1. Garicano 2000 assume time spent helping can be nonstochastic because of LLN - note that time spent helping is a function of $F(k^r)$. Note also that Garicano 2000 assumes linear utility by the firm because the firm is just optimizing output. I will also assume the nonstochasticity of helping, so that t_h^w is a deterministic constant

Difference between $u(P(t_p^r)F(k^r))$ and $P(t_p^r)u(F(k^r))$ is twofold.

1. The marginal benefit of t_p^r in $u(P(t_p^r)F(k^r))$ is affected by the curvature of u while the marginal benefit of t_p^r in $P(t_p^r)u(F(k^r))$ is affected by the shape of u
2. In $u(t_p^r F(k^r))$, our marginal utility is affected by the number of problems we solve correctly, $P(t_p^r)F(k^r)$. In $P(t_p^r)u(F(k^r))$, our marginal utility is affected by effort and knowledge separately.
3. It seems like it's reasonable to separate % of problems solved from number of problems attempted. Even omitting costs from the conversation, we don't want to treat the satisfaction someone gets from solving 1 problem (after trying to solve 10) the same way someone gets from solving 1 problem (after trying to solve 1).

Let P_s be the proportion solved, with $P_s \sim N(F(k^r), \frac{F(k^r)(1-F(k^r))}{t_p^r})$. Since

$$E[P(t_p^r)u(P_s))] = P(t_p^r)E[u(P_s)]$$

and we can rewrite

$$E[u(P_s)] \approx u(F(k^r)) + \frac{u''(F(k^r))F(k^r)(1-F(k^r))}{2t_p^r}$$

then the read rank programmer solves

$$\max_{k^r, t_p^r} P(t_p^r)\left(u(F(k^r)) + \frac{u''(F(k^r))F(k^r)(1-F(k^r))}{2t_p^r}\right) - c_r(t_p^r, k^r)$$

with first order conditions

$$[k^r] : P(t_p^r)f(k^r)\left(u'(F(k^r)) + \frac{u'''(F(k^r))F(k^r)(1-F(k^r)) + u''(F(k^r))(1-2F(k^r))}{t_p^r}\right) = \frac{\partial c_r}{\partial k^r}$$

$$[t_p^r] : P'(t_p^r)u(F(k^r)) + \frac{(2P'(t_p^r)t_p^r + P(t_p^r))(F(k^r)(1-F(k^r)))}{4(t_p^r)^2}u''(F(k^r)) = \frac{\partial c_r}{\partial t_p^r}$$

Assuming risk neutrality about u_r, then $u'' = u''' = 0$ so

$$P(t_p^r)f(k^r)\alpha_r = \frac{\partial c_r}{\partial k^r} \qquad P'(t_p^r)\alpha_r F(k^r) = \frac{\partial c_r}{\partial t_p^r}$$

- Suppose $u_r = \alpha_r F(k^r)$
- Suppose $P(t_p^r) = Pt_p^r$

Example $c_r(k^r, t_p^r) = \alpha_k k^r + \alpha_p t_p^r$

$$\frac{t_p^r f(k^r)}{F(k^r)} = \frac{\frac{\partial c_r}{\partial k^r}}{\frac{\partial c_r}{\partial t_p^r}} \iff \frac{t_p^r f(k^r)}{F(k^r)} = \frac{\alpha_k}{\alpha_p} \iff t_p^r = \frac{\alpha_k F(k^r)}{\alpha_p f(k^r)}$$

Then, the problem becomes

$$\begin{aligned}
&\max_{t_p^r, k^r} Pt_p^r u(F(k^r)) - \alpha_k k^r - \alpha_p t_p^r \\
&= \max_{k^r} P\frac{\alpha_k \alpha_r (F(k^r))^2}{\alpha_p f(k^r)} - \alpha_k k^r - \frac{\alpha_k F(k^r)}{f(k^r)} \\
&= \max_{k^r} P\frac{\alpha_k \alpha_r}{\alpha_p}(e^{k^r} - 1)(1 - e^{-k^r}) - \alpha_k k^r - \alpha_k(e^{k^r} - 1)
\end{aligned}$$

