Cooperative and Competitive Pricing in Collaborative Edge Computing
Appendix

Assumption 1. *The lower bound* $\mu_{\min}$ *satisfies*

$$\mu_{\min}/r - \lambda > 1/T. \tag{1}$$

Assumption 2. *The lower bound* $B_{\min}$ *satisfies*

$$B_{\min}/s - \lambda > 1/T. \tag{2}$$

Assumption 3. *The capacity of the local computation capability* $\hat{\mu}$ *satisfies*

$$1/(\hat{\mu}/r - \lambda) > T, \tag{3a}$$

$$r/\hat{\mu} < T. \tag{3b}$$

We introduce some necessary notations. Define a weighted sum of network and computation prices $g(p_n, p_c) \triangleq sp_n + rp_c$. The weight associated with network price p_n is the expected data sizes s to transmit for a single task. The weight associated with computation price p_c is the expected processor cycles r to process for a single task. Define $\hat{I} \triangleq 4c_l r^2 / \left(T(1-(1-2r/\hat{\mu}T)^2)\right)$ in which $2r \geq \hat{\mu}T$ holds. We also define some notations that will appear in the user's optimal decisions. Let $\beta \triangleq \sqrt{rp_c/(sp_n)}$, $\gamma \triangleq g(p_n,p_c)/(2c_l r)$. Define $B_1 \triangleq s\left((1+\beta)/T + \lambda\right)$, $B_2 \triangleq s((2+\beta)/T + \lambda - \gamma/r)$, $B_3 \triangleq s((2+\beta)/T + \lambda - \hat{\mu}/r)$, $\mu_1 \triangleq r(1/T + 1/(\beta T) + \lambda)$, $\mu_2 \triangleq r(2/T + 1/(\beta T) + \lambda - \gamma)$, and $\mu_3 \triangleq r\left(2/T + 1/(\beta T) + \lambda - \hat{\mu}/r\right)$.

Theorem 1. *Under Assumption 3, given the prices* p_n *and* p_c*, the user's optimal decisions depend on the following two scenarios:*
Scenario 1*:* $\hat{\mu} > 2r/T$.

- *Case A: When* $g(p_n,p_c) \leq 4c_l r^2/T$*, then we have*

$$\alpha^*(\boldsymbol{p}) = 1, \quad \mu_l^*(\boldsymbol{p}) = 0, \quad B^*(\boldsymbol{p}) = B_1, \quad \mu^*(\boldsymbol{p}) = \mu_1. \tag{4}$$

- *Case B: When* $g(p_n,p_c) > 4c_l r^2/T$ *and* $g(p_n,p_c) \leq 2c_l r\hat{\mu}$*, then we have*

$$\begin{aligned} &\alpha^*(\boldsymbol{p}) = 1 - \left(\frac{rp_c + sp_n}{2c_l r^2 \lambda} - \frac{1}{T\lambda}\right), \mu_l^*(\boldsymbol{p}) = \frac{rp_c + sp_n}{2c_l r}, \\ &B^*(\boldsymbol{p}) = B_2, \quad \mu^*(\boldsymbol{p}) = \mu_2. \end{aligned} \tag{5}$$

- *Case C: When $g(p_n, p_c) > 2c_l r\hat{\mu}$, we have*

$$\begin{aligned} &\alpha^*(\boldsymbol{p}) = 1 - \left(\frac{\hat{\mu}}{r\lambda} - \frac{1}{T\lambda}\right), \mu_l^*(\boldsymbol{p}) = \hat{\mu}, \\ &B^*(\boldsymbol{p}) = B_3, \quad \mu^*(\boldsymbol{p}) = \mu_3. \end{aligned} \tag{6}$$

Scenario 2: *$\hat{\mu} \le 2r/T$.*

- *Case A: When $g(p_n, p_c) \le \hat{I}$, we have*

$$\alpha^*(\boldsymbol{p}) = 1, \quad \mu_l^*(\boldsymbol{p}) = 0, \quad B^*(\boldsymbol{p}) = B_1, \quad \mu^*(\boldsymbol{p}) = \mu_1. \tag{7}$$

- *Case B: When $g(p_n, p_c) > \hat{I}$, we have*

$$\begin{aligned} &\alpha^*(\boldsymbol{p}) = 1 - \left(\frac{\hat{\mu}}{r\lambda} - \frac{1}{T\lambda}\right), \mu_l^*(\boldsymbol{p}) = \hat{\mu}, \\ &B^*(\boldsymbol{p}) = B_3, \quad \mu^*(\boldsymbol{p}) = \mu_3. \end{aligned} \tag{8}$$

Assumption 4. *The capacity of the local computation capability $\hat{\mu}$ satisfies $\hat{\mu} \le 2r/T$.*

Assumption 5. *The upper bounds of the prices $\hat{p}_n$ and $\hat{p}_c$ satisfy $g(\hat{p}_n, \hat{p}_c) > \hat{I}$.*

Theorem 2. *Under Assumptions 1 to 5, the pricing strategy (p_n^*, p_c^*) is the optimal one that solves the PPM problem if and only if it satisfies one of the following two conditions:*

a) $g(p_n^*, p_c^*) = \hat{I}$;

b) $g(p_n^*, p_c^*) > \hat{I}$ and at least one of the following two conditions holds: (i) $p_n^* = \hat{p}_n$; (ii) $p_c^* = \hat{p}_c$.

Define

$$R_{LA} \triangleq \max\left\{\frac{T(\mu_{\min} - r/T - \lambda r)}{\sqrt{rs}}, \frac{\sqrt{rs}}{T(B_{\max} - s/T - \lambda s)}\right\}, \tag{9}$$

$$R_{HA} \triangleq \min\left\{\frac{T(\mu_{\max} - r/T - \lambda r)}{\sqrt{rs}}, \frac{\sqrt{rs}}{T(B_{\min} - s/T - \lambda s)}\right\}. \tag{10}$$

Define a polynomial $f_A(t)$ as follows:

$$f_A(t) \triangleq \frac{2\hat{I}(st^2 - r)\sqrt{sr}}{T(t^2 s + r)^2} - c_n\left(\frac{2sr}{T^2 t^3} + \frac{s(2 + 2\lambda T)\sqrt{rs}}{T^2 t^2}\right) + c_c\left(\frac{2tsr}{T^2} + \frac{r(2 + 2\lambda T)\sqrt{rs}}{T^2}\right). \tag{11}$$

The polynomial $f_A(t)$ is closely related to the service provider's network cost c_n and computation cost c_c.

Proposition 1. *Under Assumptions 1 to 5, if the optimal prices satisfy $g(p_n^*, p_c^*) = \hat{I}$, then we have*

i) if $f_A(t) > 0$ for $t \in [R_{LA}, R_{HA}]$, then $p_n^*/p_c^* = R_{LA}^2$;

ii) if $f_A(t) < 0$ for $t \in [R_{LA}, R_{HA}]$, then $p_n^*/p_c^* = R_{HA}^2$;

iii) if $f_A(t) = 0$ for $t \in [R_{LA}, R_{HA}]$ has a unique solution $\tilde{t}$, and $f_A(t) < 0$ for $t \in [R_{LA}, \tilde{t}]$ and $f_A(t) > 0$ for $t \in [\tilde{t}, R_{HA}]$, then $p_n^*/p_c^* = \tilde{t}^2$.

Let

$$R_{LB} \triangleq \max\left\{ \frac{T(\mu_{\min} - 2r/T - \lambda r + \hat{\mu})}{\sqrt{rs}}, \frac{\sqrt{rs}}{T(B_{\max} - 2s/T - \lambda s + s\hat{\mu}/r)} \right\},$$

$$R_{HB} \triangleq \min\left\{ \frac{T(\mu_{\max} - 2r/T - \lambda r + \hat{\mu})}{\sqrt{rs}}, \frac{\sqrt{rs}}{T(B_{\min} - 2s/T - \lambda s + s\hat{\mu}/r)} \right\},$$

We define a polynomial $f_A(t)$ whose coefficients are related to the network cost c_n and computation cost c_c:

$$\begin{aligned} f_B(t) \triangleq \; & -2\hat{p}_c t\left(\frac{2}{T} + \lambda - \frac{\hat{\mu}}{r}\right) - \frac{2\sqrt{sr}\hat{p}_c}{T} \\ & - c_n\left(\frac{2rs}{T^2 t^3} + \frac{2s(\frac{2}{T} + \lambda - \frac{\hat{\mu}}{r})\sqrt{rs}}{Tt^2}\right) \\ & + c_c\left(\frac{2tsr}{T^2} + \frac{2r(\frac{2}{T} + \lambda - \frac{\hat{\mu}}{r})\sqrt{rs}}{T}\right). \end{aligned} \tag{12}$$

Proposition 2. *Under Assumptions 1 to 5, if the optimal prices satisfy $g(p_n^*, p_c^*) > \hat{I}$, $p_n^* < \hat{p}_n$, and $p_c^* = \hat{p}_c$, then we have*

i) if $f_B(t) > 0$ for $t \in [R_{LB}, R_{HB}]$, then $p_n^*/p_c^* = R_{LB}^2$;

ii) if $f_B(t) < 0$ for $t \in [R_{LB}, R_{HB}]$, then $p_n^*/p_c^* = R_{HB}^2$.

Let $p_{n,B}^*$ and $p_{c,B}^*$ be the optimal network price and computation price, respectively, in Case B. Let $p_{n,A}^*$ and $p_{c,A}^*$ be the optimal network price and computation price, respectively, in Case A.

Proposition 3. *Under Assumptions 1 to 5, if*

$$\begin{aligned} &\left(\frac{1}{T} + \lambda\right)(g(p_{n,B}^*, p_{c,B}^*) - \hat{I}) + \left(\frac{1}{T} - \frac{\hat{\mu}}{r}\right) g(p_{n,B}^*, p_{c,B}^*) - \frac{c_n s^2}{T}\left(1 + \sqrt{\frac{rp_{c,B}^*}{sp_{n,B}^*}}\right)\left(\frac{3}{T} + 2\lambda + \frac{2\sqrt{rp_{c,B}^*}}{T\sqrt{sp_{n,B}^*}} + \frac{\sqrt{rp_{c,A}^*}}{T\sqrt{sp_{n,A}^*}}\right) \\ &- \frac{c_c r^2}{T}\left(1 + \sqrt{\frac{sp_{n,B}^*}{rp_{c,B}^*}}\right)\left(\frac{3}{T} + 2\lambda + \frac{2\sqrt{sp_{n,B}^*}}{T\sqrt{rp_{c,B}^*}} + \frac{\sqrt{sp_{n,A}^*}}{T\sqrt{rp_{c,A}^*}}\right) > 0, \end{aligned} \tag{13}$$

then the pricing strategy $(p_{n,B}^*, p_{c,B}^*)$ is the optimal one that solves the PPM problem.

