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A Unified Predictive Display Scheme for Teleoperation Systems with Multi-time-delay Multi-rate Measurements and Missing Data

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Abstract: Predictive display is a promising approach to tackle large time delays that greatly degrade the performance of teleoperation systems. In this paper, a unified predictive display scheme is proposed to deal with multi-time-delay multi-rate measurements and missing data, which is applicable for real systems. Firstly, a formal description of predictive display problem is introduced. Next the method principle is presented : through an equivalent transformation the original problem is converted into a standard filter problem and a prediction problem, which can be tackled by an observer-predictor structure; then the re-organized innovation analysis is utilized in the observer and is extended to deal with multi-time-delay multi-rate measurements as well as missing data with the help of indication functions. Finally simulations show the effectiveness of this approach.

Key Words: Predictive Display, Teleoperation, Multi-time-delay Measurements, Multi-rate Measurements

1 Introduction

Teleoperation[1] extends the human operator's sensing and manipulation capability to remote workspace by incorporating the human into the control loop of teleoperators. It has been applied to space exploring[2], deep sea manipulation[3], nuclear material handling[4], robotic-assisted medical interventions[5], etc. Time delay resulted from the long-distance communication is one of the key challenges posed to teleoperation systems[6][7]. Depending on the medium of communication and the distance between the master and slave sites, data transmission delay in teleoperation varies from less than a millisecond to several minutes.The time delay will significantly increases the operation time, degrade the operator's sense of immersion and even destabilize the system when force feedback is presented to the operator[8].

Predictive display, aiding the operator by predicting and displaying the future states of the remote system, is considered to be a promising method to compensate for the communication delays[9][10][11]. There are two types of predictive display[12]. One is a Taylor-series extrapolation upon current state and time derivatives, which is suitable for short predictions and uses only the initial conditions of the state. Another approach involves a system model that is run many times faster than the actual process, which needs the current initial conditions as well as input signals and accounts for longer predictions as well as properties of the process.

To tackle time delays over one second, the second type of predictive display is usually employed. G. Hirzinger[13][14] et al. proposed an estimator-predictor structure, in which the extended Kalman filter was used in the estimator to give current initial conditions and then in the predictor propagate them forwards the time-delay to obtain the future state. Jun Kikuchi[15] et al. utilized a similar technique to remove visual delays during operation of a robot in a static environment. Tetsuo Kotoku[16] designed a predictive display system with force feedback and applied it to a remote manipulation system with transmission time delay. By combining predictive robot models with a model-mediated method of haptic interaction, Ryder C. Winck[17] et al. put forward a teleoperation framework that is able to predict environment motion and permits haptic interactions with the environment. In these methods, predictive displays only took into account single delayed measurement or a set of measurements with the same time delay. What's more, none of them considered the multi-rate measurements and missing data.

However, in the real applications, teleoperation systems are often confronted with multi-time-delay multi-rate measurements, and sometimes even with missing data. The remote system is usually equipped with multiple sensors, and different sensors may have different measurement rates and sensor data may be transmitted to the local site through different communication channels. In addition, different sensor data may need different preprocessing time. So the measurements utilized in the local site may have multiple time delays as well as multiple rates. What's more, due to the limited working range of sensors, sensing noises and package dropouts in transmission, data may go missing, which should be taken into account when developing predictive display algrithms. To meet these challenges posed to teleoperation systems, a unified predictive display scheme is proposed in this paper which is capable of tackling multi-time-delay multi-rate measurements as well as data loses.

Several approaches have been put forward to deal with the estimation problem in discrete-time systems with delayed measurements. A traditional one is the augmented-state method, the basic idea of which is to expand the delayed states into the system states, resulting to delay-free system state equations. However, it costs large storage space and expensive calculation especially when the measurement delays are large and the dimension of the system is high. The other one is the re-organized innovation analysis[18], which re-organizes the innovation sequence such that it compares each delayed measurement with its corresponding backward

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time-shifted estimate. It consists of several standard Kalman filters with the same dimension as the original system so it is more efficient in computation than the augmented-state method. The multi-rate optimal filtering problem has also been addressed by many researchers[19][20][21]. Some unified the rate of all measurements by down-sampling the high-rate measurements or interpolating the low-rate one. In[22], the multi-rate fusion problem with data loses was transformed into equivalent single rate estimation problem by using a "dummy" measurement. In [23], the re-organized innovation analysis was extended to the multi-rate case by down-sampling high-rate measurements, but it may cause information loss. Although the above issues are not new, the re-organized innovation analysis hasn't been extended to deal with multi-time-delay multi-rate measurements and data loses, and there are no formal contributions on tackling multi-time-delay multi-rate measurements in predictive displays, to the authors' knowledge.