Thus, at the optimal k^r,

$$\begin{aligned}
&\frac{\partial}{\partial k^r}\left(\frac{P\alpha_k\alpha_r}{\alpha_p}(e^{k^r} - 2 + e^{-k^r}) - \alpha_k k^r - \alpha_k(e^{k^r} - 1)\right) = 0 \\
&\iff \frac{P\alpha_k\alpha_r}{\alpha_p}(e^{k^r} - e^{-k^r}) = \alpha_k(e^{k^r} + 1) \\
&\iff \frac{P\alpha_r}{\alpha_p} = \frac{e^{k^r} + 1}{e^{k^r} - e^{-k^r}}
\end{aligned}$$

and we find that

$$k^r = \log\left(\frac{P\alpha_r}{P\alpha_r - \alpha_p}\right) \tag{1}$$

$$t_p^r = \frac{\alpha_k}{P\alpha_r - \alpha_p} \tag{2}$$

When the cost of knowledge acquisition α_k goes up, then production effort increases, per 2. Knowledge acquired stays the same, per 1. This occurs because the decrease in knowledge caused by its increased marginal cost is precisely balanced out by the need for more knowledge because it complements increased production effort.
I suspect this is because the cost function is linear. Next, I test how this changes with

a cost function with marginally increasing costs. Also I think the linear cost function is fucked...

Example $c_r(k^r, t_p^r) = \alpha_k (k^r)^2 + \alpha_p (t_p^r)^2$

$$\frac{t_p^r f(k^r)}{F(k^r)} = \frac{\frac{\partial c_r}{\partial k^r}}{\frac{\partial c_r}{\partial t_p^r}} \iff \frac{t_p^r f(k^r)}{F(k^r)} = \frac{\alpha_k k^r}{\alpha_p t_p^r} \iff t_p^r = \sqrt{\frac{\alpha_k}{\alpha_p} k^r (e^{k^r} - 1)}$$

Thus, the problem becomes

$$\begin{aligned}
&\max_{t_p^r, k^r} P t_p^r u(F(k^r)) - \alpha_k (k^r)^2 - \alpha_p (t_p^r)^2 \\
&= \max_{k^r} P\alpha_r (1 - e^{-k^r}) \sqrt{\frac{\alpha_k}{\alpha_p} k^r (e^{k^r} - 1)} - \alpha_k (k^r)^2 - \alpha_k k^r (e^{k^r} - 1)
\end{aligned}$$

Thus, at the optimal k^r,

$$\begin{aligned}
&\frac{\partial}{\partial k^r} \left(P\alpha_r (1 - e^{-k^r}) \sqrt{\frac{\alpha_k}{\alpha_p} k^r (e^{k^r} - 1)} \right) = \frac{\partial}{\partial k^r} \left(\alpha_k (k^r)^2 - \alpha_k k^r (e^{k^r} - 1) \right) = 0 \\
&\iff P\alpha_r \left(\frac{1}{\alpha_k \alpha_p} \right)^{\frac{1}{2}} (1 - e^{-k^r}) = 2\sqrt{k^r (e^{k^r} - 1)}
\end{aligned}$$

and we find that

$$\frac{2\sqrt{k^r (e^{k^r} - 1)}}{1 - e^{-k^r}} = \frac{P\alpha_r}{\sqrt{\alpha_k \alpha_p}} \tag{3}$$

$$t_p^r = \frac{P\alpha_r}{2\alpha_p} (1 - e^{-k^r}) \tag{4}$$

I've omitted the algebra for the optimal k^r calculation and included it in quadratic.pdf When α_k, the cost of knowledge acquisition increases, k^r decreases as $\frac{2\sqrt{k^r (e^{k^r} - 1)}}{1 - e^{-k^r}}$ is increasing in k^r, per 3. Production effort decreases because while the marginal rate of substitution means we shift effort towards production, the negative effect of the increased cost in knowledge and the reduction in production's marginal benefit (because of a reduction in knowledge) is greater.

Differences in t_p^r functional form Let us consider an example where the marginal value of t_p^r is increasing in t_p^r.

