Obstacle detection and avoidance module for the blind

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***Abstract*—Assistive technology enables people to achieve independence when performing daily tasks and it enhances their overall quality of life. Visual information is the basis for most navigational tasks, so visually impaired individuals are at disadvantage due to the lack of sufficient information about their surrounding environment. With recent advances in inclusive technology it is possible to extend the support given to people with visual disabilities in terms of their mobility. In this context we present and describe a wearable system (Blavigator project), whose global objective is to assist visually impaired people in their navigation on indoor and outdoor environments. This paper is focused mainly on the Computer Vision module of the Blavigator prototype. We propose an object collision detection algorithm based on stereo vision. The proposed algorithm uses Peano-Hilbert Ensemble Empirical Mode Decomposition (PH-EEMD) for disparity image processing and a two layer disparity image segmentation to detect nearby objects. Using the adaptive ensemble empirical mode decomposition (EEMD) image analysis real time is not achieved, with PH-EEMD results on a fast implementation suitable for real time applications.**

***Index Terms*— Assistive technologies; Obstacle detection; Stereo vision; Empirical Mode Decomposition, Peano Hilbert curves.**

I. Introduction

Assistive technology enables people with disabilities to accomplish daily tasks and assists them in communication, education, work and recreational activities. In general, it helps them to achieve greater independence and enhance their overall quality of life. From the different assistive technologies available, a special focus was put on those that enhance the mobility of blind or visually impaired people.

The World Health Organization estimates that 285 million people are visually impaired worldwide: 39 million are blind and 246 have low vision. Blind or visually impaired people have a considerable disadvantage as they lack information for bypassing obstacles and have relatively little information about landmarks, heading, and self-velocity. The main objective when designing this kind of assistive technology is to provide useful additional information to blind people during their mobility process, i.e. walking.

Human mobility comprises orientation and navigation. Orientation can be thought of as knowledge of the basic spatial relationship between objects in the environment. Information about position, direction, desired location, route, route planning etc, are all bound up with the concept of orientation. Navigation, in contrast, suggests an ability to move within the local environment. Navigation implies the knowledge of immediate objects and obstacles, features present on the floor (holes, stairs, flooring etc.), and of dangers coming from moving and/or stationary objects.

The present work proposes an object collision detection algorithm to be used in the computer vision (CV) module that will be integrated with the Blavigator prototype, described later in this paper. The new algorithm works in conjunction with previously developed algorithms [1] to provide secure navigation along a predefined route, in the presence of obstacles. Using a pre-calibrated stereo imaging system from Point Grey Research the computer vision module obtains disparity images and calculates the depth information required to perform layer image segmentation. The images are segmented at predefined distances to find obstacles nearby and to inform the user on how to avoid the detected objects. The algorithm we propose uses Peano-Hilbert Ensemble Empirical mode decomposition (PH-EEMD) to process the disparity image and a two layer image segmentation algorithm to detect obstacles along a predefined route. The information of the obstacles detected on the pathway is passed to the Decision Module of the Blavigator and then to the User Interface via vibrotactile interface.

The paper is organized as follows. Section II presents a classification of recent navigation systems to assist visually impaired people. Some projects that represent the state of the art are presented. Section III explains the proposed algorithm and the related techniques used. Section IV presents and discusses some results obtained. Finally, Section V concludes the paper with some final remarks.

II. Navigation system to assist visually impaired people

An Electronic Travel Assistant (ETA) has to supply the visually impaired with the necessary routing information to overcome obstacles in the near environment, with minimum errors. A distinction must be made between primary support systems such as the guide cane and the guide dog, and the secondary support systems, which use the most recent technologies.

These secondary support systems are the focus of the current study and typically consist of a wearable or handheld computer with Global Positioning System (GPS) responsible for macro navigation. In order to prevent collision with obstacles (micro navigation) these secondary systems also make use of the services provided by primary support systems. In the mid nineties, Collins and Loomis independently proposed the use of GPS to assist navigation for the visually impaired, in their navigation systems [2].

According to the model proposed by Loomis [2] a system to assist navigation for visually impaired people is composed by three main basic components: 1) the position and orientation unit is responsible for supplying the navigation system with the user's location in the form of local and/or global coordinates. Due to the strong dependence on the environment in which the system is being used, this is the functional block that more strongly characterizes the navigation system; 2) the Geographic Information System (GIS) contains geo-referenced data, stored in a database. This functional block is an essential component of the navigation system. Its main function is to store additional information about user's possible position, maps, object locations and possible dangers; 3) the user interface is the most critical component in the navigation system for assisting the visually impaired because it acts as a substitute for natural vision sensing (or so attempts to). The user interface must be user-friendly never reveal to be an 'obstacle' itself. Typically, interaction with the visually impaired user is made through audio interfaces, like Text-To-Speech (TTS) or virtual audio (sonification) and tactile displays like Braille keyboards or vibrotactile devices.

