Brand Reallocation, Concentration, and Growth

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Abstract

This paper studies the macroeconomic implications of firm-branding activity. We document how firms build market share by creating new brands, exploiting the maturity of their existing brands, and buying brands from other firms, with large firms relying disproportionately more on buying brands than small firms. Brands are heterogeneous, as selection in reallocation and brand consolidation lead to significant market concentration. Further, when a brand is reallocated from small to large firms, it leads to an increase in both the sales and prices of the brand. To interpret these findings and quantify the implications of brand reallocation on efficiency, we introduce an endogenous growth model where brand innovation, maturity, and reallocation activity determine both the growth rate and market concentration. In net, brand reallocation improves efficiency even as it increases concentration, and shutting down brand reallocation reduces welfare by 1.93%. A tax on reallocation decreases efficiency, as it alleviates the concentration distortion but decreases growth; a subsidy on entry improves efficiency, as it alleviates the distortion and increases growth. A 10% entry subsidy increases welfare by 5.84% relative to the baseline. Effective policy must take into account the speed of brand maturity and degree of brand heterogeneity.

Key Words: Endogenous Growth, Firm Dynamics, Productivity, Market Concentration, Product Innovation, Reallocation, Mergers & Acquisitions, Brands, Trademarks, Intangible Assets

JEL Code: O31, O32, O34, O41, D22, D43, L11, L13, L22

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1 Introduction

Brands are an essential intangible asset for firms. According to recent estimations of brand value, the top 100 brands in the US economy were worth \$4.14 Trillion in 2021 and the relative value of brands to traditional capital has been growing over time (Bronnenberg et al., 2022).[1] Brands allow firms to differentiate their products from competitors, and naturally affect the pricing power a firm has on its branded products. Firms build brand capital through introducing new brands, the growth of their existing brands, and the reallocation of brands from other firms. Brands, like technology, drive both economic growth through new product creation and significant market concentration. Despite rising interest in product market concentration and intangible assets, the macroeconomic implications of branding activity is less understood. This motivates us to ask the following: how do different branding activities, such as brand creation and reallocation, affect market concentration and economic growth? How do they affect the efficiency of the aggregate economy?

To answer these questions, we present a brand new dataset on the merge between products, brands, and trademarks, linking the intellectual property of firms to the sales and prices of their products. This dataset is a merge of the universe of registered trademarks from the US Patent and Trademark Office (UPSTO) and price and quantity-level data at the product (defined as brand by product-group) and brand-level from RMS Nielsen retail data. we document the following three facts regarding firms' branding activities:

Fact 1 *Large firms disproportionately have more brands and more market share, and build their market share through brand reallocation more than brand creation.*

Fact 2 *Brands build sales over time, and better and more mature brands are more likely to be reallocated across firms.*

Fact 3 *When a brand is reallocated across firms, sales and prices both increase.*

The patterns in the data are striking. Both the market share distribution and brand holdings distribution are skewed. The largest firm in a given product market is over 1000-times larger than the median firm in terms of market share, and hold 27-times more brands than the median firm, which holds only 1 brand. The market share distribution at the brand-level is highly skewed as well. The largest brand in a given product market is 1000-times the size of the median brand. Branding activity is highly dynamic, both in terms of creation and reallocation. On average, there are more brands created (10% per year) than reallocated (1.9% per year). However, newly created brands are smaller than reallocated brands. As a result, brand reallocation activity rivals entry activity in terms of market shares (2.2% v. 1.1% per year respectively).

[1]This \$4.14 Trillion represents 0.47 of the size of the total value of Property, Plant, and Equipment at the same firms.

To connect these facts together and answer our questions on the role of branding, we introduce an endogenous growth model with brand creation, maturity, and reallocation, allowing for both welfare gains through love-of-variety and welfare losses due to markups. In the model, consumers spend on imperfectly substitutable products distinguished by their brand. The imperfect substitution among products mirrors the role of brands in reality: all firms brand their products to some degree, and firms apply early in their life cycle for trademarks to protect their brand capital. In our model, each product group category has a multi-product leader and endogenous measure of single-product fringe firms, mirroring the disproportionate size of large firms in the market. The multi-product leader charges a higher markup than fringe firms because, as it holds many brands, it internalizes its impact on group-level prices.

Firms build their market shares through two types of activities. They can either introduce new brands through *product innovation*, or search and acquire existing brands from other firms, *product reallocation*. Firms seek to maximize their bilateral gains from trade when making search decisions. In our model, the gains from trade originate from two sources. First, the buying firm can be more efficient in operating a brand, which we refer to as *efficient reallocation*. Second, the buying firm may use the brand to exert a higher markup, which we refer to as *strategic reallocation*.

The efficiency of the decentralized equilibrium depends on both the number of products created and the level and distribution of markups across firms. Firms do not fully internalize their activities on both consumer love-of-variety and distortion from markups. These externalities imply that the economy faces an *efficiency-markup tradeoff*. With efficient exchange, the buying firm is better equipped to sell and distribute the product and hold ownership, either due to firm-specific advantage or firm-product fit. In this case, the interests of the firms and the aggregate economy align. On the other hand, strategic exchange occurs because the buying firm can exert market power due to limited consumer substitution across products. Here, the firm's interest in consolidation may not align with overall economic efficiency. While we classify these two types of transactions as benchmarks, many product exchanges can (and do) exhibit evidence of both efficient and strategic components.

We focus on *products* in our data, where we define a product as the consumer product good associated with a trademarked brand name (in USPTO) in a given product group category, e.g. "Cheerios" in "Cereal", as defined in RMS Nielsen. Most products are associated with a core trademark that the initial owning firm applies for upon entry. The USPTO defines a trademark as the following: "A trademark can be any word, phrase, symbol, design, or a combination of these things that identifies your goods or services. It's how customers recognize you in the marketplace and distinguish you from your competitors." Trademarks allow firms to protect the consumer appeal of their brand capital by providing a mechanism by which firms can profit through distinguishing their product. This type of intangible asset has a direct link to the pricing power of a firm, and estimates indicate brand value is a huge component of the US economy.

Whether a brand ownership reallocation is efficient or strategic has different predictions on prices and sales. In efficient exchange, we expect sales to increase and prices to decrease. In strategic exchanges, we expect prices to increase and sales to stay flat or decline. If exchanges are a mix of strategic and efficient, we may see a mix of both features. Bringing these predictions to data, we indeed find evidence of both types of exchanges. In an average exchange from a small to large firm, sales go up by 47% but prices also increase by 5%. As a result, these exchanges exhibit a mix of strategic and productive interests for large firms.

To better match the reality of consumer product markets, we further augment the model with following features: (1). we assume products are born heterogeneous and experience life cycles (e.g., as noted by Argente et al., 2020a) which may vary by product group; (2) we assume firms differ both by their fixed characteristics and their match quality with products, which leads to the reallocation of products in both directions among large and small firms; (3) we introduce a tax/subsidy on both innovation and reallocation activities to evaluate the policy implications. With these additions, the growth rate, concentration, and policy implications in our model are driven by both firm dynamics and brand dynamics, as well as their joint movement.

We use the estimated model to decompose the sources of concentration and growth and study policy counterfactuals. First, we find that reallocation explains over 30% of the market concentration in the typical product group. Shutting down reallocation leads to welfare losses of 1.93%, while a 10% tax in net reduces welfare by 0.43%. This occurs due to both the immediate reallocation effect and declining entry (as firms do not have the option to develop and sell their brands). Subsidizing entry is more efficient than the baseline case, as it increases growth and reduces concentration, a 10% subsidy leads to 5.84% increase in welfare in the balanced growth equilibrium. However, there is significant heterogeneity across product groups when it comes to effectiveness of policies.

Brand maturity and reallocation have an important interaction. In markets with fast maturity, e.g., firms can build brand capital quickly, product reallocation is less costly and induces more entry. Even strategic exchanges may be efficient, and shutting down reallocation is very costly. In markets with slower maturity, we find product reallocation can often be inefficient, as the strategic effect dominates. The findings suggest that optimal policy should incorporate the group-level fundamentals. Pharmaceuticals, cereal, and beer all face different market dynamics and thus coarse policies may not fit well specific markets. The lessons from this paper link the underlying fundamentals to optimal policy and provide important lessons for policymakers thinking about the distribution of brands across firms. This points to relevant trends in the US economy as we observe the rising value of intangibles, the rise of niche consumption, and concerns about market concentration.

The remainder of this section reviews the literature, while the rest of the paper is structured as follows. Section 2 introduces the USPTO Trademark Dataset and RMS Nielsen Scanner Data, and discusses our merge and some augmented datasets. Section 3 documents the key empirical facts that frame our investigation at the firm and brand-level. Section 4 introduces the model of product creation, maturity, and reallocation with variable firm productivity and variable markups. Section 5 estimates the model. Section 6 uses the quantified model to understand the contribution of specific margins and perform policy counterfactuals. Section 7 concludes.

Related Literature

This paper builds on and contributes to several literatures: the study of firm dynamics and product dynamics, the study of concentration, innovation, and firm profitability, the macroeconomics of M&A and technology transfers, and the study of brands and branding.

Firm performance is inextricably linked to its products. Hottman et al. (2016) study multi-product firms and find the scope of products explains a large share of sales variations across firms. Argente et al. (2018, 2020a) explore how product creation and destruction are pervasive in product markets. Further, Argente et al. (2021) and Einav et al. (2021) document the expansion of product sales is largely due to expansion of the customer base, making the distinguishing factors of products essential. We connect these important empirical insights to a firm's decision when they hold many products. Atkeson and Burstein (2008) introduce oligopolistic competition into a model with multi-product large firms, which is a building block of our model. Our paper links these papers through linking the product or brand life cycle to product innovation and reallocation, an in environment where firms have variable markups as in Atkeson and Burstein (2008) and product substitution shapes firms' decisions, productivity, and market power (e.g., as noted by Syverson, 2004a,b; Melitz and Ottaviano, 2008).

By linking product innovation and reallocation, we speak to a literature where product innovation is at the center of economic growth dating back to Romer (1990) and Grossman and Helpman (1991). This has also been found empirically, as product creation plays an essential role in both economic growth and the gains from trade, as noted by Bils and Klenow (2001), Broda and Weinstein (2006), Argente et al. (2018), and Jaravel (2018). Yet, this significant literature has not allocated attention to the role of brands in the macroeconomy, even though it has huge firm value and growth implications. Brands shape markets, and the ability to build a brand shapes brand entry and brand growth.

This paper addresses market concentration and innovation theoretically and empirically. Recent work has focused on rising markups (e.g., De Loecker and Eeckhout, 2018; De Loecker et al., 2020) and rising concentration (Gutiérrez and Philippon, 2017; Eggertsson et al., 2018; Hall, 2018) and the rise of superstar firms (Autor et al., 2020). To connect these discussions to growth, our model builds on the long literature of endogenous growth through creative destruction (Aghion and Howitt, 1992, Aghion et al., 2001, Akcigit and Kerr, 2018, Peters, 2020, and Liu et al., 2022), augmenting this with literature on entry and firm development (Jovanovic, 1982; Hopenhayn, 1992). Some papers in this tradition focus on the links between factor or labor reallocation and growth (Acemoglu et al., 2018; Garcia-Macia et al., 2019). Edmond et al. (2015) focuses on the markup channel, as large firms can leverage their large market share to charge high markups. Akcigit and Ates (2019, 2021) focus on the knowledge diffusion gaps between leaders and followers driving rising concentration and falling business dynamism. De Ridder (2019) focuses on intangible capital as a barrier to entry. Cavenaile and Roldan-Blanco (2021) study the interaction between firms advertising activities and growth, while Greenwood et al. (2021) focus on the macroeconomic effects of targeting advertisement.

This paper combines methodologies from the labor search and matching literature to topics in economic growth. Theoretically, we build on Menzio and Shi (2011) by embedding a directed search model into a growth framework. This differs in method from Lentz and Mortensen (2008), who embed random search into a growth framework to study reallocation and innovation. Empirically, we build on frameworks that study reallocation, mostly in the labor context, relating to work dating back to Davis and Haltiwanger (1992) and Davis et al. (1996). We note that similar measures can be used with intangible assets.

The reallocation of intangibles connects to questions on the aggregate implications of mergers and acquisitions (M&A) and patent reallocation. David (2020) studies the aggregate implications of M&A through the lens of a random search model, and finds M&A increases overall efficiency. Akcigit et al. (2016) study intellectual property misallocation and the market for patents and find this secondary market increases efficiency. Eaton and Kortum (1996) and Shi and Hopenhayn (2017) study how the appropriability of innovation, the ability to license or sell intellectual property, induces upstream incentives. Abrams et al. (2019) find evidence that intermediaries in intellectual property transfers exhibit both positive and negative effects on downstream innovation, while Cunningham et al. (2021) find "killer acquisitions" to have an important role in pharmaceuticals. In our model, similar mechanisms are present that we discuss in detail in Section 4. Two recent papers discuss the role of antitrust policies on growth, from the perspective of technological innovation (Cavenaile et al., 2021 and Fons-Rosen et al., 2021). Our theoretical framework relates to these papers in integrating the dynamic effects of transactions, but differs in (1) the focus on brand capital and (2) more importantly in an environment with endogenous variable markups. In our model, market concentration and growth are both endogenous and impact household welfare, which allows us to discuss the benefits and costs of various antitrust and innovation policies.

Lastly, we bring important insights from the literature on brands and branding to the macroeconomic debates on concentration and growth. Brands have long been known to be an important source of firm values (e.g., Braithwaite, 1928 on advertising and Brown, 1953 on trademarks). Bain (1956) noted that "(t)he advantage to established sellers accruing from buyer preferences for their products as opposed to potential-entrant products is on the average larger and more frequent in occurrence at large values than any other barrier to entry." Theoretically, brands have been shown to generate persistent profits in markets with imperfect information (Shapiro, 1983) and have value in exchange (Tadelis, 1999). The power of branding has been detailed empirically as consumer brand preferences are quite persistent (e.g., in Bronnenberg et al., 2009, 2012) and thus provide firms significant value. Gourio and Rudanko (2014) note how consumer capital is a relevant state variable for firms and products in attempting to bring customer capital into firms' dynamic behavior. Heath and Mace (2019) show empirically how this competition over customer capital generates strategic behavior in the market for trademarks, and is consistent with the significant degree of activity in the market (noted by Kost et al., 2019). This current paper builds on these papers in two respects. First, we link brand capital to the product market shares of firms and the aggregate economy. Second, we study how brands can be reallocated across firms, which makes the distribution of brand capital across the economy a key state variable of interest. We first turn to the underlying data regarding brands to establish the links between brands and firms.

2 Data

This project studies the connection between products, brands, and firms. In this paper, we define a product as a trademarked brand within a specific product group, for example "Cheerios" in "Cereal". From now on, our reference to product and brand will interchangeably refer to a brand $\times$ product group pair. Products are always held by firms. Some firms are small and carry one brand, but large firms in consumer packaged goods (CPG) and most industries carry multiple products. We provide new evidence in line with this finding, while jointly studying products and firms. This section links the products and firms in the two datasets.

We first motivate our empirical framework and then turn to the two datasets that serve as the bedrock for our empirical analysis. In terms of the framework, we separate the product or brand performance into three components. A product's performance of good i in firm j at time t (sales c_{ijt}) could be a function of the core product, the organization producing it (the firm), and a match-specific component (the firm-product fit):

$$c_{ijt}(product_{ijt}) = \alpha(firm_{jt}) + \beta(brand_{it}) + \gamma(brand_{it} \times firm_{jt}).$$

The most appropriate dataset to understand these forces would be at the product $\times$ firm-level, and would provide detail on brand history, including the prices, sales, and age of each brand. For the firms, data on the brands and sales of firms is needed. In this section, we describe each of the three ingredients through two datasets. This paper applies USPTO trademarks and RMS Nielsen Scanner Data to track the creation, distribution, and prices and quantities of products.

The trademark data provide the history of each brand and parent firm in terms of registrations, cancellations, and transactions. To focus on the dynamics of prices and quantities, we connect these firm-product-level data to specific information on product prices and quantities sold by stores in RMS Nielsen Scanner Data. To separately identify the effects of products and firms, we rely on transactions of products across firms. The following two sections discuss these datasets in turn.

2.1 USPTO Trademark Data

USPTO Trademark data provide a unique and comprehensive insight into the distribution and history of brands across firms. Trademarks are a central and dynamic arena of the economy, as firms register for trademarks whenever they want their brand legally protected. Trademarks are common, and more firms participate in trademarking than patenting.

In this paper, we direct attention to how trademark creation and exchange interact with the growth and concentration of firms. When firms create new products, they apply to the USPTO to protect the consumer appeal associated with the product. Further, when firms buy the rights to product ownership from other firms, the trademark is reassigned across firms.

To register for a trademark, a firm must undergo the following process. First, an individual who applies must pay a fee that ranges from $225 to $400. Within three months of filing, an examining attorney checks for compliance, and if the application is approved, it "publishes for opposition." A 30-day period follows, during which third parties affected by the trademark registration can step forward to file an "Opposition Proceeding" to stop the registration. This process is again evaluated by an examiner. If it clears this process, the trademark is registered.

The owner of a registered trademark has exclusive rights to use the mark within the sphere of activity designated by the legal process. Such rights include indefinite renewal conditional on continued use and the rights to exchange. Dinlersoz et al. (2018) and Kost et al. (2019) discuss the institutional aspects of trademarks in greater detail. Further, Appendix A presents some examples of firms with multiple brands and the firms' motivations for brand reallocation from press releases. Here, we turn to summary statistics on the number of trademarks and their distribution across firms. Table 1 provides general details on the number of firms and trademarks and the distribution of trademarks across firms.

We focus on two relevant features of the data from Table 1. First, the number of unique brand transactions is almost as large as the number of registered brands, indicating constant flow of ownership across firms. There are multiple types of transactions, which we discuss in detail in Appendix B.4. Another striking feature of the data is the skewness of firm size. The 99th percentile firm is over 80-times larger than the median firm in terms of the stock of trademarks. We note even more stark patterns in terms of sales, and this recurrent pattern of concentration is a central feature of our analysis. Without detailed price and sales-level data, the efficiency implications of this concentrated ownership is unclear. Linking

Table 1: Summary Statistics on Trademarks from USPTO

Data Object	Count
# unique firms	1.35M
# unique registrations	5.36M
# unique transactions by bundle	915076
# unique transactions by ID	4.46M
# unique cancels	2.12M
99th percentile firm size	83
75th percentile firm size	5
Median firm size	1
Mean firm size	5

Notes: Variables taken over entire sample of data, with size variables taken in 2010. Firm size is defined as the number of trademarks within a firm. Source: USPTO Trademark Data.

brands to prices and sales is the next step in uncovering these forces.

2.2 RMS Nielsen Scanner Data

Detailed product-level data are central to our analysis. We apply detailed store-product-level data that come from Kilts-Nielsen Retail Measurement Services Data from the University of Chicago Booth School of Business. The data are large and comprehensive in the consumer product space from years 2006-2018. Although we apply historical use of trademark analysis to understand the age and evolution of brands, 2006-2018 is our primary focus.

We observe more than 100 billion observations at the product $\times$ store $\times$ time level. Product is defined by a UPC identifier, 12 digits that are uniquely assigned to each specific good. The store is defined at the local level, with over 40,000 total; time is defined weekly but we collapse to annual data for our analysis. Total sales are approximately \$300 billion per year, covering around half of consumption in the grocery, drug, and merchandise stores (Argente et al., 2020a), which itself covers approximately 8% of total consumption in GDP. We apply a dataset from GS1 US, the official source for UPCs, to link parent firms to products through each UPC.

The UPC barcodes provide a unique identifier for each product. Changes in any attribute of a good corresponds to a new barcode. Barcodes are widespread and thus cover a large amount of the CPG industry. However, the unique identifying feature of the barcodes may not be as relevant for our analysis. For instance, the parent trademark associated with “Coca-Cola Christmas Edition” is the original “Coca-Cola”.

This dataset has been used widely for product analysis. One important departure from the literature in this paper is focusing on *brands*, that is, brand names and IDs listed in trademark and Nielsen data, rather than *products*, that is, UPC codes. We discuss three reasons for focusing on brands rather than products. First, consumer goodwill tends to be brand rather than product-specific. Coke 12oz relies on the same core branding as Coke 20oz. Thus, the brand is a more central indicator relevant to customer capital and firm valuations. Second, when firms exchange brand ownership, or the right to sell a specific brand, they tend to systematically transfer the full rights on the consumer goodwill, making the specific product differentiation within the brand less relevant. Third, our data enable identification at the brand level in both the Nielsen data and USPTO trademark data. Nielsen provides brand identifiers in addition to product identifiers. We collapse this information into brand $\times$ product group $\times$ year. We do not focus on geographical variation in this paper. While GS1 links to most parent companies, the USPTO trademark dataset helps complement GS1 to ensure the parent company is allocated to the correct brand and augments the data by delivering parent companies.

2.3 Data Merge

To link brand age, brand exchange, and product evolution, we employ a fuzzy merge to connect product names in RMS Nielsen scanner data to USPTO trademark data. Whereas this merge is the first we know of that links USPTO trademark data to Nielsen Scanner data, Argente et al. (2020b) link USPTO patent data to RMS Nielsen data. We follow a similar method but get greater coverage in our merge, likely due to the different nature of patents and trademarks. In particular, we are able to identify specific products connected to their brand name as long as the trademarked brand name is similar enough to the store brand name.

We start by normalizing names in each dataset at both the firm and product-level. For example, we want to capture heterogeneous naming at the firm (e.g., General Mill Holdings + General Mills Minnesota Op.) and connect it to the parent company. We then turn to the brands themselves. We employ a similar fuzzy match with brands. We start by linking observations at the firm $\times$ brand level, but for observations for which we directly observe the brand, we connect the brand independently and assign ownership through trademark data. We discuss the mechanics of the merging process in Appendix A.2.

Both USPTO and Nielsen scanner data contain a firm $\times$ brand of observation of interest. The identification of firms and brands provides what is ideally a many-to-1 matching between products (which rely on the same goodwill) and brands. In reality, we must rely on many brands and many product matches. For these matches, we focus on the most reliable name match. If the same brand has multiple matches, we take the "active" brand. For instance, if a brand is reassigned across firms, we assume this represents the focal brand. Once this match is complete, we treat products (or brand $\times$ product group code) as the relevant margin for the concepts in this paper. Overall, we take the leading firm in selling each brand and use Nielsen data and the holding firm in trademark data. If these are not aligned, we take the Nielsen data unless the trademark indicates a *transfer* of ownership, which cover approximately 20% of the exchanges in our data.[2] As a result, both datasets are combined to deliver our indicator of brand ownership.

We next turn to the quality of the match between brands and products. We focus in the USPTO case on brands, and note the products they connect to in Nielsen. Table 2 provides information on the match between products and trademarks.

Table 2: Summary Statistics on Trademark Nielsen Merge

	Unique Count	Years Active	Share Match (%)
USPTO Trademark Data			
Brands	5.36M	1870-2020	1.9%
Firms	371021	1870-2020	15%
Canceled Brands	2.12M	1970-2020	
Transactions	915076	1970-2020	
RMS Nielsen Scanner Data			
Brands	1.64M	2006-2018	57%
Firms	23232	2006-2018	54%
Brand $\times$ sales		2006-2018	82%

Notes: Summary statistics on share of merge brands in both datasets. Source: USPTO Trademark Data and RMS Nielsen Scanner Data

We stress a couple points from the Table 2. First, when we take brands merged with sales weights, we are capturing a significant share of sales in the data at 82%. Without sales weights, we capture fewer brands unsurprisingly. Some small firms may choose not to protect their intellectual property via legal means.

We find in this merge that not only are multiple brands associated with single firms, but also that multiple products are connected to a single brand. On average, we observe 13 unique products per brand. As a result, brands are an important source of firm income and, and noted in the following section, provide the central focal point for firm growth.[3] We now turn to the empirics of products, brands, and firms.

