Detailed Computational Results for Exact Optimal Designs

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The following shortcuts are used in the tables:

- B: Boscia.
- C: CoBnB.
- M: Mosek.
- R: Relaxed (i.e., only one continuous relaxation problem is solved by Mosek).
- P: PNOD.
- Sum: $\sum_i^m x_i$. This is used to check wether the returned solution is feasible (highlighted in red color). For B, C, M, and P, all the returned solutions are feasible and we ignore the assiocated results.

	Nodes				CPU Time [s]					Objective Values					Sum
m	B	C	M	P	B	C	M	P	R	B	C	M	P	R	R
50	81	61	48	57	3.05E+00	3.37E-01	4.10E-01	6.71E-02	2.05E-01	1.434762	1.434762	1.434762	1.434762	1.934456	5
	21	25	24	25	9.20E-02	3.27E-01	3.01E-01	1.37E-02	1.63E-01	1.44263	1.44263	1.44263	1.44263	1.44263	**7**
	325	221	182	245	1.43E+00	1.65E+00	4.63E-01	1.32E-01	1.50E-01	1.599911	1.599911	1.599911	1.599911	1.599911	**7**
	227	153	210	195	1.10E+00	1.05E+00	4.14E-01	5.90E-02	1.50E-01	1.598618	1.576827	1.576827	1.576827	1.434382	8
	33	31	22	31	1.69E-01	2.12E-01	2.56E-01	2.30E-02	1.57E-01	1.568592	1.568592	1.568592	1.568592	1.568592	**7**
60	303	267	199	247	2.35E+00	2.23E+00	6.29E-01	8.28E-02	1.88E-01	1.605237	1.605237	1.605237	1.605237	1.497796	10
	199	273	211	281	2.65E+00	2.33E+00	6.18E-01	8.48E-02	1.93E-01	1.66301	1.605736	1.605736	1.605736	1.636376	**9**
	255	235	375	223	1.72E+00	1.85E+00	7.26E-01	7.71E-02	2.28E-01	1.648783	1.648783	1.648783	1.648783	1.745134	**9**
	259	513	397	537	2.18E+00	6.10E+00	7.95E-01	1.85E-01	1.82E-01	1.702935	1.684411	1.684411	1.684411	1.725957	**9**
	195	621	420	545	1.28E+00	4.56E+00	6.94E-01	1.44E-01	1.82E-01	1.622783	1.622783	1.622783	1.622783	1.419951	11
