for trademark exchange. Some previous work has stressed the importance of reallocation and labor market frictions in driving economic growth. For instance, Lentz and Mortensen (2008) apply a random-search framework to uncover the importance of entry, exit, and reallocation in how labor markets interact with firm dynamics. This current paper considers the frictions in the market for intellectual property, applying competitive search theory as developed in Menzio and Shi (2011). In applied search and the market for ownership, our paper is closest to David (2020), who uses a search model to understand the market for mergers and acquisitions.

A discussion of the reallocation of products naturally connects to a rich empirical literature on firm dynamics. Further, many researchers have noted a declining reallocation in the economy. For example, the reallocation rate of jobs has been decreasing, and the entry and exit rate of firms has been decreasing (Decker et al., 2014; Davis and Haltiwanger, 2014; Decker et al., 2020). Our reallocation measure relates to the work of Davis and Haltiwanger (1992) and Davis et al. (1996). Acemoglu et al. (2018) study reallocation and aggregate innovation, focusing on selection as a key force in reallocation, whereas here we note the added ingredient of product dynamics and ownership transfer.

A lot of product reallocation is due to exchanges from small firms to large firms. As such, patterns in ownership exchange relate to work on increasing concentration and markups, which have been studied extensively, both empirically (Barkai, 2020; De Loecker et al., 2020; Traina, 2018) and theoretically (Edmond et al., 2018; Peters, 2020; Akcigit and Ates, 2019, 2021). Some papers have deployed detailed methods to focus on the transfer of products and firms. Cunningham et al. (2021) focus on killer acquisitions, where incumbents purchase small firms to keep concentration high. Killer acquisitions, however, do not match the observation that large firms pay high premiums and often deploy the products from the firms they buy, as noted by David (2020). This current paper integrates these two viewpoints by informing new empirical facts and a theoretical perspective that links these facts to current hypotheses on product market concentration. Further, we connect this framework to time trends on brand evolution in the data and apply it to antitrust policies. Two recent papers discuss the role of antitrust policies on growth, from the perspective of technological innovation (Cavenaile et al., 2021; Fons-Rosen et al., 2021).

Our theoretical framework relates to these papers in integrating the dynamic effects of transactions, but differs in the focus on the life cycle of products, market shares, and the reallocation flows across firms.

Lastly, we extend a new literature on the role of trademarks in marketing and strategy to a macroeconomic context. Graham et al. (2013) provide a general overview of the dataset and insights on the uses of trademarks. Schautschick and Greenhalgh (2016), who document the importance of trademarks to firms, review other literature that confirms the growing recognition of the importance of trademarks. Dinlersoz et al. (2018) document the newly available USPTO bulk dataset on trademarks and document facts about trademarks over a firm's life cycle. Heath and Mace (2019) focus on the role of trademarks in the strategic interaction of firms. Castaldi (2019) discusses the potential of this rich dataset in providing empirical analogs of a host of subjects in management research. Kost et al. (2019) introduce trademarks in the context of macroeconomics, focusing on markups through the lens of trademarks. In this current paper, we leverage a trademark merge to prices and sales to understand in more granular detail the distribution of products across firms.

2 Data

This project studies the connection between products, brands, and firms. Products and brands are always held by firms. It is well known in the literature that some firms are small and carry one brand, but large firms in consumer packaged goods (CPG) and most industries carry more than one product. We provide new evidence in line with this finding. This section links products to firms to provide a foundation to understand firm dynamics, product dynamics, and the transaction of ownership.

We first motivate our empirical framework and then turn to the two datasets that serve as the bedrock for our empirical analysis. In terms of the framework, we separate the product or brand performance into three components. A product's performance (sales s) could be a function of the core product, the organization producing it (the firm), and a match-specific component (the firm-product fit):

$$s(product) = V(product) + V(firm) + V(product \times firm).$$

To incorporate this equation, data from the firm, the product, and the product interacted with the firm are required. The most appropriate dataset to manage these questions at the product level would be data that have brand history, including the prices, sales, and age of each brand. For the firms and the interactions, we want data on firm history and the transfer of ownership of brands across firms. To separately identify the effects of brand and firm dynamics, we rely on transactions of products across firms to obtain estimates of firm fixed effects and product fixed effects.

In this section, we describe each of the three ingredients through through two datasets. This paper applies USPTO trademarks and RMS Nielsen Scanner Data to track the creation, distribution, and prices and quantities of products.

The trademark data provide the history of each brand and parent firm in terms of registrations, cancellations, and transactions. To focus on the dynamics of prices and quantities, we connect these firm-product-level data to specific information on product prices and quantities sold by stores in RMS Nielsen Scanner Data. The following two sections discuss these datasets in turn.

2.1 USPTO Trademark Data

USPTO Trademark data provide a unique and comprehensive insight into brand-building. Trademarks are a central and dynamic arena of the economy, as firms register for trademarks whenever they want their brand legally protected. Trademarks are common, and many more firms participate in trademarking than patenting.

In this paper, we direct attention to how trademark creation and exchange interact with the growth and concentration of firms. When firms create new products, they apply to the USPTO to protect the brand related to the customer capital of the product. Further, when firms buy the rights to product ownership from other firms, the trademark is reassigned across firms.[1]

To register for a trademark, a firm must undergo the following process. First, an individual who applies must pay a fee that ranges from \$225 to \$400. Within three months of filing, an examining attorney checks for compliance, and if the application is approved, it "publishes for opposition." A 30-day period follows, during which third parties affected by the trademark registration can step forward to file an "Opposition Proceeding" to stop the registration. This process is again evaluated by an examiner. If it clears this process, the trademark is registered.

The owner of a registered trademark then has exclusive rights to use the mark within the sphere of activity designated by the legal process. Such rights include indefinite renewal conditional on continued use and the rights to exchange. Dinlersoz et al. (2018) and Kost et al. (2019) discuss the institutional aspects of trademarks in greater detail. Further, Appendix A presents some examples of firms with multiple brands. Here, we turn to summary statistics on the number of trademarks and their distribution across firms. Even in trademarks, one can observe common patterns that are relevant for our analysis. Table 1 provides general details on the number of firms and trademarks and the distribution of trademarks across firms.

We focus on two relevant features of the data from Table 1. First, the number of transacted brands is almost as large as the number of registered brands, indicating constant flow of ownership across firms. Another striking feature of the data is the skewness of firm size. The 99th percentile firm is over 80-times

[1]If the selling firm does not remain an ongoing concern or becomes a direct subsidiary of the buying firm, it may not be reassigned, because no direct legal concern exists.

Table 1: Summary Statistics on Trademarks from USPTO

	Overall
# unique firms	1.35M
# unique registrations	5.36M
# unique transactions by bundle	915076
# unique transactions by ID	4.46M
# unique cancels	2.12M
99th percentile firm size	83
75th percentile firm size	5
Median firm size	1
Mean firm size	5

Notes: Firm size is defined as the number of trademarks within a firm. Source: USPTO Trademark Data.

larger than the median firm in terms of the stock of trademarks. We note similar patterns in terms of sales, and this recurrent pattern of concentration is a central feature of our analysis. Upon looking at this result, whether large firms are simply more efficient at building brands or selling products, or have strategic interest in building brand holdings, is unclear. Linking brands to prices and sales is the next step in uncovering these forces.

2.2 Nielsen Scanner Data

Detailed product-level data are central to our analysis. We apply detailed store-product-level data that come from Kilts-Nielsen Retail Measurement Services Data from the University of Chicago Booth School of Business. The data are large and comprehensive in the consumer product space from years 2006-2018. Although we apply historical use of trademark analysis to understand the age and evolution of brands, 2006-2018 is our primary focus.

We observe more than 100 billion observations at the product $\times$ store $\times$ time level. Product is defined by a UPC identifier, 12 digits that are uniquely assigned to each specific good. The store is defined at the local level, with over 40,000 total; time is defined weekly. Total sales are approximately \$300 billion per year, covering around half of consumption in the consumer goods industry, which itself covers approximately 8% of total consumption in GDP.

The UPC barcodes provide a unique identifier for each product. Changes in any attribute of a good corresponds to a new barcode. Barcodes are widespread and thus cover a large amount of the

CPG industry. However, the unique identifying feature of the barcodes may not be as relevant for our analysis. For instance, the parent trademark associated with "Coca-Cola Christmas Edition" is the original "Coca-Cola".

This dataset has been used widely for product analysis. A key departure from the literature in our case is focusing on *brands*, that is, brand names listed in trademark and Nielsen data, rather than *products*, that is, UPC codes. We discuss three reasons for focusing on brands rather than products. First, consumer goodwill tends to be brand rather than product-specific. Coke 12oz relies on the same core branding as Coke 20oz. Thus, regarding how the consumer interacts with the product, brand is the more core indicator. Second, when firms exchange product ownership, or the right to sell a specific brand, they systematically transfer the full rights on the consumer goodwill, making the specific product differentiation within the brand less relevant. Third, our data enable identification at the brand level in both the Nielsen data and USPTO trademark data. Nielsen provides brand identifiers in addition to product identifiers. We collapse this information into brand sales by product group by year. We do not focus on geographical variation in this paper.

The volume represents over half of all transactions in grocery stores and drug stores, and slightly less than half in convenience and mass merchandise stores (as also noted by Argente et al., 2020a). We apply a dataset from GS1 US to link parent firms to products through UPCs. Whereas GS1 links to most parent companies, the USPTO trademark dataset helps complement GS1 to ensure the correct company is allocated to the correct brand. We focus on this step next.

2.3 Data Merge

To build a bridge between brand age, brand exchange, and product evolution, we employ a fuzzy merge to connect product names in RMS Nielsen scanner data to USPTO Trademark data. Whereas this merge is the first we know of that links USPTO Trademark data to Nielsen Scanner data, Argente et al. (2020b) link USPTO patent data to RMS Nielsen product-level data. We follow a similar method but get greater coverage in our merge, likely due to the different nature of patents and trademarks. In particular, we are able to identify specific products connected to their brand name as long as the trademarked brand name is similar enough to the store brand name.

We start by normalizing names in each dataset at both the firm and product-level. For example, we want to capture heterogeneous naming at the firm (e.g., General Mill Holdings + General Mills Minnesota Op.) and connect it to the parent company. We then turn to the brands themselves. We employ a similar fuzzy match with brands. We start by linking observations at the brand $\times$ firm level, but for observations for which we directly observe the brand, we connect the brand independently and assign ownership through trademark data. At times, Nielsen data are purely linked to distributors, so USPTO data may be a more reliable indicator of ownership.

Both USPTO and Nielsen scanner data contain a firm $\times$ brand of observation of interest. The identification of firms and brands provides what is ideally a many-to-1 matching between products (which rely on the same goodwill) and brands. In reality, we must rely on many brands and many product matches. For these matches, we focus on the most reliable name match. If the same brand has multiple matches, we take the "active" brand. For instance, if a brand is reassigned across firms, we assume this represents the focal group of brands. Once this match is complete, we treat brands as the relevant margin for the concepts in this paper.

We next turn to the quality of the match between brands and products. We focus in the USPTO case on brands, and note the products they connect to in Nielsen. Table 2 provides information on the match between products and trademarks.

Table 2: Summary Statistics on Trademark Nielsen Merge

	Unique Count	Years Active	Share Match (%)
USPTO Trademark Data			
Brands	5.36M	1870-2020	
Firms	371021	1870-2020	
Canceled Brands	2.12M	1970-2020	
Transactions	915076	1970-2020	
RMS Nielsen Scanner Data			
Brands	1.64M	2006-2018	57%
Firms	23232	2006-2018	54%
Brand $\times$ sales		2006-2018	86%

Notes: Summary statistics on share of merge brands in both datasets. Source: USPTO Trademark Data and RMS Nielsen Scanner Data

We stress a couple points from the table. First, when we take brands merged with sales weights, we are capturing a significant share of sales in the data at 86%. Without sales weights, we are unsurprisingly capturing fewer brands. Some small firms may choose not to protect their intellectual property via legal means.

We find in this merge that not only are multiple brands associated with single firms, but also that multiple products are connected to a single brand. On average, we observe 13 unique products per brand. As a result, brands are an important source of firm income and, and noted in the following section, provide the central focal point for firm growth.[2] We now turn to the empirics of products and firms.

[2]We consider the brand to be the core product a firm is producing, so in terms of the main takeaways of the paper, we treat product and brand as interchangeable.

3 Empirics of Reallocation and Concentration

This section focuses on the motivating empirical observations that inform our model and quantitative analysis. Following the structure discussed above, we focus on three main margins. We focus first on the firm-level margin, and discuss firm concentration and firm dynamics. We then turn to the product-level margin, where we focus on product heterogeneity and product dynamics. We then turn to the firm $\times$ product heterogeneity and dynamics, focusing on product-ownership transactions and the gross and net flows of products across firms. Section 3.4 focuses in particular on the firm $\times$ product interaction and the evidence for both efficient and strategic transactions, which is an essential question tackled by our quantitative framework.

This section proceeds in four steps. Section 3.1 motivates the empirical analysis by discussing a model of firm dynamics with consumer preferences for brands. The simplified framework provides essential infrastructure to understand the market for product reallocation. Further, it previews the model which will be used to quantify the interactions. The next three steps focus on the three components discussed in the accounting framework.

Section 3.2 starts by focusing on firms. First, we document the degree of dominance of firms in product markets, illustrating the role of market leaders and their persistence. Second, we decompose the forces that contribute to firm-level market share. We focus on three core drivers of concentration: (i) product creation and destruction, (ii) existing product sales growth, and (iii) product transactions across firms.

Section 3.3 unpacks products more directly, turning attention to the three core forces contributing to concentration – product creation, growth, and transactions of ownership across firms. The product life cycle exhibits striking patterns in the data. Older products tend to take up the largest share of sales. Products exhibit a similar pattern with transactions, as older and larger brands are more likely to be transacted.

Section 3.4 focuses on the rate and nature of product transfer across firms. We document evidence of efficiency gains from product transfer as well as strategic gains (through pricing). We further point out the amount and direction of product-ownership flows. The interaction of these three forces and the aggregate implications are discussed further in Section 6.

3.1 Accounting Framework

This framework motivates the empirical facts we uncover in this section through the lens of the model that allows us to quantify these forces. We start with a simple constant elasticity of substitution (CES) aggregator in the utility function, aggregating over k product groups, where each product group has internal competition. We highlight the two layers here, and direct particular attention to the value of individual products in this setting.

