

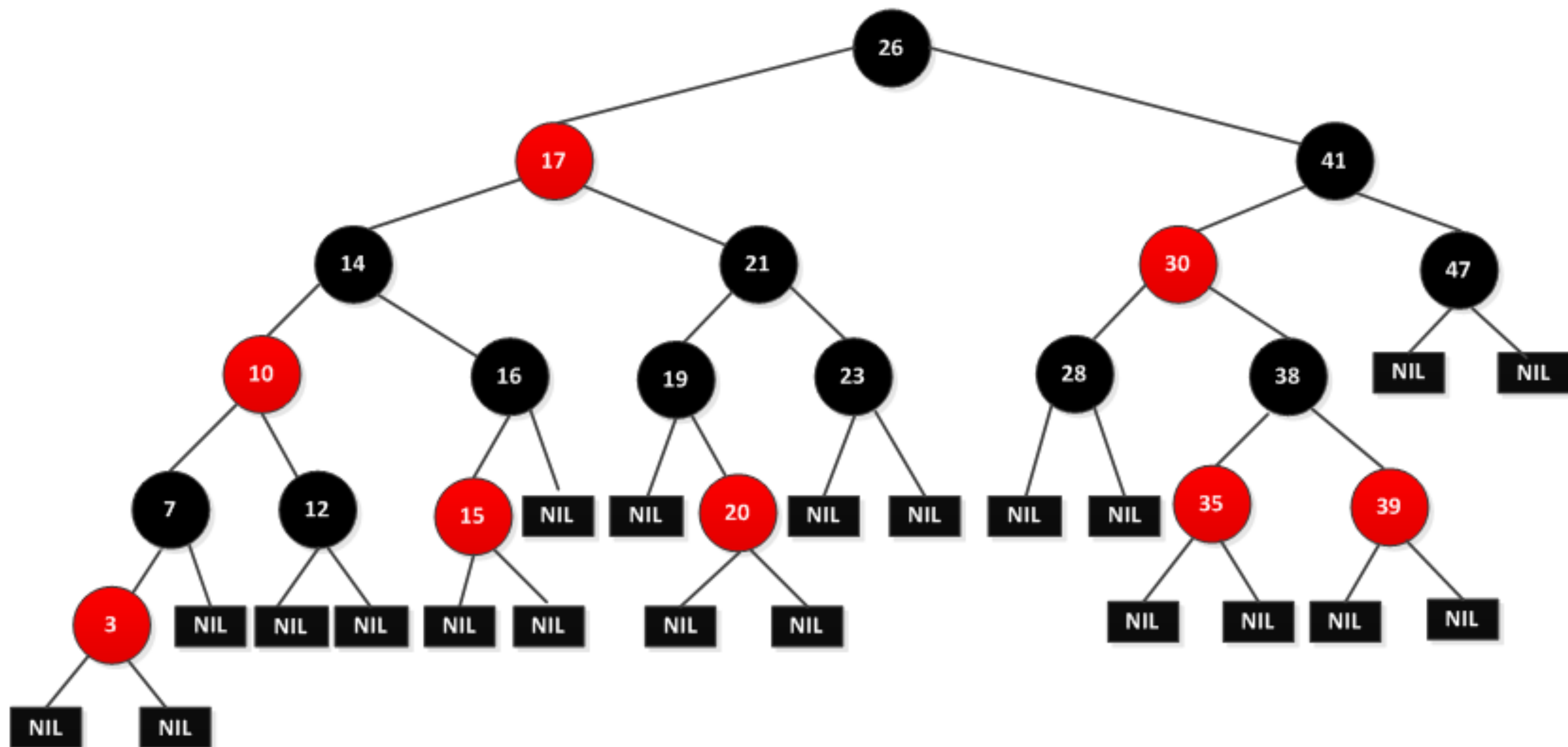
# 玩儿转数据结构

liuyubobobo

# 红黑树

# 什么是红黑树

# 红黑树



# 《算法导论》 中的红黑树

A red-black tree is a binary tree that satisfies the following *red-black properties*:

1. Every node is either red or black.
2. The root is black.
3. Every leaf (NIL) is black.
4. If a node is red, then both its children are black.
5. For each node, all simple paths from the node to descendant leaves contain the same number of black nodes.

# 《算法导论》 中的红黑树

1. 每个节点或者是红色的，或者是黑色的
2. 根节点是黑色的
3. 每一个叶子节点（最后的空节点）是黑色的
4. 如果一个节点是红色的，那么他的孩子节点都是黑色的
5. 从任意一个节点到叶子节点，经过的黑色节点是一样的



# 算法4



Robert Sedgwick

红黑树的发明人



# 算法4



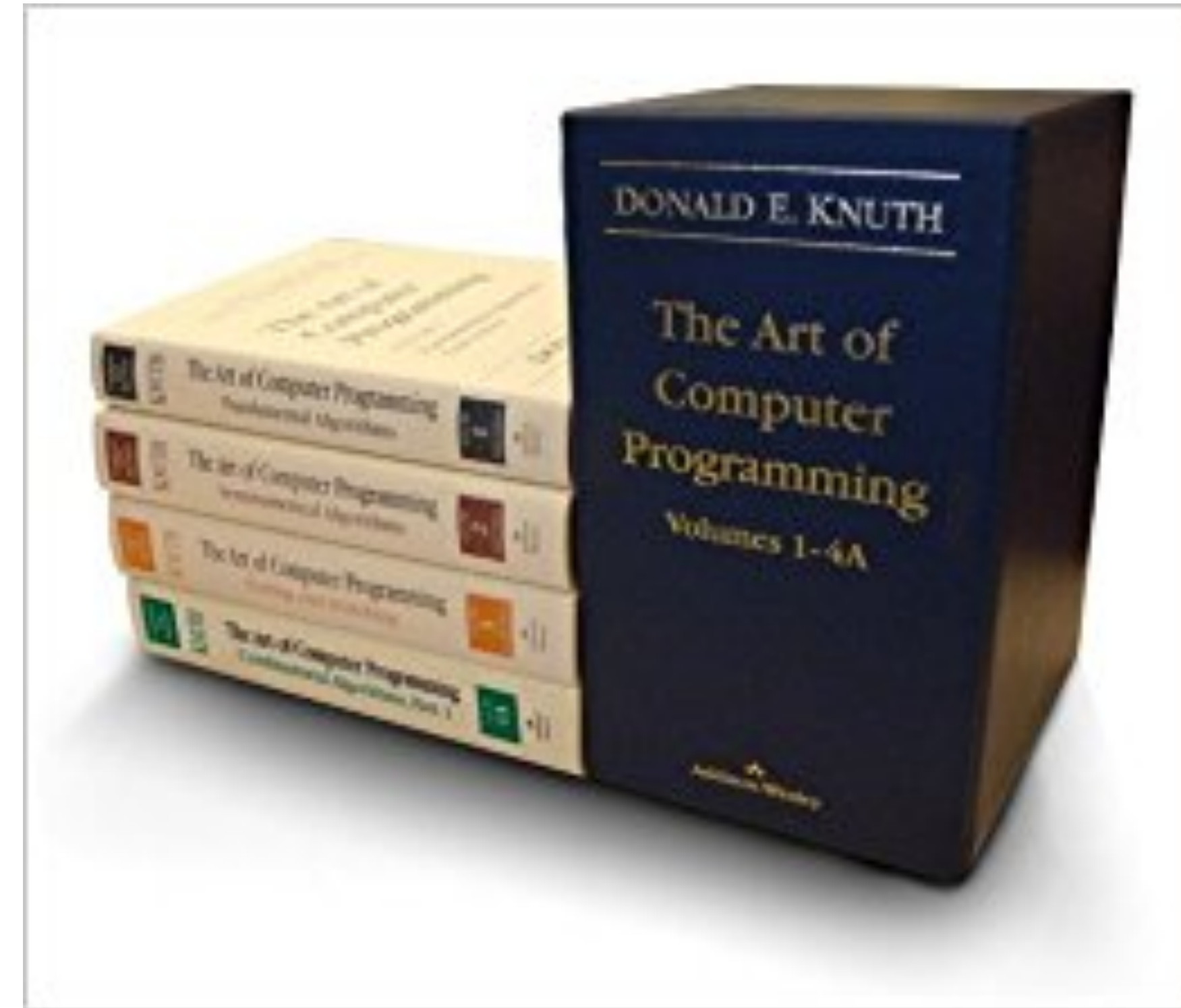
Robert Sedgwick

红黑树的发明人



Donald Knuth

现代计算机科学的前驱





# 红黑树



Robert Sedgwick

红黑树的发明人

红黑树与2-3树的等价性

# 红黑树

红黑树与2-3树的等价性

理解了2-3树和红黑树之间的关系

红黑树并不难！

# 2-3 树

# 2-3 树

学习2-3树，不仅对于理解红黑树有帮助

对于理解B类树，也是有巨大帮助的！



# 2-3树

满足二分搜索树的基本性质

节点可以存放一个元素或者两个元素

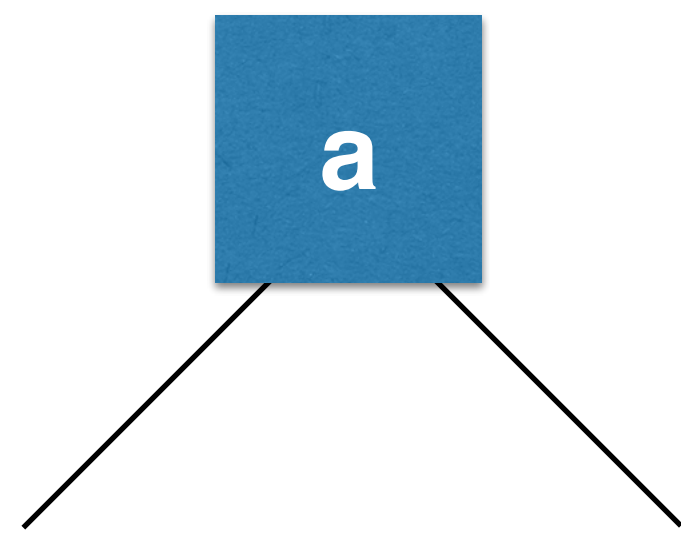


每个节点有2个或者3个孩子 —— 2-3树

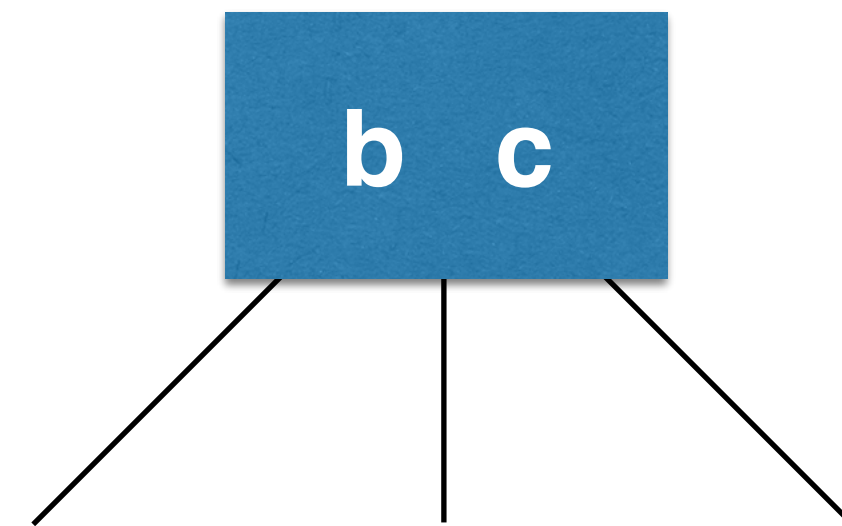
# 2-3树

满足二分搜索树的基本性质

节点可以存放一个元素或者两个元素

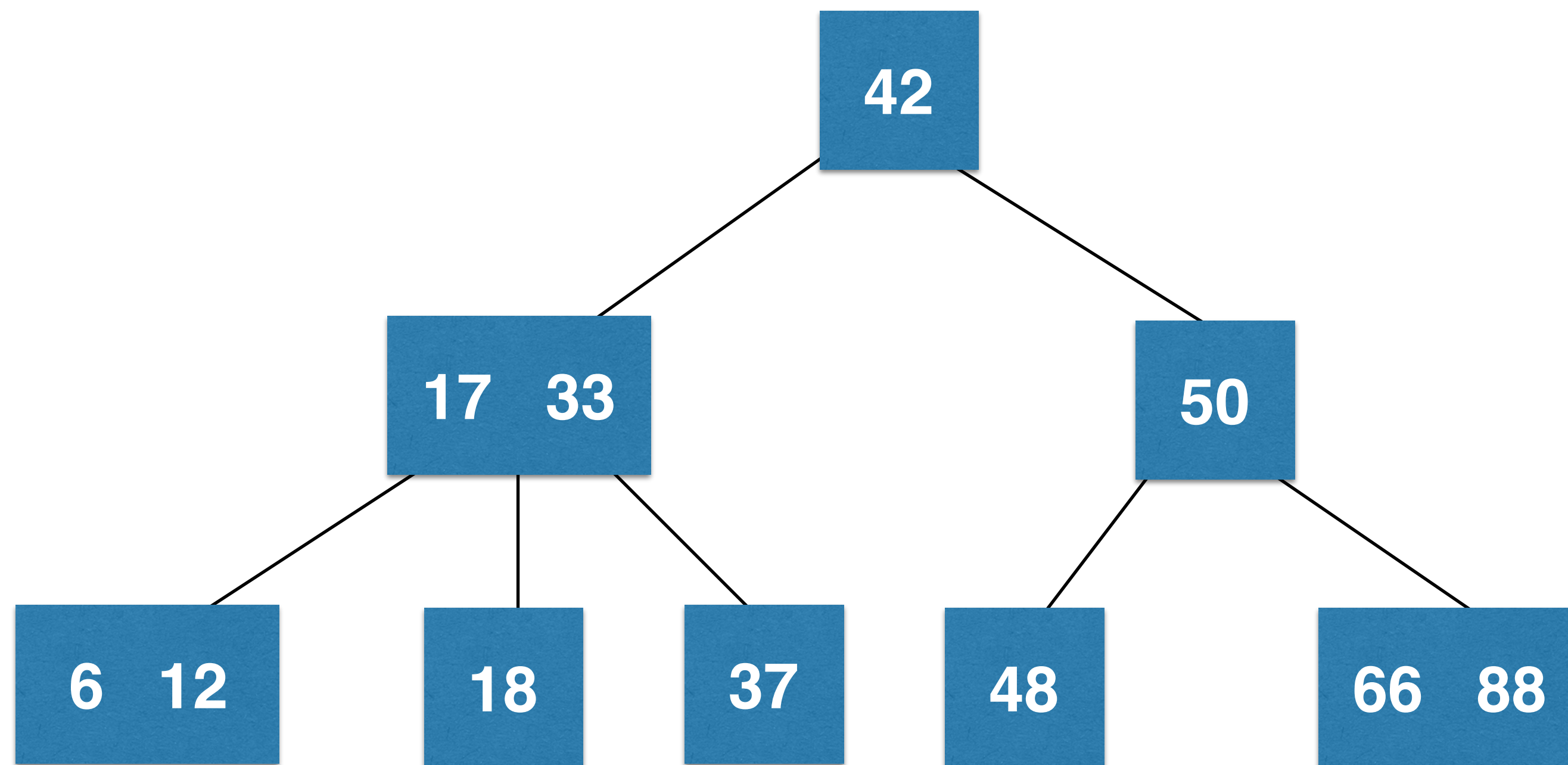


2节点



3节点

# 2-3树



2-3树是一棵绝对平衡的树

## 2-3 树如何维持绝对的平衡



# 2-3树

37

42

# 2-3树

12

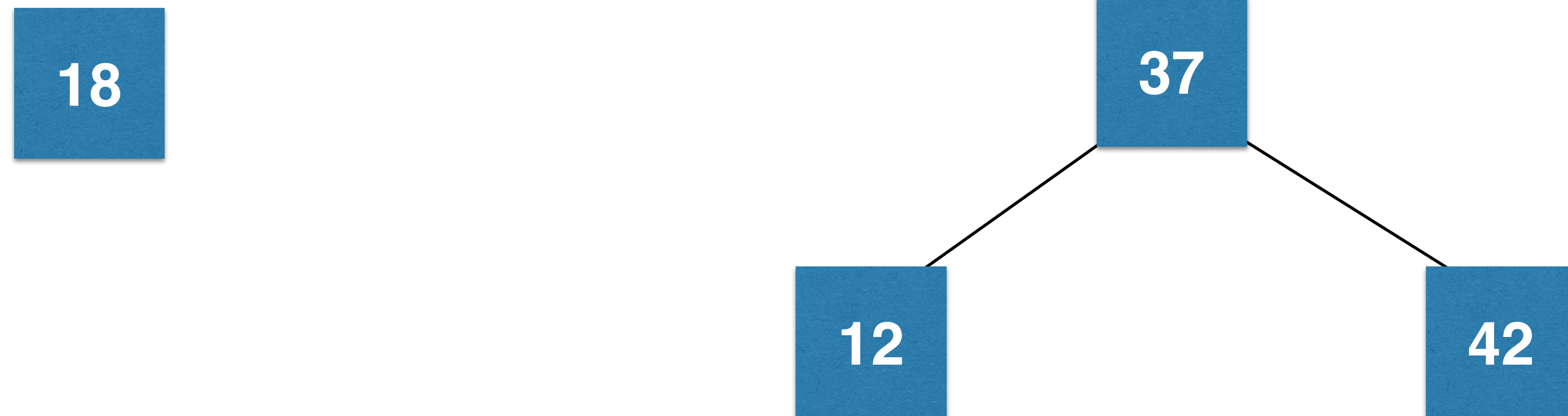
37

42

# 2-3树

12	37	42
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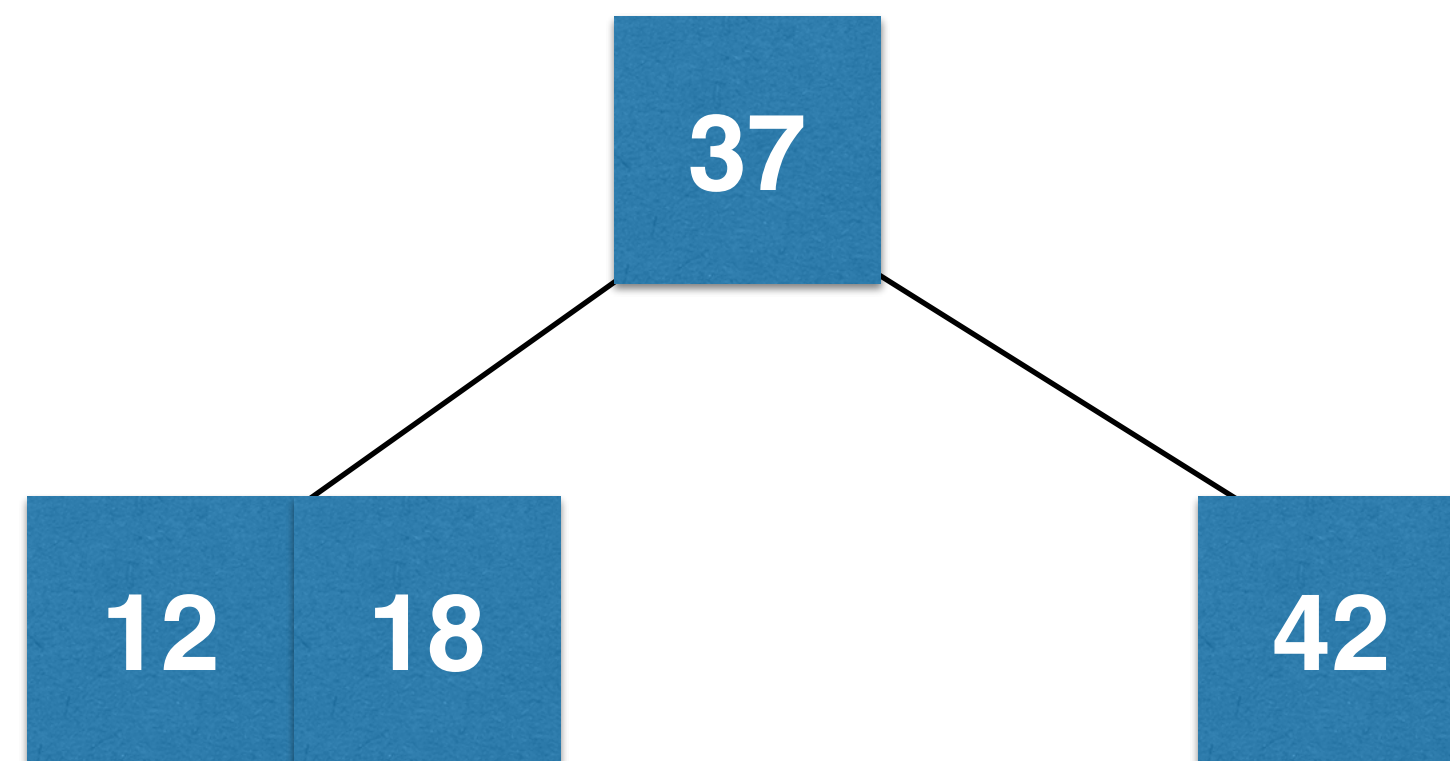
# 2-3树



