MCEN90028
Robotic Systems

Assignment 2:

Constructing the Jacobian matrix of a 5-DOF Jenga Tower Construction Robot

Assignment Group 1

Jiawei Liao	756560
Harfi Dharma Santoso	772503
Omar Sakkal	757372

Department of Mechanical Engineering

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1 Purpose of Task

The purpose of this exercise is to derive the Jacobian for our 5 DoF robot. The Jacobian is comprised of two components; the translational and angular velocity components. This will be completed using two elementary means, one via using the definition of the Jacobian (as outlined in the lecture by completing the cross product of $\hat{Z}_i$ and $\vec{r}_w - \vec{r}_i$ for each joint) and the other by differentiating the spatial position of the robot wrist joint ${}^0\vec{r}_{ow}$. Method 2 serves to verify our solutions obtained via Method 1.

2 Deriving the Jacobian

2.1 Define Frame W

Frame $\{W\}$ is defined to be aligned to frame $\{E\}$ with the origin located at the origin of Frame $\{4\}$, where it is regarded as point W to represent the robot's wrist.
The schematics in Assignment 1 was changed such that Frame $\{4\}$ and $\{5\}$ share the same origin and in order to allow for the computation of the Jacobian to be much simpler.

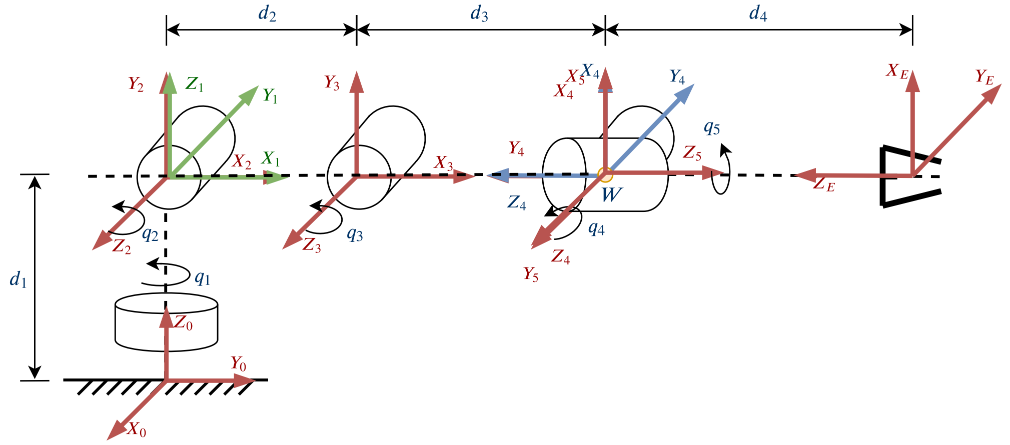

Figure 1: Schematic of the 5 DoF Robot

DH table					
i		$\mathbf{a_{i-1}}$	$\alpha_{\mathbf{i-1}}$	$\mathbf{d_{i-1}}$	$\theta_{\mathbf{i-1}}$
1		**0**	0°	$\mathbf{d_1}$	$90^\circ + \mathbf{q_1}$
2		**0**	90°	**0**	$\mathbf{q_2}$
3		$\mathbf{d_2}$	0°	**0**	$\mathbf{q_3}$
4		$\mathbf{d_3}$	0°	**0**	$90^\circ + \mathbf{q_4}$
5		**0**	90°	**0**	$\mathbf{q_5}$
6		**0**	180°	$-\mathbf{d_4}$	0°

Table 1: DH table for the 5 DoF robot illustrated in Figure (1)

The definition of Frame $\{W\}$ allows us to simplify the derivation of the Jacobian due to its alignment with Frame $\{E\}$.

$$J_E = \begin{bmatrix} V_E \\ \omega_E \end{bmatrix} = V_W + \omega_W \times \vec{r}_{WE} = \begin{bmatrix} I_{3x3} & skew(\vec{r}_{WE}) \\ 0_{3x3} & I_{3x3} \end{bmatrix} \begin{bmatrix} V_W \\ \omega_W \end{bmatrix} \tag{1}$$

