Online Operation of Renewable Energy and Battery Integrated Electric Vehicle Parking Lots with an Improved Priority Rule

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***Abstract*—Deploying renewable energy generation and battery storage can reduce the operation cost of electric vehicle parking lots. Given uncertainties arising from vehicle arrival times, charging demands and renewable output sequence, operating a parking lot without exact forecasts of uncertainty is a challenging problem. This paper proposes an improved priority rule based on vehicle charging deadlines and remaining energy needs. If the charging power can be continuously adjusted, the proposed priority rule outperforms various well-known charging policies, such as the earliest-deadline-first policy and the modified least-laxity-longer-processing-time-first policy. It is proven that there exists an optimal charging policy that follows this priority rule. During the operation process, the total vehicle charging demand and battery dispatch are firstly determined from an experience-driven K-nearest neighbor (KNN) method, and then the gross charging power is allocated to each charger according to the proposed priority rule. We further suggest a simulation procedure to evaluate the remaining cost given the currently observed state and some promising candidate actions. This could improve the prediction-free KNN policy. Experimental results on real-world data demonstrate that when integrated with KNN and online simulation, the proposed method outperforms existing charging strategies, and exhibits a cost reduction by $13\% \sim 19\%$.**

***Index Terms*—electric vehicle, energy storage, parking lot, priority rule, renewable generation.**

I. Introduction

THE widespread deployment of charging infrastructure is crucial for advancing transportation electrification. Parking lots with slow charging piles are most widely seen charging facilities in residential, business and office areas, where electric vehicles (EVs) will stay for a certain period, so the charging schedule has some degree of freedom[1]. To alleviate grid impacts and reduce operating costs, renewable energy and energy storage device can be built in parking lots [1], and economic scheduling of parking lots has been extensively studied in the existing research.

The biggest challenge in the economic scheduling of parking lots or charging stations is the uncertainty arising from EV arrival rates, charging demands and renewable output sequence. To address this issue, various stochastic models for EV arrival rates and charging demands have been established, including stochastic agent-based modeling [2], probability distribution-based modeling [3], [4], random forest [5], multivariate copula [6], and Gaussian mixture model [7]. In the context of renewable energy forecasting, numerous studies have focused on predicting photovoltaic power [8] and wind power [9], [10].

There has been extensive research on the optimal operation of EV parking lots and charging stations. Techniques such as stochastic programming [11] and robust optimization [12] have been used to solve day-ahead and two-stage optimization problems. To handle the non-anticipative issue of uncertainty, approaches like stochastic dual dynamic programming [13] and model predictive control [14] are frequently employed. However, the performance of these methods depends heavily on the quality of forecasts for the uncertain information. However, unlike meteorological condition which is governed by atmospheric physics, vehicle arrival rates and energy demands in parking lots are more difficult to predict.

With the rise of deep learning and reinforcement learning, numerous data-driven methodologies within this field have been applied to the EV charging scheduling problem, such as deep Q-network [15], [16], deep deterministic policy gradient [17], [18], proximal policy optimization [19], and soft actor-critic [20]. Since EV charging problem can be seen as a multi-agent decision making process, the multi-agent reinforcement learning framework has also been adopted in [21], [22]. The EV fleet is assumed to be homogeneous and the decentralized learning framework is used in [21] to overcome the curse of dimensionality. The multi-agent proximal policy optimization algorithm is developed in [22] to solve the pricing game of fast charging stations. Nevertheless, the need for large amount of data and the lack of performance guarantee present challenges to applying these methods in real-world systems.

Another major area of research is the priority rule-based scheduling strategies, where the charging order of EVs is determined by specific criteria, and the charging demand is met according to the priority rule set by this order. For instance, Subramanian et al. [23] examine several resource allocation strategies for deferable loads, such as earliest-deadline-first (EDF) and least-laxity-first. A well-known result in this field is presented in [24], which demonstrates the existence of an optimal charging strategy that adheres to the least-laxity-longer-processing-time-first principle (LLLP). As LLLP only establishes a partial charging order among EVs, a modified LLLP (mLLLP) is introduced, which extends the partial order to a total order among EVs. Wu et al. [25] indicate that under

[1]Charging stations with slow charging piles are called parking lots in this paper, to distinguish them with fast charging stations where vehicles wish to finish charging as soon as possible; therefore there is no opportunity to optimize the charging power.

some stringent conditions, an optimal charging strategy can be achieved by adhering to mLLLP. However, in [24], [25], it is assumed that the charging power of each EV is either maximum or zero. In practice, even if the charger output can only take a binary value, the charging time in each hour can still be adjusted to obtain a continuous demand. The smoothed least-laxity-first algorithm is proposed in [26] to improve the online feasibility of charging strategies, but the performance is not guaranteed to be optimal.

In addition to the priority rule-based charging strategies discussed previously, some research has focused on threshold-based charging strategies. In cases where only a single vehicle is being charged and electricity can be either bought from or sold to the grid, Jeon et al. [27] show that the optimal charging strategy follows a piece-wise function based on a few threshold parameters. For multiple EVs with identical arrival and departure times, Jin et al. [28] demonstrate that the optimal charging strategy can be achieved by charging and discharging EVs towards the same threshold. However, the assumption on identical arrival and departure times is restrictive. Hao et al. [29] establish the existence of an optimal charging strategy where EVs with the same deadline are charged towards the same threshold at any given time. While the conditions for this property are more general than those in previous studies, the relationship between thresholds for EVs with different deadlines is not explored. Due to the significant variation in deadlines among individual EVs in actual scheduling processes, this property remains challenging to be applied in broader instances.

In summary, exploring the structure of the optimal operation strategy that is generally applicable to renewable energy and battery integrate EV parking lots remains an open problem. In this article, we propose an improved priority rule that does not restrict the charging action to binary values, and prove its appealing property. An operation policy for EV parking lots that utilizes the KNN algorithm [30] is proposed to determine the total charging power of EVs without the need for large amount of historical data; then the proposed priority rule is applied to allocate the total charging power to individual vehicles. We further adopt the rollout algorithm [31] to improve the proposed charging algorithm with online simulation, and finally obtain a real-time dispatch method for renewable energy and battery integrated EV parking lots. The main contributions are summarized as follows.

(1) An improved priority rule for the charging order of EVs. It relaxes the default setting in LLLP and mLLLP which assumes the charging power is either maximum or zero. We prove that there exists an optimal charging policy that complies with the proposed priority rule. Since the feasible charging power is less restrictive, the optimality can be improved.

(2) A KNN algorithm to determine the gross EV charging power and dispatched power of battery from experiences. It does not require a large amount of historical data. An online simulation procedure is further developed to evaluate remaining costs corresponding to the current state and candidate actions. Through numerical experiments on real-world data, we show that the proposed method outperforms mLLLP and deadline-based policies, and the KNN-based strategy can be further enhanced by deploying the best candidate action.

The remainder of this article is organized as follows. In Section II, we introduce the mathematical model of parking lot dispatch. The proposed priority rule is defined and illustrated in Section III. The KNN based operation strategy and online simulation based action selection are given in Section IV. Numerical experiments are conducted in Section V. Finally, Section VI concludes this article.

II. Problem Formulation

A. System Model

We consider a parking lot equipped with N chargers, and the charging decisions are determined on each timeslot $t \in \mathcal{T} = \{1, 2, \cdots, T\}$. In timeslot t, a total number of $|\mathcal{N}_t|$ EVs come to the parking lot, and the set of arriving EVs is denoted as $\mathcal{N}_t$. For an EV $i \in \mathcal{N}_t$, its charging deadline (i.e., departing time) is denoted as d_i, and its total charging demand is denoted as E_i. Each EV will be assigned to one charger, and we assume that maximum charging rates for all chargers are the same and denoted as p_c^{max}. Then the **normalized** total charging demand of EV i is defined as

$$e_{i,a_i} = \frac{E_i}{p_c^{max} \Delta t}, \tag{1}$$

where a_i is the arriving time of vehicle i and Δt is the length of each timeslot. In another timeslot t', we define $e_{i,t'}$ as the normalized remaining charging demand for EV i in this timeslot. e_{i,a_i} can be viewed as the total number of timeslots needed to serve vehicle i at the maximum charging rates, but different from [24], [25], we don't assume e_{i,a_i} to be an integer.

In timeslot t, the set of EVs that have arrived at the parking lot but not departed yet (including the newly arrived EVs in $\mathcal{N}_t$) is denoted as I_t. For each vehicle $i \in \mathcal{I}_t$, it can be charged with power $p_{i,t} \in [0, p_c^{max}]$, and the **normalized**[2] charging rate is defined as

$$v_{i,t} = \frac{p_{i,t}}{p_c^{max}}, \ \forall t. \tag{2}$$

Then the state transition of EVs can be formulated as

$$e_{i,t+1} = e_{i,t} - v_{i,t}, \ \forall t, \ \forall i \in \mathcal{I}_t, \tag{3}$$

with the following constraints:

$$0 \leq v_{i,t} \leq 1, \ \forall t, \ \forall i \in \mathcal{I}_t, \tag{4}$$

$$e_{i,d_i} = 0, \ \forall i. \tag{5}$$

For the energy storage device, its storage level in timeslot t is u_t, and the charging power in timeslot t is p_t (positive for charging and negative for discharging). Then the state transition of storage level can be formulated as follows:

$$u_{t+1} = u_t + \mathbf{1}_{p_t \geq 0} \eta_c p_t \Delta_t + \mathbf{1}_{p_t < 0} \frac{p_t \Delta_t}{\eta_d}, \ \forall t, \tag{6}$$

[2]Throughout the subsequent text, unless explicitly stated otherwise, the terms "charging power" and "remaining charging demand" are used to denote "normalized charging power" and "normalized remaining charging demand," respectively.

where $\mathbf{1}_{p_t\geq 0}/\mathbf{1}_{p_t\leq 0}$ are indicator functions, and η_c/η_d denote the charging/discharging efficiency. Charging power and storage level must be bounded:

$$0 \leq u_t \leq u^{\max},\ \forall t, \tag{7}$$

$$p_s^{\min} \leq p_t \leq p_s^{\max},\ \forall t. \tag{8}$$

Let $g_t \in \mathbb{R}_+$ be the renewable power generated in timeslot t, and then the net power demand l_t from the power grid is:

$$l_t = p_c^{max}\sum_{i\in\mathcal{I}_t} v_{i,t} + p_t - g_t,\ \forall t. \tag{9}$$

The electricity bill in timeslot t is

$$c_t = \lambda_t[l_t]_+,\ \forall t, \tag{10}$$

where $[x]_+ = \max(x,0)$, and λ_t is the electricity price.

If we have perfect knowledge of the stochastic information (i.e., the g_t, λ_t and $\mathcal{N}_t$), then the deterministic optimization problem for parking lot dispatch would be

$$\min_{\boldsymbol{v}_t,p_t} \quad \sum_{t=1}^{T} c_t - \beta u_{T+1} \qquad \text{s.t.} \quad (1)-(10) \tag{11}$$

where $\beta > 0$ is constant such as the daily average electricity price, and βu_{T+1} represents the salvage value of remaining energy in the storage unit; the vector $\boldsymbol{v}_t$ represents the charging rates $v_{i,t}$ for each EV $i \in \mathcal{I}_t$.

However, in practice, the stochastic information in the future is unknown. Therefore, we formulate the problem as a Markov decision process (MDP) [32] as follows:

- State: Let $x_{i,t} = (e_{i,t}, d_i)$ be the state of charger[3] i, and $\boldsymbol{x}_t$ denote the state for all chargers. Then $\boldsymbol{s_t} = (\boldsymbol{x}_t, u_t)$ denotes the endogenous state. The exogenous state containing the stochastic renewable energy generation and the electricity price is denoted by $\boldsymbol{w_t} = (g_t, \lambda_t)$.
- Action: In each timeslot, the action $\boldsymbol{a}_t$ contains the charging rate of all chargers $\boldsymbol{v}_t$ as well as the energy storage charging decision p_t, i.e., $\boldsymbol{a}_t = (\boldsymbol{v}_t, p_t)$.
- Cost: The MDP cost function is

$$C_t(\boldsymbol{s}_t, \boldsymbol{w}_t, \boldsymbol{a}_t) = \begin{cases} c_t & t = 1:T-1 \\ c_T - \beta u_{T+1} & t = T \end{cases}$$

The endogenous state transition function is given by $\boldsymbol{s}_{t+1} = f_t(\boldsymbol{s}_t, \boldsymbol{a}_t)$ through (3)(6), and the feasible action space $\mathcal{A}_t$ is restricted by constraints (4)(5)(7)(8). The exogenous states evolve following an unknown stochastic distribution.

Denote $\pi = (\pi_1, \cdots, \pi_T)$ as the charging policy, which maps the endogenous and exogenous state to the action, i.e., $\boldsymbol{a}_t = \pi_t(\boldsymbol{s}_t, \boldsymbol{w}_t)$. Then we aim to find an optimal policy that minimize the expected accumulated cost:

$$\min_{\pi} \mathbb{E}_\pi \sum_{\tau=1}^{T}[C_\tau(\boldsymbol{s}_\tau, \boldsymbol{w}_\tau, \pi_\tau(\boldsymbol{s}_\tau, \boldsymbol{w}_\tau))]. \tag{12}$$

[3]Unless specifically indicated, the term "charger" mentioned hereinafter refers to a charger that is currently occupied by an EV. And when we refer to the state $(e_{i,t}, d_i)$ of this charger, we mean the (normalized) remaining charging demand and charging deadline of the EV it is currently serving.

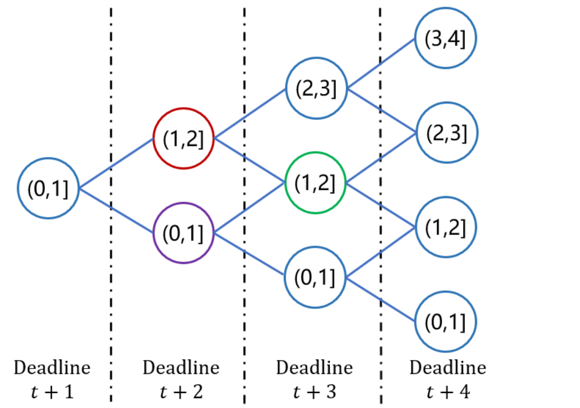

Fig. 1. An illustration of the Definition 1. Each circle stands for a set of chargers with the normalized remaining charging need specified in the interval at the center of the circle, and the charging deadline specified in the bottom. The charging order is represented by blue lines.

We will solve this problem by first introduce a priority rule that the optimal charging policy should adhere to, and then propose an effective priority rule-compliant policy.

III. The Priority Rule for Vehicle Charging

The priority rule is based on a charging order, determining the charging priority of different chargers. In this section, we first define the charging order. Since we do not restrict the charging power to take a binary value, a charging policy that complies with the priority rule exhibits more complex behavior, which is demonstrated by an illustrative example. Finally, the definition of priority rule-compliant policy and its property are given.

Definition 1. *For two chargers i and j with state (e_i, d_i) and (e_j, d_j), we define the charging order $\prec$ as follows:*

1) *if $d_i = d_j$, then $(e_i, d_i) \prec (e_j, d_j) \iff e_i < e_j$,*
2) *if $d_i < d_j$, then $(e_i, d_i) \prec (e_j, d_j) \iff \lceil e_i \rceil \leq \lceil e_j \rceil - (d_j - d_i)$,*
3) *if $d_i > d_j$, then $(e_i, d_i) \prec (e_j, d_j) \iff \lceil e_i \rceil \leq \lceil e_j \rceil$,*

and $(e_i, d_i) \nprec (e_j, d_j)$ means $(e_i, d_i) \prec (e_j, d_j)$ does not hold.

An illustration of the Definition 1 is shown in Fig. 1. In the figure, each circle stands for a set of chargers with the normalized remaining charging need specified in the interval at the center of the circle, and the charging deadline specified in the bottom. For example, if the state of charger i is $e_i = 1.4, d_i = t+2$, then this charger belongs to the red circle. For chargers in the same circle, the larger its e_i, the higher priority it has. For two chargers belonging to different circles, if these two circles are connected by a blue line (e.g., the red circle and the green circle), then the charger belonging to the circle with higher position (e.g., the red circle) has higher charging priority than the other one (e.g., the green circle). Also, the defined charging order is transitive, so chargers belonging to red circle have higher charging priority than those in the purple circle. Indeed, the defined charging order is a partial order.

In Definition 1, we use expressions like (e_i, d_i) instead of $(e_{i,t}, d_i)$ to indicate that e_i may represent the virtual remaining

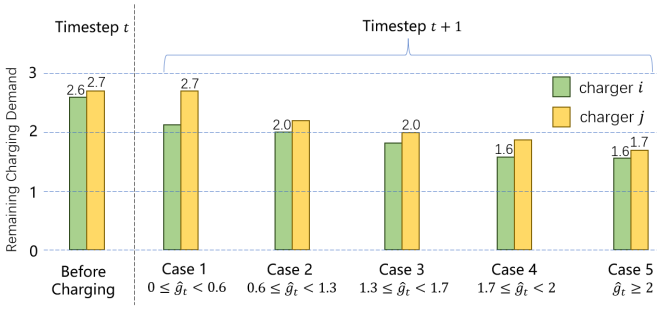

Fig. 2. Demonstration of the priority rule-compliant policy with two EVs.

charging demand of **virtual state** that corresponds to intermediate power allocation result instead of any actual system state. For example, if $(e_{i,t}, d_i) = (3, 5)$ and $(e_{j,t}, d_j) = (3.3, 6)$, then $(e_{i,t}, d_i) \prec (e_{j,t}, d_j)$, while $(e_{i,t}, d_i) \not\prec (e_{j,t} - 0.5, d_j)$, where $(e_{j,t} - 0.5, d_j)$ is a virtual state. The role of virtual state will become clearer in the following discussion.

Intuitively, if $(e_i, d_i) \prec (e_j, d_j)$, it means that the charging priority of charger j is higher than that of charger i. However, when the charging rate can vary continuously in the $[0, 1]$ interval, the charging behavior determined by this priority will exhibit a complex pattern. Therefore, before giving the formal description in Definition 2, we present an example in Fig. 2 to facilitate understanding. In this example, we assume there are only two chargers i and j that are currently occupied, with $d_j = d_i + 1$. In this setting, we have

$$(e_i, d_i) \prec (e_j, d_j) \iff \lceil e_i \rceil \le \lceil e_j \rceil - 1, \tag{13}$$

$$(e_j, d_j) \prec (e_i, d_i) \iff \lceil e_i \rceil \ge \lceil e_j \rceil. \tag{14}$$

Additionally, we assume current timeslot t is far earlier than d_i and d_j, so the parking lot is inclined to consume renewable energy instead of purchasing electricity from the power grid. The current charger state is fixed at $e_{i,t} = 2.6, e_{j,t} = 2.7$, while the normalized renewable generation $\hat{g}_t = g_t / p_c^{\max}$ varies. According to the value of $\hat{g}_t$, the charging decision complying with the priority rule is discussed in five cases:

Case 1: $0 \le \hat{g}_t < 0.6$. In this case, the origin state satisfies $(e_{j,t}, d_j) \prec (e_{i,t}, d_i)$ as $\lceil e_{i,t} \rceil = \lceil e_{j,t} \rceil$, which suggests that the charger i has higher charging priority than j before charging. In fact, even if all power of $\hat{g}_t$ is assigned to charger i, we still have $(e_{j,t}, d_j) \prec (e_{i,t} - \hat{g}_t, d_i)$ as $\lceil e_{i,t} - \hat{g}_t \rceil = \lceil e_{j,t} \rceil$. Consequently, the priority rule-compliant action should be $v_{i,t} = \hat{g}_t$ and $v_{j,t} = 0$.