Assumption 6. *The lower bounds of provided capabilities $B_{\min}$ and $\mu_{\min}$ satisfy*

$$T^2\left(\frac{\mu_{\min}}{r} - \lambda + \frac{\hat{\mu}}{r} - \frac{2}{T}\right)\left(\frac{B_{\min}}{s} - \lambda + \frac{\hat{\mu}}{r} - \frac{2}{T}\right) < 1. \tag{14}$$

We define some necessary notations. Let

$$\tilde{R}_{LA} \triangleq \frac{(T\mu_{\min} - r - T\lambda r)^2}{rs},$$
$$\tilde{R}_{HA} \triangleq \frac{rs}{(TB_{\min} - s - T\lambda s)^2},$$

$$\tilde{R}_{LB} \triangleq \frac{(T\mu_{\min} - T\lambda r + T\hat{\mu} - 2r)^2}{sr},$$

$$\tilde{R}_{HB} \triangleq \frac{sr}{(TB_{\min} - T\lambda s + Ts\hat{\mu}/r - 2s)^2}.$$

Theorem 3. *Under Assumptions 1 to 6, the NE* (p_n^{NE}, p_c^{NE})*, where* $p_n^{NE} \leq \hat{p}_n$*,* $p_c^{NE} \leq \hat{p}_c$*, of the Price Competition Game satisfy at least one of the following two conditions.*

a) $g(p_n^{NE}, p_c^{NE}) = \hat{I}$*, and*

$$\tilde{R}_{LA} \leq \frac{p_n^{NE}}{p_c^{NE}} \leq \tilde{R}_{HA}. \tag{15}$$

b) $g(p_n^{NE}, p_c^{NE}) > \hat{I}$*, and*

$$(p_n^{NE}, p_c^{NE}) = \begin{cases} \left(\hat{p}_n, \frac{\hat{p}_n}{\tilde{R}_{LB}}\right), & \text{if } \frac{\hat{p}_n}{\hat{p}_c} \leq \tilde{R}_{LB}, \quad ① \\ (\hat{p}_n, \hat{p}_c), & \text{if } \tilde{R}_{LB} < \frac{\hat{p}_n}{\hat{p}_c} < \tilde{R}_{HB}, ② \\ \left(\hat{p}_c \tilde{R}_{HB}, \hat{p}_c\right), & \text{if } \frac{\hat{p}_n}{\hat{p}_c} \geq \tilde{R}_{HB}. \quad ③ \end{cases} \tag{16}$$

Proposition 4. *Under Assumptions 1 to 6, if the following conditions hold,*

$$\left(\frac{1}{T} + \lambda\right)(g(\hat{p}_n, \hat{p}_c) - \hat{I}) + s\hat{p}_n\left(\frac{1}{T} - \frac{\hat{\mu}}{r}\right) - \frac{c_n s^2}{T}(1 + \sqrt{\frac{r\hat{p}_c}{\hat{I} - s\hat{p}_n}})\left(\frac{3}{T} + 2\lambda + \frac{2\sqrt{r\hat{p}_c}}{T\sqrt{\hat{I} - r\hat{p}_c}} + \frac{\sqrt{r\hat{p}_c}}{T\sqrt{s\hat{p}_n}}\right) > 0,$$

$$\left(\frac{1}{T} + \lambda\right)(g(\hat{p}_n, \hat{p}_c) - \hat{I}) + r\hat{p}_c\left(\frac{1}{T} - \frac{\hat{\mu}}{r}\right) - \frac{c_c r^2}{T}(1 + \sqrt{\frac{s\hat{p}_n}{\hat{I} - r\hat{p}_c}})\left(\frac{3}{T} + 2\lambda + \frac{2\sqrt{s\hat{p}_n}}{T\sqrt{\hat{I} - s\hat{p}_n}} + \frac{\sqrt{s\hat{p}_n}}{T\sqrt{r\hat{p}_c}}\right) > 0,$$

then the strategy profile $(\hat{p}_n, \hat{p}_c)$ *is the unique NE of the Price Competition Game.*

1 Proof of Theorem 1

Proof. Solve the CCM problem can be divided into two cases: $\alpha = 1$ (the user offload all the tasks to the collaborative server) and $0 < \alpha < 1$ (the user offload partial the tasks). We compute the minimal costs under both cases and then make comparison to find the optimal solution.

1.1 Calculating optimal solutions for $\alpha = 1$ and $0 < \alpha < 1$

1.1.1 Case 1: $\alpha = 1$

In this case, the CCM problem is as follows.

$$\begin{aligned} \min_{B,\mu} \quad & p_n B + p_c \mu \\ s.t. \quad & \frac{1}{B/s - \lambda} + \frac{1}{\mu/r - \lambda} \leq T, \\ & B/s > \lambda, \\ & \mu/r > \lambda, \end{aligned} \tag{17}$$

Lemma 1. *The solutions to the problem in (17) are*

$$B^* = \frac{s}{T}\left(1 + \sqrt{\frac{rp_c}{sp_n}}\right) + \lambda s, \tag{18}$$

$$\mu^* = \frac{r}{T}\left(1 + \sqrt{\frac{sp_n}{rp_c}}\right) + \lambda r. \tag{19}$$

Proof. The Lagrangian of the problem (17) is as follows:

$$\begin{aligned} L = \; & p_n B + p_c \mu + \beta_1 \left(\frac{1}{B/s - \lambda} + \frac{1}{\mu/r - \lambda} - T \right) \\ & + \beta_2(\lambda - B/s) + \beta_3(\lambda - \mu/r). \end{aligned} \tag{20}$$

The KKT conditions for the optimality are as follows.

$$\frac{\partial L}{\partial B} = p_n - \frac{\beta_1^*}{s(B^*/s - \lambda)^2} - \beta_2^*/s = 0, \tag{21}$$

$$\frac{\partial L}{\partial \mu} = p_c - \frac{\beta_1^*}{r(\mu^*/r - \lambda)^2} - \beta_3^*/r = 0, \tag{22}$$

$$\beta_1^* \left(\frac{1}{B^*/s - \lambda} + \frac{1}{\mu^*/r - \lambda} - T \right) = 0, \tag{23}$$

$$\beta_2^*(\lambda - B^*/s) = 0, \quad \beta_3^*(\lambda - \mu^*/r) = 0, \tag{24}$$

We can check that the following primal and dual variables satisfy the KKT conditions.

$$B^* = \frac{s}{T}\left(1 + \sqrt{\frac{rp_c}{sp_n}}\right) + \lambda s, \tag{25}$$

$$\mu^* = \frac{r}{T}\left(1 + \sqrt{\frac{sp_n}{rp_c}}\right) + \lambda r, \tag{26}$$

$$\beta_1^* = \frac{(\sqrt{sp_n} + \sqrt{rp_c})^2}{T^2}, \beta_2^* = 0, \beta_3^* = 0. \tag{27}$$

To see this, we have

$$B^*/s - \lambda = \frac{1}{T}\left(1 + \sqrt{\frac{rp_c}{sp_n}}\right), \tag{28}$$

$$\mu^*/r - \lambda = \frac{1}{T}\left(1 + \sqrt{\frac{sp_n}{rp_c}}\right), \tag{29}$$

Then, we have

$$
\begin{aligned}
& p_n - \frac{\beta_1^*}{s(B^*/s-\lambda)^2} - \beta_2^*/s \\
= \quad & p_n - \frac{(\sqrt{sp_n}+\sqrt{rp_c})^2}{sT^2} \cdot \frac{T^2}{\left(1+\sqrt{\frac{rp_c}{sp_n}}\right)^2} \\
= \quad & p_n - \frac{(\sqrt{sp_n}+\sqrt{rp_c})^2 \cdot \sqrt{sp_n}^2}{s\left(1+\sqrt{\frac{rp_c}{sp_n}}\right)^2 \cdot \sqrt{sp_n}^2} \\
= \quad & p_n - \frac{sp_n}{s} = 0,
\end{aligned} \tag{30}
$$

and

$$
\begin{aligned}
& p_c - \frac{\beta_1^*}{s(\mu^*/s-\lambda)^2} - \beta_3^*/r \\
= \quad & p_c - \frac{(\sqrt{sp_n}+\sqrt{rp_c})^2}{rT^2} \cdot \frac{T^2}{\left(1+\sqrt{\frac{sp_n}{rp_c}}\right)^2} \\
= \quad & p_c - \frac{(\sqrt{sp_n}+\sqrt{rp_c})^2 \cdot \sqrt{rp_c}^2}{s\left(1+\sqrt{\frac{sp_n}{rp_c}}\right)^2 \cdot \sqrt{rp_c}^2} \\
= \quad & p_c - \frac{rp_c}{r} = 0,
\end{aligned} \tag{31}
$$

Thus, conditions in (21) and (22) hold. Meanwhile,

$$
\begin{aligned}
& \frac{1}{B^*/s-\lambda} + \frac{1}{\mu^*/r-\lambda} - T \\
= & \frac{T}{1+\sqrt{\frac{rp_c}{sp_n}}} + \frac{T}{1+\sqrt{\frac{sp_n}{rp_c}}} - T \\
= & \frac{T\sqrt{sp_n}}{\sqrt{sp_n+\sqrt{rp_c}}} + \frac{T\sqrt{rp_c}}{\sqrt{rp_c}+\sqrt{sp_n}} - T = 0
\end{aligned} \tag{32}
$$

Thus, condition in (23) hold. Finally, condition in (24) hold as $\beta_2^* = \beta_3^* = 0$. In conclusion, we have proved the optimality of the primal and dual variables. □

We plug the expressions of B^* and μ^* in Lemma 1 into the objective function of the problem in (17) and obtain the corresponding cost

$$
C_1 = \frac{1}{T}(\sqrt{sp_n}+\sqrt{rp_c})^2 + \lambda(sp_n + rp_c). \tag{33}
$$

1.1.2 Case 2: $\alpha < 1$

In this case, the CCM problem is as follows.

$$
\begin{aligned}
\min_{\mu_l,\alpha,B,\mu} \quad & c_l\mu_l^2 + p_nB + p_c\mu \\
s.t. \quad & \mu_l/r - (1-\alpha)\lambda \geq 1/T, \\
& \frac{1}{B/s - \alpha\lambda} + \frac{1}{\mu/r - \alpha\lambda} \leq T, \\
& 0 \leq \mu_l \leq \hat{\mu}, \\
& B/s > \alpha\lambda, \\
& \mu/r > \alpha\lambda, \\
& 0 < \alpha < 1.
\end{aligned}
\tag{34}
$$

Lemma 2. *The solutions to the problem in (34) are*

$$
\alpha^* = \begin{cases} 1 - \left(\frac{rp_c+sp_n}{2c_lr^2\lambda} - \frac{1}{T\lambda}\right), & if \;\; \hat{\mu} > \frac{rp_c+sp_n}{2c_lr}, \\ 1 - \left(\frac{\hat{\mu}}{r\lambda} - \frac{1}{T\lambda}\right), & if \;\; \hat{\mu} \leq \frac{rp_c+sp_n}{2c_lr}, \end{cases} \tag{35}
$$

$$
\mu_l^* = \begin{cases} \frac{rp_c+sp_n}{2c_lr}, & if \;\; \hat{\mu} > \frac{rp_c+sp_n}{2c_lr}, \\ \hat{\mu}, & if \;\; \hat{\mu} \leq \frac{rp_c+sp_n}{2c_lr}. \end{cases} \tag{36}
$$

$$
B^* = \begin{cases} \frac{s}{T}\left(2 + \sqrt{\frac{rp_c}{sp_n}}\right) + \lambda s - \frac{s(rp_c+sp_n)}{2c_lr^2}, & if \;\; \hat{\mu} > \frac{rp_c+sp_n}{2c_lr}, \\ \frac{s}{T}\left(2 + \sqrt{\frac{rp_c}{sp_n}}\right) + \lambda s - \frac{\hat{\mu}s}{r}, & if \;\; \hat{\mu} \leq \frac{rp_c+sp_n}{2c_lr}, \end{cases} \tag{37}
$$

$$
\mu^* = \begin{cases} \frac{r}{T}\left(2 + \sqrt{\frac{sp_n}{rp_c}}\right) + \lambda r - \frac{rp_c+sp_n}{2c_lr}, & if \;\; \hat{\mu} > \frac{rp_c+sp_n}{2c_lr}, \\ \frac{r}{T}\left(2 + \sqrt{\frac{sp_n}{rp_c}}\right) + \lambda r - \hat{\mu}, & if \;\; \hat{\mu} \leq \frac{rp_c+sp_n}{2c_lr}. \end{cases} \tag{38}
$$