3 Empirical Analysis

This section focuses on the motivating empirical observations that inform our model and quantitative analysis. Following the structure discussed above, we focus on three main margins. We focus first on the

[2]For aggregate activity measures applied in the quantitative section, we apply transaction data when the buyer and seller are different, but may have an unidentified buyer or seller in our data.

[3]We consider the brand to be the core product a firm is producing, so in terms of the main takeaways of the paper, we treat product and brand as interchangeable.

firm-level margin and discuss firm concentration and firm dynamics. We then turn to the product-level margin, where we focus on product heterogeneity and product dynamics. We then turn to firm × product analysis, focusing on brand-ownership transactions and the outcomes of reallocation events.

This section proceeds in three steps. Section 3.1 starts by focusing on firms. We start by documenting the degree of dominance of firms in product markets, illustrating the role of market leaders and their persistence. Second, we decompose the forces that contribute to firm-level market share. We focus on three core drivers of concentration: (i) product creation and destruction, (ii) existing product maturity and sales growth, and (iii) product reallocation across firms.

Section 3.2 unpacks products more directly, turning attention to the three core forces contributing to concentration – product creation, maturity, and reallocation of ownership across firms. The product life cycle exhibits striking patterns in the data. Older products tend to take up the largest share of sales. Products exhibit a similar pattern with transactions, as older and larger brands are more likely to be transacted. Both transaction rates and sales exhibit declines later in life.

Section 3.3 focuses on the rate and nature of product reallocation across firms. We document evidence of efficiency gains from product reallocation as sales increase, as well as strategic gains, as prices increase. The interaction of these three forces and the aggregate implications are discussed further in the estimation and quantitative analysis in Section 5 and Section 6 respectively.

3.1 Firm-Level Analysis

All product sales accrue to firms. We focus in this section on the level, persistence, and sources of market shares. We first focus on overall concentration and then turn to the persistence of market leadership. Lastly, we turn an analysis of the dynamic elements driving concentration, decomposing the sources of concentration into product entry/creation, product maturity/growth, and product reallocation.

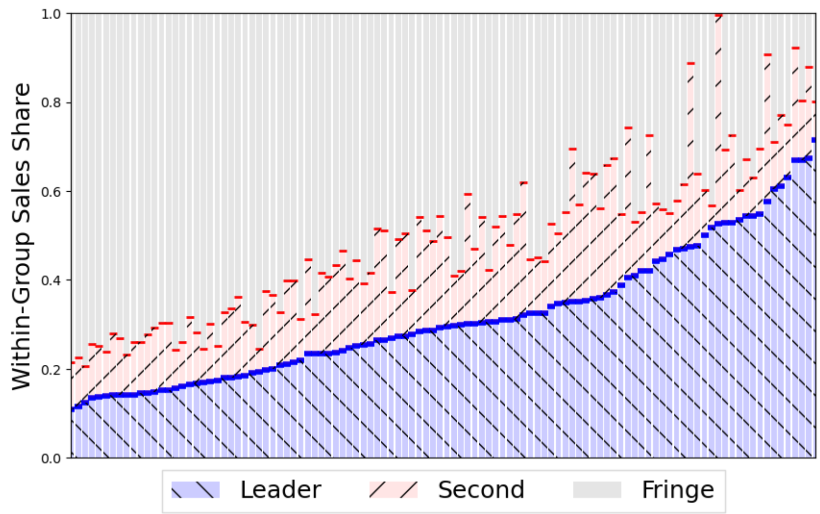
Figure 1 maps out the sales share of the product leader, the second firm, and the remaining firms in the market. This split by product group contains 116 unique product-group categories (e.g., “ICE CREAM" or “BEER”). The average top firm share is 32% of the total market, while the top two firms hold nearly half of the market.

Table 3: Firm Market Shares

Top firm share by group	Top 2 firm share	median share
31.6%	48.1%	0.01%

Table 3 shows the average leader (and second firm) share, as well as the share from the median firm. The top two firms hold on about half the sales in a given market. We also note the presence of a host of small firms (median share as 0.01%), and in our framework, we think of these firms as “fringe” in the

Figure 1: Sales Share of Leader, by Product Group



Note: *This figure shows the sales share by product group (ordered by % share of leader) in 2010.* Source: *RMS Kilts-Nielsen Data Center & GS1 firm-product merge.*

sense they hold few products and small market share.

Leading firms are also quite persistent. Across all categories, the leading firm in one period has a 97% chance of being among the top two firms in the product group in the next period. Concentration in product markets is real and persistent, yet it is not made up of single products. On average, market leaders hold 27 unique brands within the product group they lead. Variation in concentration will be closely connected to how firms develop and hold market shares with their brands.

Product market dominance does not happen in a day. Concentration is an outcome of long-run competition in the market shares of products across firms. The product life cycle is intertwined with firm growth and decline through three core channels. First, and most noted within the innovation literature, is product creation and destruction. Second, once products are born, they mature (and decay) over time. Third, product ownership is reallocated across firms.

The structure of our data allows us to characterize these three forces in detail. We set up three regressions where the coefficients add up to 1 (following a beta decomposition), and each coefficient is linked to the amount of variation of firm growth and decline the margin explains. We run the following regression of three distinct margins of change, y_{it}, on the change of sales in each period $\Delta sales_{it}$:

$$y_{it} = \alpha + \beta \Delta sales_{it} + \epsilon_{it}. \tag{1}$$

Equation (1) focuses on three different margins for y_{it}. We substitute each of the three margins discussed above as y_{it} (y_{it}=creation, maturity, reallocation). Product maturity can refer to both increases and declines in sales over time. We present the results of the three separate regressions in Table 4.

Table 4: Sources of Market Share Reallocation

	Leader			
	Innovation	Incumbency	Reallocation	Entry/Reallocation
Value	0.033***	0.84***	0.13***	**0.25**
	(0.315)	(0.000)	(0.000)	
Observations	383	383	383	
	Fringe			
	Entry	Incumbency	Reallocation	Entry/Reallocation
Value	0.091***	0.89***	0.021***	**4.33**
	(0.000)	(0.000)	(0.000)	
Observations	95353	95353	95353	

p-values in parentheses, clustered at product-group level.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: Market share reallocation is measured across different firm types, following Equation (1). Source: RMS Nielsen.

We stress two main takeaways from Table 4. At the firm level, variation from entry is much more important for fringe firms (e.g., small firms) than for large firms (0.091 versus 0.033). Reallocation is relatively much more important for sales variation for large firms (about 6-times larger). For both fringe firms and market leaders, incumbency/maturity of products drive a significant amount of firm-level variation. Some variation from incumbent products may come directly from the life cycle, whereas others may be due to idiosyncratic shocks. We focus directly on the role of life cycle variation in Table B1 in Appendix B. We find that the fitted life cycle explains a significant amount of variation at the firm-level, with approximately 20% left as a residual.

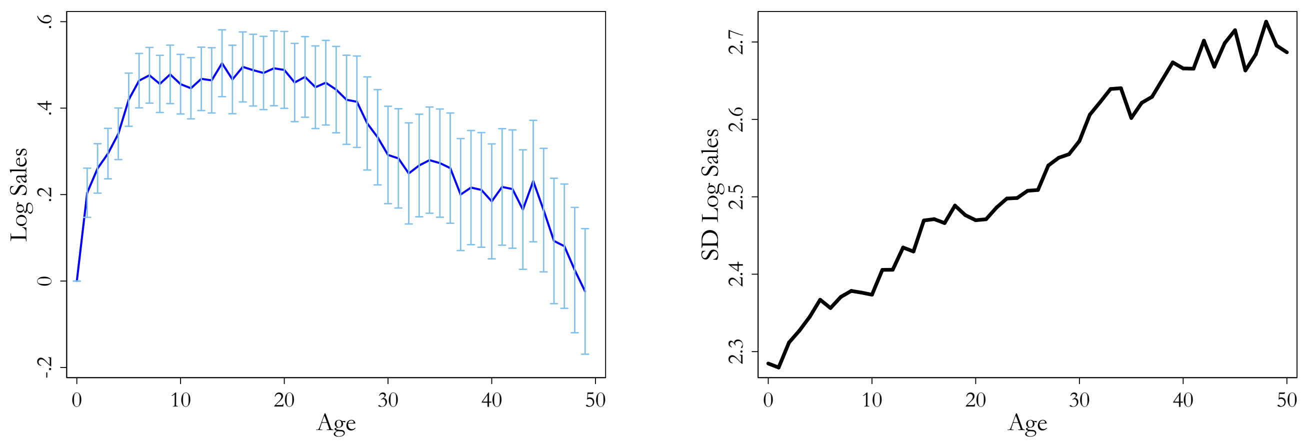
3.2 Brand-Level Analysis

Products are both a significant source of firm concentration (Hottman et al., 2016), and highly dynamic (Argente et al., 2020a), thus affecting the overall sales at the firm and product group level. All products are produced within a brand umbrella. Our goal in this section is to focus on the interaction of brands, the life cycle, and brand reallocation. We now turn to study the life cycle, where we leverage age data from trademarks registrations from the USPTO and sales data from RMS Nielsen. We use a method to study the life cycle of brands, following similar work from Altonji and Shakotko (1987), Fitzgerald et al. (2016), and Argente et al. (2018). Figure 2 plots a regression that illustrates the nature of the product life cycle in sales. This graph plots the coefficients to the following regression:

$$\log y_{it} = \alpha + \sum_{a=1}^{50} \beta_a D_a + \gamma_b + \lambda_t + \theta_i + \epsilon_{it} \tag{2}$$

The regression in Equation (2) considers the sales of brand i at time t, $\log y_{it}$ as a function of a constant (α), brand age indicators from 1 to 50, D_a, and fixed effects for cohort (γ_b) and time (λ_t).[4] The θ_i indicates a brand fixed-effect. Figure 2a plots the regressions by age coefficient β_a. The standard error bars indicate the 95-percent confidence interval for the point estimates for the regression with clustered standard errors at the brand-level. Figure 2b tracks the within-age log sales dispersion across brands.

Figure 2: Brand-Level Dynamics



(a) Log Sales × Age Regression

(b) Log Sales Dispersion × Age

Notes: This figure plots a regression of log sales on age from Equation 2 (panel a), and the standard deviation of log sales within age (panel b). 95% confidence interval standard errors clustered at the brand-group level. Source: USPTO Trademark Data and RMS Nielsen

There are two main takeaways from Figure 2. First, we see brands themselves exhibit an inverted-U pattern in sales over their life cycle. This is consistent with Argente et al. (2020a, 2021), yet in brands the product life cycle is much longer and peaks far later. Second, the dispersion of log sales is increasing in age, which is relevant in the model. As brands mature, the dispersion of sales varies more drastically. The life cycle itself drives not only level differences but within age heterogeneity.[5] Even at age 0, products show large sales dispersion, so this heterogeneity does not simply emerge through the life cycle. In Appendix B, we explore these differences in greater detail.

Product sales do not only change over their life cycle, so do their transaction rates. Few products are reallocated when very young, because they need time to build customer capital and exposure to other firms. Later, as products decay they also experience a decline in the rate of reallocation. We focus on the correlation between transactions and sales and transactions and age in the following figure.

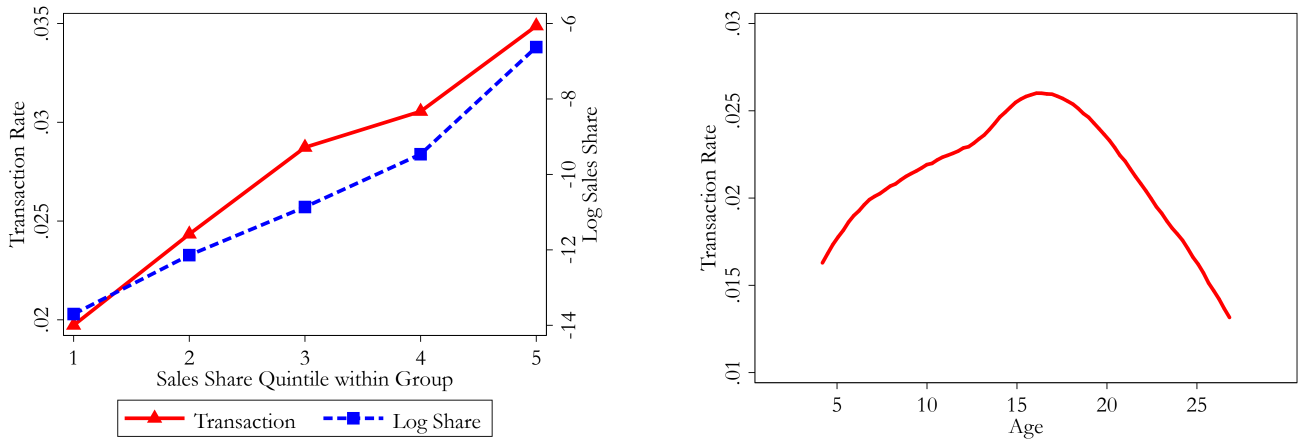
Figure 3 focuses on the interaction of transaction rates, sales share, and age. Figure 3a focuses on the interaction of sales share and transaction, finding the positive selection on transaction. Figure 3b

[4]Given the linear relationship between age, time, and cohort, we follow a method developed by Deaton (1997) to correct for this issue. The normalization orthogonalizes the cohort trends such that growth components move with age and time effects.

[5]This is consistent with the theoretical predictions from brands learning their type as well, as noted in firm dynamics by Jovanovic (1982) and Hopenhayn (1992).

leverages the history of USPTO trademark data to understand the interaction of age and transaction.

Figure 3: Transactions, Age, and Sales



(a) Transaction Rate by Quintile

(b) Transaction Rate by Age

Notes: Panel (a): Transaction rate by sales share. Panel (b): Transaction rate by age. Source: USPTO Trademark and RMS Nielsen.

Figure 3a splits the products into quintiles (truncating products with less than $1000 in sales in a year), and plots the transaction rate against product quintile and the log sales share. We see transaction rates are higher for products with larger market shares. This result can be rationalized in various ways, but we inform the patterns in transactions through directed search in Section 4. Firms select on searching for products with higher value to transact.

Similarly, Figure 3b shows how the transaction rate changes with age. Here, we plot a standard smoothed hazard function to ask at what age a brand that has not yet been transacted becomes transacted. We find the age peaks around age 15-20, around when sales peaks. We next turn to the role of the transactions across firms, particularly looking at the interaction between leading and fringe firms.

3.3 Firm × Brand Analysis

The interaction of firms and products, through in particular the firm-product fit, can be informative for studying the implications of the transactions of brands across firms. In this section, we focus on the differences in sales and prices between fringe and leading firms.

We ask what is, conditional on being held by both a large and small firm, the effect of being held by a larger firm on log sales and log prices?[6] We use the regression in Equation (3) for our analysis:

$$y_{ikt} = \alpha_0 + \alpha_1 \mathbb{I}\{j(i) = \text{T10 firm}\} + \Gamma_{ik} + \Lambda_t + a_{t-b(i)} + \epsilon_{ikt}. \tag{3}$$

[6]Firm size is defined over the entire horizon of the data, but results are similar if firm is defined in only the first period. We define a large firm in our sample as a firm in the top 10 firms in a product group in order to include a broader set of transactions.

The regression evaluates an outcome variable y_{ikt} (e.g., log sales or log prices with product share weights), as a function of whether it's held by a market leader. We include a product (brand-group) fixed effect in Γ_{ik}, a year fixed effect (Λ_t), and an age fixed effect $a_{t-b(i)}$.[7]

Table 5: Log Price and Sales Conditional on Holding Firm

	(1) Log Sales	(2) Log Price
Top 10 Firm Holding	0.391***	0.069**
	(0.000)	(0.019)
R^2	0.843	0.983
N	485261	485261

p-values in parentheses,clustered at brand × group level

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands. Data from 2007-2018. Source: USPTO and RMS Nielsen.

The results are striking on both counts, and hold true with various specifications. If a top 10 firm holds a brand, sales on average are 0.39 log points higher, whereas prices are on average 0.069 log points higher. Both sales and prices are higher at leading firms, indicating large firms could have both strategic and efficiency reasons to buy brands. We discuss these results further in the quantitative section when we ask about the overall effects of this brand reallocation on market activity.

Event Study: Impact of Reallocation on Sales and Prices. We observe transactions in the data and ask how prices and sales respond. To ensure a relevant comparison group, we link transacted brands to never transacted brands with a similar sales profile. Figure 4 plots two separate regressions on one graph with different outcome variables of interest: prices and sales.

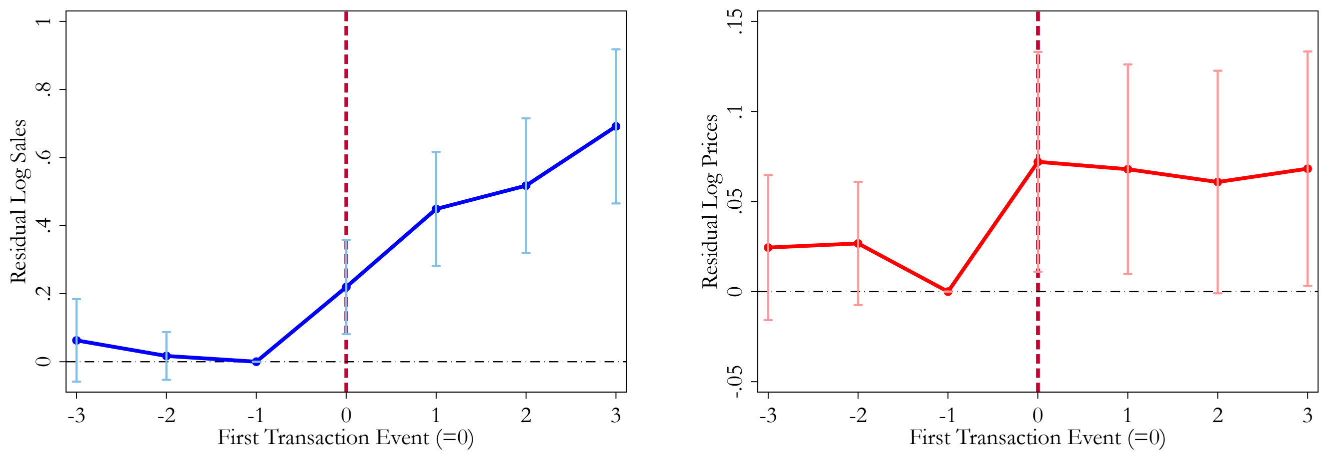
$$\log y_{it} = \alpha + \sum_{t=-3}^{3} \beta_t D_t \times exposed + \lambda_t + \theta_i + \Lambda_a + \epsilon_{it} \tag{4}$$

Equation (4) illustrates the regression we run with the matched sample (with β_t as our variable of interest), which is the term associated with the transacted brand, controlling for age (Λ), year (λ), and brand (θ). We plot each coefficient with the clustered standard error at the brand-group level in Figure 4.

After the event, both prices and sales move strongly, with sales moving significantly more so. With the increase in prices, the results in Figure 4 provide evidence that after adding additional brands, firms may increase their market power over time. Combining this with the rising rate of transfer from small

[7]In this specification, we use UPC age but evaluate the robustness of these results to different specifications in Appendix B.3.

Figure 4: Coarsened Exact Match and Brand Transaction, Small to Large



(a) Sales × Transaction (b) Prices × Transaction

Notes: Coarsened exact match coefficients. Match is made on pre-trend sales change, brand age, and year. 95% confidence interval standard errors clustered at the brand-group level.

to large firms can help connect the importance of brand dynamism with the aggregate distribution of markups across firms. Further, the change in markups is a key outcome of our model.

While fringe to fringe firm transactions are almost 10 times as common as fringe-to-leader transactions, they take up a much smaller market share per transaction. Leaders buy larger brands on average. Further, as noted earlier in this section, leaders tend to increase the sales of brands they have acquired. Thus, although reallocation is a common feature of this market, certain patterns of reallocation are especially important for thinking about market concentration. We now turn to a summary of the three main facts discussed in this section.

Fact 1: Firm Level *Large firms disproportionately have more brands and more market share, and build their market share through brand reallocation more than brand creation.*

Fact 2: Brand Level *Brands build sales over time, and better and more mature brands are more likely to be reallocated across firms.*

Fact 3: Firm × Brand Level *When a brand is reallocated across firms, sales and prices both increase.*

Markets are concentrated, concentration is persistent over time, and concentration is built through reallocation and brand maturity (Fact 1). In line with findings on the product life cycle, we find patterns of higher profile and more mature brands being more likely to be transacted (Fact 2). Directing more specific attention to brands across firms, we find movement in prices and sales upon transaction (Fact 3).

These results motivate a model that can incorporate these forces and develop counterfactuals. The model will incorporate each of these ingredients to understand the drivers of concentration and the dynamic effects of brand reallocation. We turn to the model next.

4 Model

We introduce a firm dynamics model with brand capital and reallocation of brand ownership. Leading firms hold a bundle of products with brand capital and compete against fringe firms, each with imperfectly substitutable products.[8] Firms create products, charge variable markups, and product ownership flows across firms. The model incorporates some standard features of an endogenous growth framework driven by product variety and product innovation. In addition to these standard features, we include three new ingredients: i) brand capital, ii) a brand/product life cycle, and iii) firms can buy and sell brand ownership in the open market. The model interprets the observed skewed distribution of firm size and product reallocation flows across firms. The goal of the model is to provide a quantitative framework to incorporate the empirical facts to provide an understanding both the efficiency of the market for brands and enable policy counterfactuals.

Consumers choose varieties and supply labor to firms. Consumers have CES preferences across imperfectly substitutable product varieties. The brand capital of each product is driven by three sources of heterogeneity. Heterogeneity at the firm-level determines the scope of the brand (e.g. through distribution or marketing). Heterogeneity at the brand-level determines the consumer preference for the brand (e.g. through brand/product enjoyment). Heterogeneity at the firm $\times$ product level determines the fit of the brand with the parent firm. The model incorporates these layers of heterogeneity in the reallocation market in a parsimonious way by leveraging tools from search theory (as in Menzio and Shi, 2011), which simplifies the joint decision problem of firms. The key objects of interest in our model will be the overall brand capital/product scope and the distribution of ownership across firms. These will have important implications for both economic growth and market power.

Section 4.1 characterizes consumer demand for products, directing attention to the sources of product appeal and how it interacts with consumer preferences. Section 4.2 focuses on the leading firms in each product group and the competition in each market. We then introduce the concept of the trademark and discuss the value of the trademark to a firm. This connects to a discussion of product innovation and reallocation. Section 4.3 discusses the value of holding products to each firm and the innovation and reallocation decisions, with particular interest to the conditions when reallocation is efficient versus strategic. Section 4.4 closes the model by completing the household problem. Section 4.5 characterizes

[8]We refer to products and brands interchangeably in our setting. A product is defined as a brand by product group in RMS Nielsen, and we identify sales associated with these brand $\times$ group entities.

the aggregation across product groups, the growth rate, concentration, and overall efficiency.

4.1 Product Demand

Time is continuous and there is a representative household that endogenously supplies labor $\mathbf{L}_t$ and spends on products to maximize its discounted utility. We focus on the consumer's problem in this section and complete the full household problem in Section 4.4. At instant t, the real consumption of the household $\mathbf{C}_t$ is given by a Cobb-Douglas aggregator across a unit measure of product groups, indexed by $k \in [0, 1]$:

$$\ln \mathbf{C}_t = \int_0^1 \xi_k \ln C_{kt} dk, \tag{5}$$

where C_{kt} is the real consumption from product group k and ξ_k is the appeal of product group k to the household.

