Resource Allocation Among Competing Innovators*

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May 18, 2020

Abstract

Many innovative products are designed to satisfy the demand of specific target consumers, and thus innovators with new products will inevitably compete with each other in the post-innovation market. We investigate how a profit-maximizing principal should properly allocate her limited resources to support the innovations of multiple potentially competing innovators. We find that as the available resources increase, the number of agents receiving resources may first increase and then decrease. This interesting nonmonotone pattern is driven by a trade-off between the risk of innovation failure and rent dissipation due to competition. The results are robust to incorporating an endogenous profit-sharing rule and costly resources. Using the framework, we also analyze a nonprofit principal seeking to maximize the total number of successful innovations, the probability of at least one innovator succeeding, consumer surplus, and total social welfare.

Key words: innovation, competition, incentives, resource allocation

1 Introduction

Innovation plays an essential role in economic growth by creating new products, developing new business models, and improving production processes. Governments, nongovernmental organizations, and private companies invest billions of dollars and other resources every year in research and development to foster innovation. Often, there are many innovators targeting the demand of a given group of consumers. For example, several pharmaceutical companies develop their new medicines in order to cure a disease. A government funds knowledge transfer projects on industrial robots to fulfill manufacturing firms' need for automation. A venture capital firm invests in a new business model such as ride-sharing apps to reap profits from the transportation market.

*We greatly appreciate the comments and suggestions from Robin Gong, Xinyu Hua, Kohei Kawaguchi, Edwin Lai, Yang Lu, Wooyoung Lim, Albert Park, and Qinggong Wu.

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In this paper, we consider a particular dilemma faced by investors: how to invest in potentially competing innovators. If the investor invests in only a small number of startups, there is a risk of missing a "star" company. However, investing in many innovators not only requires more resources but also may intensify the competition in the post-innovation market. Igami (2017) characterizes an example in the hard disk industry. When more firms decide to innovate and develop the next generation of hard disks, each firm has a smaller market share and earns less profit. The bike-sharing market in China is also an outstanding example to illustrate the investor's predicament.[1] By 2015, 162 bike-sharing companies had opened for business across China, attracting more than CNY 2 billion of investment. These companies provided similar bike-sharing services, which led to fierce market competition that drove the price to almost zero. Following a huge wave of bankruptcies, fewer than 10 companies remained active in 2019, with most cities having fewer than three active companies.

In reality, we observe that investors may apply different investment strategies in different industries. Figure 1 illustrates part of Alibaba's investment in innovation. In the real estate sector, Alibaba invested in Shengong 007 and Easyhome, which are competing in the areas of building materials and home furnishing. In the finance sector, Alibaba invested in Ant Financial and Paytm. They do not compete directly because their main businesses are in different regions (China and India, respectively). However, Alibaba invested in only one company in each of the sports, marketing and agriculture sectors. This phenomenon motivates us to investigate the following research questions: How should one determine the optimal investment strategies among competing innovators? What are the factors affecting investors' adoption of different investment approaches?

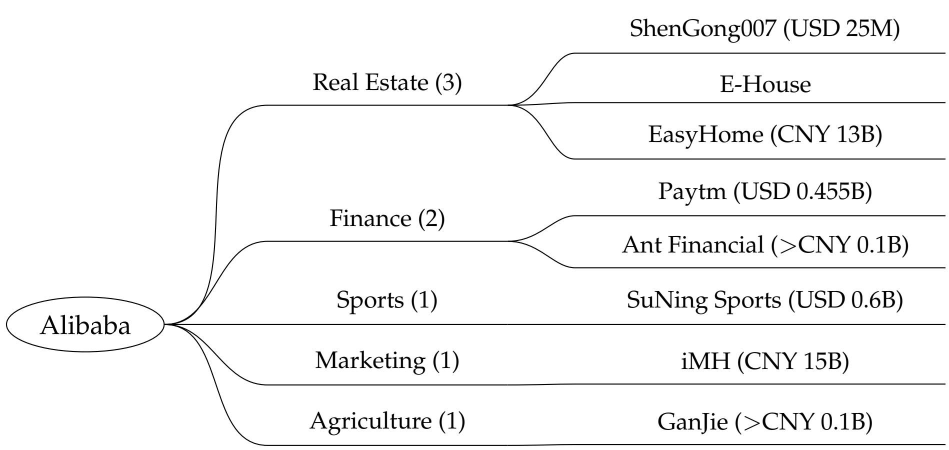


Figure 1: Alibaba Investment in Different Industries (Source: `m.gelonghui.com/p/264672`)

We explore the investor's (principal, she) optimal resource allocation strategy among multiple innovators (agents, he) who are potential competitors in the post-innovation market. The principal, who owns the essential resources for innovation, intends to maximize the investment returns

[1]See Cao et al. (2018) for a study of the Chinese bike-sharing industry.

from sharing the profit with the agents. We model the principal's resource allocation decision as a sequential game. The principal first decides how to allocate its resources to competing agents. Then, given the resource allocation, each agent decides how much effort to exert on innovative activity. The success rate of innovation depends on both the amount of resources and effort. Agents who successfully innovate will launch products in the post-innovation market. If more than one product is launched, these products will compete with one another and affect the profit.

From the agent's perspective, we find that the agent's equilibrium effort level first increases and then decreases in the amount of resources allocated. This nonmonotone relationship is rooted in the pattern of complementarity between the resources and the agent's effort in determining the success probability. When there is only a very small amount of resources, these need to be complemented by a substantial amount of effort to achieve a reasonably high success rate. As a result, the agent is reluctant to exert effort or even chooses not to participate if resources are insufficient.[2] However, when resources become excessively abundant, because of the substitutability between effort and resources, the agent tends to reduce his effort. This effort-reduction effect is even stronger if the other agent also receives resources and actively innovates. The potential competition in the post-innovation market discourages agents from exerting effort *ex ante*.

In terms of the principal's resource allocation, we identify a key trade-off between the risk of innovation failure and rent dissipation due to competition among agents. When facing uncertainty in innovation, a profit-oriented principal can lower the risk of having no successful innovation by investing in more agents. However, if multiple similar innovative products are successfully developed, the profit from the post-innovation market will shrink. This trade-off, together with the agents' strategic choices of effort levels, leads to an interesting pattern in the optimal resource allocation strategy: as the available resources increase, the principal will first invest in one agent, then switch to investing in multiple agents and finally switch back to investing in one agent. In other words, the diversification of investment is not monotone in all resources.

When there is a shortage of resources, concentrating all resources to invest in one agent will guarantee the agent's active participation in his innovative project. Dividing the resources may cause each agent to receive too little investment to be incentivized. As the resources become more abundant, diverse investment leads to an increase in the overall success rate of innovation. Each agent conducts innovation independently, so the probability of at least one agent succeeding increases. However, if diverse investment also increases the probability of more than one agent launching products on the post-innovation market, the business-stealing effect may erode the total profit. As a result, the principal tends to concentrate her investment in order to avoid *ex post* competition when resources are abundant and the competition is intense.

We derive the main results of optimal resource allocation in a baseline model with two agents. These results are robust to the cases of multiple agents, costly resources and an endogenous profit-

[2]For example, many startups need venture capital investment to carry out their innovative ideas. Researchers conduct research projects conditional on receiving sufficient grants from funding authorities. In the absence of sufficient resources, it is almost impossible for these projects to succeed.

sharing rule. However, the resource allocation pattern is slightly different if the principal is not profit-oriented. We consider that the principal seeks to maximize the number of successful innovations, the probability of at least one innovator succeeding, consumer surplus, or total social welfare. We find the principal diversify her investment as resources become abundant. The nonmonotone pattern of a profit-maximizing principal disappears. Intuitively, diversifying investment is likely to lead to more successful innovations, and having more innovative products always improves consumer and social welfare.

2 Literature Review

Since the groundbreaking work of Schumpeter (1942), a large body of literature has explored the relationship between innovation and competition. One central question is how competition affects innovators' incentives. The classical Schumpeterian view emphasizes that the essential driving force of innovation is the monopoly rent. Aghion and Howitt (1992) and Caballero and Jaffe (1993), among many others, have shown that excessive competition discourages innovation. However, empirical studies, such as Nickell (1996) and Blundell et al. (1999), found positive effects of competition on innovation, followed by several theoretical works, such as Aghion et al. (2001). Aghion et al. (2005) combines both theoretical and empirical analysis and shows that the relationship between competition and innovation exhibits an inverted U-shape. Most previous papers focus on how firms choose their innovation strategies under various market structures. This paper emphasizes that *ex ante* innovation activities are affected by potential *ex post* market competition, and the principal needs to anticipate this effect when making investment decisions.[3]

Our paper contributes to three strands of literature. First, our study sheds light on how financial resources affect operational outcomes.[4] Nelson (1961) notes the importance of parallel R&D efforts by multiple agents in uncertain innovation tasks. He suggests that resource allocation should be gradually concentrated as more information becomes available. Schlapp et al. (2015) further investigate the interaction among agents in effort exerting and information sharing. They show that the resource allocation decisions should align the incentives of agents to pursue a common goal and reveal their findings. Chao et al. (2009) show that resource allocation affects the incentives in pursuing incremental versus radical innovation projects. Ning and Babich (2017) find that firms obtaining resources by debt financing lead to risk shifting, which results in more investment in high-risk R&D projects (lower free-riding incentives). These examples demonstrate that the allocation of financial resources has strong impacts on various aspects of innovation outcomes. Most previous studies in the literature emphasize interaction among innovative agents during the pro-

[3] This is a popular operational problem faced by investors in reality. For example, Alibaba provides financial support to help retailers deal with demand uncertainty. When making the operational decision, the platform needs to consider the implications on competition intensity among retailers (Dong et al., 2018).

[4] Babich and Kouvelis (2018) provides a survey of studies that combine financial decisions with operations choices and risk management.

cess of R&D such as information sharing (Schlapp et al., 2015) and free riding (Ning and Babich, 2017). In contrast, we consider the scenario in which agents conduct independent R&D projects but their profits from the post-innovation market are correlated. We provide a clear message to funding authorities and investors that the innovation outcomes depend not only on the competition during R&D but also the competition after R&D.

Second, our model provides new insight into the literature on the design of innovation contests. Contests are a widely used mechanism to stimulate research or procure innovative products (Che and Gale, 2003). In a contest, agents exert costly effort to compete for one or several prizes granted based on the ranking of their performance. Most previous studies focus on the design of the prize scheme (Terwiesch and Xu, 2008), information disclosure policy (Zhang and Zhou, 2015; Bimpikis et al., 2019), or other contest rules to induce agents to actively participate and exert effort. Typical objectives of the contest designer (principal) include maximizing total effort (Moldovanu and Sela, 2001), the highest or average performance outcome (Hu and Wang, 2017; Ales et al., 2017; Körpeoğlu and Cho, 2018) and K-best outputs (Ales et al., 2014). In a contest, competition occurs during the process of innovation, and agents receive monetary transfers after exerting innovative effort. Our model is based on the scenario in which resources are first allocated to agents, and then the agents carry out innovation and compete for the "prize" from the post-innovation market. This setting suggests a new tool of contest design: the principal can divide her budget for post-innovation prizes and pre-innovation resources. The pre-innovation resource allocation among agents can be used to handicap some advantageous agents (Pérez-Castrillo and Wettstein, 2016), encourage participation, and maintain a proper competitive intensity.

Our study also provides a theoretical explanation of some empirical findings on investment in innovation and entrepreneurship. For example, Klingebiel and Rammer (2014) find that managers of innovation portfolios benefit from allocating resources broadly in the early stages of the development process but being more selective in later stages. This is consistent with the implications of our model. Diversifying resources at first reduces the risk of failure of all projects, while concentrating resources in the end can avoid launching too many products onto the market.

3 The Model

3.1 Model Setup

We study a one-principal-two-agents model with a resource-allocation stage followed by an innovation stage. A principal has a finite amount of resources with capacity $\overline{B} > 0$ that can be allocated to two agents indexed by $i = 1, 2$. The resources are key inputs for innovation. In the resource-allocation stage, the principal chooses a resource allocation, (b_1, b_2), subject to the budget constraint $b_1 + b_2 \leq \overline{B}$ with $b_i \geq 0$.

In the innovation stage, given (b_1, b_2), two agents simultaneously choose their effort levels (x_1, x_2). By exerting effort $x_i \geq 0$, agent i incurs a cost x_i. For each agent, the outcome of innovation is binary: either success or failure. Agent i's success rate of innovation is

$$p(b_i, x_i) = 1 - e^{-b_i x_i}, \tag{1}$$

which is increasing and concave in both b_i and x_i.[5] This functional form implies that having more resources or exerting more effort will lead to a higher probability of successful innovation, but their marginal effects are decreasing. The resource is essential for innovation. If an agent does not receive any resource, his probability of success is zero no matter how much effort he exerted.

Once an agent successfully innovates, he can launch a new product in the market. The product can be a drug, a cell phone app, or a new business model. The profit from the product is shared between the agent and the principal. The agent obtains a $\gamma \in (0,1)$ fraction of the profit. The profit-sharing rate γ is exogenously determined.[6] The agent's payoff is the expected profit from the innovative product market less his effort cost.

The principal receives the remaining $1-\gamma$ fraction of the profit from both agents. In the baseline model, the resources are not costly to the principal. This setting resembles nonprofit funding authorities in reality. The funds of research foundations such as the National Science Foundation (`www.nsf.org`) and the National Institutes of Health (`www.nih.gov`) come from government budgets and donations, so allocating more research grants does not directly incur costs. For nonprofit organizations such as university technology transfer offices and business incubators, the supporting resources (e.g., training, legal advisory, office space) typically have designated purposes (Rothaermel and Thursby, 2005). Thus, these organizations usually do not consider how to save the resources but rather how to use the resources efficiently. We consider costly resources in Section 4, which resembles the case of venture capital firms.

The innovation activities of two agents are independent, but their profits from innovative products are interdependent due to possible post-innovation competition.[7] We normalize the profit from one innovative product in the market without competitors to 1. Hence, when only one agent successfully innovates, he earns a profit of γ from the product market. If the agent fails to innovate, his profit is zero. If both agents succeed, two competing products are launched. Each agent earns a profit $\gamma\alpha$, where $\alpha \in [0,1]$ captures the intensity of competition between the two innovative products. When $\alpha = 1$, the two innovative products target two separate groups of consumers and are not competing. However, when $\alpha < 1$, the two products are substitutes and

[5] The exponential functional form can be interpreted in a dynamic way. Suppose that the innovation is conducted over a period of duration 1. The event that the innovation succeeds follows a Poisson process with a rate of $b_i x_i$. At the end of the period, the probability of success will be $1 - e^{-b_i x_i}$. The exponential form of the success rate has been widely used in the literature. See, for example, Reinganum (1983); Parra (2019). Using alternative functional forms such as $p(b_i, x_i) = 1 - e^{-\sqrt{b_i x_i}}$ and $p(b_i, x_i) = 1 - e^{-c b_i x_i}$ ($c > 0$) does not qualitatively distort our main results.

[6] In reality, the profit-sharing rate is usually determined by the funding authority's technology transfer policy or the venture capital market. We endogenize the principal's choice of γ in Section 4.

[7] Alternatively, the profit from the post-innovation market may be affected by the patent thicket problem when there are multiple innovators. Shapiro (2000) shows that having multiple complementary patents causes excessively high royalty rates and holdup problems that jeopardize the interests of all license holders. Agreement on cross-licenses or patent pools can mitigate this problem.

compete for the same group of consumers.[8] This is called rent dissipation (Choi, 1996) and the strength of this effect is measured by α. Appendix A demonstrates how values of α can be directly mapped to the institutional details regarding market competition.

In this paper, we restrict the principal to adopt a simple profit-sharing contract form. This setting resembles the practice of innovation investment in reality. The investment contracts between the funding agencies and startups typically assign certain equity shares to the investors. We do not analyze more complicated contract forms for two reasons. First, it can be extremely challenging to adopt the mechanism design framework to analyze an environment with competing agents. For this reason, we do not consider a model in which agents have private information. Second, if we grant the principal with more authority or control rights, she can simply shut down one agent's product when both agents succeed in innovation. Then, the key trade-off between the risk of innovation failure and rent dissipation would not exist.

