



All weight = 1

$$\text{ReLU} = \max(0, x)$$

All bias = -0.5
learning rate = 0.01

① name Hidden Layer 1, Hidden Layer 2.

H1, H2

H3, H4

$$H_1 = w_1 x_1 + b = 1 \times 2 - 0.5 = 1.5$$

$$H_2 = w_2 x_1 + b = 1 \times 2 - 0.5 = 1.5$$

$$H_3 = w_{13} \cdot H_1 + w_{23} \cdot H_2 + b = 1 \times 1.5 + 1 \times 1.5 - 0.5 = 2.5$$

$$H_4 = w_{14} \cdot H_1 + w_{24} \cdot H_2 + b = 1 \times 1.5 + 1 \times 1.5 - 0.5 = 2.5$$

$$y_1 = w_3 \cdot H_3 + w_4 \cdot H_4 + b = 1 \times 2.5 + 1 \times 2.5 - 0.5 = 4.5$$

forward pass

BP

② 逆計算誤差

$$E = \sum \frac{1}{2} (\text{target} - \text{output})^2$$

$$= \frac{1}{2} (1 - 4.5)^2 = 6.125$$

$$\begin{aligned} & \text{更新 } w_3, w_4 \\ & b_3, b_4 \quad \begin{matrix} w_3 \\ w_4 \end{matrix} \quad \begin{matrix} \frac{\partial E}{\partial b} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial b} = t - y = 3.5 \\ \frac{\partial E}{\partial w} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial w} = 0.5 - 0.035 = -0.535 \end{matrix} \end{aligned}$$

$$\frac{\partial E}{\partial w_3} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial w_3} \quad \because E = \frac{1}{2} (t - y)^2$$

$$\begin{aligned} \frac{\partial E}{\partial w_4} &= \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial w_4} \\ &\therefore \frac{\partial E}{\partial y} = 2 \cdot \frac{1}{2} (t - y) \cdot -1 \\ &= y - t \\ &= 4.5 - 1 = 3.5 \end{aligned}$$

$$\therefore y = w_3 \cdot h_3 + w_4 \cdot h_4 + b$$

$$\therefore \frac{\partial y}{\partial w_3} = h_3 - 2.5 \quad \frac{\partial y}{\partial w_4} = h_4 - 2.5$$

$$3.5 \times 2.5 = 8.75$$

$$\text{new } w_3 \leftarrow w_3 - \eta \times 8.75 \Rightarrow 1 - 0.0875 \\ = 0.9125$$

$$w_4 \leftarrow w_4 - \eta \times 8.75 = 0.9125$$

重り $w_{13}, w_{14}, w_{23}, w_{24}$ $b_{13}, b_{14}, b_{23}, b_{24}$

$$\frac{\partial E}{\partial w_{13}} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial h_3} \cdot \frac{\partial h_3}{\partial w_{13}}$$

$$\frac{\partial y}{\partial h_3} = w_3$$

$$(f-y) \cdot w_3 \cdot 1$$

$$= (f-y) \times w_3 \cdot H_1 = 5.25 = 3.5$$

$$\rightarrow -0.535$$

② 理 $= 5.25$

$$w_{14} = (f-y) \times w_2 f \cdot H_1 = 5.25$$

$$w_{23} = (f-y) \times w_3 H_2 = 5.25$$

$$w_{24} = (f-y) \times w_4 H_2 = 5.25$$

$$w_{13}^* \leftarrow w_{13} - \eta \times 5.25 = 1 - 0.0525 \\ = 0.9475$$

w_{14}^*

w_{23}^*

w_{24}^*

$$b_1, b_2 \rightarrow \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial h_3} \dots$$

$$\Rightarrow f_y \cdot w_3 \cdot w_{13} - 1 + 2 \\ = -0.57$$

$$\frac{\partial E}{\partial w_1} = \frac{\partial E}{\partial y} \sum_{i=1}^3 \frac{\partial y}{\partial h_i} \frac{\partial h_i}{\partial w_1}$$

$$= \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial h_3} \cdot \frac{\partial h_3}{\partial w_1} + 2 \quad (\because \text{the same})$$

$$= f_y \cdot w_3 \cdot w_{13} - x_1$$

$$= 3.5 \cdot 1 - 1 \cdot 2 \times 2 = 1$$

$$\frac{\partial f}{\partial w_2} = f_y \cdot w_4 \cdot w_{14} \cdot x_1 + 2 \\ = 1$$

$$w_1^* = 1 - 0.09 = 0.86$$

$$w_2^* = 0.86$$

After 1 Bp new

$$\begin{cases} w_1 = 0.86 \\ w_2 \end{cases})$$

$$\begin{cases} w_3 \\ w_4 = 0.9475 \\ w_5 \\ w_6 \end{cases})$$

$$\begin{cases} w_3 = 0.9125 \\ w_4 \end{cases})$$

forward pass [→]

$$H_1 = w_1 x_1 + b = 0.86 \cdot 2 - 0.57 = 1.15$$

$$H_2 = w_2 x_1 + b = 0.86 \cdot 2 - 0.57 = 1.15$$

$$H_3 = w_{13} \cdot H_1 + w_{23} \cdot H_2 + b = (0.9475 \cdot 1.15) - 0.535 = 1.6443$$

