HW4-2

min $z = 2x_1 + 3x_2 + 4x_3$

s.t. $x_1 - x_2 + x_3 \geq 10$

$x_1 - 2x_2 + 3x_3 \geq 6$

$3x_1 - 4x_2 + 5x_3 \geq 15$

$x_1, x_2, x_3 \geq 0$

to standard form →

standard form

max $z = -2x_1 - 3x_2 - 4x_3$

s.t. $-x_1 + x_2 - x_3 \leq -10$

$-x_1 + 2x_2 - 3x_3 \leq -6$

$-3x_1 + 4x_2 - 5x_3 \leq -15$

$x_1, x_2, x_3 \geq 0$

to slack form →

slack form

max $z = -2x_1 - 3x_2 - 4x_3$

s.t. $s_1 = -10 + x_1 - x_2 + x_3$

$s_2 = -6 + x_1 - 2x_2 + 3x_3$

$s_3 = -15 + 3x_1 - 4x_2 + 5x_3$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

4-2-2 : It's not appropriate to use (primal) simplex method since all $b_i < 0$, meaning that it's infeasible.

Instead, the LP is in optimality condition because all c_js are less than 0.

Use dual simplex method

max $-z = -2x_1 - 3x_2 - 4x_3$

s.t. $s_1 = -10 + x_1 - x_2 + x_3$

$s_2 = -6 + x_1 - 2x_2 + 3x_3$

$s_3 = -15 + 3x_1 - 4x_2 + 5x_3$ → pivot s_3. min$_j$ $\left(\frac{-2}{-3}, \frac{-4}{-5}\right)$ choose x_1

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

$x_1 = 5 + \frac{1}{3}s_3 + \frac{4}{3}x_2 - \frac{5}{3}x_3$

max $-z = -10 - \frac{2}{3}s_3 - \frac{1}{3}x_2 - \frac{2}{3}x_3$

s.t. $s_1 = -5 + \frac{1}{3}s_3 + \frac{1}{3}x_2 - \frac{2}{3}x_3$ ⇒ pivot s_1 min$_j$ $(2, 1)$ choose s_3

$s_2 = -1 + \frac{1}{3}s_3 - \frac{2}{3}x_2 + \frac{4}{3}x_3$

$x_1 = 5 + \frac{1}{3}s_3 + \frac{4}{3}x_2 - \frac{5}{3}x_3$

$s_3 = 15 + 3x_1 - x_2 + 2x_3$

$\max -z = -10 - \frac{2}{3}(15+3S_1-X_2+2X_3) - \frac{17}{3}X_2 - \frac{2}{3}X_3$

$= -10 - 10 - 2S_1 + \frac{2}{3}X_2 - \frac{4}{3}X_3 - \frac{17}{3}X_2 - \frac{2}{3}X_3$

$= -20 - 2S_1 - 5X_2 - 2X_3$

s.t. $S_3 = 15 + 3S_1 - X_2 + 2X_3$

$S_2 = -1 + \frac{1}{3}(15+3S_1-X_2+2X_3) - \frac{2}{3}X_2 + \frac{4}{3}X_3$

$= -1 + 5 + S_1 - \frac{1}{3}X_2 + \frac{2}{3}X_3 - \frac{2}{3}X_2 + \frac{4}{3}X_3$

$= 4 + S_1 - X_2 + 2X_3$

$X_1 = 5 + \frac{1}{3}(15+3S_1-X_2+2X_3) + \frac{4}{3}X_2 - \frac{5}{3}X_3$

$= 5 + 5 + S_1 - \frac{1}{3}X_2 + \frac{2}{3}X_3 + \frac{4}{3}X_2 - \frac{5}{3}X_3$

$= 10 - S_1 + X_2 - X_3$

$X_1, X_2, X_3, S_1, S_2, S_3 \geq 0$

Now we stop, it's optimal. $\max -z = 20$ $\min z = 20$, when $(X_1, X_2, X_3) = (10, 0, 0)$

Use 2-Phase method

Phase I obj. $\max\{-X_0\}$ $\max -z = -2X_1 - 3X_2 - 4X_3$

$S_1 = X_0 - 10 + X_1 - X_2 + X_3$

$S_2 = X_0 - 6 + X_1 - 2X_2 + 3X_3$

$S_3 = X_0 - 15 + 3X_1 - 4X_2 + 5X_3$ → pivot on X_0 $X_0 = 15 - 3X_1 + 4X_2 - 5X_3 + S_3$

$X_1, X_2, X_3, S_1, S_2, S_3 \geq 0$

$\max -\{X_0\} \rightarrow \max\{-15 + 3X_1 - 4X_2 + 5X_3 - S_3\}$

s.t.

$S_1 = 15 - 3X_1 + 4X_2 - 5X_3 + S_3 - 10 + X_1 - X_2 + X_3$

$= 5 - 2X_1 + 3X_2 - 4X_3 + S_3$ $\quad X_3 \leq \frac{5}{4}$ ✓

$S_2 = 15 - 3X_1 + 4X_2 - 5X_3 + S_3 - 6 + X_1 - 2X_2 + 3X_3$

$= 9 - 2X_1 + 2X_2 - 2X_3 + S_3$ $\quad X_3 \leq \frac{9}{2}$

$X_0 = 15 - 3X_1 + 4X_2 - 5X_3 + S_3$ $\quad X_3 \leq 3$

$X_1, X_2, X_3, S_1, S_2, S_3 \geq 0$

pivot X_3

→ $X_3 = \frac{5}{4} - \frac{1}{4}X_1 + \frac{3}{4}X_2 + \frac{1}{4}S_3 - \frac{1}{4}S_1$

$\max\{-15 + 3X_1 - 4X_2 + 5(\frac{5}{4} - \frac{1}{4}X_1 + \frac{3}{4}X_2 + \frac{1}{4}S_3 - \frac{1}{4}S_1) - S_3\}$

$= \{-15 + 3X_1 + 4X_2 + \frac{25}{4} - \frac{5}{4}X_1 + \frac{15}{4}X_2 + \frac{5}{4}S_3 - \frac{5}{4}S_1 - S_3\}$

$= \{\frac{-35}{4} + \frac{1}{4}X_1 - \frac{1}{4}X_2 + \frac{1}{4}S_3 - \frac{5}{4}S_1\}$

se.

