

Compiler

--- Machine Independent Optimization

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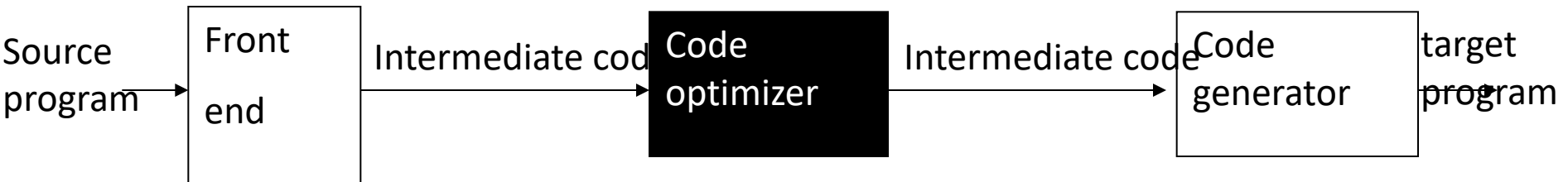
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Southeast University

8.4, 8.5, 9.1 in Textbook

Role

1. Position of code optimizer



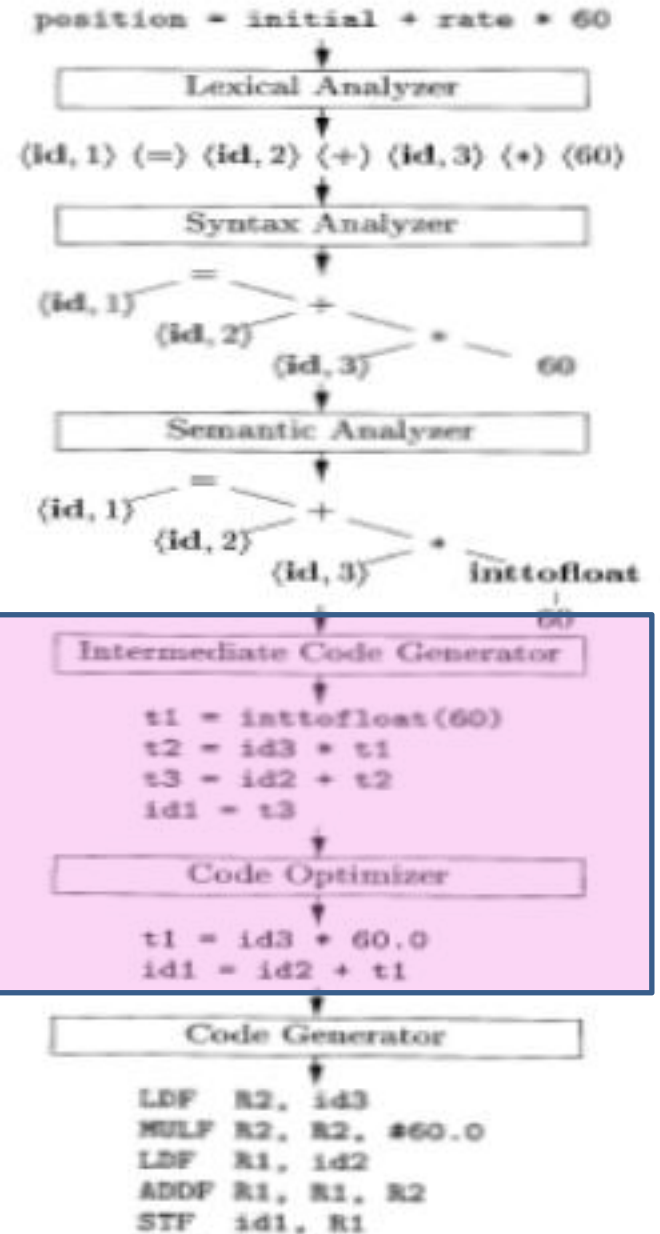
2. Purpose of code optimizer

- to get better efficiency
 - Run faster
 - Take less space

Example

1	position	...
2	initial	...
3	rate	...

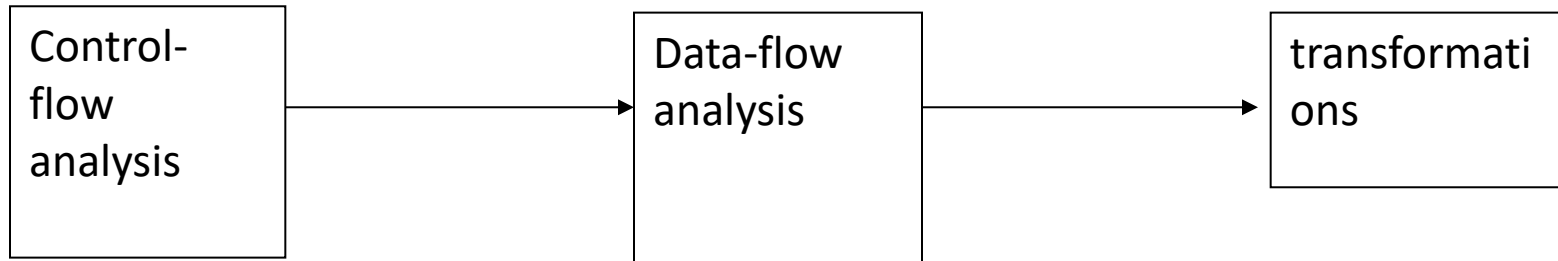
SYMBOL TABLE



Three Types of Optimizations

- Source code
 - User can profile the program, change algorithm or transform loops.
- Intermediate code (**Machine-independent**)
 - Compiler can improve loops, procedure calls or address calculations
- Target code
 - Compiler can use registers, select instructions or do peephole transformations

Machine-independent Optimization Procedure



Control-flow analysis

- Identify blocks and loops in the flow graph of a program

Data-flow analysis

- Collect information about the program as a whole and to distribute this information to each block in the flow graph.

Basic Blocks and flow graphs

1.Flow graph

- A directed graph that are composed of the set of basic blocks making up a program

2.Basic blocks

- A sequence of consecutive statements in which flow of control enters at the beginning and leaves at the end without halt or possibility of branching except at the end.

- E.g. this is a basic block

$T1 = a * a$

$T2 = a * b$

$T3 = 2 * T2$

$T4 = T1 + T3$

$T5 = b * b$

$T6 = T4 + T5$

Notes: A name in a basic block is said to be live at a given point if its value is used after that point in the program, perhaps in another basic block.