80	615	2207	1058	2239	8.47E+00	2.60E+01	1.79E+00	8.52E-01	2.59E-01	1.657053	1.645455	1.645455	1.645455	1.645588	**12**
	717	1433	946	1321	1.50E+01	1.83E+01	1.80E+00	6.00E-01	3.00E-01	1.842224	1.807147	1.807147	1.807147	2.042939	10
	469	5009	1810	4965	6.61E+00	6.46E+01	2.38E+00	1.93E+00	2.55E-01	1.780265	1.780265	1.780265	1.780265	1.820409	**12**
	695	2981	1372	3029	9.00E+00	3.31E+01	2.07E+00	1.05E+00	2.63E-01	1.753716	1.751043	1.751043	1.751043	1.681496	13
	1193	2109	1741	2053	1.48E+01	3.01E+01	2.47E+00	9.99E-01	2.55E-01	1.764607	1.764607	1.764607	1.764607	1.845029	**12**
100	1401	46529	10897	46069	4.81E+01	1.07E+03	2.06E+01	2.79E+01	4.02E-01	1.861931	1.861931	1.861931	1.861931	1.727438	18
	1195	22897	7519	23011	4.56E+01	6.21E+02	1.41E+01	1.57E+01	3.41E-01	1.932406	1.92995	1.92995	1.92995	1.999706	**15**
	55733	113965	20830	104773	3.86E+03	3.01E+03	3.78E+01	7.69E+01	3.40E-01	1.939204	1.901243	1.901243	1.901243	2.51106	13
	28649	205181	43246	205277	2.33E+03	6.14E+03	7.65E+01	1.45E+02	3.90E-01	1.912867	1.87387	1.87387	1.87387	2.768428	12
	10631	58137	20466	58557	4.66E+02	1.62E+03	3.76E+01	4.39E+01	3.40E-01	1.873536	1.864863	1.864863	1.864863	2.045004	14
120	20077	940673	278632	942827	5.62E+03	4.92E+04	7.42E+02	1.08E+03	4.50E-01	1.947026	1.908998	1.908998	1.908998	2.567878	14
	19747	1294837	186031	1282173	5.01E+03	7.53E+04	4.91E+02	1.77E+03	4.86E-01	2.037346	1.983385	1.983385	1.983385	2.045698	**18**
	41301	1317025	457997	1313579	7.20E+03	1.01E+05	1.20E+03	2.50E+03	4.34E-01	1.953046	1.918192	1.918192	1.918192	36.62351	11
	40249	1259741	567505	1260691	7.20E+03	1.09E+05	1.49E+03	2.62E+03	4.33E-01	2.017118	1.992516	1.992516	1.992516	1.792986	22
	22767	104443	36509	103843	4.76E+03	4.93E+03	9.78E+01	1.24E+02	4.84E-01	1.927206	1.872662	1.872662	1.872662	2.161492	15

