

3.1

a.

$q_10, \sqcup q_2\sqcup, \sqcup\sqcup q_{accept}$

c.

$q_1000, \sqcup q_200, \sqcup xq_30, \sqcup x0q_4\sqcup, \sqcup x0\sqcup q_{reject}$

d.

$q_1000000, \sqcup q_200000, \sqcup xq_30000, \sqcup x0q_4000, \sqcup x0xq_300, \sqcup x0x0q_40, \sqcup x0x0xq_3\sqcup, \sqcup x0x0q_5x\sqcup, \sqcup x0xq_50x\sqcup, \sqcup x0q_5x0x\sqcup, \sqcup xq_50x0x\sqcup, \sqcup q_5x0x0x\sqcup, \sqcup q_5\sqcup x0x0x\sqcup, \sqcup q_2x0x0x\sqcup, \sqcup xq_20x0x\sqcup, \sqcup xxq_3x0x\sqcup, \sqcup xxxq_30x\sqcup, \sqcup xxx0q_4x\sqcup, \sqcup xxx0xq_4\sqcup, \sqcup xxx0x\sqcup q_{reject}$

00000000

$q_100000000 \rightarrow \sqcup q_20000000 \rightarrow \sqcup xq_3000000 \rightarrow \sqcup x0q_400000 \rightarrow \sqcup x0xq_30000 \rightarrow \sqcup x0x0q_4000 \rightarrow \sqcup x0x0xq_300 \rightarrow \sqcup x0x0x0q_40 \rightarrow \sqcup x0x0x0xq_3\sqcup \rightarrow \sqcup x0x0x0q_5x\sqcup \rightarrow \sqcup x0x0xq_50x\sqcup \rightarrow \sqcup x0x0q_5x0x\sqcup \rightarrow \sqcup x0xq_50x0x\sqcup \rightarrow \sqcup xq_50x0x0x\sqcup \rightarrow \sqcup q_5x0x0x0x\sqcup \rightarrow \sqcup q_5\sqcup x0x0x0x\sqcup \rightarrow \sqcup q_2x0x0x0x\sqcup \rightarrow \sqcup xq_20x0x0x\sqcup \rightarrow \sqcup xxq_3x0x0x\sqcup \rightarrow \sqcup xxxq_30x0x\sqcup \rightarrow \sqcup xxx0q_4x0x\sqcup \rightarrow \sqcup xxx0xq_40x\sqcup \rightarrow \sqcup xxx0xxq_3x\sqcup \rightarrow \sqcup xxx0xxxq_3\sqcup \rightarrow \sqcup xxx0xxq_5x\sqcup \rightarrow \sqcup xxx0xq_5xx\sqcup \rightarrow \sqcup xxx0q_5xxx\sqcup \rightarrow \sqcup xxxq_50xxx\sqcup \rightarrow \sqcup xxxq_5x0xxx\sqcup \rightarrow \sqcup xq_5xx0xxx\sqcup \rightarrow \sqcup q_5xxx0xxx\sqcup \rightarrow \sqcup q_5\sqcup xxx0xxx\sqcup \rightarrow \sqcup q_2xxx0xxx\sqcup \rightarrow \sqcup xq_2xx0xxx\sqcup \rightarrow \sqcup xxq_2x0xxx\sqcup \rightarrow \sqcup xxxq_20xxx\sqcup \rightarrow \sqcup xxxxxq_3xxx\sqcup \rightarrow \sqcup xxxxxq_3xx\sqcup \rightarrow \sqcup xxxxxq_3x\sqcup \rightarrow \sqcup xxxxxxxq_3\sqcup \rightarrow \sqcup xxxxxxxq_5x\sqcup \rightarrow \sqcup xxxxxxxq_5xx\sqcup \rightarrow \sqcup xxxxxxxq_5xxx\sqcup \rightarrow \sqcup xxxq_5xxxxx\sqcup \rightarrow \sqcup xxq_5xxxxxx\sqcup \rightarrow \sqcup xq_5xxxxxxx\sqcup \rightarrow \sqcup q_5xxxxxxx\sqcup \rightarrow \sqcup q_5\sqcup xxxxxxxx\sqcup \rightarrow \sqcup q_2xxxxxxx\sqcup \rightarrow \sqcup xq_2xxxxxx\sqcup \rightarrow \sqcup xxq_2xxxxx\sqcup \rightarrow \sqcup xxxq_2xxxx\sqcup \rightarrow \sqcup xxxxxq_2xxx\sqcup \rightarrow \sqcup xxxxxq_2xx\sqcup \rightarrow \sqcup xxxxxq_2x\sqcup \rightarrow \sqcup xxxxxxxq_2\sqcup \rightarrow \sqcup xxxxxxxx\sqcup q_{accept}$