Each product group k at time t contains N_{kt} measure of imperfectly differentiable products which are identified with a brand. The real consumption from product group k, C_{kt}, is a CES aggregator across these varieties:

$$C_{k,t} = \left(Q_{kt}^{\frac{\nu-1}{\sigma_k}} \int_0^{N_{kt}} \psi_{ikt}^{\frac{1}{\sigma_k}} c_{ikt}^{\frac{\sigma_k - 1}{\sigma_k}} di \right)^{\frac{\sigma_k}{\sigma_k - 1}}, \tag{6}$$

where c_{ikt} is the consumption on variety i, ψ_{it} is the appeal of product i, Q_{kt} is the total quality of products within the group (defined in Equation 9), ν is the measure of love-of-variety, and σ_k is the substitution elasticity across products, which we allow to vary by product group.[9] There are two key objects of interest from our model. First, we are concerned with the total number of products, N_{kt}, as more products contribute positively to consumer welfare. Second, we are concerned with the appeal, ψ_{ikt}, of each product. The joint distribution of these two objects connects directly to consumer welfare. Mirroring the empirical decomposition of product-level sales, we assume the appeal ψ_{ikt} is a combination of three components:

$$\log \psi_{ikt} = \alpha_{jk(t)} + \beta_{ikt} + \gamma_{ijkt}. \tag{7}$$

Equation (7) closely resembles our empirical framework. $\alpha_{jk(i)}$ is the appeal of product-owning firm j in group k (which it may expand through marketing/distribution), $\beta_{ik}(t)$ is the specific product appeal which links to consumer's taste for the brand in group k, and $\gamma_{ijk(t)}$ is the match-specific quality between product i and its owning firm j in group k. The optimal consumption decision within group k gives the

[9]We introduce the love-of-variety ν so the model can target flexible levels of real consumption growth. This parameter does not directly affect the qualitative characterization of the equilibrium. For the model discussion, we set $\nu = 1$.

demand curve for variety i, given the appeal ψ and price p:

$$c_{kt}(p, \psi) = \psi \times p^{-\sigma_k} \times P_{kt}^{\sigma_k - 1}, \tag{8}$$

where the group-level price index P_{kt} is given by $P_{kt} = \left(\int_0^{N_{kt}} \psi_{ikt} p_{ikt}^{1-\sigma_k} di \right)^{\frac{1}{1-\sigma_k}}$. We normalize the aggregate price index to be 1 for any t. Given price p, the demand for product i is increasing in the appeal ψ. Given appeal ψ, product demand is decreasing in the price p to the degree that the price diverts from the group-level price index. The elasticity of substitution σ_k determines the consumer sensitivity to price. In a market with high elasticity of substitution ($\sigma_k \to \infty$), individuals are very price responsive to price and less responsive to appeal. In markets with low elasticity of substitution ($\sigma_k \to 1$), firms have lots of pricing leverage through consumer appeal.

In the full quantitative section, we allow all parameters to vary by group k. For the rest of the theoretical discussion we perform two adjustments to aid exposition. First, we omit the group k index, as the group index does not change how we characterize the equilibrium. Second, we set $\xi_k = 1$ to study a general group k with group appeal 1.

4.2 Firms

Each product group contains one large multi-product firm and endogenous measure of single-product firms. We refer to the multi-product firm as the group leader and single-product firms as the fringe. The leader and fringe firms are different in the following aspects: (1) *Capacity*. The leaders are able to own and operate many varieties, whereas the fringe firms are only able to operate one product. We denote the leader's basket of products at time t as $\mathcal{I}_t^L$ and the fringe basket of products as $\mathcal{I}_t^F$; (2) *Entry*. The leaders are not subject to firm entry and exit, while there is free entry of fringe firms. (3) *Productivity*. All varieties are produced using a linear technology in labor. The leader has productivity $e^{z/(\sigma-1)}$, and fringe firms have the same firm-level productivity 1. (4) *Pricing*. The leaders are big relative to their product group, and they internalize their impact on the group-level price index. The fringe firms are small relative to the market, and they behave as monopolistically competitive firms. Each firm can charge a markup through branding, but large firms have more pricing power through their larger consumer appeal.

Given the appeal of products and the productivity of firms, the sales of product i at instant t is determined by the following composite quality index if it is operated by the leader j:

$$\log q_{it} = z + \alpha + \beta_{it} + \gamma_{ijt},$$

and the following composite quality index if it is operated by a fringe firm:

$$\log q_{it} = \beta_{it} + \gamma_{it},$$

We define the quality index for leader as the sum of quality indices across all of its products $Q_t^L = \int_{i \in \mathcal{I}_t^L} q_{it} di$, and the quality index of fringe firms as the sum of quality indices across all of their products $Q_t^F = \int_{i \in \mathcal{I}_t^F} q_{it} di$. Two group-level indices are welfare relevant in our model. The first index is the group-level composite quality index Q_t, because it increases the marginal utility of consumption through the love-of-variety:

$$Q_t = Q_t^L + Q_t^F.$$

We denote the growth rate of this object $g_t = \frac{\dot{Q}_t}{Q_t}$. The second is the ratio of the leader's quality index and the fringe firms' quality index:

$$\phi_t = \frac{Q_t^L}{Q_t^F}. \tag{9}$$

Competition. We assume the firms compete through price and price their products jointly. For each product, firms internalize their impact on the group-level price index according to their market shares as in Atkeson and Burstein (2008).

Product Innovation. New products are created through product innovation, and can be done by both fringe firms and market leaders. The leader can choose its innovation intensity η by paying labor cost $D(\eta)$. $D(\eta)$ is increasing and convex in η, and $D(0) = 0$. The fringe firms can endogenously enter with an entry cost $\frac{\kappa^e}{Q_t}$. Having entered, the fringe creates a new product which it trademarks. New products draw an initial product-specific quality β from exogenous distribution $F_\beta(\beta)$ and match-specific quality γ from distribution $F_\gamma(\gamma)$.

Product Reallocation. The ownership of existing products can be reallocated across firms within the same product group. This reallocation process is modeled as a market with search and matching frictions (consistent with observed gains from trade in ownership exchange as in David, 2020). We assume search is directed (consistent with the fact that brands are not hard to find) as in Menzio and Shi (2011). At each instant, buyers can create vacancies with a constant cost; Sellers post the transfers they would require in exchange for their products; Buyers, observing all posted transfers, direct search to their preferred sellers. The matching between sellers and buyers is frictional, modeled by a matching function that moves with the number of vacancies and the number of sellers. This matching function captures natural features of the market for brand exchange. Mathematically, we assume the number of matches is given by $m(v, u)$, where v is the number of vacancies and u is the number of sellers. We assume m is increasing and concave in both arguments. It is useful to define the selling rate as $\lambda(\theta) = m(\theta, 1)$, where θ is the ratio between the number of buyers and the number of sellers.

Trademarks. Whenever a firm introduces a new product, it has an incentive to trademark the product. Without a trademark, there is not a mechanism for consumers to have imperfect substitution over your product. Further, there is evidence that firms trademark early in the firm life cycle (as noted by Dinlersoz et al., 2018), and trademark application costs are low. The corresponding market power accrues to both leaders and fringe firms, but leaders can potentially markup higher than fringe firms due to the reasoning discussed above. This pricing power is one force generating product-ownership reallocation from small to large firms. The other force is the natural efficiencies of the brands matching with the right firms.

Product Life Cycle. When a brand is initially introduced, the brand has low brand capital as consumers are not broadly aware of it. This is consistent with evidence on the long-run development of consumer demand presented earlier (and noted by, among others, Bronnenberg et al., 2009; Einav et al., 2021) and the previous evidence presented on low reallocation rates early in a brand's life cycle. Brands then develop consumer capital through a dynamic process. This both induces increases in sales and increases in visibility for reallocation of brand ownership. As discussed earlier, new products draw initial brand capital $\beta_0 \sim F_\beta$. We model the brand development process as the following, $d\beta = \iota(\bar{\beta} + \beta_0 - \beta)$, where we match ι and $\bar{\beta}$ to the life cycle in the data.

Implications of Product Reallocation. In the next section, we discuss the characterization of product reallocation. Here, we briefly discuss two extreme cases of product reallocation: *efficient* and *strategic* reallocation. The intuition of these two polar cases captures important features of the market for brands. Firms transfer the intellectual capital associated with product β, which is the fixed component of the product. However, reallocation may lead to different appeal through α (firm effect) or γ (firm $\times$ product effect). In the case where the leading firm has large advantages (e.g., in distribution or marketing, $\alpha >> 0$), we expect transactions to exhibit efficiency gains as brands have been allocated to a better firm. If leading firms do not have large advantages in distribution and marketing, (e.g., $\alpha \approx 0$), then the brand ownership transfer simply exacerbates an appeal gap between leaders and fringe firms. We study this further in the characterization of the market equilibrium.

4.3 Firm's Problem

Firms face a static pricing and production problem and a dynamic innovation and reallocation problem. We discuss these choices in turn.

Static Pricing Problem. Fringe firms are infinitesimal relative to the market, and thus do not internalize their own impact on group-level price indices. In the equilibrium, they charge a constant markup $\frac{\sigma}{\sigma-1}$. The leaders have a different pricing problem because they are large relative to the market and internalize their impact on the group-level price index. Given the demand curve for each variety in Equation (8), the product-group leader's pricing decision is:

$$\max_{p_i} \int_{i \in \mathcal{I}_t^L} (p_i - e^{-z/(\sigma-1)} \mathbf{w}_t) c_t(p_i, \psi_i) di$$

s.t.

$$c_t(p, \psi) \text{ given by Equation (8).}$$

As in the models in patent race literature, the competition of a group can be summarized by the gap between leaders' quality and fringe firms' quality as noted in Equation (9). Given the gap ϕ, the equilibrium markup charged by the leader is increasing in its market share s, given by

$$\mu = \frac{\sigma(1-s)+s}{\sigma(1-s)+s-1}, \tag{10}$$

and the market share depends on both the gap between leader and fringe firms and the markup as follows,

$$s = \frac{\mu^{1-\sigma}}{\mu^{1-\sigma} + \phi^{-1} \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}}. \tag{11}$$

Equation (10) and Equation (11) constitute two equations in two variables (s, μ). Given any gap ϕ, we denote the solution to this equation system as $s(\phi)$ and $\mu(\phi)$. Due to the assumption of Cobb-Douglas aggregation across product groups, the leader's profit can written as $\Pi(\phi(t))\mathbf{C}(t)$, where $\Pi(\phi)$ is the share of aggregate expenditure accruing to the leader is:

$$\Pi(\phi) = \frac{s(\phi)}{\sigma\left[1-s(\phi)\right]+s(\phi)}. \tag{12}$$

Similarly, the profit share that accrues in aggregate to fringe firms is

$$\pi(\phi) = \frac{s(\phi)}{\sigma}. \tag{13}$$

In Equation (12), Given a unit of expenditure, $s(\phi)$ accrues to the leader, while $1-s(\phi)$ accrues to fringe firms, who receive $\frac{1}{\sigma}$ profit margin. The leader, due to its collection of appeal, "perceives" a more inelastic consumer, $\sigma\left(1-s(\phi)\right)+s(\phi)$, than fringe firms, σ. The inverse of the leader's perceived elasticity is its profit margin.

A leading firm's incentive to engage in product innovation and product reallocation derives from the firm's ability to increase its profits. We characterize the marginal increase of a leader's profit when it increases its quality gap from the fringe firms. We find this marginal profit has a closed-form solution in terms of market share, which we discuss in the following lemma.

Lemma 1 *The elasticity of profit with respect a change in quality gap ϕ is*

$$\frac{\partial \log \Pi(\phi)}{\partial \log \phi} = 1 - s(\phi) \tag{14}$$

Two extreme cases are helpful in understanding the result in Equation (14). When the leader has 0 market share, the profit elasticity is 1. At this point, the leader has an infinitesimal share of the market and charges the same markup as the fringe firms. Thus a 1% increase in its market share translates into 1% increase in profits without losing in markups. When the leader has 100% of the market, the profit elasticity is 0. When the leader has taken over the whole market, a marginal increase in its quality gap only cannibalizes its own market shares without changing firm-level profit.

Dynamic Innovation and Reallocation Problem. To characterize the incentive of innovation and reallocation, we need to first write out the value of heterogeneous products to leaders and fringe firms. For notational simplicity, we denote the vector of product characteristics as $\mathbf{x}_{it} = \left(\alpha_{ij(i,t)k}, \beta_0, \beta_{it}, \gamma_{ij(i,t)}\right)$. Consider a product that is currently operated by a fringe firm, with characteristics $\mathbf{x}$. To the fringe firm, this product has value $u_t(\mathbf{x})$ that solves the following Hamilton-Jacobi-Bellman equation:

$$\begin{aligned}(\rho + g_t)u_t(\mathbf{x}) = & \underbrace{e^{\beta+\gamma}\,(1+\phi_t)\,\frac{1}{\phi_t}\pi(\phi_t)}_{\text{Operating Profit}} + \underbrace{\iota(\bar{\beta}-\beta)\frac{\partial u}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} \\ & + \underbrace{\max_{\theta} \lambda(\theta)\mathbb{E}_{\gamma'}\Omega_t(\mathbf{x}',\mathbf{x}) - \theta\kappa_s^{FL}\frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Value of Selling}} + \dot{u}_t(\mathbf{x})\end{aligned} \tag{15}$$

The value to a fringe firm in group k with product appeal β and firm product fit γ has two components. The first component is the instantaneous return which moves positively with β and γ. The second component is the option value in the search market, where the fringe firm chooses search intensity θ to generate an arrival rate $\lambda(\theta)$ at which the firm receives the surplus from transferring their brand if the gains from trade are positive. If leaders have a large firm appeal advantage, α, they will demand a higher quantity of products, shifting $\lambda(\theta)$, and inducing a higher value for the fringe firm through this channel.

The product group leader chooses its innovation and reallocation activity to maximize its discounted profit. At time t, given the innovation intensity η_t^L and the reallocation decision $\theta_t^{LF}(\mathbf{x})$ and $\theta_t^{FL}(\mathbf{x})$, the density of products with characteristics $\mathbf{x}$ that are operated by the leader evolves according to:

$$\dot{n}_t^L(\mathbf{x}) = \underbrace{\eta_t^L f(\beta)\mathbb{I}_{\gamma=0}}_{\text{Innovation}} - \underbrace{\iota(\bar{\beta}-\beta)\frac{\partial n_t^L}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} \tag{16}$$
$$- \underbrace{\lambda\left(\theta_t^{LF}(\mathbf{x})\right) n_t^L(\mathbf{x})}_{\text{L-t-F Reallocation}} + \underbrace{\int_{\Omega(\mathbf{x},\mathbf{x}')>0} f_\gamma(\gamma)\lambda\left(\theta_t^{FL}(\mathbf{x}')\right) n_t^F(\mathbf{x}')d\gamma'}_{\text{F-t-L Reallocation}}.$$

Similarly, the density of products with characteristics $\mathbf{x}$ that are operated by fringe firms evolves according to:

$$\dot{n}_t^F(\mathbf{x}) = \underbrace{\eta_t^F f(\beta)\mathbb{I}_{\gamma=0}}_{\text{Innovation}} - \underbrace{\iota(\bar{\beta}-\beta)\frac{\partial n_t^F}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} \tag{17}$$
$$- \underbrace{\lambda\left(\theta_t^{FL}(\mathbf{x})\right) n_t^F(\mathbf{x})}_{\text{F-t-L Reallocation}} + \underbrace{\int_{\Omega(\mathbf{x}',\mathbf{x})<0} f_\gamma(\gamma)\lambda\left(\theta_t^{LF}(\mathbf{x}')\right) n_t^L(\mathbf{x}')d\gamma'}_{\text{L-t-F Reallocation}}.$$

A product group leader, taking as given the entry decision of fringe firms, chooses its own innovation and reallocation activity to maximize the discounted net profit,

$$\max_{\eta_t,\theta_t^{LF}(\mathbf{x}),\tau_t^{LF}(\mathbf{x})} \int_0^\infty e^{-\int_0^t \mathbf{r}(t')dt'}\left[\Pi(\phi_t) - D(\eta_t) - B_t + S_t\right]dt,$$

s.t.

$$\phi_t = \frac{\int e^{z+\alpha+\beta+\gamma} n_t^L(\mathbf{x})d\mathbf{x}}{\int e^{\beta+\gamma} n_t^F(\mathbf{x})d\mathbf{x}},$$

$$B_t = \int \left[\lambda(\theta_t^{FL}(\mathbf{x}))\tau^{FL}(\mathbf{x}) - \theta_t^{FL}(\mathbf{x})\kappa_s^{FL}\frac{\mathbf{w}_t}{\mathbf{C}_t}\right] n_t^F(\mathbf{x})d\mathbf{x},$$

$$S_t = \int \lambda\left(\theta_t^{LF}(\mathbf{x})\right)\tau^{LF}(\mathbf{x}) n_t^L(\mathbf{x})d\mathbf{x}.$$

The leader's problem is a complicated problem that involves the joint distribution of product quality and firm ownership. We show the full details of characterization of this problem in the Appendix C.1. It turns out this complicated problem can be characterized by calculating the discounted values of products to different firms (leader or fringe). This result comes from two features of our model: (1). the product qualities can be linearly added into a firm-level quality index $\int_{\mathcal{I}_t^L} q_{ijt}$; (2) the love-of-variety and competition of each product group are fully characterized by the quality indices.

For a product with state $\mathbf{x}$ that is currently operated by the group leader, its discounted value to the leader is

$$(\rho+g_t)v_t(\mathbf{x}) = \underbrace{e^{z+\alpha+\beta+\gamma}\left(1+\phi_t\right)\Pi'(\phi_t)\phi_t}_{\text{Operating Profit}} + \underbrace{\iota(\bar{\beta}-\beta)\frac{\partial v}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} \\ + \underbrace{\max_{\theta}\lambda(\theta)\mathbb{E}_{\gamma'}\left[-\Omega_t(\mathbf{x}',\mathbf{x})\right]^{+} - \theta\kappa_s^{LF}\frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Value of Selling}} + \dot{v}_t(\mathbf{x}) \tag{18}$$

where $\Pi'(\phi_t)(1+\phi_t)$ is the flow marginal value and $U_k^L(\beta,\gamma)$ is the optimal value of selling to fringe firms. The (negative) value of a similar product operated by the fringe to the leader is:

$$(\rho+g_t)y_t(\mathbf{x}) = -\underbrace{e^{\beta+\gamma}\left(1+\phi_t\right)\Pi'(\phi_t)}_{\text{Operating Profit}} + \underbrace{\iota(\bar{\beta}-\beta)\frac{\partial y}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} + \dot{y}_t(\mathbf{x}) \tag{19}$$

The value functions of the leader and fringe link to the reallocation decisions through the joint surplus, which we define as $\Omega_t\left(\mathbf{x}_L,\mathbf{x}_F\right) = v_t(\mathbf{x}_L) - y_t(\mathbf{x}_F) - u_t(\mathbf{x}_F)$. $\Omega_t\left(\mathbf{x}_L,\mathbf{x}_F\right)$ measures the joint surplus from trade for a product reallocated from a fringe firm to a leader, with product appeal β, fringe match quality γ', and leader match quality γ. Correspondingly, $-\Omega_t\left(\mathbf{x}_F,\mathbf{x}_L\right)$ is the joint surplus of reallocating the product from a leader to a fringe firm. The joint surplus $\Omega_t\left(\beta,\gamma,\gamma'\right)$ satisfies the following Bellman equation:

$$\begin{aligned}(\rho+g_t)\,\Omega_t\left(\mathbf{x}_L,\mathbf{x}_F\right) = \; & \underbrace{\omega(\beta,\gamma_L,\gamma_F)}_{\text{Flow Gains from Trade}} + \dot{\Omega}_t\left(\mathbf{x}_L,\mathbf{x}_F\right) \\ & + \underbrace{\iota(\bar{\beta}-\beta_L)\frac{\partial \Omega}{\partial \beta_L}(\mathbf{x}_L,\mathbf{x}_F)}_{\text{Leader Maturity}} + \underbrace{\iota(\bar{\beta}-\beta_F)\frac{\partial \Omega}{\partial \beta_F}(\mathbf{x}_L,\mathbf{x}_F)}_{\text{Fringe Maturity}} \\ & - \underbrace{\max_{\theta^{FL}(\mathbf{x})}\lambda\left(\theta^{FL}(\mathbf{x})\right)\mathbb{E}_{\gamma'}\Omega_t(\mathbf{x}',\mathbf{x}_F)^{+} - \theta^{FL}(\mathbf{x})\kappa_s^{FL}\frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Fringe's Value of Selling}} \\ & + \underbrace{\max_{\theta^{LF}(\mathbf{x})}\lambda\left(\theta^{LF}(\mathbf{x})\right)\mathbb{E}_{\gamma'}\left[-\Omega_t(\mathbf{x}_L,\mathbf{x}')\right]^{+} - \theta^{LF}(\mathbf{x})\kappa_s^{LF}\frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Leader's Value of Selling}}\end{aligned} \tag{20}$$

where

$$\omega(\beta,\gamma_L,\gamma_F) = \left(\frac{e^{\alpha+\gamma_L-\gamma_F}+\phi}{1+\phi}\frac{\Pi}{\frac{\phi}{\sigma_k(1+\phi)}} - 1\right)\frac{\pi}{z_F}e^{\beta+\gamma_F}.$$

This equation provides the basis for gains from trade. We take note of two important polar cases in this environment. Table 6 focuses on the conditions under which reallocation would be *only efficient* or *only strategic*. We take the extreme cases in the parameter set that would induce completely efficient and completely strategic transactions.

Table 6: The Efficient and Strategic Gains from Reallocation

Case	Condition	Gains from Trade	Discussion
Efficient:	$\phi \to 0$	$\left(e^{\alpha+\gamma_L-\gamma_F}-1\right)\frac{\pi}{z_F}$	Gains from only $\alpha > 0$ or $\gamma_L - \gamma_F > 0$, sales $\uparrow$
Strategic:	$\alpha + \gamma_L - \gamma_F = 0$	$\left(\frac{\Pi}{\frac{\phi}{\sigma(1+\phi)}} - 1\right)\frac{\pi}{z_F}e^{\beta+\gamma_F}$	Gains from higher concentration, markup $\uparrow$

Efficient reallocation occurs when leaders have no market concentration. In this case, gains from trade emerge only from the leader expanding brand appeal or having a good fit with the brand (α or γ). The strategic reallocation occurs when the leader has no advantage in marketing or distributing the brand (they may even have a disadvantage, e.g., $\alpha + \gamma_L - \gamma_F < 0$), and thus the reallocation simply increases concentration. These two benchmarks provide important conceptual extremes to characterize efficiency in the market. We now turn to the decisions at the firms on innovation and reallocation.

Innovation Decisions. If there is positive product entry for fringe firms, the expected value of product entry must equal the entry cost κ^e adjusted by the wage-consumption ratio:

$$\mathbb{E}_\beta\left[u(\beta,0)\right] = \kappa_e^F \frac{\mathbf{w}_t}{\mathbf{C}_t}. \tag{21}$$

The optimal innovation by the leader requires that the marginal cost of innovation equals the marginal benefit of having an additional new product:

$$\mathbb{E}_\beta[v(\beta,0)] = D'(\eta)\frac{\mathbf{w}_t}{\mathbf{C}_t}.$$

Reallocation Decisions. A central focus of our model is the product reallocation flows across different firms. From Equation (15), the equilibrium buyer-seller ratio for a product with quality (β,γ), where buyers and sellers are both fringe firms, equalizes the marginal value of trade and the marginal cost,

$$\lambda'(\theta_t^{FL}(\beta,\gamma))\mathbb{E}_{\gamma'}\Omega(\mathbf{x}',\mathbf{x})^+ = \kappa_s^{FL}\frac{\mathbf{w}_t}{\mathbf{C}_t}. \tag{22}$$

The result is similar for leader-to-fringe reallocation,

$$\lambda'(\theta_t^{LF}(\beta,\gamma))\mathbb{E}_{\gamma'}\left[-\Omega(\mathbf{x},\mathbf{x}')\right]^{+}=\kappa_s^{LF}\frac{\mathbf{w}_t}{\mathbf{C}_t}. \tag{23}$$

We expand on these details in Appendix C. Having characterized the innovation and reallocation decisions, we turn to closing the model with the household problem.