Proof. The Lagrangian of the problem (34) is as follows:

$$
\begin{aligned}
L = \; & c_l\mu_l^{*2} + p_nB^* + p_c\mu^* + \beta_1^*(1/T - \mu_l^*/r + (1-\alpha^*)\lambda) \\
& + \beta_2^*\left(\frac{1}{B^*/s - \alpha^*\lambda} + \frac{1}{\mu^*/r - \alpha^*\lambda} - T\right) + \beta_3^*(\mu_l^* - \hat{\mu}).
\end{aligned}
\tag{39}
$$

Here, we drop the constraints $\mu_l \geq 0$, $B/s > \alpha\lambda$, $\mu/r > \alpha\lambda$, and $0 < \alpha < 1$. Later we will show that the solutions actually satisfy these constraints. The KKT conditions for the optimality are as follows.

$$
\frac{\partial L}{\partial \mu_l} = 2c_l\mu_l^* - \beta_1^*/r + \beta_3^* = 0, \tag{40}
$$

$$
\frac{\partial L}{\partial \alpha} = -\beta_1^*\lambda + \frac{\beta_2^*\lambda}{(B^*/s - \alpha^*\lambda)^2} + \frac{\beta_2^*\lambda}{(\mu^*/r - \alpha^*\lambda)^2} = 0, \tag{41}
$$

$$
\frac{\partial L}{\partial B} = p_n - \frac{\beta_2^*}{s(B^*/s - \alpha^*\lambda)^2} = 0, \tag{42}
$$

$$
\frac{\partial L}{\partial \mu} = p_c - \frac{\beta_2^*}{r(\mu^*/r - \alpha^*\lambda)^2} = 0, \tag{43}
$$

$$
\beta_1^*(1/T - \mu_l^*/r + (1-\alpha^*)\lambda) = 0, \tag{44}
$$

$$\beta_2^*\left(\frac{1}{B^*/s-\alpha^*\lambda}+\frac{1}{\mu^*/r-\alpha^*\lambda}-T\right)=0, \tag{45}$$

$$\beta_3^*(\mu_l^*-\hat{\mu})=0, \tag{46}$$

We can check that the primal variables in Lemma 2 and the dual variables as follows satisfy the KKT conditions.

$$\beta_1^*=rp_c+sp_n \tag{47}$$

$$\beta_2^*=\frac{(\sqrt{sp_n}+\sqrt{rp_c})^2}{T^2} \tag{48}$$

$$\beta_3^*=\begin{cases}0 & \text{if } \ \hat{\mu}>\frac{rp_c+sp_n}{2c_l r},\\ \frac{rp_c+sp_n}{r}-2c_l\hat{\mu}, & \text{if } \ \hat{\mu}\le\frac{rp_c+sp_n}{2c_l r}.\end{cases} \tag{49}$$

To see this, we divide the discussions into two cases: $\hat{\mu}>\frac{rp_c+sp_n}{2c_l r}$ and $\hat{\mu}\le\frac{rp_c+sp_n}{2c_l r}$.

- Case 1: $\hat{\mu}>\frac{rp_c+sp_n}{2c_l r}$. We have

$$B^*/s-\alpha^*\lambda=\frac{1}{T}\left(2+\sqrt{\frac{rp_c}{sp_n}}\right)+\lambda-\frac{rp_c+sp_n}{2c_l r^2}-\lambda+\left(\frac{rp_c+sp_n}{2c_l r^2}-\frac{1}{T}\right)=\frac{1}{T}\left(1+\sqrt{\frac{rp_c}{sp_n}}\right)=\frac{\sqrt{sp_n}+\sqrt{rp_c}}{T\sqrt{sp_n}} \tag{50}$$

$$\mu^*/r-\alpha^*\lambda=\frac{1}{T}\left(2+\sqrt{\frac{sp_n}{rp_c}}\right)+\lambda-\frac{rp_c+sp_n}{2c_l}-\lambda+\left(\frac{rp_c+sp_n}{2c_l r^2}-\frac{1}{T}\right)=\frac{1}{T}\left(1+\sqrt{\frac{sp_n}{rp_c}}\right)=\frac{\sqrt{sp_n}+\sqrt{rp_c}}{T\sqrt{rp_c}} \tag{51}$$

Next, we show that the KKT conditions are satisfied:

$$\frac{\partial L}{\partial \mu_l}=2c_l\mu_l^*-\beta_1^*/r+\beta_3^*=2c_l\cdot\frac{rp_c+sp_n}{2c_l r}-\frac{rp_c+sp_n}{r}+0=0, \tag{52}$$

$$\begin{aligned}\frac{\partial L}{\partial \alpha}&=-\beta_1^*\lambda+\frac{\beta_2^*\lambda}{(B^*/s-\alpha^*\lambda)^2}+\frac{\beta_2^*\lambda}{(\mu^*/r-\alpha^*\lambda)^2}\\&=-(rp_c+sp_n)\lambda+\frac{(\sqrt{sp_n}+\sqrt{rp_c})^2\lambda}{T^2}\left(\frac{T^2sp_n}{\left(\sqrt{sp_n}+\sqrt{rp_c}\right)^2}+\frac{T^2rp_c}{\left(\sqrt{sp_n}+\sqrt{rp_c}\right)^2}\right)\\&=0\end{aligned} \tag{53}$$

$$\frac{\partial L}{\partial B}=p_n-\frac{\beta_2^*}{s(B^*/s-\alpha^*\lambda)^2}=p_n-\frac{(\sqrt{sp_n}+\sqrt{rp_c})^2}{T^2s}\cdot\frac{T^2sp_n}{(\sqrt{sp_n}+\sqrt{rp_c})^2}=0, \tag{54}$$

$$\frac{\partial L}{\partial \mu}=p_c-\frac{\beta_2^*}{r(\mu^*/r-\alpha^*\lambda)^2}=p_c-\frac{(\sqrt{sp_n}+\sqrt{rp_c})^2}{T^2r}\cdot\frac{T^2rp_c}{(\sqrt{sp_n}+\sqrt{rp_c})^2}=0, \tag{55}$$

$$\beta_1^*(1/T-\mu_l^*/r+(1-\alpha^*)\lambda)=(rp_c+sp_n)\cdot\left(\frac{1}{T}-\frac{rp_c+sp_n}{2c_l r^2}+\left(\frac{rp_c+sp_n}{2c_l r^2\lambda}-\frac{1}{T\lambda}\right)\cdot\lambda\right)=0, \tag{56}$$

$$\beta_2^*\left(\frac{1}{B^*/s-\alpha^*\lambda}+\frac{1}{\mu^*/r-\alpha^*\lambda}-T\right)=\frac{(\sqrt{sp_n}+\sqrt{rp_c})^2}{T^2}\left(\frac{T\sqrt{sp_n}}{\sqrt{sp_n}+\sqrt{rp_c}}+\frac{T\sqrt{rp_c}}{\sqrt{sp_n}+\sqrt{rp_c}}-T\right)=0, \tag{57}$$

$$\beta_3^*(\mu_l^*-\hat{\mu})=0\cdot(\mu_l^*-\hat{\mu})=0. \tag{58}$$

It is also obvious that $\mu_l^*\geq 0$, $B^*/s>\alpha^*\lambda$, $\mu^*/r>\alpha^*\lambda$, and $0<\alpha^*<1$.

- Case 2: $\hat{\mu}\leq\frac{rp_c+sp_n}{2c_lr}$. We have

$$B^*/s-\alpha^*\lambda=\frac{1}{T}\left(2+\sqrt{\frac{rp_c}{sp_n}}\right)+\lambda-\frac{\hat{\mu}}{r}-\lambda+\left(\frac{\hat{\mu}}{r}-\frac{1}{T}\right)=\frac{1}{T}\left(1+\sqrt{\frac{rp_c}{sp_n}}\right)=\frac{\sqrt{sp_n}+\sqrt{rp_c}}{T\sqrt{sp_n}} \tag{59}$$

$$\mu^*/r-\alpha^*\lambda=\frac{1}{T}\left(2+\sqrt{\frac{sp_n}{rp_c}}\right)+\lambda-\frac{\hat{\mu}}{r}-\lambda+\left(\frac{\hat{\mu}}{r}-\frac{1}{T}\right)=\frac{1}{T}\left(1+\sqrt{\frac{sp_n}{rp_c}}\right)=\frac{\sqrt{sp_n}+\sqrt{rp_c}}{T\sqrt{rp_c}} \tag{60}$$

Next, we show that the KKT conditions are satisfied:

$$\frac{\partial L}{\partial\mu_l}=2c_l\mu_l^*-\beta_1^*/r+\beta_3^*=2c_l\cdot\hat{\mu}-\frac{rp_c+sp_n}{r}+\frac{rp_c+sp_n}{r}-2c_l\hat{\mu}=0, \tag{61}$$

$$\begin{aligned}\frac{\partial L}{\partial\alpha}&=-\beta_1^*\lambda+\frac{\beta_2^*\lambda}{(B^*/s-\alpha^*\lambda)^2}+\frac{\beta_2^*\lambda}{(\mu^*/r-\alpha^*\lambda)^2}\\&=-(rp_c+sp_n)\lambda+\frac{(\sqrt{sp_n}+\sqrt{rp_c})^2\lambda}{T^2}\left(\frac{T^2sp_n}{\left(\sqrt{sp_n}+\sqrt{rp_c}\right)^2}+\frac{T^2rp_c}{\left(\sqrt{sp_n}+\sqrt{rp_c}\right)^2}\right)\\&=0\end{aligned} \tag{62}$$

$$\frac{\partial L}{\partial B}=p_n-\frac{\beta_2^*}{s(B^*/s-\alpha^*\lambda)^2}=p_n-\frac{(\sqrt{sp_n}+\sqrt{rp_c})^2}{T^2s}\cdot\frac{T^2sp_n}{(\sqrt{sp_n}+\sqrt{rp_c})^2}=0, \tag{63}$$

$$\frac{\partial L}{\partial\mu}=p_c-\frac{\beta_2^*}{r(\mu^*/r-\alpha^*\lambda)^2}=p_c-\frac{(\sqrt{sp_n}+\sqrt{rp_c})^2}{T^2r}\cdot\frac{T^2rp_c}{(\sqrt{sp_n}+\sqrt{rp_c})^2}=0, \tag{64}$$

$$\beta_1^*(1/T-\mu_l^*/r+(1-\alpha^*)\lambda)=(rp_c+sp_n)\cdot\left(\frac{1}{T}-\frac{\hat{\mu}}{r}+\left(\frac{\hat{\mu}}{r\lambda}-\frac{1}{T\lambda}\right)\cdot\lambda\right)=0, \tag{65}$$

$$\beta_2^*\left(\frac{1}{B^*/s-\alpha^*\lambda}+\frac{1}{\mu^*/r-\alpha^*\lambda}-T\right)=\frac{(\sqrt{sp_n}+\sqrt{rp_c})^2}{T^2}\left(\frac{T\sqrt{sp_n}}{\sqrt{sp_n}+\sqrt{rp_c}}+\frac{T\sqrt{rp_c}}{\sqrt{sp_n}+\sqrt{rp_c}}-T\right)=0, \tag{66}$$

$$\beta_3^*(\mu_l^*-\hat{\mu})=\frac{rp_c+sp_n}{r}-2c_l\hat{\mu}\cdot(\hat{\mu}-\hat{\mu})=0. \tag{67}$$

It is also obvious that $\mu_l^*\geq 0$, $B^*/s>\alpha^*\lambda$, $\mu^*/r>\alpha^*\lambda$, and $0<\alpha^*<1$.