Table 1: A-design with independent data.

	Nodes				CPU Time [s]					Objective Values					Sum
m	B	C	M	P	B	C	M	P	R	B	C	M	P	R	R
50	2811	93	156	77	8.83E+00	2.31E+00	5.12E-01	2.70E-02	1.50E-01	-2.36392	-2.36392	-2.36392	-2.36392	-2.34669	**7**
	1081	67	61	71	2.93E+00	3.66E-01	3.62E-01	4.25E-02	1.49E-01	-2.37421	-2.37421	-2.37421	-2.37421	-2.16978	6
	2193	61	29	55	5.85E+00	4.61E-01	2.94E-01	3.95E-02	1.51E-01	-2.41029	-2.41029	-2.41029	-2.41029	-2.40013	**7**
	1321	81	126	89	3.86E+00	6.38E-01	4.48E-01	5.09E-02	1.48E-01	-2.51503	-2.51503	-2.51503	-2.48533	-2.66614	8
	2927	73	79	77	6.52E+00	4.61E-01	4.58E-01	6.04E-02	1.92E-01	-2.73926	-2.77613	-2.77613	-2.77613	-2.61934	6
60	5425	531	261	61	2.29E+01	3.63E+00	6.12E-01	2.38E-02	1.82E-01	-2.41066	-2.41066	-2.4507	-2.4507	-1.7977	7
	14431	1153	517	1135	6.56E+01	8.81E+00	1.11E+00	4.82E-01	1.83E-01	-2.55836	-2.55836	-2.55836	-2.55836	-2.29289	7
	2609	97	65	101	1.01E+01	1.15E+00	5.07E-01	5.40E-02	1.82E-01	-2.60824	-2.66836	-2.66836	-2.66836	-2.51049	8
	16851	897	506	843	8.05E+01	6.92E+00	9.13E-01	3.37E-01	1.82E-01	-2.64358	-2.64358	-2.64358	-2.64358	-2.0949	7
	12501	697	407	765	4.94E+01	4.37E+00	1.07E+00	3.18E-01	2.27E-01	-2.65236	-2.65236	-2.65236	-2.65236	-2.32183	7
80	7807	1185	564	1337	5.04E+01	1.05E+01	1.46E+00	5.88E-01	2.59E-01	-2.69676	-2.77985	-2.77985	-2.77985	-2.70762	**12**
	13871	3125	1889	3161	8.02E+01	3.43E+01	2.61E+00	1.39E+00	2.56E-01	-2.36783	-2.36783	-2.36783	-2.36783	-2.43442	13
	16875	1193	682	1175	1.32E+02	1.71E+01	1.61E+00	6.63E-01	2.59E-01	-2.37587	-2.46305	-2.46305	-2.46305	-2.3291	11
	1643	235	165	235	1.25E+01	3.16E+00	8.33E-01	3.81E-01	3.00E-01	-2.57955	-2.88952	-2.88952	-2.88952	-2.88952	**12**
	21779	2099	1580	2289	1.67E+02	2.53E+01	2.33E+00	9.78E-01	2.65E-01	-2.59732	-2.63299	-2.63299	-2.63299	-2.55385	**12**
100	7109	33173	6440	33005	1.09E+02	5.96E+02	1.33E+01	1.86E+01	3.51E-01	-2.69328	-2.82602	-2.82602	-2.82602	-2.15026	13
	108199	53481	13284	51969	2.20E+03	1.21E+03	2.64E+01	3.30E+01	3.88E-01	-2.601	-2.61868	-2.61868	-2.61868	-2.43943	16
	25961	11015	3731	8245	2.94E+02	1.92E+02	8.04E+00	4.60E+00	3.45E-01	-2.7306	-2.76679	-2.76679	-2.76679	-2.72698	**15**
	28291	63833	16233	64085	3.53E+02	1.30E+03	3.09E+01	3.73E+01	3.43E-01	-2.91438	-2.93653	-2.93653	-2.93653	-2.64279	13
	80909	37729	10488	37633	1.32E+03	7.42E+02	2.03E+01	2.23E+01	3.42E-01	-2.68003	-2.71881	-2.71881	-2.71881	-2.93001	18
120	76641	89805	15592	86529	2.24E+03	2.61E+03	4.50E+01	5.93E+01	4.94E-01	-2.93726	-2.95759	-2.95759	-2.95759	-2.60868	15
	61661	297535	89839	263489	2.14E+03	1.18E+04	2.49E+02	2.59E+02	4.46E-01	-2.74685	-2.80627	-2.80627	-2.80627	-2.35115	16
	100013	501067	39708	525355	6.51E+03	1.91E+04	1.10E+02	4.68E+02	4.76E-01	-2.75848	-2.89198	-2.89198	-2.89198	-2.6358	16
	41	124181	44892	133445	1.69E-01	5.30E+03	1.27E+02	1.45E+02	4.35E-01	-2.48923	-3.00432	-3.00432	-3.00432	-2.51322	14
	19981	90543	25299	89753	1.68E+03	3.72E+03	7.18E+01	9.00E+01	4.34E-01	-2.7466	-2.9391	-2.9391	-2.9391	-2.91939	**18**

