$$\frac{P(t_p^r)\, f(k^r)\, \alpha_r}{P'(t_p^r)\, \alpha_r\, F(k^r)} = \frac{\frac{\partial C}{\partial k^r}}{\frac{\partial C_r}{\partial t_p^r}}$$

$$\frac{t_p^r \cdot f(k^r)}{F(k^r)} = \frac{\frac{\partial C}{\partial k^r}}{\frac{\partial C_r}{\partial t_p^r}} \qquad C_r = \alpha_k {k^r}^2 + \alpha_p {t_p^r}^2$$

$$= \frac{\alpha_k k_r}{\alpha_p t_p}$$

$$t_p^r = \sqrt{\frac{\alpha_k}{\alpha_p} k_r \left(e^{k_r} - 1\right)} \qquad \frac{F(k^r)}{f(k^r)} = e^{k_r} - 1$$

$$\max\; P\, t_p^r\, \alpha_r F(k^r) - \alpha_k k_r^2 - \alpha_p {t_p^r}^2$$

$$P \sqrt{\frac{\alpha_k}{\alpha_p} k_r \left(e^{k_r} - 1\right)}\; \alpha_r F(k^r) - \alpha_k k_r^2 - \cancel{\alpha_p} \frac{\alpha_k}{\cancel{\alpha_p}} k_r \left(e^{k_r} - 1\right)$$

$$\frac{\partial}{\partial k_r}\left(P d_r (1-e^{-k_r})\sqrt{\frac{d_k}{d_p} k_r(e^{k_r}-1)} - d_k\left(k_r^2 + k_r(e^{k_r}-1)\right)\right) = 0$$

$$\Downarrow$$

$$\frac{P d_r (1-e^{-k_r})\left(\frac{d_k}{d_p}\right)^{\frac{1}{2}}(e^{k_r}-1+k_r e^{k_r})}{2\sqrt{k_r(e^{k_r}-1)}} + P d_r e^{-k_r}\sqrt{\frac{d_k}{d_p} k_r(e^{k_r}-1)} = d_k\left(2k_r + e^{k_r}(k_r+1)-1\right)$$

$$P d_r \left(\frac{d_k}{d_p}\right)^{\frac{1}{2}}\left(\frac{(1-e^{-k_r})(e^{k_r}-1+k_r e^{k_r})}{2\sqrt{k_r(e^{k_r}-1)}} + e^{-k_r}\sqrt{k_r(e^{k_r}-1)}\right) = \quad \Downarrow$$

$$P d_r \left(\frac{1}{d_k d_p}\right)^{\frac{1}{2}}\left(\frac{(1-e^{-k_r})(e^{k_r}-1+k_r e^{k_r}) + 2k_r(1-e^{-k_r})}{2\sqrt{k_r(e^{k_r}-1)}}\right) = 2k_r + e^{k_r}(k_r+1)-1$$

$$P d_r \left(\frac{1}{d_k d_p}\right)^{\frac{1}{2}}\left(\frac{(1-e^{-k_r})\cancel{(e^{k_r}-1+k_r e^{k_r}+2k_r)}}{2\sqrt{k_r(e^{k_r}-1)}}\right) = \cancel{2k_r + e^{k_r}(k_r+1)-1}$$

$$P d_r \left(\frac{1}{d_k d_p}\right)^{\frac{1}{2}}(1-e^{-k_r}) = 2\sqrt{k_r(e^{k_r}-1)}$$

d_k increases,

smaller $\left\{ \frac{P d_r}{\sqrt{d_k d_p}} \right. = \frac{2\sqrt{k_r(e^{k_r}-1)}}{1-e^{-k_r}}$ So k_r decreases

$$\frac{P d_r}{\sqrt{d_p}}\,\frac{(1-e^{-k_r})}{2} = \sqrt{k_r(e^{k_r}-1)\,d_k}$$

$$t_p^r = \int \frac{d_k}{d_p} k r \left(e^{kr} - 1\right) = \frac{P d_r}{2 d_p}\left(1 - e^{-kr}\right)$$