- Set up the scenario - Interested in understanding whether knowledge decreases more or less, and whether production decreases more or less 1) When knowledge acquisition costs increase, knowledge decreases in both cases, no matter what. Basic economic principle - when costs increase, consumption decreases 2) How much it decreases depends

on how production levels respond. Since $P'e^{t_p^r} = Pt_p^r$, if production didn't respond to knowledge, the two cases have identical marginal benefit curves 3) The thing is that production does respond to knowledge because production and knowledge are complements. Consequently, decreases in knowledge also decrease production. What we're interested in is the magnitude of the decrease when $P(t_p^r) = P'e^{t_p^r}$ vs. $P(t_p^r) = Pt_p^r$.
Suppose $P'e^{t_p^r}$ is above Pt_p^r; that is, for all $1 < t < t_p^r$, $P'e^t > Pt$. Then, the individual with $P(t') = P'e^{t'}$ can attain the same benefit from production as someone with $P(t'') = Pt''$ at much lower levels of production effort so that $t' < t''$. But now the problem is I don't know whether $P'e^{t'} \gtreqless Pt''$ even if I know $t' < t''$. The same is true with the opposite in the below curve case
The intuition is helpful but I think I need to do the math to actually show what's going on. Using a marginally decreasing functional form is similar - it's pinned down by the above/below curve case
Then, I think we can describe how this would change with u'', u''' that are non zero
First of all, I really just want to assume away the u'' term. I can assume quadratic utility, or I can assume that t_p^r is sufficiently large (which I prefer).
Define $k^r, t_p^r, k^{r*}, t_p^{r*}$, where k^r, t_p^r are optimal knowledge and production for $P(t_p^r) = Pt_p^r$ and k^r, t_p^r are optimal knowledge and production for $P^*(t_p^r) = P^*e^{t_p^r}$. Now, suppose we define P^* such that $k^r = k^{r*}$, $t_p^r = t_p^{r*}$ and $P(t_p^r) = P^*(t_p^r)$ (assuming $\alpha_r, \alpha_p, \alpha_k$ are shared). When α_k decreases, **change depends on the shape of the lines**
we expect production to decrease more in the case of P^* because the relative loss of losing production is less (think e^x above $x + 1$ at $x = 0$). This also decreases the marginal value of knowledge in the case of P^* more than in the case of P so knowledge will also decrease more.
I don't have the intuition for production and should try some functional forms... I also don't have intuition for knowledge and should try some functional forms... but this one will be harder (try quadratic of some form) Ask Jesse if the $k^r < 1$ is a problem - resolved by using $1 - e^{-(x-1)}$ imo

1 Old

Suppose that $F(k^r)$ is distributed exponentially, following the literature[1]. When $\frac{\partial c_r}{\partial k^r}$ decreases, k^r responds by increasing. This will cause $P'(t_p^r)$ to either decrease or increase,

[1] see how varying this changes the way people behave

depending on whether $\frac{\partial}{\partial t_p^r}(\frac{\partial c_r}{P'(t_p^r)\partial t_p^r}) > 0$. When $\frac{\partial}{\partial t_p^r}(\frac{\partial c_r}{P'(t_p^r)\partial t_p^r}) > 0$ ($\frac{\partial}{\partial t_p^r}(\frac{\partial c_r}{P'(t_p^r)\partial t_p^r}) < 0$), then t_p^r increases (decreases) in response to increases in knowledge acquisition.
I test three different representative examples for $P(t_p^r)$:
Constant marginal value: $P(t_p^r) = \alpha t_p^r$

$$\alpha\alpha_r t_p^r f(k^r) = \frac{\partial c_r}{\partial k^r} \qquad \alpha\alpha_r F(k^r) = \frac{\partial c_r}{\partial t_p^r}$$

Since $\frac{\partial}{\partial t_p^r}(\frac{\partial c_r}{\alpha_r \partial t_p^r}) > 0$, t_p^r increases in response to increases in knowledge acquisition.
Increasing marginal value: $P(t_p^r) = e^{t_p^r}$

$$\frac{f(k^r)}{F(k^r)} = \frac{\frac{\partial c_r}{\partial k^r}}{\frac{\partial c_r}{\partial t_p^r}}$$