4.4 Closing the Model: Household Problem

To close the model, we detail the household's consumption-saving and labor supply decision. The household can freely borrow or save by investing in a representative portfolio of firms in the economy, taking as given the interest rate and prices. This assumption means that the profits of firms are all accrued back to the household. As discussed earlier, we normalize the aggregate price index to be 1 and express other prices in their real units. Denote $\mathbf{w}_t$ as the real wage and $\mathbf{r}_t$ as the real interest rate. The household takes these prices as given and chooses its real consumption $\mathbf{C}_t$ and labor supply $\mathbf{L}_t$ to maximize:

$$\max_{c_{ikt},\mathbf{L}_t}\int_0^\infty e^{-\rho t}\left[\ln\mathbf{C}_t-\varphi_0\frac{\mathbf{L}_t^{1+1/\varphi}}{1+1/\varphi}\right]dt,$$

s.t.

$$\dot{a}=\mathbf{r}_t a_t-\mathbf{C}_t+\mathbf{w}_t\mathbf{L}_t,$$

$$\mathbf{C}_t \text{ given by (5) and (6).}$$

The optimal saving decision implies the Euler equation must hold:

$$\frac{\dot{\mathbf{C}}}{\mathbf{C}}=\mathbf{r}-\rho,$$

and the optimal labor supply decision requires that the marginal rate of substitution between leisure and consumption equals the real wage:

$$\varphi_0\mathbf{L}_t^{1/\varphi}=\frac{\mathbf{w}}{\mathbf{C}}.$$

Discussion of Cobb-Douglas Assumption.- One key assumption from the household side leads to the simplification of the environment. By assuming the consumption from different product groups is aggregated through a Cobb-Douglas utility function, we assume that evolution within each product group does not lead to reallocation of market shares across product groups. In addition to being the standard assumptions in the patent race literature such as Liu et al. (2022) and in the product dynamics literature such as Hottman et al. (2016) and Argente et al. (2021), this assumption is without apology given our context is the product market at an annual frequency where this reallocation across groups is small in magnitude.

We discuss the evolution of the distribution in Appendix C. The main finding from the distribution is that the holdings of both the fringe and leading firms can be expressed in closed form, linking the flows across firms and innovation to aggregate shares.

4.5 General Equilibrium and Aggregation

The goal of this model is to provide a conceptual and quantitative framework to link branding activity to macroeconomic outcomes to discuss the implications for efficiency and welfare. We do so in this section by discussing how innovation and reallocation in product markets lead to overall growth, concentration, and market efficiency.

Within-Group Equilibrium. We now reintroduce the notation for each product group k to discuss the equilibrium and aggregation. From the previous sections, we know the firm's optimal innovation and reallocation decisions and maturity process shape the overall aggregate appeal.

$$Q_{kt} = \int_{i \in \mathcal{I}_{kt}^L} q_{ikt} di + \int_{i \in \mathcal{I}_{kt}^F} q_{ikt} di$$

Given the detrended distribution of product quality and the innovation and reallocation rates, the growth rate of total quality within product group k is:

$$g_{kt} = \underbrace{\eta_{kt}^L + \eta_{kt}^F}_{\text{Innovation}} + \underbrace{\iota_{kt}^L + \iota_{kt}^F}_{\text{Maturity}} + \underbrace{\Lambda_{kt}^{FL} + \Lambda_{kt}^{LF}}_{\text{Reallocation}}. \tag{24}$$

where

$$\Lambda_{kt}^{FL} = \int_{\Omega_t(\mathbf{x}', \mathbf{x})>0} \exp(\alpha_k + z_k + \gamma' - \gamma) \lambda_t^{FL}(\mathbf{x}) n_t^F(\mathbf{x}) d\mathbf{x}$$

$$\Lambda_{kt}^{LF} = \int_{\Omega_t(\mathbf{x}, \mathbf{x}')<0} \exp(\gamma' - \gamma - \alpha_k - z_k) \lambda_t^{LF}(\mathbf{x}) n_{kt}^L(\mathbf{x}) d(\mathbf{x})$$

$$\iota_{kt}^L = \iota \int \exp(\bar{\beta} - \beta) n_{kt}^L(\mathbf{x}) d\mathbf{x}$$

$$\iota_{kt}^F = \iota \int \exp(\bar{\beta} - \beta) n_{kt}^F(\mathbf{x}) d\mathbf{x}$$

η_{lt} and η_{ft} are the endogenous leader and fringe innovation decisions. ι_{kt} is the maturity process at the group level. Λ is the respective flows in each direction (FtL, LtF), which is a function of the distributions of α, β, and γ. We are now ready to define the following group-level equilibrium.

Definition 2 *(Group Equilibrium) A group equilibrium in group k given the aggregate wage-GDP ratio $\frac{\mathbf{w}_t}{\mathbf{C}_t}$ is $\{\phi_{kt}, g_{kt}\}$ and $\{v_{kt}(\mathbf{x}), u_{kt}(\mathbf{x}), \Omega_{kt}(\mathbf{x}_L, \mathbf{x}_F), \theta^{LF}_{kt}(\mathbf{x}), \eta^L_{kt}, \eta^F_{kt}\}$, and $\{n^L_t(\mathbf{x}), n^F_t(\mathbf{x})\}$ such that*
1. Given (ϕ_{kt}, g_{kt}), $\{v_{kt}(\mathbf{x}), u_{kt}(\mathbf{x}), \Omega_{kt}(\mathbf{x}_L, \mathbf{x}_F), \theta^{LF}_{kt}(\mathbf{x}), \eta^L_{kt}, \eta^F_{kt}\}$ solve firms' optimization;
3. Given Step 1, $\{n^F_t(\mathbf{x}), n^L_t(\mathbf{x})\}$ solves equations (16) and (17);
4. (ϕ_{kt}, g_{kt}) are consistent with equations (18) and (24)

The group-level equilibrium can be solved in isolation from the aggregate variables working from step 1 to step 3. With the group-level equilibrium characterized, we turn to the overall brand appeal in the economy and consumer welfare.

This section links the outcomes in each market k to the overall efficiency in the economy. The results in this section inform the eventual discussion of aggregate efficiency and the social planner's problem. Given the partial equilibrium within each market, specifically $\{\phi_k, Z_k\}$, the following proposition summarizes the general equilibrium of the economy.

Proposition 3 *Given the product-group equilibria, the general equilibrium of the economy is characterized as follows:*
1. Given $\{\phi_k, z_k\}$, calculate the following productivity, markup, and misallocation indices:
(Productivity)

$$\mathbf{Z}(t) = \mathbf{Z}(0)\exp(\mathbf{g}t), \qquad \mathbf{g} = \int_0^1 \frac{\xi_k \nu}{\sigma_k - 1} g_k dk; \tag{25}$$

(Markup)

$$\mathbf{M} = \exp\left(\int_0^1 \xi_k \log M_k dk\right), \tag{26}$$

$$M_k = \left[\frac{\phi_k}{1+\phi_k}\mu_k(\phi_k)^{1-\sigma_k} + \frac{1}{1+\phi_k}\left(\frac{\sigma_k}{\sigma_k - 1}\right)^{1-\sigma_k}\right]^{\frac{1}{1-\sigma_k}}; \tag{27}$$

(Misallocation)

$$\mathbf{A} = \int_0^1 \xi_k \left(\frac{M_k}{\mathbf{M}}\right)^{-1} A_k dk, \tag{28}$$

$$A_k = \frac{\phi_k}{1+\phi_k}\left(\frac{\mu_k(\phi_k)}{M_k}\right)^{-\sigma_k} + \frac{1}{1+\phi_k}\left(\frac{\sigma_k}{\sigma_k - 1}\right)^{-\sigma_k}. \tag{29}$$

2. The aggregate objects $\mathbf{C}, \mathbf{L}_P, \mathbf{L}_S, \mathbf{L}_D$ are given by

$$\mathbf{C} = \mathbf{Z}\mathbf{A}\mathbf{L}_P \tag{30}$$

$$\mathbf{w} = \frac{\mathbf{Z}}{\mathbf{A}\mathbf{M}} \tag{31}$$

$$\varphi_0(\mathbf{L}_S + \mathbf{L}_P + \mathbf{L}_D)^{1/\varphi} = \frac{1}{\mathbf{A}\mathbf{M}\mathbf{L}_\mathbf{P}}. \tag{32}$$

This aggregation result follows a similar structure to recent literature on the aggregate implications of firm-level markups (or markdowns) such as Edmond et al. (2015) and Berger et al. (2019).

Definition 4 *(General Equilibrium) A general equilibrium is* $\frac{\mathbf{w}_t}{\mathbf{C}_t}$ *such that:*

1. All product groups are in equilibrium as defined in Definition 2;

2. Given the group equilibrium, the aggregation holds as defined in Proposition 3.

Our welfare metric is the discounted utility of the representative household:

$$\mathcal{W} = \int_0^\infty e^{-\rho t} \left(\ln \mathbf{C}_t - \varphi_0 \frac{\mathbf{L}^{1+1/\varphi}}{1+1/\varphi} \right) dt. \tag{33}$$

The aggregation in this section points to some key ingredients for our quantitative analysis. Overall, this project focuses on two main components in the development of overall output. First, we look at the productivity of the economy, **Z**, which moves positively with social welfare. Then, we evaluate the misallocation, **A**, and firm markups, **M**, which moves negatively with social welfare. This general framework operates in the background of our estimation and quantitative analysis.

4.6 Discussion of the Model

Before turning to the quantitative analysis, we stress some specific contributions of the model. One salient feature of product market competition is multi-product firms and heterogeneous products (e.g., as noted in Hottman et al., 2016). In our quantitative analysis, we indeed find these features matter for our conclusions. Here, we highlight the connections and departures of our theoretical model with prior literature in two domains.

Relationship to Endogenous Growth Theory. Dealing within a firm's holding of heterogeneous products in a variable markup environment is a daunting task because it involves a multi-dimensional portfolio choice decision. This is different from the step-by-step innovation models widely used in the literature (Aghion et al., 2001; Klette and Kortum, 2004; Akcigit and Kerr, 2018; Peters, 2020; Cavenaile et al., 2021), where the relevant firm-level technology measure is the summation of past innovation steps. By making this assumption, this class of models assumes that once an innovation is incorporated into the firm, the specific innovation quality is no longer relevant. This feature is not well suited in our context, because products are born heterogeneous and experience a long life cycle (see Figure 2); further they have different chances of being reallocated depending on their age and sales (see Figure 3).

Once we consider this heterogeneity and its impact on firms' dynamic decisions in a discrete step-by-step innovation model, the firm's problem involves tracking an endogenous distribution of product characteristics. This is an infinite dimensional object and is well known to be complicated to tackle (e.g., noted in finance as in Merton, 1973 and macroeconomics as in Krusell and Smith, 1998). Our assumption of infinitesimal products greatly simplifies this problem while maintaining the rich heterogeneity. In our setting, the decision regarding each product can be separately analyzed, with the firm's evaluation of these products summarized by the quality gap ϕ.

Killer Acquisitions. As noted earlier, brand reallocation has important connections to market concentration. Further, in the model, large firms have an incentive to buy brands even without efficiency gains, as they can clear out competitors and increase markups. We consider the *protective* incentive of a leader to maintain market concentration in this section. Our model thus speaks to the literature on how this protective incentive leads to killer acquisitions (Cunningham et al., 2021). A killer acquisition is a case where large firms buy up-and-coming brands (or firms) in order to kill the product. Our model provides an interesting threshold that links this current paper to this general phenomena.

We define a "killer acquisition threshold", $\bar{s}$. This threshold $\bar{s}$, defined in Equation (34), is the market share beyond which the protective incentive of the leader implies they have an incentive to engage in acquisitions *even if the fringe firm is arbitrarily more efficient at holding the brand.*[10] For any $s > \bar{s}$ there will be acquisitions of any fringe brand regardless of the fringe firm's efficiency. Further, a leader will *never* sell their brand to a fringe firm regardless of how much more efficient the fringe firm is if their market share is greater than in Equation (34),

$$\bar{s} = \frac{\sigma}{2\sigma - 1}. \tag{34}$$

Equation (34) provides some interesting benchmarks. In the case where products are highly substitutable ($\sigma \to \infty$), the threshold $\bar{s}$ converges to 1/2. This means that if the leading firm has over 50% of the market, they will *never* sell a brand, regardless of the efficiency differential. This is due to firms internalizing the lost market share of efficient competitors. This substitution elasticity is likely closer than the other extreme to the situation noted in Cunningham et al. (2021), who study the market for pharmaceuticals, where the question about up-and-coming drugs is more about the underlying biochemistry than brand image.

In the case where products are highly differentiable ($\sigma \to 1$), the role for killer acquisitions disappears ($\bar{s} \to 1$). This highlights an important gain from product differentiation. When there is significant product differentiation, firms have an interest in sorting brands to efficient firms, and leading firms would never want to kill the brand image of a product. These forces speak to a central general tension in our environment.

[10]Appendix C.3 discusses the derivation of this threshold in detail.

5 Estimation

We estimate the model parameters employing the empirical moments of product innovation, maturity, and reallocation. The model delivers simple objects that enable identification and estimation. We primarily explore two methods of estimation.

5.1 Estimation Procedure

In the baseline estimation, we assume all product groups are identical in their parameters, and we estimate the model to match the aggregate moments. We refer to this as the *homogeneous group estimation*. The homogeneous group estimation provides a natural benchmark, and our primary results will be understood through this lens.[11]

We then estimate the model assuming product groups are heterogeneous in their substitution elasticity, the search cost, and the entry cost. We refer to this estimation as *heterogeneous group estimation*. This approach provides a more granular analysis of particular product groups. In the following paragraphs, we detail the homogeneous group estimation procedure.[12] Table 7 provides the link between moments in the data that are independently calibrated and jointly estimated to the relevant parameters in the homogenous group case.

Externally Calibrated Parameters. We set the discount rate to be the annual risk-free rate of $\rho = 0.02$. We set the labor supply elasticity to be $\varphi = 0.5$ (Berger et al., 2019), and the innovation elasticity to be 1 $(D(\eta) = \kappa_e^L \eta^2)$, as in Akcigit and Kerr (2018). Hottman et al. (2016) estimate the substitution elasticities in a demand system that is similar to our setting. We thus directly take the estimates of UPC-level substitution elasticities from Hottman et al. (2016). In the homogeneous group estimation, we set the substitution elasticity $\sigma = 6.9$, which is the median of UPC-level substitution elasticity from Hottman et al. (2016). For the heterogeneous group case, we take Hottman et al. (2016) group-level substitution elasticities.

Estimation of Search and Innovation Costs. We estimate the innovation and search costs to match observed innovation rates and reallocation rates. Three cost shifters exist that we allow to vary by product group: the innovation cost shifter d_k, the entry cost κ_k^e, and the search cost κ_k^s. Our model provides a direct link from observed market shares and new product creation rate at the group level to these costs.

[11] We discuss the details and estimation methods in Appendix D.

[12] The estimation process is very similar for heterogeneous groups, but simply takes different inputs for each ingredient. We include the heterogeneous group analysis in our results and discuss the procedure further in Appendix D.

Estimation of Leader Appeal Advantage. We reference the regression from Equation (3) in Section 3.3 to estimate the sales difference between a leader and fringe firm in holding a brand, which is 0.391. Controlling for brand $\times$ group, age, and year fixed effects, top 10 firms have 0.391 log points higher sales than non-top 10 firms.

Estimation of Gains from Trade. We evaluate event studies when brands flow from large to small firms to identify the distribution of the fit between brand and firm when there are gains from trade. We assume the distribution of γ, F_{γ}, is exponential with the mean identified off of the event studies. We find this average to be 0.17.

Table 7: Estimation Moments and Parameters

Parameter		Value	Moment	Data (p.p.)	Model (p.p.)
Independently Calibrated					
Household Parameters					
Discount Rate	ρ	0.02	Annual Risk-free Rate	**Exact Match**	
Substitution Elasticity	σ	6.90	Hottman et al. (2016)		
Firm Parameters					
Leader Advantage	α	0.39	Leader Advantage (Table 5)		
Product Quality				**Exact Match**	
Age Profile - Growth Rate	ι	0.04	Sales growth (Figure 2a)		
Age Profile - Peak Appeal	$\bar{\beta}$	0.45	Sales peak (Figure 2a)		
Distribution at Entry - $\mathbf{N}(0, \varsigma_{\beta_0})$	ς_{β_0}	2.31	SD sales, age 0 (Figure 2b)		
Innov. + Reall. Elasticities				**Exact Match**	
Matching Elasticity	m	0.21	Sales-Reallocation Profile		
Innovation Elasticity	d	1.00	Akcigit and Kerr (2018)		
Jointly Estimated					
Leader Innovation Cost	κ_e^L	5232.20	Leader New Product Share	0.25	0.25
Fringe Innovation Cost	κ_e^F	7.01	Fringe New Product Share	0.83	0.83
F-t-L Reallocation Cost	k_s^{FL}	624.67	F-t-L Flows	0.62	0.62
L-t-F Reallocation Cost	κ_s^{LF}	14.29	L-t-F Flows	0.53	0.53
Match Quality	Exp. Dist $\bar{\gamma}$	0.13	L-t-F Sales Effect	0.17	0.17

Notes: parameters estimated separately (top panel) and jointly (bottom panel). Source: RMS Nielsen, USPTO and author calculations.

We rely on the optimality conditions for innovation, entry, and acquisition to recover these parameters. First, we note the marginal value of the state variables can be written as functions of quality gap ϕ and growth rate g. Both variables have data counterparts. Specifically, the quality gap ϕ has one-to-one mapping to the observed market share given σ_k; the growth rate g is linked to the new-product innovation rate by the fringe firms. With these two variables, we can directly calculate the marginal value of products to the group leader. For each product group, we find the set of parameters (κ_k^e, κ_k^s) that minimize the distance between data prediction of the leader's innovation rate, average selling rate, and innovation rate of fringe firms.

Estimation of Matching Elasticity. We estimate the innovation and matching elasticity using indirect inference. The targeted moment for this elasticity is the age profile of a product getting transacted. In our model, the difference between the transaction rate for a new product and for a matured product is governed by the difference in marginal benefits as well as the matching elasticity. The difference is in the matching elasticity. In the extreme, if the matching function is inelastic with respect to tightness, no differential in the sales-transaction rates exists. Our estimation yields a matching elasticity of 0.292.

5.2 Comparison of Untargeted Moments

We now compare the predictions of the model regarding the data moments that are not targeted in the estimation procedure to inform the overall model fit. We discuss five different moments of interest. which are detailed in Table 8, and discussed in turn below.

Out-of-Sample Summary. Table 8 summarizes the untargeted moments in our analysis and the sign of the response if there is not a point estimate. Qualitatively, our out-of-sample moments match the data and the literature.

Table 8: Untargeted Moments, Summary

Outcome of Interest	Model	Data/Literature
Leader Market Share	0.327	0.316
Event Study Log Prices	0.066	0.057
M&A Premium	0.42	0.47 (David, 2020)
Age Distribution of Brands	see Figure 5	see Figure 5
Entry Response to Transactions	Qualitative Match (+)	0.028
Entry Response to Transaction Age	Qualitative Match (-)	-0.083

Market Concentration. We do not directly target market concentration, e.g. the sales share of the market leader. Our model focuses on the observed innovation, reallocation, and maturity in the data to predict a given market share for the leader. However, there are many forces determining the market share of a market leader. In the estimated model, we project the leader to hold 32.7% of the market, which is close to the observed empirical shares. In the data, the leader holds on average 31.6% of the market in each product group, which is close to the model prediction. Further, in our heterogeneous group estimation, we find the correlation between the model project concentration and the observed concentration to be 0.2.

Fringe-to-Leader Event Study: Prices. The model delivers an out-of-sample prediction on the change in prices upon transaction from a fringe to a leading firm. We follow Hottman et al. (2016) who find that average marginal costs for leading firms are similar to fringe firms, and set leader labor productivity

$\exp(z) = 1$. We then predict the change in brand prices upon transaction, and find a similar response in the quantitative framework (0.066) to our event study (0.057).

Entry, Reallocation, and Maturity. While we separately used moments in entry, reallocation, and maturity in generating the model, we did not use any features of the *correlation* between these movements. Here, we discuss briefly some qualitative out-of-sample results. First, we would expect firms to be more likely to enter if the firm expects a brand reallocation from a leading firm. We perform the following logistic regression to understand the contribution of transaction age and transaction rate at the group-year level to the effect of brand entry, regressing an indicator on whether a brand is entering a product group on the mean transactions in the group by year $\bar{M}_{jt}$, and the average age of transactions $\bar{D}_{jt}$ by group-year.

$$\eta_{ijt} = \beta_0 + \beta_1 \bar{M}_{jt} + \beta_2 \bar{D}_{jt} + \epsilon_{ijt} \tag{35}$$

Table 9: Logistic Regression of Probability of Holding Entering Brand and Transaction Rate/Age

	(1) Brand Entry	(2) Brand Entry	(3) Brand Entry
Transaction Rate (Standardized)	0.011		0.028***
	(0.0074)		(0.0075)
Transaction Age (Standardized)		-0.079***	-0.084***
		(0.0067)	(0.0068)
N	367421	367421	367421
Pseudo-R2	0.0135	0.0142	0.0142

Standard errors in parentheses.

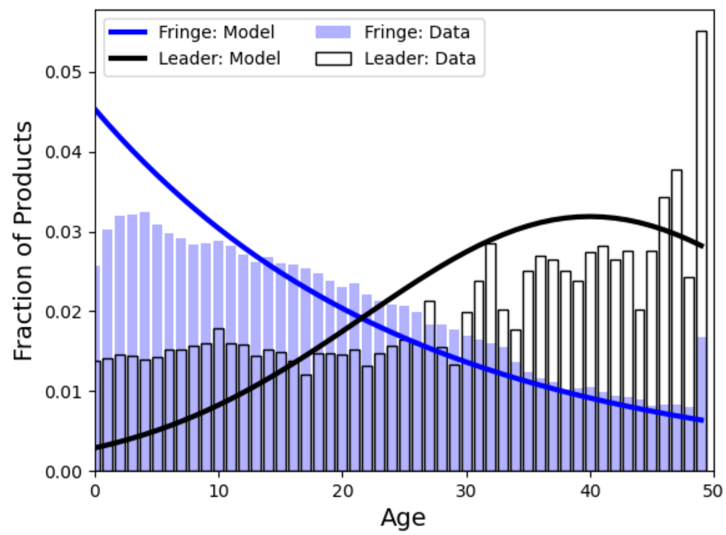
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: Brand entry logistic regressions on reallocation rates and age following Equation (35). Source: RMS Nielsen and USPTO.

We find a weak correlation between entry and reallocation, seen in column (1) in Table 9. Part of the reason for the weak connection links to maturity. In markets where older brands are transacted, we would expect the amount of transactions to have less of an impact on entry. When we control for both, we find a strong effect of each. We direct attention to column (3), which includes both the transaction rate at the group-year level and transaction age at the group-year level. We find that both forces independently matter for entry, consistent with the model predictions.

Age Distribution of Brands. Due to selective reallocation and differences in innovation intensities, the age distribution of brands held by leaders and fringe firms are likely to be different. While untargeted, this moment would provide affirmation that the reallocation and innovation margins are correctly generating the age distribution of brand holdings. We find qualitatively the model and data indicate a similar pattern. In the data, we see the age distribution of holdings. Leaders tend to hold older brands, as indicated in Figure 5.