3.2 Equilibrium Analysis of the Innovation Stage

The equilibrium analysis is conducted via backward induction. We divide the analysis into two cases: the principal investing in only one agent and investing positive amounts in both agents. In the latter case, the resources allocated to the two agents can be asymmetric, although both agents are *ex ante* symmetric.

One agent receiving resources

Suppose that the principal only invests in agent i with $b_i > 0$. By (1), the other agent lacking the essential resources has zero probability of success. If agent i successfully innovates, he receives a profit γ from the product market. Given b_i, agent i chooses his effort x_i by maximizing his expected payoff $\gamma p(b_i, x_i) - x_i$. It is easy to show that the equilibrium effort level is

$$x_i^*(b_i) = \begin{cases} 0, & \gamma b_i < 1 \\ \frac{\ln \gamma b_i}{b_i}, & \gamma b_i \geq 1. \end{cases} \tag{2}$$

The red solid curve in Figure 2 illustrates $x_i^*(b_i)$. When $\gamma b_i \leq 1$, the profit share and the allocated resources are too small to encourage the agent to exert costly effort on innovation. Only when $\gamma b_i > 1$ does exerting positive effort lead to a positive expected payoff for the agent. Note that when $\gamma b_i \geq 1$, the equilibrium effort level x^* follows a hump shape in b_i. Intuitively, when a small amount of resources is allocated, the agent does not have a strong incentive to exert effort because the probability of success is low. However, when abundant resources are allocated, the agent can rely on these resources and need not exert much effort. Note that $x_i^*(\cdot)$ reaches its

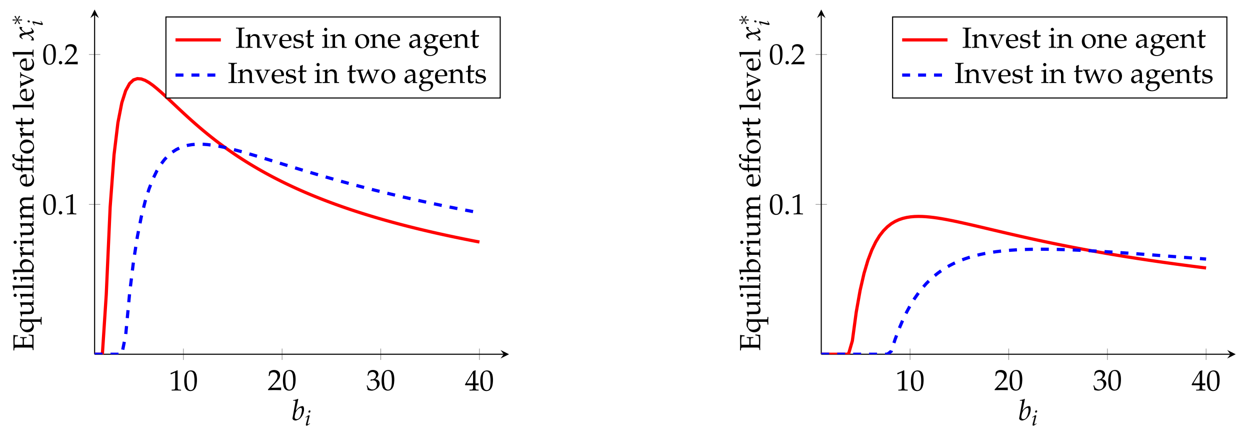
[8] For example, two drugs are developed for the same disease, or two bike-sharing companies launch similar mobile apps and overlapping bike networks.

maximum at $b_i = \frac{e}{\gamma}$. This means that if the agent's profit share γ is higher, the maximal effort can be induced by investing less resources.

Although the effort level is nonmonotone in b_i, the equilibrium success rate,

$$p_i(b_i, x_i^*(b_i)) = 1 - e^{-b_i x_i^*} = \begin{cases} 0 & \gamma b_i < 1, \\ 1 - \frac{1}{\gamma b_i} & \gamma b_i \geq 1, \end{cases} \tag{3}$$

is increasing in b_i. Therefore, investing more resources stimulates innovation.



(a) $\gamma = 0.5$

(b) $\gamma = 0.25$

Figure 2: Agent's Equilibrium Effort Level $x_i^*(b_i)$ With $\alpha = 0.6$.

Both agents receiving resources

Given $b_1 > 0$ and $b_2 > 0$, the equilibrium effort levels, $(x_1^*(b_1, b_2), x_2^*(b_1, b_2))$, are determined by a static game between two agents. Let $y_i \equiv 1 - p(b_i, x_i)$ denote the failure probability; then, $x_i = -\frac{\ln y_i}{b_i}$. Because there is a one-to-one correspondence between x_i and y_i, it is equivalent to considering the agent choosing y_i instead of x_i. This transformation simplifies the algebra in the equilibrium analysis. We restrict $y_i > 0$ because achieving a zero failure probability ($y_i = 0$) requires an infinitely large effort. The boundary condition, $y_i = 1$, indicates that an agent exerts zero effort and thus has zero probability of success. Without loss of generality, let $b_1 \geq b_2$, and hence in equilibrium, $y_1^* \leq y_2^*$.

Given resources b_i and the other agent's failure probability y_{-i}, agent i's best response is determined by solving the following problem:

$$\begin{aligned} Y(b_i, y_{-i}) :=& \text{argmax}_{1 \geq y_i > 0} \{\gamma(1 - y_i)(\alpha(1 - y_{-i}) + y_{-i}) + \frac{\ln y_i}{b_i}\} \\ =& \text{argmax}_{1 \geq y_i > 0} \left\{1 - y_i + \frac{\ln y_i}{\gamma[\alpha(1 - y_{-i}) + y_{-i}]b_i}\right\}. \end{aligned} \tag{4}$$

We first consider the case of an interior solution in which both agents exert positive effort. Lemma 1 characterizes the condition under which both agents exert positive effort.

Lemma 1. *The sufficient and necessary condition for* $0 < y_1^* \le y_2^* < 1$ *is*

$$\gamma b_2 > 1 \quad \textit{and} \quad 1 \le \frac{b_1}{b_2} < 1 + \alpha(\gamma b_1 - 1). \tag{5}$$

Under this condition, y_1^* *and* y_2^* *are unique.*

To encourage both agents to actively participate in innovation, the amount of resources allocated to each agent cannot be too low ($\gamma b_2 > 1$). Moreover, the allocation cannot be too imbalanced ($\frac{b_1}{b_2} < 1 + \alpha(\gamma b_1 - 1)$). Otherwise, the business-stealing effect will discourage the less resourceful agent from exerting any effort because the expected profit generated from the post-innovation market cannot compensate for his effort cost.

Under condition (5), the equilibrium failure probabilities, y_1^* and y_2^*, are determined by the first-order conditions (FOC) of (4), which can be rearranged as

$$\frac{1}{\gamma b_1} = \alpha(1 - y_2^*)y_1^* + y_1^* y_2^* \quad \text{and} \quad \frac{1}{\gamma b_2} = \alpha(1 - y_1^*)y_2^* + y_1^* y_2^*. \tag{6}$$

The solution, $y_1^*(b_1, b_2)$ and $y_2^*(b_1, b_2)$, exhibits the following comparative statistics:

Lemma 2. *Given condition (5),* y_i^* *decreases in* b_i *and increases in* b_{-i}.

The agent's failure probability decreases in the amount of his own resources but increases in that of the other agent. Intuitively, as an agent obtains more resources, his success probability will increase. The other agent's motivation to exert effort decreases because he is more likely to face post-innovation competition.

There is a qualitative difference between the pattern of effort exertion in the cases of investing in one and two agents. When investing in only one agent, a smaller amount of resources always leads to a lower probability of successful innovation. However, in the latter case, because reducing b_i will motivate agent $-i$ to work harder, reducing all resources allocated to two agents may weaken competition and better incentivize the agents. Therefore, the principal needs to carefully decide how to allocate resources when faced with multiple agents with possible post-innovation competition.

3.3 The Optimal Resource Allocation Strategy

Given a resource capacity, $\overline{B}$, how should the principal allocate her resources to the two agents? Based on the equilibrium failure probability in (4), the principal's optimization problem is

$$\begin{aligned} \Pi(\overline{B}) := \max_{b_1 \ge 0, b_2 \ge 0} \ & (1-\gamma)[2\alpha(1-y_1^*)(1-y_2^*) + (1-y_1^*)y_2^* + (1-y_2^*)y_1^*] \\ \text{s.t.} \ & b_1 + b_2 \le \overline{B}, \ y_1^* = Y(b_1, y_2^*), \ y_2^* = Y(b_2, y_1^*). \end{aligned} \tag{7}$$

The term $2\alpha(1-y_1^*)(1-y_2^*)$ is the total expected profit when both agents successfully innovate. This total profit decreases in α.

We start the analysis by first fixing a total amount of investment $B = b_1 + b_2 \leq \overline{B}$ with $b_1 \geq b_2$. Given B, the principal's investment decision degenerates to being one dimensional. Without loss of generality, let b_2 be the principal's choice variable. The principal's problem can be rewritten as

$$\max_{b_2 \in [0, \frac{B}{2}]} \tilde{\Pi}(B-b_2, b_2) := (1-\gamma)[2\alpha(1-y_1^*)(1-y_2^*) + (1-y_1^*)y_2^* + (1-y_2^*)y_1^*].$$

We can show that the solution satisfies the following proposition.

Proposition 1. *The optimal resource allocation scheme must exhibit one of the following properties: either* $b_2^* = 0$ *or* $b_1^* = b_2^*$.

Figure 3 plots the expected profit of each agent and the total profit of two agents as functions of b_2/B. From panels (a) and (c), we see that $\tilde{\Pi}(B-b_2, b_2)$ is quasiconvex on the domain $b_2 \in [0, \frac{B}{2}]$. The total profit reaches its maximum at either the left boundary as in panel (a) or at the right boundary as in panel (c). Therefore, under the proportional profit-sharing rule, the principal will use either a symmetric or highly asymmetric investment strategy, namely, investing in only one agent or investing an equal amount in two agents. As a result, the ratio between the amounts of resources allocated to two agents does not change continuously when the parameters change. At the breakpoints, the ratio can suddenly change from 0 to 1 or from 1 to 0. Based on Proposition 1, we compute and compare the principal's profits under the two investment strategies.

Investing in one agent

Given a fixed total investment $B > 0$, let $\Pi_m(B)$ denote the profit from investing in one agent. The subscript m indicates that the resource allocation strategy induces a monopoly product in the post-innovation market. By (3), the principal's profit is

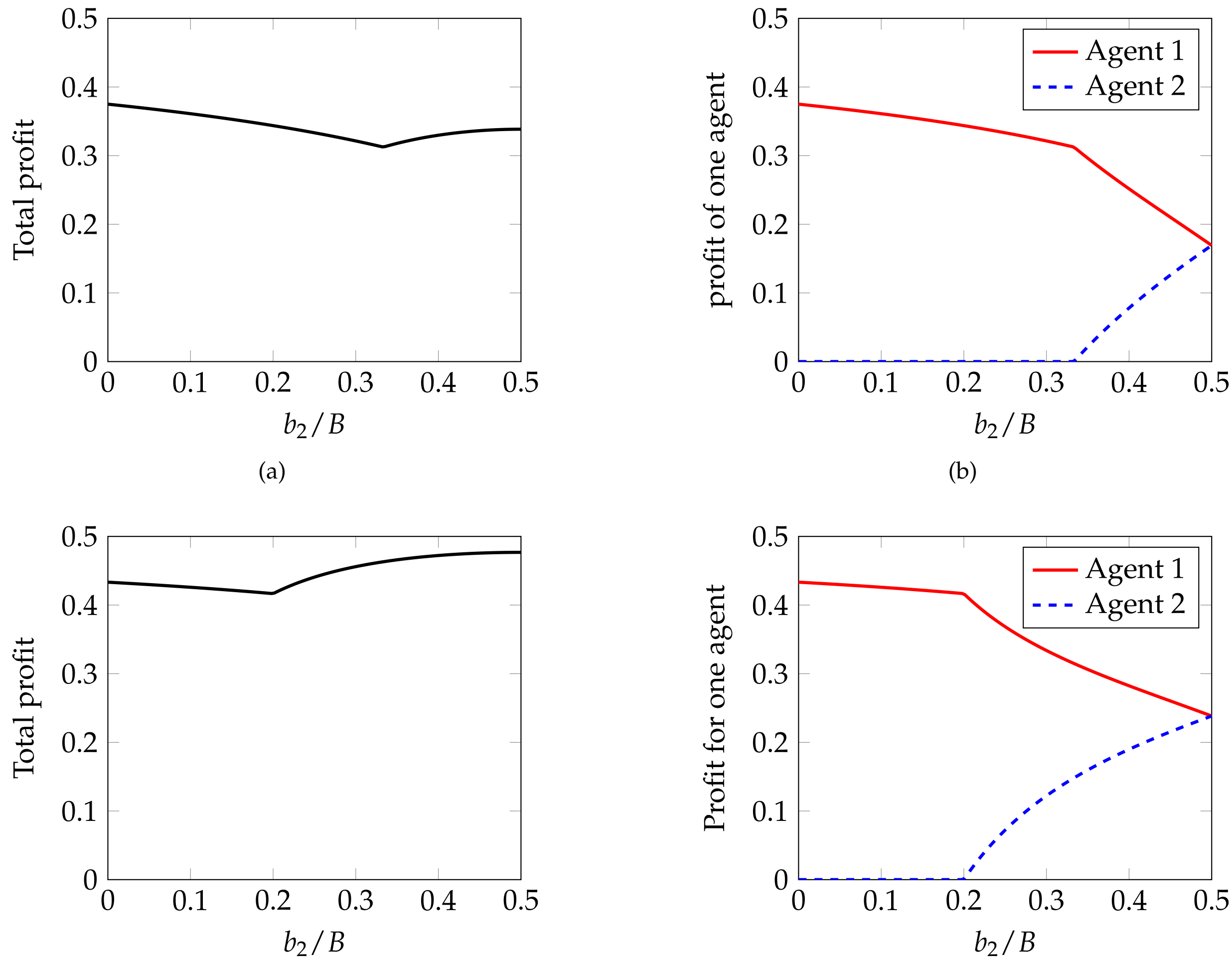
$$\Pi_m(B) = \begin{cases} 0 & \gamma B \leq 1, \\ (1-\gamma)(1-\frac{1}{\gamma B}) & \gamma B > 1. \end{cases} \tag{8}$$

Because $\Pi_m(B)$ increases in B, the principal will exhaust her resources. We reach the following result.

Proposition 2. *When the principal only invests in one agent, the principal chooses* $(b_1, b_2) = (\overline{B}, 0)$.

Accordingly, the principal's expected payoff when investing in one agent is $\Pi_m(\overline{B})$ and the expected payoff of the agent who receives investment is

$$U_m(\overline{B}) = \begin{cases} 0 & \gamma\overline{B} < 1, \\ \gamma - \frac{1+\ln\gamma\overline{B}}{\overline{B}} & \gamma\overline{B} \geq 1. \end{cases} \tag{9}$$



(a) (b) (c) (d)

Note: We set $\gamma = 0.5$ and $\alpha = 0.6$. For panels (a) and (b), $B = 8$. For panel (c) and (d), $B = 15$.

Figure 3: Illustration of Proposition 1

U_m is also nondecreasing in $\overline{B}$. Thus, both the principal and the agent who receives investment are always better off under a larger resource endowment.