Time is continuous. A representative household endogenously supplies labor $\mathbf{L}_t$ and spends on consumption goods to maximize its discounted utility. At instant t, the real consumption of the household $\mathbf{C}_t$ is given by a CES aggregator across a unit measure of product groups, indexed by $k \in [0,1]$. Each product group k at time t contains N_{kt} measure of imperfectly differentiable products. The real consumption from product group k, C_{kt}, is a CES aggregator across these varieties. Specifically, the real consumption is given by

$$\mathbf{C}_t = \left(\int_0^1 \varphi_k^{\frac{1}{\theta}} C_{kt}^{\frac{\theta-1}{\theta}} dk \right)^{\frac{\theta}{\theta-1}},$$

where C_{kt} is the real consumption from product group k, ϕ_k is the appeal of product group k to the household, and θ is the substitution elasticity across product groups. The real consumption C_{kt} is given by:

$$C_{kt} = \left(\int_0^{N_{kt}} z_{it}^{\frac{1}{\sigma}} c_{it}^{\frac{\sigma_k-1}{\sigma_k}} di \right)^{\frac{\sigma_k}{\sigma_k-1}},$$

where N_{kt} is the measure of products that are available in group k at instant t, z_{it} is the appeal of product i, and σ_k is the substitution elasticity across products, which we allow to vary by product groups. z_{it} is a core object of interest from our model. We assume the appeal of products is driven by both fixed characteristics of products (product components), the characteristics of owning firms (firm components), and a goodness of match between product and owning firms (match specific):

$$\log z_{it} = \underbrace{\alpha_{j(i,t)}}_{\text{firm}} + \underbrace{\beta_i}_{\text{product}} + \underbrace{\gamma_{ij(i,t)}}_{\text{firm} \times \text{product}}, \tag{1}$$

In our empirical and model framework, we distinguish between three main forces at play: firm heterogeneity (α), product heterogeneity (β) and firm $\times$ product heterogeneity (γ). This is at the core of our empirical and theoretical analysis.

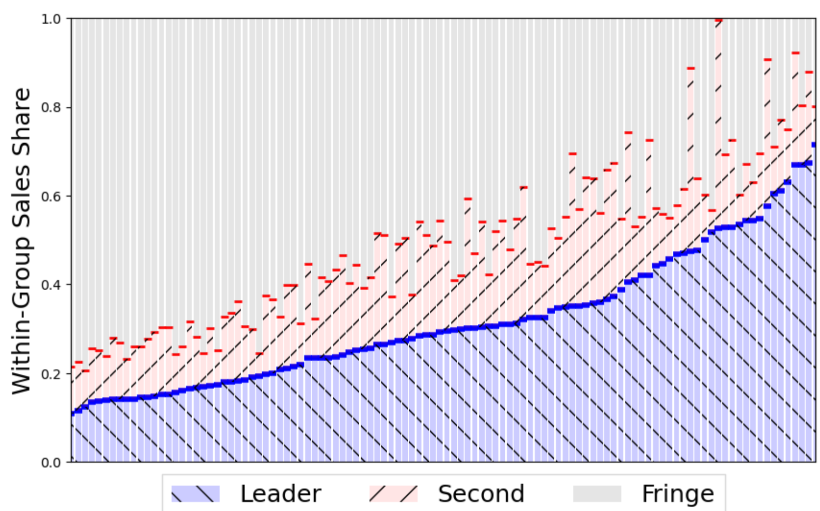
Some readers may notice the connection between the main ingredients of this framework and standard models of the labor market. Substituting *labor* for *product* theoretically connects to this literature. We are motivated by this framework, and thus, our theoretical analysis employs components from the labor search literature, as firms search for products to buy and the market churns. In this section, we discuss the three margins separately. We now turn our attention to the empirical facts in the firm, product, and firm $\times$ product dynamics.

3.2 Firm-Level Analysis

To focus on the firm component, we first look at the nature of concentration and drivers of firm advantage, which supplements our interest in $\alpha_{j(i,t)}$ in equation (1). Our accounting framework stresses that market concentration is not a purely static force but is driven by dynamism in firm ownership. Companies such as Coca-Cola, Procter & Gamble, and General Mills at one time did not have large and persistent market shares. Market share must be built through slow-moving customer capital and brand acquisitions. Although concentration can be dynamic, concentration is also persistent. Coca-Cola was a powerful firm 10 years ago and still is today. We focus in this section on the level, persistence, and sources of market shares. We first focus on overall concentration and then turn to the persistence of market leadership. Lastly, we turn an analysis of the dynamic elements driving concentration.

Figure 1 maps out the sales share of the product leader, the second firm, and the remaining firms in the market. This split by product group contains 116 unique product-group categories (e.g., “ICE CREAM" or “BEER”). The average top firm share is 34% of the total market, though in many markets, the top firm holds a significantly larger share. Thus, understanding how large and fringe firms interact is essential to understanding concentration.

Figure 1: Sales Share of Leader, by Product Group



Note: *This figure shows the sales share by product group (ordered by % share of leader) in 2010.* Source: *RMS Kilts-Nielsen Data Center & GS1 firm-product merge*

Table 3: Firm Market Shares

Top firm share by group	Top 2 firm share	median share
33.6%	50.3%	0.04%

Table 3 shows the average leader (and second firm) share, as well as the share from the median firm.

The top two firms control on average more than half the sales in a given market. We note also the presence of a host of small firms (median share as 0.04%), and in our framework, we think of these firms as "fringe" in the sense they hold few products and small market share.

We also note these leading firms are quite persistent. Across all categories, the leading firm in one period has a 97% chance of being among the top two firms in the product group in the next period. Concentration in product markets is real and persistent, yet it is not made up of single products. On average, market leaders hold 27 unique brands within the product group they lead. As a result, variation in concentration will be closely connected to how firms interact with their brands.

Product market dominance does not happen in a day. Concentration is the result of the progressive reallocation of market shares of products across firms. The product life cycle is intertwined with firm growth and decline through three core channels. First, and most noted within the innovation literature, is product creation and destruction. Second, once products are born, they grow and decay over time. Third, product ownership is transacted across firms.

The format of our data allows us to characterize these three forces. We set up three regressions where the coefficients add up to 1 (following a beta decomposition), and each coefficient is linked to the amount of variation of firm growth and decline the margin explains. We run the following regression of three distinct margins of change, y_{it}, on the change of sales in each period $\Delta sales_{it}$:

$$y_{it} = \alpha + \beta \Delta sales_{it} + \epsilon_{it}. \tag{2}$$

Equation (2) focuses on three different margins for y_{it}. We substitute each of the three margins discussed above as y_{it} (y_{it}=creation, growth, transaction). We refer to product growth as incumbency because it can include decline as well. We present the results of the three separate regressions in Table 4.

We stress two main takeaways from Table 4. At the firm level, variation from entry is much more important for fringe firms (e.g., small firms) than for large firms. Acquisition is relatively much more important for sales variation for large firms. Relative to acquisition, entry is almost 30-times more important for small firms. For both fringe firms and market leaders, incumbent products drive a significant amount of firm-level variation. Some variation from incumbent products may come directly from the life cycle, whereas others may be due to idiosyncratic shocks. We focus directly on the role of life-cycle variation in Table B1 in Appendix B.

3.3 Product-Level Analysis

Products are both a significant source of firm concentration (Hottman et al., 2016), and highly dynamic (Argente et al., 2020a), thus affecting the overall sales as in β_i from equation (1). The concentration implies a rich heterogeneity, but the dynamic nature implies that heterogeneity changes over time. The

Table 4: Sources of Market Share Reallocation

	Leader			
	Entry	Incumbency	Acquisition	Entry/Acquisition
Value	0.0026	0.89***	0.11***	**0.023**
	(0.315)	(0.000)	(0.000)	
Observations	364	364	364	
	Fringe			
	Entry	Incumbency	Acquisition	Entry/Acquisition
Value	0.12***	0.69***	0.19***	**0.626**
	(0.000)	(0.000)	(0.000)	
Observations	100408	100408	100408	

p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: Market share reallocation is measured across different firm types, following equation (2). Source: RMS Nielsen

change in products can come from development of a product line or transactions of products from worse to better firms.

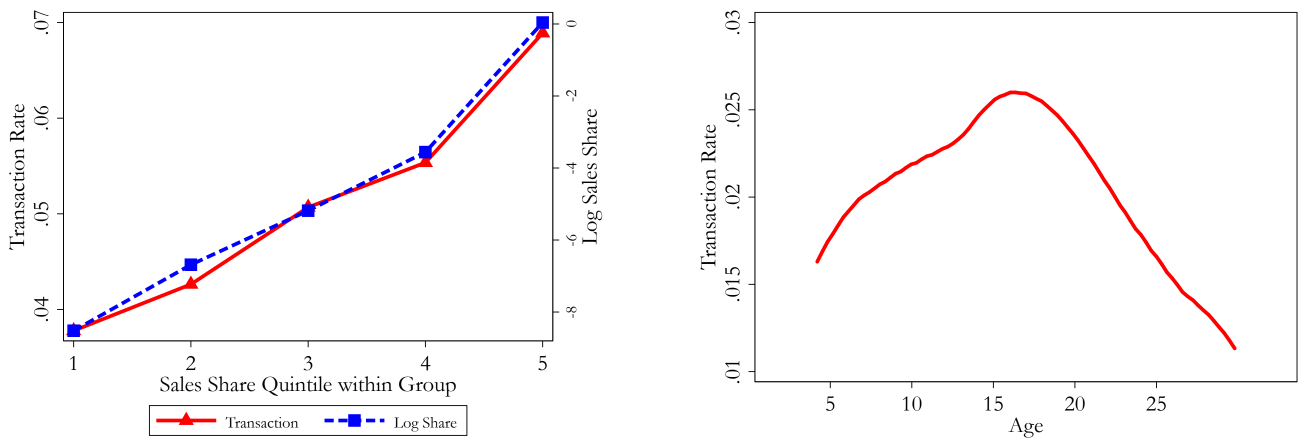
Our goal in this section is to focus on the interaction of products and reallocation. Although we also link trademark data to product sales to expand on the work related to rich product life cycles (e.g. Argente et al., 2020a, 2021), we focus more directly on the interaction with reallocation. We focus on the age profile of sales in Appendix B.

Products evolve over their life cycle. Gourio and Rudanko (2014) document the slow development of customer capital, whereas Foster et al. (2016) note the importance of learning about demand for firms. The same is true for transactions. Few products are transacted when very young, because they need time to build customer capital and exposure to other firms. We focus on the correlation between transactions and sales and transactions and age in the following figures.

Figure 2 focuses on the interaction of transaction rates, sales share, and age. Figure 2a focuses on the interaction of sales share and transaction, whereas Figure 2b leverages the history of USPTO trademark data to understand the interaction of age and transaction.

Figure 2a splits the products into quintiles (truncating products with less than \$1000 in sales in a year), and plots the transaction rate against product quintile and the log sales share. We see transaction rates are higher for products with larger market shares. This result can be rationalized in various ways, but we inform the patterns in transaction through directed search in Section 4. Firms select on searching for products with higher value to transact.

Figure 2: Transactions, Age, and Sales



(a) Transaction Rate by Quintile (b) Transaction Rate by Age

Notes: Panel (a): Transaction rate by sales share. Panel (b): Transaction rate by age. Source: USPTO Trademark and RMS Nielsen.

Similarly, Figure 2b shows how the transaction rate changes with age. Here, we plot a standard smoothed hazard function to ask at what age a brand that has not yet been transacted becomes transacted. We find the age peaks close to 20, around when log sales share peaks. We next turn to the role of the transactions across firms, particularly looking at the interaction between leading and fringe firms.

3.4 Firm $\times$ Product Analysis

Firm ($\alpha_{j(i,t)}$ from equation (1)) and product ($\beta_{i,t}$ from equation (1)) dynamics evolve to determine market outcomes. However, the interaction of firms and products, through in particular the firm-product fit ($\gamma_{i,j(t)}$ from equation (1)) is also an important force in market dynamics, especially when it comes to the transaction of products across firms.

We focus in this section on the *rate* of exchange and the change in sales and prices upon exchange. Given our initial motivation of market concentration, we focus on the heterogeneous effect of being held by a large versus a small firm. We ask what is, conditional on being held by both a large and small firm, the effect of being held by a larger firm on log sales and log prices?[3] We use the regression in equation (3) for our analysis:

$$y_{ikt} = \alpha_0 + \alpha_1 \mathbb{I}\{j(i) = \text{T10 firm}\} + \Gamma_{ik} + \Lambda_t + a_{t-b(i)} + \epsilon_{ikt}. \tag{3}$$

The regression evaluates an outcome variable y_{ikt} (e.g., log sales or log prices), as a function of whether it's held by a market leader. We include a product (brand-group) fixed effect in Γ_{ik}, a year fixed

[3]Firm size is defined over the entire horizon of the data, but results are similar if firm is defined in only the first period.

effect (Λ_t), and an age fixed effect $a_{t-b(i)}$.[4]

Table 5: Log Price and Sales Conditional on Holding Firm

	(1)	(2)
	Log Sales	Log Price
Top 10 Firm Holding	0.65***	0.058***
	(0.000)	(0.000)
R^2	0.810	0.916
N	4430	4430

p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands.

The results are striking on both counts, and hold true with various specifications. If a top 10 firm holds a brand, sales on average are 0.56 log points higher, whereas prices are on average 0.058 log points higher. Both sales and prices are higher at leading firms, indicating large firms could have both strategic and efficiency reasons to buy products. These results find greater texture in the quantitative section when we ask about the overall effects of these purchases on market activity. This section motivates our further interest in understanding the market for transactions.

We briefly turn to the flows to understand the rates of product movement in each direction. The persistence of transactions suggests match-specific components are likely at work. We focus on three types of flows. Brands may move from fringe firms to other fringe firms, from fringe firms to market leaders, and from market leaders to fringe firms. We consider a leader a top 10 firm in a product group. Table 6 documents the flows into each group (leader from fringe, etc.) in terms of the number of observations and the sales-weighted share.

Table 6: Flows across Firms

Transaction Type	**Num. Obs**	**Share of obs (sales-wt)**
Fringe to Leader	1,123	26%
Leader to Fringe	1,076	18%
Fringe to Fringe	10,223	56%

Notes: This documents the count of observations (and corresponding market share) of flows to leader and to fringe firms. Source: USPTO and RMS Nielsen.

[4]Due to a colinearity problem with age and year, we adjust for age fixed effects following Deaton (1997).

While fringe to fringe firm transactions are almost 10 times as common as fringe-to-leader transactions, they take up a much smaller market share per transaction. Leaders buy larger brands on average. Further, as noted earlier in this section, leaders tend to increase the sales of brands they have acquired. Thus, although reallocation is a common feature of this market, certain patterns of reallocation are especially important for thinking about market concentration. We now turn to a summary of the three main facts discussed in this section.

