Product Reallocation and Market Concentration

Jeremy G. Pearce* Liangjie Wu† ‡

October 10, 2022

Abstract

This paper studies the macroeconomic implications of firm branding activity. We document how firms build market share through both creating new brands and buying existing ones, with large firms relying more on buying brands than small firms. When a brand is reallocated from small to large firms, it leads to an increase in both the sales and prices of the brand. To interpret these findings and quantify the implications of brand reallocation on efficiency, we introduce an endogenous growth model where both the growth rate and market concentration are determined by brand innovation and reallocation activity. In net, brand reallocation increases efficiency.

Key Words: Market Concentration, Product Innovation, Firm Dynamics, Endogenous Growth, Reallocation, Mergers & Acquisitions, Brands

JEL Code: D22, D43, L11, L13, L22, O31, O34

*Department of Economics, University of Chicago, jgpearce@uchicago.edu

†EIEF, liangjie.wu@eief.it;

‡Researcher(s) own analyses calculated (or derived) based in part on data from Nielsen Consumer LLC and marketing databases provided through the NielsenIQ Datasets at the Kilts Center for Marketing Data Center at The University of Chicago Booth School of Business.; The conclusions drawn from the NielsenIQ data are those of the researcher(s) and do not reflect the views of NielsenIQ. NielsenIQ is not responsible for, had no role in, and was not involved in analyzing and preparing the results reported herein.

1 Introduction

Brands are an essential intangible asset for firms. According to estimations of brand value, the top 100 brands in the US economy were worth 4.14 Trillion USD in 2021, and the relative value of brands to traditional capital has been growing over time (Bronnenberg et al., 2022). Brands allow firms to differentiate their products from competitors, and naturally affect the pricing power a firm has on its branded products. Firms build brand capital through both introducing new brands and the transfer, or reallocation, of existing brands from other firms. Despite rising interest in product market concentration and intangible assets, the macroeconomic implications of brands is less understood.

This paper links together the empirical patterns of brand-ownership allocation to aggregate concentration and efficiency. Utilizing a linked dataset of the universe of registered trademarks from the US Patent and Trademark Office (UPSTO) and price and quantity-level data at the product and brand-level from RMS Nielsen retail data, we document the following facts regarding firms' branding activities:

Fact 1 *Large firms disproportionately have more brands and more market share, and build their market share through brand reallocation more than brand creation.*

Fact 2 *Better and more mature brands are more likely to be reallocated across firms.*

Fact 3 *When a brand is reallocated across firms, sales and prices both increase.*

The above facts motivate questions about the interaction between firm dynamics and firm branding activity. How do different branding activities, such as brand creation and reallocation, affect firm productivity and the distribution of market shares across firms? How do they affect the efficiency of the aggregate economy?

To answer these questions, we introduce an endogenous growth model with brand creation and brand reallocation, allowing for both welfare gains through love of variety and welfare losses due to markups. In the model, consumers spend on imperfectly substitutable products distinguished by their brand. The imperfect substitution among products mirrors the role of brands in reality: all firms brand their products to some degree, and firms apply early in their life cycle for trademarks to protect their brand capital. In our model, each product group category has a multi-product leader and endogenous measure of single-product fringe firms, mirroring the disproportionate size of large firms in the market. The multi-product leader charges a higher markup than fringe firms because, as it holds many brands, it internalizes its impact on group-level prices.

Firms build their market shares through two types of activities. They can either introduce new brands through *product innovation*, or search to acquire existing brands from other firms, *product reallocation*. Firms seek to maximize their bilateral gains from trade when making search decisions. In our model, the gains from trade originate from two sources. First, the buying firm can be more efficient in operating a brand, which we refer to as *efficient reallocation*. Second, the buying firm may use the brand to exert a higher markup, which we refer to as *strategic reallocation*.

The efficiency of the decentralized equilibrium depends on both the number of products created and the level and distribution of markups across firms. Firms do not fully internalize their activities on both consumer love of variety and distortion from markups. These externalities imply that the economy faces a *efficiency-markup tradeoff*. With efficient exchange, the buying firm is better equipped to sell and distribute the product and hold ownership, either due to firm-specific advantage or firm-product fit. In this case, the interests of the firms and the aggregate economy align. On the other hand, strategic exchange occurs because the buying firm can exert market power due to limited consumer substitution across products. Here, the firm's interest in consolidation may not align with overall economic efficiency. While we classify these two types of transactions as benchmarks, many product exchanges can (and do) exhibit evidence of both efficient and strategic components.

We focus on *products* in our data, where we define a product as the consumer product good associated with a trademarked brand name (in USPTO) in a given product group category, e.g. "Cheerios" in "Cereal", as defined in RMS Nielsen. Most products are associated with a core trademark that the initial owning firm applies for upon entry. The USPTO defines a trademark as the following: "A trademark can be any word, phrase, symbol, design, or a combination of these things that identifies your goods or services. It's how customers recognize you in the marketplace and distinguish you from your competitors." Trademarks allow firms to protect the consumer appeal of their brand capital by providing a mechanism by which firms can profit through distinguishing their product. This type of intangible asset has a direct link to the pricing power of a firm, and estimates indicate brand value is a huge component of the US economy.

Whether a brand ownership reallocation is efficient or strategic has different predictions on prices and sales. In efficient exchange, we expect sales to increase and prices to decrease. In strategic exchanges, we expect prices to increase and sales to stay flat or decline. If exchanges are a mix of strategic and efficient, we may see a mix of both features. Bringing these predictions to data, we indeed find evidence of both types of exchanges. In an average exchange from a small to large firm, sales go up by 60% but prices also increase by 5%. As a result, these exchanges exhibit a mix of strategic and productive interests for large firms.

To better match the reality of consumer product market, we further augment the model with following features: (1). we assume products are heterogeneous and experience life cycles (Argente et al., 2020a) which may vary by product group; (2) we assume firms differ both by their fixed characteristics and their match quality with products, which leads to both reallocation of products in both directions among large and small firms; (3) we introduce a tax/subsidy on both innovation and reallocation activities to evaluate the policy implications. With these additions, the growth rate, concentration, and policy implications in our model are driven by both firm dynamics and brand dynamics, as well as their joint movement.

We use the estimated model to study policy counterfactuals. We start by studying a counterfactual of shutting down reallocation from small to large firms, and find significant welfare losses of 6%. This occurs due to both the immediate reallocation effect and declining entry (as firms do not have the option to develop and sell their brands). We then test a combination of policies focusing on how optimal policy differs by product group and the welfare gains and losses change over time. We find that in markets with fast maturity, e.g., firms can build brand capital quickly, product reallocation is less costly and induces more entry. Even exchange that looks strategic may be efficient. In markets with slower maturity, we find product reallocation can often be inefficient, as the strategic effect dominates. This significant heterogeneity across groups indicates that optimal policy should incorporate the group-level fundamentals. Pharmaceuticals, cereal, and beer all face different market dynamics and thus coarse policies may not fit well specific markets. The lessons from this paper link the underlying fundamentals to optimal policy and provide important lessons for policymakers thinking about the distribution of brands across firms. This points to relevant trends in the US economy as we observe the rising value of intangibles, the rise of niche consumption, and concerns about market concentration.

The remainder of this section reviews the literature, while the rest of the paper is structured as follows. Section 2 introduces the USPTO Trademark Dataset and RMS Nielsen Scanner Data, and discusses our merge and some augmented datasets. Section 3 documents the key empirical facts that frame our investigation at the firm and brand-level. Section 4 introduces the model of product creation, maturity, and reallocation with variable firm productivity and variable markups. Section 5 estimates the model. Section 6 uses the quantified model to understand the contribution of specific margins and perform policy counterfactuals. Section 7 concludes.

Related Literature

This paper builds on and contributes to several literatures: the study of firm dynamics and product dynamics, the macroeconomics of M&A and technology transfers, the study of concentration and firm profitability, and the study of brands and branding.

Firm performance is inextricably linked to its products. Hottman et al. (2016) study multi-product firms and find the scope of products explains a large share of sales variations across firms. Argente et al. (2018, 2020a) explore how product creation and destruction are pervasive in product markets. Further, Argente et al. (2021) and Einav et al. (2021) document the expansion of product sales is largely due to expansion of the customer base, making the distinguishing factors of products essential. Foster et al. (2016) discuss how plants grow also through building a consumer base, as entrants start out well behind incumbents and converge to them. We connect this important empirical insight to a firm's decision when they hold many products. Atkeson and Burstein (2008) introduce oligopolistic competition into a model with multi-product large firms, which is the building block of our model. Our paper links these papers through linking the product or brand life cycle to product innovation and reallocation, an in environment where firms have variable markups as in Atkeson and Burstein (2008).

By linking product innovation and reallocation, we speak to a literature where product innovation is at the center of economic growth dating back to Romer (1990) and Grossman and Helpman (1991). This has also been found empirically, as product creation plays an essential role in both economic growth and the gains from trade, as noted by Bils and Klenow (2001), Broda and Weinstein (2006), Argente et al. (2018), and Jaravel (2018). Yet, this significant literature has not allocated attention to the role of brands in the macroeconomy, even though it has huge firm value and growth implications. Brands shape markets, and the ability to build a brand shapes brand entry and brand growth.

In addition to product innovation, product ownership reallocation plays an important role in macroeconomics and economic growth. Further, ownership reallocation closely connects to questions on the aggregate implications of mergers and acquisitions (M&A) and intellectual property transfer. David (2020) studies the aggregate implications of M&A, and finds gains from trade on both sides of exchange, albeit with some failures in overall market efficiency. Akcigit et al. (2016) study intellectual property misallocation and the market for patents and find this secondary market increases efficiency. (Eaton and Kortum, 1996) and Shi and Hopenhayn (2017) study how the appropriability of innovation, the ability to license or sell intellectual property, induces upstream incentives. Abrams et al. (2019) find evidence that intermediaries in intellectual property transfers exhibit both positive and negative effects on downstream innovation, while Cunningham et al. (2021) find "killer acquisitions" to be have an important role in pharmaceuticals. Two recent papers discuss the role of antitrust policies on growth, from the perspective of technological innovation (Cavenaile et al., 2021 and Fons-Rosen et al., 2021). Our theoretical framework relates to these papers in integrating the dynamic effects of transactions, but differs in the focus on brand capital, the product life cycle, and endogenous market shares. In our model, market concentration and growth are both endogenous and impact household welfare, which allows us to discuss the benefits and costs of various antitrust and innovation policies.

The reallocation of product ownership touches on important debates in both market concentration and innovation. Recent work has focused on rising markups (e.g., De Loecker and Eeckhout, 2018; De Loecker et al., 2020) and rising concentration (Gutiérrez and Philippon, 2017; Eggertsson et al., 2018; Hall, 2018). To connect these discussions to growth, our model builds on the long literature of endogenous growth through creative destruction (Aghion and Howitt, 1992, Aghion et al., 2001, Garcia-Macia et al., 2019, Liu et al., 2019, and Peters, 2020). Edmond et al., 2015 focuses on the markup channel, as large firms can leverage their large market share to charge high markups. Akcigit and Ates (2019, 2021) focus on the knowledge diffusion gaps between leaders and followers driving rising concentration and falling business dynamism. De Ridder (2019) focuses on intangible capital as a barrier to entry. Empirically, we build on frameworks that study reallocation, mostly in the labor context, relating to work dating back to Davis and Haltiwanger (1992) and Davis et al. (1996). We note that similar measures can be used with intangible assets. We complement these papers by focusing on the sources of concentration but differ primarily in two respects. First, we put brand capital ownership at the center and focus on the reallocation of brands. Second, we focus on endogenous market shares and concentration that emerge from the imperfect substitution across brands.

Lastly, we bring important insights from the literature on brands and branding to the macroeconomic debates on concentration. Brands have long been known to be an importance source of firm values (e.g., Brown, 1953). Bain (1956) noted that "(t)he advantage to established sellers accruing from buyer preferences for their products as opposed to potential-entrant products is on the average larger and more frequent in occurrence at large values than any other barrier to entry." This has been detailed empirically as the stickiness of brand preferences is quite persistent (e.g., in Bronnenberg et al., 2009, 2012) and provides firms significant value. Bronnenberg et al. (2022) find that the top 100 brands alone accounted for over $4 Trillion USD in 2021, and brand value is increasing over time relative to traditional capital assets. Thus, brands are an essential aspect of a firm's portfolio and important in the macroeconomy. Gourio and Rudanko (2014) note how consumer capital is a relevant state variable for firms and products. Heath and Mace (2019) show empirically how this generates strategic behavior in the market for trademarks, and is consistent with the significant degree of activity in the market (noted by Kost et al., 2019). This current paper builds on these papers in two respects. First, we link brand capital to the aggregate economy and the product market shares of firms. Second, we study how brands can be reallocated across firms, which makes the distribution of brand capital across the economy an essential state variable of interest at not just the firm-level but economy-wide.

2 Data

This project studies the connection between products, brands, and firms. For the purposes of this paper we treat as interchangeable trademarked products and brands at the product-group level and the focal point of customer capital. In the paper, we define a product as a trademarked brand within a specific product group, for example "Cheerios" in "Cereal". Products are always held by firms. It is well known in the literature that some firms are small and carry one brand, but large firms in consumer packaged goods (CPG) and most industries carry more than one product. We provide new evidence in line with this finding, and then study the nature of products and product ownership exchange. This section links products to firms in our two datasets.

We first motivate our empirical framework and then turn to the two datasets that serve as the bedrock for our empirical analysis. In terms of the framework, we separate the product or brand performance into three components. A product's performance of good i in firm j at time t (sales c_{ijt}) could be a function of the core product, the organization producing it (the firm), and a match-specific component (the firm-product fit):

$$c_{ijt}(product_{ij}) = c(product_i) + c(firm_j) + c(product_i \times firm_j).$$

To incorporate this equation, data from the firm, the product, and the product interacted with the firm are required. The most appropriate dataset to understand these forces would be at the product-level, and would provide detail on brand history, including the prices, sales, and age of each brand. For the firms, data on the brands and sales of firms is essential. To separately identify the effects of products and firm dynamics, we rely on transactions of products across firms to obtain estimates of firm fixed effects and product fixed effects.

In this section, we describe each of the three ingredients through through two datasets. This paper applies USPTO trademarks and RMS Nielsen Scanner Data to track the creation, distribution, and prices and quantities of products.

The trademark data provide the history of each brand and parent firm in terms of registrations, cancellations, and transactions. To focus on the dynamics of prices and quantities, we connect these firm-product-level data to specific information on product prices and quantities sold by stores in RMS Nielsen Scanner Data. The following two sections discuss these datasets in turn.

2.1 USPTO Trademark Data

USPTO Trademark data provide a unique and comprehensive insight into brand-building. Trademarks are a central and dynamic arena of the economy, as firms register for trademarks whenever they want their brand legally protected. Trademarks are common, and many more firms participate in trademarking than patenting.

In this paper, we direct attention to how trademark creation and exchange interact with the growth and concentration of firms. When firms create new products, they apply to the USPTO to protect the consumer appeal associated with the product. Further, when firms buy the rights to product ownership from other firms, the trademark is reassigned across firms.[1]

To register for a trademark, a firm must undergo the following process. First, an individual who

[1] If the selling firm does not remain an ongoing concern or becomes a direct subsidiary of the buying firm, it may not be reassigned, because no direct legal concern exists. We supplement this data with transfers observed in RMS Nielsen and M&A from Refinitiv.

applies must pay a fee that ranges from $225 to $400. Within three months of filing, an examining attorney checks for compliance, and if the application is approved, it "publishes for opposition." A 30-day period follows, during which third parties affected by the trademark registration can step forward to file an "Opposition Proceeding" to stop the registration. This process is again evaluated by an examiner. If it clears this process, the trademark is registered.

The owner of a registered trademark then has exclusive rights to use the mark within the sphere of activity designated by the legal process. Such rights include indefinite renewal conditional on continued use and the rights to exchange. Dinlersoz et al. (2018) and Kost et al. (2019) discuss the institutional aspects of trademarks in greater detail. Further, Appendix A presents some examples of firms with multiple brands. Here, we turn to summary statistics on the number of trademarks and their distribution across firms. Even in trademarks, one can observe common patterns that are relevant for our analysis. Table 1 provides general details on the number of firms and trademarks and the distribution of trademarks across firms.

Table 1: Summary Statistics on Trademarks from USPTO

Data Object	**Count**
# unique firms	1.35M
# unique registrations	5.36M
# unique transactions by bundle	915076
# unique transactions by ID	4.46M
# unique cancels	2.12M
99th percentile firm size	83
75th percentile firm size	5
Median firm size	1
Mean firm size	5

Notes: Firm size is defined as the number of trademarks within a firm. Source: USPTO Trademark Data.

We focus on two relevant features of the data from Table 1. First, the number of transacted brands is almost as large as the number of registered brands, indicating constant flow of ownership across firms. Another striking feature of the data is the skewness of firm size. The 99th percentile firm is over 80-times larger than the median firm in terms of the stock of trademarks. We note similar patterns in terms of sales, and this recurrent pattern of concentration is a central feature of our analysis. Upon looking at this result, whether large firms are simply more efficient at building brands or selling products, or have strategic interest in building brand holdings, is unclear. Linking brands to prices and sales is the next step in uncovering these forces.

2.2 Nielsen Scanner Data

Detailed product-level data are central to our analysis. We apply detailed store-product-level data that come from Kilts-Nielsen Retail Measurement Services Data from the University of Chicago Booth School of Business. The data are large and comprehensive in the consumer product space from years 2006-2018. Although we apply historical use of trademark analysis to understand the age and evolution of brands, 2006-2018 is our primary focus.

We observe more than 100 billion observations at the product $\times$ store $\times$ time level. Product is defined by a UPC identifier, 12 digits that are uniquely assigned to each specific good. The store is defined at the local level, with over 40,000 total; time is defined weekly. Total sales are approximately \$300 billion per year, covering around half of consumption in the consumer goods industry, which itself covers approximately 8% of total consumption in GDP.

The UPC barcodes provide a unique identifier for each product. Changes in any attribute of a good corresponds to a new barcode. Barcodes are widespread and thus cover a large amount of the CPG industry. However, the unique identifying feature of the barcodes may not be as relevant for our analysis. For instance, the parent trademark associated with "Coca-Cola Christmas Edition" is the original "Coca-Cola".

This dataset has been used widely for product analysis. A key departure from the literature in our case is focusing on *brands*, that is, brand names listed in trademark and Nielsen data, rather than *products*, that is, UPC codes. We discuss three reasons for focusing on brands rather than products. First, consumer goodwill tends to be brand rather than product-specific. Coke 12oz relies on the same core branding as Coke 20oz. Thus, regarding how the consumer interacts with the product, brand is the more core indicator. Second, when firms exchange product ownership, or the right to sell a specific brand, they systematically transfer the full rights on the consumer goodwill, making the specific product differentiation within the brand less relevant. Third, our data enable identification at the brand level in both the Nielsen data and USPTO trademark data. Nielsen provides brand identifiers in addition to product identifiers. We collapse this information into brand sales by product group by year. We do not focus on geographical variation in this paper.

The volume represents over half of all transactions in grocery stores and drug stores, and slightly less than half in convenience and mass merchandise stores (as also noted by Argente et al., 2020a). We apply a dataset from GS1 US to link parent firms to products through UPCs. Whereas GS1 links to most parent companies, the USPTO trademark dataset helps complement GS1 to ensure the correct company is allocated to the correct brand. We focus on this step next.

2.3 Data Merge

To link brand age, brand exchange, and product evolution, we employ a fuzzy merge to connect product names in RMS Nielsen scanner data to USPTO Trademark data. Whereas this merge is the first we know of that links USPTO Trademark data to Nielsen Scanner data, Argente et al. (2020b) link USPTO patent data to RMS Nielsen product-level data. We follow a similar method but get greater coverage in our merge, likely due to the different nature of patents and trademarks. In particular, we are able to identify specific products connected to their brand name as long as the trademarked brand name is similar enough to the store brand name.

We start by normalizing names in each dataset at both the firm and product-level. For example, we want to capture heterogeneous naming at the firm (e.g., General Mill Holdings + General Mills Minnesota Op.) and connect it to the parent company. We then turn to the brands themselves. We employ a similar fuzzy match with brands. We start by linking observations at the brand $\times$ firm level, but for observations for which we directly observe the brand, we connect the brand independently and assign ownership through trademark data.

Both USPTO and Nielsen scanner data contain a firm $\times$ brand of observation of interest. The identification of firms and brands provides what is ideally a many-to-1 matching between products (which rely on the same goodwill) and brands. In reality, we must rely on many brands and many product matches. For these matches, we focus on the most reliable name match. If the same brand has multiple matches, we take the "active" brand. For instance, if a brand is reassigned across firms, we assume this represents the focal group of brands. Once this match is complete, we treat brands as the relevant margin for the concepts in this paper. At times, Nielsen data are purely linked to distributors, so USPTO data may be a more reliable indicator of ownership. Overall, we take the leading firm in selling each brand and use Nielsen data and the holding firm in trademark data. If these are not aligned, we take the Nielsen data unless the trademark indicates a *transfer* of ownership, which cover approximately 35% of the exchanges in our data. Thus, both datasets are combined to deliver our indicator of brand ownership.

We next turn to the quality of the match between brands and products. We focus in the USPTO case on brands, and note the products they connect to in Nielsen. Table 2 provides information on the match between products and trademarks.

We stress a couple points from the table. First, when we take brands merged with sales weights, we are capturing a significant share of sales in the data at 82%. Without sales weights, we are unsurprisingly capturing fewer brands. Some small firms may choose not to protect their intellectual property via legal means.