Investing in two agents

By Proposition 1, we only need to consider the case in which the principal allocates an equal amount of resources to each agent, i.e., $b_1 = b_2 = B/2$. By Lemma 1, when $\gamma B > 2$, each of the two agents will exert positive effort. By (6), the equilibrium failure probability of each agent is

$$y_1^* = y_2^* = y^* := \frac{4}{\alpha B\gamma + \sqrt{B\gamma}\sqrt{8 - 8\alpha + \alpha^2 B\gamma}}. \tag{10}$$

The subscript d indicates that the resource allocation strategy may induce a duopoly post-innovation market structure. The principal's expected profit is

$$\Pi_d(B) = (1-\gamma)(2\alpha(1-y^*)^2 + 2y^*(1-y^*))$$

$$= \begin{cases} 0, & \gamma B \le 2, \\ (1-\gamma)(\alpha - \frac{4}{\gamma B} + \sqrt{\frac{\gamma\alpha^2 B - 8\alpha + 8}{\gamma B}}), & \gamma B > 2. \end{cases} \tag{11}$$

The expected payoff of each agent is

$$U_d(B) = \gamma(1-y^*)(\alpha(1-y^*) + y^*) + \frac{\ln y_*}{B/2}$$

$$= \begin{cases} 0, & \gamma B \le 2, \\ \frac{1}{2B}\left[\alpha B\gamma + \sqrt{B\gamma}\sqrt{\gamma\alpha^2 B - 8\alpha + 8} + 4\ln\frac{4}{\alpha B\gamma + \sqrt{B\gamma}\sqrt{8-8\alpha+\alpha^2 B\gamma}} - 4\right], & \gamma B > 2. \end{cases} \tag{12}$$

The following lemma summarizes the properties of y^*, $\Pi_d(B)$, and $U_d(B)$.

Lemma 3. *If the principal adopts the equal investment strategy, $b_1 = b_2 = B/2$, and $\gamma B > 2$, then*

(i) The equilibrium failure probability of each agent, $y^*(B)$, decreases in B, while the equilibrium effort levels $x^*(B)$ first increase and then decrease in B.

(ii) The agent's expected payoff, $U_d(B)$, is always increasing in $[\frac{2}{\gamma}, B_*]$, where $B_* = \underset{B}{\operatorname{argmax}}\, \Pi_d(B)$.

(iii) When $\alpha \ge 1/2$, the principal's profit $\Pi_d(\cdot)$ always increases in B. When $\alpha < 1/2$, the principal's expected profit first increases and then decreases in B. It reaches the maximum at $B = \frac{8(1-\alpha)}{\gamma(1-2\alpha)}$.

Figure 2 illustrates Lemma 3-(i). The blue dashed curve is the equilibrium effort level of one agent when two agents obtain the same amount of resources. It has a similar humped shape but it is higher than the red curve when b_i is sufficiently large. That is, when resources are abundant, competition may be beneficial to induce agents to exert effort.

Lemma 3-(ii) states that an agent's expected payoff increases in the available resources when $B \le B_*$. Intuitively, the principal's interest is aligned with those of the agents, so it must be suboptimal for the principal to adopt an investment strategy that hurts the agents.

Lemma 3-(iii) considers two cases. When $\alpha \ge 1/2$, the business-stealing effect is weak. The total profits from the post-innovation market with two successful agents are larger than that with only one successful agent ($2\alpha \ge 1$). Therefore, $\Pi_d(B)$ is always increasing in B. In contrast, when $\alpha < 1/2$, rent dissipation occurs, so $\Pi_d(B)$ decreases in B when B becomes large. Therefore, investing too many resources can backfire for the principal even though the investments are not costly per se.

Mathematically, with equal investments, the principal's profit is proportional to $2\alpha(1-y^*)^2 + 2y^*(1-y^*)$, which is maximized at $y^* = \max\{0, \frac{1-2\alpha}{2-2\alpha}\}$. Therefore, when $\alpha < 1/2$, the principal is better off ensuring that the failure probability of each agent is not smaller than $\frac{1-2\alpha}{2-2\alpha}$. Because y^* decreases in B, the principal may be better off not allocating all her resources.

Comparison between two strategies

Now, we compare the principal's expected profit in (8) and (11) under the two investment strategies. Because $\Pi_d(B) = 0$ when $\gamma B < 2$, we will focus on the case in which $\gamma B \geq 2$.

Proposition 3. *$\Pi_m(B)$ and $\Pi_d(B)$ exhibit the following properties:*

(i) $\max\{\Pi_m(B), \Pi_d(B)\}$ *is increasing in B.*

(ii) $\Pi_m(B) - \Pi_d(B)$ *is quasiconvex in $B \in [\frac{2}{\gamma}, \infty)$.*

Proposition 3-(i) implies that the principal will always exhaust her resources, i.e., $b_1^* + b_2^* = \overline{B}$. Note that as $B \to 2/\gamma$, $\Pi_d(B) \to 0$, so when B is low, $\Pi_m(B) - \Pi_d(B) > 0$ and the principal invests in only one agent. Proposition 3-(ii) implies that when $B \geq 2/\gamma$, $\Pi_m(B) - \Pi_d(B)$ may first decrease and become negative (depending on α and γ); thus, investing in two agents may become optimal. The quasiconvexity implies that there only exists at most one continuous region of $\overline{B}$ where investing in two agents is optimal.

Based on the results above, we obtain the optimal resource allocation strategy as follows.

Theorem 1. *Given α, γ, and $\overline{B}$, the optimal resource allocation strategy is as follows:*

(i) If $\gamma\overline{B} \leq 1$, the principal invests nothing and obtains zero profit, i.e., $(b_1^*, b_2^*) = (0, 0)$.

(ii) If $1 < \gamma\overline{B} \leq 2$, the principal invests all of her resources in one agent, i.e., $(b_1^*, b_2^*) = (\overline{B}, 0)$.

(iii) If $\gamma\overline{B} > 2$, the principal uses all of her resources. As shown in Figure 4,

a. When $\alpha \leq 6\sqrt{2} - 8$ (region I), the principal invests in one agent, i.e., $(b_1^*, b_2^*) = (\overline{B}, 0)$.

b. When $6\sqrt{2} - 8 < \alpha < 1/2$ (region II) and $\gamma\overline{B} \in [\frac{1-\alpha-\sqrt{\alpha^2+16\alpha-8}}{1-2\alpha}, \frac{1-\alpha+\sqrt{\alpha^2+16\alpha-8}}{1-2\alpha}]$ (shaded area), the principal invests equally in two agents, i.e., $(b_1^*, b_2^*) = (\overline{B}/2, \overline{B}/2)$; otherwise the principal invests in one agent, i.e., $(b_1^*, b_2^*) = (\overline{B}, 0)$.

c. When $\alpha \geq 1/2$ (region III) and $\gamma\overline{B} \geq \frac{1-\alpha-\sqrt{\alpha^2+16\alpha-8}}{1-2\alpha}$ (shaded area), the optimal allocation strategy is $(b_1^*, b_2^*) = (\overline{B}/2, \overline{B}/2)$; otherwise, it should be $(b_1^*, b_2^*) = (\overline{B}, 0)$.

The term $\gamma\overline{B}$ measures the abundance of resources from the agents' perspective. Intuitively, when $\gamma\overline{B}$ is too small ($\gamma\overline{B} \leq 1$), even if an agent obtains all the available resources, the effort cost still outweighs the potential benefit. Hence, the agent will exert zero effort; thus, the principal will hold her resources. This is the situation depicted in Theorem 1-(i). When there are a slightly more resources ($1 < \gamma\overline{B} \leq 2$), Theorem 1-(ii) shows that the principal can earn a positive profit by granting the resources to a single agent. The resources are still not sufficient for diversification

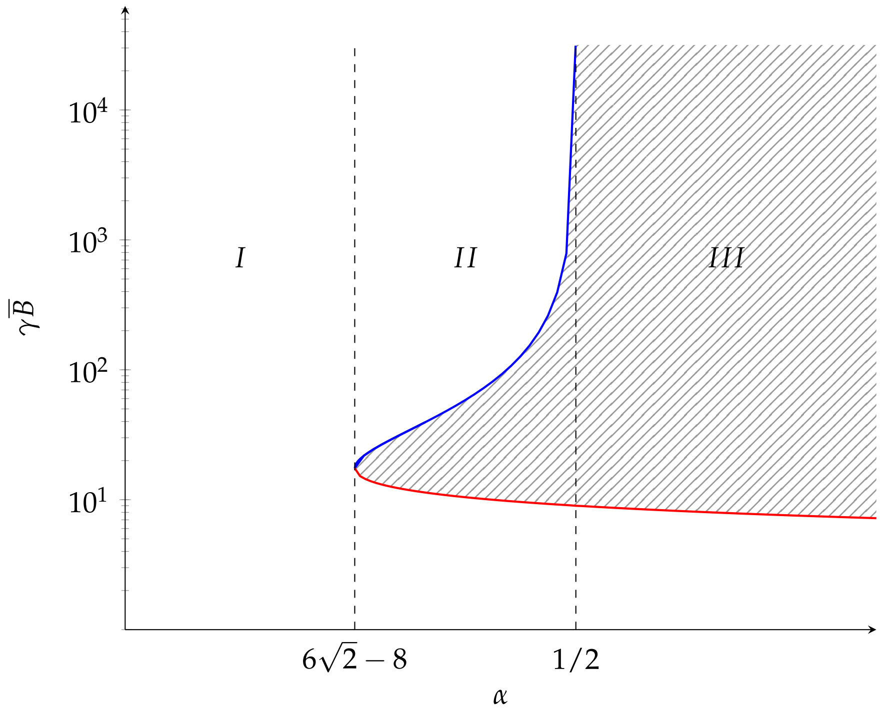


Figure 4: Principal's Optimal Resource Allocation Rule ($\gamma\bar{B} \geq 2$)

because investing in two agents will lead to insufficient resources to incentivize any one of them to exert effort (Lemma 1).

When the amount of resources is sufficiently large ($\gamma\overline{B} > 2$), the key trade-off between the risk of innovation failure and rent dissipation emerges. Figure 5 shows that, for large $\overline{B}$, the probability that at least one agent succeeds ($1 - y_1^* y_2^*$) is higher than that of investing in one agent ($1 - y_1^*$). Hence, diverse investment reduces the risk of having no product in the market. However, diverse investment also leads to a higher probability of both agent succeeding ($(1 - y_1*)(1 - y_2^*)$), which leads to dissipation of profit. The interplay between innovation success rate and the rent dissipation plays an essential role in determining the optimal resource allocation strategy.

The strength of rent dissipation effect is determined by the competition intensity α. When competition is extremely intense (Theorem 1-(iii)-a), the potentially fierce product market competition between the two successful agents will lead to very low profits. As a result, the principal cares about the success rate of one agent since it generates the highest profit ($1 > 2\alpha$). From the dotted line ($2(1 - y_1^*)y_2^*$) in Figure 5, we observe that when resources are abundant, one agent's success rate is extremely low under the strategy of investing in two agents. That is to say, diverse investment not only causes profit reduction but also leads to the success rate of one agent being significantly low by discouraging agents from exerting effort. Hence, the principal will invest in only one agent.

Theorem 1-(iii)-c characterizes the case in which the total profits from two competing firms are

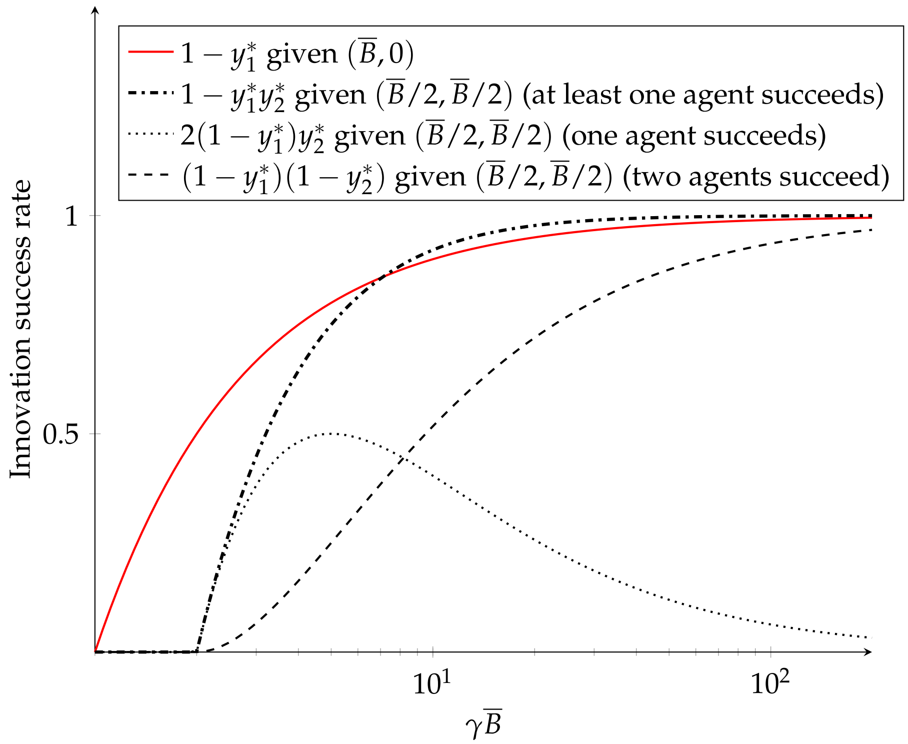


Figure 5: Success rates under two investment strategies

comparable to or greater than the profit from one successful agent. When resources are sufficiently abundant, diverse investment not only improves the potential total profit from innovation but also reduces the risk of having no product in the market. Accordingly, the principal will invest in two agents when $\overline{B}$ is large.

Theorem 1-(iii)-b demonstrates the most interesting result when the competition intensity is at an intermediate level. As $\overline{B}$ increases, the optimal resource allocation rule switches from investing in one agent to investing in two agents, and then back to investing in one agent. The combined profit from two successful agents is lower than that from one. Figure 5 shows that, the difference between $1 - y_1^* y_2^*$ and $1 - y_1^*$ converges to zero when $\overline{B}$ becomes large, which suggests that the benefit from diverse investment decreases in $\overline{B}$. Consequently, when resources are limited, the benefit from innovation risk reduction under the diverse investment dominates the profit loss. However, when the resources become abundant, the benefit from innovation risk reduction under the diverse investment shrinks and is dominated by the potential profit loss since $2\alpha < 1$. Therefore, when resources are abundant, the principal is willing to switch from investing in two agents back to investing in one agent in order to avoid the profit loss from competition by sacrificing a small amount of success rate.

4 Extensions

4.1 Endogenous Profit Sharing

In the baseline model, we assume that the principal will charge an exogenously fixed share $1-\gamma$ of the profit. In addition, our analysis shows that given γ, when the amount of resources is larger than $1/\gamma$, the principal should exhaust her resources by investing all of them in one agent or investing an equal amount in each of two agents. Because the resources are essential for successful innovation, the principal may have the power to strategically determine the profit-sharing rate γ.

In this extension, we let the principal endogenously choose a profit-sharing rate γ in addition to the resource allocation. Given any γ, the optimal resource allocation will still follow Theorem 1. Similar to the analysis of the baseline model, we first separately investigate the case of investing in one agent and the case of investing in two agents, and then we compare the profits between these two cases to derive the optimal resource allocation rule.

Based on Theorem 1, we know that given γ, when $\overline{B} \geq \frac{1}{\gamma}$ and the principal invests in only one agent, the principal should grant all resources $\overline{B}$ to this agent. The agent's equilibrium failure probability y_i^* is determined by maximizing his expected payoff function with $b_i = \overline{B}$:

$$\max_{1 \geq y_i \geq 0} \gamma(1-y_i) + \frac{\ln y_i}{\overline{B}}.$$

We can easily show that $y_i^* = \min\{\frac{1}{\overline{B}\gamma}, 1\}$. Therefore, the principal's problem is

$$\max_{\gamma}(1-\gamma)(1-\min\{\frac{1}{\overline{B}\gamma}, 1\}).$$

Proposition 4. *Given $\overline{B} > 1$, when the principal only invests in one agent, the optimal profit-sharing rate is $\gamma^* = \frac{1}{\sqrt{\overline{B}}}$. Accordingly, the principal's expected profit is $(1-\frac{1}{\sqrt{\overline{B}}})^2$, and the agent's expected payoff is $\frac{-1+\sqrt{\overline{B}}-\ln \overline{B}/2}{\overline{B}}$.*

Note that the optimal profit-sharing rate γ^* decreases in the total amount of available resources $\overline{B}$. In other words, as $\overline{B}$ increases, she will extract an increasing share of profits. Intuitively, when granting an agent more resources, the agent exerts less effort (Figure 2) and mainly relies on the abundant resources to conduct innovation. Hence, the principal deserves to enjoy a larger share of the total profit due to her significant contribution to the success of innovation.