3.Partition into basic blocks

- **Input.** A sequence of three-address statements
- **Output.** A list of basic blocks with each three-address statement in exactly one block.
- **Method.**
 - 1) We first determine the set of leaders, the first statements of basic blocks.The rules we use are the following:

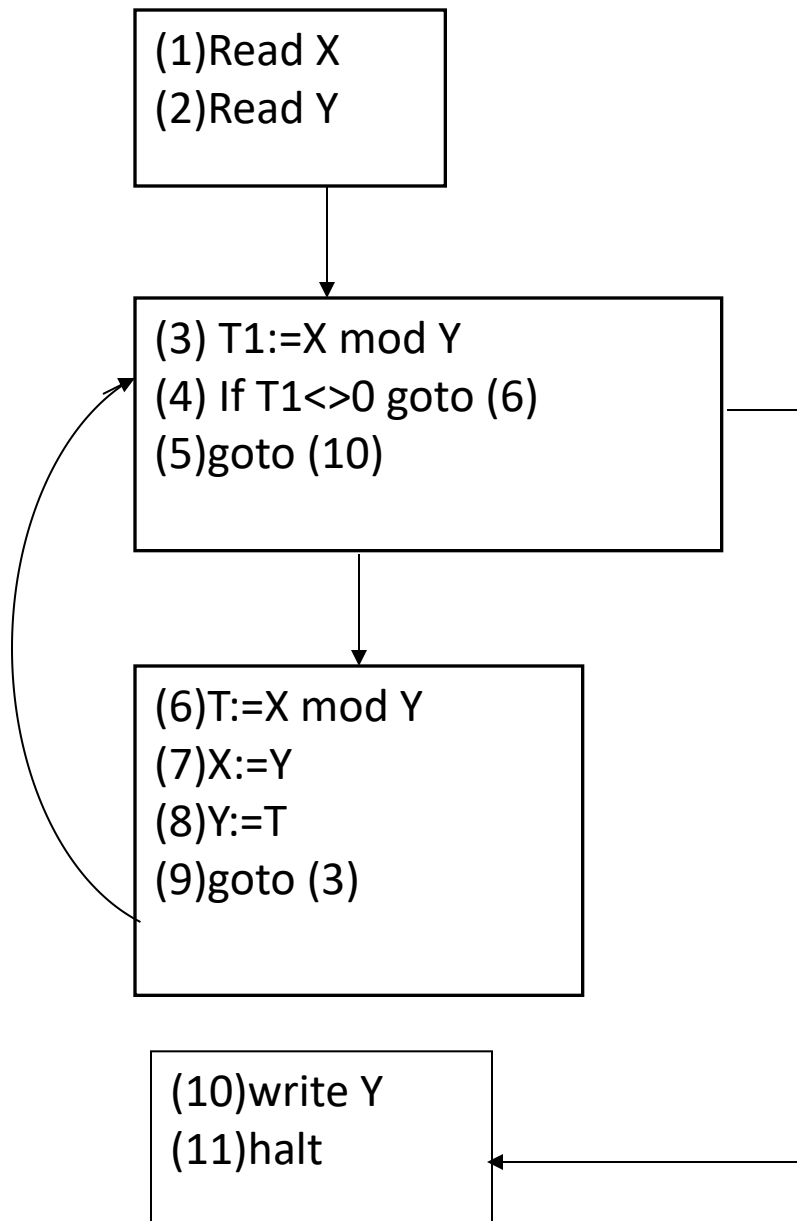
- (1) The first statement is a leader.
- (2) Any statement that is the target of a conditional or unconditional goto is a leader.
- (3) Any statement that immediately follows a goto or conditional goto statement is a leader.

2) For each leader, its basic block consists of the leader and all statements up to but not including the next leader or the end of the program.

EXAMPLE 1.

```
begin
  read X;
  read Y;
  while (X mod Y<>0) do
    begin
      T:=X mod Y;
      X:=Y;
      Y:=T
    end;
  write Y
end
```

```
(1)Read X
(2)Read Y
(3) T1:=X mod Y
(4) If T1<>0 goto (6)
(5)goto (10)
(6)T:=X mod Y
(7)X:=Y
(8)Y:=T
(9)goto (3)
(10)write Y
(11)halt
```

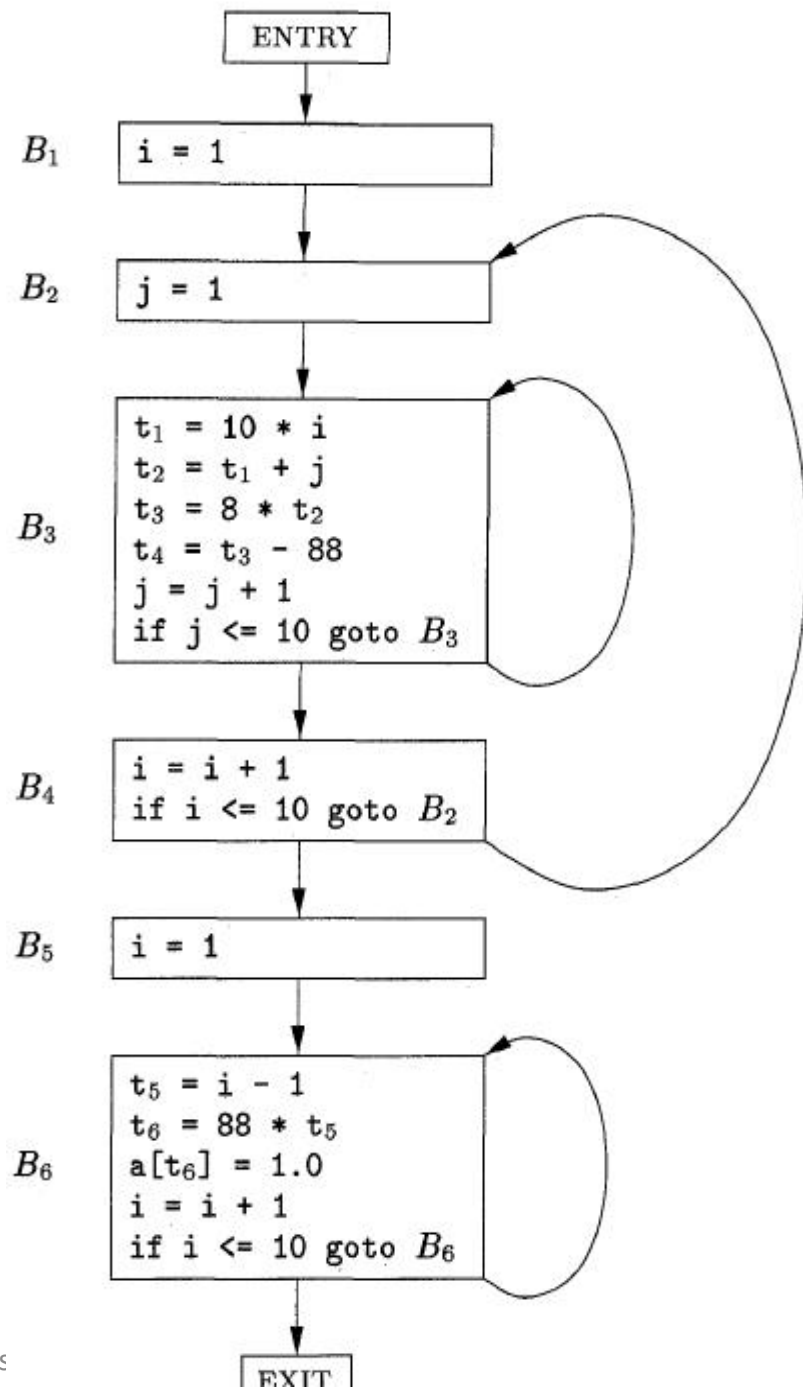


Blocks and Flow graph

```

1)  i = 1
2)  j = 1
3)  t1 = 10 * i
4)  t2 = t1 + j
5)  t3 = 8 * t2
6)  t4 = t3 - 88
7)  a[t4] = 0.0
8)  j = j + 1
9)  if j <= 10 goto (3)
10) i = i + 1
11) if i <= 10 goto (2)
12) i = 1
13) t5 = i - 1
14) t6 = 88 * t5
15) a[t6] = 1.0
16) i = i + 1
17) if i <= 10 goto (13)

```



Next-Use Information

Knowing when the value of a variable will be used next is essential for generating good code. If the value of a variable that is currently in a register will never be referenced subsequently, then that register can be assigned to another variable.