We find in this merge that not only are multiple brands associated with single firms, but also that multiple products are connected to a single brand. On average, we observe 13 unique products per brand.

Table 2: Summary Statistics on Trademark Nielsen Merge

	Unique Count	Years Active	Share Match (%)
USPTO Trademark Data			
Brands	5.36M	1870-2020	
Firms	371021	1870-2020	
Canceled Brands	2.12M	1970-2020	
Transactions	915076	1970-2020	
RMS Nielsen Scanner Data			
Brands	1.64M	2006-2018	57%
Firms	23232	2006-2018	54%
Brand × sales		2006-2018	82%

Notes: Summary statistics on share of merge brands in both datasets. Source: USPTO Trademark Data and RMS Nielsen Scanner Data

As a result, brands are an important source of firm income and, and noted in the following section, provide the central focal point for firm growth.[2] We now turn to the empirics of products and firms.

3 Empirics of Reallocation and Concentration

This section focuses on the motivating empirical observations that inform our model and quantitative analysis. Following the structure discussed above, we focus on three main margins. We focus first on the firm-level margin, and discuss firm concentration and firm dynamics. We then turn to the product-level margin, where we focus on product heterogeneity and product dynamics. We then turn to the firm × product heterogeneity and dynamics, focusing on product-ownership transactions and the gross and net flows of products across firms.

This section proceeds in three steps. Section 3.1 starts by focusing on firms. First, we document the degree of dominance of firms in product markets, illustrating the role of market leaders and their persistence. Second, we decompose the forces that contribute to firm-level market share. We focus on three core drivers of concentration: (i) product creation and destruction, (ii) existing product sales growth, and (iii) product transactions across firms.

Section 3.2 unpacks products more directly, turning attention to the three core forces contributing to concentration – product creation, growth, and transactions of ownership across firms. The product life cycle exhibits striking patterns in the data. Older products tend to take up the largest share of sales. Products exhibit a similar pattern with transactions, as older and larger brands are more likely to be transacted.

Section 3.3 focuses on the rate and nature of product reallocation across firms. We document evidence

[2]We consider the brand to be the core product a firm is producing, so in terms of the main takeaways of the paper, we treat product and brand as interchangeable.

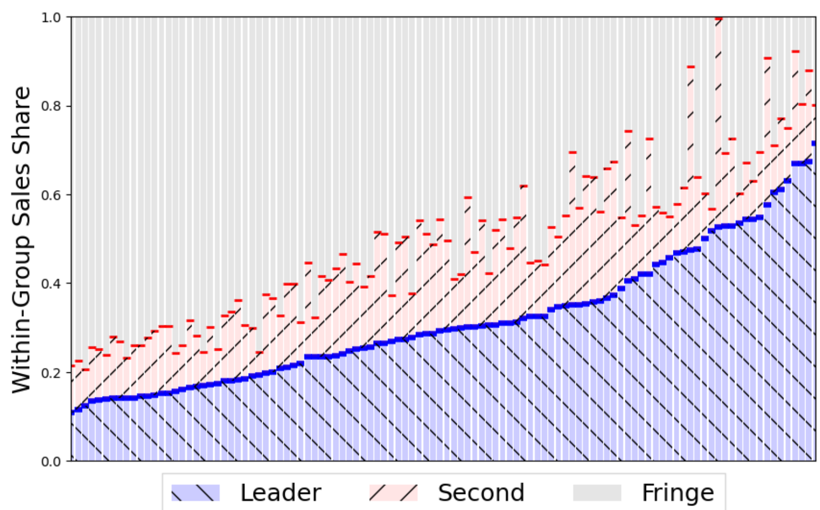
of efficiency gains from product transfer as well as strategic gains (through pricing). We further point out the amount and direction of product-ownership flows. The interaction of these three forces and the aggregate implications are discussed further in Section 6.

3.1 Firm-Level Analysis

To focus on the firm component, we first look at the nature of concentration and drivers of firm advantage. Market concentration is not a purely static force but is driven by dynamism in firm ownership. We focus in this section on the level, persistence, and sources of market shares. We first focus on overall concentration and then turn to the persistence of market leadership. Lastly, we turn an analysis of the dynamic elements driving concentration.

Figure 1 maps out the sales share of the product leader, the second firm, and the remaining firms in the market. This split by product group contains 116 unique product-group categories (e.g., “ICE CREAM" or “BEER”). The average top firm share is 34% of the total market, though in many markets, the top firm holds a significantly larger share. Thus, understanding how large and fringe firms interact is essential to understanding concentration.

Figure 1: Sales Share of Leader, by Product Group



Note: *This figure shows the sales share by product group (ordered by % share of leader) in 2010.* Source: *RMS Kilts-Nielsen Data Center & GS1 firm-product merge*

Table 3: Firm Market Shares

Top firm share by group	Top 2 firm share	median share
33.6%	50.3%	0.04%

Table 3 shows the average leader (and second firm) share, as well as the share from the median firm.

The top two firms control on average more than half the sales in a given market. We note also the presence of a host of small firms (median share as 0.04%), and in our framework, we think of these firms as "fringe" in the sense they hold few products and small market share.

We also note these leading firms are quite persistent. Across all categories, the leading firm in one period has a 97% chance of being among the top two firms in the product group in the next period. Concentration in product markets is real and persistent, yet it is not made up of single products. On average, market leaders hold 27 unique brands within the product group they lead. As a result, variation in concentration will be closely connected to how firms interact with their brands.

Product market dominance does not happen in a day. Concentration is the result of the progressive reallocation of market shares of products across firms. The product life cycle is intertwined with firm growth and decline through three core channels. First, and most noted within the innovation literature, is product creation and destruction. Second, once products are born, they grow and decay over time. Third, product ownership is transacted across firms.

The format of our data allows us to characterize these three forces. We set up three regressions where the coefficients add up to 1 (following a beta decomposition), and each coefficient is linked to the amount of variation of firm growth and decline the margin explains. We run the following regression of three distinct margins of change, y_{it}, on the change of sales in each period $\Delta sales_{it}$:

$$y_{it} = \alpha + \beta \Delta sales_{it} + \epsilon_{it}. \tag{1}$$

Equation (1) focuses on three different margins for y_{it}. We substitute each of the three margins discussed above as y_{it} (y_{it}=creation, growth, transaction). We refer to product growth as incumbency because it can include decline as well. We present the results of the three separate regressions in Table 4.

We stress two main takeaways from Table 4. At the firm level, variation from entry is much more important for fringe firms (e.g., small firms) than for large firms. Acquisition is relatively much more important for sales variation for large firms. Relative to acquisition, entry is almost 30-times more important for small firms. For both fringe firms and market leaders, incumbent products drive a significant amount of firm-level variation. Some variation from incumbent products may come directly from the life cycle, whereas others may be due to idiosyncratic shocks. We focus directly on the role of life-cycle variation in Table B1 in Appendix B.

3.2 Product-Level Analysis

Products are both a significant source of firm concentration (Hottman et al., 2016), and highly dynamic (Argente et al., 2020a), thus affecting the overall sales at the firm and product group level. Our goal in this section is to focus on the interaction of products and reallocation. Although we also link trademark

Table 4: Sources of Market Share Reallocation

	Leader			
	Innovation	Incumbency	Reallocation	Entry/Reallocation
Value	0.033***	0.84***	0.13***	**0.25**
	(0.315)	(0.000)	(0.000)	
Observations	383	383	383	
	Fringe			
	Entry	Incumbency	Reallocation	Entry/Reallocation
Value	0.091***	0.89***	0.021***	**4.33**
	(0.000)	(0.000)	(0.000)	
Observations	95353	95353	95353	

p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: Market share reallocation is measured across different firm types, following equation (1). Source: RMS Nielsen

data to product sales to expand on the work related to rich product life cycles (e.g. Argente et al., 2020a, 2021), we focus more directly on the interaction with reallocation. We focus on the age profile of sales in Appendix B.[3] The transaction profile has some similarities to the age-sales profile, as brands are born unlikely to be transacted and are more likely to be transacted over time.

Products evolve over their life cycle. Gourio and Rudanko (2014) document the slow development of customer capital, whereas Foster et al. (2016) note the importance of learning about demand for firms. The same is true for transactions. Few products are transacted when very young, because they need time to build customer capital and exposure to other firms. We focus on the correlation between transactions and sales and transactions and age in the following figure.

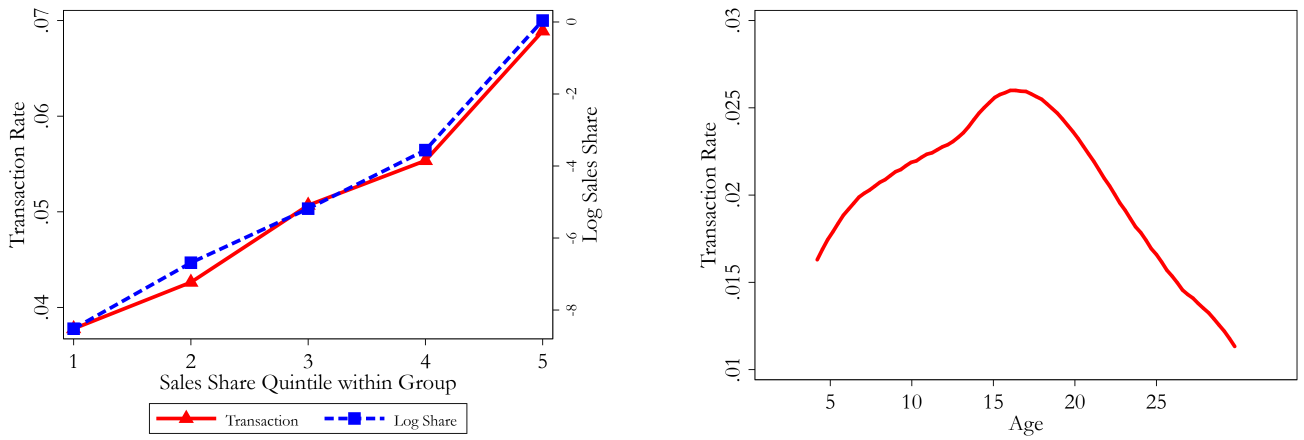
Figure 2 focuses on the interaction of transaction rates, sales share, and age. Figure 2a focuses on the interaction of sales share and transaction, finding the positive selection on transaction. Figure 2b leverages the history of USPTO trademark data to understand the interaction of age and transaction.

Figure 2a splits the products into quintiles (truncating products with less than $1000 in sales in a year), and plots the transaction rate against product quintile and the log sales share. We see transaction rates are higher for products with larger market shares. This result can be rationalized in various ways, but we inform the patterns in transaction through directed search in Section 4. Firms select on searching for products with higher value to transact.

Similarly, Figure 2b shows how the transaction rate changes with age. Here, we plot a standard smoothed hazard function to ask at what age a brand that has not yet been transacted becomes transacted.

[3] In Appendix B, we add to the discussion by linking sales to the overall age of brands. This procedure is only enabled by looking at trademark histories, as RMS Nielsen does not extend as far back as the birth of most brands in the data.

Figure 2: Transactions, Age, and Sales



(a) Transaction Rate by Quintile (b) Transaction Rate by Age

Notes: Panel (a): Transaction rate by sales share. Panel (b): Transaction rate by age. Source: USPTO Trademark and RMS Nielsen.

We find the age peaks close to 20, around when log sales share peaks. We next turn to the role of the transactions across firms, particularly looking at the interaction between leading and fringe firms.

3.3 Firm × Product Analysis

Firm and product dynamics evolve to determine market outcomes. However, the interaction of firms and products, through in particular the firm-product fit is also an important force in market dynamics, especially when it comes to the transaction of products across firms.

We focus in this section on the *rate* of exchange and the change in sales and prices upon exchange. Given our initial motivation of market concentration, we focus on the heterogeneous effect of being held by a large versus a small firm. We ask what is, conditional on being held by both a large and small firm, the effect of being held by a larger firm on log sales and log prices?[4] We use the regression in equation (2) for our analysis:

$$y_{ikt} = \alpha_0 + \alpha_1 \mathbb{I}\{j(i) = \text{T10 firm}\} + \Gamma_{ik} + \Lambda_t + a_{t-b(i)} + \epsilon_{ikt}. \tag{2}$$

The regression evaluates an outcome variable y_{ikt} (e.g., log sales or log prices), as a function of whether it's held by a market leader. We include a product (brand-group) fixed effect in Γ_{ik}, a year fixed effect (Λ_t), and an age fixed effect $a_{t-b(i)}$.[5]

The results are striking on both counts, and hold true with various specifications. If a top 10 firm holds a brand, sales on average are 0.57 log points higher, whereas prices are on average 0.044 log points

[4] Firm size is defined over the entire horizon of the data, but results are similar if firm is defined in only the first period.
[5] Due to a colinearity problem with age and year, we adjust for age fixed effects following Deaton (1997).

Table 5: Log Price and Sales Conditional on Holding Firm

	(1) Log Sales	(2) Log Price
Top 10 Firm Holding	0.57^{***}	0.044^{**}
	(0.000)	(0.032)
N	441300	441300
R^2	0.844	0.945

p-values in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands.

higher. Both sales and prices are higher at leading firms, indicating large firms could have both strategic and efficiency reasons to buy products. These results find greater texture in the quantitative section when we ask about the overall effects of these purchases on market activity. This section motivates our further interest in understanding the market for transactions.

We briefly turn to the flows to understand the rates of product movement in each direction. The persistence of transactions suggests match-specific components are likely at work. We focus on three types of flows. Brands may move from fringe firms to other fringe firms, from fringe firms to market leaders, and from market leaders to fringe firms. We consider a leader a top 10 firm in a product group. Table 6 documents the flows into each group (leader from fringe, etc.) in terms of the number of observations and the sales-weighted share.

Price and Quantity Impact of Transaction

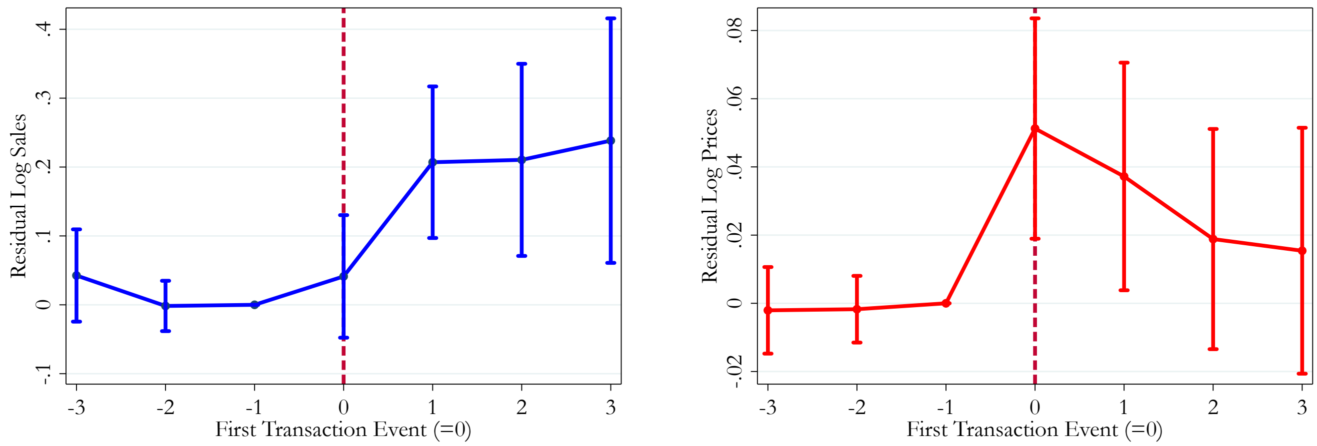
We observe transactions in the data and ask how prices and sales response. To ensure a relevant comparison group, we link transacted brands to never transacted brands with a similar sales profile. Figure B12 plots two separate regressions on one graph with different outcome variables of interest: prices and sales. Equation 3 illustrates the regression we run with the matched sample (with β_t as our variable of interest), which is the term associated with the transacted brand, controlling for age (Λ), year (λ), and brand (θ).

$$\log y_{it} = \alpha + \sum_{t=-3}^{3} \beta_t D_t \times exposed + \lambda_t + \theta_i + \Lambda_a + \epsilon_{it} \tag{3}$$

We plot each coefficient with the clustered standard error at the brand-group level in Figure B12.

After the event, both prices and sales move strongly, with sales moving significantly more so. With the increase in prices, the results in Figure B12 provide evidence that after adding additional brands, firms may increase their market power over time. Combining this with the rising rate of transfer from small to large firms can help connect the importance of brand dynamism with the aggregate distribution

Figure 3: Coarsened Exact Match and Brand Transaction



(a) Sales × Transaction

(b) Prices × Transaction

Notes: Coarsened exact match coefficients. Match is made on pre-trend sales and prices respectively, brand age profile, and year.

of markups across firms. Further, the change in markups is a key outcome of our model.

Table 6: Flows across Firms

Transaction Type	**Num. Obs**	**Share of obs (sales-wt)**
Fringe to Leader	1,123	26%
Leader to Fringe	1,076	18%
Fringe to Fringe	10,223	56%

Notes: This documents the count of observations (and corresponding market share) of flows to leader and to fringe firms. Source: USPTO and RMS Nielsen.

While fringe to fringe firm transactions are almost 10 times as common as fringe-to-leader transactions, they take up a much smaller market share per transaction. Leaders buy larger brands on average. Further, as noted earlier in this section, leaders tend to increase the sales of brands they have acquired. Thus, although reallocation is a common feature of this market, certain patterns of reallocation are especially important for thinking about market concentration. We now turn to a summary of the three main facts discussed in this section.

Fact 1: Firm Level *Large firms disproportionately have more brands and more market share, and build their market share through brand reallocation more than brand creation.*

Fact 2: Product Level *Better and mature brands are more likely to be reallocated across firms.*

Fact 3: Firm × Product Level *When a brand is reallocated across firms, sales and prices both increase.*

In this section, we find that markets are concentrated, concentration is persistent over time, and concentration is built through acquisition and product maturity (Fact 1). In line with findings on the product life cycle, we find patterns of higher profile and more mature brands being more likely to be transacted (Fact 2). Directing more specific attention to products across firms, we find interesting patterns of transaction and outcomes upon transaction (Fact 3). These results motivate a model that can incorporate these forces and develop counterfactuals. The model will incorporate each of these ingredients to understand the drivers of concentration and the dynamic effects of product reallocation. We turn to the model next.

4 Model

We introduce a firm dynamics model with brand capital and transfer of brand ownership. Leading firms hold a bundle of products with brand capital and compete against fringe firms, each with imperfectly substitutable products.[6] Firms create products, charge variable markups, and product ownership flows across firms. The model incorporates some standard features of an endogenous growth framework driven by product variety and product innovation. In addition to these standard features, we include three new ingredients: i) brand capital, ii) a brand/product life cycle, and iii) firms can buy and sell brand ownership in the open market. The model interprets the observed skewed distribution of firm size and product reallocation flows across firms. The goal of the model is to provide a quantitative framework to incorporate the empirical facts to provide an understanding both the efficiency of the market for brands and enable policy counterfactuals.

Consumers choose varieties and supply labor to firms. Consumers have CES preferences across imperfectly substitutable product varieties. The brand capital of each product is driven by three sources of heterogeneity. Heterogeneity at the firm-level determines the scope of the brand (e.g. through distribution or marketing). Heterogeneity at the product-level determines the consumer preference for the brand (e.g. through pure enjoyment). Heterogeneity at the firm $\times$ product level determines the fit of the brand with the parent firm. The model incorporates these layers of heterogeneity in a parsimonious way by leveraging tools from search theory (as in Menzio and Shi, 2011), which simplifies the joint decision problem of firms. The key objects of interest in our model will be the overall brand capital/product scope and the distribution of ownership across firms. These will have important implications for both economic growth and market power.

Section 4.1 characterizes consumer demand for products, directing attention to the sources of product appeal and how it interacts with consumer preferences. Section 4.2 focuses on the leading firms in each

[6] We refer to products and brands interchangeably in our setting. A product is defined as a brand by product group in RMS Nielsen, and we identify sales associated with these brand $\times$ group entities.

product group and the competition in each market. We then introduce the concept of the trademark and discuss the value of the trademark to a firm. This connects to a discussion of product innovation and reallocation. Section 4.3 discusses the value of holding products to each firm, and the innovation and reallocation decisions, with particular interest to the conditions when reallocation is efficient versus strategic. Section 4.4 closes the model by completing the household problem. Section 4.5 characterizes the aggregation across product groups, the growth rate, concentration, and overall efficiency.

4.1 Product Demand

Time is continuous and there is a representative household that endogenously supplies labor $\mathbf{L}_t$ and spends to maximize its discounted utility. We focus on the consumer's problem in this section and complete the full household problem in Section 4.4. At instant t, the real consumption of the household $\mathbf{C}_t$ is given by a Cobb-Douglas aggregator across a unit measure of product groups, indexed by $k \in [0, 1]$:

$$\ln \mathbf{C}_t = \int_0^1 \xi_k \ln C_{kt} dk, \tag{4}$$

where C_{kt} is the real consumption from product group k, ξ_k is the appeal of product group k to the household.

