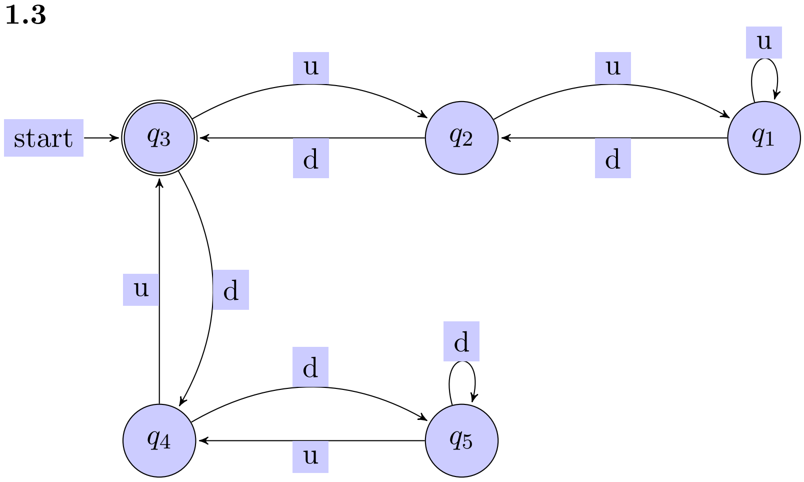
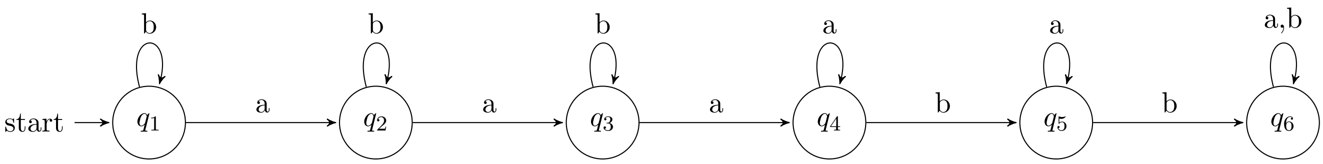
Wen Liang(Wen_Liang@student.uml.edu) 01724877

1.3

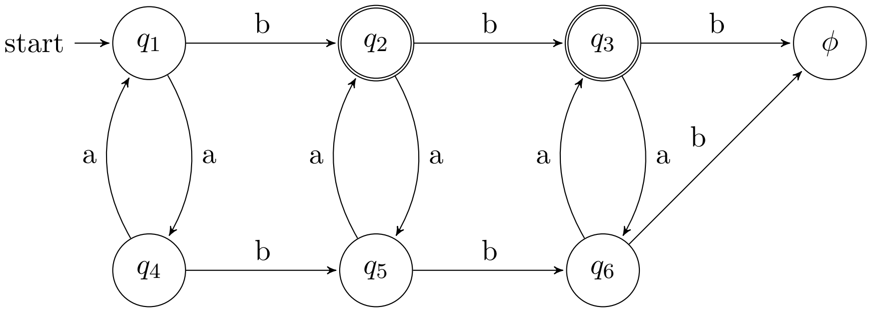


1.4

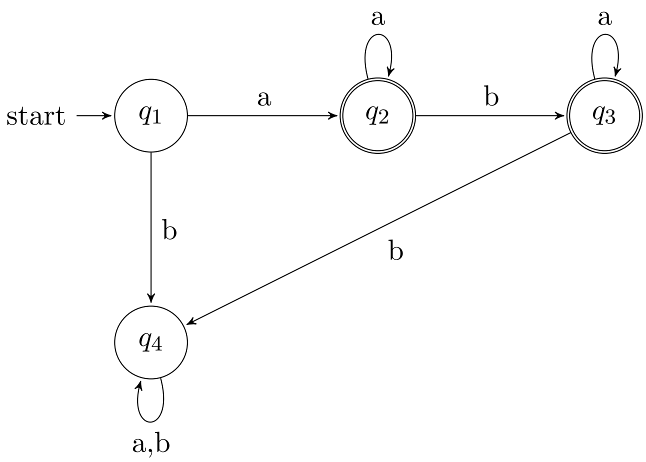
a.



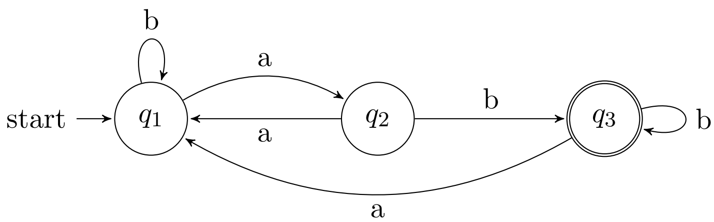
c.



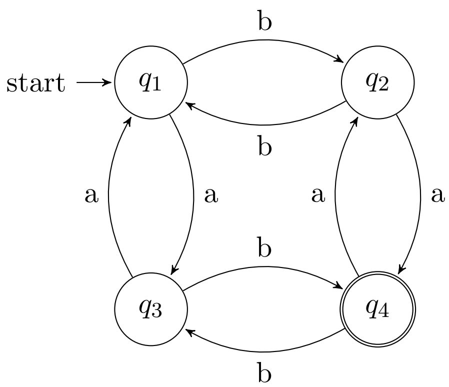
e.



f.

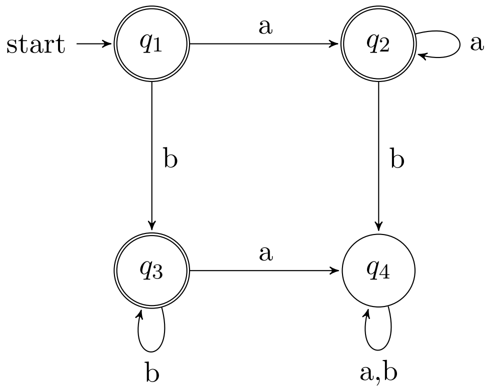


g.

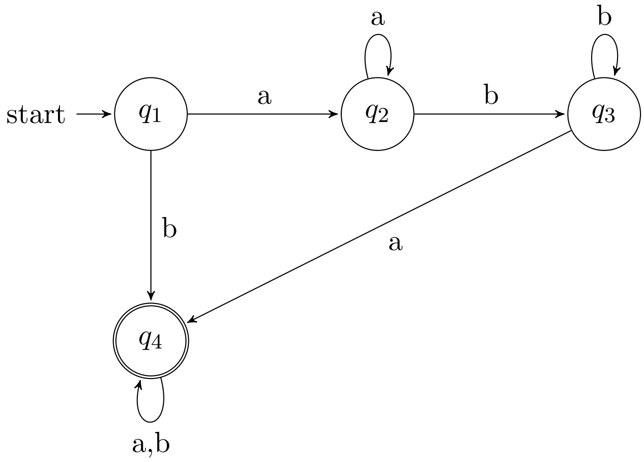


1.5

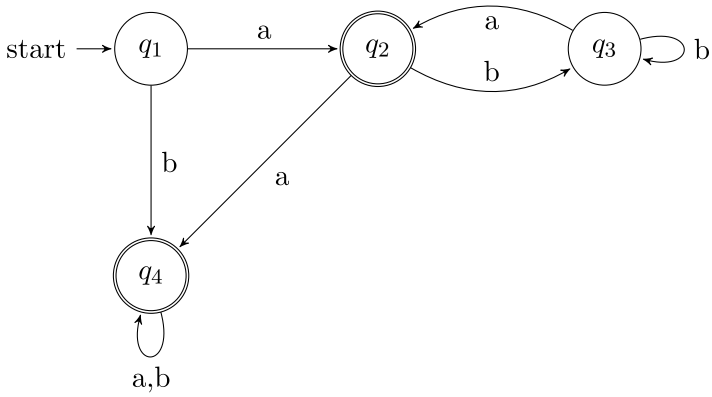
c.


