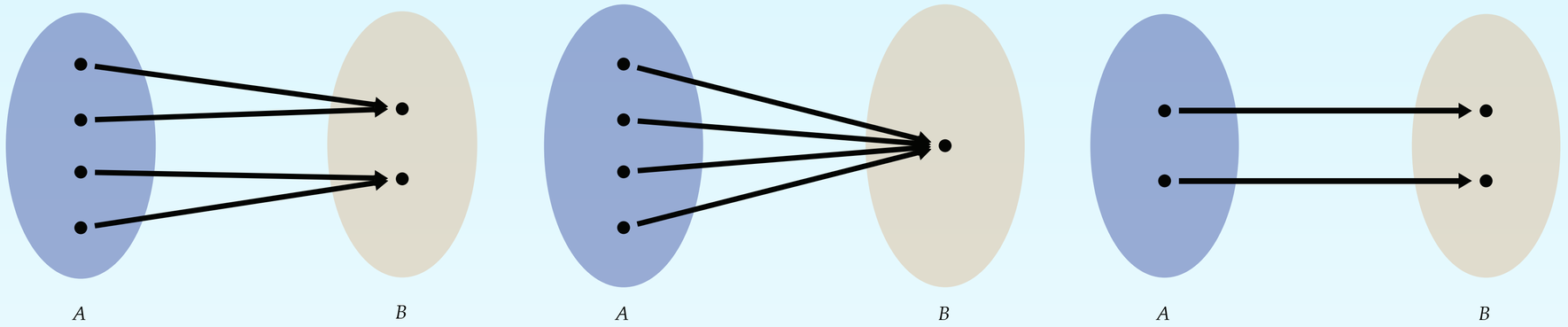



```
Definition Surjection(Map f, Set X, Set Y):
    forall y : Y | exists x : X | f(x) = y
f: A -> B
Set A, B
Surjection(f, A, B)
```

Substance and Style: domain-specific languages for mathematical diagrams

Wode "Nimo" Ni
Columbia University
with Katherine Ye, Joshua Sunshine, Jonathan Aldrich, Keenan Crane

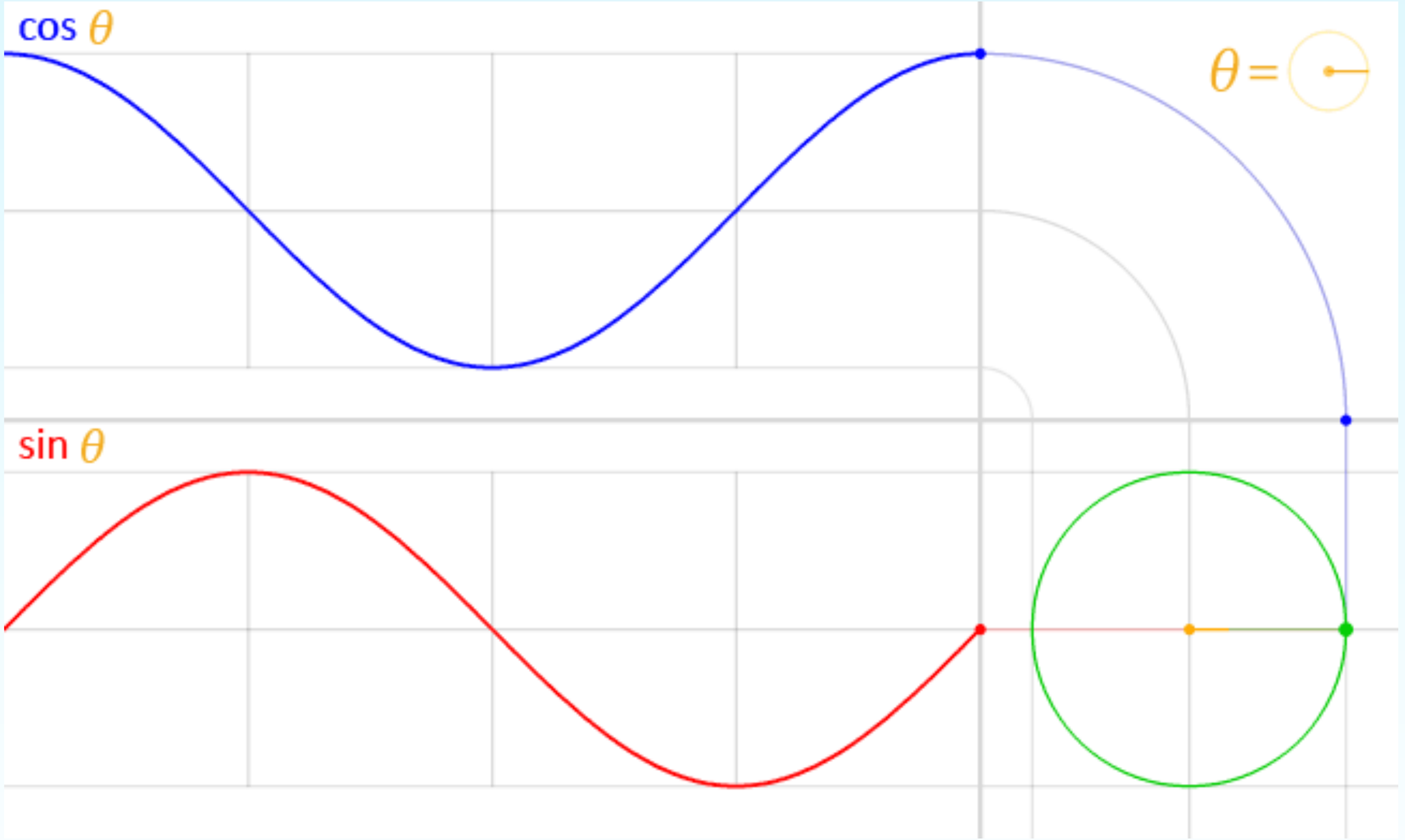
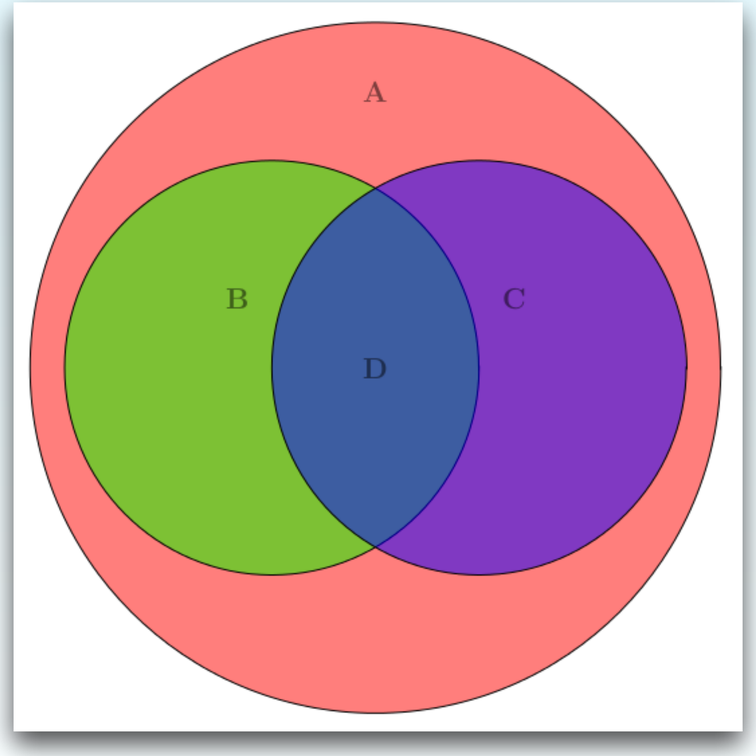


Diagram from Wikipedia page for “sine”. https://commons.wikimedia.org/wiki/File:Circle_cos_sin.gif

Illustrating Venn diagram in TikZ

```
\documentclass{article}

\usepackage{tikz}
\begin{document}
\pagestyle{empty}
\begin{tikzpicture}
    \begin{scope}[shift={(3cm,-5cm)}, fill opacity=0.5]
    \draw[fill=red, draw = black] (0,0) circle (5);
    \draw[fill=green, draw = black] (-1.5,0) circle (3);
    \draw[fill=blue, draw = black] (1.5,0) circle (3);
    \node at (0,4) (A) {\large\textbf{A}};
    \node at (-2,1) (B) {\large\textbf{B}};
    \node at (2,1) (C) {\large\textbf{C}};
    \node at (0,0) (D) {\large\textbf{D}};
    \end{scope}
\end{tikzpicture}
\end{document}
```



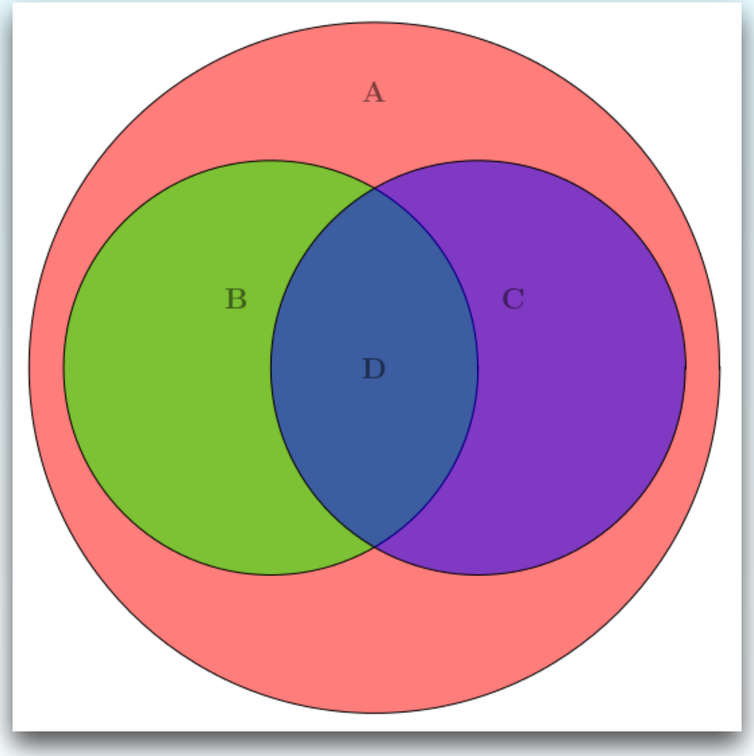
Illustrating Venn diagram in TikZ

```
\documentclass{article}

\usepackage{tikz}
\begin{document}
\pagestyle{empty}
\begin{tikzpicture}
```

1. Specify the colors, sizes, and locations of the circles
2. Create “nodes” for the labels and determine the locations of the labels

```
\end{tikzpicture}
\end{document}
```



