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Misaligned Spherical-Mirror Waveguide Resonators

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The effect of misaligning the spherical mirror of a hollow waveguide laser resonator on the diffraction loss is investigated on the assumption that the waveguide acts as an EH_{11} signel-mode filter. Theoretical and experimental results are obtained and compared by means of the formula for homogeneous line broadening.

§1. Introduction

Marcatili and Schmeltzer first proposed the use of hollow dielectric waveguide to confine the the discharge and simultaneously to guide the laser radiation.[1] One of the advantages of laser devices in this manner is the increased gain per unit length with decreasing tube radius, which in turn leads to a decrease in the physical size of the device. This situation motivated us to undertake a study of the misalignment of waveguide laser resonators. This is a problem concerned with the estimation of the limit tolerance in waveguide laser engineering and has been investigated experimentally by Jensen and Tobin.[2] For conventional laser resonators, the effect of mirror misalignment has been discussed by many investigators (see, *e.g.*, reference 3 and the references quoted therein).

§2. Theory

A waveguide laser has two reflecting mirrors of the same diameter $2b$. This guide is a hollow dielectric pipe of length l and with circular cross section of radius a. These two mirrors ave the radii of curvature C_1 and C_2 and are separated from the two ends of the guide by distances d_1 and d_2, respectively. Figure 1 is the end view of this configuration, in which we introduce three cartesian coordinate systems: viz (ξ, η, z) and (x', y', z') which are parallel, and system (x, y, z) which is tilted. The relation between the latter two systems is

$$x' = x\cos\theta - z\sin\theta, \quad y' = y, \quad z' = x\sin\theta + z\cos\theta. \tag{2}$$

Here θ is a small displacement angle obtained by rotating the mirror (see Fig. 1) with respect to the y-axis. The equation of this mirror is known. For a sphere of radius C_1, it is

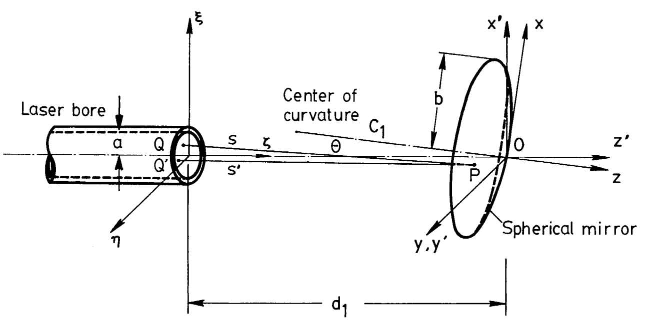

Fig. 1. An end view of the misaligned waveguide resonator. Illustration of the notations.

$$x^2 + y^2 + (z + C_1)^2 = C_1^2. \tag{3}$$

Instead of cartesian systems, we substituted two cylindrical systems, i.e.,

$$\left.\begin{aligned} \xi &= \rho\cos\phi, \quad x = r\cos\psi, \\ \eta &= \rho\sin\phi, \quad y = r\sin\psi. \end{aligned}\right\} \tag{4}$$

We then calculated the misalignment loss due to the rotation of one of the two reflecting mirrors. Our subsequent calculations were made under the assumption that the waveguide acts as an EH_{11} single-mode filter. This means that the light disturbance at a point Q on the end plane of the guide can be expressed as[1]

$$U(Q) = \begin{cases} J_0(u_{01}\rho/a)/\sqrt{\pi}\,a\,|J_1(u_{01})|, & \rho \leqslant a, \\ 0, & \rho > a, \end{cases} \tag{5}$$

where u_{01} is the first zero of Bessel function J_0.

