Optical-axis perturbation singularity in an out-of-plane ring resonator

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I use a matrix approach to determine the optical-axis perturbation sensitivity of an out-of-plane stable ring resonator. It is found to have a singularity for certain design parameters. On one side of the singularity the cavity behaves abnormally: the longer the mirror radius, the larger the mode volume and, surprisingly, the less the perturbation sensitivity. This is opposite the behavior of conventional stable resonators, a finding that is important to the cavity designs of out-of-plane ring lasers.

For investigation of the optical-axis behavior in an arbitrary multielement out-of-plane resonator, it is necessary to find the general round-trip propagation matrix that includes the perturbation to any of its optical elements. The ray matrix of a general optical component with angular misalignment as a perturbation source[1] can be expressed as

$$\begin{bmatrix} r_0 \\ r_0' \\ 1 \end{bmatrix} = \begin{bmatrix} A & B & 0 \\ C & D & \S \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_i \\ r_i' \\ 1 \end{bmatrix}, \qquad (1)$$

where r_i and r_1' are the input ray transverse position and slope, respectively, and r_0 and r_0' are the output ray transverse position and slope, respectively. A, B, C, and D are the standard ray-matrix elements, and § is the angular perturbation induced in this optical component. In the case of a mirror, $\S = 2\tan\Delta$, where Δ is the mirror misalignment angle.

Other types of perturbation, such as length and refractive-index variations, can be represented in a modified form of Eq. (1). In this Letter I focus on mirror misalignment as the main source of perturbation.

A general resonator is made up of many segments. Each segment, as shown in Fig. 1, contains free-space propagation L_i, a reflection off one apex mirror with radius R_i (+ for concave, − for convex) and incidence angle $\oslash_i$, and a coordinate rotation θ_i. The input ray for this leg at input point P_{i-1} is expressed in a local right-hand coordinate system (X_i, Y_i, Z_i) as $\hat{Y}_i \,//\, \mathbf{P}_{i-1}\mathbf{P}_i \times \mathbf{P}_i\mathbf{P}_{i+1}$, with Z_i in the direction of propagation. After reflection off the ith mirror, the output coordinates (X_i', Y_i', Z_i') need to be rotated at an angle θ_i to align the Y_i' axis with $\mathbf{P}_i\mathbf{P}_{i+1} \times \mathbf{P}_{i+1}\mathbf{P}_{i+2}$ such that it is matched to the next $i+1$ segment propagation. The matrix for such a single segment is computed as

$$M_i = \begin{bmatrix} -\cos\theta_i & -L_i\cos\theta_i & 0 & \sin\theta_i & L_i\sin\theta_i & 0 \\ \dfrac{2\cos\theta_i}{R_i\cos\oslash_i} & \dfrac{2L_i\cos\theta_i}{R_i\cos\oslash_i} - \cos\theta_i & -\mathrm{d}x_i\cos\theta_i & \dfrac{-2\sin\theta_i\cos\oslash_i}{R_i} & \dfrac{-2L_i\sin\theta_i\cos\oslash_i}{R_i} + \sin\theta_i & \mathrm{d}y_i\sin\theta_i \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \sin\theta_i & L_i\sin\theta_i & 0 & \cos\theta_i & L_i\cos\theta_i & 0 \\ \dfrac{-2\sin\theta_i}{R_i\cos\oslash_i} & \dfrac{-2L_i\sin\theta_i}{R_i\cos\oslash_i} + \sin\theta_i & \mathrm{d}x_i\sin\theta_i & \dfrac{-2\cos\theta_i\cos\oslash_i}{R_i} & \dfrac{-2L_i\cos\theta_i\cos\oslash_i}{R_i} + \cos\theta_i & \mathrm{d}y_i\cos\theta_i \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (2)$$

where $\mathrm{d}x_i = 2\tan\Delta_x, \mathrm{d}y_i = 2\tan\Delta_y$, and Δ_x and Δ_y are the misalignment angles of mirror P_i in its local tangential and sagittal planes, respectively. If necessary, M_i can be written in a 5×5 matrix with a single row containing the element 1 for incorporation of the misalignment calculation.[2]