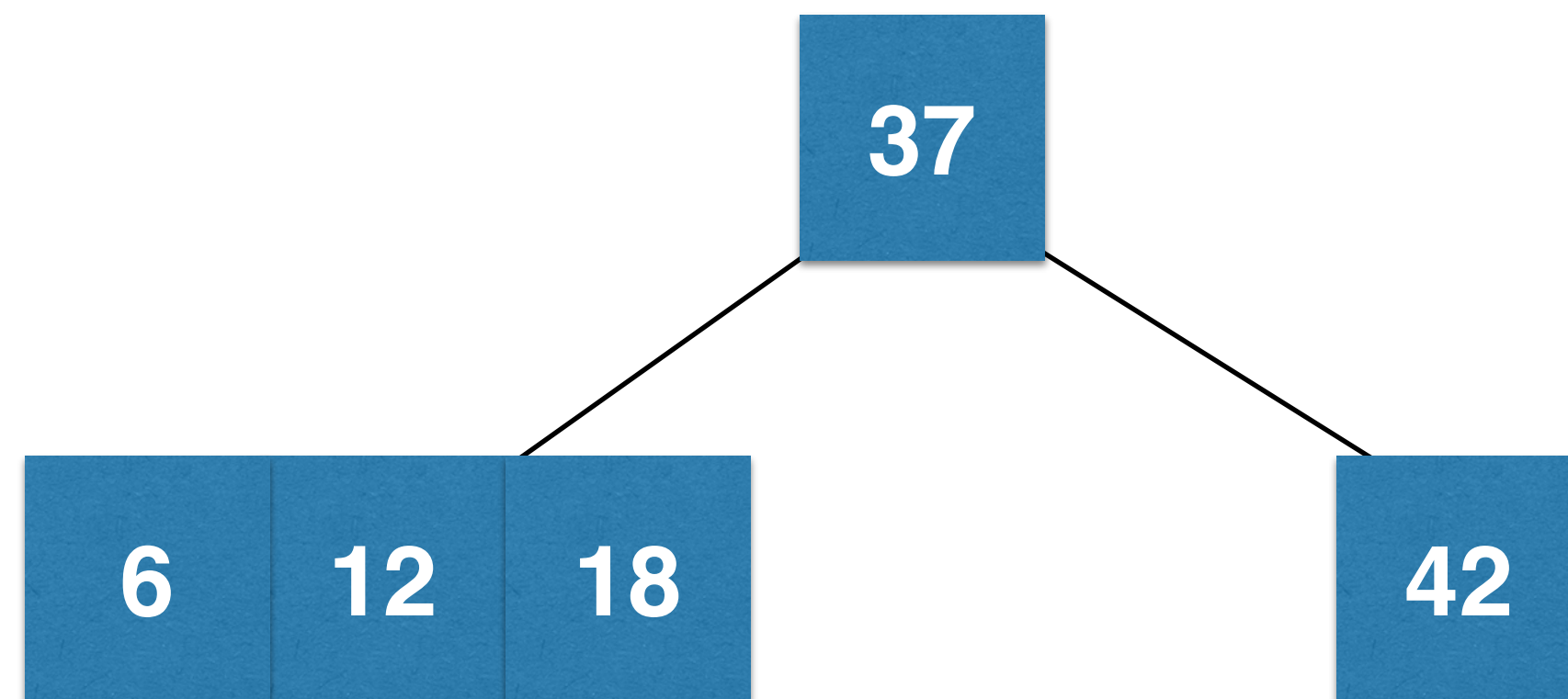


# 2-3树

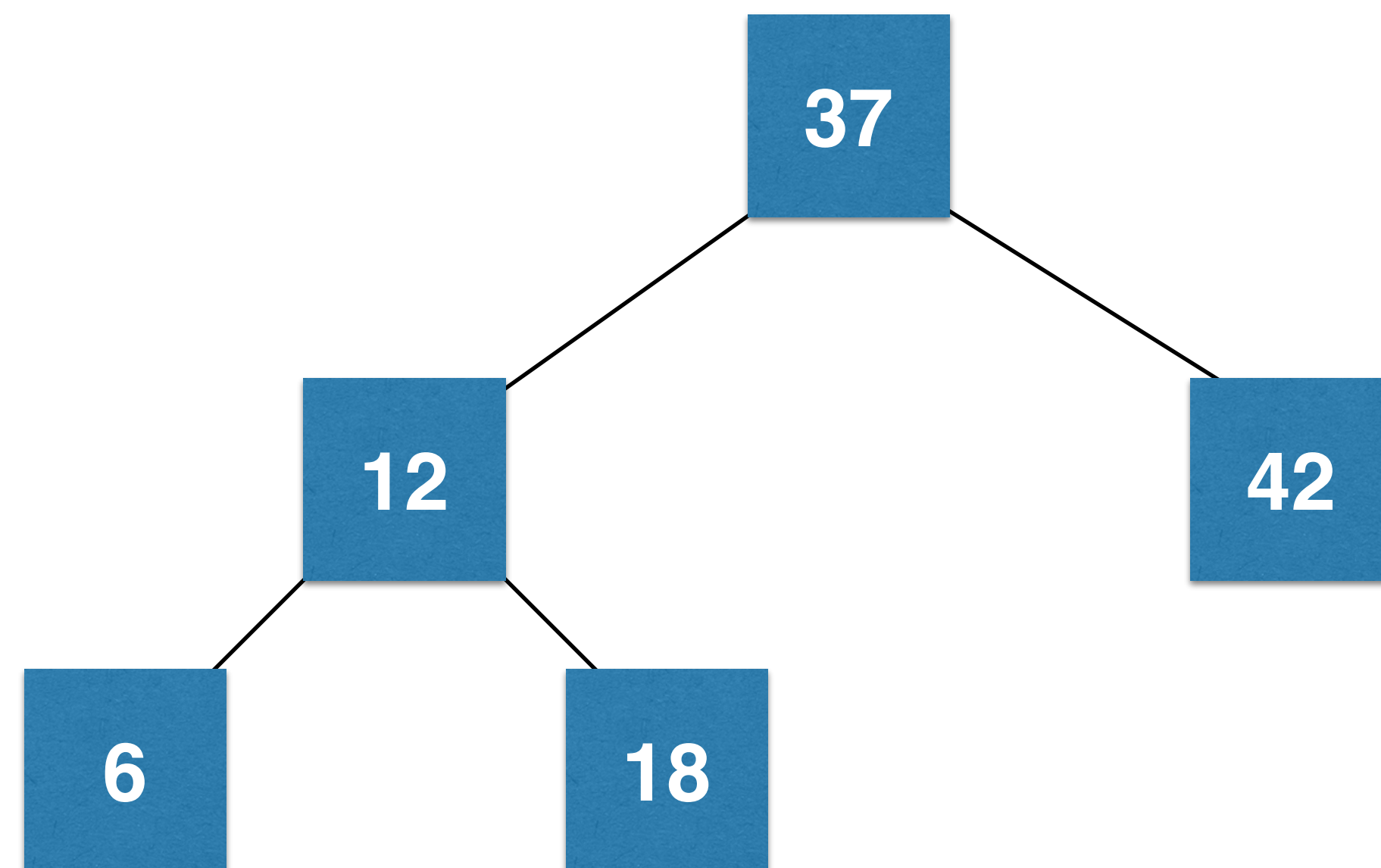
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# 2-3树

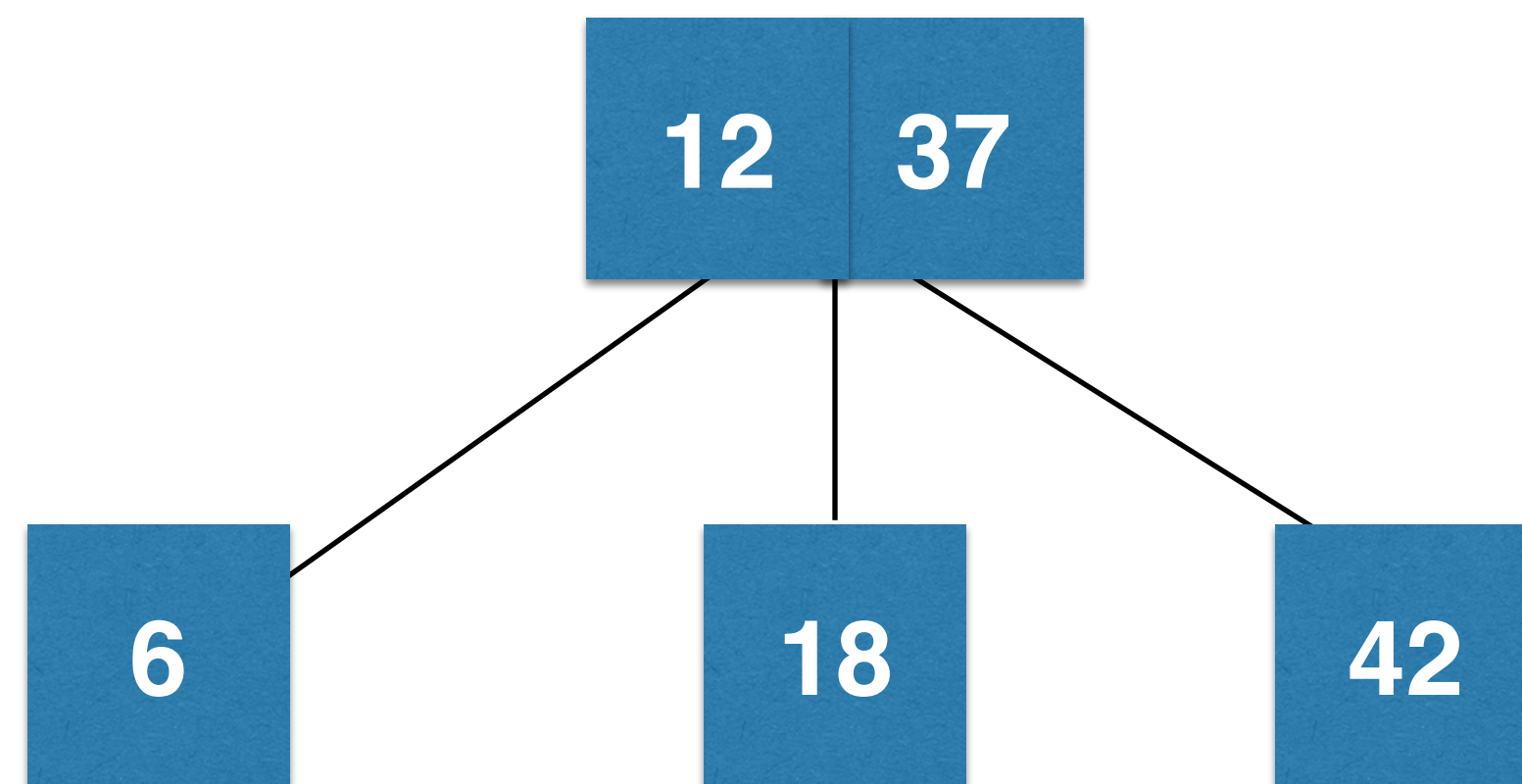


# 2-3树



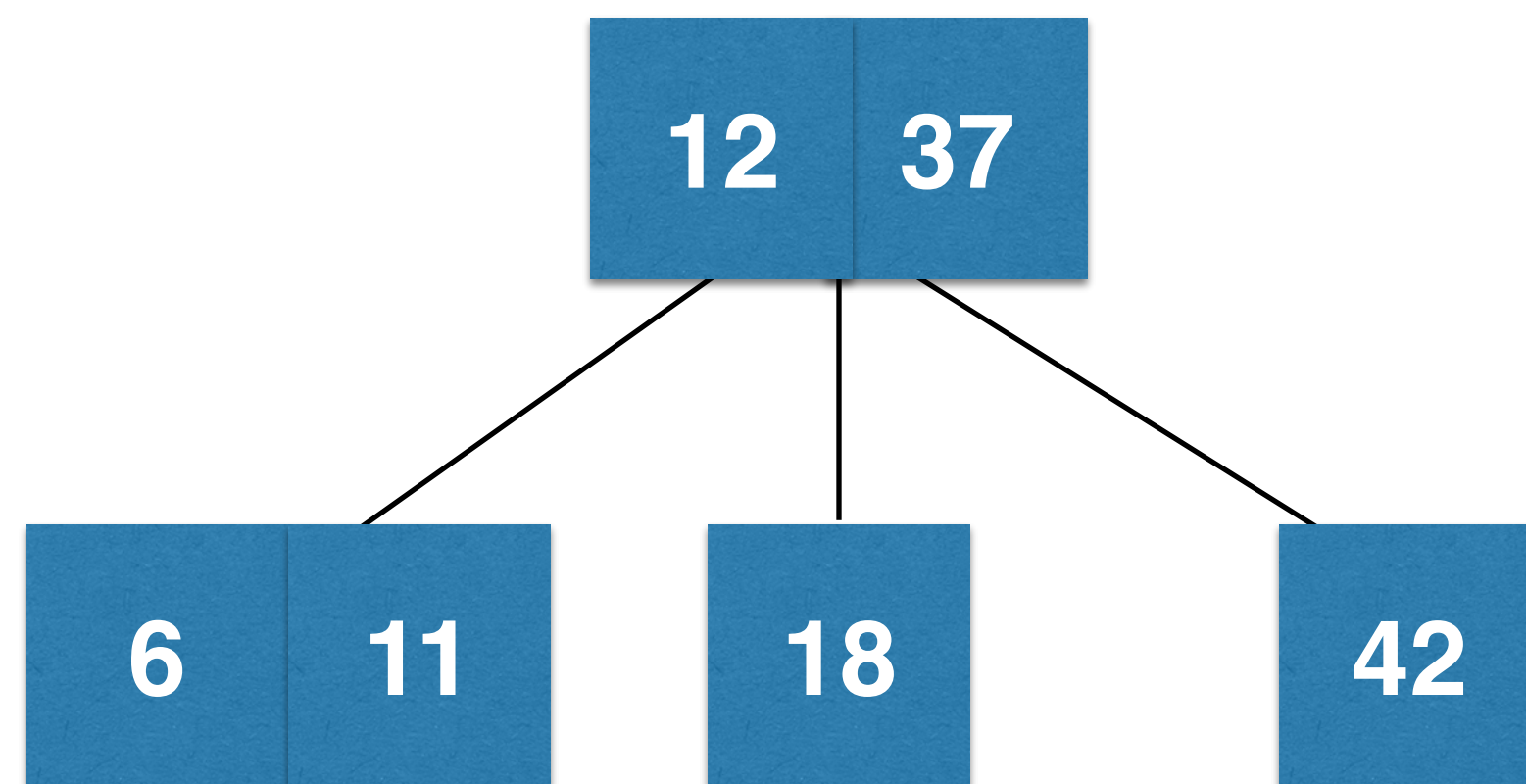
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11

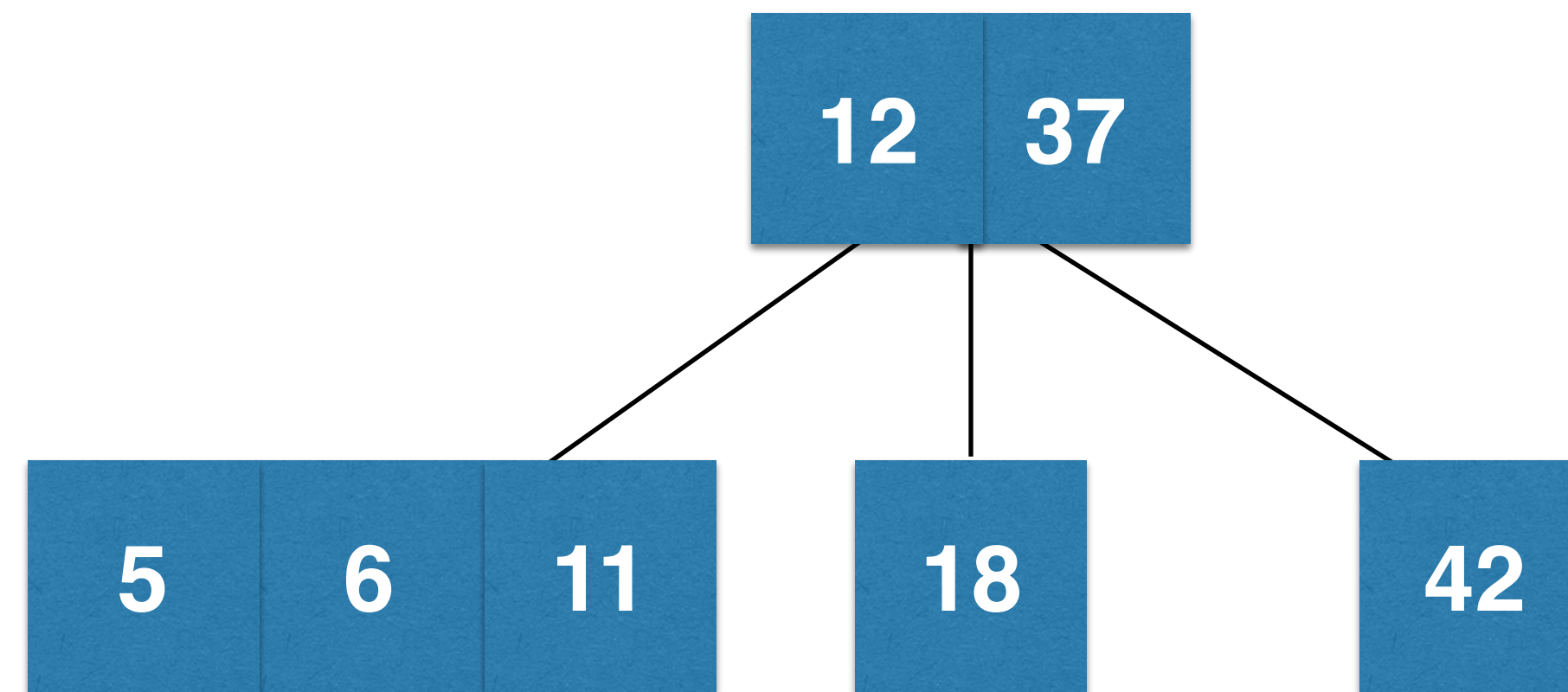




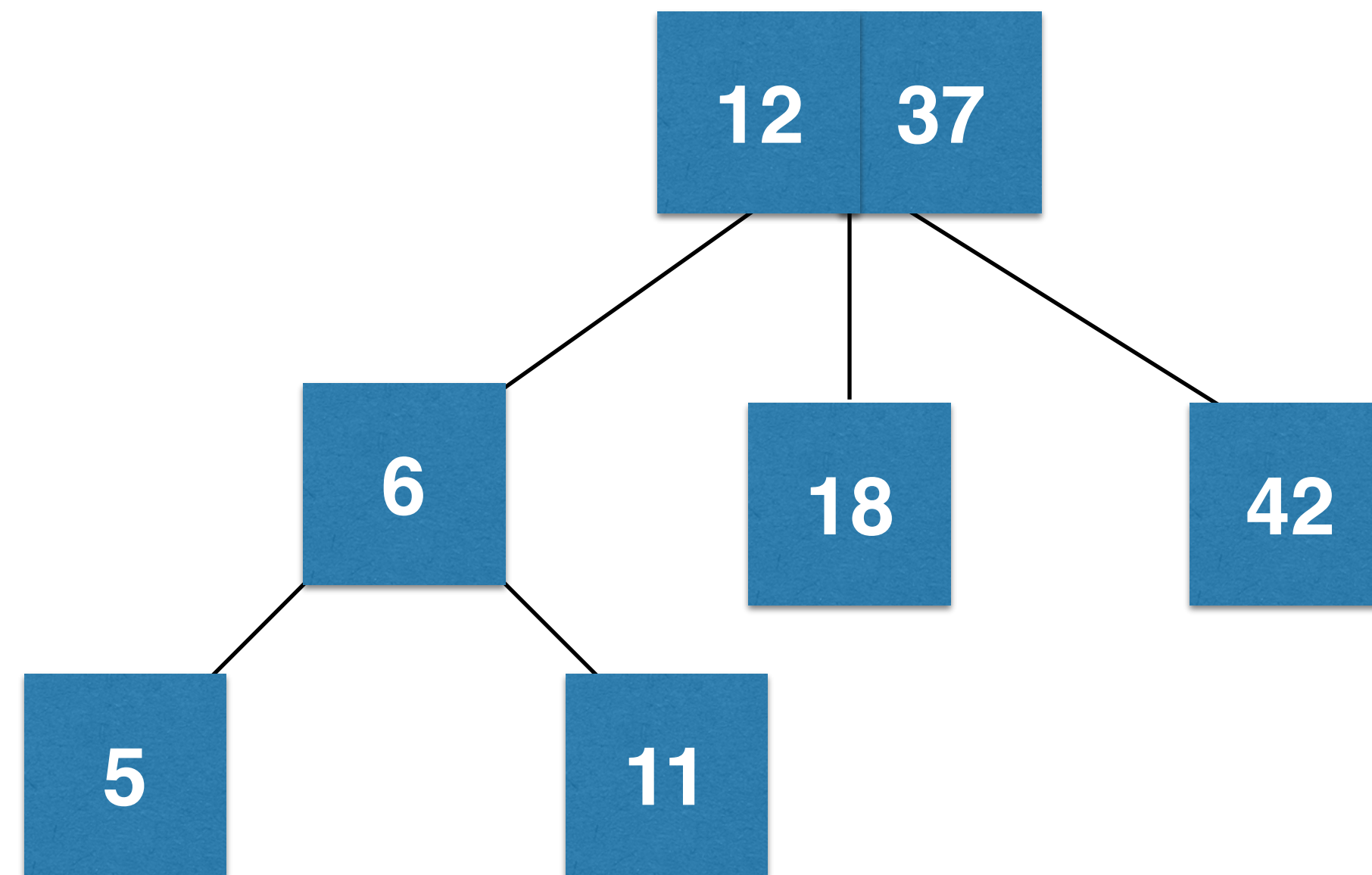
# 2-3树



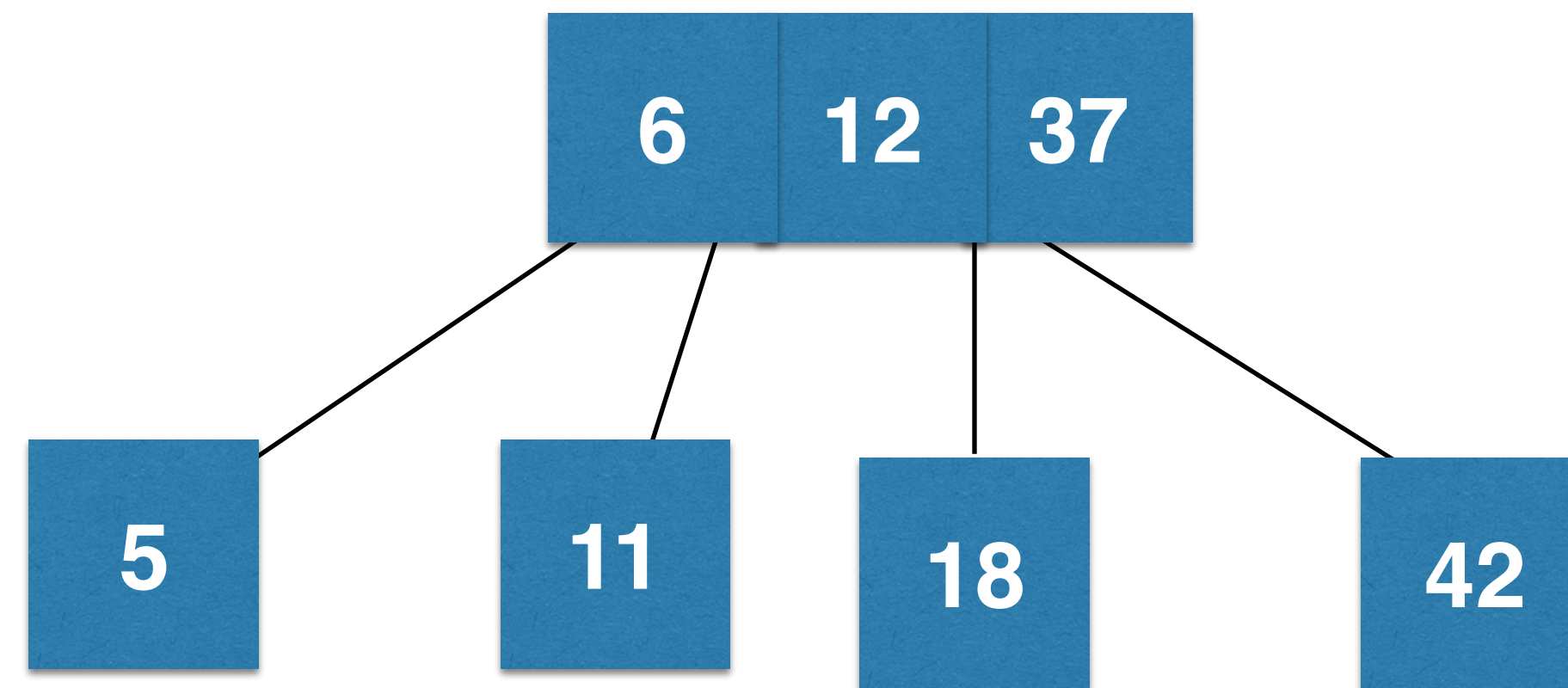
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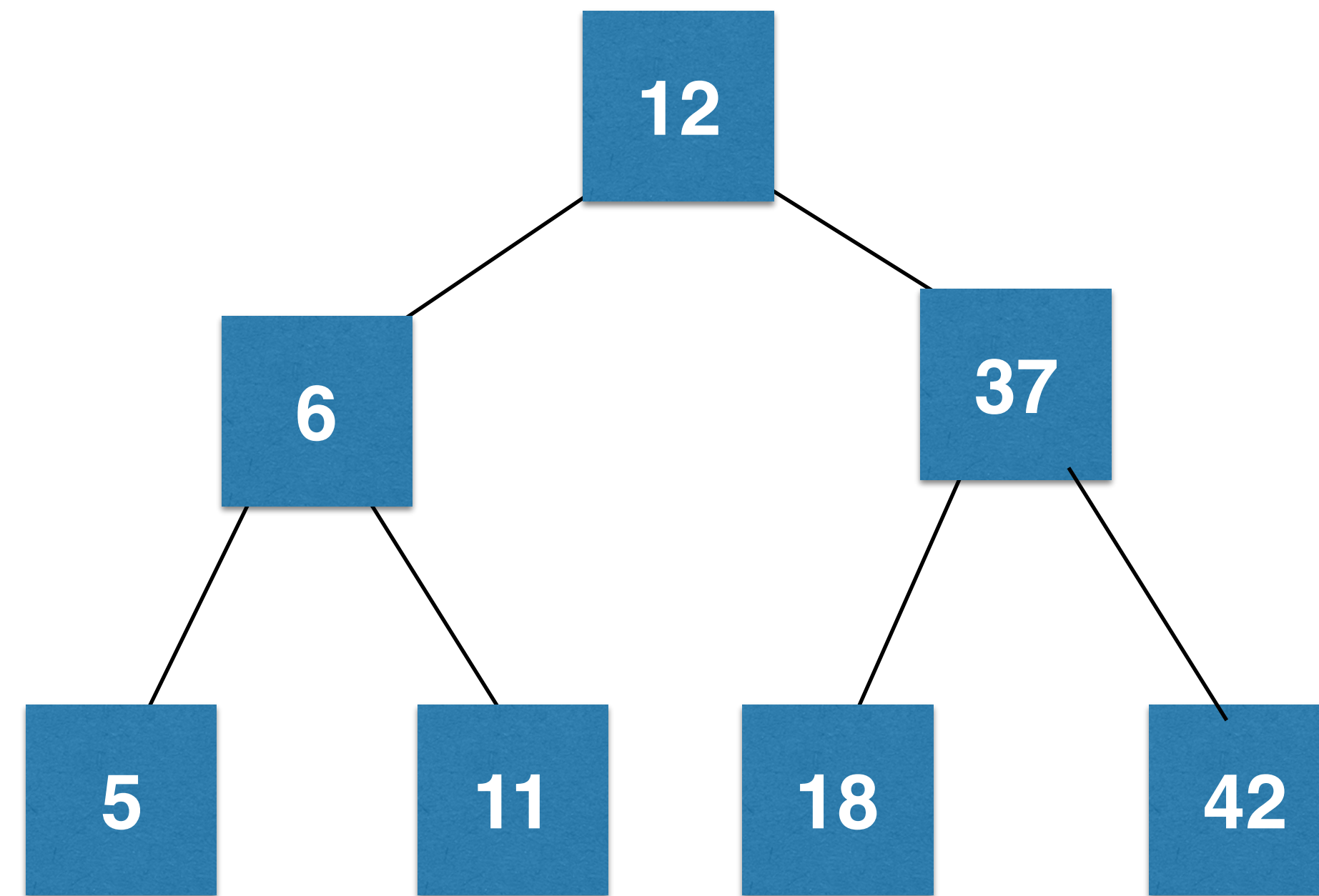
# 2-3树