start
q_1
q_2
q_3
q_4
a
a
b
b
a
b
a,b

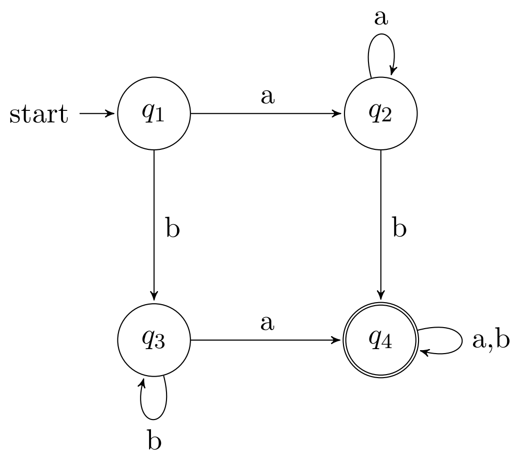
d.


start
q_1
q_2
q_3
q_4
a
b
a
b
b
a
a,b

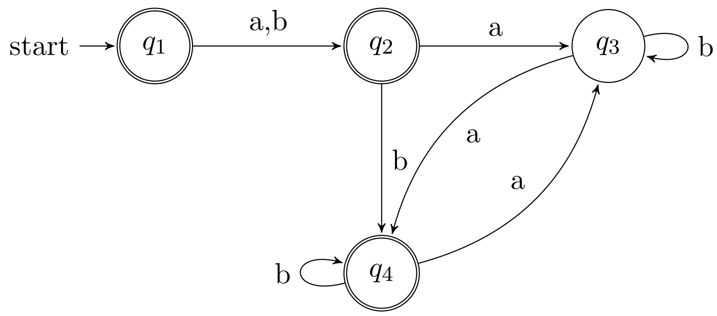
e.


start
q_1
q_2
q_3
q_4
a
a
b
b
b
a
a,b

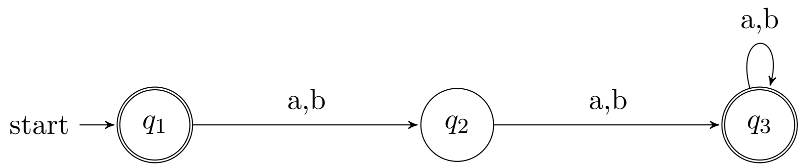
f.



g.

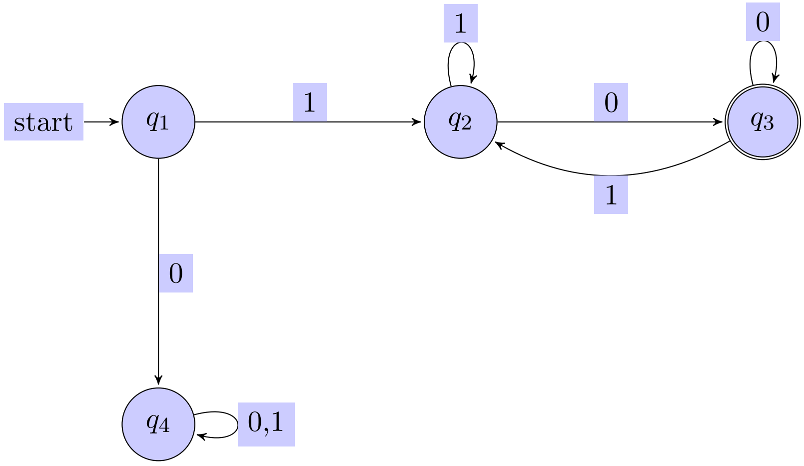


h.

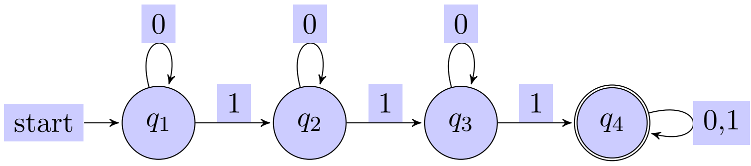


1.6

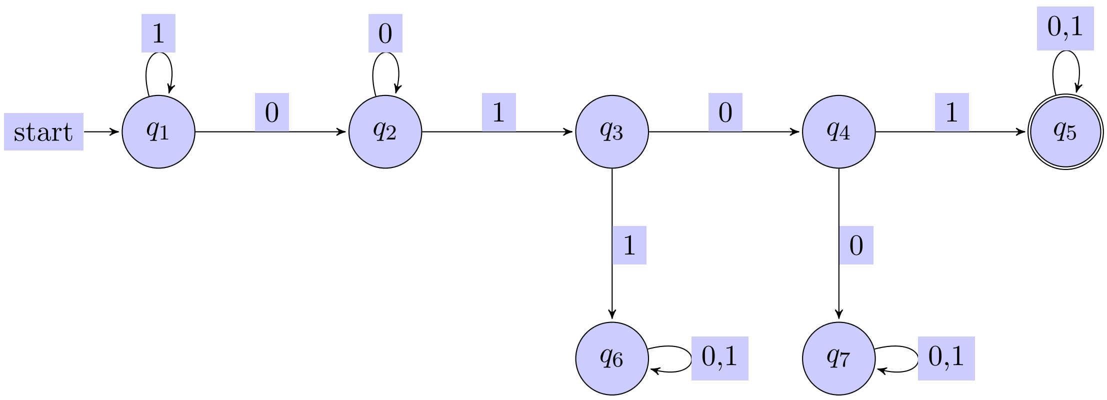
a.


start
q1
q2
q3
q4
1
1
0
0
1
0
0,1

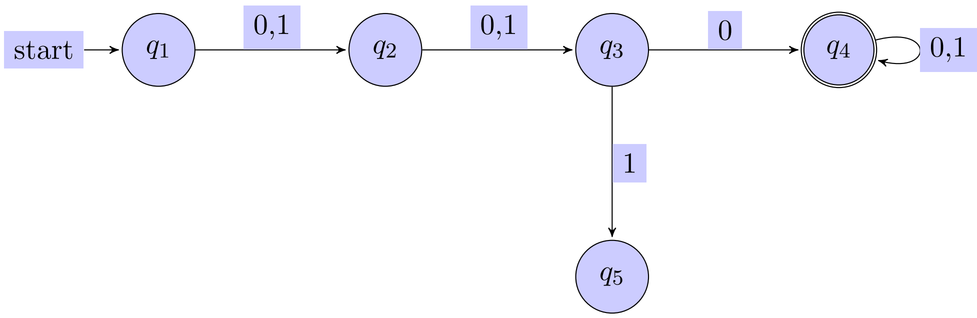
b.