Furthermore, observing the principal's and the agent's expected profits shown in Proposition 4, we find that when the principal invests in one agent, the agent's expected payoff first increases and then decreases in $\overline{B}$, while the investor's payoff is always increasing in it. This result is different from the case of exogenous profit sharing where both the principal and agent's expected payoffs are increasing in $\overline{B}$. This is because, in the endogenous case, the share of the profits owned by the agent is decreasing in $\overline{B}$. Therefore, when $\overline{B}$ is high enough, although the total expected profit $1 - \frac{1}{\overline{B}\gamma}$ increases in $\overline{B}$, the agent's expected profit is lower due to his small share γ.

Next, by Theorem 1, if the principal finds it optimal to invest in two agents, the resource allocation will be $(\overline{B}/2, \overline{B}/2)$. Applying a similar analysis as in Section 3.2, we obtain that the equilibrium failure probabilities are

$$y_1^* = y_2^* = y^* = \begin{cases} 1, & \gamma\overline{B} \leq 2 \\ \frac{4}{\alpha\overline{B}\gamma + \sqrt{\overline{B}\gamma}\sqrt{8-8\alpha+\alpha^2\overline{B}\gamma}}, & \gamma\overline{B} > 2. \end{cases} \tag{13}$$

The principal's optimization problem is

$$\max_{1>\gamma>0} (1-\gamma)[2\alpha(1-y^*)^2 + 2(1-y^*)y^*]. \tag{14}$$

When $\overline{B} \leq 2$, an agent is unwilling to exert positive effort; hence, the innovation activity fails for sure, i.e., $y^* = 1$. Then, the principal always obtains a zero payoff when $\overline{B} \leq 2$, which is independent of the profit-sharing rule γ. Having understood the case of $\overline{B} \leq 2$, we now focus on the case of $\overline{B} > 2$. By solving the optimization problem in (14), we have

Proposition 5. *When $\overline{B} > 2$ and the principal exhausts her resources and invests equally in two agents, the principal's profit is quasiconcave in $\gamma \in (0,1)$. Therefore, the optimal sharing rule γ^* uniquely exists. Furthermore, we find that it is decreasing in $\overline{B}$ and increasing in α.*

The explanation for γ^*'s monotonicity on $\overline{B}$ is the same as that for the case of investing in one agent. The intuition for the monotonicity on α is as follows. The total expected profit generated from two agents increases in α. Therefore, to encourage each agent to exert more effort, the principal will set a more generous profit-sharing rate. Interestingly, in the case of endogenous profit sharing, the principal will always use up resources when investing in two agents equally. This is different from Lemma 3 in which the principal may retain some resources to avoid fierce post-innovation competition. Based on (13), we know that given γB, the failure probability y^* is fixed. Therefore, when the available $\overline{B}$ increases, the principal can always increase her profit by reducing the sharing rule γ and increasing the resource input B to keep γB unchanged. Hence, it is optimal for the principal to exhaust her resources under endogenous profit sharing.

Based on Propositions 4 and 5, we reach the following result:

Theorem 2. *In the case of endogenous profit sharing, the optimal resource allocation rule is as follows:*

(i) When $\alpha \leq 6\sqrt{2} - 8$, the principal always invests in one agent, i.e., $(b_1^*, b_2^*) = (\overline{B}, 0)$, and the optimal sharing rule is $\gamma^* = \frac{1}{\sqrt{\overline{B}}}$.

(ii) When $6\sqrt{2} - 8 < \alpha < 1/2$, as $\overline{B}$ increases, the investor will switch from investing in one agent to investing equally in two agents, and then back to investing in one.

(iii) When $\alpha \geq 1/2$, as $\overline{B}$ increases, the investor will first invest in one agent and then invest equally in two agents.

Therefore, when the principal has the power to determine the "tax" rate, the optimal allocation rule is qualitatively similar to that under exogenous profit sharing.

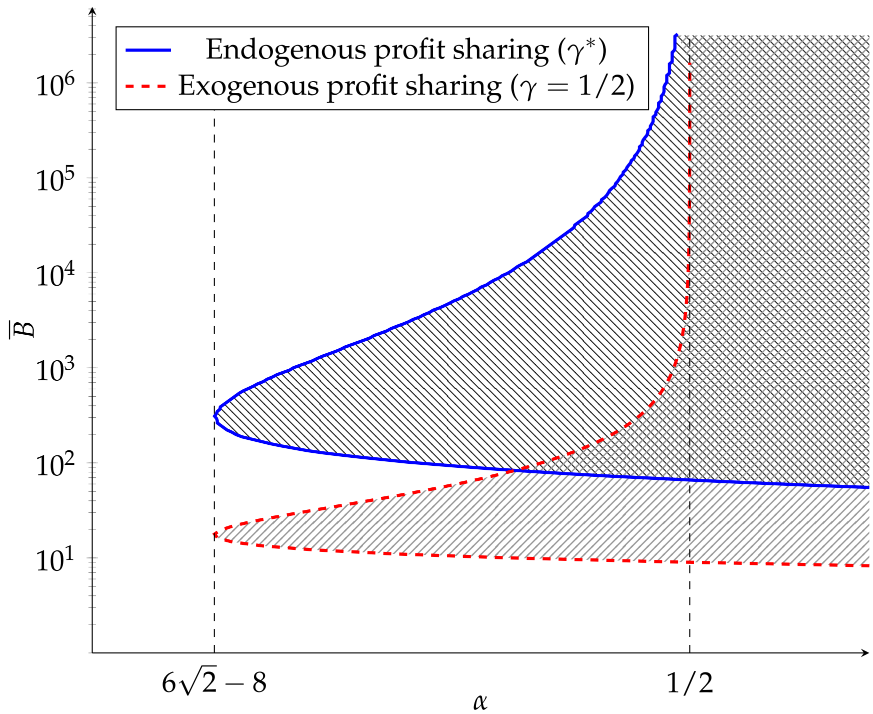


Figure 6: Optimal Resource Allocation with Endogenous and Exogenous γ.

4.2 Nonprofit Principal

Many research grants and entrepreneurship supporting funds are provided by government and nonprofit organizations. These funding authorities may have objectives other than profit maximization. In this extension, we consider four nonprofit objectives: the total number of successful innovations, the probability that at least one innovation succeeds, consumer surplus, and total welfare. Because the principal is not profit-oriented, we set $\gamma = 1$, so agents obtain all profits from sales.

Number of successful innovations

Suppose that the principal seeks to maximize the expected total number of successful innovations. The principal's problem is

$$N(\overline{B}) := \max_{b_1 \geq 0, b_2 \geq 0} \; 1 - y_1^* + 1 - y_2^* \quad \text{s.t.} \quad b_1 + b_2 \leq \overline{B},\; y_1^* = Y(b_1, y_2^*),\; y_2^* = Y(b_2, y_1^*). \tag{15}$$

The optimal resource allocation has the following property:

Proposition 6. *To maximize $N(\overline{B})$, there exists a threshold $B_{\#1}(\alpha)$ such that when $\overline{B} \leq B_{\#1}(\alpha)$, the principal's optimal resource allocation is $(b_1^*, b_2^*) = (\overline{B}, 0)$; when $\overline{B} > B_{\#1}(\alpha)$, the principal's optimal resource allocation is $(b_1^*, b_2^*) = (\overline{B}/2, \overline{B}/2)$. Moreover, $B_{\#1}(\alpha)$ is decreasing in α.*

Probability that at least one innovator succeeds

Suppose that the principal seeks to maximize the probability that at least one innovator succeeds. The principal's problem is

$$P(\overline{B}) := \max_{b_1 \geq 0, b_2 \geq 0} \; 1 - y_1^* y_2^* \quad \text{s.t.} \quad b_1 + b_2 \leq \overline{B},\; y_1^* = Y(b_1, y_2^*),\; y_2^* = Y(b_2, y_1^*). \tag{16}$$

The following proposition describes the solution to (16).

Proposition 7. *To maximize $P(\overline{B})$, there exists a threshold $B_{\#2}(\alpha)$ such that when $\overline{B} \leq B_{\#2}(\alpha)$, the principal's optimal resource allocation is $(b_1^*, b_2^*) = (\overline{B}, 0)$; when $\overline{B} > B_{\#2}(\alpha)$, the principal's optimal resource allocation is $(b_1^*, b_2^*) = (\overline{B}/2, \overline{B}/2)$. Moreover, $B_{\#2}(\alpha)$ is decreasing in α.*

Consumer surplus

Now assume that the principal seeks to maximize consumer surplus. Suppose that the post-innovation market is characterized by the quantity competition model in Appendix A. In line with Vives (1984), consumer surplus is

$$CS(q_1, q_2, p_1, p_2) = A(q_1 + q_2) - (q_1^2 + 2gq_1q_2 + q_2^2)/2 - p_1q_1 - p_2q_2,$$

where $g \in (0,1)$ measures the intensity of competition, q_i is the quantity supplied by agent i, and $p_i = A - gq_{-i} - q_i$ is the equilibrium price of product i. Note that $q_i = 0$ if agent i fails at innovation. We set $A = 2$. In this case, the profit for a monopolistic product is 1, and for two successfully launched products, each agent earns $\alpha = \frac{4}{(2+g)^2}$.

When the market has only one monopolistic agent, the equilibrium quantity is $\frac{A}{2} = 1$, the equilibrium price is 1, and consumer surplus is $\frac{1}{2}$. When both agents successfully launch products, the equilibrium quantity of each agent is $\frac{A}{2+g} = \frac{2}{2+g}$, the equilibrium price is $\frac{2}{2+g}$, and the consumer surplus is $\frac{4(1+g)}{(2+g)^2}$. Therefore, the problem of the principal can be formulated as follows

$$\begin{aligned} CS(\overline{B}) := \max_{b_1 \geq 0, b_2 \geq 0} \ & \frac{4(1+g)}{(2+g)^2}(1 - y_1^* y_2^*) + \frac{1}{2}[(1-y_1^*)y_2^* + (1-y_2^*)y_1^*] \\ \text{s.t.} \ & b_1 + b_2 \leq \overline{B},\ y_1^* = Y(b_1, y_2^*),\ y_2^* = Y(b_2, y_1^*). \end{aligned} \tag{17}$$

The following proposition characterizes the solution to (17):

Proposition 8. *To maximize $CS(\overline{B})$, there exists a threshold $B_{\#3}(\alpha)$ such that when $\overline{B} \leq B_{\#3}(\alpha)$, the principal's optimal resource allocation is $(b_1^*, b_2^*) = (\overline{B}, 0)$; when $\overline{B} > B_{\#3}(\alpha)$, the principal's optimal resource allocation is $(b_1^*, b_2^*) = (\overline{B}/2, \overline{B}/2)$. Moreover, $B_{\#3}(\alpha)$ is decreasing in α.*

Total welfare

Using the same quantity competition model as above, when there is one successful innovation, the total welfare (profit plus consumer surplus) is $1 + \frac{1}{2} = \frac{3}{2}$; when there are two, the welfare is $2\alpha + \frac{4(1+g)}{(2+g)^2} = \frac{4(3+g)}{(2+g)^2}$. To maximize the total welfare, the principal solves

$$\begin{aligned} W(\overline{B}) := \max_{b_1 \geq 0, b_2 \geq 0} \ & \frac{4(3+g)}{(2+g)^2}(1 - y_1^* y_2^*) + \frac{3}{2}[(1-y_1^*)y_2^* + (1-y_2^*)y_1^*] \\ \text{s.t.} \ & b_1 + b_2 \leq \overline{B},\ y_1^* = Y(b_1, y_2^*),\ y_2^* = Y(b_2, y_1^*). \end{aligned} \tag{18}$$

The optimal resource allocation strategy is as follows.

Proposition 9. *To maximize $W(\overline{B})$, there exists a threshold $B_{\#4}(\alpha)$ such that when $\overline{B} \leq B_{\#4}(\alpha)$, the principal's optimal resource allocation is $(b_1^*, b_2^*) = (\overline{B}, 0)$; when $\overline{B} > B_{\#4}(\alpha)$, the principal's optimal resource allocation is $(b_1^*, b_2^*) = (\overline{B}/2, \overline{B}/2)$. Moreover, $B_{\#4}(\alpha)$ is decreasing in α.*

Propositions 6, 7, 8, and 9 share some common features. First, for all these objectives, the principal's optimal allocation strategy is either investing in one agent or investing equally in two agents. Interestingly, even though the principal is not profit-oriented from the post-innovation market, she will sometimes invest in only one agent under limited resources and intense market competition. The reason is that the agents are still profit-maximizing, so the potential rent dissipation can discourage them from exerting effort.

Second, we do not observe the nonmonotone investment strategy as derived in our baseline model. As the total amount of resources becomes abundant, the principal's investment strategy switches from investing in one agent to two agents but does not switch back. This is because for all four objectives above, the principal always prefers having two successful innovations than one. Hence, the optimal resource allocation strategy switches only once.

In Figure 7, we plot the thresholds $B_{\#1}(\alpha)$, $B_{\#2}(\alpha)$, $B_{\#3}(\alpha)$, and $B_{\#4}(\alpha)$, which characterize the optimal investment strategies under different objectives of a nonprofit principal. We find that

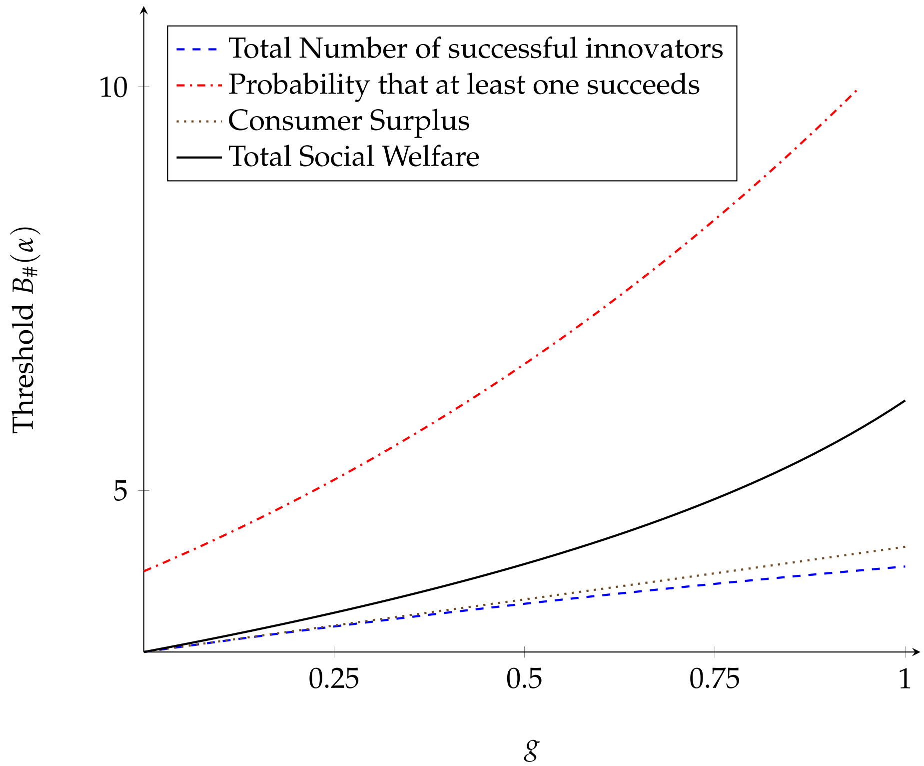


Figure 7: Switch Points of Different Objectives

the principal is more reluctant to invest in two agents when maximizing the probability of at least one innovator succeeding. In other words, when the principal pursues successful innovation (e.g., development and production of vaccines and pharmaceuticals), concentrated investment is preferred when abundant resources are not available.

4.3 Multiple Agents

In this subsection, we relax the restriction of having only two agents. Suppose there are a large or infinite number of agents who can receive the principal's resources and engage in innovation. All agents are symmetric and share the same success rate function (1). A principal with limited resources $\overline{B}$ can choose to invest in n agents with a desired total amount of B. If $m \leq n$ agents successfully innovate and launch their products on the market, each agent obtains α^{m-1} profit with $\alpha \in (0, 1)$.[9]

We focus on the symmetric case in which the principal allocates a total amount of resources $B \leq \overline{B}$ evenly to n agents. Each agent receives resources $b = B/n$. Under the resource allocation

[9] This profit structure simplifies our analysis. A quantity competition model can provide the foundation of this assumption. See Appendix A.

