HW4 Problem 2

$D = \{(0, -1), (\sqrt{2}, +1\} = (x^1, y^1)$ and (x^2, y^2)

Kernel: $\phi(x) = (1, \sqrt{2}x, x^2)$

$\phi(x^1) = (1, \sqrt{2(0)}, 0^2) = (1, 0, 0)$

$\phi(x^2) = (1, \sqrt{2(2)}, \sqrt{2}^2) = (1, 2, 2)$

Want to minimize $\frac{1}{2}\|\theta\|^2$ such that

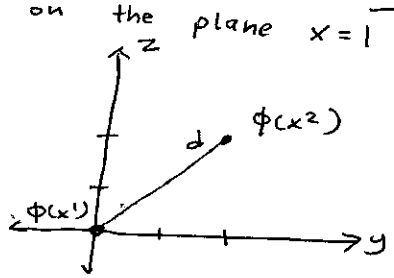
$y^1(\theta^T\phi(x^1) + \theta_0) \geq 1$

$y^2(\theta^T\phi(y^2) + \theta_0) \geq 1$

A vector parallel to the optimal vector θ is the distance vector between the two closest points of each class

$d = \phi(x^1) - \phi(x^2) = \boxed{(0, -2, -2)}$

Visualizing on the plane $x = 1$



This distance makes up the slab (2x the margin)

$\text{slab} = \sqrt{(0)^2 + (-2)^2 + (-2)^2} = \sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$

$\text{margin} = \frac{\text{slab}}{2} = \frac{2\sqrt{2}}{2} = \boxed{\sqrt{2}}$

$\text{margin} = \frac{1}{\|\theta\|} = \sqrt{2}$

$\|\theta\| = \frac{1}{\sqrt{2}} = \sqrt{\theta_1^2 + \theta_2^2 + \theta_3^2}$

$\theta_1^2 + \theta_2^2 + \theta_3^2 = \frac{1}{2}$

θ must be parallel to $d\ (0, -2, -2)$ and therefore takes on the form $(0, m, m)$

$0^2 + m^2 + m^2 = \frac{1}{2} = 2m^2$

$m = \sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

$\boxed{\theta = (0, \frac{1}{2}, \frac{1}{2})}$ or $(0, -\frac{1}{2}, -\frac{1}{2})$

using $\theta = (0, \frac{1}{2}, \frac{1}{2})$:

$y^1(\theta^T\phi(x^1) + \theta_0) \geq 1 \longrightarrow -1([0, \frac{1}{2}, \frac{1}{2}]^T[1, 0, 0] + \theta_0) \geq 1$ ①

$y^2(\theta^T\phi(x^2) + \theta_0) \geq 1 \longrightarrow +1([0, \frac{1}{2}, \frac{1}{2}]^T[1, 2, 2] + \theta_0) \geq 1$ ②

From ① $-1(0(1) + \frac{1}{2}(0) + \frac{1}{2}(0) + \theta_0) \geq 1$

$$-1(\theta_0) \geq 1$$

$$\theta_0 \leq -1$$

From ② $+1(0(1) + \frac{1}{2}(2) + \frac{1}{2}(2) + \theta_0) \geq 1$

$$+1(2 + \theta_0) \geq 1$$

$$\theta_0 \geq -1$$

$$\therefore \boxed{\theta_0 = -1}$$

Equation of Decision Boundary

$$\boxed{\theta^T x + \theta_0 = 0}$$

$$(0, \tfrac{1}{2}, \tfrac{1}{2})^T x + -1 = 0$$

$$x = (x, y, z)$$

$$0x + \tfrac{1}{2}y + \tfrac{1}{2}z = 1$$

$$\boxed{y + z = 2}$$

using $\theta = (0, -\frac{1}{2}, -\frac{1}{2})$

$$-1(0(1) - \tfrac{1}{2}(0) - \tfrac{1}{2}(0) + \theta_0) \geq 1$$

$$-\theta_0 \geq 1$$

$$\theta_0 \leq -1$$

$$+1(0(1) + -\tfrac{1}{2}(2) + -\tfrac{1}{2}(2) + \theta_0) \geq 1$$

$$+1(-2 + \theta_0) \geq 1$$

$$\theta_0 \geq 3$$

$$\therefore \theta \neq (0, -\tfrac{1}{2}, -\tfrac{1}{2})$$