$x_3 = \frac{5}{4} - \frac{1}{2}x_1 + \frac{3}{4}x_2 + \frac{1}{4}s_3 - \frac{1}{4}s_1$ $\quad x_1 \le \frac{5}{2}$ ✓

$s_2 = 9 - 2x_1 + 2x_2 + s_3 - \frac{5}{2} + x_1 - \frac{3}{2}x_2 - \frac{1}{2}s_3 + \frac{1}{2}s_1$

$= \frac{13}{2} - x_1 + \frac{1}{2}x_2 + \frac{1}{2}s_3 + \frac{1}{2}s_1$ $\quad x_1 \le \frac{13}{2}$

$x_6 = 15 - 3x_1 + 4x_2 - 5x_3 + s_3 = 15 - 3x_1 + 4x_2 - 5\left(\frac{5}{4} - \frac{1}{2}x_1 + \frac{3}{4}x_2 + \frac{1}{4}s_3 - \frac{1}{4}s_1\right) + s_3$

$= 15 - 3x_1 + 4x_2 - \frac{25}{4} + \frac{5}{2}x_1 - \frac{15}{4}x_2 + \frac{5}{4}s_3 + \frac{5}{4}s_1 + s_3$

$= \frac{35}{4} - \frac{1}{2}x_1 + \frac{1}{4}x_2 + \frac{5}{4}s_1 - \frac{1}{4}s_3$ $\quad x_1 \le \frac{35}{2}$

pivot x_1 $\quad x_1 = \frac{5}{2} + \frac{3}{2}x_2 - 2x_3 - \frac{1}{2}s_1 + \frac{1}{2}s_3$

$\max\left\{\frac{-35}{4} + \frac{1}{2}\left(\frac{5}{2} + \frac{3}{2}x_2 - 2x_3 - \frac{1}{2}s_1 + \frac{1}{2}s_3\right) - \frac{1}{4}x_2 + \frac{1}{4}s_3 - \frac{5}{4}s_1\right\}$

$= \left\{\frac{-35}{4} + \frac{5}{4} + \frac{3}{4}x_2 - x_3 - \frac{1}{4}s_1 + \frac{1}{4}s_3 - \frac{1}{4}x_2 + x_2 + \frac{1}{4}s_3 - \frac{5}{4}s_1\right\}$

$= \left\{\frac{-15}{2} + \frac{1}{2}x_2 - x_3 - \frac{3}{2}s_1 + \frac{1}{2}s_3\right\}$

$$\begin{cases} x_1 = \frac{5}{2} + \frac{3}{2}x_2 - 2x_3 - \frac{1}{2}s_1 + \frac{1}{2}s_3 & x \ge -5 \\ s_2 = \frac{13}{2} - \frac{5}{2} - \frac{3}{2}x_2 + 2x_3 + \frac{1}{2}s_1 - \frac{1}{2}s_3 + \frac{1}{2}x_2 + \frac{1}{2}s_3 + \frac{1}{2}s_1 = 4 - x_2 + 2x_3 + s_1 & x \ge \frac{-5}{3} \\ x_6 = \frac{35}{4} - \frac{1}{2}\left(\frac{5}{2} + \frac{3}{2}x_2 - 2x_3 - \frac{1}{2}s_1 + \frac{1}{2}s_3\right) + \frac{1}{4}x_2 + \frac{5}{4}s_1 - \frac{1}{4}s_3 & \\ \quad = \frac{15}{2} - \frac{1}{2}x_2 + x_3 + \frac{3}{2}s_1 - \frac{1}{2}s_3 & x \le 15 \end{cases}$$

pivot s_3 $\quad s_3 = 15 - x_2 + 2x_3 + 3s_1 - x_6$

$$\begin{cases} x_1 = \frac{5}{2} + \frac{3}{2}x_2 - 2x_3 - \frac{1}{2}s_1 + \frac{1}{2}(15 - s_3 + x_2 - 2x_3 + 3s_1 - x_6) \\ \quad = 10 + x_2 - x_3 + s_1 - x_6 \\ s_2 = 4 - x_2 + 2x_3 + s_1 \\ s_3 = 15 - x_2 + 2x_3 + 3s_1 - x_6 \end{cases}$$

$\max\left\{\frac{-15}{2} + \frac{1}{2}x_2 - x_3 - \frac{3}{2}s_1 + \frac{1}{2}(15 - x_2 + 2x_3 + 3s_1 - x_6)\right\} = \{-x_6\}$ Done opt=0

phase II

$\max -z = -2x_1 - 3x_2 - 4x_3 = -2(10 + x_2 - x_3 + s_1 - x_6) - 3x_2 - 4x_3 = -20 - x_2 - 2x_3 - 2s_1 + 2x_6$

$x_1 = 10 + x_2 - x_3 + s_1$

$s_2 = 4 - x_2 + 2x_3 + s_1$

$s_3 = 15 - x_2 + 2x_3 + 3s_1 - x_6$

when $(x_1, x_2, x_3) = (10, 0, 0)$

$(x_1, x_2, x_3, s_1, s_2, s_3) = (10, 0, 0, 0, 4, 15)$ $\quad \max -z = -20$ $\quad \min z = 20$

Par-Dual