SSE5107 Optimization Theory and Algorithms
Homework 2
Due: Nov. 23, 2021, in class

Problem 1

Formulate the following problems as LPs, where $A \in \mathbf{R}^{m \times n}$ and $b \in \mathbf{R}^m$ are given.

1. Minimize $\|Ax - b\|_\infty$.

2. Minimize $\|Ax - b\|_1$.

3. Minimize $\|Ax - b\|_1$ subject to $\|x\|_\infty \leq 1$.

4. Minimize $\|x\|_1$ subject to $\|Ax - b\|_\infty \leq 1$.

5. Minimize $\|Ax - b\|_1 + \|x\|_\infty$.

Problem 2

Suppose $A : \mathbf{R}^n \to \mathbf{S}^m$ is affine, i.e.,

$$A(x) = A_0 + x_1 A_1 + \cdots + x_n A_n$$

where $A_i \in \mathbf{S}^m$. Let $\lambda_1(x) \geq \lambda_2(x) \geq \cdots \geq \lambda_m(x)$ denote the eigenvalues of $A(x)$. Show how to formulate the following problems as SDPs.

1. Minimize the maximum eigenvalue $\lambda_1(x)$.

2. Minimize $\lambda_1(x) - \lambda_m(x)$.