Case 2: $0.6 \le \hat{g}_t < 1.3$. In this case, $(e_{i,t} - 0.6, d_i) \prec (e_{j,t}, d_j)$, but $\forall \epsilon > 0$, $(e_{j,t}, d_j) \prec (e_{i,t} - 0.6 + \epsilon, d_i)$. Hence, before (virtually) allocating 0.6 normalized power to charger i, the charging priority of charger i is higher than j. But after (virtually) allocating 0.6 normalized power to charger i, the charging priority of j becomes higher than i. In fact, even if all power of $\hat{g}_t - 0.6$ is assigned to charger j, we still have $(e_{i,t} - 0.6, d_i) \prec (e_{j,t} - (\hat{g}_t - 0.6), d_j)$ as $\lceil e_{i,t} - 0.6 \rceil = \lceil e_{j,t} - (\hat{g}_t - 0.6) \rceil - 1$. Hence, the priority rule-compliant action should be $v_{i,t} = 0.6$ and $v_{j,t} = \hat{g}_t - 0.6$.

Case 3: $1.3 \le \hat{g}_t < 1.7$. In this case, $(e_{j,t} - 0.7, d_j) \prec (e_{i,t} - 0.6, d_i)$, but $\forall \epsilon > 0$, $(e_{i,t} - 0.6, d_j) \prec (e_{j,t} - 0.7 + \epsilon, d_j)$. Hence, we should first (virtually) allocate 0.6 normalized power to charger i, and then (virtually) allocate 0.7 normalized power to charger j, and finally (virtually) allocate the remaining power to charger i. In fact, even if all power of $\hat{g}_t - 0.7$ is assigned to charger i, we still have $(e_{j,t} - 0.7, d_j) \prec (e_{i,t} - (\hat{g}_t - 0.7), d_i)$. So the priority rule-compliant action should be $v_{i,t} = \hat{g}_t - 0.7$ and $v_{j,t} = 0.7$.

Case 4: $1.7 \le \hat{g}_t < 2$. In this case, when $\hat{g}_t = 1.7$, as discussed in Case 3, the decision is $v_{i,t} = 1$ and $v_{j,t} = 0.7$. At this point, we still have $(e_{j,t} - 0.7, d_j) \prec (e_{i,t} - 1, d_i)$. However, since charger i has reached the maximum charging constraint ($v_{i,t} \le 1$), we have no choice but to allocate the remaining power to charger j. As a result, the priority rule-compliant action should be $v_{i,t} = 1$ and $v_{j,t} = \hat{g}_t - 1$.

Case 5: $\hat{g}_t \ge 2$. In this case, the renewable energy generation is sufficient to charging both charger i and charger j at maximum charging power. As a consequence, the priority rule-compliant action should be $v_{i,t} = 1$ and $v_{j,t} = 1$.

Based on the intuition in this example, we are ready to give the formal definition of priority rule-compliant policy.

Definition 2. *A charging policy π is said to be priority rule-compliant, if for any two chargers i and j, in any state $(\boldsymbol{s}_t, \boldsymbol{w}_t)$ where $t < \min(d_i, d_j)$, the following conditions hold. Let $v_{i,t}$ and $v_{j,t}$ denote the charging decisions for i and j under policy π. If there exists $\epsilon > 0$ such that*

$$(e_{i,t+1} + \epsilon, d_i) \prec (e_{j,t+1}, d_j), \tag{15}$$

where $e_{i,t+1} = e_{i,t} - v_{i,t}$ and $e_{j,t+1} = e_{j,t} - v_{j,t}$, then at least one of the following conditions must be satisfied:

1) $v_{i,t} = 0$,
2) $v_{j,t} = \min(1, e_{j,t})$.

The intuition behind Definition 2 is that after two chargers i and j get charged in one timeslot, if j has a higher priority than i, then either i does not get charged, or j has been charged at the maximum rate. The following theorem shows that to find the optimal policy, we only need to consider priority rule-compliant policies.

Theorem 1. *There exists an optimal charging policy π^* which is priority rule-compliant.*

The proof can be found in the appendix.

IV. Priority Rule-based KNN Policy

Theorem 1 establishes the fact that we can confine the strategies to the priority rule-compliant policy, which greatly reveals the structure of the optimal policy and simplifies the problem. In this section, we will establish a charging policy that is not only priority rule-compliant, but also effectively and reliably utilizes historical data, even when the dataset is relatively small in scale.

Our proposed charging policy is divided into two stages. In the first stage, we use the KNN algorithm to determine the total charging power of all chargers as well as the charging power of energy storage unit. In the second stage, based on the proposed priority rule, the total charging power is allocated among individual chargers.

A. Determining the Total Charging Power: KNN Algorithm

The optimal charging policy is influenced by the distribution of stochastic information (e.g., the renewable energy generation). Since the actual distribution is difficult to obtain, it is imperative to exploit historical data to construct effective policy. Existing methods include forecasting the future information with techniques like neural networks [8], [9] and utilizing reinforcement learning [21], [22]. However, these methods requires large amount of data to achieve good performance, while large-scale dataset may be hard to obtain for the considered parking lot. To still achieve good performance without much data, we adopt the KNN algorithm [30], which is a non-parametric learning technique that works well in small to medium sized dataset [33].

We first collect historical trajectories of electricity prices, renewable energy generation and EV charging demand. For each scenario (i.e., one day) of trajectories, we solve linear program (11), obtaining the optimal charging decisions of each charger as well as the storage dispatch p_t. The total charging power of all EVs, denoted as V_t, can be further calculated. The optimal trajectories in each scenario are added to the scenario set Ω as dataset in KNN algorithm.

For each scenario $\omega \in \Omega$, we denote the electricity price and renewable energy generation in timeslot t as $\lambda_{\omega,t}$ and $g_{\omega,t}$. To characterize the EV charging need in each scenario, we calculate the average charging need $\{h_{\omega,t}\}_{t=1}^{T}$ as follows:

$$h_{\omega,t} = \sum_{i \in \mathcal{I}_t} \frac{e_{i,a_i}}{d_i - a_i}. \tag{16}$$

To determine V_t and p_t in timeslot t using revealed information $\boldsymbol{x}_t$, $\{g_\tau\}_{\tau=1}^{t}$ and $\{\lambda_\tau\}_{\tau=1}^{t}$, we first calculate the average charging need $\{h_\tau\}_{\tau=1}^{t}$ like in (16), and then calculate the similarity score between current state and each scenario ω in the dataset:

$$\begin{aligned} S_\omega =& \alpha_g \sum_{\tau=1}^{t} |g_\tau - g_{\omega,\tau}| + \alpha_\lambda \sum_{\tau=1}^{t} |\lambda_\tau - \lambda_{\omega,\tau}| \\ &+ \alpha_h \sum_{\tau=1}^{t} |h_\tau - h_{\omega,\tau}|. \end{aligned} \tag{17}$$

where α_g,α_λ,α_h are hyper-parameters with $\alpha_g + \alpha_\lambda + \alpha_h = 1$.

Finally, we retrieve K scenarios in the dataset with largest similarity score, and use the averaged V_t and p_t over these K scenarios in this timeslot. We further devise a set of simple rules to prevent obviously unreasonable charging behaviors. For instance, if the current renewable energy generation is abundant, all EVs and energy storage units will be charged at the maximum rates.

B. Allocating the Charging Power to Individual Charger

After determining the total charging rate of chargers with the KNN algorithm, we still need to solve two problems to obtain the charging rates of individual chargers. First, the charging order we defined is a partial order; for chargers that are incomparable with respect to the partial order, we still need to decide their charging priority. Second, as explained in the example in Section III, even if a charger has the highest charging priority before allocating any power, its priority may be lower than other chargers after allocating some power, thus potentially violating Definition 2. We need to address this issue appropriately in the charging algorithm.

For the first problem, we define the interval-based priority score of a virtual state (e_i, d_i) as follows:

$$Score_i = T(\lceil e_i \rceil - d_i) + d_i. \tag{18}$$

$Score_i$ determines the charging priority of EVs belonging to different circles in Fig. 1, without violating the charging order in Definition 1. Then we propose Algorithm 1, which not only solves the second problem, but also takes care of the charging priority of EVs belonging to the same circle in Fig. 1.

Algorithm 1 EV Charging Allocation Algorithm

```
1: Input: State of all chargers x_t, the total EV charging power V_t
2: Initialize the remaining power r_t ← V_t
3: Initialize the dictionary with interval-based priority score as the key, and the chargers with the corresponding score as the value: D ← {}
4: for each charger i ∈ I_t do
5:    if e_i > (d_i − t − 1) then
6:       Charge i to (d_i − t − 1)
7:       Update r_t
8:    end if
9: end for
10: for each charger i ∈ I_t do
11:    Calculate Score_i according to (18)
12:    Add i to D[Score_i]
13: end for
14: while D ≠ ∅ and r_t > 0 do
15:    s ← max{key in D}
16:    μ ← the lower bound of the corresponding remaining charging need interval of s
17:    if r_t can charge all chargers in D[s] to target μ then
18:       Charge all chargers in D[s] to target μ
19:       Update r_t
20:       Move all chargers in D[s] that have not reached maximum charging rate to D[s − T]
21:       Remove D[s] from D
22:    else
23:       Charge all chargers in D[s] towards some target μ̄ that will consume r_t altogether
24:       break
25:    end if
26: end while
27: Return the charging action of each charger
```

Lines 1-3 initialize the parameters and a dictionary D, which maps an interval-based priority score to the chargers with the corresponding score. Lines 4-9 warrant that the total charging need of any EV can be satisfied at the departing time, by charging minimum amount of power.

In line 10-13, the priority scores of all chargers are calculated. In line 14-26, the remaining charging power (if any) is iteratively allocated to individual vehicles according to the priority scores. In each iteration, we aim to charge all chargers with the maximum priority score s to the lower bound of the corresponding remaining charging need interval. By charging charger i to target μ, we mean

$$v_{i,t} = \min(\max(0, e_{i,t} - \mu), 1). \tag{19}$$

For example, if s corresponds to remaining charging need interval $e_i \in (2, 3]$ (with a specific charging deadline d), we charge all corresponding chargers to 2. Specifically, in line 15-16, we retrieve the maximum priority score s and the corresponding chargers, denoted as $\mathcal{J}$. In line 17-21, if the remaining charging power can charge all chargers in $\mathcal{J}$ to the lower bound of the remaining charging need interval, then the priority score of these chargers will become $s - T$, as shown in (18). Otherwise, in line 22-24, we seek a target remaining need $\bar{\mu}$ through binary search such that all the remaining charging power r_t is exactly consumed after we charge all chargers in $\mathcal{J}$ to target $\bar{\mu}$. In the algorithm, by introducing charging target in the chargers with the same priority score, the charging order defined by 1) of Definition 1 is met. By allocating charging power via the order indicated by the interval-based priority score, the action generated by Algorithm 1 complies with Definition 2. Combining methods described above, the integrated charging algorithm is named as KNN-PRA (where PRA is the abbreviation of Priority Rule-base Allocation).

C. Rollout-based Policy Improvement

Although appearing to be robust and effective, KNN algorithm also exhibits drawbacks on small dataset. For example, the current state is unlikely to be seen in the dataset, hence averaging the optimal actions of experiences is often sub-optimal. To this end, we adopt the rollout algorithm [31].

The basic idea is to improve the performance of a base policy through online simulation. It is proved in [31] that rollout policy will perform no worse than the base policy under mild assumptions. Here, the proposed KNN-PRA policy serves as the base policy π, and then the rollout algorithm determines the charging decision as follows:

$$\begin{aligned} \boldsymbol{a}_t^* = &\arg\min_{\boldsymbol{a}_t \in \mathcal{A}_t} [C_t(\boldsymbol{s}_t, \boldsymbol{w}_t, \boldsymbol{a}_t) \\ &+ \mathbb{E} \sum_{\tau=t+1}^{T} C_\tau(\boldsymbol{s}_\tau, \boldsymbol{w}_\tau, \pi_t(\boldsymbol{s}_\tau, \boldsymbol{w}_\tau))], \end{aligned} \tag{20}$$

where $C_t(\boldsymbol{s}_t, \boldsymbol{w}_t, \boldsymbol{a}_t)$ is the immediate cost by taking an action $\boldsymbol{a}_t$ in the current state, and $\mathbb{E} \sum_{\tau=t+1}^{T} C_\tau(\boldsymbol{s}_\tau, \boldsymbol{w}_\tau, \pi_t(\boldsymbol{s}_\tau, \boldsymbol{w}_\tau))$ is the expected future cost by conducting the base policy.

Without the distribution of the stochastic information in the future, to calculate the expectation in (20), we retrieve the M instances from the dataset with the highest similarity scores to the current state, using them as the trajectories for future exogenous states:

$$\begin{aligned} \boldsymbol{a}_t^* = &\arg\min_{\boldsymbol{a}_t \in \mathcal{A}_t} [C_t(\boldsymbol{s}_t, \boldsymbol{w}_t, \boldsymbol{a}_t) \\ &+ \frac{1}{M} \sum_{\omega=1}^{M} \sum_{\tau=t+1}^{T} C_\tau(\boldsymbol{s}_{\omega,\tau}, \boldsymbol{w}_{\omega,\tau}, \pi_t(\boldsymbol{s}_{\omega,\tau}, \boldsymbol{w}_{\omega,\tau}))]. \end{aligned} \tag{21}$$

To speed up the rollout algorithm, rather than determining the charging rate of the individual chargers, we determine the total charging power V_t (and the dispatch power of energy storage), so the dimension of action space is greatly reduced from $\mathbb{R}^{N+1}$ to $\mathbb{R}^2$. Also, the discretization strategy is applied to solve the minimization problem in (21), which will be further discussed in Section V-B. Finally, V_t is re-distributed according to Algorithm 1. The simulation-assist algorithm is referred to as Rollout-KNN-PRA.

V. Numerical Results

A. Experiment Setup

We conduct numerical experiments to compare the proposed KNN-PRA and Rollout-KNN-PRA with the following widely used methods:

(1) Lazy policy: This is a straightforward rule where an EV will be charged only in two situations. One is that the required demand cannot be met before departure if charging does not start immediately. The other is that if there is surplus renewable energy. The energy storage device is charged when there is surplus renewable energy after charging all EVs, and discharged when renewable generation falls short of meeting the minimum EV charging requirements.

(2) Stochastic Programming (SP) [11]: in each timeslot, retrieve M scenarios with top-M highest similarity scores. Then, a two-stage stochastic program is solved. The planning horizon covers the remaining timeslots of the day.

(3) Model Predictive Control (MPC) [14]: In each timeslot, the planning horizon is set to 10 instead of the remaining timeslots of the day. In this sense, the SP method is a special case of MPC with a dynamic look-ahead window.

(4) Order-based allocation methods, including the well-known EDF policy [23], the latest-deadline-first (LDF) policy and mLLLP [24]. They allocate V_t among individual vehicles, and V_t and p_t are determined by the KNN algorithm described in Section IV-A. For the state-of-the-art mLLLP, we also evaluate its performance by integrating it with the KNN algorithm, yielding the KNN-mLLLP policy.

In the tests, $T = 48$ and $\Delta t = 0.5$ hour are used. The photovoltaic generation is adopted as the renewable resource, and the dataset released by Sheffield Solar [34] is employed. The real-time electricity price of Midwest Independent System Operator in [35] is adopted. The EV charging information, including the arriving/departing time and the charging need, is obtained from ACN-Data [36]. We consider typical level 2 single-phase AC chargers with maximum charging rate 7kW [29]. The capacity of the battery storage in the parking lot is 600kWh, the maximum charging rate is 40kW, and the charging/discharging efficiency is 0.95. For the KNN method, the dataset contains 90 days and $K = 9$.

B. Performance Comparison

We test each algorithm with 30 days (not included in the KNN dataset) and calculate the total cost, as shown in Table I. The KNN-based methods outperform existing approaches such as SP and MPC, demonstrating the effectiveness of the KNN algorithm with medium-sized historical data.

TABLE I
PERFORMANCE ACROSS 30 TEST CASES

Method Type	Method	Cost($)
Existing Methods	Lazy	1774.8
	SP	1432.5
	MPC	1514.4
KNN-based Methods	KNN-EDF	1261.2
	KNN-LDF	1186.8
	KNN-mLLLP	1166.7
	KNN-PRA	1132.8
Methods with Rollout	Rollout-KNN-mLLLP	1040.7
	Rollout-KNN-PRA	1014.3

Among the KNN-based methods, the proposed KNN-PRA achieves superior performance compared to other allocation methods, including mLLLP and EDF, verifying the advantages of the proposed priority rule. While KNN-mLLLP ranks as the second-best KNN-based method, the assumption on binary charging actions limits its effectiveness compared to KNN-PRA. Furthermore, applying the rollout method with KNN-PRA and KNN-mLLLP as base policies further enhances performance in both cases. As KNN-PRA serves as a stronger base policy, the corresponding Rollout-KNN-PRA achieves superior results compared to Rollout-KNN-mLLLP and delivers the best overall performance among all competing methods.

In KNN-based methods, a pivotal parameter is the selection of the value of K. To investigate the impact of K, we change its value from 1 to 13, and the corresponding costs of KNN-PRA, KNN-mLLLP and KNN-EDF are shown in Fig. 3. For each K, the KNN-PRA consistently achieves the best performance among the different allocation methods. For each method, it is observed that the charging cost initially decreases and then increases as K increases. Specifically, when K is too small (i.e., 1-2), the number of neighbors, which serve as references for the current charging decision, is insufficient, leading to poor performance. Gradually increasing the value of K to 9 will adopt more reference scenarios and overcome the above issue. However, when K is further increased from 9, the KNN algorithm incorporates more historical scenarios that might be significantly different from the current state, which also degrades the optimality performance. As the conclusion, $K = 9$ is a proper choice.

When further implementing the rollout algorithm based on KNN-PRA, it is essential to employ a discretization strategy to solve the minimization problem in (21). Specifically, we sample N_{EV} and N_{ES} points within the feasible range of V_t and p_t, respectively. To increase the sample efficiency, the range of the sampling is centered at the V_t and p_t predicted by KNN-PRA base policy. These points are combined to form $N_{EV} \cdot N_{ES}$ candidate actions, and the minimizer of (21) is deployed. To explore the impact of N_{EV} and N_{ES}, we vary their values from 1 to 6, and the corresponding costs of Rollout-KNN-PRA are shown in Fig. 4.

As the number of samples increases, the cost tends to decrease. Moreover, when N_{EV} is fixed and N_{ES} is increased, or

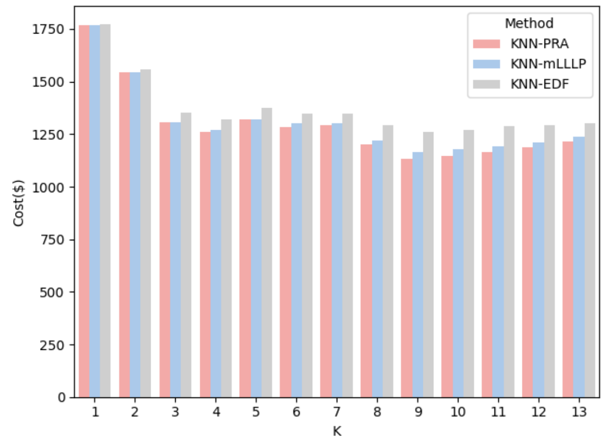

Fig. 3. The impact of K on the performance of KNN-based methods.