# 2-3树

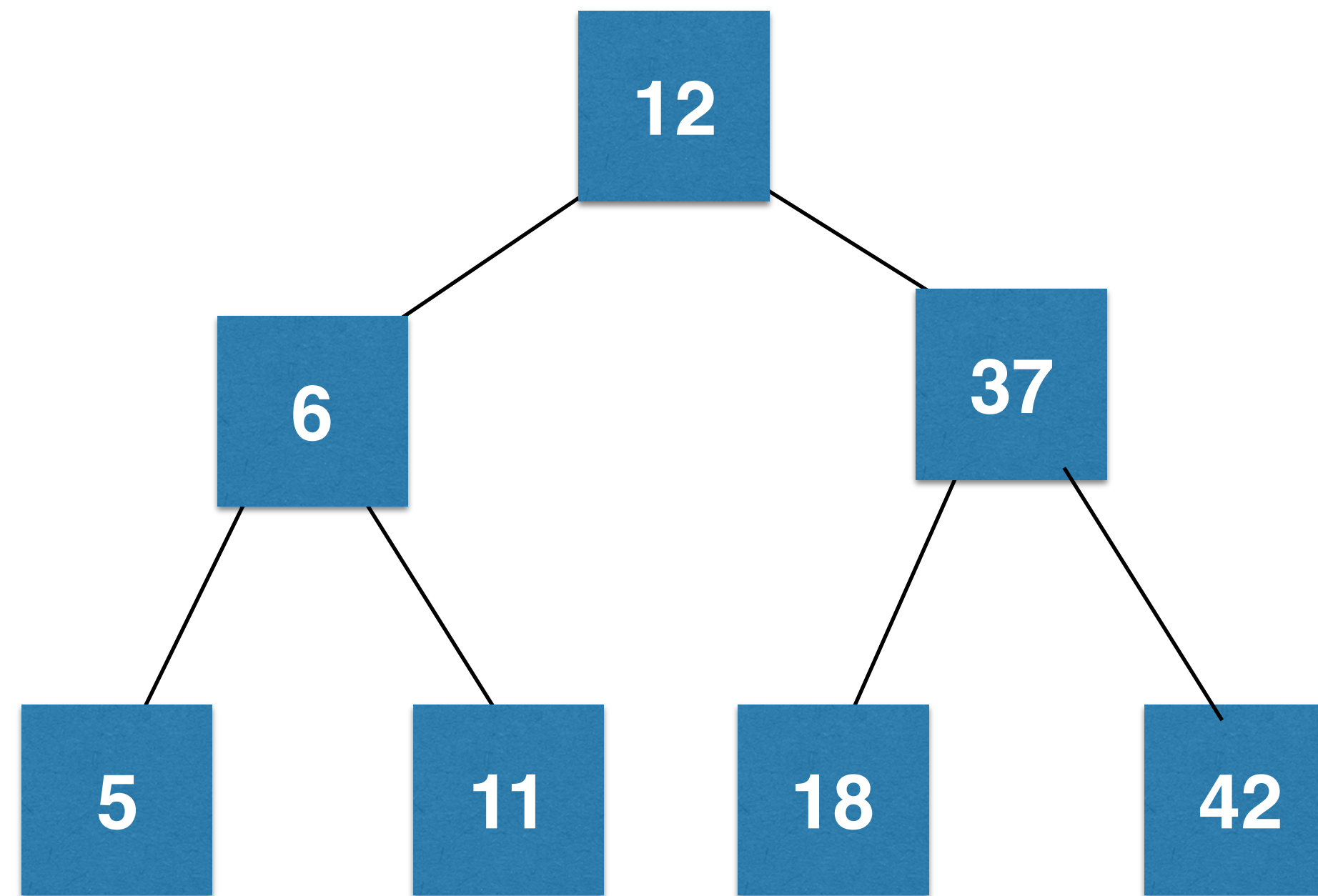


# 2-3树





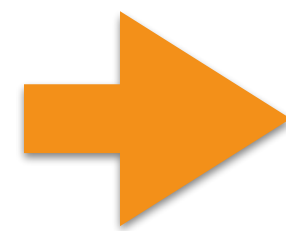
# 2-3树



# 2-3树

如果插入2-节点

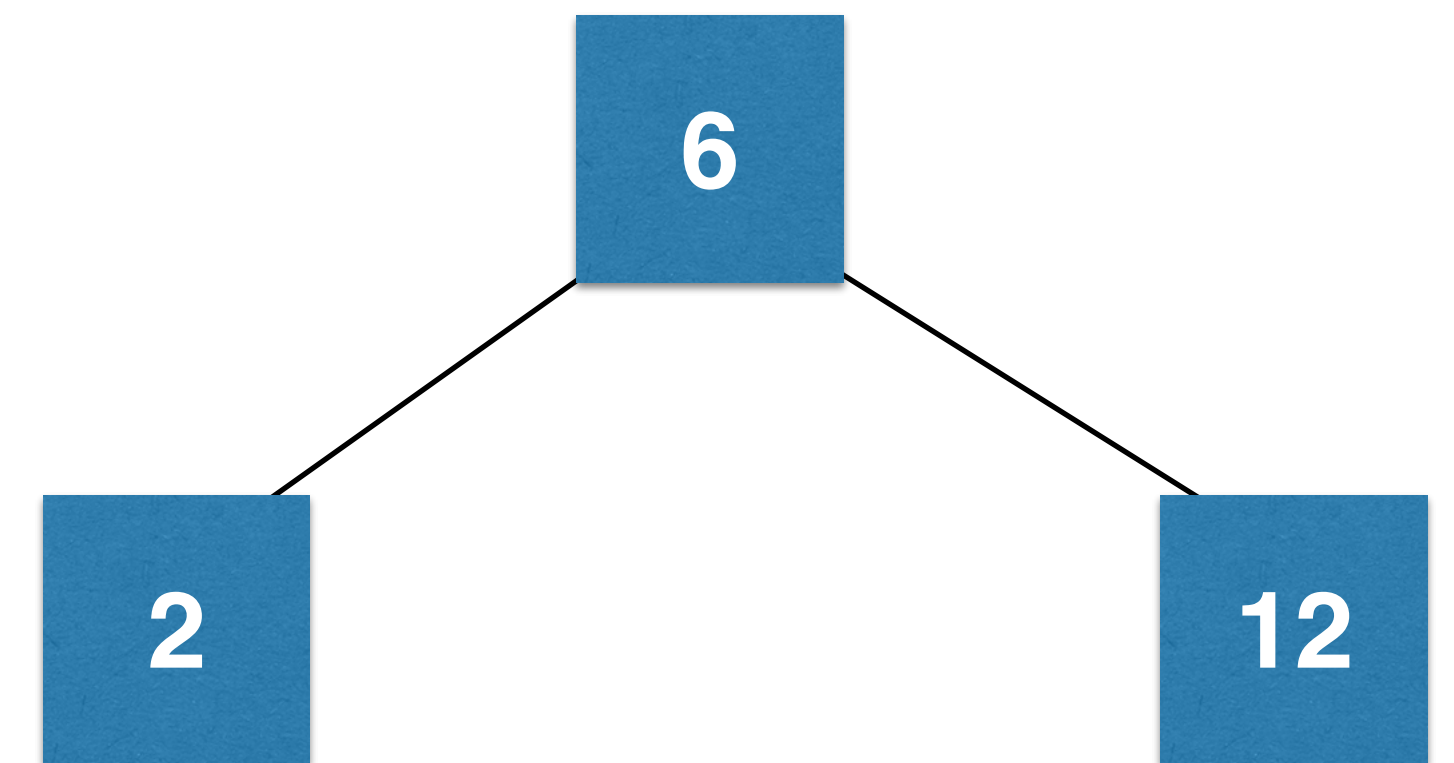
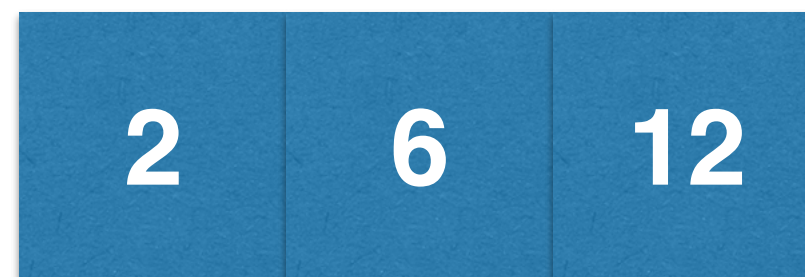
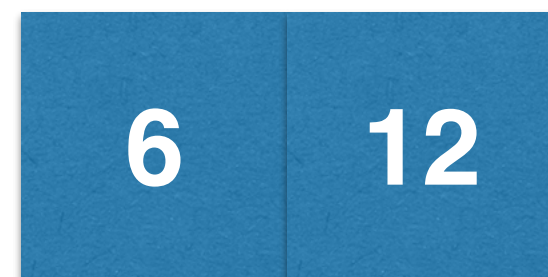
12



6	12
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# 2-3树

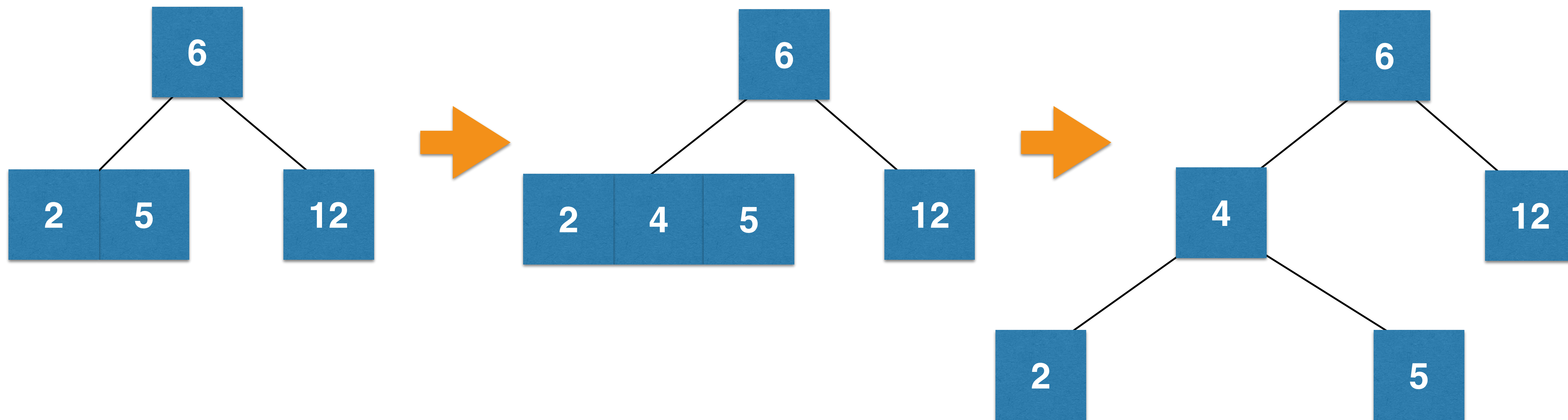
如果插入3-节点



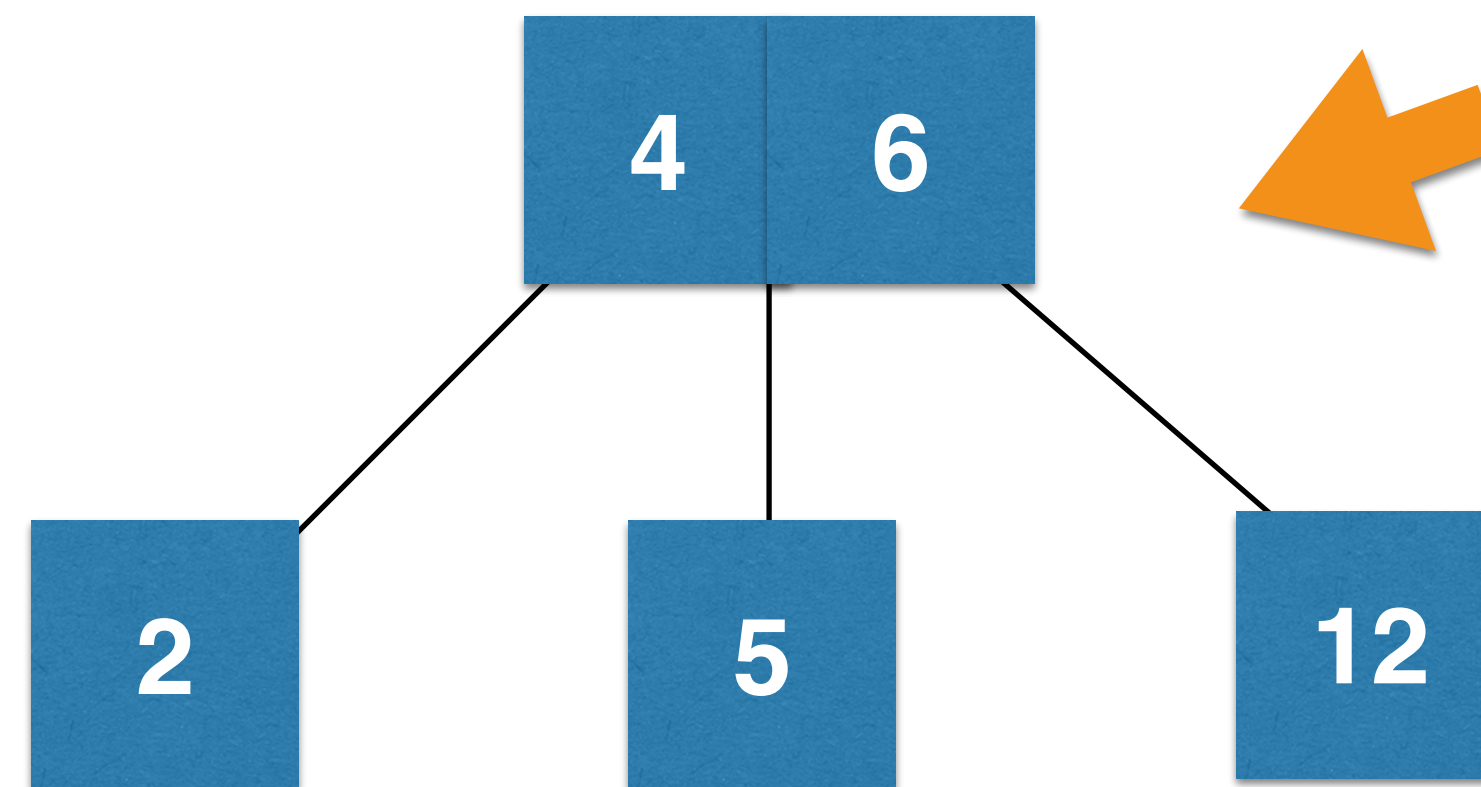
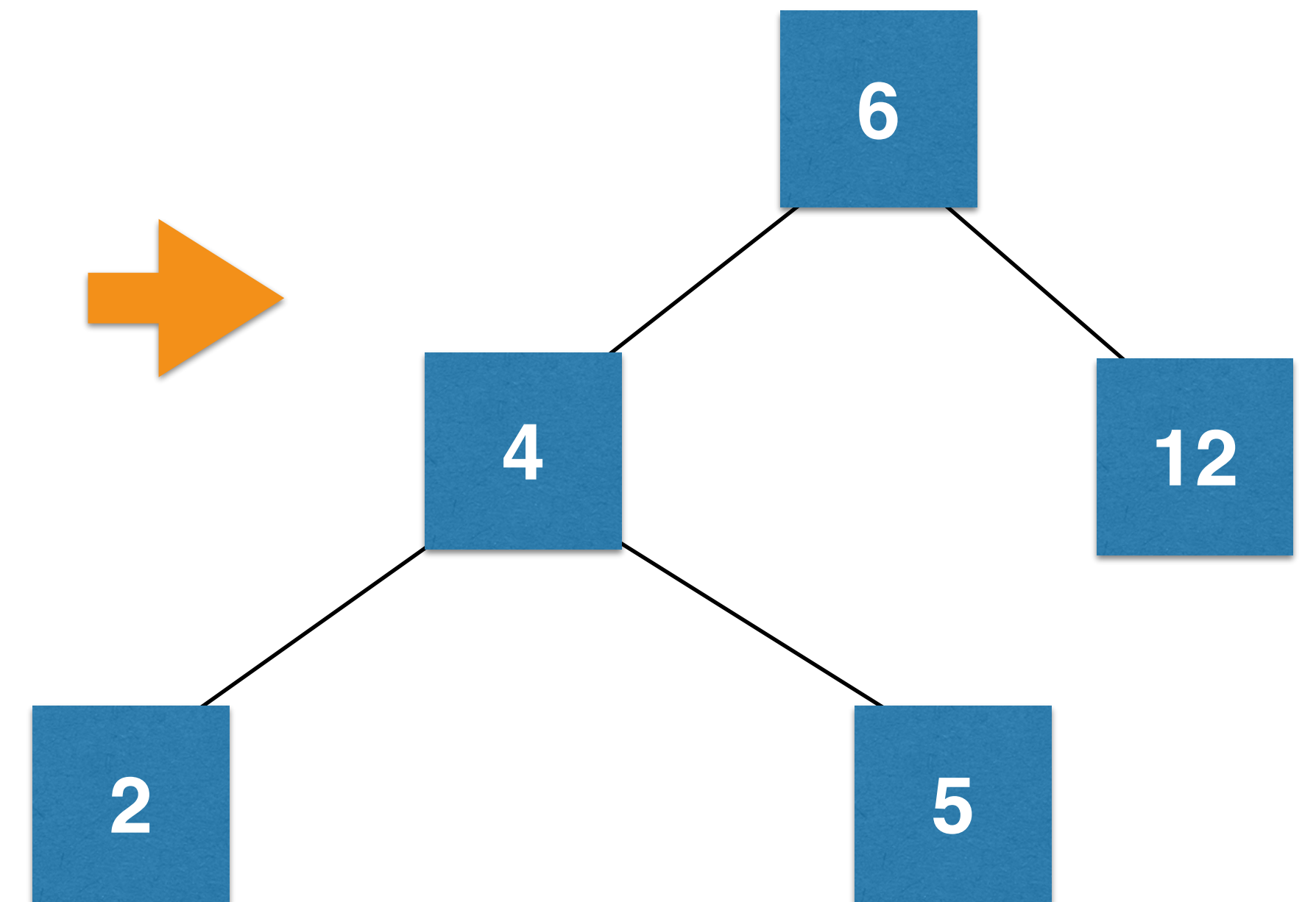
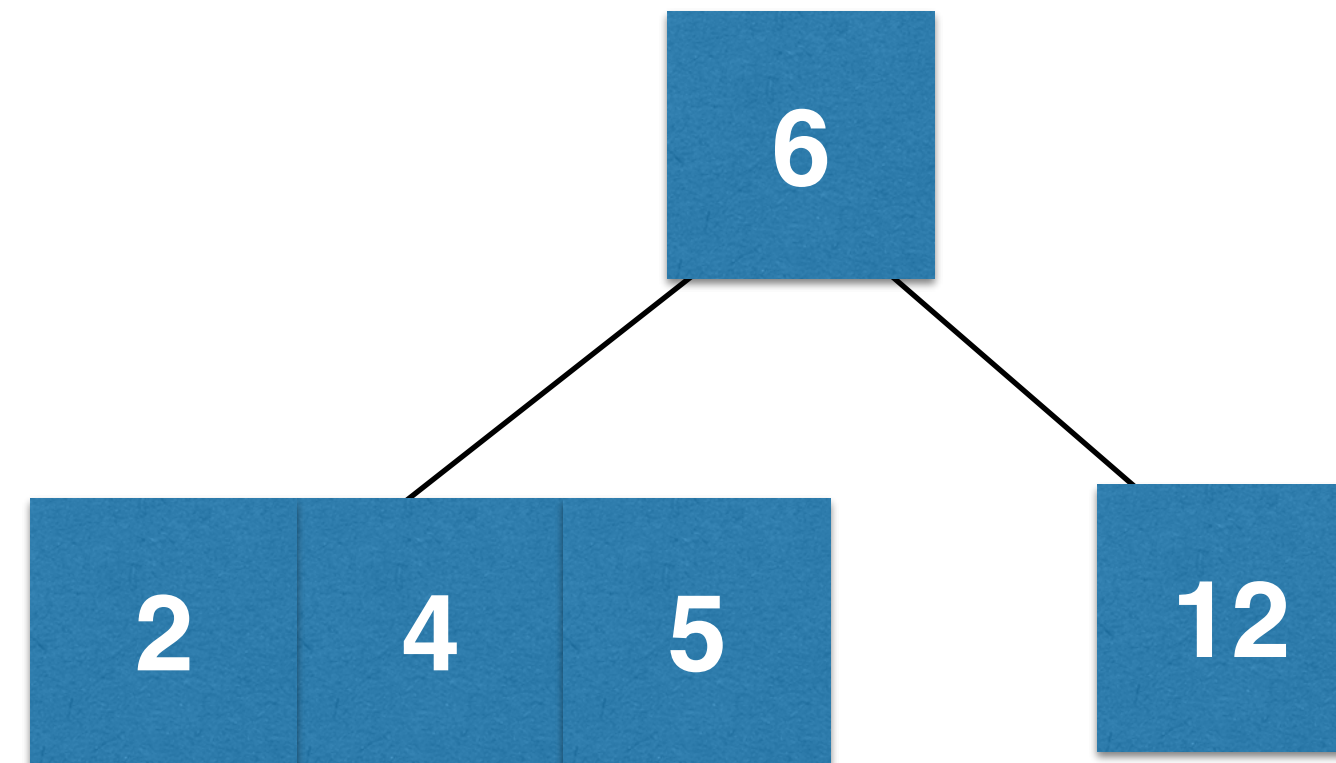
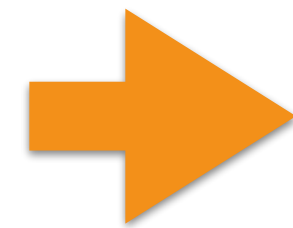
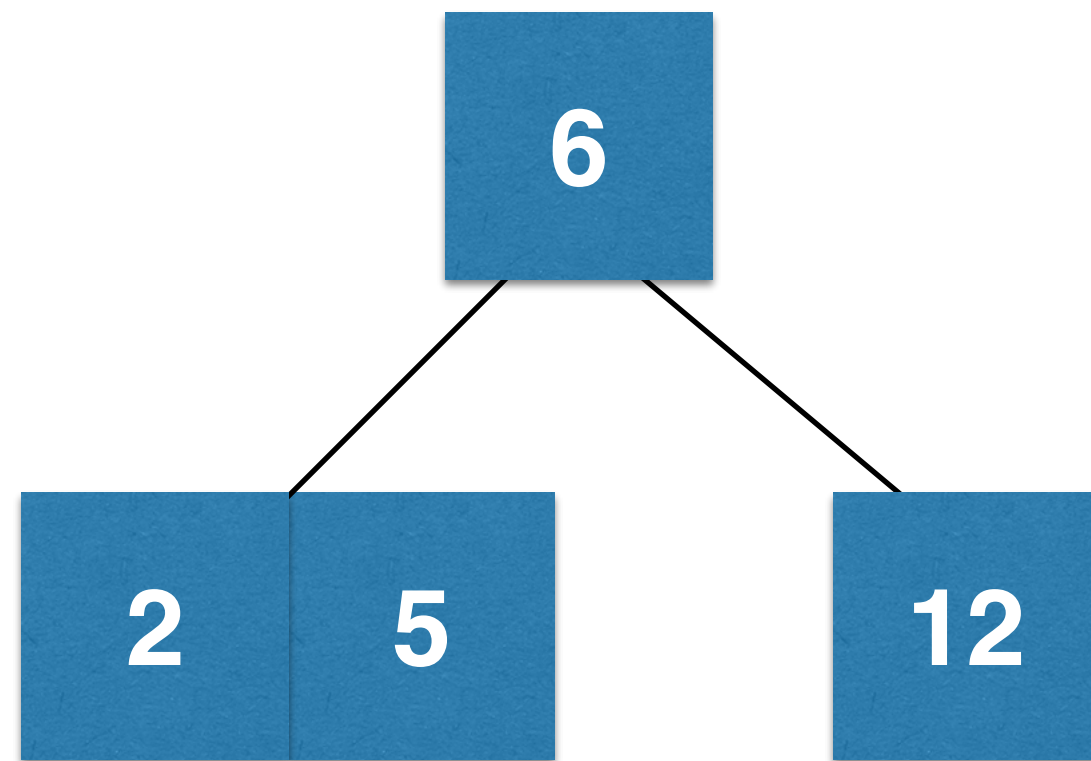
# 2-3树

