① 虚拟流动性 Virtual Reserves

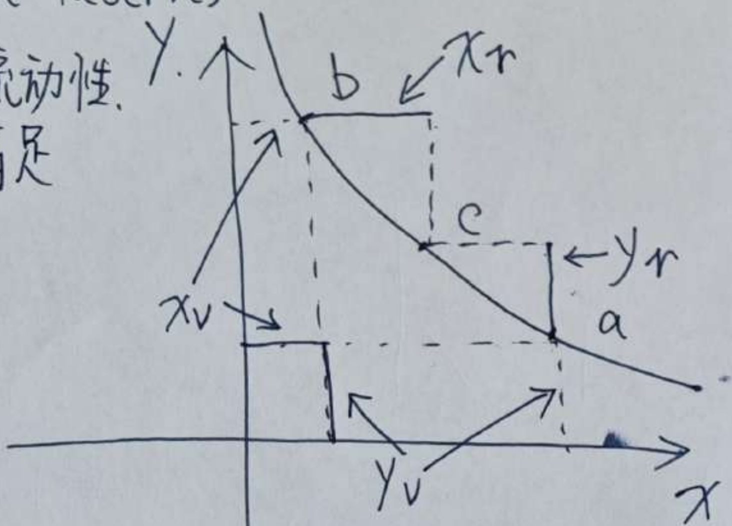
用户在a点和b点之间添加流动性.
在任意 a, b 之间的 c点, 满足

$x \cdot y = k = L^2$.

其中 $x = x_r + x_v$
$y = y_r + y_v$.

在 a. b 两点确定不变的情况下.
x_v 和 y_v 为固定值. x_r, y_r 为变量

$(x_r + x_v)(y_r + y_v) = L^2$



$x \cdot y = k = L^2 \Rightarrow$ $x = \frac{L}{\sqrt{P}}$
$P = \frac{y}{x}$ $y = L\sqrt{P}$.