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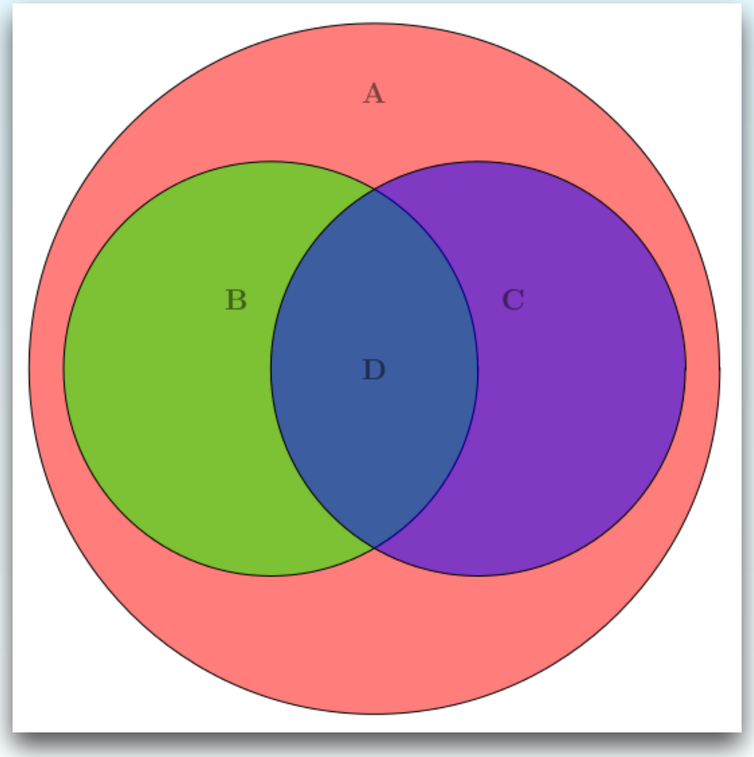
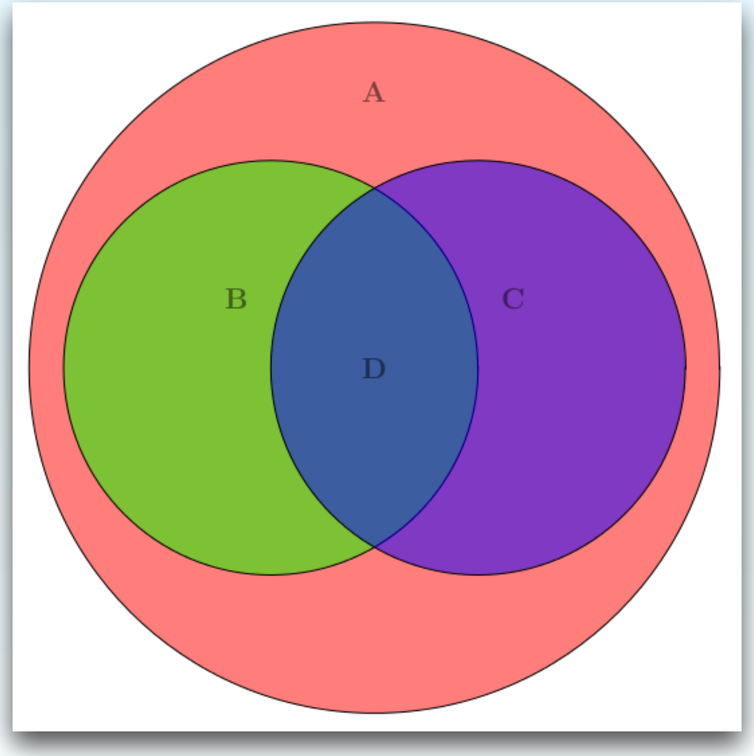
    \node at (2,1) (C) {\large\textbf{C}};
    \node at (0,0) (D) {\large\textbf{D}};
    \end{scope}
\end{tikzpicture}
\end{document}
```



Illustrating Venn diagram in TikZ

Low-level manipulation down to pixels.

Semantics of the diagram is completely lost.



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    \node at (-2,1) (B) {\large\textbf{B}};
    \node at (2,1) (C) {\large\textbf{C}};
    \node at (0,0) (D) {\large\textbf{D}};
    \end{scope}
\end{tikzpicture}
\end{document}
```
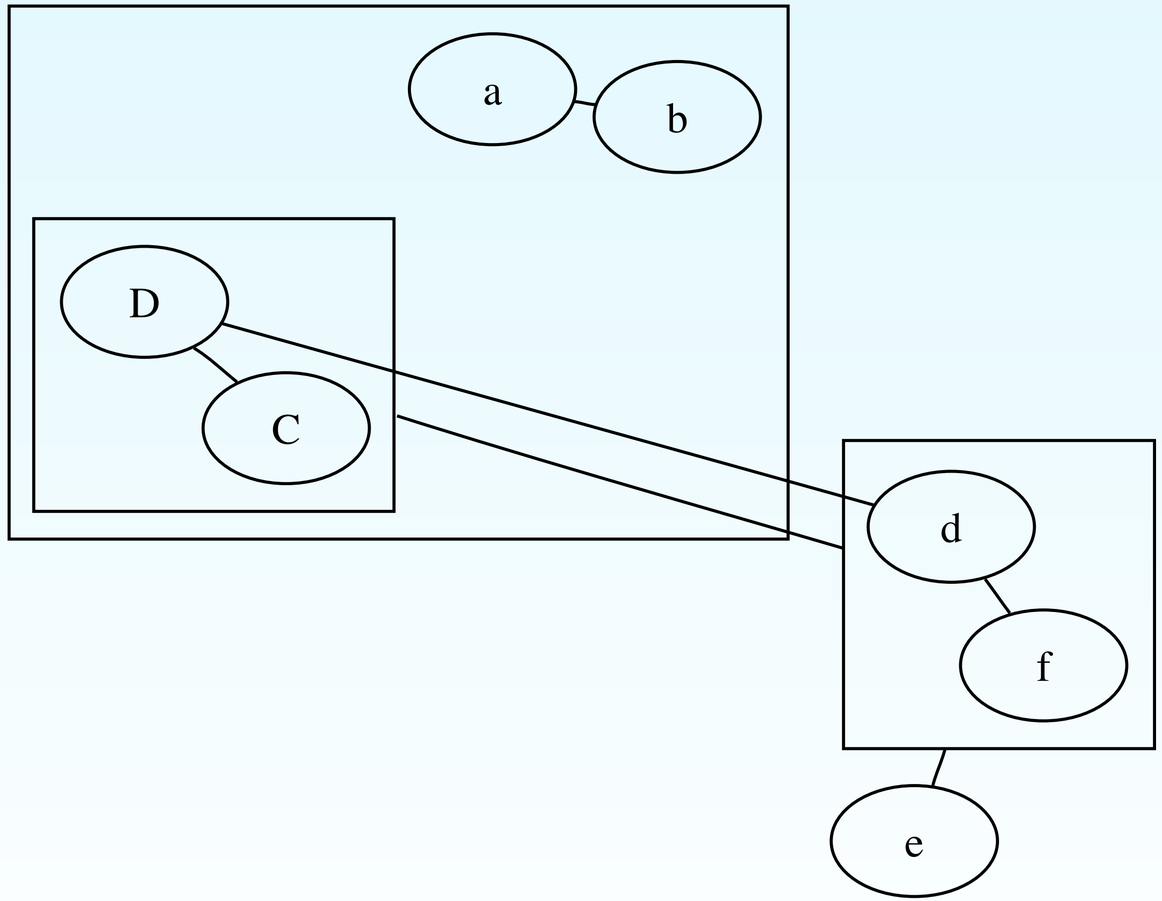

What were we trying to illustrate originally? Circles and text labels?

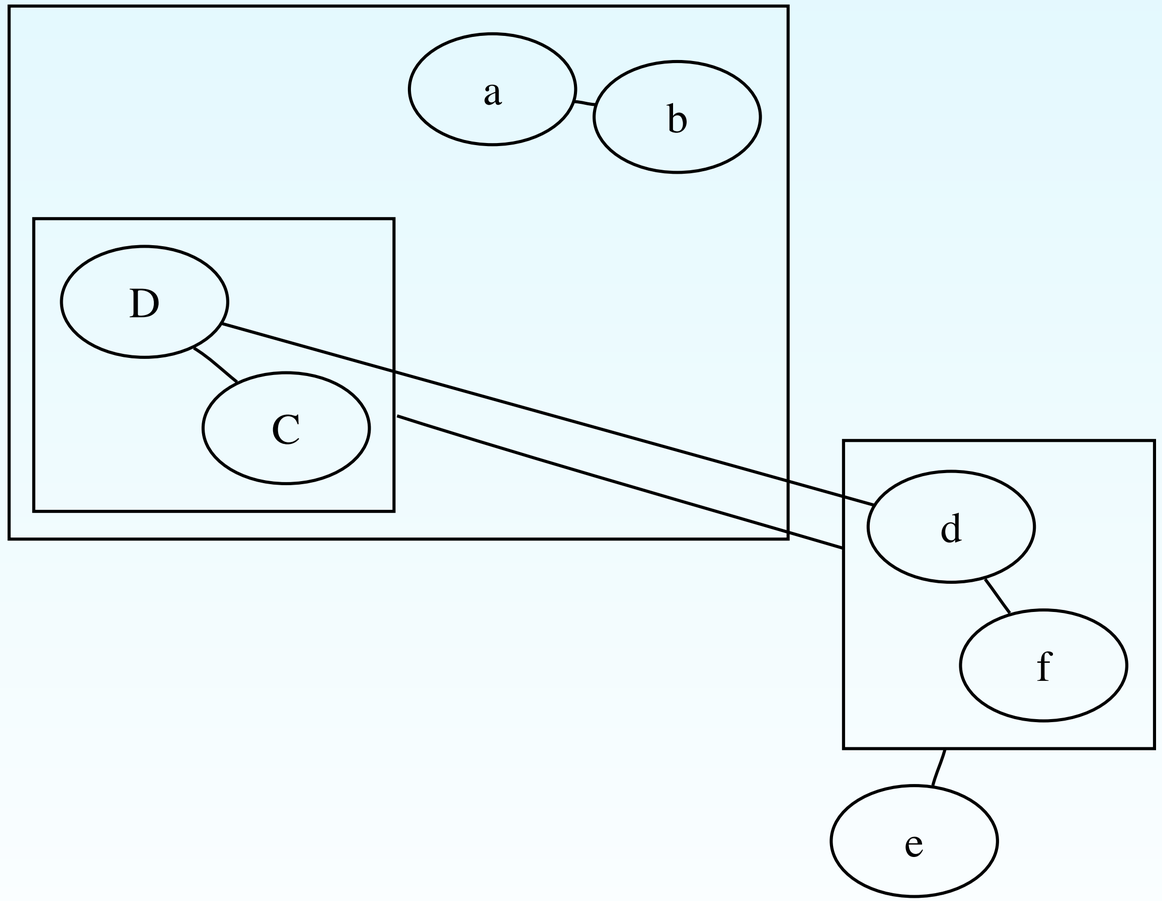
Graph visualization using Graphviz

```
graph G {
  e
  subgraph clusterA {
    a -- b;
    subgraph clusterC {
      C -- D;
    }
  }
  subgraph clusterB {
    d -- f
  }
  d -- D
  e -- clusterB
  clusterC -- clusterB
}
```