> **Fact 1, Firm Level** *Trademark and consumer product markets are dominated by large firms that persistently lead their market group. Existing product growth and product-ownership acquisitions play a significant role in firm concentration.*

> **Fact 2, Product Level** *Products are (a) heterogeneous and (b) dynamic; products start out different and over time become transacted and make up a large share of sales in the market.*

> **Fact 3, Firm $\times$ Product Level** *Firms frequently exchange product ownership, we find evidence for strategic (increasing markups) and efficiency (increasing sales) effects of ownership exchange.*

This section showed first that markets are concentrated, and this concentration is persistent over time and built through acquisition and product maturity (Fact 1). In line with findings on the product life cycle, we find patterns of higher profile and more mature brands being more likely to be transacted (Fact 2). Directing more specific attention to products across firms, we find interesting patterns of transaction and outcomes upon transaction (Fact 3). These results motivate a model that can incorporate these forces and develop counterfactuals. We turn to the model next.

4 Model

We introduce a firm dynamics model with endogenous (1) product creation, (2) variable markups, and (3) transfer of brand ownership. The model incorporates some standard features of an endogenous growth framework driven by product variety. In addition to these standard features, we include market power (e.g., as in Atkeson and Burstein, 2008) and directed search for transfer of product ownership as in the labor literature.

The model interprets the observed product reallocation flows in efficiency and provides a framework for counterfactual analysis. The model contains a representative household that consumes imperfectly substitutable varieties from different product groups, and supplies labor to firms. The consumption from different product groups is aggregated through a Cobb-Douglas utility function across different product groups and a CES aggregator among imperfectly substitutable varieties within product groups.

Incorporating heterogeneity at the product-group, firm, and product levels in an environment with ownership transactions and variable markups is challenging, because the decisions of firms depend on the joint distribution of the heterogeneity at the firm and product level. One contribution of our model is to provide an environment where the equilibrium is block recursive (as in Menzio and Shi, 2011), where the equilibrium objects can be easily characterized while sill capturing the relevant mechanisms.

Section 4.1 characterizes the environment, directing particular attention to the household, firms, and products. Section 4.2 characterizes the steady-state equilibrium with a balanced growth. This equilibrium links the decisions of households to firm optimization and the evolution of products and their flows across firms. Section 4.3 focuses on how the outcomes that emerge from separate product markets aggregate and the welfare implications from an overall perspective. Each of these sections pays particular attention to how product innovation and reallocation interact with firm decisions and consumer welfare.

4.1 Environment

Representative Household

Time is continuous and there is a representative household that endogenously supplies labor $\mathbf{L}_t$ and spends to maximize its discounted utility. At instant t, the real consumption of the household $\mathbf{C}_t$ is given by a Cobb-Douglas aggregator across a unit measure of product groups, indexed by $k \in [0, 1]$:

$$\ln \mathbf{C}_t = \int_0^1 \phi_k \ln C_{kt} dk, \tag{4}$$

where C_{kt} is the real consumption from product group k, ϕ_k is the appeal of product group k to the household. Each product group k at time t contains N_{kt} measure of imperfectly differentiable products. The real consumption from product group k, C_{kt}, is a CES aggregator across these varieties:

$$C_{kt} = \left(\int_0^{N_{kt}} z_{it}^{\frac{1}{\sigma_k}} c_{it}^{\frac{\sigma_k - 1}{\sigma_k}} di \right)^{\frac{\sigma_k}{\sigma_k - 1}}, \tag{5}$$

where c_{ikt} is the consumption on variety i, z_{it}^D is the appeal of product i, and σ_k is the substitution elasticity across products, which we allow to vary by product groups. z_{ikt}^D is a core object of interest from our model. Mirroring the empirical decomposition of product-level sales, we assume the appeal z_{ikt} is a combination of three components:

$$\log z_{it}^D = \alpha_{j(i,t)}^D + \beta_i + \gamma_{ij(i,t)}. \tag{6}$$

Equation (6) closely resembles our empirical framework. $\alpha^D_{j(i,t)}$ is the appeal of owning firms, β_i is the product fixed effect, and $\gamma_{ij(i,t)}$ is the match-specific quality between product i and its owning firm $j(i,t)$.

The household can freely borrow or save by investing in a representative portfolio of firms in the economy, taking as given the interest rate and prices. Throughout the paper, we normalize the aggregate price index to be 1 and express other prices in their real units. Denote $\mathbf{w}_t$ as the real wage and $\mathbf{r}_t$ as the real interest rate. The household takes these prices as given and chooses its real consumption $\mathbf{C}_t$ and labor supply $\mathbf{L}_t$ to maximize:

$$\max_{c_{ikt},\mathbf{L}_t} \int_0^\infty e^{-\rho t}\left[\ln \mathbf{C}_t - \varphi_0 \mathbf{L}_t\right] dt, \tag{7}$$

s.t.

$$\dot{a} = \mathbf{r}_t a_t - \mathbf{C}_t + \mathbf{w}_t \mathbf{L}_t, \tag{8}$$

$$\mathbf{C}_t \text{ given by (4) and (5).} \tag{9}$$

Discussion of Assumptions.- Two key assumptions from the household side lead to the simplification of the environment. First, by assuming the consumption from different product groups is aggregated through a Cobb-Douglas utility function, we assume that evolution within each product group does not lead to reallocation of market shares across product groups. In addition to being the standard assumptions in the patent race literature such as Liu et al. (2019) and in the product dynamics literature such as Argente et al. (2021) and Hottman et al. (2016), this assumption is without apology given our context is the product market at an annual frequency where this reallocation across groups is small in magnitude. Second, by assuming the labor disutility is linear in labor supply, we assume the wage-GDP ratio is a constant. This assumption eliminates the general equilibrium wage effect through the endogenous innovation and search inputs of firms. This assumption is motivated by the fact that product market innovation is a relatively small part of aggregate innovation (e.g., as in Garcia-Macia et al., 2019). We may reasonably believe this general equilibrium impact is small. We also highlight our main results do not come from the labor costs.

Firms

Each product group contains one large multi-product firm and endogenous measure of single-product firms. We refer to the multi-product firm as the group leader and single-product firms as the fringe. The leader and fringe firms are different in the following aspects: (1) *Capacity in operating varieties*. The leaders are able to own and operate positive measure of products, whereas the fringe firms are only able to operate an infinitesimal product; (2) *Entry*. The leaders are not subject to entry and exit, with free entry of fringe firms. (3) *Productivity*. All varieties are produced using a linear technology in labor. The leader has productivity $e^{\alpha^L_{kt}/(\sigma_{kt}-1)}$, and fringe firms have the same productivity 1. (4) *Pricing*. The leaders are big relative to their product groups, and they internalize their impact on the group-level price index. The fringe firms are small relative to the market, and they behave as monopolistic competitive firms.

Competition. We assume the firms compete through price, where firms internalize their impact on group-level price indices according to their market shares as in Atkeson and Burstein (2008). As the fringe firms are infinitesimal relative to the market, they do not internalize their own impacts on the price indices. In the equilibrium, they charge a constant markup $\frac{\sigma_k}{\sigma_k - 1}$. Given the demand curve for each variety in equation (12), the product-group leader's pricing decision is:

$$\max_{p_i} \int_{i \in \mathcal{I}_{kt}} (p_i - \mathbf{w}_t) c_{kt}(p_i, z_{ikt}) di$$

s.t.

$$c_{kt}(p, z) \text{ given by equation (12).}$$

Product Reallocation. The ownership of existing products can be reallocated across firms within the same product group. This reallocation process is modeled as a market with search and matching frictions. We assume search is directed. At each instant, the market for product ownership is organized as follows: (1) product group leaders and fringe firms can all be either buyers or sellers; (2) becoming a buyer incurs a constant search cost $\frac{\kappa_k^s}{Z_{kt}}$ in the units of labor, where κ_k^s is a parameter we refer to as search cost;[5](3) buyers post the transfers they are willing to make in exchange for the ownership of existing products; (4) sellers observe all posted terms of trades and direct search to different buyers; and (5) the contact rate for the buyers is $\frac{\lambda(\theta)}{\theta}$ and the contact rate for buyers is $\lambda(\theta)$ if the buyer is a fringe firm, where θ is the expected number of sellers per buyer; the contact rate for a seller is $\lambda(\theta)$, whereas the contact rate for the buyer is $\frac{\lambda(\theta)}{\theta} Z_{kt}$ if the buyer is a group leader.[6] We assume λ is increasing and concave in θ, and $\frac{\lambda(\theta)}{\theta}$ is decreasing and convex in θ; (6) when a contact occurs between seller and buyer, the buyer draws a new match quality $\gamma' \sim F_\gamma(\gamma)$, and decide whether to finalize the trade. Once the deal is finalized, the ownership and operation of the focal product is transferred to the buyer, with new quality (β, γ').

Innovation. New products are created through product innovation. The leader can choose its innovation intensity η_t by paying labor cost $D_k(\eta)$. $D_k(\eta)$ is increasing and convex in η, and $D_k(0) = 0$. The fringe firms can endogenously enter with entry cost $\frac{\kappa_k^e}{Z_{kt}}$. Having entered, the fringe creates a new product. New products draw their permanent quality β from exogenous distribution $F_\beta(\beta)$ and match-specific quality γ from distribution $F_\gamma(\gamma)$.

Discussion of Assumptions. Two assumptions lead to tractability of this model. First, by assuming competitive search, we isolate the reallocation decisions from the within-group distribution of products.

[5]We assume the search cost decreases proportionally to the quality of a group. This assumption is important to guarantee balanced growth path, as made in standard product innovation models such as Grossman and Helpman (1991b).

[6]Implicitly, we assume that as the number of varieties within the product group is growing, it also linearly scales up the chances of leaders meeting others. This assumption is similar to the assumption of labor augmenting technological change from Uzawa's Theorem. It is an important assumption to guarantee a balanced growth path in our environment.

This assumption is well motivated by our empirical finding that the reallocation rate is correlated with quality of products. Second, we assume that the entry and search cost scale with innovation, as is standard in the endogenous growth literature (Grossman and Helpman, 1991a).

4.2 Characterization

In most of the paper, we look for an empirically relevant steady-state equilibrium with positive entry in all groups and with positive product reallocation from fringe firms to leaders, from leaders to the fringe, and among fringe firms. Focusing on the steady state, we omit the time subscript whenever we refer to the aggregate variables. Hereafter, time subscripts only summarize the time evolution within each product group, given constant aggregates. We start by breaking down the core interactions in the economy; the household decision, the firm's pricing decision, and the reallocation of products across firms. We then turn to the innovation decisions and value functions, focusing on the driving forces behind product reallocation from the firm's point of view. We then discuss the growth rate within group, the evolution of the distribution, and the within-group equilibrium.

Household Decision

The optimal saving decision implies the Euler equation must hold:

$$\frac{\dot{\mathbf{C}}}{\mathbf{C}} = \mathbf{r} - \rho = 0, \tag{10}$$

and the optimal labor supply decision requires that the marginal rate of substitution between leisure and consumption equals the real wage:

$$\varphi_0 = \frac{\mathbf{w}}{\mathbf{C}}. \tag{11}$$

The optimal consumption decision within group k gives the demand curve for variety i, given the appeal z and price p:

$$c_{kt}(p, z) = z \times p^{-\sigma_k} \times P_{kt}^{\sigma_k - \theta}, \tag{12}$$

where the group-level price index P_{kt} is given by $P_{kt} = \left(\int_0^{N_{kt}} z_{ikt} p_{ikt}^{1-\sigma_k} di \right)^{\frac{1}{1-\sigma_k}}$.

Pricing Equilibrium

As in the models in patent race literature, the competition of a group can be summarized by the gap between leaders' quality and fringe firms' quality. In our context, the relevant measure is the quality gap between the leader's basket of products and fringe firms' basket of products, adjusted by the relative productivity α_k^L:

$$\phi_k = \frac{\int_{i \in \mathcal{I}_L} \tilde{z}_{ikt} di}{\int_{i \in \mathcal{I}_F} z_{ikt} di}, \tag{13}$$

where

$$\log z_{ikt} = \underbrace{\alpha_k^L + \alpha_k^D}_{=\alpha_k^L} + \beta_{ik} + \gamma_{ikt}^L.$$

This equation is identical to our empirical decomposition of product heterogeneity in sales, as in equation (1). The pricing equilibrium within product group is summarized by the following lemma:

Lemma 1 *Given ϕ, the within-group market share of leader and markup (s, μ) in product group k jointly solves*

$$s = \frac{\mu^{1-\sigma_k}}{\mu^{1-\sigma_k} + \phi^{-1}[\frac{\sigma_k}{\sigma_k - 1}]^{1-\sigma_k}}, \tag{14}$$

$$\mu = \frac{\sigma_k(1-s)+s}{\sigma_k(1-s)+s-1}. \tag{15}$$

Denote the solution as $s_k(\phi)$ and $\mu_k(\phi)$.

The profit for a product-group leader and fringe firms given any (ϕ, Z), as a fraction of aggregate expenditure $\mathbf{C}$, can thus be written as

(Leader Profit)

$$\Pi_{kt}(\phi) = s_k(\phi) \times \frac{1}{\sigma_k\Big(1-s_k(\phi)\Big)+s_k(\phi)} \times \varphi_k, \tag{16}$$

(Fringe Profit)

$$\pi_{kt}(\phi) = \Big(1-s_k(\phi)\Big) \times \frac{1}{\sigma_k} \times \varphi_k. \tag{17}$$

In equation (16), φ_k share of aggregate expenditure $\mathbf{C}$ is spent on group k. Within this expenditure, $s_k(\phi)$ is accrued to the leader. Because the leader has a perceived elasticity $\sigma_k\Big(1-s_k(\phi)\Big)+s_k(\phi)$, the inverse of this perceived elasticity is its profit margin. The product of the three components yields the leader's profit. The total profit that is accrued to the fringe is similarly derived in equation (17), with two differences. (a) The market share accrued to fringe totals to $1-s_k(\phi)$; (b) the profit margin of fringe firms is $\frac{1}{\sigma_k}$.

A leading firm's incentive to engage in product innovation and product reallocation derives from the increase in its profits. We thus ask: what is the marginal increase of leader's profit when it increases its quality gap from the fringe? We find this marginal profit has a closed-form solution in terms of market share:

Lemma 2 *The elasticity of profit with respect a change in quality gap ϕ in group k is*

$$\frac{\partial \log \Pi_k(\phi)}{\partial \log \phi} = 1 - s_k(\phi) \tag{18}$$

Two extreme cases are helpful in understanding the result in equation (2). When the leader has 0 market share, the profit elasticity is 1. At this point, the leader has an infinitesimal share of the market and charges the same markup as the fringe firms. Thus a 1% increase in its market share translates into 1% increase in profits without losing in markups. When the leader has 100% of the market, the profit elasticity is 0. When the leader has taken over the whole market, a marginal increase in its quality gap only cannibalizes its own market shares without changing firm-level profit.