According to the Huygens-Fresnel Principle, the field at any point P on the mirror can be expressed by

$$U(P) = \frac{1}{i\lambda}\int_0^{2\pi} d\phi \int_0^a U(Q)\frac{\exp(iks)}{s}\rho\, d\rho, \quad (k = 2\pi/\lambda). \tag{6}$$

Here λ is the wavelength, and $s = \overline{QP} = \sqrt{(\xi - x')^2 + (\eta - y')^2 + (z' + d_1)^2}$ is the distance between point Q and point P. Making use of eqs. (2)–(4), the following expansion for s is obtained:

$$\begin{aligned} s \simeq d_1 &+ \frac{\rho^2}{2d_1} - \frac{\rho r\cos(\psi - \phi)}{d_1} + g\frac{r^2}{2d_1} \\ &+ \left(r\cos\psi - \frac{\rho r^2\cos\phi}{2d_1C_1}\right)\sin\theta \\ &+ \left(\frac{r^2}{2C_1} + \frac{\rho r\cos\psi\cos\phi}{d_1}\right)(1 - \cos\theta), \end{aligned} \tag{7}$$

where $g = 1 - d_1/C_1$. It was then found necessary to determine the field from the mirror to the guide and back into the latter. Again, by the application of the Huygens-Fresnel Principle, we reached the equation

$$U(Q') = \frac{1}{i\lambda}\int_0^{2\pi} d\psi \int_0^b U(P)\frac{\exp(iks')}{s'} r\, dr. \tag{8}$$

in which $s' = \overline{PQ'}$ denotes the distance between point P and a new point $Q' = Q'(\rho', \phi', 0)$ on the end plane of the guide. If we change the notation (ρ, ϕ) into (ρ', ϕ'), the expression for s' is the same as eq. (7).

We next considered what portion of the returning radiation is coupled into the EH_{11} mode in the hollow waveguide. The expected portion is the cross-correlation of the emerging field [eq. (5)] and the returning field [eq. (8)]. This means that the fraction of power returned to the guide in the EH_{11} mode is determined by the equation

$$|F(\theta)|^2 = \left| \frac{1}{\sqrt{\pi} a J_1(u_{01})} \int_0^{2\pi} d\psi' \times \int_0^a J_0(u_{01}\rho'/a) U(Q') \rho' \, d\rho' \right|^2. \tag{9}$$

We substitute eqs. (5)–(9) into the above equation and use the simpler approximations $s \simeq$, $s \simeq d_1$ in the less critical denominator terms of eq. (6) and eq. (8). After a lengthy derivation, omitted here, the result obtained was

$$|F(\theta)|^2 = |F(0)|^2 \left| 1 - \theta^2 (\pi N)^2 \frac{\int_0^{\gamma^2} G(w_1) \exp(i2\pi g N w_1) \, dw_1}{H(N, g)} + \cdots \right|^2. \tag{10}$$

Here $N = a^2/\lambda d_1$ is the Fresnel number, $\gamma = b/a$ and $w_1 = (r/a)^2$. In the above expression, $F(0)$ is the coupling factor of an aligned system (i.e., system for $\theta = 0$) which takes the following form[4)]

$$F(0) = -\exp(i2kd_1)(\pi N)^2 H(N, g)/J_1^2(u_{01}), \tag{11}$$

where

$$H(N, g) = \int_0^{\gamma^2} G_1^2(w_1) \exp(i2\pi g N w_1) \, dw_1$$

and

$$G_1(w_1) = \int_0^1 J_0(u_{01}\sqrt{w}) J_0(2\pi N \sqrt{w_1 w}) \exp(i\pi N w) \, dw, \tag{12a}$$