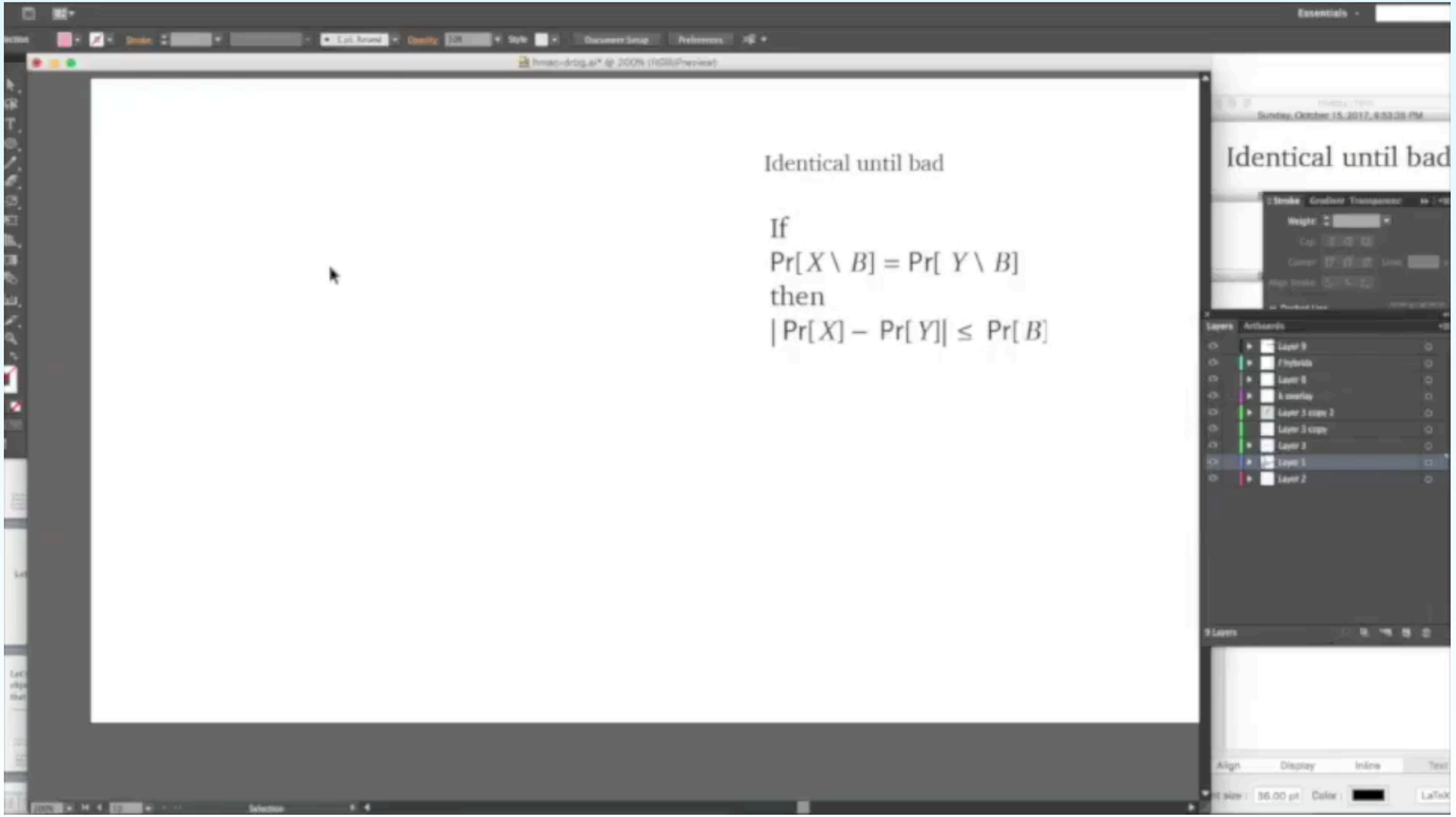


Graph visualization using Graphviz

High-level and clean, but only works for graphs!



Direct manipulation tools



Penrose

Language design

Penrose has two extensible DSLs

Substance

High-level declaration of mathematical objects

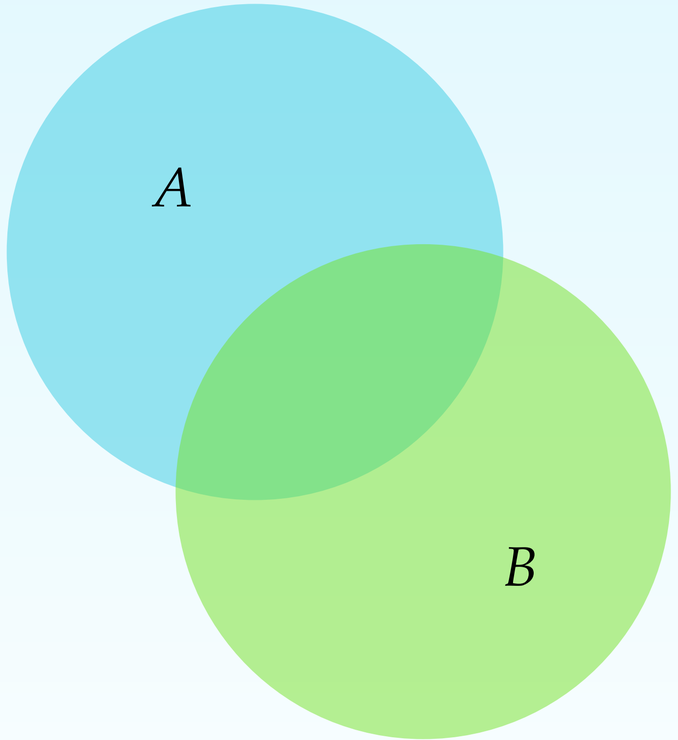
```
Set A, B
Intersect A B
```

Style

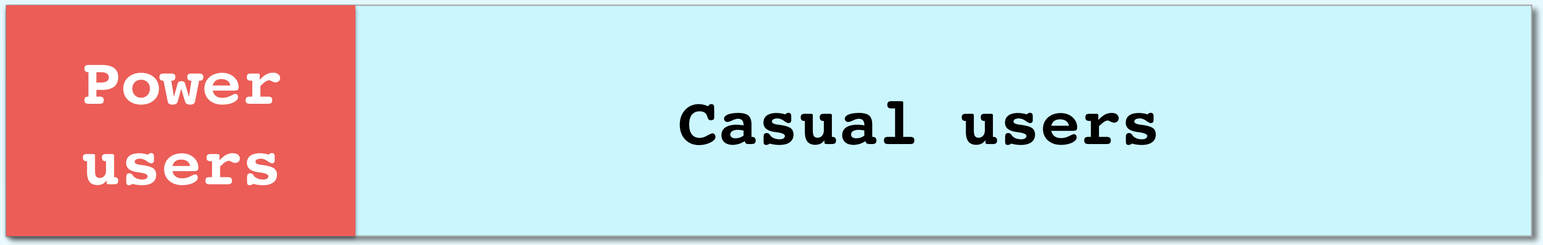
Specifies the visual *presentation* of the objects

```
Set A {
  shape = Circle{ }
}
Intersect X Y {
  ensure X overlapping Y
}
```

```
Set A, B
Intersect A B

Set A {
   shape = Circle{ }
}
Intersect X Y {
   ensure X overlapping Y
}
```

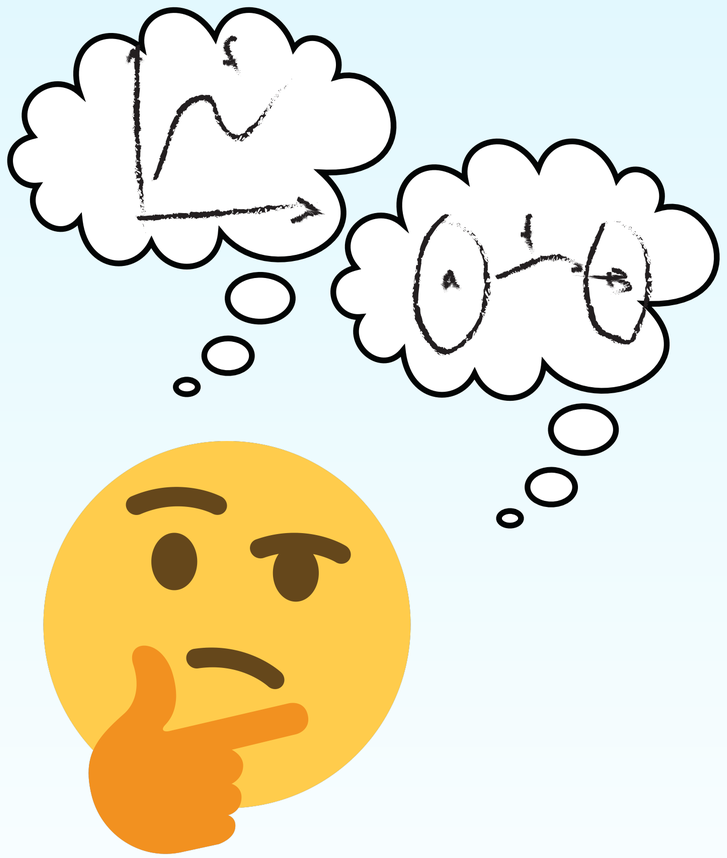


Substance and Style



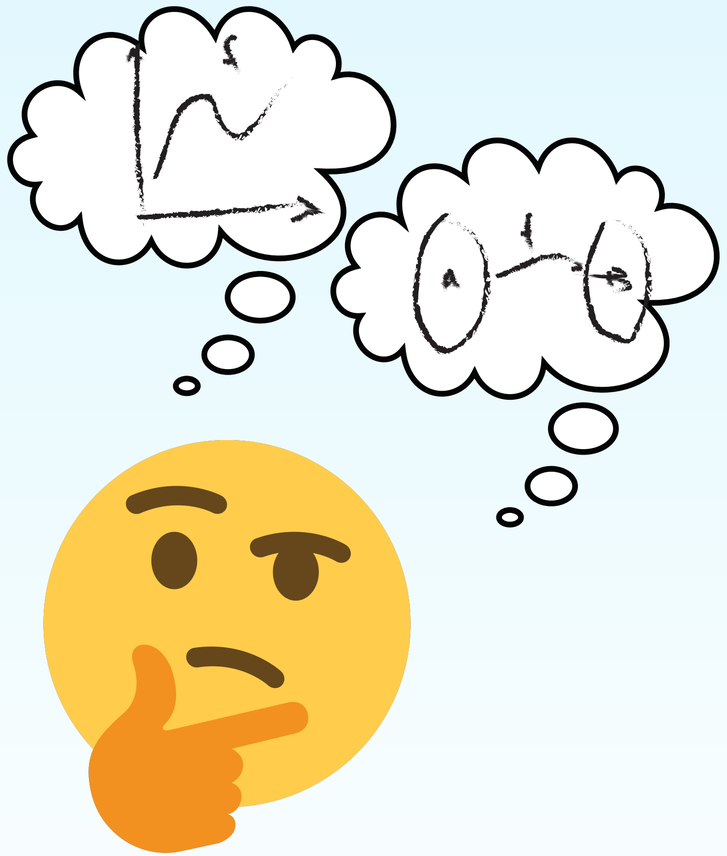
- Power users encode domain expertise in Style
- Casual users create diagrams declaratively using Substance

Dr. M wants to visualize a function…



How should I draw $f : X \to Y$?

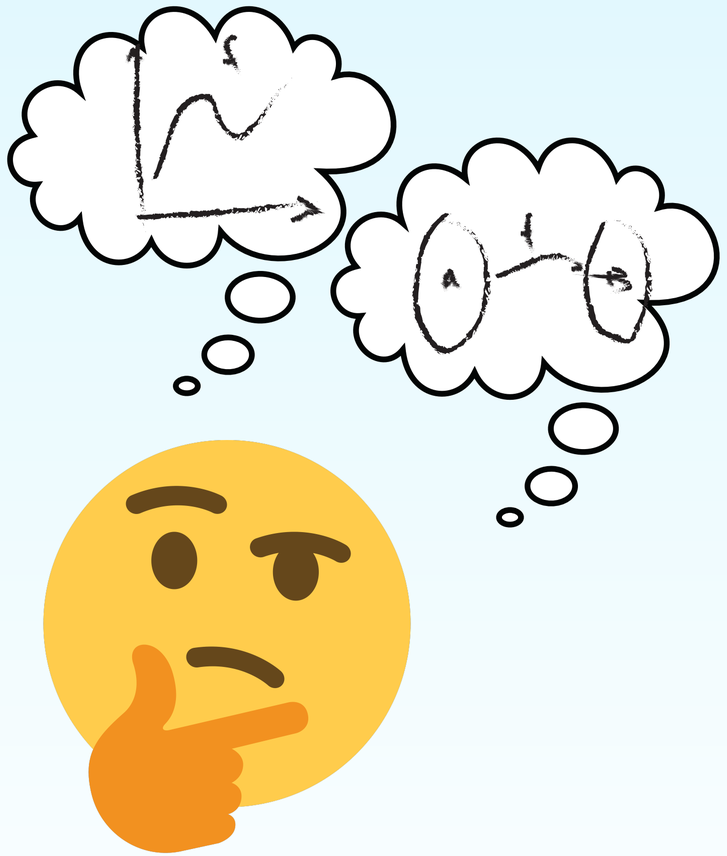
Dr. M wants to visualize a function…



```
f: X -> Y
```

How should I draw $f : X \to Y$?