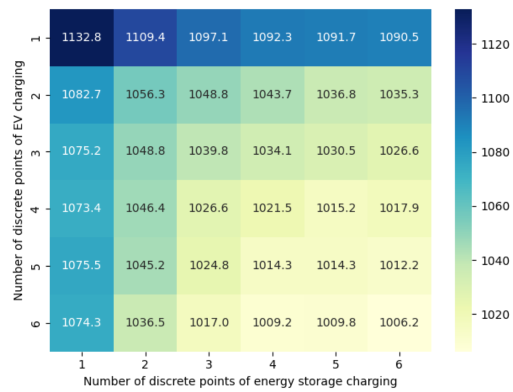

Fig. 4. The impact of N_{EV} and N_{ES} on the performance of Rollout-KNN-PRA algorithm.

vice versa, the cost decreases in both scenarios. This indicates that the rollout algorithm can improve the charging strategies for both EVs and energy storage device. When N_{EV} and N_{ES} is relatively small (i.e., from 1 to 3), increasing the number of samples significantly enhances the algorithm's performance. However, once N_{EV} and N_{ES} exceed 4, the performance improvement resulting from further increasing the number of samples becomes less significant, suggesting that the sampling density is already sufficient. Since the runtime of the rollout algorithm is proportional to $N_{EV} \cdot N_{ES}$, an excessive number of sampling points imposes significant computational burden. Therefore, $N_{EV} = N_{ES} = 5$ is an appropriate choice.

To demonstrate how the proposed priority rule differs from mLLLP and how the rollout algorithm can further enhance performance, the charging strategies of Rollout-KNN-PRA, KNN-PRA, and KNN-mLLLP for a single day (selected from the 30 test days) are shown in Fig. 5. In this case, the costs($) for Rollout-KNN-PRA, KNN-PRA, and KNN-mLLLP are 10.5, 15.6, and 16.7, respectively. From Fig. 5, it is evident that the battery dispatch strategies of the three methods on the selected day follow a similar pattern: charging the energy storage device during periods of surplus renewable generation and discharging it when renewable generation is insufficient.

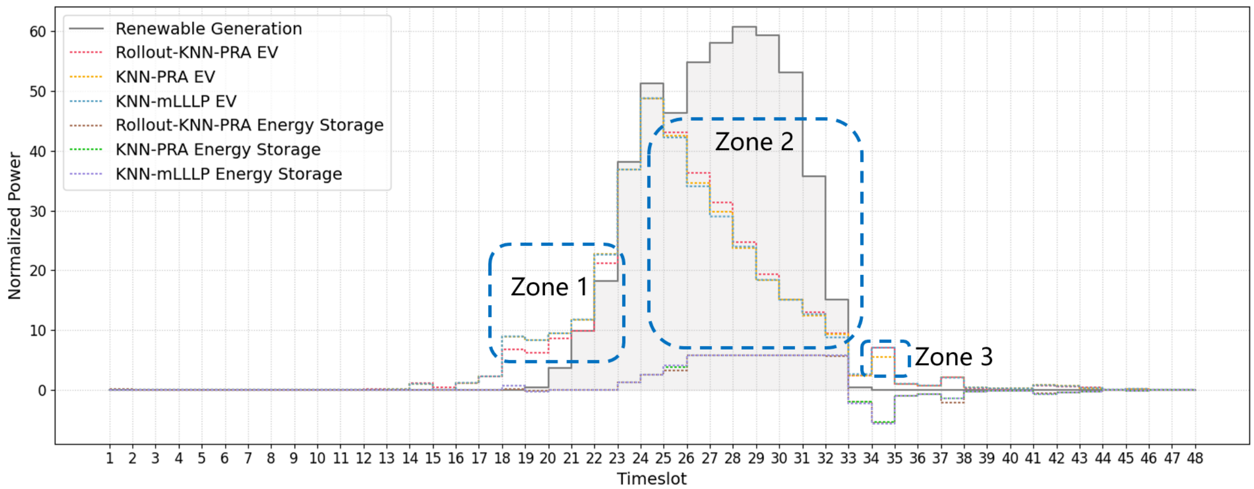

Fig. 5. Comparison of charging strategies of Rollout-KNN-PRA, KNN-PRA and KNN-mLLLP.

Therefore, the analysis focuses on differences in EV charging behavior, which are highlighted in zones 1–3 of the figure.

For Rollout-KNN-PRA and KNN-PRA, the key differences lie in zones 1 and 2. In zone 1, renewable generation is temporarily insufficient to meet the charging demands of EVs. Using historical charging behavior as a reference, KNN-PRA purchases additional energy from the grid to charge the EVs. However, through online simulation, Rollout-KNN-PRA identifies the possibility of postponing the charging of some EVs to zone 2, where sufficient renewable generation is available. As a result, it purchases less power from the grid in zone 1, leading to a lower total charging cost.

For KNN-PRA and KNN-mLLLP, the key differences are observed in zones 2 and 3. In zone 2, renewable generation is sufficient to meet the charging demands of all EVs in the parking lot. During these timeslots, KNN-PRA allocates more power to EVs, achieving better utilization of renewable generation; the underlying mechanism behind this behavior will be explained later. As a result, in zone 3 where renewable generation is absent, KNN-mLLLP requires more energy from the grid compared to KNN-PRA, resulting in higher costs.

To further explain the differences between the proposed priority rule and mLLLP, the charging strategies for two EVs (denoted as EV1 and EV2) are illustrated in Fig. 6. EV1/EV2 arrives at the parking lot in timeslot 20 with normalized demands of 11.61/12.33, respectively, and departs at timeslot 35/34. The key difference between the two charging strategies occurs at timeslot 21. As shown in Fig. 5, during timeslot 21, both KNN-PRA and KNN-mLLLP allocate the same total amount of energy to the EVs. However, the two methods distribute the energy differently. As depicted in Fig. 6(a), KNN-mLLLP prioritizes EV2 over EV1, reducing EV2's demand from 11.33 to 10.33 in timeslot 21, while EV1 is not charged. In contrast, the KNN-PRA method, shown in Fig. 6(b), (virtually) charges EV2 from 11.33 to 11.00, then charges EV1 from 11.61 to 11.00, and finally (virtually) charges EV2 from 11.00 to 10.94.

As illustrated in Fig. 6, between timeslots 22 and 31, both methods charge EV1 and EV2 at the maximum charging rate.

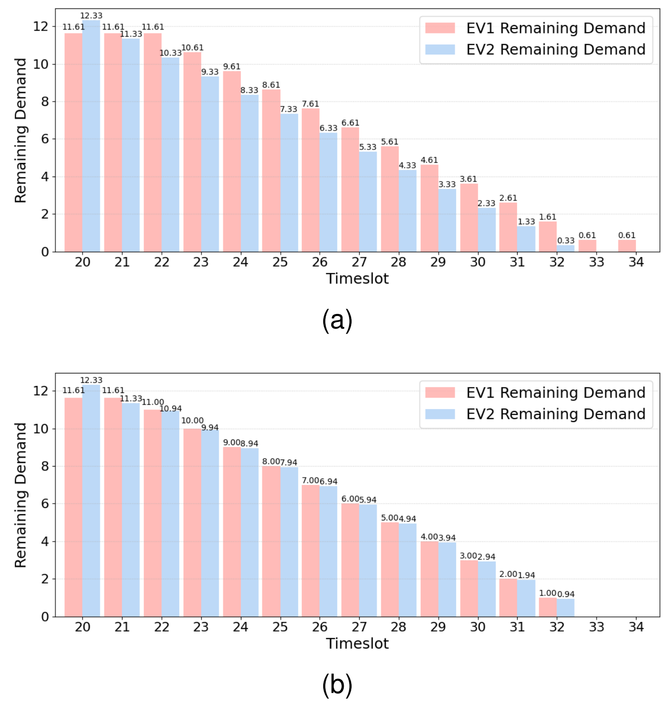

Fig. 6. Charging process of EV1 and EV2 in case study. (a) KNN-mLLLP. (b) KNN-PRA.

In timeslot 32, both methods meet the remaining charging needs of EV2 (i.e., 0.33 and 0.94) using renewable energy, without purchasing power from the grid, as shown in Fig. 5. As a result, despite KNN-PRA allocating less energy to EV2 in timeslot 21, it does not result in a higher cost. Furthermore, KNN-PRA fulfills the remaining charging demand of EV1 in timeslot 32 using renewable generation, while KNN-mLLLP fails to do so because the remaining demand of 1.61 exceeds the maximum charging rate. Consequently, in timeslot 34, when there is no renewable generation, KNN-mLLLP must purchase energy from the grid to meet the remaining charging demand of EV1, leading to a higher cost. To conclude, this case study demonstrates that the proposed priority rule allows for a more reasonable allocation of power among EVs, enabling the optimal utilization of renewable energy.

VI. Conclusion

This paper proposes a generally applicable priority rule to determine the charging order of electric vehicles in a parking lot and establishes its desirable property. It improves existing methods by allowing charging actions to take continuous rather than binary values. On-site battery storage is also considered, and a KNN method is employed to determine the total power for vehicle charging and battery dispatch. Empirical tests demonstrate that the proposed priority rule outperforms the existing mLLLP and deadline based policies. Through a simulation based policy improvement, the charging cost can be further reduced. Since the simulation based implementation is convenient to include factors that are not easily represented by a mathematical model, we can consider charging site recommendation for logistics vehicle fleet and the coordination between charging stations and the power grid in future works.

Appendix
Proofs

We now conduct the proof for Theorem 1. The core idea of the proof is that we can modify an optimal policy to comply with the priority rule, while preserving optimality throughout the adjustment process. In problem (12), the optimal value is influenced by the total charging power V_t and dispatched power p_t of energy storage (ES), so we write the optimal policy as $\pi^* = (\pi^*_{EV}, \pi^*_{ES})$. For simplicity of notation, in the following proof, we focus on modifying π^*_{EV} to compliy with the priority rule and omit π^*_{ES}. However, if π^*_{ES} is considered, the proof process remains essentially the same. Also, we slightly change the MDP defined in Section II: We relax the constraint (5), and therefore the charging demand of EVs are not necessarily satisfied. However, for each EV departing with unsatisfied charging demand e, we will add a penalty term γe in the cost of the departing timeslot. We set the γ to be greater than the maximum possible electricity price to ensure that optimal policy will satisfy all charging demands.

We first introduce a few notations. The state value function and the state-action value function are defined as:

$$V_t^{\pi}(\boldsymbol{x}_t, \boldsymbol{w_t}) = \mathbb{E}_{\pi} \sum_{\tau=t}^{T} [C_\tau(\boldsymbol{x}_\tau, \boldsymbol{w}_\tau, \pi_\tau(\boldsymbol{x}_\tau, \boldsymbol{w}_\tau))], \tag{22}$$

$$V_t(\boldsymbol{x}_t, \boldsymbol{w_t}) = \mathbb{E}_{\pi^*} \sum_{\tau=t}^{T} [C_\tau(\boldsymbol{x}_\tau, \boldsymbol{w}_\tau, \pi^*_\tau(\boldsymbol{x}_\tau, \boldsymbol{w}_\tau))], \tag{23}$$

$$\begin{aligned} Q_t(\boldsymbol{x}_t, \boldsymbol{w_t}, \boldsymbol{v}_t) = C_t(\boldsymbol{x}_t, \boldsymbol{w_t}, \boldsymbol{v}_t) + \\ \mathbb{E}_{\pi^*} \sum_{\tau=t+1}^{T} [C_\tau(\boldsymbol{x}_\tau, \boldsymbol{w}_\tau, \pi^*_\tau(\boldsymbol{x}_\tau, \boldsymbol{w}_\tau))], \end{aligned} \tag{24}$$

where π^* is an optimal policy. Let $\mathcal{D}_t$ denote the set of chargers for which the EVs they charge will leave the parking lot at the beginning of timeslot t. Let $\mathcal{G}_{d,t}$ denote the set of chargers with charging deadline d at timeslot t, i.e.,

$$\mathcal{G}_{d,t} = \{i \in \mathcal{I}_t | d_i = d\}. \tag{25}$$

Lemma 1. *Consider two charger states $\boldsymbol{x}_t = \{(e_{i,t}, d_i)\}_{i=1}^N$ and $\hat{\boldsymbol{x}}_t = \{(\hat{e}_{i,t}, d_i)\}_{i=1}^N$, which are identical except for the remaining charging demand at charger i, where*

$$\hat{e}_{i,t} = e_{i,t} + \epsilon, \tag{26}$$

where $\epsilon > 0$. Then, the following holds:

$$0 \le V_t(\hat{\boldsymbol{x}}_t, \boldsymbol{w_t}) - V_t(\boldsymbol{x}_t, \boldsymbol{w_t}) \le \gamma\epsilon. \tag{27}$$

Proof. Since $0 \le V_t(\hat{\boldsymbol{x}}_t, \boldsymbol{w_t}) - V_t(\boldsymbol{x}_t, \boldsymbol{w_t})$ is obvious, we only need to prove $V_t(\hat{\boldsymbol{x}}_t, \boldsymbol{w_t}) - V_t(\boldsymbol{x}_t, \boldsymbol{w_t}) \le \gamma\epsilon$.

Let π^* denote the optimal policy, and starting from state $\hat{\boldsymbol{x}}_t$, we construct another policy $\hat{\pi}$ that mimics the charging behavior of π^* starting from state $\boldsymbol{x}_t$:

$$\hat{\pi}(\hat{\boldsymbol{x}}_{t+\tau}, \boldsymbol{w}_{t+\tau}) = \pi^*(\boldsymbol{x}_{t+\tau}, \boldsymbol{w}_{t+\tau}), \ \forall \tau = 1, ..., T-t. \tag{28}$$

We can see that the difference of $V_t^{\hat{\pi}}(\hat{\boldsymbol{x}}_t, \boldsymbol{w_t})$ and $V_t(\boldsymbol{x}_t, \boldsymbol{w_t})$ merely comes from the penalty term in timeslot d_i:

$$V_t^{\hat{\pi}}(\hat{\boldsymbol{x}}_t, \boldsymbol{w_t}) - V_t(\boldsymbol{x}_t, \boldsymbol{w_t}) = \gamma(e_{i,d_i} + \epsilon) - \gamma(e_{i,d_i}) = \gamma\epsilon. \tag{29}$$

Then we conclude that

$$V_t(\hat{\boldsymbol{x}}_t, \boldsymbol{w_t}) \le V_t^{\hat{\pi}}(\hat{\boldsymbol{x}}_t, \boldsymbol{w_t}) = V_t(\boldsymbol{x}_t, \boldsymbol{w_t}) + \gamma\epsilon. \tag{30}$$

□

Lemma 2. *Consider any charger state $\boldsymbol{x}_t = \{(e_{i,t}, d_i)\}_{i=1}^N$ where two chargers i and j satisfy $(e_{i,t}, d_i) \prec (e_{j,t}, d_j)$. Let $m^* = \inf\{m | (e_{i,t} + m, d_i) \not\prec (e_{j,t} - m, d_j)\}$. $\forall m \in [0, m^*]$, if we construct another charger state $\tilde{\boldsymbol{x}}_t = \{(\tilde{e}_{i,t}, d_i)\}_{i=1}^N$ that is identical with $\boldsymbol{x}_t$, except for chargers i and j where the following conditions hold:*

$$\tilde{e}_{i,t} = e_{i,t} + m, \tag{31}$$
$$\tilde{e}_{j,t} = e_{j,t} - m, \tag{32}$$

then we have

$$V_t(\tilde{\boldsymbol{x}}_t, \boldsymbol{w_t}) \le V_t(\boldsymbol{x}_t, \boldsymbol{w_t}). \tag{33}$$

Proof. Since Lemma 1 implies that the value function $V_t(\boldsymbol{x}_t, \boldsymbol{w_t})$ is continuous with respect to the remaining demand of any charger, it is enough to prove (33) for $m \in [0, m^*)$. Note that in this case we have

$$(\tilde{e}_{i,t}, d_i) \prec (\tilde{e}_{j,t}, d_j). \tag{34}$$

Then we proceed with proof using backward induction with the following two steps:

Step 1. When $t = \min(d_i, d_j)$, we discuss the following cases:

Case 1. If $d_i < d_j$, and $t = d_i$, then by (34) and Definition 1 we have

$$\lceil \tilde{e}_{i,t} \rceil \le \lceil \tilde{e}_{j,t} \rceil - (d_j - d_i). \tag{35}$$

If $m = 0$, then $\tilde{\boldsymbol{x}}_t = \boldsymbol{x}$, and (33) holds trivially. If $m > 0$, then $\tilde{e}_{i,t} > 0$, and by (35) we have

$$\tilde{e}_{j,t} > d_j - d_i, \tag{36}$$

which means charger j is destined to fail to fulfill the charging demand at timeslot d_j. Let π^* denote the optimal policy, and starting from state $\tilde{\boldsymbol{x}}_t$, we construct another policy $\tilde{\pi}$ that mimics the charging behavior of π^* starting from state $\boldsymbol{x}_t$:

$$\tilde{\pi}(\tilde{\boldsymbol{x}}_{t+\tau}, \boldsymbol{w}_{t+\tau}) = \pi^*(\boldsymbol{x}_{t+\tau}, \boldsymbol{w}_{t+\tau}),\ \forall \tau = 1, ..., T-t. \quad (37)$$

The feasibility of policy $\tilde{\pi}$ is guaranteed by (36), as there is no chance that $\tilde{v}_{j,t+\tau} = v_{j,t+\tau} > \tilde{e}_{j,t+\tau}$. We can see that the difference of $V_t^{\tilde{\pi}}(\hat{\boldsymbol{x}}_t, \boldsymbol{w_t})$ and $V_t(\boldsymbol{x}_t, \boldsymbol{w_t})$ merely comes from the penalty term in timeslot d_i and d_j:

$$\begin{aligned} & V_t^{\tilde{\pi}}(\tilde{\boldsymbol{x}}_t, \boldsymbol{w_t}) - V_t(\boldsymbol{x}_t, \boldsymbol{w_t}) \\ =& [\gamma \tilde{e}_{i,d_i} + \gamma \tilde{e}_{j,d_j}] - [\gamma e_{i,d_i} + \gamma e_{j,d_j}] \\ =& [\gamma(e_{i,d_i} + \epsilon) + \gamma(e_{j,d_j} - \epsilon)] - [\gamma e_{i,d_i} + \gamma e_{j,d_j}] \\ =& 0. \end{aligned} \quad (38)$$

Then we conclude that

$$V_t(\tilde{\boldsymbol{x}}_t, \boldsymbol{w_t}) \leq V_t^{\tilde{\pi}}(\tilde{\boldsymbol{x}}_t, \boldsymbol{w_t}) = V_t(\boldsymbol{x}_t, \boldsymbol{w_t}). \quad (39)$$