Figure 5: Age Distribution of Brand Holdings



Notes: Age Distribution of leading (top firm) and fringe firms (all else), in data and model. Source: Author calculations and USPTO.

M&A Premium. The way gains from trade are split between buyers and sellers of product ownership is important for the counterfactual analysis. We thus compare our model's prediction regarding the rent splitting with the ones observed in data. In the models with random search (David, 2020), the rent split between buyers and sellers is primarily determined by the bargaining powers of both parties in the Nash bargaining step. In our model, due to the assumption of competitive search, the rent split is a by-product of the matching process, and thus primarily determined by the estimated matching elasticity m. David (2020) estimated the average premium to be 0.47. Our model predicts a weighted premium of 0.42, which is in line with the empirical finding.

6 Quantitative Analysis

With the estimated model, we are ready to explore the quantitative implications of our framework. We do this in two steps. First, we decompose the main driving sources of the variation in growth and concentration and discuss welfare implications. Second, we explore various policy counterfactuals related to innovation, reallocation, and antitrust policies.

On the first point, we start by discussing the sources of growth (e.g., innovation, maturity, reallocation) through the lens of our model. We then turn to the sources of market concentration. The discussion of these two forces presents an important tension in the economy, and our policy analysis will explore this tension. On the second point, we analyze standard policies (e.g. blocking acquisitions, acquisition taxes and subsidies, entry subsidies) through the lens of our quantitative framework and evaluate their joint effect on growth, concentration, and consumer welfare. We also expand the discussion to address size-dependent policies, e.g. taxes that scale with market concentration. We focus on the main characterization under the homogeneous group estimation.

6.1 Sources of Growth: Innovation, Maturity, and Reallocation

Most models of growth from expanding product variety focus on product entry as the central driver of economic growth. Motivated by empirical evidence from Section 3, we depart from this standard, and note that reallocation and maturity are also essential components of growth. We return to our growth equation from the model to explore the interaction between these margins, inputting in the relevant data counterpoints. From the model, the growth rate of real consumption can be decomposed into the following margins:

$$g_{\mathbf{C}} = \int_0^1 \frac{\xi_k \nu}{\sigma_k - 1} \left(\underbrace{\eta_{kt}^L + \eta_{kt}^F}_{\text{Innovation}} + \underbrace{\iota_{kt}^L + \iota_{kt}^F}_{\text{Maturity}} + \underbrace{\Lambda_{kt}^{FL} + \Lambda_{kt}^{LF}}_{\text{Reallocation}} \right) dk,$$

where ξ_k is the appeal of group k, σ_k is the substitution elasticity of group k, and ν is the love-of-variety of the representative consumer. Our goal is to use the growth equation to decompose the variation driven by the three main processes in the data. First, there are the innovation rates of the leader and the fringe. Second, there is the brand maturity. The last three are the reallocation flows: fringe-to-leader and leader-to-fringe.[13]

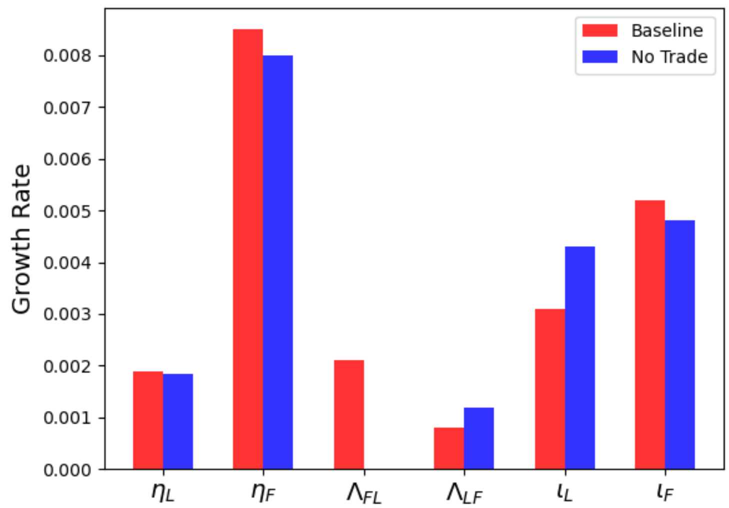
Figure 6 focuses on the contributions to growth in three different scenarios applying the homogeneous group calibrated model.[14] First, we evaluate the baseline economy (red), then we turn off fringe-to-leader exchange (blue), then we shut down trade completely (green).

We stress three main findings from Figure 6. First, as can be seen in the baseline economy, fringe entry η_F, leader brand maturity ι_L, and fringe-to-leader reallocation, Λ_{FL}, are the three main sources of growth. Second, when we shut down reallocation across firms, we find that the steady state responses of each force is small. However, this does not include the transitional dynamics, as the economy is operating

[13] We apply the definitions of leader and fringe discussed in Section 3 and the theoretical specifications from Sections 4 and D.

[14] Due to the log-linear structure of the utility function, the growth rates at product group level can be decomposed and linearly aggregated.

Figure 6: Sources of Growth, Baseline and No Reallocation Equilibrium



Notes: Sources of growth from fringe F, leader L; innovation (η), reallocation (Λ), and maturity (ι). Source: Author calculations.

at lower capacity. We discuss the implications for overall welfare in Section 6.4.

6.2 Sources of Concentration: Innovation, Maturity, and Reallocation

As this paper has stressed, concentration emerges from three dynamic forces. Within the model, concentration can be decomposed into the dynamic components driving it,

$$\phi_k = \frac{\eta_L + \iota_L + \alpha \Lambda^{FL} - \Lambda^{LF}}{\eta_F + \iota_F - \Lambda^{FL} + \gamma \Lambda^{LF}}. \tag{36}$$

For each type of flow, we define a counterfactual concentration as the ratio of appeals in the counterfactual, and calculate the ratio between the predicted concentration and baseline concentration. This force has the potential to limit output in the market through pricing power as discussed in Section 4. Table 10 focuses on the contribution of innovation and reallocation to concentration.

Table 10 compares the leader's market share in baseline economy to the counterfactual case where there is no reallocation and no maturity, on the balanced growth path in the homogeneous group estimation. We thus report the leader's market share and the contribution of innovation, maturity, and reallocation to this market share by assuming the corresponding elements to be zero. Ignoring the maturity margin, our model predicts a higher concentration (from 32.70% to 36.83%). This comes from the fact leaders tend to hold more mature products that grow slower while fringes tend to hold new products that grow faster, matching the pattern in Figure 2a. Shutting down reallocation and maturity altogether decreases concentration to 27.37%. Reallocation contributes to one-forth of the baseline leader's market share.

Table 10: Concentration – Innovation vs. Reallocation

	All	Innovation + Reallocation	Innovation Only
a. Homogeneous Groups			
Baseline			
Leader's Market Share (%)	32.70	36.83	27.37
No Reallocation			
Leader's Market Share (%)	21.59	24.27	28.44
a. Heterogeneous Groups			
Baseline			
Leader's Market Share (%)	32.70	35.92	27.12
No Reallocation			
Leader's Market Share (%)	21.33	22.78	27.87

Notes: Each cell reports the leader's market share. For the column All, we report the leader's market share in the baseline case and the case without reallocation, allowing all components of Equation (36) to change; For the column Innovation+Reallocation, we assume maturity rate $\iota = 0$, allowing innovation and reallocation to change; In the column Innovation Only, we only allow innovation rate to change. Source: Author calculations.

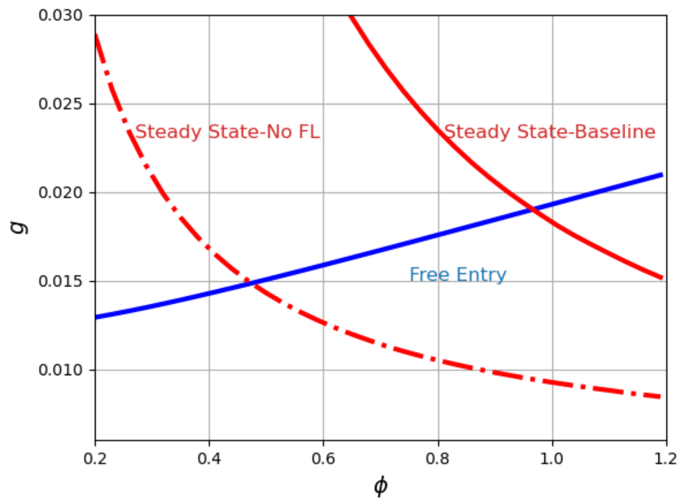
Simply using Equation (36) ignores the equilibrium responses of firms' strategies. In the second row of Table 10, we report the counterfactual leader's market share by solving the new balanced growth path without reallocation. Comparing to the baseline level, in the balanced growth path without reallocation, the leader's market share falls to 21.59%. However, if we simply focus on the predicted concentration due to innovation, concentration increases without reallocation. This comes from the rent sharing effect of reallocation. As reallocation is shut down, fringe firms are less compensated by the option value of selling, and fringes' entry rate falls. This fall increases concentration on the balanced growth path.

6.3 Good Concentration or Bad Concentration?

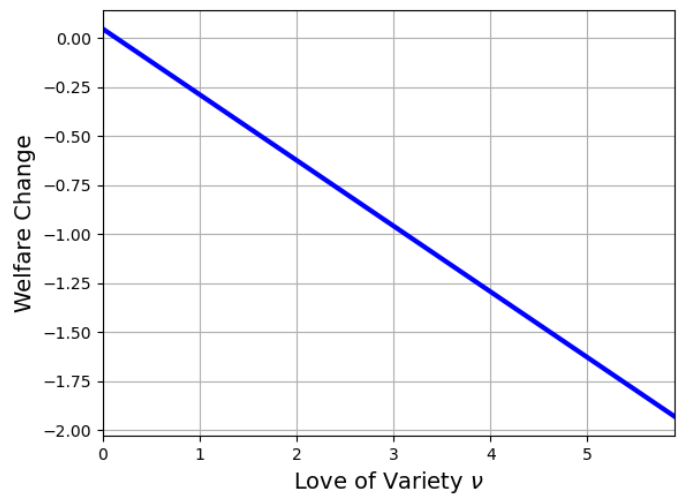
Recent literature (e.g., Covarrubias et al., 2019) point out that concentration can be "good" in the sense that more productive firms take larger market shares and increase consumer welfare, but it can also be "bad" in the sense that larger firms amass pricing power and restrict consumer substitution. Our quantitative results indeed point to the coexistence of both effects. Although we find in our baseline estimation the growth effect dominates and the reallocation of product ownership tends to increase aggregate efficiency, in this section we show this conclusion hinges on the magnitude of the love-of-variety. By increasing the love-of-variety elasticity ν, we up-weight the importance of growth in consumption. An informative threshold to note is the level of love-of-variety elasticity such that the growth effect and the concentration effect exactly offset each other. Figure 7 illustrates this tradeoff, plotting the equilibrium growth rate against the concentration calibrated from the homogeneous group model. In this scenario, we plot two steady state curves: one in the benchmark equilibrium, and one without any fringe-to-leader brand exchange.

There are countervailing effects on growth from rising concentration. A more concentrated industry $\phi \uparrow$ is associated with more opportunity from entry through reallocation and the higher markups from

Figure 7: Welfare Comparison to No-Reallocation BGP



(a) S.S. Comparison, Growth (g) and Concentration (ϕ)



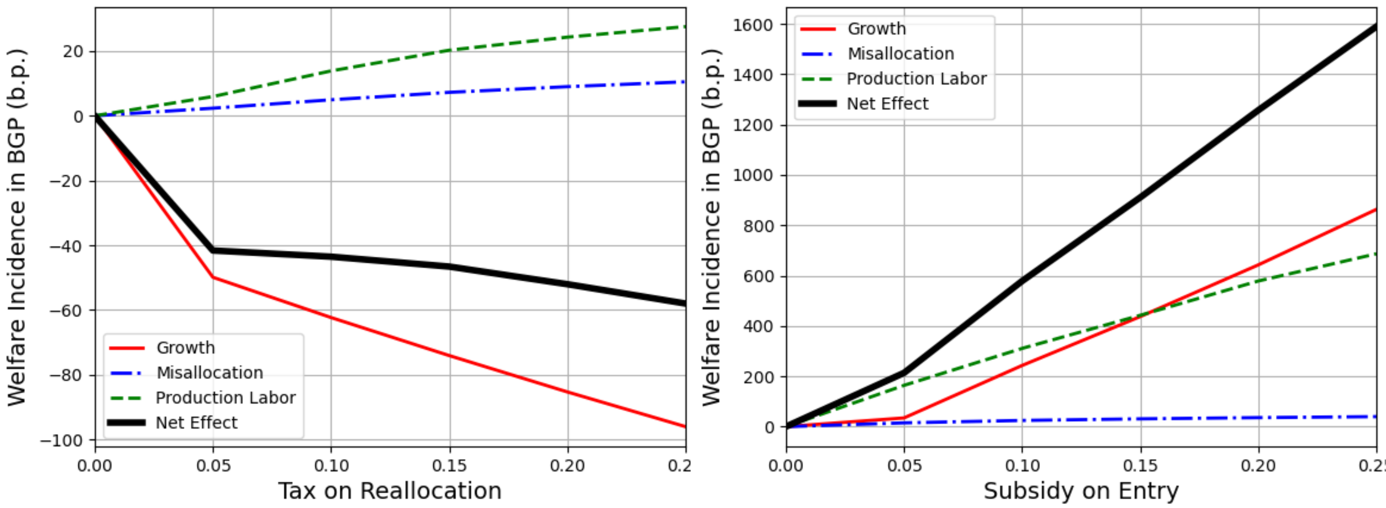
(b) Varying Love-of-Variety ν

Notes: Panel (a) plots the counterfactual scenarios for concentration and growth; Panel (b) plots the welfare loss with shutting down reallocation, through varying love-of-variety. Source: Author calculations.

incumbents. However, given a level of brand appeal in the economy, a higher concentration will dampen growth through limiting the ability of entrants to build brand appeal. Further, higher concentration gives leaders more pricing power and inefficiency from markups. We find that in net, shutting down brand reallocation decreases steady state welfare by 1.93% given the estimate of love-of-variety (5.9) that links expenditure growth to consumption growth.

The consolidation of brands connects to classic debates in the role of brands and marketing in the economy (e.g., the role of advertising and branding as a costly or informative expenditure, Galbraith, 1958 versus Stigler, 1961), an issue we mostly sidestep in this paper. As brand capital continues to play a large role in the macroeconomy, this issue has important implications for thinking about the interaction of firm product ownership acquisitions. If an increase in sales at the consumer-level is due to the consumers love-of-variety, this leads to more efficient outcomes when leading firms acquire brands. If one takes the view that brand creation and brand development is simply an activity that poaches customer capital from other firms, the costs of concentration are higher than we find here. In this paper, we work mostly in line with the significant empirical literature that points to large consumer benefits from new products and product maturity, but believe these questions will be important in further research on the macroeconomic implications of brands.

Figure 8: Effects of Reallocation Tax/Entry Subsidy on BGP



(a) Reallocation Tax (b) Entry Subsidy

Notes: Welfare response to taxes and subsidy outcomes. Source: Author calculations.

6.4 Policy Analysis

This section explores various policy tools that may be implemented in markets where policymakers either have an interest in the costs of market concentration or the benefits of innovation. We apply some standard tools (e.g., transaction tax or entry subsidy) and ask about the economic implications of these policies for concentration, growth, and welfare. Our framework provides a general equilibrium setting where both growth and concentration of the market are endogenously determined by firms' innovation and reallocation activities, and have welfare consequences.

Reallocation Taxes. Figure 8 focuses on the effects of reallocation taxes (panel a) and entry subsidies (panel b) on welfare along the balanced growth path in the homogeneous group estimation. We note that welfare responds positively to entry subsidies as it both alleviates the distortion from concentration and increases growth.

Table 11 focuses on the effects of different taxes and subsidies on reallocation. We perform these policies in one world where all markets have the same structure, panel (a), and one world where we allow the parameters to vary by group, panel (b). We ask what the corresponding change is to policy implementation in terms of concentration (leader share), growth rate, and welfare (in both the steady state BGP and including the transitional dynamics).

First, we note that taxes on reallocation have the expected effects on leader market share. Against a baseline of 33% market share, taxes on reallocation can reduce the leader's steady state share significantly,

Table 11: Counterfactual – Reallocation Tax and Entry Subsidy

	10 % Reallocation Tax	10% Entry Subsidy
a. Homogeneous Groups		
Leader's Market Share (p.p)	24.21	20.30
Δ Growth rate (p.p) (%)	-0.012	0.046
Δ Welfare (BGP, p.p)	-0.432	5.839
Δ Welfare (Transition, p.p)	-0.511	5.230
a. Heterogeneous Groups		
Leader's Market Share (p.p)	26.94	21.56
Δ Growth rate (p.p) (%)	-0.009	0.022
Δ Welfare (BGP, p.p)	-2.932	4.852
Δ Welfare (Transition, p.p)	-0.133	4.211

Notes: Reallocation tax and entry subsidies, outcomes in counterfactual. Source: Author calculations.

to 24%. However, the net welfare effects are negative, as the overall growth rate also exhibits a significant negative decline. This decline in growth occurs through both the static loss from reallocation of products to better firms and the dynamic loss from entry. Shutting down trade completely is a costly policy.

The efficiency gains from reallocation overall outweigh strategic gains. This implies antitrust policies such as taxing transactions may not be efficient, if done at the *aggregate*. However, when we look across groups, there may be a set of groups where taxing transactions would be efficient. Thus, a key question for policy is at what level it is implemented. A coarser policy that does not take into account the rich market dynamics of each sub-market may induce efficiency losses.

Innovation Subsidies. Figure 8 and Table 11 also presents the effects of subsidies on entry. As in the previous table, we perform these policies in one world where all markets have the same structure, panel (a), and one world where we allow the parameters to vary by group, panel (b). We ask what the corresponding change is to policy implementation in terms of concentration (leader share), growth rate, and welfare (in both the steady state BGP and including the transitional dynamics).

We find a strong effect of innovation subsidies on both concentration and growth. As fringe firms have an easier time engaging in innovation than incumbents, the subsidy induces a lot of fringe firm entry. This also increases growth and welfare significantly. This message abstracts away from budget balancing at the government level, so policies for increasing government revenue to support subsidy policies would be important to evaluate. However, it does send a clear message relative to the taxes on reallocation. Subsidizing entry is a better means of reducing market concentration and increasing growth more so than focusing on taxes or blocking of reallocation. Large firms engage in less product entry than fringe firms, and this allows for within-fringe reallocation and declines in inefficient concentration. As in standard growth models, the entry margin is on net quite positive.

6.5 Discussion

This section discusses two aspects of the quantitative framework that deserve some additional emphasis in their relevance for counterfactual analysis.[15]

The Role of Maturity. The downstream innovation response to brand reallocation is a function of the interaction of brand maturity and reallocation. As a result, as a first order, policymakers can ignore the innovation effects of antitrust policy *when transactions are of mature products*, because the discounted value of transactions to entering firms is low. However, there is a rising tendency for products to exhibit shorter life cycles over time and become transacted earlier in their life cycle. For transactions early in the product life cycle, the dynamic effects of reallocation become more relevant, as the option value of selling for a fringe firm becomes more relevant.

We discuss these results quantitatively here. Recall ι measures the speed of product maturity. We evaluate the policy of shutting down reallocation with three benchmarks in Table 12.

Table 12: Counterfactual – Maturity and Efficiency w/ Shutting Down Reallocation

	Baseline	Fast Maturity ($\iota \times 10$)	Slow Maturity ($\iota/10$)
Change in Leader's Market Share (p.p)	-11.11	-9.23	-17.29
Change in Growth rate (p.p) (%)	-0.321	-0.982	-0.141
Welfare (BGP, p.p)	-1.930	-21.75	-0.023
Welfare (Transition, p.p)	-1.332	-19.36	-0.001

Notes: Three counterfactual maturity scenarios. Source: Author calculations.

From column (1) to column (3), we consider how a different maturity rate of products (with $\iota = 4\%$ as the estimated baseline) leads to different market concentration and welfare incidence. We add two extreme cases in columns (1) and (3), one where a product grows at an average 0.4% per year until peak, and another where a product grows at an average of 40% per year until peak. We then compare changes in the innovation cost for entrants (κ_e) and changes in the search cost for product reallocation (κ_s, as a stand-in for an ownership transaction tax). The results are striking, and suggest the maturity channel cannot be ignored in innovation and antitrust policies.

When markets mature quickly (e.g. average growth of 40% to peak), there is a large growth and welfare cost to shutting down reallocation (22% welfare cost). This is because the policy has a larger effect on both entry and reallocation. When maturity is slower (e.g., average growth of 0.4% to peak), shutting down trade decreases the leader's market share with a minimal impact on welfare (close to 0%). This occurs because the decline in reallocation has very little effect on entry, but significantly reduces concentration.

As for policy recommendations, both across industries and over time it is essential for policymakers to understand the life cycle profile and the age distribution of transactions. If older brands are much

[15] We discuss the robustness of our main results to theoretical and empirical specifications in Appendix Section E.

more likely to be sold, then the focus on transactions will weigh the markup and efficiency effects. The framework in our model is a useful method to link market shares, efficiency, and markups.

Yet, in markets where young brands are being transacted, policymakers should note the interaction between entry and reallocation. Entry responds positively to reallocation, in particular if reallocation rates are linked to young products (see Table 9). If policymakers focus only on the predictions on sales and prices, they may miss the dynamic effects and induce efficiency losses by simply looking at the problem in a static setting.

The Demand System. Our demand system with nested CES structure follows a host of papers that study product markets, and we think this framework captures well the competition for product market share. One might wonder, what is the effect of changing the demand system to a different type of system (e.g. an aggregator as in Kimball, 1995)? We find that our results qualitatively go through. The key distinguishing aspect is that when products move from fringe to leading firms it has similar outcomes as when products move from smaller to larger firms more generally. Both Kimball (1995) and Atkeson and Burstein (2008) generate pricing power gaps that emerge through market concentration. We find nested CES the most demand system to discuss the role of multi-product firms and the mechanisms through which firms build their market power through product innovation and reallocation, because this is a demand system where the role of product scope is natural.

7 Conclusion

Brand capital is a central component of the modern economy and brand reallocation plays a central role in sales concentration, firm dynamics, and efficiency. We employ a novel dataset on the universe of brands to unpack the role of brand reallocation and brand dynamics in the macroeconomy. We find that brand entry plays a much larger role for small firms than large firms, while brand reallocation plays an essential role for large firms. For both, the life cycle of brands they hold is a key ingredient to their market shares.

After developing the facts that relate the firm-brand dynamics to aggregate concentration and innovation, we introduce a model of multi-product firms that innovate and acquire brands with productive and strategic incentives. These large firms both tend to be more efficient than smaller fringe firms, but have more pricing power through amassing brand capital. This leads to a natural tension between growth and concentration, which we study with the estimated model. We estimate the model using detailed price and quantity-level data at the brand and product-level. We use the estimated model to study a set of relevant policy counterfactuals: How does taxing brand reallocation affect consumer welfare? How does subsidizing brand entry? How does shutting down trade impact the economy?

We find taxes and blocking reallocation to large firms tend to reduce both concentration and growth, leading to lower welfare. Subsidizing entry is a policy that can be used to target the same level of concentration with lower growth cost. Further, there is significant heterogeneity in optimal policy across product groups. If policy is coarse, taxes and subsidies on reallocation may decrease economic efficiency. If policy can be applied by group, there may be gains from subsidizing reallocation in some groups and taxing reallocation in others. However, for the same level of concentration entry subsidies persistently appear to induce more growth and be more welfare-enhancing.