Dr. M wants to visualize a function…



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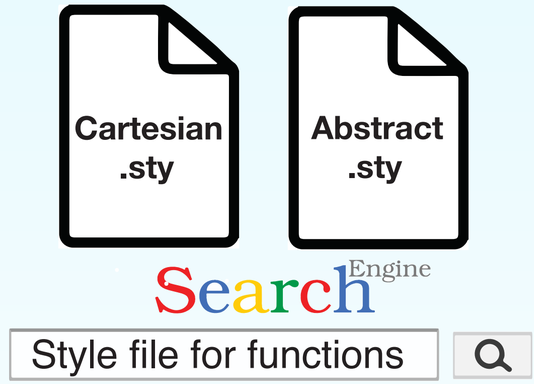

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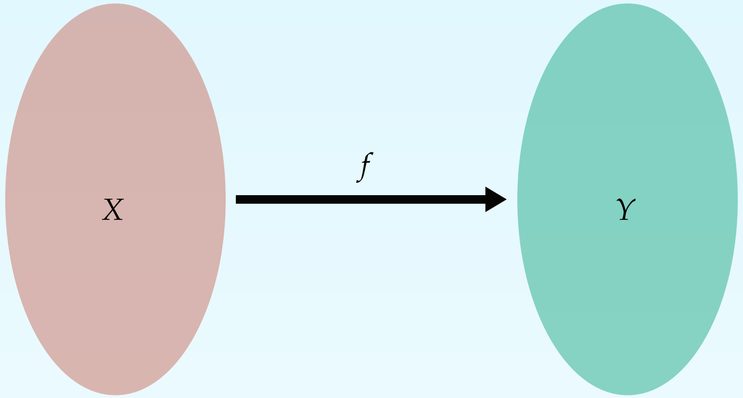
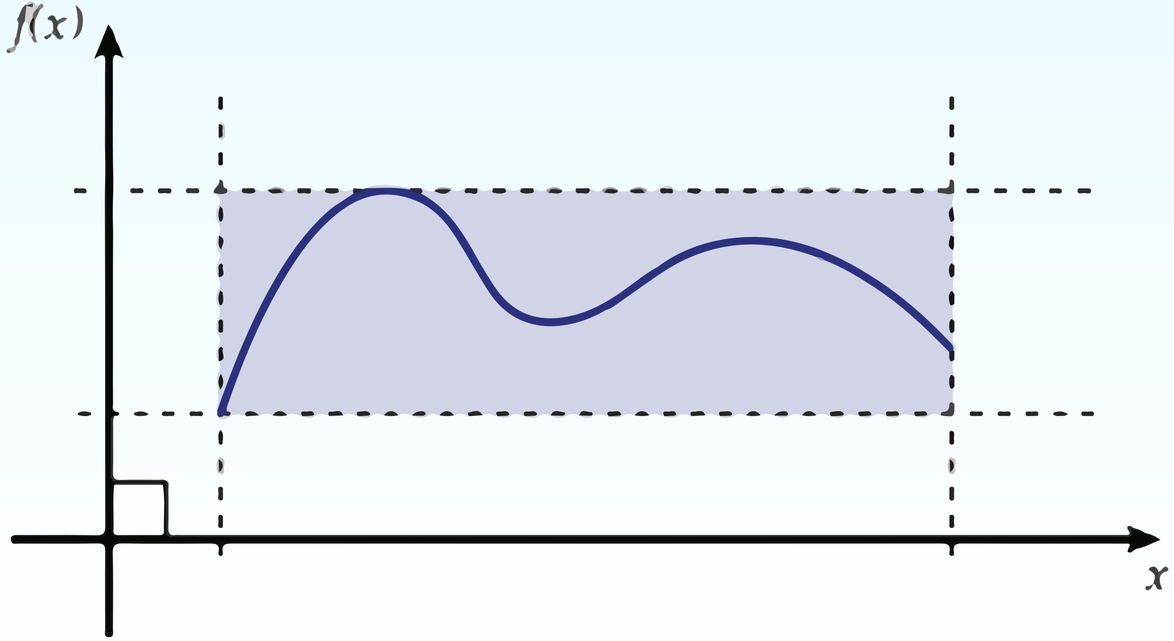
How should I draw f: X -> Y?

Dr. M wants to visualize a function…

Abstract
.sty

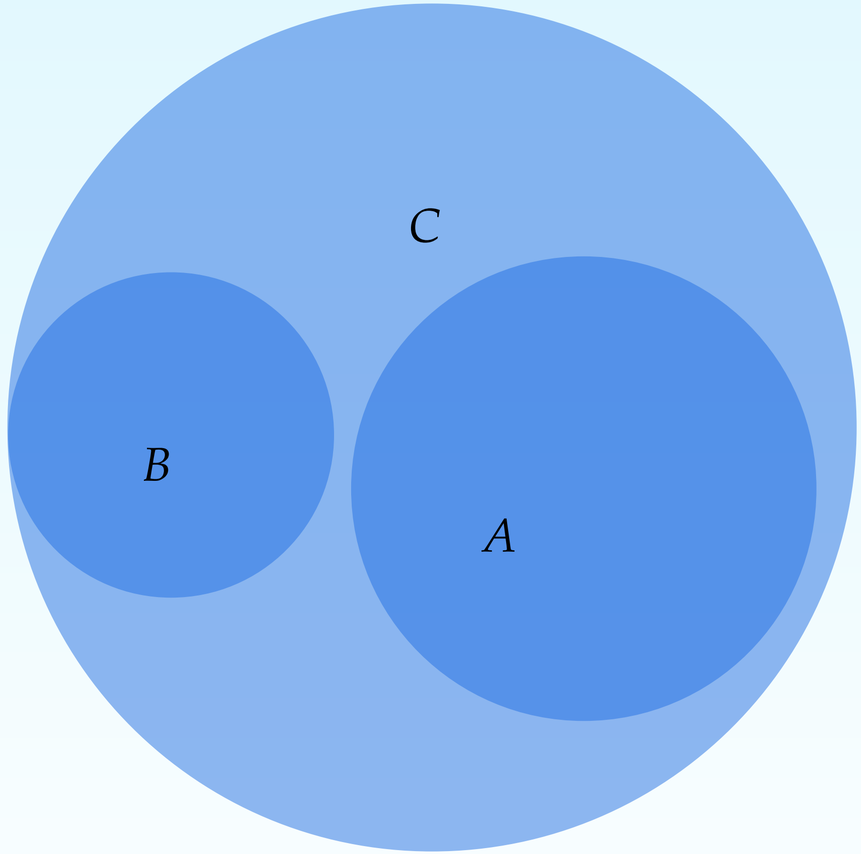
Cartesian
.sty





Style language

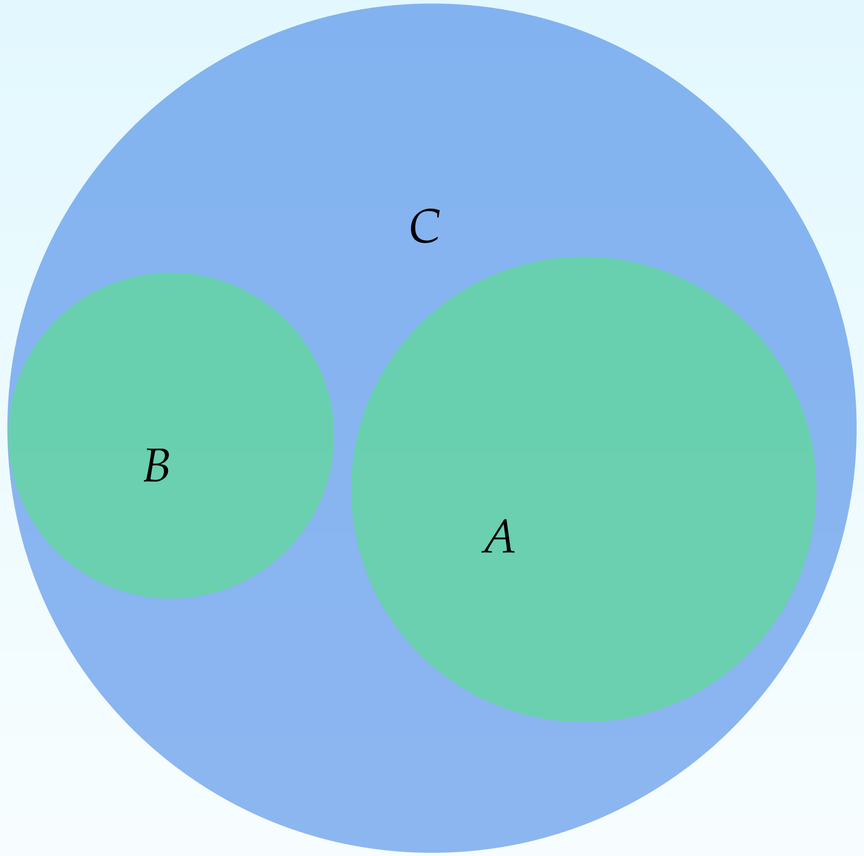
```
Set X {
   shape = Circle {
      color = blue
   }
}
```



Selectors using pattern matching

Style language

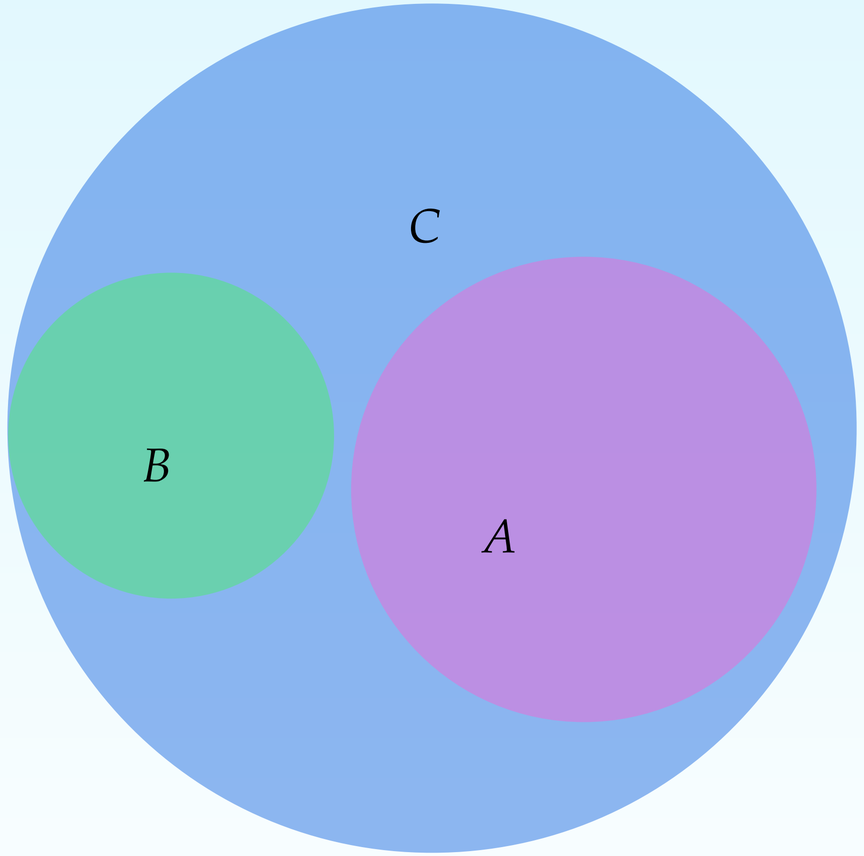
```
Set X {
   shape = Circle {
      color = blue
   }
}
Subset X Y {
   X.color = green
}
```