A. Navigation systems and related work

Navigation systems designed to assist visually impaired people can be classified in three main groups, based on their usage. Indoor systems are to be used in structured environments with less complex scenes, typically inside buildings or in controlled environments. Outdoor navigation systems are intended to be used in outdoor open space, typically on the street. Indoor/outdoor systems can be used in both indoor and outdoor environments, switching functionalities based on the surrounding environment.

Following paragraphs present some commercial and research (R&D) projects that currently describe the state of the art in outdoor navigation systems for assisting visually impaired people.

1) Navigation systems without local obstacle information, like NOPPA [3], Navigator [4] and Drishti [5] are GPS based systems to assist the navigation of visually impaired people. Their primary components are a PDA or Laptop specifically designed/adapted for people with visual disabilities, a Bluetooth GPS receiver and specially developed software for orientation, route mapping and configuration. User interaction can be made with a Braille display or a speech synthesizer.

2) Navigation systems with local obstacle information provide better knowledge of the local scenario, increasing the amount and the quality of the information provided to the blind user, which is very helpful to overcome local obstacles.

In the context of assisting visually impaired people, the computer vision model must deal with large amounts of image data (high bandwidth process) and provide useful information to the user (Human Computer Interaction - HCI) which is typically a low bandwidth process.

Some computer vision techniques have been used in navigation systems to assist people with visual disabilities. The iCare system [6] is a wearable face recognition system for individuals with visual impairments. This system uses computer vision algorithms for helping the individuals who are visually impaired during social interactions. The work presented by Zelek in [7] presents a portable system consisting of a stereo camera and haptic gloves. Obstacles and the ground plane are adequately extracted for navigation using computer vision algorithms. The intensity of haptic motor vibration was found to be appropriate for providing depth information. Conveying selective camera image content via haptic feedback can help way finding and provides the foundation for seeing by touch. A research team at the University of Wollongong [8] developed a vision system for providing 3D perception of the environment via transcutaneous electro-neural stimulation. This system uses computer vision to detect obstacles in the environment. The user uses his mental maps to locate himself based on the outputs of the system. The SmartEyes project [9] developed an enhanced orientation and navigation system for blind or visually impaired people. This system uses computer vision algorithms combined with a GPS receiver and a geographic information system to locate the user within a known environment. Neural Networks were used in NAVI [10], and later the same authors also tested Fuzzy Learning Vector Quantification (FLVQ) to classify objects in the scene.

B. Blavigator system

A system to assist the navigation of blind or visually impaired people is currently being developed at the University of Trás-os-Montes and Alto Douro (UTAD). This project is named Blavigator, which is the successor of the previously developed SmartVision project [1], and its main objective is to develop a cheap and easy to use mobile wearable navigation system that helps visually impaired people to navigate, providing ways to get to a desired location and, while doing so, providing contextual information about obstacles and points-of-interest (POI) like zebra-crossings, building entrances, etc. The system is built in a modular structure, combining several technologies.

III. Mathematical Background and Proposed Method

This section introduces the techniques used in our proposed method. Empirical Mode Decomposition (EMD) was used in image processing due to its properties, explained later. Real time two dimensions EMD applications are extremely hard to achieve. Thus we use one dimension EMD with a space-filing curve (SFC) to process images and to reduce the computing demands.

In the real world, data from natural phenomena like life science, social and economic systems are mostly non-linear and non-stationary. Fourier and wavelet transform (built upon predefined basis functions) are traditional methods that sometimes have difficulty in reveal the nature of complex real life data. The adoption of adaptive base functions introduced by Huang et al. [11] provided the means for creating intrinsic a posteriori base functions with meaningful instantaneous frequency in the form of Hilbert spectrum expansion [11]. This approach is embedded into a decomposition algorithm, namely Empirical Mode Decomposition (EMD) [11], which provides a powerful tool for adaptive multi-scale analysis of non-linear and non-stationary signals. EMD is a method of breaking down the signal without leaving the time domain. It filters out functions which form a complete and nearly orthogonal basis for the signal being analysed. These functions, known as Intrinsic Mode Functions (IMFs), are sufficient to describe the signal, even though they are not necessarily orthogonal. IMFs computed via an iterative 'sifting process' (SP), are functions with zero local mean having symmetric upper and lower envelopes. The SP depends both on an interpolation method and on a stop criteria that ends the procedure. Some updates to the 1D-EMD have been proposed which address mode mixing effects that sometimes occur in the EMD domain. In this sense, 1D-Ensemble EMD (1D-EEMD) has been proposed [12], where the objective is to obtain a mean ensemble of IMFs with mixed mode cancelation due to input signal noise addition.