Empirically, one avenue for further research is to understand the long-run evolution of the market for brands and long-run changes in ownership structure. This touches on important topical economic questions in innovation, concentration and the role of intangibles in firm dynamics. Understanding the brand-firm or product-firm interaction is essential to understanding the trends in market shares and market dynamics. Theoretically, as the importance of brand capital continues to rise, we believe frameworks that address the connection between brands and growth will be essential for academic and policy discussions. We expect to see brands playing an important role in linking firm dynamics to market shares and the aggregate economy. Placing brands into an endogenous growth framework provides a new foundation for understanding the joint determinants of markups, concentration, and growth.

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Appendix

Our Appendix is in five sections, mirroring the structure of the text. Appendix A discusses the data background and general points about large firms and brand acquisitions. Appendix B discusses the empirical analysis connections to the literature and robustness. Appendix C discusses the theoretical proofs and expands on the firm's dynamic problem. Appendix D discusses the estimation. Appendix E discusses the general robustness of the quantitative results. For an updated Online Appendix discussing technical details, please see liangjiewu.com/files/tm_pw_apx_oct22.pdf.

A Data Appendix

This section addresses the set of data sources relevant for the analysis and the data examples that motivate our investigation. Section A.1 motivates the general setting by evaluating examples of market concentration and brand-building at the firm-level. Section A.2 expands on the details of the merge across datasets.

A.1 Product Market Concentration: Large Firms

A salient feature of markets for intellectual property is the common rate of exchange, in particular with multi-product firms. We find this in both our data sources (USPTO and RMS Nielsen), and additionally from investigating firms in company reports of acquisition. Firms detail their major acquisitions and reasons for acquisitions in press releases. Firms claim different reasons for acquisition. Often the motivating claims for acquisition are close to the two main theoretical mechanisms in the paper, with many acquisitions claiming often synergies or good product-firm fit (e.g., with the General Mills purchase of Annie's in 2014)[16] and others focusing on the importance of market leadership (e.g., Nestle acquiring Dreyer in 2006).[17]

These persistent transactions lead to the observed skewed distribution of firms, and the fact that many firms hold brands they did not originally introduce. Both of these forces can be seen in Figure A1, where many brands that individuals associate with only the brand are held by larger parent firms.

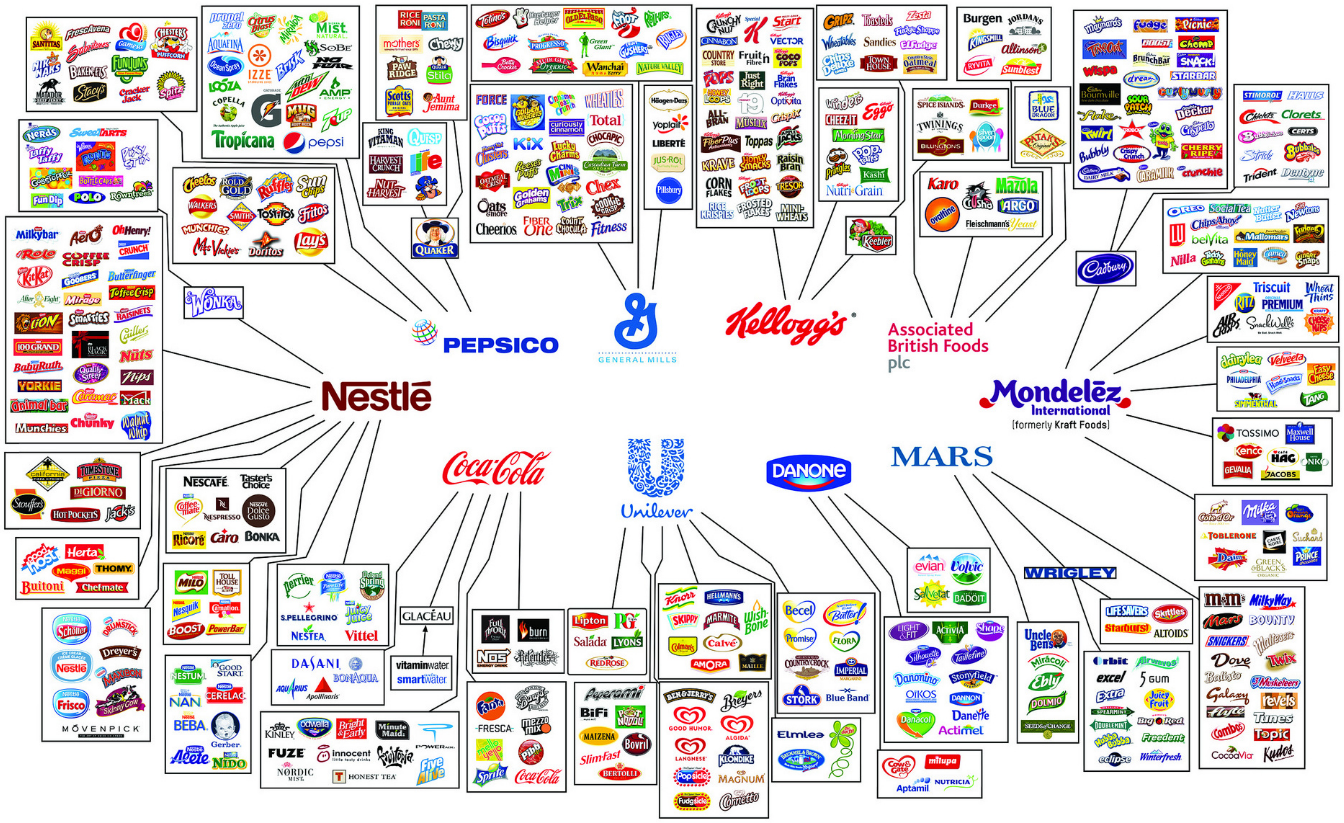
Figure A1 illustrates how many distinct brands are owned by the same firm. In Figure A1, for example, around half of the brands originally started at a different firm from the one it is currently linked to. Further, most brands are mature and took some time to build customer capital. Both findings complement the key ingredients of brand maturity and reallocation in our framework.

This general pattern is true across an array of industries, but the empirical section of this paper directs our attention to the Consumer Packaged Goods (CPG) industry, with the limited consumer substitution.

[16] Source: https://investors.generalmills.com/press-releases/press-release-details/2014/General-Mills-To-Acquire-Annies/default.aspx

[17] Source: https://www.nestle.com/media/pressreleases/allpressreleases/dreyersandworldleadericecream-19jan06

Figure A1: Brands at Major Firms



Notes: Parent Companies for each brand. Source: The Independent, 2017.[a]

[a]https://www.independent.co.uk/life-style/companies-control-everything-you-buy-kelloggs-nestle-unilever
html, Apr 2017, accessed September 2022

We further illustrate this force by showing the progressive increase of brands in Procter and Gamble (P&G) and Johnson & Johnson (J&J), two large companies that hold many brands. We see that their stock of live trademarks is increasing over time, and the hundreds of trademarks seen in Figure A2 represent a host of brands.

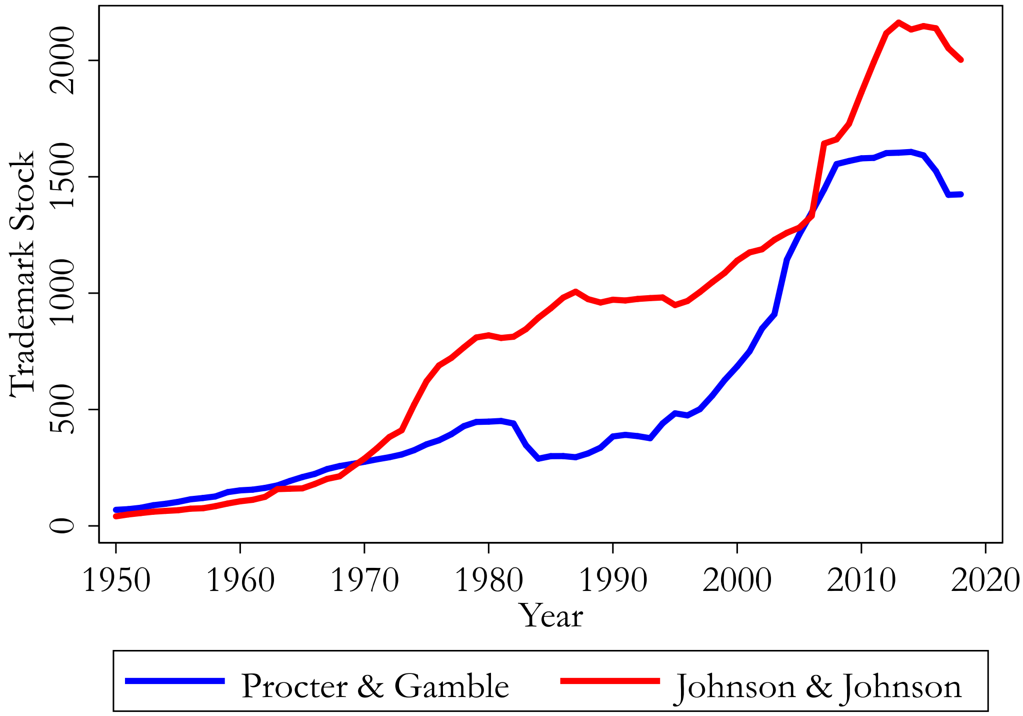
A.2 Data Merge Details

As discussed previously, our main merge links USPTO Trademark data with RMS Nielsen Scanner data. We proceed by linking firms and products separately. Our merge matches over 80% of sales-weighted products. Some problems still emerge with short-names. We use "tokens" and fuzzy matches to deal with the names. Firms and products follow similar procedures and we discuss them in turn.

Firms. For matching firms, we first standardize on a large set of firm tags, eliminating common firm words, e.g. "CORP", "INC", "ESTABLISHMENT").[18] We then take the cleaned and standardized name

[18]The full list is here ('AB', 'AG', 'BV', 'CENTER', 'CO', 'COMPANY', 'COMPANIES', 'CORP', 'CORPORATION', 'DIV', 'GMBH', 'GROUP', 'INC', 'INCORPORATED', 'KG', 'LC', 'LIMITED', 'LIMITEDPARTNERSHIP',

Figure A2: The brands of P&G and J&J over time



Notes: This collects the total stock of trademarks held by P&G and J&J in each year since 1950. Includes trademarks held through registration and assignment. Source: USPTO.

and match according to a tokenized bigram matching procedure.

Brands. By focusing on brands, we direct our attention to long-running products held by firms. USPTO Trademark data provides the "tm_name" or the name associated with a registered trademark. RMS Nielsen follows a similar format, which has a "brand_name". We join the two by employing a token name matching. For brand names, there are no further removals of tokens beyond the firm-level analysis.[19] For brand age, we focus on the "prior" brand, as in the broader brand umbrella of the production. For transacted brands, we observe the level of the transaction and focus on this.

Transactions. As we discussed previously, we leverage evidence from transactions in both USPTO and RMS Nielsen Scanner data. Overall, we get 20% of brand transactions from USPTO Trademark data and 80% of transactions from Nielsen. While there are more transactions observed in trademark data, there are some within firm transactions we drop, as we generate a text similarity threshold above which we do not consider transactions.

'LLC', 'LP', 'LTD', 'NV', 'PLC', 'SA', 'SARL', 'SNC', 'SPA', 'SRL', 'TRUST', 'USA', 'KABUSHIKI', 'KAISHA', 'AKTIENGESELLSCHAFT', 'AKTIEBOLAG', 'SE', 'CORPORATIN', 'GROUP', 'GRP', 'HLDGS', 'HOLDINGS', 'COMM', 'INDS', 'HLDG', 'TECH', 'GAISHA', 'AMERICA', 'AMERICAN', 'NORTH', 'OPERATIONS', 'OPERATION', 'DIVISION', 'COMPAGNIE','INTERNATIONAL', 'NORTH AMERICA', 'InBev').

[19]Standardizations include removing any relevant firm names as discussed in the firms section, but does not do any further standardizations and tracks the token grams within each brand name.

B Empirical Appendix

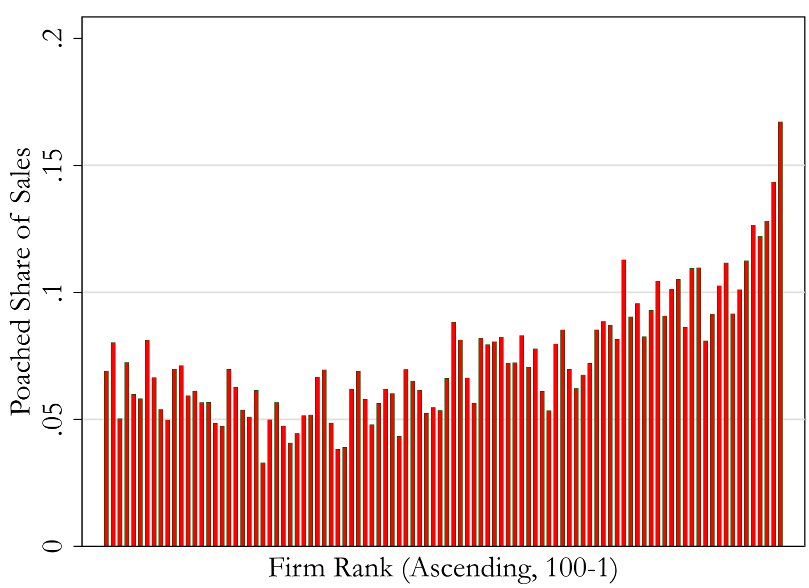
This section explores some additional evidence on a couple core messages from the paper, focusing in particular on the firm, brand, and firm $\times$ brand analysis. We apply broader data from the USPTO to indicate the fact that large firms build large portfolios of brands and their acquired brands drive a larger share of their portfolio. We then discuss the product life cycle with reference to the literature and discuss the integration of the product life cycle with our firm-level analysis. In each case, we explore the robustness of our results to varying definitions.

We start by expanding on the main elements of firm analysis in Section B.1, returning to the study of the sources of concentration, and evaluate the robustness of the empirical results on firms. We expand on the product life cycle in Section B.2, focusing on the interaction of age and sales, and the evidence for the importance of product maturity and sales dispersion over time. We further discuss our connection to the literature on the product life cycle and then turn to the robustness of product-level results. We then explore the event studies and the interaction of reallocation flows across firms in Section B.3. Lastly, we discuss the types of reassignment in the trademark data in Section B.4, which is in part a plea for further research to investigate further the sources and implications of IP reallocation.

B.1 Firm-Level Analysis

In Figure 1, we showed how buying of brands contributes significantly to large firms market share. Figure B3 shows this pattern with respect to sales in Nielsen Scanner Data. We plot the share of sales from bought brands against the percentile (running from 1-100) of the firm size in sales.

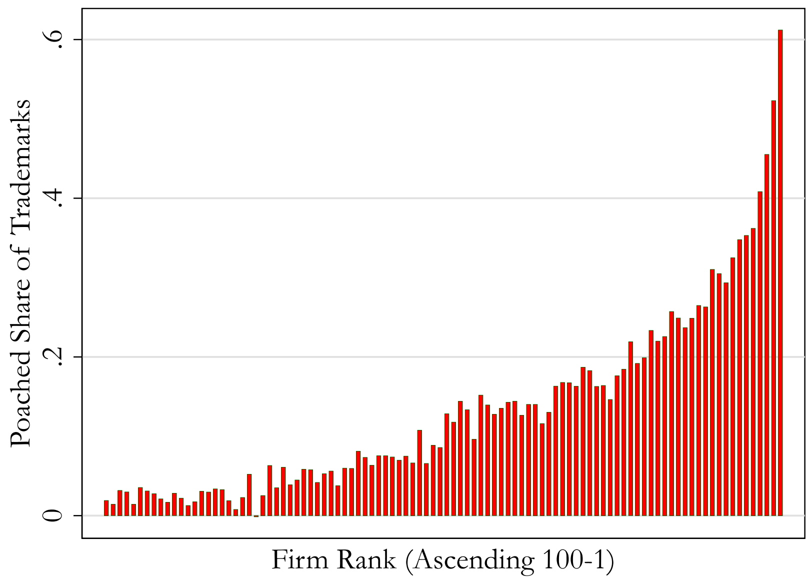
Figure B3: Contribution of Buying to Sales Share



Notes: Share of total sales from poached brands. Source: RMS Nielsen and USPTO Trademark

We find that the highest-selling firms have almost 4-times as much poached share of sales that a median firm, indicating that the pattern we find in the Trademark data on its own is consistent in the sales-share data. We observe this in both RMS Nielsen Scanner data and in USPTO Trademark data. Turning to USPTO data, we find the results are even more stark. Large firms tend to carry bought trademarks as a much larger share of their portfolio. This is noted in Kost et al. (2019), and can be seen in Figure B4.

Figure B4: Contribution of Buying to Trademark Stock



Notes: Market share reallocation measures across different firm types, following Equation (1). Source: RMS Nielsen

In Table 4, we explored the concentration due to incumbent products versus entering or reallocated products. However, incumbent variation may be driven by many forces outside of the life cycle of the product. Here, we combine our life cycle analysis with the variance decomposition in Equation (1).

we run the following regression of three distinct margins of change, y_{it}, on the change of sales in each period $\Delta sales_{it}$:

$$y_{it} = \alpha + \beta \Delta sales_{it} + \epsilon_{it}. \tag{37}$$

Equation (37) focuses on three different margins for y_{it}, with the entry and reallocation following the same structure as Table 4. We substitute y_{it} as the fitted value of sales on maturity ("Fitted Maturity") to understand how much of the observed variation from maturity is due to predictable life cycle growth. Table B1 evaluates the contribution of each force in the equation.

We find similar patterns in Table 4 and Table B1. When we take fitted values from product-level age regressions, as discussed in the next section, the general pattern stays the same. We note that each force has a non-negligible contribution to the distribution of market shares, and our empirical model explains around 85% of the variation for large firms, and 70% of the variation for fringe firms.

Table B1: Sources of Reallocation

	(1) Entry	(2) Fitted Maturity	(3) Reallocation	(4) Unexplained
Leader	0.033*	0.70*	0.13*	0.14
Fringe	0.091*	0.57*	0.021*	0.32

* $p < 0.001$

Note: Market share reallocation measures across different firm types, following Equation (37). Source: RMS Nielsen.

B.1.1 Empirical Robustness: Firm Measures

In our robustness, we return to look at the qualitative similarities between the results in the main text and results depending on the definition of the firm and the main data used. For the main paper, we maintain the same dataset, focusing on brands with at least \$1000 sales in a given year and brands that successfully merge to a trademark. Furthermore, given the nature of ownership we keep only the primary owner of a brand. This robustness section focuses on the empirical facts at the firm-level addressing some changes to these definitions.

The results in Section 3.1 delivered two main messages. First, markets are highly concentrated, as the largest firm is more than 1000-times larger than the median firm across most product groups. This is noted in Figure 1 and Table 3. Second, large firms' outcomes are more driven by product maturity and product reallocation, while smaller firms rely much more on product entry. This is noted in Table 4 and Table B1.

This current section explores varying the definitions of the firm-product relationship. We explore the differences in both the RMS Nielsen data on its own and USPTO Trademark data. When exploring RMS Nielsen data, we expand our sales to include unmerged brands and brands with sales less than \$1000. As a result, we revisit Table 3 with different definitions of firm ownership. In overview, the results are qualitatively similar. We turn to the two main departures in our definition of the firm. We first look exclusively to Nielsen scanner data and then USPTO Trademark data.

RMS Nielsen Scanner. Our first fact primarily employed RMS Nielsen Scanner data, but we only included the successfully merged products in order to maintain a consistent sample. Given the success of the merge, one should expect the general results to be similar. In this section, we confirm that intuition.

The average top firm share is 32% of the total market in the main part of the manuscript. When we expand our set, we find that the results are similar. We visit the shares in Table B2, as below, where we drop any external observed reassignments:

Without adjusting the weights by the overall sales of a group, we find a similar skewness in firm size albeit with the top 2 firms having a larger share, as well as the median firm. This can be seen in Table B3.

Table B2: Firm Market Shares in 2010, Restrict to Merged w/o Adjustment

Top firm share by group	Top 2 firm share	median share
30.4%	45.4%	0.01%

Table B3: Firm Market Shares, unadjusted weights

Top firm share by group	Top 2 firm share	median share
33.6%	50.3%	0.06%

USPTO Trademarks. We now define firms at the USPTO level rather than the Nielsen level to explore different patterns in share holdings. For unidentified transfers[20], we maintain the originator as the parent company.

B.2 Brand-Level Analysis

In this section, we expand on the brand-level discussion in the main text, referring to brands and products interchangeably unless specifically indicated. Products are both a significant source of firm concentration (Hottman et al., 2016), yet highly dynamic (Argente et al., 2020a). The concentration implies a rich heterogeneity, but the dynamic nature implies that heterogeneity changes over time. The change in products can come from development of a product line or transactions of products from worse to better firms. Our goal in this section is to isolate the product element of the life cycle and show how even separate from the firms that hold them, products exhibit rich life cycles. This general point has been shown before (e.g. Argente et al., 2020a, 2021), but by integrating with USPTO Trademark data we are able to examine the longer brand life cycle and control for the transactions across firms.

Some products charge to dominance quickly, others rise gradually but maintain leadership, whereas others survive but remain in obscurity. Yet all brands must build consumer capital to build market share. We direct our attention to brand *age* as a key ingredient to product market shares. We first focus on a snapshot of the distribution of sales by age, and then turn to an analysis of the life cycle to understand the more granular dynamics.

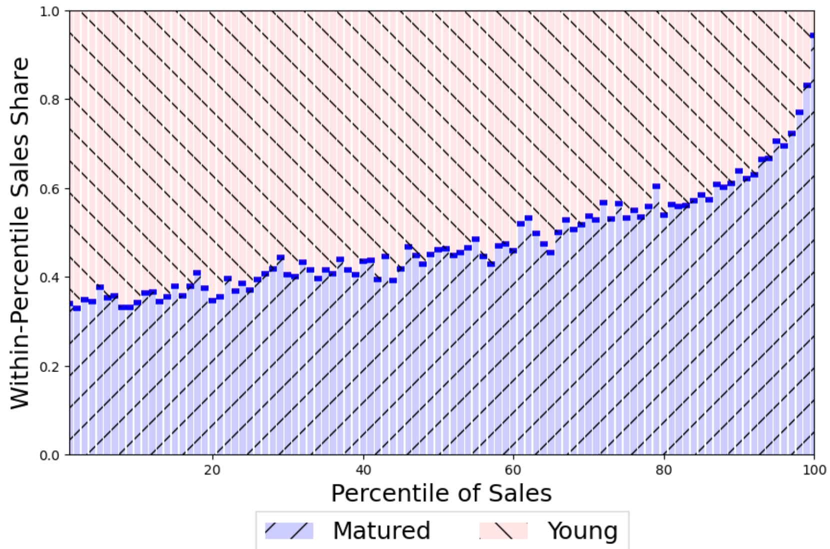
Products evolve over their life cycle. Gourio and Rudanko (2014) and Foster et al. (2016), among many others, have noted that customer capital is not built in a day. By looking at trademark data and Nielsen data, one can observe the importance of senior brands. Figure B5 takes data from 2016. We plot the brand percentile in terms of overall sales on the x-axis. On the y-axis, we plot the share of sales in this group that belongs to brands older than 10 years and brands younger than 10 years.[21]

For brands created in 2006 and earlier, they maintain large sales share into the future. By 2016, those brands are still dominant in the top 1% of brands. Within the top 1% of brands, brands created before 2006 make up 92% of sales. Overall, old brands make up over 70% of sales, but only about 1/3rd of

[20] There are cases where trademarks are reallocated to unidentified firms, and we limit our use of these observations.

[21] We omit brands with less than $1000 in sales over an entire year, to have only brands that at least have a product line.

Figure B5: Brand Percentile and Maturity



Note: This figure shows the sales share within a percentile bin of products, split by those born before 2006 ("Matured") and after 2006 ("Young").