□

We plug the expressions of α^*, μ_l, B^* and μ^* into the objective function of the problem in (34) and obtain the corresponding cost. When $\hat{\mu} > \frac{rp_c+sp_n}{2c_l r}$, the cost is

$$C_{2,1} = \frac{2\sqrt{sp_n rp_c} + 2sp_n + 2rp_c}{T} + (rp_c + sp_n)(\lambda - \frac{rp_c + sp_n}{4c_l r^2}). \tag{68}$$

When $\hat{\mu} \leq \frac{rp_c+sp_n}{2c_l r}$, the cost is

$$C_{2,2} = c_l \hat{\mu}^2 + \frac{2\sqrt{sp_n rp_c} + 2sp_n + 2rp_c}{T} + (rp_c + sp_n)(\lambda - \frac{\hat{\mu}}{r}). \tag{69}$$

1.2 Considering two scenarios

Next, we consider two scenarios, $\hat{\mu} > 2r/T$ and $\hat{\mu} \leq 2r/T$. For each scenario, we compare the minimum costs in two cases to characterize the optimal decisions. We define

$$g(p_n, p_c) \triangleq sp_n + rp_c \tag{70}$$

and

$$\hat{I} \triangleq \frac{4c_l r^2}{T\left(1 - \left(1 - \frac{2r}{\hat{\mu}T}\right)^2\right)}, \tag{71}$$

in which $2r \geq \hat{\mu}T$ holds.

$h(p_n, p_c) = 2g(p_n, p_c)r(1 - \sqrt{1 - 1/Tg(p_n, p_c)})$, where we need $1 < Tg(p_n, p_c)$ to well define $h(p_n, p_c)$.

1.2.1 Scenario 1: $\hat{\mu} > 2r/T$

- When $g(p_n, p_c) \leq 4c_l r^2/T$, we have $g(p_n, p_c) \leq 4c_l r^2/T < 2c_l r\hat{\mu}$, i.e., $\frac{rp_c+sp_n}{2c_l r} < \hat{\mu}$. Thus, the minimum cost in the case of $\alpha < 1$ is $C_{2,1}$. We calculate

$$\begin{aligned}
&C_{2,1} - C_1 \\
=&\frac{2\sqrt{sp_n rp_c} + 2sp_n + 2rp_c}{T} + (rp_c + sp_n)(\lambda - \frac{rp_c + sp_n}{4c_l r^2}) - \frac{1}{T}(\sqrt{sp_n} + \sqrt{rp_c})^2 - \lambda(sp_n + rp_c) \\
=&\frac{sp_n + rp_c}{T} - \frac{(rp_c + sp_n)^2}{4c_l r^2} \\
=&(rp_c + sp_n)\left(\frac{1}{T} - \frac{rp_c + sp_n}{4c_l r^2}\right) \\
\geq&0.
\end{aligned} \tag{72}$$

  The above inequality holds as $g(p_n, p_c) = (sp_n + rp_c) \leq 4c_l r^2/T$. In conclusion, choosing $\alpha = 1$ gives the minimum cost. Thus, the optimal decisions are presented in Lemma 1 with $\alpha^* = 1$ and $\mu_l^* = 0$.

- When $g(p_n, p_c) > 4c_l r^2/T$ and $g(p_n, p_c) < 2c_l r\hat{\mu}$, i.e., $\frac{rp_c+sp_n}{2c_l r} < \hat{\mu}$, the minimum cost in the case of $\alpha < 1$ is $C_{2,1}$. We calculate

$$\begin{aligned}
&C_{2,1} - C_1 \\
=&\frac{sp_n + rp_c}{T} - \frac{(rp_c + sp_n)^2}{4c_l r^2} \\
=&(rp_c + sp_n)\left(\frac{1}{T} - \frac{rp_c + sp_n}{4c_l r^2}\right) \\
<&0.
\end{aligned} \tag{73}$$

The above inequality holds as $g(p_n, p_c) = (sp_n + rp_c) > 4c_l r^2/T$. In conclusion, choosing $\alpha < 1$ gives the minimum cost. Thus, the optimal decisions are presented in Lemma 2 when $\hat{\mu} < \frac{rp_c+sp_n}{2c_l r}$.

- When $g(p_n, p_c) \geq 2c_l r\hat{\mu}$, i.e., $\frac{rp_c+sp_n}{2c_l r} \geq \hat{\mu}$, the minimum cost in the case of $\alpha < 1$ is $C_{2,2}$. We calculate

$$
\begin{aligned}
&C_{2,2} - C_1 \\
=&c_l\hat{\mu}^2 + \frac{2\sqrt{sp_n rp_c} + 2sp_n + 2rp_c}{T} + (rp_c + sp_n)(\lambda - \frac{\hat{\mu}}{r}) - \frac{1}{T}(\sqrt{sp_n} + \sqrt{rp_c})^2 - \lambda(sp_n + rp_c) \\
=&c_l\hat{\mu}^2 - \frac{(sp_n + rp_c)\hat{\mu}}{r} + \frac{sp_n + rp_c}{T} \\
=&c_l\left(\hat{\mu} - \frac{sp_n + rp_c}{2c_l r}\right)^2 + \frac{sp_n + rp_c}{T} - \frac{(sp_n + rp_c)^2}{4c_l r^2} \\
=&c_l\left(\hat{\mu} - \frac{sp_n + rp_c}{2c_l r}\right)^2 + (sp_n + rp_c)\left(\frac{1}{T} - \frac{sp_n + rp_c}{4c_l r^2}\right)
\end{aligned}
\tag{74}
$$

Define a function

$$
f(x) = c_l\left(x - \frac{sp_n + rp_c}{2c_l r}\right)^2 + \frac{sp_n + rp_c}{T} - \frac{(sp_n + rp_c)^2}{4c_l r^2}. \tag{75}
$$

As $g(p_n, p_c) \geq 2c_l r\hat{\mu} > 4c_l r^2/T$, we get that

$$
f\left(\frac{sp_n + rp_c}{2c_l r}\right) = (sp_n + rp_c)\left(\frac{1}{T} - \frac{sp_n + rp_c}{4c_l r^2}\right) < 0. \tag{76}
$$

Notice that $x_1 = \frac{2r}{T\left(1+\sqrt{1-\frac{4c_l r^2}{(rp_c+sp_n)T}}\right)}$ is the left zero point of $f(x)$. As $rp_c + sp_n > 4c_l r^2/T$, we have $\frac{4c_l r^2}{(rp_c+sp_n)T} < 1$, which gives

$$
x_1 = \frac{2r}{T\hat{\mu}} \cdot \frac{\hat{\mu}}{\left(1 + \sqrt{1 - \frac{4c_l r^2}{(rp_c+sp_n)T}}\right)} < \hat{\mu} \tag{77}
$$

The above inequality holds due to $2r/(T\hat{\mu}) < 1$. That is, $x_1 < \hat{\mu}$. According to the shape of the quadratic function $f(x)$, we obtain $f(\hat{\mu}) < 0$, i.e., $C_{2,2} - C_1 < 0$. In conclusion, choosing $\alpha < 1$ gives the minimum cost. Thus, the optimal decisions are presented in Lemma 2 when $\hat{\mu} \leq \frac{rp_c+sp_n}{2c_l r}$.

1.2.2 Scenario 2: $\hat{\mu} \leq 2r/T$

- When $g(p_n, p_c) \leq \hat{I}$, as $\hat{I} > \frac{4c_l r^2}{T}$, this case can be further divided into two subcases: $g(p_n, p_c) < \frac{4c_l r^2}{T}$ and $\frac{4c_l r^2}{T} \leq g(p_n, p_c) \leq \hat{I}$.
  - $g(p_n, p_c) < \frac{4c_l r^2}{T}$: the minimum cost in the subcase of $g(p_n, p_c) < \frac{4c_l r^2}{T}$ is lower than that in the case of $\alpha < 1$. To see this, we calculate

$$
\begin{aligned}
&C_{2,1} - C_1 \\
=&\frac{sp_n + rp_c}{T} - \frac{(rp_c + sp_n)^2}{4c_l r^2} \\
=&(rp_c + sp_n)\left(\frac{1}{T} - \frac{rp_c + sp_n}{4c_l r^2}\right) \\
>&0.
\end{aligned}
\tag{78}
$$

The above inequality holds as $rp_c + sp_n < \frac{4c_l r^2}{T}$. And we also have

$$
\begin{aligned}
& C_{2,2} - C_1 \\
=& c_l\hat{\mu}^2 - \frac{(sp_n + rp_c)\hat{\mu}}{r} + \frac{sp_n + rp_c}{T} \\
=& c_l\left(\hat{\mu} - \frac{sp_n + rp_c}{2c_l r}\right)^2 + \frac{sp_n + rp_c}{T} - \frac{(sp_n + rp_c)^2}{4c_l r^2} \\
=& c_l\left(\hat{\mu} - \frac{sp_n + rp_c}{2c_l r}\right)^2 + (sp_n + rp_c)\left(\frac{1}{T} - \frac{rp_c + sp_n}{4c_l r^2}\right) \\
>& 0.
\end{aligned} \tag{79}
$$

The above inequality holds as $rp_c + sp_n < \frac{4c_l r^2}{T}$. In conclusion, choosing $\alpha = 1$ in the subcase of $g(p_n, p_c) < \frac{4c_l r^2}{T}$ gives the minimum cost. Thus, the optimal decisions in the subcase of $g(p_n, p_c) < \frac{4c_l r^2}{T}$ are presented in Lemma 1 with $\alpha^* = 1$ and $\mu_l^* = 0$.

– $\frac{4c_l r^2}{T} \le g(p_n, p_c) \le \hat{I}$: As $2r/T \ge \hat{\mu}$, we have as $g(p_n, p_c) \ge \frac{4c_l r^2}{T} \ge 2c_l\hat{\mu}r$. Thus, the minimum cost in the case of $\alpha < 1$ is $C_{2,2}$. We calculate

$$
C_{2,2} - C_1 = c_l\hat{\mu}^2 - \frac{(sp_n + rp_c)\hat{\mu}}{r} + \frac{sp_n + rp_c}{T}. \tag{80}
$$

From $g(p_n, p_c) \le \hat{I}$, we have

$$
\begin{aligned}
& \frac{4c_l r^2}{T\left(1 - \left(1 - \frac{2r}{\hat{\mu}T}\right)^2\right)} \ge sp_n + rp_c \\
\Rightarrow & 1 - \left(1 - \frac{2r}{\hat{\mu}T}\right)^2 \le \frac{4c_l r^2}{T(sp_n + rp_c)} \\
\Rightarrow & \frac{2r}{\hat{\mu}T} - 1 \ge \sqrt{1 - \frac{4c_l r^2}{T(sp_n + rp_c)}} \\
\Rightarrow & \frac{2r}{\hat{\mu}T} \ge 1 + \sqrt{1 - \frac{4c_l r^2}{T(sp_n + rp_c)}} \\
\Rightarrow & \frac{2r}{T\left(1 + \sqrt{1 - \frac{4c_l r^2}{(rp_c + sp_n)T}}\right)} \ge \hat{\mu}.
\end{aligned} \tag{81}
$$

That is, $x_1 \ge \hat{\mu}$. As $f(x_1) = 0$, we have $f(\hat{\mu}) \ge 0$. That is, $C_{2,2} - C_1 \ge 0$. In conclusion, choosing $\alpha = 1$ in the subcase of $\frac{4c_l r^2}{T} \le g(p_n, p_c) \le \hat{I}$ gives the minimum cost. Thus, the optimal decisions are presented in Lemma 1 with $\alpha^* = 1$ and $\mu_l^* = 0$.

In summary, when $g(p_n, p_c) \le \hat{I}$, the optimal decisions are presented in Lemma 1 with $\alpha^* = 1$ and $\mu_l^* = 0$.