2.2 Symbolic components of $\hat{Z}_i$ and $(\vec{r}_w - \vec{r}_i)$

2.2.1 Symbolic components of $\hat{Z}_i$

Using Figure (1), we observe the following:

- $\hat{Z}_1$ aligns with $\hat{Z}_0$.
- $\hat{Z}_2$, $\hat{Z}_3$ and $\hat{Z}_4$ align with each other.
- $\hat{Z}_E$ aligns with $\hat{Z}_W$ and are in the opposite direction of $\hat{Z}_5$.

Some useful rotation matrices to be used during the calculations are computed using Table (1).

$$
{}^1_0R = \begin{pmatrix} -\sin(q_1) & -\cos(q_1) & 0 \\ \cos(q_1) & -\sin(q_1) & 0 \\ 0 & 0 & 1 \end{pmatrix},
$$

$$
{}^2_0R = \begin{pmatrix} -\cos(q_2)\sin(q_1) & \sin(q_1)\sin(q_2) & \cos(q_1) \\ \cos(q_1)\cos(q_2) & -\cos(q_1)\sin(q_2) & \sin(q_1) \\ \sin(q_2) & \cos(q_2) & 0 \end{pmatrix},
$$

$$
{}^5_0R = \begin{pmatrix} \cos(q_1)\sin(q_5) + \sigma_1\cos(q_5)\sin(q_1) & \cos(q_1)\cos(q_5) - \sigma_1\sin(q_1)\sin(q_5) & -\sigma_2\sin(q_1) \\ \sin(q_1)\sin(q_5) - \sigma_1\cos(q_1)\cos(q_5) & \cos(q_5)\sin(q_1) + \sigma_1\cos(q_1)\sin(q_5) & \sigma_2\cos(q_1) \\ \sigma_2\cos(q_5) & -\sigma_2\sin(q_5) & \sigma_1 \end{pmatrix}
$$

$$
\text{where } \sigma_1 = \sin(q_2+q_3+q_4), \qquad \sigma_2 = \cos(q_2+q_3+q_4)
$$

Using the above observations and the 3 rotation matrices, we can now express $z_i s$ in Frame $\{0\}$:

$$
{}^0\hat{Z}_1 = {}^0\hat{Z}_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},
$$

$$
{}^0\hat{Z}_2 = {}^0\hat{Z}_3 = {}^0\hat{Z}_4 = {}^2_0R \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos(q_1) \\ \sin(q_1) \\ 0 \end{pmatrix}
$$

$$
{}^0\hat{Z}_5 = {}^5_0R \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\cos(q_2+q_3+q_4)\sin(q_1) \\ \cos(q_2+q_3+q_4)\cos(q_1) \\ \sin(q_2+q_3+q_4) \end{pmatrix}
$$

$$
{}^0\hat{Z}_E = {}^0\hat{Z}_W = -{}^0\hat{Z}_5 = \begin{pmatrix} \cos(q_2+q_3+q_4)\sin(q_1) \\ -\cos(q_2+q_3+q_4)\cos(q_1) \\ -\sin(q_2+q_3+q_4) \end{pmatrix}
$$

2.2.2 Symbolic components of $\vec{r}_w - \vec{r}_i$

The right column of ${}^4_0T$ could give us $\vec{r}_w$ in Frame $\{0\}$ as point W is the origin of Frame $\{4\}$.

$$
{}^0\vec{r}_{0W} = {}^4_0T(1:3,4) = \begin{pmatrix} -\cos(q_2+q_3+q_4)\sin(q_1) \\ \cos(q_2+q_3+q_4)\cos(q_1) \\ \sin(q_2+q_3+q_4) \end{pmatrix}
$$

Similarly using the results of the homogeneous transformation matrices (T) derived in Assignment 1, we can compute the symbolic expressions of ${}^0\vec{r}_1, {}^0\vec{r}_2, {}^0\vec{r}_3, {}^0\vec{r}_5, {}^0\vec{r}_E$.