如果插入3-节点

父亲节点为2-节点



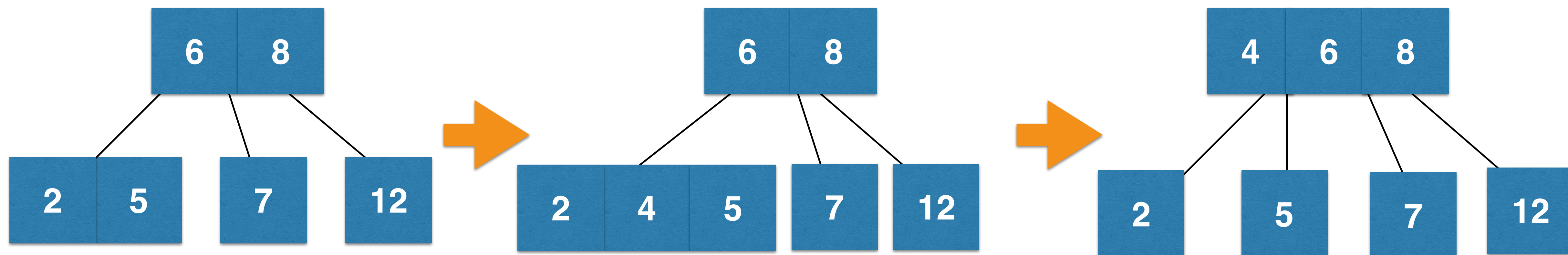
# 2-3树



# 2-3树

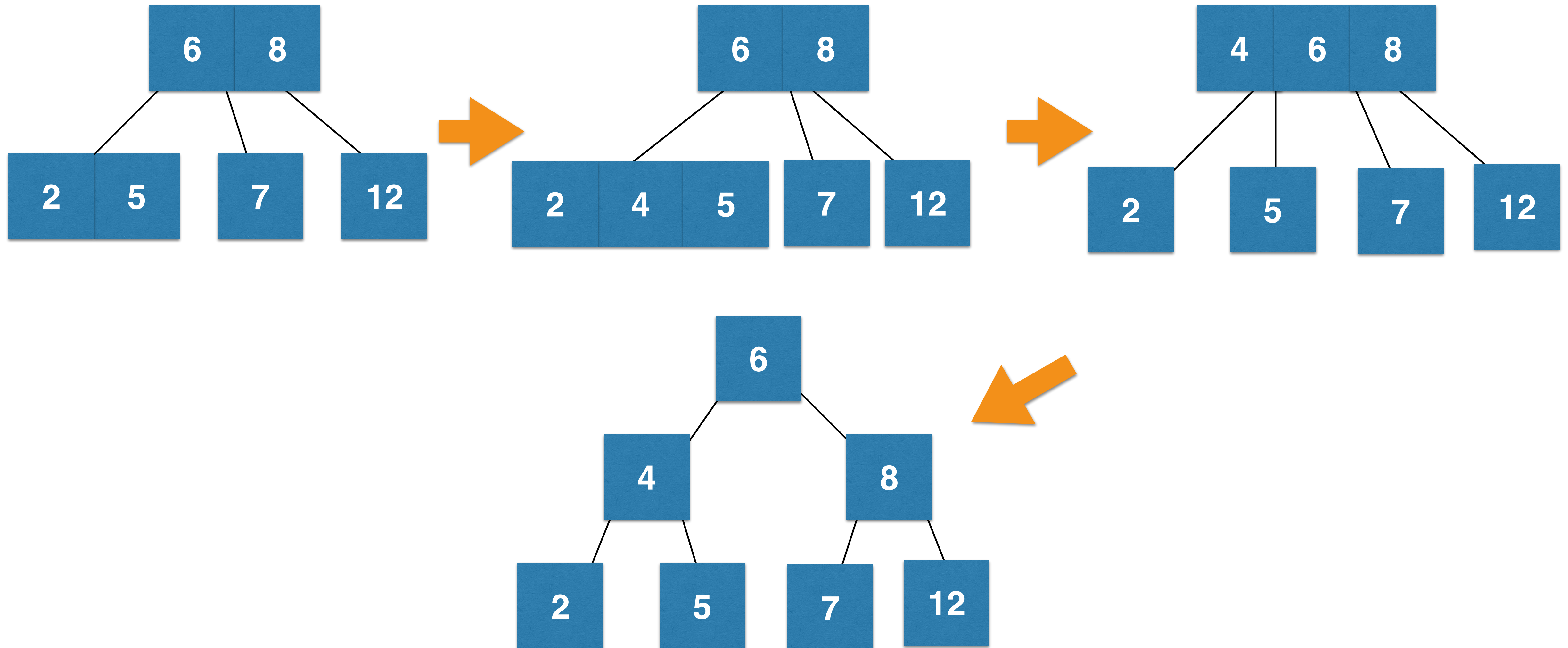
如果插入3-节点

父亲节点为3-节点





# 2-3树

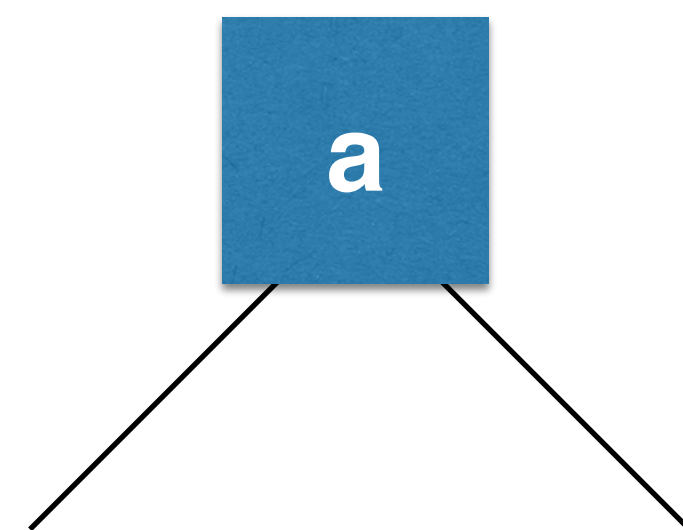




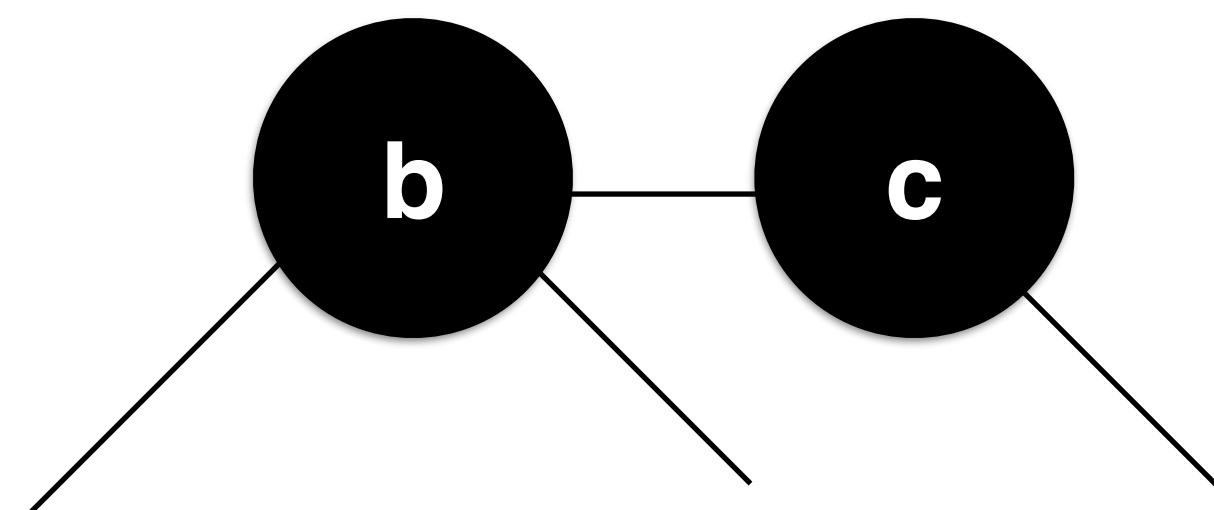
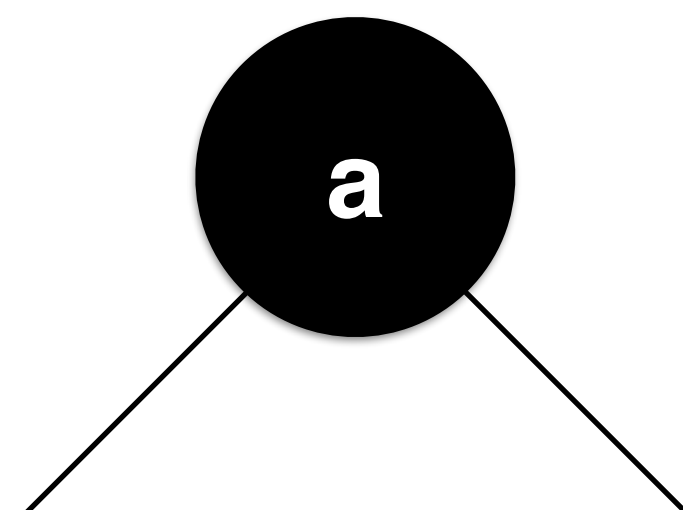
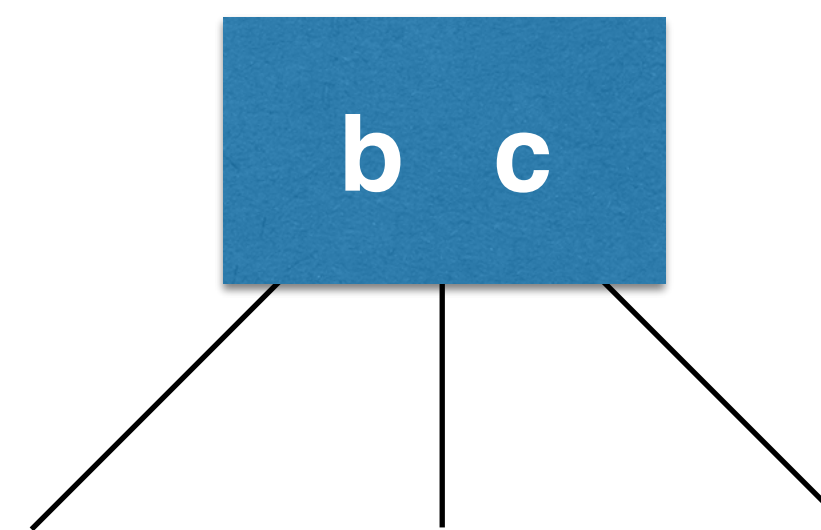
# 红黑树和 2-3 树的等价性

# 红黑树和2-3树

2节点

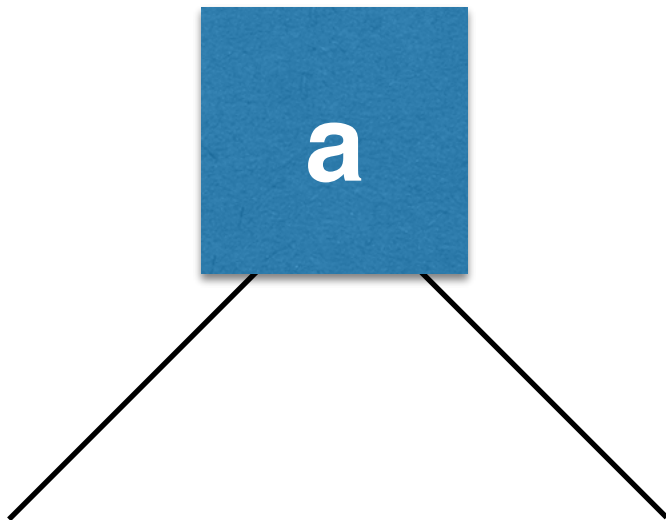


3节点

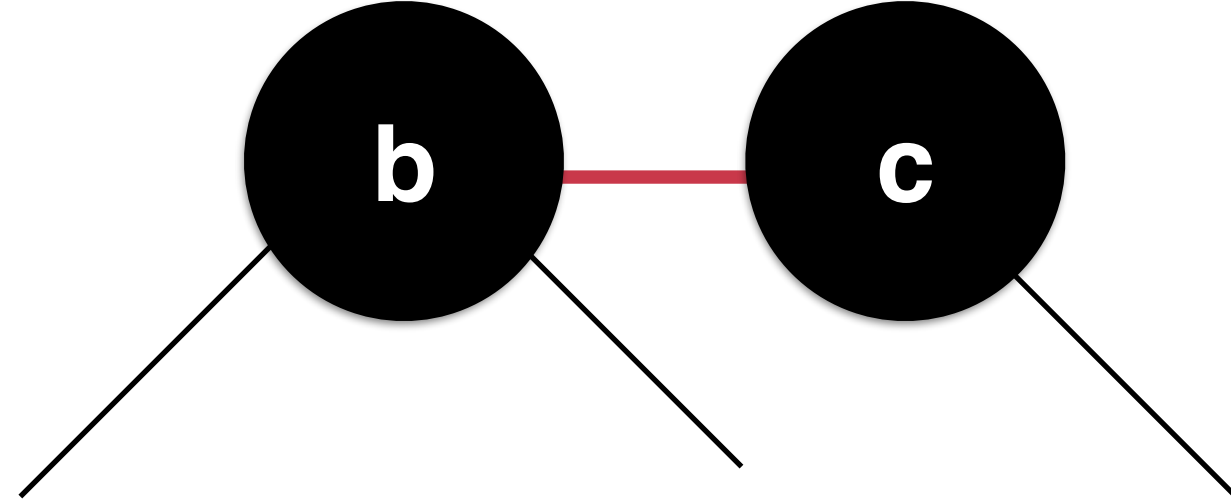
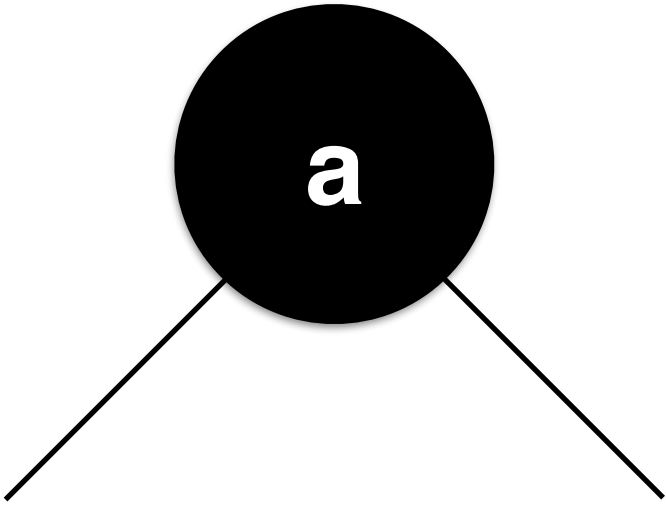
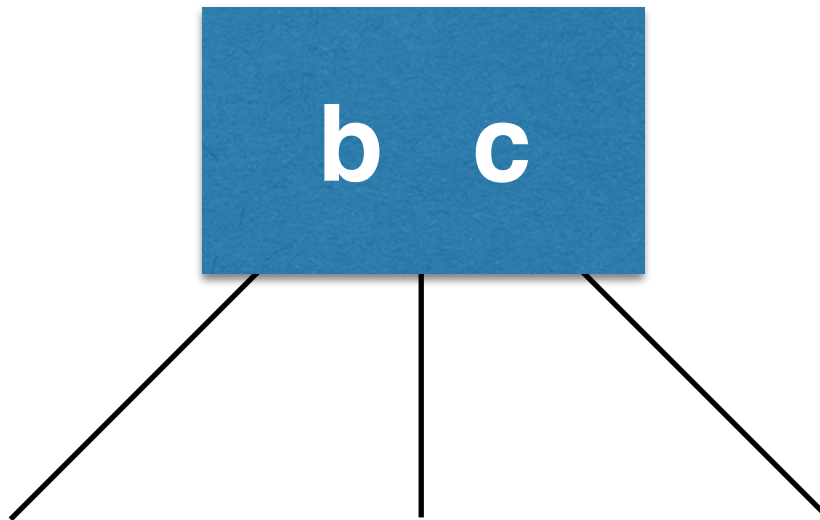