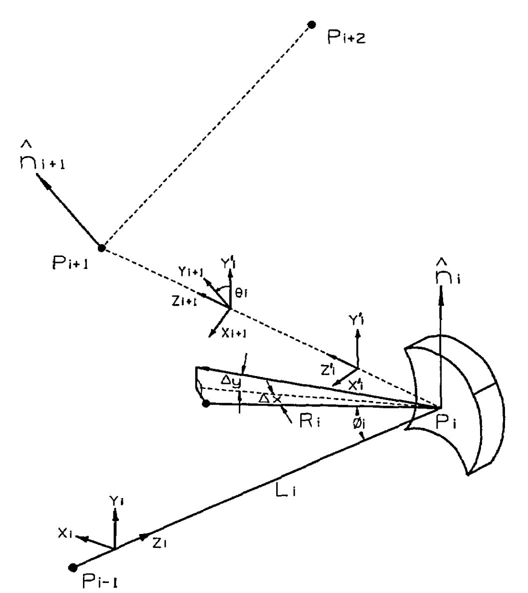

Fig. 1. One single segment of a general resonator.

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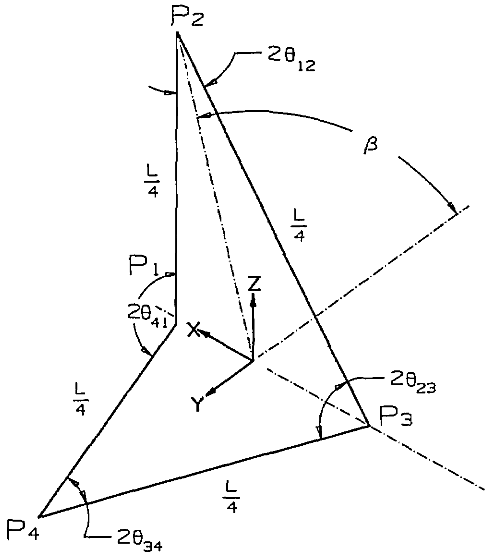

Fig. 2. Four-equal-sided out-of-plane ring cavity used in the analysis, where the fold angle is $\beta = 104°$ and all the incidence angles are equal, $\theta_{12} = \theta_{23} = \theta_{34} = \theta_{41} = 31.62°$

The total round-trip matrix of a resonator is the product of each individual segment matrix in proper sequential order:

$$M = \prod M_i\,. \tag{3}$$

The resonator optical axis that is invariant under the round-trip propagation coincides with the eigenvector of M with eigenvalue 1:

$$\begin{bmatrix} r_x \\ r_x' \\ 1 \\ r_y \\ r_y' \\ 1 \end{bmatrix} = M \begin{bmatrix} r_x \\ r_x' \\ 1 \\ r_y \\ r_y' \\ 1 \end{bmatrix}. \tag{4}$$

Matrix equation (4) can be solved to produce a new set of linear equations that determine the optical-axis location (r_x, r_y) and slope (r_x', r_y') for a round trip:

$$\begin{bmatrix} M_{11}-1 & M_{12} & M_{14} & M_{15} \\ M_{21} & M_{22}-1 & M_{24} & M_{25} \\ M_{41} & M_{42} & M_{44}-1 & M_{45} \\ M_{51} & M_{52} & M_{54} & M_{55}-1 \end{bmatrix} \times \begin{bmatrix} r_x \\ r_x' \\ r_y \\ r_y' \end{bmatrix} = -\begin{bmatrix} M_{13}+M_{16} \\ M_{23}+M_{26} \\ M_{43}+M_{46} \\ M_{53}+M_{56} \end{bmatrix}. \tag{5}$$

Once the relationship between the optical axis and the round-trip matrix is established, one may find the optical-axis movements under the influence of any combination of optical component misalignments by solving Eq. (5).

This approach is utilized to study the mirror misalignment-induced optical-axis movements of the out-of-plane stable ring resonator. There has been extensive research on this type of resonator[3–7] and its applications in the ring laser gyro[8] and the monolithic nonplanar ring laser.[9,10]

The numerical results presented here are for the case of a tetragonally shaped four-equal-sided ring cavity with a fold angle of $\beta = 104°$ and the incidence angles on all four mirrors equal, $\theta_{12} = \theta_{23} = \theta_{34} = \theta_{41} = 31.62°$ (Fig. 2). This cavity has a round-trip image rotation of 270° counterclockwise if viewed along the direction of propagation.