$$
\begin{aligned}
{}^0\vec{r}_1 = {}^0\vec{r}_2 = {}^1_0T(1:3,4) &= \begin{pmatrix} 0 \\ 0 \\ d_1 \end{pmatrix} \\
{}^0\vec{r}_3 = {}^3_0T(1:3,4) &= \begin{pmatrix} -d_2\cos(q_2)\,\sin(q_1) \\ d_2\cos(q_1)\,\cos(q_2) \\ d_1 + d_2\sin(q_2) \end{pmatrix} \\
{}^0\vec{r}_4 = {}^0\vec{r}_5 = {}^4_0T(1:3,4) &= \begin{pmatrix} -\sin(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2)\big) \\ \cos(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2)\big) \\ d_1 + d_3\sin(q_2+q_3) + d_2\sin(q_2) \end{pmatrix} \\
{}^0\vec{r}_E = {}^E_0T(1:3,4) &= \begin{pmatrix} -\sin(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)\big) \\ \cos(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)\big) \\ d_1 + d_3\sin(q_2+q_3) + d_2\sin(q_2) + d_4\sin(q_2+q_3+q_4) \end{pmatrix}
\end{aligned}
$$

Now the expressions of $\vec{r}_w - \vec{r}_i$ can be obtained via:

$$
{}^0\vec{r}_{1W} = {}^0\vec{r}_w - {}^0\vec{r}_1 = \begin{pmatrix} -\sin(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2)\big) \\ \cos(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2)\big) \\ d_3\sin(q_2+q_3) + d_2\sin(q_2) \end{pmatrix} \tag{2}
$$

$$
{}^0\vec{r}_{2W} = {}^0\vec{r}_w - {}^0\vec{r}_2 = \begin{pmatrix} -\sin(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2)\big) \\ \cos(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2)\big) \\ d_3\sin(q_2+q_3) + d_2\sin(q_2) \end{pmatrix} \tag{3}
$$

$$
{}^0\vec{r}_{3W} = {}^0\vec{r}_w - {}^0\vec{r}_3 = \begin{pmatrix} d_2\cos(q_2)\,\sin(q_1) - \sin(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2)\big) \\ \cos(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2)\big) - d_2\cos(q_1)\,\cos(q_2) \\ d_3\sin(q_2+q_3) \end{pmatrix} \tag{4}
$$

$$
{}^0\vec{r}_{4W} = {}^0\vec{r}_w - {}^0\vec{r}_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{5}
$$

$$
{}^0\vec{r}_{5W} = {}^0\vec{r}_w - {}^0\vec{r}_5 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{6}
$$

(7)

The expression ${}^0\vec{r}_{5W}$ is useful in the calculations for obtaining the Jacobian (J_E) via the method of decoupling.

$$
{}^0\tilde{\mathrm{r}}_{\mathrm{WE}} = {}^0\tilde{\mathrm{r}}_{\mathrm{E}} - {}^0\tilde{\mathrm{r}}_{\mathrm{W}} = \begin{pmatrix} \sin(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2)\big) - \sin(q_1)\,\sigma_1 \\ \cos(q_1)\,\sigma_1 - \cos(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2)\big) \\ d_4\sin(q_2+q_3+q_4) \end{pmatrix} \tag{8}
$$

where

$$
\sigma_1 = d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)
$$

2.3 The Jacobian

2.3.1 J_W

Using the results obtained from the previous section, we can now compute the Jacobian at the wrist point (W) by using the definition as illustrated below:

$$\begin{bmatrix} V_W \\ \omega_W \end{bmatrix} = \begin{bmatrix} {}^0\hat{Z}_1 \times {}^0\vec{r}_{1W} & {}^0\hat{Z}_2 \times {}^0\vec{r}_{2W} & {}^0\hat{Z}_3 \times {}^0\vec{r}_{3W} & 0_{3\times1} & 0_{3\times1} \\ {}^0\hat{Z}_1 & {}^0\hat{Z}_2 & {}^0\hat{Z}_3 & {}^0\hat{Z}_4 & {}^0\hat{Z}_5 \end{bmatrix}_{J_W} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} \tag{9}$$