# 红黑树和2-3树

2节点

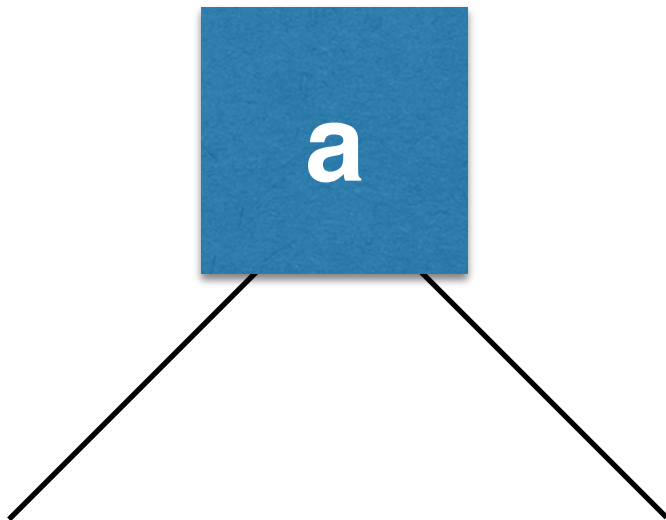


3节点

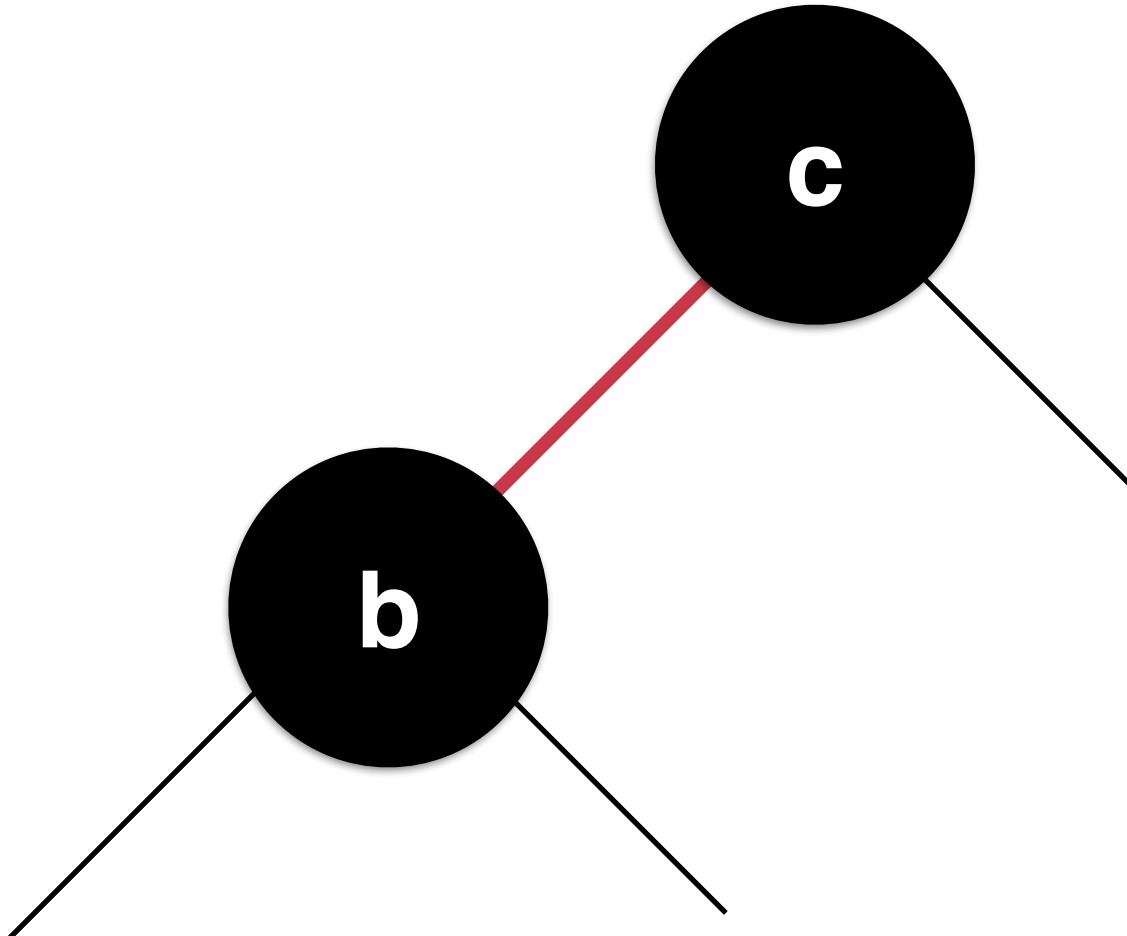
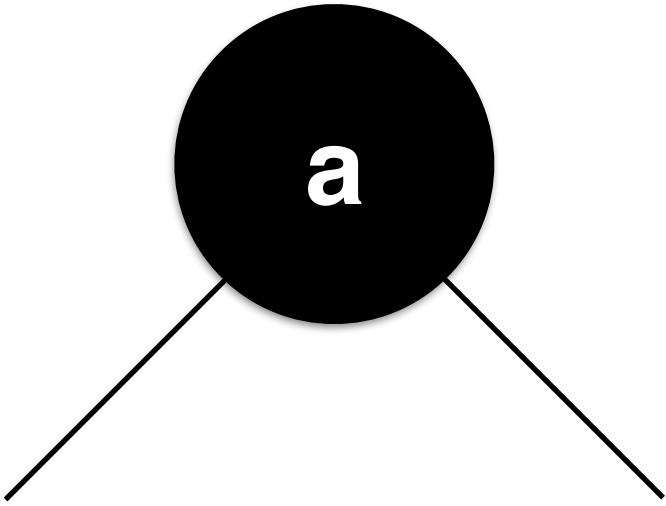
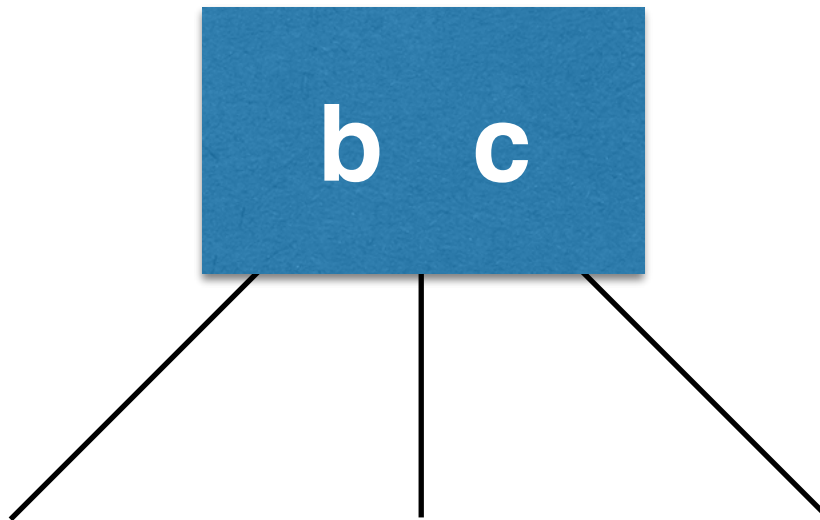


# 红黑树和2-3树

2节点

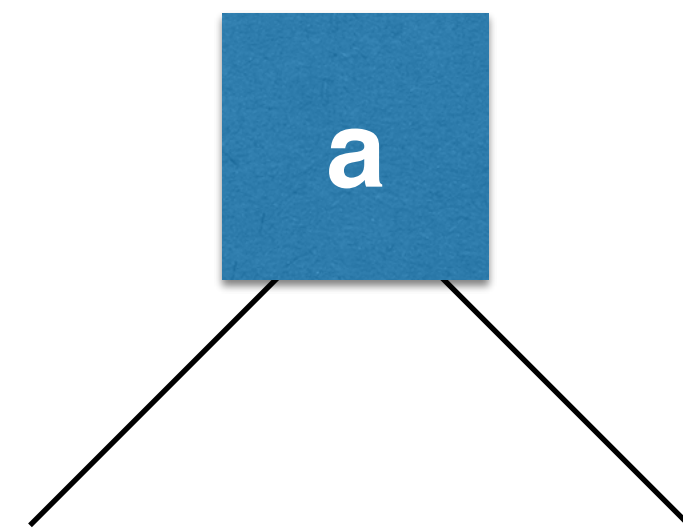


3节点

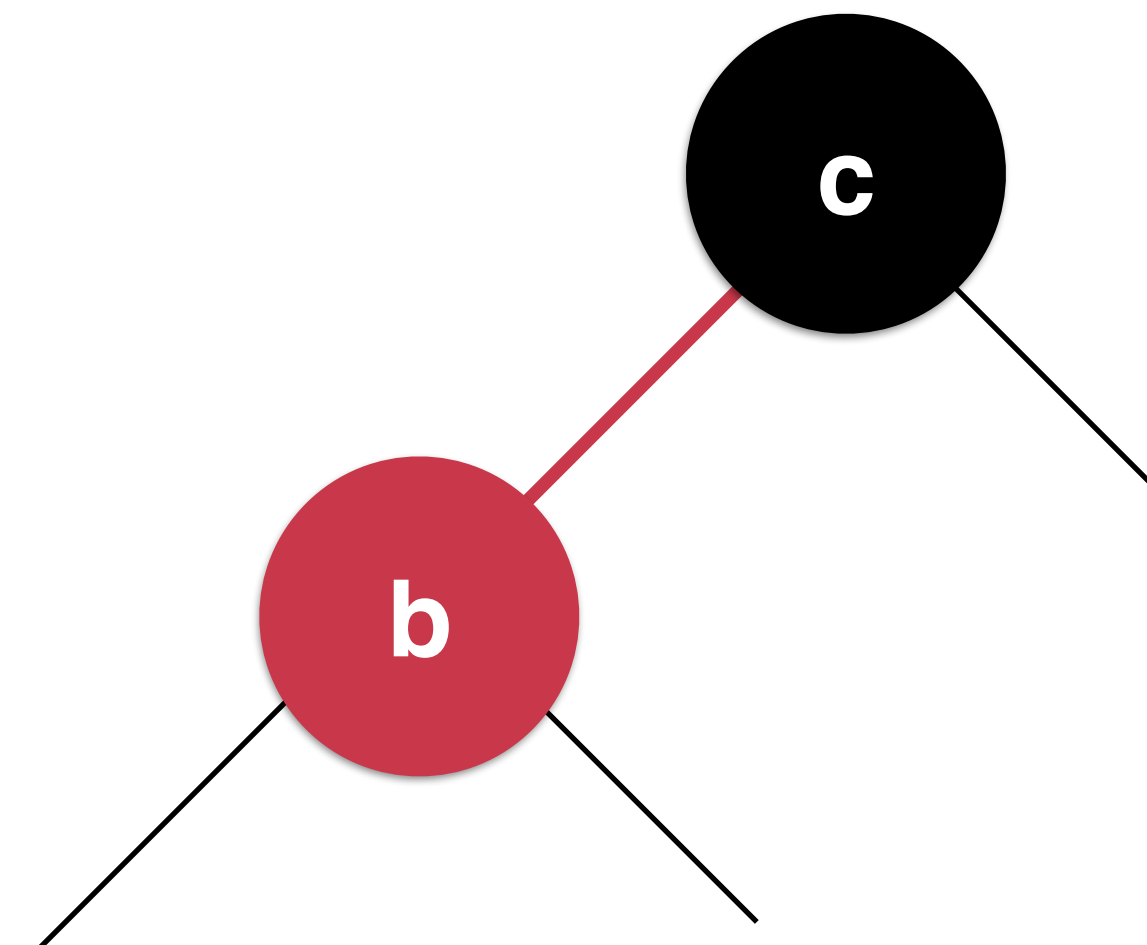
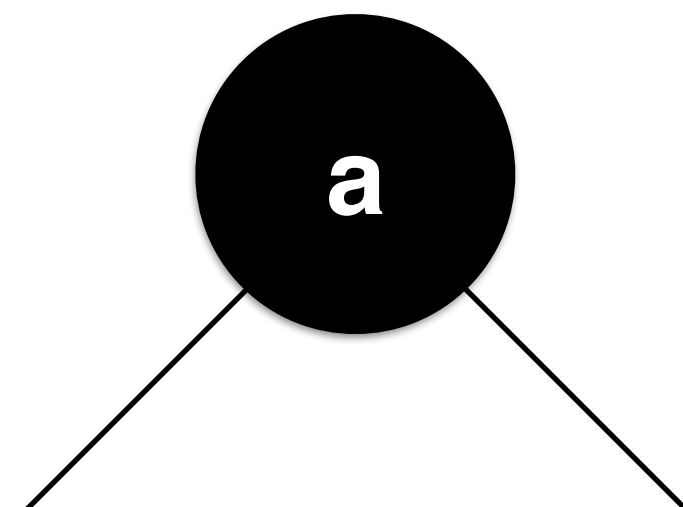
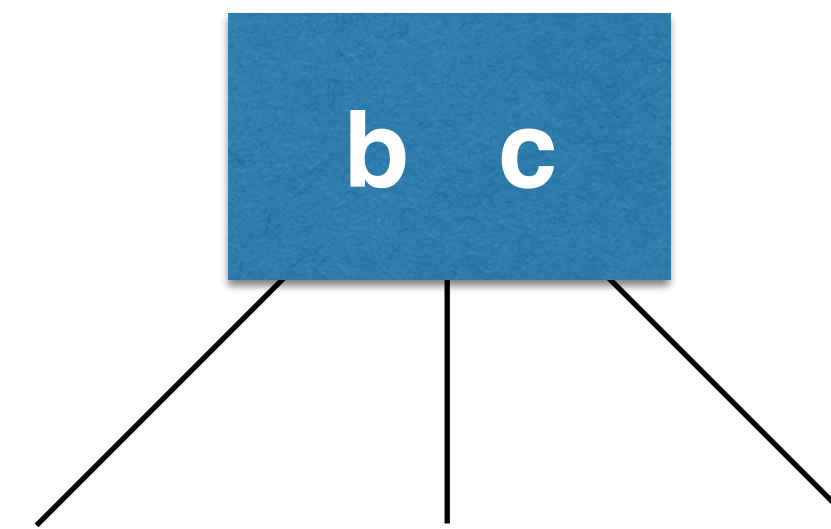


# 红黑树和2-3树

2节点

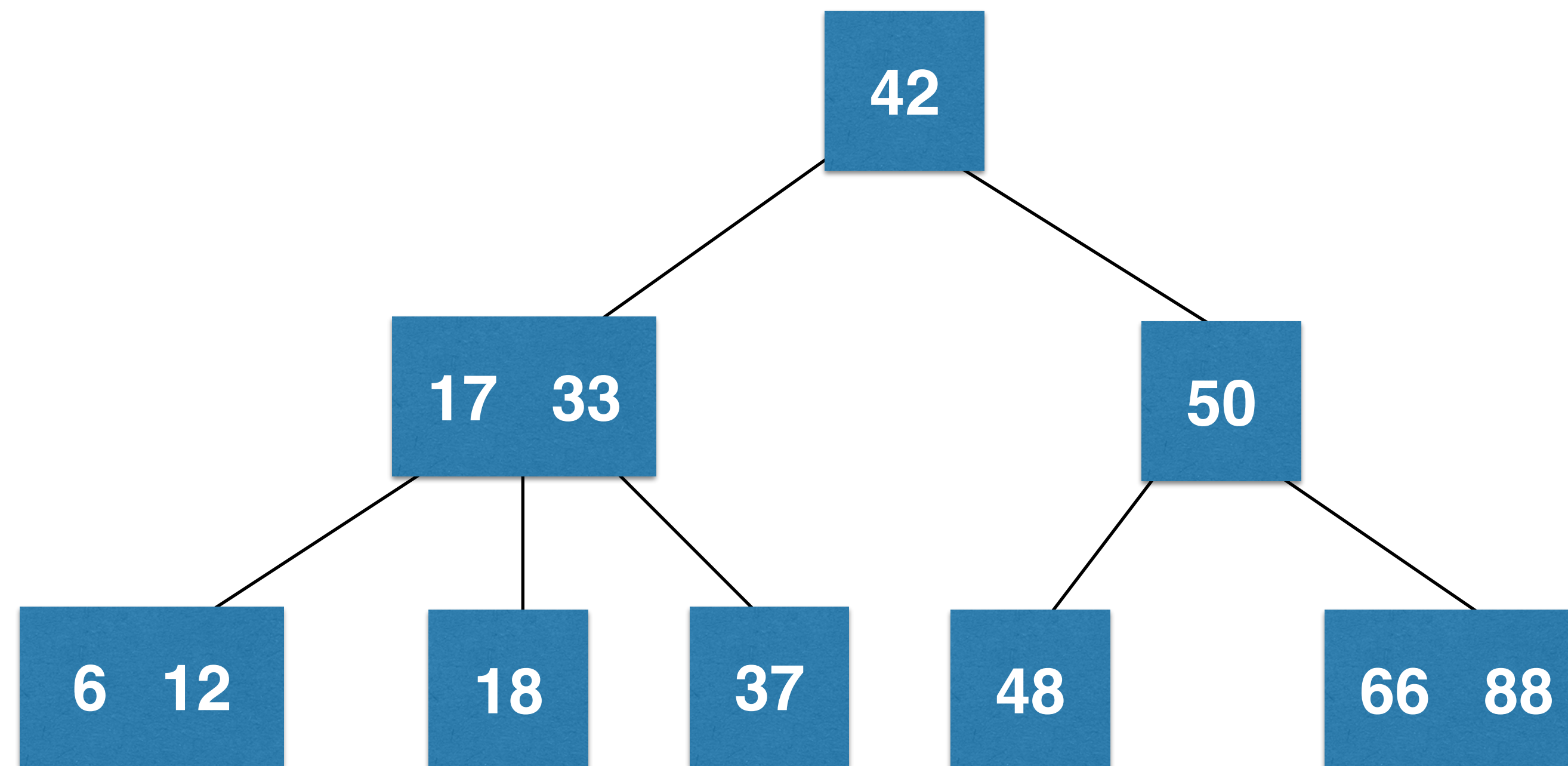


3节点

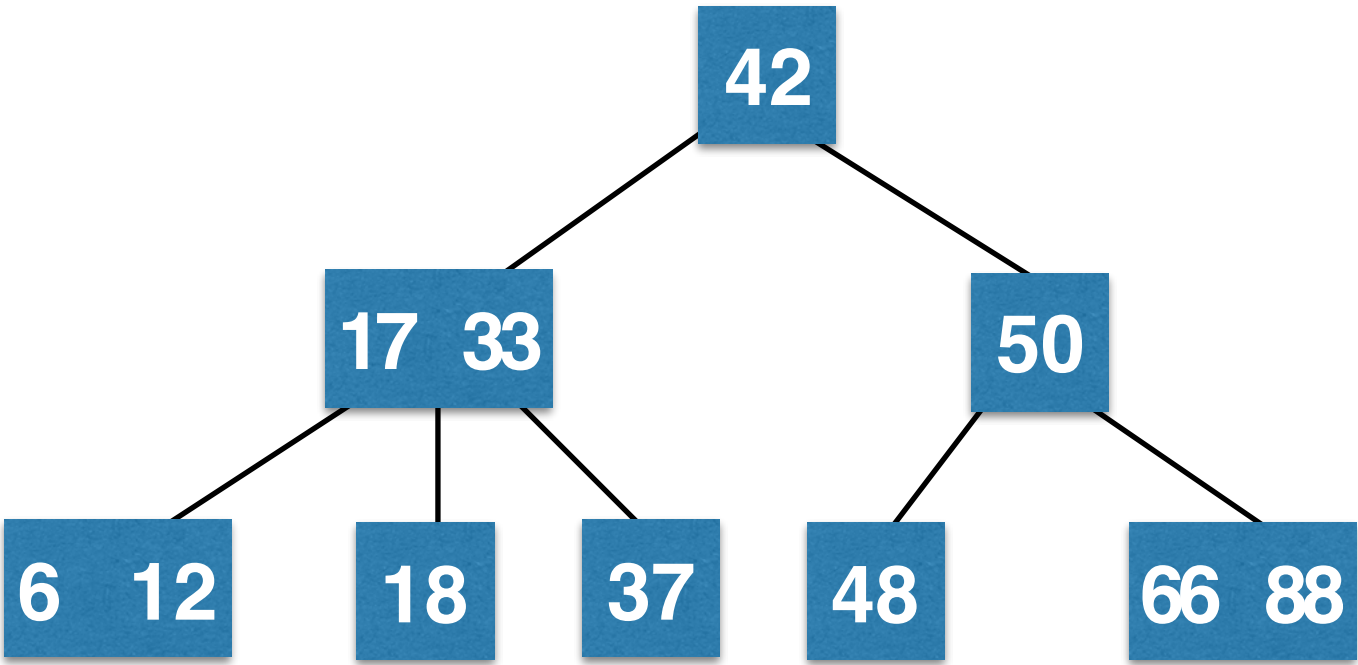


所有的红色节点都是左倾斜的

# 红黑树和2-3树

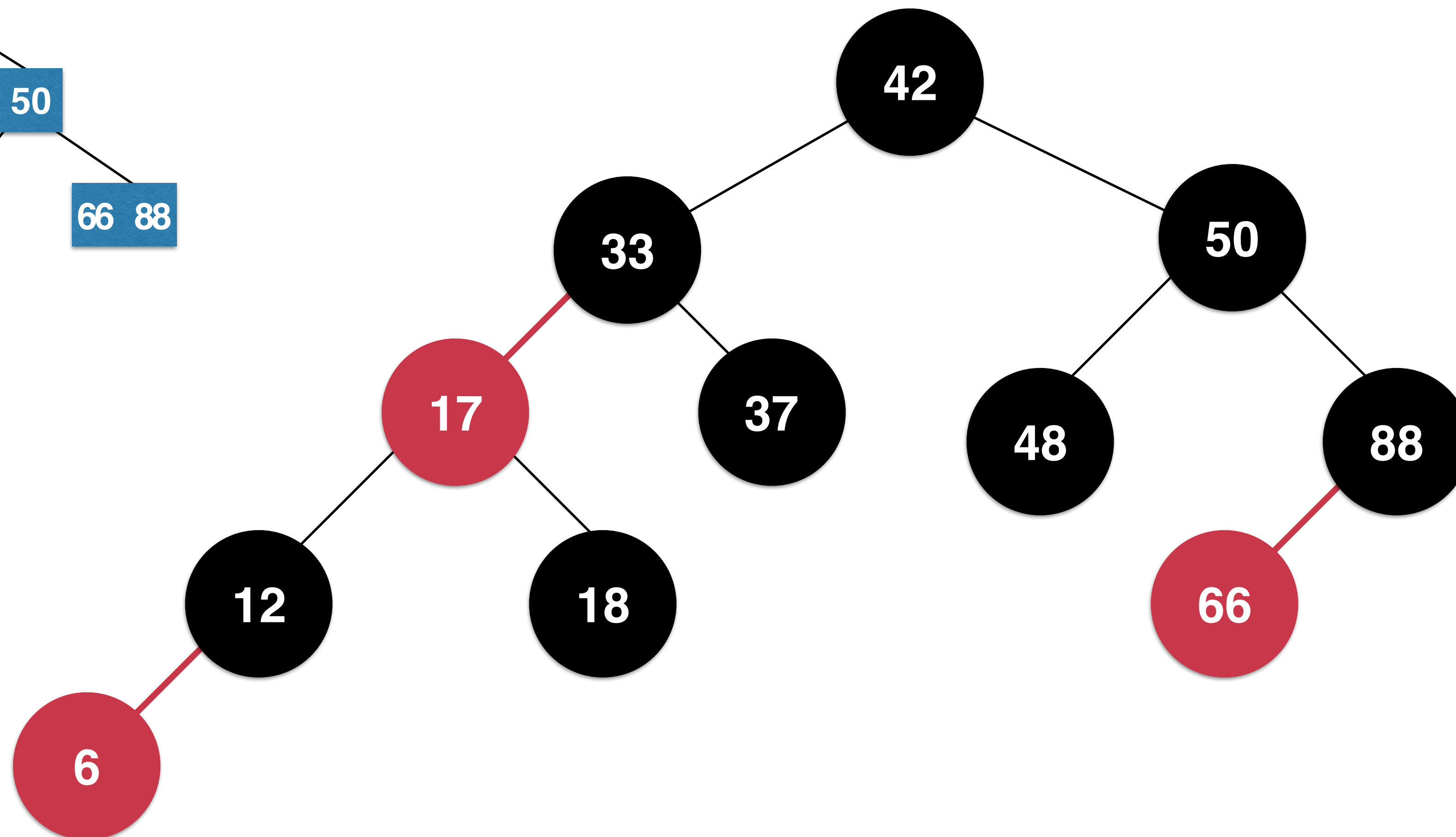
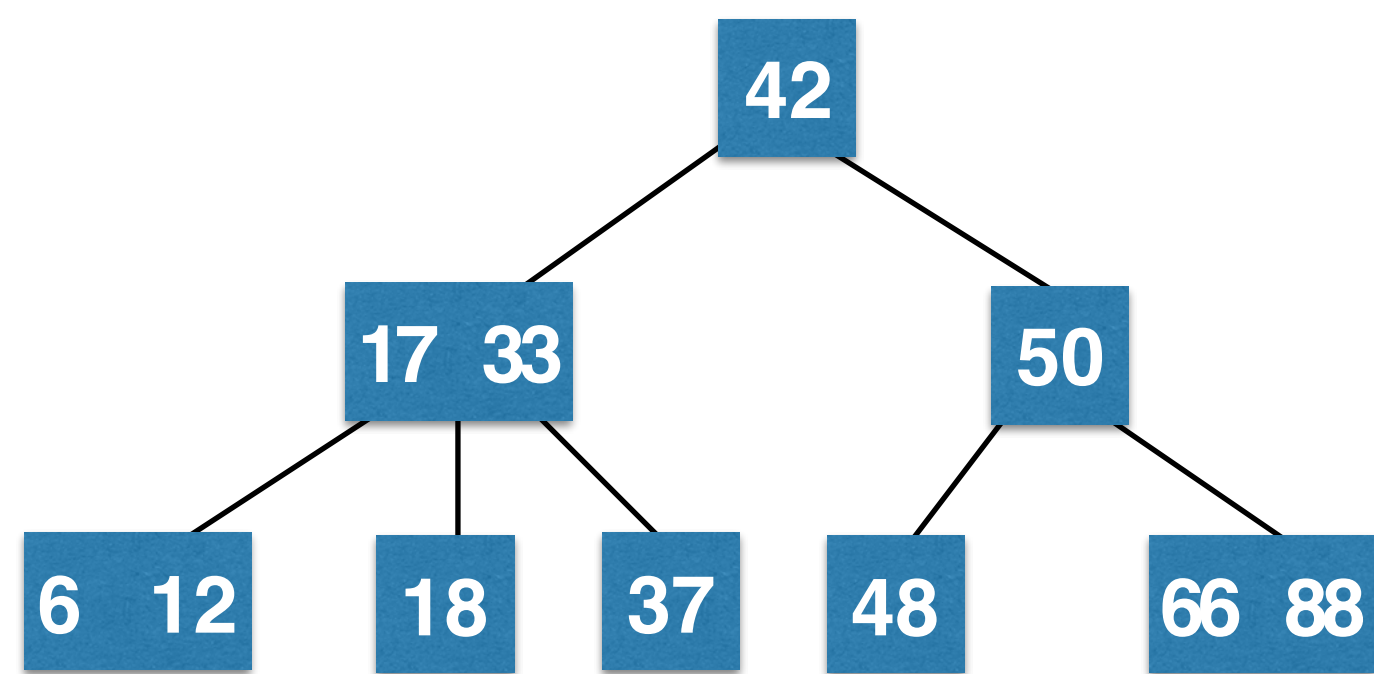