The novel scheme proposed in the paper is in the category of the second type of predictive display. A formal description of predictive displays problem is introduced and the problem is transformed into a standard filter problem with multiple time-delayed measurements and a prediction problem, which can be tackled with an observer-predictor structure. The re-organized innovation analysis is utilized in the observer and is extended to consider multi-time-delay multi-rate measurements and missing data with the help of indication functions indicating whether measurements are available and valid. This approach is a significant extension of the ideas presented in early works and is more practical and appropriate for real teleoperation applications.

The rest of the paper is arranged as follows. Problems formulation is presented in section 2. Then the details of our predictive display scheme are introduced in section 3. Simulations section 4 show the effectiveness of our approach. Finally conclusions are drawn in section 5.

2 Problem Formulation

Consider the linear discrete-time system with l set of delayed measurements

$$\boldsymbol{x}(k+1) = \boldsymbol{A}(k)\boldsymbol{x}(k) + \boldsymbol{B}(k)\boldsymbol{u}(k) + \boldsymbol{w}(k) \tag{1}$$

$$\boldsymbol{y}_i(k) = \begin{cases} \boldsymbol{H}_i(k)\boldsymbol{x}(k-d_i) + \boldsymbol{v}_i(k) & \text{if } F_i(k) = 1 \\ \text{Null} & \text{if } F_i(k) = 0 \end{cases} \tag{2}$$

$$\boldsymbol{F}_i(k) = \begin{cases} 0 & \text{if mod}(k-d_i, r_i) \neq 0 \text{ or data go missing} \\ 1 & \text{else} \end{cases}$$

$$i = 0, 1, \dots l-1 \tag{3}$$

$$0 < d_0 < d_1 < \dots < d_{l-1}$$

where $\boldsymbol{x}(k) \in \boldsymbol{R}^n$ is the states of the tele-operators and remote environment, $\boldsymbol{u}(k) \in \boldsymbol{R}^r$is the control input, $\boldsymbol{w}(k) \in \boldsymbol{R}^n$ is the system noise, $\boldsymbol{y}_i \in \boldsymbol{R}^{m_i}, i = 0, 1, \dots l-1$, are delayed measurements, $\boldsymbol{v}_i(k), \in \boldsymbol{R}^{m_i}, i = 0, 1, \dots l-1$,are measurement noises. $\boldsymbol{A}(k)$, $\boldsymbol{B}(k)$ and $\boldsymbol{H}_i(k)$ are time-varying matrices with suitable dimensions. $F_i(k), i = ,1, \dots l-1$ are indication functions that indicate whether the measurements $\boldsymbol{y}_i(k), i =, 1, \dots l-1$ are sampled and whether they go missing due to the sensing noises or package dropouts. $r_i, i =, 1, \dots l-1$ are the sampling intervals for measurements $\boldsymbol{y}_i(k), i =, 1, \dots l-1$. $\boldsymbol{w}(k)$and $\boldsymbol{v}_i(k)$ are uncorrected white noises with zero means and known covariance matrixes, $E[\boldsymbol{w}(k)\boldsymbol{w}^T(j)] = \boldsymbol{Q}_w(k)\delta_{ij}$ and $E[\boldsymbol{v}_i(k)\boldsymbol{v}_i^T(j)] = \boldsymbol{Q}_{v_i(k)}\delta_{ij}$ respectively. In this paper, $E[*]$ denotes the expectation.

Due to the uplink delay d_u, when the tele-operators act on the control input $\boldsymbol{u}(k)$, the operator generates the future control input $\boldsymbol{u}(k+d_u)$. In order to help the operator generate suitable control commands, the predictive display system is supposed to estimate the future state $\boldsymbol{x}(k+d_u)$ based on delayed measurements $\boldsymbol{y}_i(k), i =, 1, \dots l-1$ and then display it to the operator. The schematic of predictive display is shown in Fig. 1.

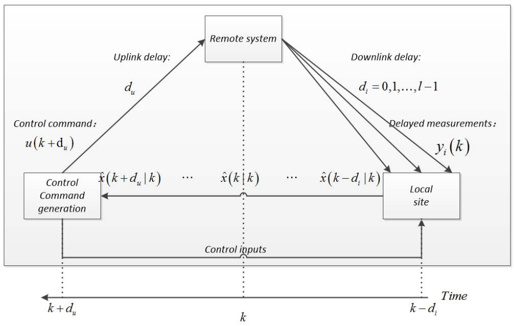

Fig. 1: Schematic of the predictive display

In (2), $\boldsymbol{y}_i(k), i = 0, 1 \cdots l-1$ mean the delayed observations of the state $\boldsymbol{x}(k-d_i)$ received at time k. Let $\boldsymbol{Y}(k)$ denotes all the delayed measurements received at time k, then we have

$$\boldsymbol{Y}(k) = [\boldsymbol{y}_0^T(k) \cdots \boldsymbol{y}_{i-1}^T(t)], \tag{4}$$

$$\text{if } d_{i-1} \leq k < d_i,\ i = 1, 2 \dots l-1$$

$$\boldsymbol{Y}(k) = [\boldsymbol{y}_0^T(k) \cdots \boldsymbol{y}_{l-1}^T(k)], \quad \text{if } k \geq d_{l-1} \tag{5}$$