The *use* of a name in a three-address statement is defined as follows. Suppose three-address statement i assigns a value to x . If statement j has x as an operand, and control can flow from statement i to j along a path that has no intervening assignments to x , then we say statement j *uses* the value of x computed at statement i . We further say that x is *live* at statement i .

Optimization of basic blocks

1. Function-preserving transformations

1) Methods

- Constant folding
 - The evaluation at compile-time of expressions whose operands are known to be constant
- Common sub-expression elimination
- Copy propagation
- Dead-code elimination

2)Constant folding & Common sub-expression elimination

- An expression E is called a common sub-expression if an expression E was previously computed, and the values of variables in E have not changed since the previous computation.

Notes: We can avoid re-computing the expression if we can use the previously computed value

e.g: A source code

$Pi := 3.14$

$A := 2 * Pi * (R + r);$

$B := A;$

$B := 2 * Pi * (R + r) * (R - r)$

(1) $Pi := 3.14$

(2) $T1 := 2 * Pi$

(3) $T2 := R + r$

(4) $A := T1 * T2$

(5) $B := A$

(6) $T3 := 2 * Pi$

(7) $T4 := R + r$

(8) $T5 := T3 * T4$

(9) $T6 := R - r$

(10) $B := T5 * T6$

(1) $Pi := 3.14$

(2) $T1 := 2 * Pi$

(3) $T2 := R + r$

(4) $A := T1 * T2$

(5) $B := A$

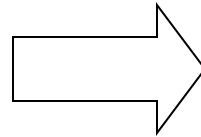
(6) $T3 := 2 * Pi$

(7) $T4 := R + r$

(8) $T5 := T3 * T4$

(9) $T6 := R - r$

(10) $B := T5 * T6$



(1) $Pi := 3.14$

(2) $T1 := 6.28$

(3) $T2 := R + r$

(4) $A := T1 * T2$

(5) $B := A$

(6) $T3 := T1$

(7) $T4 := T2$

(8) $T5 := T3 * T4$

(9) $T6 := R - r$

(10) $B := T5 * T6$

3)Copy Propagation

- One concerns assignments of the form $f:=g$ called copy statements

(1) $\text{Pi} := 3.14$
(2) $\text{T1} := 6.28$
(3) $\text{T2} := \text{R} + \text{r}$
(4) $\text{A} := \text{T1} * \text{T2}$
(5) $\text{B} := \text{A}$
(6) $\text{T3} := \text{T1}$
(7) $\text{T4} := \text{T2}$
(8) $\text{T5} := \text{T3} * \text{T4}$
(9) $\text{T6} := \text{R} - \text{r}$
(10) $\text{B} := \text{T5} * \text{T6}$

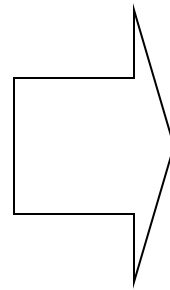
(1) $\text{Pi} := 3.14$
(2) $\text{T1} := 6.28$
(3) $\text{T2} := \text{R} + \text{r}$
(4) $\text{A} := \text{T1} * \text{T2}$
(5) $\text{B} := \text{A}$
(6) $\text{T3} := \text{T1}$
(7) $\text{T4} := \text{T2}$
(8) $\text{T5} := \text{T1} * \text{T2}$
(9) $\text{T6} := \text{R} - \text{r}$
(10) $\text{B} := \text{T5} * \text{T6}$

4) Dead-code elimination

- A variable is **live** at a point in a program if its value can be used subsequently; otherwise it is **dead** at that point.

Notes: A related idea is dead or useless code, statements that compute values that never get used.

(1) $\text{Pi} := 3.14$
(2) $\text{T1} := 6.28$
(3) $\text{T2} := \text{R} + \text{r}$
(4) $\text{A} := \text{T1} * \text{T2}$
(5) $\text{B} := \text{A}$
(6) $\text{T3} := \text{T1}$
(7) $\text{T4} := \text{T2}$
(8) $\text{T5} := \text{T1} * \text{T2}$
(9) $\text{T6} := \text{R} - \text{r}$
(10) $\text{B} := \text{T5} * \text{T6}$



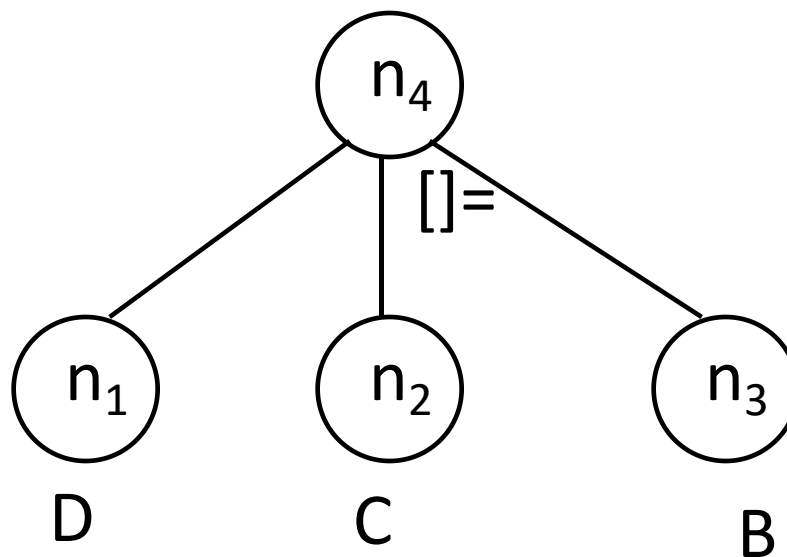
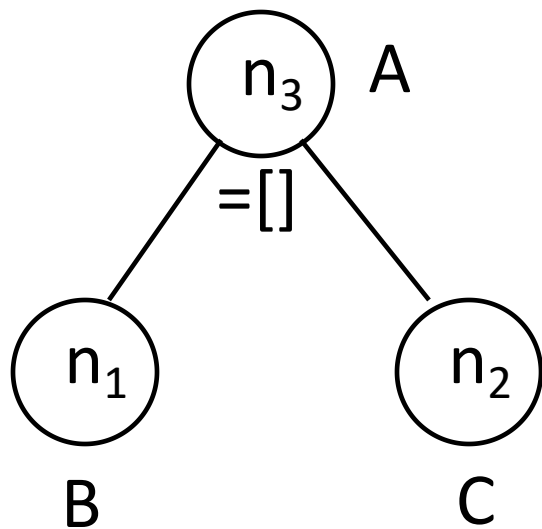
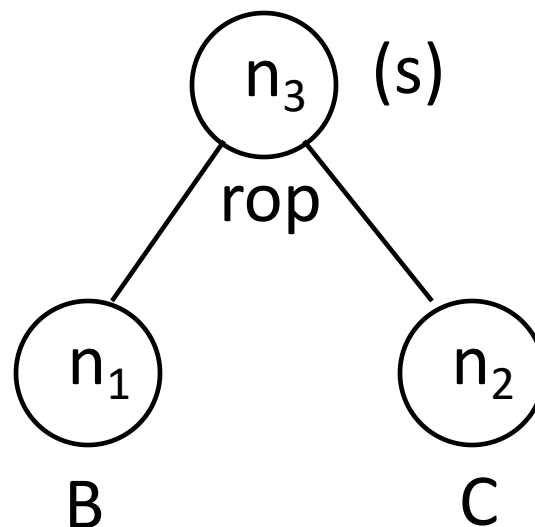
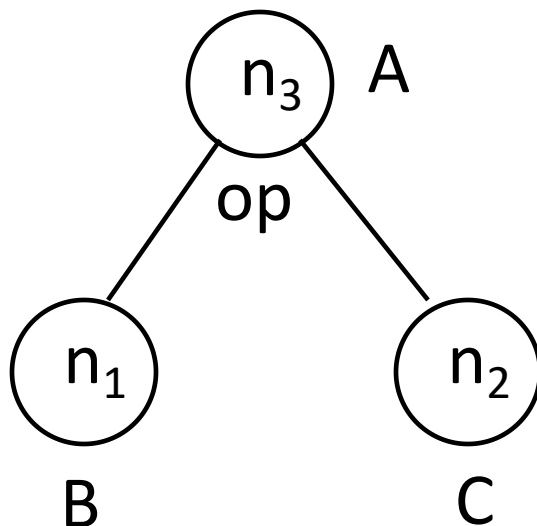
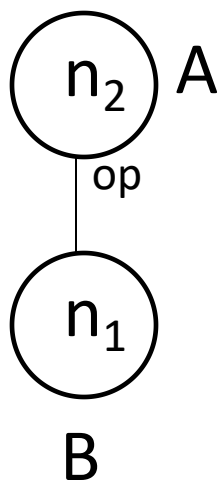
(1) $\text{Pi} := 3.14$
(2) $\text{T1} := 6.28$
(3) $\text{T2} := \text{R} + \text{r}$
(4) $\text{A} := \text{T1} * \text{T2}$
(8) $\text{T5} := \text{T1} * \text{T2}$
(9) $\text{T6} := \text{R} - \text{r}$
(10) $\text{B} := \text{T5} * \text{T6}$

