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Research Statement

My current research focus, in general terms, is about inverse problems of the form $y = \tau(v)$, where one aims to recover the underlying signal v given the function τ and the measurement data y. Problems of this kind arise in signal processing, machine learning, and computer vision.

I consider such problems from two viewpoints. The first one is *algebraic-geometric*. This perhaps less-known perspective is better appreciated through an example, where τ is a matrix. Let it be recalled that v can be uniquely recovered from τ and y, provided that τ is of full column rank. With this well known fact comes the well-posedness of the problem and the mathematical wisdom of the following choice: if τ is of full column rank then solve the linear equations $y = \tau(v)$ for v via Gaussian elimination or whatever, otherwise take the solution of minimum norm, in the Euclidean or ℓ_0 sense[1]. Far more complicated than solving linear equations are multiple modern machine learning applications such as *unlabeled sensing*, *real phase retrieval*, *linear regression without correspondences*, where τ is known only up to a finite set of linear transformations. As a consequence, non-linearity and discreteness come into picture. Algebraic geometry coming into play, it was in my work (with my co-authors) that a characterization of the well-posedness for *homomorphic sensing of subspace arrangements* was established, with which the characterizations for the above-mentioned applications can be easily derived in a unified way, coinciding with the results independently established in different fields via different approaches and stated in different mathematical languages. Moreover, this work also provided a series of new conditions for sparse variants of the aforementioned applications. My future work could be establishing characterizations for other inverse problems not mentioned here.

The next one is *algorithmic*. Despite theoretical understanding, the ultimate goal should be actually obtaining a solution to the inverse problem of interest — correctly and economically. Correctness alone is rarely a concern: any kind of brute force search would do the job. For NP-hard problems it is possible to obtain solutions efficiently via solving polynomial equations and branch-and-bound; see my recent two papers on linear regression without correspondences. The issue is the scalability. An alternative route towards solving NP-hard problems, that I wish to follow in the future, is via semi-definite programming (SDP). A recent paper (see *scalable semi-definite programming* on arXiv) presented an algorithm that solves SDP at large scale. I have an interest in further developing this idea, or specializing it to solve linear programming or the linear assignment problem, or applying it to the problem of linear regression without correspondences. Finally, I am also interested in developing algorithms for non-convex optimization, which in my eyes involves developing statistical guarantees for approaches such as (sub)-gradient descent, alternating minimization, and reweighted least-squares.

[1] It is perhaps the consideration of taking the solution with minimal ℓ_0 norm cultivates what is now known as *compressed sensing*.