A. Ensemble Empirical Mode Decomposition

One of the major drawbacks of the original 1D-EMD approach is the appearance of mode mixing, which is defined as a single IMF consisting of signals of widely disparate scales, or a signal of similar scale residing in different IMF components. Adding white noise scales uniformly through the whole time-scale or time-frequency space, which provides a reference distribution that facilitates the decomposition. The white noise added may also help extracting the true signals in the data, a truly Noise-Assisted Data Analysis [[12]]. The 1D-EEMD is implemented as follows:

1) Add white noise series w(t) to the data x(t), X(t) = x(t) + w(t);

2) Decompose X(t) data (with added white noise) into IMFs, $X(t) = \sum_{j=1}^{N} c_j(t) + r_N(t)$;

3) Repeat step 1 and step 2 several times with different noise series wi(t), Xi(t) = x(t) + wi(t), and obtain corresponding IMFs, $X_i(t) = \sum_{j=1}^{N} c_{ij}(t) + r_{iN}(t)$;

4) Finally, the ensemble mean of the corresponding IMFs that result from the decomposition are

$$c_j(t) = \frac{1}{N}\sum_{i=1}^{N} c_{ij}(t), \qquad (1)$$

where N is the number of ensemble members.

The sifting notion is essentially identical in 1D and 2D cases. Nevertheless, due to the nature of the 2D data of the images, some issues should be handled with care.

In particular, in 1D space the number of local extrema and zero crossings of an IMF must be the same, or differ by one [11]. In a 2D space, the IMFs typically use the definition of symmetry of upper and lower envelopes related to local mean [13]. There are many ways of defining the extrema in use; hence, different local extrema detection algorithms could be applied. Fast algorithms use the comparison of candidate extrema with its nearest 8-connected neighbours [14], while more sophisticated methods, like morphological reconstruction, are based on geodesic operators [15]. Furthermore, the interpolation method should rely on proper 2D spline interpolation of the scattered extrema points. In [13] the thin-plate smoothing spline interpolation is used. In Bi-dimensional Empirical Mode Decomposition (BEMD) [15] Radial Basis Functions are used for surface interpolation. This combination of 2D extrema extraction and 2D surface interpolation requires very heavy computational power, suitable neither for real-time implementations nor for use in mobile devices.

B. Peano Hilbert-Ensemble Empirical Mode Decomposition (PH-EEMD)

A space-filing curve is a continuous scan that passes through every pixel of the image only once. In order to transform an image (2D data) on a signal (1D), the space filing curve must preserve the neighborhood properties of the pixel [16]. These curves were first studied by Peano [17] and later by Hilbert [18] and corresponding algorithms are described in [19]. The Peano-Hilbert curve has three main interesting properties: (i) the curve is continuous; (ii) a scanning curve is continuous almost everywhere; and (iii) some parts of the curve are similar with whole curve suggesting a fractal structure. The Peano-Hilbert curve is the most popular recursive SFC and is used in many applications. Figure 1 represents three Peano-Hilbert space filing square curves of area one.

The advantages of space filing curves were combined with the 1D-EEMD algorithm proposed by Huang for image processing. Some extensions to 2D-EMD were proposed to deal with the 2D nature of the data, but these fully 2D-EMD approaches are very time consuming processes. An algorithm for 2D application was proposed with relatively low trend in processing time.

This algorithm is based on three phases:

1) Decompose the image using the Peano-Hilbert curve and get the equivalent 1D signal.

For the Peano-Hilbert algorithm, a recursive function is used to get the n-th order area one curve.

This procedure converts 2D data into 1D signal maintaining the local pixel spatial relations between neighbors.

2) Apply the 1D Ensemble Empirical Mode Decomposition (1D-EEMD) to the linear signal in order to compute the 1D Intrinsic Mode Functions that carry multi-scale space-frequency information. For the 1D-EEMD white noise with amplitude of 0.1, standard deviation of the original data is added and the process is repeated 8 times.