start
q1
q2
q3
q4
0
0
0
1
1
1
0,1

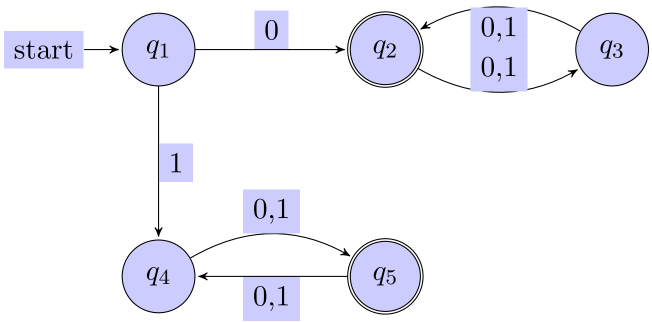
c.


start
q1
q2
q3
q4
q5
q6
q7
1
0
0,1
0
1
0
1
1
0
0,1
0,1

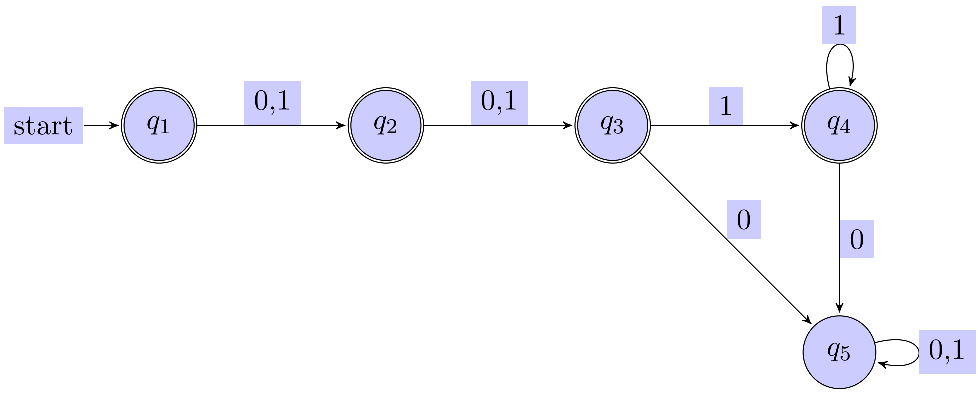
d.


start
0,1
0,1
0
0,1
1
q_1
q_2
q_3
q_4
q_5

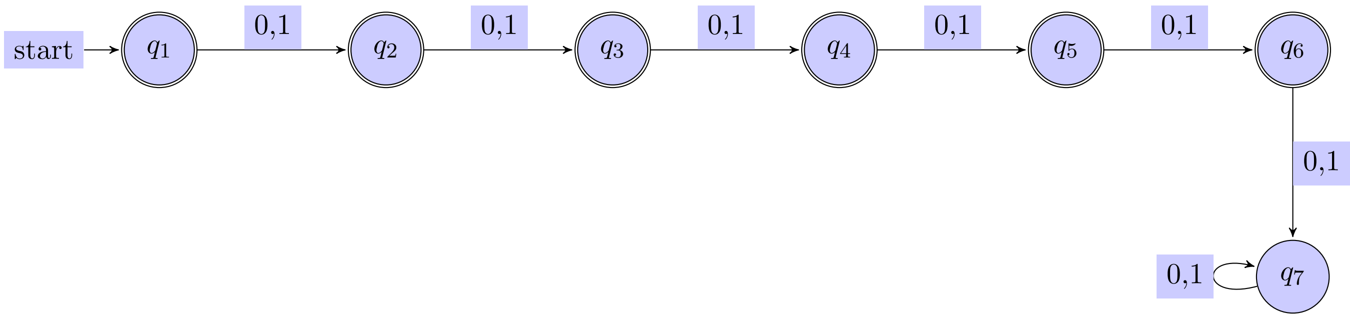
e.


start
0
0,1
0,1
1
0,1
0,1
q_1
q_2
q_3
q_4
q_5

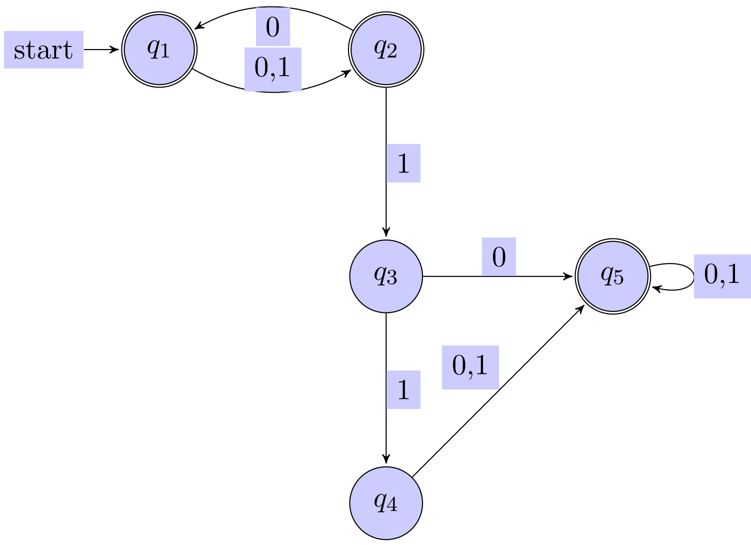
f.


1
start
0,1
0,1
1
0
0
0,1
q_1
q_2
q_3
q_4
q_5

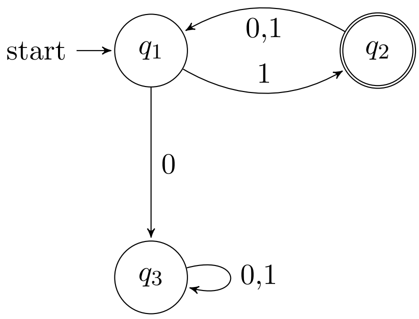
g.


start
q_1
0,1
q_2
0,1
q_3
0,1
q_4
0,1
q_5
0,1
q_6
0,1
0,1
q_7

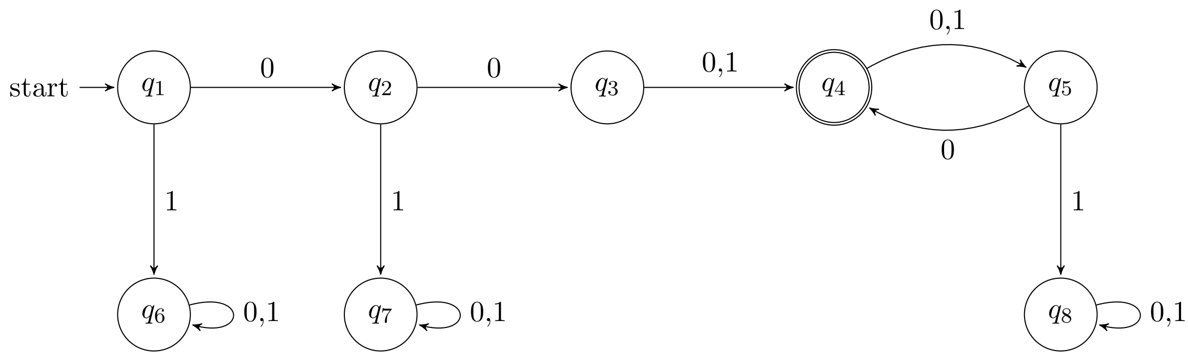
h.