Here I introduce a dimensionless sensitivity factor:

$$S_{\left(\substack{x\\y}\right)i\left(\substack{x\\y}\right)j} = \frac{1}{L}\left[\frac{\partial r_{\left(\substack{x\\y}\right)i}}{\partial \Delta_{\left(\substack{x\\y}\right)j}}\right], \tag{6}$$

where $r_{\left(\substack{x\\y}\right)i}$ is the induced x or y component (in local coordinates) of axis movement on the ith mirror as the result of the jth mirror tangential (x) or sagittal (y) direction angular misalignment (in local coordinates) of an amount of $\Delta_{\left(\substack{x\\y}\right)j}$ (in radians). L is the total cavity length, which serves as a normalization factor.

Figure 3 shows various sensitivity factors S versus L/R of this special ring resonator, where R is the common radius of curvature of concave mirrors P_1 and P_4. Mirrors P_2 and P_3 are chosen to be flat. The source of perturbation in this example is the misalignment of mirror P_2 in its local sagittal plane. It is interesting to note that there is a singular point at approximately $L/R = 0.57$ where the axis movements diverge. Misalignment of mirrors other than P_2 in both the tangential and the sagittal directions has shown similar sensitivity characteristics with a singular point located at $L/R = 0.57$.

For a further understanding this out-of-plane stable ring resonator, the Gaussian beam sizes at the midpoint between the two curved mirrors, P_2 and P_4, of this specific resonator are calculated with an elliptical coordinates system.[8] The results are shown

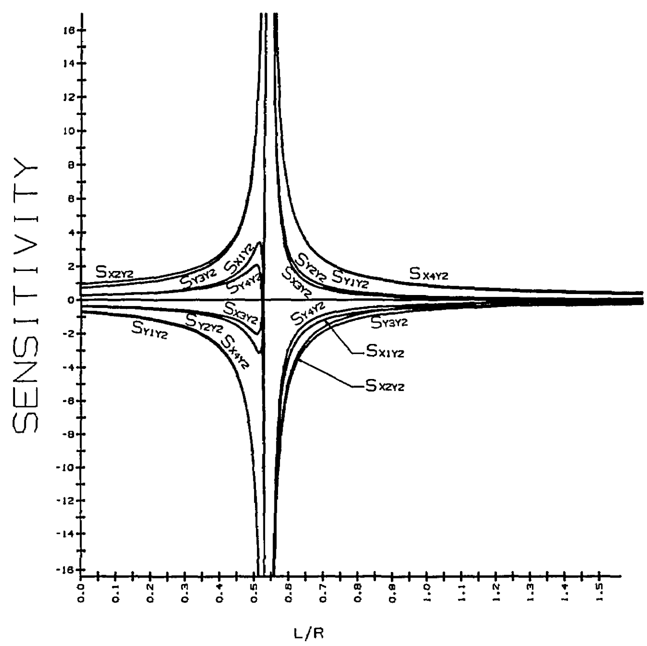

Fig. 3. Optical-axis perturbation sensitivity S versus L/R for misalignment of mirror P_2 in its sagittal plane.

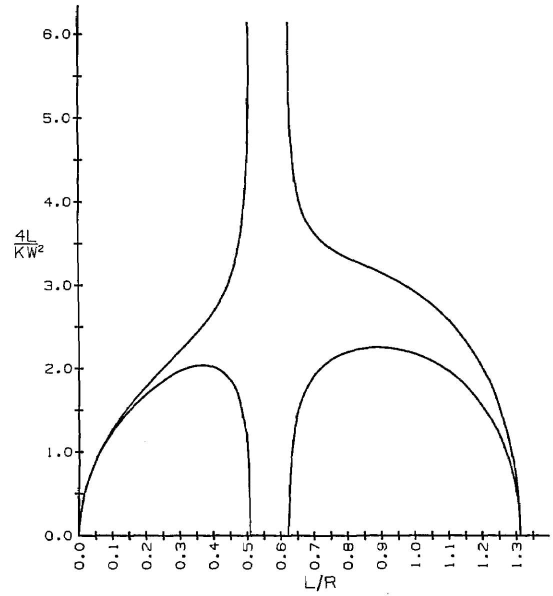

Fig. 4. Normalized Gaussian beam size parameter $4L/kw^2$ versus L/R; w is the Gaussian beam radius and $k = 2\pi/\lambda$.