Case 2. If $d_j < d_i$ and $t = d_j$, let $\boldsymbol{v}_t$ denote the optimal charging decisions at state $(\boldsymbol{x}_t, \boldsymbol{w}_t)$, then we have

$$\begin{aligned} V_t(\boldsymbol{x}_t, \boldsymbol{w_t}) =& \gamma e_{j,t} + \gamma \sum_{k \in \mathcal{D}_t \setminus \{j\}} e_{k,t} + \lambda_t [\mathbf{1}^T \boldsymbol{v}_t p_c^{\max} - g_t]_+ \\ &+ \mathop{\mathbb{E}}_{\boldsymbol{w_{t+1}}} [V_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{w}_{t+1})], \end{aligned} \quad (40)$$

and

$$\begin{aligned} V_t(\tilde{\boldsymbol{x}}_t, \boldsymbol{w_t}) \leq& \gamma \tilde{e}_{j,t} + \gamma \sum_{k \in \mathcal{D}_t \setminus \{j\}} e_{k,t} + \lambda_t [\mathbf{1}^T \boldsymbol{v}_t p_c^{\max} - g_t]_+ \\ &+ \mathop{\mathbb{E}}_{\boldsymbol{w_{t+1}}} [V_{t+1}(\tilde{\boldsymbol{x}}_{t+1}, \boldsymbol{w}_{t+1})], \end{aligned} \quad (41)$$

where $\tilde{\boldsymbol{x}}_{t+1}$ is identical to $\boldsymbol{x}_{t+1}$ except for the charger i, where

$$\tilde{e}_{i,t+1} = e_{i,t+1} + m. \quad (42)$$

According to Lemma 1, we have

$$\mathop{\mathbb{E}}_{\boldsymbol{w_{t+1}}} [V_{t+1}(\tilde{\boldsymbol{x}}_{t+1}, \boldsymbol{w}_{t+1})] \leq \mathop{\mathbb{E}}_{\boldsymbol{w_{t+1}}} [V_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{w}_{t+1})] + \gamma m. \quad (43)$$

By (32)(40)(41)(43) we have

$$V_t(\tilde{\boldsymbol{x}}_t, \boldsymbol{w_t}) \leq V_t(\boldsymbol{x}_t, \boldsymbol{w_t}). \quad (44)$$

Case 3. If $t = d_j = d_i$, the only difference between $\boldsymbol{x}_t$ and $\tilde{\boldsymbol{x}}_t$ is the difference remaining demands of the EVs that are going to depart from chargers i and j. As $\tilde{e}_{i,t} + \tilde{e}_{j,t} = e_{i,t} + e_{j,t}$, we have

$$\gamma \sum_{k \in \mathcal{D}_t} e_{k,t} = \gamma \sum_{k \in \mathcal{D}_t} \tilde{e}_{k,t}. \quad (45)$$

Then we conclude that

$$V_t(\tilde{\boldsymbol{x}}_t, \boldsymbol{w_t}) = V_t(\boldsymbol{x}_t, \boldsymbol{w_t}). \quad (46)$$

Step 2. When $t < \min(d_i, d_j)$, suppose (33) holds for timeslot $t+1$. Let $\boldsymbol{v}_t$ denote the optimal charging decisions at state $(\boldsymbol{x}_t, \boldsymbol{w}_t)$, and $\boldsymbol{x}_{t+1}$ the next state after the charging action $\boldsymbol{v}_t$. Specifically, we have

$$e_{i,t+1} = e_{i,t} - v_{i,t}, \quad (47)$$
$$e_{j,t+1} = e_{j,t} - v_{j,t}. \quad (48)$$

For the convenience of the following discussion, we first list some properties that the mentioned variables satisfy:

$$e_{i,t} + e_{j,t} = \tilde{e}_{i,t} + \tilde{e}_{j,t}, \quad (49)$$
$$0 \leq v_{i,t} \leq \min(e_{i,t}, 1), \quad (50)$$
$$0 \leq v_{j,t} \leq \min(e_{j,t}, 1), \quad (51)$$
$$\tilde{e}_{i,t} \geq e_{i,t}, \quad (52)$$
$$\tilde{e}_{j,t} \leq e_{j,t}. \quad (53)$$

We will complete the proof by discussing the following two cases.

Case 1. If $\tilde{e}_{j,t} \geq e_{j,t+1}$, for state $\tilde{\boldsymbol{x}}_t$ (and $\boldsymbol{w}_t$), we will construct the charging decision $\tilde{\boldsymbol{v}}_t$ that is identical to $\boldsymbol{v}_t$ except for charger i and j, where:

$$\tilde{v}_{i,t} = \min(\tilde{e}_{i,t}, v_{i,t} + v_{j,t} - (\tilde{e}_{j,t} - e_{j,t+1}), 1), \quad (54)$$
$$\tilde{v}_{j,t} = v_{i,t} + v_{j,t} - \tilde{v}_{i,t}. \quad (55)$$

First, we check the feasibility of $\tilde{v}_{i,t}$ and $\tilde{v}_{j,t}$ defined in (54) and (55). Note that by (48) and (53) we have

$$\tilde{e}_{j,t} - e_{j,t+1} \leq v_{j,t}. \quad (56)$$

Combining (54) and (56) we have

$$0 \leq \tilde{v}_{i,t} \leq \min(\tilde{e}_{i,t}, 1). \quad (57)$$

The non-negative charging constraint for $\tilde{v}_{j,t}$ holds trivially by (54) and (55):

$$\tilde{v}_{j,t} \geq 0. \quad (58)$$

As we will demonstrate, the maximum charging constraint for $\tilde{v}_{j,t}$ also holds, and the proof will be given later in the discussion:

$$\tilde{v}_{j,t} \leq \min(\tilde{e}_{j,t}, 1). \quad (59)$$

We now proceed to verify the correctness of (33), where

$$\begin{aligned} V_t(\boldsymbol{x}_t, \boldsymbol{w_t}) = & \gamma \sum_{k \in \mathcal{D}_t} e_{k,t} + \lambda_t [\mathbf{1}^T \boldsymbol{v}_t p_c^{\max} - g_t]_+ \\ &+ \mathop{\mathbb{E}}_{\boldsymbol{w_{t+1}}} [V_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{w}_{t+1})], \end{aligned} \quad (60)$$

and

$$\begin{aligned} V_t(\tilde{\boldsymbol{x}}_t, \boldsymbol{w_t}) \leq & \gamma \sum_{k \in \mathcal{D}_t} e_{k,t} + \lambda_t [\mathbf{1}^T \tilde{\boldsymbol{v}}_t p_c^{\max} - g_t]_+ \\ &+ \mathop{\mathbb{E}}_{\boldsymbol{w_{t+1}}} [V_{t+1}(\tilde{\boldsymbol{x}}_{t+1}, \boldsymbol{w}_{t+1})], \end{aligned} \quad (61)$$

where $\tilde{\boldsymbol{x}}_{t+1}$ is identical to $\boldsymbol{x}_{t+1}$ except for the chargers i and j which satisfies

$$\tilde{e}_{i,t+1} = \tilde{e}_{i,t} - \tilde{v}_{i,t}, \quad (62)$$
$$\tilde{e}_{j,t+1} = \tilde{e}_{j,t} - \tilde{v}_{j,t}. \quad (63)$$

By (49)(55)(62)(63) we have

$$\tilde{e}_{i,t+1} + \tilde{e}_{j,t+1} = e_{i,t+1} + e_{j,t+1}. \quad (64)$$

By (55) we have $\mathbf{1}^T \boldsymbol{v}_t = \mathbf{1}^T \tilde{\boldsymbol{v}}_t$, and therefore to prove (33) we only need to prove the following property:

$$\mathop{\mathbb{E}}_{\boldsymbol{w_{t+1}}} [V_{t+1}(\tilde{\boldsymbol{x}}_{t+1}, \boldsymbol{w}_{t+1})] \leq \mathop{\mathbb{E}}_{\boldsymbol{w_{t+1}}} [V_{t+1}(\boldsymbol{x}_{t+1}, \boldsymbol{w}_{t+1})]. \quad (65)$$

According to the value of $\tilde{v}_{i,t}$, Case 1 can be further divided into 3 sub-cases. In each sub-case, we will check the correctness of (59) and (65).

Case 1.1. If $\tilde{v}_{i,t} = \tilde{e}_{i,t}$, then we have

$$\tilde{e}_{i,t} \le v_{i,t} + v_{j,t} - (\tilde{e}_{j,t} - e_{j,t+1}). \tag{66}$$

By (48)(49)(66) we have

$$v_{i,t} \ge e_{i,t}. \tag{67}$$

Combining with (50) we have

$$v_{i,t} = e_{i,t}. \tag{68}$$

Then we have

$$\begin{aligned} \tilde{v}_{j,t} &= v_{i,t} + v_{j,t} - \tilde{v}_{i,t} \\ &= e_{i,t} + v_{j,t} - \tilde{e}_{i,t} \\ &\le v_{j,t} \\ &\le 1, \end{aligned} \tag{69}$$

and

$$\begin{aligned} \tilde{e}_{j,t} - \tilde{v}_{j,t} &= (e_{i,t} + e_{j,t} - \tilde{e}_{i,t}) - (v_{i,t} + v_{j,t} - \tilde{v}_{i,t}) \\ &= (e_{i,t} - v_{i,t}) + (e_{j,t} - v_{j,t}) \\ &\ge 0. \end{aligned} \tag{70}$$

From (69) and (70) we can see (59) holds.

Since $\tilde{v}_{i,t} = \tilde{e}_{i,t}$ we have

$$\tilde{e}_{i,t+1} = 0 = e_{i,t+1}. \tag{71}$$

Then from (64) we have

$$\tilde{e}_{j,t+1} = e_{j,t+1}. \tag{72}$$

Therefore (65) holds as $\tilde{\boldsymbol{x}}_{t+1} = \boldsymbol{x}_{t+1}$.

Case 1.2. If $\tilde{v}_{i,t} = v_{i,t} + v_{j,t} - (\tilde{e}_{j,t} - e_{j,t+1})$, then combining (55) we have

$$\tilde{v}_{j,t} = \tilde{e}_{j,t} - e_{j,t+1}, \tag{73}$$

Combining (73) with (48)(51)(53) we can see (59) holds. Alao, by (73) and (63) we have

$$\tilde{e}_{j,t+1} = e_{j,t+1}. \tag{74}$$

Then from (64) we have

$$\tilde{e}_{i,t+1} = e_{i,t+1}. \tag{75}$$

Therefore (65) holds as $\tilde{\boldsymbol{x}}_{t+1} = \boldsymbol{x}_{t+1}$.

Case 1.3. If $\tilde{v}_{i,t} = 1$, then $\tilde{e}_{i,t} \ge 1$. We can assume $\tilde{e}_{i,t} > 1$ as the case for $\tilde{e}_{i,t} = 1$ can be covered by Case 1.1. Then from (34) and Definition 1 we can see $\tilde{e}_{j,t} > 1$. On the other hand, combining $\tilde{e}_{i,t} > 1$ with (50)(51)(55) we have

$$\tilde{v}_{j,t} \le 1 < \tilde{e}_{j,t}, \tag{76}$$

and therefore (59) holds.

Recall that $(\tilde{e}_{i,t}, d_i) \prec (\tilde{e}_{j,t}, d_j)$ from (34). According to the Definition 1, the fact $\tilde{v}_{i,t} = 1$, and the relationship (62)(63), we can see that

$$(\tilde{e}_{i,t+1}, d_i) \prec (\tilde{e}_{j,t+1}, d_j). \tag{77}$$

On the other hand, combining (54)(55)(63) we have

$$\tilde{e}_{j,t+1} \le e_{j,t+1}. \tag{78}$$

Then from (64) we can see that there exists $\hat{m} \in [0, \hat{m}^*]$ such that

$$\tilde{e}_{i,t+1} = e_{i,t+1} + \hat{m}, \tag{79}$$

$$\tilde{e}_{j,t+1} = e_{j,t+1} - \hat{m}, \tag{80}$$

where $\hat{m}^* = \inf\{m | (e_{i,t+1} + m, d_i) \nprec (e_{j,t+1} - m, d_j)\}$. Then according to the property (33) for timeslot $t+1$ we can see (65) holds.

Case 2. If $\tilde{e}_{j,t} < e_{j,t+1}$, for state $(\tilde{\boldsymbol{x}}_t, \boldsymbol{w}_t)$, we will construct the charging decision $\tilde{\boldsymbol{v}}_t$ that is identical to $\boldsymbol{v}_t$ except for charger i and j which satisfies:

$$\tilde{v}_{i,t} = \min(v_{i,t} + v_{j,t}, 1), \tag{81}$$

$$\tilde{v}_{j,t} = v_{i,t} + v_{j,t} - \tilde{v}_{i,t}. \tag{82}$$

For the feasibility of $\tilde{v}_{i,t}$ and $\tilde{v}_{j,t}$, it is obvious that

$$0 \le \tilde{v}_{i,t} \le 1, \tag{83}$$

$$0 \le \tilde{v}_{j,t}. \tag{84}$$

We prove $\tilde{v}_{i,t} \le \tilde{e}_{i,t}$ by showing that $\tilde{e}_{i,t} \ge v_{i,t} + v_{j,t}$. Otherwise,

$$\begin{aligned} \tilde{e}_{i,t} + \tilde{e}_{j,t} &< v_{i,t} + v_{j,t} + e_{j,t+1} \\ &\le e_{i,t} + e_{j,t}, \end{aligned} \tag{85}$$

which contradicts with (49). Then similar to Case 1, we only need to prove (59) and (65) in 2 sub-cases:

Case 2.1. If $\tilde{v}_{i,t} = v_{i,t} + v_{j,t}$, then combining (82) we have

$$\tilde{v}_{j,t} = 0, \tag{86}$$

and then (59) holds trivially. Recall that $(\tilde{e}_{i,t}, d_i) \prec (\tilde{e}_{j,t}, d_j)$ from (34). According to the Definition 1, the fact $\tilde{v}_{j,t} = 0$, and the relationship (62)(63), we can see that

$$(\tilde{e}_{i,t+1}, d_i) \prec (\tilde{e}_{j,t+1}, d_j). \tag{87}$$

On the other hand, Combining $\tilde{e}_{j,t} < e_{j,t+1}$ with (63) we have

$$\tilde{e}_{j,t+1} < e_{j,t+1}. \tag{88}$$

Then from (64) we can see that there exists $\hat{m} \in [0, \hat{m}^*]$ such that

$$\tilde{e}_{i,t+1} = e_{i,t+1} + \hat{m}, \tag{89}$$

$$\tilde{e}_{j,t+1} = e_{j,t+1} - \hat{m}, \tag{90}$$

where $\hat{m}^* = \inf\{m | (e_{i,t+1} + m, d_i) \nprec (e_{j,t+1} - m, d_j)\}$. Then according to the property (33) for timeslot $t+1$ we can see (65) holds.

Case 2.2. If $\tilde{v}_{i,t} = 1$, then $\tilde{e}_{i,t} \ge 1$. If $\tilde{e}_{i,t} = 1$, then

$$1 = \tilde{v}_{i,t} \le v_{i,t} + v_{j,t} \le \tilde{e}_{i,t} = 1, \tag{91}$$

and therefore

$$v_{i,t} + v_{j,t} = 1, \tag{92}$$

and this case can be covered by Case 2.1. Then we consider the case $\tilde{e}_{i,t} > 1$. From (34) and Definition 1 we can see $\tilde{e}_{j,t} > 1$. On the other hand, combining $\tilde{e}_{i,t} > 1$ with (50)(51)(82) we have

$$\tilde{v}_{j,t} \le 1 < \tilde{e}_{j,t}, \tag{93}$$

and therefore (59) holds.

Recall that $(\tilde{e}_{i,t}, d_i) \prec (\tilde{e}_{j,t}, d_j)$ from (34). According to the Definition 1, the fact $\tilde{v}_{i,t} = 1$, and the relationship (62)(63), we can see that

$$(\tilde{e}_{i,t+1}, d_i) \prec (\tilde{e}_{j,t+1}, d_j). \tag{94}$$

On the other hand, we have

$$\tilde{e}_{j,t+1} \leq \tilde{e}_{j,t} < e_{j,t+1}. \tag{95}$$

Then from (64) we can see that there exists $\hat{m} \in [0, \hat{m}^*]$ such that

$$\tilde{e}_{i,t+1} = e_{i,t+1} + \hat{m}, \tag{96}$$
$$\tilde{e}_{j,t+1} = e_{j,t+1} - \hat{m}, \tag{97}$$

where $\hat{m}^* = \inf\{m|(e_{i,t+1}+m, d_i) \not\prec (e_{j,t+1}-m, d_j)\}$. Then according to the property (33) for timeslot $t+1$ we can see (65) holds.

□

Next, we introduce the concept of differentiated threshold charging (DTC) policy, which has been discussed in [29] and is very important for our following proofs. In a DTC-type policy, at each timeslot t, there is a (normalized) charging threshold $\mu_{d,t} \geq 0$ for each deadline d, and the normalized charging rate for charger i with deadline d is determined by

$$v_{i,t} = \min(\max(e_{i,t} - \mu_{d,t}, 0), 1), \ \forall t, \ \forall i \in \mathcal{G}_{d,t}. \tag{98}$$

It is shown in [29] that there exists an optimal DTC-type charging policy. Therefore, in the following proofs, we focus on the DTC-type policies. First, we will define the concept of charging order-compliance for thresholds in a DTC-type policy.

Definition 3. *In a DTC-type charging policy, for two charging deadlines $d_1 < d_2$, we say that charging thresholds $\mu_{d_1,t}$ and $\mu_{d_2,t}$ are charging order-compliant, if the following two conditions hold:*

1) $\forall \epsilon > 0, \ (\mu_{d_1,t} + \epsilon, d_1) \not\prec (\mu_{d_2,t}, d_2)$,
2) $\forall \epsilon > 0, \ (\mu_{d_2,t} + \epsilon, d_2) \not\prec (\mu_{d_1,t}, d_1)$.

Furthermore, let $d_1 < d_2 < \cdots < d_m$ be m different charging deadlines, then we say $\mu_{d_1,t}, \mu_{d_2,t}, \cdots, \mu_{d_m,t}$ are charging order-compliant, if $\mu_{d_k,t}$ and $\mu_{d_{k+1},t}$ are charging order-compliant for all $1 \leq k \leq m-1$.

Lemma 3. *In a DTC-type charging policy, let $d_1 < d_2$ be two charging deadlines and $\mu_{d_1,t}, \mu_{d_2,t}$ be corresponding charging thresholds. Then the following conditions are equivalent:*

1) *$\mu_{d_1,t}, \mu_{d_2,t}$ are charging order-compliant,*
2) $\lceil \mu_{d_1,t} \rceil \leq \mu_{d_2,t} \leq \lfloor \mu_{d_1,t} \rfloor + (d_2 - d_1)$,
3) $\lceil \mu_{d_2,t} \rceil - (d_2 - d_1) \leq \mu_{d_1,t} \leq \lfloor \mu_{d_2,t} \rfloor$.