2.DAG Representation(**8.5.1 section**)

A Dag for a basic block is a directed acyclic graph with the following labels on nodes:

1. Leaves are labeled by unique identifiers, either variable names or constants.
2. There is a node associated with each statement s within the block.
3. Interior nodes are labeled by an operator symbol.

- The children of n are those nodes corresponding to statements that are the last definitions prior to s of the operands used by s .
- Nodes are also optionally given a sequence of identifiers for labels. The intension is that interior nodes represent computed values, and the identifiers labeling a node are deemed to have the value.

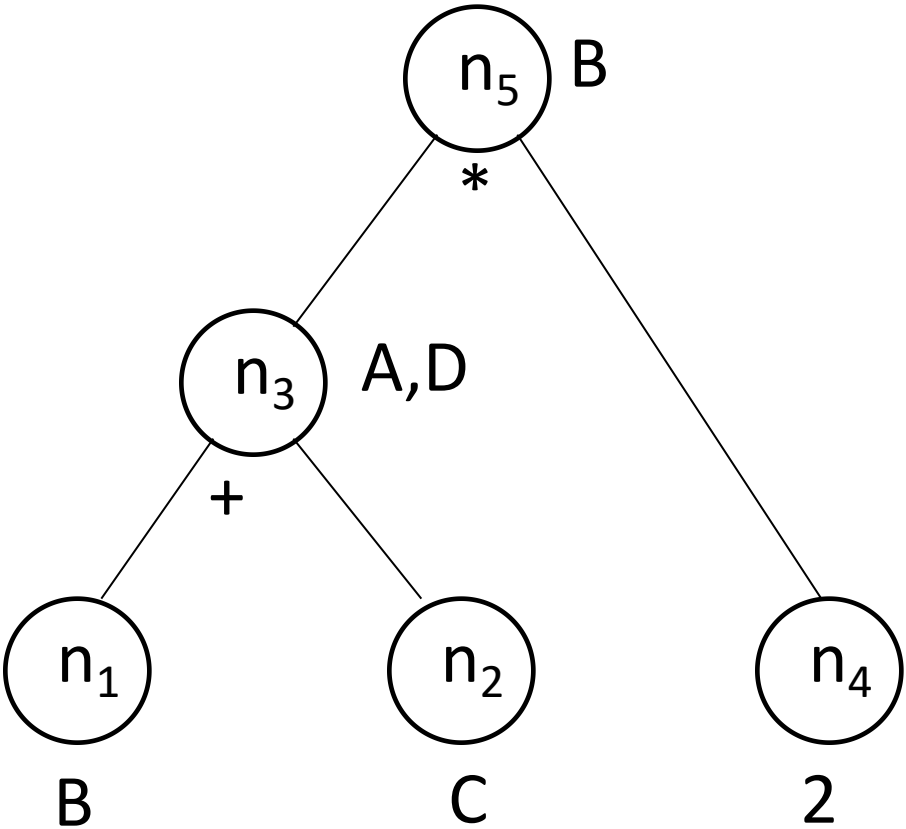


e.g.

$A := B + C$

$B := 2 * A$

$D := A$



(1)t1:=4*I

(2)t2:=a[t1]

(3)t3:=4*I

(4)t4:=b[t3]