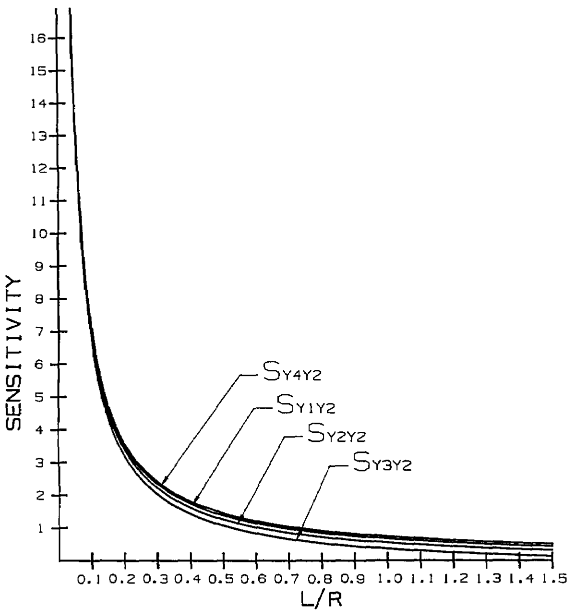

Fig. 5. Planar ring cavity optical-axis perturbation sensitivity versus L/R for misalignment of mirror P_2 in its sagittal plane.

in Fig. 4. Not surprisingly, the axis perturbation instability point near $L/R = 0.57$ does correspond to an unstable region of $0.62 > L/R > 0.51$, within which there is no stable Gaussian mode solution. As L/R approaches this unstable region from above 0.62 or below 0.51, the mode sizes in the two principal beam axes diverge. The beam shape on either side of this singular region is highly elliptical. It is interesting to note that in the region $L/R < 0.51$ the mode size (or volume) increases as R increases. Interestingly, the axis perturbation sensitivity decreases with increasing R. This behavior is opposite that of conventional stable resonators.

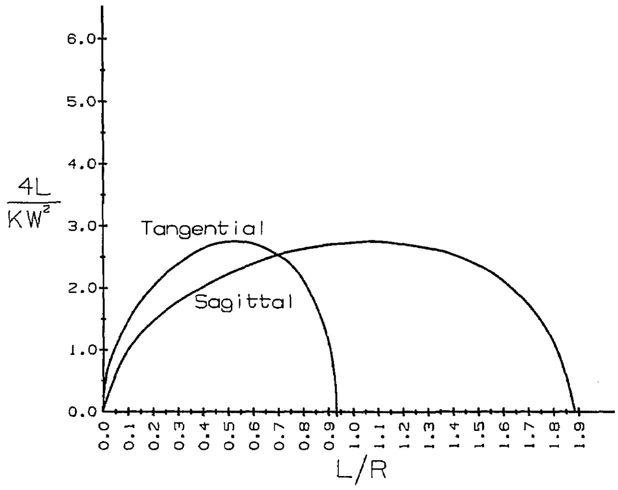

Fig. 6. Planar cavity spot size parameter versus L/R.

As a comparison, an in-plane square ring cavity of the same design as in Fig. 2 but with $\beta = 0°$ and $\theta_{12} = \theta_{23} = \theta_{34} = \theta_{41} = 45°$ is tested. Figure 5 shows the axis perturbation sensitivity versus L/R. The beam axis movements diverge monotonically as R increases, which is expected for an ordinary stable resonator. Figure 6 shows the Gaussian beam sizes versus L/R. Strong astigmatism separates the sagittal and tangential branches, but this cavity does not have an unstable region sandwiched between two stable domains as in the case of an out-of-plane ring cavity.

In conclusion, I have calculated the cavity misalignment characteristics and the Gaussian mode sizes of an out-of-plane ring resonator. Its singular behavior at certain cavity design parameters is different from that of the usual in-plane ring resonators. This finding is useful in the cavity designs of the out-of-plane ring lasers, in which cavity misalignment insensitivity, fabrication tolerances, mode sizes, astigmatism, etc. can be optimized by choice of the proper cavity parameters with a singular point carefully taken into consideration.

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