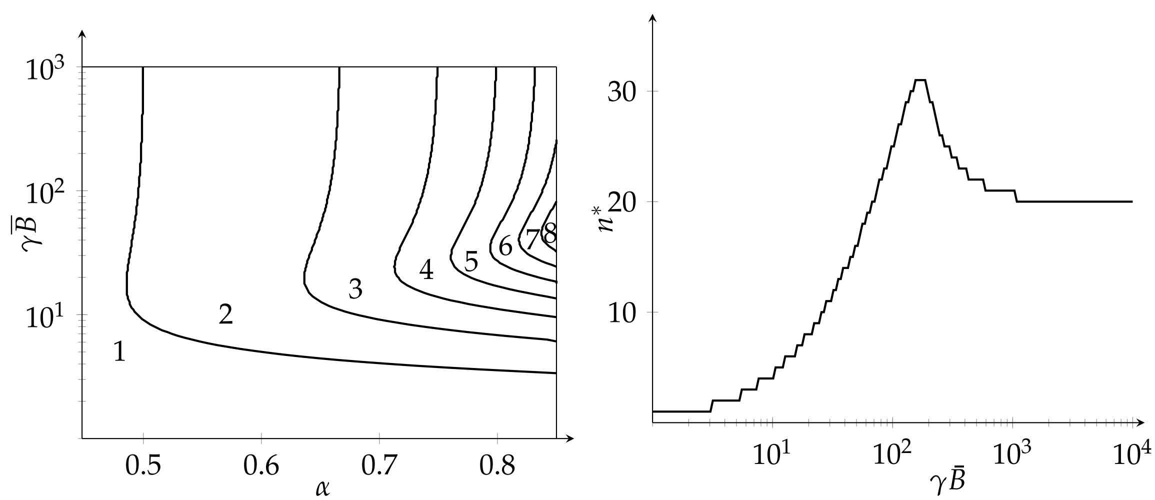
rule $(b, b, ..., b)$, a generic agent chooses his failure probability by solving the following problem.

$$\begin{aligned}
y^* &= \underset{0<y\leq 1}{\text{argmax}}\{\gamma(1-y)\sum_{m=0}^{n-1} C_{n-1}^m (1-y^*)^m (y^*)^{n-1-m}\alpha^m + \frac{\ln y}{b}\} \\
&= \underset{0<y\leq 1}{\text{argmax}}\{\gamma(1-y)(\alpha + y^* - \alpha y^*)^{n-1} + \frac{\ln y}{b}\}.
\end{aligned}$$

Based on the FOC, we find that when $n \leq \lfloor \gamma B \rfloor$, $1 > y^*$, the investment decision satisfies: $\gamma B y^*(\alpha + y^* - \alpha y^*)^{n-1} = n$, which implies that y^* is unique and decreasing in B. Since the profit for the principal is $(1-\gamma)n(1-y^*)(\alpha + y^* - \alpha y^*)^{n-1}$, the principal's optimization problem is

$$\begin{aligned}
\max_{1\leq n\leq \lfloor \gamma B \rfloor} \quad & (1-\gamma)n(1-y^*)(\alpha + y^* - \alpha y^*)^{n-1} \\
s.t. \quad & \gamma B y^*(\alpha + y^* - \alpha y^*)^{n-1} = n \text{ and } B \leq \overline{B}.
\end{aligned}$$

Since the above problem is analytically intractable, we solve it numerically and obtain the optimal number of agents receiving positive investment, $n^*(\alpha, \gamma\overline{B})$. Figure 8 panel (a) illustrates $n^*(\alpha, \gamma\overline{B})$ for a relatively large α. We observe that as the post-innovation market becomes less competitive (or as α increases), the principal tends to invest in more agents. Panel (b) displays a slice of $n^*(\alpha, \gamma\overline{B})$ at $\alpha = 0.95$. It shows that for a large α, n^* first increases and then decreases in $\overline{B}$. This pattern resembles Theorem 1, which states that with more resources, the principal may reduce the number of invested agents. Therefore, the main results in the baseline model with n capped at two are robust to removing the cap.



(a) $n^*(\alpha, \gamma\bar{B})$

(b) $n^*(0.95, \gamma\bar{B})$

Figure 8: Optimal Number of Agents with Positive Investment

4.4 Costly Resources

In this extension, we consider the case in which the resource is costly to the principal. For example, the investments of venture capital firms are costly per se. Additionally, when offering the right to use an asset (e.g., office space, equipment, lab) to innovators, the principal needs to bear the cost of depreciation. We use a parameter $k \in [0,1]$ to capture the cost of using resources. If the principal allocates $B = b_1 + b_2 \leq \overline{B}$ resources, she incurs a cost kB. In this setting, the principal's payoff becomes

$$R(k) = \max_{b_1, b_2} \{\Pi(b_1, b_2) - k(b_1 + b_2)\} \; s.t. \; b_1 + b_2 \leq \overline{B}, \tag{19}$$

where $\Pi(b_1, b_2)$ is given by (7). We can show the following result:

Proposition 10. *As the cost parameter k increases, the optimal amount of total resources $B^* = b_1^* + b_2^*$ allocated to two agents decreases.*

Given the optimally allocated B^*, the resource allocation rule follows Theorem 1. Hence, our main results hold in the case of costly resources.

In addition to the extensions above, our model can be extended in other aspects. For example, we can allow agents to have asymmetric abilities in innovation. Agent i's success rate of innovation is $p(b_i, x_i) = 1 - e^{-c_i b_i x_i}$, and $c_1 \neq c_2$. Moreover, with endogenous profit sharing, we can allow the principal to offer different profit-sharing rates to different agents. These extensions will likely lead to unequal investment for the two agents, but the key trade-off captured in the paper should remain the important factor in determining the optimal resource allocation strategy.

It is also interesting to consider agents with correlated innovation success rates. If the successes of multiple agents are positively correlated, each agent will have a lower incentive to exert effort because he is more likely to face competition in the post-innovation market. This resembles the case with knowledge spillovers between innovators in Ning and Babich (2017). In this case, optimal resource allocation should be more concentrated. If the successes of agents are negatively correlated, competition in the post-innovation market rarely occurs, and diverse investment would be more desirable.

5 Conclusion

This paper advocates a holistic view of innovation from the upstream resource allocation to the downstream competition. The interplay between allocated resources and market competition plays a pivotal role in crafting an innovator's strategy. Specifically, an innovative agent will exert effort and actively innovate only if he anticipates the profit from the post-innovation market can cover his costs, after taking into account the uncertainty of innovation. Thus, competition factors in through agents' strategic considerations, and this propagates to the principal's resource arrangement.

In particular, we find the resource allocation strategy exhibits an interesting pattern that the level of diversification first increases and then decreases in the amount of resources. When the amount of resources is low, disseminating resources to multiple agents will discourage all agents from exerting effort in innovation process. As resources become abundant, although investing in more agents increases the probability that at least one agent successfully innovates, the post-innovation competition will erode profits when more than one agent succeeds. As a result, the optimal resource allocation strategy is nonmonotone in the resource capacity.

In this regard, our analytical framework and results offer some important policy and managerial implications to both profit-oriented investors (such as venture capital firms) and nonprofit funding authorities (such as research foundations and incubators). Although the principal herself may not be profit-oriented, the principal needs to be mindful of agents' incentives because the ultimate success of innovation relies on their efforts. The resource capacity subsequently dictates the number of agents that the principal can incentivize effectively. When resources are limited, some degree of concentration is necessary to guarantee the overall success rate of innovation.

Our analytical characterizations also provide guidelines for diversification or concentration strategies that are institution-specific. For various industries and different stages of R&D, diversifying innovation investment can lead to vastly different outcomes. As funding authorities devote more resources to innovation, it is possible that funding more innovators will lead to worse outcomes in innovation and knowledge transfer because too many similar startups enter the same market. In establishing innovation policy, the policymaker must carefully consider this possibility. For example, China currently has at least 1,500 incubators supported by the Ministry of Science and Technology, funding approximately 80,000 companies.[10] Most of these companies are in several thrust areas such as advanced equipment manufacturing, new materials, and new-energy vehicles. The post-innovation competition intensity should be an important factor in determining the resource distribution and investment diversity in different areas. Concentrating resources in some areas while diversifying in others may substantially improve the performance of these incubators.

We also find that profit-oriented funding authorities will invest resources in a more concentrated way than non-profit counterparts. When resources are abundant, non-profit funding authorities will induce more successful innovations and higher social surplus but a lower profit. Depending on the policymaker's goal, some areas of innovation should be directed by nonprofit funding authorities, while others by private venture capitals. For example, to develop a COVID-19 vaccine, funding decisions should be made by nonprofit entities because the success rate and social welfare outweigh the profit during the pandemic. However, the development of bike-sharing

[10] Besides financial resources, these incubators also provide management services, policy support, opportunities to access university technologies and resources such as funds and work space. See `www.scmp.com/article/topics/invest-china/1727504/china-nurturing-start-ups-cash-incentives-boost-economy`.

apps should be directed by private entities to guarantee the best use of financial resources and avoid over-investment in this area.

For future research, we also plan to explore a model with multiple principals. In reality, the innovation activities are funded by multiple funding authorities including profit-oriented and nonprofit entities. This analysis will help the policymaker compare different ways of stimulating innovation and evaluate how efficient is the decentralized innovation investment outcome.

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Appendix

A Product Market Competition

A.1 Quantity competition

Suppose that two successful agents engage in quantity competition. The price of agent i's product is determined by an inverse demand function (Vives, 1984), that is, $P_i(q_i, q_{-i}) = A - gq_{-i} - q_i$. Here, A measures the size of the market, q_i is the quantity produced by agent i, and $g \in (0,1)$ characterizes the degree of substitution between two agents' products. Note that an agent is able to launch and produce his product only when his innovation is successful. If there is only one agent succeeding in innovation, he becomes a monopolist with the profit

$$\pi_m = \max_{q_i} \gamma(A - q_i)q_i = \frac{\gamma A^2}{4}.$$

If both agents succeed in the innovation stage, it is straightforward to show that the equilibrium quantities are $q_1^* = q_2^* = \frac{A}{2+g}$. The profit for each agent is

$$\pi_d = \frac{\gamma A^2}{(2+g)^2}$$

As a result, the ratio between the monopoly profit and the duopoly profit can be derived as

$$\alpha = \frac{\pi_d}{\pi_m} = \frac{4}{(2+g)^2}.$$

One can show that when $g \in (0,1)$, quantity competition leads to $\alpha \in (\frac{4}{9}, 1)$.

The quantity competition model can be easily generalized into the case with m competing agents. When there are m agents succeeding in innovation, each will earns a profit $\frac{\gamma a^2}{(2+g(m-1))^2}$, where a is the market potential level and g measures the competition intensity among agents. Specifically, $g = 0$ indicates that these m products are fully independent, and $g = 1$ means that they are homogeneous.

In Section 4.3, we use α^{m-1} to capture the profit structure for successful agents. The main results are consistent irrespective of using g or α to measure the competition intensity.

A.2 Price competition

Post-innovation competition can also be modeled as price competition based on a multinomial-logit (MNL) discrete choice model (Gallego and Topaloglu, 2019). Suppose that two agents engage in innovation to create a specified product for a market of size 1. Let the base quality of a product relative to the outside option be v_0. Apart from the base quality, consumers have heterogeneous preferences on each agent or the outside option. The idiosyncratic random utility terms of consumers follow the i.i.d. Gumbel distribution. When there are k products, consumers make a discrete choice based on the MNL model. The choice probability of product i is $\frac{e^{v_0-p_i}}{1+\sum_{j=1}^{k} e^{v_0-p_j}}$.

When there is only one successful agent in the market, he chooses p by maximizing his profit. The profit of the agent under the monopoly case can be derived as follows:

$$\pi_m = \max_p \frac{\gamma e^{v_0-p} p}{1+e^{v_0-p}} = \gamma \mathcal{L}(e^{v_0-1}),$$

where $\mathcal{L}(y)$ is the root of $xe^x = y$. However, when both agents successfully launch their products, they have to compete with each other on their prices. The optimal prices of the two agents are symmetric, which satisfies the following condition:

$$p^* = \operatorname*{argmax}_p \frac{\gamma e^{v_0-p} p}{1+e^{v_0-p^*}+e^{v_0-p}}.$$

Then, we can obtain $e^{v_0} = \frac{e^{p^*}(p^*-1)}{2-p^*}$ with $1 \le p^* \le 2$. Accordingly, the profit for each agent in the competitive case is

$$\pi_d = \frac{\gamma e^{v_0-p^*} p^*}{1+2e^{v_0-p^*}} = \gamma(p^*-1).$$

As a result, the ratio between the monopoly profit and the duopoly profit is captured by:

$$\alpha = \frac{\pi_d}{\pi_m} = \frac{p^*-1}{\mathcal{L}(\frac{e^{p^*-1}(p^*-1)}{2-p^*})}.$$

One can show that when $v_0 \in (-\infty, \infty)$, namely, $p^* \in (1,2)$, it leads to $\alpha \in (0,1)$.

B Proofs

Proof of Lemma 1 By equation (6), for $y_1^* > 0$ and $y_2^* > 0$, we have

$$y_{1(2)}^* = \frac{\sqrt{4(1-\alpha)\alpha^2\gamma b_1 b_2^2 + (b_2(\alpha-1)+b_1(\gamma\alpha^2 b_2 - \alpha + 1))^2} - \alpha^2\gamma b_1 b_2 \pm (b_2-b_1)(1-\alpha)}{2\alpha(1-\alpha)\gamma b_1 b_2}.$$

Then letting $0 < y_{1(2)}^* < 1$, we can yield the conditions shown in Lemma 1. ■

Proof of Lemma 2 Given b_{-i}, differentiating equation (6) with respect to b_i yields

$$[\alpha(1-y_{-i})+y_{-i}]\frac{dy_i}{db_i} + (1-\alpha)y_i\frac{dy_{-i}}{db_i} = -\frac{1}{\gamma b_i^2},$$

and

$$[\alpha(1-y_i)+y_i]\frac{dy_{-i}}{db_i}+(1-\alpha)y_{-i}\frac{dy_i}{db_i}=0.$$

By solving the two equations above, we have

$$\frac{dy_i}{db_i}=-\frac{1}{\alpha b_i^2\gamma}\frac{\alpha+(1-\alpha)y_i}{\alpha+(1-\alpha)(y_1+y_2)}<0,$$

and

$$\frac{dy_{-i}}{db_i}=\frac{1}{\alpha b_i^2\gamma}\frac{(1-\alpha)y_{-i}}{\alpha+(1-\alpha)(y_1+y_2)}>0.$$

■

Proof of Proposition 1 We divide the proof into several cases.

Case 1: $\gamma B \leq 2$

Note that when $\gamma B \leq 2$, then $\gamma b_2 \leq 1$. Due to the insufficient amount of resources to incentivize both agents, $y_2^* = 1$ for sure. Now, the profit of the principal only comes from a potentially successful outcome from agent 1's innovation. Similar to Proposition 2, $\tilde{\Pi}(b_2; B)$ is nonincreasing in $b_2 \in [0, \frac{B}{2}]$.

Case 2: $\gamma B > 2$

For $\gamma B > 2$, we do a special transformation to simplify the exposition of our proof: we let $b_2 = \frac{B}{2}[1-\sqrt{1-\frac{1-\alpha}{z}}]$ with $z \in [1-\alpha, \infty)$. This guarantees $b_2 \leq b_1$. Note that $\gamma b_1 > 1$ for sure.

If $\frac{b_1(z)}{b_2(z)} = 1+\alpha(\gamma b_1(z)-1)$, then

$$16z^2+4(\alpha^2 B\gamma-\alpha^2+4\alpha-4)z-(1-\alpha)\alpha^2B^2\gamma^2=0.$$

As $0<\alpha<1$, the two roots to the equation above satisfy that $z_1z_2 = -(1-a)\alpha^2B^2\gamma^2/16 < 0$. Hence, there is a unique non-negative root denoted as $\hat{z}$. It is easy to verify that $\hat{z} \geq 1-\alpha$. Besides, it can be verified that the condition in Lemma 1 holds only when $z < \hat{z}$.