Competitive Search Equilibrium: Product Reallocation Flows

A central focus of our model is the product reallocation flows across different firms. In this section, we characterize the partial equilibrium in the search and matching markets, given (ϕ_k, Z_k) and the gains from reallocation across firms. Specifically, let $u_k(\beta, \gamma)$ be the discounted value of a fringe firm with product quality β and match quality γ, let $v_k(\beta, \gamma)$ be the discounted value of an additional product to the leader, and let $x_k(\beta, \gamma)$ be the discounted loss of an additional product operated by the leader in the calculation of leaders. When positive buying flows into fringe firms occur, the optimal buying decision of a fringe firm with (β, γ) is as follows:

$$\kappa_k^s \varphi_0 = \max_{\tau} \frac{\lambda(\theta)}{\theta} \mathbb{E}_{\Delta} \Big[u(\beta, \gamma_L + \Delta) - \tau \Big]^+, \tag{19}$$

s.t.

$$\lambda(\theta) \mathbb{E}_{\gamma_L} \Big[u(\beta, \gamma_L) - \tau \Big]^+ = U_k^F(\beta, \gamma).$$

Definition 3 *(Equilibrium Fringe-to-Fringe [FtF] Flows) Given the value of fringe firms $u_k(\beta, \gamma)$ and $\frac{\mathbf{w}}{\mathbf{C}}$, the search equilibrium in FtF flow is $\{\theta^{FF}(\beta, \gamma), U_k^F(\beta, \gamma)\}$ such that equation (19) holds.*

It is straightforward to show equation (19) is equivalent to the following problem in terms of solutions:

$$U_k^F(\beta, \gamma) = \max_{\theta} \lambda(\theta) \mathbb{E}_{\gamma_L'} \Big[u(\beta, \gamma + \Delta) - u(\beta, \gamma) \Big]^+ - \theta \kappa_k^s \varphi_0 \tag{20}$$

Equation (20) provides an intuitive interpretation of the reallocation process: due to directed search and the competition on the buyer side, the terms of trade aims to maximize the net benefit of reallocating products from fringe firms to other fringe firms, taking into consideration of the search friction and the cost of search. It is also worth noting that for each (β, γ), equation (20) can be independently solved without referring to the distribution of products across firms. This mechanism is the block recursivity highlighted in Menzio and Shi (2011).

Similarly, due to free entry of fringe buyers, the leader-to-fringe (LtF) flows can be characterized in the same way. For notation simplicity, we define the joint surplus of reallocating a product from fringe to leader as $\Omega(\beta, \gamma_L, \gamma_F)$. The equilibrium in the LtF market is characterized by $\{U^L(\beta, \gamma), \theta_k^{LF}(\beta, \gamma)\}$ that jointly solve the following problem:

$$U_k^L(\beta, \gamma) = \max_{\theta} \lambda(\theta) \mathbb{E}_{\Delta} \Big[-\Omega(\beta, \gamma, \gamma + \Delta) \Big]^+ - \theta \kappa_k^s \varphi_0. \tag{21}$$

The reallocation flow from the fringe to leaders is more complicated because there is no longer free entry on both sides of the market. However, the leader as a buyer faces competitive pressure from fringe buyers. In an equilibrium where both FtF and FtL flows are observed, the leader must offer the same expcted value of selling as the fringe buyers. Thus, the optimal buying decision of the leader is

$$\kappa_k^s \varphi_0 \le \max_{\tau} \frac{\lambda(\theta)}{\theta} \mathbb{E}_{\gamma_L} \Omega(\beta, \gamma', \gamma_F) - \frac{1}{\theta} U_k^F(\beta, \gamma_F). \tag{22}$$

Definition 4 *(Competitive Search Equilibrium) Given the values $u_k(\beta, \gamma)$ and $\Omega(\beta, \gamma_L, \gamma_F)$, the competitive search equilibrium for each (β, γ) is $\{U_k^L(\beta, \gamma), U_k^F(\beta, \gamma)\}$ and $\{\theta^{FF}(\beta, \gamma), \theta^{FL}(\beta, \gamma), \theta^{LF}(\beta, \gamma)\}$ that jointly solve (20), (21), and (22).*

Innovation Decisions

For positive entry by innovation to exist, the expected value of product entry by new firms must equal the entry cost κ_k^e adjusted by the wage-consumption ratio:

$$\mathbb{E}_{\beta} \left[u_k(\beta, 0) \right] = \kappa_k^e \varphi_0. \tag{23}$$

The optimal innovation by the leader requires that the marginal cost of innovation equals the marginal benefit of having an additional new product:

$$\mathbb{E}_{\beta}[v_k(\beta, 0)] = D'(\eta) \varphi_0. \tag{24}$$

Value Functions

For a fringe firm whose product has quality (β, γ), its value follows the Bellman equation:

$$\rho u_k(\beta,\gamma) = e^{\beta+\gamma}\Big(1+\phi_k\Big)\pi_k(\phi_k) + \max_{\theta} \lambda(\theta)\mathbb{E}_{\gamma'_L}\Big[u(\beta,\gamma+\Delta) - u(\beta,\gamma)\Big]^+ - \theta\kappa_k^s\varphi_0. \tag{25}$$

Together with the free entry condition in equation (23), equation (25) implies the equilibrium quality gap ϕ_k within product group k can be solved without referring to any other equilibrium objects. As a result, no transitional path in the quality gap exists. This finding is similar to the ones in Grossman and Helpman (1991b) and Menzio and Shi (2011). It is a result of the following assumptions: (a) The search cost and entry cost are both scaled by the total quality Z_{kt}; (b) the direct search in reallocation market and (c) free entry from both innovation and reallocation.

The value of a product of quality (β, γ) to the group leader is:

$$\rho v_k(\beta,\gamma) = e^{\alpha_k+\beta+\gamma}\Big(1+\phi_k\Big)\Pi_\phi(\phi, Z) + U_k^L(\beta,\gamma), \tag{26}$$

where $\Big(1+\phi_k\Big)\Pi_\phi(\phi, Z)$ is the flow marginal value and $U_k^L(\beta,\gamma)$ is the optimal value of selling to fringe firms, as defined in equation (21). The value of a similar product operated by the fringe to the leader is:

$$(\rho+\delta)x_k(\beta,\gamma) = -e^{\beta+\gamma}\frac{\phi_k(\phi_k+1)}{Z_k}\Pi_\phi(\phi_k) + \lambda(\theta^{FF}(\theta,\gamma))\int_\gamma^\infty \left[x_k(\beta,\gamma') - x_k(\beta,\gamma')\right] dF_\gamma(\gamma') \tag{27}$$

To understand the buying and selling decisions, we define the joint surplus as $\Omega_t(\beta,\gamma_L,\gamma_F) = v_t(\beta,\gamma_L) - x_t(\beta,\gamma_F) - u_t(\beta,\gamma_F)$. $\Omega_t(\beta,\gamma_L,\gamma_F)$ measures the joint surplus from trade when a product is transferred from a fringe firm to a leader, when the product has quality β, the fringe has match quality γ_F, and the leader has match quality γ_L. Correspondingly, $-\Omega_t(\beta,\gamma_L,\gamma_F)$ is the joint surplus of transferring a product from a leader to a fringe firm. With the definition, $\Omega_t(\beta,\gamma_L,\gamma_F)$ must satisfy the following Bellman equation:

$$\begin{aligned}(\rho+\delta)\Omega_k(\beta,\gamma_L,\gamma_F) =& \omega_k(\beta,\gamma_L,\gamma_F) + U_k^L(\beta,\gamma_L) - U_k^F(\beta,\gamma_F) \\ &+\lambda(\theta_k^{FF}(\beta,\gamma^F))\mathbb{E}_{\gamma'_F>\gamma_F}\Big[\Omega_k(\beta,\gamma_L,\gamma'_F) - \Omega_k(\beta,\gamma_L,\gamma_F)\Big],\end{aligned} \tag{28}$$

where

$$\omega(\beta,\gamma_L,\gamma_F) = \left(\frac{e^{\alpha+\gamma_L-\gamma_F}+\phi}{1+\phi}\frac{\Pi}{\frac{\phi}{\sigma_k(1+\phi)}} - 1\right)\frac{\pi}{z_F}e^{\beta+\gamma_F}. \tag{29}$$

To explore the key mechanisms governing transactions, Table 7 focuses on the conditions under which transactions would be *only efficient* or *only strategic*. We take the extreme cases in the parameter set that would induce completely efficient and completely strategic transactions.

Table 7: Two Cases of Gains from Trade

Case	Condition	Gains from Trade	Note
Efficiency:	$\phi_k \to 0$	$\left(e^{\alpha+\gamma_L-\gamma_F}-1\right)\frac{\pi}{z_F}$	Gains from only α or $\gamma_L-\gamma_F$, sales $\uparrow$
Strategy:	$\alpha_k+\gamma_L-\gamma_F=0$	$\left(\frac{\Pi}{\frac{\phi}{\sigma_k(1+\phi)}}-1\right)\frac{\pi}{z_F}e^{\beta+\gamma_F}$	Gains from higher concentration, markup $\uparrow$

We discuss the evolution of the distribution in Appendix C. The main finding from the distribution is that the holdings of both the fringe and leading firms can be expressed in closed form, linking the flows across firms and innovation to aggregate shares.

Growth Rate within Group

Given the detrended distribution of product quality and the innovation and reallocation rates, the growth rate of total quality within product group k is:

$$g_C = \frac{1}{\sigma_k - 1}\Bigg(\underbrace{\eta_l + \eta_f}_{\text{Innovation}} + \underbrace{\Lambda_{FL} + \Lambda_{FF} + \Lambda_{LF}}_{\text{Reallocation}}\Bigg). \tag{30}$$

Within-Group Equilibrium

We define the following group-level equilibrium. With the normalization on innovation and search cost, the equilibrium structure is block recursive: given any parameter set, one can solve the group-level equilibrium independently of the aggregate variables. Because solving general equilibrium involving variable markups and many product groups is costly, this simplification is an important step towards a computationally tractable model for our purposes.

Definition 5 *(Group Equilibrium) A group equilibrium in group k is* $\{\phi_k, g_k\}$ $\{u_k(\beta,\gamma), \theta_k^{FF}(\beta,\gamma), \theta_k^{FL}(\beta,\gamma)\}$, $\{\Omega_{kt}(\beta,\gamma_L,\gamma_F), \theta_k^{LF}(\beta,\gamma), \eta\}$ *such that*

1. ϕ_k, $\{u_k(\beta,\gamma), \theta_k^{FF}(\beta,\gamma)\}$ *solve the free entry condition equations (23) and (20);*

2. Given step 1, $\{\Omega_k(\beta,\gamma_L,\gamma_F), \theta_k^{FL}(\beta,\gamma), \theta_k^{LF}(\beta,\gamma)\}$ *solve equations (22) and (28);*

3. Given step 2, $\{g^F(\beta,), g^L(\beta,\gamma), g_k\}$ *solve equation and ().*

The group-level equilibrium can be solved in isolation from the aggregate variables and in the order from step 1 to step 3.

4.3 Aggregation and Welfare

This section builds a bridge from the outcomes in each market k to the overall efficiency in the economy. The results in this section inform the eventual discussion of aggregate efficiency and the social planner's problem. Given the partial equilibrium within each market, specifically $\{\phi_k, Z_k\}$, the following proposition summarizes the general equilibrium of the economy.

Proposition 6 *Given the product-group equilibria, the general equilibrium of the economy is characterized as follows:*

1. Given $\{\phi_k, z_k\}$, calculate the following productivity, markup, and misallocation indices:

(Productivity)

$$\mathbf{Z}(t) = \mathbf{Z}(0)\exp(\mathbf{g}t), \qquad \mathbf{g} = \int_0^1 \frac{\varphi_k}{\sigma_k - 1} g_k dk; \tag{31}$$

(Markup)

$$\mathbf{M} = \exp\left(\int_0^1 \varphi_k \log M_k dk\right), \tag{32}$$

$$M_k = \left[\frac{\phi_k}{1+\phi_k}\mu_k(\phi_k)^{1-\sigma_k} + \frac{1}{1+\phi_k}\left(\frac{\sigma_k}{\sigma_k - 1}\right)^{1-\sigma_k}\right]^{\frac{1}{1-\sigma_k}}; \tag{33}$$

(Misallocation)

$$\mathbf{A} = \int_0^1 \varphi_k \left(\frac{M_k}{\mathbf{M}}\right)^{-1} dk, \tag{34}$$

$$A_k = \frac{\phi_k}{1+\phi_k}\left(\frac{\mu_k(\phi_k)}{M_k}\right)^{-\sigma_k} + \frac{1}{1+\phi_k}\left(\frac{\sigma_k}{\sigma_k - 1}\right)^{-\sigma_k}. \tag{35}$$

2. The aggregate objects $\mathbf{C}, \mathbf{L}_P, \mathbf{L}_S, \mathbf{L}_D$ are given by

$$\mathbf{C} = \mathbf{Z}\mathbf{A}\mathbf{L}_P \tag{36}$$

$$\mathbf{w} = \frac{\mathbf{Z}}{\mathbf{M}} \tag{37}$$

$$\varphi_0 = \frac{1}{\mathbf{AML_P}}. \tag{38}$$

The demand side of this economy resembles the ones in Edmond et al. (2015); thus, the welfare metric of our model follows the aggregation in the literature.

Definition 7 *(General Equilibrium) A general equilibrium is $(\mathbf{L}_P, \mathbf{w})$ such that:*

1. All product groups are in equilibrium as defined in (5);

2. Given the group equilibrium, the aggregation holds as defined in (6).

More specifically, the discounted utility of the representative household in the steady-state equilibrium is summarized by the aggregate labor supply $\mathbf{L}$, the aggregate labor supply utilized in production $\mathbf{L}^P$, a measure of aggregate mark-up $\mathbf{M}$, and a measure of labor productivity $\mathbf{A}$:

$$\mathcal{W} = \frac{1}{\rho}\left(\log \frac{\mathbf{Z}}{\varphi_0 \mathbf{M}} - \frac{1}{\mathbf{AM}}\right). \tag{39}$$

The aggregation is this section points to key ingredients for our quantitative analysis. Overall, this project focuses on two main components in the development of overall output. First, we look at the productivity of the economy, $\mathbf{Z}$, which correlates positively with social welfare. Then, we evaluate the misallocation, $\mathbf{A}$, and firm markups, $\mathbf{M}$, which tend to correlate negatively with social welfare. This general framework operates in the background of our estimation and quantitative analysis.

5 Estimation

We estimate the model parameters to the empirical moments of product reallocation and innovation. The model is parsimonious in a way that delivers simple objects that enable identification and simultaneous estimation. We primarily explore two methods of estimation.