Defining the following definitions:

$$\begin{aligned}
&S_1 \equiv \sin q_1, \quad C_1 \equiv \sin q_1, \quad S_2 \equiv \sin q_2, \quad C_2 \equiv \sin q_2, && .\text{etc} \\
&S_{23} \equiv \sin(q_2 + q_3), \quad C_{23} \equiv \cos(q_2 + q_3), && .\text{etc} \\
&S_{234} \equiv \sin(q_2 + q_3 + q_4), \quad C_{234} \equiv \cos(q_2 + q_3 + q_4) && .\text{etc}
\end{aligned}$$

Utilizing this definition and change of notation to simplify the visual representation of the Jacobian yields the following:

$$\mathrm{J_W} = \begin{pmatrix} -C_1\,(C_2\,d_2 + C_{23}\,d_3) & S_1\,(S_2\,d_2 + S_{23}\,d_3) & S_1\,S_{23}\,d_3 & 0 & 0 \\ -S_1\,(C_2\,d_2 + C_{23}\,d_3) & -C_1\,(S_2\,d_2 + S_{23}\,d_3) & -C_1\,S_{23}\,d_3 & 0 & 0 \\ 0 & C_2\,d_2 + C_{23}\,d_3 & C_{23}\,d_3 & 0 & 0 \\ 0 & C_1 & C_1 & C_1 & -C_{234}\,S_1 \\ 0 & S_1 & S_1 & S_1 & C_1\,C_{234} \\ 1 & 0 & 0 & 0 & S_{234} \end{pmatrix} \tag{10}$$

2.3.2 J_E via decoupling using J_W and ${}^0\vec{r}_{WE}$

$$\mathrm{skew}({}^0\tilde{\mathrm{r}}_{\mathrm{WE}}) = \begin{pmatrix} 0 & d_4\sin(q_2+q_3+q_4) & \sigma_3 - \cos(q_1)\,\sigma_1 \\ -d_4\sin(q_2+q_3+q_4) & 0 & \sigma_2 - \sin(q_1)\,\sigma_1 \\ \cos(q_1)\,\sigma_1 - \sigma_3 & \sin(q_1)\,\sigma_1 - \sigma_2 & 0 \end{pmatrix}$$

where

$$\sigma_1 = d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)$$

$$\sigma_2 = \sin(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2)\big)$$

$$\sigma_3 = \cos(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2)\big)$$

Using the value of $skew({}^0\vec{r}_{WE})$ and the Jacobian J_W at point W, we can now compute the Jacobian (J_E) at the end-effector using the formula described in equation 1.

$$J_E = \begin{bmatrix} I_{3x3} & skew(\vec{r}_{WE}) \\ 0_{3x3} & I_{3x3} \end{bmatrix} \begin{bmatrix} V_W \\ \omega_W \end{bmatrix} \tag{11}$$

$$= \begin{pmatrix} -C_1\,(C_2\,d_2 + C_{23}\,d_3 + C_{234}\,d_4) & S_1\,(S_2\,d_2 + S_{23}\,d_3) + S_1\,S_{234}\,d_4 & S_1\,(S_{23}\,d_3 + S_{234}\,d_4) & S_1\,S_{234}\,d_4 & 0 \\ -S_1\,(C_2\,d_2 + C_{23}\,d_3 + C_{234}\,d_4) & -C_1\,(S_2\,d_2 + S_{23}\,d_3) - C_1\,S_{234}\,d_4 & -C_1\,(S_{23}\,d_3 + S_{234}\,d_4) & -C_1\,S_{234}\,d_4 & 0 \\ 0 & C_2\,d_2 + C_{23}\,d_3 + C_{234}\,d_4 & C_{23}\,d_3 + C_{234}\,d_4 & C_{234}\,d_4 & 0 \\ 0 & C_1 & C_1 & C_1 & -C_{234}\,S_1 \\ 0 & S_1 & S_1 & S_1 & C_1\,C_{234} \\ 1 & 0 & 0 & 0 & S_{234} \end{pmatrix} \tag{12}$$

3 Verifying the Jacobian solution:

It is possible to compute the translational velocity component of the Jacobian JV_E using two other methods to verify the solution.