Proof. The equivalence can be established by

$$\forall \epsilon > 0, (\mu_{d_1,t} + \epsilon, d_1) \not\prec (\mu_{d_2,t}, d_2) \tag{99}$$
$$\Longleftrightarrow \forall \epsilon > 0, \lceil \mu_{d_1,t} + \epsilon \rceil > \lceil \mu_{d_2,t} \rceil - (d_2 - d_1) \tag{100}$$
$$\Longleftrightarrow \mu_{d_1,t} \geq \lceil \mu_{d_2,t} \rceil - (d_2 - d_1) \tag{101}$$
$$\Longleftrightarrow \lfloor \mu_{d_1,t} \rfloor \geq \lceil \mu_{d_2,t} \rceil - (d_2 - d_1) \tag{102}$$
$$\Longleftrightarrow \mu_{d_2,t} \leq \lfloor \mu_{d_1,t} \rfloor + (d_2 - d_1), \tag{103}$$

and

$$\forall \epsilon > 0, (\mu_{d_2,t} + \epsilon, d_2) \not\prec (\mu_{d_1,t}, d_1) \tag{104}$$
$$\Longleftrightarrow \forall \epsilon > 0, \lceil \mu_{d_2,t} + \epsilon \rceil > \lceil \mu_{d_1,t} \rceil \tag{105}$$
$$\Longleftrightarrow \mu_{d_2,t} \geq \lceil \mu_{d_1,t} \rceil \tag{106}$$
$$\Longleftrightarrow \lfloor \mu_{d_2,t} \rfloor \geq \lceil \mu_{d_1,t} \rceil \tag{107}$$
$$\Longleftrightarrow \mu_{d_1,t} \leq \lfloor \mu_{d_2,t} \rfloor. \tag{108}$$

□

Lemma 4. *In a DTC-type charging policy, let $d_1 < d_2 < d_3$ be three different charging deadlines, if $\mu_{d_1,t}$ and $\mu_{d_2,t}$ are charging order-compliant, and $\mu_{d_2,t}$ and $\mu_{d_3,t}$ are charging order-compliant, then $\mu_{d_1,t}$ and $\mu_{d_3,t}$ are charging order-compliant.*

Proof. Because $\mu_{d_1,t}$ and $\mu_{d_2,t}$ are charging order-compliant, and $\mu_{d_2,t}$ and $\mu_{d_3,t}$ are charging order-compliant, according to Lemma 3 we have

$$\lceil \mu_{d_1,t} \rceil \leq \mu_{d_2,t} \leq \lfloor \mu_{d_1,t} \rfloor + (d_2 - d_1), \tag{109}$$
$$\lceil \mu_{d_2,t} \rceil \leq \mu_{d_3,t} \leq \lfloor \mu_{d_2,t} \rfloor + (d_3 - d_2), \tag{110}$$

and therefore

$$\lceil \mu_{d_1,t} \rceil \leq \mu_{d_3,t} \leq \lfloor \mu_{d_1,t} \rfloor + (d_3 - d_1). \tag{111}$$

then according to Lemma 3, $\mu_{d_1,t}$ and $\mu_{d_3,t}$ are charging order-compliant. □

Next, we will show that we can modify charging thresholds to be charging order-compliant without increasing the state-action value function. We first discuss a simple case, where two charging thresholds are adjusted.

Lemma 5. *For any charging threshold $\boldsymbol{\mu}_t$, and any deadline d, $d+\tau$ satisfying $t < d < d+\tau \leq T$, there is another charging threshold $\tilde{\boldsymbol{\mu}}_t$ that is identical to $\boldsymbol{\mu}_t$ except for $\tilde{\mu}_{d,t}$ and $\tilde{\mu}_{d+\tau,t}$ which are charging order-compliant, and $Q_t(\boldsymbol{x}_t, \boldsymbol{w}_t, \tilde{\boldsymbol{\mu}}_t) \leq Q_t(\boldsymbol{x}_t, \boldsymbol{w}_t, \boldsymbol{\mu}_t)$.*

Proof. We only need to consider the following cases where $\mu_{d,t}$ and $\mu_{d+\tau,t}$ are not charging order-compliant:

Case 1: $\exists \epsilon > 0$ such that $(\mu_{d,t} + \epsilon, d) \prec (\mu_{d+\tau,t}, d+\tau)$. From Lemma 3, we have the following properties:

$$\mu_{d,t} < \lceil \mu_{d+\tau,t} \rceil - \tau, \tag{112}$$
$$\mu_{d+\tau,t} > \lfloor \mu_{d,t} \rfloor + \tau. \tag{113}$$

Let $A = \lceil \mu_{d+\tau,t} \rceil - \lfloor \mu_{d,t} \rfloor - \tau$, we have $A \geq 1$ by (113). Then we define two sequences $\{\hat{\mu}_{d,t}^{(i)}\}_{i=0}^{A}$ and $\{\hat{\mu}_{d+\tau,t}^{(i)}\}_{i=0}^{A}$ as follows:

$$\hat{\mu}_{d,t}^{(i)} = \begin{cases} \mu_{d,t}, & i = 0, \\ \lfloor \mu_{d,t} \rfloor + i, & i = 1, 2, ..., A. \end{cases} \tag{114}$$

$$\hat{\mu}_{d+\tau,t}^{(i)} = \begin{cases} \lfloor \mu_{d,t} \rfloor + i + \tau, & i = 0, 1, ..., A-1, \\ \mu_{d+\tau,t}, & i = A. \end{cases} \tag{115}$$

We define the (normalized) total charging power of EVs with deadline d and $d+\tau$ with charging threshold $\bar{\mu}_d$ and $\bar{\mu}_{d+\tau}$ as:

$$C_t^{d,d+\tau}(\bar{\mu}_d, \bar{\mu}_{d+\tau}) = \sum_{i\in\mathcal{G}_{d,t}} \min(\max(e_{i,t}-\bar{\mu}_d, 0), 1) + \sum_{i\in\mathcal{G}_{d+\tau,t}} \min(\max(e_{i,t}-\bar{\mu}_{d+\tau}, 0), 1). \tag{116}$$

Specifically, we define

$$c = C_t^{d,d+\tau}(\mu_{d,t}, \mu_{d+\tau,t}), \tag{117}$$
$$\hat{c}^{(i)} = C_t^{d,d+\tau}(\hat{\mu}_{d,t}^{(i)}, \hat{\mu}_{d+\tau,t}^{(i)}), \qquad i = 0, 1, ..., A. \tag{118}$$

Then we can verify that $\hat{c}^{(0)} \geq \hat{c}^{(1)} \geq \cdots \geq \hat{c}^{(A)}$ and $\hat{c}^{(0)} \geq c \geq \hat{c}^{(A)}$. Then we let

$$k = \min\{0 \leq k' \leq A-1 | \hat{c}^{(k')} \geq c \text{ and } \hat{c}^{(k'+1)} \leq c\}. \tag{119}$$

If $C_t^{d,d+\tau}(\hat{\mu}_{d,t}^{(k+1)}, \hat{\mu}_{d+\tau,t}^{(k)}) \leq c$, then we define

$$\tilde{\mu}_{d,t} = \min\{\hat{\mu}_{d,t}^{(k)} \leq \bar{\mu}_d \leq \hat{\mu}_{d,t}^{(k+1)} | C_t^{d,d+\tau}(\bar{\mu}_d, \hat{\mu}_{d+\tau,t}^{(k)}) = c\}, \tag{120}$$

$$\tilde{\mu}_{d+\tau,t} = \hat{\mu}_{d+\tau,t}^{(k)}. \tag{121}$$

Otherwise, we define

$$\tilde{\mu}_{d,t} = \hat{\mu}_{d,t}^{(k+1)}, \tag{122}$$

$$\tilde{\mu}_{d+\tau,t} = \min\{\hat{\mu}_{d+\tau,t}^{(k)} \leq \bar{\mu}_{d+\tau} \leq \hat{\mu}_{d+\tau,t}^{(k+1)} | C_t^{d,d+\tau}(\hat{\mu}_{d,t}^{(k+1)}, \bar{\mu}_{d+\tau}) = c\}. \tag{123}$$

Note that the total charging power remains the same after changing $\boldsymbol{\mu}_t$ to $\tilde{\boldsymbol{\mu}}_t$. Now we will show that $\tilde{\mu}_{d,t}$ and $\tilde{\mu}_{d+\tau,t}$ are charging order-compliant, which is equivalent to

$$\lceil \tilde{\mu}_{d,t} \rceil \leq \tilde{\mu}_{d+\tau,t} \leq \lfloor \tilde{\mu}_{d,t} \rfloor + \tau. \tag{124}$$

If $C_t^{d,d+\tau}(\hat{\mu}_{d,t}^{(k+1)}, \hat{\mu}_{d+\tau,t}^{(k)}) \leq c$, assume $1 \leq k \leq A-1$ (the case for $k=0$ can be proved likewise), according to (114)(115)(120)(121) we have

$$\lfloor \mu_{d,t} \rfloor + k \leq \tilde{\mu}_{d,t} \leq \lfloor \mu_{d,t} \rfloor + k + 1, \tag{125}$$
$$\tilde{\mu}_{d+\tau,t} = \lfloor \mu_{d,t} \rfloor + k + \tau. \tag{126}$$

From (125)(126) we know (124) holds.

If $C_t^{d,d+\tau}(\hat{\mu}_{d,t}^{(k+1)}, \hat{\mu}_{d+\tau,t}^{(k)}) > c$, assume $0 \leq k \leq A-2$ (the case for $k = A-1$ can be proved likewise), according to (114)(115)(122)(123) we have

$$\tilde{\mu}_{d,t} = \lfloor \mu_{d,t} \rfloor + k + 1, \tag{127}$$
$$\lfloor \mu_{d,t} \rfloor + k + \tau \leq \tilde{\mu}_{d+\tau,t} \leq \lfloor \mu_{d,t} \rfloor + k + 1 + \tau. \tag{128}$$

From (127)(128) we know (124) holds.

Next we will prove that $Q_t(\boldsymbol{x}_t, \boldsymbol{w}_t, \tilde{\boldsymbol{\mu}}_t) \leq Q_t(\boldsymbol{x}_t, \boldsymbol{w}_t, \boldsymbol{\mu}_t)$. Note that by (125)(126)(127)(128) we have

$$(\tilde{\mu}_{d,t} - \epsilon, d) \prec (\tilde{\mu}_{d+\tau,t} + \epsilon, d+\tau), \ \forall \epsilon > 0. \tag{129}$$

Also, it is obvious that

$$\tilde{\mu}_{d,t} \geq \mu_{d,t}, \tag{130}$$
$$\tilde{\mu}_{d+\tau,t} \leq \mu_{d+\tau,t}. \tag{131}$$

We denote all chargers in $\mathcal{G}_{d,t}$ as $k_1, k_2, ..., k_p$ and all chargers in $\mathcal{G}_{d+\tau,t}$ as $l_1, l_2, ..., l_q$. Then the sequence $\mathcal{E} = \{e_{k_1,t+1}, ..., e_{k_p,t+1}, e_{l_1,t+1}, ..., e_{l_q,t+1}\}$ can be calculated by

$$e_{k_n,t+1} = \begin{cases} e_{k_n,t}, & e_{k_n,t} \leq \mu_{d,t}, \\ \mu_{d,t}, & \mu_{d,t} < e_{k_n,t} \leq \mu_{d,t}+1, \\ e_{k_n,t}-1, & e_{k_n,t} > \mu_{d,t}+1. \end{cases} \tag{132}$$

$$e_{l_n,t+1} = \begin{cases} e_{l_n,t}, & e_{l_n,t} \leq \mu_{d+\tau,t}, \\ \mu_{d+\tau,t}, & \mu_{d+\tau,t} < e_{l_n,t} \leq \mu_{d+\tau,t}+1, \\ e_{l_n,t}-1, & e_{l_n,t} > \mu_{d+\tau,t}+1. \end{cases} \tag{133}$$

Similarly, we can calculate the the sequence $\tilde{\mathcal{E}} = \{\tilde{e}_{k_1,t+1}, ..., \tilde{e}_{k_p,t+1}, \tilde{e}_{l_1,t+1}, ..., \tilde{e}_{l_q,t+1}\}$ by

$$\tilde{e}_{k_n,t+1} = \begin{cases} e_{k_n,t}, & e_{k_n,t} \leq \tilde{\mu}_{d,t}, \\ \tilde{\mu}_{d,t}, & \tilde{\mu}_{d,t} < e_{k_n,t} \leq \tilde{\mu}_{d,t}+1, \\ e_{k_n,t}-1, & e_{k_n,t} > \tilde{\mu}_{d,t}+1. \end{cases} \tag{134}$$

$$\tilde{e}_{l_n,t+1} = \begin{cases} e_{l_n,t}, & e_{l_n,t} \leq \tilde{\mu}_{d+\tau,t}, \\ \tilde{\mu}_{d+\tau,t}, & \tilde{\mu}_{d+\tau,t} < e_{l_n,t} \leq \tilde{\mu}_{d+\tau,t}+1, \\ e_{l_n,t}-1, & e_{l_n,t} > \tilde{\mu}_{d+\tau,t}+1. \end{cases} \tag{135}$$

Combining with (130)(131) we know

$$\tilde{e}_{k_n,t+1} \geq e_{k_n,t+1}, \ \forall k_n, \tag{136}$$
$$\tilde{e}_{l_n,t+1} \leq e_{l_n,t+1}, \ \forall l_n. \tag{137}$$

If $\hat{\mathcal{E}} = \mathcal{E}$, then the changing of $\boldsymbol{\mu}_t$ to $\tilde{\boldsymbol{\mu}}_t$ does not alter the actual changing power of any charger, then naturally $Q_t(\boldsymbol{x}_t, \boldsymbol{w}_t, \tilde{\boldsymbol{\mu}}_t) \leq Q_t(\boldsymbol{x}_t, \boldsymbol{w}_t, \boldsymbol{\mu}_t)$.

Otherwise, from (120)(123)(130)(131) we know both set $\{k_n | \tilde{e}_{k_n,t+1} > e_{k_n,t+1}\}$ and $\{l_n | \tilde{e}_{l_n,t+1} < e_{l_n,t+1}\}$ are not empty. Then we recursively define multiple sequences $\hat{\mathcal{E}}^{(0)}, \hat{\mathcal{E}}^{(1)}, \hat{\mathcal{E}}^{(2)}, ...$ as follows:

1) We define $\hat{\mathcal{E}}^{(0)} = \mathcal{E}$,
2) If $\hat{\mathcal{E}}^{(i)} = \{\hat{e}_{k_1,t+1}^{(i)}, ..., \hat{e}_{k_p,t+1}^{(i)}, \hat{e}_{l_1,t+1}^{(i)}, ..., \hat{e}_{l_q,t+1}^{(i)}\}$ is not identical to $\tilde{\mathcal{E}}$, then we select any $k^{(i)}$ and $l^{(i)}$ that satisfy:

$$k^{(i)} \in \{k_n | \hat{e}_{k_n,t+1}^{(i)} < \tilde{e}_{k_n,t+1}\}, \tag{138}$$
$$l^{(i)} \in \{l_n | \hat{e}_{l_n,t+1}^{(i)} > \tilde{e}_{l_n,t+1}\}. \tag{139}$$

Let $m = \min(\tilde{e}_{k^{(i)},t+1} - \hat{e}_{k^{(i)},t+1}^{(i)}, \hat{e}_{l^{(i)},t+1}^{(i)} - \tilde{e}_{l^{(i)},t+1})$, and we construct $\hat{\mathcal{E}}^{(i+1)}$ which is identical to $\hat{\mathcal{E}}^{(i)}$ except for $k^{(i)}$ and $l^{(i)}$:

$$\hat{e}_{k^{(i)},t+1}^{(i+1)} = \hat{e}_{k^{(i)},t+1}^{(i)} + m, \tag{140}$$
$$\hat{e}_{l^{(i)},t+1}^{(i+1)} = \hat{e}_{l^{(i)},t+1}^{(i)} - m. \tag{141}$$

From the construction process of $\hat{\mathcal{E}}^{(i)}$, we have

$$\tilde{e}_{k^{(i)},t+1} > e_{k^{(i)},t+1}, \tag{142}$$
$$\tilde{e}_{l^{(i)},t+1} < e_{l^{(i)},t+1}. \tag{143}$$

Combining (142) with (132)(134) we can infer that

$$\tilde{e}_{k^{(i)},t+1} \leq \tilde{\mu}_{d,t}, \tag{144}$$

and therefore

$$\hat{e}_{k^{(i)},t+1}^{(i+1)} \leq \tilde{\mu}_{d,t}. \tag{145}$$

Similarly, we have

$$\hat{e}^{(i+1)}_{l^{(i)},t+1} \geq \tilde{\mu}_{d+\tau,t}. \tag{146}$$

Let $\hat{\boldsymbol{v}}^{(i)}_t$ and $\hat{\boldsymbol{v}}^{(i+1)}_t$ denote the corresponding charging action of $\hat{\mathcal{E}}^{(i)}$ and $\hat{\mathcal{E}}^{(i+1)}$, then $\hat{\boldsymbol{v}}^{(i)}$ and $\hat{\boldsymbol{v}}^{(i+1)}$ are identical except for $k^{(i)}$ and $l^{(i)}$, with the following property:

$$\mathbf{1}^T\hat{\boldsymbol{v}}^{(i)}_t = \mathbf{1}^T\hat{\boldsymbol{v}}^{(i+1)}_t. \tag{147}$$

On the other hand, combining (140)(141)(145)(146) with (129) we have

$$(\hat{e}^{(i)}_{k^{(i)},t+1}+m-\epsilon, d) \prec (\hat{e}^{(i)}_{l^{(i)},t+1}-m+\epsilon, d+\tau),\ \forall\epsilon>0, \tag{148}$$

and therefore

$$m \leq \inf\{m'|(\hat{e}^{(i)}_{k^{(i)},t+1}+m', d) \not\prec (\hat{e}^{(i)}_{l^{(i)},t+1}-m', d+\tau)\}. \tag{149}$$

Then according to Lemma 2 we know

$$\mathop{\mathbb{E}}_{\boldsymbol{w}_{t+1}}[V_{t+1}(\hat{\boldsymbol{x}}^{(i+1)}_{t+1}, \boldsymbol{w}_{t+1})] \leq \mathop{\mathbb{E}}_{\boldsymbol{w}_{t+1}}[V_{t+1}(\hat{\boldsymbol{x}}^{(i)}_{t+1}, \boldsymbol{w}_{t+1})]. \tag{150}$$

By (147)(150) we have

$$Q_t(\boldsymbol{x}_t, \boldsymbol{w}_t, \hat{\boldsymbol{v}}^{(i+1)}_t) \leq Q_t(\boldsymbol{x}_t, \boldsymbol{w}_t, \hat{\boldsymbol{v}}^{(i)}_t). \tag{151}$$

Note that $\hat{\mathcal{E}}^{(i+1)}$ and $\tilde{\mathcal{E}}$ share at least one more identical element than $\hat{\mathcal{E}}^{(i)}$ and $\tilde{\mathcal{E}}$. Then we know there is some $\tilde{i}$ such that $\hat{\mathcal{E}}^{(\tilde{i})} = \tilde{\mathcal{E}}$, which means $\hat{\boldsymbol{v}}^{(\tilde{i})}$ is the charging action corresponding to $\tilde{\boldsymbol{\mu}}_t$. Repeatedly applying Lemma 2 to the recursive definition of $\hat{\mathcal{E}}^{(i+1)}$ we know $Q_t(\boldsymbol{x}_t, \boldsymbol{w}_t, \tilde{\boldsymbol{\mu}}_t) \leq Q_t(\boldsymbol{x}_t, \boldsymbol{w}_t, \boldsymbol{\mu}_t)$.