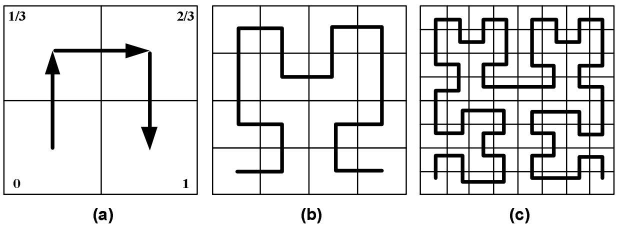

Fig. 1 Peano-Hilbert Curve: a) basic curve; b) 2 interactions; c) 3 interactions.

Boundaries problems are associated to most of the data processing algorithms due to finite data samples. In the 1D-EMD algorithm this is particularly true in the interpolation procedure on the sifting process; to solve this, an even data extension method is used [11].

3) To get the 2D data decomposition the inverse procedure must be taken to reconstruct the image from the data, using the Peano-Hilbert pixel spatial relations to process the 1D IMFs back to 2D IMFs, according Figure 1.

C. Proposed computer vision approach

In this section we propose an algorithm to detect and avoid obstacles that appears in front of the blind. The principal and mandatory characteristic is that the method must be robust and secure. The blind could be in danger and may cause severe injuries if the method fails, i.e. don't detect and avoid an obstacle in front of the blind. Take this into account, the proposed algorithm must detect obstacles in front of the blind in real time and pass the information to the Decision Module of the Blavigator. With this algorithm a safe walking area (SWA) of two meters long is created.

Nearby objects need to be detected and for practical walking conditions, two distance detected ranges were chosen. At one meter deep is considered as near proximity and represent critical security detection and the two meter threshold is used for general detection.

According to camera settings (f=0,765731 and B=0,120049) disparity maps are computed for one meter range and two meters range, originating two layers that correspond to disparities of 27 and 16, respectively. Remember that larger disparity values correspond to closer objects. Depending on depth resolution, object size and orientation, objects will generally appear at two or more depth planes.

The proposed algorithm has four steps, as follows:

1) Decompose the disparity image with Peano-Hilbert Ensemble Empirical Mode Decomposition.

Figure 2 shows the result of a typical UTAD campus disparity image decomposition with five IMFs and the residue;

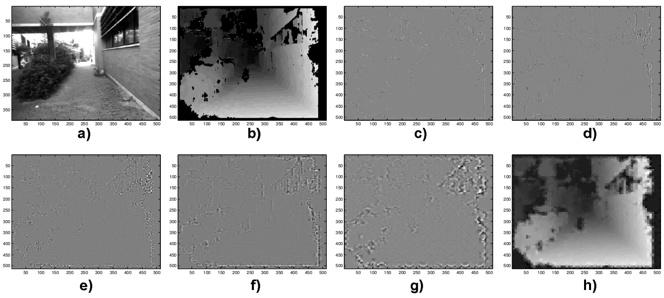

Fig. 2 Peano-Hilbert EEMD disparity image decomposition. Figure a) is the real image, b) is the disparity image, c) to g) are the IMFs and h) is the Residue.

2) The disparity image is filtered to eliminate higher frequencies containing noise and fine details. This process is achieved in the reconstruction of Ensemble Empirical Mode Decomposition by eliminating the first two IMFs and using root mean square error minimization according to (1),

$$x(t) = \sum_{i=3}^{N} c_i(t) + r_N(t), \text{ were i start at 3 IMF;}$$

EEMD, due to its adaptive and data driven process, has advantages over traditional filtering processes. Because different types of images appear in UTAD campus the process is adapted to each type of image. In situations where images are affected by heterogeneous illuminations, by removing the last IMFs and residue of EEMD that contains low frequencies the image equalization is achieved. For the last, EEMD produce space-frequency multi-scale decomposition which we will use later (step 4) for objet recognition;

3) To minimize the data to be processed two regions of interests (ROIs) were created near the blind. Data outside of ROIs are far from the blind and are not critical to ensure safe walk thought possible obstacles and can be set to zero.

In our case we chose to analyse the depth information at one meter which is used to prevent critical situations of possible obstacle collision and two meters for the guiding process;

4) Finally, we will apply range image segmentation by combining the information from depth maps with real images to extract object information at those distances. This procedure will be useful for future object recognition.

IV. Experimental results and discussion

The first test is the capability of the proposed algorithm to run in real time which will ensure the safe walking of the blind. The EEMD processing image cannot run in real time due to its computing demands. In order to test computational cost gain of the proposed algorithm with PH-EEMD method when compared with EEMD a set of different representative images with different resolutions were fully decomposed and decomposition times were. Table 1 shows measured times compared with the algorithm using EEMD as a reference.