In the baseline estimation, we assume all product groups are identical in their parameters, and we estimate the model to match the aggregate moments, which we refer to as *homogeneous group estimation*. The homogeneous group estimation provides the natural estimation benchmark. We then estimate the model assuming product groups are heterogeneous in their substitution elasticity, search cost, entry cost, level of leader's productivity advantage, and product quality distribution, which we refer to as *heterogeneous group estimation*. This approach provides a more granular analysis of product-group interaction. In the following paragraphs, we detail the procedure of estimation.

Externally Calibrated Parameters

We set the discount rate to be the annual risk-free rate: $\rho = 0.03$. We calibrate φ_0 to match the aggregate labor income as a share of aggregate consumption. Following the literature, we set the innovation elasticity to be 1. By doing so, we set the innovation cost function to be quadratic. Hottman et al. (2016), estimate the substitution elasticities in a demand system that is similar to our setting. We thus directly take the estimates of UPC-level substitution elasticities from Hottman et al. (2016). In the homogeneous group estimation, we set the substitution elasticity $\sigma = 6.9$, which is the median of UPC-level substitution elasticity.

Table 8: Externally Calibrated Parameters

Name	Symbol	Value	Target
Discount Rate	ρ	0.03	Annual Risk-free Rate
Substitution Elasticity	$\{\sigma_k\}$	Varies by k	Hottman et al. (2016)
Innovation Elasticity	χ	1.0	Akcigit and Kerr (2018)
Disutility of Work	φ_0	5.46×10^{-9}	Wage - PCE Ratio

Notes: Parameters are separately estimated or reference the literature.

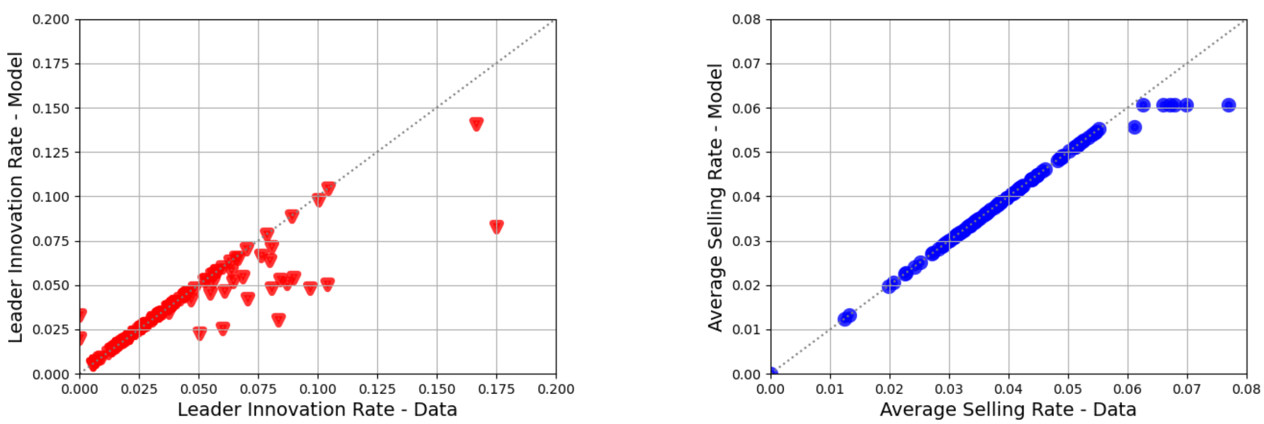
Estimation of Search and Innovation Costs

We estimate the innovation and search costs to match observed innovation rates and reallocation rates. Three cost shifters exist that we allow to vary by product group: the innovation cost shifter d_k, the entry cost κ_k^e, and the search cost κ_k^s. Our model provides a direct link from observed market share and new product creation rate at the group level to these costs.

Table 9: Search Cost and Innovation Cost: Homogeneous Estimation

Name	Symbol	Target	Data (p.p.)	Model (p.p.)
Leader Innovation Cost	d_0	Leader New Product Share	1.02	1.02
Fringe Innovation Cost	κ^e	Median Leader Share	33.6	33.6
Leader Matching Efficiency	m^L	F-t-L Flows	0.59	0.59
Fringe Matching Efficiency	m^F	F-t-F Flows	1.26	1.26

Figure 3: Innovation and Selling Rate: Model v. Data



(a) Innovation Rate

(b) Selling Rate

Notes: The left panel plots the group-level leader innovation rate (red triangle points) from model (y-axis) and from data (x-axis). The right panel plots the group-level average selling (red triangle points) from model (y-axis) and from data (x-axis). In both panel, the grey dotted line is the 45-degree line.

We rely on the optimality conditions for innovation, entry, and acquisition to recover these parameters. First, we note the marginal value of the state variables can be written as functions of quality gap ϕ and growth rate g. Both variables have data counterparts. Specifically, the quality gap ϕ has one-to-one mapping to the observed market share given σ_k; the growth rate g is linked to the new-product innovation rate by the fringe firms. With these two variables, we can directly calculate the marginal value of products to the group leader. For each product group, we find the set of parameters (κ_k^e, κ_k^s) that minimize the distance between data prediction of the leader's innovation rate, average selling rate, and innovation rate of fringe firms.

Estimation of Matching Elasticity

We estimate the innovation and matching elasticity using indirect inference. The targeted moment for this elasticity is the age profile of a product getting transacted. In our model, the difference between the transaction rate for a new product and for a matured product is governed by the difference in marginal benefits as well as the matching elasticity. The difference is in the matching elasticity. In the extreme, if the matching function is inelastic with respect to tightness, no differential in the young-old transaction rates exists. Our estimation yields a matching elasticity of 0.292.

5.1 Comparison of Untargeted Moments

We now compare the predictions of the model regarding the data moments that are not targeted in the estimation procedure.

M&A Premium

The way gains from trade are split between buyers and sellers of product ownership is important for the counterfactual analysis. We thus compare our model's prediction regarding the rent splitting with the ones observed in data. Specifically, we use the premium in mergers and acquisitions as our variable of interest.

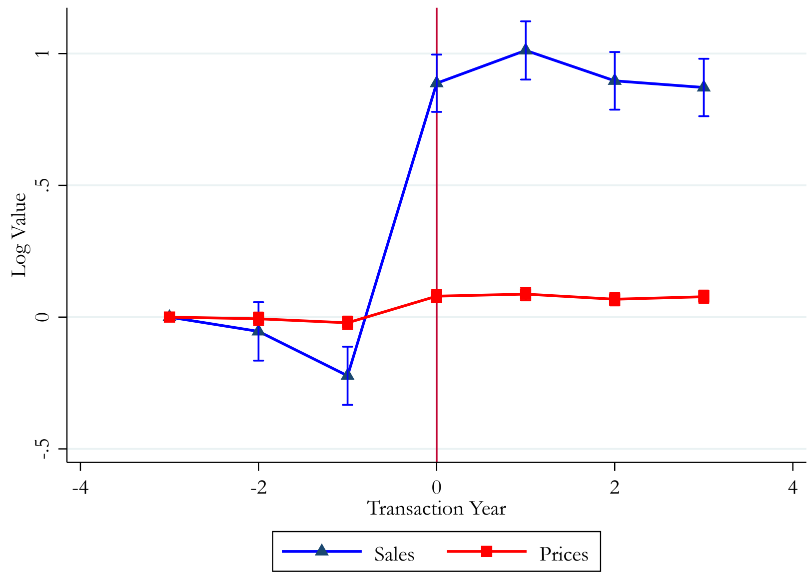
In the data, the premium is calculated as the transfer paid to the merger target as a ratio of accounting book value. The accounting book value from our model is defined as the flow profits adjusted by the real interest rate:

$$U^{book}(\phi, g) = \frac{(1+\beta q)\pi(\phi, g)}{\rho + g}. \tag{40}$$

The transfer from buyer to seller on the balanced growth path is

$$\text{Premium}_{New} = \frac{\gamma \frac{\kappa_s}{\lambda'(\theta_N)} + \kappa_e}{\pi(\phi, g)/(\rho + g)} \tag{41}$$

Figure 4: Event in Nielsen



Note: This figure shows the regression coefficients on a brand transaction across firms in RMS Nielsen.

$$\text{Premium}_{Mature} = \frac{\left(\frac{\gamma}{\lambda'(\theta_M)} + \frac{\gamma}{1-\gamma}\frac{\theta_M}{\rho+g}\right)\kappa_s + q\frac{\pi(\phi,g)}{\rho+g}}{q\pi(\phi,g)/(\rho+g)} \tag{42}$$

Price and Quantity Impact of Transaction

We use log sales and look at the first event of trademarking. Figure 4 plots two separate regressions on one graph with different outcome variables of interest: prices and sales. We plot each coefficient with the clustered standard error.

After the event, both prices and sales move strongly, with sales moving significantly more so. With the increase in prices, the results in Figure 4 provide evidence that after adding additional brands, firms may increase their market power over time. Combining this with the rising rate of transfer from small to large firms can help connect the importance of brand dynamism with the aggregate distribution of markups across firms. Further, the change in markups is a key outcome of our model.

6 Quantitative Analysis

(In Progress)

Table 10: Role of Three Reallocation Margins

	(1)		(2)		(3)	
	$\beta = 0.33$		$\beta = 0.00$		$\beta = 10.00$	
a. varying fixed cost holding search cost constant						
	Leader Share	Welfare	Leader Share	Welfare	Leader Share	Welfare
Baseline	0.35	1.00	0.12	0.93	0.43	1.01
$0.1 \times \kappa_f$	0.14	1.05	0.12	0.95	0.51	1.08
$10 \times \kappa_f$	0.37	0.92	0.13	0.91	0.62	1.02
b. varying search cost holding fixed cost constant						
Baseline	0.35	1.00	0.12	0.93	0.43	1.01
$0.1 \times \kappa_s$	0.42	0.91	0.23	0.95	0.53	1.07
$10 \times \kappa_s$	0.32	0.94	0.12	0.92	0.39	1.04

6.1 Growth: Innovation v.s. Reallocation

6.2 Sources of Concentration: Innovation v.s. Reallocation

6.3 Efficiency

6.4 Policy Analysis: Transaction Tax and Entry Subsidy

Entry

Reallocation

6.5 Reallocation and Concentration

With the estimated model, we ask how different reallocation margins affect concentration in the consumer product market. We first compare the average leaders' market shares in these years.

Role of Entry.- In panel (a) of Table 10, we conduct counterfactuals regarding the role of fixed cost. In the first set of comparisons, we fix both the search cost κ_s and aging rate β, and vary the fixed cost κ_f. In the baseline case, the fixed cost is the estimated value $\kappa_f = 132$. We consider a case where the fixed cost is lowered to one-tenth of the benchmark level, and another where the fixed cost is increased by 10-times the benchmark level. Compared to the benchmark, a lowered fixed cost leads to lower concentration and higher welfare in the steady state. Mirroring the comparative statics in the steady state, as the fixed cost falls. Both leader innovation and fringe firm entry increase. In this specific numerical case, the effect from entry dominates the effect from innovation on concentration. Thus the average concentration of the economy falls. The welfare of the representative household at the steady state increases by 5%. On the contrary, an increase in the fixed cost increases concentration and decreases welfare.

Role of Acquisition.- In panel (b) of Table 10, we conduct counterfactuals regarding the role of search cost. We consider a case where the search cost is lowered to one-tenth of the benchmark level, and another where the search cost is increased by 10-times the benchmark level. Compared to the benchmark, a lowered search cost enables product group leaders to increase their acquisitions. As a result, more products are reallocated toward group leaders, and the concentration of a representative product group increases. This leads to a lower welfare at the steady state for the representative household. On the contrary, a higher search cost decreases concentration. One striking difference between the comparison of varying search cost and varying fixed cost is that the counterfactual on search cost shows non-monotonicity. Both a higher and lower search cost results in lower welfare compared to the baseline case. This monotonicity comes from the entry effect of acquisitions. As the group leaders acquires more often, it becomes more profitable for fringe firms to enter the market, which increases welfare.

Role of Aging.- From column (1) to column (3), we consider how a different aging rate of products leads to different market concentration and welfare incidence. We consider two extreme cases, one where a product always stays young ($\beta = 0$) and another where a product becomes mature almost upon entry ($\beta = 10$). We note that a lower maturity rate of products leads to lower concentration of markets but also leads to lower welfare. A lower aging rate means on average products are of lower quality. Although concentration falls, the representative household is consuming on average lower quality products. Not only does aging itself have implications on welfare, it also interacts with entry cost and search cost. For instance, under the case of immediate maturity, a lower search cost leads to very high concentration, but higher welfare for the representative household. This result highlights that ignoring the life cycle of products can lead to miscalculation of the welfare incidence of different mechanisms that lead to concentration. A higher concentration can be welfare-enhancing or welfare-reducing, depending on how fast new products can catch up to existing ones.

6.6 Policy Analysis: Transaction Tax and Entry Subsidy

In Table 11, we consider a proportional taxation on the search cost. This can be interpreted as tighter antitrust law enforcement, that makes it more costly for the group leaders to set up recruitment teams. We consider a case where the trade of product ownership is completely shut down, and different levels of tax rate and subsidies. We again notice a non-monotonicity of the effect of transaction on welfare. A taxation on transactions decreases welfare, and a mild subsidy increases welfare. However, a big transaction subsidy eventually decreases welfare. These forces can change over time, and we turn to an analysis of the overall trends next.

Table 11: Counterfactual Analysis $\kappa_s(1+\tau)$

	Baseline	No Trade	50% Tax	10% Subsidy	50% Subsidy
Welfare					
Discounted	1.00	0.95	0.96	1.01	0.98
Steady State	1.00	0.94	0.94	1.03	1.01
Decomposition					
Consumption	1.00	0.89	0.89	1.08	1.08
Markup	1.00	0.90	0.91	1.07	1.15
Research Inputs	1.00	0.85	0.85	1.10	1.12

7 Conclusion

Product reallocation plays a central role in sales concentration, firm dynamics, and efficiency. We employ a novel dataset on the universe of brands to unpack this interaction. After illustrating key facts related to the dynamism of firms and products, we develop a model of multi-product firms with pricing power, product innovation and evolution, and product transaction. The model incorporates a matching process between firms and brands to incorporate search into endogenous growth, as products sort to firms.

We direct particular attention to *efficient* versus *strategic* brand acquisitions. Firms may have efficient reasons for engaging in transactions (the acquiring firm is more efficient overall, or a better fit for the product), or strategic reasons, for example, engaging in pricing power. We find both of these mechanisms confirmed in the data. We calibrate the model to understand the interaction between these forces and aggregate concentration and economic growth.