Cascading: later rules override the earlier ones

Style language

```
Set X {
  shape = Circle {
    color = blue
  }
}
Subset X Y {
  X.color = green
}
Set `A` {
  A.color = pink
}
```



Style language

```
Set X {
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  }
}
Subset X Y {
  X.color = green
}
Set `A` {
  A.color = pink
}
```

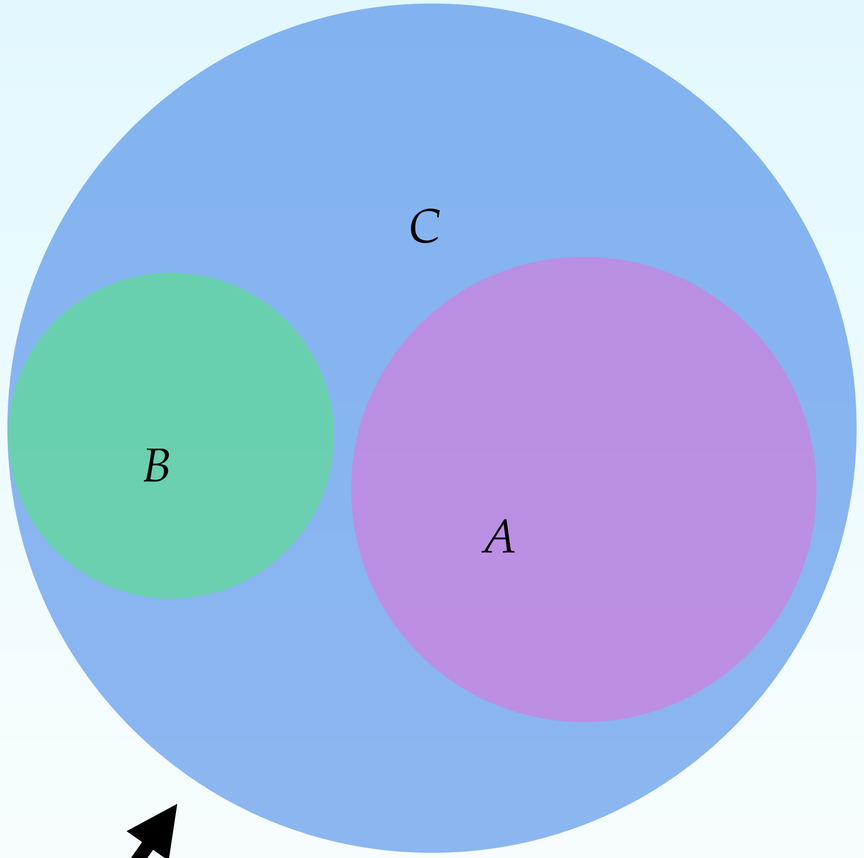

Wait. How does Penrose choose the positions and sizes of them?

Style language

```
Set X {
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}
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```
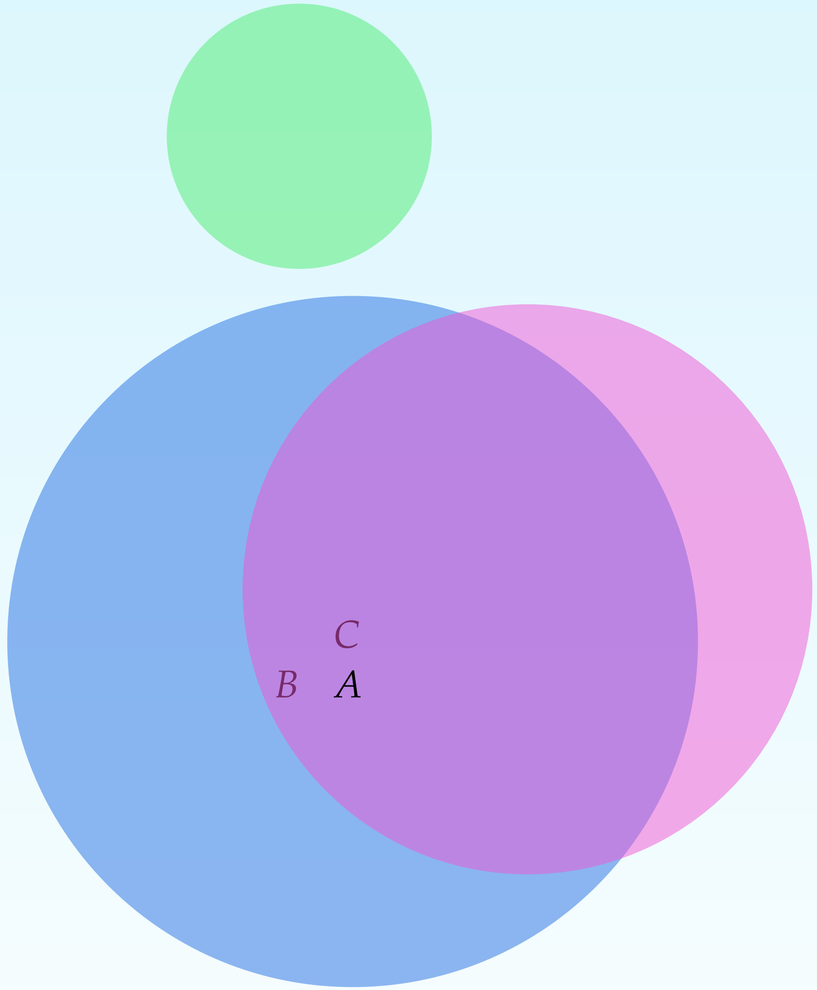
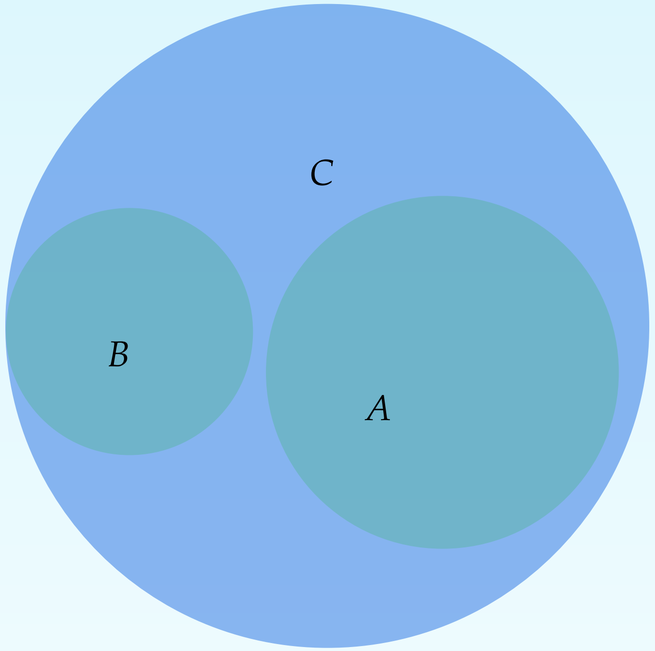



Wait. How does Penrose choose the positions and sizes of them?

Optimization-based layout

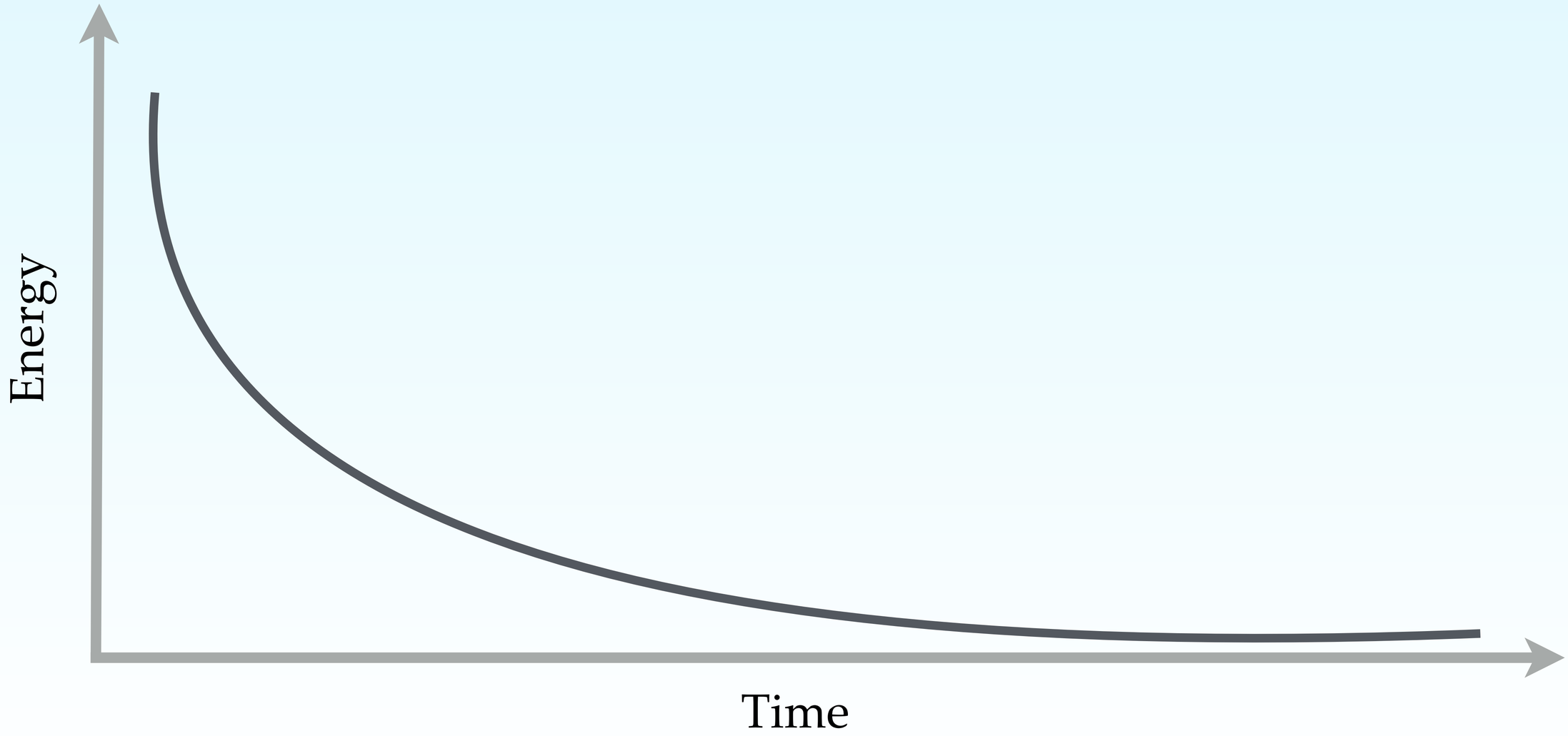
Specifying Layout in Style