求 x_v、y_v: (参考右边公式)
在b点, $x_r = 0$

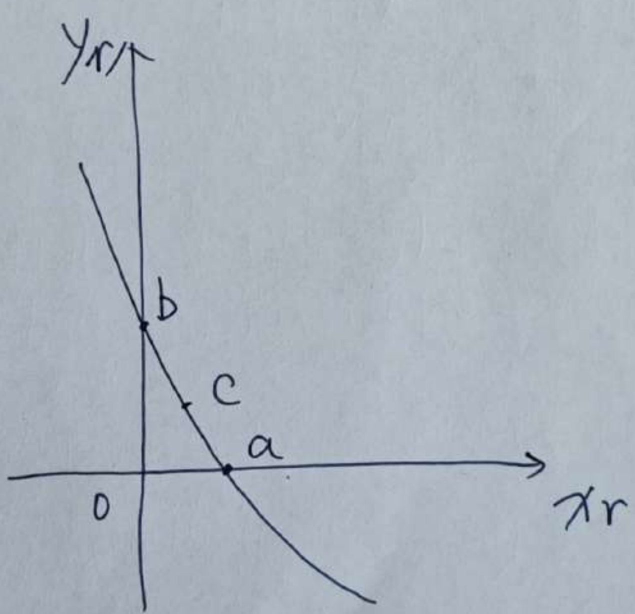
$\cancel{x_v}$

$x = \frac{L}{\sqrt{P_b}} = x_v + x_r = x_v$
⇓
$x_v = \frac{L}{\sqrt{P_b}}$

在a点, $y_r = 0$
$y = L\sqrt{P_a} = y_v + y_r = y_v$.
⇓
$y_v = L\sqrt{P_a}$

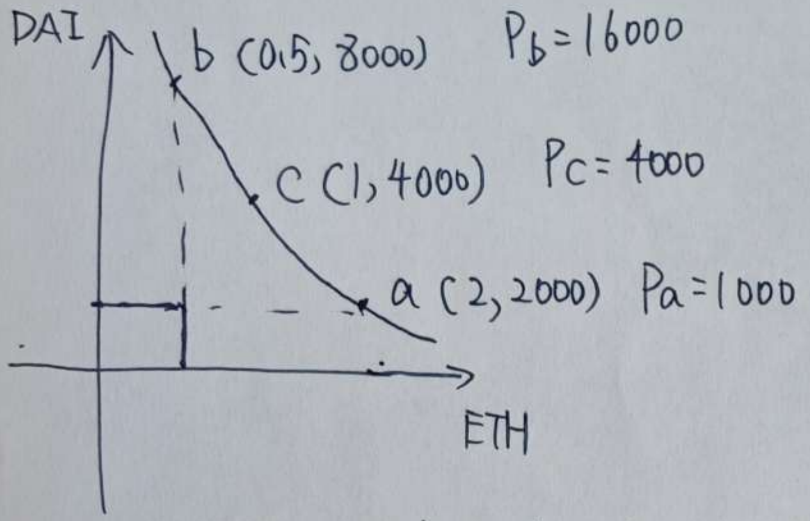
将 x_v、y_v 代入上面公式.

$(x_r + \frac{L}{\sqrt{P_b}})(y_r + L\sqrt{P_a}) = L^2$



② 虚拟流动性案例分析 一1

$x \cdot y = L^2 = 4000 \Rightarrow L = 20\sqrt{10}$



求当前场景下的虚拟流动性 x_v 和 y_v.

$$x_v = \frac{L}{\sqrt{P_b}} = \frac{20\sqrt{10}}{\sqrt{16000}} = \frac{20\sqrt{10}}{40\sqrt{10}} = 0.5$$

$$y_v = L\sqrt{P_a} = 20\sqrt{10} \cdot \sqrt{1000} = 20\sqrt{10} \cdot 10\sqrt{10} = 2000.$$

在 a、b 之间添加流动性、以 C点为例

$(x_r + 0.5) \cdot (y_r + 2000) = L^2 = 4000.$

在 C点价格 $= y_r / x_r = 4000/1 = 4000 \Rightarrow y_r = 4000\, x_r$

$(x_r + 0.5) \cdot (4000\, x_r + 2000) = 4000$

$(x_r + 0.5)^2 \cdot 4000 = 4000$

$x_r + 0.5 = 1 \Rightarrow x_r = 0.5.$

将 $x_r = 0.5$ 代入上述公式. 得 $y_r = 2000$.

在 C点, 用户实际添加流动性为 (0.5, 2000).

虚拟流动性为 (0.5, 2000).

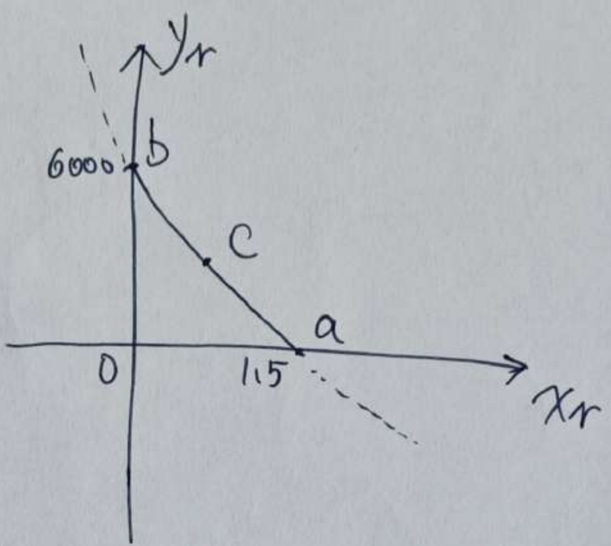
$x = x_r + x_v = 0.5 + 0.5 = 1$

$y = y_r + y_v = 2000 + 2000 = 4000.$

其曲线与 Uniswap v2 中 $x \cdot y = 4000$ 一致

③ 虚拟流动性案例分析 —2.

$(x_r+0.5)(y_r+2000)=4000$，画出该曲线



在a点，$y_r=0$

$(x_r+0.5)(0+2000)=4000$

$x_r+0.5=2 \Rightarrow x_r=1.5$

在b点，$x_r=0$

$(0+0.5)(y_r+2000)=4000$

$y_r+2000=8000 \Rightarrow y_r=6000$

在b点添加流动性.(0, 6000)

如果降到a点，移除流动性.会得到多少x.和y?

在a点.(x_r, y_r)为(1.5, 0)

相当于以均价 $\frac{6000}{1.5}=4000$ 的价格把6000个DAI换成了1.5个ETH.