Case 2: $\exists\epsilon>0$ such that $(\mu_{d+\tau,t}+\epsilon, d+\tau) \prec (\mu_{d,t}, d)$. From Lemma 3, we have the following properties:

$$\mu_{d,t} > \lfloor \mu_{d+\tau,t}, t\rfloor, \tag{152}$$
$$\mu_{d+\tau,t} < \lceil \mu_{d,t}\rceil. \tag{153}$$

Let $A = \lceil\mu_{d,t}\rceil - \lfloor\mu_{d+\tau,t}\rfloor$, we have $A \geq 1$ by (153). Then we define two sequences $\{\hat{\mu}^{(i)}_{d,t}\}_{i=0}^A$ and $\{\hat{\mu}^{(i)}_{d+\tau,t}\}_{i=0}^A$ as follows:

$$\hat{\mu}^{(i)}_{d,t} = \begin{cases} \lfloor\mu_{d+\tau,t}\rfloor + i, & i=0,1,...,A-1,\\ \mu_{d,t}, & i=A. \end{cases} \tag{154}$$

$$\hat{\mu}^{(i)}_{d+\tau,t} = \begin{cases} \mu_{d+\tau,t}, & i=0,\\ \lfloor\mu_{d+\tau,t}\rfloor + i, & i=1,...,A. \end{cases} \tag{155}$$

We can also verify that $\hat{c}^{(0)} \geq \hat{c}^{(1)} \geq \cdots \geq \hat{c}^{(A)}$ and $\hat{c}^{(0)} \geq c \geq \hat{c}^{(A)}$. Then we let

$$k = \min\{0 \leq k' \leq A-1 | \hat{c}^{(k')} \geq c \text{ and } \hat{c}^{(k'+1)} \leq c\}. \tag{156}$$

If $C^{d,d+\tau}_t(\hat{\mu}^{(k)}_{d,t}, \hat{\mu}^{(k+1)}_{d+\tau,t}) \leq c$, then we define

$$\tilde{\mu}_{d,t} = \hat{\mu}^{(k)}_{d,t}, \tag{157}$$

$$\tilde{\mu}_{d+\tau,t} = \min\{\hat{\mu}^{(k)}_{d+\tau,t} \leq \mu_{d+\tau} \leq \hat{\mu}^{(k+1)}_{d+\tau,t} | C^{d,d+\tau}_t(\hat{\mu}^{(k)}_{d,t}, \mu_{d+\tau}) = c\}. \tag{158}$$

Otherwise, we define

$$\tilde{\mu}_{d,t} = \min\{\hat{\mu}^{(k)}_{d,t} \leq \mu_d \leq \hat{\mu}^{(k+1)}_{d,t} | C^{d,d+\tau}_t(\mu_d, \hat{\mu}^{(k+1)}_{d+\tau,t}) = c\}, \tag{159}$$

$$\tilde{\mu}_{d+\tau,t} = \hat{\mu}^{(k+1)}_{d+\tau,t}. \tag{160}$$

The rest of the proof is similar to Case 1 and is therefore omitted. □

With Lemma 5, we denote the mapping from $\mu_{d_1,t}, \mu_{d_2,t}$ to $\tilde{\mu}_{d_1,t}, \tilde{\mu}_{d_2,t}$ as $\mathcal{F}^{d_1,d_2}_{2,t}$:

$$(\tilde{\mu}_{d_1,t}, \tilde{\mu}_{d_2,t}) = \mathcal{F}^{d_1,d_2}_{2,t}(\mu_{d_1,t}, \mu_{d_2,t}). \tag{161}$$

Combining (114)(114)(125)(126)(127)(128)(154)(155) in proof of Lemma 5 with Lemma 3, we can verify the following corollaries:

Corollary 1. *Suppose $\mu_{d_1,t}, \mu_{d_2,t}$ are not charging order-compliant. Let*

$$(\tilde{\mu}_{d_1,t}, \tilde{\mu}_{d_2,t}) = \mathcal{F}^{d_1,d_2}_{2,t}(\mu_{d_1,t}, \mu_{d_2,t}). \tag{162}$$

If $\tilde{\mu}_{d_1,t} > \mu_{d_1,t}$, then $\tilde{\mu}_{d_2,t} \leq \mu_{d_2,t}$.

Corollary 2. *Suppose $\mu_{d_1,t}, \mu_{d_2,t}$ are not charging order-compliant. Let*

$$(\tilde{\mu}_{d_1,t}, \tilde{\mu}_{d_2,t}) = \mathcal{F}^{d_1,d_2}_{2,t}(\mu_{d_1,t}, \mu_{d_2,t}). \tag{163}$$

Then $\exists j \in \{1,2\}$ such that $\tilde{\mu}_{d_j,t} \neq \mu_{d_j,t}$ and also $\tilde{\mu}_{d_j,t}$ is an integer.

Corollary 3. *Suppose $\mu_{d_1,t}, \mu_{d_2,t}$ are charging order-compliant, if $\hat{\mu}_{d_2,t} > \mu_{d_2,t}$ is another charging threshold for deadline d_2 with $\mu_{d_1,t}, \hat{\mu}_{d_2,t}$ not charging order-compliant. Let*

$$(\tilde{\mu}_{d_1,t}, \tilde{\mu}_{d_2,t}) = \mathcal{F}^{d_1,d_2}_{2,t}(\mu_{d_1,t}, \hat{\mu}_{d_2,t}), \tag{164}$$

then

$$\tilde{\mu}_{d_1,t} \geq \mu_{d_1,t}, \tag{165}$$
$$\mu_{d_2,t} \leq \tilde{\mu}_{d_2,t} \leq \hat{\mu}_{d_2,t}, \tag{166}$$
$$\tilde{\mu}_{d_1,t} \leq \lfloor\tilde{\mu}_{d_2,t}\rfloor - (d_2-d_1) + 1, \tag{167}$$
$$\tilde{\mu}_{d_2,t} \geq \lceil\tilde{\mu}_{d_1,t}\rceil + (d_2-d_1) - 1, \tag{168}$$
$$\tilde{\mu}_{d_1,t} \leq \lceil\hat{\mu}_{d_2,t}\rceil - (d_2-d_1), \tag{169}$$
$$\tilde{\mu}_{d_2,t} \geq \lfloor\mu_{d_1,t}\rfloor + (d_2-d_1). \tag{170}$$

Corollary 4. *Suppose $\mu_{d_1,t}, \mu_{d_2,t}$ are charging order-compliant, if $\hat{\mu}_{d_1,t} < \mu_{d_1,t}$ is another charging threshold for deadline d_1 with $\hat{\mu}_{d_1,t}, \mu_{d_2,t}$ not charging order-compliant. Let*

$$(\tilde{\mu}_{d_1,t}, \tilde{\mu}_{d_2,t}) = \mathcal{F}^{d_1,d_2}_{2,t}(\hat{\mu}_{d_1,t}, \mu_{d_2,t}), \tag{171}$$

then

$$\tilde{\mu}_{d_2,t} \leq \mu_{d_2,t}, \tag{172}$$
$$\hat{\mu}_{d_1,t} \leq \tilde{\mu}_{d_1,t} \leq \mu_{d_1,t}, \tag{173}$$
$$\tilde{\mu}_{d_1,t} \leq \lfloor\tilde{\mu}_{d_2,t}\rfloor - (d_2-d_1) + 1, \tag{174}$$
$$\tilde{\mu}_{d_2,t} \geq \lceil\tilde{\mu}_{d_1,t}\rceil + (d_2-d_1) - 1, \tag{175}$$
$$\tilde{\mu}_{d_1,t} \leq \lceil\mu_{d_2,t}\rceil - (d_2-d_1), \tag{176}$$
$$\tilde{\mu}_{d_2,t} \geq \lfloor\hat{\mu}_{d_1,t}\rfloor + (d_2-d_1). \tag{177}$$

Lemma 6. *For any charging threshold $\boldsymbol{\mu}_t$, and any m deadlines $d_1, \cdots, d_{m-1}, d_m$, satisfying $t < d_1 < \cdots < d_{m-1} < d_m \le T$, there is another charging threshold $\tilde{\boldsymbol{\mu}}_t$ that is identical to $\boldsymbol{\mu}_t$ except for $\tilde{\mu}_{d_1,t}, \cdots, \tilde{\mu}_{d_{m-1},t}, \tilde{\mu}_{d_m,t}$ which are charging order-compliant, and $Q(\boldsymbol{x}_t, \boldsymbol{w}_t, \tilde{\boldsymbol{\mu}}_t) \le Q(\boldsymbol{x}_t, \boldsymbol{w}_t, \boldsymbol{\mu}_t)$.*

Proof. We prove this lemma by induction. Notice that the case for $m = 2$ have been proved in Lemma 5.

Suppose the lemma holds for any m deadlines $d_1, \cdots, d_{m-1}, d_m$. Then we denote the mapping from $(\mu_{d_1,t}, \cdots, \mu_{d_{m-1},t}, \mu_{d_m,t})$ to $(\tilde{\mu}_{d_1,t}, \cdots, \tilde{\mu}_{d_{m-1},t}, \tilde{\mu}_{d_m,t})$ as $\mathcal{F}_{m,t}^{d_1,\cdots,d_m}$:

$$(\tilde{\mu}_{d_1,t}, \cdots, \tilde{\mu}_{d_m,t}) = \mathcal{F}_{m,t}^{d_1,\cdots,d_m}(\mu_{d_1,t}, \cdots, \mu_{d_m,t}). \tag{178}$$

Now we will prove the case for $m+1$ charging thresholds $\mu_{d_1,t}, \cdots, \mu_{d_{m-1},t}, \mu_{d_m,t}, \mu_{d_{m+1},t}$. We first assume that $\mu_{d_1,t}, \cdots, \mu_{d_{m-1},t}, \mu_{d_m,t}$ are charging order-compliant. Otherwise, we replace it with $\mathcal{F}_{m,t}^{d_1,\cdots,d_m}(\mu_{d_1,t}, \cdots, \mu_{d_m,t})$ without increasing $Q(\boldsymbol{x}_t, \boldsymbol{w}_t, \boldsymbol{\mu}_t)$.

We need two additional auxiliary properties to complete the inductive proof, which are described as follows.

Property 1: Suppose $\mu_{d_1,t}, \cdots, \mu_{d_{m-1},t}, \mu_{d_m,t}$ are charging order-compliant, if $\hat{\mu}_{d_m,t} > \mu_{d_m,t}$ is another charging threshold for deadline d_m with $\mu_{d_{m-1},t}, \hat{\mu}_{d_m,t}$ not charging order-compliant. Let

$$(\tilde{\mu}_{d_1,t}, \cdots, \tilde{\mu}_{d_m,t}) = \mathcal{F}_{m,t}^{d_1,\cdots,d_m}(\mu_{d_1,t}, \cdots, \mu_{d_{m-1},t}, \hat{\mu}_{d_m,t}), \tag{179}$$

then

$$\begin{aligned}
&\tilde{\mu}_{d_j,t} \ge \mu_{d_j,t}, \ j = 1, \cdots, m-1, && (180)\\
&\mu_{d_m,t} \le \tilde{\mu}_{d_m,t} \le \hat{\mu}_{d_m,t}, && (181)\\
&\tilde{\mu}_{d_{m-1},t} \le \lfloor \tilde{\mu}_{d_m,t} \rfloor - (d_m - d_{m-1}) + 1, && (182)\\
&\tilde{\mu}_{d_m,t} \ge \lceil \tilde{\mu}_{d_{m-1},t} \rceil + (d_m - d_{m-1}) - 1, && (183)\\
&\tilde{\mu}_{d_{m-1},t} \le \lceil \hat{\mu}_{d_m,t} \rceil - (d_m - d_{m-1}), && (184)\\
&\tilde{\mu}_{d_m,t} \ge \lfloor \mu_{d_{m-1},t} \rfloor + (d_m - d_{m-1}). && (185)
\end{aligned}$$

Property 2: Suppose $\mu_{d_1,t}, \cdots, \mu_{d_m,t}$ are not charging order-compliant. Let

$$(\tilde{\mu}_{d_1,t}, \cdots, \tilde{\mu}_{d_m,t}) = \mathcal{F}_{m,t}^{d_1,\cdots,d_m}(\mu_{d_1,t}, \cdots, \mu_{d_m,t}). \tag{186}$$

Then $\exists j \in \{1, \cdots, m\}$ such that $\tilde{\mu}_{d_j,t} \ne \mu_{d_j,t}$ and also $\tilde{\mu}_{d_j,t}$ is an integer.

From the Corollary 3 and Corollary 2 we know the two properties above hold for $m = 2$. Now we suppose these two properties hold for any deadline $d_1, \cdots, d_{m-1}, d_m$, and we will also prove these two properties for the case of $m+1$ charging thresholds. We will perform our proof in multiple steps.

Step 1: if $\mu_{d_m,t}, \mu_{d_{m+1},t}$ are charging order-compliant, then $\mu_{d_1,t}, \cdots, \mu_{d_{m-1},t}, \mu_{d_m,t}, \mu_{d_{m+1},t}$ have been charging order-compliant, and the desired properties hold naturally. Otherwise, we define $\tilde{\boldsymbol{\mu}}_t^{(1)}$ as follows[4]:

$$\begin{aligned}
&(\tilde{\mu}_{d_1,t}^{(1)}, \cdots, \tilde{\mu}_{d_{m-1},t}^{(1)}) = (\mu_{d_1,t}, \cdots, \mu_{d_{m-1},t}), && (187)\\
&(\tilde{\mu}_{d_m,t}^{(1)}, \tilde{\mu}_{d_{m+1},t}^{(1)}) = \mathcal{F}_{2,t}^{d_m,d_{m+1}}(\mu_{d_m,t}, \mu_{d_{m+1},t}). && (188)
\end{aligned}$$

According to the induction hypothesis for 2 charging thresholds, we have

$$Q(\boldsymbol{x}_t, \boldsymbol{w}_t, \tilde{\boldsymbol{\mu}}_t^{(1)}) \le Q(\boldsymbol{x}_t, \boldsymbol{w}_t, \boldsymbol{\mu}_t). \tag{189}$$

If $\tilde{\mu}_{d_{m-1},t}^{(1)}, \tilde{\mu}_{d_m,t}^{(1)}$ are charging order-compliant, then $\tilde{\mu}_{d_1,t}^{(1)}, \cdots, \tilde{\mu}_{d_{m-1},t}^{(1)}, \tilde{\mu}_{d_m,t}^{(1)}, \tilde{\mu}_{d_{m+1},t}^{(1)}$ are charging order-compliant, and $\tilde{\boldsymbol{\mu}}_t = \tilde{\boldsymbol{\mu}}_t^{(1)}$ satisfy the desired properties. Otherwise, since $\tilde{\mu}_{d_{m-1},t}^{(1)}, \mu_{d_m,t}$ are charging order-compliant, we have

$$\tilde{\mu}_{d_m,t}^{(1)} \ne \mu_{d_m,t}. \tag{190}$$

In the following discussion, we will assume

$$\tilde{\mu}_{d_m,t}^{(1)} > \mu_{d_m,t}. \tag{191}$$

In fact, the case for $\tilde{\mu}_{d_m,t}^{(1)} < \mu_{d_m,t}$ can be proved likewise. Then according to Corollary 1, we have

$$\tilde{\mu}_{d_{m+1},t}^{(1)} \le \mu_{d_{m+1},t}. \tag{192}$$

Then we define $\bar{\boldsymbol{\mu}}_t^{(1)}$ as follows:

$$\begin{aligned}
(\bar{\mu}_{d_1,t}^{(1)}, \cdots, \bar{\mu}_{d_m,t}^{(1)}) &= \mathcal{F}_{m,t}^{d_1,\cdots,d_m}(\tilde{\mu}_{d_1,t}^{(1)}, \cdots, \tilde{\mu}_{d_m,t}^{(1)}), && (193)\\
\bar{\mu}_{d_{m+1},t}^{(1)} &= \tilde{\mu}_{d_{m+1},t}^{(1)}. && (194)
\end{aligned}$$

According to the induction hypothesis for m charging thresholds, we have

$$Q(\boldsymbol{x}_t, \boldsymbol{w}_t, \bar{\boldsymbol{\mu}}_t^{(1)}) \le Q(\boldsymbol{x}_t, \boldsymbol{w}_t, \tilde{\boldsymbol{\mu}}_t^{(1)}) \le Q(\boldsymbol{x}_t, \boldsymbol{w}_t, \boldsymbol{\mu}_t). \tag{195}$$

According to (191) and Property 1, we have

$$\begin{aligned}
&\bar{\mu}_{d_j,t}^{(1)} \ge \tilde{\mu}_{d_j,t}^{(1)} = \mu_{d_j,t}, \ j = 1, \cdots, m-1, && (196)\\
&\mu_{d_m,t} \le \bar{\mu}_{d_m,t}^{(1)} \le \tilde{\mu}_{d_m,t}^{(1)}, && (197)\\
&\bar{\mu}_{d_{m-1},t}^{(1)} \le \left\lfloor \bar{\mu}_{d_m,t}^{(1)} \right\rfloor - (d_m - d_{m-1}) + 1, && (198)\\
&\bar{\mu}_{d_m,t}^{(1)} \ge \left\lceil \bar{\mu}_{d_{m-1},t}^{(1)} \right\rceil + (d_m - d_{m-1}) - 1, && (199)\\
&\bar{\mu}_{d_{m-1},t}^{(1)} \le \left\lceil \tilde{\mu}_{d_m,t}^{(1)} \right\rceil - (d_m - d_{m-1}), && (200)\\
&\bar{\mu}_{d_m,t}^{(1)} \ge \left\lfloor \tilde{\mu}_{d_{m-1},t}^{(1)} \right\rfloor + (d_m - d_{m-1}). && (201)
\end{aligned}$$

If $\bar{\mu}_{d_m,t}^{(1)}, \bar{\mu}_{d_{m+1},t}^{(1)}$ are charging order-compliant, then $\bar{\mu}_{d_1,t}^{(1)}, \cdots, \bar{\mu}_{d_{m-1},t}^{(1)}, \bar{\mu}_{d_m,t}^{(1)}, \bar{\mu}_{d_{m+1},t}^{(1)}$ are charging order-compliant, and $\tilde{\boldsymbol{\mu}}_t = \bar{\boldsymbol{\mu}}_t^{(1)}$ satisfy the desired properties. Otherwise, since $\tilde{\mu}_{d_m,t}^{(1)}, \bar{\mu}_{d_{m+1},t}^{(1)}$ are charging order-compliant, we have

$$\bar{\mu}_{d_m,t}^{(1)} < \tilde{\mu}_{d_m,t}^{(1)}. \tag{202}$$

[4]In the following proof, we will only change the charging thresholds for deadlines $d_1, \cdots, d_m, d_{m+1}$ and keep other charging thresholds unchanged.