We use the estimated model to study a relevant policy counterfactual: How does taxing brand exchange affect consumer welfare? We find wide heterogeneity in terms of optimal taxation. If policy is coarse, antitrust is ineffective because it will overall decrease the economic efficiency. If policy can be applied by group, there are gains from subsidizing exchange in some groups and taxing exchange in others. We find that, on average, the significant gains in sales from transactions to leaders outweighs the strategic cost.

Empirically, one avenue for further research is to understand the long-run evolution of the market for products. Various works show the rise of niche consumption, yet rising concerns about market concentration. We believe understand the product-firm interaction is essential to understanding this trend. Theoretically, we hope to continue to see a literature develop that relates the search market to the market for innovation and growth, as we feel many advances can be made in this arena.

References

Abrams, David S, Ufuk Akcigit, Gokhan Oz, and Jeremy G Pearce, "The Patent Troll: Benign Middleman or Stick-Up Artist?," Working Paper 25713, National Bureau of Economic Research March 2019.

Acemoglu, Daron, Ufuk Akcigit, Harun Alp, Nicholas Bloom, and William Kerr, "Innovation, Reallocation, and Growth," *American Economic Review*, 2018, *108* (11), 3450–3491.

Akcigit, Ufuk and Sina T Ates, "What Happened to U.S. Business Dynamism?," Working Paper 25756, National Bureau of Economic Research April 2019.

_ and Sina T. Ates, "Ten Facts on Declining Business Dynamism and Lessons from Endogenous Growth Theory," *American Economic Journal: Macroeconomics*, January 2021, *13* (1), 257–98.

_ and William R. Kerr, "Growth through Heterogeneous Innovations," *Journal of Political Economy*, 2018, *126* (4), 1374–1443.

_ , Murat Alp Celik, and Jeremy Greenwood, "Buy, Keep, or Sell: Economic Growth and the Market for Ideas," *Econometrica*, 2016, *84* (3), 943–984.

Altonji, Joseph G. and Robert A. Shakotko, "Do Wages Rise with Job Seniority?," *The Review of Economic Studies*, 1987, *54* (3), 437–459.

Argente, David, Doireann Fitzgerald, Sara Moreira, and Anthony Priolo, "How Do Firms Build Market Share?," Technical Report 2021. SSRN Working Paper.

_ , Munseob Lee, and Sara Moreira, "Innovation and Product Reallocation in the Great Recession," *Journal of Monetary Economics*, 2018, *93*, 1–20. Carnegie-Rochester-NYU Conference on Public Policy held at the Stern School of Business at New York University.

_ , _ , and _ , "The Life Cycle of Products: Evidence and Implications," Technical Report 2020.

_ , Salome Baslandze, Douglas Hanley, and Sara Moreira, "Patents to Products: Product Innovation and Firm Dynamics," *SSRN Working Paper*, 2020.

Atkeson, Andrew and Ariel Burstein, "Pricing-to-Market, Trade Costs, and International Relative Prices," *American Economic Review*, December 2008, *98* (5), 1998–2031.

_ and _ , "Innovation, Firm Dynamics, and International Trade," *Journal of Political Economy*, 2010, *118* (3), 433–484.

Autor, David, David Dorn, Lawrence F Katz, Christina Patterson, and John Van Reenen, "The Fall of the Labor Share and the Rise of Superstar Firms*," *The Quarterly Journal of Economics*, 02 2020, *135* (2), 645–709.

Bain, Joe S., *Barriers to New Competition: Their Character and Consequences in Manufacturing Industries*, Harvard University Press, 1956.

Barkai, Simcha, "Declining Labor and Capital Shares," *The Journal of Finance*, 2020, *75* (5), 2421–2463.

Becker, Gary S. and Kevin M. Murphy, "A Simple Theory of Advertising as a Good or Bad*," *The Quarterly Journal of Economics*, 11 1993, *108* (4), 941–964.

Berger, David W, Kyle F Herkenhoff, and Simon Mongey, "Labor Market Power," Working Paper 25719, National Bureau of Economic Research March 2019.

Berry, Steven, Martin Gaynor, and Fiona Scott Morton, "Do Increasing Markups Matter? Lessons from Empirical Industrial Organization," *Journal of Economic Perspectives*, August 2019, *33* (3), 44–68.

Berry, Steven T., "Estimation of a Model of Entry in the Airline Industry," *Econometrica*, 1992, *60* (4), 889–917.

Bils, Mark and Peter J. Klenow, "The Acceleration of Variety Growth," *American Economic Review*, May 2001, *91* (2), 274–280.

Bresnahan, Timothy F., "Chapter 17 Empirical studies of industries with market power," in "in," Vol. 2 of *Handbook of Industrial Organization*, Elsevier, 1989, pp. 1011–1057.

_ **and Peter C. Reiss**, "Entry and Competition in Concentrated Markets," *Journal of Political Economy*, 1991, *99* (5), 977–1009.

Broda, Christian and David E. Weinstein, "Globalization and the Gains From Variety*," *The Quarterly Journal of Economics*, 05 2006, *121* (2), 541–585.

Bronnenberg, Bart J., Jean-Pierre Dubé, and Matthew Gentzkow, "The Evolution of Brand Preferences: Evidence from Consumer Migration," *American Economic Review*, May 2012, *102* (6), 2472–2508.

_ **, Jean-Pierre Dube, and Sridhar Moorthy**, "Chapter 6 - The economics of brands and branding," in Jean-Pierre Dube and Peter E. Rossi, eds., *Handbook of the Economics of Marketing, Volume 1*, Vol. 1 of *Handbook of the Economics of Marketing*, North-Holland, 2019, pp. 291 – 358.

_ **, Sanjay K. Dhar, and Jean-Pierre Dubé**, "Brand History, Geography, and the Persistence of Brand Shares," *Journal of Political Economy*, 2009, *117* (1), 87–115.

Brown, GH, "Brand Loyalty: Fact or Fiction," *The Trademark Reporter*, 1953, *43* (1), 251–258.

Castaldi, Carolina, "All the great things you can do with trademark data: Taking stock and looking ahead," *Strategic Organization*, 2019, p. 1476127019847835.

Cavenaile, Laurent, Murat Alp Celik, and Xu Tian, "The Dynamic Effects of Antitrust Policy on Growth and Welfare," *Journal of Monetary Economics*, 2021, *121*, 42–59.

Cunningham, Colleen, Florian Ederer, and Song Ma, "Killer Acquisitions," *Journal of Political Economy*, 2021, *129* (3), 649–702.

David, Joel M, "The Aggregate Implications of Mergers and Acquisitions," *The Review of Economic Studies*, 11 2020, *88* (4), 1796–1830.

Davis, Steven J. and John Haltiwanger, "Gross Job Creation, Gross Job Destruction, and Employment Reallocation*," *The Quarterly Journal of Economics*, 08 1992, *107* (3), 819–863.

Davis, Steven J and John Haltiwanger, "Labor Market Fluidity and Economic Performance," Working Paper 20479, National Bureau of Economic Research September 2014.

Davis, Steven, John Haltiwanger, and Scott Schuh, *Job Creation and Destruction*, Cambridge, MA: MIT Press, 1996.

Deaton, Angus, *The analysis of household surveys: a microeconometric approach to development policy*, World Bank Publications, 1997.

Decker, Ryan A., John Haltiwanger, Ron S. Jarmin, and Javier Miranda, "Changing Business Dynamism and Productivity: Shocks versus Responsiveness," *American Economic Review*, December 2020, *110* (12), 3952–90.

Decker, Ryan, John Haltiwanger, Ron Jarmin, and Javier Miranda, "The Role of Entrepreneurship in US Job Creation and Economic Dynamism," *Journal of Economic Perspectives*, September 2014, *28* (3), 3–24.

Dhingra, Swati, "Trading Away Wide Brands for Cheap Brands," *American Economic Review*, October 2013, *103* (6), 2554–84.

Dinlersoz, Emin M, Nathan Goldschlag, Amanda Myers, and Nikolas Zolas, "An Anatomy of U.S. Firms Seeking Trademark Registration," Working Paper 25038, National Bureau of Economic Research September 2018.

Dubé, Jean-Pierre, Günter J. Hitsch, and Peter E. Rossi, "State dependence and alternative explanations for consumer inertia," *The RAND Journal of Economics*, 2010, *41* (3), 417–445.

Eaton, Jonathan and Samuel Kortum, "Trade in ideas Patenting and productivity in the OECD," *Journal of International Economics*, 1996, *40* (3), 251–278. Symposium on Growth and International Trade: Empirical Studies.

Edmond, Chris, Virgiliu Midrigan, and Daniel Yi Xu, "Competition, Markups, and the Gains from International Trade," *American Economic Review*, October 2015, *105* (10), 3183–3221.

_ , _ , and _ , "How Costly Are Markups?," Working Paper 24800, National Bureau of Economic Research July 2018.

Einav, Liran, Peter Klenow, Jonathan D Levin, and Murciano-Goroff Ravid, "Customers and Retail Growth," 2021. Working Paper.

Fitzgerald, Doireann, Stefanie Haller, and Yaniv Yedid-Levi, "How Exporters Grow," Working Paper 21935, National Bureau of Economic Research January 2016.

Fons-Rosen, Christian, Pau Roldan-Blanco, and Tom Schmitz, "The Aggregate Effects of Acquisitions on Innovation and Economic Growth," *SSRN Working Paper*, 2021.

Foster, Lucia, John Haltiwanger, and Chad Syverson, "The Slow Growth of New Plants: Learning about Demand?," *Economica*, 2016, *83* (329), 91–129.

Garcia-Macia, Daniel, Chang-Tai Hsieh, and Peter J. Klenow, "How Destructive Is Innovation?," *Econometrica*, 2019, *87* (5), 1507–1541.

Gourio, François and Leena Rudanko, "Customer Capital," *The Review of Economic Studies*, 03 2014, *81* (3), 1102–1136.

Graham, Stuart J.H., Galen Hancock, Alan C. Marco, and Amanda Fila Myers, "The USPTO Trademark Case Files Dataset: Descriptions, Lessons, and Insights," *Journal of Economics & Management Strategy*, 2013, *22* (4), 669–705.

Grossman, Gene and Elhanan Helpman, *Innovation and Growth in the Global Economy*, 1 ed., Vol. 1, The MIT Press, 1991.

Grossman, Gene M and Elhanan Helpman, "Quality Ladders in the Theory of Growth," *Review of Economic Studies*, 1991, *58* (1), 43–61.

Heath, Davidson and Christopher Mace, "The Strategic Effects of Trademark Protection," *The Review of Financial Studies*, 08 2019, *33* (4), 1848–1877.

Hottman, Colin J., Stephen J. Redding, and David E. Weinstein, " Quantifying the Sources of Firm Heterogeneity *," *The Quarterly Journal of Economics*, 03 2016, *131* (3), 1291–1364.

Jaravel, Xavier, "The Unequal Gains from Product Innovations: Evidence from the U.S. Retail Sector*," *The Quarterly Journal of Economics*, 12 2018, *134* (2), 715–783.

Kost, Kyle, Jeremy Pearce, and Liangjie Wu, "Market Power through the Lens of Trademarks," *Working Paper*, 2019.

Lentz, Rasmus and Dale Mortensen, "An Empirical Model of Growth through Product Innovation," *Econometrica*, 2008, *76* (6), 1317–1373.

Liu, Ernest, Atif Mian, and Amir Sufi, "Low Interest Rates, Market Power, and Productivity Growth," 2019. Becker Friedman Institute Working Papers Series 2019-09.

Loecker, Jan De, Jan Eeckhout, and Gabriel Unger, "The Rise of Market Power and the Macroeconomic Implications*," *The Quarterly Journal of Economics*, 01 2020, *135* (2), 561–644.

Melitz, Marc J. and Gianmarco I. P. Ottaviano, "Market Size, Trade, and Productivity," *The Review of Economic Studies*, 01 2008, *75* (1), 295–316.

Menzio, Guido and Shouyong Shi, "Efficient Search on the Job and the Business Cycle," *Journal of Political Economy*, 2011, *119* (3), 468–510.

Nelson, Phillip, "Information and Consumer Behavior," *Journal of Political Economy*, 1970, *78* (2), 311–329.

_ , "Advertising as Information," *Journal of Political Economy*, 1974, *82* (4), 729–754.

Peters, Michael, "Heterogeneous Markups, Growth, and Endogenous Misallocation," *Econometrica*, 2020, *88* (5), 2037–2073.

Romer, Paul M, "Endogenous Technological Change," *Journal of Political Economy*, 1990, *98* (5 pt 2).

Schautschick, Philipp and Christine Greenhalgh, "Empirical studies of trade marks–The existing economic literature," *Economics of Innovation and New Technology*, 2016, *25* (4), 358–390.

Shi, Liyan and Hugo A. Hopenhayn, "Knowledge Creation and Diffusion with Limited Appropriation," 2017 Meeting Papers 1501, Society for Economic Dynamics 2017.

Sutton, John, *Sunk Costs and Market Structure: Price Competition, Advertising, and the Evolution of Concentration*, MIT Press, 1991.

Syverson, Chad, "Market Structure and Productivity: A Concrete Example," *Journal of Political Economy*, 2004, *112* (6), 1181–1222.

_ , "Product Substitutability and Productivity Dispersion," *The Review of Economics and Statistics*, 2004, *86* (2), 534–550.

Traina, James, "Is Aggregate Market Power Increasing? Production Trends Using Financial Statements," 2018. Stigler Center New Working Paper Series No. 17, University of Chicago Booth School of Business.