```
Set X {
  shape = Circle {
    color = blue
  }
  ensure X contains X.label
}
Subset X Y {
  X.color = green
  ensure Y contains X
  ensure X smallerThan Y
  ensure Y.label outsideOf X
}
```

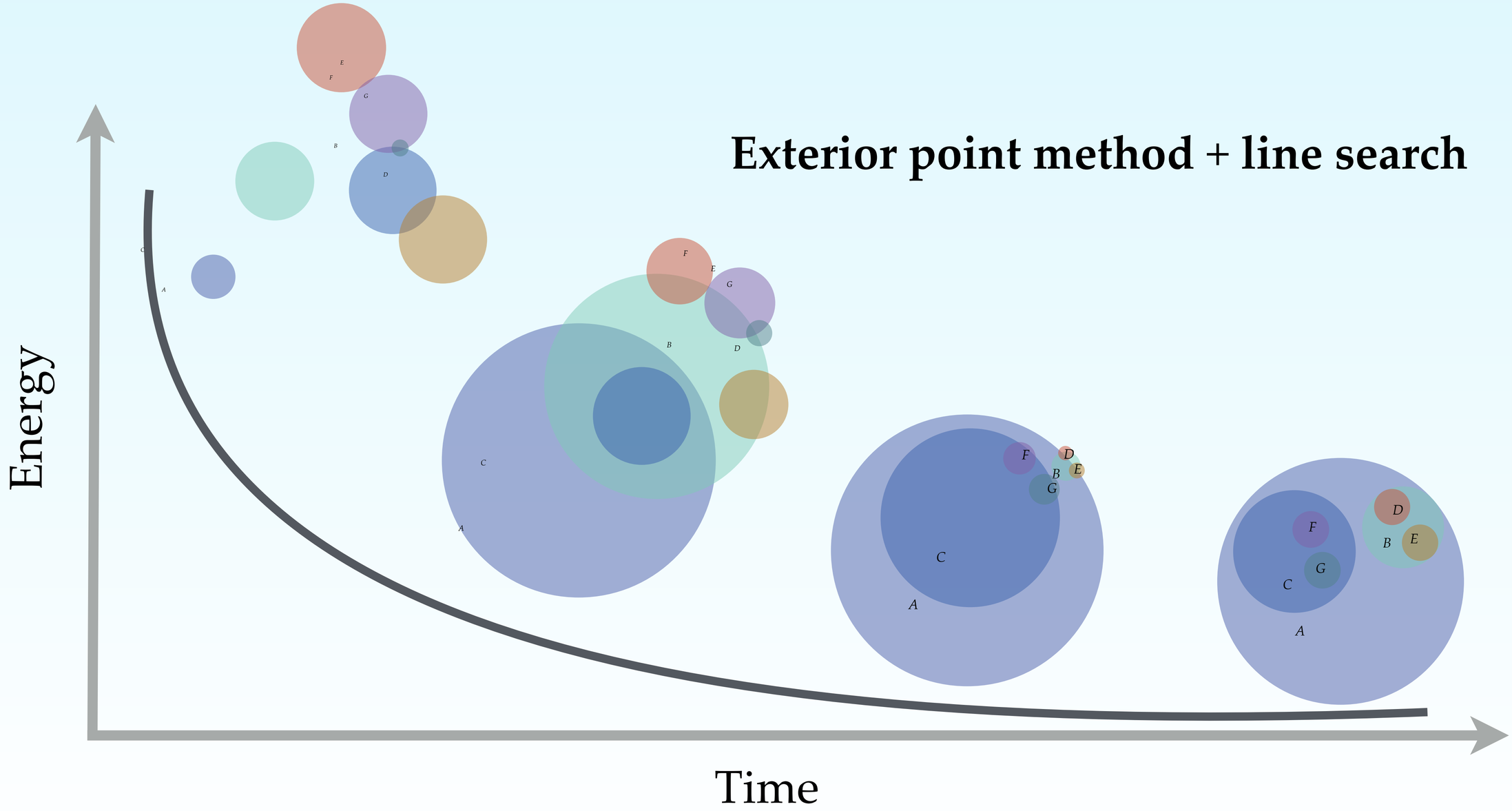


Declaratively specify optimization requirements!

Optimizing diagram layout



Optimizing diagram layout



Illustrating abstract function definitions

Injective functions

A function is **injective**, or "**one-to-one**," if every element of its codomain is mapped from *at most* one element of the domain.

$$\forall x, x' \in X, f(x) = f(x') \rightarrow x = x'.$$

```
Definition Injection(Map f, Set A):
    forall a1, a2 : A | f(a1) = f(a2) implies a1 = a2
```

Extend with external tools

- Penrose doesn't have to *understand* first order logic!
- Extend the system to integrate with an **external tool,** *Alloy***,** to generate concrete instances given the definition of injective functions.

```
Definition Injection(Map f, Set A):
    forall a1, a2 : A | f(a1) = f(a2) implies a1 = a2
f: A -> B
Set A, B
Injection(f, A)
```

Penrose program

Extend with external tools

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```
sig A {
    f : B
}
sig B {
}
fact {
  all a1,a2 : A | a1.f
= a2.f implies a1 = a2
}
pred show() { }
run show for 5
```

Extend with external tools

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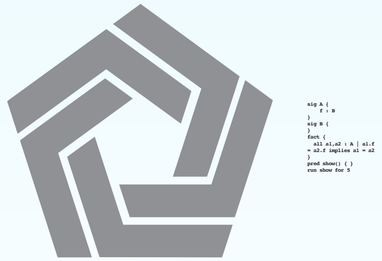

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sig B {
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= a2.f implies a1 = a2
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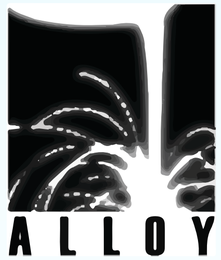

Extend with external tools

- Penrose doesn't have to *understand* first order logic!
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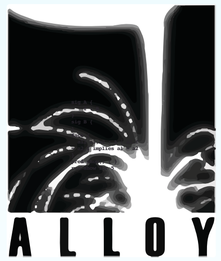
```
A: {A$0}
B: {B$0, B$1, B$2, B$3, B$4}
f: {A$0->B$4}
```

Alloy instances



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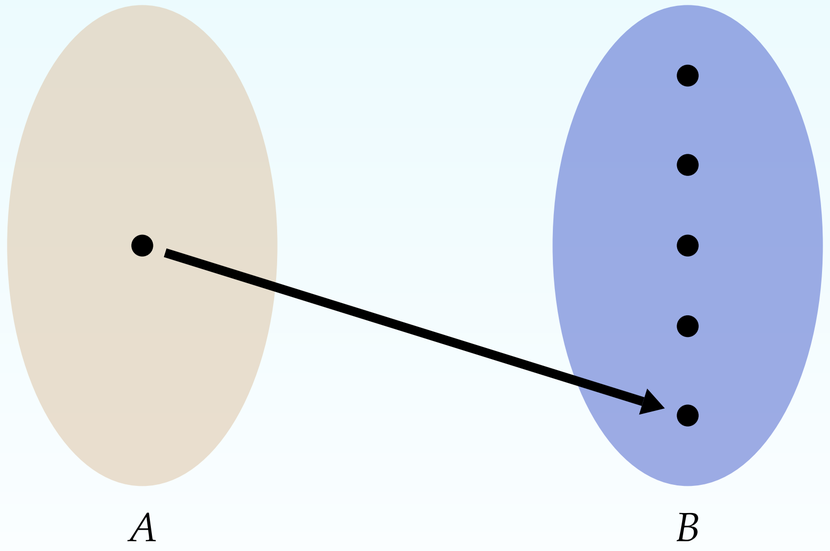
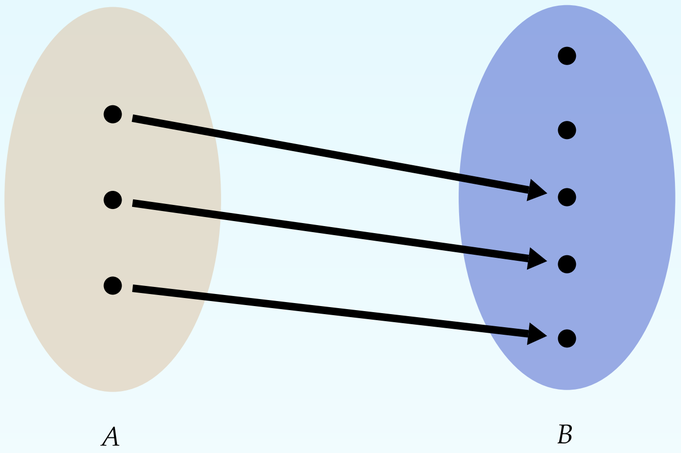
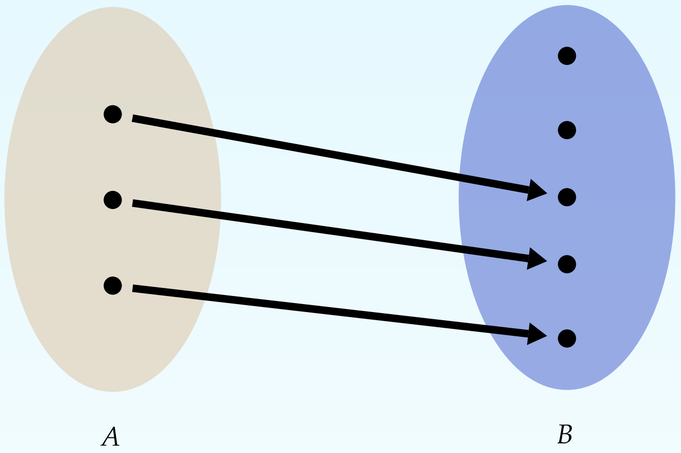
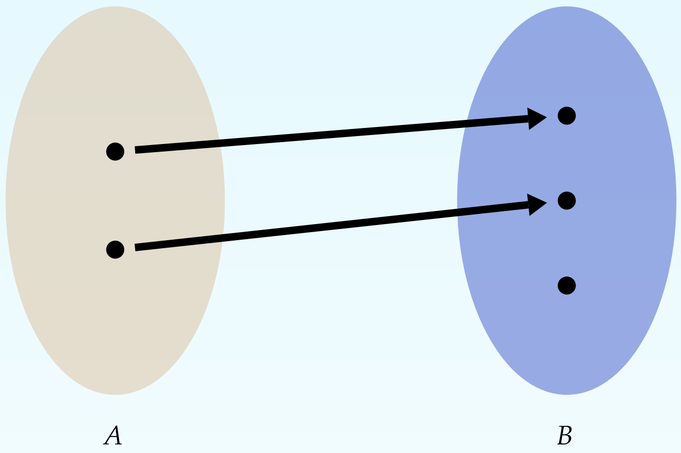


