Soviet Journal of Quantum Electronics

Analysis of misalignment sensitivity of ring laser resonators

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(Submitted September 21, 1976)
Kvantovaya Elektron. (Moscow) **4**, 1517–1521 (July 1977)

A new type of ring laser resonator is proposed. In this resonator a self-conjugate ray exists for an arbitrary distribution of the mirrors. An estimate is made of the displacement of such a ray due to mirror misalignment.

PACS numbers: 42.60.Da

Lasing action takes place in a linear resonator of the Fabry–Perot type only if the mirrors are parallel to one another to a high degree of precision. However, in solid-state lasers a high precision of the mirror alignment does not guarantee operation of the resonator because inhomogeneity of the active and other elements as well as the optical wedge effect act in the same way as mirror misalignment. Estimates of these effects can be found in Refs. 1–3.

This feature of linear resonators has been assumed to be a property of all plane-mirror systems, including ring resonators. In fact, planar ring resonators, i.e., resonators in which the normals to the mirrors lie in the same plane, are very sensitive to mirror misalignment and to aberrations in the resonator. We shall consider a ring resonator in which there is always a self-conjugate ray, i.e., a ray which coincides with itself after a reflection from all the resonator mirrors, so that it is insensitive to misalignment and aberrations.

1. ANALYSIS OF THE CONDITIONS FOR THE EXISTENCE OF A SELF-CONJUGATE RAY IN A RING RESONATOR WITH PLANE REFLECTING SURFACES

Clearly, the image of an axis passing through any section of a self-conjugate ray between two adjacent resonator mirrors, obtained successively in all the mirrors, coincides with the axis itself. Therefore, an analysis of the conditions for the existence of a self-conjugate ray in a ring resonator reduces to the identification of the axes in space whose image in a system of mirrors forming the resonator coincides with the axes themselves.

We shall consider an optical system with N plane mirrors (P1, P2, ..., PN) distributed in an arbitrary manner in space (Fig. 1). If there is some object in front of the mirror P1, for example, a right-handed coordinate system $O_0X_0Y_0Z_0$, it follows that after successive specular reflections of this system in the mirrors P1, P2, ..., PN we can find its image created by all the mirrors, which is the system $O_nX_nY_nZ_n$. The construction of an image by an arbitrary mirror PJ can be described by a specular reflection operator $\hat{p}_j$ relative to the plane coinciding with the mirror. Then, the system $O_nX_nY_nZ_n$ is obtained from the system $O_0X_0Y_0Z_0$ as a result of the following operation:

$$(O_nX_nY_nZ_n)=p_n\cdot p_{n-1}\cdots\ p_1(O_0X_0Y_0Z_0).$$

In general, none of the pairs of axes O_nX_n and O_0X_0, O_nY_n and O_0Y_0, and O_nZ_n and O_0Z_0 coincide with one another. We shall now determine whether there is an axis **l** in space whose image coincides with itself.

Since the operator $\hat{K}=\hat{p}_n\cdot\hat{p}_{n-1}\cdots\hat{p}_1$ is an element of the Euclidean system, it can always be represented uniquely in the following form[4]:

$$\hat{K}=\hat{T}_{a\gamma}\cdot\hat{C}_{\gamma}\quad\text{for even } N, \tag{1}$$

$$\hat{K}=\hat{T}_{\mathbf{r}}\cdot\hat{p}_A\cdot\hat{C}_{\gamma_A}\quad\text{for odd } N, \tag{2}$$

where $\hat{C}_\gamma$ is the operator representing rotation through an angle $\boldsymbol{\gamma}$; $T_{a\gamma}$ is the operator representing translation by a vector $a\boldsymbol{\gamma}$; $\hat{p}_A$ is the operator of specular reflection in a plane p_A; $\hat{C}_{\gamma A}$ is the operator representing rotation through an angle $\boldsymbol{\gamma}_A$, where $\boldsymbol{\gamma}_A\perp p_A$; $\hat{T}_{\mathbf{r}}$ is the operator of translation by a vector $\mathbf{r}\in p_A$.

The application of Eqs. (1) and (2) gives the following equations for finding the required axes:

$$\hat{T}_{a\gamma}\cdot\hat{C}_{\gamma}\cdot\mathbf{l}=\mathbf{l}\quad\text{for even } N, \tag{3}$$

$$\hat{T}_{\mathbf{r}}\cdot\hat{p}_A\cdot\hat{C}_{\gamma_A}\,\mathbf{l}=\mathbf{l}\quad\text{for odd } N. \tag{4}$$

The form of the operator for transforming the space by a system with an even number of mirrors shows that Eq. (3) has a solution: this solution is the axis coinciding with the vector $\boldsymbol{\gamma}$. It is worth considering specially the case of a planar four-mirror resonator. The rotation vector $\boldsymbol{\gamma}$ of this resonator is perpendicular to the resonator plane and, therefore, in this case a self-conjugate ray exists only on condition that $\boldsymbol{\gamma}=0$, and then the operator K reduces to the operator of translation in the resonator plane and the solutions of Eq. (3) are the axes **l** parallel to the translation vector.