Then we define $\tilde{\boldsymbol{\mu}}_t^{(2)}$ as follows:

$$(\tilde{\mu}_{d_1,t}^{(2)},\cdots,\tilde{\mu}_{d_{m-1},t}^{(2)})=(\bar{\mu}_{d_1,t}^{(1)},\cdots,\bar{\mu}_{d_{m-1},t}^{(1)}), \tag{203}$$

$$(\tilde{\mu}_{d_m,t}^{(2)},\tilde{\mu}_{d_{m+1},t}^{(2)})=\mathcal{F}_{2,t}^{d_m,d_{m+1}}(\bar{\mu}_{d_m,t}^{(1)},\bar{\mu}_{d_{m+1},t}^{(1)}). \tag{204}$$

According to the induction hypothesis for 2 charging thresholds, we have

$$Q(\boldsymbol{x}_t,\boldsymbol{w}_t,\tilde{\boldsymbol{\mu}}_t^{(2)})\le Q(\boldsymbol{x}_t,\boldsymbol{w}_t,\bar{\boldsymbol{\mu}}_t^{(1)})\le Q(\boldsymbol{x}_t,\boldsymbol{w}_t,\boldsymbol{\mu}_t). \tag{205}$$

According to (202) and Corollary 4, we have

$$\tilde{\mu}_{d_{m+1},t}^{(2)}\le\bar{\mu}_{d_{m+1},t}^{(1)}, \tag{206}$$

$$\bar{\mu}_{d_m,t}^{(1)}\le\tilde{\mu}_{d_m,t}^{(2)}\le\tilde{\mu}_{d_m,t}^{(1)}, \tag{207}$$

$$\tilde{\mu}_{d_m,t}^{(2)}\le\left\lfloor\tilde{\mu}_{d_{m+1},t}^{(2)}\right\rfloor-(d_{m+1}-d_m)+1, \tag{208}$$

$$\tilde{\mu}_{d_{m+1},t}^{(2)}\ge\left\lceil\tilde{\mu}_{d_m,t}^{(2)}\right\rceil+(d_{m+1}-d_m)-1, \tag{209}$$

$$\tilde{\mu}_{d_m,t}^{(2)}\le\left\lceil\bar{\mu}_{d_{m+1},t}^{(1)}\right\rceil-(d_{m+1}-d_m), \tag{210}$$

$$\tilde{\mu}_{d_{m+1},t}^{(2)}\ge\left\lfloor\bar{\mu}_{d_m,t}^{(1)}\right\rfloor+(d_{m+1}-d_m). \tag{211}$$

Step 2: Now we repeat the process in Step 1 for $i=2,3,\cdots$ to define $\bar{\boldsymbol{\mu}}_t^{(i)}$ and $\tilde{\boldsymbol{\mu}}_t^{(i+1)}$:

$$(\bar{\mu}_{d_1,t}^{(i)},\cdots,\bar{\mu}_{d_m,t}^{(i)})=\mathcal{F}_{m,t}^{d_1,\cdots,d_m}(\tilde{\mu}_{d_1,t}^{(i)},\cdots,\tilde{\mu}_{d_m,t}^{(i)}), \tag{212}$$

$$\bar{\mu}_{d_{m+1},t}^{(i)}=\tilde{\mu}_{d_{m+1},t}^{(i)}, \tag{213}$$

$$(\tilde{\mu}_{d_1,t}^{(i+1)},\cdots,\tilde{\mu}_{d_{m-1},t}^{(i+1)})=(\bar{\mu}_{d_1,t}^{(i)},\cdots,\bar{\mu}_{d_{m-1},t}^{(i)}), \tag{214}$$

$$(\tilde{\mu}_{d_m,t}^{(i+1)},\tilde{\mu}_{d_{m+1},t}^{(i+1)})=\mathcal{F}_{2,t}^{d_m,d_{m+1}}(\bar{\mu}_{d_m,t}^{(i)},\bar{\mu}_{d_{m+1},t}^{(i)}). \tag{215}$$

According to the induction hypothesis for 2 charging thresholds, we have

$$Q(\boldsymbol{x}_t,\boldsymbol{w}_t,\bar{\boldsymbol{\mu}}_t^{(i)})\le Q(\boldsymbol{x}_t,\boldsymbol{w}_t,\boldsymbol{\mu}_t), \tag{216}$$

$$Q(\boldsymbol{x}_t,\boldsymbol{w}_t,\tilde{\boldsymbol{\mu}}_t^{(i+1)})\le Q(\boldsymbol{x}_t,\boldsymbol{w}_t,\boldsymbol{\mu}_t). \tag{217}$$

Besides, we have the following properties:

$$\bar{\mu}_{d_j,t}^{(i)}\ge\tilde{\mu}_{d_j,t}^{(i)},\ j=1,\cdots,m-1, \tag{218}$$

$$\bar{\mu}_{d_m,t}^{(i-1)}\le\bar{\mu}_{d_m,t}^{(i)}\le\tilde{\mu}_{d_m,t}^{(i)}, \tag{219}$$

$$\tilde{\mu}_{d_{m+1},t}^{(i+1)}\le\bar{\mu}_{d_{m+1},t}^{(i)}, \tag{220}$$

$$\bar{\mu}_{d_m,t}^{(i)}\le\tilde{\mu}_{d_m,t}^{(i+1)}\le\tilde{\mu}_{d_m,t}^{(i)}, \tag{221}$$

$$\bar{\mu}_{d_m,t}^{(i)}\ge\left\lfloor\tilde{\mu}_{d_{m-1},t}^{(i)}\right\rfloor+(d_m-d_{m-1}), \tag{222}$$

$$\tilde{\mu}_{d_m,t}^{(i+1)}\le\left\lceil\bar{\mu}_{d_{m+1},t}^{(i)}\right\rceil-(d_{m+1}-d_m). \tag{223}$$

If for some $i\in\{2,3,\cdots\}$, $\bar{\boldsymbol{\mu}}_t^{(i)}$ or $\tilde{\boldsymbol{\mu}}_t^{(i+1)}$ is order-compatible, then we can finish our proof. Otherwise, we have the following infinite sequences:

$$\mu_{d_j,t}=\tilde{\mu}_{d_j,t}^{(1)}\le\bar{\mu}_{d_j,t}^{(1)}=\tilde{\mu}_{d_j,t}^{(2)}\le\bar{\mu}_{d_j,t}^{(2)}=\cdots,\ 1\le j\le m-1, \tag{224}$$

$$\mu_{d_m,t}\le\bar{\mu}_{d_m,t}^{(1)}\le\bar{\mu}_{d_m,t}^{(2)}\le\cdots, \tag{225}$$

$$\tilde{\mu}_{d_m,t}^{(1)}\ge\tilde{\mu}_{d_m,t}^{(2)}\ge\cdots, \tag{226}$$

$$\mu_{d_{m+1},t}\ge\tilde{\mu}_{d_{m+1},t}^{(1)}=\bar{\mu}_{d_{m+1},t}^{(1)}\ge\tilde{\mu}_{d_{m+1},t}^{(2)}=\bar{\mu}_{d_{m+1},t}^{(2)}\ge\cdots. \tag{227}$$

We can verify that all the infinite sequences are bounded by 0 and $\mu_{d_{m+1},t}$. Since monotonic sequence converges if it is bounded [37], we have

$$\lim_{i\to\infty}\tilde{\mu}_{d_j,t}^{(i)}=\lim_{i\to\infty}\bar{\mu}_{d_j,t}^{(i)}=\mu_{d_j,t}^{(\infty)},1\le j\le m-1, \tag{228}$$

$$\lim_{i\to\infty}\bar{\mu}_{d_m,t}^{(i)}=\bar{\mu}_{d_m,t}^{(\infty)}, \tag{229}$$

$$\lim_{i\to\infty}\tilde{\mu}_{d_m,t}^{(i)}=\tilde{\mu}_{d_m,t}^{(\infty)}, \tag{230}$$

$$\lim_{i\to\infty}\tilde{\mu}_{d_{m+1},t}^{(i)}=\lim_{i\to\infty}\bar{\mu}_{d_{m+1},t}^{(i)}=\mu_{d_{m+1},t}^{(\infty)}. \tag{231}$$

$$\tag{232}$$

And according to (219), we have

$$\bar{\mu}_{d_m,t}^{(\infty)}\le\tilde{\mu}_{d_m,t}^{(\infty)}. \tag{233}$$

Step 3: In this step, we will show that $\bar{\mu}_{d_m,t}^{(\infty)}<\tilde{\mu}_{d_m,t}^{(\infty)}$, and both $\bar{\mu}_{d_m,t}^{(\infty)}$ and $\tilde{\mu}_{d_m,t}^{(\infty)}$ are integers.

Let $k'_{d_{m+1}}=\min\{i|\left|\tilde{\mu}_{d_{m+1},t}^{(i+1)}-\mu_{d_{m+1},t}^{(\infty)}\right|<1\}$, then there is at most one integer $k''_{d_{m+1}}>k'_{d_{m+1}}$ such that $\tilde{\mu}_{d_{m+1},t}^{(k''_{d_{m+1}}+1)}$ is an integer and different from $\bar{\mu}_{d_{m+1},t}^{(k''_{d_{m+1}})}$. Otherwise, if two numbers $k'''_{d_{m+1}}>k''_{d_{m+1}}>k'_{d_{m+1}}$ satisfy

1) $\tilde{\mu}_{d_{m+1},t}^{(k''_{d_{m+1}}+1)}$ is an integer and different from $\bar{\mu}_{d_{m+1},t}^{(k''_{d_{m+1}})}$,
2) $\tilde{\mu}_{d_{m+1},t}^{(k'''_{d_{m+1}}+1)}$ is an integer and different from $\bar{\mu}_{d_{m+1},t}^{(k'''_{d_{m+1}})}$,

then according to the monotonicity described in (227), we have

$$\left|\tilde{\mu}_{d_{m+1},t}^{(k'''_{d_{m+1}}+1)}-\tilde{\mu}_{d_{m+1},t}^{(k'_{d_{m+1}}+1)}\right|>1, \tag{234}$$

which contradicts with the definition of $k'_{d_{m+1}}$.

Then we let $k_{d_{m+1}}=k''_{d_{m+1}}$(or $k_{d_{m+1}}=k'_{d_{m+1}}$ if $k''_{d_{m+1}}$ does not exist). Then $\forall i>k_{d_{m+1}}$, the situation where $\tilde{\mu}_{d_{m+1},t}^{(i+1)}$ is an integer different from $\bar{\mu}_{d_{m+1},t}^{(i)}$ will not happen. Therefore, according to Corollary 2, $\forall i>k_{d_{m+1}}$, $\tilde{\mu}_{d_m,t}^{(i+1)}$ is an integer different from $\bar{\mu}_{d_m,t}^{(i)}$.

Similarly, we can find k_{d_j} for each $j\in\{1,\cdots,m-1\}$ such that $\forall i>k_{d_j}$, the situation where $\bar{\mu}_{d_j,t}^{(i)}$ is an integer different from $\tilde{\mu}_{d_j,t}^{(i)}$ will not happen. Let $k'=\max\{k_{d_1},\cdots,k_{d_{m-1}}\}$, according to Property 2, $\forall i>k'$, $\bar{\mu}_{d_m,t}^{(i)}$ is an integer different from $\tilde{\mu}_{d_m,t}^{(i)}$.

Therefore, $\forall i>\max(k_{d_{m+1}},k')$, $\bar{\mu}_{d_m,t}^{(i)}$ is an integer different from $\tilde{\mu}_{d_m,t}^{(i)}$, and $\tilde{\mu}_{d_m,t}^{(i+1)}$ is an integer different from $\bar{\mu}_{d_m,t}^{(i)}$. Considering the monotonicity described in (225) and (226), there is some integer K such that $\forall i\ge K$, $\tilde{\mu}_{d_m,t}^{(i)}=\tilde{\mu}_{d_m,t}^{(\infty)}$, and $\bar{\mu}_{d_m,t}^{(i)}=\bar{\mu}_{d_m,t}^{(\infty)}$. This shows that $\tilde{\mu}_{d_m,t}^{(\infty)}$ and $\bar{\mu}_{d_m,t}^{(\infty)}$ are two different integers. Combining (233), we have

$$\bar{\mu}_{d_m,t}^{(\infty)}<\tilde{\mu}_{d_m,t}^{(\infty)}. \tag{235}$$

Step 4: In this step, we will prove that changing the charging threshold of deadline d_m from $\bar{\mu}_{d_m,t}^{(\infty)}$ to $\tilde{\mu}_{d_m,t}^{(\infty)}$ will not alter actual charging power of any chargers. In other word, $\forall l\in\mathcal{G}_{d_m,t}$, we have $e_{l,t}\le\bar{\mu}_{d_m,t}^{(\infty)}$ or $e_{l,t}\ge\tilde{\mu}_{d_m,t}^{(\infty)}+1$.

We will prove this by contradiction. First we define the (normalized) total charging rate that is needed to charge chargers with deadline d to charging threshold μ_d as:

$$C_t^d(\mu_d) = \sum_{i\in\mathcal{G}_{d,t}} \min(\max(e_{i,t}-\mu_d,0),1). \tag{236}$$

With this definition, if the claim does not hold, then $\exists B>0$, $\forall i\in\mathbb{N}$, such that

$$\left|C_t^{d_m}(\tilde{\mu}_{d_m}^{(i+1)}) - C_t^{d_m}(\bar{\mu}_{d_m}^{(i)})\right| \geq B. \tag{237}$$

Then by (120)(123) we have

$$\left|C_t^{d_{m+1}}(\tilde{\mu}_{d_{m+1}}^{(i+1)}) - C_t^{d_{m+1}}(\bar{\mu}_{d_{m+1}}^{(i)})\right| \geq B. \tag{238}$$

However, by (231) we have

$$\begin{aligned}&\left|C_t^{d_{m+1}}(\tilde{\mu}_{d_{m+1}}^{(i+1)}) - C_t^{d_{m+1}}(\bar{\mu}_{d_{m+1}}^{(i)})\right|\\ \leq&\left|\mathcal{G}_{d_{m+1},t}\right|\left|\tilde{\mu}_{d_{m+1}}^{(i+1)}-\bar{\mu}_{d_{m+1}}^{(i)}\right|\\ \to&0,\ i\to\infty,\end{aligned} \tag{239}$$

which contradicts with (238).

Step 5: Recall that in Step 3, we prove that there is some integer K such that

1) $\bar{\mu}_{d_m,t}^{(K)} = \bar{\mu}_{d_m,t}^{(\infty)} < \tilde{\mu}_{d_m,t}^{(\infty)} = \tilde{\mu}_{d_m,t}^{(K)}$,
2) $\bar{\mu}_{d_1,t}^{(K)},\cdots,\bar{\mu}_{d_{m-1},t}^{(K)},\bar{\mu}_{d_m,t}^{(K)}$ are charging order-compliant,
3) $\tilde{\mu}_{d_m,t}^{(K)},\tilde{\mu}_{d_{m+1},t}^{(K)}$ are charging order-compliant.

In the rest of Step 5, we will prove that $\bar{\mu}_{d_{m-1},t}^{(K)}$ and $\tilde{\mu}_{d_{m+1},t}^{(K)}$ are not charging order-compliant.

We will prove this by contradiction. If $\bar{\mu}_{d_{m-1},t}^{(K)}$ and $\tilde{\mu}_{d_{m+1},t}^{(K)}$ are charging order-compliant, by Lemma 3 we have

$$\tilde{\mu}_{d_{m+1},t}^{(K)} \leq \left\lfloor \bar{\mu}_{d_{m-1},t}^{(K)}\right\rfloor + d_{m+1}-d_{m-1}, \tag{240}$$

and therefore

$$\left\lceil \tilde{\mu}_{d_{m+1},t}^{(K)}\right\rceil - \left\lfloor \bar{\mu}_{d_{m-1},t}^{(K)}\right\rfloor \leq d_{m+1}-d_{m-1}. \tag{241}$$

On the other hand, by (214)(222) we have

$$\bar{\mu}_{d_m,t}^{(K+1)} - \left\lfloor \bar{\mu}_{d_{m-1},t}^{(K)}\right\rfloor \geq d_m-d_{m-1}. \tag{242}$$

By (213)(223) we have

$$\left\lceil \tilde{\mu}_{d_{m+1},t}^{(K)}\right\rceil - \tilde{\mu}_{d_m,t}^{(K+1)} \geq d_{m+1}-d_m. \tag{243}$$

Since $\bar{\mu}_{d_m,t}^{(K+1)}$ and $\tilde{\mu}_{d_m,t}^{(K+1)}$ are two different integers, we have

$$\tilde{\mu}_{d_m,t}^{(K+1)} - \bar{\mu}_{d_m,t}^{(K+1)} \geq 1. \tag{244}$$

Combining (242)(243)(244) we have

$$\left\lceil \tilde{\mu}_{d_{m+1},t}^{(K)}\right\rceil - \left\lfloor \bar{\mu}_{d_{m-1},t}^{(K)}\right\rfloor \geq d_{m+1}-d_{m-1}+1, \tag{245}$$

which contradicts with (241).

Step 6: Now we define $\tilde{\mu}_{d_1,t},\cdots,\tilde{\mu}_{d_{m-1},t},\tilde{\mu}_{d_{m+1},t}$ as follows:

$$\begin{aligned}&(\tilde{\mu}_{d_1,t},\cdots,\tilde{\mu}_{d_{m-1},t},\tilde{\mu}_{d_{m+1},t})\\ =&\mathcal{F}_{m,t}^{d_1,\cdots,d_{m-1},d_{m+1}}(\bar{\mu}_{d_1,t}^{(K)},\cdots,\bar{\mu}_{d_{m-1},t}^{(K)},\tilde{\mu}_{d_{m+1},t}^{(K)}).\end{aligned} \tag{246}$$

Note that we have not defined the value of $\tilde{\mu}_{d_m,t}$. Instead, we will first find the range of $\tilde{\mu}_{d_m,t}$ such that $\tilde{\mu}_{d_1,t},\cdots,\tilde{\mu}_{d_{m-1},t},\tilde{\mu}_{d_m,t},\tilde{\mu}_{d_{m+1},t}$ is order compliant. Since $\tilde{\mu}_{d_1,t},\cdots,\tilde{\mu}_{d_{m-1},t},\tilde{\mu}_{d_{m+1},t}$ have been charging order-compliant, all we need is that $\tilde{\mu}_{d_{m-1},t},\tilde{\mu}_{d_m,t},\tilde{\mu}_{d_{m+1},t}$ are charging order-compliant. According to Lemma 3, this is equivalent to

$$\begin{aligned}\tilde{\mu}_{d_m,t}\in&\left[\left\lceil\tilde{\mu}_{d_{m-1},t}\right\rceil,\left\lfloor\tilde{\mu}_{d_{m-1},t}\right\rfloor+(d_m-d_{m-1})\right]\\ &\cap\left[\left\lceil\tilde{\mu}_{d_{m+1},t}\right\rceil-(d_{m+1}-d_m),\left\lfloor\tilde{\mu}_{d_{m+1},t}\right\rfloor\right].\end{aligned} \tag{247}$$

In the rest of step 6, we will prove that (247) is equivalent to the following condition:

$$\begin{aligned}&\tilde{\mu}_{d_m,t}\\ \in&\left[\left\lceil\tilde{\mu}_{d_{m+1},t}\right\rceil-(d_{m+1}-d_m),\left\lfloor\tilde{\mu}_{d_{m-1},t}\right\rfloor+(d_m-d_{m-1})\right].\end{aligned} \tag{248}$$

First, by (182) we have

$$\begin{aligned}\tilde{\mu}_{d_{m-1},t}&\leq\left\lfloor\tilde{\mu}_{d_{m+1},t}\right\rfloor-(d_{m+1}-d_{m-1})+1\\ &\leq\left\lceil\tilde{\mu}_{d_{m+1},t}\right\rceil-d_{m+1}+(d_{m-1}+1)\\ &\leq\left\lceil\tilde{\mu}_{d_{m+1},t}\right\rceil-(d_{m+1}-d_m),\end{aligned} \tag{249}$$

and therefore

$$\left\lceil\tilde{\mu}_{d_{m-1},t}\right\rceil\leq\left\lceil\tilde{\mu}_{d_{m+1},t}\right\rceil-(d_{m+1}-d_m). \tag{250}$$

Second, by Lemma 3 we have

$$\tilde{\mu}_{d_{m+1},t}\leq\left\lfloor\tilde{\mu}_{d_{m-1},t}\right\rfloor+(d_{m+1}-d_{m-1}), \tag{251}$$

and therefore

$$\left\lceil\tilde{\mu}_{d_{m+1},t}\right\rceil-(d_{m+1}-d_m)\leq\left\lfloor\tilde{\mu}_{d_{m-1},t}\right\rfloor+(d_m-d_{m-1}). \tag{252}$$

Third, by (183) we have

$$\begin{aligned}\tilde{\mu}_{d_{m+1},t}&\geq\left\lceil\tilde{\mu}_{d_{m-1},t}\right\rceil+(d_{m+1}-d_{m-1})-1\\ &\geq\left\lfloor\tilde{\mu}_{d_{m-1},t}\right\rfloor+(d_{m+1}-1)-d_{m-1}\\ &\geq\left\lfloor\tilde{\mu}_{d_{m-1},t}\right\rfloor+(d_m-d_{m-1}),\end{aligned} \tag{253}$$

and therefore

$$\left\lfloor\tilde{\mu}_{d_{m-1},t}\right\rfloor+(d_m-d_{m-1})\leq\left\lfloor\tilde{\mu}_{d_{m+1},t}\right\rfloor. \tag{254}$$

From (250)(252)(254) we know (247) and (248) are equivalent.