- Direct derivation using the definition without the aid of decoupling
- Functional derivative of the ${}^0\vec{r}_E$ with respect to q_is

The angular velocity component of the Jacobian $J\omega_E$ could also be verified using a second method.

- Functional derivative of the ${}^0\vec{w}_E$ with respect to $\dot{q}_i$s

3.1 Verifying JV_E using definition without the aid of decoupling

Similarly to Section (2.2.2), the expressions of $\vec{r}_E - \vec{r}_i$ are obtained as follows:

$$
{}^0\vec{r}_{1E} = {}^0\vec{r}_E - {}^0\vec{r}_1 = \begin{pmatrix} -\sin(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)\big) \\ \cos(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)\big) \\ d_3\sin(q_2+q_3) + d_2\sin(q_2) + d_4\sin(q_2+q_3+q_4) \end{pmatrix}
$$

$$
{}^0\vec{r}_{2E} = {}^0\vec{r}_E - {}^0\vec{r}_2 = \begin{pmatrix} -\sin(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)\big) \\ \cos(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)\big) \\ d_3\sin(q_2+q_3) + d_2\sin(q_2) + d_4\sin(q_2+q_3+q_4) \end{pmatrix}
$$

$$
{}^0\vec{r}_{3E} = {}^0\vec{r}_E - {}^0\vec{r}_3 = \begin{pmatrix} d_2\cos(q_2)\,\sin(q_1) - \sin(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)\big) \\ \cos(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)\big) - d_2\cos(q_1)\,\cos(q_2) \\ d_3\sin(q_2+q_3) + d_4\sin(q_2+q_3+q_4) \end{pmatrix}
$$

$$
{}^0\vec{r}_{4E} = {}^0\vec{r}_E - {}^0\vec{r}_4 =
$$

$$
\begin{pmatrix} \sin(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2)\big) - \sin(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)\big) \\ \cos(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)\big) - \cos(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2)\big) \\ d_4\sin(q_2+q_3+q_4) \end{pmatrix}
$$

$$
{}^0\vec{r}_{5E} = {}^0\vec{r}_E - {}^0\vec{r}_5 =
$$

$$
\begin{pmatrix} \sin(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2)\big) - \sin(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)\big) \\ \cos(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)\big) - \cos(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2)\big) \\ d_4\sin(q_2+q_3+q_4) \end{pmatrix}
$$

We can then use the definition of the Jacobian to directly compute J_E:

$$
\begin{bmatrix} V_E \\ \omega_E \end{bmatrix} = \begin{bmatrix} {}^0\hat{Z}_1 \times {}^0\vec{r}_{1E} & {}^0\hat{Z}_2 \times {}^0\vec{r}_{2E} & {}^0\hat{Z}_3 \times {}^0\vec{r}_{3E} & {}^0\hat{Z}_4 \times {}^0\vec{r}_{4E} & {}^0\hat{Z}_5 \times {}^0\vec{r}_{5E} \\ {}^0\hat{Z}_1 & {}^0\hat{Z}_2 & {}^0\hat{Z}_3 & {}^0\hat{Z}_4 & {}^0\hat{Z}_5 \end{bmatrix}_{J_E} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} \tag{13}
$$

Utilizing MATLAB to verify the computation reveals consistent results for JV_E.