When $\frac{\partial}{\partial t_p^r}(\frac{\partial c_r}{e^{t_p^r}\partial t_p^r}) > 0$, t_p^r increases in response to increases in knowledge acquisition. It's possible in this case for production effort t_p^r to decrease, but it must be accompanied by increases in knowledge k^r (or otherwise the read-rank contributor would be worse off).
Decreasing marginal value: $P(t_p^r) = \log(t_p^r)$

$$\frac{\log(t_p^r) f(k^r)}{t_p^r F(k^r)} = \frac{\frac{\partial c_r}{\partial k^r}}{\frac{\partial c_r}{\partial t_p^r}}$$

When $\frac{\partial}{\partial t_p^r}(t_p^r \frac{\partial c_r}{\partial t_p^r}) > 0$, t_p^r increases in response to increases in knowledge acquisition. More likely,

While it may seem interesting that production increases when the cost ratio of production to the marginal value of production increases, note the following few facts

1. If the ratio decreased, since $\alpha_r F(k^r) > \frac{\frac{\partial c_r}{\partial t_p^r}}{P'(t_p^r)}$, production would occur forever
2. If the ratio decreased as we increased t_p^r, the previous equilibrium would not have been an equilibrium as we could have increased t_p^r, and we would have had

$$P(t_p^r) f(k^r)\alpha_r > \frac{\partial c_r}{\partial k^r} \qquad P'(t_p^r)\alpha_r F(k^r) > \frac{\partial c_r}{\partial t_p^r}$$

When contributors are risk averse, compared to risk neutral contributors, their knowledge to production effort ratios are higher. Mathematically, this is because $u'' < 0$

and $u''' > 0$. As such, $u_k^3 = \frac{u'''(F(k^r))F(k^r)(1-F(k^r))}{t_p^r}$ is positive, $u_2^k = \frac{u''(F(k^r))(1-2F(k^r))}{t_p^r}$ is positive for $k^r > \frac{1}{2}$ and $u_2^{t_p^r} = \frac{(2P'(t_p^r)t_p^r + P(t_p^r))(F(k^r)(1-F(k^r)))}{4(t_p^r)^2} u''(F(k^r))$ is negative. How to handle $k^r < \frac{1}{2}$??

Moving these expressions to the marginal costs side of the expression, we get

$$[k^r] : P(t_p^r)f(k^r)u'(F(k^r)) = \frac{\partial c_r}{\partial k^r} - P(t_p^r)(f(k^r)u_k^3 + u_2^k)$$
$$[t_p^r] : P'(t_p^r)u(F(k^r)) = \frac{\partial c_r}{\partial t_p^r} - u_2^{t_p^r}$$

The marginal costs side (RHS) of knowledge is less than in the risk neutral case. On the other hand, regardless of k^r's magnitude, the marginal costs side (LHS) of production effort is higher than in the risk neutral case. An increase knowledge acquired k^r, and decreasing production effort t_p^r fits with this narrative. Intuitively, this makes sense; risk-averse individuals want to increase how much of the uncertain good, knowledge, they acquire, which comes at the expense of production.

On the other hand, we can flip the signs for the calculation above to show that risk seeking individuals are comfortable with lower levels of knowledge acquisition and consequently spend more time on production. This also makes economic sense - they derive higher utility from the same level of knowledge acquisition than a risk neutral or risk averse individual in expectation and since costs are concave, they expend additional time on production. [2]

Two further notes: First, risk neutrality about knowledge acquisition may not always be a reasonable assumption. For someone who contributes to OSS only because they might want feedback/to share their solutions for a bug, OSS contribution has a small enough impact on their daily routine that we can consider them risk neutral. For someone like a student, whose hoping to acquire programming knowledge through contributing to OSS, the potential benefits of knowledge acquisition through OSS contribution might be large enough that they are affected by their personal risk neutral/risk favouring preferences.

Second, for write rank individuals, following Garicano 2000, we will assume that the value of unsolved problems $\beta_w t_h^w$ is non stochastic by the large of law numbers. Since they divide responsibility equally in deterministic fashion, then their solution does not need to be calculated with expectations in mind.

[2] Work more on both of these sections in the appendix