PROCEEDINGS OF SPIE

SPIEDigitalLibrary.org/conference-proceedings-of-spie

Optical Resonator Axis Stability And Instability From First Principles

Irl W. Smith

Irl W. Smith, "Optical Resonator Axis Stability And Instability From First Principles," Proc. SPIE 0412, Fiber Optic and Laser Sensors I, (19 September 1983); doi: 10.1117/12.935817

SPIE.

Event: 1983 Technical Symposium East, 1983, Arlington, United States

Optical resonator axis stability and instability from first principles

Irl W. Smith

Raytheon Research Division
131 Spring Street, Lexington, Massachusetts 02173

Abstract

It is shown that optical resonators may be broadly divided into two classes based on their mirror count and other properties, and that the two classes exhibit very different axis stability properties. When a resonator of one class is slightly misaligned, its optic axis (and hence its input axis for rotation sensing) remains close to its nominal position independently of the focussing action of the resonator optics. In contrast, when a resonator of the other class is slightly misaligned, the displacement and reorientation of its optic axis is restrained only by the focussing optics. Hence resonators of this second class are restricted in the range of mirror radii of curvatures they may employ. This has implications in noise and lockin performance for rotation sensors.

Introduction

This paper relates to a particular property, alignment stability, of all optical resonators and, in particular, to those used in ring laser gyroscopes (RLGs). Certain types of RLGs, namely four-frequency, or "multioscillator", RLGs[1], require an optical resonator which has intracavity circular birefringence. One way to achieve this circular birefringence (polarization rotation) is to use the image-rotating properties of nonplanar traveling-wave resonators. Such resonators are defined as having mirrors not all of whose surface normals lie in (or parallel to) a single plane. As will be shown below, a nonplanar resonator having an even number of mirrors and a resulting image rotation (per round trip) not equal to zero modulo 360 degrees has an additional property of value to RLG performance, namely, optic axis stability independent of any focussing optics within the resonator. As used here, stability refers to the robustness of constraint of the position of the optic axis when the resonator is perturbed. We will not consider the other meaning of the word, which has to do with mode size and diffraction loss.

When a nonplanar resonator is slightly misaligned, e.g. when one of its mirrors is slightly tilted, its optic axis (and hence its input axis for rotation sensing) remains close to its nominal position independently of the focussing action of the resonator optics. In contrast, when a resonator of one of the other, planar, classes is slightly misaligned, the displacement and reorientation of its optic axis is restrained only by the focussing optics. Hence planar resonators are restricted in the range of mirror radii of curvatures they may employ: the focussing action of the curved mirrors will be too weak to compensate for misalignments if the mirror radius is too long.

Discussion

An optical resonator, whether for a linear or ring laser or for other purposes (such as a passive rotation sensor), consists of a train of elements, usually mirrors, arranged so that there exists a ray (which, in the geometric discussion below, will be taken to be a directed line), called the optic axis, which repeats itself after a round trip through the resonator. In most applications it is of interest to have the spatial position of the axis, measured with respect to the resonator substrate structure, remain stable under the sort of small perturbations in mirror alignment and position which may be engendered either by environmental changes, such as thermal gradients, or by imperfect motions of moveable resonator mirrors.

I will show that optical resonators may be broadly divided into classes based on essentially their mirror count (odd or even), and that the classes exhibit very different axis stability properties. In this discussion, resonators will be divided into three classes: "nonplanar" resonators, which (as will be shown) must have an even number of mirrors; "planar even" resonators, which have an even number of mirrors and a mirror arrangement which results in substantially zero (modulo 360 degrees) image rotation; and "odd" resonators, which have an odd number of mirrors.

First let me dispose of the curved mirrors, at least one of which is present in real resonators to assure a finite mode size and acceptably low diffraction losses. The optic axis intersects and is reflected from the mirror at some particular point on the mirror. The static geometric properties of the resonator may be considered by assuming that the mirror is plane and tangent to the real mirror at this point of intersection. The dynamic aspects (stability) of the resonator may be fundamentally determined by the curvature of the mirror or may not; This latter dichotomy is, in fact, the subject at hand. In the discussion which follows, we will usually consider the idealized resonator in which all mirrors are taken as plane, and then add in the curvature afterwards.

Definitions and Lemmas. The following relies on concepts developed very nicely in Reference 2. First, some definitions:

An isometry is a transformation or mapping (of three-dimensional Euclidean space onto itself) which preserves distances. Any isometry is either direct or opposite. A direct isometry preserves "handedness", an opposite one reverses it.