3.2

b. $1\#1. \quad q_11\#1 \rightarrow xq_3\#1 \rightarrow x\#q_51 \rightarrow xq_6\#x \rightarrow q_7x\#x \rightarrow xq_1\#x \rightarrow x\#q_8x \rightarrow x\#q_8\sqcup \rightarrow x\#x\sqcup q_{accept}.$

c. $1\#\#1. \quad q_11\#\#1 \rightarrow xq_3\#\#1 \rightarrow x\#q_5\#1 \rightarrow x\#\#q_{reject}1.$

d. $10\#11. \quad q_110\#11 \rightarrow xq_30\#11 \rightarrow x0q_3\#11 \rightarrow x0\#q_511 \rightarrow x0q_6\#x1 \rightarrow xq_70\#x1 \rightarrow q_7x0\#x1 \rightarrow xq_10\#x1 \rightarrow xxq_2\#x1 \rightarrow xx\#q_4x1 \rightarrow xx\#xq_41 \rightarrow xx\#x1q_{reject}\sqcup.$

e. $10\#10$. $q_110\#10 \rightarrow xq_30\#10 \rightarrow x0q_3\#10 \rightarrow x0\#q_510 \rightarrow x0q_6\#x0 \rightarrow$
 $xq_70\#x0 \rightarrow q_7x0\#x0 \rightarrow xq_10\#x0 \rightarrow xxq_2\#x0 \rightarrow xx\#q_4x0 \rightarrow xx\#xq_40 \rightarrow$
 $xx\#q_6xx \rightarrow xxq_6\#xx \rightarrow xq_7x\#xx \rightarrow xq_7x\#xx \rightarrow xxq_1\#xx \rightarrow xx\#q_8xx \rightarrow$
 $xx\#xq_8x \rightarrow xx\#xxq_8\sqcup \rightarrow xx\#xx\sqcup q_{accept}$.

