MATHEMATICS

What is the name of an algebraic structure which is both a tensor space and a module.

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I am studying a set of high-dimensional arrays of complex numbers, which is both a tensor space (defined with the notion of addition and complex scalar product) and a module (defined with the same notion of addition and the notion of the product with a ring element). What is the formal name of this kind of structure? Is there some literature that discusses this structure in more detail?

I think I can even embed a subset of this structure in it. The subset has a topology and is locally homeomorphic to a small open set of the unnamed structure, and has properties analogous to a canonical manifold. Since the unnamed structure is a module, can I call the constrained set a generalized manifold? Since the unnamed structure is also a vector space (i.e., a tensor space), the subset is also a canonical manifold.

What is the name of this topological subset? The topological subset is established over a new commutative algebra called T-Algebra. The commutative t-algebra is both a finite-dimensional commutative ring and a vector space. I leave the URL of the t-algebra as follows if somebody wants to know more about the algebra.
https://github.com/liaoliang2020/talgebra

Thanks for any help.

abstract-algebra terminology manifolds

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edited yesterday
J. W. Tanner
54.3k 3 31 69

asked yesterday
Liang Liao
31 3

I'm confused why one would distinguish between "with a complex scalar product" and "a $\mathbb{C}$ module." Is your ring a $\mathbb{C}$ algebra, so that this thing is just a module over a $\mathbb{C}$-algebra? – rschwieb yesterday

"tensor space" doesn't look like a proper algebraic structure to me, just a particular shorthand for a particular construction of a vector space based on a vector space V. Algebraically it looks like it is simply a vector space. – rschwieb yesterday

The above-mentioned module G over C is equipped with a scalar product, then is a vector space (or the so-called tensor space in my case). The module G over the ring C can be conveniently considered as a set of fixed-sized matrices with entries from C. On the other hand, my ring C is also a $\mathbb{C}-$algebra because C is equipped with a product with complex numbers, addition, and multiplication. A module G over the $\mathbb{C}$-algebra C is also a complex vector space. Due to this versatility of G, I prefer to think the C-module G might deserve a unique name. – Liang Liao 14 hours ago

One can say that the "tensor space" is a vector space. However, the vector space is equipped with the notion of multi-mode multiplication with a proper complex matrix. Therefore, I prefer to call this vector space a tensor space. The term "tensor space" is not the core issue of my question. It's okay to reduce the term to "vector space" but my question still remains the same. – Liang Liao 14 hours ago

A vector space over complex numbers is also a module over the complex numbers since the notion of module generalizes the notion of vector space. The above-mentioned structure G is a module over a complex algebra C and at the same time a vector space over $\mathbb{C}$. The structure G can not be characterized by the beginning sentence of this paragraph. I hope to make this issue clearer. – Liang Liao 13 hours ago

I've never heard the terms “multi mode multiplication” or “proper matrix” so I can't really follow your description. Does the $\mathbb{C}$ scalar operation match that of the image of $\mathbb{C}$ in the algebra? – rschwieb 13 hours ago

1 I'm trying to eliminate the possibility it is just a module, or at worst maybe a bimodule. – rschwieb 13 hours ago

Let's forget the terms multi-mode multiplications, which is something talked about by engineers. The term "tensor" is not the issue. Replace "tensor" with "vector" if it makes things clearer. I am not sure the meaning of "the $\mathbb{C}$ scalar operation match the image of $\mathbb{C}$ is the algebra". However, the algebra C is closed with the $\mathbb{C}$ scalar multiplication, since C is a complex algebra and, $C \neq \mathbb{C}$. – Liang Liao 13 hours ago

From what you said above, C is isomorphic to $M_n(\mathbb{C})$ for some n. You say G is a C module. But C contains a copy of $\mathbb{C}$ (constant diagonal matrices). I'm asking if your "complex scalar product on G" is no different than taking the constant diagonal matrix from C and using the module action to multiply. – rschwieb 2 hours ago

The multiplication on C is defined via circular convolution rather than the usual matrix multiplication. To make things clearer via a concrete example, please see another post on this issue. math.stackexchange.com/questions/4022580/…. It is different than taking the constant diagonal matrix from C and using the module action to multiply. By definition of my algebra C, the constant diagonal matrix has no particular meaning to C. The algebra C has no connection to $M_n(\mathbb{C})$. – Liang Liao 1 hour ago

The identity of C is not in the form of a const diagonal matrix. The identity is in the form of a matrix (more accurately order-n array) whose only non-zero entity is the inception scalar entry, with other entries zeros. I call an array in this form an inception array. It is no different than taking a constant inception array from C and using the module action to multiply. I have discussed the isomorphism of C in equation (2.4) of my paper with URL arxiv.org/pdf/2011.00307.pdf – Liang Liao 50 mins ago

Ok, sorry about that, but even so my question still stands: does the image of $\mathbb{C}$ in C (whatever it looks like, if it is not the constant diagonal matrices!) acting like the scalar action of $\mathbb{C}$? – rschwieb 37 mins ago

Consider C as a set of fixed-sized complex matrices, the product $\alpha \cdot A$ where $\alpha \in \mathbb{C}$ and $A \in C$ acts like the usual multiplication of α with the complex matrix A. Our t-algebra is a finite-dimensional semi-simple algebra which is a direct product of a finite number of factor algebras, each one is isomorphic the algebra $\mathbb{C}$. – Liang Liao 16 mins ago

No, not the action of $\mathbb{C}$ on A, the action of $\mathbb{C}$ on G. As I've been asking, want to know if C's copy of $\mathbb{C}$ acts the same way as your $\mathbb{C}$ acts on G. – rschwieb 14 mins ago

If I understand you correctly, given an algebra element $A \in C$ and C-module element $B \in G$, their multiplication $A \circ B$ generally is NOT equivalent to a multiplication $\alpha \cdot B$ where $\alpha \in \mathbb{C}$. – Liang Liao 3 mins ago Edit