A central inversion in a point O carries each point P into a point P' such that POP' is a straight line and PO = PO'. A central inversion is an opposite isometry.

A reflection in a plane passing through a point R and normal to a line RS carries each point P into a point P' on the other side of the plane such that the foot F of the perpendicular from P to the reflection plane is the same as the foot of the perpendicular from P' to the reflection plane, and FP = FP'. Points in the reflection plane are invariant. A reflection is an opposite isometry.

A rotation about an axis OA carries each point P in a plane perpendicular to OA into a point P' also in that plane such that the angle (the rotation angle) between the lines from P and P' to the point where OA intersects the plane is the same for all points P in space. Points on OA are invariant. A rotation is a direct isometry.

A translation carries each point P into a point P' such that the distance and direction of PP' is the same for all points P in space. The vector PP' is called the translation vector. There are no invariant points (except the one at infinity). A translation is a direct isometry.

The operation resulting from the successive applications of any number of isometries is referred to as the "product" of that (ordered) chain of isometries and is direct or opposite as the number of opposite isometries in the chain is even or odd.

A screw (or screw displacement) is the product of a rotation about an axis OA and a translation whose vector is parallel to (or "along") OA.

A glide reflection is the product of a reflection in a plane and a translation along a vector parallel to (or "in") the plane.

A rotary reflection is the product of a reflection in a plane and a rotation about an axis perpendicular to the plane.

Any isometry may be expressed as one of the three preceding compound isometries. This statement will be illustrated below. In the compound isometries the two elements commute, that is, for example, a translation followed by a reflection in a plane containing the translation vector (a glide reflection) produces the same effect as the same translation preceded by the same reflection.

The above are essentially definitions. The following are theorems which may be easily proved but which are here merely stated. They seem to be intuitively obvious -- especially if one has spent the last seven years of his life thinking about such topics!

The product of two rotations is a screw (which degenerates into a rotation if the two original rotations have intersecting axes). Admitting that a rotation is a screw, the product of any number of screws is a screw. An essential step in recognizing the truth of this last step is to note that a rotation about an axis OA is equal to a rotation about a parallel axis O'A' followed by a translation perpendicular to OA. Hence any "residual" translation may be resolved into a part perpendicular to the axis, which is eliminated by a parallel displacement of the choice of axis (which thereby becomes unique), and a part along the axis. The latter part remains as the displacement part of the screw. From this result it follows that the action of a resonator having an even number of mirrors upon the electromagnetic field is precisely that of a screw. This screw may be degenerate, having either zero rotation or zero translation or both.

The product of two reflections is either: (a) a rotation about the line OA along which the two planes intersect, or (b) a translation perpendicular to the two planes if they are parallel. From (b) it follows that any translation followed by a reflection can be expressed as a glide reflection, by using a similar technique as in reducing an arbitrary translation plus a rotation to a screw and employing the fact that the square of a given reflection is the identity.

A reflection is the product of: a rotation of 180 degrees (a "half-turn") about an axis perpendicular to the reflection plane; and a central inversion in the point where the axis intersects the plane. These two operations commute.

The product of a reflection and a rotation about an axis in its plane is a reflection, in a plane which has the rotation axis in common with the original reflection plane. The angle between the final and original reflection planes is one-half the rotation angle.

The product of a reflection and a rotation about an axis parallel to the reflection plane is a glide reflection.

The product of a reflection and a rotation about an axis not parallel to its plane is a rotary reflection (whose rotation angle may be shown bo be nonzero unless the input rotation is zero). This may be seen by decomposing the reflection into a half-turn and an inversion about the point where the rotation axis intersects the reflection plane. The half-turn and the original rotation combine into a rotation about some axis (still passing through the inversion point) by an angle A, which may be written as a rotation of A - 180 degrees about that axis followed by a half-turn. This latter half-turn may be combined with the leftover inversion to give a reflection in a plane which is now, by construction, perpendicular to the rotation axis (of the rotation by A - 180 degrees).

Now that this foundation has finally been constructed, we are in a position to essentially state the desired result: An odd resonator has an optic axis only in singular cases. We write the odd number of reflections as a single one of the reflections, times a screw. If the screw axis is not parallel to the reflection plane, the result is a rotary reflection, which clearly possesses no ray which is invariant; for if the ray be taken to lie in the plane, it is moved by the rotation, while if it be taken to intersect the plane, its component perpendicular to the plane is reversed in sign by the reflection.