The predictive display problem can be stated as

Problem 1 *Given the measurements* $\left\{\{\boldsymbol{Y}(i)\}_{i=d_0}^k\right\}$ *and control input history* $\left\{\{\boldsymbol{u}(i)\}_{i=0}^{k+d_u-1}\right\}$*, find the linear least mean square error estimat* $\hat{\boldsymbol{x}}(k+d_u|k)$ *for* $\boldsymbol{x}(k+d_u)$.

3 Predictive Display Scheme

3.1 Equivalent Transformation and the Observer-Predictor Structure

For the predictive display system at the local site, whether the time delays locate in the uplink or the downlink makes no difference. In other words, some part of the downlink delay can be "moved" to the uplink for the convenience of further discussion. Inspired by this idea, we make the following transformation

$$t = k - d_0 \tag{6}$$

and we denote

$$D_i \equiv d_i - d_0, i = 0, 1 \cdots, l-1 \tag{7}$$

$$\boldsymbol{z}_i(t) \equiv y_i(k), i = 0, 1 \cdots, l-1 \tag{8}$$

$$\boldsymbol{H}_i^*(t) \equiv \boldsymbol{H}_i(k), i = 0, 1 \cdots, l-1 \tag{9}$$

$$\boldsymbol{v}_i^*(t) \equiv \boldsymbol{v}_i(k), i = 0, 1 \cdots, l-1 \tag{10}$$

then an equivalent system for (1)(2)(3) can be described as

$$\boldsymbol{x}(t+1) = \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{B}(t)\boldsymbol{u}(t) + \boldsymbol{w}(t) \tag{11}$$

$$\boldsymbol{z}_i(t) = \begin{cases} \boldsymbol{H}_i^*(t)\boldsymbol{x}(t-D_i) + \boldsymbol{v}_i^*(t) & \text{if } F_i(t) = 1 \\ \text{Null} & \text{if } F_i(t) = 0 \end{cases} \tag{12}$$

$$\boldsymbol{F}_i(t) = \begin{cases} 0 & \text{if mod}(t - D_i, r_i) \neq 0 \text{ or data go missing} \\ 1 & \text{else} \end{cases} \tag{13}$$

$$i = 0, 1, \ldots l-1$$

$$0 = D_0 < D_1 < \cdots < D_{l-1}$$

In (12), $\boldsymbol{z}_i(t), i = 1, 2 \cdots l-1$ mean the delayed observations of the state $\boldsymbol{x}(t - D_i)$ at time t. Let $\boldsymbol{Z}(t)$ denotes all the delayed measurements at time t, then we have

$$\boldsymbol{Z}(t) = [\boldsymbol{z}_0^T(t) \cdots \boldsymbol{z}_{i-1}^T(t)], \tag{14}$$

$$\text{if } D_{i-1} \leq t < D_i,\ i = 1, 2 \ldots l-1$$

$$\boldsymbol{Z}(t) = [\boldsymbol{z}_0^T(t) \cdots \boldsymbol{z}_{l-1}^T(t)], \quad \text{if } t \geq D_{l-1} \tag{15}$$

The equivalent system is shown in Fig. 2. When the tele-operators act on the control input $\boldsymbol{u}(t)$, the operator generates the future control input $\boldsymbol{u}(t + d_u + d_0)$. In order to help the operator generate suitable control commands, the predictive display system is supposed to estimate the future state $\hat{\boldsymbol{x}}(k + d_u + d_0)$ using measurements $\boldsymbol{z}_i(t), i =, 1, \ldots l-1$ with delay D_i and then display it to the operator. The equivalent problem for Problem. 1 can be stated as

Problem 2 *Given the measurements* $\left\{\{\boldsymbol{Z}(i)\}_{i=0}^t\right\}$ *and control input history* $\left\{\{\boldsymbol{u}(i)\}_{i=0}^{t+d_u+d_0-1}\right\}$, *find the linear least mean square error estimat* $\hat{\boldsymbol{x}}(t + d_u + d_0|t)$ *for* $\boldsymbol{x}(t + d_u + d_0)$.