$$
JV_E(\mathrm{m2}) = \begin{pmatrix} -C_1\,(C_2\,d_2 + C_{23}\,d_3 + C_{234}\,d_4) & S_1\,(S_2\,d_2 + S_{23}\,d_3 + S_{234}\,d_4) & S_1\,(S_{23}\,d_3 + S_{234}\,d_4) & S_1\,S_{234}\,d_4 & 0 \\ -S_1\,(C_2\,d_2 + C_{23}\,d_3 + C_{234}\,d_4) & -C_1\,(S_2\,d_2 + S_{23}\,d_3 + S_{234}\,d_4) & -C_1\,(S_{23}\,d_3 + S_{234}\,d_4) & -C_1\,S_{234}\,d_4 & 0 \\ 0 & C_2\,d_2 + C_{23}\,d_3 + C_{234}\,d_4 & C_{23}\,d_3 + C_{234}\,d_4 & C_{234}\,d_4 & 0 \end{pmatrix} \tag{14}
$$

3.2 Verifying JV_E by obtaining the functional derivative of ${}^0\vec{r}_E$

The JV_E component can be directly computed by differentiating the expression of ${}^0\vec{r}_E$ with respect to q_1, q_2, q_3, q_4, q_5.

$$
{}^0\vec{r}_E = {}^E_0T(1:3,4) = \begin{pmatrix} -\sin(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)\big) \\ \cos(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)\big) \\ d_1 + d_3\sin(q_2+q_3) + d_2\sin(q_2) + d_4\sin(q_2+q_3+q_4) \end{pmatrix}
$$

Performing functional partial derivatives of ${}^0\vec{r}_E$ with respect to each q_i yields:

$$
\frac{\partial({}^0\vec{r}_E)}{\partial(q_1)} = \begin{pmatrix} -\cos(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)\big) \\ -\sin(q_1)\,\big(d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4)\big) \\ 0 \end{pmatrix} \tag{15}
$$

$$
\frac{\partial({}^0\vec{r}_E)}{\partial(q_1)} = \begin{pmatrix} \sin(q_1)\,\big(d_3\sin(q_2+q_3) + d_2\sin(q_2) + d_4\sin(q_2+q_3+q_4)\big) \\ -\cos(q_1)\,\big(d_3\sin(q_2+q_3) + d_2\sin(q_2) + d_4\sin(q_2+q_3+q_4)\big) \\ d_3\cos(q_2+q_3) + d_2\cos(q_2) + d_4\cos(q_2+q_3+q_4) \end{pmatrix} \tag{16}
$$

$$
\frac{\partial({}^0\vec{r}_E)}{\partial(q_1)} = \begin{pmatrix} \sin(q_1)\,\big(d_3\sin(q_2+q_3) + d_4\sin(q_2+q_3+q_4)\big) \\ -\cos(q_1)\,\big(d_3\sin(q_2+q_3) + d_4\sin(q_2+q_3+q_4)\big) \\ d_3\cos(q_2+q_3) + d_4\cos(q_2+q_3+q_4) \end{pmatrix} \tag{17}
$$

$$
\frac{\partial({}^0\vec{r}_E)}{\partial(q_1)} = \begin{pmatrix} d_4\sin(q_2+q_3+q_4)\,\sin(q_1) \\ -d_4\sin(q_2+q_3+q_4)\,\cos(q_1) \\ d_4\cos(q_2+q_3+q_4) \end{pmatrix} \tag{18}
$$

$$
\frac{\partial({}^0\vec{r}_E)}{\partial(q_1)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{19}
$$

The results obtained here are consistent with the results obtained in Sections 2 and 3.2.

$$
JV_E(\mathrm{m3}) = \begin{pmatrix} -C_1\,(C_2\,d_2 + C_{23}\,d_3 + C_{234}\,d_4) & S_1\,(S_2\,d_2 + S_{23}\,d_3 + S_{234}\,d_4) & S_1\,(S_{23}\,d_3 + S_{234}\,d_4) & S_1\,S_{234}\,d_4 & 0 \\ -S_1\,(C_2\,d_2 + C_{23}\,d_3 + C_{234}\,d_4) & -C_1\,(S_2\,d_2 + S_{23}\,d_3 + S_{234}\,d_4) & -C_1\,(S_{23}\,d_3 + S_{234}\,d_4) & -C_1\,S_{234}\,d_4 & 0 \\ 0 & C_2\,d_2 + C_{23}\,d_3 + C_{234}\,d_4 & C_{23}\,d_3 + C_{234}\,d_4 & C_{234}\,d_4 & 0 \end{pmatrix} \tag{20}
$$