The $G(w_1)$ function in eq. (10) is determined by the following combination of the G-integrals:

$$G(w_1) = w_1 \left[\frac{2d_1}{a} G_1(w_1) + i \left(\frac{a}{C_1} \right) \sqrt{w_1} G_2(w_1) \right]^2 - \sqrt{w_1} \frac{G_1(w_1) G_2(w_1)}{\pi N} + \frac{w_1^2}{2} \left(\frac{a}{C_1} \right)^2 \times \left[G_1(w_1) G_3(w_1) + G_2^2(w_1) \right], \tag{13}$$

where

$$G_2(w_1) = \int_0^1 J_0(u_{01}\sqrt{w}) J_1(2\pi N \sqrt{w_1 w}) \times \exp(i\pi N w) \sqrt{w} dw, \tag{12b}$$

$$G_3(w_1) = \int_0^1 J_0(u_{01}\sqrt{w}) J_0(2\pi N \sqrt{w_1 w}) \times \exp(i\pi N w) w \, dw, \quad [w = (\rho/a)^2]. \tag{12c}$$

The behaviour of the G-integrals are shown in Fig. 2 for the Fresnel number $N = 0.25$. The spiralled curves in this figure reveal that when $w_1 \lesssim 1$ integral G_1 is larger than the other integrals. It is known that $w_1 \lesssim 1$ is the range in which the value of the numerator on the right-hand side of eq. (10) is determined. This is because that the G-integrals are rapidly oscillatory functions. Alternatively, we found that we were dealing with the computation of stationary phase integrals.[5)] Moreover, a close examination of the mathematical structure of eq. (13) shows that G_1 is multiplied by $(2d_1/a)$, a factor which is much larger than the other multiplying factors in eq. (13). Based on these reasons, we kept only the first term and neglected all the other terms in eq. (13). This means that we set

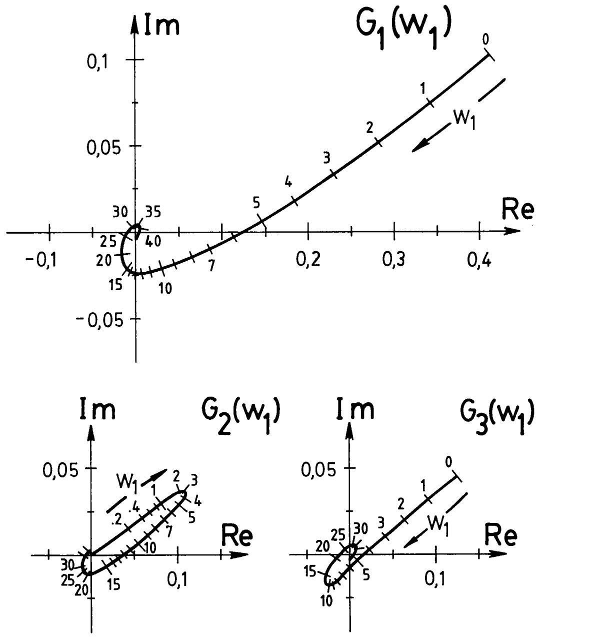

Fig. 2. Oscillatory behaviour of the G-integrals for the Fresnel number $N = 0.25$.

$$G(w_1) \simeq w_1 \left(\frac{2d_1}{a} \right)^2 G_1^2(w_1). \tag{14}$$

This approximation leads to a considerable reduction of the numerical works. Using this approximation, numerical experiments indicated that in a very large range of w_1 the relative error between eq. (13) and eq. (14) is less than 0.1% when $a/C_1 = 0.01$, $2d_1/a = 100$ and $N = 0.25$. Thus, by inserting eq. (14) into eq. (10) and neglecting the terms which are higher than θ^2, we obtained

$$\left| \frac{F(\theta)}{F(0)} \right|^2 \simeq 1 - \left(\frac{\theta}{\theta_m} \right)^2, \tag{15}$$

in which

$$\theta_m = \frac{\lambda}{2\sqrt{2}\pi a} \Big/ Re\,[I(N, g)] \tag{16}$$

and

$$I(N, g) = \int_0^{\gamma^2} G_1^2(w_1) \exp(i2\pi g N w_1) w_1 \, dw_1 \Big/ \int_0^{\gamma^2} G_1^2(w_1) \exp(i2\pi g N w_1) \, dw_1. \tag{17}$$

The correctness of the above results was then checked experimentally, as may be seen in the following section of this paper.