- When $g(p_n, p_c) > \hat{I}$, as $2r/T \ge \hat{\mu}$, we have as $g(p_n, p_c) > \hat{I} > \frac{4c_l r^2}{T} \ge 2c_l\hat{\mu}r$. Thus, the minimum cost in the case of $\alpha < 1$ is $C_{2,2}$. We calculate

$$
C_{2,2} - C_1 = c_l\hat{\mu}^2 - \frac{(sp_n + rp_c)\hat{\mu}}{r} + \frac{sp_n + rp_c}{T}. \tag{82}
$$

From $g(p_n, p_c) > \hat{I}$, we have $x_1 < \hat{\mu}$. As $f(x_1) = 0$, we have $f(\hat{\mu}) < 0$. That is, $C_{2,2} - C_1 < 0$. In conclusion, choosing $\alpha < 1$ gives the minimum cost. Thus, the optimal decisions are presented in Lemma 2 when $\hat{\mu} \le \frac{rp_c + sp_n}{2c_l r}$.

The proof of Theorem 1 ends.

□

2 Proof of Theorem 2

Under Assumption 4, i.e., $\hat{\mu} \le 2r/T$, there are two cases: Case A, $g(p_n, p_c) \le \hat{I}$ and Case B, $g(p_n, p_c) > \hat{I}$. We discuss these two cases separately.

2.1 Case A: $g(p_n, p_c) \le \hat{I}$

By plugging the users' decisions (B^*, μ^*) in Theorem 1 into the providers' PPM problem, we obtain the following optimization problem P1,

$$\begin{aligned}
\text{P1}\ \ \max_{p_n, p_c}\ & \left(\frac{1}{T} + \lambda\right) \cdot (rp_c + sp_n) + \frac{2}{T}\sqrt{rsp_cp_n} - \frac{sr}{T^2}\left(\frac{c_np_c}{p_n} + \frac{c_cp_n}{p_c}\right) - \frac{\sqrt{rs}(2+2\lambda T)}{T^2}\left(\sqrt{\frac{p_c}{p_n}} \cdot c_ns + \sqrt{\frac{p_n}{p_c}} \cdot c_cr\right) \\
s.t.\ \ & B_{\min} \le \frac{s}{T}\left(1 + \sqrt{\frac{rp_c}{sp_n}}\right) + \lambda s \le B_{\max}, \\
& \mu_{\min} \le \frac{r}{T}\left(1 + \sqrt{\frac{sp_n}{rp_c}}\right) + \lambda r \le \mu_{\max}, \\
& sp_n + rp_c \le \hat{I}.
\end{aligned} \tag{83}$$

Define an auxiliary variable $t \triangleq \sqrt{\frac{p_n}{p_c}}$. Obviously, we have $p_n = p_ct^2$. We can transform the above optimization problem P1 with decisions variables p_n and p_c to an equivalent P2 with decisions variables t and p_c as follows:

$$\begin{aligned}
\text{P2}\ \ \min_{t, p_c}\ & -\left(\frac{1}{T} + \lambda\right) \cdot (rp_c + sp_ct^2) - \frac{2}{T}\sqrt{rs}p_ct + \frac{sr}{T^2}\left(\frac{c_n}{t^2} + c_ct^2\right) + \frac{\sqrt{rs}(2+2\lambda T)}{T^2}\left(\frac{c_ns}{t} + c_crt\right) \\
s.t.\ \ & R_{LA} \le t \le R_{HA}, \\
& sp_ct^2 + rp_c \le \hat{I}.
\end{aligned} \tag{84}$$

Recall that the constants R_{LA} and R_{HA} are given by

$$R_{LA} = \max\left\{\frac{T}{\sqrt{rs}}\left(\mu_{\min} - \frac{r}{T} - r\lambda\right), \frac{\sqrt{rs}}{T\left(B_{\max} - \frac{s}{T} - s\lambda\right)}\right\}, \tag{85}$$

$$R_{HA} = \min\left\{\frac{T}{\sqrt{rs}}\left(\mu_{\max} - \frac{r}{T} - r\lambda\right), \frac{\sqrt{rs}}{T\left(B_{\min} - \frac{s}{T} - s\lambda\right)}\right\}. \tag{86}$$

The first constraint in P2 is derived from the first and the second constraints in P1.

It suffices to analyze P2 to obtain the providers' optimal decisions. The Lagragian of P2 is as follows:

$$\begin{aligned}
L = & -\left(\frac{1}{T} + \lambda\right) \cdot (rp_c + sp_ct^2) - \frac{2}{T}\sqrt{rs}p_ct + \frac{sr}{T^2}\left(\frac{c_n}{t^2} + c_ct^2\right) + \frac{\sqrt{rs}(2+2\lambda T)}{T^2}\left(\frac{c_ns}{t} + c_crt\right) \\
& + \beta_1(R_{LA} - t) + \beta_2(t - R_{HA}) + \beta_3(sp_ct^2 + rp_c - \hat{I})
\end{aligned} \tag{87}$$

The derivatives of the Lagragian with respect to t and p_c are as follows

$$\frac{\partial L}{\partial t} = -2sp_c\left(\frac{1}{T}+\lambda\right)t - \frac{2}{T}\sqrt{rs}p_c + \frac{sr}{T^2}\left(-\frac{2c_n}{t^3}+2c_ct\right) + \frac{\sqrt{rs}(2+2\lambda T)}{T^2}\left(-\frac{c_ns}{t^2}+c_cr\right) - \beta_1+\beta_2+2\beta_3p_cst, \tag{88}$$

$$\frac{\partial L}{\partial p_c} = -\left(\frac{1}{T}+\lambda\right)\cdot(r+st^2) - \frac{2}{T}\sqrt{rs}t + \beta_3(st^2+r). \tag{89}$$

From $\frac{\partial L}{\partial p_c}=0$, we can obtain

$$\beta_3 = \frac{\left(\frac{1}{T}+\lambda\right)\cdot(r+st^2)+\frac{2}{T}\sqrt{rs}t}{st^2+r} > 0. \tag{90}$$

Complementary slackness $\beta_3\cdot(sp_ct^2+rp_c-\hat{I})=0$ gives $sp_ct^2+rp_c=\hat{I}$, i.e., $sp_n+rp_c\le\hat{I}$. That is, in the Case A ($g(p_n,p_c)\le\hat{I}$), the optimal solution satisfies $g(p_n^*,p_c^*)=\hat{I}$. This concludes the proof for Case A.

2.2 Case B: $g(p_n,p_c)>\hat{I}$

By plugging the users' decisions (B^*,μ^*) in Theorem 1 into the providers' PPM problem, we obtain the following optimization problem P1,

$$\begin{aligned}
\text{P1}\quad \max_{p_n,p_c}\quad & (sp_n+rp_c)\left(\frac{2}{T}+\lambda-\frac{\hat{\mu}}{r}\right)+\frac{2}{T}\sqrt{rsp_cp_n}-\frac{sr}{T^2}\left(\frac{c_np_c}{p_n}+\frac{c_cp_n}{p_c}\right)-\frac{2\sqrt{rs}}{T}\left(\frac{2}{T}+\lambda-\frac{\hat{\mu}}{r}\right)\left(\sqrt{\frac{p_c}{p_n}}\cdot c_ns+\sqrt{\frac{p_n}{p_c}}\cdot c_cr\right)\\
s.t.\quad & B_{\min}\le\frac{s}{T}\left(2+\sqrt{\frac{rp_c}{sp_n}}\right)+\lambda s-\frac{\hat{\mu}s}{r}\le B_{\max},\\
& \mu_{\min}\le\frac{r}{T}\left(2+\sqrt{\frac{sp_n}{rp_c}}\right)+\lambda r-\hat{\mu}\le\mu_{\max},\\
& sp_n+rp_c>\hat{I},\\
& 0\le p_c\le\hat{p}_c, 0\le p_n\le\hat{p}_n.
\end{aligned} \tag{91}$$

We can transform the above optimization problem with decisions variables p_n and p_c to an equivalent one with decisions variables $t\triangleq\sqrt{\frac{p_n}{p_c}}$ and p_c as follows:

$$\begin{aligned}
\min_{t,p_c}\quad & -\left(\frac{2}{T}+\lambda-\frac{\mu}{r}\right)\cdot(rp_c+sp_ct^2)-\frac{2}{T}\sqrt{rs}p_ct+\frac{sr}{T^2}\left(\frac{c_n}{t^2}+c_ct^2\right)+\frac{2\sqrt{rs}}{T}\left(\frac{2}{T}+\lambda-\frac{\hat{\mu}}{r}\right)\left(\frac{c_ns}{t}+c_crt\right)\\
s.t.\quad & R_{LB}\le t\le R_{HB},\\
& sp_ct^2+rp_c>\hat{I},\\
& 0\le p_c\le\hat{p}_c, 0\le p_ct^2\le\hat{p}_n.
\end{aligned} \tag{92}$$

Recall that the constants R_{LB} and R_{HB} are given by

$$R_{LB} = \max\left\{\frac{T}{\sqrt{rs}}\left(\mu_{\min}-\frac{2r}{T}-r\lambda+\hat{\mu}\right), \frac{\sqrt{rs}}{T\left(B_{\max}-\frac{2s}{T}-s\lambda+\frac{\hat{\mu}s}{r}\right)}\right\}, \tag{93}$$

$$R_{HB} = \min\left\{\frac{T}{\sqrt{rs}}\left(\mu_{\max} - \frac{2r}{T} - r\lambda + \hat{\mu}\right), \frac{\sqrt{rs}}{T\left(B_{\min} - \frac{2s}{T} - s\lambda + \frac{\hat{\mu}s}{r}\right)}\right\}. \tag{94}$$

It suffices to analyze the transformed problem to obtain the providers' optimal decisions. The Lagragian of P2 is as follows:

$$\begin{aligned} L = &-\left(\frac{2}{T} + \lambda - \frac{\mu}{r}\right)\cdot(rp_c + sp_ct^2) - \frac{2}{T}\sqrt{rs}p_ct + \frac{sr}{T^2}\left(\frac{c_n}{t^2} + c_ct^2\right) + \frac{2\sqrt{rs}}{T}\left(\frac{2}{T} + \lambda - \frac{\hat{\mu}}{r}\right)\left(\frac{c_ns}{t} + c_crt\right) \\ &+ \beta_1(R_{LA} - t) + \beta_2(t - R_{HA}) + \beta_3(p_c - \hat{p}_n) + \beta_4(p_ct^2 - \hat{p}_n). \end{aligned} \tag{95}$$

Here, we drop the constraint $sp_ct^2 + rp_c > \hat{I}$. Later, we will show that the solution indeed satisfies this constraint. The derivatives of the Lagragian with respect to t and p_c are as follows

$$\frac{\partial L}{\partial t} = -2sp_c\left(\frac{2}{T} + \lambda - \frac{\mu}{r}\right)t - \frac{2}{T}\sqrt{rs}p_c + \frac{sr}{T^2}\left(-\frac{2c_n}{t^3} + 2c_ct\right) + \frac{2\sqrt{rs}}{T}\left(\frac{2}{T} + \lambda - \frac{\hat{\mu}}{r}\right)\left(-\frac{c_ns}{t^2} + c_cr\right) - \beta_1 + \beta_2 + 2\beta_4p_ct, \tag{96}$$

$$\frac{\partial L}{\partial p_c} = -\left(\frac{2}{T} + \lambda - \frac{\mu}{r}\right)\cdot(r + st^2) - \frac{2}{T}\sqrt{rs}t + \beta_3 + \beta_4t^2. \tag{97}$$

By KKT condition $\frac{\partial L}{\partial p_c} = 0$, we have

$$\beta_3 + \beta_4t^2 = \left(\frac{2}{T} + \lambda - \frac{\mu}{r}\right)\cdot(r + st^2) + \frac{2}{T}\sqrt{rs}t > 0, \tag{98}$$

which indicates at least one of β_3 and β_4 is greater than zero. Combining complementary slackness ($\beta_3(p_c - \hat{p}_n) = 0$ and $\beta_4(p_ct^2 - \hat{p}_n)$, we can see that at least one of $p_c - \hat{p}_n = 0$ and $p_ct^2 - \hat{p}_n = 0$ holds. This concludes the proof for Case B.