start
q_1
0
0,1
q_2
1
q_3
0
q_5
0,1
1
0,1
q_4

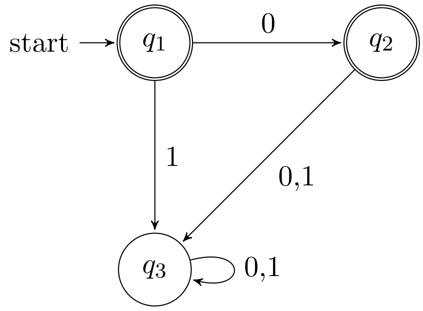
i.


start
q_1
0,1
1
q_2
0
q_3
0,1

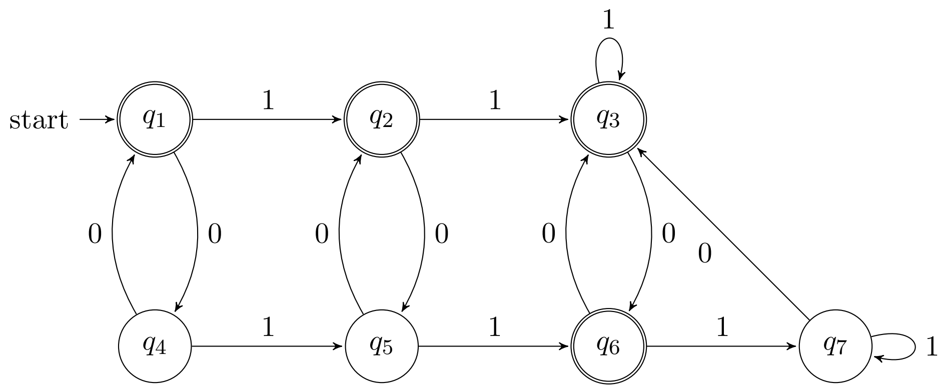
j.


start
q_1 q_2 q_3 q_4 q_5 q_6 q_7 q_8
0
0
0,1
0,1
0
1
1
1
0,1
0,1
0,1

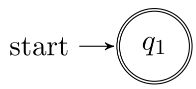
k.


start
q_1 q_2 q_3
0
1
0,1
0,1

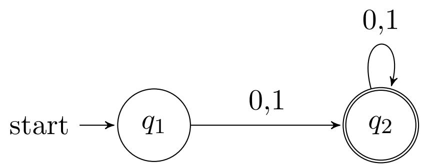
l.


start
q_1 q_2 q_3 q_4 q_5 q_6 q_7
1
1
1
0 0 0 0 0 0 0
1 1 1 1

m.

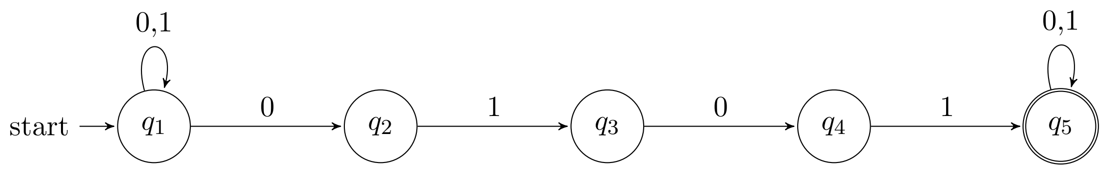

start
q_1

n.

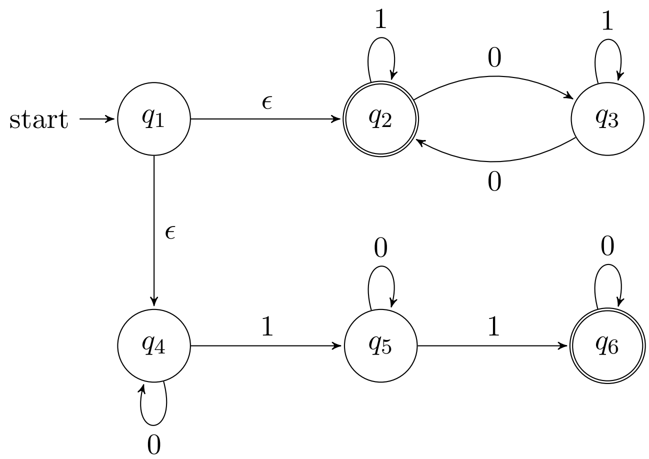

0,1
start
q_1
0,1
q_2

1.7

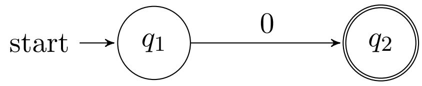
b.


0,1
0,1
start
q_1
0
q_2
1
q_3
0
q_4
1
q_5

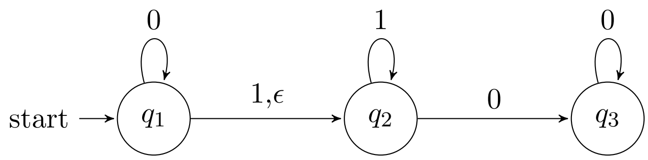
c.


1
1
0
start
q_1
ϵ
q_2
q_3
0
ϵ
0
0
q_4
1
q_5
1
q_6
0

d.

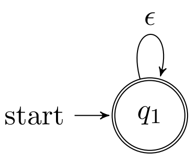

start
q_1
0
q_2

e.



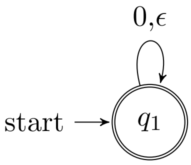
start
q_1
q_2
q_3
0
1
0
1,ϵ
0

g.



start
q_1
ϵ

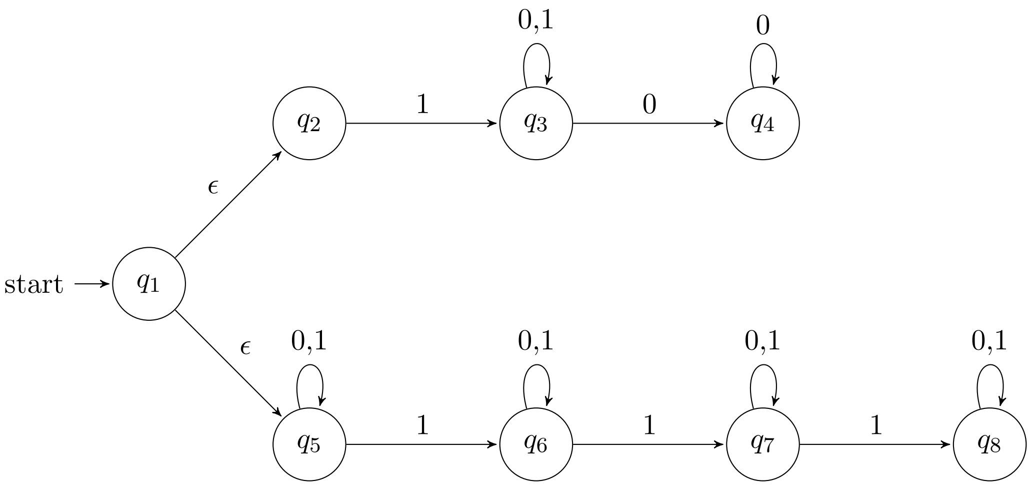
h.