Source: RMS Nielsen Scanner Data.

products. For the median brand in terms of sales, older brands make up less than half (38%) of total sales.

The dominance of mature brands could come from two forces. First, if few brands achieve such large sales, there may be a selection process. Young brands have less of a chance than old brands to have high consumer capital, as the brands that survive to maturity must have a high quality draw. The composition only selects for the best. Second, brands could increase their sales over the life cycle such that only mature brands have significant sales share. We aim to understand this by linking a brand to its specific age. This is noted in the main text in Equation (2) and Figure 2a. Yet, we are not the first to focus on this life cycle so we review the current literature benchmarks here.

Literature Benchmark: The Product Life Cycle. As discussed in the main text, our findings on the brand life cycle are significantly longer than the life cycle discussed in recent work (e.g. Argente et al., 2018). Here, we crosswalk our results to existing work on the product life cycle to benchmark where we diverge. Argente et al. (2018) focus on the life cycle of products applying Nielsen Scanner Data. This work is able to identify new products and brands and document their life cycle patterns. However, it is not able to link brands and products to their history, and is thus unable to speak to the longer time horizon of persistent brands. We perform similar life cycle regressions to the main text and compare them to a relevant current paper in the literature, in particular focusing on defining age in two different ways, to ensure the differences in the age profile does not simply come from applying a dataset with different age measures. Equation (38) presents the regression:

$$\log y_{it} = \alpha + \sum_{a=0}^{4} \beta_a D_a + \gamma_b + \lambda_t + \epsilon_{it} \tag{38}$$

Where the coefficients of interest are the coefficients on age (β_a) with controls for cohort and time effects (and an adjustment on cohort from Deaton, 1997). Table B4 engages in the same specification as Argente et al. (2018) in the UPC data (panels 1 and 2) and Trademark merged data (panels 3 and 4) respectively.

Table B4: Log Sales, by Nielsen and Trademark Age

	(1) Log Sales	(2) Log Sales	(3) Log Sales	(4) Log Sales
Age 1	0.939***	1.095***	0.917***	0.953***
	(0.00)	(0.00)	(0.00)	(0.00)
Age 2	0.857***	1.159***	1.019***	1.060***
	(0.00)	(0.00)	(0.00)	(0.00)
Age 3	0.632***	1.016***	0.834***	0.832***
	(0.00)	(0.00)	(0.00)	(0.00)
Age 4	0.169***	0.644***	0.412*	0.488***
	(0.00)	(0.00)	(0.00)	(0.00)
N	668993	89203	3402	4136
R^2	0.138	0.179	0.256	0.050
Variation	UPC	Brand-Group	TM Brand	TM Brand-Group

p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: Balanced Panel Life Cycle Regressions of Log Sales on Age, utilizing different age sources and different variation.
Source: USPTO Trademark and RMS Nielsen

Note that while at the level of brands and trademarks there are significantly fewer observations, the same general pattern holds. This indicates how age is picking up something similar in our context, yet due to the broader horizon of historical data we are able to connect brands to their histories, indicating a significantly longer brand life cycle than found in Argente et al. (2018). We also show here similar general trends as in the main text when we evaluate the life cycle of products, controlling for brand-firm-group level.

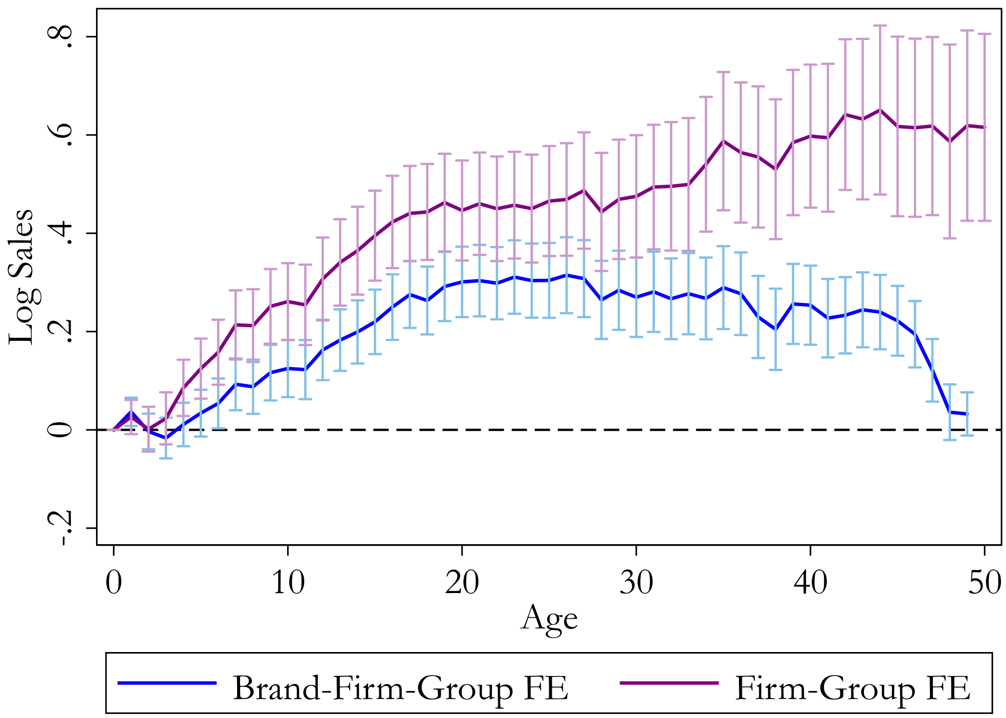
Figure B7 evaluates the life cycle profile within a given product group code. We follow the regression in the main text, except in prices we weight by sales share. Equation (39) illustrates the structure of the regression.

$$\log y_{ijkt} = \alpha + \sum_{a=1}^{50} \beta_a D_a + \gamma_b + \lambda_t + \theta_{ikj(i)} + \epsilon_{ijkt} \tag{39}$$

The regression in Equation (39) considers the sales and prices of brand i with firm j in group k at time t, $\log y_{ijkt}$ as a function of a constant (α), brand age indicators from 1 to 50, D_a, and fixed effects for cohort (γ_b) and time (λ_t).[22] The $\theta_{ikj(i)}$ indicates a brand-group or firm-group fixed-effect. Figure B7 plots the regressions by age coefficient β_a.

[22]Given the linear relationship between age, time, and cohort, we follow a method developed by Deaton (1997) to correct for this issue. The normalization orthogonalizes the cohort trends such that growth components move with age and time effects.

Figure B6: Life Cycle Regressions



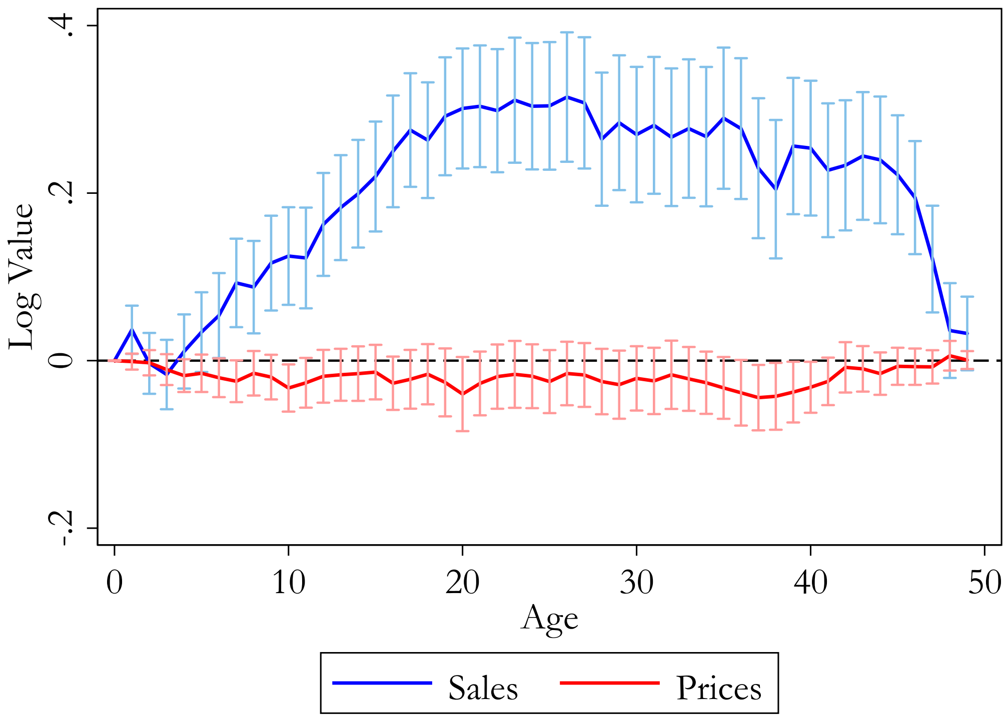
Note: Plots of log sales on age regression coefficients, controlling for brand-group and controlling only for firm-group. 95% confidence intervals plotted alongside coefficients. Source: RMS Nielsen and USPTO.

Figure B7 is consistent with the main facts from Section 3.2. We note that the inverted-U profile is still persistent within group, though with a slightly lower peak than in the brand's overall life cycle. We also note that the life cycle of prices shows on average somewhat minimal activity for the brand across age. This means that the strategic pricing firms engage in does not appear to be correlated with age, though as we have noted from events there are shifts in prices, consistent with previous evidence in the literature.

Coarsened Exact Match: Details. In this section, we expand on the coarsened exact match procedure in Section 3.3, discussing the method we use to link brands to their counterfactual brands prior to the event. We link brands that are reallocated to matched brands that have the same change in log sales in the previous two periods to the event period of the reallocated brand, the same year, and the same age bin (where we define age bin in 4 groups: 0-7, 8-19, 20-36, 37 or older). We perform the weighting following Blackwell et al. (2009), and take a synthetic control that we compare with the treated brands.

Definitions. In this section, we return to look at the qualitative similarities between the results in the main text and results depending on the definition of the firm and the main data used. For the main paper, we maintain the same dataset, focusing on brands with at least $1000 sales in a given year and brands that successfully merge to a trademark. Furthermore, given the nature of ownership we keep only the primary

Figure B7: Life Cycle Regressions, Prices and Sales within Group



Note: Plots of log sales and prices on age regression coefficients, controlling for brand-group, as in Equation (39). 95% confidence intervals plotted alongside coefficients. Source: RMS Nielsen and USPTO.

owner of a brand. This robustness section focuses on the empirical facts at the firm-level addressing some changes to these definitions.

Product Definition. We focus on the product life cycle in our data, but aggregate across all brands in the main maturity specification to avoid brand $\times$ product group features. The life cycle peaks around the same time in both specifications (see the peak age in Figure 2a and Figure B7). However, when we analyze brand $\times$ group, the life cycle peaks at a slightly younger level (0.35 versus 0.45). This should not change the qualitative implications of our results.

Transaction Definition. Transactions are defined at both the Nielsen and USPTO level. The reason we define transactions using both is as follows. We note that when we plot the results applying only USPTO transaction information we find as follows. Multiple serial numbers per brand.

B.3 Empirical Robustness: Firm $\times$ Brand Analysis

In the main paper, we focused on the responsiveness of sales and prices to both events and allocation to top firms. Here, we discuss different definitions of top firms and events to understand the general robustness of our results. We find qualitatively very similar results, which would not change the main messages of our analysis.

Prices and Sales at Top Firms. In this section, we explore varying the definition of a top firm in order to understand the differences in predicted sales. Table B5 focuses on the robustness of the higher log sales at larger firms. We see that larger firms tend to show higher sales of the same brand.

Table B5: Log Sales Conditional on Holding Firm, Trademark Age Fixed Effects

	(1) Log Sales	(2) Log Sales	(3) Log Sales	(4) Log Sales	(5) Log Sales	(6) Log Sales
Top 10 Overall	0.57***	0.55***				
	(0.000)	(0.000)				
Last Period Top 10			0.59***	0.69***		
			(0.000)	(0.000)		
Top 10 in 2006					0.47***	0.53***
					(0.000)	(0.000)
N	441300	3972	441300	3972	441300	3972
R^2	0.844	0.741	0.844	0.735	0.844	0.740
Weights	No	No	No	No	No	No
Restrictions	No	Only trans.	No	Only trans.	No	Only trans.

p-values in parentheses, clustered at brand-group level.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands.

Table B6 focuses on the robustness of the higher log prices at larger firms, focusing only on the merged sample. We note that the results directionally hold, but exhibit a higher variace.

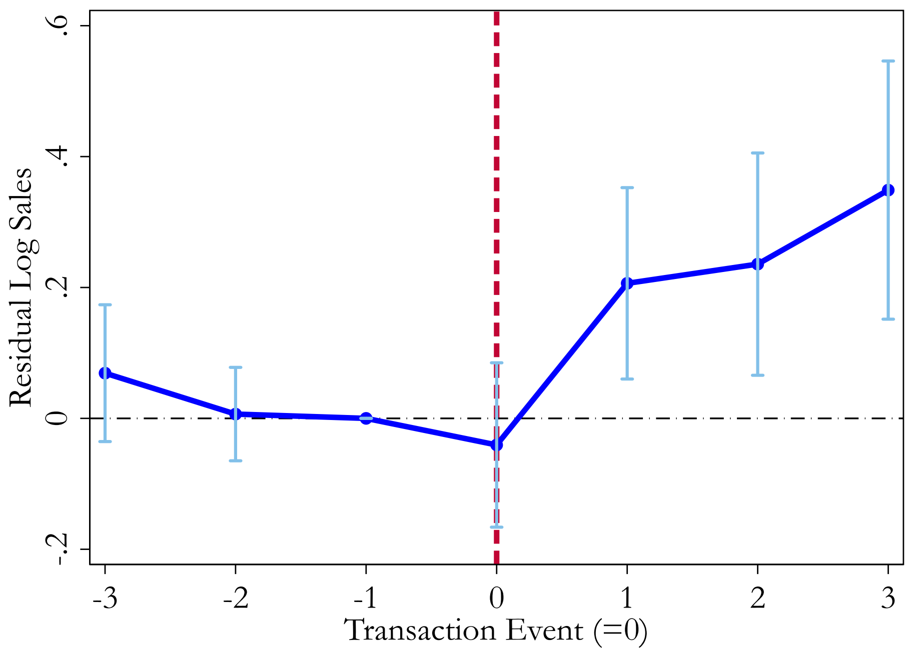
Table B6: Log Price Conditional on Holding Firm, TM age FE (limited sample)

	(1) Log Price	(2) Log Price	(3) Log Price	(4) Log Price	(5) Log Price	(6) Log Price	(7) Log Price	(8) Log Price
Top 10 Firm	0.33	0.26*			0.057	0.036		
	(0.177)	(0.062)			(0.170)	(0.403)		
Top 10 Firm in 2006			0.14	0.34*			-0.0091	0.029
			(0.350)	(0.088)			(0.869)	(0.516)
N	441300	3972	441300	3972	441300	3972	441300	3972
R^2	0.967	0.881	0.967	0.882	0.983	0.983	0.983	0.983
Weights	Total Wt.	Total Wt.	Total Wt.	Total Wt.	Period Wt.	Period Wt.	Period Wt.	Period Wt.
Restrictions	No	Only trans.	No	Only trans.	No	Only trans.	No	Only trans.

p-values in parentheses, clustered at brand-group level.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table documents two separate regressions on brands that are held by both market leaders (top 10 firm overall) and fringe firms, looking at the effect of leaders holding brands.

Figure B8: Coarsened Exact Match and Brand Transaction Leader-to-Fringe, Log Sales



Notes: Coarsened exact match coefficients. Match is made on pre-trend sales change, brand age bin, and year.

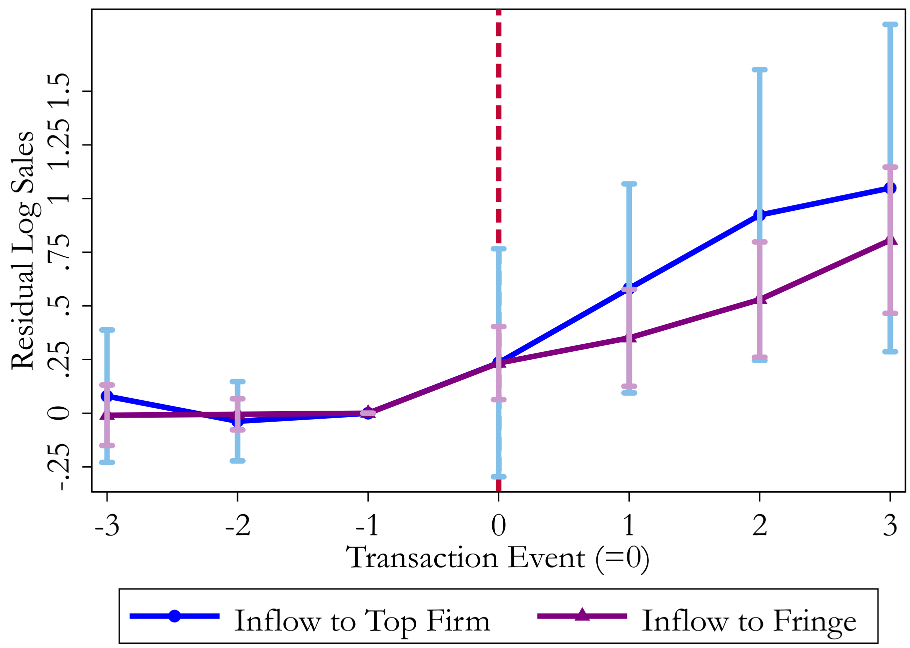
Gross Flows and Net Flows. One of the main aspects of our paper focuses on the reallocation of products across firms. We identify this reallocation by jointly using RMS Nielsen Scanner data and USPTO Trademark data.

Event Studies. Our event studies focus on transactions across firms in the data. For an observed transaction, both the buyer and the seller must exist in the data. In order to identify the overall event study, we employ a balanced panel with seven periods. Given we use data from 2006-2018, we must restrict our event study analysis to brand transactions from 2009-2015. Due to some of the restrictions on our data, we focus on a broader definition of leading firms and flows from low-type to high-type firms. We explore the robustness of event studies depending on our characterization of an event study and definition of firm type.

To characterize flows that link fringe and leader buyers and sellers, we evaluate exchanges that move from smaller sellers to larger buyers, defined over the horizon of the sample. We make a couple of adjustments to the definition of a large firm to evaluate the robustness of our event study results.

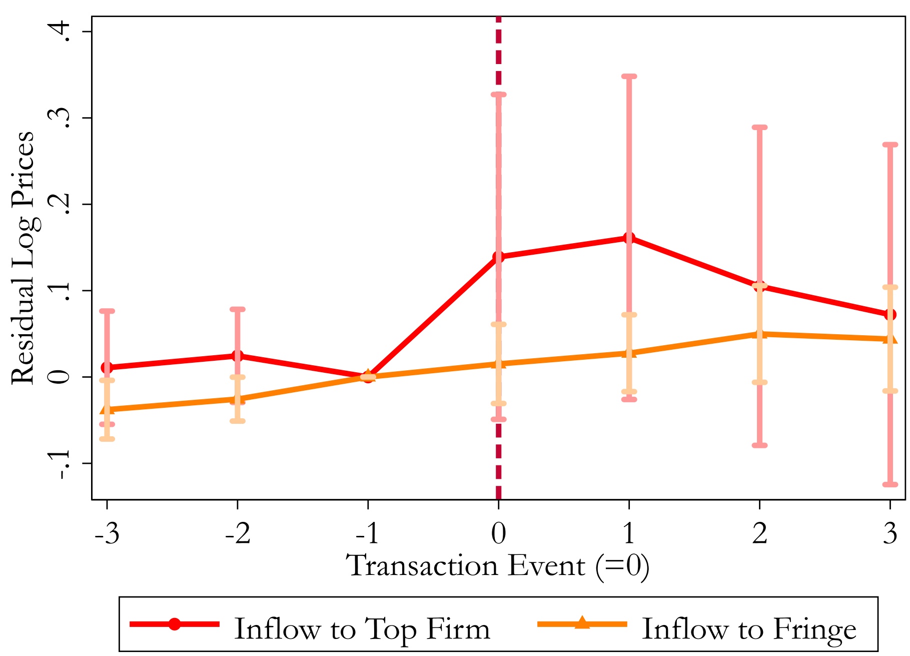
Lastly, we compare more broadly the change in prices and sales upon the inflow of a brand to a large and small firm. We consider a large firm to be a top 10 firm within the product group code, and a small firm to be all other firms. We ask how prices and sales respond by doing the same analysis here. Limiting attention only to brands that move between firms, we also evaluate the price and sales differences depending on the holding firm in Table B6.

Figure B9: Inflow to Top Firm and Fringe, Sales



Notes: Coarsened exact match coefficients. Match is made on pre-trend sales change, brand age bin, and year.

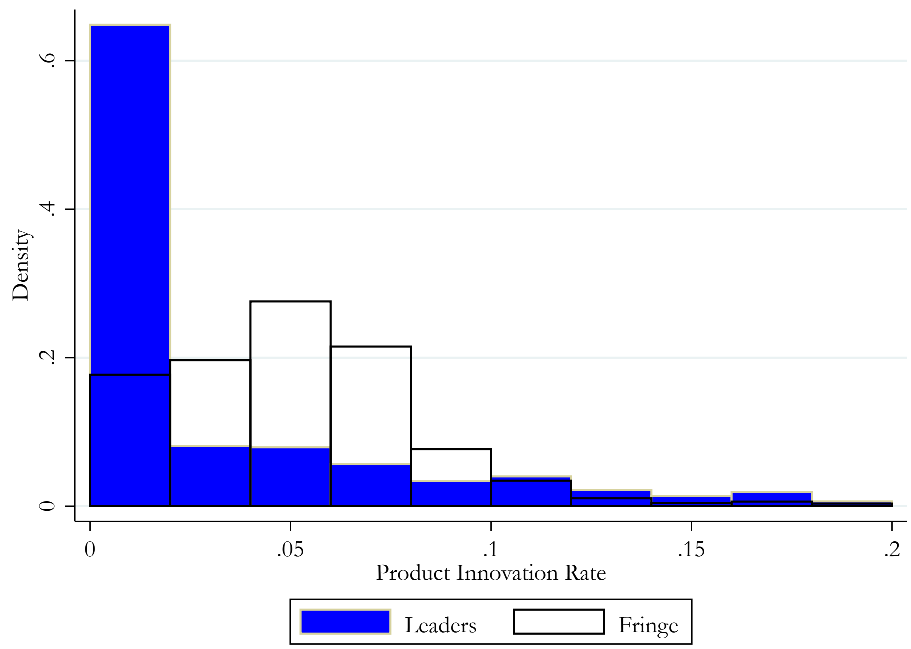
Figure B10: Inflow to Top Firm and Fringe, Prices



Notes: Coarsened exact match coefficients. Match is made on pre-trend sales change, brand age bin, and year.

Figure B11 focuses on the different product innovation rates (entry as share of overall firm sales), and we note the much stronger entry rate of fringe firms than leaders.

Figure B11: Product Innovation by Type



Notes: This looks at the product innovation distribution by firm type, *Source:* RMS Nielsen and USPTO Trademark. Fringe and leader defined as in text.