3 Proof of Proposition 1

The proof of Proposition 1 follows the analysis of Case A regarding the proof for Case A of Theorem 2 in Section 2.1. We put the expression of β_3 in equation (90) and the expression of $p_c = \hat{I}/(st^2 + r)$ to the derivative of Lagragian with respect to the variable t in equation (88). We can obtain

$$\begin{aligned} \frac{\partial L}{\partial t} =& \frac{2\hat{I}(st^2 - r)\sqrt{sr}}{T(t^2s + r)^2} - c_n\left(\frac{2sr}{T^2t^3} + \frac{s(2 + 2\lambda T)\sqrt{rs}}{T^2t^2}\right) + c_c\left(\frac{2tsr}{T^2} + \frac{r(2 + 2\lambda T)\sqrt{rs}}{T^2}\right) - \beta_1 + \beta_2 \\ =& f_A(t) - \beta_1 + \beta_2. \end{aligned} \tag{99}$$

We have the following three cases depending on the sign of the function $f_A(t)$.

i) if $f_A(t) > 0$ for $t \in [R_{LA}, R_{HA}]$, by KKT condition $\frac{\partial L}{\partial t} = 0$, we have $\beta_1 = f_A(t) + \beta_2 > 0$. And complementary slackness $\beta_1(R_{LA} - t) = 0$ gives $t = R_{LA}$. That is $p_n^*/p_c^* = R_{LA}^2$.

ii) if $f_A(t) < 0$ for $t \in [R_{LA}, R_{HA}]$, by KKT condition $\frac{\partial L}{\partial t} = 0$, we have $\beta_2 = -f_A(t) + \beta_2 > 0$. And complementary slackness $\beta_2(R_{HA} - t) = 0$ gives $t = R_{HA}$. That is $p_n^*/p_c^* = R_{HA}^2$.

iii) if $f_A(t) = 0$ for $t \in [R_{LA}, R_{HA}]$ has a unique solution $\tilde{t}$, and $f_A(t) < 0$ for $t \in [R_{LA}, \tilde{t}]$ and $f_A(t) > 0$ for $t \in [\tilde{t}, R_{HA}]$, suppose the NCR $p_n^*/p_c^* \in (R_{LA}, R_{HA})$. We will show that is indeed the case. By complimentary slackness ($\beta_1(R_{LA}-t)=0$ and $\beta_2(R_{HA}-t)=0$), we have both dual variables β_1 and β_2 to be zeros. Then $\frac{\partial L}{\partial t}=f_A(t)$. As $f_A(t)<0$ for $t\in[R_{LA},\tilde{t}]$ and $f_A(t)>0$ for $t\in[\tilde{t},R_{HA}]$, we can find that L is decreasing in $[R_{LA},\tilde{t}]$ and then increasing in $[R_{LA},\tilde{t}]$. The minimum point is at $t=\tilde{t}\in(R_{LA},R_{HA})$. That is, we have $p_n^*/p_c^*=\tilde{t}^2$.

This concludes the proof of Proposition 1.

4 Proof of Proposition 2

The proof of Proposition 2 follows the analysis of Case B regarding the proof for Case B of Theorem 2 in Section 2.2. Combining $p_n=p_ct^2<\hat{p}_n$ and complimentary slackness $\beta_4(p_ct^2-\hat{p}_n)$, we have $\beta_4=0$. We put the expression of $p_c=\hat{p}_c$ to the derivative of Lagragian with respect to the variable t in equation (96). We can obtain

$$\begin{aligned}\frac{\partial L}{\partial t}=&-2s\hat{p}_c\left(\frac{2}{T}+\lambda-\frac{\mu}{r}\right)t-\frac{2}{T}\sqrt{rs}\hat{p}_c+\frac{sr}{T^2}\left(-\frac{2c_n}{t^3}+2c_ct\right)+\frac{2\sqrt{rs}}{T}\left(\frac{2}{T}+\lambda-\frac{\hat{\mu}}{r}\right)\left(-\frac{c_ns}{t^2}+c_cr\right)-\beta_1+\beta_2\\=&f_B(t)-\beta_1+\beta_2.\end{aligned}\tag{100}$$

We have the following three cases depending on the sign of the function $f_B(t)$.

i) if $f_B(t)>0$ for $t\in[R_{LB},R_{HB}]$, by KKT condition $\frac{\partial L}{\partial t}=0$, we have $\beta_1=f_B(t)+\beta_2>0$. And complementary slackness $\beta_1(R_{LB}-t)=0$ gives $t=R_{LB}$. That is, $p_n^*/p_c^*=R_{LB}^2$.

ii) if $f_B(t)<0$ for $t\in[R_{LB},R_{HB}]$, by KKT condition $\frac{\partial L}{\partial t}=0$, we have $\beta_2=-f_B(t)+\beta_2>0$. And complementary slackness $\beta_2(R_{HB}-t)=0$ gives $t=R_{HB}$. That is, $p_n^*/p_c^*=R_{HB}^2$.

This concludes the proof of Proposition 2.

5 Proof of Proposition 3

The optimal profit in Case A ($g(p_{n,A}^*,rp_{c,A}^*)=\hat{I}$) is

$$\begin{aligned}V_A=&(sp_{n,A}^*+rp_{c,A}^*)\left(\frac{1}{T}+\lambda\right)+\frac{2}{T}\sqrt{srp_{n,A}^*p_{c,A}^*}-c_ns^2\left(\frac{1}{T}\left(1+\sqrt{\frac{rp_{c,A}^*}{sp_{n,A}^*}}\right)+\lambda\right)^2-c_cr^2\left(\frac{1}{T}\left(1+\sqrt{\frac{sp_{n,A}^*}{rp_{c,A}^*}}\right)+\lambda\right)^2\\=&\hat{I}\left(\frac{1}{T}+\lambda\right)+\frac{2}{T}\sqrt{srp_{n,A}^*p_{c,A}^*}-c_ns^2\left(\frac{1}{T}\left(1+\sqrt{\frac{rp_{c,A}^*}{sp_{n,A}^*}}\right)+\lambda\right)^2-c_cr^2\left(\frac{1}{T}\left(1+\sqrt{\frac{sp_{n,A}^*}{rp_{c,A}^*}}\right)+\lambda\right)^2\end{aligned}\tag{101}$$

The optimal profit in Case B ($g(p_{n,B}^*,p_{c,B}^*)>\hat{I}$) is

$$\begin{aligned}V_B=&(sp_{n,B}^*+rp_{c,B}^*)\left(\frac{2}{T}+\lambda-\frac{\hat{\mu}}{r}\right)+\frac{2}{T}\sqrt{srp_{n,B}^*p_{c,B}^*}-c_ns^2\left(\frac{1}{T}\left(2+\sqrt{\frac{rp_{c,B}^*}{sp_{n,B}^*}}\right)+\lambda-\frac{\hat{\mu}}{r}\right)^2\\&-c_cr^2\left(\frac{1}{T}\left(2+\sqrt{\frac{sp_{n,B}^*}{rp_{c,B}^*}}\right)+\lambda-\frac{\hat{\mu}}{r}\right)^2.\end{aligned}\tag{102}$$

If

$$\left(\frac{1}{T}+\lambda\right)(g(p_{n,B}^*,p_{c,B}^*)-\hat{I})+\left(\frac{1}{T}-\frac{\hat{\mu}}{r}\right)g(p_{n,B}^*,p_{c,B}^*)-\frac{c_ns^2}{T}\left(1+\sqrt{\frac{rp_{c,B}^*}{sp_{n,B}^*}}\right)\left(\frac{3}{T}+2\lambda+\frac{\sqrt{rp_{c,B}^*}}{T\sqrt{sp_{n,B}^*}}+\frac{\sqrt{rp_{c,A}^*}}{T\sqrt{sp_{n,A}^*}}\right)$$
$$-\frac{c_cr^2}{T}\left(1+\sqrt{\frac{sp_{n,B}^*}{rp_{c,B}^*}}\right)\left(\frac{3}{T}+2\lambda+\frac{\sqrt{sp_{n,B}^*}}{T\sqrt{rp_{c,B}^*}}+\frac{\sqrt{sp_{n,A}^*}}{T\sqrt{rp_{c,A}^*}}\right)+\frac{2\sqrt{st}}{T}(\sqrt{p_{c,B}^*p_{n,B}^*}-\sqrt{p_{c,A}^*p_{n,A}^*})>0, \tag{103}$$

then

$$\begin{aligned}&\left(\frac{1}{T}+\lambda\right)(g(p_{n,B}^*,p_{c,B}^*)-\hat{I})+\left(\frac{1}{T}-\frac{\hat{\mu}}{r}\right)g(p_{n,B}^*,p_{c,B}^*)+\frac{2\sqrt{st}}{T}(\sqrt{p_{c,B}^*p_{n,B}^*}-\sqrt{p_{c,A}^*p_{n,A}^*})\\&-c_ns^2\left(\frac{1}{T}+\frac{1}{T}\sqrt{\frac{rp_{c,B}^*}{sp_{n,B}^*}}-\frac{1}{T}\sqrt{\frac{rp_{c,A}^*}{sp_{n,A}^*}}-\frac{\hat{\mu}}{r}\right)\left(\frac{3}{T}+2\lambda+\frac{\sqrt{rp_{c,B}^*}}{T\sqrt{sp_{n,B}^*}}+\frac{\sqrt{rp_{c,A}^*}}{T\sqrt{sp_{n,A}^*}}-\frac{\hat{\mu}}{r}\right)\\&-c_cr^2\left(\frac{1}{T}+\frac{1}{T}\sqrt{\frac{sp_{n,B}^*}{rp_{c,B}^*}}-\frac{1}{T}\sqrt{\frac{rp_{c,A}^*}{sp_{n,A}^*}}-\frac{\hat{\mu}}{r}\right)\left(\frac{3}{T}+2\lambda+\frac{\sqrt{sp_{n,B}^*}}{T\sqrt{rp_{c,B}^*}}+\frac{\sqrt{sp_{n,A}^*}}{T\sqrt{rp_{c,A}^*}}-\frac{\hat{\mu}}{r}\right)>0,\end{aligned} \tag{104}$$

i.e., $V_B-V_A>0$. So the pricing strategy $(p_{n,B}^*,p_{c,B}^*)$ is the optimal one that solves the PPM problem. This concludes the proof of Proposition 3.

6 Proof of Theorem 3

Under Assumption 4, i.e., $\hat{\mu}\le 2r/T$, there are two cases: Case A, $g(p_n,p_c)\le\hat{I}$ and Case B, $g(p_n,p_c)>\hat{I}$. We discuss these two cases separately.

6.1 Case A: $g(p_n,p_c)\le\hat{I}$

We will analyze the best responses of network service provider (NSP) and computation service provider (CSP).