start
q_1
0,ϵ

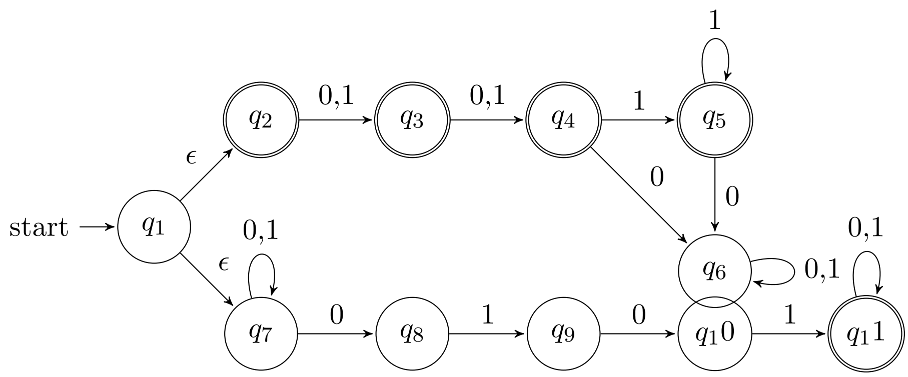
1.8

a.



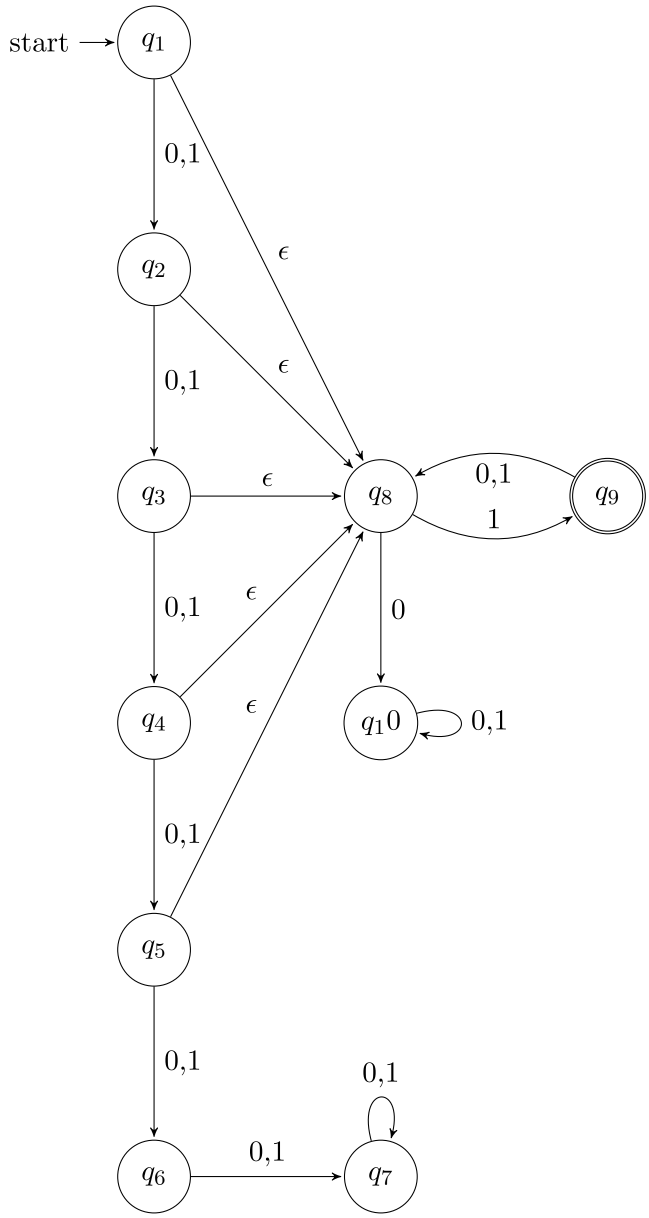
start
q_1
q_2
q_3
q_4
q_5
q_6
q_7
q_8
ϵ
ϵ
1
0
0,1
0
0,1
0,1
0,1
0,1
1
1
1

b.

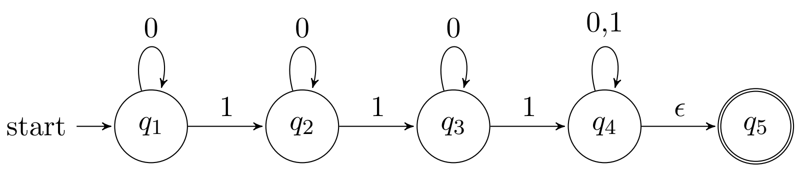

start
q_1
ϵ
ϵ
q_2
0,1
q_3
0,1
q_4
1
q_5
1
0
0
q_6
0,1
0,1
q_7
0
q_8
1
q_9
0
q_10
1
q_11
0,1

1.9

a.

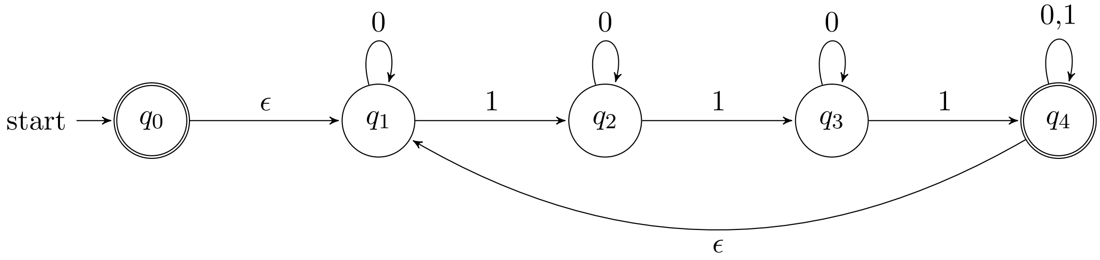

start
q_1
q_2
q_3
q_4
q_5
q_6
q_7
q_8
q_9
q_10
0,1
ϵ
1
0

b.

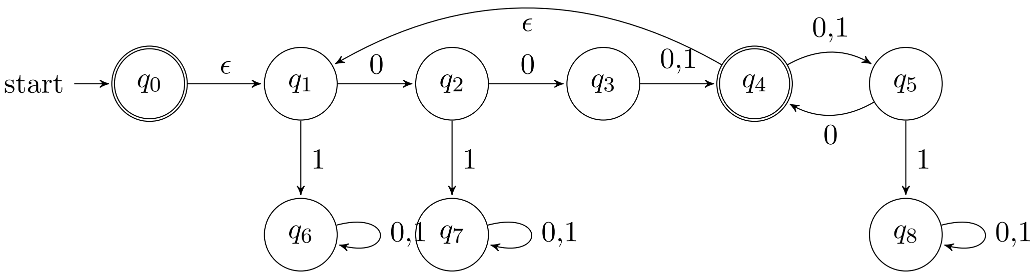

start
q_1
q_2
q_3
q_4
q_5
0
1
0,1
ϵ

1.10

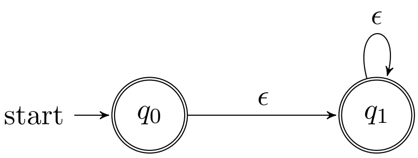
a.