We shall now analyze Eq. (4), which represents a system with an odd number of mirrors. It follows from this equation that the solution exists if the axes **l** and $\hat{p}_A\cdot\hat{C}_{\gamma_A}\cdot\mathbf{l}$ can be made to coincide by translation along the vector **r** which lies in the p_A plane, i.e., the lines

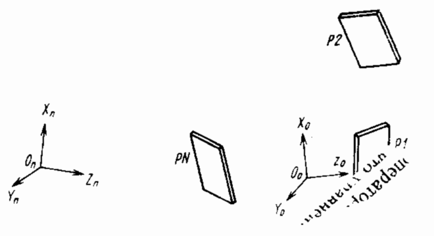

FIG. 1.

1 and $\hat{p}_A \cdot \hat{C}_{\gamma_A} \cdot 1$ should be parallel. This is possible only if $\hat{C}_{\gamma_A} \equiv 1$.

Thus, not every system with an odd number of mirrors has an axis 1 which retains its position in space after the transformation $\hat{\mathbf{K}}$: this is possible only if

$$\hat{K} = \hat{T}_r \cdot \hat{p}_A. \tag{5}$$

In this case any axis lying in the p_A plane and parallel to the vector **r** satisfies Eq. (4).

The application of the results obtained by solving Eqs. (3) and (4) to ring resonators yields two conclusions: 1) a closed ray path always exists in nonplanar resonators with an even number of mirrors; 2) a closed ray path in resonators with an odd number of mirrors can exist only if mirrors are suitably aligned.

2. MISALIGNMENT SENSITIVITY OF RING RESONATORS

As shown above, a self-conjugate ray in a ring resonator with an even number of mirrors exists for any misalignment of the mirrors but its position then varies in space.

Let us consider Fig. 2 which shows one of the resonator mirrors P*N* and a right-handed coordinate system $O_0X_0Y_0Z_0$ in which the O_0Z_0 axis coincides with a self-conjugate ray and the O_0X_0 axis lies in the plane of incidence of the ray and its image in the resonator mirrors is the system $O_nX_nY_nZ_n$. It follows from the above analysis that the O_nZ_n axis coincides with O_0Z_0, whereas the O_nX_n and O_nY_n axes are rotated relative to the O_0X_0 and O_0Y_0 axes through an angle γ about the O_0Z_0 axis.

Rotation of the mirror P*N* through a certain angle φ about the O_0Y_0 axis, perpendicular to the plane of incidence, rotates the $O_nX_nY_nZ_n$ system about the same axis through an angle 2φ. The new image of the $O_0X_0Y_0Z_0$ system is denoted by $O_n'X_n'Y_n'Z_n'$. The systems $O_n'X_n'Y_n'Z_n'$ and $O_0X_0Y_0Z_0$ can be made to coincide by rotation about a certain axis 1 and translation along this axis. The direction vector of this new axis can be found in the $O_0X_0Y_0Z_0$ system by identifying the eigenvector of the operator describing successive rotation first about the O_0Z_0 axis through an angle γ and then about the O_0Y_0 axis through an angle 2φ.

The matrix form of this operator is[5]

$$\begin{pmatrix} \cos 2\varphi & 0 & \sin 2\varphi \\ 0 & 1 & 0 \\ -\sin 2\varphi & 0 & \cos 2\varphi \end{pmatrix} \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \tag{6}$$

where

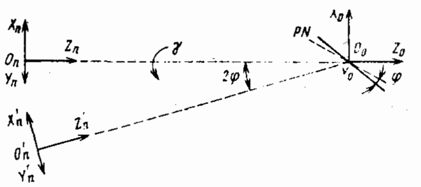

FIG. 2.

$a_{11}=\cos 2\varphi \cos\gamma$; $a_{12}=-\cos 2\varphi \sin\gamma$; $a_{13}=\sin 2\varphi$; $a_{21}=\sin\gamma$; $a_{22}=\cos\gamma$; $a_{23}=0$; $a_{31}=\sin 2\varphi \cos\gamma$; $a_{32}=\sin 2\varphi \sin\gamma$; $a_{33}=\cos 2\varphi$.

The eigenvector of the operator (6) is found from

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \tag{7}$$

It is evident that Eq. (7) can readily be reduced to a system of two equations with three unknowns:

$$\begin{cases} (1-\cos 2\varphi\cos\gamma)x + \cos 2\varphi \sin\gamma\, y - \sin 2\varphi\, z = 0, \\ -\sin\gamma\, x + (1-\cos\gamma)\, y = 0. \end{cases} \tag{8}$$

Each of these equations defines a plane passing through the origin and their intersection gives a line whose direction vector is an eigenvector of the operator (6). Solving the system (8), we find the angles between the new direction of the self-conjugate ray 1 and the axes of the coordinate system $O_0X_0Y_0Z_0$. In particular, the angle α_z between the O_0Z_0 axis (the old direction of the self-conjugate ray) and the vector 1 is given by

$$\cos\alpha_z = \cos\varphi\,(1+\sin^2\varphi\,\mathrm{ctg}^2\gamma/2)^{-1/2}. \tag{9}$$