01100#01100

$q_101100\#01100 \rightarrow xq_21100\#01100 \rightarrow x1q_2100\#01100 \rightarrow x11q_200\#01100 \rightarrow$
 $x110q_20\#01100 \rightarrow x1100q_2\#01100 \rightarrow x1100\#q_401100 \rightarrow x1100q_6\#x1100 \rightarrow$
 $x110q_70\#x1100 \rightarrow x11q_700\#x1100 \rightarrow x1q_7100\#x1100 \rightarrow xq_71100\#x1100 \rightarrow$
 $q_7x1100\#x1100 \rightarrow xq_11100\#x1100 \rightarrow xxq_3100\#x1100 \rightarrow xx1q_300\#x1100 \rightarrow$
 $xx10q_30\#x1100 \rightarrow xx100q_3\#x1100 \rightarrow xx100\#q_5x1100 \rightarrow xx100\#xq_51100 \rightarrow$
 $xx100\#q_6xx100 \rightarrow xx100q_6\#xx100 \rightarrow xx10q_70\#xx100 \rightarrow xx1q_700\#xx100 \rightarrow$
 $xxq_7100\#xx100 \rightarrow xq_7x100\#xx100 \rightarrow xxq_1100\#xx100 \rightarrow xxxq_300\#xx100 \rightarrow$
 $xxx0q_30\#xx100 \rightarrow xxx00q_3\#xx100 \rightarrow xxx00\#q_5xx100 \rightarrow xxx00\#xq_5x100 \rightarrow$
 $xxx00\#xxq_5100 \rightarrow xxx00\#xq_6xx00 \rightarrow xxx00\#q_6xxx00 \rightarrow xxx00q_6\#xxx00 \rightarrow$
 $xxx0q_70\#xxx00 \rightarrow xxxq_700\#xxx00 \rightarrow xxq_7x00\#xxx00 \rightarrow xxxq_100\#xxx00 \rightarrow$
 $xxxxq_20\#xxx00 \rightarrow xxx0q_2\#xxx00 \rightarrow xxx0\#q_4xxx00 \rightarrow xxx0\#xq_4xx00 \rightarrow$
 $xxxx0\#xxq_4x00 \rightarrow xxx0\#xxxq_400 \rightarrow xxx0\#xxq_6xx0 \rightarrow xxx0\#xq_6xxx0 \rightarrow$
 $xxxx0\#q_6xxxx0 \rightarrow xxx0q_6\#xxxx0 \rightarrow xxxq_70\#xxxx0 \rightarrow xxxq_7x0\#xxxx0 \rightarrow$
 $xxxxq_10\#xxxx0 \rightarrow xxxxxq_2\#xxxx0 \rightarrow xxxxx\#q_4xxxx0 \rightarrow xxxxx\#xq_4xxx0 \rightarrow$
 $xxxxx\#xxq_4xx0 \rightarrow xxxxx\#xxxq_4x0 \rightarrow xxxxx\#xxxq_40 \rightarrow xxxxx\#xxxq_6xx \rightarrow$
 $xxxxx\#xxq_6xxx \rightarrow xxxxx\#xq_6xxxx \rightarrow xxxxx\#q_6xxxxx \rightarrow xxxxxq_6\#xxxxx \rightarrow$
 $xxxxxq_7x\#xxxxx \rightarrow xxxxxq_1\#xxxxx \rightarrow xxxxx\#q_8xxxxx \rightarrow xxxxx\#xq_8xxxx \rightarrow$
 $xxxxx\#xxq_8xxx \rightarrow xxxxx\#xxxq_8xx \rightarrow xxxxx\#xxxq_8x \rightarrow xxxxx\#xxxxq_8\sqcup \rightarrow$
 $xxxxx\#xxxxx\sqcup q_{accept}$