如果缩小a, b点的距离.一定程度上起到了限价订单的作用

问题：在Uniswap V2中.如果想在C点.添加流动性.需要提供(1, 4000).

在Uniswap V3中.同样的流动性曲线.只需要真实提供(0.5, 2000).

便可以在a, b点之间达到同样的k值($k=x \cdot y$).问.如果在V3中.

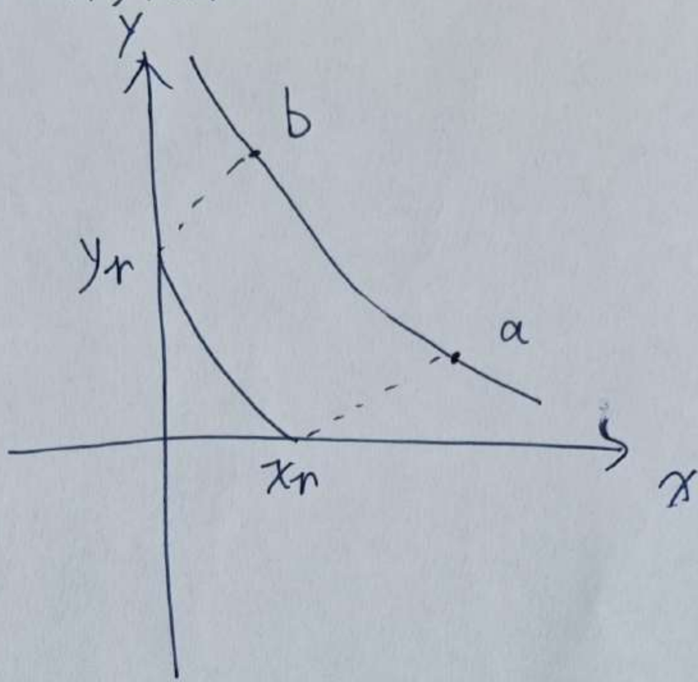
真实提供(1, 4000).那么k值(L值. $k=L^2$)会如何变化呢?

如果在价格 $P \leq P_a$ 时.提供流动性. k值(L值会如何变化?)

如果在价格 $P \geq P_b$ 时提供流动性. k值(L值会如何变化?)

④ 不同价格区间.流动性L和x_r, y_r的关系

$$(x_r+\frac{L}{\sqrt{P_b}})(y_r+L\sqrt{P_a})=L^2$$



1、当$P\le P_a$时. $y_r=0$.

$$(x_r+\frac{L}{\sqrt{P_b}})\cdot L\sqrt{P_a}=L^2$$

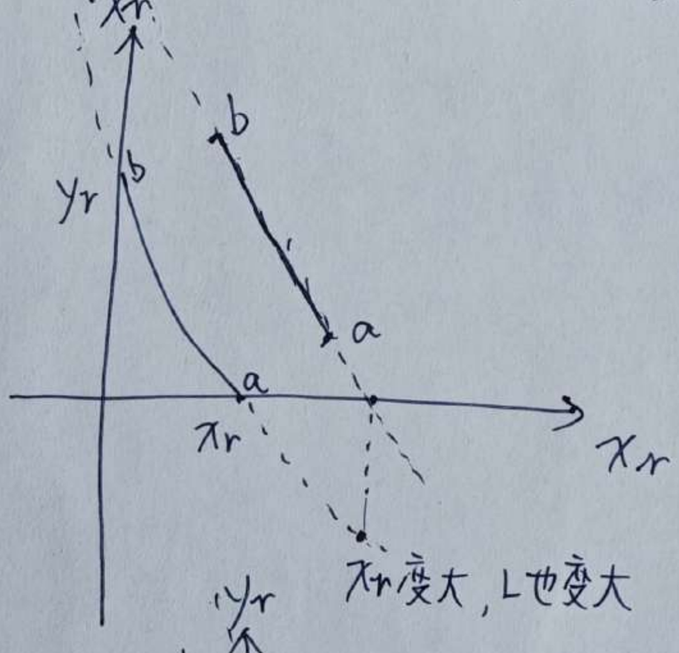
$$x_r+\frac{L}{\sqrt{P_b}}=\frac{L}{\sqrt{P_a}}$$

$$x_r=L(\frac{1}{\sqrt{P_a}}-\frac{1}{\sqrt{P_b}})=L\frac{\sqrt{P_b}-\sqrt{P_a}}{\sqrt{P_a}\cdot\sqrt{P_b}}$$

$$L=x_r\cdot\frac{\sqrt{P_a}\cdot\sqrt{P_b}}{\sqrt{P_b}-\sqrt{P_a}}$$

(此处网页演示)