Each product group k at time t contains N_{kt} measure of imperfectly differentiable products which are identified with a brand. The real consumption from product group k, C_{kt}, is a CES aggregator across these varieties:

$$C_{k,t} = \left(\int_0^{N_{kt}} \psi_{ikt}^{\frac{1}{\sigma_k}} c_{ikt}^{\frac{\sigma_k - 1}{\sigma_k}} di \right)^{\frac{\sigma_k}{\sigma_k - 1}}, \tag{5}$$

where c_{ikt} is the consumption on variety i, ψ_{it} is the appeal of product i, and σ_k is the substitution elasticity across products, which we allow to vary by product group. There are two main objects of interest from our model. First, we are concerned with the total number of products, N_{kt}, as more products contribute positively to consumer welfare. Second, we are concerned with the appeal, ψ_{ikt}, of each product. The joint distribution of these two objects connects directly to consumer welfare. Mirroring the empirical decomposition of product-level sales, we assume the appeal ψ_{ikt} is a combination of three components:

$$\log \psi_{ikt} = \alpha_{jk(t)} + \beta_{ikt} + \gamma_{ijkt}. \tag{6}$$

Equation (6) closely resembles our empirical framework. $\alpha_{jk(i)}$ is the appeal of product-owning firm j in group k (which it may expand through marketing/distribution), $\beta_{ik}(t)$ is the specific product appeal which links to consumer's taste for the brand in group k, and $\gamma_{ijk(t)}$ is the match-specific quality between product i and its owning firm j in group k. The optimal consumption decision within group k gives the demand curve for variety i, given the appeal ψ and price p:

$$c_{kt}(p,\psi) = \psi \times p^{-\sigma_k} \times P_{kt}^{\sigma_k - 1}, \tag{7}$$

where the group-level price index P_{kt} is given by $P_{kt} = \left(\int_0^{N_{kt}} \psi_{ikt} p_{ikt}^{1-\sigma_k} di \right)^{\frac{1}{1-\sigma_k}}$. Given price p, the demand for product i is increasing in the appeal ψ. Given appeal ψ, product demand is decreasing in the price p to the degree that the price diverts from the group-level price index. The elasticity of substitution σ_k determines the sensitivity to price. In a market with high elasticity of substitution ($\sigma_k \to \infty$), individuals are very price responsive to price and less responsive to appeal. In markets with low elasticity of substitution ($\sigma_k \to 1$), firms have lots of pricing leverage through consumer appeal. We normalize the aggregate price index to be 1 for any t.

In the full quantitative section, we allow all parameters to vary by group k. For the rest of the theoretical discussion we perform two adjustments to aid exposition. First, we omit group k index, as the group index does not change how we characterize the equilibrium. Second, we set $\xi_k = 1$ to study a general group k with group appeal 1.

4.2 Firms

Each product group contains one large multi-product firm and endogenous measure of single-product firms. We refer to the multi-product firm as the group leader and single-product firms as the fringe. The leader and fringe firms are different in the following aspects: (1) *Capacity*. The leaders are able to own and operate many varieties, whereas the fringe firms are only able to operate one product. We denote the leader's basket of products at time t as $\mathcal{I}_t^L$ and the fringe basket of products as $\mathcal{I}_t^F$; (2) *Entry*. The leaders are not subject to firm entry and exit, while there is free entry of fringe firms. (3) *Productivity*. All varieties are produced using a linear technology in labor. The leader has productivity $e^{z/(\sigma-1)}$, and fringe firms have the same firm-level productivity 1. (4) *Pricing*. The leaders are big relative to their product group, and they internalize their impact on the group-level price index. The fringe firms are small relative to the market, and they behave as monopolistically competitive firms. Each firm can charge a markup through branding, but large firms have more pricing power through their larger consumer appeal. Given the appeal of products and the productivity of firms, the sales of product i at instant t is determined by the following composite quality index if it is operated by the leader j:

$$\log q_{it} = z + \alpha + \beta_{it} + \gamma_{ijt}, \tag{8}$$

and the following composite quality index if it is operated by a fringe firm:

$$\log q_{it} = \beta_{it} + \gamma_{it}, \tag{9}$$

We define the quality index for leader as the sum of quality indices across all of its products $Q_t^L = \int_{i \in \mathcal{I}_t^L} q_{it} di$, and the quality index of fringe firms as the sum of quality indices across all of their products $Q_t^F = \int_{i \in \mathcal{I}_t^F} q_{it} di$. Two group-level indices are welfare relevant in our model. The first index is the group-level composite quality index Q_t, because it increases the marginal utility of consumption through the love of variety:

$$Q_t = Q_t^L + Q_t^F. \tag{10}$$

We denote the growth rate of this object $g_t = \frac{\dot{Q}_t}{Q_t}$. The second is the ratio of the leader's quality index and the fringe firms' quality index:

$$\phi_t = \frac{Q_t^L}{Q_t^F}. \tag{11}$$

Competition. We assume the firms compete through price and price their products jointly. For each product, firms internalize their impact on the group-level price index according to their market shares as in Atkeson and Burstein (2008).

Product Innovation. New products are created through product innovation, and can be done by both fringe firms and market leaders. The leader can choose its innovation intensity η by paying labor cost $D(\eta)$. $D(\eta)$ is increasing and convex in η, and $D(0) = 0$. The fringe firms can endogenously enter with entry cost $\frac{\kappa^e}{Q_t}$. Having entered, the fringe creates a new product which it trademarks. New products draw an initial product-specific quality β from exogenous distribution $F_\beta(\beta)$ and match-specific quality γ from distribution $F_\gamma(\gamma)$.

Product Reallocation. The ownership of existing products can be reallocated across firms within the same product group. This reallocation process is modeled as a market with search and matching frictions (consistent with observed gains from trade in ownership exchange as in David, 2020). We assume search is directed (consistent with the fact that brands are not hard to find) as in Menzio and Shi (2011). At each instant, buyers can create vacancies with constant costs; Sellers post the transfers they would require in exchange for their products; Buyers, observing all posted transfers, direct search to their preferred sellers. The matching between sellers and buyers is frictional, modeled by a matching function that moves with the number of vacancies and the number of sellers. This matching function captures natural features of the market for brand exchange. Mathematically, we assume the number of matches is given by $m(v, u)$, where v is the number of vacancies and u is the number of sellers. We assume m is increasing and concave in both arguments. It is useful to define the selling rate as $\lambda(\theta) = m(\theta, 1)$, where θ is the ratio between the number of buyers and the number of sellers.

Trademarks. Whenever a firm introduces a new product, it has an incentive to trademark the product. Without a trademark, there is not a mechanism for consumer to have imperfect substitution over your product. Further, there is evidence firms trademark early in the firm lifecycle (as noted by Dinlersoz et al., 2018), and trademark application costs are low. The corresponding market power accrues to both leaders and fringe firms, but leaders can potentially markup higher than fringe firms due to the reasoning discussed above. This pricing power is one force generating product-ownership reallocation from small to large firms. The other force is the natural efficiencies of the brands matching with the right firms.

Product Life cycle. When a brand is initially introduced, the brand has low brand capital as consumers are not deeply aware of it. This is consistent with evidence on the long-run development of consumer demand presented earlier (and noted by, among others, Bronnenberg et al., 2009; Einav et al., 2021) and the previous evidence presented on low transaction rates early in a brand's life cycle. Brands then develop consumer capital through a dynamic process. This both induces increases in sales and increases in visibility for reallocation of brand ownership. As discussed earlier, new products draw initial brand capital $\beta_0 \sim F_\beta$. These brands however develop over time. We model the brand development process as the following, $d\beta = \iota(\bar{\beta} + \beta_0 - \beta)$, where we match ι and $\bar{\beta}$ to the life cycle in the data.

Implications of Product Reallocation. In the next section, we discuss the characterization of product reallocation. Here, we briefly discuss two extreme cases of product reallocation: *efficient* and *strategic* reallocation. The intuition of these two polar cases captures important features of the market for brands. Firms transfer the intellectual capital associated with product β, which is the fixed component of the product. However, reallocation may lead to different appeal through α (firm effect) or γ (firm $\times$ product effect). In the case where the leading firm has large advantages (e.g. in distribution or marketing, $\alpha >> 0$), we expect transactions to exhibit efficiency gains as brands have been allocated to a better firm. If leading firms do not have large advantages in distribution and marketing, (e.g., $\alpha \approx 0$), then the brand ownership transfer simply exacerbates an appeal gap between leaders and fringe firms. We study this further in the characterization of the market equilibrium.

4.3 Firm's Problem

Static Pricing Problem. Fringe firms are infinitesimal relative to the market, they do not internalize their own impact on group-level price indices. In the equilibrium, they charge a constant markup $\frac{\sigma_k}{\sigma_k - 1}$. The leaders have a different pricing problem because they are large relative to the market and internalize their impact on the group-level price index. Given the demand curve for each variety in equation (7), the product-group leader's pricing decision is:

$$\max_{p_i} \int_{i\in\mathcal{I}_t^L} (p_i - e^{-z/(\sigma-1)}\mathbf{w}_t) c_t(p_i, \psi_i) di$$

s.t.

$$c_t(p, \psi) \text{ given by equation (7).}$$

As in the models in patent race literature, the competition of a group can be summarized by the gap between leaders' quality and fringe firms' quality as noted in equation (11). Given the gap ϕ, the equilibrium markup charged by the leader is increasing in its market share s, given by

$$\mu = \frac{\sigma(1-s)+s}{\sigma(1-s)+s-1}. \tag{12}$$

The market share depends on both the gap between leader and fringe firms and the markup as follows,

$$s = \frac{\mu^{1-\sigma}}{\mu^{1-\sigma} + \phi^{-1}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}. \tag{13}$$

Equation (12) and (13) constitute two equations in two variables (s, μ). Given any gap ϕ, we denote the solution to this equation system as $s_k(\phi)$ and $\mu_k(\phi)$. Due to the assumption of Cobb-Douglas aggregation across product groups, the leader's profit can written as $\Pi_k(\phi_k(t))\mathbf{C}(t)$, where $\Pi_k(\phi)$ is the share of aggregate expenditure accruing to the leader in group k:

$$\Pi(\phi) = \frac{s(\phi)}{\sigma\left[1-s(\phi)\right]+s(\phi)}. \tag{14}$$

Similarly, the profit share that accrues in aggregate to fringe firms is

$$\pi(\phi) = \frac{s(\phi)}{\sigma}. \tag{15}$$

In equation (14), ξ_k share of aggregate expenditure $\mathbf{C}(t)$ is spent on group k.

Within this expenditure, $s_k(\phi)$ is accrues to the leader, while $1-s(\phi)$ accrues to fringe firms, who receive $\frac{1}{\sigma}$ profit margin. The leader, due to its collection of appeal, perceives the consumer elasticity $\sigma(1-s(\phi))+s(\phi)$ to be more inelastic than the fringe firms. The inverse of its perceived elasticity is its profit margin.

A leading firm's incentive to engage in product innovation and product reallocation derives from a firm's ability to increase in its profits. We characterize the marginal increase of leader's profit when it increases its quality gap from the fringe firms. We find this marginal profit has a closed-form solution in terms of market share, which we discuss in the following lemma.

Lemma 1 *The elasticity of profit with respect a change in quality gap ϕ is*

$$\frac{\partial \log \Pi(\phi)}{\partial \log \phi} = 1 - s(\phi) \tag{16}$$

Two extreme cases are helpful in understanding the result in equation (16). When the leader has 0 market share, the profit elasticity is 1. At this point, the leader has an infinitesimal share of the market and charges the same markup as the fringe firms. Thus a 1% increase in its market share translates into 1% increase in profits without losing in markups. When the leader has 100% of the market, the profit elasticity is 0. When the leader has taken over the whole market, a marginal increase in its quality gap only cannibalizes its own market shares without changing firm-level profit.

Dynamic Innovation and Reallocation Problem. To characterize the incentive of innovation and reallocation, we need to first write out the value of heterogeneous products to leaders and fringe firms. First consider a fringe firm holding a product with appeal (β, γ), which has the following Hamilton-Jacobi-Bellman:

$$(\rho + g)u(\beta, \gamma) = e^{\beta+\gamma}\left(1 + \phi\right)\pi(\phi) + \max_{\theta} \lambda(\theta)\mathbb{E}_{\gamma'_L}\left[u(\beta, \gamma + \Delta) - u(\beta, \gamma)\right]^{+} - \theta\kappa^{s}\varphi_0. \tag{17}$$

The value to a fringe firm in group k with product appeal β and firm product fit γ has two components. The first component is the instantaneous return which moves positively with β and γ. The second component is the option value in the search market, where the fringe firm chooses search intensity θ to generate arrival rate $\lambda(\theta)$ at which the firm receives the surplus from transferring their brand if the gains from trade are positive. The fringe firm does not internalize the α from the leader because the market for brands is pinned down by the fringe-to-fringe transaction. If leaders have large firm advantage, they will demand a higher quantity of products, shifting $\lambda(\theta)$, and inducing a higher value for the fringe firm through this channel.

The product group leader chooses its innovation and reallocation activity to maximize its discounted profit. At time t, given the innovation intensity η_t^L and the reallocation decision $\theta_t^{LF}(\beta, \gamma)$, $v_t^{FL}(\beta, \gamma)$, and $\theta_t^{FL}(\beta, \gamma)$, the density of products with characteristics (β, γ) that are operated by the leader evolves according to:

$$\dot{n}_t^L(\beta,\gamma) = \underbrace{\eta_t f(\beta)\mathbb{I}_{\gamma=0}}_{\text{Innovation}} - \underbrace{\iota(\beta_0+\bar{\beta}-\beta)\frac{\partial n_t^L}{\partial \beta}(\beta,\gamma)}_{\text{Maturity}} \tag{18}$$
$$- \underbrace{\lambda_t^{LF}(\beta,\gamma)n_t^L(\beta,\gamma)}_{\text{L-t-F Reallocation}} + \underbrace{\int_{\Omega(\beta,\gamma,\gamma')>0} f_\gamma(\gamma)\lambda_t^{FL}(\beta,\gamma')n_t^F(\beta,\gamma')d\gamma'}_{\text{F-t-L Reallocation}}.$$

Similarly, the density of products with characteristics (β,γ) that are operated by fringe firms evolves according to:

$$\dot{n}_t^F(\beta,\gamma) = \underbrace{\eta_t^F f(\beta)\mathbb{I}_{\gamma=0}}_{\text{Innovation}} - \underbrace{\iota(\beta_0+\bar{\beta}-\beta)\frac{\partial n_t^L}{\partial \beta}(\beta,\gamma)}_{\text{Maturity}} \tag{19}$$
$$- \underbrace{\lambda_t^{FL}(\beta,\gamma)n_t^F(\beta,\gamma)}_{\text{F-t-L Reallocation}} + \underbrace{\int_{\Omega(\beta,\gamma',\gamma)<0} f_\gamma(\gamma)\lambda_t^{LF}(\beta,\gamma')n_t^L(\beta,\gamma')d\gamma'}_{\text{L-t-F Reallocation}}$$
$$\underbrace{- \lambda_t^{FF}(\beta,\gamma)n_t^F(\beta,\gamma) + \int_{\gamma'>\gamma} f_\gamma(\gamma)\lambda_t^{FF}(\beta,\gamma')n_t^F(\beta,\gamma')d\gamma'}_{\text{F-t-F Reallocation}}.$$

A product group leader, taking as given the entry decision of fringe firms, chooses its own innovation and reallocation activity to maximize the discounted net profit,

$$\max_{\eta_t, v(\beta,\gamma), \tau^{LF}(\beta,\gamma)} \int_0^\infty e^{-\int_0^t \mathbf{r}(t')dt'}\left[\Pi(\phi_t) - D(\eta_t) - B_t + S_t\right]dt, \tag{20}$$

s.t.

$$\phi_t = \frac{\int e^{z+\alpha+\beta+\gamma} n_t^L(\beta,\gamma)d(\beta,\gamma)}{\int e^{\beta+\gamma} n_t^F(\beta,\gamma)d(\beta,\gamma)}, \tag{21}$$

$$B_t = \int \left[M(v_t(\beta,\gamma), u_t(\beta,\gamma))\tau^{FL}(\beta,\gamma) - v_t(\beta,\gamma)\right] d(\beta,\gamma), \tag{22}$$

$$S_t = \int \lambda_t(\theta_t(\beta,\gamma))\tau^{LF}(\beta,\gamma)n_t(\beta,\gamma)d(\beta,\gamma), \tag{23}$$

The leader's problem is a complicated problem that involves the joint distribution of product quality and firm ownership. We show the full details of characterization of this problem in the Appendix C. It turns out this complicated problem can be characterized by calculating the discounted values of products to different firms (leader or fringe). This result comes from two features of our model: (1). the product qualities can be linearly added into a firm-level quality index $\int_{\mathcal{I}_t^L} q_{ijt}$; (2) the love of variety and competition of each product group are fully characterized by the quality indices.

For a product with state (β, γ) that is currently operated by the group leader, its discounted value is

$$(\rho + g_t)\, v_t(\beta, \gamma) = e^{z+\alpha+\beta+\gamma}\Pi'(\phi_t)(1+\phi_t) + \max_{\theta} \lambda(\theta)\mathbb{E}_{\gamma_F}\Omega_t(\beta, \gamma, \gamma_F)^+ - \theta\kappa_s^{LF} + \dot{v}_t(\beta, \gamma). \tag{24}$$

where $\Pi'(\phi_t)(1+\phi_t)$ is the flow marginal value and $U_k^L(\beta, \gamma)$ is the optimal value of selling to fringe firms. The (negative) value of a similar product operated by the fringe to the leader is:

$$(\rho + g_t)\, x_t(\beta, \gamma) = -e^{\beta+\gamma}\Pi'(\phi_t)(1+\phi_t)\phi_t + \lambda(\theta^{FF}(\beta, \gamma))\mathbb{E}_{\gamma'>\gamma}\left(x_t(\beta, \gamma') - x_t(\beta, \gamma)\right) + \dot{x}_t(\beta, \gamma). \tag{25}$$

The value functions of the leader and fringe link to the reallocation decisions through the joint surplus, which we define as $\Omega_t\,(\beta, \gamma, \gamma') = v_t(\beta, \gamma) - x_t(\beta, \gamma') - u_t(\beta, \gamma')$. $\Omega_t\,(\beta, \gamma, \gamma')$ measures the joint surplus from trade for a product reallocated from a fringe firm to a leader, with product appeal β, fringe match quality γ', and leader match quality γ. Correspondingly, $-\Omega_t\,(\beta, \gamma, \gamma')$ is the joint surplus of reallocating the product from a leader to a fringe firm. The joint surplus $\Omega_t\,(\beta, \gamma, \gamma')$ satisfies the following Bellman equation:

$$\begin{aligned}(\rho + g_t)\, \Omega_t(\beta, \gamma_L, \gamma_F) = {} & \underbrace{\omega(\beta, \gamma_L, \gamma_F)}_{\text{Flow Gains from Trade}} + \dot{\Omega}_t(\beta, \gamma_L, \gamma_F) \\ & + \underbrace{\lambda(\theta^{FL}(\beta, \gamma_L))\mathbb{E}_{\gamma'}\Omega_t(\beta, \gamma_L, \gamma')^+ - \theta^{FL}(\beta, \gamma_L)\kappa_s^{LF}}_{\text{Leader's Value of Selling}} \\ & + \underbrace{\lambda(\theta^{FF}(\beta, \gamma_F))\mathbb{E}_{\gamma'>\gamma_F}\Big[\Omega(\beta, \gamma_L, \gamma') - \Omega(\beta, \gamma_L, \gamma_F)\Big] - \theta^{FF}(\beta, \gamma_F)\kappa_s^{FF}}_{\text{Fringe's Value of Selling + Competition Value for Leader}},\end{aligned} \tag{26}$$

where

$$\omega(\beta, \gamma_L, \gamma_F) = \left(\frac{e^{\alpha+\gamma_L-\gamma_F} + \phi}{1+\phi} \frac{\Pi}{\frac{\phi}{\sigma_k(1+\phi)}} - 1 \right) \frac{\pi}{z_F} e^{\beta+\gamma_F}. \tag{27}$$

This equation provides the basis for gains from trade. We take note of two important polar cases in this environment. Table 7 focuses on the conditions under which reallocation would be *only efficient* or *only strategic*. We take the extreme cases in the parameter set that would induce completely efficient and completely strategic transactions.

Efficient reallocation occurs when leaders have no market concentration. In this case, gains from trade emerge only from the leader expanding brand appeal or having a good fit with the brand (α or γ). The strategic reallocation occurs when the leader has no advantage in marketing or distributing the brand (they may even have a disadvantage, e.g., $\alpha + \gamma_L - \gamma_F < 0$), and thus the reallocation simply increases concentration. These two benchmarks provide important conceptual extremes to characterize efficiency

Table 7: Two Cases of Gains from Trade

Case	Condition	Gains from Trade	Discussion
Efficient:	$\phi \to 0$	$\left(e^{\alpha+\gamma_L-\gamma_F}-1\right)\frac{\pi}{z_F}$	Gains from only $\alpha > 0$ or $\gamma_L - \gamma_F > 0$, sales $\uparrow$
Strategic:	$\alpha + \gamma_L - \gamma_F = 0$	$\left(\frac{\Pi}{\frac{\phi}{\sigma(1+\phi)}} - 1\right)\frac{\pi}{z_F}e^{\beta+\gamma_F}$	Gains from higher concentration, markup $\uparrow$

in the market. We now turn to the decisions at the firms on innovation and reallocation.