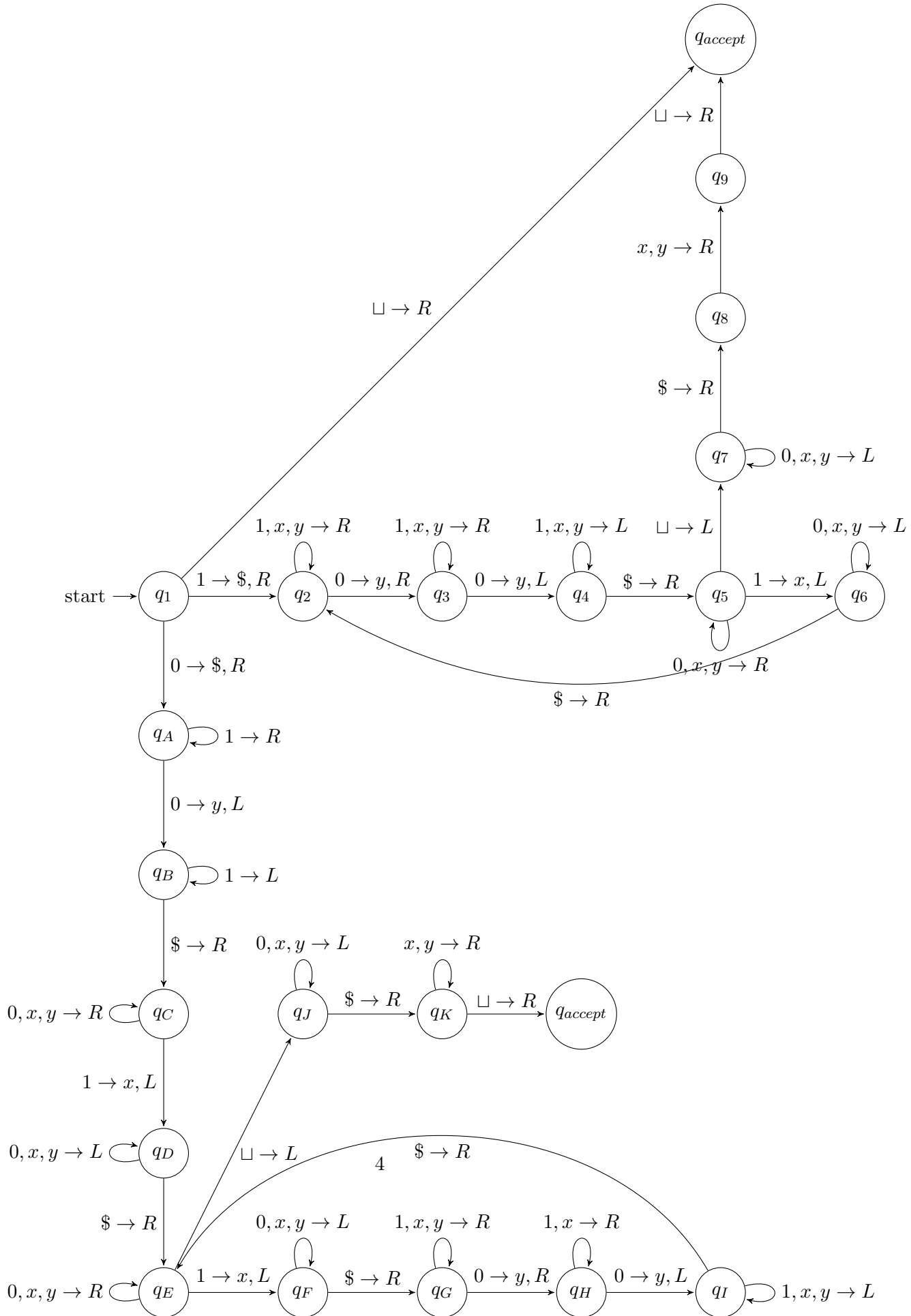
01101#01100

$q_101101\#01100 \rightarrow xq_21101\#01100 \rightarrow x1q_2101\#01100 \rightarrow x11q_201\#01100 \rightarrow$
 $x110q_21\#01100 \rightarrow x1101q_2\#01100 \rightarrow x1101\#q_401100 \rightarrow x1101q_6\#x1100 \rightarrow$
 $x110q_71\#x1100 \rightarrow x11q_701\#x1100 \rightarrow x1q_7101\#x1100 \rightarrow xq_71101\#x1100 \rightarrow$
 $q_7x1101\#x1100 \rightarrow xq_11101\#x1100 \rightarrow xxq_3101\#x1100 \rightarrow xx1q_301\#x1100 \rightarrow$
 $xx10q_31\#x1100 \rightarrow xx101q_3\#x1100 \rightarrow xx101\#q_5x1100 \rightarrow xx101\#xq_51100 \rightarrow$
 $xx101\#q_6xx100 \rightarrow xx101q_6\#xx100 \rightarrow xx10q_71\#xx100 \rightarrow xx1q_701\#xx100 \rightarrow$
 $xxq_7101\#xx100 \rightarrow xq_7x101\#xx100 \rightarrow xxq_1101\#xx100 \rightarrow xxxq_301\#xx100 \rightarrow$