当x_r变大、L也变大



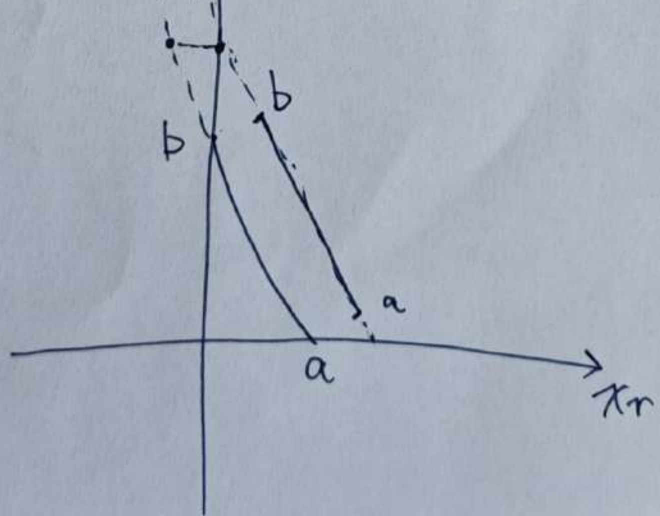
② 当$P\ge P_b$时. $x_r=0$

$$\frac{L}{\sqrt{P_b}}\cdot(y_r+L\sqrt{P_a})=L^2$$

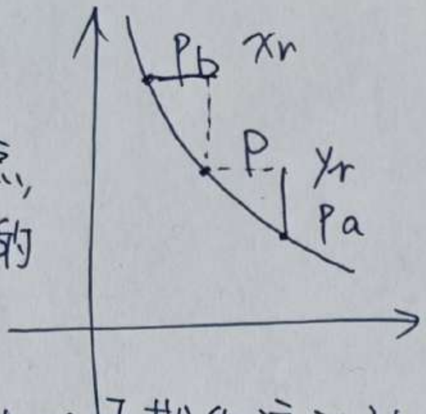
$$y_r+L\sqrt{P_a}=L\sqrt{P_b}.$$

$$y_r=L(\sqrt{P_b}-\sqrt{P_a})$$

$$L=\frac{y_r}{\sqrt{P_b}-\sqrt{P_a}}$$



3. 当 $P_a < P < P_b$ 时.



当P处于Pa和Pb之间的任意一点，xr和yr对于维护[Pa, Pb]之间的流动性贡献是一样的.

先计算(P, Pb)价格之间.单纯由xr贡献的流动性.

$$L = x_r \cdot \frac{\sqrt{P}\cdot\sqrt{P_b}}{\sqrt{P_b}-\sqrt{P}}$$

再计算(Pa, P)价格之间.单纯由yr贡献的流动性.

$$L = \frac{y_r}{\sqrt{P}-\sqrt{P_a}}$$

$$L = \frac{x_r\cdot\sqrt{P}\cdot\sqrt{P_b}}{\sqrt{P_b}-\sqrt{P}} = \frac{y_r}{\sqrt{P}-\sqrt{P_a}}$$

案例解析(以第②页的例子).

$P_a = 1000$, ~~$(x_r, y_r) = (1.5, 0)$~~ $P_b = 16000$, $P = 4000$

1. 当 $P = P_a$ 时. $(x_r, y_r) = (1.5, 0)$

$$L = x_r\frac{\sqrt{P_a}\cdot\sqrt{P_b}}{\sqrt{P_b}-\sqrt{P_a}} = 1.5\times\frac{\sqrt{1000}\cdot\sqrt{16000}}{\sqrt{16000}-\sqrt{1000}} = 1.5\times\frac{\sqrt{16000}}{\sqrt{16}-1} = 0.5\times40\sqrt{10} = 20\sqrt{10}$$