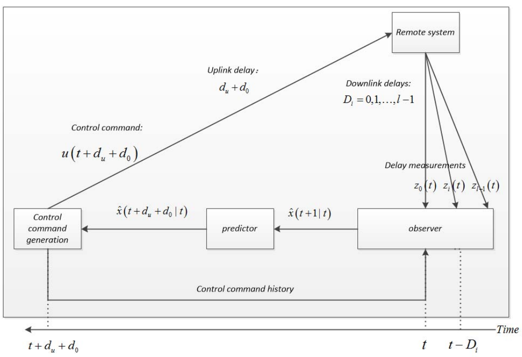

Fig. 2: Schematic of the equivalent system

Problem.2 then can be divided into a standard filter problem with multiple delayed measurements(Problem. 3) and a prediction problem(Problem. 4), which can be tackled by an observer-predictor structure. To be specific, the observer focus on solving Problem.3, while the predictor deals with Problem.4.

Problem 3 *Given the measurements* $\left\{\{\boldsymbol{Z}(i)\}_{i=0}^t\right\}$ *and control input history* $\left\{\{\boldsymbol{u}(i)\}_{i=0}^t\right\}$, *find the linear least mean square error estimat* $\hat{\boldsymbol{x}}(t+1|t)$ *for* $\boldsymbol{x}(t+1)$.

Problem 4 *Given the estimator* $\hat{\boldsymbol{x}}(t+1|t)$ *and control input* $\left\{\{\boldsymbol{u}(i)\}_{i=t+1}^{t+d_u+d_0}\right\}$, *find the linear least mean square error estimat* $\hat{\boldsymbol{x}}(t + d_u + d_0|t)$ *for* $\boldsymbol{x}(t + d_u + d_0)$.

3.2 Observer

The re-organized innovation analysis approach is employed in the observer and is extended to tackle multi-time-delay multi-rate measurements and missing data with the help of indication functions $F_i(k), i =, 1, \ldots l-1$. At time $t \geq D_{l-1}$, the measurement sequence $\left\{\{\boldsymbol{Z}(i)\}_{i=0}^t\right\}$ is expressed by
$\{\boldsymbol{Z}(0), \boldsymbol{Z}(1) \cdots \boldsymbol{Z}(t)\} =$

$$\left\{ \begin{array}{cccccc} \boldsymbol{z}_0(0) & \cdots & \boldsymbol{z}_0(D_i - 1) & \boldsymbol{z}_0(D_i) & \cdots & \boldsymbol{z}_0(t) \\ * & \cdots & \boldsymbol{z}_1(D_i - 1) & \boldsymbol{z}_1(D_i) & \cdots & \boldsymbol{z}_1(t) \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \cdots & \boldsymbol{z}_{i-1}(D_i - 1) & \boldsymbol{z}_{i-1}(D_i) & \cdots & \vdots \\ \vdots & \cdots & * & \boldsymbol{z}_{i-1}(D_i) & \cdots & \vdots \\ \vdots & \cdots & \vdots & * & \cdots & \vdots \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ * & * & * & * & \cdots & \boldsymbol{z}_{l-1}(t) \end{array} \right\} \tag{16}$$

It can be found that all the valid elements in (16) are located in the upper-right corner. In the re-organized innovation analysis approach, the elements in (16) are re-arranged by left-aligning the valid elements.

$$\left\{ \begin{array}{cccccc} \boldsymbol{z}_0(0) & \cdots & \boldsymbol{z}_0(t-D_i) & \boldsymbol{z}_0(t-D_i+1) & \cdots & \boldsymbol{z}_0(t) \\ \vdots & \cdots & \vdots & \vdots & \cdots & * \\ \vdots & \cdots & \vdots & \boldsymbol{z}_{i-1}(t+D_{i-1}-D_i+1) & \cdots & \vdots \\ \vdots & \cdots & \boldsymbol{z}_i(t) & * & \cdots & \vdots \\ \vdots & \cdots & * & \vdots & \cdots & \vdots \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ \boldsymbol{z}_{l-1}(D_{l-1}) & \cdots & * & * & \cdots & * \end{array} \right\}$$

$$\equiv \{\boldsymbol{M}_{l-1}(0) \cdots \boldsymbol{M}_i(t - D_i)\ \boldsymbol{M}_{i-1}(t - D_i - 1) \cdots \boldsymbol{M}_0(t)\} \tag{17}$$

where for $i = l-1; \quad 0 \leq \tau \leq t - D_i$

$$\boldsymbol{M}_i(\tau) = [\boldsymbol{z}_0^T(\tau), \boldsymbol{z}_0^T(\tau + D_1) \cdots \boldsymbol{z}_{l-1}^T(\tau + D_{l-1})]^T \tag{18}$$

for $i = 0, 1 \cdots l-2; \quad t - D_{i+1} < \tau \leq t - D_i$

$$\boldsymbol{M}_i(\tau) = [\boldsymbol{z}_0^T(\tau), \boldsymbol{z}_0^T(\tau + D_1) \cdots \boldsymbol{z}_i^T(\tau + D_i)]^T \tag{19}$$