 $xxx0q_31\#xx100 \rightarrow xxx01q_3\#xx100 \rightarrow xxx01\#q_5xx100 \rightarrow xxx01\#xq_5x100 \rightarrow$
 $xxx01\#xxq_5100 \rightarrow xxx01\#xq_6xx00 \rightarrow xxx01\#q_6xxx00 \rightarrow xxx01q_6\#xxx00 \rightarrow$
 $xxx0q_71\#xxx00 \rightarrow xxxq_701\#xxx00 \rightarrow xxq_7x01\#xxx00 \rightarrow xxxq_101\#xxx00 \rightarrow$
 $xxxxq_21\#xxx00 \rightarrow xxx1q_2\#xxx00 \rightarrow xxx1\#q_4xxx00 \rightarrow xxx1\#xq_4xx00 \rightarrow$
 $xxxx1\#xxq_4x00 \rightarrow xxx1\#xxxq_400 \rightarrow xxx1\#xxq_6xx0 \rightarrow xxx1\#xq_6xxx0 \rightarrow$
 $xxxx1\#q_6xxxx0 \rightarrow xxx1q_6\#xxxx0 \rightarrow xxxq_71\#xxxx0 \rightarrow xxxq_7x1\#xxxx0 \rightarrow$
 $xxxxq_11\#xxxx0 \rightarrow xxxxxq_3\#xxxx0 \rightarrow xxxxx\#q_5xxxx0 \rightarrow xxxxx\#xq_5xxx0 \rightarrow$
 $xxxxx\#xxq_5xx0 \rightarrow xxxxx\#xxxq_5x0 \rightarrow xxxxx\#xxxq_50 \rightarrow xxxxx\#xxxxq_{reject}0$

