STABILITY OF A RING RESONATOR WITH A NONPLANE AXIAL CONTOUR

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Ring resonators with a nonplane axial contour (RRNC) have a number of advantages in comparison with plane ring resonators [1-4]. In this connection an appreciable number of papers [5-9] have been devoted to the investigation of RRNC. It has been shown that one of the parameters which determine the properties of RRNC is the turning angle of the field in the resonator.

Rotation of the field should affect the stability of a resonator. The stability of an RRNC with an astigmatic focusing element is investigated in this paper on the basis of an analysis of the eigenvalues of the ray matrix of the resonator.

We shall assume that the resonator is formed by an even number of plane mirrors, the normals to which do not lie in the same plane (Fig. 1). We shall denote the turning angle of the field in the resonator by γ. The value of the angle γ is determined by the angles between the planes of incidence of radiation on adjacent mirrors [7]. Let an astigmatic lens with focal lengths f_x and f_y be mounted in the resonator. The ray matrix of the resonator $\hat{M}$ in the xyz coordinate system is equal to $\hat{F}\hat{L}\hat{T}(\gamma)$, where $\hat{F}$ is the matrix of the astigmatic lens, $\hat{L}$ is the matrix of translation by the distance L, and $\hat{T}(\gamma)$ is the matrix of turning by an angle γ. An RRNC is a nonorthogonal optical system [10, 11]; therefore one should use matrices 4 × 4 in size to describe the properties of the optical elements [8, 9]. It is necessary for finding the eigenvalues of a matrix to solve the equation $|\hat{M} - \lambda\hat{I}| = 0$, which reduces to the fourth-degree equation

$$\lambda^4 - a\lambda^3 + b\lambda^2 - c\lambda + |\hat{M}| = 0, \tag{1}$$

where a is the trace of the matrix.

The determinant of the ray matrix $|M|$ is equal to unity [10]; in addition one can show by using the results of [10] that $a = c$, due to which Eq. (1) is transformed to the form

$$(\lambda + \lambda^{-1})^2 - a(\lambda + \lambda^{-1}) + b - 2 = 0. \tag{2}$$

Expressions for a and b are determined by the ray matrix. It is not difficult to show that for the resonator under discussion Eq. (2) is of the form

$$(\lambda + \lambda^{-1})^2 - 2\cos\gamma\,(g_x + g_y)\,(\lambda + \lambda^{-1}) + 4g_x g_y - 4\sin^2\gamma = 0, \tag{3}$$

where $g_x = 1 - \frac{L}{2f_x}$, $g_y = 1 - \frac{L}{2f_y}$.

It follows from Eq. (3) that if λ is a root, λ^{-1} is also a root of the equation. It is well known that in a stable resonator all the eigenvalues λ_k of the ray matrix are simple and equal to unity in absolute value (for example, see [12]); therefore for the roots of the characteristic equation of a stable resonator the following expressions are valid: $e^{i\theta_1}$, $e^{-i\theta_1}$, $e^{i\theta_2}$, $e^{-i\theta_2}$, where $\theta_{1,2}$ are real numbers. We shall denote $1/2(\lambda + \lambda^{-1})$ as $\cos\theta$, and we shall assume that the resonator is stable in the case in which both solutions of the equation

$$\cos^2\theta - \cos\gamma\,(g_x + g_y)\cos\theta + g_x g_y - \sin^2\gamma = 0 \tag{4}$$

for $\cos\theta$ are real and do not exceed unity in absolute magnitude. Equation (4) determines two functions of γ: $\cos\theta_1(\gamma)$ and $\cos\theta_2(\gamma)$. It follows from (4) that by replacing γ by $-\gamma$ the value of the functions is not altered. In addition one can assume that the functions only

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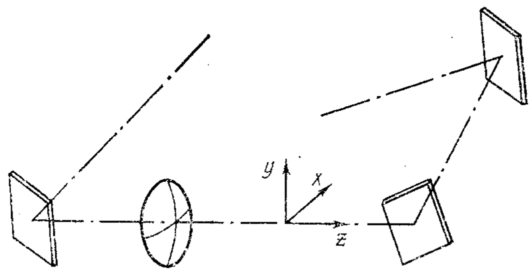

Fig. 1. Ring resonator with a nonplane axial contour.

change sign when γ is replaced by $\pi - \gamma$. Thus it is sufficient for analysis of the functions $\cos\theta_1(\gamma)$ and $\cos\theta_2(\gamma)$, and consequently the stability of the resonator, to investigate these functions in the interval 0-$\pi/2$. It also follows from Eq. (4) that upon the simultaneous replacement of g_x by $-g_x$ and g_y by $-g_y$ the absolute value of the functions is not changed.