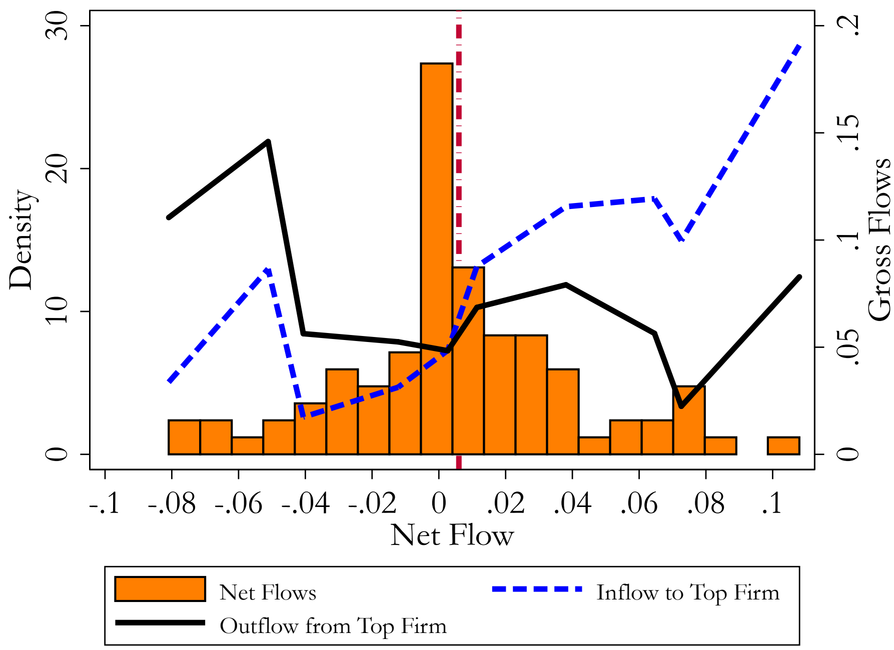
Further, we note that overall in the transfer of brand ownership there are more flows from small to large firms. This can be seen in firm press releases, as we observe many inflows and outflows of brand ownership for large firms, with inflows being more common. This can be seen in Figure B12, which collapsed the total brand flows (as share of sales) in both directions, with a histogram of net flows and line plots of gross flows.

B.4 USPTO Trademarks: Reassignment

The most reliable long-term data source for brand reallocation is USPTO Trademark data. Our focus in this paper is particularly on reallocation due to either pure reassignment (e.g. ownership transfer) or mergers & acquisitions. In this section, we discuss the general contours of the trademark data when it comes to reallocation of ownership. There is significant reallocation in the data, but some reallocation does not fall under the specific "merger" or "reassignment", but instead is linked to name changing, collateral, and other corrections and adjustments.

Table B7 splits the different transactions in the data into their different groupings. Most transactions in the data are available from 1970-2018. We order the transaction type by largest share of transactions. However, each transaction may contain a bundle of trademarks (e.g., transfer of ownership of "Odwalla"

Figure B12: Gross and Net Flows, Fringe and Leader



Notes: This looks at the market shares transferred across firms in market shares by year, averaged by product group code. *Source:* RMS Nielsen and USPTO Trademark. Fringe and leader (top 10 firm) defined as in text.

may be bundled with various sub-brands of the core brand Odwalla). For example, in the case of "Security Interest" (or collateral), note that on average a larger number of brands are involved in the pledged bundle.

Table B7: Summary Statistics on Trademarks from USPTO

	Transaction Count	Trademark (TM) Count	TM/Transaction	Transaction Share	TM Share
Reassignment	478442	1.54M	3.21	**0.523**	0.345
Name Change	200767	795465	3.96	**0.219**	0.178
Security Interest	101280	1.10M	10.91	**0.111**	0.248
Merger	46610	287001	6.16	**0.051**	0.064
Correction	23500	119017	5.06	**0.026**	0.027
Other	64456	615334	9.55	**0.070**	0.138
Total	915055	4457996	4.87	1	1

Note: This table describes the category of each transaction in USPTO and orders them by their share of total transactions. Source: USPTO.

While our main focus in this paper has been mergers and reassignments, we note the richness of the data on multiple margins. Name changes are frequent, as firms may attempt to retool but maintain brand loyalty. Further, as noted previously, trademarks are often used as collateral. While Security Interest transactions are a small share of overall exchanges (around 10%), they make up almost 25% of all trademarks in exchanges. However, without transfer the firm may continue to operate these product lines. The benefit of focusing on mergers and reassignments is the reallocation of ownership and management across firms, but we hope to see further research on these margins.

C Theoretical Appendix

This section expands on some model discussion in the main text. Section C.1 expands on the full leader's problem in the main text, while Section C.2 expands on the equations and proofs in the main text.

C.1 Leader's Dynamic Problem

The leader chooses an innovation intensity (η), vacancies (o) and terms of trade (τ) to maximize the dynamic returns as follows,

$$\max_{\eta_t, o(\mathbf{x}), \tau^{LF}(\mathbf{x})} \int_0^\infty e^{-\int_0^t \mathbf{r}(t')dt'} \left[\Pi(\phi_t) - D(\eta_t) - B_t + S_t\right] dt, \tag{40}$$

s.t.

$$\phi_t = \frac{\int e^{z+\alpha+\beta+\gamma} n_t^L(\mathbf{x}) d\mathbf{x}}{\int e^{\beta+\gamma} n_t^F(\mathbf{x}) d\mathbf{x}},$$

$$B_t = \int \left[M(v_t(\mathbf{x}), u_t(\mathbf{x}))\tau^{FL}(\mathbf{x}) - o_t(\mathbf{x})\right] d\mathbf{x},$$

$$S_t = \int \lambda(\theta_t(\mathbf{x}))\tau^{LF}(\mathbf{x}) n_t^L(\mathbf{x}) d\mathbf{x},$$

To characterize the optimal solution, we start by setting up the full Lagrangian:

$$
\begin{aligned}
&\mathcal{L}\Big(\eta_t, \nu_t(\mathbf{x}), \tau_t^{LF}(\mathbf{x}), v_t(\mathbf{x}), q_t(\mathbf{x}), \zeta_t\Big) \\
&= \int_0^\infty e^{-\int_0^t \mathbf{r}(t')dt'} \left[\Pi(\phi_t) - D(\eta_t) - B_t + S_t\right] dt + \int_0^\infty \zeta_t \left(\phi_t - \frac{\int e^{z+\alpha+\beta+\gamma} n_t^L(\mathbf{x}) d\mathbf{x}}{\int e^{\beta+\gamma} n_t^F(\mathbf{x}) d\mathbf{x}}\right) dt \\
&+ \int_0^\infty e^{-\rho t} v_t(\mathbf{x}) [\dot{n}_t^L(\mathbf{x}) - \underbrace{\eta_t f(\beta) \mathbb{I}_{\gamma=0}}_{\text{Innovation}} + \underbrace{\iota(\beta_0 + \bar{\beta} - \beta) \frac{\partial n_t^L}{\partial \beta}(\mathbf{x})}_{\text{Maturity}} \\
&+ \qquad \underbrace{\lambda\left(\theta_t^{LF}(\mathbf{x})\right) n_t^L(\mathbf{x})}_{\text{L-t-F Reallocation}} + \underbrace{\int_{\Omega(\mathbf{x},\tilde{\mathbf{x}})>0} f_\gamma(\gamma) \lambda\left(\theta_t^{FL}(\tilde{\mathbf{x}})\right) n_t^F(\tilde{\mathbf{x}}) d\tilde{\mathbf{x}}}_{\text{F-t-L Reallocation}} - g_t n_t^L(\mathbf{x})] dt \\
&+ \int_0^\infty e^{-\rho t} y_t(\mathbf{x}) [\dot{n}_t^L(\mathbf{x}) - \underbrace{\eta_t^F f(\beta) \mathbb{I}_{\gamma=0}}_{\text{Innovation}} + \underbrace{\iota(\beta_0 + \bar{\beta} - \beta) \frac{\partial n_t^F}{\partial \beta}(\mathbf{z})}_{\text{Maturity}} \\
&+ \qquad \underbrace{\lambda\left(\theta_t^{FL}(\mathbf{x})\right) n_t^F(\mathbf{x})}_{\text{L-t-F Reallocation}} + \underbrace{\int_{\Omega(\mathbf{x},\tilde{\mathbf{x}})<0} f_\gamma(\gamma) \lambda\left(\theta_t^{LF}(\tilde{\mathbf{x}})\right) n_t^L(\tilde{\mathbf{x}}) d\tilde{\mathbf{x}}}_{\text{F-t-L Reallocation}} - g_t n_t^F(\mathbf{x})] dt
\end{aligned}
$$

We rewrite the integral with $\dot{n}_t^L$ using integration by part:

$$
\int_0^\infty \int_{\mathbf{x}} e^{-\rho t} v_t(\mathbf{x}) \dot{n}_t^L(\mathbf{x}) d\mathbf{x} dt = \int_{\mathbf{x}} \Big(v_\infty(\mathbf{x}) n_\infty^L(\mathbf{x}) - v_0(\mathbf{x}) n_0^L(\mathbf{x}) + \rho e^{-\rho t} v_t(\mathbf{x}) n_t^L(\mathbf{x}) - e^{-\rho t} n_t^L(\mathbf{x}) \dot{v}_t(\mathbf{x})\Big) d\mathbf{x}
$$

Similarly

$$
\int_0^\infty \int_{\mathbf{x}} e^{-\rho t} y_t(\mathbf{x}) \dot{n}_t^F(\mathbf{x}) d\mathbf{x} dt = \int_{\mathbf{x}} \Big(y_\infty(\mathbf{x}) n_\infty^F(\mathbf{x}) - y_0(\mathbf{x}) n_0^F(\mathbf{x}) + \rho e^{-\rho t} y_t(\mathbf{x}) n_t^F(\mathbf{x}) - e^{-\rho t} n_t^F(\mathbf{x}) \dot{y}_t(\mathbf{x})\Big) d\mathbf{x}
$$

For the choices to be optimal, any perturbation to distribution $n_t^L(\mathbf{z})$ must yields no change to the Lagrangian. This implies:

$$
\begin{aligned}
(\rho + g_t) v_t(\mathbf{x}) =& e^{z+\alpha+\beta} \zeta_t \underbrace{\frac{Q_t}{Q_t^F}}_{=1+\phi_t} + \iota(\bar{\beta} - \beta) \frac{\partial v_t}{\partial \beta}(\mathbf{x}) + \dot{v}_t(\mathbf{x}) && (41) \\
&+ \max_{\theta,\tau} \lambda(\theta) \mathbb{E}_{\gamma'} \Big[u_t(\mathbf{x}') + y_t(\mathbf{x}') - v_t(\mathbf{x})\Big] - \theta \kappa_s \frac{\mathbf{w}_t}{\mathbf{C}_t} && (42) \\
& && (43)
\end{aligned}
$$

For the choices to be optimal, any perturbation to distribution $n_t^F(\mathbf{z})$ must yields no change to the

Lagrangian. This implies:

$$(\rho + g_t)\, y_t(\mathbf{x}) = -\, e^{z+\beta}\zeta_t\phi_t(1+\phi_t) + \iota(\bar{\beta}-\beta)\frac{\partial y_t}{\partial \beta}(\mathbf{x}) + \dot{y}_t(\mathbf{x}) \tag{44}$$

The other choices follow its first order condition:

$[\eta_t]$

$$D'(\eta_t)\frac{\mathbf{w}_t}{\mathbf{C}_t} = \mathbb{E}_{\beta_0} v\Big((\beta_0, 0, 0)\Big) \tag{45}$$

$[\phi_t]$

$$\zeta_t = \Pi'(\phi_t) \tag{46}$$

Combining these equations we reach the results in main text.

C.2 Model Proofs and Discussion

Proof of Proposition 3. To reach the aggregation result, we aim to write the real consumption as a function of production labor input $\mathbf{L}_P$, aggregate appeal $\mathbf{Z}$, and an aggregate efficiency of labor allocation $\mathbf{A}$. We start from the aggregation within product group k. Within product group k, the total group-level expenditure is $\alpha_k\mathbf{C}(t)$. Using the formula for sales shares, the expenditure for leader is

$$\frac{\phi\mu(\phi)^{1-\sigma_k}}{\phi\mu(\phi)^{1-\sigma_k} + \bar{\mu}^{1-\sigma_k}}\alpha_k\mathbf{C}(t) \tag{47}$$

Using the accounting equation for profit $\alpha_k\mathbf{C}(t) = \mu(\phi)\mathbf{w}(t)L_P$, we write that:

$$L_k(t) = \frac{\phi_k\mu(\phi_k)^{-\sigma_k}\frac{1}{Z_L(t)} + \bar{\mu}_k^{-\sigma_k}\frac{1}{Z_F(t)}}{\phi_k\mu(\phi_k)^{1-\sigma_k} + \bar{\mu}_k^{1-\sigma_k}}\frac{\alpha_k\mathbf{C}(t)}{\mathbf{w}(t)}. \tag{48}$$

$$Z_k(t)^{\frac{1}{\sigma_k-1}}L_k(t) = \frac{\phi_k\mu(\phi_k)^{-\sigma_k}\frac{Z_k(t)^{\frac{1}{\sigma_k-1}}}{Z_L(t)^{\frac{1}{\sigma_k-1}}} + \bar{\mu}_k^{-\sigma_k}\frac{Z_k(t)^{\frac{1}{\sigma_k-1}}}{Z_F(t)}^{\frac{1}{\sigma_k-1}}}{\phi_k\mu(\phi_k)^{1-\sigma_k} + \bar{\mu}_k^{1-\sigma_k}}\frac{\alpha_k\mathbf{C}(t)}{\mathbf{w}(t)}. \tag{49}$$

Adding across all product groups

$$\mathbf{L}_P(t) = \frac{\mathbf{C}(t)}{\mathbf{w}(t)}\int_0^1 \alpha_k\frac{\phi_k\mu(\phi_k)^{-\sigma_k} + \bar{\mu}_k^{-\sigma_k}}{\phi_k\mu(\phi_k)^{1-\sigma_k} + \bar{\mu}_k^{1-\sigma_k}}dk. \tag{50}$$

Search Process Discussion. In this section, we characterize the partial equilibrium in the search and matching markets, given (ϕ_k, Z_k) and the gains from reallocation across firms. Specifically, let $u_k(\beta,\gamma)$ be the discounted value of a fringe firm with product quality β and match quality γ, let $v_k(\beta,\gamma)$ be the discounted value of an additional product to the leader, and let $x_k(\beta,\gamma)$ be the discounted loss of an additional product operated by the leader in the calculation of leaders.

When positive buying flows into fringe firms occur, the optimal buying decision of a fringe firm with (β, γ) is as follows:

$$\kappa^s \varphi_0 = \max_{\tau} \frac{\lambda(\theta)}{\theta} \mathbb{E}_{\Delta} \Big[u(\beta, \gamma_L + \Delta) - \tau \Big]^+, \tag{51}$$

s.t.

$$\lambda(\theta) \mathbb{E}_{\gamma_L} \Big[u(\beta, \gamma_L) - \tau \Big]^+ = U^F(\beta, \gamma).$$

It is straightforward to show Equation (51) is equivalent to the following problem in terms of solutions:

$$U^F(\beta, \gamma) = \max_{\theta} \lambda(\theta) \mathbb{E}_{\gamma_L'} \Big[u(\beta, \gamma + \Delta) - u(\beta, \gamma) \Big]^+ - \theta \kappa^s \varphi_0 \tag{52}$$

Equation (52) provides an intuitive interpretation of the reallocation process: due to directed search and the competition on the buyer side, the terms of trade aims to maximize the net benefit of reallocating products from fringe firms to other fringe firms, taking into consideration of the search friction and the cost of search. It is also worth noting that for each (β, γ), Equation (52) can be independently solved without referring to the distribution of products across firms. This mechanism is the block recursivity highlighted in Menzio and Shi (2011).

Similarly, due to free entry of fringe buyers, the leader-to-fringe (LtF) flows can be characterized in the same way. For notational simplicity, we define the joint surplus of reallocating a product from fringe to leader as $\Omega(\beta, \gamma_L, \gamma_F)$. The equilibrium in the LtF market is characterized by $\{U^L(\beta, \gamma), \theta^{LF}(\beta, \gamma)\}$ that jointly solve the following problem:

$$U^L(\beta, \gamma) = \max_{\theta} \lambda(\theta) \mathbb{E}_{\Delta} \Big[- \Omega(\beta, \gamma, \gamma + \Delta) \Big]^+ - \theta \kappa^s \varphi_0. \tag{53}$$

The reallocation flow from the fringe to leaders is more complicated because there is no longer free entry on both sides of the market. However, the leader as a buyer faces competitive pressure from fringe buyers. In an equilibrium where flows are observed, the leader must offer the same expected value of selling as the fringe buyers. Thus, the optimal buying decision of the leader is

$$\kappa^s \varphi_0 \leq \max_{\tau} \frac{\lambda(\theta)}{\theta} \mathbb{E}_{\gamma_L} \Omega(\beta, \gamma', \gamma_F) - \frac{1}{\theta} U^F(\beta, \gamma_F). \tag{54}$$

C.3 Derivation of Killer Acquisition Threshold

The killer acquisition threshold occurs when the value of selling a brand to the leader is negative, regardless of the efficiency differential between leader and fringe firm. This same intuition delivers a situation where leaders want to buy brands regardless of how efficient they would be at deploying the brand.

To theoretically study this situation, we focus on the relationship the leader and fringe have to a fringe firm's brand. We first take the difference between $u_t(\mathbf{x})$ (the value of brand to fringe firm) and $y_t(\mathbf{x})$ (the value of a *fringe's* brand to leader):

$$(\rho + g_t)\left[-y_t(\mathbf{x}) - u_t(\mathbf{x})\right] = \underbrace{e^{\beta+\gamma}\left(1+\phi_t\right)\left[\Pi'(\phi) - \pi(\phi_t)\right]}_{\text{Operating Profit}} + \underbrace{\iota(\bar{\beta} - \beta)\left[-\frac{\partial y_t}{\partial \beta}(\mathbf{x}) - \frac{\partial u_t}{\partial \beta}(\mathbf{x})\right]}_{\text{Maturity}} \tag{55}$$

$$+ \underbrace{\max_{\theta} \lambda(\theta)\mathbb{E}_{\gamma'}\left[\Omega_t(\mathbf{x}', \mathbf{x})\right]^{+} - \theta\kappa_s^{FL}\frac{\mathbf{w}_t}{\mathbf{C}_t}}_{\text{Value of Selling}} + \left[-\dot{y}_t(\mathbf{x}) - \dot{u}_t(\mathbf{x})\right]$$

First we find a threshold of ϕ such that $\Pi'(\phi) > \pi(\phi)\frac{1}{\phi}$:

$$(1 - s(\phi))\frac{\Pi(\phi)}{\phi} > \frac{1 - s(\phi)}{\sigma\phi}$$

$$\iff$$

$$\frac{s(\phi)}{\sigma(1 - s(\phi)) + s(\phi)} > \frac{1}{\sigma}$$

$$\iff$$

$$s(\phi) > \frac{\sigma}{2\sigma - 1}$$

Whenever the market share of leader is above this threshold, it must be $-y_t(\mathbf{x}) - u_t(\mathbf{x}) > 0$. Thus the gains from trade $\Omega(\mathbf{x}', \mathbf{x})$ is positive for any combination $(\mathbf{x}', \mathbf{x})$. As a result, there is never gains from trade of reallocating a product from leaders to fringes.

D Estimation Appendix

In this section, we discuss in greater detail the estimation process, starting generally and then discussing the different ingredients central to our estimation. 1. We directly calibrate the substitution elasticity σ_k to the ones estimated in the literature; 2. Given ϕ_k, we jointly estimate $\{\kappa_s, \kappa_e, d_k\}$ that minimize the distance between the observed reallocation rate, ϕ_k, and leader's innovation rate, as well as fringe firms' innovation rate.

D.1 Solving Equilibrium Given Parameters

Given any set of parameters, we take the following steps to solve the equilibrium, working at the group and aggregate level:

G1. *(Group Loop - Value Function)* For a fixed $\left(\frac{\mathbf{w}}{\mathbf{C}}, \phi, g\right)$, we solve the balanced growth path value functions and decisions according to the Bellman equations discussed in the main text. This can be done using any PDE solvers. We used the finite difference method;

G2. *(Group Loop - Aggregation)* Given the decisions, we solve for the BGP distribution, scaling the fringes' entry rate such that the BGP quality gap is consistent with the imputed ϕ. With this distribution, we calculate the residual in the free-entry condition and the residual in growth decomposition.

G3. *(Group Loop - Equilibrium)* We repeat step 1 and 2 such that the residuals on free entry condition and growth decomposition are both close enough to zeros.

A1. We repeat G1 - G3 for all groups, given a guess $\frac{\mathbf{w}}{\mathbf{C}}$. Using the aggregation results, we solve for the aggregate search labor, innovation labor, and production labor.

A2. Repeat A1 until the guessed $\frac{\mathbf{w}}{\mathbf{C}}$ are close enough to the one implied by labor supply curve.

D.2 Estimating Parameters

There are four parameters to estimate for each product group: two innovation costs and two search costs. For each set of parameter values, we solve the equilibrium, and calculate the aggregate innovation rate and reallocation rate for leaders and fringes. We find the parameters by the method of moments by minimizing the absolute norm between the model predicted rates and rates from data.

Elasticities, Shares, and Markups. At the inner layer, we need to establish the value functions of each agent and do value function iteration to link the shares and elasticities with the optimization problem of the leader and the fringe entry and selling decisions.

We start by specifying the leader's perceived elasticity, as discussed in the model, and in Equation (56),

$$\epsilon(s) = (\sigma(1-s) + s). \tag{56}$$

This simultaneously delivers a markup, of a leader with share s and a standard markup $\bar{\mu}$ for the fringe firm in Equation (57),

$$\mu(s) = \frac{\epsilon(s)}{\epsilon(s) - 1} \;\; ; \;\; \bar{\mu} = \frac{\sigma}{\sigma - 1}. \tag{57}$$

We also can specify the share as a function of the leader quality advantage ϕ, as follows:

$$s(\phi) = \max(1 + \phi^{-1}(\sigma/(\sigma-1)/\mu(x))^{1-\sigma}), (0,1)) \tag{58}$$

$$\Pi_{fringe}(\phi) = 1/\sigma(1+\phi)(1-s(\phi)) \tag{59}$$

As a result, we can link the leader concentration ϕ to market shares and the elasticities firms face. This will represent the inner layer of our model, which occurs inside each iteration.

Full Discussion of Estimation. For more granular details of estimation, please see liangjiewu.com/files/tm_pw_apx_oct22.pdf

E Quantitative/Policy Discussion

In our quantitative exercises, we focus on different policies that seem to send a general message on brand reallocation. First, due to significant leader appeal and sales movement after exchange, we expect brand reallocation to show efficiency gains. We find this in the model, and find that downstream innovation also responds positively. Second, due to the age profile, the reallocation has less of an effect on growth than in markets with a faster age profile. Third, subsidizing entry is a more effective means of pursuing a reduction in concentration, as it simultaneously solves the growth and concentration externalities.

We believe that these results are robust to various specifications. First, on shutting down or taxing the reallocation of brands, we observe the responsiveness of brands to leader appeal is consistently larger than 0.4, while the marginal cost of leaders appear to be similar with fringe firms (Hottman et al., 2016). Leaders do engage in strategic behavior, but policy that shuts down reallocation will lose out on these gains and the forward looking behavior of fringe firms. This is attenuated if the leader appeal advantage declines or the brand maturity slows.

Second, as we see in Table 12, varying the maturity of brands has significant effects on policies, as faster maturity links innovation and reallocation more tightly together. This comes from directed search, and is consistent regardless of the life cycle characteristics, as long as there is some time to maturity, which is consistent with our paper and other work in the literature (e.g., Bronnenberg et al., 2009).

Third, subsidizing entry is a good policy for both attenuating concentration and increasing growth. This should hold as long as fringe firms have an innovation advantage (relative to their size) to leaders. If policy subsidizes product entry, both fringe and leading firms response, but fringe firms are able to respond more strongly. Even with reallocation, the steady state share of fringe firms holdings are higher because reallocation occurs later in life. As a result, we feel the main messages of policy are robust to different specifications, but we look forward to further empirical and quantitative work to further explore these mechanisms.