Visualizing injective functions

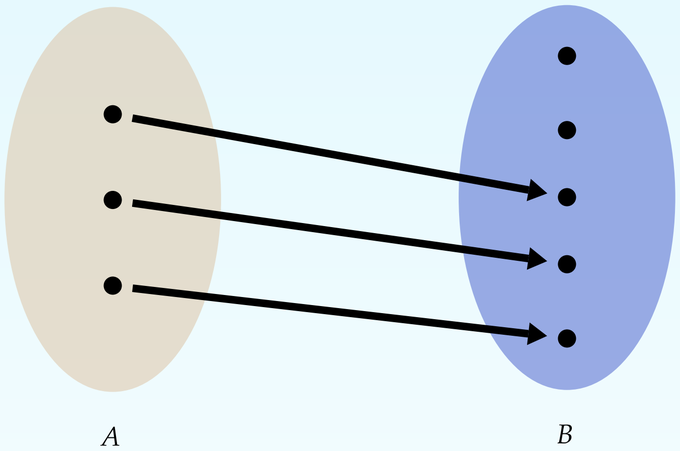


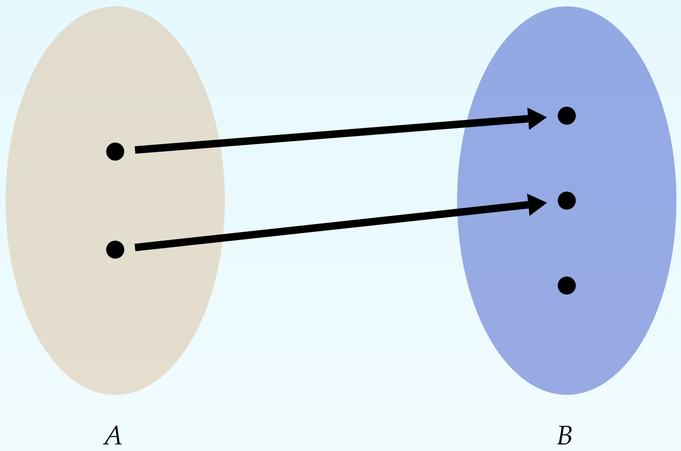
Visualizing injective functions

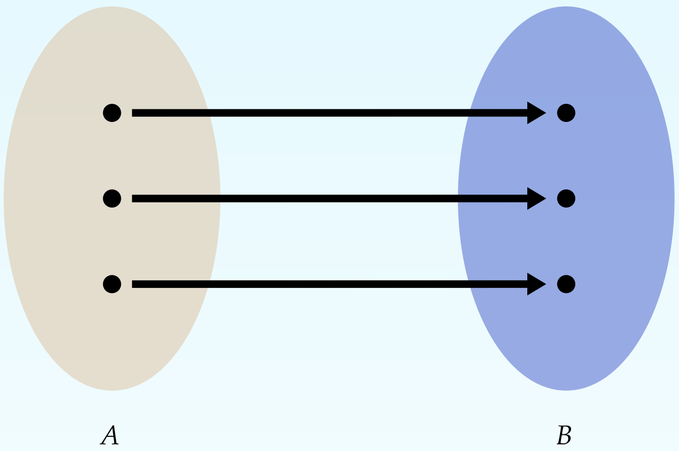




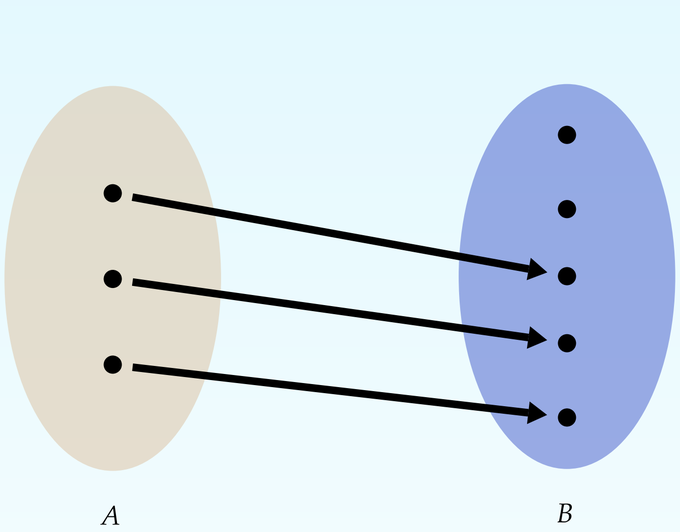
Visualizing injective functions

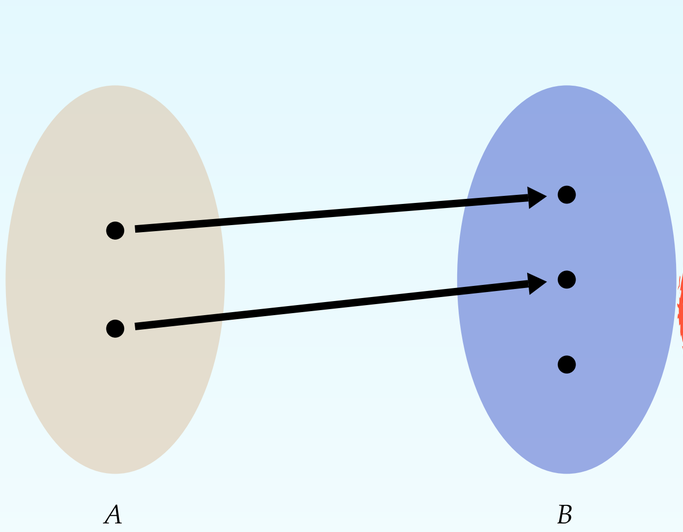


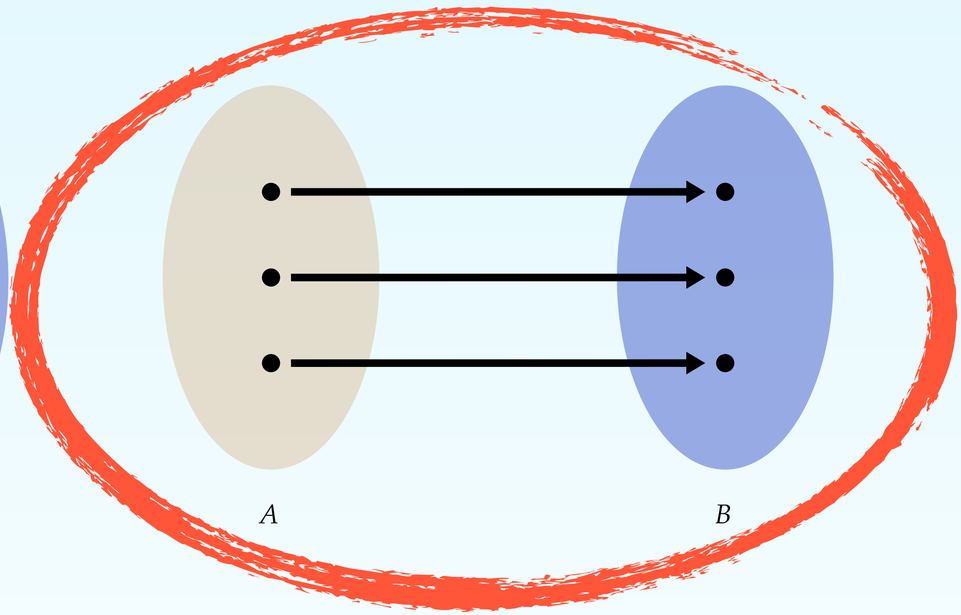




Visualizing injective functions



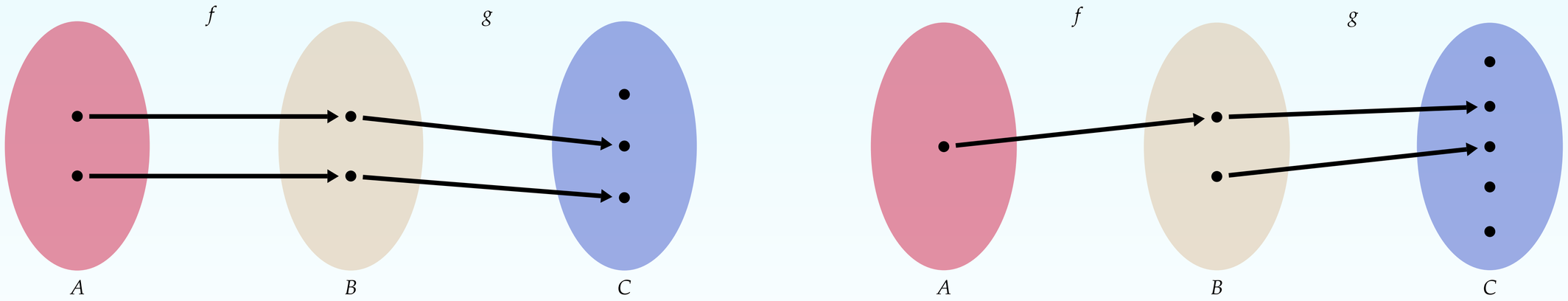




A **bijection** is a special case of injection!

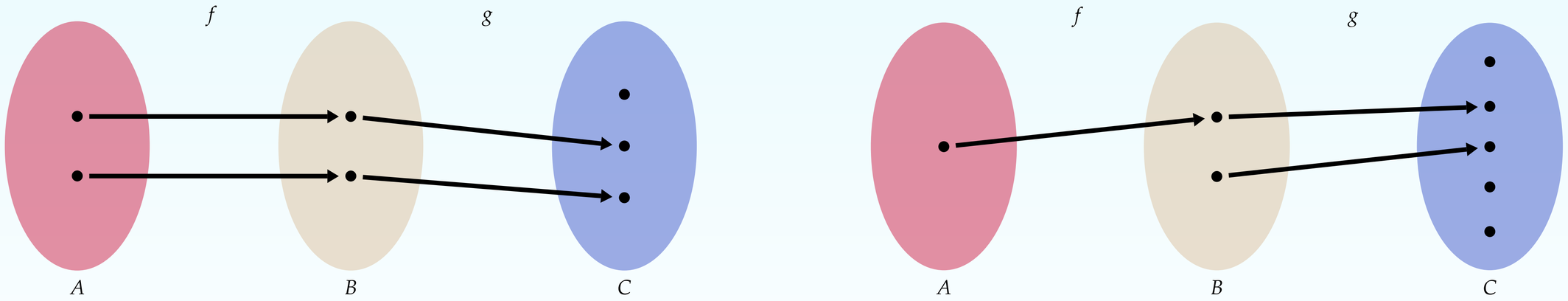
Visualizing composition of functions

```
Set A, B, C
f: A -> B
g: B -> C
Injection(f, A)
Injection(g, B)
```



Visualizing composition of functions

```
Set A, B, C
f: A -> B
g: B -> C
Injection(f, A)
Injection(g, B)
```



The **composition** of injections is still an injection!