2. 当 $P = P_b$ 时. $(x_r, y_r) = (0, 6000)$

$$L = \frac{y_r}{\sqrt{P_b}-\sqrt{P_a}} = \frac{6000}{\sqrt{16000}-\sqrt{1000}} = \frac{6000}{30\sqrt{10}} = \frac{200}{\sqrt{10}} = 20\sqrt{10}$$

3. 当 $P = P_c$ 时. $(x_r, y_r) = (0.5, 2000)$

$$L = \frac{x_r\cdot\sqrt{P}\cdot\sqrt{P_b}}{\sqrt{P_b}-\sqrt{P}} = \frac{0.5\times\sqrt{4000}\cdot\sqrt{16000}}{\sqrt{16000}-\sqrt{4000}} = \frac{0.5\times\sqrt{16000}}{\sqrt{4}-1} = 0.5\times40\sqrt{10} = 20\sqrt{10}$$

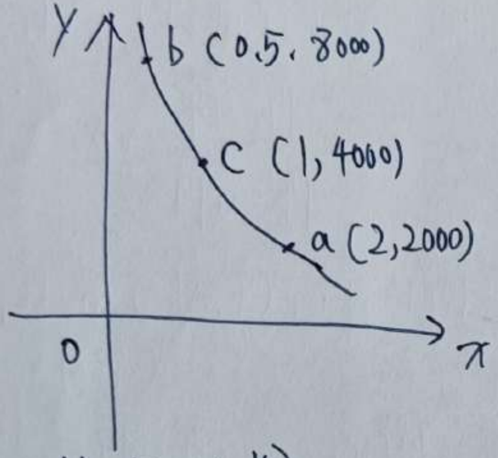
$$L = \frac{y_r}{\sqrt{P}-\sqrt{P_a}} = \frac{2000}{\sqrt{4000}-\sqrt{1000}} = \frac{2000}{20\sqrt{10}-10\sqrt{10}} = \frac{200}{\sqrt{10}} = 20\sqrt{10}$$

⑥ 在 $(P_a, P_b) = (1000, 16000)$，$(x_r, y_r) = (1, 4000)$ 的情况下，求 L.

$p = 4000$

$$L = \frac{x_r \cdot \sqrt{p} \cdot \sqrt{p_b}}{\sqrt{p_b} - \sqrt{p}} = \frac{1 \cdot \sqrt{4000} \cdot \sqrt{16000}}{\sqrt{16000} - \sqrt{4000}} = \frac{1 \cdot \sqrt{16000}}{\sqrt{4} - 1} = 40\sqrt{10}$$

此案例中无常损失的计算.



Uniswap V2

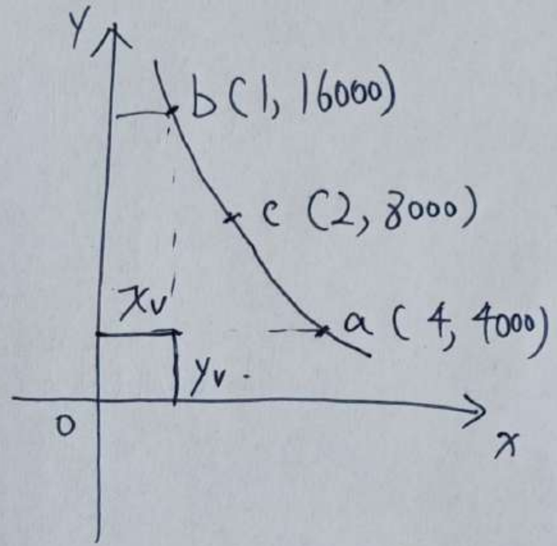
当价格从 C 变到 a.

$V_0 = 1 \times 4000 + 4000 = 8000$

$V_{hodl} = 1 \times 1000 + 4000 = 5000$

$V_2 = 2 \times 1000 + 2000 = 4000$

在 Uniswap V2 中. 无常损失为 $(V_{hodl} - V_2) = 1000$.



