

$$\min Z = 2x_1 + 3x_2 + 4x_3 = \max -Z = -2x_1 - 3x_2 - 4x_3$$

s.t.

$$x_1 - x_2 + x_3 \geq 10$$

$$-x_1 + x_2 - x_3 \leq -10$$

$$x_1 - 2x_2 + 3x_3 \geq 6$$

$$\rightarrow -x_1 + 2x_2 - 3x_3 \leq -6$$

slack form

$$S_1 = -10 + x_1 - x_2 + x_3$$

$$3x_1 - 4x_2 + 5x_3 \geq 15$$

$$-3x_1 + 4x_2 - 5x_3 \leq -15$$

$$S_2 = -6 + x_1 - 2x_2 + 3x_3$$

$$x_1, x_2 \geq 0$$

$$S_3 = -15 + 3x_1 - 4x_2 + 5x_3$$

此時 $(x_1, x_2, x_3, S_1, S_2, S_3) = (0, 0, 0, -10, -6, -15) \rightarrow$ 不能採用 Simplex method.

$$\max(-x_0) \quad \max \bar{Z} = -Z = -2x_1 - 3x_2 - 4x_3 \quad \text{當 } S_1, S_2, S_3 < 0.$$

$$S_1 = x_0 - 10 + x_1 - x_2 + x_3$$

most (-) basic variable. \rightarrow 2-Phase Method.

$$S_2 = x_0 - 6 + x_1 - 2x_2 + 3x_3$$

$$S_3 = x_0 - 15 + 3x_1 - 4x_2 + 5x_3 \rightarrow x_0 = 15 - 3x_1 + 4x_2 - 5x_3 + S_3$$

$$S_1 = 5 - 2x_1 + 3x_2 - 4x_3 + S_3$$

$$\max(-15 + 3x_1 - 4x_2 + 5x_3 - S_3)$$

$$\max \max(Z) = -2x_1 - 3x_2 - 4x_3$$

$$S_2 = 9 - 2x_1 + 2x_2 - 2x_3 + S_3$$

$$\rightarrow \text{pivot on } x_3 = \frac{5}{4} - \frac{1}{2}x_1 + \frac{3}{4}x_2 + \frac{1}{4}S_3 - \frac{1}{4}S_1$$

$$x_0 = 15 - 3x_1 + 4x_2 - 5x_3 + S_3$$

$$\rightarrow \text{pivot on } x_1 \rightarrow x_1 = \frac{5}{2} + \frac{3}{2}x_2 - 2x_3 + \frac{1}{2}S_3 - \frac{1}{2}S_1$$

$$x_1 = \frac{5}{2} + \frac{3}{2}x_2 - 2x_3 + \frac{1}{2}S_3 - \frac{1}{2}S_1$$

$$\max(-\frac{15}{2} + \frac{1}{2}x_2 - x_3 + \frac{1}{2}S_3 - \frac{3}{2}S_1)$$

$$\max \bar{Z} = -5 - 6x_2 - 3x_3 + S_1$$

$$S_2 = 4 - x_2 + 2x_3 + S_1$$

pivot on $x_2 \rightarrow$

$$x_0 = \frac{15}{2} - \frac{1}{2}x_2 + x_3 - \frac{1}{2}S_3 + \frac{3}{2}S_1$$

$$\max(-\frac{35}{4} + \frac{1}{2}x_1 - \frac{1}{4}x_2 + \frac{1}{4}S_3 - \frac{5}{4}S_1)$$

$\max(-Z)$

$$= -5 - 6x_2 - 3x_3 + S_1$$

$$x_3 = \frac{5}{4} - \frac{1}{2}x_1 + \frac{3}{4}x_2 + \frac{1}{4}S_3 - \frac{1}{4}S_1$$

pivot on x_1 pivot on x_1

$$S_2 = \frac{13}{2} - x_1 + \frac{1}{2}x_2 + \frac{1}{2}S_3 + \frac{1}{2}S_1$$

$$x_0 = \frac{35}{4} - \frac{1}{2}x_1 + \frac{1}{4}x_2 - \frac{1}{4}S_3 + \frac{5}{4}S_1$$