It is clear that

$$\boldsymbol{M}_i(\tau) = \mathbb{H}_i(\tau)\boldsymbol{x}(\tau) + \mathbb{V}_i(\tau) \tag{20}$$

$i = 0, 1 \cdots l-1; \quad t - D_{i+1} + 1 < \tau \leq t - D_i$
where

$$\mathbb{H}_i(\tau) \triangleq [\boldsymbol{H}_0^{*T}(\tau)\ \boldsymbol{H}_1^{*T}(\tau) \cdots \boldsymbol{H}_i^{*T}(\tau)] \tag{21}$$

$$\mathbb{V}_i(\tau) \triangleq [\boldsymbol{v}_0^{*T}(\tau)\ \boldsymbol{v}_1^{*T}(\tau) \cdots \boldsymbol{v}_i^{*T}(\tau)] \tag{22}$$

According to (22), it can be derived that $\mathbb{V}_i(\tau)$ are white noises with covariance matrixes

$$\boldsymbol{Q}_{\mathbb{V}_i}(\tau) = \text{diag}\left\{\boldsymbol{Q}_{v_0}(\tau+d_0), \boldsymbol{Q}_{v_i}(\tau+d_i) \cdots \boldsymbol{Q}_{v_i}(\tau+d_i)\right\} \tag{23}$$

According to (12) and (18), it can been found that some elements in $\boldsymbol{M}_i(\tau)$ are null. So in our method, $\boldsymbol{M}_i(\tau)$, $\mathbb{H}_i(\tau)$ and $\mathbb{V}_i(\tau)$ are adjusted as follows:
if $F_i(\tau+D_i)) = 0$, then $\boldsymbol{z}_i^T(\tau+D_i)$ in $\boldsymbol{M}_i(\tau)$ is canceled, so do its corresponding entries in $\mathbb{H}_i(\tau)$ and $\mathbb{V}_i(\tau)$. We denote the $\boldsymbol{M}_i(\tau)$, $\mathbb{H}_i(\tau)$ and $\mathbb{V}_i(\tau)$ after adjusted as $\boldsymbol{M}_i^*(\tau)$, $\mathbb{H}_i^*(\tau)$ and $\mathbb{V}_i^*(\tau)$ respectively. Then we have

$$\boldsymbol{M}_i^*(\tau) = \mathbb{H}_i^*(\tau)\boldsymbol{x}(\tau) + \mathbb{V}_i^*(\tau) \tag{24}$$

We denote

$$\left\{\boldsymbol{M}_{l-1}^*(0) \cdots \boldsymbol{M}_i^*(t-D_i)\ \boldsymbol{M}_{i-1}^*(t-D_i-1) \cdots \boldsymbol{M}_0^*(t)\right\} \tag{25}$$
$$\equiv \mathbb{M}^*(t)$$

Note that $\mathbb{M}^*(t)$ and $\left\{\{\boldsymbol{Z}(i)\}_{i=0}^t\right\}$ carry the same information, so estimating $\boldsymbol{x}(t+1)$ based on $\left\{\{\boldsymbol{Z}(i)\}_{i=0}^t\right\}$ is equivalent to on $\mathbb{M}^*(t)$. Since the measurements $\boldsymbol{M}_i^*(\tau)$ in (24) are no longer with any delay, the standard Kalman filter can be applied to system (11),(24). The detailed deduction is omitted in this paper, which can be referred to in [19].

The following definition is made the same as that in [18].

Definition 1 *For* $i = l-1, 0 < \tau \le t - D_{l+1}$, $\boldsymbol{x}(\tau, i)$ *is the optimal estimation of* $\boldsymbol{x}(\tau)$ *given the observation* $\{\boldsymbol{M}_{l-1}(0) \cdots \boldsymbol{M}_{l-1}(\tau-1)\}$;

For $i = l-2 \cdots 0, t - D_{i+1} + 1 < \tau \le t - D_i$, $\boldsymbol{x}(\tau, i)$ *is the optimal estimation of* $\boldsymbol{x}(\tau)$ *given the observation* $\{\boldsymbol{M}_{l-1}(0) \cdots \boldsymbol{M}_{l-1}(t-D_{l-1});\ \boldsymbol{M}_{l-2}(t-D_{l-1}+1) \cdots \cdots \boldsymbol{M}_{l-2}(t-D_{l-2}); \cdots; \boldsymbol{M}_i(t-D_{i+1}+1) \cdots \boldsymbol{M}_i(\tau-1)\}$;

For $i = l-2 \cdots 0, \tau = t - D_{i+1} + 1$, $\boldsymbol{x}(\tau, i)$ *is the optimal estimation of* $\boldsymbol{x}(\tau)$ *given the observation* $\{\boldsymbol{M}_{l-1}(0) \cdots \boldsymbol{M}_{l-1}(t-D_{l-1}); \cdots \cdots; \boldsymbol{M}_{i+1}(t-D_{i+2}+1) \cdots \boldsymbol{M}_{i+1}(t-D_{i+1})\}$.