start
q_0
ϵ
q_1
0
1
q_2
0
1
q_3
0
1
q_4
0,1
ϵ

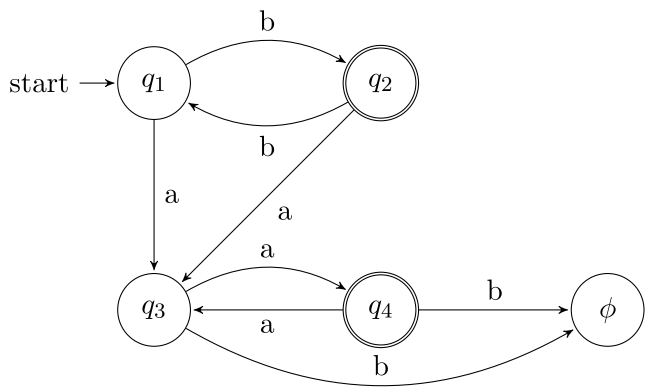
b.


start
q_0
ϵ
q_1
0
q_2
0
q_3
0,1
q_4
ϵ
0,1
q_5
0
1
1
1
q_6
0,1
q_7
0,1
q_8
0,1

c.


start
q_0
ϵ
q_1
ϵ

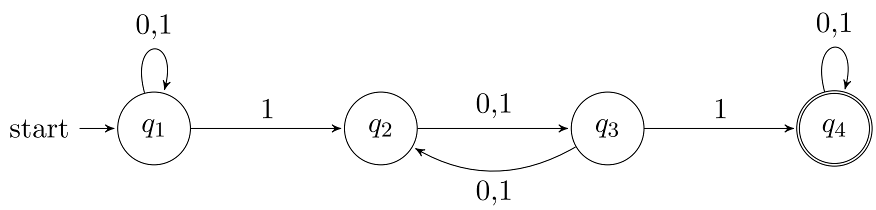
1.12



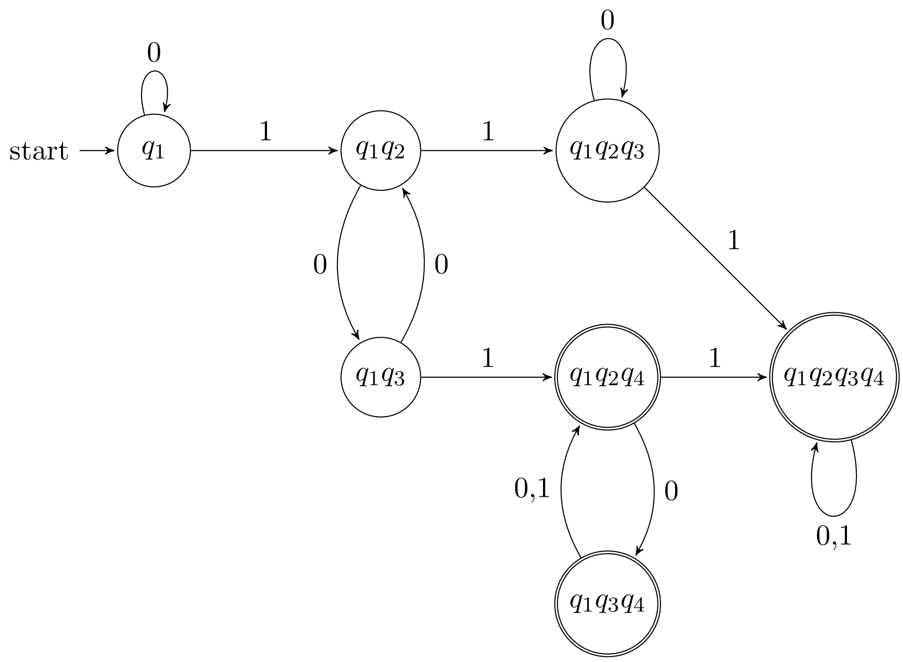
regular expression: $b(bb)^*(aa)^*$

1.13

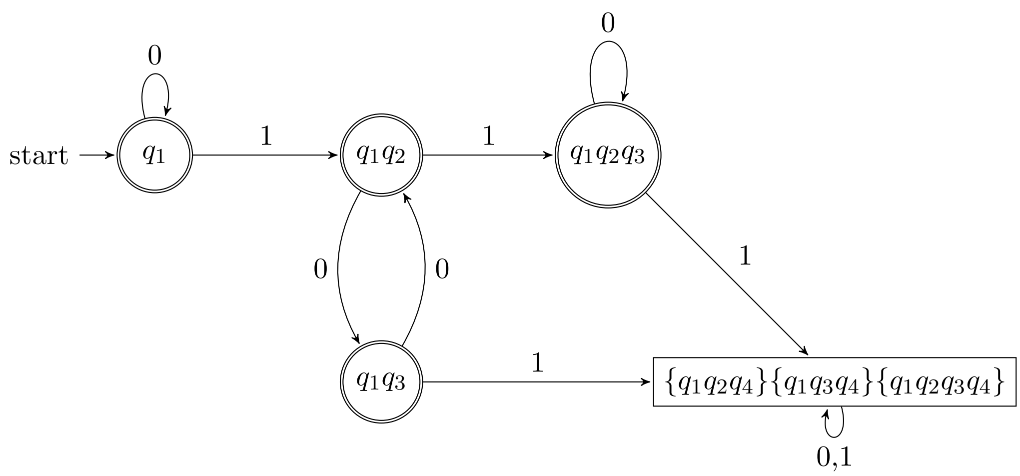
step 1) 4-state NFA recognizing a pair of 1 with an odd number of symbols in between



step 2) convert the NFA to the corresponding DFA

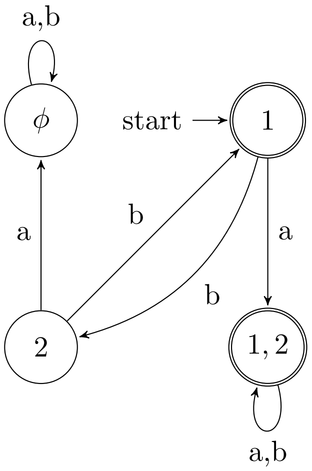


step 3) complement and minimize the states of the DFA above

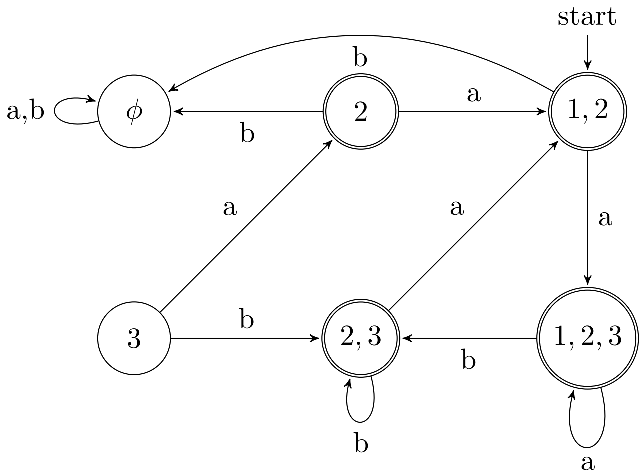


1.16

a.