Table 2: A-design with correlated data.

	Nodes				CPU Time [s]					Objective Values					Feasibility
m	B	C	M	P	B	C	M	P	R	B	C	M	P	R	R
50	151	101	82	101	1.32E+00	1.06E+00	1.60E+00	2.78E-02	8.11E-01	-0.04164	-0.04164	-0.04164	-0.04164	-0.05315	8
	143	117	141	95	3.81E-01	1.69E+00	1.93E+00	1.55E-02	8.01E-01	-0.04024	-0.04024	-0.04024	-0.04024	-0.00867	5
	341	237	196	269	8.99E-01	3.02E+00	1.92E+00	9.95E-02	8.08E-01	-0.03469	-0.03469	-0.03469	-0.03469	-0.04798	8
	81	155	201	163	1.92E-01	2.22E+00	1.81E+00	4.15E-02	7.61E-01	-0.03425	-0.0346	-0.0346	-0.0346	-0.04993	8
	49	33	35	33	1.54E-01	3.28E-01	1.35E+00	7.28E-03	8.24E-01	-0.03698	-0.03698	-0.03698	-0.03698	-0.02154	6
60	511	451	418	439	2.45E+00	5.30E+00	3.36E+00	1.21E-01	1.18E+00	-0.04989	-0.04989	-0.04989	-0.04989	-0.07051	11
	201	131	138	87	9.10E-01	2.55E+00	2.33E+00	3.33E-02	1.18E+00	-0.05109	-0.05318	-0.05318	-0.05318	-0.06269	10
	377	345	291	365	1.56E+00	4.86E+00	3.29E+00	1.18E-01	1.18E+00	-0.04463	-0.04463	-0.04463	-0.04463	-0.05541	10
	383	139	279	139	1.58E+00	1.81E+00	2.56E+00	3.76E-02	1.18E+00	-0.04351	-0.04509	-0.04509	-0.04509	-0.04104	**9**
	361	295	224	291	1.37E+00	5.31E+00	2.97E+00	7.46E-02	1.19E+00	-0.04945	-0.04945	-0.04945	-0.04945	-0.04945	**9**
80	195	639	614	529	2.01E+00	1.05E+01	8.14E+00	2.08E-01	2.25E+00	-0.071	-0.07287	-0.07287	-0.07287	-0.06293	11
	455	1197	765	1179	4.85E+00	2.10E+01	8.11E+00	4.24E-01	2.30E+00	-0.05646	-0.05737	-0.05737	-0.05737	-0.03628	10
	877	1185	839	1289	6.92E+00	2.36E+01	7.95E+00	3.52E-01	2.31E+00	-0.0634	-0.06433	-0.06433	-0.06433	-0.06997	13
	255	369	570	373	1.69E+00	5.85E+00	7.03E+00	1.04E-01	2.30E+00	-0.06737	-0.06791	-0.06791	-0.06791	-0.05662	11
	285	239	178	263	1.69E+00	4.04E+00	5.33E+00	8.67E-02	2.33E+00	-0.06415	-0.06415	-0.06415	-0.06415	-0.06415	**12**
100	1415	14429	7866	14685	2.27E+01	2.64E+02	8.00E+01	5.68E+00	3.88E+00	-0.07649	-0.07692	-0.07692	-0.07692	-0.09734	18
	725	6115	4196	5643	1.09E+01	1.10E+02	4.65E+01	1.88E+00	3.90E+00	-0.07013	-0.07074	-0.07074	-0.07074	-0.06284	14
	2597	24535	8846	23587	4.34E+01	4.39E+02	9.06E+01	1.06E+01	3.90E+00	-0.07136	-0.07136	-0.07136	-0.07136	-0.05362	14
	1573	28735	16628	28707	2.63E+01	3.94E+02	1.64E+02	1.12E+01	3.91E+00	-0.07397	-0.07397	-0.07397	-0.07397	-0.06087	14
	9659	35089	19786	35581	1.54E+02	6.19E+02	1.93E+02	1.63E+01	3.97E+00	-0.07416	-0.07416	-0.07416	-0.07416	-0.03475	11
	5795	125891	70712	126391	4.08E+02	3.59E+03	1.09E+03	7.30E+01	6.19E+00	-0.08515	-0.08648	-0.08648	-0.08648	-0.08548	**18**
120	5837	30855	18979	33535	3.81E+02	8.94E+02	2.99E+02	1.97E+01	6.28E+00	-0.0804	-0.08211	-0.08211	-0.08211	-0.07456	17
	11137	176673	84959	181135	7.73E+02	5.42E+03	1.27E+03	1.14E+02	6.34E+00	-0.08549	-0.08688	-0.08688	-0.08688	-0.05816	15
	5575	289467	156473	290131	4.56E+02	1.07E+04	2.38E+03	2.13E+02	6.29E+00	-0.07739	-0.07905	-0.07905	-0.07905	-0.07142	17
	3011	23133	15441	22913	7.81E+01	7.58E+02	2.49E+02	1.49E+01	6.33E+00	-0.08943	-0.09016	-0.09016	-0.09016	-0.07462	16

Table 3: D-design with independent data.