The linear least mean square error estimator $\hat{\boldsymbol{x}}(t+1|t)$ of $\boldsymbol{x}(t+1)$ can be obtained by calculate (26-30) recursively for $\tau = 0, 1, \cdots t$

$$n = \begin{cases} l-1 & \text{if } \tau+1 \le t - D_{l-1} \\ i & \text{if } t - D_{i+1} < \tau+1 \le t - D_i, \\ & \quad i = 0, 1, \cdots l-2 \\ 0 & \text{if } \tau = t \end{cases} \tag{26}$$

$$m = \begin{cases} l-1 & \text{if } \tau \le t - D_{l-1} \\ i & \text{if } t - D_{i+1} < \tau \le t - D_i, \\ & \quad i = 0, 1, \cdots l-2 \end{cases} \tag{27}$$

$$\begin{aligned} &\boldsymbol{x}(\tau+1, n) = \boldsymbol{B}(\tau)\boldsymbol{u}(\tau) + \\ &\boldsymbol{A}(\tau)\Big\{\boldsymbol{x}(\tau, m) + \boldsymbol{K}(\tau)\big[\boldsymbol{M}_m^*(\tau) - \mathbb{H}_m^*(\tau)\boldsymbol{x}(\tau, m)\big]\Big\} \end{aligned} \tag{28}$$

$$\boldsymbol{K}(\tau) = \boldsymbol{P}(\tau)\mathbb{H}_m^{*T}(\tau)\Big[\mathbb{H}_m^*(\tau)\boldsymbol{P}(\tau)\mathbb{H}_m^{*T}(\tau) + \boldsymbol{Q}_{V_m}(\tau)\Big]^{-1} \tag{29}$$

$$\begin{aligned} &\boldsymbol{P}(\tau) = \boldsymbol{A}(\tau-1)\boldsymbol{P}(\tau-1)\boldsymbol{A}^T(\tau-1) + \boldsymbol{Q}_W(\tau-1) - \\ &\boldsymbol{A}(\tau-1)\boldsymbol{K}(\tau-1)\mathbb{H}_m^*(\tau-1)\boldsymbol{P}(\tau-1)\boldsymbol{A}^T(\tau-1) \end{aligned} \tag{30}$$

then we get $\hat{\boldsymbol{x}}(t+1|t)$ by

$$\hat{\boldsymbol{x}}(t+1|t) = \boldsymbol{x}(t+1, 0) \tag{31}$$

If the initial conditions are unknown, they can be set to

$$\boldsymbol{x}(0, l-1) = \boldsymbol{0} \tag{32}$$

$$\boldsymbol{P}(0) = p\boldsymbol{I} \tag{33}$$

where p is a positive constant large enough.

Remark 1 *Since* $\boldsymbol{x}(t - D_{l-1} + 1, l-2)$ *calculated at time* t *can be re-used at time* $t+1$ *to compute* $\boldsymbol{x}(t - D_{l-1} + 2, l-2)$, *so the re-organized innovation approach can be implemented in a recursive manner.*

3.3 Predictor

In the predictor, the estimation $\hat{\boldsymbol{x}}(t+d_u+d_0|t)$ of $\boldsymbol{x}(t+d_u+d_0)$ is obtained by calculating (34) recursively for $\tau = t+1, t+2, \ldots, t+d_u+d_0-1$

$$\hat{\boldsymbol{x}}(\tau+1|t) = \boldsymbol{A}(\tau)\hat{\boldsymbol{x}}(\tau|t) + \boldsymbol{B}(\tau)\boldsymbol{u}(\tau) \tag{34}$$

3.4 Computation Procedure of our Predictive Display Scheme

The overall computation procedure of our predictive display scheme is shown in Fig. 3, which can be summarized as following steps.

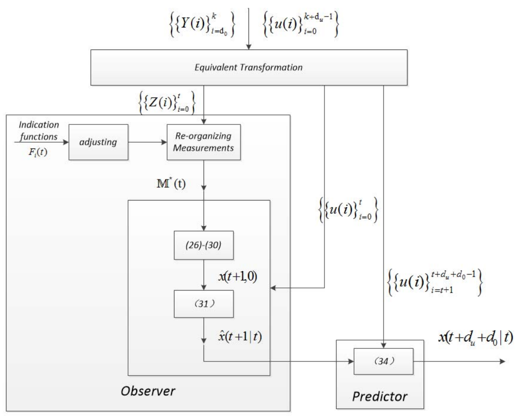

Fig. 3: Computation procedure of our predictive display scheme

Step 1 *Make the equivalent transformation presented in section 3.1 and then* $\left\{\{\boldsymbol{Z}(i)\}_{i=0}^t\right\}$ *and* $\left\{\{\boldsymbol{u}(i)\}_{i=0}^{t+d_u+d_0-1}\right\}$ *can be obtained.*