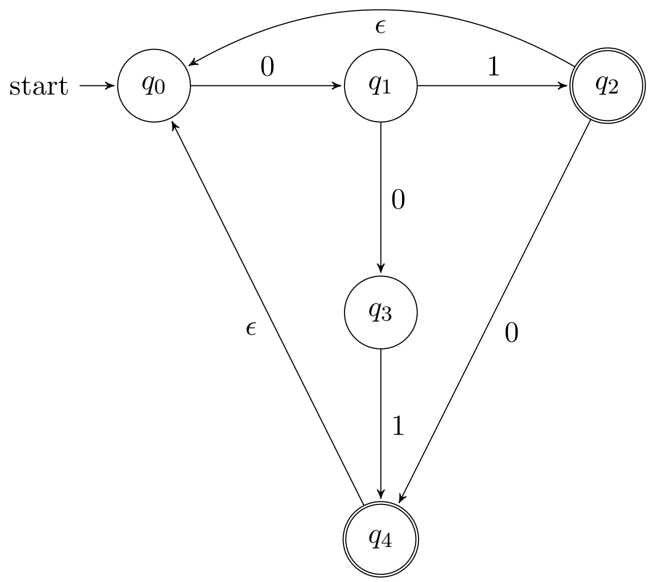


b.



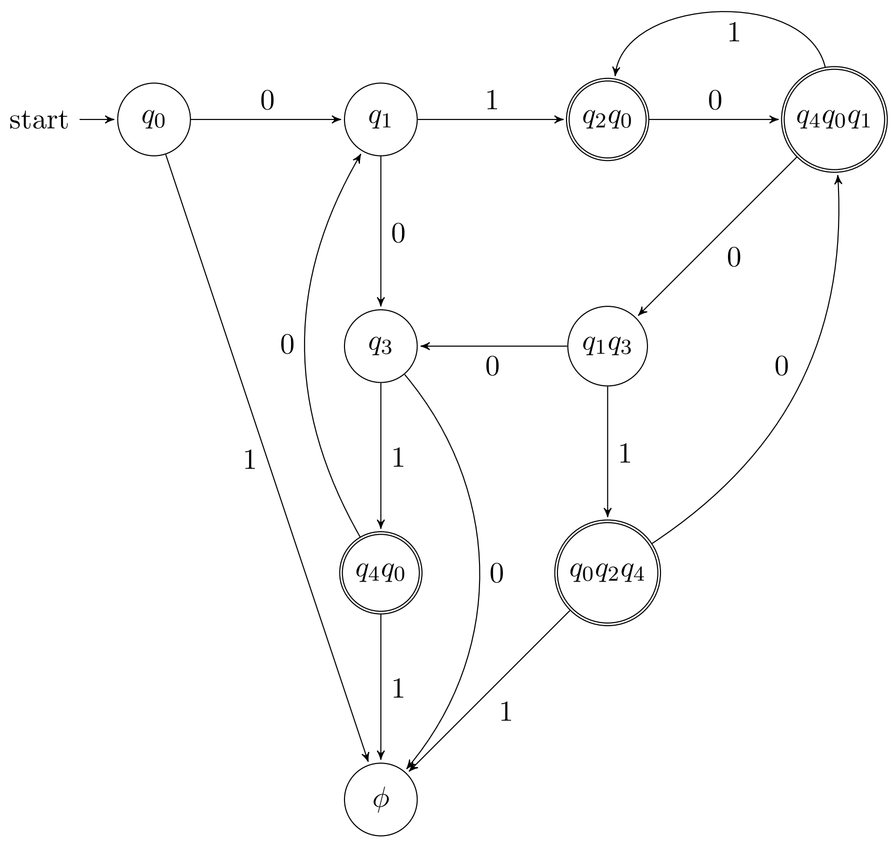
1.17

a.



start
q_0
q_1
q_2
q_3
q_4
0
1
0
1
0
ϵ
ϵ

b.



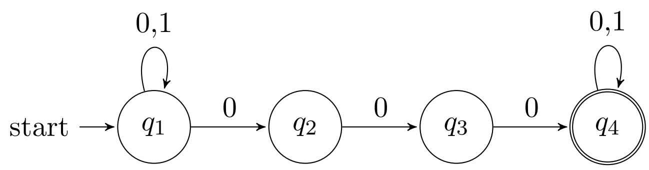
start
q_0
q_1
q_2q_0
$q_4q_0q_1$
q_3
q_1q_3
q_4q_0
$q_0q_2q_4$
ϕ
0
1
0
1
0
0
0
0
1
1
1
0
0
1
1

1.18

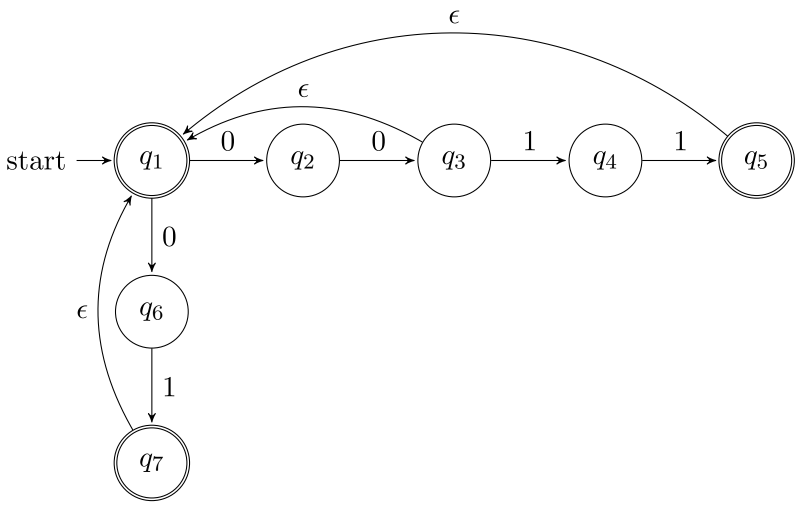
a. $1\sum^*0$
b. $\sum^*1\sum^*1\sum^*1\sum^*$
c. $\sum^*0101\sum^*$
d. $\sum\sum0\sum^*$
e. $(0\cup1\sum)(\sum\sum)^*$
f. $0^*(10^+)^*1^*$
g. $\sum\sum\sum\sum\sum$
h. $\sum^*\sum\sum\sum\sum\sum^*\cup\sum\cup0\sum\cup10\cup0\sum\sum\cup10\sum\cup110$
i. $(1\sum)^*$
j. $0^+\sum0^+\cup\sum0^+0^+\cup0^+0^+\sum$
k. $\epsilon\cup0$
l. $(1^*01^*01^*)^*\cup0^*10^*10^*$
m. ϕ
n. $\sum^*\sum^+$

1.19

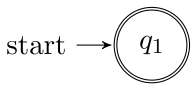
a.



b.



c.