§3. Experimental Investigations

The effect of mirror misalignment on the laser output power at wavelength 10.6 μm was investigated in a Brewster-windows sealed system. Great care was taken to cut the Brewster-angle of 67.4° for the ZnSe window plates. The cathode of the discharge tube was a section of stainless steel tube and the tungsten-rod anode was applied. The tube consisted of a 1 mm-bore Pyrex capillary 10 cm in length with a concentric cooling jacket. The laser was operated in a longitudinally excited mode with non-flowing gas mixture of CO_2, N_2 and He in the ratio of 4:2:8. The total pressure was a little less than 70 Torr. The initial alignment of laser optics was accomplished with the aid of a He–Ne laser, meanwhile, the angle of misalignment was calibrated by means of adjusting a micrometer mounted behind the mirror. The output power was $P_0=0.29$ W measured by a Coherent 210 power meter under discharge conditions of 2 mA current and 20 kV anode voltage.

In our experiments, a 10 cm radius of curvature mirror was placed 9.5 cm from one end of the 1 mm-diameter capillary ($N=0.248$). The reflectance of this mirror was 99% ($T_1=0.01$). A second mirror was placed at the other end of the guide at a distance of 7 cm. The second mirror, which in our experiments was fixed, had a 7.5 cm radius of curvature and its reflectance was 95% ($T_2=0.05$). Both mirrors were made by ZnSe material obtained from Two-Six Co. and had the external diameter of (3/4)in.

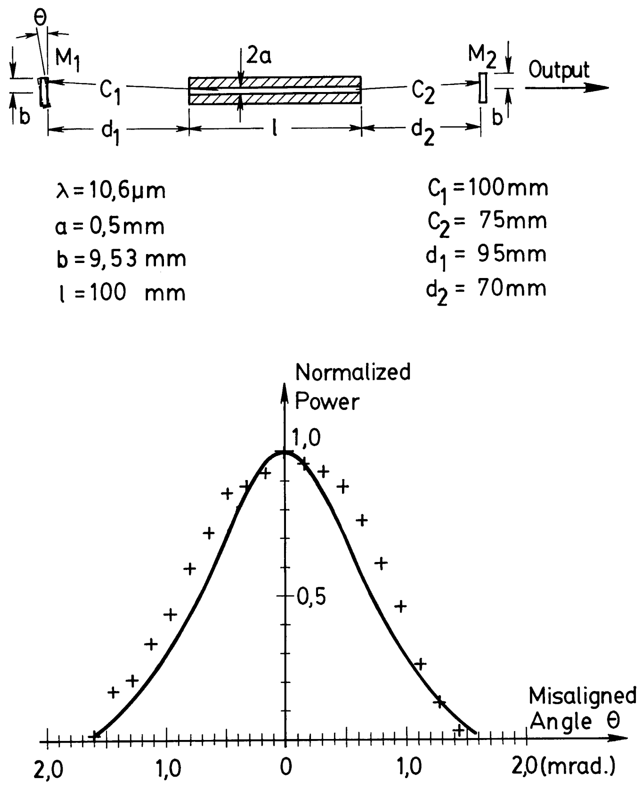

Fig. 3. Output power as a function of the angle of misalignment. Theoretical results (solid curve) are calculated from eq. (21), experimental data are shown by points.

The output power versus the angle of misalignment was measured and shown in Fig. 3. The results obtained were normalized by P_0, the output of the aligned system. The mode patterns were documented on a slip of thermo-paper placed 3 cm from the output mirror and exposed for two seconds (see Table 1). Based on the materials in this table, it may be concluded that there is no evidence to show that as the mirror is misaligned, the alternation of the working mode in the waveguide. The only change observable was the size of the spot, a phenomenon which can be explained by the fact that the emission of the waveguide laser decreases monotonically with the increasing values of the angle of misalignment.