It follows from Eq. (9) that the angular displacement of the self-conjugate ray depends on the rotation of the mirrors and on their initial positions. The sensitivity of the resonator to the angular misalignment of the mirrors is given by the quantity $d\alpha_z/d\varphi\,|_{\varphi=0}$, which represents the rate of displacement of the self-conjugate ray. It follows from Eq. (9) that

$$\left.\frac{d\alpha_z}{d\varphi}\right|_{\varphi=0} = \left(\sin\frac{\gamma}{2}\right)^{-1}. \tag{10}$$

Equation (10) describes the sensitivity of a resonator with an even number of mirrors to the misalignment of a mirror relative to the axis perpendicular to the plane of incidence of a closed ray. Similarly, we can show that the sensitivity of a ring resonator with an even number of mirrors to the angular misalignment of a mirror relative to the axis lying in the plane of incidence is given by

$$\left.\frac{d\alpha_z}{d\varphi}\right|_{\varphi=0} = \cos\theta_i \left(\sin\frac{\gamma}{2}\right)^{-1}, \tag{11}$$

where θ_i is the angle of incidence of the self-conjugate ray on the mirror in question. A comparison of Eqs. (10) and (11) shows that the resonators with even numbers of mirrors rotating the cross section of a self-conjugate ray through 180° are least sensitive to the mirror positions.

An example of such a resonator is one with an odd number of mirrors in two mutually perpendicular planes. After passing through this resonator the cross section of a self-conjugate ray is rotated about its central point by 180° (Fig. 3).

As pointed out before, planar resonators occupy a special place among the resonators with even numbers of mirrors. We can easily show that in the case of a closed planar resonator we have $\gamma = 0°$ and, consequently, $d\alpha_z/d\varphi\,|_{\varphi=0} = \infty$. Therefore, planar resonators with

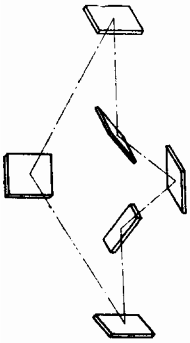

FIG. 3.

even numbers of mirrors are very sensitive to the mirror positions.

CONCLUSIONS

An analysis of ring laser resonators from the point of view of their sensitivity to the mirror positions shows that in the case of nonplanar resonators with an even number of mirrors there is a self-conjugate ray irrespective of what the mirror positions are. When the resonator becomes misaligned under the action of external perturbations, the closed trajectory becomes shifted and the shift depends strongly on the mirror positions. The resonators characterized by $\gamma = 180°$ are least sensitive to misalignment. This feature of ring resonators with an even number of mirrors makes it possible to construct a resonator which is insensitive to misalignments and there is no need then to attach mirrors to a special massive block. Resonators of this kind are also less sensitive to aberrations of the laser elements.

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Translated by A. Bryl

Investigation of submillimeter-wave amplification in optically pumped molecular gases

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(Submitted July 30, 1976)

Kvantovaya Elektron. (Moscow) 4, 1522–1528 (July 1977)

The results are presented of measurements of the gain for certain emission lines of cw optically pumped submillimeter lasers utilizing HCOOH, CH_3OH, $C_2H_2F_2$, CH_3I, and CD_3I molecules. The gain was measured as a function of the intensity of the pump radiation, gas pressure, and polarization of the input signal. The degree of polarization anisotropy of the excited medium depended on the type of pumping transition. The largest gain, 3.15 dB/m, was obtained for the $\lambda = 432.6\mu$ transition in the HCOOH molecule.

PACS numbers: 42.55.Hq

1. INTRODUCTION

Around 60 papers have been published to date on optically pumped submillimeter lasers. In addition to emitting a wide range of frequencies ($0.03 < \lambda < 2.65$ mm), lasers of this type are characterized by a comparatively high radiated power in cw operation (0.1–100 mW). More than 40 different molecules have been proposed for obtaining submillimeter-wave generation and the number of known emission lines is already approaching a thousand. In a number of papers, experiments have been described on obtaining the maximum pulse power,[1,2] on frequency measurements,[3–5] and also on the identification of the emission lines.[6,7] However, one of the most important parameters of an active medium, the gain, has not been studied in detail for any of the emission lines.

The aim of the present publication is a study of amplification in molecular gases, with optical pumping by CO_2 laser radiation. Such studies are important both for understanding the processes going on in optically pumped submillimeter lasers and for designing lasers with optimized output power.

2. METHOD OF MEASUREMENT

Measurements of the gain of laser transitions can be made directly using a submillimeter oscillator and amplifier combination. A schematic diagram of the apparatus used for measuring the gain is shown in Fig. 1. A submillimeter gas laser 1, optically pumped by CO_2 laser radiation, served as the source of the amplifier input signal. The submillimeter radiation was introduced into the amplifier through a crystalline quartz window 4 by means of a concave mirror 3. The amplifier consisted of a glass cell of length 1.1 m and inter-