Step 7: In this step, with the following definition:

$$\mathcal{S}_1=\left[\left\lceil\tilde{\mu}_{d_{m+1},t}\right\rceil-(d_{m+1}-d_m),\left\lfloor\tilde{\mu}_{d_{m-1},t}\right\rfloor+d_m-d_{m-1}\right], \tag{255}$$

$$\mathcal{S}=\mathcal{S}_1\cap[\bar{\mu}_{d_m,t}^{(\infty)},\tilde{\mu}_{d_m,t}^{(\infty)}], \tag{256}$$

we will show that $\mathcal{S}$ is not empty.

We prove this by contradiction. If $\mathcal{S}$ is empty, and one of (257) and (258) must hold:

$$\tilde{\mu}_{d_m,t}^{(\infty)}<\left\lceil\tilde{\mu}_{d_{m+1},t}\right\rceil-(d_{m+1}-d_m), \tag{257}$$

$$\bar{\mu}_{d_m,t}^{(\infty)}>\left\lfloor\tilde{\mu}_{d_{m-1},t}\right\rfloor+d_m-d_{m-1}. \tag{258}$$

However, since $\tilde{\mu}_{d_m,t}^{(\infty)}(=\tilde{\mu}_{d_m,t}^{(K)}),\tilde{\mu}_{d_{m+1},t}^{(K)}$ are charging order-compliant, by Lemma 3 we have

$$\tilde{\mu}_{d_m,t}^{(\infty)}\geq\left\lceil\tilde{\mu}_{d_{m+1},t}^{(K)}\right\rceil-(d_{m+1}-d_m). \tag{259}$$

On the other hand, since $\bar{\mu}^{(K)}_{d_{m-1},t}, \bar{\mu}^{(K)}_{d_m,t}$ are charging order-compliant, if we let $\mathring{\mu}_{d_{m+1},t} = \bar{\mu}^{(K)}_{d_m,t}$ (for charging deadline d_{m+1}), from Lemma 3 we know $\bar{\mu}^{(K)}_{d_{m-1},t}$ and $\mathring{\mu}_{d_{m+1},t}$ will be charging order-compliant. Since by Step 5 we know $\bar{\mu}^{(K)}_{d_{m-1},t}, \tilde{\mu}^{(K)}_{d_{m+1},t}$ are not charging order-compliant and $\mathring{\mu}_{d_{m+1},t} < \tilde{\mu}^{(K)}_{d_m,t} \le \tilde{\mu}^{(K)}_{d_{m+1},t}$, by (181) in Property 1 we have

$$\tilde{\mu}_{d_{m+1},t} \le \tilde{\mu}^{(K)}_{d_{m+1},t}, \tag{260}$$

and therefore

$$\tilde{\mu}^{(\infty)}_{d_m,t} \ge \lceil \tilde{\mu}_{d_{m+1},t} \rceil - (d_{m+1} - d_m), \tag{261}$$

which contradicts with (257).

Similarly, we have

$$\begin{aligned}\bar{\mu}^{(\infty)}_{d_m,t} &\le \left\lfloor \bar{\mu}^{(K)}_{d_{m-1},t} \right\rfloor + (d_m - d_{m-1}) \\ &\le \lfloor \tilde{\mu}_{d_{m-1},t} \rfloor + (d_m - d_{m-1}),\end{aligned} \tag{262}$$

which contradicts with (258). And we can conclude that $\mathcal{S}$ is not empty.

Step 8: Now we can choose $\tilde{\mu}_{d_m,t} \in \mathcal{S}$, then $\tilde{\mu}_{d_{m-1},t}, \tilde{\mu}_{d_m,t}, \tilde{\mu}_{d_{m+1},t}$ are charging order-compliant. In the proof process above, we can see that change charging thresholds $\mu_{d_1,t}, \cdots, \mu_{d_{m-1},t}, \mu_{d_m,t}, \mu_{d_{m+1},t}$ to $\bar{\mu}^{(K)}_{d_1,t}, \cdots, \bar{\mu}^{(K)}_{d_{m-1},t}, \bar{\mu}^{(K)}_{d_m,t}, \tilde{\mu}^{(K)}_{d_{m+1},t}$, and further to $\tilde{\mu}_{d_1,t}, \cdots, \tilde{\mu}_{d_{m-1},t}, \bar{\mu}^{(K)}_{d_m,t}, \tilde{\mu}_{d_{m+1},t}$ will do no harm to the charging performance. From the results of Step 4, we know that changing $\bar{\mu}^{(K)}_{d_m,t}$ to $\tilde{\mu}_{d_m,t} \in [\bar{\mu}^{(\infty)}_{d_m,t}, \tilde{\mu}^{(\infty)}_{d_m,t}]$ will not alter charging behavior of any chargers. As the conclusion, $\tilde{\mu}_{d_1,t}, \cdots, \tilde{\mu}_{d_m,t}, \tilde{\mu}_{d_{m+1},t}$ are charging order-compliant, and $Q(\boldsymbol{x}_t, \boldsymbol{w}_t, \tilde{\boldsymbol{\mu}}_t) \le Q(\boldsymbol{x}_t, \boldsymbol{w}_t, \boldsymbol{\mu}_t)$.

The only thing we need to do next is to prove Property 1 and Property 2 for the case of $m+1$ charging thresholds.

Step 9: We now prove Property 1 for $m+1$ charging thresholds. For the consistency of notations, we suppose $\mu_{d_1,t}, \cdots, \mu_{d_{m-1},t}, \mu_{d_m,t}, \mu'_{d_{m+1},t}$ are charging order-compliant, $\mu_{d_{m+1},t} > \mu'_{d_{m+1},t}$ is another charging threshold for deadline d_{m+1} with $\mu_{d_m,t}, \mu_{d_{m+1},t}$ not charging order-compliant. Let

$$(\tilde{\mu}^{(1)}_{d_m,t}, \tilde{\mu}^{(1)}_{d_{m+1},t}) = \mathcal{F}^{d_m,d_{m+1}}_{2,t}(\mu_{d_m,t}, \mu_{d_{m+1},t}), \tag{263}$$

then by Corollary 3 we have

$$\tilde{\mu}^{(1)}_{d_m,t} \ge \mu_{d_m,t}. \tag{264}$$

If $\tilde{\mu}^{(1)}_{d_m,t} = \mu_{d_m,t}$, then $\tilde{\mu}^{(1)}_{d_1,t}, \cdots, \tilde{\mu}^{(1)}_{d_{m-1},t}, \tilde{\mu}^{(1)}_{d_m,t}, \tilde{\mu}^{(1)}_{d_{m+1},t}$ are charging order-compliant, and Property 1 can be established by Corollary 3. Otherwise, we have

$$\tilde{\mu}^{(1)}_{d_m,t} > \mu_{d_m,t}, \tag{265}$$

which satisfies the assumption (191) in Step 1. Therefore, all the properties established in steps above can be used in current step.

Note that the final $\tilde{\boldsymbol{\mu}}_t$ can be generated in Step1/Step2, where $\tilde{\mu}^{(i)}_{d_1,t}, \cdots, \tilde{\mu}^{(i)}_{d_m,t}, \tilde{\mu}^{(i)}_{d_{m+1},t}$ or $\bar{\mu}^{(i)}_{d_1,t}, \cdots, \bar{\mu}^{(i)}_{d_m,t}, \bar{\mu}^{(i)}_{d_{m+1},t}$ is order-compatible for some finite i, or in Step6 and Step8 otherwise. We now prove the more complex case where $\tilde{\boldsymbol{\mu}}_t$ is generated in Step6 and Step8, and the case where $\tilde{\boldsymbol{\mu}}_t$ is generated in Step1/Step2 can be proved likewise.

Now we establish some more properties that $\tilde{\mu}_{d_1,t}, \cdots, \tilde{\mu}_{d_{m-1},t}, \tilde{\mu}_{d_{m+1},t}$ in Step 6 should satisfy. Note that if we define $\hat{\mu}^{(K)}_{d_{m+1},t} = \left\lfloor \bar{\mu}^{(K)}_{d_{m-1},t} \right\rfloor + d_{m+1} - d_{m-1}$, then $\bar{\mu}^{(K)}_{d_{m-1},t}, \hat{\mu}^{(K)}_{d_{m+1},t}$ are charging order-compliant. Also by (245) we have

$$\tilde{\mu}^{(K)}_{d_{m+1},t} > \hat{\mu}^{(K)}_{d_{m+1},t}. \tag{266}$$

Then according to Property 1 for m thresholds, we have

$$\tilde{\mu}_{d_j,t} \ge \bar{\mu}^{(K)}_{d_j,t}, \ j = 1, \cdots, m-1, \tag{267}$$

$$\hat{\mu}^{(K)}_{d_{m+1},t} \le \tilde{\mu}_{d_{m+1},t} \le \tilde{\mu}^{(K)}_{d_{m+1},t}, \tag{268}$$

$$\tilde{\mu}_{d_{m+1},t} \ge \left\lfloor \bar{\mu}^{(K)}_{d_{m-1},t} \right\rfloor + (d_{m+1} - d_{m-1}), \tag{269}$$

$$\tilde{\mu}_{d_{m-1},t} \le \lfloor \tilde{\mu}_{d_{m+1},t} \rfloor - (d_{m+1} - d_{m-1}) + 1. \tag{270}$$

Then by (224)(267) we know

$$\tilde{\mu}_{d_j,t} \ge \mu_{d_j,t}, \ j = 1, \cdots, m-1. \tag{271}$$

Also, by (225) and $\tilde{\mu}_{d_m,t} \in \mathcal{S}$ we have

$$\tilde{\mu}_{d_m,t} \ge \mu_{d_m,t}. \tag{272}$$

Since $\mu_{d_{m-1},t}, \mu_{d_m,t}, \mu'_{d_{m+1},t}$ is order compliant, by Lemma 3 we have

$$\mu'_{d_{m+1},t} \le \lfloor \mu_{d_{m-1},t} \rfloor + (d_{m+1} - d_{m-1}), \tag{273}$$

$$\mu_{d_m,t} \le \lfloor \mu_{d_{m-1},t} \rfloor + (d_m - d_{m-1}). \tag{274}$$

Then by (224)(269)(273) we have

$$\tilde{\mu}_{d_{m+1},t} \ge \mu'_{d_{m+1},t}. \tag{275}$$

By (227)(268) we have

$$\tilde{\mu}_{d_{m+1},t} \le \mu_{d_{m+1},t}. \tag{276}$$

By (224)(269)(274) we have

$$\tilde{\mu}_{d_{m+1},t} \ge \lfloor \mu_{d_m,t} \rfloor + (d_{m+1} - d_m). \tag{277}$$

Since $\mu_{d_{m+1},t} > \mu'_{d_{m+1},t}$, according to (188) and Corollary 3 we have

$$\tilde{\mu}^{(1)}_{d_m,t} \le \lceil \mu_{d_{m+1},t} \rceil - (d_{m+1} - d_m). \tag{278}$$

Then by (226)(278) and $\tilde{\mu}_{d_m,t} \in \mathcal{S}$ we have

$$\tilde{\mu}_{d_m,t} \le \lceil \mu_{d_{m+1},t} \rceil - (d_{m+1} - d_m). \tag{279}$$

Since $\tilde{\mu}_{d_m,t} \in \mathcal{S}$, we have

$$\tilde{\mu}_{d_m,t} \le \lfloor \tilde{\mu}_{d_{m-1},t} \rfloor + d_m - d_{m-1}. \tag{280}$$

Combining (270)(280) we have

$$\tilde{\mu}_{d_m,t} \le \lfloor \tilde{\mu}_{d_{m+1},t} \rfloor - (d_{m+1} - d_m) + 1, \tag{281}$$

and equivalently

$$\tilde{\mu}_{d_{m+1},t} \ge \lceil \tilde{\mu}_{d_m,t} \rceil + (d_{m+1} - d_m) - 1. \tag{282}$$

Now we have finished the proof for Property 1 for $m+1$ thresholds.

Step 10: We now prove Property 2 for $m+1$ thresholds. According to how the final modified charging thresholds $\tilde{\boldsymbol{\mu}}_t$ are generated, we can discuss in following 3 cases:

1) $\tilde{\boldsymbol{\mu}}_t$ is generated in Step 1 or Step 2, and $\tilde{\boldsymbol{\mu}}_t = \tilde{\boldsymbol{\mu}}_t^{(i)}$ for some $i \in \{1, 2, \cdots\}$. In this case, if $\tilde{\mu}^{(i)}_{d_m,t}$ is an integer different from $\bar{\mu}^{(i-1)}_{d_m,t}$, then by (221)(225) we know $\tilde{\mu}^{(i)}_{d_m,t}$ is an integer different from $\mu_{d_m,t}$. If $\tilde{\mu}^{(i)}_{d_{m+1},t}$ is an integer different from $\bar{\mu}^{(i-1)}_{d_{m+1},t}$, then by (220)(227) we know $\tilde{\mu}^{(i)}_{d_{m+1},t}$ is an integer different from $\mu_{d_{m+1},t}$.
2) $\tilde{\boldsymbol{\mu}}_t$ is generated in Step 1 or Step 2, and $\tilde{\boldsymbol{\mu}}_t = \bar{\boldsymbol{\mu}}_t^{(i)}$ for some $i \in \{1, 2, \cdots\}$. In this case, if for some $j \in \{1, \cdots, m-1\}$, $\bar{\mu}^{(i)}_{d_j,t}$ is an integer different from $\tilde{\mu}^{(i)}_{d_j,t}$. Then by (224) we know $\bar{\mu}^{(i)}_{d_j,t}$ is an integer different from $\mu_{d_j,t}$.
Otherwise, according to Property 2 for m thresholds, $\bar{\mu}^{(i)}_{d_m,t}$ is an integer different from $\tilde{\mu}^{(i)}_{d_m,t}$. By Corollary 3 we know that
$$\tilde{\mu}^{(i)}_{d_m,t} \le \left\lfloor \tilde{\mu}^{(i)}_{d_{m+1},t} \right\rfloor - (d_{m+1} - d_m) + 1, \tag{283}$$
and since $\bar{\mu}^{(i)}_{d_m,t}$ is an integer different from $\tilde{\mu}^{(i)}_{d_m,t}$, with $\bar{\mu}^{(i)}_{d_{m+1},t} = \tilde{\mu}^{(i)}_{d_{m+1},t}$ we have
$$\bar{\mu}^{(i)}_{d_m,t} \le \left\lfloor \bar{\mu}^{(i)}_{d_{m+1},t} \right\rfloor - (d_{m+1} - d_m). \tag{284}$$
However, since $\bar{\mu}^{(i)}_{d_m,t}, \bar{\mu}^{(i)}_{d_{m+1},t}$ are charging order-compliant, and by Lemma 3 we know
$$\bar{\mu}^{(i)}_{d_m,t} \ge \left\lceil \bar{\mu}^{(i)}_{d_{m+1},t} \right\rceil - (d_{m+1} - d_m). \tag{285}$$
From (284)(285) we know $\bar{\mu}^{(i)}_{d_{m+1},t}$ is an integer. Since $\mu_{d_m,t}, \mu_{d_{m+1},t}$ is not order compliant, we conclude that either $\bar{\mu}^{(i)}_{d_m,t}$ is an integer different from $\mu_{d_m,t}$, or $\bar{\mu}^{(i)}_{d_{m+1},t}$ is an integer different from $\mu_{d_{m+1},t}$.
3) $\tilde{\boldsymbol{\mu}}_t$ is generated in Step 6 and Step 8. Then applying Property 2 to (246) we know there is some $j \in \{1 \cdots, m-1, m+1\}$ such that $\tilde{\mu}_{d_j,t}$ is an integer different from $\bar{\mu}^{(K)}_{d_j,t}$. Then according to (224) or (227) we know $\tilde{\mu}_{d_j,t}$ is an integer different from $\mu_{d_j,t}$.

□

Now we can establish the proof for Theorem 1.

Proof. It is shown in [29] that there exists an optimal DTC-type charging policy, and we denote it as π. Then according to Lemma 6, we can convert π into an DTC-type optimal charging policy π^*, such that for any state $(\boldsymbol{x}_t, \boldsymbol{w}_t)$, let $\boldsymbol{\mu}_t = \pi^*_t(\boldsymbol{x}_t, \boldsymbol{w}_t)$, then $\mu_{t+1,t}, \cdots, \mu_{T,t}$ are charging order-compliant.

We will prove that policy π^* is priority rule-compliant. We prove it by contradiction. Suppose π^* is not priority rule-compliant, then there is a timeslot t and two chargers i and j with $t < \min(d_i, d_j)$, whose charging decisions given by π^* are $v_{i,t}$ and $v_{j,t}$ satisfying

1) $\exists \epsilon > 0$ such that $(e_{i,t+1} + \epsilon, d_i) \prec (e_{j,t+1}, d_j)$,
2) $v_{i,t} > 0$,
3) $v_{j,t} < \min(1, e_{j,t})$.

Then by 2) we know $\mu_{d_i,t} \le e_{i,t+1}$, by 3) we know $\mu_{d_j,t} \ge e_{j,t+1}$, and combining with 1) we know $\exists \epsilon > 0$ such that
$$(\mu_{d_i,t} + \epsilon, d_i) \prec (\mu_{d_j,t}, d_j), \tag{286}$$
which contradicts with the fact that $\mu_{d_i,t}$ and $\mu_{d_j,t}$ are charging order-compliant. This suggests that π^* is priority rule-compliant.

□

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