(5)t5:=t2*t4

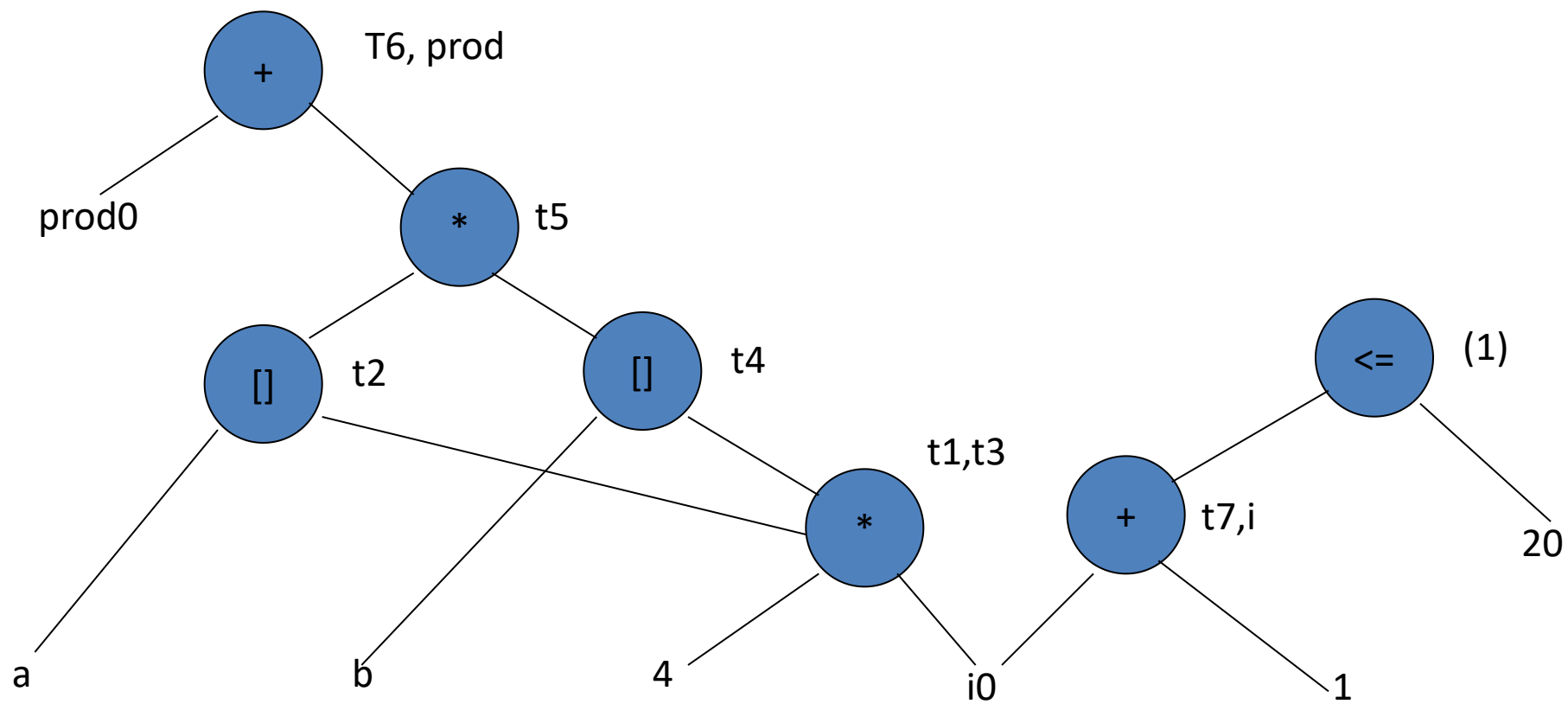
(6)t6:=prod+t5

(7)prod:=t6

(8)t7:=i+1

(9)i:=t7

(10)If i<=20 goto(1)



DAG based Optimization by Rewriting TAC

- a) a leave without additional label, no TAC
- b) a leave with label B, and additional label A, then we can get assignment statement $A=B$, if it has several additional labels, then we can get several assignment statements with the form $X=B$.
- c) for a Interior node with additional label A and label op, we can get $A:=B \text{ op } C$, $A:=\text{op } B$, $A:=B[C]$ or if $B \text{ rop } C \text{ goto } (s)$, if it has more than one additional labels, then we can also get assignment statements with the form $X=A$
- d) for a Interior node without additional label, add a new temp additional label S_i , and goto c)
- e) Dead code Elimination
- f) Using Algebraic Identities

Note.

Algebraic Identities

$$x + 0 = 0 + x = x$$

$$x \times 1 = 1 \times x = x$$

$$x - 0 = x$$

$$x/1 = x$$

EXPENSIVE

$$x^2$$

=

$$x \times x$$

$$2 \times x$$

=

$$x + x$$

$$x/2$$

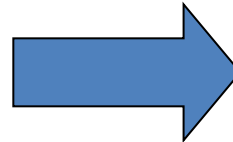
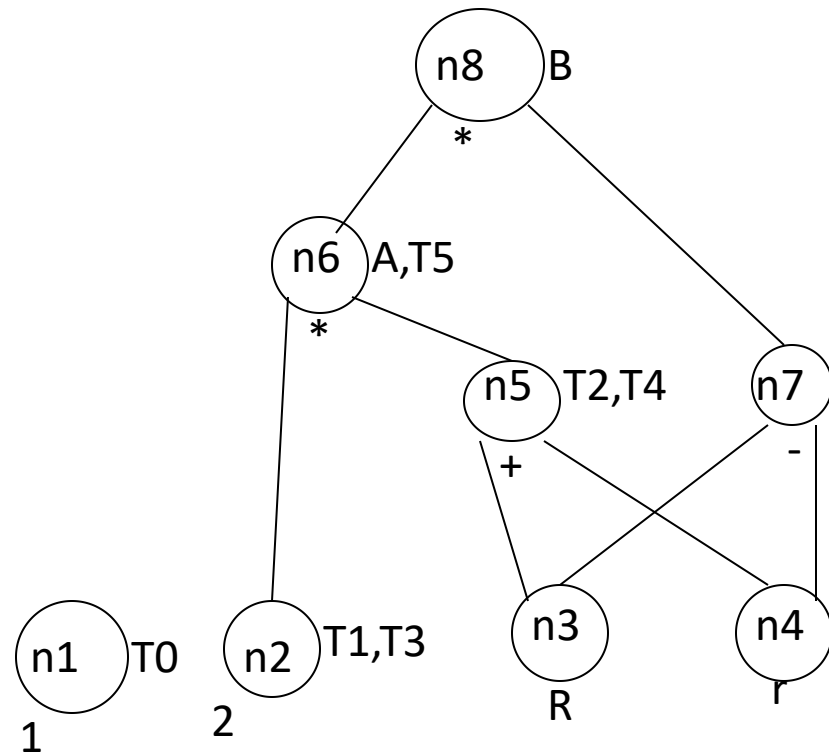
=