3.8

b

1. Scan the tape and mark the first 1 which has not been marked. If no unmarked 1s are found go to stage 5. Otherwise move the head back to the start of the tape
2. Scan the tape until an unmarked 0 is found, mark the 0, if no 0s are found reject
3. Scan the tape once more until an unmarked zero is found, mark the 0, if no 0s are found reject
4. Move the head back to the start of the tape and go to stage 1
5. Move the head back to the start of the tape. Scan the tape to see if any unmarked 0s are found. If none are found accept, otherwise reject.



Note: Because the space of the page, I can not label the transition to the reject state, all the input string can not reach the accept state or stuck in some state because of the next input symbol can not match the transition function here will directly go to the reject state, aka q_{reject} .

010100

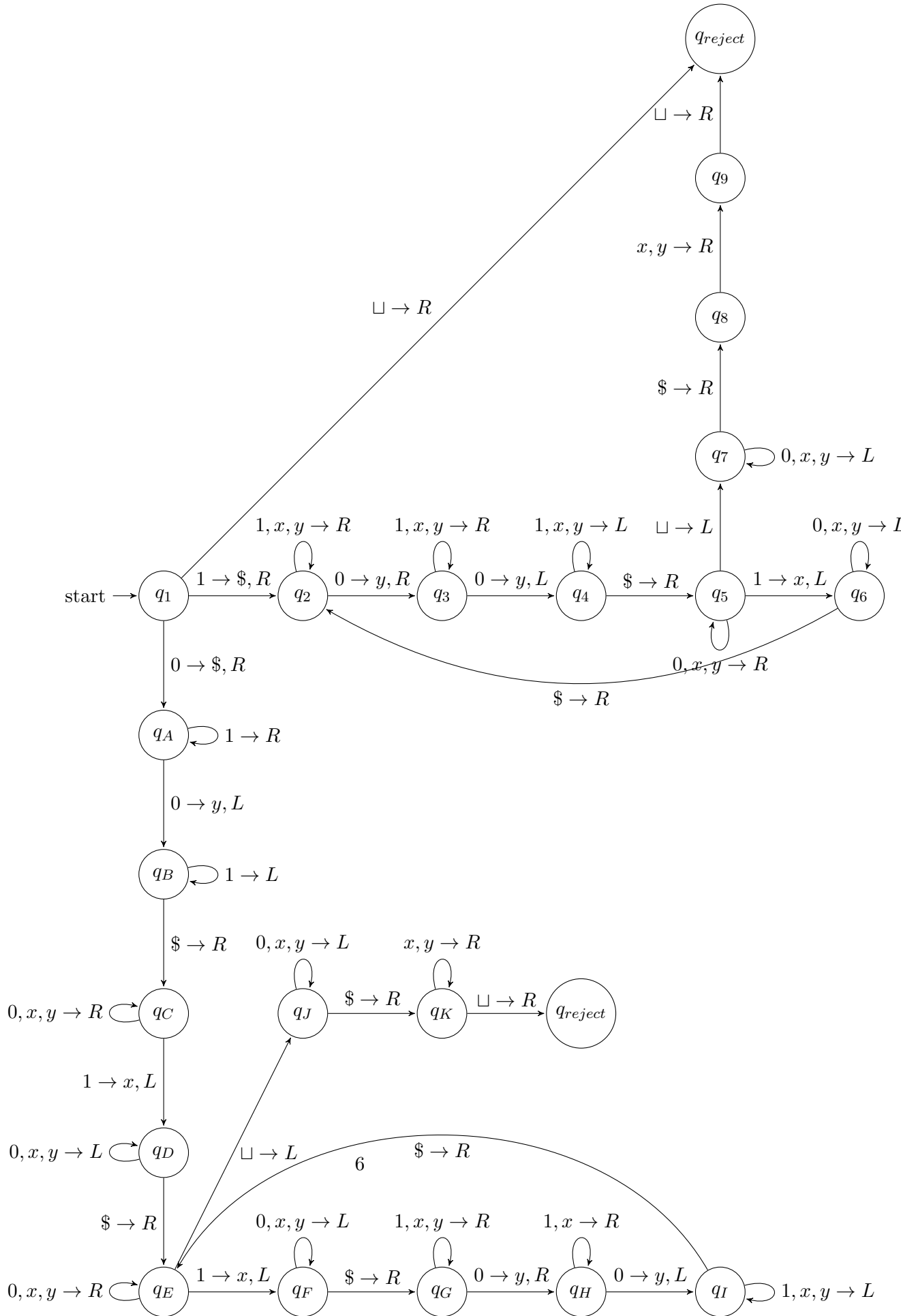
$q_1010100 \rightarrow \$q_A10100 \rightarrow \$1q_A0100 \rightarrow \$q_B1y100 \rightarrow q_B\$1y100 \rightarrow \$q_C1y100 \rightarrow$
 $\$q_Dxy100 \rightarrow q_D\$xy100 \rightarrow \$q_Exy100 \rightarrow \$xq_Ey100 \rightarrow \$xyq_E100 \rightarrow \$xq_Fyx00 \rightarrow$
 $\$q_Fxyx00 \rightarrow q_F\$xyx00 \rightarrow \$q_Gxyx00 \rightarrow \$xq_Gyx00 \rightarrow \$xyq_Gx00 \rightarrow \$xyxq_G00 \rightarrow$
 $\$xyxyq_H0 \rightarrow \$xyxq_Iyy \rightarrow \$xyq_Ixyy \rightarrow \$xq_Ixyy \rightarrow \$q_Ixyxyy \rightarrow q_I\$xyxyy \rightarrow$
 $\$q_Exyxyy \rightarrow \$xq_Exyy \rightarrow \$xyq_Exyy \rightarrow \$xyxq_Eyy \rightarrow \$xyxyq_Ey \rightarrow \$xyxyyq_E\sqcup \rightarrow$
 $\$xyxyq_Jy\sqcup \rightarrow \$xyxq_Jyy\sqcup \rightarrow \$xyq_Jxyy\sqcup \rightarrow \$xq_Jyxyy\sqcup \rightarrow \$q_Jxyxyy\sqcup \rightarrow$
 $q_J\$xyxyy\sqcup \rightarrow \$q_Kxyxyy\sqcup \rightarrow \$xq_Kxyy\sqcup \rightarrow \$xyq_Kxyy\sqcup \rightarrow \$xyxq_Kyy\sqcup \rightarrow$
 $\$xyxyq_Ky\sqcup \rightarrow \$xyxyyq_K\sqcup \rightarrow \$xyxyy\sqcup q_{accept}$