	Nodes				CPU Time [s]					Objective Values					Sum
m	B	C	M	P	B	C	M	P	R	B	C	M	P	R	R
	13	21	22	27	0.886	0.201825	1.119034	0.006255	0.762374	-0.43895	-0.43895	-0.43895	-0.43895	-0.43895	**7**
	15	47	29	51	0.022	1.371286	1.493615	0.016216	1.063007	-0.45256	-0.45256	-0.45256	-0.45256	-0.45256	**7**
50	19	35	46	39	0.028	0.720113	1.406321	0.016018	0.801904	-0.44527	-0.44527	-0.44527	-0.44527	-0.44464	**7**
	15	67	68	57	0.027	1.573053	1.62692	0.017668	0.809261	-0.47219	-0.47219	-0.47219	-0.47219	-0.45801	6
	15	45	81	47	0.02	0.403668	1.363342	0.014135	0.761245	-0.46982	-0.46982	-0.46982	-0.46982	-0.48343	8
	3	55	108	67	0.023	0.574702	2.601325	0.022973	1.199975	-0.44193	-0.4546	-0.4546	-0.4546	-0.4434	8
	13	113	210	141	0.041	1.28259	3.257189	0.098944	1.179643	-0.47399	-0.47477	-0.47477	-0.47477	-0.44588	7
60	27	275	251	239	0.078	6.164294	3.341732	0.059914	1.203427	-0.48015	-0.48015	-0.48015	-0.48015	-0.49061	10
	33	393	338	381	0.094	6.573542	3.683303	0.119217	1.210182	-0.50081	-0.50112	-0.50112	-0.50112	-0.51257	10
	37	385	326	449	0.09	7.517614	3.459445	0.209295	1.207465	-0.48485	-0.48568	-0.48568	-0.48568	-0.4729	8
	19	311	435	361	0.101	5.871775	6.930081	0.123518	2.236059	-0.52573	-0.52632	-0.52639	-0.52639	-0.50868	10
	37	717	726	767	0.151	12.00395	7.601306	0.283598	2.275386	-0.49282	-0.49314	-0.49314	-0.49314	-0.51026	14
80	29	689	375	697	0.128	10.64573	7.507315	0.301807	2.269305	-0.4972	-0.49904	-0.49904	-0.49904	-0.49768	**12**
	15	887	656	867	0.088	17.56696	9.041049	0.258951	2.286159	-0.53928	-0.53981	-0.53981	-0.53981	-0.53969	**12**
	11	467	420	459	0.046	9.422584	7.036418	0.152531	2.281927	-0.51478	-0.51703	-0.51703	-0.51703	-0.52583	13
	23	2329	2542	2471	0.135	43.06324	31.42052	1.258657	3.889728	-0.55414	-0.55539	-0.55539	-0.55539	-0.56637	17
	61	1943	1472	1943	0.387	40.24302	21.35093	1.075315	4.009676	-0.53209	-0.53383	-0.53383	-0.53383	-0.53351	**15**
100	71	7353	5309	6917	0.467	126.1221	56.94396	3.483215	3.947227	-0.54092	-0.54181	-0.54181	-0.54181	-0.54824	16
	37	4753	3794	4769	0.225	99.62108	42.13473	2.041134	3.900112	-0.56192	-0.56213	-0.56213	-0.56213	-0.57476	17
	37	1035	1093	991	0.243	20.71913	17.42868	0.406109	3.912569	-0.541	-0.54256	-0.54256	-0.54256	-0.5419	**15**
	37	31501	19204	29937	0.353	794.3156	298.5706	16.94365	6.097007	-0.58158	-0.58262	-0.58262	-0.58262	-0.57338	17
	57	70237	39885	71165	0.49	1301.24	603.623	34.31921	6.176917	-0.56528	-0.56528	-0.56528	-0.56528	-0.56528	**18**
120	99	25741	14440	22717	1.242	532.3967	233.3591	11.06108	6.19401	-0.58171	-0.58207	-0.58207	-0.58207	-0.57499	17
	25	12003	9017	12347	0.229	233.8313	146.1561	5.63249	6.202873	-0.58544	-0.58617	-0.58617	-0.58617	-0.56905	16
	41	2351	1582	2419	0.422	43.76389	36.08488	1.019536	6.174791	-0.57535	-0.57793	-0.57793	-0.57793	-0.5647	16

Table 4: D-design with correlated data.