

# Operation Research

# Homework4

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1.

```
sys.path.append('C:/Users/amy874944/Desktop/pycharm/08_hw4')
Python 2.7.15 [Anaconda, Inc.] | default, May 1 2018, 10:37:05
In [3]: import Fib2 as f1
In [4]: f1.fibo(3)
Out[4]: 1
In [5]: f1.fibo(1)
Out[5]: 1
In [6]: f1.fibo(8)
Out[6]: 1, 1, 2, 3, 5, 8, 13, 21
```

2. 此題不適用 Primal Simplex Method，可以使用 2-Phase Method 或著是 Dual Simplex Method。以下有這兩種方法的手算稿。最後結果證實結果相同。

## 2-Phase Method

$\text{Min } Z = 2x_1 + 3x_2 + 4x_3$ $\text{s.t. } x_1 - x_2 - x_3 \geq 10$ $x_1 - 2x_2 + 3x_3 \geq 6$ $2x_1 + 4x_2 + 5x_3 \geq 5$ $x_1, x_2, x_3 \geq 0$	$\text{Standard Form}$ $\text{Max } -Z = -2x_1 - 3x_2 - 4x_3$ $\text{s.t. } -x_1 + x_2 + x_3 \leq -10$ $-x_1 + 2x_2 - 3x_3 \leq -6$ $-2x_1 - 4x_2 - 5x_3 \leq -5$ $x_1, x_2, x_3 \geq 0$
$\text{Slack Form}$ $\text{Max } -Z = -2x_1 - 3x_2 - 4x_3$ $\text{s.t. } x_1 = 10 + x_2 + x_3$ $S_1 = -6 + x_2 - 1x_3 + 1x_4$ $S_2 = -5 + 2x_1 - 4x_2 + 3x_3$ $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$	$\text{Use 2-Phase method}$ $\text{Max } -P = -2x_1 - 3x_2 - 4x_3$ $\text{s.t. } x_1 = -10 + x_2 + x_3$ $S_1 = -6 + x_2 - 1x_3 + 1x_4$ $S_2 = -5 + 2x_1 - 4x_2 + 3x_3$ $x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

$$\text{Max } \{-3 \cdot S_1 - 3x_1 - 4x_2 + 5x_3\} \rightarrow \text{pivot } x_1$$

$$\text{Max } \{-Z = -2x_1 - 3x_2 - 4x_3\}$$

$$\text{s.t. } S_1 = 5 + S_2 - 1x_1 + 1x_2 - 4x_3 \rightarrow x_1 = \frac{5}{-1} - \frac{1}{-1}S_2 + \frac{1}{-1}x_2 - 1x_3$$

$$S_2 = 7 + S_3 - 2x_1 - 1x_2 + 1x_3$$

$$x_1 = 13 + S_3 - 1x_2 - 4x_3 - 3x_4$$

$$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$$

$$\text{Max } \{-\frac{15}{2} - \frac{3}{2}S_1 + \frac{1}{2}S_2 + \frac{1}{2}x_2 - 1x_3\} \rightarrow \text{pivot } S_2$$

$$\text{Max } \{-Z = -15 + 3 - S_1 - 1x_2\}$$

$$\text{s.t. } x_1 = S_1 - \frac{1}{2}S_2 + \frac{1}{2}x_2 - \frac{1}{2}x_3$$

$$S_1 = 1 - \frac{1}{2}S_2 + \frac{1}{2}x_2 - \frac{1}{2}x_3 \rightarrow S_2 = 2 - 2S_1 + x_2 - x_3$$

$$S_1, S_2, x_1, x_2, x_3 \geq 0$$

$$\text{Max } \{-x_1\}$$

$$\text{Max } \{-Z = -70 - 2x_1 + 2x_2 - 5x_3 - 2x_4\}$$

$$\text{s.t. } x_1 = 10 + S_1 - x_2 + x_3$$

$$S_2 = 4 + S_1 - x_2 + 2x_3$$

$$S_3 = 15 + 3S_1 - 2x_2 - x_3 + 2x_4$$

$$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$$

$$\Rightarrow x_1 \text{ is in DHE } (x_1 = 0) \Rightarrow \text{Phase I Done!}$$

Phase - II

$$\max \{-Z = -20 - 2S_1 - 5X_2 - 2X_3\}$$

$$\text{s.t. } \begin{aligned} X_1 &= 10 + S_1 + X_2 - X_3 \\ S_2 &= 4 + S_1 - X_2 + 2X_3 \\ S_3 &= 15 + 3S_1 - X_2 + 2X_3 \end{aligned}$$

$$X_1, X_2, X_3, S_1, S_2, S_3 \geq 0$$

$\Rightarrow \because$  Obj. func.  $\geq$  RHS 係取皆  $< 0$

$\Rightarrow$  Phase 2 DOVE!!