$$H_4 = w_{14} \cdot H_1 + w_{24} \cdot H_2 + b = (0.9475 \cdot 1.15) - 0.535 = 1.6443$$

$$y_1 = w_3 \cdot H_3 + w_4 \cdot H_4 + b = (0.9125 \cdot 1.6443) - 0.535 = 2.4658$$

$$E = \frac{1}{2} (1 - 2.4658)^2 = 1.0743$$

Bsp 2 :

suchen w_3, w_4

$$\frac{\partial E}{\partial w_3} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial w_3}$$

$$\frac{\partial E}{\partial w_4} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial w_4}$$

$$\frac{\partial E}{\partial b} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial b}$$

$$\frac{\partial E}{\partial b} = -0.5 - 0.035 = -0.535$$

$$\therefore E = \frac{1}{2} (t - y)^2$$

$$\therefore \frac{\partial E}{\partial y} = 2 \cdot \frac{1}{2} (t - y) \cdot -1$$

$$= y - t$$

$$= 2.4658^{-1} = 1.4658$$

$$\therefore y = w_3 \cdot h_3 + w_4 \cdot h_4 + b$$

$$\therefore \frac{\partial y}{\partial w_3} = h_3 = \frac{\partial y}{\partial w_4} = h_4 = 1.6443$$

$1.4658 \times 1.6443 \rightarrow 2.41$

$$\text{new } w_3 \leftarrow w_3 - \eta^{2.41} \Rightarrow 0.9125 - 0.241 \\ = 0.8884$$

$$w_4 \leftarrow w_4 - \eta^{2.41} = 0.8884$$

重 $\frac{\partial E}{\partial w_3}$ $w_{13}, w_{14}, w_{23}, w_{24}$ $b_{13}, b_{14}, b_{23}, b_{24}$

$$\frac{\partial E}{\partial w_{13}} = \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial h_3} \cdot \frac{\partial h_3}{\partial w_{13}}$$

$$\frac{\partial y}{\partial h_3} = w_3 \\ (t-y) \cdot 0.9125$$

$$= (t-y) \times w_3 \cdot h_1 = 1.5428 = 1.47$$

$$1.47 \times 0.9125 < 1.15 \rightarrow -0.535$$

同理

$$w_{14} = (t-y) \times w_4 \cdot h_1 = -0.0134$$

$$= 0.5484$$

$$w_{23} = (t-y) \times w_3 \cdot h_2 =$$

$$w_{24} = (t-y) \times w_4 \cdot h_2 =$$

$$W_{13}^* \leftarrow W_{13} - \eta \times \cancel{S_{13}} = 0.9475 - 0.015426$$

(1.5426)

$$= 0.93207$$

W_{14}^*

W_{23}^*

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W_{24}^*

$$b_1, b_2 \rightarrow \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial H_3} \dots$$

$$\Rightarrow f_y \cdot W_3 \cdot W_3 - 1 + 2$$

$$1.47 \times 0.9125 \times 0.9475 \times 2$$

$$= 2.542 - 0.59$$

$$\frac{\partial E}{\partial W_1} = \frac{\partial E}{\partial y} \sum_{i=3}^4 \frac{\partial y}{\partial H_i} \frac{\partial H_i}{\partial W_1} = -0.0254$$

$$= \frac{\partial E}{\partial y} \cdot \frac{\partial y}{\partial H_3} \cdot \frac{\partial H_3}{\partial W_1} \cdot \frac{\partial H_1}{\partial W_1} + 2 \quad (\because \text{the same})$$

$$= f_y \cdot W_3 \cdot W_{13} - x_1$$

$$= 1.47 \times 0.9125 \times 0.9475 \times 2 \times 2 = 5.084$$

$$f_{W2}^* = f(y) \cdot w_4 \cdot w_{14} \cdot x_1 + 2 \\ = f$$

$$w_1^* = 0.86 - 0.05084 \cdot 0.8092 \\ - 0.09 = 0.86 \\ 0.8092 \\ w_2^* = 0.86 -$$

after 2 sg

$$(w_1^*, w_2^*, 0.8092)$$

$$\begin{cases} w_1 \\ w_1^* = 0.93207 \\ w_2 \\ w_3 \\ w_4 \end{cases}$$

$$\begin{cases} w_3 \\ w_4^* = 0.8884 \end{cases}$$

$$H_1 = w_1 x_1 + b = x_2 - \frac{0.8092}{0.5154} = 1.023$$

$$H_2 = w_2 x_1 + b = x_2 - \underline{= 1.023}$$

$$H_3 = w_{13} \cdot H_1 + w_{23} \cdot H_2 + b = (0.93207 \times 1.023) \times 2 - 0.5484 =$$

$$H_4 = w_{14} \cdot H_1 + w_{24} \cdot H_2 + b = \underline{= 1.3586}$$

$$y_1 = w_3 \cdot H_3 + w_4 \cdot H_4 + b = (0.8884 \times 1.3586) \times 2 - 0.5491 = 1.8643$$

$$E = \frac{1}{2} (1 - 1.8643)^2 = \underline{0.3735}$$

Line ✎