$$x \times 0.5$$

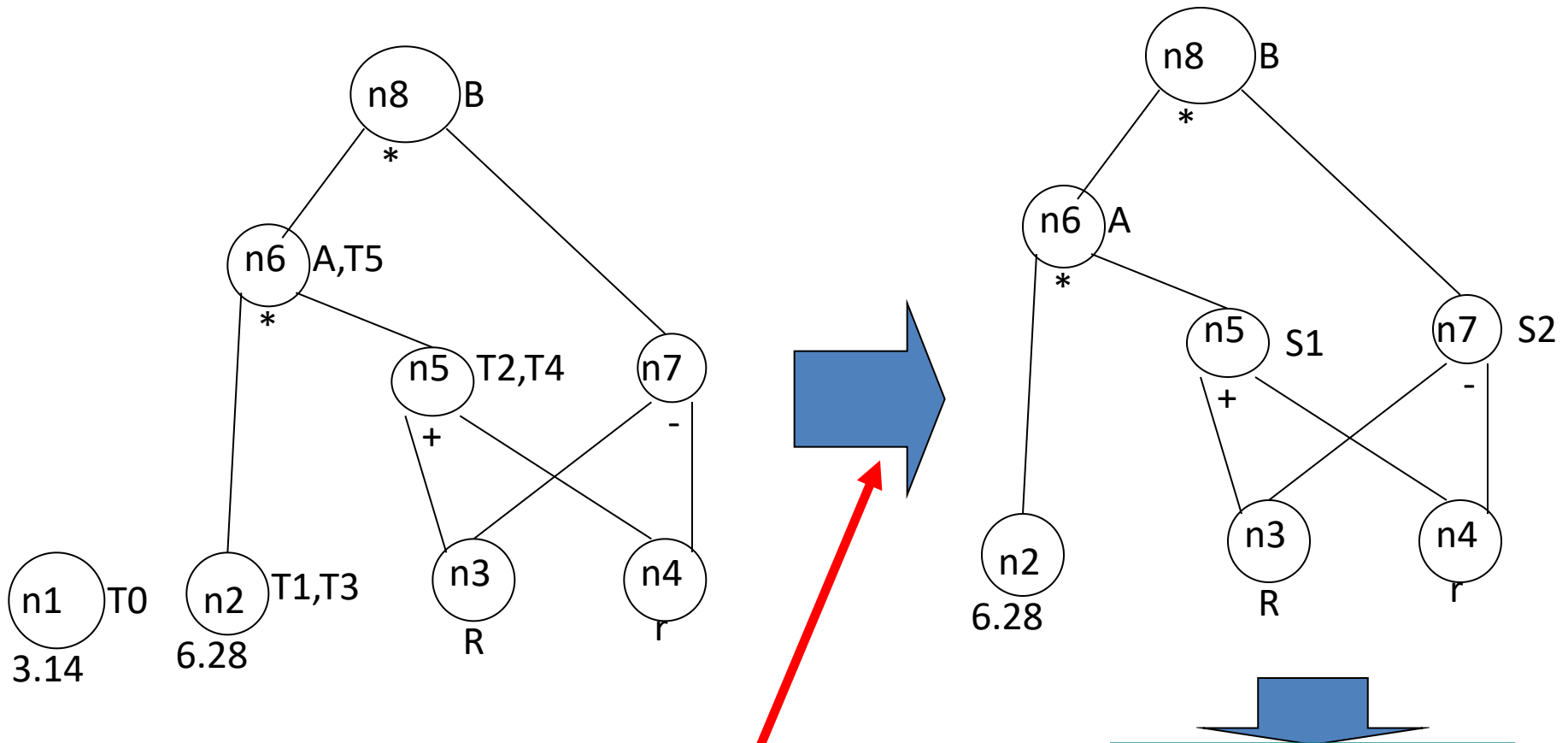
CHEAPER

Example

(1) $T0 := 1$ (2) $T1 := 2 * T0$ (3) $T2 := R + r$
(4) $A := T1 * T2$ (5) $B := A$ (6) $T3 := 2 * T0$
(7) $T4 := R + r$ (8) $T5 := T3 * T4$ (9) $T6 := R - r$
(10) $B := T5 * T6$



(1) $T0 := 1$
 (2) $T1 := 2$
 (3) $T3 := 2$
 (4) $T2 := R + r$
 (5) $T4 := T2$
 (6) $A := 2 * T2$
 (7) $T5 := A$
 (8) $T6 := R - r$
 (9) $B := A * T6$



When T0,T1,T2,T3,T4,T5, T6
are dead

(1) $S1 := R + r$

(2) $A := 2 * s1$

(3) $s2 := R - r$

(4) $B := A * s2$



(1) $S1 := R + r$

(2) $A := s1 + s1$

(3) $s2 := R - r$

(4) $B := A * s2$

Please **construct the DAG** for the following basic block, and we assume that only variable “M” would be used later, please optimize the block and **rewrite the block** in optimized code form.(10%)

T1=1*20

T2=T1+1

T3=T2*4

T4=addrA-T3

T5=i*20

T6=T5+j

T7=T6*2

T8=1*20

T9=T8+1

T10=T9*4

T11=addrB-T10

T12=i*20

T13=T12+j

T14=T13*2

T15=T11[T14]

A=3

T16=T15*A

T4[T7]=T16

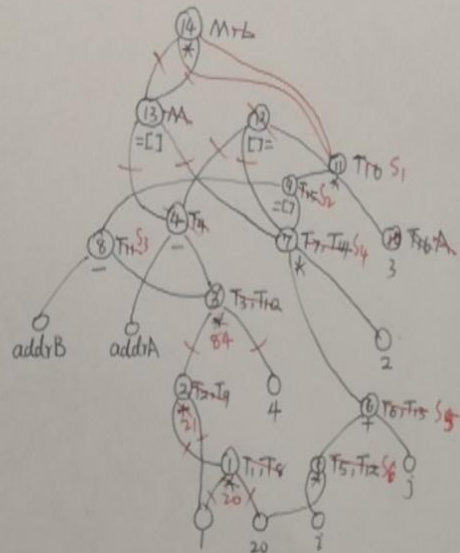
M=T4[T7]

M=M*M

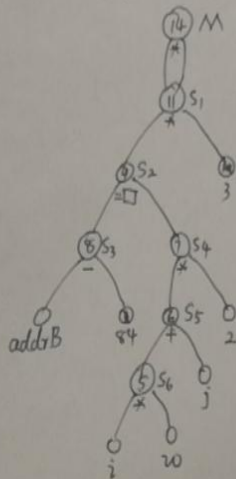
L=M

8.

1) Construct the DAG



2) Optimizing the DAG



3) Rewriting the TAC

Topology order: 14 11 9 8 7 6 5

$$S_6 = i * 20$$

$$S_5 = S_6 + j$$

$$S_4 = S_5 * 2$$

$$S_3 = \text{addrB} - 84$$

$$S_2 = S_3 [S_4]$$

$$S_1 = S_2 * 3$$

$$M = S_1 * 20$$

Generate a quadruple sequence
for the following expression,
and optimize and rewrite it
using DAG techniques.

$y := b + c$

$x := 0 * b$

$a := b + c$

$z := a * a$

$w := y * y$

$u := x + 3$

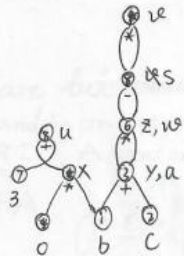
$v := z - w$

$v := v * v$

Assume that the only variables
that are live at the exit of this
block are v and z .

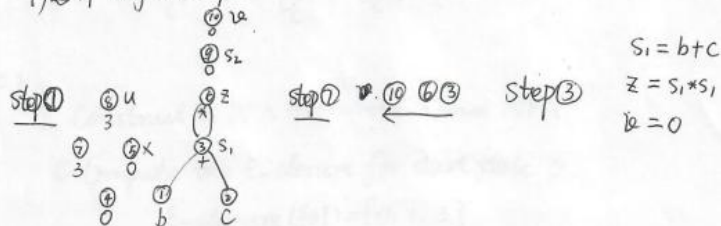
9. Answer.

(1) Construct a DAG

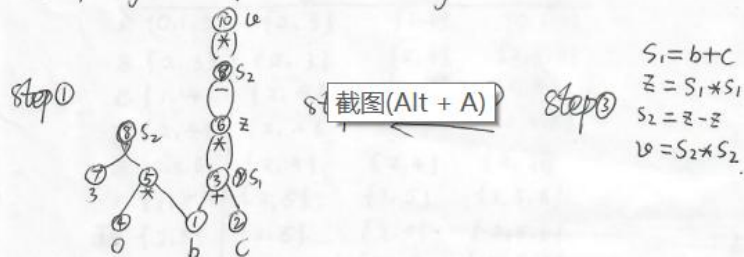


(2) Show the result DAG and rewrite its TAC

1) if algebraic optimization is allowed



2) if algebraic optimization is ignored



Creating an Topology Order:

(1) Depth first traversal.

(2) Ancestor node first traversal.

Creating an Topology Order:

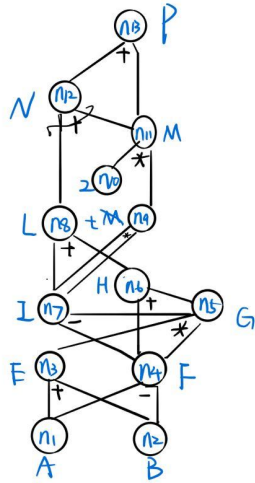
(1) Depth first traversal.

(2) Ancestor node first traversal.