$\Rightarrow \min \{Z\} = 20$  as  $(X_1, X_2, X_3, S_1, S_2, S_3) = (10, 0, 0, 0, 4, 15)$

## Dual - Simplex Method

$\max Z = 2X_1 + 3X_2 + 4X_3$ $\text{s.t. } \begin{aligned} X_1 - X_2 + X_3 &\geq 10 \\ X_1 - 2X_2 + 3X_3 &\geq 6 \\ 3X_1 - 4X_2 + 5X_3 &\geq 15 \end{aligned}$ $X_1, X_2, X_3 \geq 0$	<p>Slack Form</p> $\max -Z = -2X_1 - 3X_2 - 4X_3$ $\text{s.t. } \begin{aligned} S_1 &= -10 - X_1 - X_2 + X_3 \\ S_2 &= -6 - X_1 - 2X_2 + 3X_3 \\ S_3 &= -15 + 3X_1 - 4X_2 + 5X_3 \end{aligned}$ $X_1, X_2, X_3, S_1, S_2, S_3 \geq 0$
<p>Dual Problem</p> $\min -Z_d = -10Y_1 - 6Y_2 - 15Y_3$ $\text{s.t. } \begin{aligned} -Y_1 - Y_2 - 3Y_3 &\geq -2 \\ Y_1 + 2Y_2 + 4Y_3 &\geq -3 \\ -Y_1 - 3Y_2 - 5Y_3 &\geq -4 \end{aligned}$ $Y_1, Y_2, Y_3 \geq 0$	<p>Slack Form</p> $\max -Z_d = 10Y_1 + 6Y_2 + 15Y_3 \rightarrow \text{pivot } Y_3$ $\text{s.t. } \begin{aligned} W_1 &= 2 - Y_1 - Y_2 - 3Y_3 \rightarrow Y_3 = \frac{2}{3} - \frac{1}{3}Y_1 - \frac{1}{3}Y_2 \\ W_2 &= 3 + Y_1 + 2Y_2 + 4Y_3 \\ W_3 &= 4 - Y_1 - 3Y_2 - 5Y_3 \end{aligned}$ $Y_1, Y_2, Y_3, W_1, W_2, W_3 \geq 0$
$\max \{-Z_d = 20 - 5W_1 + 5Y_1 + Y_2\} \rightarrow \text{pivot } Y_1$ $\text{s.t. } \begin{aligned} Y_3 &= \frac{2}{3} - \frac{1}{3}W_1 - \frac{1}{3}Y_2 - \frac{1}{3}Y_3 \rightarrow Y_1 = 2 - W_1 - Y_2 - 3Y_3 \\ W_2 &= 5 - W_1 - Y_2 + Y_3 \\ W_3 &= 2 + W_1 - 2Y_2 - 2Y_3 \end{aligned}$ $Y_1, Y_2, Y_3, W_1, W_2, W_3 \geq 0$	
$\max \{-Z_d = 20 - 10W_1 - 4Y_2 - 15Y_3\} \rightarrow \text{Obj. func. } \geq \text{RHS 係取皆 } < 0$ $\text{s.t. } \begin{aligned} Y_1 &= 2 - W_1 - Y_2 - 3Y_3 \\ W_2 &= 5 - W_1 - Y_2 + Y_3 \\ W_3 &= 2 + W_1 - 2Y_2 - 2Y_3 \end{aligned}$ $Y_1, Y_2, Y_3, W_1, W_2, W_3 \geq 0$	$\Rightarrow \min \{Z_d\} = -20$ as $(Y_1, Y_2, Y_3, W_1, W_2, W_3) = (2, 0, 0, 0, 5, 2)$ $\Rightarrow Z_d = -Z = -20$ $\Rightarrow Z = 20$

計算結束後我有用 pivot 檢驗 Dual Simplex Method 的正確性，選擇檢驗 Dual Simplex Method 的原因是因為 2-Phase Method 的步驟較為繁瑣，比較不適合用電腦檢驗。

## Bonus

我有設計一個自動計算 Simplex Method 的程式在 simplex method.py 檔中，這個程式只能解最基礎的 Simplex Method，意即 LP 化為 Slack Form 時在 RHS 的常數必須皆大於0，不然不能計算。此程式也不行計算 Dual Simplex Method，因為 Dual Simplex Method 在選擇 pivot 的思維跟 Simplex Method 有所不同，Dual Simplex Method 的選擇較為嚴格，因此不能通用。而其最後的輸出結果為計算到最後的矩陣跟 BV List。