$$x_1 = \frac{5}{2} + \frac{3}{2}x_2 - 2x_3 + \frac{1}{2}S_3 - \frac{1}{2}S_1$$

$$x_1 = \frac{5}{2} + \frac{3}{2}x_2 - 2x_3 + \frac{1}{2}S_3 - \frac{1}{2}S_1$$

$$x_1 = \frac{5}{2} + \frac{3}{2}x_2 - 2x_3 + \frac{1}{2}S_3 - \frac{1}{2}S_1$$

$$\max(-\frac{15}{2} + \frac{1}{2}x_2 + \frac{1}{2}S_3 - \frac{3}{2}S_1)$$

$\max(-Z)$

$$= -5 - 6x_2 - 3x_3 + S_1$$

$$S_2 = 4 - x_2 + 2x_3$$

pivot on S_2

$$x_0 = \frac{15}{2} - \frac{1}{2}x_2 + \frac{1}{2}S_3 + \frac{3}{2}S_1$$

$$S_3 = 15 - x_2 + 2x_3 + 3S_1 - x_0$$

$$S_1 = 15 - x_2 + 2x_3 + 3S_1 - x_0$$

$$\max\{-x_0\}$$

$$\max(-Z) = -20 - 3S_1 - 5x_2 - 2x_3 + x_0.$$

$$S_2 = 4 - x_2 + 2x_3 + S_1$$

此時結束 1 phase.

$$S_3 = 15 - x_2 + 2x_3 + 3S_1 - 2x_0$$

goes to 2 phase (拿掉全部的 x_0)

此時 $\max(-Z)$ 有最大值 -20 在 $(x_1, x_2, x_3, S_1, S_2, S_3)$
 Z 有最大值 20 在 $(10, 0, 0, 0, 4, 15)$

$$\min Z = 2x_1 + 3x_2 + 4x_3$$

s.t.

$$x_1 - x_2 + x_3 \geq 10$$

$$-x_1 + y_2 - x_3 \leq -10$$

$$x_1 - 2x_2 + 3x_3 \geq 6$$

$$\rightarrow -x_1 + 2x_2 - 3x_3 \leq -6$$

$$3x_1 - 4x_2 + 5x_3 \geq 15$$

$$\rightarrow -3x_1 + 4x_2 - 5x_3 \leq -15$$

$$x_1, x_2, x_3 \geq 0.$$

$$\text{slack form } S_1 = -10 + x_1 - x_2 + x_3$$

$$S_2 = -6 + x_1 - 2x_2 + 3x_3$$

$$S_3 = -15 + 3x_1 - 4x_2 + 5x_3$$

此時

全為負數，不能使用 Simplex method (primal)

→ 改用 Dual method

$$(y_1 + y_2 + 3y_3)x_1 + (-y_1 - 2y_2 - 4y_3)x_2 + (y_1 + 3y_2 + 5y_3)x_3 \leq 10y_1 + 6y_2 + 15y_3$$

$$y_1 + y_2 + 3y_3 \leq 2$$

$$-y_1 - 2y_2 - 4y_3 \leq 3$$

$$y_1 + 3y_2 + 5y_3 \leq 4$$

$$Z = \max (10y_1 + 6y_2 + 15y_3)$$

pivot on y_3

$$(10 + 5y_1 + y_2 - 5S_1)$$

$$Z_{\max} (20 - 5y_1 - 5y_2 - 5S_1)$$

$$S_1 = 2 - y_1 - y_2 - 3y_3 \quad \checkmark$$

$$S_2 = 3 + y_1 + 2y_2 + 4y_3$$

$$S_3 = 4 - y_1 - 3y_2 - 5y_3$$

$y_3 =$

$$y_3 = \frac{2}{3} - \frac{1}{3}y_1 - \frac{1}{3}y_2 - \frac{1}{3}S_1 \quad \checkmark$$

$$S_2 = \frac{17}{3} - \frac{1}{3}y_1 + \frac{2}{3}y_2 - \frac{4}{3}S_1$$

$$S_3 = \frac{2}{3} + \frac{2}{3}y_1 - \frac{4}{3}y_2 + \frac{5}{3}S_1$$

pivot on y_1

$$Z_{\max} (20 - 4y_2 - 15y_3 - 10S_1)$$

$$y_1 = 2 - y_2 - 3y_3 - S_1$$

$$S_2 = 5 + y_2 + 4y_3 - S_1$$

$$S_3 = 2 - 2y_2 - 2y_3 + S_1$$

此時有 $Z_{\max} 20$.

→

at $(y_1 = 2, y_2 = 0, y_3 = 0, S_1 = 0$

$S_2 = 5, S_3 = 2)$.