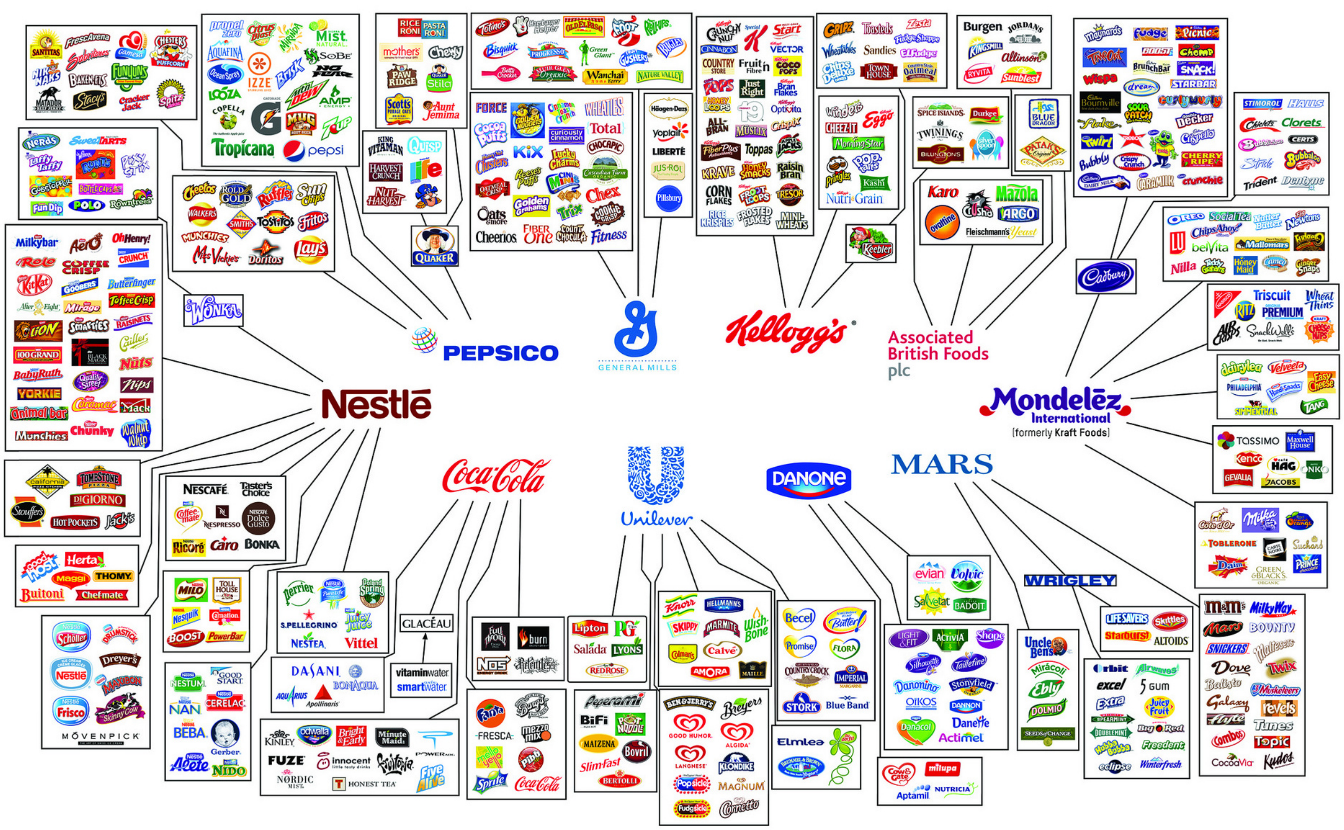
Appendix

A Data Appendix

A.1 Example of Product Market Concentration

As we discuss in the main text, product markets are dominated by large firms. We see this clearly in the following illustration, where many brands that individuals associate with only the brand are held by larger parent firms that aggregate brands.

Figure A1: Brands at Major Firms



This general pattern is true across an array of industries, but the empirical section of this paper directs our attention to the Consumer Packaged Goods (CPG) industry. As such, we turn to a specific example of a large firm in the CPG space.

A.2 Example of Brand-Building: Procter & Gamble

Figure A2 illustrates how many firms that rely specifically on their brand relationships are held by P&G.

Figure A3 shows how P&G's trademark holdings have grown over time. Much of this trademark increase has come through poaching trademarks from other firms or purchasing other firms.

Figure A2: Example of P&G Brands

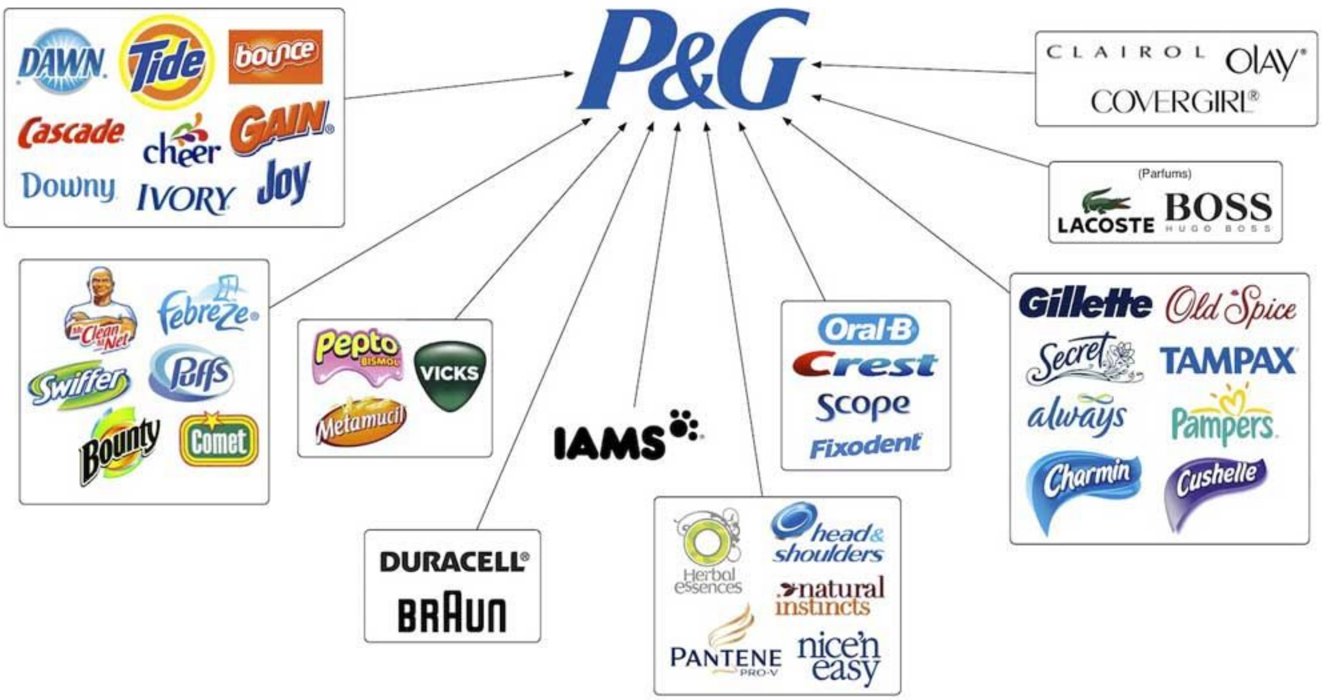


Figure A3: Tracing the brands of P&G over time

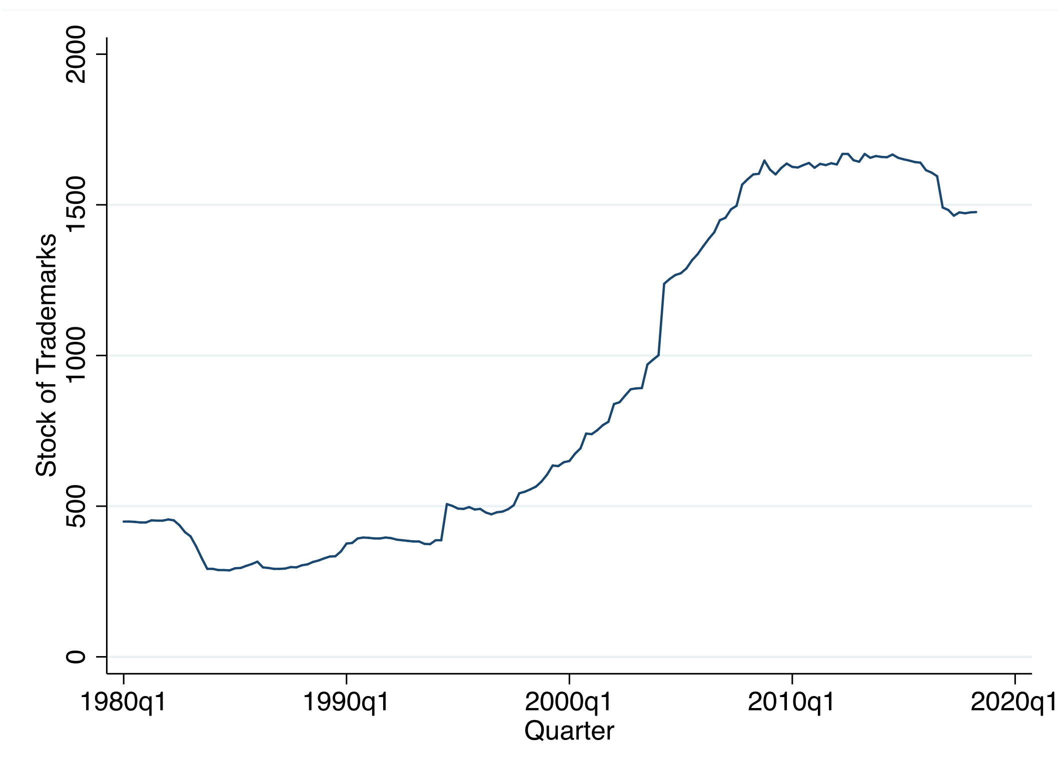
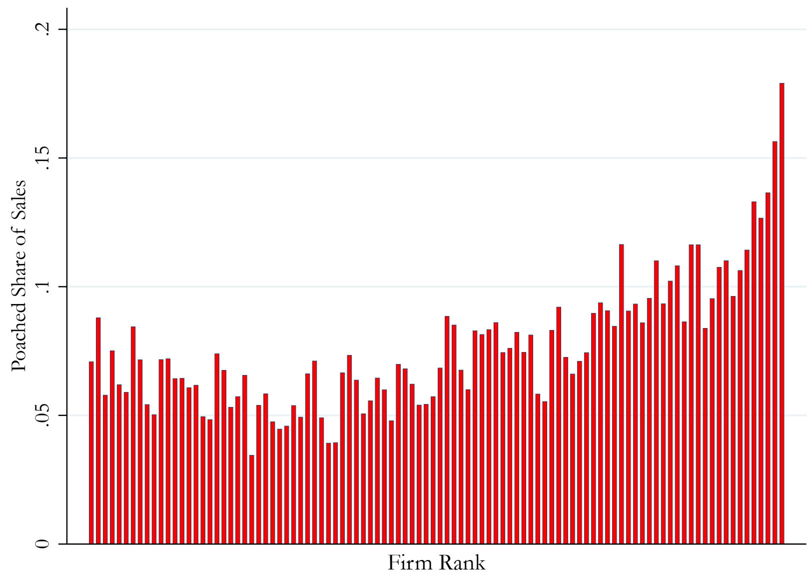


Figure B4: Contribution of Buying to Sales Share



B Empirical Appendix

B.1 Firm Concentration and Brand Buying

In Figure 1, we showed how buying of brands contributes significantly to large firms market share. Figure B4 shows this pattern with respect to sales in Nielsen Scanner Data. We plot the share of sales from bought brands against the percentile (running from 1-100) of the firm size in sales.

We find that the highest-selling firms have almost 4-times as much poached share of sales that a median firm, indicating that the pattern we find in the Trademark data on its own is consistent in the sales-share data.

In Table 4, we explored the concentration due to incumbent products versus entrant products. However, incumbent variation may be driven by many forces outside of the life cycle of the product. In Table B1, we evaluate the contribution for each firm attempting to fit the product-level predicted values (to eliminate the noise), and split the firms into three groups. Again we apply Equation (2).

We find a similar pattern in Tables 4 and B1. When we take fitted values from product-level age regressions, as discussed in the next section, the general pattern stays the same. We note that each force has a non-negligible contribution to the distribution of market shares.

B.2 Product-Level Analysis

Products are both a significant source of firm concentration (Hottman et al., 2016), yet highly dynamic (Argente et al., 2020a). This affects the overall sales as in β_i from Equation (1). The concentration implies a rich heterogeneity, but the dynamic nature implies that heterogeneity changes over time. The

Table B1: Sources of Reallocation

	(1) Entry	(2) Fitted Incumbents	(3) Acquisition
Top 1%	0.041*	0.063*	0.12*
Average	0.0013	-0.013	-0.001
Top 1-25 %	0.065*	0.14*	0.10*
Average	0.0002	-0.018	-0.004
The Rest	0.14*	0.077*	0.19*
Average	-0.012	0.078	-0.029
Overall	0.13*	0.084*	0.18*
Average	-0.009	0.053	-0.022

p-values in parentheses
* $p < 0.001$

Notes: Market share reallocation measures across different firm types, following Equation (2). Source: RMS Nielsen

change in products can come from development of a product line or transactions of products from worse to better firms. Our goal in this section is to isolate the product element of the life cycle and show how even separate from the firms that hold them products exhibit rich life cycles. This general point has been shown before (e.g. Argente et al., 2020a, 2021), but by integrating with USPTO Trademark data we are able to examine the longer brand life cycle and control for the transactions across firms.

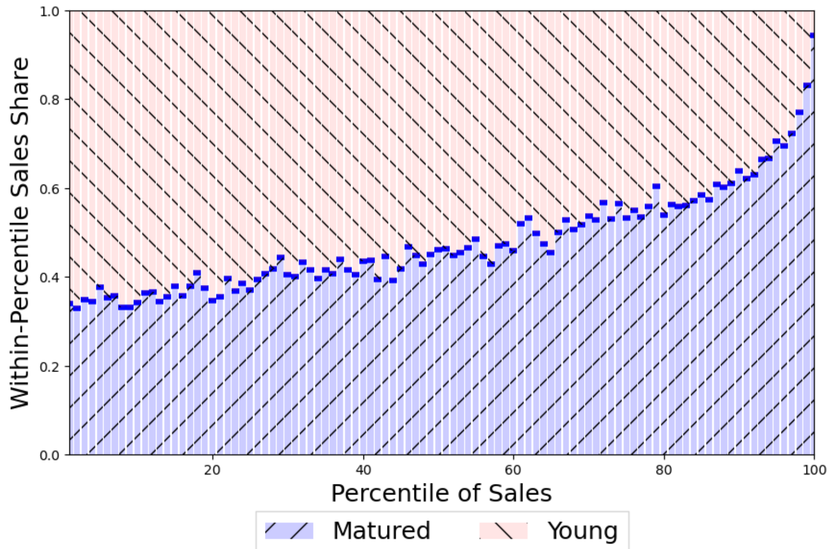
Some products charge to dominance quickly, others rise gradually but maintain leadership, whereas others survive but remain in obscurity. Yet all brands must build consumer capital to build market share. We direct our attention to brand *age* is as an ingredient to product market shares. We first focus on a snapshot of the distribution of sales by age, and then turn to an analysis of the life cycle to understand the more granular dynamics.

Products evolve over their life cycle. Gourio and Rudanko (2014) and Foster et al. (2016), among many others, have noted that customer capital is not built in a day. By looking at trademark data and Nielsen data, one can observe the importance of senior brands. Figure B5 takes data from 2016. We plot the brand percentile in terms of overall sales on the x-axis. On the y-axis, we plot the share of sales in this group that belongs to brands older than 10 years and brands younger than 10 years.[7]

For brands created in 2006 and earlier, they maintain large sales share into the future. By 2016, those brands are still dominant in the top 1% of brands. Within the top 1% of brands, brands created before 2006 make up 92% of sales. Overall, old brands make up over 70% of sales, but only about 1/3rd of

[7]We omit brands with less than $1000 in sales over an entire year, to have only brands that at least have a product line.