6.1.1 NSP's best response

By plugging the users' decisions (B^*,μ^*) in Theorem 1 into the NSP's profit, we obtain the NSP's optimization problem as follows,

$$\begin{aligned}\max_{p_n}\ \ & V_n(p_n,p_c)=\left(\frac{s}{T}\left(1+\sqrt{\frac{rp_c}{sp_n}}\right)+\lambda s\right)p_n-c_n\left(\frac{s}{T}\left(1+\sqrt{\frac{rp_c}{sp_n}}\right)+\lambda s\right)^2\\ s.t.\ \ & B_{\min}\le\frac{s}{T}\left(1+\sqrt{\frac{rp_c}{sp_n}}\right)+\lambda s\le B_{\max}\\ & rp_c+sp_n\le\hat{I}.\end{aligned} \tag{105}$$

The derivative of V_n with respect to p_n is

$$\frac{\partial V_n}{\partial p_n}=s\left(\frac{1}{T}+\lambda\right)+\frac{\sqrt{srp_c}}{2T\sqrt{p_n}}+c_ns^2\left(\frac{rp_c}{T^2s^2p_n^2}+\frac{1+\lambda}{T^2}\sqrt{\frac{rp_c}{s}}\left(\frac{1}{\sqrt{p_n}}\right)^3\right)>0. \tag{106}$$

Thus, the NSP's profit is increasing in his price p_n. From the first constraint of the NSP's optimization problem, we can obtain

$$p_n \leq \frac{rp_c}{s\left(T(\frac{B_{\min}}{s} - \lambda) - 1\right)^2}. \tag{107}$$

From the second constraint of the NSP's optimization problem, we can obtain

$$p_n \leq \frac{\hat{I} - rp_c}{s}. \tag{108}$$

So the NSP's best response given the CSP's price p_c is

$$p_n^*(p_c) = \min\left\{\frac{rp_c}{s\left(T(\frac{B_{\min}}{s} - \lambda) - 1\right)^2}, \frac{\hat{I} - rp_c}{s}\right\} \tag{109}$$

Define $X \triangleq \left(T(\frac{B_{\min}}{s} - \lambda) - 1\right)^2$. Then,

$$p_n^*(p_c) = \begin{cases} \frac{rp_c}{sX}, & \text{if } \ p_c < \frac{X\hat{I}}{rX+r}, \\ \frac{\hat{I}-rp_c}{s}, & \text{if } \ p_c \geq \frac{X\hat{I}}{rX+r}. \end{cases} \tag{110}$$

6.1.2 CSP's best response

By plugging the users' decisions (B^*, μ^*) in Theorem 1 into the CSP's profit, we obtain the CSP's optimization problem as follows,

$$\begin{aligned} \max_{p_c} \ \ & V_c(p_n, p_c) = \left(\frac{r}{T}\left(1 + \sqrt{\frac{sp_n}{rp_c}}\right) + \lambda r\right) p_c - c_c\left(\frac{r}{T}\left(1 + \sqrt{\frac{sp_n}{rp_c}}\right) + \lambda r\right)^2 \\ s.t. \ \ & \mu_{\min} \leq \frac{r}{T}\left(1 + \sqrt{\frac{sp_n}{rp_c}}\right) + \lambda r \leq \mu_{\max} \\ & rp_c + sp_n \leq \hat{I}. \end{aligned} \tag{111}$$

The derivative of V_c with respect to p_c is

$$\frac{\partial V_c}{\partial p_c} = r\left(\frac{1}{T} + \lambda\right) + \frac{\sqrt{srp_n}}{2T\sqrt{p_c}} + c_c r^2\left(\frac{sp_n}{T^2 r^2 p_c^2} + \frac{1+\lambda}{T^2}\sqrt{\frac{sp_n}{r}}\left(\frac{1}{\sqrt{p_c}}\right)^3\right) > 0. \tag{112}$$

Thus, the CSP's profit is increasing in his price p_c. From the first constraint of the NSP's optimization problem, we can obtain

$$p_c \leq \frac{sp_n}{r\left(T(\frac{\mu_{\min}}{r} - \lambda) - 1\right)^2}. \tag{113}$$

From the second constraint of the NSP's optimization problem, we can obtain

$$p_c \leq \frac{\hat{I} - sp_n}{r}. \tag{114}$$

So the CSP's best response given the NSP's price p_n is

$$p_c^*(p_n) = \min\left\{\frac{sp_n}{r\left(T(\frac{\mu_{\min}}{r} - \lambda) - 1\right)^2}, \frac{\hat{I} - sp_n}{r}\right\} = \begin{cases} \frac{sp_n}{r\left(T(\frac{\mu_{\min}}{r} - \lambda) - 1\right)^2}, & \text{if } \ p_n \leq \frac{\left(T(\frac{\mu_{\min}}{s} - \lambda) - 1\right)^2 \hat{I}}{s\left(T(\frac{\mu_{\min}}{s} - \lambda) - 1\right)^2 + s}, \\ \frac{\hat{I} - sp_n}{r}, & \text{if } \ p_n > \frac{\left(T(\frac{\mu_{\min}}{s} - \lambda) - 1\right)^2 \hat{I}}{s\left(T(\frac{\mu_{\min}}{s} - \lambda) - 1\right)^2 + s}. \end{cases} \tag{115}$$

Define $Y \triangleq \left(T(\frac{\mu_{\min}}{r} - \lambda) - 1\right)^2$.

$$p_c^*(p_n) = \begin{cases} \frac{sp_n}{rY}, & \text{if } \ p_n < \frac{Y\hat{I}}{sY+s}, \\ \frac{\hat{I}-sp_n}{r}, & \text{if } \ p_n \geq \frac{Y\hat{I}}{sY+s}. \end{cases} \tag{116}$$

According to Assumption 6, i.e.,

$$T^2\left(\frac{\mu_{\min}}{r} - \lambda + \frac{\hat{\mu}}{r} - \frac{2}{T}\right)\left(\frac{B_{\min}}{s} - \lambda + \frac{\hat{\mu}}{r} - \frac{2}{T}\right) < 1. \tag{117}$$

we have

$$T^2\left(\frac{\mu_{\min}}{r} - \lambda - \frac{1}{T}\right)\left(\frac{B_{\min}}{s} - \lambda - \frac{1}{T}\right) < 1, \tag{118}$$

as $\frac{\hat{\mu}}{r} > \frac{1}{T}$ by Assumption 3. And by Assumptions 1 and 2, we have $\frac{\mu_{\min}}{r} - \lambda - \frac{1}{T} > 0$ and $\frac{B_{\min}}{s} - \lambda - \frac{1}{T} > 0$. That is, $X \cdot Y < 1$, which gives $\frac{Y}{Y+1} < \frac{1}{X+1}$.

Lemma 3. *In the case of $g(p_n, p_c) \leq \hat{I}$, any strategy profile that satisfies $p_n^{NE} \in [\frac{Y\hat{I}}{sY+s}, \frac{\hat{I}}{Xs+s}]$ and $rp_c^{NE} + sp_n^{NE} = \hat{I}$ is the Nash Equilibrium of the Price Competition Game.*

Proof. Notice that $\frac{Y\hat{I}}{sY+s} < \frac{\hat{I}}{Xs+s}$ according to Assumption 6.

To prove Lemma 3, it suffices to show that the strategy profile presented in Lemma 3 satisfies the best responses of the NSP and the CSP.

We firstly show that the NSP is indeed making best response. To see this, as $p_n^{NE} \leq \frac{\hat{I}}{Xs+s}$, then

$$p_c^{NE} = \frac{\hat{I} - sp_n^{NE}}{r} \geq \frac{\hat{I} - s\frac{\hat{I}}{Xs+s}}{r} = \frac{X\hat{I}}{rX + r}, \tag{119}$$

i.e., $p_c^{NE} \geq \frac{X\hat{I}}{rX+r}$. Thus, $p_n^{NE} = \frac{\hat{I}-rp_c^{NE}}{s}$ is indeed making best response according to (110).

We then show that the CSP is indeed making best response. To see this, as $p_n^{NE} \geq \frac{Y\hat{I}}{sY+s}$, then $p_c^{NE} = \frac{\hat{I}-sp_n^{NE}}{r}$ is indeed making best response according to (116). □

From Lemma 3, it is easy to see that the Nash Equilibrium satisfies $rp_c^{NE} + sp_n^{NE} = \hat{I}$. And when $p_n^{NE} = \frac{Y\hat{I}}{sY+s}$, $p_c^{NE} = \frac{\hat{I}-sp_n^{NE}}{r} = \frac{\hat{I}}{rY+r}$. Then

$$\frac{p_n^{NE}}{p_c^{NE}} = \frac{\frac{Y\hat{I}}{sY+s}}{\frac{\hat{I}}{rY+r}} = \frac{rY}{s} = \frac{\left(T\mu_{\min} - r - T\lambda r\right)^2}{rs}. \tag{120}$$

When $p_n^{NE} = \frac{\hat{I}}{Xs+s}$, then $p_c^{NE} = \frac{X\hat{I}}{rX+r}$. Then

$$\frac{p_n^{NE}}{p_c^{NE}} = \frac{\frac{\hat{I}}{Xs+s}}{\frac{X\hat{I}}{rX+r}} = \frac{r}{sX} = \tilde{R}_{HA} \triangleq \frac{rs}{\left(TB_{\min} - s - T\lambda s\right)^2}. \tag{121}$$

In conclusion, we have

$$\frac{\left(T\mu_{\min} - r - T\lambda r\right)^2}{rs} = \frac{\left(T\mu_{\min} - r - T\lambda r\right)^2}{rs} \leq \frac{rs}{\left(TB_{\min} - s - T\lambda s\right)^2}. \tag{122}$$

This concludes the proof for Case A.

6.2 Case B: $g(p_n, p_c) > \hat{I}$

We will analyze the best responses of network service provider (NSP) and computation service provider (CSP).

6.2.1 NSP's best response

By plugging the users' decisions (B^*, μ^*) in Theorem 1 into the NSP's profit, we obtain the NSP's optimization problem as follows,

$$\begin{aligned}
\max_{p_n} \ & V_n(p_n, p_c) = \left(\frac{s}{T}\left(2+\sqrt{\frac{rp_c}{sp_n}}\right)+\lambda s-\frac{\hat{\mu}s}{r}\right)p_n - c_n\left(\frac{s}{T}\left(2+\sqrt{\frac{rp_c}{sp_n}}\right)+\lambda s-\frac{\hat{\mu}s}{r}\right)^2 \\
s.t. \ & B_{\min} \leq \frac{s}{T}\left(2+\sqrt{\frac{rp_c}{sp_n}}\right)+\lambda s-\frac{\hat{\mu}s}{r} \leq B_{\max} \\
& rp_c + sp_n > \hat{I}, \\
& p_n \leq \hat{p}_n.
\end{aligned} \tag{123}$$

The NSP's profit is increasing in his price p_n. From the first constraint of the optimization problem, we can see that

$$p_n \leq \frac{rp_c}{s\left(T\frac{B_{\min}}{s} - T\lambda + \frac{T\hat{\mu}}{r} - 2\right)^2}. \tag{124}$$

So the NSP's best response given the CSP's price p_c is

$$p_n^*(p_c) = \min\left\{\frac{rp_c}{s\left(T\frac{B_{\min}}{s} - T\lambda + \frac{T\hat{\mu}}{r} - 2\right)^2}, \hat{p}_n\right\}. \tag{125}$$

Recall that we have defined

$$\tilde{R}_{HB} \triangleq \frac{sr}{(TB_{\min} - T\lambda s + Ts\hat{\mu}/r - 2s)^2}.$$

Thus,

$$p_n^*(p_c) = \min\left\{\tilde{R}_{HB}p_c, \hat{p}_n\right\}. \tag{126}$$

6.2.2 CSP's best response

By plugging the users' decisions (B^*, μ^*) in Theorem 1 into the CSP's profit, we obtain the CSP's optimization problem as follows,

$$\begin{aligned}
\max_{p_c} \ & V_c(p_n, p_c) = \left(\frac{r}{T}\left(2+\sqrt{\frac{sp_n}{rp_c}}\right)+\lambda r-\hat{\mu}\right)p_c - c_c\left(\frac{r}{T}\left(2+\sqrt{\frac{sp_n}{rp_c}}\right)+\lambda r-\hat{\mu}\right)^2 \\
s.t. \ & \mu_{\min} \leq \frac{r}{T}\left(2+\sqrt{\frac{sp_n}{rp_c}}\right)+\lambda r-\hat{\mu} \leq \mu_{\max} \\
& rp_c + sp_n > \hat{I}, \\
& p_c \leq \hat{p}_c.
\end{aligned} \tag{127}$$

The CSP's profit is increasing in his price p_c. From the first constraint of the optimization problem, we can see that

$$p_c \leq \frac{sp_n}{r\left(T\frac{\mu_{\min}}{r} - T\lambda + \frac{T\hat{\mu}}{r} - 2\right)^2}. \tag{128}$$

So the NSP's best response given the CSP's price p_c is

$$p_c^*(p_n) = \min \left\{ \frac{sp_n}{r\left(T\frac{\mu_{\min}}{r} - T\lambda + \frac{T\hat{\mu}}{r} - 2\right)^2}, \hat{p}_c \right\}. \tag{129}$$

Recall that we have defined

$$\tilde{R}_{LB} \triangleq \frac{(T\mu_{\min} - T\lambda r + T\hat{\mu} - 2r)^2}{sr},$$

Thus,

$$p_c^*(p_n) = \min \left\{ \frac{p_n}{\tilde{R}_{LB}}, \hat{p}_c \right\}. \tag{130}$$

According to Assumption 6, we have $\frac{\tilde{R}_{LB}}{<}\tilde{R}_{HB}$.