Case 2.a: $z \geq \hat{z}$

In the case, $y_2^* = 1$, and the profit of the principal will only come from a potentially successful outcome from agent 1's innovation. Then the profit of the principal is increasing in $z \in [\hat{z}, \infty)$ as agent 1 gets more resources.

Case 2.b: $z \in [1-\alpha, \hat{z})$

Using the expressions of y_1^* and y_2^* in the proof of Lemma 1, the principal's objective becomes as follows,

$$\tilde{\Pi}(z)=(1-\gamma)(\alpha-\frac{4z}{(1-\alpha)\gamma B}+\frac{\sqrt{\alpha^4B^2\gamma^2+8z(\alpha^2B\gamma+2\alpha-2)+16z^2}}{\alpha B\gamma}).$$

Denote $d(z) := \alpha^4B^2\gamma^2+8z(\alpha^2B\gamma+2\alpha-2)+16z^2$. As $\gamma B > 2$, $z \geq 1-\alpha$ and $0<\alpha<1$, we have

$d(z) \geq 0$. Moreover, we get

$$\frac{\tilde{\Pi}''(z)}{1-\gamma} = \frac{64(1-\alpha)}{\sqrt{d^3(z)}\alpha B\gamma}(\alpha^2 B\gamma + \alpha - 1).$$

Case 2.b.1: $\alpha^2 B\gamma + \alpha - 1 \geq 0$

In this case, $\Pi(z)$ is convex in $z \in [1-\alpha, \hat{z})$.

Case 2.b.2: $\alpha^2 B\gamma + \alpha - 1 < 0$

$\tilde{\Pi}(z)$ is concave in $z \in [1-\alpha, \hat{z})$. If $\tilde{\Pi}(z)$ is maximized at z_* with $z_* \in (1-\alpha, \hat{z})$, by the FOC, we have

$$0 = \frac{\tilde{\Pi}'(z_*)}{1-\gamma} = \frac{4}{B\gamma}[\frac{1}{\alpha-1} + \frac{-2+2\alpha+\alpha^2 B\gamma + 4z_*}{\alpha\sqrt{d(z_*)}}].$$

By rearranging the terms, we have

$$\begin{aligned}0 = &-4(1-\alpha)^4 + 4(1-\alpha)^3\alpha^2 B\gamma + (2\alpha-1)\alpha^4 b^2\gamma^2 \\ &+ 8(2\alpha-1)(\alpha^2 B\gamma + 2\alpha - 2)z_* + (32\alpha - 16)z_*^2.\end{aligned}$$

If there is a real solution of z_*, then the following inequation should be satisfied

$$\begin{aligned}&(8(2\alpha-1)(\alpha^2 B\gamma + 2\alpha - 2))^2 \\ &-4(-4(1-\alpha)^4 + 4(1-\alpha)^3\alpha^2 B\gamma + (2\alpha-1)\alpha^4 B^2\gamma^2)(32\alpha-16) \\ =&256\alpha^2(1-3\alpha+2\alpha^2)(\alpha^2 B + \alpha - 1) \geq 0,\end{aligned}$$

which means that $1 > \alpha \geq 1/2$. However, together with $\alpha^2 B\gamma + \alpha - 1 < 0$ and $\gamma B > 2$, there is a contradiction. Therefore $\tilde{\Pi}(z)$ is monotone in $z \in [1-\alpha, \hat{z})$.

In summary, $\tilde{\Pi}(z)$ is convex or monotone in $z \in [1-\alpha, \hat{z})$ and $\tilde{\Pi}(z)$ is increasing in $z \in [\hat{z}, \infty)$. As a result, $\tilde{\Pi}(z)$ is quasi-convex in $z \in [1-\alpha, \infty)$, which implies the claim in the proposition. ■

Proof of Lemma 3 For (i), from the expression $y_1^* = y_2^* = y^* := \frac{4}{\alpha B\gamma + \sqrt{B\gamma}\sqrt{8-8\alpha+\alpha^2 B\gamma}}$, it is easy to show that y^* decreases when B increases.

Note that $x^* = -\frac{2\ln y^*}{B}$. Denote $\mathcal{D}(B) := B^2\frac{dx^*}{dB} = 1 + \frac{\alpha\sqrt{B\gamma}}{\sqrt{8-8\alpha+\alpha^2 B\gamma}} + 2\ln\frac{4}{\alpha B\gamma + \sqrt{B\gamma}\sqrt{8-8a+a^2 B\gamma}}$. Then $Sign(\mathcal{D}(B)) = Sign(\frac{dx^*}{dB})$. Moreover,

$$\mathcal{D}(B = 2/\gamma) = \frac{2}{2-a} > 0, \quad \lim_{B\to\infty}\mathcal{D}(B) = -\infty < 0,$$

and

$$\frac{d\mathcal{D}(B)}{dB} = -\frac{4\alpha\sqrt{B\gamma}(1-\alpha) + \alpha^3(B\gamma)^{3/2} + (8-8\alpha+\alpha^2 B\gamma)^{3/2}}{B(8-8\alpha+\alpha^2 B\gamma)^{3/2}} < 0.$$

As a result, when B increases, $\mathcal{D}(B)$ and $\frac{dx^*}{dB}$ are first positive and then negative.

We fist show (iii). Note that $\frac{B^2\Pi_d'(B)}{4} = 1 - \frac{(1-\alpha)\sqrt{B}}{\sqrt{8-8a+a^2B}}$, and results can be easily derived.

For (ii), we focus on the case of $\gamma B > 2$.

Denote $\tilde{\mathcal{D}}(B) := B^2\frac{dU_d(B)}{dB} = 1 + \frac{(\alpha-2)\sqrt{B\gamma}}{\sqrt{8-8\alpha+\alpha^2B\gamma}} - 2\ln\frac{4}{\alpha B\gamma+\sqrt{B\gamma}\sqrt{8-8a+a^2B\gamma}}$. Then $Sign(\tilde{\mathcal{D}}(B)) = Sign(\frac{dU_d(B)}{dB})$. Moreover, $\tilde{\mathcal{D}}(B=\frac{2}{\gamma}) = 0$, $\frac{d\tilde{\mathcal{D}}(B)}{dB}|_{B=\frac{2}{\gamma}} = \frac{\gamma}{(2-\alpha)^2} > 0$ and

$$\frac{d\tilde{\mathcal{D}}(B)}{dB} = \frac{1}{B}(1 - \frac{4(2-5\alpha+3\alpha^2)\sqrt{B\gamma} - \alpha^3(B\gamma)^{3/2}}{(8-8\alpha+\alpha^2B\gamma)^{3/2}}).$$

Solving $\frac{d\tilde{\mathcal{D}}(B)}{dB} = 0$ directly gives two solutions as $\frac{1-\alpha}{2\alpha^3\gamma}[4-12\alpha-3\alpha^2 \pm (2-a)\sqrt{(2-\alpha)(2-9\alpha)}]$. Then we have the following findings:

(1) When $\alpha \geq 2/9$, there does not exist real solution of B satisfying $\frac{d\tilde{\mathcal{D}}(B)}{dB} = 0$. Then as $\tilde{\mathcal{D}}(B = \frac{2}{\gamma}) = 0$ and $\frac{d\tilde{\mathcal{D}}(B)}{dB}|_{B=\frac{2}{\gamma}} > 0$, $U_d(B)$ is always increasing in B.

(2) When $\alpha < 2/9$, the two solutions are real. Note that $B_* = \frac{8(1-\alpha)}{\gamma(1-2\alpha)}$ is smaller than the larger solution. Therefore when $B \in [\frac{2}{\gamma}, B_*]$ increases, as $\frac{d\tilde{\mathcal{D}}(B)}{dB}|_{B=\frac{2}{\gamma}} > 0$, $\frac{d\tilde{\mathcal{D}}(B)}{dB}$ is either a) first positive, and then negative, or b) always non-negative. Case b) leads to $\tilde{\mathcal{D}}(B) > 0$, namely $\frac{dU_d(B)}{dB} > 0$ for all $B \in [\frac{2}{\gamma}, B_*]$. For case a), when $B \in [\frac{2}{\gamma}, B_*]$ increases, $\tilde{\mathcal{D}}(B)$ first increases and then decreases. As $\tilde{\mathcal{D}}(B = \frac{2}{\gamma}) = 0$, $\mathcal{D}$ is always positive for the increasing part. If $\tilde{\mathcal{D}}(B) < 0$ for some $B \in [\frac{2}{\gamma}, B_*]$, we must have $\tilde{\mathcal{D}}(B_*) = \tilde{\mathcal{D}}(\frac{8(1-\alpha)}{\gamma(1-2\alpha)}) = -\frac{1}{1-\alpha} + 2\ln\frac{2(1-\alpha)}{1-2\alpha} < 0$, leading to a contradiction when $\alpha < 2/9$. ■

Proof of Proposition 3 For (i), as $\Pi_m(B)$ is always increasing in B, we only need to show that $\Pi_d(B)$ is increasing in B when $\Pi_d(B) > \Pi_m(B)$.

(1) If $\alpha \geq 1/2$, $\Pi_d(B)$ is increasing in B for sure by Lemma 3.

(2) If $\alpha < 1/2$, we need to consider two cases. The condition $\Pi_m(B) = \Pi_d(B)$ implies

$$(1-2\alpha)\gamma^2B^2 - 2(1-2\alpha)\gamma B + 9 = 0. \tag{20}$$

When $\alpha < 6\sqrt{2} - 8$, there is no solution of $\gamma B > 2$ to the equation above. Because $\Pi_m(2/\gamma) > \Pi_d(2/\gamma)$, for any given B, $\Pi_m(B) > \Pi_d(B)$.

When $6\sqrt{2} - 8 \leq \alpha < 1/2$, there are two solutions for (20). At optimality, $\Pi_d(B) \geq \Pi_m(B)$ if and only if $\gamma B \in [\frac{1-\alpha-\sqrt{\alpha^2+16\alpha-8}}{1-2\alpha}, \frac{1-\alpha+\sqrt{\alpha^2+16\alpha-8}}{1-2\alpha}]$. Recalling Lemma 3 that $\Pi_d(B)$ is increasing in $B \in \frac{1}{\gamma}[2, \frac{8(1-\alpha)}{1-2\alpha}]$. As $\frac{1-\alpha+\sqrt{\alpha^2+16\alpha-8}}{1-2\alpha} < \frac{8(1-\alpha)}{1-2\alpha}$ for $\alpha \in [6\sqrt{2}-8, 1/2]$, $\Pi_d(B)$ is increasing in B when $\Pi_m(B) < \Pi_d(B)$.

As a result, $\max\{\Pi_m(B), \Pi_d(B)\}$ is always increasing in B.

For (ii), let $\Delta(B) = \frac{\Pi_m(B)-\Pi_d(B)}{1-\gamma} = 1 - \frac{1}{\gamma B} - (\alpha - \frac{4}{\gamma B} + \sqrt{\frac{\gamma\alpha^2B-8\alpha+8}{\gamma B}})$. There is only one point satisfying $\Delta'(B) = 0$, which is $\tilde{B} = \frac{72(1-\alpha)}{\gamma(7\alpha^2-32\alpha+16)}$. As a result, $\Delta(B)$ is either quasi-convex or quasi-concave. If $\Delta(B)$ is quasi-convex, the claim holds for sure. If $\Delta(B)$ is quasi-concave, as $\Delta'(B)|_{B=2/\gamma} = \frac{(2+\alpha)\gamma}{4(\alpha-2)} < 0$, then $\Delta(B)$ is decreasing in $B \in [2/\gamma, \infty)$, which also implies it is quasi-convex in $B \in [2/\gamma, \infty)$. ■

Proof of Theorem 1 By Proposition 3, $b_1^* + b_2^* = \overline{B}$. Applying similar analysis as that of Proposition 3, we can can easily derive the result. ■

Proof of Proposition 5 At optimality, $\gamma^*\overline{B} > 2$, otherwise $y^* = 1$. So we focus on $\gamma \in (\frac{2}{\overline{B}}, 1]$.

Step 0: Auxiliary functions.

Note that y^* only depends on $\gamma\overline{B}$. We denote

$$\pi(x) := 2\alpha(1-y^*)^2 + 2(1-y^*)y^*|_{\gamma\overline{B}=x} = \alpha - \frac{4}{x} + \sqrt{\frac{8-8\alpha+\alpha^2 x}{x}},$$

where $x > 2$. It can be shown that $\pi(2) = 0$ and $\pi'(2) = \frac{1}{2-a} > 0$.

Step 0.a: $\pi(x)$ is quasi-concave in $x \in [2, \infty)$.

Similar to Lemma 3, $\pi(x)$ is increasing in x if $\alpha \geq 1/2$; $\pi(x)$ is quasi-concave in x and maximized at $x = \frac{8(1-\alpha)}{1-2\alpha}$, when $\alpha < 1/2$.

Step 0.b: $k(x) := \frac{\pi(x)}{\pi'(x)} + x$ is increasing in $x \in [2, x_*)$, where

$$x^* = \begin{cases} \frac{8(1-\alpha)}{1-2\alpha}, & 0 < \alpha < 1/2 \\ \infty, & 1 \geq \alpha \geq 1/2. \end{cases}.$$

Solving $k'(x) = 0$ directly gives one solution as $\frac{18(1-\alpha)}{1-\alpha-2\alpha^2}$. we observe that when $\alpha \geq 1/2$ the solution is negative; when $0 < \alpha < 1/2$, $\frac{18(1-\alpha)}{1-\alpha-2\alpha^2} > \frac{8(1-\alpha)}{1-2\alpha}$. Then $k'(x) \neq 0$ for all $x \in [2, x_*)$. Then $k'(2) > 0$, we have the claim.

Step 1: $\Pi_d(\gamma; \overline{B})$ is strictly quasi-concave in $\gamma \in (\frac{2}{\overline{B}}, 1)$.

We can write the profit of the principal as follows,

$$\Pi_d(\gamma, B) := (1-\gamma)\pi(\gamma B).$$

Step 1.a: $\Pi_d(\gamma; \overline{B})$ is strictly quasi-concave in $\gamma \in (\frac{2}{\overline{B}}, \frac{x_*}{\overline{B}})$, where x_* is defined in Step 0.b.

Note that

$$\frac{\Pi'(\gamma; \overline{B})}{\pi'(\gamma\overline{B})} = -k(\gamma\overline{B}) + \overline{B},$$

where function $k(\cdot)$ is defined in Step 0.b. From Step 0.a, $\pi'(x) > 0$ when $x < x_*$. Then when $\gamma \in (\frac{2}{\overline{B}}, \frac{x_*}{\overline{B}})$, $sign(\frac{\Pi'(\gamma;\overline{B})}{\pi'(\gamma\overline{B})}) = sign(\Pi'(\gamma; \overline{B}))$. From Step 0.b, $\frac{\Pi'(\gamma;\overline{B})}{\pi'(\gamma\overline{B})}$ is decreasing in $\gamma \in (\frac{2}{\overline{B}}, \frac{x_*}{\overline{B}})$, namely, $\frac{\Pi'(\gamma;\overline{B})}{\pi'(\gamma\overline{B})}$ is first positive and then negative when γ increases. Therefore, $\Pi(\gamma; \overline{B})$ is strictly quasi-concave in $\gamma \in (\frac{2}{\overline{B}}, \frac{x_*}{\overline{B}})$ and there is a unique γ satisfying $\Pi'(\gamma; \overline{B}) = 0$.

Step 1.b: $\Pi_d(\gamma; \overline{B})$ is decreasing in $\gamma \in (\frac{x_*}{\overline{B}}, 1)$, where x_* is defined in Step 0.b.

From Step 0.a, $\pi'(x) \leq 0$ when $x \geq x_*$. Then when $\gamma \in (\frac{2}{\overline{B}}, 1)$, we have $\Pi'(\gamma; \overline{B}) = -\pi(\gamma\overline{B}) + \pi'(\gamma\overline{B})\overline{B}(1-\gamma) < 0$.