Uniswap V3

在 C 点 $(x_r, y_r) = (1, 4000)$

先求 x_v, y_v.

$$x_v = \frac{L}{\sqrt{p_b}} = \frac{40\sqrt{10}}{\sqrt{16000}} = 1$$

$$y_v = L\sqrt{p_a} = 40 \cdot \sqrt{10}\sqrt{1000} = 4000$$

得出

在 a 点 $(x_r, y_r) = (4-1, 4000-4000) = (3, 0)$

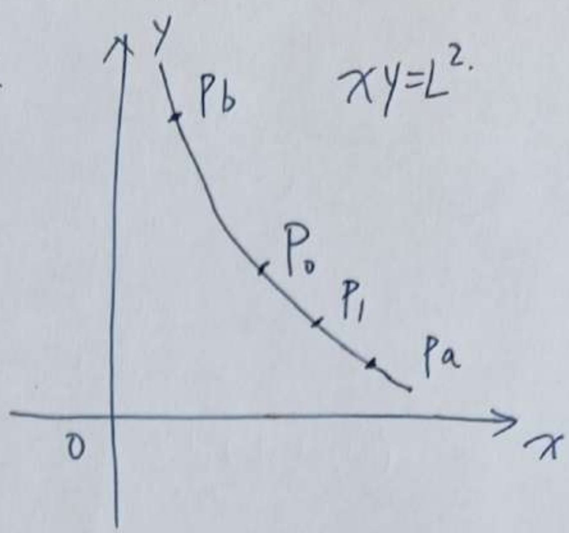
$$\begin{pmatrix} x = x_r + x_v \\ y = y_r + y_v \end{pmatrix} \Rightarrow \begin{pmatrix} 4 = x_r + 1 \\ 4000 = y_r + 4000 \end{pmatrix}$$

$V_3 = 3 \times 1000 = 3000$

在 Uniswap V3 中. 无常损失为 $(V_{hold} - V_3) = 2000$

在 Uniswap V3 中. 无常损失更大.

⑦ 在使用Δx或Δy参与swap时引起的价格变化.



前提：流动性足够. swap之后的P_1满足

$P_a < P_1 < P_b$.

$$xy=k=L^2 \quad p=\frac{y}{x} \Rightarrow x=\frac{L}{\sqrt{p}} \quad y=L\sqrt{p}$$

初始状态、$x_0=\frac{L}{\sqrt{P_0}} \cdot x_1=\frac{L}{\sqrt{P_1}}$

$$\Delta x = x_1 - x_0 = \frac{L}{\sqrt{P_1}} - \frac{L}{\sqrt{P_0}} = L\left(\frac{1}{\sqrt{P_1}}-\frac{1}{\sqrt{P_0}}\right) = \Delta\frac{1}{\sqrt{P}}\cdot L$$ 白皮书

$$\Delta y = y_1 - y_0 = L\sqrt{P_1} - L\sqrt{P_0} = L(\sqrt{P_1}-\sqrt{P_0}) = \Delta\sqrt{P}\cdot L$$

$$\sqrt{P_1} = \frac{L\sqrt{P_0}}{\Delta x\cdot\sqrt{P_0}+L}$$

$$\sqrt{P_1} = \frac{\Delta y}{L}+\sqrt{P_0}$$

案例解读

以第②页案例为例、从C点到a点、

$P_0=4000,\ P_1=1000,\ L=20\sqrt{10},\ \Delta y=-2000,\ \Delta x=1$

$$\frac{L\sqrt{P_0}}{\Delta x\sqrt{P_0}+L} = \frac{20\sqrt{10}\cdot\sqrt{4000}}{1\cdot\sqrt{4000}+20\sqrt{10}} = 10\sqrt{10}.$$

$$\frac{\Delta y}{L}+\sqrt{P_0} = \frac{-2000}{20\sqrt{10}}+\sqrt{4000} = -10\sqrt{10}+20\sqrt{10} = 10\sqrt{10}$$

$\sqrt{P_1}=10\sqrt{10}$.

⑧ 复习：Uniswap V2 无常损失的计算

1. 每个交易对都由两种币组成
2. 交易对里两种币的价值相等. 各自组成了50%的池子价值.

$$x\cdot y=k,\quad p=\frac{y}{x} \Rightarrow \begin{cases} x=\sqrt{k}/\sqrt{p} \\ y=\sqrt{k}\cdot\sqrt{p} \end{cases}$$

t_0的时候: x_0, y_0, $p_0=\frac{y_0}{x_0}$, $x_0\cdot y_0=k_0$

t_0:

- t_1: (x_1, y_1). $p_1=\frac{y_1}{x_1}$ $x_1\cdot y_1=k_1$
  $p_1=p_0\cdot\alpha$. α为价格变化因子.
- t_{hold} (x_0, y_0), $p_1=\frac{y_1}{x_1}$

