Tut 8: PCA Dimensional Reduction

May/June 2022

When variances $\text{Var}(x_{\cdot j})$ for features/columns $x_{\cdot j}$ differ a lot, we need to perform scaling:

`pca$scale`: $\sqrt{\frac{\sum_i (x_{ij}-\overline{x}_{\cdot j})^2}{n-1}}$

However, you do not need to scale the data unless it is stated in the question.

Original data: X; Data shifted to centre: $\widetilde{X}$

`pca$center`: $\overline{x}_{\cdot j}$

`pca$sdev`: $\sqrt{\lambda_i}$

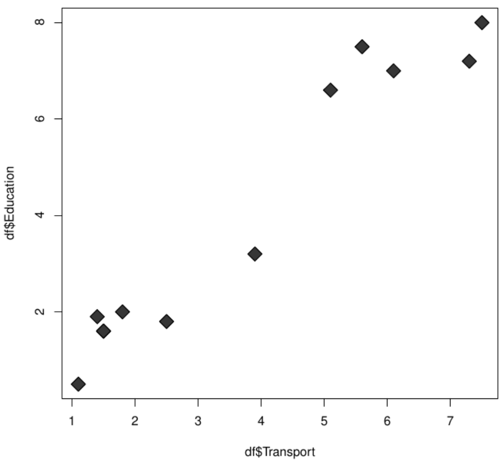
`pca$rotation`: $[\boldsymbol{e}_1, \boldsymbol{e}_2, \cdots]$

`pca$x`: $[\widetilde{X}\boldsymbol{e}_1, \widetilde{X}\boldsymbol{e}_2, \cdots]$

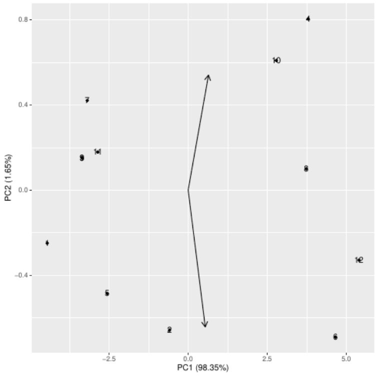
1. You are given 12 communities that were rated according to transportation and education — the higher the score the better. For example, a better transportation system will score higher. Higher education facilities will score higher as well. The table below shows the score for 12 communities in the two criteria:

Obs	Transportation	Education
1	1.1	0.5
2	3.9	3.2
3	1.5	1.6
4	5.6	7.5
5	2.5	1.8
6	7.3	7.2
7	1.4	1.9
8	6.1	7.0
9	1.5	1.6
10	5.1	6.6
11	1.8	2.0
12	7.5	8.0

(a) Plot a scatterplot to visualize your data.

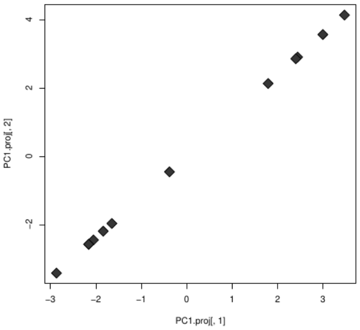


(b) Generate two principal components for the data.



(c) Choose one suitable principal component to represent the data.

(d) Plot your data with the principal component you chose in (c).



(e) With the eigenvalues computed in (b), calculate the proportion of variance explained by each component and the cumulative proportion.

(f) With a targeted explained variation of 95%, how many principal components should be considered? State the total variation explained.

2. (May 2020 Final Q4(a)) Given the following data with 8 observations in Table 4.1:

Table 4.1: Data with 2 features.

Obs	x	y
A	5.51	5.35
B	20.82	24.03
C	-0.77	-0.57
D	19.30	19.39
E	14.24	12.77
F	9.74	9.68
G	11.59	12.06
H	-6.08	-5.22

Find the first principle component and project the data $(5.51, 5.35)$ to the space span by the first principal component. (4 marks)

3. (Jan 2021 Final Q3(a)) Given the following data with 11 observations in Table 3.1:

Table 3.1: Data with two features.

Obs	x	y
1	-5.79	4.91
2	-3.73	4.87
3	-3.25	3.98
4	-2.61	4.09
5	-2.76	4.90
6	2.81	-5.34
7	2.92	-6.15
8	1.97	-4.51
9	5.17	-5.29
10	2.66	-7.10
11	3.47	-4.70

Find the proportions of variance and the principle components. (5 marks)