从根节点按深度优先遍历，遇到有父节点尚未遍历的则先遍历其父节点。无附标的叶子节点不遍历。



1. $E = A + B$
 $F = A - B$
 $G = E * F$
 $H = F + G$
 $I = F - G$
 $L = I + H$
 $M = I * I$
 $N = 2 * M$
 $N = L + M$
 $P = N + M$



n13 n12 n8 n11 n9 n7 n6 n5 n3 n4

n4 $F = A - B$

n3 $E = A + B$

n5 $G = E * F$

n6 $H = F + G$

n7 $I = F - G$

n9 $t = I * I$

n11 $M = 2 * t$

n8 $L = I + H$

n12 $N = L + M$

n13 $P = N + M$

Exercises

- Please **construct the DAG** for the following basic block, and we assume that only variable “M” is live on exit, please optimize the block and **rewrite the block** in optimized code form

$E = A + B$

$F = E - C$

$G = F * F$

$H = E - C$

$I = H - G$

$J = I + G$

$M = J * J$

Loops in flow graphs

1. Dominators

We say node d of a flow graph dominates node n , written $d \text{ dom } n$, if every path from the initial node of the flow graph to n goes through d

2.Back edge

- The edges whose heads dominate their tails.

3.Find all the loops in a flow graph

- Input. A flow graph G and a back edge $n \rightarrow d$.
- Output. The set loop consisting of all nodes in the natural loop of $n \rightarrow d$.

```
void insert (m){  
    if m is not in loop {  
        loop=loop  $\cup$  {m};  
        push m onto stack }}
```

```
Main( )
```

```
{ Stack=empty;  
    Loop={d};  
    Insert(n);  
    While stack is not empty {  
        pop m, the first element of stack, off stack;  
        for each predecessor p of m do insert(p) }  
}
```

4.Pre-Headers

- Move statements before the header
- Create a new block, preheader. It has only the header as successor, and all edges which formerly entered the header of L from outside L instead enter the preheader.

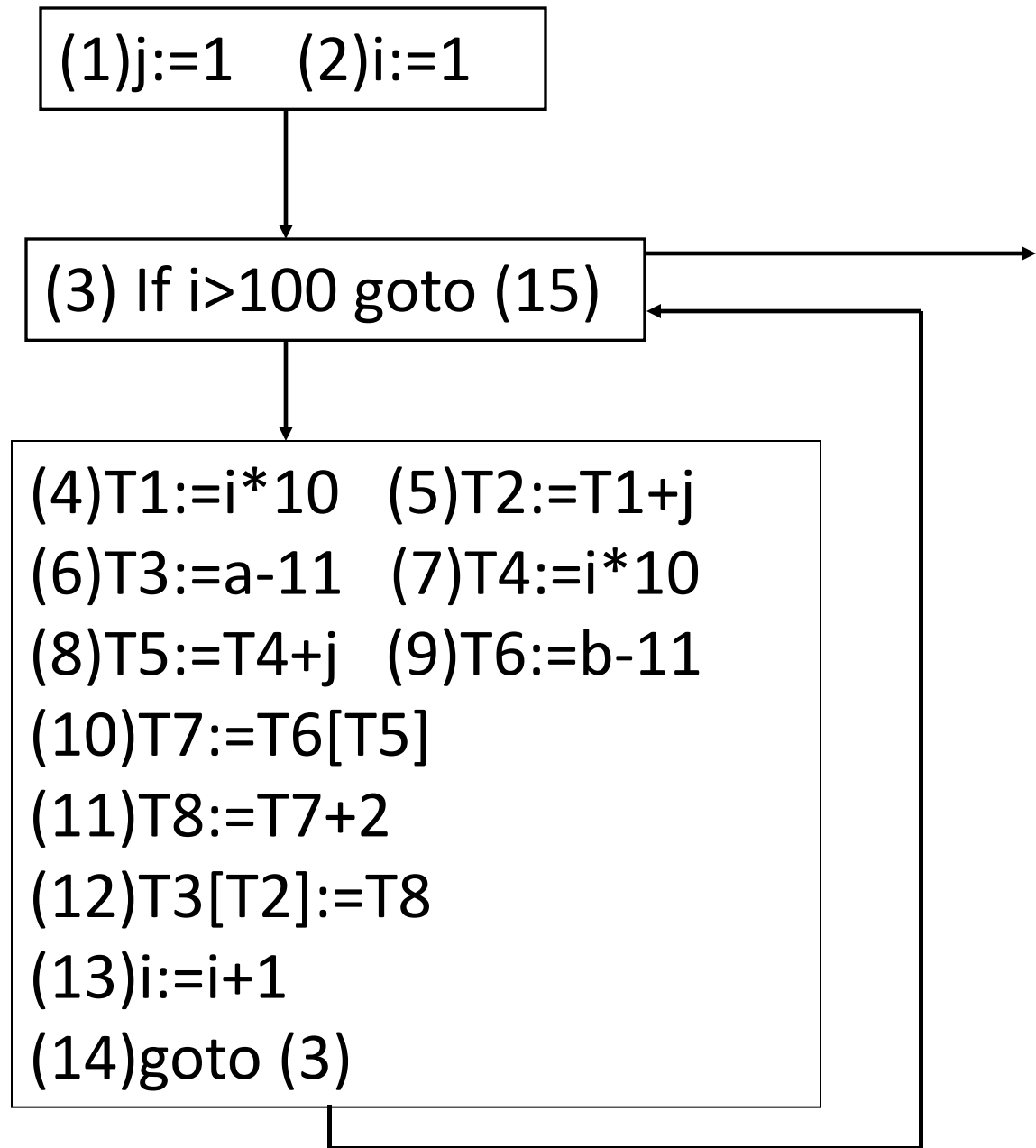
5.Code optimization in a loop

- Code motion
- Induction Variables and Reduction in Strength

e.g.

```
j:=1;  
for i=1 to 100 do  
  A[i,j]:=B[i,j] +2
```

And the array is :
A,B:array[1:100,1:
10]
m=1



(1)j:=1 (2)i:=1

(3) If i>100 goto (15)