In the case of a resonator with a plane axial contour ($\gamma = 0$) we obtain from Eq. (4) $\cos\theta_1 = g_x$ and $\cos\theta_2 = g_y$, whence the well-known stability conditions follow: $|g_x| < 1$ and $|g_y| < 1$. It is necessary in the investigation of the stability of an RRNC to find for the various regions of variation of g_x and g_y the range of angles γ in which the absolute value of the functions $\cos\theta_1$ and $\cos\theta_2$ does not exceed unity.

1. $0 \leqslant g_x < 1,\ 0 \leqslant g_y < 1.$

In this case the resonator is stable when $\gamma = 0$. We shall determine within what limits the functions $\cos\theta_1$ and $\cos\theta_2$ vary upon a variation of γ from 0 to $\pi/2$. With this aim we shall find the γ for which the functions become equal to ± 1. It is necessary for this to substitute into Eq. (4) the value $\cos\theta = \pm 1$ and to solve the equation obtained

$$\cos^2\gamma \mp (g_x + g_y)\cos\gamma + g_x g_y = 0. \tag{5}$$

The solutions of Eq. (5) are of the form

$$\cos\gamma_x = \pm g_x, \quad \cos\gamma_y = \pm g_y, \tag{6}$$

where the upper sign corresponds to the case $\cos\theta = 1$.

It follows from (6) that for positive g_x and g_y the functions being investigated in the range of angles 0-$\pi/2$ are always larger than -1 and become equal to unity when $\gamma_x = \arccos g_x$ and $\gamma_y = \arccos g_y$. We shall determine the sign of the derivative at the points γ_x and γ_y. It is not difficult to show that the derivatives of the functions specified by Eq. (4) with $\cos\theta = 1$ are equal to

$$\left.\frac{d(\cos\theta)}{d\gamma}\right|_{\gamma=\gamma_x} = \frac{(g_x - g_y)\sin\gamma_x}{2 - g_x(g_x + g_y)}, \quad \left.\frac{d(\cos\theta)}{d\gamma}\right|_{\gamma=\gamma_y} = \frac{(g_y - g_x)\sin\gamma_y}{2 - g_y(g_x + g_y)}. \tag{7}$$

Using (7), one can show that for the values of g_x and g_y indicated above the derivative is positive for the smaller of the values of γ under discussion and negative for the larger. It follows from this that one of the functions being investigated in the range of angles γ_x–γ_y becomes larger than unity, whereas the other one is always smaller than unity (Fig. 2a). Thus, the resonator is stable in the ranges $0 \leqslant \gamma < \gamma_x$ and $\gamma_y < \gamma \leqslant \pi/2\ (g_x > g_y)$.

2. $0 \leqslant g_x < 1,\ g_y > 1$.

The resonator is unstable when $\gamma = 0$. However, as follows from Eqs. (5), one of the functions becomes equal to unity when $\gamma = \gamma_x$. We shall determine the sign of the derivative at this point. It follows from (7) that when $g_x + g_y < 2$ the derivative is negative, i.e., when $\gamma = \gamma_x$, the function, which was equal to g_y for $\gamma = 0$ (Fig. 2b), becomes equal to unity; consequently, the resonator is stable in the range $\gamma_x < \gamma \leqslant \pi/2$ It also follows from (7) that the derivative at the point γ_x is negative also when $g_x + g_y > 2$ if $g_x(g_x + g_y) < 2$.

