Generalized Image Reconstruction over T-Algebra

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I. Introduction

Let the Stiefel manifold over the field $\mathbb{F}$ be

$$V_k(\mathbb{F}^n) \doteq \{X \in \mathbb{F}^{n\times k} | X^* X = I_k\} \tag{1}$$

Then, according to the standard knowledge of the Stiefel manifold, one has the following results.

$$\begin{aligned} &\dim V_k(\mathbb{R}^n) = nk - \frac{1}{2}\,k(k+1) \\ &\dim V_k(\mathbb{C}^n) = 2nk - k^2\ . \end{aligned} \tag{2}$$

The Stiefel manifolfd $V_k(\mathbb{R}^n)\,\big|_{n=2,k=1} \equiv V_k(\mathbb{C}^n)\,\big|_{1=2,k=1}$ is a circle with the radius 1 in a two-dimensional real Eucliddean space.

Thefore,

$$\begin{aligned} &\dim V_k(\mathbb{R}^n)\,\big|_{n=2,k=1} = \dim\, T_X V_k(\mathbb{R}^n) = 1 \\ &\dim V_k(\mathbb{C}^n)\,\big|_{n=1,k=1} = \dim\, T_X V_k(\mathbb{C}^n) = 1 \end{aligned} \tag{3}$$

The number $\frac{1}{2}k(k+1) = k^2 = 1$ (where $k = 1$) is the dimension of the normal space $N_X V_k(\mathbb{R}^n)$ or $N_X V_k(\mathbb{C}^n)$ for all X.