Step 2 *In the observer, re-organize* $\left\{\{\boldsymbol{Z}(i)\}_{i=0}^t\right\}$ *and adjust it with the help of indication functions* $F_i(t), i = , 1, \ldots l-1$, *after which* $\mathbb{M}^*(t)$ *can be obtained. And then get* $\hat{\boldsymbol{x}}(t+1|t)$ *by calculating (26-30) recursively.*

Step 3 *By calculating (34) recursively,* $\hat{\boldsymbol{x}}(t+d_u+d_0|t)$ *of* $\boldsymbol{x}(t+d_u+d_0)$ *can be obtained.*

Remark 2 *Since the re-organized innovation approach can be implemented in a recursive manner, our predictive display algorithm can also be implemented in a recursive manner by re-using* $\boldsymbol{x}(t-D_{l-1}+1, l-2)$ *calculated at previous times.*

4 Simulation Example

4.1 Simulation Settings

An simulation example is given below to show the effectiveness of our predictive display scheme. Consider an application that a vehicle is being tele-operated to catch an object with uniform straight line motion. The remote system can be described by (1) , where

$$\boldsymbol{A}(k) = \begin{bmatrix} 1 & d & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \boldsymbol{B}(k) = \begin{bmatrix} 0 \\ d \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{x}(k) = \begin{bmatrix} p_1(k) \\ \dot{p_1}(k) \\ p_2(k) \\ \dot{p_2}(k) \end{bmatrix} \qquad \boldsymbol{w}(k) = \begin{bmatrix} 0 \\ w_1(k) \\ 0 \\ w_2(k) \end{bmatrix}$$

$d = 0.05s$ is the system sampling interval, $p_1(k), \dot{p_1}(k)$ are the position and velocity of the tele-operated vehicle, $p_2(k), \dot{p_2}(k)$ are the position and velocity of the moving object $w_1(k), w_2(k)$ are uncorrelated white noises with zero means and known covariance matrices $E[w_1(k)w_1^T(j)] = Q_{w_1}(k)\delta_{ij} = 0.01^2\delta_{ij}$ and $E[w_2(k)w_2^T(j)] = Q_{w_2}(k)\delta_{ij} = 0.01^2\delta_{ij}$, $\boldsymbol{u}(k)$ is the control input generated by the human operator. The initial codition is set to $\boldsymbol{x}(0) = [0 \quad 0 \quad 5 \quad 0.5]^T$.

There are two sources of measurements. One is from a surveillance camera, giving the position of the vehicle and the moving object, whose measurement sampling interval is $2d$ and it won't be received by the local site until $10d$ after it is generated. The other is from a range finder mounted on the vehicle, giving the distance between the vehicle and the object if the distance is less than $3m$, whose measurement sampling interval is d and it won't be received by the local site until $12d$ after it is generated. The measurement process can be model as

$$\boldsymbol{y}_0(k) = \begin{cases} \boldsymbol{H}_0(k)\boldsymbol{x}(k-d_0) + \boldsymbol{v}_0(k) & \text{if } F_0(k) = 1 \\ \text{Null} & \text{if } F_0(k) = 0 \end{cases} \tag{35}$$

$$F_0(k) = \begin{cases} 1 & \text{if mod}(k-d_0, 2) = 0 \\ 0 & \text{else} \end{cases} \tag{36}$$

$$k = 10, 11, 12, \ldots$$

$$\boldsymbol{y}_1(k) = \begin{cases} \boldsymbol{H}_1(k)\boldsymbol{x}(k-d_1) + \boldsymbol{v}_1(k) & \text{if } F_1(k) = 1 \\ \text{Null} & \text{if } F_1(k) = 0 \end{cases} \tag{37}$$

$$F_1(k) = \begin{cases} 1 & \text{if } |p_3(k-d_1) - p_1(k-d_1)| \le 3 \\ 0 & \text{else} \end{cases} \tag{38}$$

$$k = 12, 13, 14, \ldots$$

where $\boldsymbol{H}_0(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $\boldsymbol{H}_1(k) = \begin{bmatrix} -1 & 0 & 1 & 0 \end{bmatrix}$, $d_0 = 10, d_1 = 12$, $\boldsymbol{v}_0(k)$ and $\boldsymbol{v}_1(k)$ are uncorrelated white noise with zero means and known covariance matrices $E[\boldsymbol{v_0}(k)\boldsymbol{v}_0^T(j)] = 0.3^2\delta_{ij}\boldsymbol{I}$ and $E[\boldsymbol{v_1}(k)\boldsymbol{v_1}^T(j)] = 0.2^2\delta_{ij}\boldsymbol{I}$ respectively. $\boldsymbol{I}$ is unit matrix with suitable dimensions.