1.20

a. <u>members</u>: ab, aab   <u>not members</u>: ba,baa
b. <u>members</u>: abab,ab   <u>not members</u>: aa,a
c. <u>members</u>: a,b   <u>not members</u>: ab,ba
d. <u>members</u>: aaa,aaaaaa   <u>not members</u>: b,ba
e. <u>members</u>: aba,aaba   <u>not members</u>: ba,b
f. <u>members</u>: aba,bab   <u>not members</u>: a,b
g. <u>members</u>: ab,b   <u>not members</u>: aa,a
h. <u>members</u>: a,ba   <u>not members</u>: b,ϵ

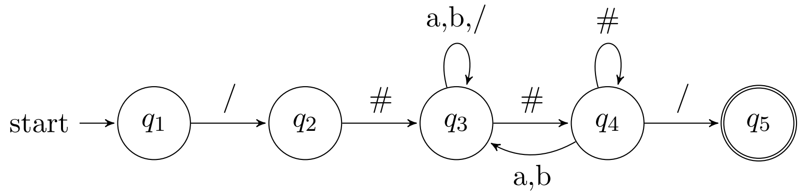
1.21

a. $a^*ba^*(ba^*ba^*)^*$

b. $\sum a^*b(ba^*b)^*(a\sum a^*b(ba^*b)^*)^*$

1.22

a.

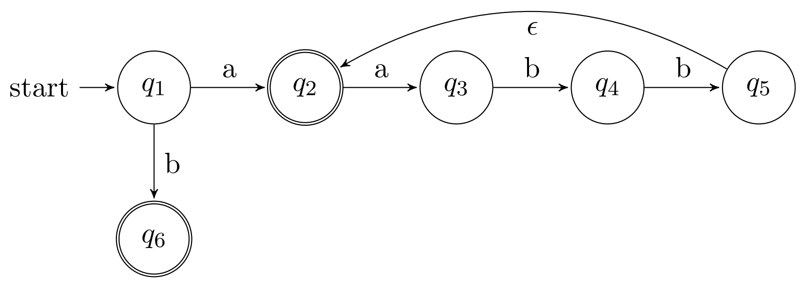


b.

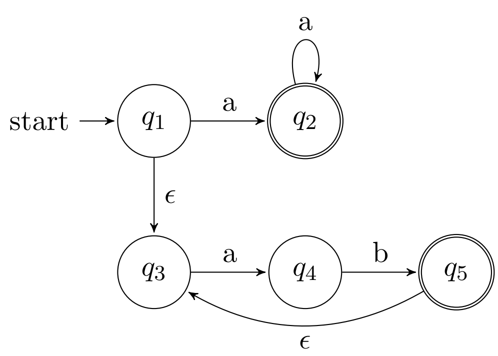
/#(a∪b∪/)*#((a∪b)(a∪b∪/)*#)*#*/

1.28

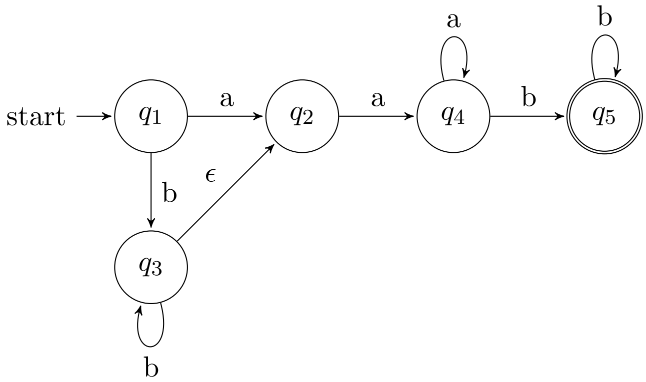
a.



b.



c.



1.29

a)

Assume that $A_1 = \{0^n1^n2^n|\text{n} \geq 0\}$ is regular. Let p be the pumping length given by the pumping lemma. Choose s to be the string $0^p1^p2^p$. Because s is a member of A_1 and s is longer than p, the pumping lemma guarantees that s can be split into three pieces, s = xyz, where for any i $\geq$ 0 the string xy^iz is in A_1. Consider two possibilities: 1. The string y consists only of 0s, only of 1s, or only of 2s. In these cases, the string xyyz will not have equal numbers of 0s, 1s, and 2s. Hence xyyz is not a member of A_1, a contradiction. 2. The string y consists of more than one kind of symbol. In this case, xyyz will have the 0s, 1s, or 2s out of order. Hence xyyz is not a member of A_1, a contradiction. Either way we arrive at a contradiction. Therefore, A_1 is not regular.

b)

Suppose that A_2 is a regular language. Let p be the pumping length of the Pumping Lemma. Consider the string s = a^pba^pba^pb. Note that s $\in A_2$,since s = $(a^p\text{b})^3$, and | s| = $3(p + 1) \geq$ p, so the Pumping Lemma will hold. Thus, we can split the string s into 3 parts $s = xyz$
satisfying the conditions:1)xy^iz$\in A_2$ for each i $\geq$ 0;2)|y|>0; 3)|xy| $\leq$p. Since the first p symbols of s are all a, the third condition implies that x and y consist only of a. So z will be the rest of the first set of a, followed by ba^pba^pb. The second condition states that |y| > 0, so y has at least one a. More precisely, we can then say that
x=a^j for some j$\geq$0,
y=a^k for some k$\geq$1,
z=a^mba^pba^pb for some m$\geq$0.
Since a^pba^pba^pb=s=xyz=$a^j a^k a^m$ba^pba^pb=a^{j+k+m}ba^pba^pb,we must have that$j + k +$

$m = p$. The first condition implies that $xy^2z \in A_2$,but xy^2z=$a^j a^k a^k a^m$ba^pba^pb
= a^{p+k}ba^pba^pb
since $j + k + m = p$. Hence,$xy^2z \notin A_2$ because k $\geq$ 1, and we get a contradiction. Therefore, A_2 is a nonregular language.

c)

Assume that $A_3 = \{a^{2^n} \mid n \geq 0\}$is regular. Let p be the pumping length given by the pumping lemma. Choose s to be the string 2^p. Because s is a member of A_3 and s is longer than p, the pumping lemma guarantees that s can be split into three pieces, s = xyz, satisfying the three conditions of the pumping lemma. The third condition tells us that $\mid xy \mid \leq p$. Furthermore, $p < 2^p$ and so $\mid y \mid < 2^p$. Therefore, $\mid xyyz \mid = \mid xyz \mid + \mid y \mid < 2^p + 2^p = 2^{p+1}$. The second condition requires $\mid y \mid > 0$ so $2^p < \mid xyyz \mid < 2^{p+1}$. The length of xyyz cannot be a power of 2. Hence xyyz is not a member of A_3, a contradiction. Therefore, A_3 is not regular.

1.31

One solution is recursively (or inductively) define a reversing operation on regular expressions, and apply that operation on the regular expression for A. In particular, given a regular expression R, reverse(R) is:

- a for some a $\in$,
- ϵ if R = ϵ ,
- ϕ if R = ϕ ,
- (reverse(R_1) $\cup$ reverse(R_2)), if R = $R_1 \cup R_2$,
- (reverse(R_2) $\circ$ reverse(R_1)) if R = $R_1 \circ R_2$, or
- (reverse(R_1)*), if R = (R_1^*)

Let M = (Q,$\sum$,δ,s,F) be a NFA that accepts L. Create a NFA R = (Q$\cup$r,$\sum$,γ,r,{s}) to accept L^R.r is a new state not in Q.γ reverse the transitions in δ,i.e. for each (q,c)$\rightarrow$ q$\prime$ in δ there is a (q$\prime$,c) $\rightarrow$ q in γ. Also γ contains ϵ transitions from r to every state in F. Thus there exists a sequence of transitions from s to a state in F in M on input w iff there exists a sequence of transitions from r to s in R on input w^R.