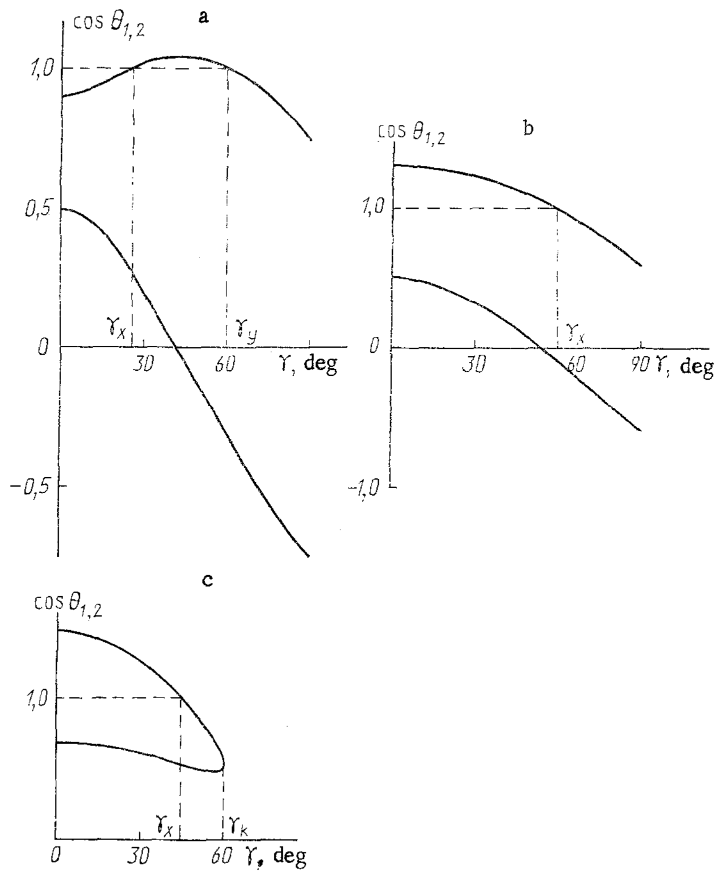

Fig. 2. Dependence of the solutions of the characteristic equation on the turning angle of the field in the resonator. (a) $g_x = 0.9$, (b) 0.5, and (c) 0.7; (a) $g_y = 0.5$, (b) 1.3, and (c) 1.5.

However, as it is not difficult to show, the discriminant of Eq. (4) becomes negative in this case when $\gamma > \gamma_k = \arcsin((g_y - g_x)/(\sqrt{(g_x + g_y)^2 - 4}))$; therefore the stability region of the resonator turns out to be smaller: $\gamma_x < \gamma < \gamma_k$ (Fig. 2c).

Investigating the solutions of Eq. (4) in a similar way, one can show that for $0 \leqslant g_x < 1$, and $-1 < g_y \leqslant 0$ the resonator is stable in the range $0 \leqslant \gamma < \gamma_x$ when $g_x > |g_y|$ and in the range $0 \leqslant \gamma < \pi - \gamma_y$ when $g_x < |g_y|$, and also that for the cases $0 \leqslant g_x < 1$, $g_y < -1$ and $g_x > 1$, $g_y > 1$ the resonator is unstable for all γ.

Keeping the properties of the solutions of Eq. (4) in mind, one can investigate the stability of an RRNC for arbitrary values of g_x, g_y, and γ on the basis of the results obtained.

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INVESTIGATION OF THE PHOTOREDUCTION OF RHODAMINE 6G IN SOLUTION BY CARBAZOLYL-CONTAINING DONORS

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The reaction involving the phototransfer of an electron onto Rhodamine 6G (R) from molecules of various different donors (hydroquinone, triphenylamine, piperidine, etc.) in solution is well known [1-4]. However, there are no data on the possibility and mechanism of this reaction with donors which contain a carbazolyl heterocycle, derivatives of which such as poly-N-vinylcarbazole (PVC) and poly-N-epoxypropylcarbazole (PEPC), sensitized by R, have found extensive application in photosensitive materials for electrophotography (EPH) [5].

It has been shown in [6] that the phototransfer of an electron is the primary act in the sensitization of layers based on PVC and PEPC with R. The present work is concerned with the investigation of this reaction in a solution of R in chloroform with additions of the carbazolyl-containing donors, N-methyl-carbazole (MC) and PEPC.

The absorption spectra and the kinetics of the short-lived intermediate particles, triplets, the reduced dye species (RD) in the form of the neutral radical $\dot{R}$, and the cation radical of the carbazolyl donor D^+ were measured by means of pulsed photolysis. It was established that phototransfer of an electron occurred via the T_1 state of the dye in the absence of and at a low concentration of the donor while, at a high concentration of the donor, it occurred via the T_1 state and possibly the S_1 state. Using the methods of fluorescence quenching and photoconductivity it was shown that the process occurring involves a complex stepwise mechanism. An exciplex formed during this process between an excited singlet state molecule R and a molecule of the donor was identified. The quantum yields for the phototransfer reaction in a deoxygenated individual solution of R and at various different concentrations of the donor were estimated. It was discovered that dissolved oxygen had a pronounced effect on the induced absorption spectra and the photoconductivity which is explained in the present paper as being due to a reaction with RD.

Rhodamine 6G of "quantum electronics" grade, chromatographically purified chloroform, "pure" grade N-methylcarbazole recrystallized from alcohol, and PEPC oligomer of commercial production with an average molecular mass M = 740, purified by reprecipitation from solution were employed.

The photoconversion of R in chloroform solutions with excitation of the S_0—S_1 transition was investigated using a pulse photolysis apparatus equipped with a shutter for isolating the scattered light [7]. Solutions of the dye which had been deaerated by pumping them down under a high vacuum were irradiated in the range from 460 to 580 nm by pulses with an energy of 450 J and a duration $\tau_{0.1}$ = 70 μsec.

The photoinduced absorption spectra of R ($C = 5\cdot10^{-6}$ mole/liter), corrected for the irreversible decomposition of the dye, are shown in Fig. 1. Immediately after the individual solution had been irradiated it became clear and the primary T—T absorption arose with maxima at 9600, 16,400, and 24,700 cm^{-1} which was replaced by the band due to the photoproduct with a maximum at around 24,200 cm^{-1}. As experiments on T—T energy transfer from naphthalene to R have shown, it is formed via the T_1 state of the dye. Additions of electron donors and

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