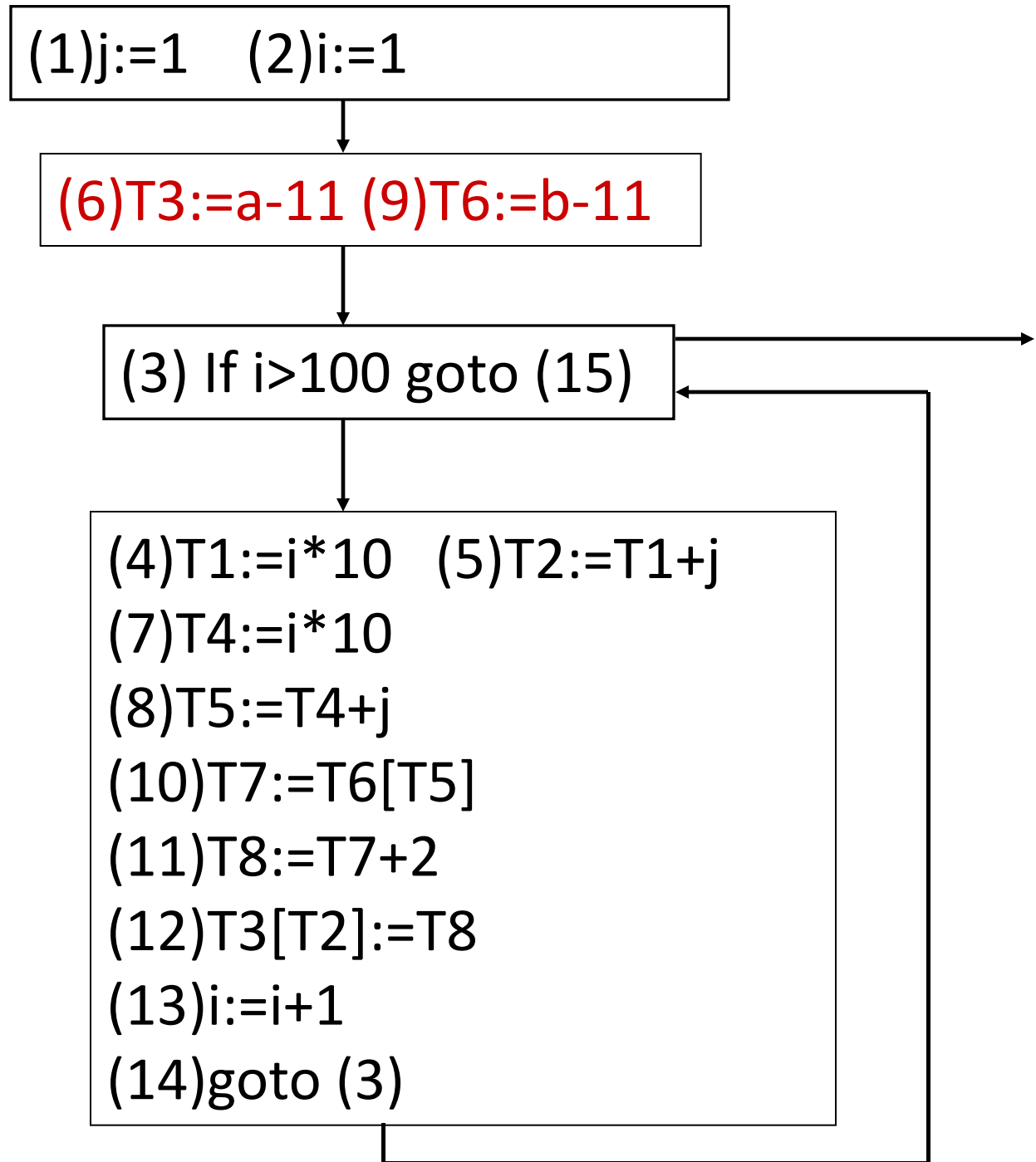
(4)T1:=i*10 (5)T2:=T1+j
(6)T3:=a-11 (7)T4:=i*10
(8)T5:=T4+j (9)T6:=b-11
(10)T7:=T6[T5]
(11)T8:=T7+2
(12)T3[T2]:=T8
(13)i:=i+1
(14)goto (3)

(1)j:=1 (2)i:=1

(6)T3:=a-11 (9)T6:=b-11

(3) If i>100 goto (15)

(4)T1:=i*10 (5)T2:=T1+j
(7)T4:=i*10
(8)T5:=T4+j
(10)T7:=T6[T5]
(11)T8:=T7+2
(12)T3[T2]:=T8
(13)i:=i+1
(14)goto (3)



(1)j:=1 (2)i:=1

(6)T3:=a-11 (9)T6:=b-11 (4)T1=10 (7)T4=10

(3) If i>100 goto (15)

(4)T1:=T1+10 (5)T2:=T1+j
(7)T4:=T4+10
(8)T5:=T4+j
(10)T7:=T6[T5]
(11)T8:=T7+2
(12)T3[T2]:=T8
(13)i:=i+1
(14)goto (3)

(1)j:=1 (2)i:=1

(6)T3:=a-11 (9)T6:=b-11 (4)T1=10 (7)T4=10

(3) If T1>1000 goto (15)

(4)T1:=T1+10 (5)T2:=T1+j
(7)T4:=T4+10
(8)T5:=T4+j
(10)T7:=T6[T5]
(11)T8:=T7+2
(12)T3[T2]:=T8
(14)goto (3)

Data-flow analysis

1.Data-flow analysis

- Analyze global data-flow information to do code optimization and a good job of code generation

Notes: Data-flow information can be collected by setting up and solving systems of equations that relate data information **at various points** in a program flow.

2. Definitions

1) Point

The label of a statement in a program

2) Path

Path from p_1 to p_n is a sequence of points p_1, p_2, \dots, p_n such that for each i between 1 and $n-1$, either

2. Definitions

2) Path

- (1) p_i is the point immediately preceding a statement and p_{i+1} is the point immediately following that statement in the same block, or
- (2) p_i is the end of some block and p_{i+1} is the beginning of a successor block.

2. Definitions

3) Reaching

- A definition of a variable x is a statement that assigns, or may assign, a value to x .
- We say a definition d reaches a point p if there is a path from the point immediately following d to p , such that d is not “killed” along that path.

2. Definitions

3) Reaching

Notes: (1) Intuitively, if a definition d of some variable a reaches point p , then d might be the place at which the value of a used at p might last have been defined.

(2) We kill a definition of a variable a if between two points along the path there is a definition of a .

3.Reaching-Definition Data-flow equations

$$OUT[B]=(IN[B]-KILL[B])\cup GEN[B]$$

$$IN[B]=\bigcup_{P\in P[B]}OUT[P]$$

$P[B]$ is the predecessor of B

$IN[B]$:Set of definitions of each reachable variable at the beginning of Block B

$KILL[B]$:Set of re-defined definitions of variables reached at the beginning of Block B

$GEN[B]$:Set of definitions of new generated variables in Block B

$OUT[B]$:Set of definition of each reachable variables at the end of Block B

4. Reaching Algorithm

- Input . A flow graph for which $kill[B]$ for which $kill[B]$ and $gen[B]$ have been computed for each block B .
- Output. $In[B]$ and $out[B]$ for each block B .
- Method

```

{for each block B do
  { IN[Bi]= $\Phi$ ;OUT[Bi]=GEN[Bi]; }
  change=TRUE;
  while change do {
    change=FALSE;
    for each block B do {
      IN[B]=  $\bigcup_{P \in P[Bi]} OUT[P]$ 
      oldout=out[B];
      OUT[Bi]=IN[Bi]-KILL[Bi] $\cup$ GEN[Bi]
      if out[B]<>oldout
        change=TRUE;
    }
  }
}

```

5.Live-Variable Analysis

- In live-variable analysis we wish to know for variable x and point p whether the value of x at p could be used along some path in the flow graph starting at p .
- If so, we say x is live at p ; otherwise x is dead at p .

5.Live-Variable Analysis

$$L.IN[B] = (L.OUT[B] - L.DEF[B]) \cup L.USE[B]$$

$$L.OUT[B] = \bigcup_{S \in S[B]} L.IN[S]$$

$S[B]$ is a successor of B

6.Live variable Algorithm

- Input. A flow graph with L.DEF and L.USE computed for each block.
- Output. L.Out[B], the set of variables live on exit from each block B of the flow graph.
- Method.

```

{for each block B do L.IN[Bi]= $\Phi$ ;
while change do {
  for each block B do {
    L.OUT[B]=  $\bigcup_{S \in S[Bi]} IN[S]$ 
    IN[B]=(OUT[B]-DEF[B]) $\cup$ USE[Bi]

  }
}}

```

7. Definition-Use Chains

- We say a variable is used at statement s if its r -value may be required.
- The du-chaining problem is to compute for a point p the set of uses s of a variable, say x , such that there is a path from p to s that does not redefine x .