010101

$q_1010101 \rightarrow \$q_A10101 \rightarrow \$1q_A0101 \rightarrow \$q_B1y101 \rightarrow q_B\$1y101 \rightarrow \$q_C1y101 \rightarrow$
 $\$q_Dxy101 \rightarrow q_D\$xy101 \rightarrow \$q_Exy101 \rightarrow \$xq_Ey101 \rightarrow \$xyq_E101 \rightarrow \$xq_Fyx01 \rightarrow$
 $\$q_Fxyx01 \rightarrow q_F\$xyx01 \rightarrow \$q_Gxyx01 \rightarrow \$xq_Gyx01 \rightarrow \$xyq_Gx01 \rightarrow \$xyxq_G01 \rightarrow$
 $\$xyxyq_H1 \rightarrow \$xyxy1q_H\sqcup \rightarrow \$xyxy1q_{reject}\sqcup$

c

1. Scan the tape and mark the first 1 which has not been marked. If no unmarked 1s are found go to stage 5. Otherwise move the head back to the start of the tape
2. Scan the tape until an unmarked 0 is found, mark the 0, if no 0s are found accept
3. Scan the tape once more until an unmarked zero is found, mark the 0, if no 0s are found accept
4. Move the head back to the start of the tape and go to stage 1
5. Move the head back to the start of the tape. Scan the tape to see if any unmarked 0s are found. If none are found reject, otherwise accept



Note: Because the space of the page, I can not lable the transition to the accept state, all the input string can not reach the reject state or stuck in some state because of the next input symbol can not match the transition function here will directly go to the accept state, aka q_{accept} .

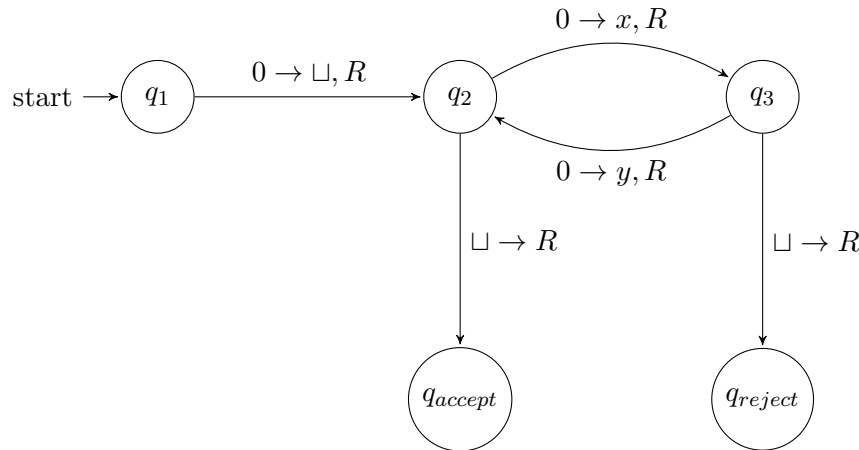
000111

$q_1000111 \rightarrow \$q_A00111 \rightarrow q_B\$y0111 \rightarrow \$q_Cy0111 \rightarrow \$yq_C0111 \rightarrow \$y0q_C111 \rightarrow$
 $\$yq_D0x11 \rightarrow \$q_Dy0x11 \rightarrow q_D\$y0x11 \rightarrow \$q_Ey0x11 \rightarrow \$yq_E0x11 \rightarrow \$y0q_Ex11 \rightarrow$
 $\$y0xq_E11 \rightarrow \$y0q_Fxx1 \rightarrow \$yq_F0xx1 \rightarrow \$q_Fy0xx1 \rightarrow q_F\$y0xx1 \rightarrow \$q_Gy0xx1 \rightarrow$
 $\$yq_G0xx1 \rightarrow \$yyq_Hxx1 \rightarrow \$yyxq_Hx1 \rightarrow \$yyxxq_H1 \rightarrow \$yyxx1q_H\sqcup \rightarrow \$yyxx1\sqcup q_{accept}$