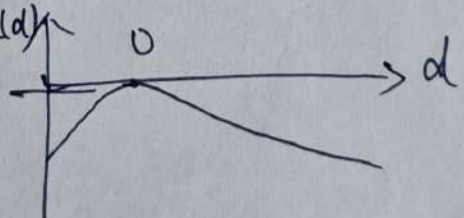
$$IL(\alpha)=f(\alpha)=\frac{V_1-V_{hold}}{V_{hold}}=\frac{V_1}{V_{hold}}-1$$

$$V_1=y_1+x_1\cdot p_1=2\sqrt{k_1}\cdot\sqrt{p_1}$$

$$V_{hold}=y_0+x_0\cdot p_1=(1+\alpha)\sqrt{k_0}\cdot\sqrt{p_0}$$

在没有手续费的情况下. $k_0=k_1$

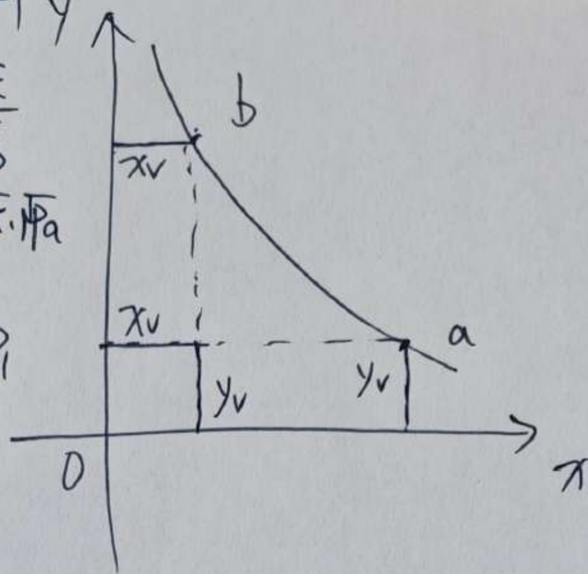
$$IL(\alpha)=f(\alpha)=\frac{V_1}{V_{hold}}-1=\frac{2\sqrt{k_1}\cdot\sqrt{p_1}}{(1+\alpha)\sqrt{k_0}\cdot\sqrt{p_0}}=\frac{2\sqrt{\alpha}}{1+\alpha}-1$$



⑨ Uniswap V3里的无常损失-1

$x\cdot y=k \Rightarrow x=\sqrt{k}/\sqrt{p}$ V3里 $x_v=\frac{L}{\sqrt{p_b}}=\frac{\sqrt{k}}{\sqrt{p_b}}$

$p=\frac{y}{x}$ $y=\sqrt{k}\cdot\sqrt{p}$. $y_v=L\sqrt{p_a}=\sqrt{k}\cdot\sqrt{p_a}$



1、假设起始价格P_0，变化后的价格P_1都满足 $P_0, P_1 \in [P_a, P_b]$、求此时的无常损失（无手续费）

t_0的时候. (x_0, y_0) $P_0=\frac{y_0}{x_0}$. $x_0\cdot y_0=k$.

$$t_0 \begin{cases} t_1:\ (x_1, y_1)\quad P_1=\frac{y_1}{x_1}. \\ t_{hodl}:\ (x_0, y_0)\quad P_1=\frac{y_1}{x_1} \end{cases}$$

$$IL(d)=f(d)=\frac{V_1-V_{hodl}}{V_{hodl}}=\frac{V_1}{V_{hodl}}-1$$

$$\begin{aligned} V_1 &= x_{real}\cdot P_1+y_{real} \\ &= (x_1-x_v)\cdot P_1+(y_1-y_v) \\ &= \left(\frac{\sqrt{k}}{\sqrt{P_1}}-\frac{\sqrt{k}}{\sqrt{P_b}}\right)\cdot P_1+(\sqrt{k}\cdot\sqrt{P_1}-\sqrt{k}\cdot\sqrt{P_a}) \\ &= 2\sqrt{k}\cdot\sqrt{P_1}-\sqrt{k}\cdot\sqrt{P_a}-\frac{\sqrt{k}}{P_b}\cdot P_1 \end{aligned}$$