The theoretical results obtained from eq. (15) were then checked by a consideration based on the relationship between the gain factor and the radiation intensity.[6] Under the condition of homogeneous line broadening, the output of a waveguide laser resonator may be expressed as

$$\frac{P(\theta)}{P_p}=\frac{T}{T+A(\theta)}\{g_0L+0.5\ln[1-T-A(\theta)]\}, \tag{18}$$

where $P(\theta)$ is the output power which is now a function of the angle of misalignment, P_p is a saturation parameter, g_0 the unsaturated gain coefficient, L the optical path length, $T=T_1+T_2$ and

$$A(\theta)=2-|F_1(0)|^2\left[1-\left(\frac{\theta}{\theta_m}\right)^2\right]-|F_2(\theta)|^2 \tag{19}$$

is the diffraction losses coefficient. In this equation we attached the suffixes 1 and 2 to the coupling coefficient $F(0)$ [see eq. (11)] to indicate that they are different quantitites relating respectively to the mirrors 1 and 2 on the two sides of the guide.

Under the condition of $\theta=0$, we have

$$\frac{P(0)}{P_p}=\frac{T}{T+A(0)}\{g_0L+0.5\ln[1-T-A(0)]\}. \tag{20}$$

By a comparison of eqs. (18) and (20), we eliminated the factor P_p and obtain

$$\frac{P(\theta)}{P(0)}=\frac{T+A(0)}{T+A(\theta)}\frac{g_0L+0.5\ln[1-T-A(\theta)]}{g_0L+0.5\ln[1-T-A(0)]}. \tag{21}$$

The resonator geometry is shown in Fig. 1 where we have $T=0.06$ and $A(0)=0.159$. If we substitute the experimental data [$\theta=1.28$ mrad., $P(\theta)/P(0)=0.068$] into eq. (7),

Table I. Near field patterns recorded on a slip of thermo-paper placed 3 cm from the output mirror. Resonator geometry is shown in Fig. 3.

Misaligned angle (mrad.)	Output power (W)	Mode pattern
0	0.292	● = 4.8 mm
0.16	0.274	● = 4.6 〃
0.32	0.242	● = 4.0 〃
0.48	0.220	● = 3.8 〃
0.64	0.188	● = 3.2 〃
0.80	0.145	● = 3.0 〃
0.96	0.120	• = 2.8 〃
1.12	0.095	• = 2.4 〃
1.28	0.068	• = 1.6 〃

we obtain $g_0L=2.43$. On this basis, theoretical results were plotted in Fig. 3 by the solid curve and compared with the output power observed in our experiments.

§4. Discussion and Conclusions

In this paper, we have seen the effect of mirror misalignment on the output power of a hollow waveguide laser resonator, which decreases rapidly with the increasing values of θ (the angle of misalignment). Therefore, the misalignment should be carefully controlled, otherwise the desired output power cannot be attained.

In this paper, only one misalignment parameter was considered. Whereas, it is known that, in general, a misalignement between two axes (*e.g.*, the axis of the guide and the axis of the mirror) is described by three parameters.[7] Consequently, exact agreement between our calculated and experimental results cannot be expected. The discrepancy between the aforementioned two results is also due to the fact that only the first order correction was considered. From Fig. 3, it is seen that a higher order correction to the resonator misalignement is necessary. We plan to consider this topic in a subsequent study.

Finally, we wish to mention that five parameters (i.e., a, b, C_1, d_1 and λ) are necessary for defining the misalignment sensitivity of waveguide laser resonators. We found that five can be expressed by four combinations, namely, the Fresnel number N, the g-parameter, (λ/a) and γ^2. From our calculations, it is seen that the first three are more important quantities than γ^2 as the latter is larger than 10, i.e., when the outer radius of the reflecting mirror is three times large than the radius of the capillary.

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