The uplink delay is $10d$. The specific predictive display problem can be stated as: Given the measurements $\left\{\{\boldsymbol{y}_0(i)\}_{i=10}^{k}\right\}, \left\{\{\boldsymbol{y}_1(i)\}_{i=12}^{k}\right\}$ and control input history $\left\{\{\boldsymbol{u}(i)\}_{i=0}^{k+9}\right\}$, find a linear least mean square error estimation $\hat{\boldsymbol{x}}(k+10)$ of $\boldsymbol{x}(k+10)$.

4.2 Simulation Results and Analysis

Measurement $\boldsymbol{y}_0(k)$ and $\boldsymbol{y}_1(k)$ are shown in Fig. 4 and Fig. 5 respectively. Both measurements are delayed and contaminated by noises. The sampling rate of $\boldsymbol{y}_1(k)$ is twice the rate of $\boldsymbol{y}_0(k)$. Due to the limited working range of the ranger finder, the data of $\boldsymbol{y}_1(k)$ are available only after $k = 88$. In other words, the system (1)(35-38) have multi-time-delay multi-rate measurements and missing data, which can not be dealt with by existing predictive display schemes.

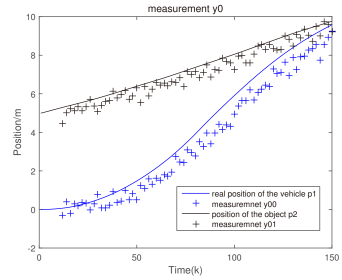

Fig. 4: Measurement $\boldsymbol{y}_0$

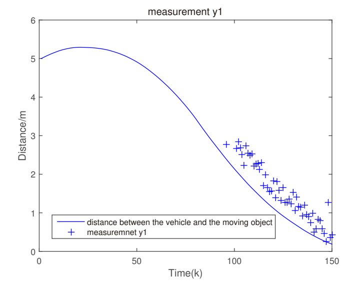

Fig. 5: Measurement $\boldsymbol{y}_1$

The initial condition of our algorithm is set to $\hat{\boldsymbol{x}}(0) = [0 \quad 0 \quad 0 \quad 0]^T$, which is different from its true value $\boldsymbol{x}(0)$. The output of our predictive display scheme is the prediction of the future state of the remote workspace $\boldsymbol{x}(k+d_u)$. The results of this example is shown in Fig. 6. At time $k < d_0 + d_u = 10 + 10 = 20$, $\hat{\boldsymbol{x}}(k) = \hat{\boldsymbol{x}}(0)$; after $k = 20$ (when the dealyed measurements become avaliable), the output of our algorithm $\hat{\boldsymbol{x}}(k)$ converges to $\boldsymbol{x}(k)$ quickly.

Fig. 7 shows the variance of the prediction of $p_1(k)$. It can be found that after $k = 10$(when the delayed measurements become available), the variance of the prediction drops quickly. It means that our predictive display scheme can predict the state of the remote workspace effectively.

All in all, the results of this simulation show that our predictive display scheme is capable of tackling multi-time-delay multi-rate measurements and missing data and predicting the future state of the remote workspace at a fast convergent rate.

5 Conclusion

Predictive display for teleoperation systems with multi-time-delay multi-rate measurements as well as missing data

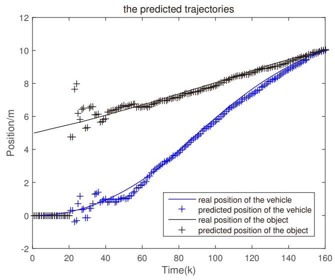

Fig. 6: Output of our predictive display scheme

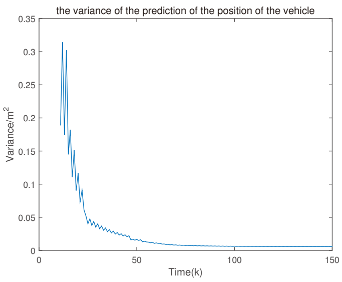

Fig. 7: Variance of the prediction of the position of the vehicle

is addressed in this paper and a unified scheme to deal with such a problem is proposed. An equivalent transformation is performed to change the original problem into a standard filter problem with multiple time-delay measurements and a prediction problem, which are tackled by an observer- predictor structure. In the observer, the re-organized innovation approach is extended to deal with multi-time-delay multi-rate measurements as well as missing data. The scheme is practical and appropriate for real teleoperation applications.

Many interesting works lies ahead, which include taking into account time-variant delays and the modeling errors. It is also interesting to incorporate other filters, such as particle filter into this scheme to deal with estimation problems in nonlinear systems.

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