TABLE 1 Relative execution times of PHEEMD to EEMD algorithms

Resolution	EEMD	PHEEMD
64x64	1.0	10
128x128	1.0	15
256x256	1.0	20
512x512	1.0	22

For our typical resolution our method runs approximately 20 times faster and is able to process tree to five images per second which is acceptable for obstacles detection and safe walking of the blind. Compared with previously developed algorithm [20] the optimization of this phase with PH-EEMD reduces substantially the computation power demands.

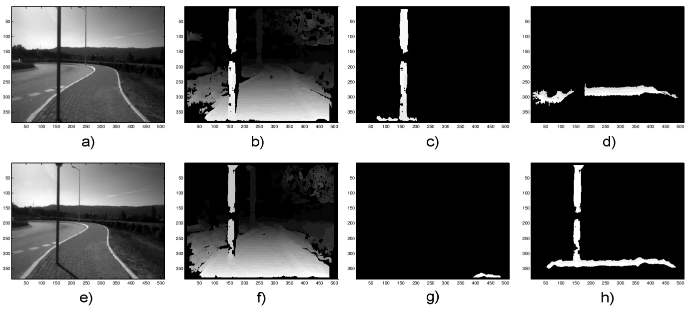

Fig. 3 Results of the proposed algorithm for obstacle detection. Figures a) and e) are real images, b) and f) are filtered EEMD disparity images, c) and g) are disparity images segmented at one meter, d) and h) are disparity images segmented at two meters.

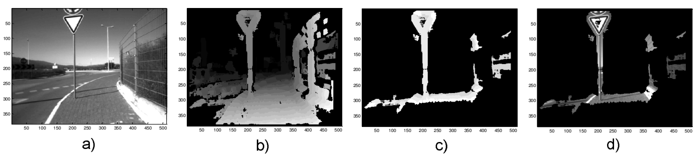

Fig. 4 Results of the proposed algorithm for obstacle detection. Figures a) real image, b) EEMD filtered disparity image, c) segmented disparity at two meters, d) real object segmentation.

In order to test the proposed algorithm, a set of different images were taken on UTAD campus. The stereo images obtained with Bumblebee have a lot of noise due to the outdoor environment characteristics. In addition, we have to deal with illumination problems which also affect the quality of stereo images.

Figure 3 represents the global results of information extraction taken from a sample image of one of the predefined routes. Figures 3a) and 3e) are sample images to be analyzed. Figures 3b) and 3f) are disparity maps processed in the first and second steps of the algorithm. Disparity maps are decomposed, filtered with PH-EEMD to remove noise, fine details and some inhomogeneous illumination present in some stereo images.

Figures 3c) and 3g) are segmented disparity maps at distances of about one meter from the stereo camera on which the remaining distance levels are set to zero. At this range in all of the images processed the ground effect does not appear.

Figures 3d) and 3h) represent the same disparity map segmentation but at a distance of two meters away from camera. The main difference to the previous segmentation (1 meter away from the user) is that the ground floor effect appears. In future implementations, this issue needs to be minimized or removed.

Figure 4 shows the described methodology applied to another typical image (4a), 4b) and 4c)). Data from segmented disparity maps is combined with real image to extract useful information from objects, as shows in Figure 4d). Using this configuration, the disparity map was segmented at two meters and, as previously, the ground floor effect also appears.

Based on the relative position of the blind user and the detection of the nearest objects in the environment it is possible to compute the trajectory correction and respective output to be given to the blind user. The user interface uses microvibrators that signals corrections corresponding to the five directions: left, left-diagonally, straight, right diagonally and right.

V. Conclusion

In the presented work a computer vision obstacle detection module for the Blavigator project was proposed. For an efficient navigation the CV module must detect accurately specific features in the environment. In outdoor navigation, specifically due to very different possible scenarios, we adopted to implement a two layer obstacle detection and avoidance. The first layer, representing information at two meters in front of the user is intended for object detection, and a second layer, one meter away from the user is used for backup and trajectory correction purposes. Using the stereo vision system it is possible to implement range image segmentation and extract useful information for object detection and recognition.

The Blavigator prototype is also composed by other modules, as seen in Section II and, at the moment, they are all being integrated. The CV module is assembled on a laptop and at the moment it is capable of real time stereo image processing (13 frames per second). In order to reduce the computation complexity in image analysis, the PH-EEMD optimization was used in the phase's one and two. At these image resolutions and without any other strategy the system is near the real time execution. In addition a set of tests are necessary to be done to the assembled system by blind users to validate and improve the system.

Further work is needed to enhance the overall accuracy, for example to remove or minimize ground floor effects and the implementation of object recognition. Future improvements will continue to use PH-EEMD for image analysis.

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