3.3 Verifying $J\omega_E$ by obtaining the functional derivative of ${}^0\vec{w}_E$

The angular velocity components of the Jacobian ($J\omega_E$) is obtained by firstly deriving the absolute angular velocity of the end-effector (${}^0\vec{w}_E$) and then taking partial derivatives of the absolute angular velocity vector (${}^0\vec{w}_E$) with respect to each of the $\dot{q}_i$s.
The absolute angular velocity vector of the end-effector (${}^0\vec{w}_E$) is computed as follows:

$$
{}^0\vec{w}_E = {}^0_1R \cdot {}^1\vec{w}_{1,0} + {}^0_2R \cdot {}^2\vec{w}_{2,1} + {}^0_3R \cdot {}^3\vec{w}_{3,2} + {}^0_4R \cdot {}^4\vec{w}_{4,3} + {}^0_5R \cdot {}^5\vec{w}_{5,4} \tag{21}
$$

$$
= \begin{pmatrix} \dot{q}_2\cos(q_1) + \dot{q}_3\cos(q_1) + \dot{q}_4\cos(q_1) - \dot{q}_5\cos(q_2+q_3+q_4)\,\sin(q_1) \\ \dot{q}_2\sin(q_1) + \dot{q}_3\sin(q_1) + \dot{q}_4\sin(q_1) + \dot{q}_5\cos(q_2+q_3+q_4)\,\cos(q_1) \\ \dot{q}_1 + \dot{q}_5\sin(q_2+q_3+q_4) \end{pmatrix} \tag{22}
$$

where

$$
{}^1\vec{w}_{1,0} = \begin{pmatrix} 0 \\ 0 \\ \dot{q}_1 \end{pmatrix}, {}^2\vec{w}_{2,1} = \begin{pmatrix} 0 \\ 0 \\ \dot{q}_2 \end{pmatrix}, {}^3\vec{w}_{3,2} = \begin{pmatrix} 0 \\ 0 \\ \dot{q}_3 \end{pmatrix}, {}^4\vec{w}_{4,3} = \begin{pmatrix} 0 \\ 0 \\ \dot{q}_4 \end{pmatrix}, {}^5\vec{w}_{5,4} = \begin{pmatrix} 0 \\ 0 \\ \dot{q}_5 \end{pmatrix}
$$

Then we could compute the partial derivative of ${}^0\vec{w}_E$ with respect to the the $\dot{q}_i$s:

$$J\omega_E = \begin{bmatrix} \frac{\partial({}^0\vec{w}_E)}{\partial(\dot{q}_1)} & \frac{\partial({}^0\vec{w}_E)}{\partial(\dot{q}_2)} & \frac{\partial({}^0\vec{w}_E)}{\partial(\dot{q}_3)} & \frac{\partial({}^0\vec{w}_E)}{\partial(\dot{q}_4)} & \frac{\partial({}^0\vec{w}_E)}{\partial(\dot{q}_5)} \end{bmatrix} \tag{23}$$

$$= \begin{pmatrix} 0 & \cos(q_1) & \cos(q_1) & \cos(q_1) & -\cos(q_2+q_3+q_4)\sin(q_1) \\ 0 & \sin(q_1) & \sin(q_1) & \sin(q_1) & \cos(q_2+q_3+q_4)\cos(q_1) \\ 1 & 0 & 0 & 0 & \sin(q_2+q_3+q_4) \end{pmatrix} \tag{24}$$

The results obtained here are also consistent with the results obtained in Section 2.

4 Conclusion

The Jacobian we have constructed is representative of our system and we have a great deal of confidence the Jacobian we derived is accurate as we were able to verify it via different elementary means.
The MATLAB live script "mcen90028_ass2_main.mlx" provided demonstrates the three methods we utilized during the course of the assignment and serves as evidence for analytically verifying the Jacobian.