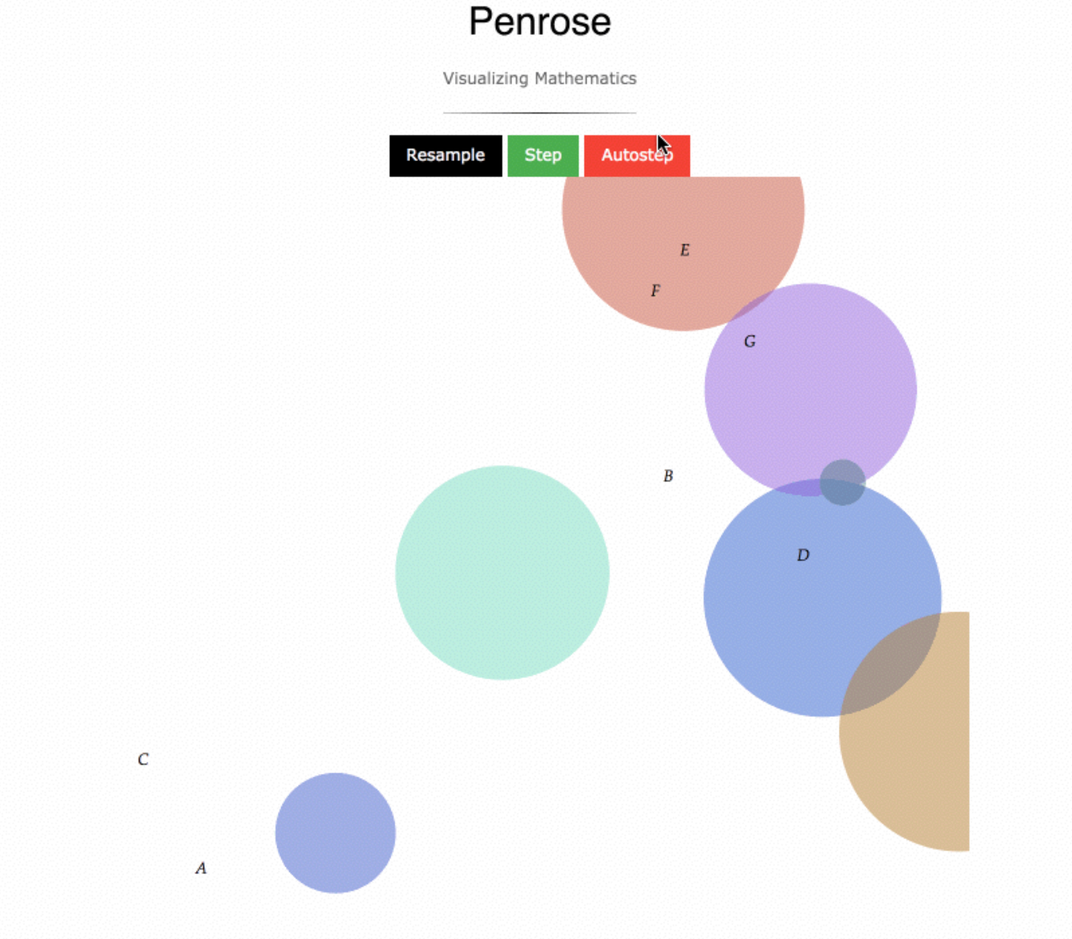
Running Penrose

Example: Venn vs Tree diagram

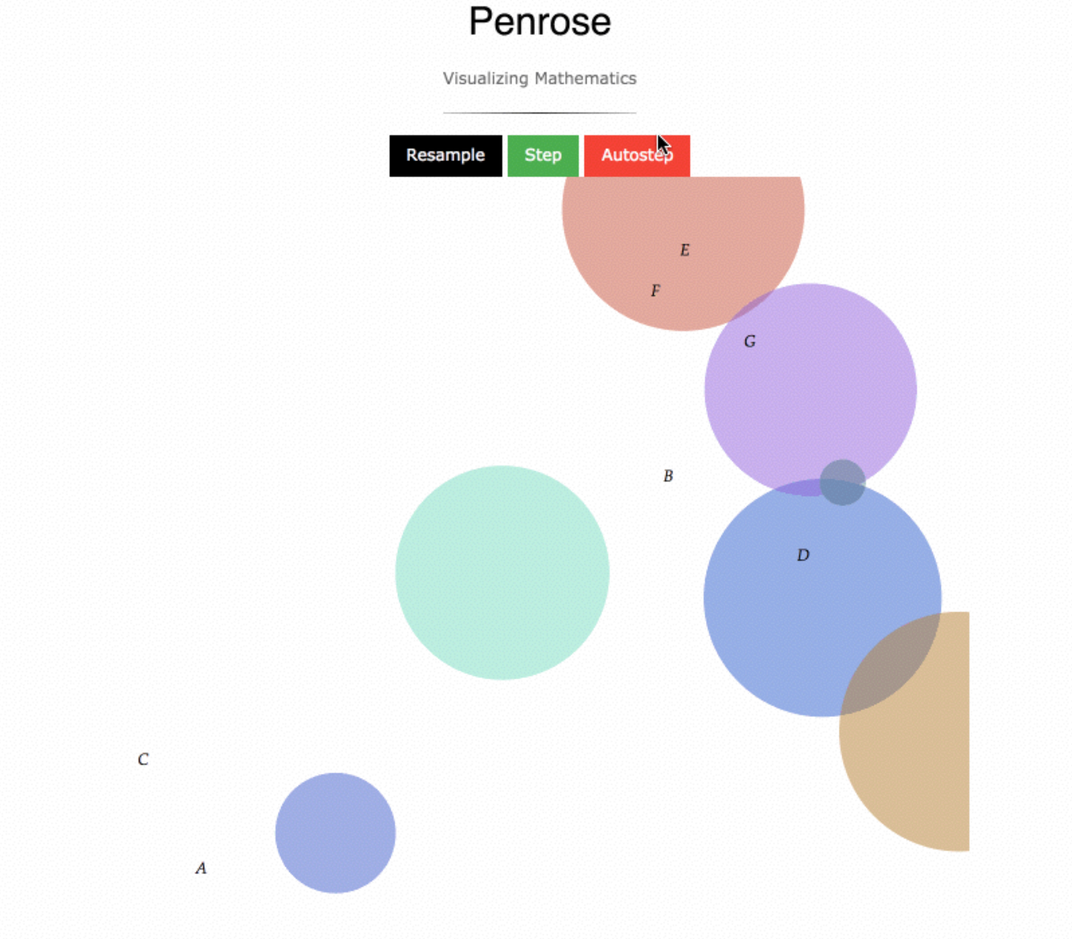
```
Set A, B, C, D, E, F, G
Subset B A
Subset C A
Subset D B
Subset E B
```

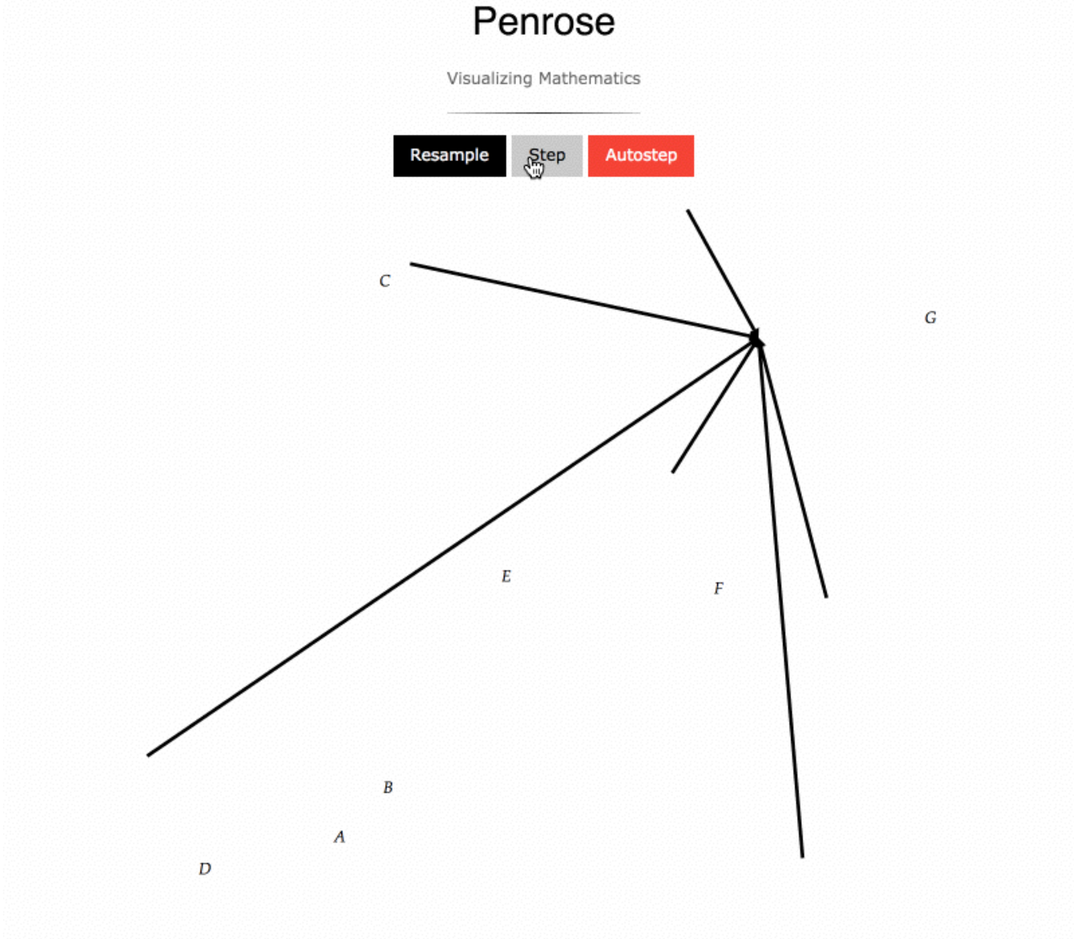
```
Subset F C
Subset G C
NoIntersect E D
NoIntersect F G
NoIntersect B C
```

Example: Venn vs Tree diagram

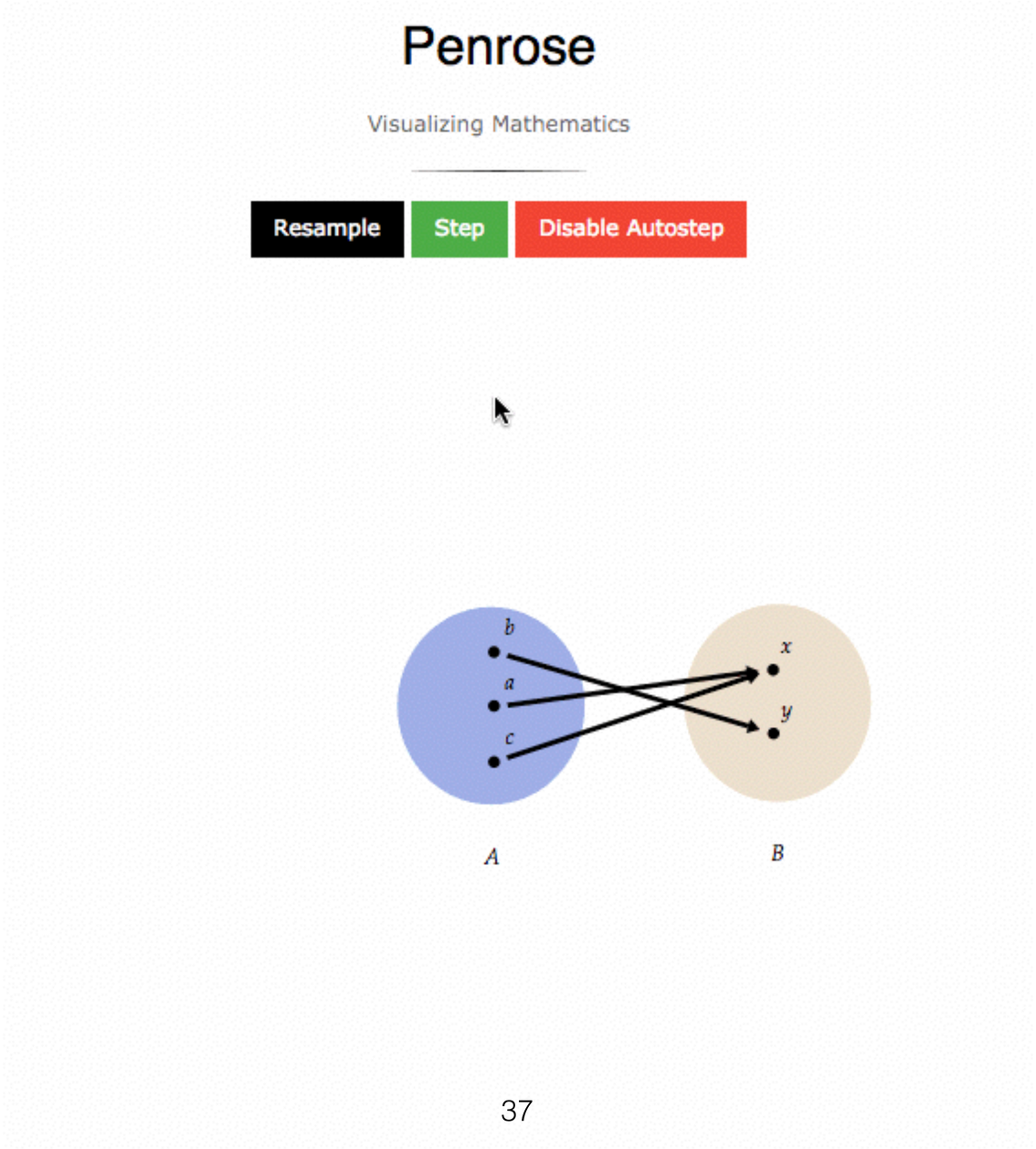


Example: Venn vs Tree diagram





Example: restarting the optimization



Conclusion

http://penrose.ink
http://github.com/penrose/penrose

- *Goal*: Enables users to make beautiful diagrams **without** design background
- **DSLs** that cleanly separate high-level semantics and styling details
- Optimizes layout declaratively
- Extensible system