# 红黑树和2-3树

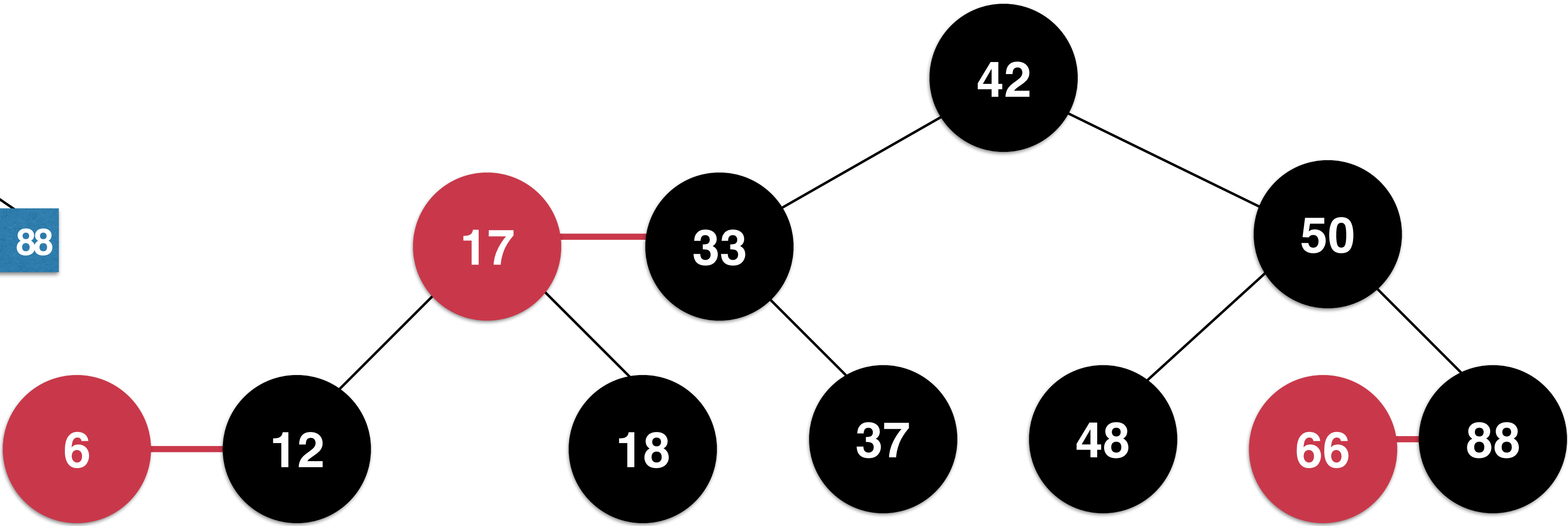
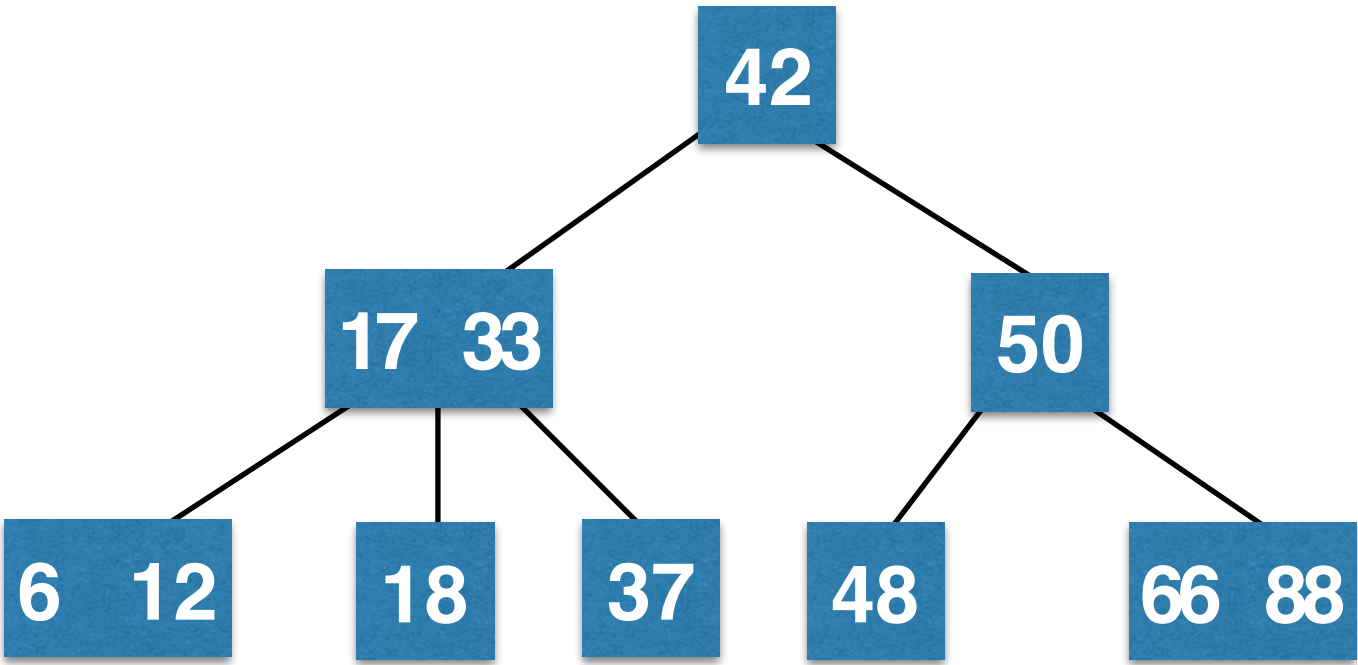




# 红黑树和2-3树



# 红黑树和2-3树

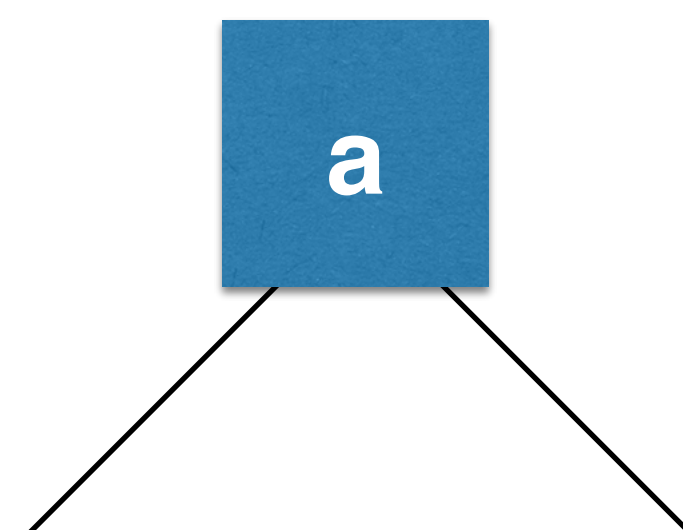


# 实践：红黑树的节点

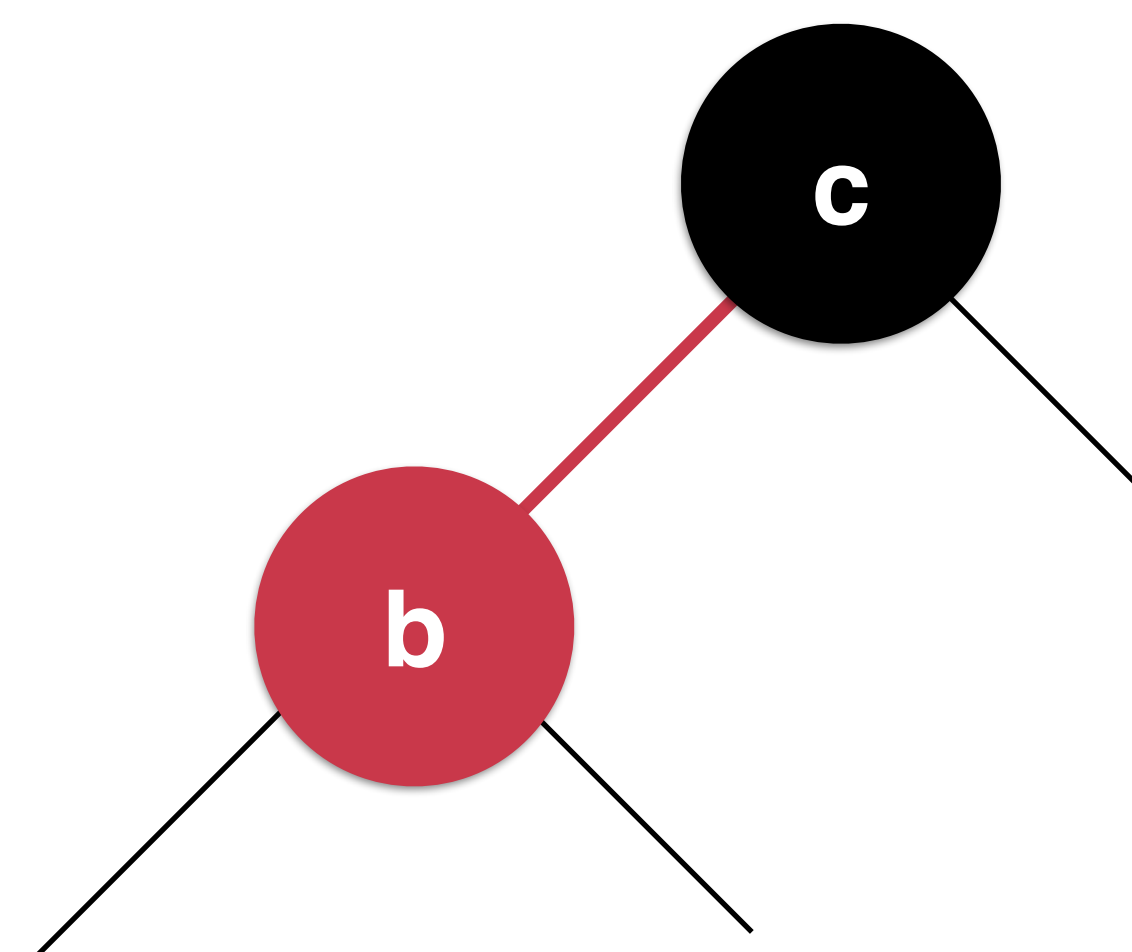
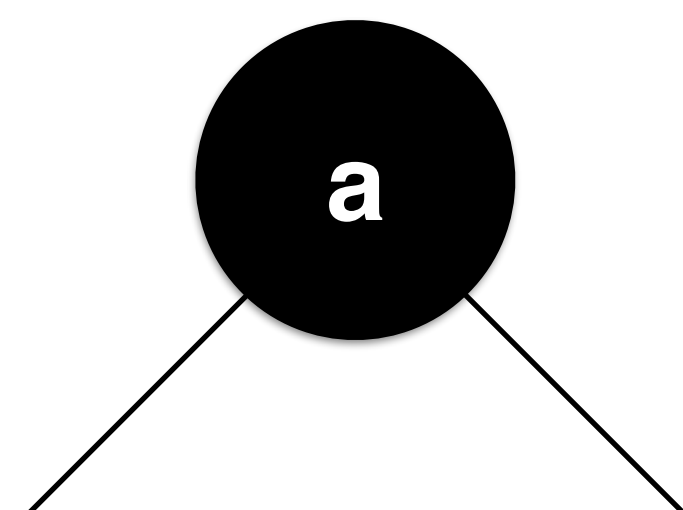
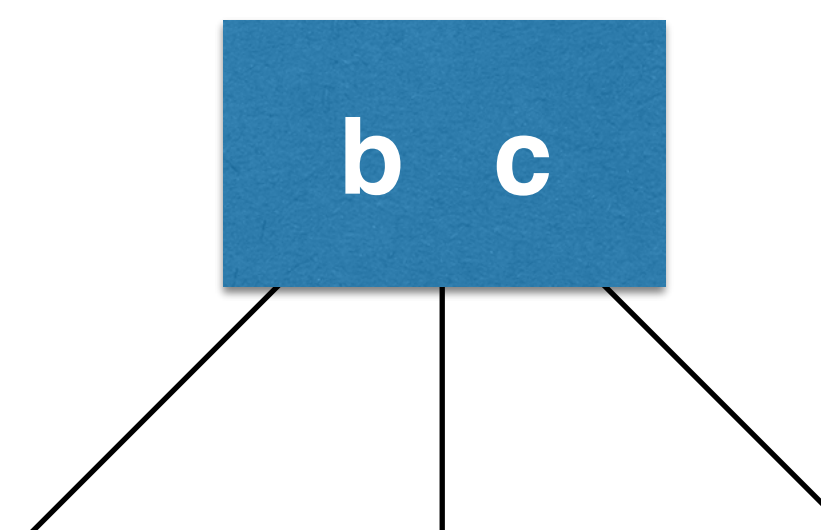
# 红黑树的重要性质

# 红黑树和2-3树的等价性

2节点



3节点



# 《算法导论》 中的红黑树

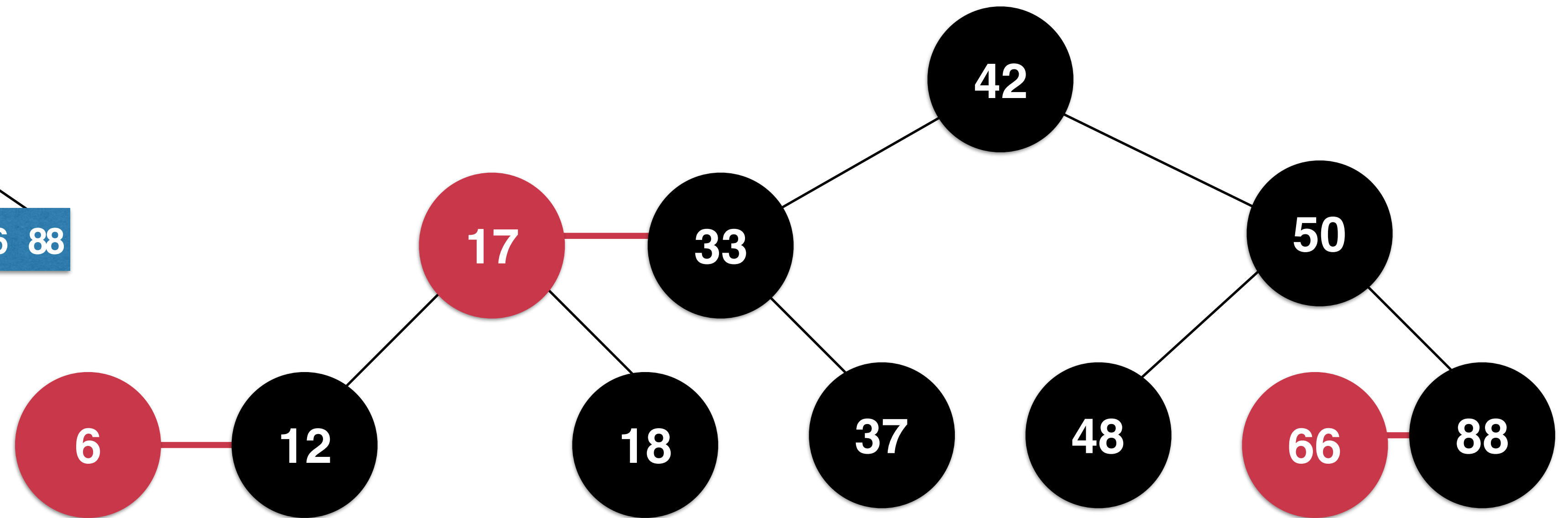
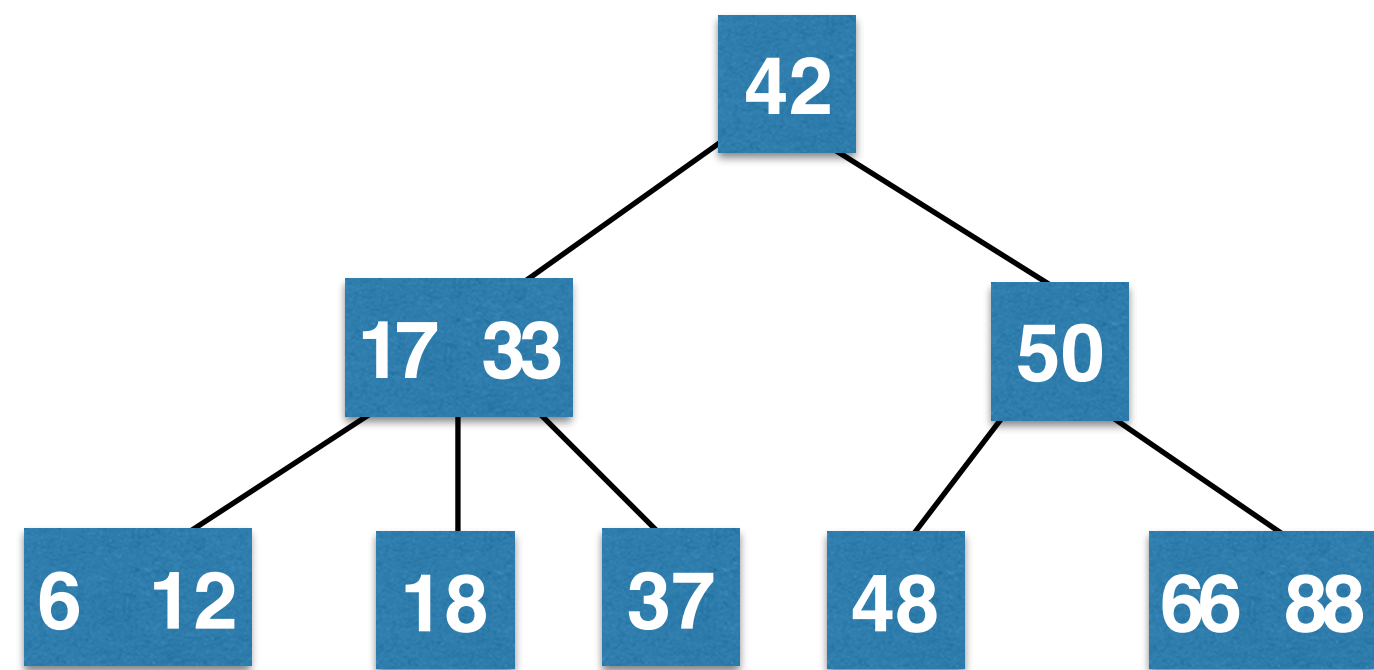
A red-black tree is a binary tree that satisfies the following *red-black properties*:

1. Every node is either red or black.
2. The root is black.
3. Every leaf (NIL) is black.
4. If a node is red, then both its children are black.
5. For each node, all simple paths from the node to descendant leaves contain the same number of black nodes.