Next, we show that the strategy profile as follows constitutes mutual best responses.

$$(p_n^{NE}, p_c^{NE}) = \begin{cases} \left(\hat{p}_n, \frac{\hat{p}_n}{\tilde{R}_{LB}}\right), & \text{if } \frac{\hat{p}_n}{\hat{p}_c} \leq \tilde{R}_{LB}, \quad ① \\ (\hat{p}_n, \hat{p}_c), & \text{if } \tilde{R}_{LB} < \frac{\hat{p}_n}{\hat{p}_c} < \tilde{R}_{HB}, ② \\ \left(\hat{p}_c\tilde{R}_{HB}, \hat{p}_c\right), & \text{if } \frac{\hat{p}_n}{\hat{p}_c} \geq \tilde{R}_{HB}. \quad ③ \end{cases} \tag{131}$$

- Case ①:$\frac{\hat{p}_n}{\hat{p}_c} \leq \tilde{R}_{LB}$. When $p_n = \hat{p}_n$, then $\frac{p_n}{\tilde{R}_{LB}} = \frac{\hat{p}_n}{\tilde{R}_{LB}} \leq \hat{p}_c$. So the CSP's price $p_c = \frac{\hat{p}_n}{\tilde{R}_{LB}}$ is the best response. On the other hand, when $p_c = \frac{\hat{p}_n}{\tilde{R}_{LB}}$, then $\tilde{R}_{HB}p_c = \tilde{R}_{HB} \cdot \frac{\hat{p}_n}{\tilde{R}_{LB}} > \hat{p}_n$, due to $\tilde{R}_{LB} < \tilde{R}_{HB}$. So the NSP's price $p_n = \hat{p}_n$ is the best response. In conclusion, $(p_n^{NE}, p_c^{NE}) = \left(\hat{p}_n, \frac{\hat{p}_n}{\tilde{R}_{LB}}\right)$ is the Nash Equilibrium.

- Case ②: $\tilde{R}_{LB} < \frac{\hat{p}_n}{\hat{p}_c} < \tilde{R}_{HB}$. When $p_n = \hat{p}_n$, then $\frac{p_n}{\tilde{R}_{LB}} = \frac{\hat{p}_n}{\tilde{R}_{LB}} > \hat{p}_c$. So the CSP's price $p_c = \hat{p}_c$ is the best response. On the other hand, when $p_c = \hat{p}_c$, then $\tilde{R}_{HB}p_c = \tilde{R}_{HB} \cdot \hat{p}_c > \hat{p}_n$. So the NSP's price $p_n = \hat{p}_n$ is the best response. In conclusion, $(p_n^{NE}, p_c^{NE}) = (\hat{p}_n, \hat{p}_n)$ is the Nash Equilibrium.

- Case ③: $\frac{\hat{p}_n}{\hat{p}_c} \geq \tilde{R}_{HB}$. When $p_n = \hat{p}_c\tilde{R}_{HB}$, then $\frac{p_n}{\tilde{R}_{LB}} = \frac{\hat{p}_c\tilde{R}_{HB}}{\tilde{R}_{LB}} > \hat{p}_c$, due to $\tilde{R}_{LB} < \tilde{R}_{HB}$. So the CSP's price $p_c = \hat{p}_c$ is the best response. On the other hand, when $p_c = \hat{p}_c$, then $\tilde{R}_{HB}p_c = \tilde{R}_{HB} \cdot \hat{p}_c \leq \hat{p}_n$. So the NSP's price $p_n = \tilde{R}_{HB} \cdot \hat{p}_c$ is the best response. In conclusion, $(p_n^{NE}, p_c^{NE}) = \left(\tilde{R}_{HB}\hat{p}_c, \hat{p}_n\right)$ is the Nash Equilibrium.

This concludes the proof for Case B.

7 Proof of Proposition 4

Recall that by Assumption 5, we have $r\hat{p}_c + s\hat{p}_n > \hat{I}$.

The NSP's profit under the strategy profile $(\hat{p}_c, \frac{\hat{I} - r\hat{p}_c}{s})$ in Case A is

$$V_{n,A} = (\hat{I} - r\hat{p}_c)\left(\frac{1}{T} + \lambda\right) + \frac{\sqrt{r\hat{p}_c(\hat{I} - r\hat{p}_c)}}{T} - c_n s^2 \left(\frac{1}{T}\left(1 + \sqrt{\frac{r\hat{p}_c}{\hat{I} - r\hat{p}_c}}\right) + \lambda\right)^2. \tag{132}$$

The NSP's profit under the strategy profile $(\hat{p}_c, \hat{p}_n)$ in Case B is

$$V_{n,B} = \hat{p}_n s\left(\frac{2}{T} + \lambda - \frac{\hat{\mu}}{r}\right) + \frac{\sqrt{sr\hat{p}_n\hat{p}_c}}{T} - c_n s^2\left(\frac{1}{T}\left(2 + \sqrt{\frac{r\hat{p}_c}{s\hat{p}_n}}\right) + \lambda - \frac{\hat{\mu}}{r}\right)^2. \tag{133}$$

If

$$\left(\frac{1}{T} + \lambda\right)(g(\hat{p}_n, \hat{p}_c) - \hat{I}) + s\hat{p}_n\left(\frac{1}{T} - \frac{\hat{\mu}}{r}\right) - \frac{c_n s^2}{T}\left(1 + \sqrt{\frac{r\hat{p}_c}{\hat{I} - s\hat{p}_n}}\right)\left(\frac{3}{T} + 2\lambda + \frac{2\sqrt{r\hat{p}_c}}{T\sqrt{\hat{I} - r\hat{p}_c}} + \frac{\sqrt{r\hat{p}_c}}{T\sqrt{s\hat{p}_n}}\right) > 0, \tag{134}$$

then

$$\begin{aligned}&\left(\frac{1}{T} + \lambda\right)(g(\hat{p}_n, \hat{p}_c) - \hat{I}) + s\hat{p}_n\left(\frac{1}{T} - \frac{\hat{\mu}}{r}\right) + \frac{\sqrt{r\hat{p}_c}}{T}\left(\sqrt{s\hat{p}_n} - \sqrt{\hat{I} - r\hat{p}_c}\right) \\ &- \frac{c_n s^2}{T}\left(1 + \sqrt{\frac{r\hat{p}_c}{\hat{I} - s\hat{p}_n}} - \sqrt{\frac{r\hat{p}_c}{s\hat{p}_n}} - \frac{\hat{\mu}}{r}\right)\left(\frac{3}{T} + 2\lambda + \frac{\sqrt{r\hat{p}_c}}{T\sqrt{\hat{I} - r\hat{p}_c}} + \frac{\sqrt{r\hat{p}_c}}{T\sqrt{s\hat{p}_n}} - \frac{\hat{\mu}}{r}\right) > 0,\end{aligned} \tag{135}$$

i.e., $V_{n,B} - V_{n,A} > 0$. So by comparing the NSP's profit in Case B and Case A, we can see that choosing price $p_n = \hat{p}_n$ gives the NSP a higher profit than choosing $p_n = \frac{\hat{I} - r\hat{p}_c}{s}$.

The CSP's profit under the strategy profile $(\frac{\hat{I} - s\hat{p}_n}{r}, \hat{p}_n)$ in Case A is

$$V_{c,A} = (\hat{I} - s\hat{p}_n)\left(\frac{1}{T} + \lambda\right) + \frac{\sqrt{s\hat{p}_n(\hat{I} - s\hat{p}_n)}}{T} - c_c r^2\left(\frac{1}{T}\left(1 + \sqrt{\frac{s\hat{p}_n}{\hat{I} - s\hat{p}_n}}\right) + \lambda\right)^2. \tag{136}$$

The CSP's profit under the strategy profile $(\hat{p}_c, \hat{p}_n)$ in Case B is

$$V_{c,B} = \hat{p}_c r\left(\frac{2}{T} + \lambda - \frac{\hat{\mu}}{r}\right) + \frac{\sqrt{sr\hat{p}_n\hat{p}_c}}{T} - c_c r^2\left(\frac{1}{T}\left(2 + \sqrt{\frac{s\hat{p}_n}{r\hat{p}_c}}\right) + \lambda - \frac{\hat{\mu}}{r}\right)^2. \tag{137}$$

If

$$\left(\frac{1}{T} + \lambda\right)(g(\hat{p}_n, \hat{p}_c) - \hat{I}) + r\hat{p}_c\left(\frac{1}{T} - \frac{\hat{\mu}}{r}\right) - \frac{c_c r^2}{T}\left(1 + \sqrt{\frac{s\hat{p}_n}{\hat{I} - r\hat{p}_c}}\right)\left(\frac{3}{T} + 2\lambda + \frac{2\sqrt{s\hat{p}_n}}{T\sqrt{\hat{I} - s\hat{p}_n}} + \frac{\sqrt{s\hat{p}_n}}{T\sqrt{r\hat{p}_c}}\right) > 0, \tag{138}$$

then

$$\begin{aligned}&\left(\frac{1}{T} + \lambda\right)(g(\hat{p}_n, \hat{p}_c) - \hat{I}) + r\hat{p}_c\left(\frac{1}{T} - \frac{\hat{\mu}}{r}\right) + \frac{\sqrt{s\hat{p}_n}}{T}\left(\sqrt{r\hat{p}_c} - \sqrt{\hat{I} - s\hat{p}_n}\right) \\ &- \frac{c_c r^2}{T}\left(1 + \sqrt{\frac{s\hat{p}_n}{\hat{I} - r\hat{p}_c}} - \sqrt{\frac{s\hat{p}_n}{r\hat{p}_c}} - \frac{\hat{\mu}}{r}\right)\left(\frac{3}{T} + 2\lambda + \frac{\sqrt{c\hat{p}_n}}{T\sqrt{\hat{I} - s\hat{p}_n}} + \frac{\sqrt{s\hat{p}_n}}{T\sqrt{r\hat{p}_c}} - \frac{\hat{\mu}}{r}\right) > 0,\end{aligned} \tag{139}$$

i.e., $V_{c,B} - V_{c,A} > 0$. So by comparing the CSP's profit in Case B and Case A, we can see that choosing price $p_c = \hat{p}_c$ gives the CSP a higher profit than choosing $p_c = \frac{\hat{I} - s\hat{p}_n}{r}$. In conclusion, the strategy profile $(\hat{p}_c, \hat{p}_n)$ is the unique Nash Equilibrium.

This concludes the proof of Proposition 4.