Thus we see that an optic axis can exist only if the chosen reflection's plane is parallel to the axis of the screw made up of the remaining (even number of) reflections. In such a case the product isometry is a glide reflection; such an isometry has an infinite number of optic axes, all parallel to the direction of the translational part of the glide and all lying in the reflection plane. (In a real resonator the optic axis is fixed by the fact that only one point on the curved mirror fulfills the orientational constraint, and the optic axis must include that point.) Note that the effect of the resonator on the polarization of light is to leave unchanged light polarized in the plane of the glide reflection but to inflict a 180 degree phase shift on light polarized orthogonally. Thus such resonators effectively incorporate 180 degrees of phase birefringence: Their polarization properties are those of a half-wave plate.

Since the resonator possesses an optic axis only if a particular geometric condition is exactly satisfied, perturbation of the mirror orientations will result in a resonator with no optic axis. (To be exact, any tilt perturbation of a given mirror which is not a rotation about precisely the axis of the screw defined by the remaining mirrors will result in disappearance of the optic axis). What this means in reality is that, in the perturbed resonator, any ray will gradually walk away from its original position, bang into the wall, and be absorbed. Such a resonator will then have large diffraction loss and be unsuitable for RLG applications. To avoid this problem, real RLGs have a mirror whose orientation is a function of where the optic axis intersects its surface, i.e. a mirror which is curved. The movement of the optic axis, then, for a small angular perturbation of one of the resonator's mirrors by an angle A, is of order AR where R is the radius of curvature of the mirror.

Consider the contrasting case of an even resonator. As we have seen, the product of an even number of reflections is a screw, the axis of which is obviously the optic axis of the resonator and the rotational angle of which is the image rotation. An angular perturbation by angle A will then displace the optic axis by a distance AL where L is related to the average distance between the axes of the rotations which the screw may be thought to be made up of. A reasonable estimate of L, as found by actual numerical calculations on our nonplanar resonators, is the circumference of the resonator.

We thus see that a nonplanar even resonator is as stable as an odd resonator having a mirror whose radius of curvature is roughly equal to the circumference of the resonator. Put differently, the motion of the optic axis in an odd resonator of length L and mirror effective radius of curvature R is of the same order of magnitude as that which would occur in a nonplanar even resonator of length R: the motion is roughly (R/L) larger than an equally-sized nonplanar resonator.

One further case must be discussed for completeness, the planar even resonator, or any even resonator with approximately zero modulo 360 degrees total image rotation. Clearly if the rotation angle of the screw (which constitutes the most general isometry possible for an even number of mirrors) is zero, the resonator isometry is a pure translation. Such a resonator has a doubly infinite set of optic axes. However, if any of the resonator's mirrors is slightly disturbed, the degeneracy is broken: the rotation is no longer zero and hence the direction of the screw axis is fixed by the fact that it is parallel to the axis of the resultant rotation. The amount of rotation engendered by the small perturbation will be very small, so the position of the axis will usually be forced to lie far away in order to annul the transverse translational components and result in a true screw, as discussed above. Hence for all practical purposes the resonator's optic axis may be found by requiring the optic axis to pass through that point on the curved mirror which makes the total rotation zero. This is reminiscent of the condition found above for odd resonators.

Conclusion

Nonplanar resonators with an even number of mirrors and significantly nonzero image rotation are inherently self-aligning, that is, they have a robustly constrained optic axis without focussing optics. Odd resonators are self-aligning against only such misalignments as lie in a particular direction in a given transverse plane; in planar odd resonators, this direction lies in the plane of the resonator, while in nonplanar odd resonators it lies in general in some other direction, different at each mirror. Finally, planar even resonators are not self-aligning at all.

References

1. Statz, H., Terry A. Dorschner, M. Holz, and Irl W. Smith, "The Multioscillator Ring Laser Gyroscope", in Laser Handbook (M. Stich, ed; North Holland; Amsterdam) (1983) (to be published). Dorschner, T. A. and I. W. Smith, "Clear-Path Four-Frequency Resonators for Ring Laser Gyros," J. Opt. Soc. Amer. 68 (1981), p. 1381; Smith, I. W. and T. A. Dorschner, "Biassing the Raytheon Four-Frequency Ring Laser Gyroscope," Laser Inertial Rotation Sensors SPIE Proceedings 157 (1978), pp. 21-29.
2. Coxeter, H. S. M., Introduction to Geometry (1961) (Wiley; New york), chapter 7.