Innovation Decisions. If there is positive product entry for fringe firms, the expected value of product entry must equal the entry cost κ^e adjusted by the wage-consumption ratio:

$$\mathbb{E}_\beta\left[u(\beta,0)\right] = \kappa_e^F \frac{\mathbf{w}_t}{\mathbf{C}_t}. \tag{28}$$

The optimal innovation by the leader requires that the marginal cost of innovation equals the marginal benefit of having an additional new product:

$$\mathbb{E}_\beta[v(\beta,0)] = D'(\eta)\frac{\mathbf{w}_t}{\mathbf{C}_t}. \tag{29}$$

Reallocation Decisions. A central focus of our model is the product reallocation flows across different firms. From equation (17), the equilibrium buyer-seller ratio for a product with quality (β,γ), where buyers and sellers are both fringe firms, equalizes the marginal value of trade and the marginal cost,

$$\lambda'(\theta_t^{FF}(\beta,\gamma))\mathbb{E}_{\gamma'}\Big[u(\beta,\gamma') - u(\beta,\gamma)\Big]^+ = \kappa_s^{FF}\frac{\mathbf{w}_t}{\mathbf{C}_t}. \tag{30}$$

The result is similar for leader-to-fringe reallocation,

$$\lambda'(\theta_t^{LF}(\beta,\gamma))\mathbb{E}_{\gamma'}\Omega_t(\beta,\gamma,\gamma')^+ = \kappa_s^{LF}\frac{\mathbf{w}_t}{\mathbf{C}_t}. \tag{31}$$

The fringe-to-leader reallocation is the solution to the following problem to the leader who chooses the buyer-seller ratio per vacancy, θ, and the number of vacancies, v:

$$\max_{\theta,v} \quad \lambda(\theta)v\mathbb{E}_{\gamma'}\Omega_t(\beta,\gamma,\gamma') - \theta v U_t^F(\beta,\gamma) - \chi(v) \tag{32}$$

We expand on these details in Appendix C. Having characterized the innovation and reallocation decisions, we turn to closing the model with the household problem.

4.4 Closing the Model: Household Problem

In this section, we detail the household's consumption-saving and labor supply decision. The household can freely borrow or save by investing in a representative portfolio of firms in the economy, taking as given the interest rate and prices. This assumption means that the profit of firms are all accrued back to the household. As discussed earlier, we normalize the aggregate price index to be 1 and express other prices in their real units. Denote $\mathbf{w}_t$ as the real wage and $\mathbf{r}_t$ as the real interest rate. The household takes these prices as given and chooses its real consumption $\mathbf{C}_t$ and labor supply $\mathbf{L}_t$ to maximize:

$$\max_{c_{ikt}, \mathbf{L}_t} \int_0^\infty e^{-\rho t} \left[\ln \mathbf{C}_t - \varphi_0 \mathbf{L}_t \right] dt, \tag{33}$$

s.t.

$$\dot{a} = \mathbf{r}_t a_t - \mathbf{C}_t + \mathbf{w}_t \mathbf{L}_t, \tag{34}$$

$$\mathbf{C}_t \text{ given by (4) and (5).} \tag{35}$$

The optimal saving decision implies the Euler equation must hold:

$$\frac{\dot{\mathbf{C}}}{\mathbf{C}} = \mathbf{r} - \rho, \tag{36}$$

and the optimal labor supply decision requires that the marginal rate of substitution between leisure and consumption equals the real wage:

$$\varphi_0 \mathbf{L}_t^{1/\varphi} = \frac{\mathbf{w}}{\mathbf{C}}. \tag{37}$$

Discussion of Assumptions.- Two key assumptions from the household side lead to the simplification of the environment. First, by assuming the consumption from different product groups is aggregated through a Cobb-Douglas utility function, we assume that evolution within each product group does not lead to reallocation of market shares across product groups. In addition to being the standard assumptions in the patent race literature such as Liu et al. (2019) and in the product dynamics literature such as Argente et al. (2021) and Hottman et al. (2016), this assumption is without apology given our context is the product market at an annual frequency where this reallocation across groups is small in magnitude. Second, by assuming the labor disutility is linear in labor supply, we assume the wage-GDP ratio is a constant. This assumption eliminates the general equilibrium wage effect through the endogenous innovation and search inputs of firms. This assumption is motivated by the fact that product market innovation is a relatively small part of aggregate innovation (e.g., as in Garcia-Macia et al., 2019). We may reasonably believe this general equilibrium impact is small. We also highlight our main results do not come from the labor costs.

We discuss the evolution of the distribution in Appendix C. The main finding from the distribution is that the holdings of both the fringe and leading firms can be expressed in closed form, linking the flows across firms and innovation to aggregate shares.

4.5 General Equilibrium and Aggregation

The goal of this model is to provide a conceptual and quantitative framework to link branding activity to macroeconomic outcomes to discuss the implications for efficiency and welfare. We do so in this section by discussing how innovation and reallocation in product markets lead to overall growth, concentration, and market efficiency.

Within-Group Equilibrium. We now reintroduce the notation for each product group k to discuss the equilibrium and aggregation. From the previous sections, we know the firm's optimal innovation and reallocation decisions and maturity process shape the overall aggregate appeal.

$$Q_{kt} = \int_{i \in \mathcal{I}^L_{kt}} q_{ikt} di + \int_{i \in \mathcal{I}^F_{kt}} q_{ikt} di$$

Given the detrended distribution of product quality and the innovation and reallocation rates, the growth rate of total quality within product group k is:

$$g_{kt} = \underbrace{\eta^L_{kt} + \eta^F_{kt}}_{\text{Innovation}} + \underbrace{\iota^L_{kt} + \iota^F_{kt}}_{\text{Maturity}} + \underbrace{\Lambda^{FL}_{kt} + \Lambda^{FF}_{kt} + \Lambda^{LF}_{kt}}_{\text{Reallocation}}. \tag{38}$$

where

$$\Lambda^{FL}_{kt} = \int_{\Omega_t(\beta,\gamma',\gamma)>0} \exp(\alpha_k + z_k + \gamma' - \gamma) \lambda^{FL}_t(\beta,\gamma) n^F_t(\beta,\gamma) d(\beta,\gamma) \tag{39}$$

$$\Lambda^{LF}_{kt} = \int_{\Omega_t(\beta,\gamma,\gamma')<0} \exp(\gamma' - \gamma - \alpha_k - z_k) \lambda^{LF}_t(\beta,\gamma) n^L_{kt}(\beta,\gamma) d(\beta,\gamma) \tag{40}$$

$$\Lambda^{FF}_{kt} = \int_{\gamma'>\gamma} \exp(\gamma' - \gamma) \lambda^{FF}_t(\beta,\gamma) n^L_{kt}(\beta,\gamma) d(\beta,\gamma) \tag{41}$$

$$\iota^L_{kt} = \int \exp(\iota(\bar{\beta} + \beta_0 - \beta)) n^L_{kt}(\beta,\gamma) d(\beta,\gamma) \tag{42}$$

$$\iota^F_{kt} = \int \exp(\iota(\bar{\beta} + \beta_0 - \beta)) n^F_{kt}(\beta,\gamma) d(\beta,\gamma) \tag{43}$$

η_{lt} and η_{ft} are the endogenous leader and fringe innovation decisions. ι_{kt} is the maturity process at the group level. Λ is the respective flows in each direction (FtL, LtF, FtF), which is a function of the distributions of α, β, and γ. We are now ready to define the following group-level equilibrium.

Definition 2 *(Group Equilibrium) A group equilibrium in group k is* $\{\phi_{kt}, g_{kt}\}$ $\{u_{kt}(\beta,\gamma), \theta^{FF}_{kt}(\beta,\gamma), \theta^{FL}_{kt}(\beta,\gamma)\}$, $\{\Omega_{kt}(\beta,\gamma_L,\gamma_F), \theta^{LF}_{kt}(\beta,\gamma), \eta_t\}$ *such that*

1. Given (ϕ_{kt}, g_{kt}), $\{u_k(\beta,\gamma), \theta^{FF}_k(\beta,\gamma)\}$ *solve the free entry condition equations (28) and (30);*

2. Given step 1, $\{\Omega_k(\beta,\gamma_L,\gamma_F), \theta^{FL}_k(\beta,\gamma), \theta^{LF}_k(\beta,\gamma)\}$ *solve equations (26), (31), and (32);*

3. Given Step 2, $\{n^F_t(\beta,), n^L_t(\beta,\gamma), g_{kt}, \phi_{kt}\}$ *solve equations (18) and (19);*

4. (ϕ_{kt}, g_{kt}) *are consistent with equation (21) and (38)*

The group-level equilibrium can be solved in isolation from the aggregate variables and in the order from step 1 to step 3. With the group-level equilibrium characterized, we turn to the overall brand appeal in the economy and consumer welfare.

This section builds a bridge from the outcomes in each market k to the overall efficiency in the economy. The results in this section inform the eventual discussion of aggregate efficiency and the social planner's problem. Given the partial equilibrium within each market, specifically $\{\phi_k, Z_k\}$, the following proposition summarizes the general equilibrium of the economy.

Proposition 3 *Given the product-group equilibria, the general equilibrium of the economy is characterized as follows:*

1. Given $\{\phi_k, z_k\}$*, calculate the following productivity, markup, and misallocation indices:*

(Productivity)

$$\mathbf{Z}(t) = \mathbf{Z}(0)\exp(\mathbf{g}t), \qquad \mathbf{g} = \int_0^1 \frac{\xi_k}{\sigma_k - 1} g_k dk; \tag{44}$$

(Markup)

$$\mathbf{M} = \exp\left(\int_0^1 \xi_k \log M_k dk\right), \tag{45}$$

$$M_k = \left[\frac{\phi_k}{1+\phi_k}\mu_k(\phi_k)^{1-\sigma_k} + \frac{1}{1+\phi_k}\left(\frac{\sigma_k}{\sigma_k - 1}\right)^{1-\sigma_k}\right]^{\frac{1}{1-\sigma_k}}; \tag{46}$$

(Misallocation)

$$\mathbf{A} = \int_0^1 \xi_k \left(\frac{M_k}{\mathbf{M}}\right)^{-1} dk, \tag{47}$$

$$A_k = \frac{\phi_k}{1+\phi_k}\left(\frac{\mu_k(\phi_k)}{M_k}\right)^{-\sigma_k} + \frac{1}{1+\phi_k}\left(\frac{\sigma_k}{\sigma_k - 1}\right)^{-\sigma_k}. \tag{48}$$

2. The aggregate objects $\mathbf{C}, \mathbf{L}_P, \mathbf{L}_S, \mathbf{L}_D$ *are given by*

$$\mathbf{C} = \mathbf{Z}\mathbf{A}\mathbf{L}_P \tag{49}$$

$$\mathbf{w} = \frac{\mathbf{Z}}{\mathbf{A}\mathbf{M}} \tag{50}$$

$$\varphi_0(\mathbf{L}_S + \mathbf{L}_P + \mathbf{L}_D)^{1/\varphi} = \frac{1}{\mathbf{AML_P}}. \tag{51}$$

The demand side of this economy resembles Edmond et al. (2015); thus, the welfare metric of our model follows the aggregation in the literature.

Definition 4 *(General Equilibrium) A general equilibrium is* $(\mathbf{L}_P, \mathbf{w})$ *such that:*
1. All product groups are in equilibrium as defined in Definition 2;
2. Given the group equilibrium, the aggregation holds as defined in Proposition 3.

More specifically, the discounted utility of the representative household in the steady-state equilibrium is summarized by the aggregate labor supply $\mathbf{L}$, the aggregate labor supply utilized in production $\mathbf{L}^P$, a measure of aggregate mark-up $\mathbf{M}$, and a measure of labor productivity $\mathbf{A}$:

$$\mathcal{W} = \int_0^\infty e^{-\rho t} \left(\ln \mathbf{C}_t - \varphi_0 \frac{\mathbf{L}^{1+1/\varphi}}{1+1/\varphi} \right) dt. \tag{52}$$

The aggregation is this section points to key ingredients for our quantitative analysis. Overall, this project focuses on two main components in the development of overall output. First, we look at the productivity of the economy, $\mathbf{Z}$, which correlates positively with social welfare. Then, we evaluate the misallocation, $\mathbf{A}$, and firm markups, $\mathbf{M}$, which tend to correlate negatively with social welfare. This general framework operates in the background of our estimation and quantitative analysis.

5 Estimation

We estimate the model parameters employing the empirical moments of product innovation, maturity, and reallocation. The model delivers simple objects that enable identification and estimation. We primarily explore two methods of estimation.[7]

5.1 Estimation Procedure

In the baseline estimation, we assume all product groups are identical in their parameters, and we estimate the model to match the aggregate moments, which we refer to as *homogeneous group estimation*. The homogeneous group estimation provides the natural estimation benchmark, and our primary results will be understood through this lens.

We then estimate the model assuming product groups are heterogeneous in their substitution elasticity, search cost, entry cost, leader productivity, and product quality distribution. We refer to this estimation

[7] We discuss the details and estimation methods in Appendix D.

as *heterogeneous group estimation*. This approach provides a more granular analysis of product-group interaction. In the following paragraphs, we detail the homogeneous group estimation procedure.[8]

Externally Calibrated Parameters. We set the discount rate to be the annual risk-free rate: $\rho = 0.02$. We set the labor supply elasticity to be $\varphi = 0.5$, and the innovation elasticity to be 1 $(D(\eta) = \kappa_e^L \eta^2)$. Hottman et al. (2016) estimate the substitution elasticities in a demand system that is similar to our setting. We thus directly take the estimates of UPC-level substitution elasticities from Hottman et al. (2016). In the homogeneous group estimation, we set the substitution elasticity $\sigma = 6.9$, which is the median of UPC-level substitution elasticity from Hottman et al. (2016).

Estimation of Search and Innovation Costs. We estimate the innovation and search costs to match observed innovation rates and reallocation rates. Three cost shifters exist that we allow to vary by product group: the innovation cost shifter d_k, the entry cost κ_k^e, and the search cost κ_k^s. Our model provides a direct link from observed market share and new product creation rate at the group level to these costs.

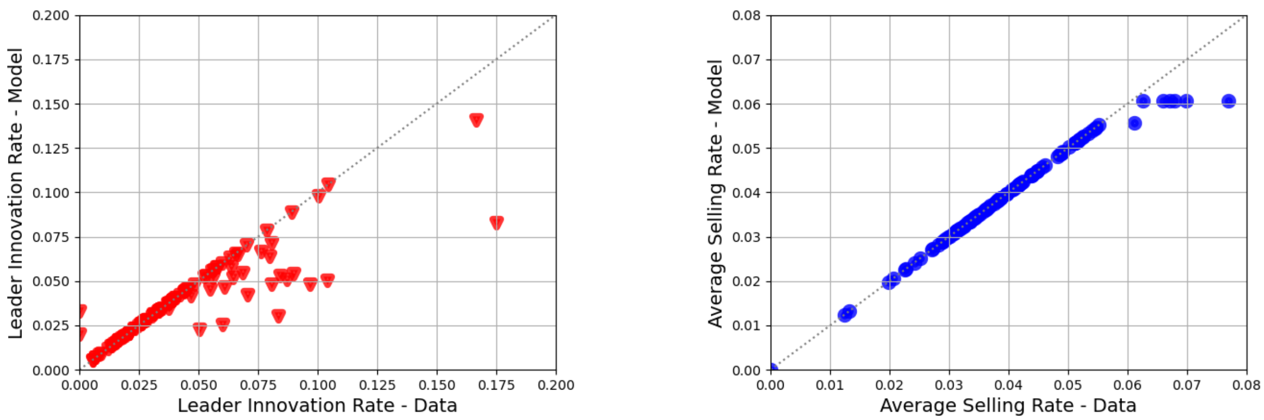
Table 8: Estimation Moments and Parameters

Parameter		**Value**	**Moment**	**Data (p.p.)**	**Model (p.p.)**
Independently Calibrated					
Household Parameters					
Discount Rate	ρ	0.03	Annual Risk-free Rate	**Exact Match**	
Substitution Elasticity	σ	6.90	Hottman et al. (2016)		
Firm Parameters					
Leader Advantage	α	0.65	Fact 1, Table 5		
Product Quality				**Exact Match**	
Distribution at Entry - $\mathbf{N}(0, \varsigma_{\beta_0})$	ς_{β_0}	2.31	Fact 2		
Age Profile - Growth Rate	ι	0.04	Fact 2		
Age Profile - Peak Appeal	$\bar{\beta}$	0.45	Fact 2		
Innov. + Reall. Elasticities				**Exact Match**	
Matching Elasticity	m	0.21	Sales-Reallocation Profile		
Innovation Elasticity	d	1.00	Akcigit and Kerr (2018)		
Jointly Estimated					
Leader Innovation Cost	κ_e^L	7574.72	Leader New Product Share	0.14	0.14
Fringe Innovation Cost	κ_e^F	5.57	Fringe New Product Share	1.02	1.02
F-t-L Reallocation Cost	κ_s^{FL}	1.44×10^{-2}	F-t-L Flows	0.77	0.74
F-t-F Reallocation Cost	κ_s^{FF}	0.426	F-t-F Flows	0.81	0.75
L-t-F Reallocation Cost	κ_s^{FL}	3.204×10^5	L-t-F Flows	0.55	0.52
Match Quality	Exp. Dist $\bar{\gamma}$	7.44×10^{-2}	F-t-L Flows	6.00	6.52
Leader Adv.	α	0.862	F-t-L Flows	56.80	53.61

We rely on the optimality conditions for innovation, entry, and acquisition to recover these parameters. First, we note the marginal value of the state variables can be written as functions of quality gap ϕ and growth rate g. Both variables have data counterparts. Specifically, the quality gap ϕ has one-to-one

[8]The estimation process is very similar for heterogeneous groups, but simply takes different inputs for each ingredient.

Figure 4: Innovation and Selling Rate: Model v. Data



(a) Innovation Rate (b) Selling Rate

Notes: The left panel plots the group-level leader innovation rate (red triangle points) from model (y-axis) and from data (x-axis). The right panel plots the group-level average selling (red triangle points) from model (y-axis) and from data (x-axis). In both panel, the grey dotted line is the 45-degree line.

mapping to the observed market share given σ_k; the growth rate g is linked to the new-product innovation rate by the fringe firms. With these two variables, we can directly calculate the marginal value of products to the group leader. For each product group, we find the set of parameters $\left(\kappa_k^e, \kappa_k^s\right)$ that minimize the distance between data prediction of the leader's innovation rate, average selling rate, and innovation rate of fringe firms.

Estimation of Matching Elasticity. We estimate the innovation and matching elasticity using indirect inference. The targeted moment for this elasticity is the age profile of a product getting transacted. In our model, the difference between the transaction rate for a new product and for a matured product is governed by the difference in marginal benefits as well as the matching elasticity. The difference is in the matching elasticity. In the extreme, if the matching function is inelastic with respect to tightness, no differential in the sales-transaction rates exists. Our estimation yields a matching elasticity of 0.292.

5.2 Comparison of Untargeted Moments

We now compare the predictions of the model regarding the data moments that are not targeted in the estimation procedure to inform the overall model fit. We discuss three different moments of interest. First, we discuss market concentration (e.g. market leader shares), which emerge from innovation, maturity, and reallocation. Second, we discuss the M&A premium with reference to the literature. Third, we return to our event studies on the prices and sales of products when transferred across firms.

Market Concentration. Our model is estimated without specific reference to a leading firm's market concentration. However, the model delivers a steady state market concentration (e.g., leading firm's share) that is consistent with the data. In the data, if we take sales-weighted leader market share across groups and years, we find the leader to have around 36.8% market share, while the model delivers a prediction of 38.5%. This result is encouraging that the dynamic patterns in the model can deliver a close fit with the concentration without specifically targeting it.

M&A Premium. The way gains from trade are split between buyers and sellers of product ownership is important for the counterfactual analysis. We thus compare our model's prediction regarding the rent splitting with the ones observed in data. In the models with random search (David, 2020), the rent split between buyers and sellers is primarily determined by the bargaining powers of both parties in the Nash bargaining step. In our model, due to the assumption of competitive search, the rent split is a by-product of the matching process, and thus primarily determined by the estimated matching elasticity m. David (2020) estimated the average premium to be 0.47. Our model predicts a weighted premium of 0.42, which is in line with the empirical finding.

Event Study. In our estimated model, we did not leverage the observed changes in prices and sales upon the reallocation of a product discussed in Section 3.3. In this section, we discuss predictions from the model on the corresponding outcome of a reallocation event and compare it with what we observe in the data. We start with a discussion of sales. In any fringe-to-leader event, the leader contributes the firm-level productivity to a given brand (α), but also may increase markups. The estimated model predicts that a leading firm acquiring a brand would be associated with a greater than 50% increase in sales and a corresponding increase in prices. This is consistent with firms having two key components of productivity (in the model α and z), that link to the ability to expand brand appeal and the cost of production. If firms expand distribution, this will more likely show up in sales. If firms produce at a lower cost, this will more likely show up in prices. Yet, the patterns in the data suggest both components are important. If we return to our event study, we find that the average increase in sales upon transaction from a fringe firm to a leader is 60% in the 3 years after the exchange, which is consistent with the model. The price increase is 5%, which implies that the cost of production declines upon acquisition. Overall, the model provides parameters that again identify these magnitudes out-of-sample, and provides an estimate of differential costs.

6 Quantitative Analysis

After estimating the model, we are now ready to explore the quantitative implications of our framework. The goal of this section is twofold. First, we aim to decompose the main driving sources of the variation in growth and concentration in our quantitative framework. Second, we explore various policy counterfactuals related to innovation, reallocation, and antitrust policies.

On the first point, we start by discussing the sources of growth (e.g., innovation, maturity, reallocation) through the lens of our model. We then turn to the sources of market concentration. The discussion of these two forces presents an important tension in the economy, and our policy analysis will explore this tension. We then analyze standard policies (e.g. transaction taxes and subsidies, entry subsidies) through the lens of our quantitative framework and evaluate their joint effect on growth, concentration, and consumer welfare. We focus on the main characterization under the homogeneous group estimation.