# 《算法导论》 中的红黑树

1. 每个节点或者是红色的，或者是黑色的
2. 根节点是黑色的
3. 每一个叶子节点（最后的空节点）是黑色的
4. 如果一个节点是红色的，那么他的孩子节点都是黑色的
5. 从任意一个节点到叶子节点，经过的黑色节点是一样的

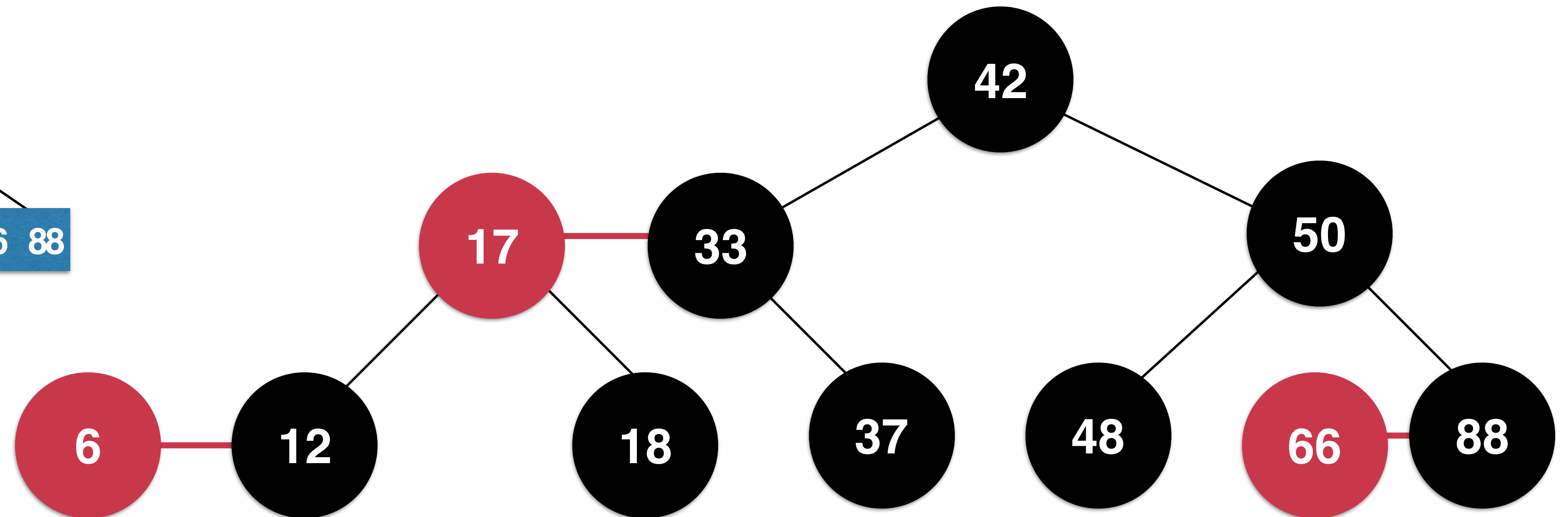
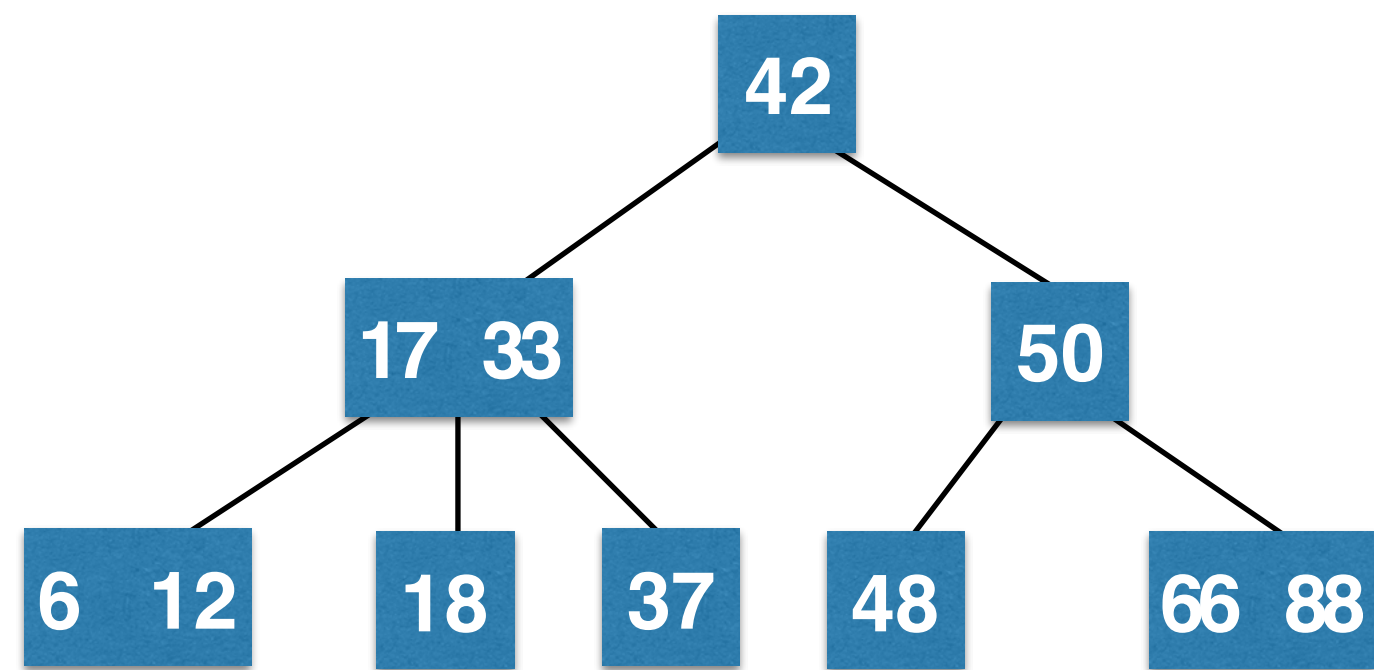
# 红黑树和2-3树



1. 每个节点或者是红色的，或者是黑色的



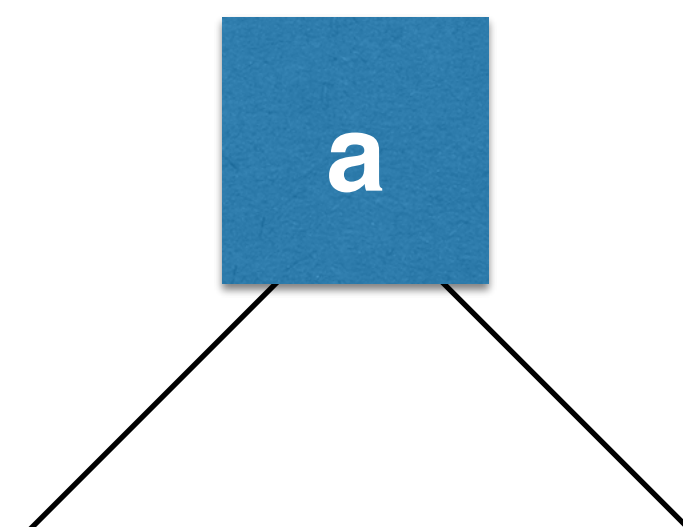
# 红黑树和2-3树



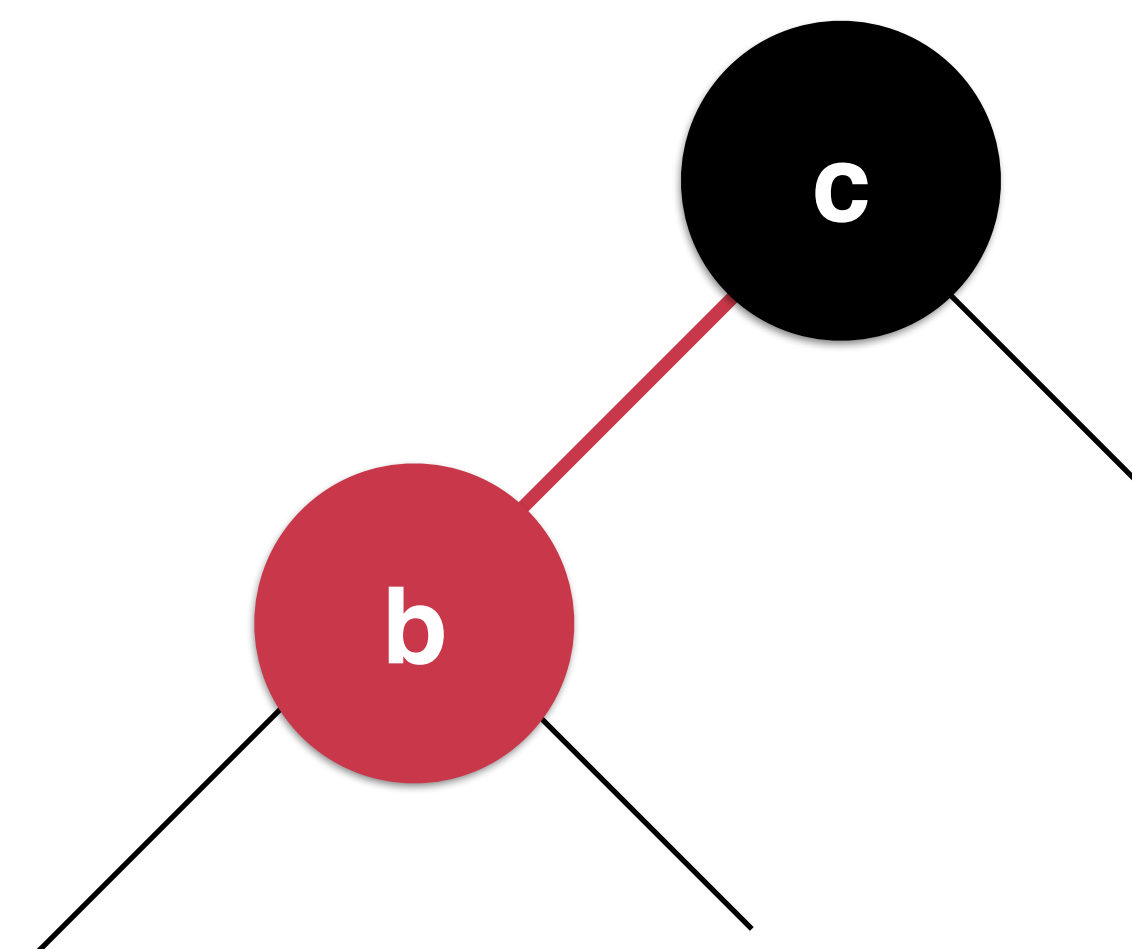
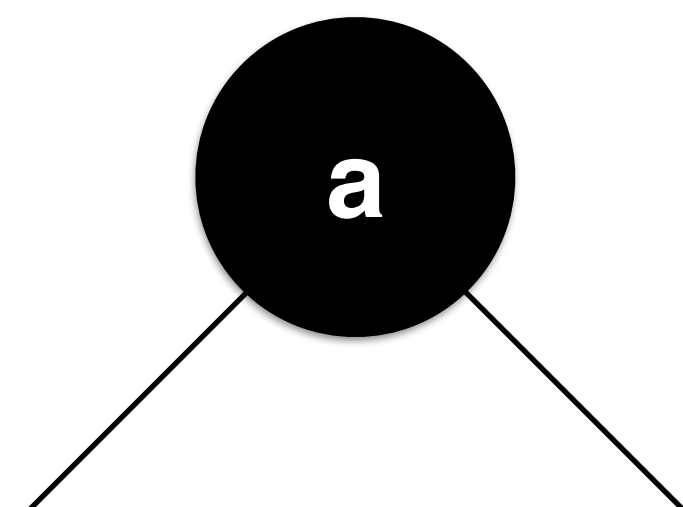
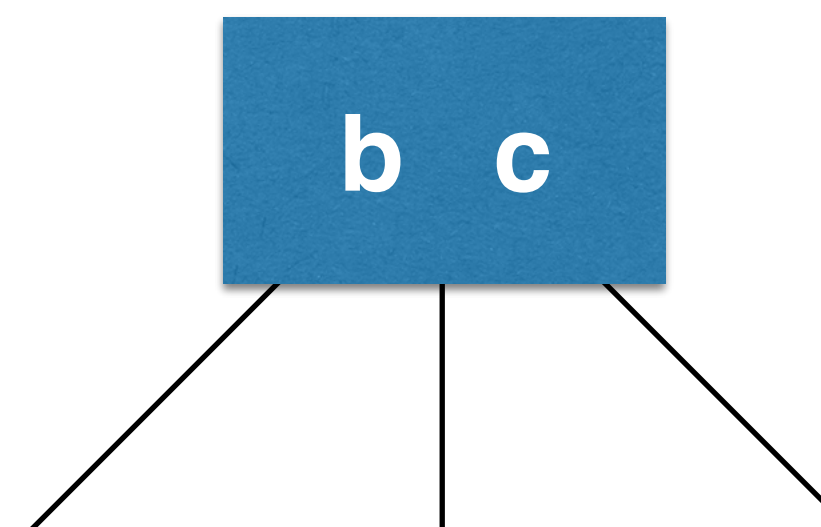
2. 根节点是黑色的

# 红黑树和2-3树

2节点

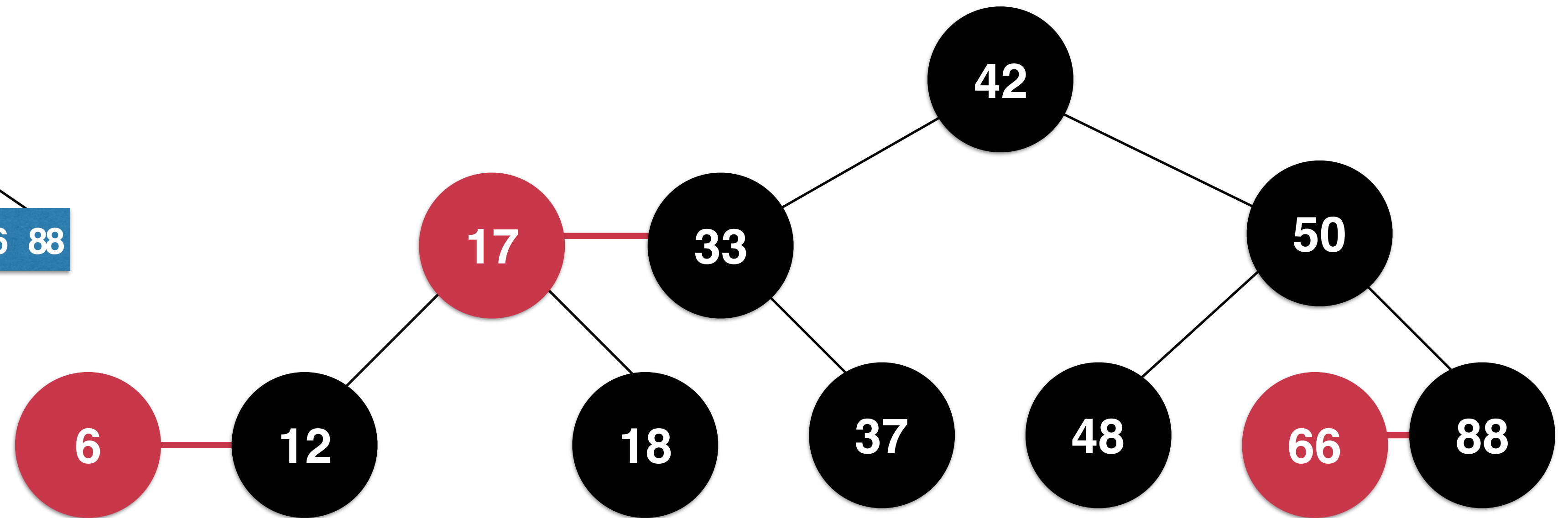
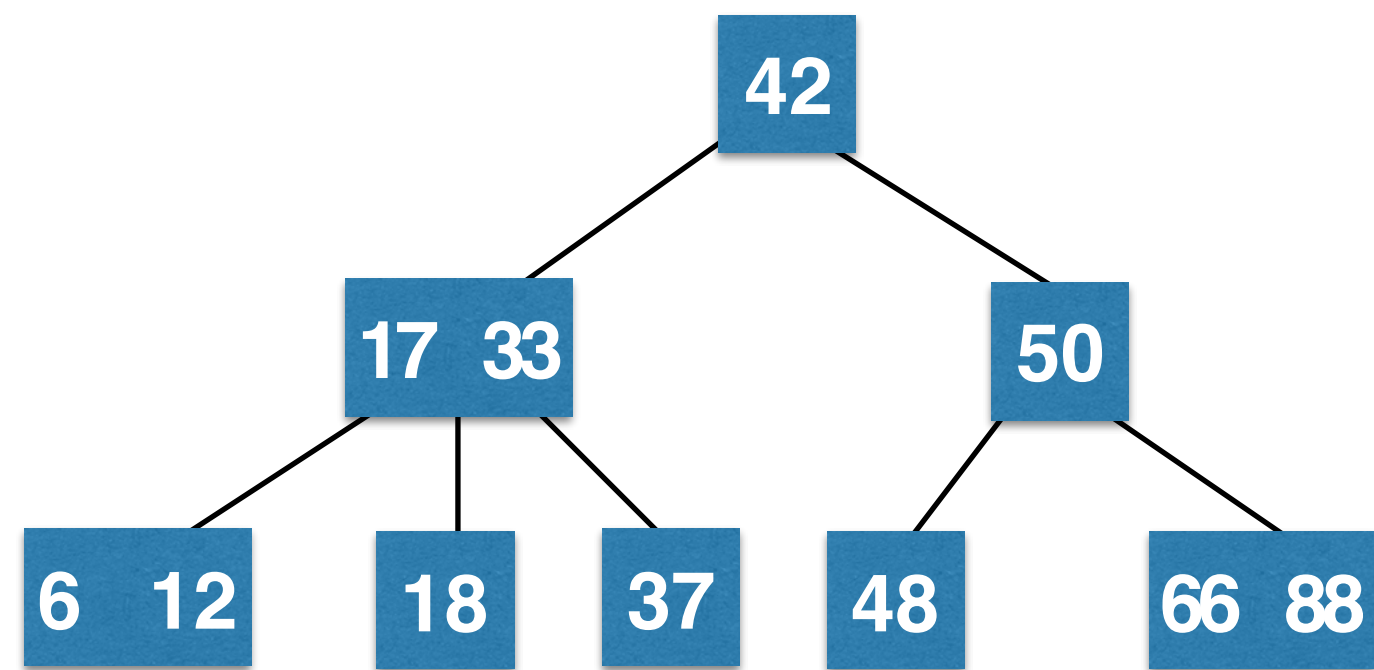


3节点



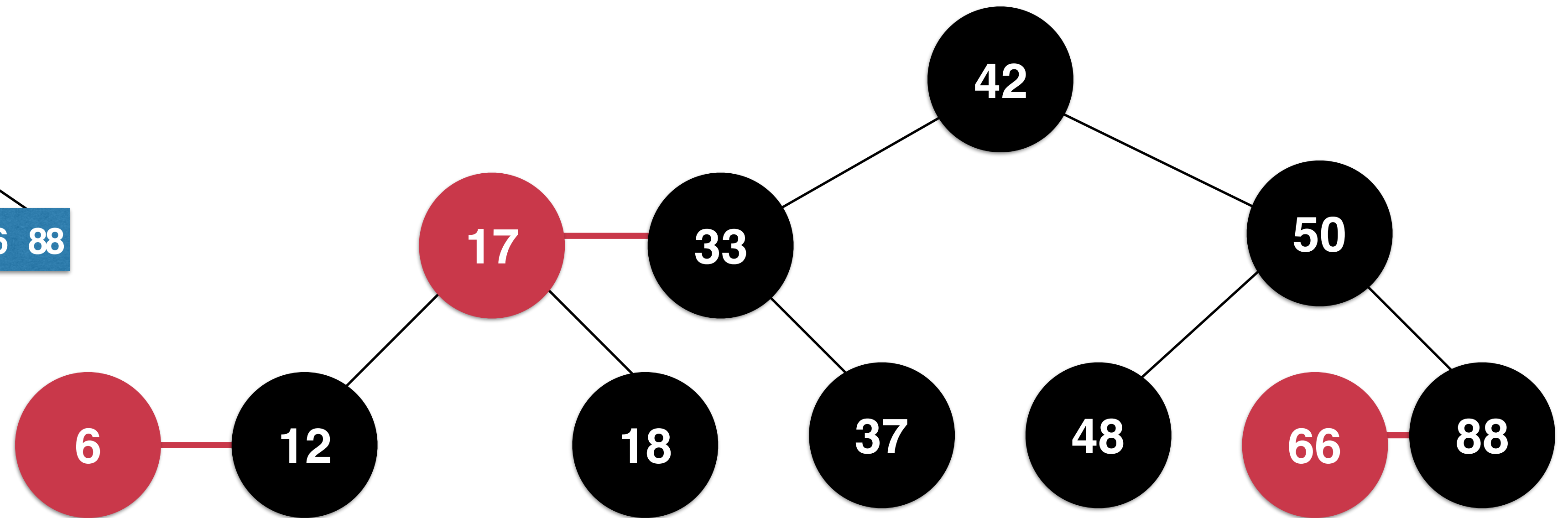
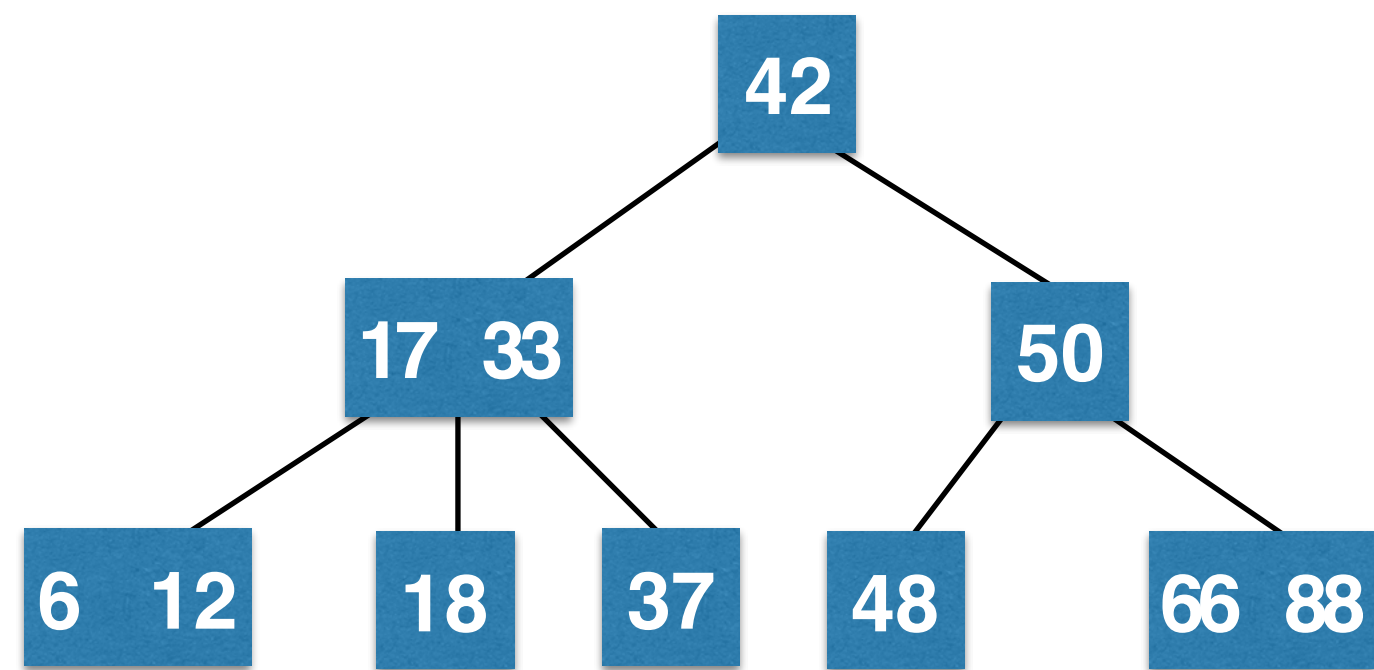
2. 根节点是黑色的