$x=x_{real}+x_v$

$y=y_{real}+y_v$

$x_1=\frac{\sqrt{k}}{\sqrt{P_1}}$. $y_1=\sqrt{k}\cdot\sqrt{P_1}$

$x_0=\frac{\sqrt{k}}{\sqrt{P_0}}$. $y_0=\sqrt{k}\cdot\sqrt{P_0}$

$$\begin{aligned} V_{hodl} &= x_{real}\cdot P_1+y_{real} \\ &= (x_0-x_v)\cdot P_1+(y_0-y_v) \\ &= \left(\frac{\sqrt{k}}{\sqrt{P_0}}-\frac{\sqrt{k}}{\sqrt{P_b}}\right)\cdot P_1+(\sqrt{k}\cdot\sqrt{P_0}-\sqrt{k}\cdot\sqrt{P_a}) \\ &= \sqrt{k}\left(\frac{1}{\sqrt{P_0}}-\frac{1}{\sqrt{P_b}}\right)\cdot P_1+(\sqrt{P_0}-\sqrt{P_a})\sqrt{k} \end{aligned}$$

$$IL(d)=f(d)=\frac{2\sqrt{k}\cdot\sqrt{P_1}-\sqrt{k}\cdot\sqrt{P_a}-\frac{\sqrt{k}}{\sqrt{P_b}}\cdot P_1}{\sqrt{k}\left(\frac{1}{\sqrt{P_0}}-\frac{1}{\sqrt{P_b}}\right)\cdot P_1+(\sqrt{P_0}-\sqrt{P_a})\sqrt{k}}=\frac{2\sqrt{P_1}-\sqrt{P_a}-\frac{P_1}{P_b}}{\left(\frac{1}{\sqrt{P_0}}-\frac{1}{\sqrt{P_b}}\right)P_1+\sqrt{P_0}-\sqrt{P_a}}-1$$

⑩ Uniswap V3里的无常损失-2.

2. 其他条件不变.假设变化后的价格 $P_1 < P_a$.计算无常损失

$$IL(d) = f(d) = \frac{V_1}{V_{hold}} - 1$$

V_1: 在 $P_1 < P_a$ 时. y_{real} 为0

$$V_1 = x_{real} \cdot P_1 + y_{real}$$

$$= x_{real} \cdot P_1 = \left(\frac{\sqrt{L}}{\sqrt{P_1}} - \frac{\sqrt{L}}{\sqrt{P_b}}\right) \cdot P_1$$

$$IL(d) = f(d) = \frac{\left(\frac{\sqrt{L}}{\sqrt{P_1}} - \frac{\sqrt{L}}{\sqrt{P_b}}\right) \cdot P_1}{\sqrt{L}\left(\frac{1}{\sqrt{P_0}} - \frac{1}{\sqrt{P_b}}\right) \cdot P_1 + (\sqrt{P_0} - \sqrt{P_a}) \cdot \sqrt{L}} - 1 = \frac{\left(\frac{1}{\sqrt{P_1}} - \frac{1}{\sqrt{P_b}}\right) \cdot P_1}{\left(\frac{1}{\sqrt{P_0}} - \frac{1}{\sqrt{P_b}}\right) P_1 + (\sqrt{P_0} - \sqrt{P_a})} - 1$$

3. 其他条件不变.假设变化后的价格 $P_1 > P_b$.计算无常损失.

(此为课后作业)

当 P_a 趋向于0. P_b 趋向于 ∞ 的时候

$$IL(d) = \frac{2\sqrt{P_1} - \sqrt{P_a} - \frac{P_1}{\sqrt{P_b}}}{\left(\frac{1}{\sqrt{P_0}} - \frac{1}{\sqrt{P_b}}\right) \cdot P_1 + \sqrt{P_0} - \sqrt{P_a}} - 1 = \frac{2\sqrt{P_1}}{\frac{P_1}{\sqrt{P_0}} + \sqrt{P_0}} - 1$$

$P_1 = P_0 \cdot d$ 则

$$IL(d) = \frac{2\sqrt{P_0 \cdot d}}{\frac{P_0 \cdot d}{\sqrt{P_0}} + \sqrt{P_0}} - 1 = \frac{2\sqrt{d}}{1+d} - 1$$ 、和V2中的一致

(图形演示)