6.1 Sources of Growth: Innovation, Maturity, and Reallocation

Most models of growth from expanding product variety focus on product entry as the central driver of economic growth. Motivated by empirical evidence from Section 3, we depart from this standard, and note that reallocation and maturity are also essential components of growth. We return to our growth equation from the model to explore the interaction between these margins, inputting in the relevant data counterpoints. From the model, the growth rate of real consumption can be decomposed into the following margins:

$$g_C = \int_0^1 \frac{\xi_k}{\sigma_k - 1} \left(\underbrace{\eta_{kt}^L + \eta_{kt}^F}_{\text{Innovation}} + \underbrace{\iota_{kt}^L + \iota_{kt}^F}_{\text{Maturity}} + \underbrace{\Lambda_{kt}^{FL} + \Lambda_{kt}^{FF} + \Lambda_{kt}^{LF}}_{\text{Reallocation}} \right) dk,$$

Our goal is to use the growth equation to decompose the variation driven by the three main processes in the data. First, there are the innovation rates of the leader and the fringe. Second, there is the brand maturity. The last three are the reallocation flows: fringe-to-fringe, fringe-to-leader, and leader-to-fringe.[9]

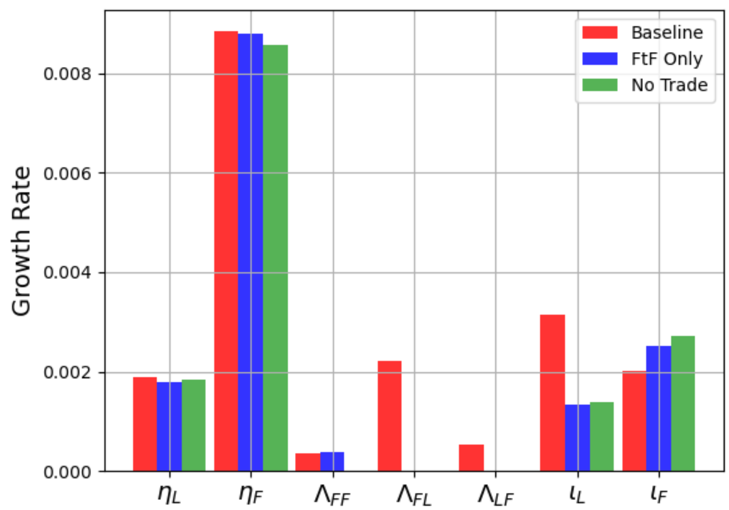
Figure 5 focuses on the contributions to growth in three different scenarios applying the homogeneous group calibrated model.[10] First, we evaluate the baseline economy (red), then we turn off fringe-to-leader exchange (blue), then we shut down trade completely (green).

We stress three main findings from Figure 5. First, as can be seen in the baseline economy, fringe entry η_F, leader brand maturity ι_L, and fringe-to-leader reallocation, $\Lambda^F L$, are the three main sources of growth. Second, when we shut down reallocation across firms, we find that the steady state responses of

[9]We apply the definitions of leader and fringe discussed in Section 3 and the theoretical specifications from Sections 4 and D.

[10]Due to the log-linear structure of the utility function, the growth rates at product group level can be decomposed and linearly aggregated.

Figure 5: Sources of Growth and Counterfactuals



each force is small. However, this does not include the transitional dynamics, as the economy is operating at lower capacity. We discuss the implications for overall welfare in Section 6.4.

6.2 Sources of Concentration: Innovation, Maturity, and Reallocation

Within the model, concentration can be decomposed into different sources:

$$\phi_k = \frac{\eta_L + \iota_L + (\alpha + \gamma)\Lambda_{FL} - \Lambda_{LF}}{\eta_F + \iota_F + \gamma\Lambda_{FF} - \Lambda_{FL} + \gamma\Lambda_{LF}} \tag{53}$$

For each type of flow, we define a counterfactual concentration as the ratio of appeals, when we assume the relevant margin is absent, and calculate the ratio between the predicted concentration and baseline concentration. This section force has the potential to limit output in the market through pricing power as discussed in Section 4. We focus now on the contribution of innovation and reallocation to concentration.

We first note that there are two relevant components of our analysis. First, leading firms and fringe firms may engage in different levels of innovation intensity which would lead to different levels of market shares on the balanced growth path and influence overall concentration. Second, there may be reallocation of product ownership from fringe to leaders and vice versa. This impacts directly the overall concentration in the market. We focus on the role of these two mechanisms and their interaction with concentration here.

Table 9: Concentration – Innovation vs. Reallocation

	All	Innovation + Reallocation	Innovation Only
a. Homogeneous Groups			
Baseline			
Leader's Market Share (%)	36.93	31.65	27.37
No Vertical Reallocation			
Leader's Market Share (%)	25.59	24.27	26.44
No Reallocation			
Leader's Market Share (%)	27.74	26.86	26.86
a. Heterogeneous Groups			
Baseline			
Leader's Market Share (%)	36.93	29.65	27.12
No Vertical Reallocation			
Leader's Market Share (%)	21.59	22.78	23.82
No Reallocation			
Leader's Market Share (%)	25.32	24.33	25.71

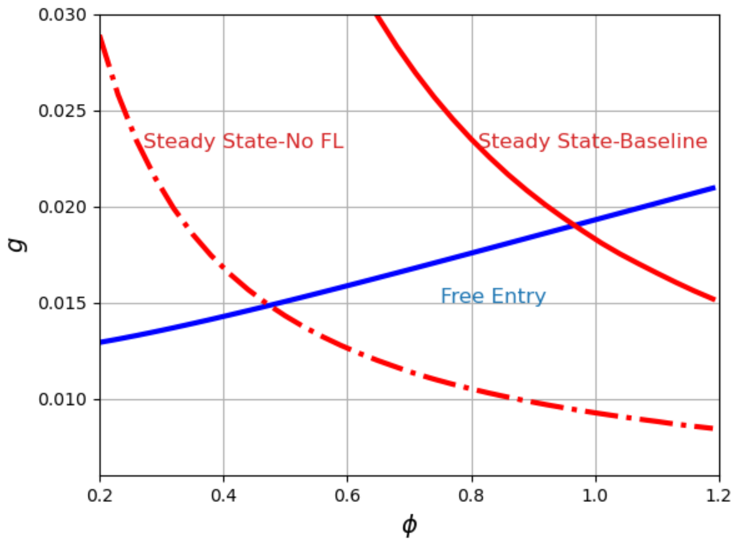
6.3 Good Concentration or Bad Concentration?

Recent literature (e.g., Covarrubias et al., 2019) point out that concentration can be "good" in the sense that more productive firms take larger market shares and increase consumer welfare, but it can also be "bad" in the sense that larger firms amass pricing power and restrict consumer substitution. Our quantitative results indeed point to the co-existence of both effects. Although we find in our baseline estimation the growth effect dominates and the reallocation of product ownership tends to increase aggregate efficiency, in this section we show this conclusion hinges on the magnitude of the love of variety. By increasing the love-of-variety elasticity ν, we up-weight the importance of growth in consumption. An informative threshold to note is the level of love-of-variety elasticity such that the growth effect and the concentration effect exactly offset each other. Figure 6 illustrates this tradeoff, plotting the equilibrium growth rate against the concentration calibrated from the homogeneous group model. In this scenario, we plot two steady state curves: one in the benchmark equilibrium, and one without any fringe-to-leader brand exchange.

There are countervailing effects on growth from rising concentration. A more concentrated industry $\phi \uparrow$ is associated with more opportunity from entry through reallocation and the higher markups from incumbents. However, given a level of brand appeal in the economy, a higher concentration will dampen growth through the ability of entrants to build brand appeal.

This connects on classic debates in the role of brands and marketing in the economy (e.g., Galbraith, 1958 versus Stigler, 1961), an issue we mostly sidestep in this paper. As brand capital continues to play a large role in the macroeconomy, this issue has important implications for thinking about the interaction of firm product ownership acquisitions. If an increase in sales at the consumer-level is due to the consumers love of variety, this leads to more efficient outcomes when leading firms acquire brands. If one takes the view that brand creation and brand development is simply an activity that poaches customer capital from

Figure 6: Concentration and Growth



other firms, the costs of concentration are higher than we find here. In this paper, we work mostly in line with the significant empirical literature that points to large consumer benefits from new products and product maturity, but believe these questions will be important in further research on the macroeconomic implications of brands.

6.4 Policy Analysis: Taxes and Subsidies

In this section, we explore various policy tools that may be implemented in markets where policymakers either have an interest in the costs of market concentration or the benefits of innovation. We apply some standard tools (e.g., transaction tax or entry subsidy) and ask about the economic implications of these policies for concentration, growth, and welfare. Our framework provides a general equilibrium setting where both growth and concentration of product market are endogenously determined by firms' innovation and reallocation activities, and have welfare consequences.

Reallocation Taxes and Subsidies. Table 10 focuses on the effects of different taxes and subsidies on reallocation, as well as government shutting it down. We perform these policies in one world where all markets have the same structure, panel (a), and one world where we allow the parameters to vary by group, panel (b). We ask what the corresponding change is to policy implementation in terms of concentration (leader share), growth rate, and welfare (in both the steady state BGP and including the transitional dynamics).

In the discussion, we focus on the homogeneous group estimation. First, we note that taxes and subsidies on reallocation have the expected effects on leader market share. Against a baseline of 37%

Table 10: Counterfactual – Tax/Subsidy on Reallocation

	50% Subsidy	10% Subsidy	10% Tax	50% Tax	No Trade
a. Homogeneous Groups					
Leader's Market Share (p.p)	51.30	40.58	31.55	28.41	25.59
Growth rate (p.p) (%)	2.142	2.044	1.741	1.581	1.472
Welfare (BGP, p.p)	-0.023	1.349	-0.241	-0.761	-1.930
Welfare (Transition, p.p)	-0.002	1.112	-0.132	-0.473	-1.332
a. Heterogeneous Groups					
Leader's Market Share (p.p)	61.21	52.93	29.19	23.04	21.5
Growth rate (p.p) (%)	3.824	2.093	1.5231	1.442	1.013
Welfare (BGP, p.p)	1.023	2.212	-0.978	-1.761	-2.177
Welfare (Transition, p.p)	1.227	2.101	-0.842	-1.511	-2.055

market share, high taxes can reduce the leader's share significantly. However, the net welfare effects are negative, as the overall growth rate also exhibits a significant negative decline. This decline in growth occurs through both the static loss from reallocation of products to better firms and the dynamic loss from entry. Shutting down trade completely is the most costly policy, while a 10% subsidy to trade increases welfare by 1.35% in the balanced growth path steady state. However, too strong a subsidy decreases welfare through too much concentration, even though both growth and concentration are higher.

While the strategic force appears to be present, the efficiency gains from reallocation overall outweigh strategic gains. This implies antitrust policies such as taxing transactions may not be efficient, if done at the *aggregate*. However, when we look across groups, we find that there are a set of groups where taxing transactions would be efficient. Thus, a key question for policy is at what level it is implemented. A coarser policy that does not take into account the rich market dynamics of each sub-market may induce efficiency losses.

Innovation Taxes and Subsidies. Table 11 focuses on the effects of taxes and subsidies on innovation. As in the previous table, we perform these policies in one world where all markets have the same structure, panel (a), and one world where we allow the parameters to vary by group, panel (b). We ask what the corresponding change is to policy implementation in terms of concentration (leader share), growth rate, and welfare (in both the steady state BGP and including the transitional dynamics).

We find a strong effect of innovation subsidies on both concentration and growth. As fringe firms have an easier time engaging in innovation than incumbents, the subsidy induces a lot of fringe firm entry. This also increases growth and welfare significantly. This message abstracts away from budget balancing at the government level, so policies for increasing government revenue to support subsidy policies would be important to evaluate. However, it does send a clear message relative to the taxes on transactions. Subsidizing entry is a means of reducing market concentration and increasing growth more so than taxing consolidation. Large firms engage in less product entry than fringe firms, and this allows for within-fringe

Table 11: Counterfactual – Tax/Subsidy on Innovation

	50% Subsidy	10% Subsidy	10% Tax	50% Tax
a. Homogeneous Groups				
Leader's Market Share (p.p)	11.04	25.02	42.34	52.01
Growth rate (p.p) (%)	3.142	2.303	1.131	0.931
Welfare (BGP, p.p)	3.204	1.783	-2.041	-3.203
Welfare (Transition, p.p)	1.592	1.529	-1.933	-2.182
a. Heterogeneous Groups				
Leader's Market Share (p.p)	13.20	31.59	52.34	60.31
Growth rate (p.p) (%)	3.824	2.093	1.5231	1.442
Welfare (BGP, p.p)	3.331	1.842	-2.589	-4.293
Welfare (Transition, p.p)	3.023	1.302	-2.224	-3.412

reallocation and declines in inefficient concentration. As in standard growth models, the entry margin is on net quite positive.

6.5 Discussion

In this section, we explore two aspects of the paper that did not play the main role in our quantitative analysis but provide essential ingredients for our study.

The Role of Maturity. The downstream innovation response to product reallocation is a function of the interaction of product maturity and reallocation. As a result, as a first order, policymakers can ignore the innovation effects of antitrust policy *when transactions are of mature products*, because the discounted value of transactions to entering firms is low. However, there is a rising tendency for products to exhibit shorter life cycles over time and become transacted earlier in their life cycle. For transactions early in the product life cycle, the dynamic effects of reallocation become more relevant. The option value of selling for a fringe firm becomes more relevant.

We discuss these results quantitatively here. We evaluate three benchmarks and three policies in Table 12. Table 12 describes the effects of policy depending on the maturity process in greater detail.

Table 12: Counterfactual – Maturity and Efficiency w/ Shutting Down Reallocation

	Baseline	Fast Maturity ($\iota \times 10$)	Slow Maturity ($\iota/10$)
Change in Leader's Market Share (p.p)	-13.21	-9.23	-17.29
Change in Growth rate (p.p) (%)	-0.321	-0.982	-0.141
Welfare (BGP, p.p)	-1.930	-2.130	-0.023
Welfare (Transition, p.p)	-1.332	-1.311	-0.001

From columns (1) to column (3), we consider how a different maturity rate of products (with $\iota = 4\%$ as the estimated baseline) leads to different market concentration and welfare incidence. We add two extreme cases in columns (1) and (3), one where a product grows at an average 0.4% per year until peak, and another where a product grows at an average of 40% per year until peak. We then compare changes in the innovation cost for entrants (κ_e) and changes in the search cost for product reallocation (κ_s, as a stand-in for an ownership transaction tax).

When markets mature quickly, there is a large growth and welfare cost to shutting down reallocation. This is because the policy has a larger effect on entry. When maturity is slower, shutting down trade decreases the leader's market share with a minimal impact on welfare. This occurs because the decrease in reallocation has very little effect on entry.

As for policy recommendations, both across industries and over time it is essential for policymakers to understand the life cycle profile and the age distribution of transactions. If older brands are much more likely to be sold, then the focus on transactions will weigh the markup and efficiency effects. The framework in our model, applying Atkeson and Burstein (2008) and Menzio and Shi (2011), is a useful method to link market shares, efficiency, and markups.

Yet, in markets where young brands are being transacted, policymakers should note the interaction between entry and reallocation. Entry responds positively to reallocation, but only if reallocation rates are linked to young products. If policymakers focus only on the predictions on sales and prices, they may miss the dynamic effects and induce efficiency losses by simply looking at the problem in a static setting.

The Demand System. Our demand system with nested CES structure follows a host of papers that study product markets, and we think this framework captures well the competition for product market share. One might wonder, what is the effect of changing the demand system to a different type of system (e.g. an aggregator as in Kimball, 1995)? We find that our results qualitatively go through. The key distinguishing aspect is that when products move from fringe to leading firms it has similar outcomes as when products move from smaller to larger firms more generally. Both Kimball (1995) and Atkeson and Burstein (2008) generate pricing power gaps that emerge through market concentration. We find nested CES the most demand system to discuss the role of multi-product firms and the mechanisms through which firms build their market power through product innovation and reallocation, because this is a demand system where the role of product scope is natural.

7 Conclusion

Brand capital is a central component of the modern economy, and brand reallocation plays a central role in sales concentration, firm dynamics, and efficiency. We employ a novel dataset on the universe of brands to unpack this interaction. After developing new facts that relate the dynamism of firms and products to overall concentration, we introduce a model of multi-product firms that innovate and acquire brands with potentially efficient and strategic incentives. These firms both tend to be more efficient than smaller fringe firms, but also have more pricing power through amassing brand capital.

We estimate the model using detailed price and quantity-level data at the brand and product-level. We find that small and large firms pursue different branding strategies and this leads to reallocation having a larger role in the large firm's portfolio. We use the estimated model to study a set of relevant policy counterfactuals: How does taxing brand exchange affect consumer welfare? We use the model to explore this question in detail.

We find tax policies tend to reduce both concentration and growth overall, leading to lower welfare. However, there is significant heterogeneity across product groups. If policy is coarse, taxes and subsidies will be ineffective because it will overall decrease the economic efficiency. If policy can be applied by group, there are gains from subsidizing exchange in some groups and taxing exchange in others.

Empirically, one avenue for further research is to understand the long-run evolution of the market for brands. This touches on essential long-run economic questions in both innovation and concentration. We believe understanding the brand-firm or product-firm interaction is essential to understanding this trend. Theoretically, as the importance of brand capital continues to rise, we believe frameworks that address the connection between brands and growth will be essential for academic and policy discussions. Putting brands into a growth model provides an important framework to understand the joint determinants of concentration and growth.

References

Abrams, David S, Ufuk Akcigit, Gokhan Oz, and Jeremy G Pearce, "The Patent Troll: Benign Middleman or Stick-Up Artist?," Working Paper 25713, National Bureau of Economic Research March 2019.

Aghion, Philippe and Peter Howitt, "A Model of Growth through Creative Destruction," *Econometrica*, 1992, *60* (2), 323–351.

_ , Christopher Harris, Peter Howitt, and John Vickers, "Competition, Imitation and Growth with Step-by-Step Innovation," *Review of Economic Studies*, 2001, *68* (3), 467–492.

Akcigit, Ufuk and Sina T Ates, "What Happened to U.S. Business Dynamism?," Working Paper 25756, National Bureau of Economic Research April 2019.

_ and Sina T. Ates, "Ten Facts on Declining Business Dynamism and Lessons from Endogenous Growth Theory," *American Economic Journal: Macroeconomics*, January 2021, *13* (1), 257–98.

_ and William R. Kerr, "Growth through Heterogeneous Innovations," *Journal of Political Economy*, 2018, *126* (4), 1374–1443.

_ , Murat Alp Celik, and Jeremy Greenwood, "Buy, Keep, or Sell: Economic Growth and the Market for Ideas," *Econometrica*, 2016, *84* (3), 943–984.

Altonji, Joseph G. and Robert A. Shakotko, "Do Wages Rise with Job Seniority?," *The Review of Economic Studies*, 1987, *54* (3), 437–459.

Argente, David, Doireann Fitzgerald, Sara Moreira, and Anthony Priolo, "How Do Firms Build Market Share?," Technical Report 2021. SSRN Working Paper.

_ , Munseob Lee, and Sara Moreira, "Innovation and Product Reallocation in the Great Recession," *Journal of Monetary Economics*, 2018, *93*, 1–20. Carnegie-Rochester-NYU Conference on Public Policy held at the Stern School of Business at New York University.

_ , _ , and _ , "The Life Cycle of Products: Evidence and Implications," Technical Report 2020.

_ , Salome Baslandze, Douglas Hanley, and Sara Moreira, "Patents to Products: Product Innovation and Firm Dynamics," *SSRN Working Paper*, 2020.

Atkeson, Andrew and Ariel Burstein, "Pricing-to-Market, Trade Costs, and International Relative Prices," *American Economic Review*, December 2008, *98* (5), 1998–2031.

Bain, Joe S., *Barriers to New Competition: Their Character and Consequences in Manufacturing Industries*, Harvard University Press, 1956.

Bils, Mark and Peter J. Klenow, "The Acceleration of Variety Growth," *American Economic Review*, May 2001, *91* (2), 274–280.

Broda, Christian and David E. Weinstein, "Globalization and the Gains From Variety*," *The Quarterly Journal of Economics*, 05 2006, *121* (2), 541–585.

Bronnenberg, Bart J., Jean-Pierre Dubé, and Matthew Gentzkow, "The Evolution of Brand Preferences: Evidence from Consumer Migration," *American Economic Review*, May 2012, *102* (6), 2472–2508.

_ **, Jean-Pierre Dubé, and Chad Syverson**, "Marketing Investment and Intangible Brand Capital," *Journal of Economic Perspectives*, August 2022, *36* (3), 53–74.

_ **, Sanjay K. Dhar, and Jean-Pierre Dubé**, "Brand History, Geography, and the Persistence of Brand Shares," *Journal of Political Economy*, 2009, *117* (1), 87–115.

Brown, GH, "Brand Loyalty: Fact or Fiction," *The Trademark Reporter*, 1953, *43* (1), 251–258.

Cavenaile, Laurent, Murat Alp Celik, and Xu Tian, "The Dynamic Effects of Antitrust Policy on Growth and Welfare," *Journal of Monetary Economics*, 2021, *121*, 42–59.

Covarrubias, Matias, Germán Gutiérrez, and Thomas Philippon, "From Good to Bad Concentration? U.S. Industries over the past 30 years," Working Paper 25983, National Bureau of Economic Research June 2019.