000110

$q_1000110 \rightarrow \$q_A00110 \rightarrow q_B\$y0110 \rightarrow \$q_Cy0110 \rightarrow \$yq_C0110 \rightarrow \$y0q_C110 \rightarrow$
 $\$yq_D0x10 \rightarrow \$q_Dy0x10 \rightarrow q_D\$y0x10 \rightarrow \$q_Ey0x10 \rightarrow \$yq_E0x10 \rightarrow \$y0q_Ex10 \rightarrow$
 $\$y0xq_E10 \rightarrow \$y0q_Fxx0 \rightarrow \$yq_F0xx0 \rightarrow \$q_Fy0xx0 \rightarrow q_F\$y0xx0 \rightarrow \$q_Gy0xx0 \rightarrow$
 $\$yq_G0xx0 \rightarrow \$yyq_Hxx0 \rightarrow \$yyxq_Hx0 \rightarrow \$yyxxq_H0 \rightarrow \$yyxq_Ixy \rightarrow \$yyq_Ixxy \rightarrow$
 $\$yq_Iyxy \rightarrow \$q_Iyyxy \rightarrow q_I\$yxy \rightarrow \$q_Eyxy \rightarrow \$yq_Eyxy \rightarrow \$yyq_Exxy \rightarrow$
 $\$yyxq_Exy \rightarrow \$yyxxq_Ey \rightarrow \$yyxxyq_E\sqcup \rightarrow \$yyxxq_Jy\sqcup \rightarrow \$yyxq_Jxy\sqcup \rightarrow \$yyq_Jxxy\sqcup \rightarrow$
 $\$yq_Jyxy\sqcup \rightarrow \$q_Jyyxy\sqcup \rightarrow q_J\$yxy\sqcup \rightarrow \$q_Kyxy\sqcup \rightarrow \$yq_Kyxy\sqcup \rightarrow$
 $\$yyq_Kxxy\sqcup \rightarrow \$yyxq_Kxy\sqcup \rightarrow \$yyxxq_Ky\sqcup \rightarrow \$yyxxyq_K\sqcup \rightarrow \$yyxxy\sqcup q_{reject}$

Modify machine M2 to recognize odd number of 0s and draw the state diagram.



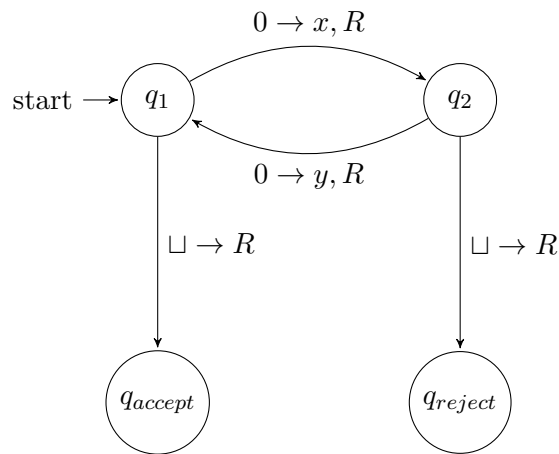
000

$q_1 000 \rightarrow \sqcup q_2 00 \rightarrow \sqcup x q_3 0 \rightarrow \sqcup x y q_2 \sqcup \rightarrow \sqcup x y \sqcup q_{accept}$

0000

$q_1 0000 \rightarrow \sqcup q_2 000 \rightarrow \sqcup x q_3 00 \rightarrow \sqcup x y q_2 0 \rightarrow \sqcup x y x q_3 \sqcup \rightarrow \sqcup x y x \sqcup q_{reject}$

Modify machine M2 to recognize even number of 0s and draw the state diagram.



000

$q_1 000 \rightarrow x q_2 00 \rightarrow x y q_1 0 \rightarrow x y x q_2 \sqcup \rightarrow x y x \sqcup q_{reject}$

0000

$q_1 0000 \rightarrow x q_2 000 \rightarrow x y q_1 00 \rightarrow x y x q_2 0 \rightarrow x y x y q_1 \sqcup \rightarrow x y x y \sqcup q_{accept}$