Based on Step 1.a and Step 1.b, we conclude that $\Pi(\gamma, \overline{B})$ is strictly quasi-concave in $\gamma \in (\frac{2}{\overline{B}}, 1)$.

Step 2: Property of γ^*.

From Step 1, at optimality,

$$\overline{B} = k(\gamma^*\overline{B}).$$

By solving this equation above, we have

$$\begin{aligned}\overline{B} =& \frac{(1+\gamma^* - \alpha\gamma^*)\sqrt{\alpha^2(2-\gamma^*)^2 + (1+\gamma^*)^2 - 2\alpha(2-\gamma^* + (\gamma^*)^2)}}{\alpha^2(\gamma^*)^2} \\ &- \frac{1 - 2\alpha + 2(1-\alpha^2)\gamma^* + (1-\alpha)^2(\gamma^*)^2}{\alpha^2(\gamma^*)^2}\end{aligned}$$

As $0 < \gamma^* < 1$ and $0 < \alpha < 1$, and we can find that the RHS is nondecreasing in α and decreasing in γ^*. Then we have the claim shown in Proposition 5. ■

Proof of Theorem 2 Recall that

$$\Pi_m^*(B) = (1 - \frac{1}{\sqrt{B}})^2.$$

By setting $z = B\gamma$, we have

$$\Pi_d^*(B) = \max_{2<z<B}\{\Pi_d(z; B) := \frac{B-z}{Bz}(\alpha z + \sqrt{z}\sqrt{8 - 8\alpha + \alpha^2 z} - 4)\}.$$

Let $z_*(B) = \underset{2<z<B}{\operatorname{argmax}}\{\Pi_d(z; B)\}$.[11] Denote $\Delta(B) := \Pi_d^*(B) - \Pi_m^*(B)$. Then we get

$$\Delta'(B) = \frac{d\Pi_d^*(z; B)}{dB}|_{z=z_*} - \frac{d\Pi_m^*(B)}{dB} = \frac{-3 - \sqrt{B} + \alpha z_* + \sqrt{z_*}\sqrt{8 + \alpha(-8 + \alpha z_*)}}{B^2}, \tag{21}$$

where in first equality, the envelop theorem is used. Note that when $B \to 2$, due to $2 < z_* < B$, $z_* \to 2$. Hence, we derive $\Delta'(2) = \frac{1}{4}(1 - \sqrt{2}) < 0$ and $\Delta(2) = -\Pi_m^*(2) < 0$.

Step 0: Auxiliary functions.

Define $g(x) := \frac{27+27\sqrt{x}-31x+9x^{3/2}}{16x(\sqrt{x}-1)}$ with $x \in [2, \infty)$. Then we have

$$g'(x) = \frac{27 - 27\sqrt{x} - 27x + 11x^{3/2}}{16x^2(\sqrt{x}-1)^2}.$$

Therefore, $g(x)$ is decreasing in $x \in [2, 9]$ and increasing in $x \in (9, \infty)$. Besides, one can verify that $g(2) \approx 2.16069 > 1$, $g(9) = \frac{1}{4}$ and $\lim_{x\to\infty} g(x) = \frac{9}{16}$. As a result, when solving $g(x) = t$, (1) if $t \leq 1/4$, there do not exist solutions of $x > 2$. (2) if $1/4 < t < \frac{9}{16}$, there exist two solutions of

[11] In the proof, all z_* will be a function of B. We omit it if there is no confusion.

$x > 2$. (3) if $\frac{9}{16} \le t \le 1$, there only exists one solution of $x > 2$.

Step 1: Main results.

Assume that at B_*, $\Delta'(B_*) = 0$. From (21), we have that

$$z_*(B_*) = \frac{-36 + 9\alpha - 24\sqrt{B_*} + 15\alpha\sqrt{B_*} - 4B_* + 7\alpha B_* + \alpha B_*^{3/2}}{-32 + 16\alpha - 2\alpha^2 + 2\alpha^2 B_*}. \tag{22}$$

Moreover, $z_*(B_*)$ satisfies the following condition,

$$\begin{aligned} 0 =& \frac{d\Pi_d(z;B)}{dz}|_{z=z_*,B=B_*} \\ =& \frac{4B_*(\sqrt{8+\alpha(-8+\alpha z_*)} - \sqrt{z_*} + \alpha\sqrt{z_*}) - z_*^{3/2}(4 - 4\alpha + \alpha^2 z_*(B_*) + \alpha\sqrt{z_*}\sqrt{8+\alpha(-8+\alpha z_*)})}{B_* z_*^2\sqrt{8+\alpha(-8+\alpha z_*)}}. \end{aligned}$$

Substituting the expression of z_* in (22) into the equation above, we can find that

$$\alpha = \frac{27 + 27\sqrt{B_*} - 31B_* + 9B_*^{3/2}}{16B_*(\sqrt{B_*} - 1)} = g(B_*). \tag{23}$$

where $g(\cdot)$ is defined in Step 0.

Step 1.a: there is at most one continuous region of B such that the principal invests in two agents.

From the result in Step 0, there is at most two points of $B > 2$, satisfying $\Delta'(B) = 0$, namely (23). Recall that $\Delta'(2) < 0$ and $\Delta(2) < 0$. Therefore there is at most one continuous region of $B > 2$ such that $\Delta(B) > 0$.[12]

Step 1.b: $\alpha \le 1/4$

When $a \le 1/4$, from step 0, there do not exist solutions of $B > 2$ satisfying $\Delta'(B) = 0$. As $\Delta'(2) < 0$ and $\Delta(2) < 0$, $\Delta(B) < 0$ for all $B > 2$. Therefore when $\alpha \le 1/4$, at optimality, the principal invests in one agent.

Step 1.c: $\alpha \ge 1/2$.

For any fixed $z > 2$, we have

$$\Pi_d(z;\infty) = \lim_{B\to\infty} \Pi_d(z;B) = \alpha + \frac{\sqrt{z}\sqrt{8 - 8\alpha + \alpha^2 z} - 4}{z}.$$

Then we get that $\lim_{z\to\infty} \Pi_d(z;\infty) = 2\alpha \ge \lim_{B\to\infty} \Pi_m(B) = 1$. Therefore, when $B \to \infty$, at optimality, the principal invests in two agents. Therefore, due to Step 1.a, when B increases, at optimality, the principal first invests in one agent and then switches to investing in two agents.

Step 1.d: $1/2 > \alpha > 1/4$.

[12] For example, suppose that there are two points of $B > 2$, satisfying $\Delta'(B) = 0$, which are denoted as $B_1 < B_2$. Since $\Delta'(2) < 0$, the sign of $\Delta'(B)$ for $B \in (2, B_1)$,(B_1, B_2),(B_2, ∞) will be one of the four cases: $(-,-,-)$,$(-,+,+)$, $(-,-,+)$ or $(-,+,-)$. As $\Delta(2) < 0$, it is easy to find that (1) for the first case, $\Delta(B) < 0$ for all $B \in [2, \infty)$, (2) for the second and third cases, if $\Delta(B) > 0$ for some $B \in [2, \infty)$, then $\Delta(\tilde{B}) > 0$ for all $\tilde{B} \ge B$ (3) for the fourth case, as $\Delta(\tilde{B}) < 0$ for all $B \in [2, B_1)$ and $\Delta(\tilde{B})$ is quasi-concave in $B \in [B_1, \infty)$, there is one continuous region of B satisfying $\Delta(B) > 0$.

From Step 0, there are two points satisfying $\Delta'(B) = 0$ (namely (23)), which are denoted as B_1 and B_2. Note that $B_1 < 9 < B_2$.

Since $\lim_{B\to\infty} \Pi_m(B) = 1$, and $\Pi_d(z; B) < 1$ for sure[13]. Then when $B \to \infty$, at optimality, the principal invests in one agent. By step 1.a, that is to say, the principal adopts the strategy of investing in two agents only in a closed region of $B > 2$. So if $\Delta(B) > 0$ for some B, the sign of $\Delta'(B)$ for $B \in (2, B_1)$, (B_1, B_2) and (B_2, ∞) must be $(-, +, -)$. That is to say, $\Delta(B)$ must be maximized at B_2 when $\Delta(B) > 0$ for some B. Based on the above discussion, to examine whether $\Delta(B) > 0$ for some B, we just need to check whether $\Delta(B_2) > 0$.

Given α, $B_2(\alpha) > 9$ will be uniquely determined by (23), which is increasing in α. By Eq.(22) and Eq.(23), we can replace α and z_* by $B_2(\alpha)$ in the expression of $\Delta(B_2, \alpha)$ and thus

$$\Delta(B_2; \alpha) = \frac{(\sqrt{B_2} - 1)(9 - 18\sqrt{B_2} + B_2)}{8(3 + \sqrt{B_2})B_2}.$$

If $\Delta(B_2; \alpha) > 0$, as $B_2 > 9$, we have that

$$B_2 > 9(17 + 12\sqrt{2}),$$

namely,

$$\alpha > g(x)|_{x=9(17+12\sqrt{2})} = 6\sqrt{2} - 8,$$

where the condition of α is due to Eq.(23). That is to say, when $\alpha \le 6\sqrt{2} - 8$, $\Delta(B) < 0$ all $B > 2$. When $1/2 > \alpha > 6\sqrt{2} - 8$, due to Eq.(23), $B_2(\alpha) > 9(17 + 12\sqrt{2})$, we get that $\Delta(B_2; \alpha) > 0$. That is to say, when $1/2 > \alpha > 6\sqrt{2} - 8$, in some region, investing in two agents can be optimal.

In summary, we have the claim shown in Theorem 2. ■

Proof of Proposition 6 Assume that $b_1 \ge b_2$.

Step 1: At optimality, the principal invests in one or two equally.

We divide the proof into several cases given $b_1 + b_2 = B$.

Case 1: $B \le 2$

In this case, a potentially successful outcome only comes from agent 1's innovation. As $N_d(B - b_2, b_2) = 1 - \frac{1}{B-b_2}$, we need to allocate all resources to agent 1.

Case 2: $B > 2$

Similar to the proof of Proposition 1, we do a special transformation to simplify the exposition of the proof: $b_2 = \frac{B}{2}[1 - \sqrt{1 - \frac{1-\alpha}{z}}]$ with $z \in [1 - \alpha, \infty)$. There is $\hat{z} \ge 1 - \alpha$, such that $y_2 < 1$ only when $z < \hat{z}$.

Case 2.a: $z \ge \hat{z}$

[13]When both agents succeed, the principal's profit is not larger than 1 (i.e., $2\alpha < 1$); When only one agent succeeds, the profit is 1.

In this case, $y_2^* = 1$ and similar to case 1, $N_d(z) := N_d(B - b_2(z), b_2(z))$ is increasing in $z \in [\hat{z}, \infty)$.

Case 2.b: $z \in [1-\alpha, \hat{z})$

Using the expressions of y_1^* and y_2^* in the proof of Lemma 1, the principal's objective becomes as follows,

$$N_d(z) = \frac{2-\alpha}{1-\alpha} - \frac{\sqrt{d(z)}}{(1-\alpha)\alpha B}$$

where $d(z) := \alpha^4 B^2 + 8z(\alpha^2 B + 2\alpha - 2) + 16z^2$. Note that

$$d'(z) = 8(2\alpha + \alpha^2 B + 4z - 2) \geq 8(2\alpha + 2\alpha^2 + 4(1-\alpha) - 2) = 16(\alpha^2 - \alpha + 1) > 0,$$

where the first inequality is due to $B \geq 2$ and $z \geq 1 - \alpha$. Then $N_d(z)$ is decreasing in $z \in [1-\alpha, \hat{z})$.

In summary, $N_d(z)$ is quasi-convex in $z \in [1-\alpha, \infty)$, which implies that the principal invests in one or invests equally in two.

Step 2: Use up resource under two strategies.

If the principal invests equally in two agents with $B = b_1 + b_2 > 2$, the equilibrium failure probability is given by (10), namely,

$$y_1 = y_2 = y^* = \frac{4}{\alpha B + \sqrt{B}\sqrt{8 - 8\alpha + \alpha^2 B}}. \tag{24}$$

The principal's objective is

$$N_d(B; \alpha) = 1 - y_1^* + 1 - y_2^* = \frac{(\alpha - 2)B + \sqrt{B(8 - 8\alpha + \alpha^2 B)}}{(\alpha - 1)B}.$$

The expected number of successful innovations, N_d, is increasing in α and B as $B > 2$ and $\alpha \in (0, 1)$. So the principal will exhaust her resources if investing in two equally, i.e., $(b_1^*, b_2^*) = (\overline{B}/2, \overline{B}/2)$.

If the principal invests in one agent with $B > 1$, by (2), the failure rate is $y^* = 1/B$. The principal chooses B by

$$\max_{B \leq \overline{B}} N_m(B) = 1 - y^* = 1 - \frac{1}{B}.$$

Obviously, $N_m(B)$ increases in B, so $(b_1^*, b_2^*) = (\overline{B}, 0)$ when the principal invests in one agent.

Step 3: Comparison between two strategies.

Next, to derive the globally optimal allocation strategy for the principal, we need to compare $N_d(\overline{B}; \alpha)$ and $N_m(\overline{B})$. Given $\overline{B} \geq 2$, the difference between $N_d(\overline{B}; \alpha)$ and $N_m(\overline{B})$ is

$$\Delta N(\overline{B}; \alpha) = N_d - N_m = \frac{\alpha - 1 - \overline{B} + \sqrt{\overline{B}(8 - 8\alpha + \alpha^2 \overline{B})}}{(\alpha - 1)\overline{B}}.$$

One can show that $\Delta N(\overline{B};\alpha)$ increases in $\overline{B}$ and α as $B > 2$ and $\alpha \in (0,1)$, which implies the proposition. Given $\overline{B} \geq 2$, if $\Delta N(\overline{B};\alpha) = 0$, $B_{\#} = \frac{3+\sqrt{\alpha^2+8}}{1+\alpha} > 3$, which is decreasing in α. ∎

Proof of Proposition 7 Assume that $b_1 \geq b_2$.

Step 1: At optimality, the principal invests in one or two equally.

We divide the proof into several cases given $b_1 + b_2 = B$.

Case 1: $B \leq 2$

In this case, a potential successful outcome only comes from agent 1's innovation. As $P_d(B - b_2, b_2) = 1 - \frac{1}{B-b_2}$, we need to allocate all resources to agent 1.

Case 2: $B > 2$

Similar to the proof of Proposition 1, we do a special transformation to simplify the exposition of the proof: $b_2 = \frac{B}{2}[1 - \sqrt{1 - \frac{1-\alpha}{z}}]$ with $z \in [1-\alpha, \infty)$. There is $\hat{z} \geq 1 - \alpha$, such that $y_2 < 1$ only when $z < \hat{z}$.

Case 2.a: $z \geq \hat{z}$

In this case, $y_2^* = 1$, and similar to case 1, $P_d(z) := P_d(B - b_2(z), b_2(z))$ is increasing in $z \in [\hat{z}, \infty)$.

Case 2.b: $z \in [1-\alpha, \hat{z})$

Using the expressions of y_1^* and y_2^* in the proof of Lemma 1, the principal's objective becomes as follows,

$$P_d(z) = \frac{2 - 4\alpha + \alpha^2}{2(1-\alpha)^2} + \frac{\sqrt{d(z)} - 4z}{2(1-\alpha)^2 B}$$

where $d(z) := \alpha^4 B^2 + 8z(\alpha^2 B + 2\alpha - 2) + 16z^2$. By the FOC, one can verify that when $0 \leq \alpha < \frac{1}{2}$ and $2 < B \leq \frac{1-\alpha}{\alpha^2}$, $\sqrt{d(z)} - 4z$ is nondecreasing in $z > 1 - \alpha$; otherwise, $\sqrt{d(z)} - 4z$ is nonincreasing in $z > 1 - \alpha$.

In summary, $P_d(z)$ is quasi-convex in $z \in [1-\alpha, \infty)$, which implies that the principal invests in one or invest in two equally.

Step 2: Use up resource under two strategies.