Cunningham, Colleen, Florian Ederer, and Song Ma, "Killer Acquisitions," *Journal of Political Economy*, 2021, *129* (3), 649–702.

David, Joel M, "The Aggregate Implications of Mergers and Acquisitions," *The Review of Economic Studies*, 11 2020, *88* (4), 1796–1830.

Davis, Steven J. and John Haltiwanger, "Gross Job Creation, Gross Job Destruction, and Employment Reallocation*," *The Quarterly Journal of Economics*, 08 1992, *107* (3), 819–863.

Davis, Steven, John Haltiwanger, and Scott Schuh, *Job Creation and Destruction*, Cambridge, MA: MIT Press, 1996.

De Loecker, Jan and Jan Eeckhout, "Global Market Power," 2018. National Bureau of Economic Research Working Paper 24768.

_ **,** _ **, and Gabriel Unger**, "The Rise of Market Power and the Macroeconomic Implications*," *The Quarterly Journal of Economics*, 01 2020, *135* (2), 561–644.

Deaton, Angus, *The analysis of household surveys: a microeconometric approach to development policy*, World Bank Publications, 1997.

Dinlersoz, Emin M, Nathan Goldschlag, Amanda Myers, and Nikolas Zolas, "An Anatomy of U.S. Firms Seeking Trademark Registration," Working Paper 25038, National Bureau of Economic Research September 2018.

Eaton, Jonathan and Samuel Kortum, "Trade in ideas Patenting and productivity in the OECD," *Journal of International Economics*, 1996, *40* (3), 251–278. Symposium on Growth and International Trade: Empirical Studies.

Edmond, Chris, Virgiliu Midrigan, and Daniel Yi Xu, "Competition, Markups, and the Gains from International Trade," *American Economic Review*, October 2015, *105* (10), 3183–3221.

Eggertsson, Gauti B., Jacob A. Robbins, and Ella Getz Wold, "Kaldor and Piketty's Facts: The Rise of Monopoly Power in the United States," 2018. National Bureau of Economic Research Working Paper 24287.

Einav, Liran, Peter Klenow, Jonathan D Levin, and Murciano-Goroff Ravid, "Customers and Retail Growth," 2021. Working Paper.

Fitzgerald, Doireann, Stefanie Haller, and Yaniv Yedid-Levi, "How Exporters Grow," Working Paper 21935, National Bureau of Economic Research January 2016.

Fons-Rosen, Christian, Pau Roldan-Blanco, and Tom Schmitz, "The Aggregate Effects of Acquisitions on Innovation and Economic Growth," *SSRN Working Paper*, 2021.

Foster, Lucia, John Haltiwanger, and Chad Syverson, "The Slow Growth of New Plants: Learning about Demand?," *Economica*, 2016, *83* (329), 91–129.

Galbraith, John Kenneth, *The Affluent Society*, Houghton Mifflin, 1958.

Garcia-Macia, Daniel, Chang-Tai Hsieh, and Peter J. Klenow, "How Destructive Is Innovation?," *Econometrica*, 2019, *87* (5), 1507–1541.

Gourio, François and Leena Rudanko, "Customer Capital," *The Review of Economic Studies*, 03 2014, *81* (3), 1102–1136.

Grossman, Gene and Elhanan Helpman, *Innovation and Growth in the Global Economy*, 1 ed., Vol. 1, The MIT Press, 1991.

Gutiérrez, Germán and Thomas Philippon, "Declining Competition and Investment in the U.S.," 2017. National Bureau of Economic Research Working Paper 23583.

Hall, Robert E., "New Evidence on Market Power, Profit, Concentration, and the Role of Mega-Firms in the U.S. Economy," 2018. Standofrd University, mimeo.

Heath, Davidson and Christopher Mace, "The Strategic Effects of Trademark Protection," *The Review of Financial Studies*, 08 2019, *33* (4), 1848–1877.

Hottman, Colin J., Stephen J. Redding, and David E. Weinstein, " Quantifying the Sources of Firm Heterogeneity *," *The Quarterly Journal of Economics*, 03 2016, *131* (3), 1291–1364.

Jaravel, Xavier, "The Unequal Gains from Product Innovations: Evidence from the U.S. Retail Sector*," *The Quarterly Journal of Economics*, 12 2018, *134* (2), 715–783.

Kimball, Miles S., "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of Money, Credit and Banking*, 1995, *27* (4), 1241–1277.

Kost, Kyle, Jeremy Pearce, and Liangjie Wu, "Market Power through the Lens of Trademarks," *Working Paper*, 2019.

Liu, Ernest, Atif Mian, and Amir Sufi, "Low Interest Rates, Market Power, and Productivity Growth," 2019. Becker Friedman Institute Working Papers Series 2019-09.

Menzio, Guido and Shouyong Shi, "Efficient Search on the Job and the Business Cycle," *Journal of Political Economy*, 2011, *119* (3), 468–510.

Peters, Michael, "Heterogeneous Markups, Growth, and Endogenous Misallocation," *Econometrica*, 2020, *88* (5), 2037–2073.

Ridder, M. De, "Market Power and Innovation in the Intangible Economy," Cambridge Working Papers in Economics 1931, Faculty of Economics, University of Cambridge March 2019.

Romer, Paul M, "Endogenous Technological Change," *Journal of Political Economy*, 1990, *98* (5 pt 2).

Shi, Liyan and Hugo A. Hopenhayn, "Knowledge Creation and Diffusion with Limited Appropriation," 2017 Meeting Papers 1501, Society for Economic Dynamics 2017.

Stigler, George J., "The Economics of Information," *Journal of Political Economy*, 1961, *69* (3), 213–225.

Appendix

A Data Appendix

This section addresses the set of data sources relevant for the analysis and the data examples that motivate our investigation. Section A.1 motivates the general setting by evaluating examples of market concentration and brand-building at the firm-level. Section A.2 expands on some notes on the merge across datasets.

A.1 Product Market Concentration: Large Firms

A salient feature of markets for intellectual property is the common rate of exchange, in particular with multi-product firms. We find this in both our data sources (USPTO and RMS Nielsen), and additionally from investigating firms in company reports of acquisition. As we discuss in the main text, product markets are dominated by large firms. We see this clearly in Figure A1, where many brands that individuals associate with only the brand are held by larger parent firms that aggregate brands.[11]

This general pattern is true across an array of industries, but the empirical section of this paper directs our attention to the Consumer Packaged Goods (CPG) industry. As such, we turn to a specific example of a large firm in the CPG space.

Brand-Building in the Data

Figure A1 illustrates how many brands are associated with the same firm. In Figure A1, for example, around half of the brands originally started at a different firm from the one it is currently linked to. The product acquisition and registration process is slow. Few of the brands in the graph were created by the same firm in the same year. This section discusses how this result can be seen in our data.

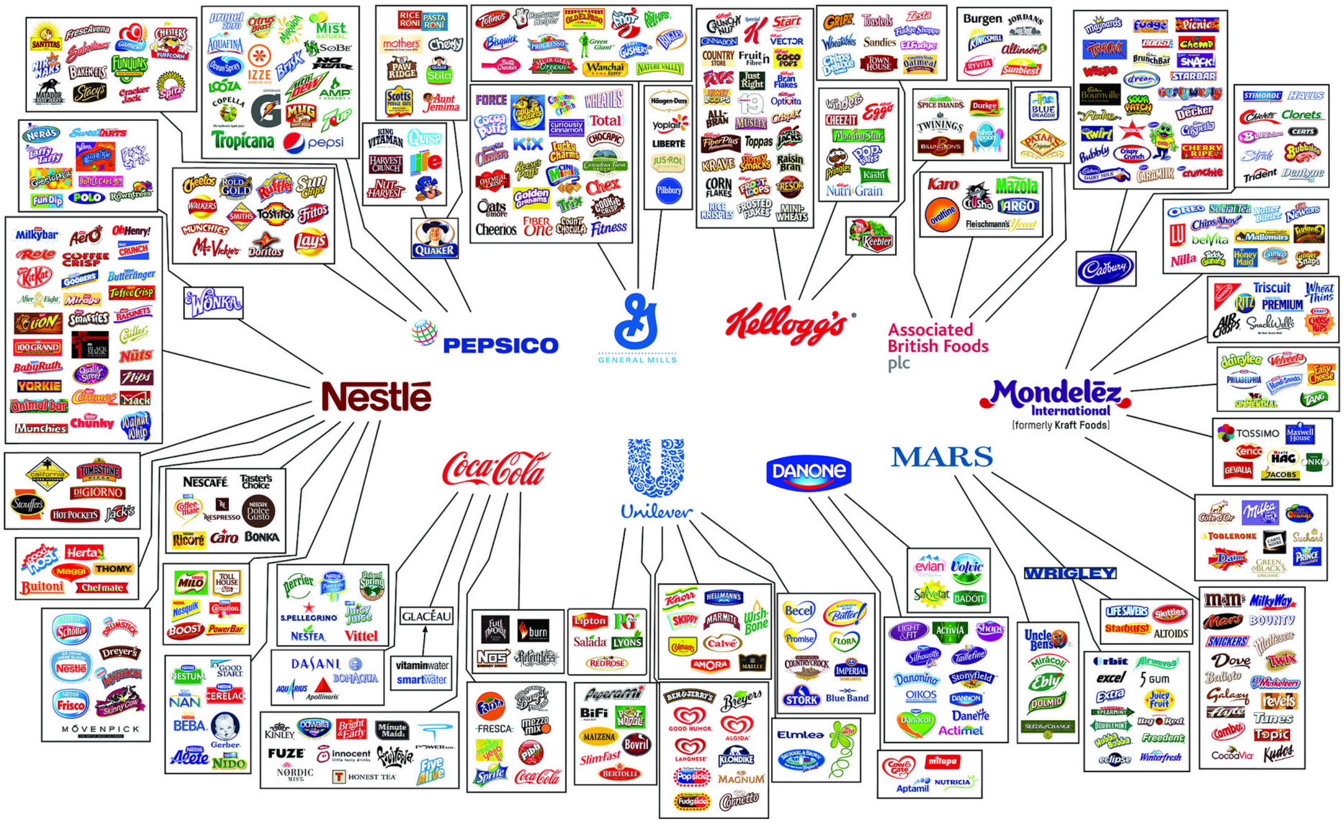
A.2 Data Merge Details

As discussed previously, our main merge links USPTO Trademark data with RMS Nielsen Scanner data. We proceed by linking firms and products separately. Our merge matches over 80% of sales-weighted products. Some problems still emerge with short-names. We use "tokens" and fuzzy matches to deal with the names. Firms and products follow similar procedures and we discuss them in turn.

Firms We employ keywords to merge firms. **Products** By focusing on brands, we direct our attention to long-running products held by firms. USPTO Trademark data provides the "tm_name" or the name associated with a registered trademark, ideal is merge by firm and product but if only product merges we merge it to the oldest brands

[11] Source: https://www.independent.co.uk/life-style/companies-control-everything-you-buy-kelloggs-nestle-unilever-a7666731.html, Apr 2017, accessed September 2022

Figure A1: Brands at Major Firms



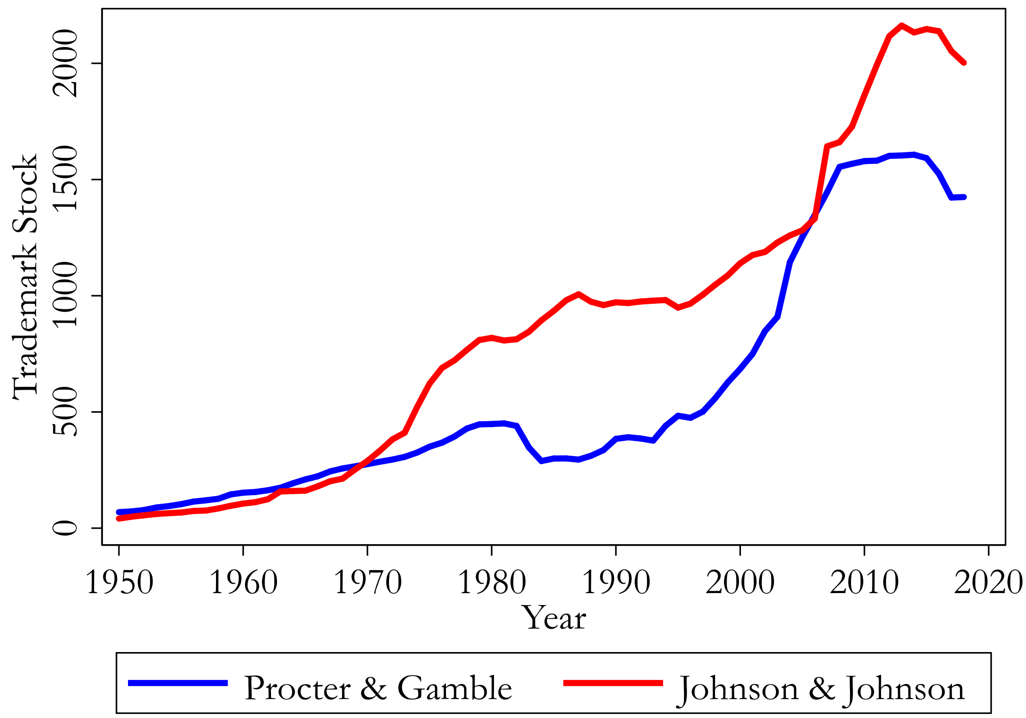
Notes: Parent Companies for each brand. Source: The Independent, 2017.

Data Sources: Transactions As we discussed previously, we leverage evidence form transactions in both USPTO and RMS Nielsen Scanner data. Overall, we get about 35% of brand transactions from USPTO Trademark data and 65% of transactions from Nielsen.

B Empirical Appendix

This section explores the main ingredients We start by expanding on the main elements of firm analysis, returning to the study of the sources of concentration in Section B.1. We expand on the product life cycle in Section B.2, focusing on the interaction of age and sales, and the evidence for the importance of product maturity and sales dispersion over time. We then turn to empirical robustness. First, we evaluate the robustness of the firm-level results in Section B.4. We turn to the robustness of product-level results in Section B.5. We then explore the event studies and the interaction of reallocation flows across firms in Section B.6.

Figure A2: The brands of P&G and J &J over time



Notes: This collects the total stock of trademarks held by P&G and J&J in each year since 1950. Includes trademarks held through registration and assignment. Source: USPTO.

B.1 Firm-Level Analysis

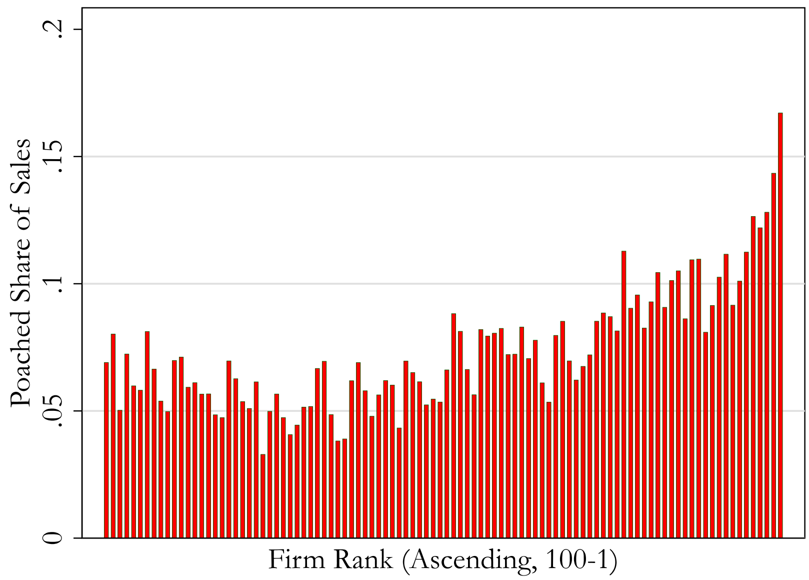
In Figure 1, we showed how buying of brands contributes significantly to large firms market share. Figure B3 shows this pattern with respect to sales in Nielsen Scanner Data. We plot the share of sales from bought brands against the percentile (running from 1-100) of the firm size in sales.

We find that the highest-selling firms have almost 4-times as much poached share of sales that a median firm, indicating that the pattern we find in the Trademark data on its own is consistent in the sales-share data. We observe this in both RMS Nielsen Scanner data and in USPTO Trademark data. Turning to USPTO data, we find the results are even more stark. Large firms tend to carry bought trademarks as a much larger share of their portfolio. This is noted in Kost et al. (2019). Figure B4 illustrates this fact as well.

In Table 4, we explored the concentration due to incumbent products versus entrant products. However, incumbent variation may be driven by many forces outside of the life cycle of the product. In Table B1, we evaluate the contribution for each firm attempting to fit the product-level predicted values (to eliminate the noise), and split the firms into three groups. Again we apply Equation (1).

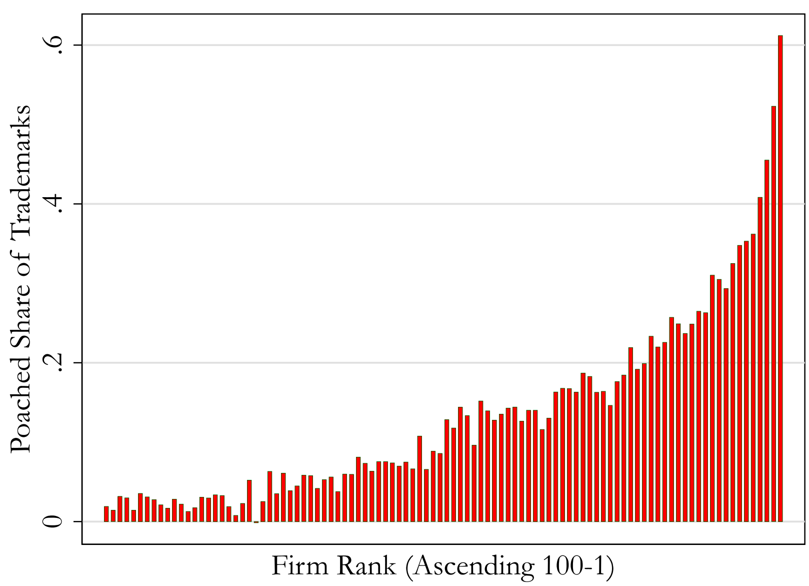
We find a similar pattern in Table 4 and Table B1. When we take fitted values from product-level age regressions, as discussed in the next section, the general pattern stays the same. We note that each force has a non-negligible contribution to the distribution of market shares.

Figure B3: Contribution of Buying to Sales Share



Notes: Market share reallocation measures across different firm types, following Equation (1). Source: RMS Nielsen

Figure B4: Contribution of Buying to Trademark Stock



Notes: Market share reallocation measures across different firm types, following Equation (1). Source: RMS Nielsen

B.2 Product-Level Analysis

Products are both a significant source of firm concentration (Hottman et al., 2016), yet highly dynamic (Argente et al., 2020a). The concentration implies a rich heterogeneity, but the dynamic nature implies that heterogeneity changes over time. The change in products can come from development of a product line or transactions of products from worse to better firms. Our goal in this section is to isolate the

Table B1: Sources of Reallocation

	(1) Entry	(2) Fitted Incumbents	(3) Acquisition
Leader	0.067*	0.62*	0.22*
Fringe	0.14*	0.47*	0.21*
Overall	0.14*	0.47*	0.21*
Average	-0.016	0.037	-0.032

* $p < 0.001$

Notes: Market share reallocation measures across different firm types, following Equation (1). Source: RMS Nielsen

product element of the life cycle and show how even separate from the firms that hold them products exhibit rich life cycles. This general point has been shown before (e.g. Argente et al., 2020a, 2021), but by integrating with USPTO Trademark data we are able to examine the longer brand life cycle and control for the transactions across firms.

Some products charge to dominance quickly, others rise gradually but maintain leadership, whereas others survive but remain in obscurity. Yet all brands must build consumer capital to build market share. We direct our attention to brand *age* is as an ingredient to product market shares. We first focus on a snapshot of the distribution of sales by age, and then turn to an analysis of the life cycle to understand the more granular dynamics.

Products evolve over their life cycle. Gourio and Rudanko (2014) and Foster et al. (2016), among many others, have noted that customer capital is not built in a day. By looking at trademark data and Nielsen data, one can observe the importance of senior brands. Figure B5 takes data from 2016. We plot the brand percentile in terms of overall sales on the x-axis. On the y-axis, we plot the share of sales in this group that belongs to brands older than 10 years and brands younger than 10 years.[12]

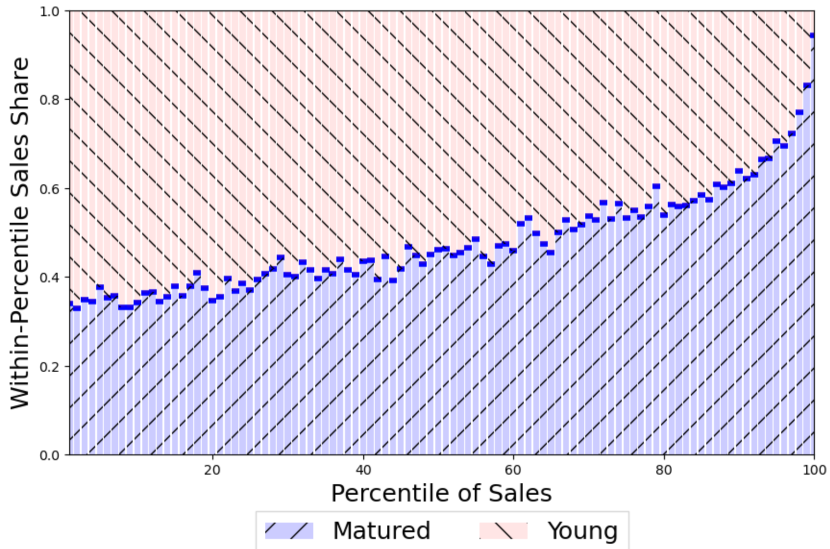
For brands created in 2006 and earlier, they maintain large sales share into the future. By 2016, those brands are still dominant in the top 1% of brands. Within the top 1% of brands, brands created before 2006 make up 92% of sales. Overall, old brands make up over 70% of sales, but only about 1/3rd of products. For the median brand in terms of sales, older brands make up less than half (38%) of total sales.

