共轭梯度法

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共轭梯度法解方程的算法流程

例 1: 设方程组

$$\begin{bmatrix} 6 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \tag{1}$$

取 $x^{(0)} = [0,0]^T$, 试用共轭梯度法求解方程组.

解: 共轭梯度法解方程组的计算程序如下取一个厨师向量 x_0, 计算 $x_0 = Ax_0 - b$, 取 $g0 = r0$

$$\begin{cases} \alpha_k = \frac{(r_k, r_k)}{(Ag_K, g_k)} \\ X_{k+1} = X_k + \alpha_k g_k, r_{k+1} = r_k + \alpha_k A g_k, k = 0, 1, 2, \cdots \\ \lambda_k = \frac{(r_{k+1}, r_{k+1})}{(r_K, r_k)} \\ g_{k+1} = r_{k+1} + \lambda_k g_k \end{cases} \tag{2}$$

具体计算过程

$$\begin{bmatrix} 6 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \tag{3}$$

其中

$$A = \begin{bmatrix} 6 & 3 \\ 3 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

取 $x_0 = x^{(0)} = [0,0]^T$, 则代入(1)式得

$$\alpha_0 = -\frac{(r_0, r_0)}{(Ag_0, g_0)} = \frac{\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}} = -\frac{1}{2}$$

$$X_1 = X_0 + \alpha_0 g_0 = [0, \frac{1}{2}]^T$$

$$r_1 = r_0 + \alpha_0 A g_0 = [-\frac{3}{2}, 0]^T$$

$$\lambda_k = \frac{(r_1, r_1)}{(r_0, r_0)} = \frac{9}{4}$$

$$g_1 = r_1 + \lambda_0 g_0 = [-\frac{3}{2}, \frac{9}{4}]^T$$

进一步计算可得

$$\alpha_1 = -\frac{(r_1, r_1)}{(A g_1, g_1)} = -\frac{2}{3}$$

$$X_2 = X_1 + \alpha_1 g_1 = [1, -2]^T$$

$$r_2 = r_1 + \alpha_1 A g_1 = [0, 0]^T$$

$$\lambda_1 = \frac{(r_2, r_2)}{(r_1, r_1)} = 0$$

$$g_2 = r_2 + \lambda_1 g_1 = [0, 0]^T$$

则可知方程组的解 $x = X_2 = [1, -2]^T$.

参考文献