If investing in two equally with resource $B > 2$, the objective of the investor is

$$P_d(B;\alpha) = 1 - y_1^* y_2^* = \frac{-4 - 4\alpha(B-1) + 2B + \alpha^2 B + \alpha\sqrt{B(8 - 8\alpha + \alpha^2 B)}}{2(1-\alpha)^2 B},$$

which can be shown that it is increasing in α and B as $B > 2$ and $\alpha \in (0,1)$. Therefore the principal will exhaust her resources if investing in two equally, i.e., $(b_1^*, b_2^*) = (\overline{B}/2, \overline{B}/2)$.

If investing in one with resource $B > 1$, the objective of the investor is

$$P_m(B) = 1 - \frac{1}{B},$$

which is increasing in $B > 1$. So $(b_1^*, b_2^*) = (\overline{B}, 0)$ when the principal invests in one agent.

Step 3: Comparison between two strategies.

Next, to derive the globally optimal allocation strategy for the principal, we need to compare $P_d(\overline{B}; \alpha)$ and $P_m(\overline{B})$. If $P_d(B_\#) = P_m(B_\#)$, $B_\# = \frac{(1+\alpha)^2}{\alpha^2} > 4$, which is decreasing in α. Moreover,

$$\frac{d[P_d(\overline{B}; \alpha) - P_m(\overline{B})]}{d\overline{B}}|_{\overline{B}=B_\#} = \frac{\alpha^4}{(3-\alpha)(1+\alpha)^3} > 0.$$

Then it implies the proposition. ■

Proof of Proposition 8 Without loss of generality, we assume that $b_1 \geq b_2$. Since $\alpha = \frac{4}{(2+g)^2}$ and $g \in (0, 1)$, we have $\alpha \in (\frac{4}{9}, 1)$ and $g = \frac{2}{\sqrt{\alpha}} - 2$.

Step 1: At optimality, the principal invests in one or two equally.

We divide the proof into several cases given $b_1 + b_2 = B$.

Case 1: $B \leq 2$

A potentially successful outcome only comes from agent 1's innovation. As $CS_d(B - b_2, b_2) = (1 - \frac{1}{B-b_2})/2$, we need to allocate all resources to agent 1.

Case 2: $B > 2$

Similar to the proof of Proposition 1, we do a special transformation to simplify the exposition of the proof: $b_2 = \frac{B}{2}[1 - \sqrt{1 - \frac{1-\alpha}{z}}]$ with $z \in [1-\alpha, \infty)$. There is $\hat{z} \geq 1 - \alpha$, such that $y_2 < 1$, only when $z < \hat{z}$.

Case 2.a: $z \geq \hat{z}$

In this case, $y_2^* = 1$, and similar to case 1, we get that $CS_d(z) := CS_d(B - b_2(z), b_2(z))$ is increasing in $z \in [\hat{z}, \infty)$.

Case 2.b: $z \in [1-\alpha, \hat{z})$

Using the expressions of y_1^* and y_2^* in the proof of Lemma 1, the principal's objective becomes

$$CS_d(z) = -\frac{\sqrt{\alpha}(4 + \sqrt{\alpha} - 3\alpha - \alpha^{3/2} + \alpha^2)}{2(\alpha^{3/2} + \alpha - \sqrt{\alpha} - 1)} - \frac{2z}{(1+\sqrt{\alpha})^2 B} + \frac{1 - 3\sqrt{\alpha} - \alpha + \alpha^{3/2}}{2(1-\sqrt{\alpha})(1+\sqrt{\alpha})^2 \alpha B}\sqrt{d(z)},$$

where $d(z) := \alpha^4 B^2 + 8z(\alpha^2 B + 2\alpha - 2) + 16z^2$. The second term is decreasing z. Due to $\alpha \in (\frac{4}{9}, 1)$, we have $\frac{1-3\sqrt{\alpha}-\alpha+\alpha^{3/2}}{2(1-\sqrt{\alpha})(1+\sqrt{\alpha})^2\alpha} < 0$ and as shown in Proposition 6, we have $d'(z) > 0$. As a result, the third term is also decreasing z. Then $CS_d(z)$ is decreasing in $z \in [1-\alpha, \hat{z})$.

In summary, $CS_d(z)$ is quasi-convex in $z \in [1-\alpha, \infty)$, which implies that the principal invests in one or invest equally in two.

Step 2: Use up resources under two strategies.

When the principal invests in one agent with resources $B > 1$, the expected consumer surplus is

$$CS_m(B) = \frac{1}{2} - \frac{1}{2B},$$

which is increasing in B. Hence, when the principal only invests in one agent, the principal will grant all available resources $\overline{B}$ to this agent, and the consumer surplus is $CS_m(\overline{B})$.

When the principal invests equally in two agents with resources B, the equilibrium failure probability is given by (24). The expected consumer surplus is

$$CS_d(B;g) = (1-y^*)^2 \frac{4(1+g)}{(2+g)^2} + (1-y^*)y^*.$$

Note that fixing g, we have

$$\frac{\partial C_d(B;g)}{\partial y^*} = -\frac{4+4g+g^2(2y_*-1)}{(2+g)^2} < -\frac{4+4g-g^2}{(2+g)^2} < 0.$$

Because $CS_d(B;g)$ decreases in y^* and y^* decreases in B, $CS_d(B;g)$ increases in B. Therefore, the principal chooses $(b_1^*, b_2^*) = (\overline{B}/2, \overline{B}/2)$.

Also, note that fixing B, we have

$$\frac{dCS_d(B;\alpha)}{dg} = -\frac{4g(1-y^*)^2}{(2+g)^3} - \frac{4+4g+g^2(2y_*-1)}{(2+g)^2}\frac{\partial y^*}{\partial g}.$$

The first term is negative and the second term is also negative as y^* decreases in α, or equivalently increases in g. Then it implies that fixing B, $CS_d(B;g)$ is decreasing in g.

Step 3: Comparison between two strategies.

Next, to derive the globally optimal allocation strategy for the principal, we need to compare $CS_d(\overline{B};\alpha)$ and $CS_m(\overline{B})$. If $CS_d(B_\#;\alpha) = CS_m(B_\#)$ and $B_\# > 2$, then $B_\#$ is unique as below,

$$B_\# = \frac{1}{2} + \frac{1}{2-8\sqrt{\alpha}+4\alpha} + \frac{6\sqrt{\alpha}}{3\sqrt{\alpha}+\alpha+\alpha^{3/2}-1} + \frac{(1-3\sqrt{\alpha}-\alpha+\alpha^{3/2})\sqrt{32\sqrt{\alpha}-16\alpha+\alpha^2-8}}{(1-4\sqrt{\alpha}+2\alpha)(3\sqrt{\alpha}+\alpha+\alpha^{3/2}-1)}.$$

Furthermore, note that $\lim_{\overline{B}\to 2} CS_d(\overline{B};\alpha) = 0 < \lim_{\overline{B}\to 2} CS_m(\overline{B})$ and $\lim_{\overline{B}\to\infty} CS_d(\overline{B};\alpha) = (1-y^*)^2\frac{4(1+g)}{(2+g)^2} + (1-y^*)y^*|_{y^*=0} = \frac{4(1+g)}{(2+g)^2} > \lim_{\overline{B}\to\infty} CS_m(\overline{B}) = \frac{1}{2}$. Combining with that $CS_d(B_\#) = CS_m(B_\#)$ has a unique solution, we conclude that: when $\overline{B} > B_\#$, $CS_d(\overline{B};\alpha) > CS_m(\overline{B})$; otherwise, $CS_d(\overline{B};\alpha) \le CS_m(\overline{B})$. Moreover, since fixing $\overline{B}$, $CS_m(\overline{B})$ does not depend on α and $CS_d(\overline{B};\alpha)$ is increasing in α (or equivalently decreasing in g). As a result, $B_\#$ will be decreasing in α. ■

Proof of Proposition 9 Assume that $b_1 \ge b_2$. Since $\alpha = \frac{4}{(2+g)^2}$ and $g \in (0,1)$, we have $\alpha \in (\frac{4}{9}, 1)$ and $g = \frac{2}{\sqrt{\alpha}} - 2$.

Step 1: At optimality, the principal invests in one or two equally.

We divide the proof into several cases given $b_1 + b_2 = B$.

Case 1: $B \le 2$

Note that when $B \le 2$, then $b_2 \le 1$. Due to the insufficient amount of resources to incentivize both agents to innovate, $y_2^* = 1$ for sure. The profit of the principal will only come from a potentially successful outcome from agent 1's innovation. Similar to Proposition 2, $W(b_2; B)$ is nonincreasing in $b_2 \in [0, \frac{B}{2}]$.

Case 2: $B > 2$

Similar to the proof of Proposition 1, we do a special transformation to simplify the exposition of the proof: $b_2 = \frac{B}{2}[1 - \sqrt{1 - \frac{1-\alpha}{z}}]$ with $z \in [1-\alpha, \infty)$. There is $\hat{z} \geq 1-\alpha$, such that $y_2 < 1$, only when $z < \hat{z}$.

Case 2.a: $z \geq \hat{z}$

In this case, $y_2^* = 1$ and the profit of the principal will only come from a successful outcome from agent 1's innovation. Then the objective of the principal is increasing in $z \in [\hat{z}, \infty)$ as agent 1 gets more resources.

Case 2.b: $z \in [1-\alpha, \hat{z})$

Using the expressions of y_1^* and y_2^* in the proof of Lemma 1, the principal's objective becomes as follows,

$$\begin{aligned} W(z) =& \frac{1}{2(1-\sqrt{\alpha})(1+\sqrt{\alpha})^2 \alpha B}[-\alpha^{3/2}(-4 - 3\sqrt{\alpha} + \alpha + 3\alpha^{3/2} + \alpha^2)B \\ & - 4(3+\sqrt{\alpha})\alpha z + (3 - \sqrt{\alpha}(1 + 3\sqrt{\alpha} + \alpha))\sqrt{d(z)}], \end{aligned}$$

where $d(z) := \alpha^4 B^2 + 8z(\alpha^2 B + 2\alpha - 2) + 16z^2 \geq 0$ and we have used $g = \frac{2}{\sqrt{\alpha}} - 2$. Denote $w(z) := -4(3+\sqrt{\alpha})\alpha z + (3 - \sqrt{\alpha}(1 + 3\sqrt{\alpha} + \alpha))\sqrt{d(z)}$. Then the problem of the investor is same as maximizing $w(z)$. Note that

$$w''(z) = \frac{64(1-\alpha)}{\sqrt{d^3(z)}}(3 - \sqrt{\alpha} - 3\alpha - \alpha^{3/2})(\alpha^2 B + \alpha - 1).$$

Case 2.b.1: $(3 - \sqrt{\alpha} - 3\alpha - \alpha^{3/2})(\alpha^2 B + \alpha - 1) \geq 0$

In this case, $w(z)$ is convex in $z \in [1-\alpha, \hat{z})$.

Case 2.b.2: $(3 - \sqrt{\alpha} - 3\alpha - \alpha^{3/2})(\alpha^2 B + \alpha - 1) < 0$

$w(z)$ is concave in $z \in [1-\alpha, \hat{z})$. If $w'(z) = 0$, we have two solutions z_1 and z_2 satisfying

$$z_1 + z_2 = 1 - \alpha - \frac{\alpha^2 B}{2},$$

and

$$z_1 - z_2 = -\frac{(3+\sqrt{\alpha})\alpha\sqrt{-(-9 + 24\sqrt{\alpha} - 13\alpha - 4\alpha^{3/2} + 2\alpha^2)(-1 + \alpha + \alpha^2 B)}}{9 - 15\sqrt{\alpha} - 2\alpha + 2\alpha^{3/2}}.$$

We will show that under Case 2.b.2, z_1 and z_2 can not be real or larger than $1-\alpha$.

To have a real solution, the following inequality should be satisfied:

$$-(-9 + 24\sqrt{\alpha} - 13\alpha - 4\alpha^{3/2} + 2\alpha^2)(-1 + \alpha + \alpha^2 b) \geq 0.$$

Combining with $[3-\sqrt{\alpha}(1+3\sqrt{\alpha}+\alpha)](\alpha^2 B+\alpha-1)<0$, $B>2$ and $\alpha\in(\frac{4}{9},1)$, the inequality above holds only when $\alpha\in(\frac{4}{9},\frac{1}{2})$ and $B\in(2,\frac{1-\alpha}{\alpha^2})$. Moreover, when $\alpha\in(\frac{4}{9},\frac{1}{2})$ and $B\in(2,\frac{1-\alpha}{\alpha^2})$, it can be verified that $z_1,z_2<1-\alpha$. Therefore, $w(z)$ is monotone in $z\in[1-\alpha,\hat{z})$.

In summary, $W(z)$ is convex or monotone in $z\in[1-\alpha,\hat{z})$ and $W(z)$ is increasing in $z\in[\hat{z},\infty)$. As a result, $W(z)$ is quasi-convex in $z\in[1-\alpha,\infty)$, which implies the claim shown in the proposition.

Step 2: Use up resources under two strategies.

When the principal invests in one agent with resources $B>1$, the expected welfare is

$$W_m(B)=\frac{3}{2}-\frac{3}{2B},$$

which is increasing in B. Hence, when the principal only invests in one agent, the principal will grant all available resources $\overline{B}$ to this agent, and the consumer surplus is $W_m(\overline{B})$.

When the principal invests equally in two agents with resources B, the equilibrium failure probability is given by (24). The expected welfare is

$$W_d(B;g)=(1-y^*)^2\frac{4(3+g)}{(2+g)^2}+3(1-y^*)y^*.$$

Note that fixing g,

$$\frac{\partial W_d(B;g)}{\partial y^*}=\frac{-12+4g+3g^2-16gy_*-6g^2y_*}{(2+g)^2}<\frac{-12+4g+3g^2}{(2+g)^2}<0.$$

Because $W_d(B;g)$ decreases in y^* and y^* decreases in B, $W_d(B;g)$ increases in B. Therefore, the principal chooses $(b_1^*,b_2^*)=(\overline{B}/2,\overline{B}/2)$.

Also, note that fixing B, we have

$$\frac{dW_d(B;\alpha)}{dg}=-\frac{4(4+g)(1-y^*)^2}{(2+g)^3}+\frac{-12+4g+3g^2-16gy_*-6g^2y_*}{(2+g)^2}\frac{\partial y^*}{\partial g}.$$

The first term is negative and the second term is also negative as y^* decreases in α, or equivalently increases in g. Then it implies that fixing B, $W_d(B;g)$ is decreasing in g.

Step 3: Comparison between two strategies.

Next, to derive the globally optimal allocation strategy for the principal, we need to compare $W_d(\overline{B};\alpha)$ and $W_m(\overline{B})$.

Denote $h=\frac{4}{\alpha\overline{B}+\sqrt{\overline{B}}\sqrt{8-8\alpha+\alpha^2\overline{B}}}$. As $\overline{B}\geq 2$, $h\in(0,1)$. Furthermore, $\overline{B}=-(2/(y(-a-y+ay)))$, then there is a one-to-one correspondence between h and $\overline{B}$. Hence, we get

$$W_d(\overline{B};\alpha)-W_m(\overline{B})=\frac{1}{4}[(8\sqrt{\alpha}+4\alpha-6)+(12-16\sqrt{\alpha}-5\alpha)h+(8\sqrt{\alpha}+\alpha-9)h^2].$$

The right hand is decreasing in h as $\alpha \in (4/9, 1)$, so $W_d(\overline{B}; \alpha) - W_m(\overline{B})$ is increasing in $\overline{B}$. Thus there is a threshold $B_{\#}$ such that when $\overline{B} > B_{\#}$, $W_d(\overline{B}; \alpha) > W_m(\overline{B})$; otherwise, $W_d(\overline{B}; \alpha) \leq W_m(\overline{B})$. Moreover, since fixing $\overline{B}$, $W_m(\overline{B})$ does not depend on α and $W_d(\overline{B}; \alpha)$ is increasing in α (or equivalently decreasing in g). Hence, $B_{\#}$ will be decreasing in α. ■

Proof of Proposition 10

$$R(k) = \max_{B \leq \overline{B}} \{\Pi(B) - kB\}$$

where $\Pi(B)$ the profit given total resources allocated is B. Clearly $R(k)$ is convex on k, as it is a supreme of affine functions. Then by the Envelope theorem, we get $\frac{dR(k)}{dk} = -B^*(k)$ and it is increasing in k. ■