The dominance of mature brands could come from two forces. First, if few brands achieve such large sales, there may be a selection process. Young brands have less of a chance than old brands to have high consumer capital, as the brands that survive to maturity must have a high quality draw. The composition only selects for the best. Second, brands could increase their sales over the life cycle such that only mature brands have significant sales share. We aim to understand this by linking a brand to its specific age.

We now turn to study the product life cycle, where we leverage age data from trademarks registrations from the USPTO and sales data from RMS Nielsen. We use a method to study the life cycle of brands,

[12]We omit brands with less than $1000 in sales over an entire year, to have only brands that at least have a product line.

Figure B5: The Power of Mature Products



Note: This figure shows the sales share within a percentile bin of products, split by those born before 2006 ("Matured") and after 2006 ("Young").

Source: RMS Nielsen Scanner Data.

following similar work from Altonji and Shakotko (1987), Fitzgerald et al. (2016), and Argente et al. (2018). Figure B6 plots a regression that illustrates the nature of the product lifecycle in sales. This graph plots the coefficients to the following regression:

$$\log y_{it} = \alpha + \sum_{a=1}^{50} \beta_a D_a + \gamma_b + \lambda_t + \theta_i + \epsilon_{it} \tag{54}$$

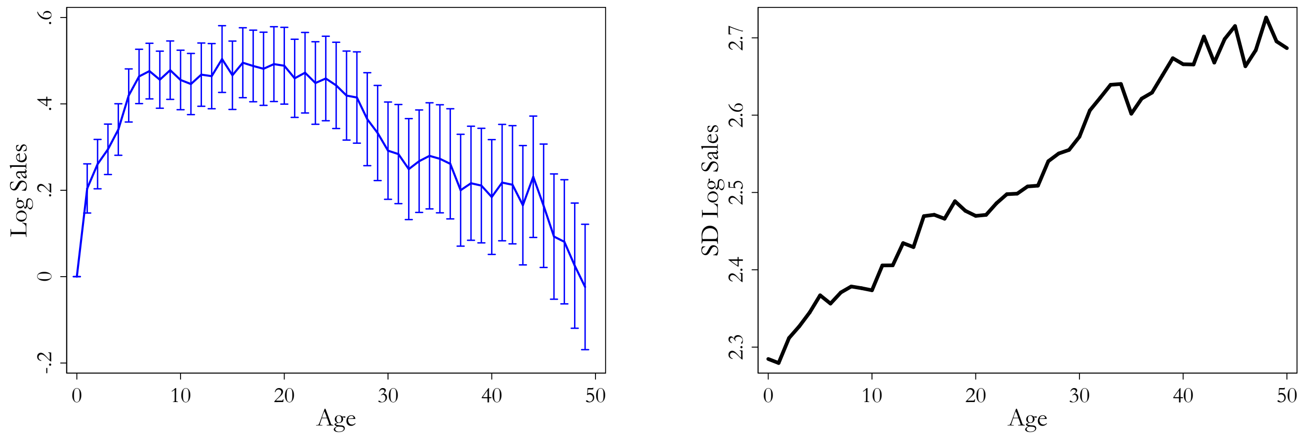
The regression in Equation (54) considers the sales of brand i at time t, $\log y_{it}$ as a function of a constant (α), brand age indicators from 1 to 50, D_a, and fixed effects for cohort (γ_b) and time (λ_t).[13] The θ_i indicates either a brand fixed-effect. Figure B6a plots the regressions by age coefficient β_a. The standard error bars indicate the 95-percent confidence interval for the point estimates for the regression with robust standard errors.

Figure B6b tracks the within-age dispersion. We note that the dispersion of log sales is increasing in age, which is relevant in our model. As brands mature, the dispersion of consumer goodwill varies more drastically.

There are two main takeaways from Figure B6. First, we see brands themselves exhibit an inverted-U pattern in sales over their life cycle. This is consistent with Argente et al. (2018, 2021), yet in brands the product life cycle is much longer and peaks far later. Second, brands exhibit much larger variance over time, indicating that the life cycle itself drives not only level differences but within age heterogeneity. Even at age 0, products show large sales dispersion, so this heterogeneity does not simply emerge through

[13]Given the linear relationship between age, time, and cohort, we follow a method developed by Deaton (1997) to correct for this issue. The normalization orthogonalizes the cohort trends such that growth components move with age and time effects.

Figure B6: Product-Level Dynamics



(a) Log Sales × Age Regression

(b) Log Sales Dispersion × Age

Notes: Source: USPTO Trademark Data and RMS Nielsen

the life cycle.

In Appendix B, we explore these differences in greater detail. The main point we make here is that when the unit of analysis is the consumer brand goodwill (e.g. the level of a trademark), and we extend the panel beyond the limited Nielsen age distribution the goodwill peaks later in the life cycle. Both of these ingredients lead to a later life cycle peak in our case.

B.2.1 Literature Benchmark: Product Life Cycle

We also aim to crosswalk some of our results to existing work on the product life cycle. We compare our benchmark against current product life cycle benchmarks in the literature. Recent work has focused on the life cycle of products applying Nielsen Scanner Data. This work is able to identify new products and brands and document their life cycle patterns. However, it is not able to link brands and products to their history, and is thus unable to speak to the longer time horizon of persistent brands. We perform similar life cycle regressions to the main text and compare them to a relevant current paper in the literature:

$$\log y_{it} = \alpha + \sum_{a=0}^{4} \beta_a D_a + \gamma_b + \lambda_t + \epsilon_{it} \tag{55}$$

Where the coefficients of interest are the coefficients on age (β_a) with controls for cohort and time effects (and an adjustment on cohort from Deaton, 1997). Table B2 engages in the same specification as Argente et al. (2018) in the UPC data (panels 1 and 2) and Trademark merged data (panels 3 and 4) respectively.

Note that while at the level of brands and trademarks there are significantly fewer observations, the same general pattern holds. This indicates how age is picking up something similar in our context, yet due to the broader horizon of historical data we are able to connect brands to their histories, indicating a

Table B2: Log Sales, by Nielsen and Trademark Age

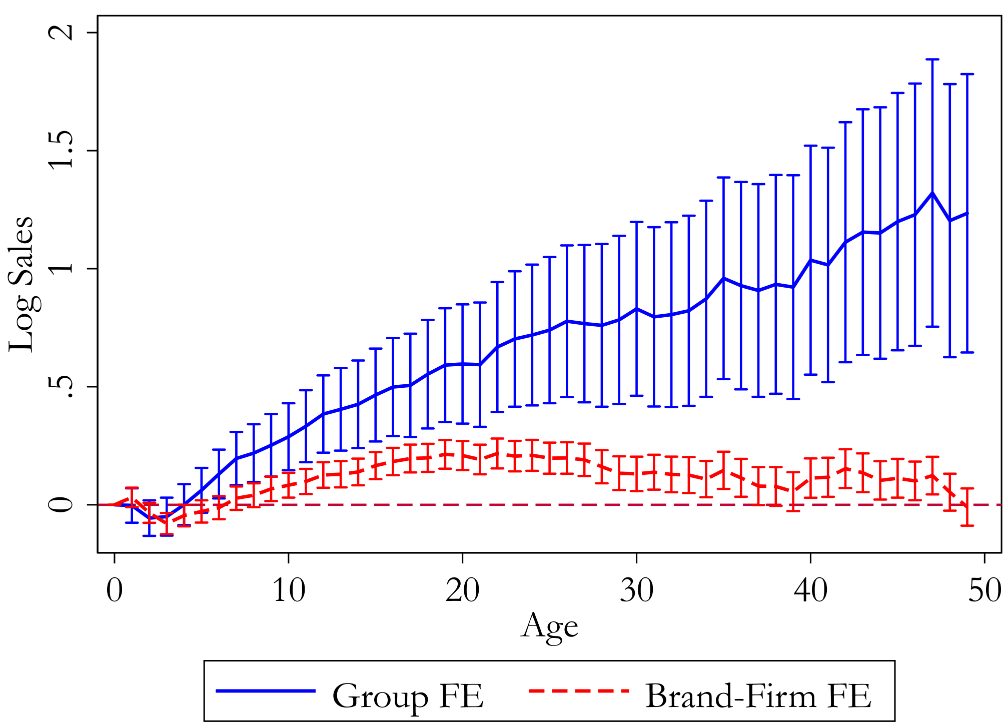
	(1) Log Sales	(2) Log Sales	(3) Log Sales	(4) Log Sales
Age 1	0.939***	1.095***	0.917***	0.953***
	(0.00841)	(0.0237)	(0.132)	(0.113)
Age 2	0.857***	1.159***	1.019***	1.060***
	(0.00876)	(0.0246)	(0.140)	(0.118)
Age 3	0.632***	1.016***	0.834***	0.832***
	(0.00914)	(0.0259)	(0.145)	(0.123)
Age 4	0.169***	0.644***	0.412*	0.488***
	(0.00995)	(0.0284)	(0.160)	(0.135)
N	668993	89203	3402	4136
R^2	0.138	0.179	0.256	0.050
Variation	UPC	Brand-Group	TM Brand	TM Brand-Group

Standard errors in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
SOurce: USPTO Trademark and RMS Nielsen

significantly longer brand life cycle than found in Argente et al. (2018).

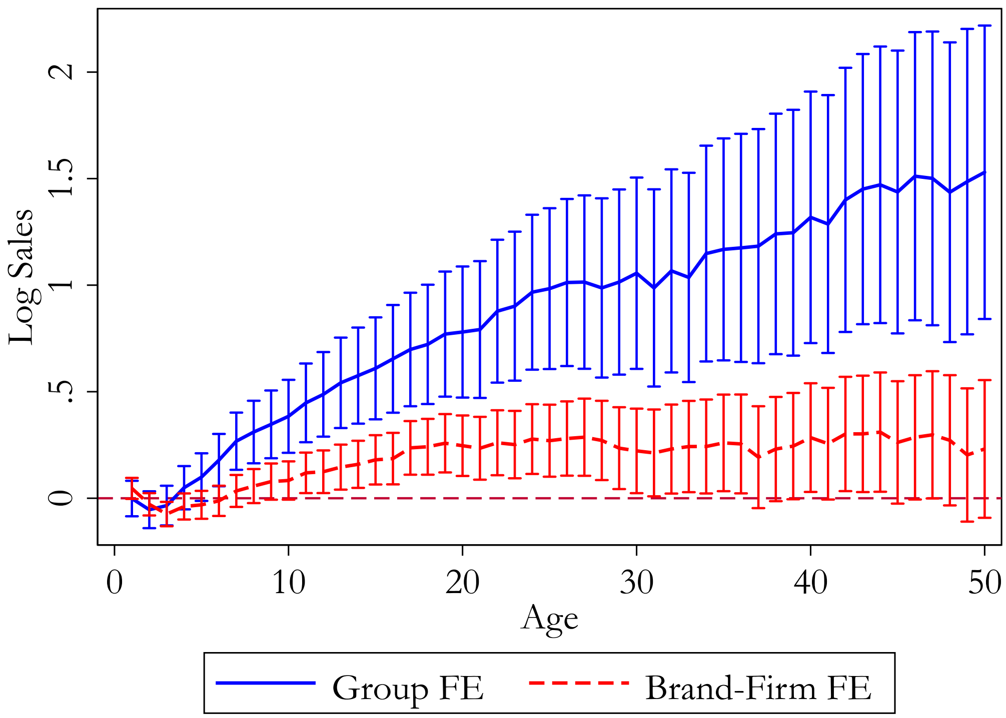
We also show here similar general trends as in the main text when we evaluate the life cycle of products, controlling for brand-firm-group level, with robust standard errors.

Figure B7: Life Cycle Regressions



When we go in the other direction and only control for brand, we find similar effects as in Figure B8.

Figure B8: Life Cycle Regressions



B.2.2 Coarsened Exact Match: Details

In this section, we expand on the coarsened exact match procedure in Section 3.3, discussing the method we use to link brands to their counterfactual brands prior to the event.

Assumptions The coarsened exact match relies on some assumptions that enable us to make a causal point on the change in sales and prices upon transaction.

- Parallel Trends

B.3 USPTO Trademarks: Reassignment

The most reliable long-term data source for product reallocation is USPTO Trademark data. As noted by Kost et al. (2019), reallocation has been steadily rising since the 1970s. Our focus in this paper is particularly on reallocation due to either pure reassignment (e.g. ownership transfer) or mergers & acquisitions. In this section, we discuss the general contours of the trademark data when it comes to reallocation of ownership. There is significant reallocation in the data, but some reallocation does not fall under the specific "merger" or "reassignment"

Table B3 splits the different transactions in the data into their different groupings. Most transactions in the data are available from 1970-2018. We order the transaction type by largest share of transactions. However, each transaction may contain a bundle of trademarks (e.g., transfer of ownership of "Odwalla" may be bundled with various sub-brands of the core brand Odwalla). As a result, we count separately transactions and trademarks. For example, in the case of "Security Interest" (or collateral), note that on average a larger number of brands are involved in the pledged bundle.

Table B3: Summary Statistics on Trademarks from USPTO

	Transaction Count	Trademark (TM) Count	TM/Transaction	Transaction Share	TM Share
Reassignment	478442	1.54M	3.21	**0.523**	0.345
Name Change	200767	795465	3.96	**0.219**	0.178
Security Interest	101280	1.10M	10.91	**0.111**	0.248
Merger	46610	287001	6.16	**0.051**	0.064
Correction	23500	119017	5.06	**0.026**	0.027
Other	64456	615334	9.55	**0.070**	0.138
Total	915055	4457996	4.87	1	1

Notes: Source:

While our main focus in this paper has been mergers and reassignments, we note the richness of the data on multiple margins. Name changes are frequent, as firms may attempt to retool but maintain brand loyalty. Further, as noted previously, trademarks are often used as collateral. While Security Interest transactions are a small share of overall exchanges (around 10%), they make up almost 25% of all trademarks in exchanges. However, without transfer the firm may continue to operate these product lines. The benefit of focusing on mergers and reassignments is the reallocation of ownership and management across firms, but we hope to see further research on these margins.

B.4 Empirical Robustness: Firm Measures

In our robustness, we return to look at the qualitative similarities between the results in the main text and results depending on the definition of the firm and the main data used. For the main paper, we maintain the same dataset, focusing on brands with at least $1000 sales in a given year and brands that successfully merge to a trademark. Furthermore, given the nature of ownership we keep only the primary owner of a brand. This robustness section focuses on the empirical facts at the firm-level addressing some changes to these definitions.

The results in Section 3.1 delivered two main messages. First, markets are highly concentrated, as the largest firm is almost 1000-times larger than the median firm across most product groups. This is noted in Figure 1 Table 3. Second, large firms outcomes are primarily driven by product maturity and product acquisition, while smaller firms rely much more on product entry. This is noted in Table 4.

This current section explores varying the definitions of the firm-product relationship. We explore the differences in both the RMS Nielsen data on its own and USPTO Trademark data. When exploring RMS Nielsen data, we expand our sales to include unmerged brands and brands with sales less than $1000.

When employing USPTO, we only keep firm assignment from USPTO Trademark data. As a result, we revisit Figure 1, Table 3, and Table 4.

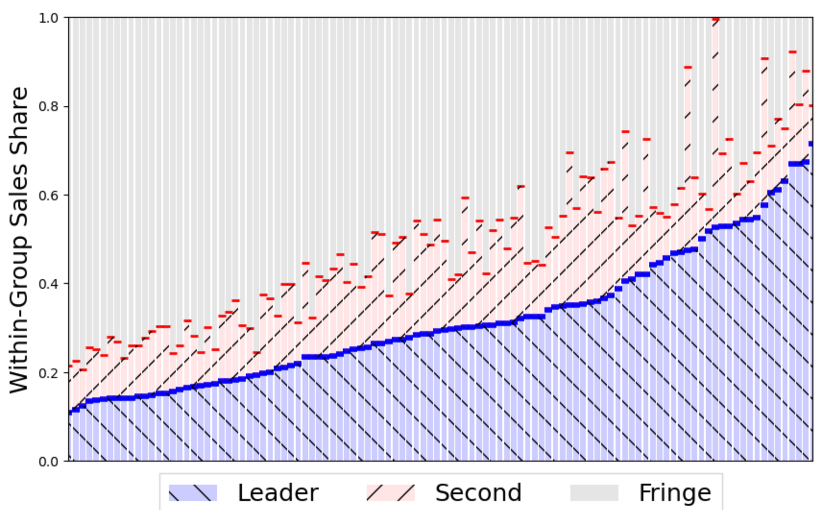
In overview, the results are qualitatively similar. We turn to the two main departures in our definition of the firm. We first look exclusively to Nielsen scanner data and then USPTO Trademark data.

RMS Nielsen Scanner

Our first fact primarily employed RMS Nielsen Scanner data, but we only included the successfully merged products in order to maintain a consistent sample. Given the success of the merge, one should expect the general results to be similar. In this section, we confirm that intuition.

We first plot the distribution of sales share of the product leader, the second firm, and the remaining firms in the market. This split by product group contains 116 unique product-group categories (e.g., ICE CREAM" or BEER). We plot Figure B9, following the same schema as Section 3.1, but including the full RMS Nielsen dataset.

Figure B9: Sales Share of Leader, by Product Group



Note: *This figure shows the sales share by product group (ordered by % share of leader) in 2010.* Source: *RMS Kilts-Nielsen Data Center & GS1 firm-product merge*

The average top firm share is 34% of the total market in the main part of the manuscript. When we expand our set, we find that the results are similar. We visit the shares in Table B4, as below:

Table B4: Firm Market Shares

Top firm share by group	Top 2 firm share	median share
30.8%	44.9%	0.05%

USPTO Trademarks

We now define firms at the USPTO level rather than the Nielsen level to explore different patterns in share holdings. For unidentified transfers[14], we maintain the originator as the parent company. We perform a similar exercise by re-evaluating Figure 1, Table 3, and Table 4.

B.5 Empirical Robustness: Product Analysis

In our robustness, we return to look at the qualitative similarities between the results in the main text and results depending on the definition of the firm and the main data used. For the main paper, we maintain the same dataset, focusing on brands with at least $1000 sales in a given year and brands that successfully merge to a trademark. Furthermore, given the nature of ownership we keep only the primary owner of a brand. This robustness section focuses on the empirical facts at the firm-level addressing some changes to these definitions.

Product Definition

We focus on the product life cycle in our data.

Transaction Definition

Transactions are defined at both the Nielsen and USPTO level. The reason we define transactions using both is as follows.

We note that when we plot the results applying only USPTO transaction information we find as follows. Multiple serial numbers per brand.

B.6 Empirical Robustness: Firm $\times$ Product Analysis

Prices and Sales at Top Firms

Our main text evaluated the differences in prices and sales in large firms versus fringe firms. We performed regressions with an indicator that provided information on the firm holding the product, including brand-group fixed effects, age fixed effects, and year. In this section, we explore varying the definition of a top firm in order to understand the differences in predicted sales.

Gross Flows and Net Flows

One of the main aspects of our paper focuses on the reallocation of products across firms. We identify this reallocation by jointly using RMS Nielsen Scanner data and USPTO Trademark data.

[14] There are cases where trademarks are reallocated to unidentified firms

Table B5: Log Price and Sales Conditional on Holding Firm

	(1) Log Sales	(2) Log Price	(3) Log Price (Sales weights)
Top 10 Firm Holding	0.57^{***}	0.044^{**}	0.057^{*}
	(0.000)	(0.032)	(0.065)
N	441300	441300	441300

p-values in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands.

Table B6: Log Sales Conditional on Holding Firm, Robustness

	(1) Log Sales	(2) Log Sales	(3) Log Sales	(4) Log Sales	(5) Log Sales	(6) Log Sales
Top 10 Overall	0.57^{***}	0.55^{***}				
	(0.000)	(0.000)				
Last Period Top 10			0.59^{***}	0.69^{***}		
			(0.000)	(0.000)		
Top 10 in 2006					0.47^{***}	0.53^{***}
					(0.000)	(0.000)
N	441300	3972	441300	3972	441300	3972
R^2	0.844	0.741	0.844	0.735	0.844	0.740

p-values in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands.