Figure B5: The Power of Mature Products



Note: This figure shows the sales share within a percentile bin of products, split by those born before 2006 ("Matured") and after 2006 ("Young").

Source: RMS Nielsen Scanner Data.

products. For the median brand in terms of sales, older brands make up less than half (38%) of total sales.

The dominance of mature brands could come from two forces. First, if few brands achieve such large sales, there may be a selection process. Young brands have less of a chance than old brands to have high consumer capital, as the brands that survive to maturity must have a high quality draw. The composition only selects for the best. Second, brands could increase their sales over the life cycle such that only mature brands have significant sales share. We aim to understand this by linking a brand to its specific age.

We now turn to study the product life cycle, where we leverage age data from trademarks registrations from the USPTO and sales data from RMS Nielsen. We use a method to study the life cycle of brands, following similar work from Altonji and Shakotko (1987), Fitzgerald et al. (2016), and Argente et al. (2018). Figure B6 plots a regression that illustrates the nature of the product lifecycle in sales. This graph plots the coefficients to the following regression:

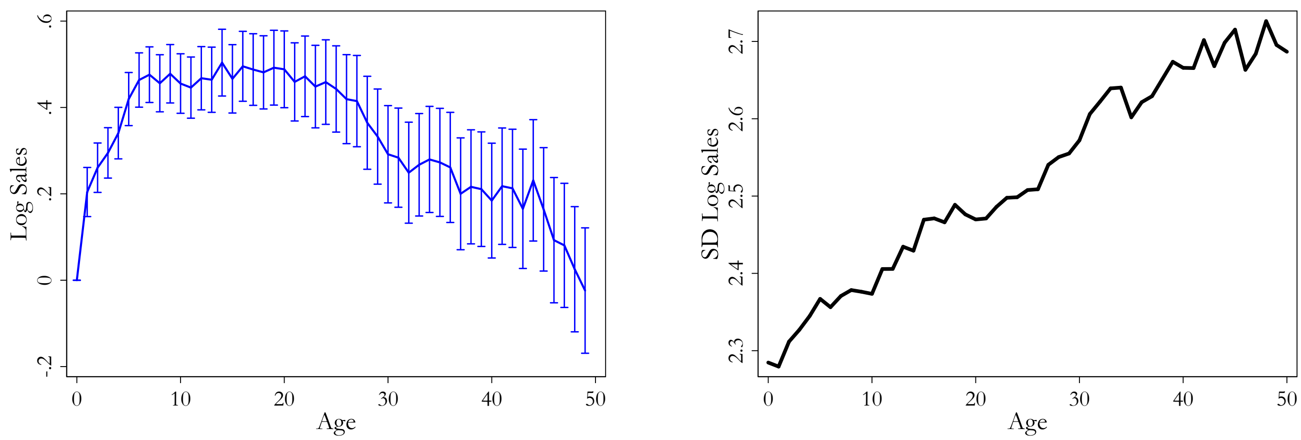
$$\log y_{it} = \alpha + \sum_{a=1}^{50} \beta_a D_a + \gamma_b + \lambda_t + \theta_i + \epsilon_{it} \tag{43}$$

The regression in Equation (43) considers the sales of brand i at time t, $\log y_{it}$ as a function of a constant (α), brand age indicators from 1 to 50, D_a, and fixed effects for cohort (γ_b) and time (λ_t).[8] The θ_i indicates either a brand fixed-effect. Figure B6a plots the regressions by age coefficient β_a. The standard error bars indicate the 95-percent confidence interval for the point estimates for the regression with robust standard errors.

[8]Given the linear relationship between age, time, and cohort, we follow a method developed by Deaton (1997) to correct for this issue. The normalization orthogonalizes the cohort trends such that growth components move with age and time effects.

Figure B6b tracks the within-age dispersion. We note that the dispersion of log sales is increasing in age, which is relevant in our model. As brands mature, the dispersion of consumer goodwill varies more drastically.

Figure B6: Product-Level Dynamics



(a) Log Sales × Age Regression

(b) Log Sales Dispersion × Age

Notes: Source: USPTO Trademark Data and RMS Nielsen

There are two main takeaways from Figure B6. First, we see brands themselves exhibit an inverted-U pattern in sales over their life cycle. This is consistent with Argente et al. (2018, 2021), yet in brands the product life cycle is much longer and peaks far later. Second, brands exhibit much larger variance over time, indicating that the life cycle itself drives not only level differences but within age heterogeneity. Even at age 0, products show large sales dispersion, so this heterogeneity does not simply emerge through the life cycle.

In Appendix B, we explore these differences in greater detail. The main point we make here is that when the unit of analysis is the consumer brand goodwill (e.g. the level of a trademark), and we extend the panel beyond the limited Nielsen age distribution the goodwill peaks later in the life cycle. Both of these ingredients lead to a later life cycle peak in our case. The rich product-level dynamics in sales can also be seen with product ownership exchange across firms. We turn to this next.

B.3 Literature Benchmark: Product Life Cycle

Here we compare our benchmark against current product life cycle benchmarks in the literature. Recent work has focused on the life cycle of products applying Nielsen Scanner Data. This work is able to identify new products and brands and document their life cycle patterns. However, it is not able to link brands and products to their history, and is thus unable to speak to the longer time horizon of persistent brands. We perform similar life cycle regressions to the main text and compare them to a relevant current paper in the literature:

$$\log y_{it} = \alpha + \sum_{a=0}^{4} \beta_a D_a + \gamma_b + \lambda_t + \epsilon_{it} \tag{44}$$

Where the coefficients of interest are the coefficients on age (β_a) with controls for cohort and time effects (and an adjustment on cohort from Deaton, 1997). Table B2 engages in the same specification as Argente et al. (2018) in the UPC data (panels 1 and 2) and Trademark merged data (panels 3 and 4) respectively.

Table B2: Log Sales, by Nielsen and Trademark Age

	(1) Log Sales	(2) Log Sales	(3) Log Sales	(4) Log Sales
Age 1	0.939***	1.095***	0.917***	0.953***
	(0.00841)	(0.0237)	(0.132)	(0.113)
Age 2	0.857***	1.159***	1.019***	1.060***
	(0.00876)	(0.0246)	(0.140)	(0.118)
Age 3	0.632***	1.016***	0.834***	0.832***
	(0.00914)	(0.0259)	(0.145)	(0.123)
Age 4	0.169***	0.644***	0.412*	0.488***
	(0.00995)	(0.0284)	(0.160)	(0.135)
N	668993	89203	3402	4136
R^2	0.138	0.179	0.256	0.050
Variation	UPC	Brand-Group	TM Brand	TM Brand-Group

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note that while at the level of brands and trademarks there are significantly fewer observations, the same general pattern holds. This indicates how age is picking up something similar in our context, yet due to the broader horizon of historical data we are able to connect brands to their histories, indicating a significantly longer brand life cycle than found in Argente et al. (2018).

We also show here similar general trends as in the main text when we evaluate the life cycle of products, controlling for brand-firm-group level, with robust standard errors.

When we go in the other direction and only control for brand, we find similar effects as in Figure B8.

Figure B7: Life Cycle Regressions

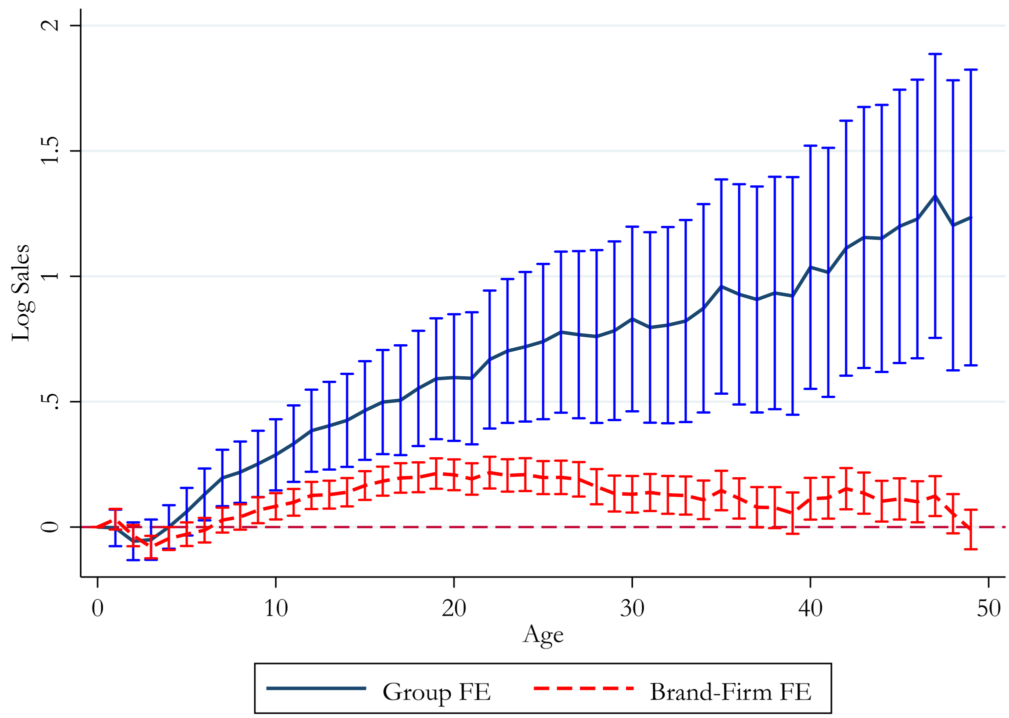
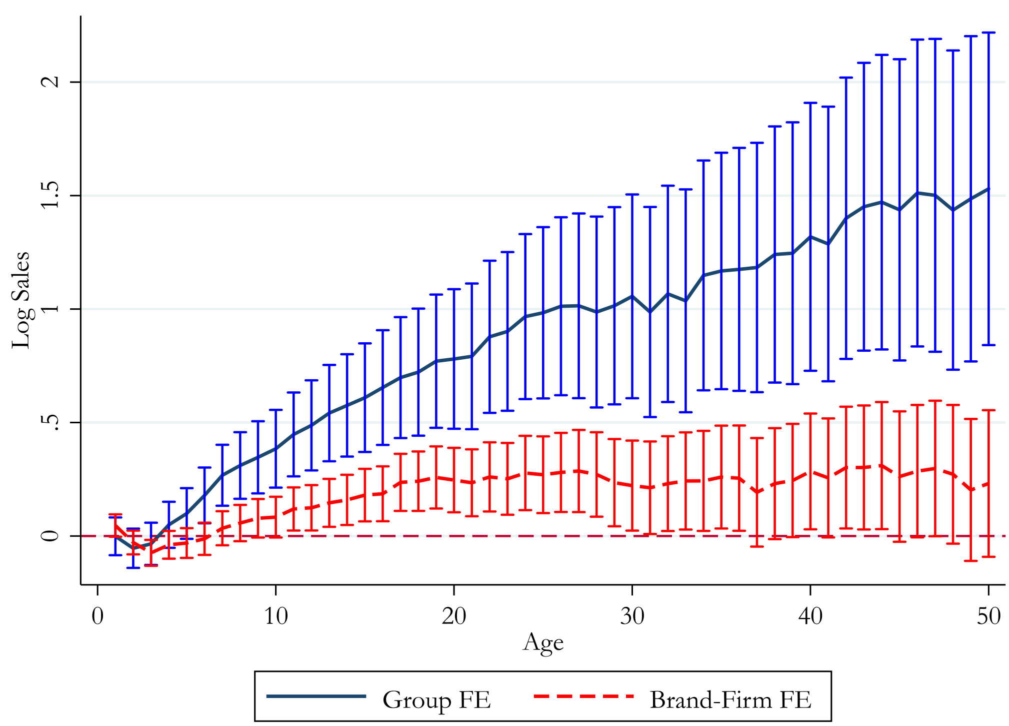


Figure B8: Life Cycle Regressions



C Model Appendix

C.1 Details on the Distribution

Distribution

Denote $n^L(\beta, \gamma)$ the density of products with quality (β, γ) currently operated by the leader and $n^F(\beta, \gamma)$ the density of products operated by fringe firms, both normalized by the total quality Z. Denote the growth rate of quality Z by g_Z. In the steady state, they have to solve the following equations:

$$n^L(\beta, \gamma) = \frac{\underbrace{\int_{\Omega(\beta,\gamma,\gamma_F)\geq 0} \frac{\lambda(\theta^{FL}(\beta, \gamma_F))}{\theta^{FL}(\beta, \gamma_F))} d\gamma_F}_{\text{Fringe-to-Leader Flow}} + \underbrace{\eta_L f_\beta(\beta)}_{\text{Leader Innovation}}}{\underbrace{g_Z}_{\text{Detrending}} + \underbrace{\lambda(\theta^{LF}(\beta, \gamma)) \int_{\Omega(\beta,\gamma,\gamma_F)<0} dF(\gamma_F)}_{\text{Leader-to-Fringe Flow}}} f_\gamma(\gamma). \tag{45}$$

Given $n^L(\beta,)$, the density for fringe firms can be accordingly solved:

$$n^F(\beta, \gamma) = \frac{\left[\underbrace{\int_{\Omega(\beta,\gamma_L,\gamma)<0} \lambda(\theta^{LF}(\beta, \gamma_L)) n^L \beta, \gamma_L d\gamma_L}_{\text{Leader-to-Fringe Flow}} + \underbrace{\eta_F f_\beta(\beta)}_{\text{Fringe Innovation}}\right] f_\gamma(\gamma) - \underbrace{\int_{\Omega(\beta,\gamma,\gamma_F)\geq 0} \frac{\lambda(\theta^{FL}(\beta, \gamma_F))}{\theta^{FL}(\beta, \gamma_F))} d\gamma_F}_{\text{Fringe-to-Leader Flow}}}{g_Z + \lambda(\theta^{FF}(\beta,)) \int_{\gamma' > \gamma} dF(\gamma')} \tag{46}$$

C.2 Estimation Details

Due to the block recursive structure of the equilibrium, the estimation procedure is done through the following steps in order:

1. We directly calibrate the substitution elasticity σ_k to the ones estimated in the literature;
2. Given the observed market share of leaders s_k and the substitution elasticity σ_k, we directly back out the quality gap ϕ_k from equation (XXX):
3. Given ϕ_k, we jointly estimate $\{\kappa_s, \kappa_e, d_k\}$ that minimize the distance between the observed reallocation rate, ϕ_k, and leader's innovation rate, as well as fringe firms' innovation rate.