Event Study

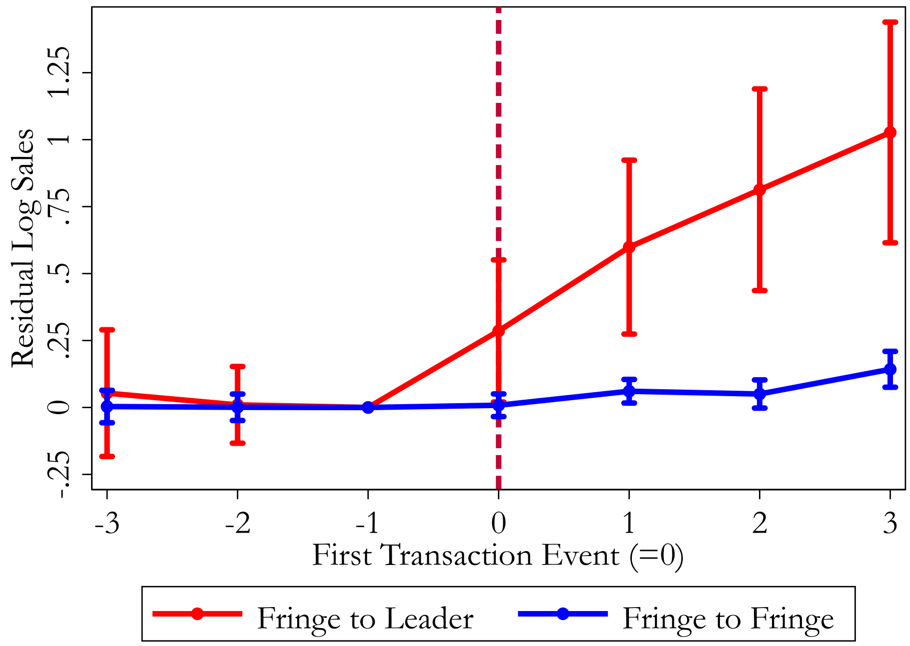
Our event studies focus on transactions across firms in the data. For an observed transaction, both the buyer and the seller must exist in the data. In order to identify the overall event study, we employ a balanced panel with seven periods. Given we use data from 2006-2018, we must restrict our event study analysis to brand transactions from 2009-2015. Due to some of the restrictions on our data, we focus on a broader definition of leading firms and flows from low-type to high-type firms. We explore the robustness of event studies depending on our characterization of an event study and definition of firm type.

To characterize flows that link fringe and leader buyers and sellers, we evaluate exchanges that move from smaller sellers to larger buyers, defined over the horizon of the sample. We make a couple of adjustments to the definition of a large firm to evaluate the robustness of our event study results.

Limiting attention only to brands that move between firms, we also evaluate the price and sales differences depending on the holding firm in Table B9.

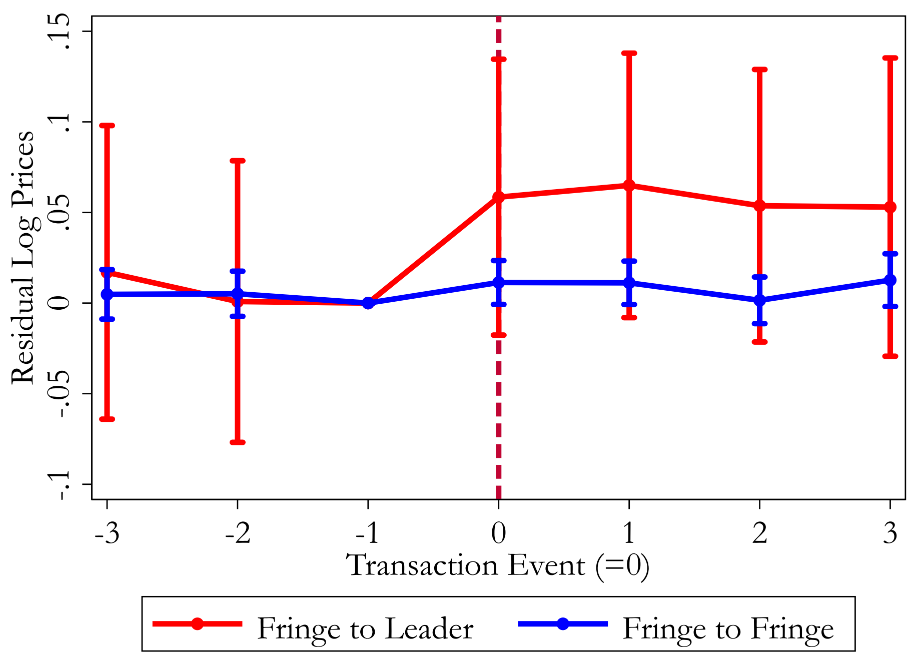
Figure B13 focuses on the different product innovation rates (entry as share of overall firm sales), and

Figure B10: Coarsened Exact Match and Brand Transaction, Sales



Notes: Coarsened exact match coefficients. Match is made on pre-trend sales and prices respectively, brand age profile, and year.

Figure B11: Coarsened Exact Match and Brand Transaction, Prices



Notes: Coarsened exact match coefficients. Match is made on pre-trend sales and prices respectively, brand age profile, and year.

Table B7: Log Price Conditional on Holding Firm, no weights

	(1) Lag Price	(2) Lag Price	(3) Lag Price	(4) Lag Price	(5) Lag Price	(6) Lag Price
Top 10 Holding (Overall)	0.044**	0.037*				
	(0.032)	(0.070)				
Lag Top 10 Holding			0.010**	0.036*		
			(0.012)	(0.082)		
Top 10 Holding (2006)					0.010	0.021
					(0.595)	(0.375)
N	441300	3972	441300	3972	441300	3972
R^2	0.945	0.884	0.945	0.884	0.945	0.884

p-values in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands.

Table B8: Log Price Conditional on Holding Firm, weight avg product share

	(1) Log Price	(2) Log Price	(3) Log Price	(4) Log Price	(5) Log Price	(6) Log Price
Top 10 Holding (Overall)	0.33**	0.26***				
	(0.013)	(0.000)				
Lag Top 10 Holding			0.053***	0.29***		
			(0.001)	(0.000)		
Top 10 Holding (2006)					0.14*	0.34***
					(0.073)	(0.001)
N	441300	3972	441300	3972	441300	3972
R^2	0.967	0.881	0.967	0.882	0.967	0.882

p-values in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands.

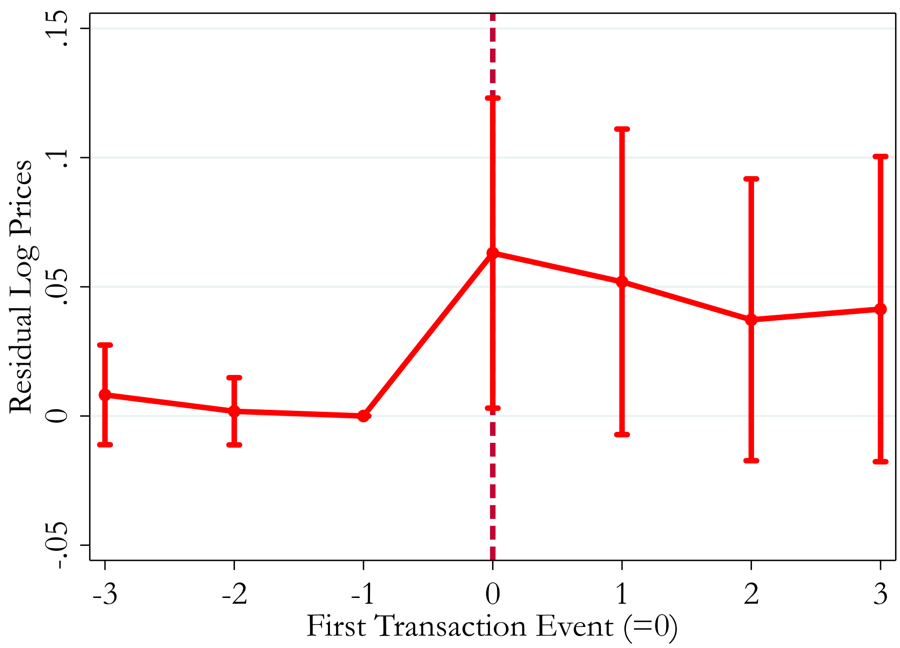
we note the much stronger entry rate of fringe firms than leaders.

Further, we note that overall in the transfer of product ownership there are more flows from small to large firms. This can be seen in Figure B14.

C Theoretical Appendix

This section expands on some model discussion in the main text. Section C.2 expands on the equations and proofs in the main text for the interested reader.

Figure B12: Defining by Fringe-Leader RMS Nielsen, CEM on Prices



Notes: Coarsened exact match coefficients. Match is made on pre-trend price changes, brand age profile, and year.

Table B9: Log Price and Sales Conditional on Holding Firm

	(1) Log Sales	(2) Log Price
Top 10 Firm Holding	0.65***	0.058***
	(0.000)	(0.000)
R^2	0.810	0.916
N	4430	4430

p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands.

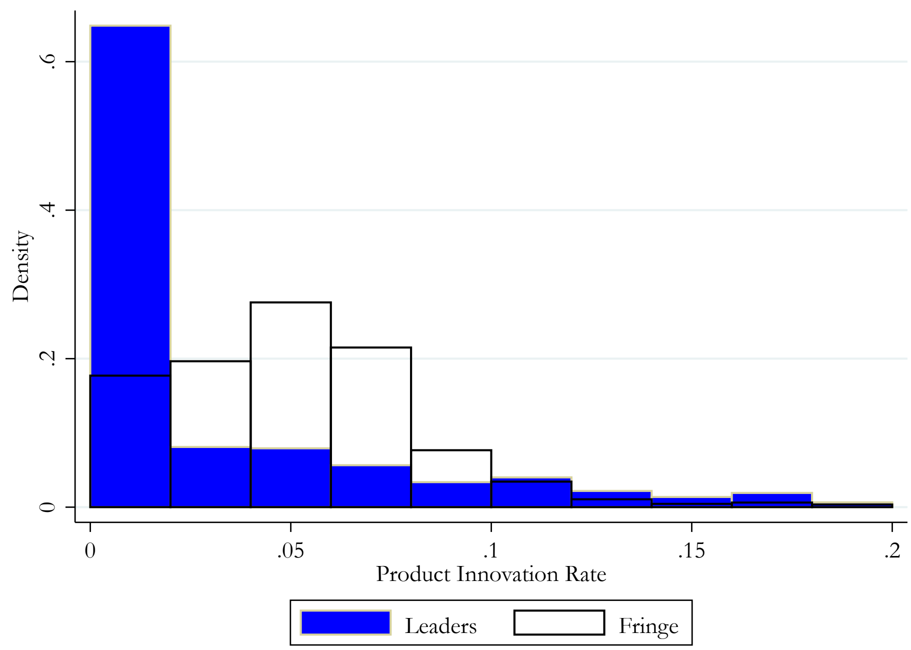
C.1 The Theory of Trademarks

Much of the essential literature in endogenous growth relates innovation to monopoly profits. The classic framework that delivers this is the *blueprint* framework, where technology-leading firms have the best blueprint for generate a consumer good product. The consumer simply chooses the best product to satisfy their utility function which aggregates across different groups, as follows:

$$C(t) = \int_0^1 z_j dj$$

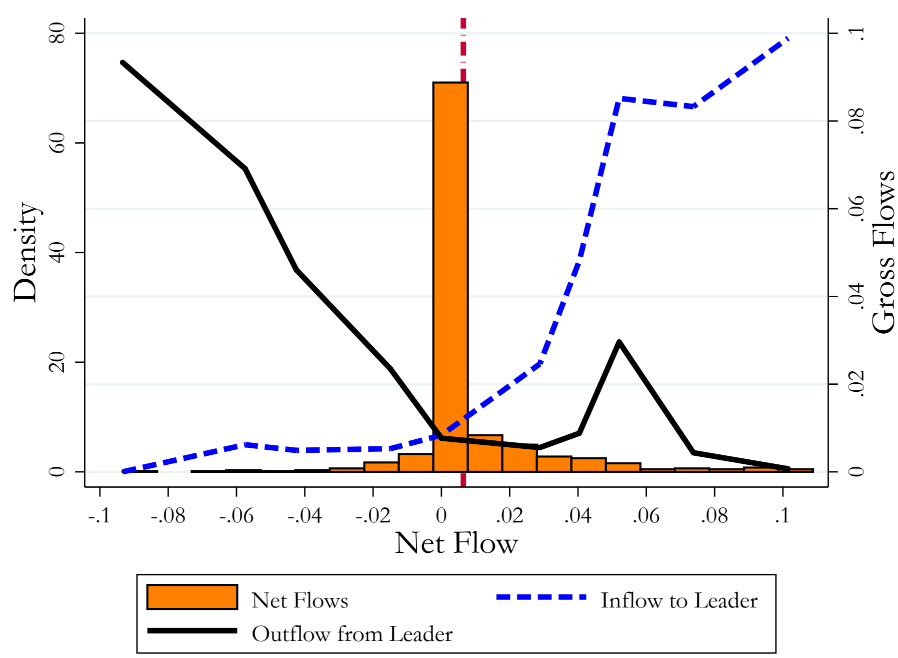
As a result, the leading firm charges a monopoly price and is the only seller in the market (as in Aghion

Figure B13: Product Innovation by Type



Notes: This looks at the product innovation distribution by firm type, *Source:* RMS Nielsen and USPTO Trademark. Fringe and leader defined as in text.

Figure B14: Gross and Net Flows, Fringe and Leader



Notes: This looks at the market shares transferred across firms in market shares by year, averaged by product group code. *Source:* RMS Nielsen and USPTO Trademark. Fringe and leader (top 10 firm) defined as in text.

and Howitt, 1992 and Akcigit and Kerr, 2018). If we imagine a fringe firm with cost ω_F and leader with cost ω_L, the leader will price at $p = \omega_F$ and capture the market:

$$\pi(Leader) = ALL$$

$$\pi(Fringe) = 0$$

Introduction of Trademarks Trademarks identify a product to a consumer and provide a means of charging above marginal cost. A trademark itself generates *imperfect substitution*. As a result, the leading technological firm may not capture the whole market, consumers may choose a brand due to its branding rather than the fundamental technology. This coheres with many markets where there are multiple competitors with brand capital and variable markups.

$$C(t) = \left(\int_0^1 z_j^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{a}{\sigma-1}}$$

C.2 Model Discussion and Proofs

Distribution

Denote $n^L(\beta, \gamma)$ the density of products with quality (β, γ) currently operated by the leader and $n^F(\beta, \gamma)$ the density of products operated by fringe firms, both normalized by the total quality Z. Denote the growth rate of quality Z by g_Z. In the steady state, they have to solve the following equations:

C.3 Proof of Proposition 3

To reach the aggregation result, we aim to write the real consumption as a function of production labor input $\mathbf{L}_P$, aggregate appeal $\mathbf{Z}$, and an aggregate efficiency of labor allocation $\mathbf{A}$. We start from the aggregation within product group k. Within product group k, the total group-level expenditure is $\alpha_k \mathbf{C}(t)$. Using the formula for sales shares, the expenditure for leader is

$$\frac{\phi \mu(\phi)^{1-\sigma_k}}{\phi \mu(\phi)^{1-\sigma_k} + \bar{\mu}^{1-\sigma_k}} \alpha_k \mathbf{C}(t) \tag{56}$$

Using the accounting equation for profit $\alpha_k \mathbf{C}(t) = \mu(\phi) \mathbf{w}(t) L_P$, we write that:

$$L_k(t) = \frac{\phi_k \mu(\phi_k)^{-\sigma_k} \frac{1}{Z_L(t)} + \bar{\mu}_k^{-\sigma_k} \frac{1}{Z_F(t)}}{\phi_k \mu(\phi_k)^{1-\sigma_k} + \bar{\mu}_k^{1-\sigma_k}} \frac{\alpha_k \mathbf{C}(t)}{\mathbf{w}(t)}. \tag{57}$$

$$Z_k(t)^{\frac{1}{\sigma_k-1}} L_k(t) = \frac{\phi_k \mu(\phi_k)^{-\sigma_k} \frac{Z_k(t)^{\frac{1}{\sigma_k-1}}}{Z_L(t)^{\frac{1}{\sigma_k-1}}} + \bar{\mu}_k^{-\sigma_k} \frac{Z_k(t)^{\frac{1}{\sigma_k-1}}}{Z_F(t)}^{\frac{1}{\sigma_k-1}}}{\phi_k \mu(\phi_k)^{1-\sigma_k} + \bar{\mu}_k^{1-\sigma_k}} \frac{\alpha_k \mathbf{C}(t)}{\mathbf{w}(t)}. \tag{58}$$

Adding across all product groups

$$\mathbf{L}_P(t) = \frac{\mathbf{C}(t)}{\mathbf{w}(t)} \int_0^1 \alpha_k \frac{\phi_k \mu(\phi_k)^{-\sigma_k} + \bar{\mu}_k^{-\sigma_k}}{\phi_k \mu(\phi_k)^{1-\sigma_k} + \bar{\mu}_k^{1-\sigma_k}} dk. \tag{59}$$

C.3.1 Search Process Discussion

In this section, we characterize the partial equilibrium in the search and matching markets, given (ϕ_k, Z_k) and the gains from reallocation across firms. Specifically, let $u_k(\beta, \gamma)$ be the discounted value of a fringe firm with product quality β and match quality γ, let $v_k(\beta, \gamma)$ be the discounted value of an additional product to the leader, and let $x_k(\beta, \gamma)$ be the discounted loss of an additional product operated by the leader in the calculation of leaders.

When positive buying flows into fringe firms occur, the optimal buying decision of a fringe firm with (β, γ) is as follows:

$$\kappa^s \varphi_0 = \max_{\tau} \frac{\lambda(\theta)}{\theta} \mathbb{E}_{\Delta} \Big[u(\beta, \gamma_L + \Delta) - \tau \Big]^+, \tag{60}$$

s.t.

$$\lambda(\theta) \mathbb{E}_{\gamma_L} \Big[u(\beta, \gamma_L) - \tau \Big]^+ = U^F(\beta, \gamma).$$

It is straightforward to show equation (60) is equivalent to the following problem in terms of solutions:

$$U^F(\beta, \gamma) = \max_{\theta} \lambda(\theta) \mathbb{E}_{\gamma_L'} \Big[u(\beta, \gamma + \Delta) - u(\beta, \gamma) \Big]^+ - \theta \kappa^s \varphi_0 \tag{61}$$

Equation (61) provides an intuitive interpretation of the reallocation process: due to directed search and the competition on the buyer side, the terms of trade aims to maximize the net benefit of reallocating products from fringe firms to other fringe firms, taking into consideration of the search friction and the cost of search. It is also worth noting that for each (β, γ), equation (61) can be independently solved without referring to the distribution of products across firms. This mechanism is the block recursivity highlighted in Menzio and Shi (2011).

Similarly, due to free entry of fringe buyers, the leader-to-fringe (LtF) flows can be characterized in the same way. For notation simplicity, we define the joint surplus of reallocating a product from fringe to leader as $\Omega(\beta, \gamma_L, \gamma_F)$. The equilibrium in the LtF market is characterized by $\{U^L(\beta, \gamma), \theta^{LF}(\beta, \gamma)\}$ that jointly solve the following problem:

$$U^L(\beta, \gamma) = \max_{\theta} \lambda(\theta) \mathbb{E}_{\Delta} \Big[-\Omega(\beta, \gamma, \gamma + \Delta) \Big]^+ - \theta \kappa^s \varphi_0. \tag{62}$$

The reallocation flow from the fringe to leaders is more complicated because there is no longer free entry on both sides of the market. However, the leader as a buyer faces competitive pressure from fringe buyers. In an equilibrium where both FtF and FtL flows are observed, the leader must offer the same expected value of selling as the fringe buyers. Thus, the optimal buying decision of the leader is

$$\kappa^s \varphi_0 \leq \max_{\tau} \frac{\lambda(\theta)}{\theta} \mathbb{E}_{\gamma_L} \Omega(\beta, \gamma', \gamma_F) - \frac{1}{\theta} U^F(\beta, \gamma_F). \tag{63}$$

D Estimation Appendix

In this section, we discuss in greater detail the estimation process, starting generally and then discussing the different ingredients central to our estimation.

Due to the block recursive structure of the equilibrium, the estimation procedure is done through the following steps in order:

1. We directly calibrate the substitution elasticity σ_k to the ones estimated in the literature;

2. Given the observed market share of leaders s_k and the substitution elasticity σ_k, we directly back out the quality gap ϕ_k: 3. Given ϕ_k, we jointly estimate $\{\kappa_s, \kappa_e, d_k\}$ that minimize the distance between the observed reallocation rate, ϕ_k, and leader's innovation rate, as well as fringe firms' innovation rate.

Elasticities, Shares, and Markups

To estimate the model, we need to establish the value functions of each agent and do value function iteration to link the shares and elasticities with the optimization problem of the leader and the fringe entry and selling decisions.

We start by specifying the leader's perceived elasticity, as discussed in the model, and in Equation 64,

$$\epsilon(s) = (\sigma(1-s) + s). \tag{64}$$

This simultaneously delivers a markup, of a leader with share s and a standard markup $\bar{\mu}$ for the fringe firm in Equation 65,

$$\mu(s) = \frac{\epsilon(s)}{\epsilon(s) - 1} \;\; ; \;\; \bar{\mu} = \frac{\sigma}{\sigma - 1}. \tag{65}$$

We also can specify the share as a function of the leader quality advantage ϕ, as follows:

$$s(\phi) = \max(1 + \phi^{-1}(\sigma/(\sigma-1)/\mu(x))^{1-\sigma}), (0,1)) \tag{66}$$

$$\Pi_{fringe}(\phi) = 1/\sigma(1+\phi)(1 - s(\phi)) \tag{67}$$

As a result, we can link the firm advantage ϕ to market shares and the elasticities firms face. This will represent the inner layer of our model, which will be determined by parameters we turn to next.

Innovation and Exchange Rates

Two key determinants of the market shares and allocations is the marginal value of innovation and the marginal value of exchange. We introduce the matching function and innovation function that firms face in the market.

‘

$$\lambda(\theta) = m_F \theta^{ematch}$$