# 红黑树和2-3树



3. 每一个叶子节点（最后的空节点）是黑色的

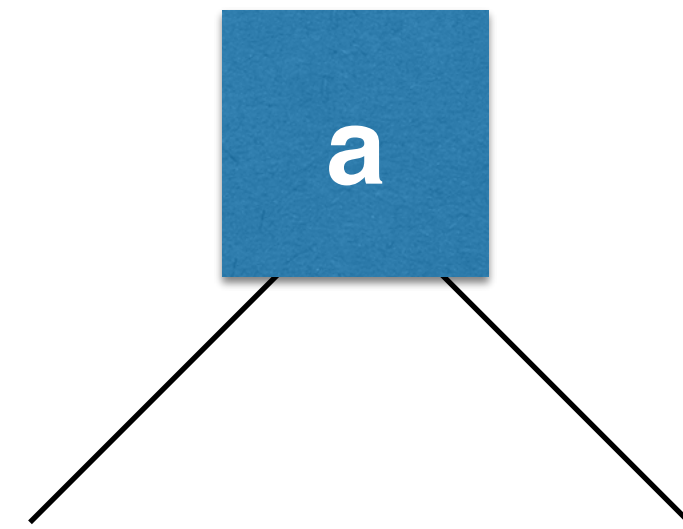
# 红黑树和2-3树



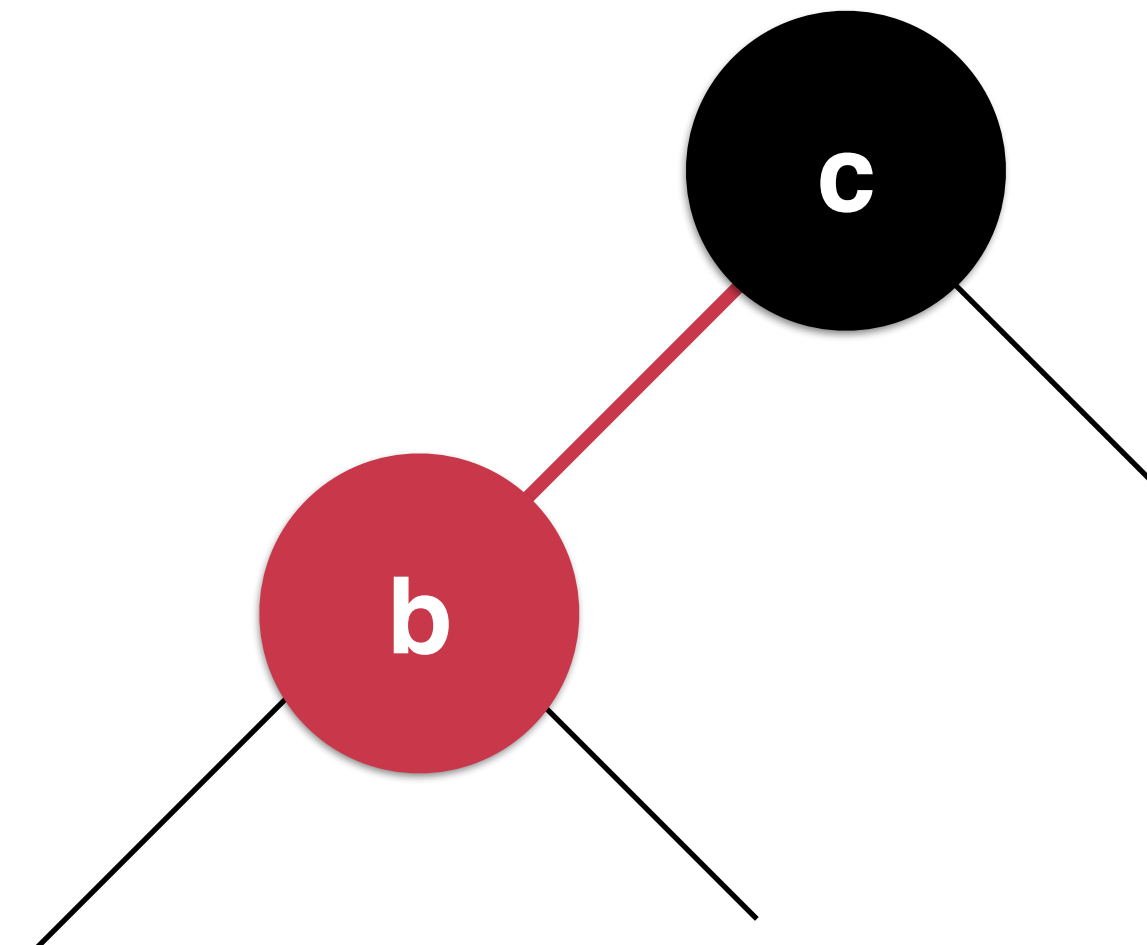
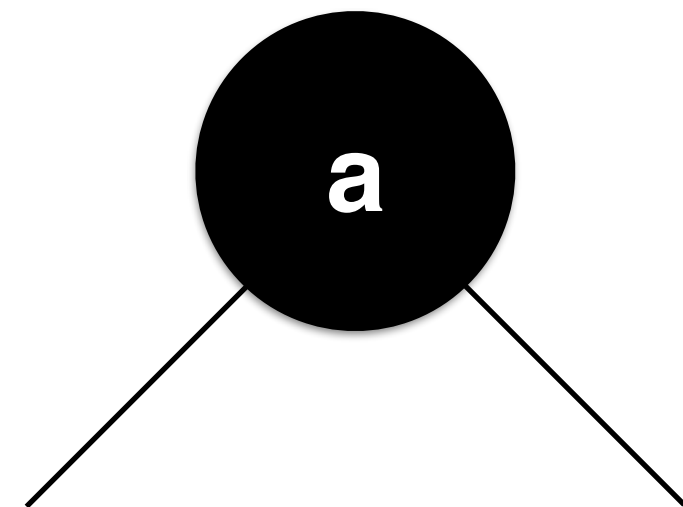
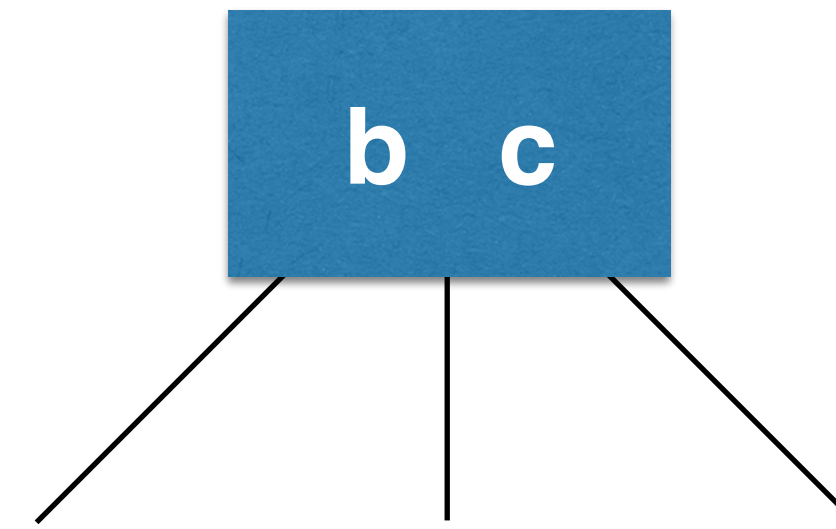
4. 如果一个节点是红色的，那么他的孩子节点都是黑色的

# 红黑树和2-3树

2节点

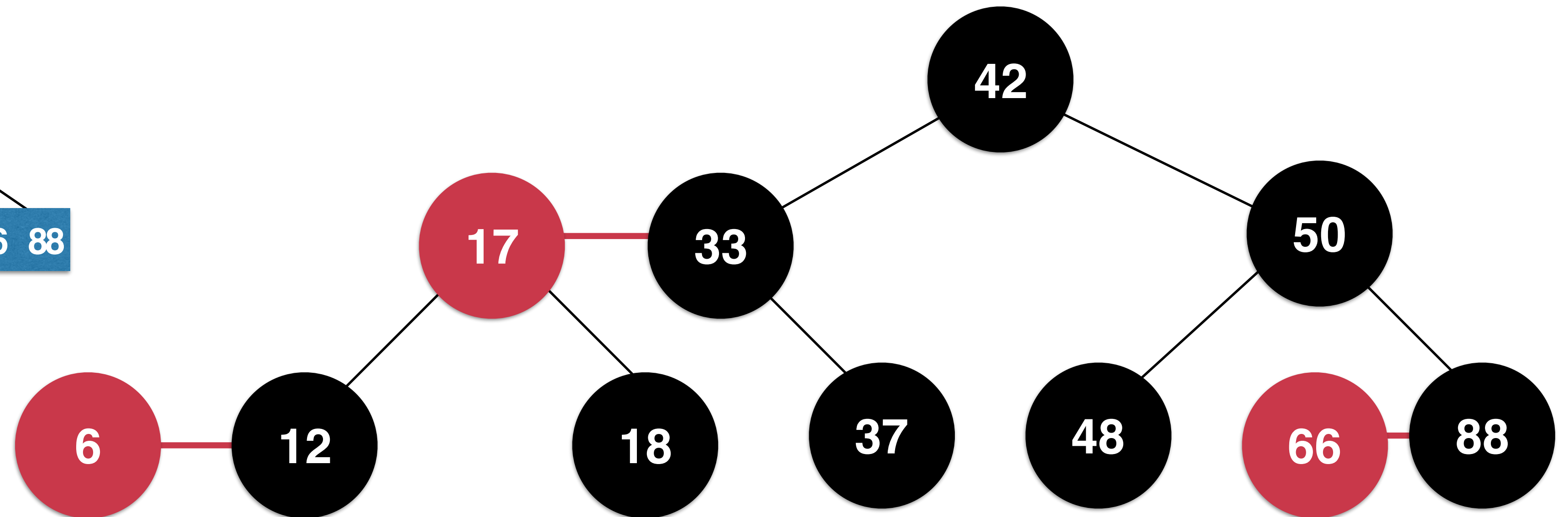
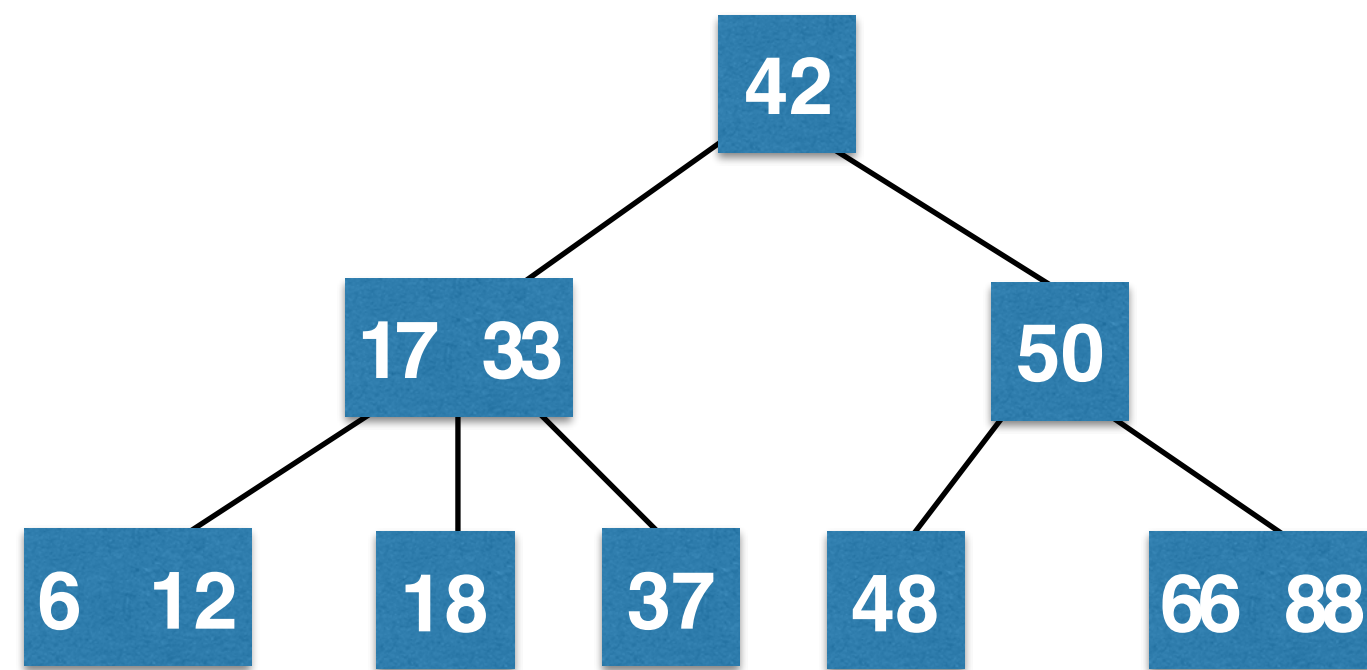


3节点



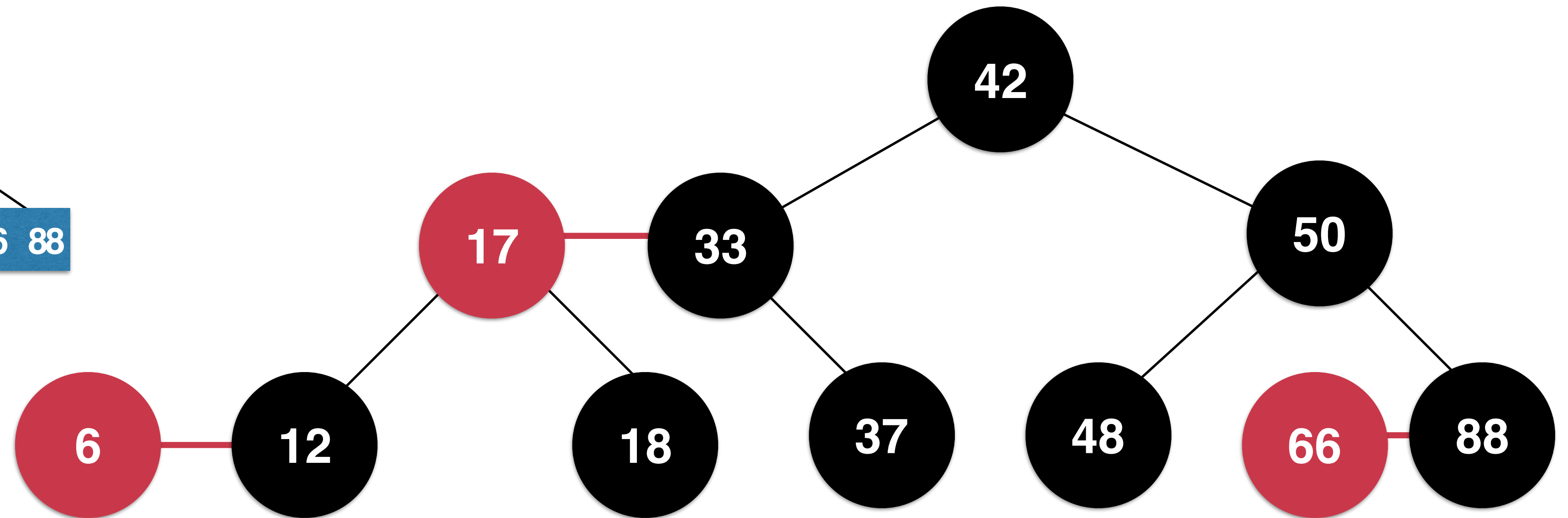
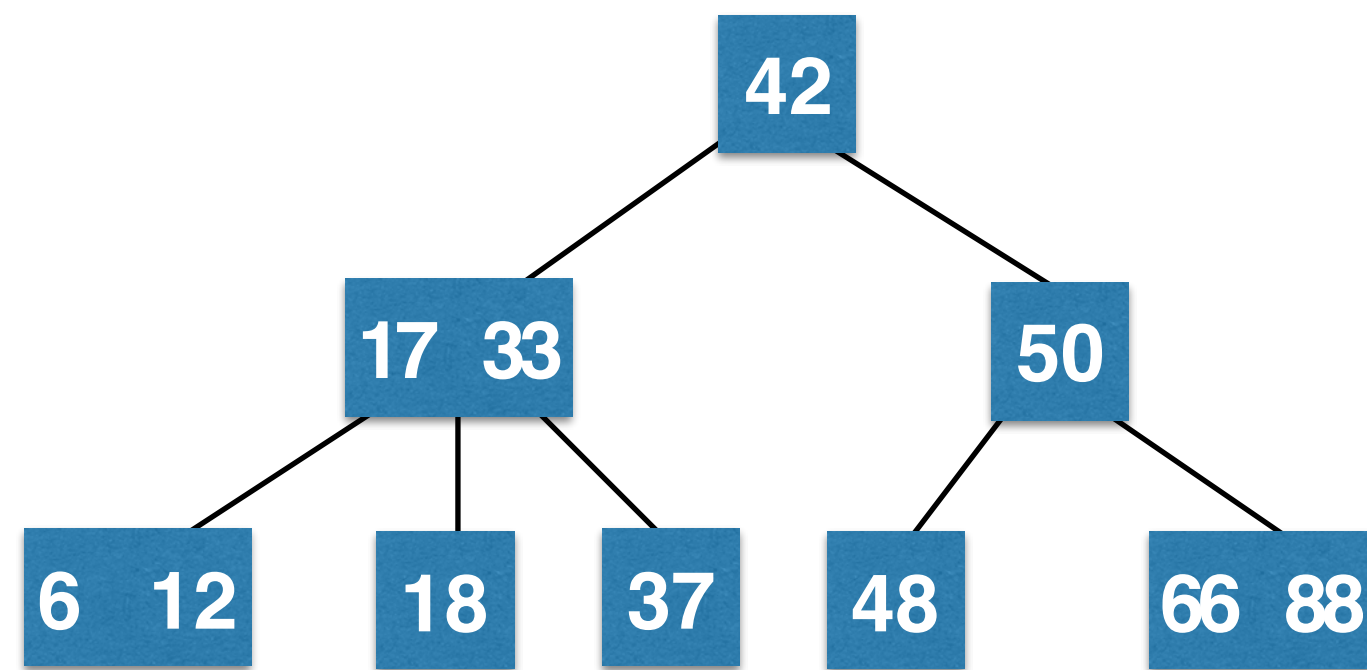
4. 如果一个节点是红色的，那么他的孩子节点都是黑色的

# 红黑树和2-3树



5. 从任意一个节点到叶子节点，经过的黑色节点是一样的

# 红黑树和2-3树



红黑树是保持“黑平衡”的二叉树

严格意义上，不是平衡二叉树

最大高度：  $2\log n$

$O(\log n)$



# 《算法导论》 中的红黑树

A red-black tree is a binary tree that satisfies the following *red-black properties*:

1. Every node is either red or black.
2. The root is black.
3. Every leaf (NIL) is black.
4. If a node is red, then both its children are black.
5. For each node, all simple paths from the node to descendant leaves contain the same number of black nodes.



# 红黑树添加新元素

保持根节点黑色和左旋转

# 红黑树添加新元素

2-3树中添加一个新元素

或者添加进2-节点，形成一个3-节点

或者添加进3-节点，暂时形成一个4-节点

永远添加红色节点

# 红黑树添加新元素

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# 红黑树添加新元素

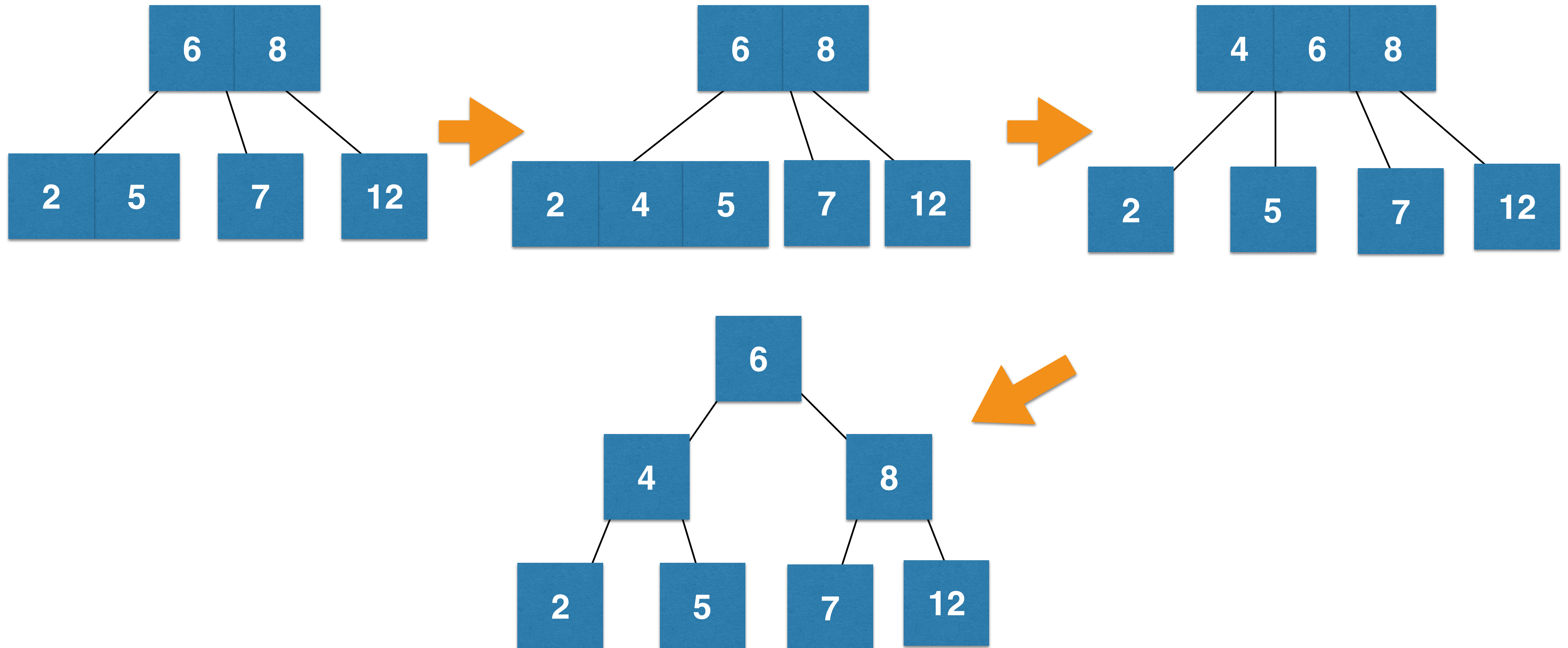


42

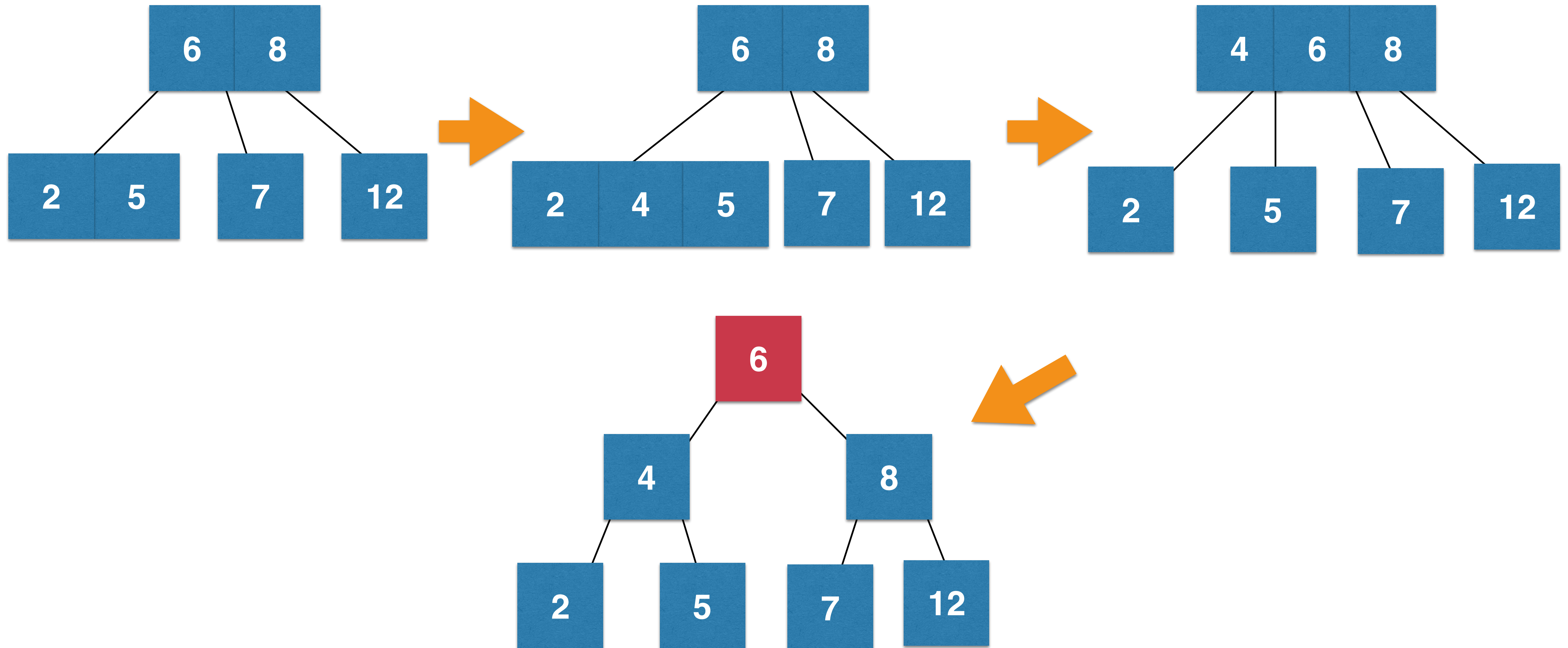
# 红黑树添加新元素

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# 2-3树



# 2-3树





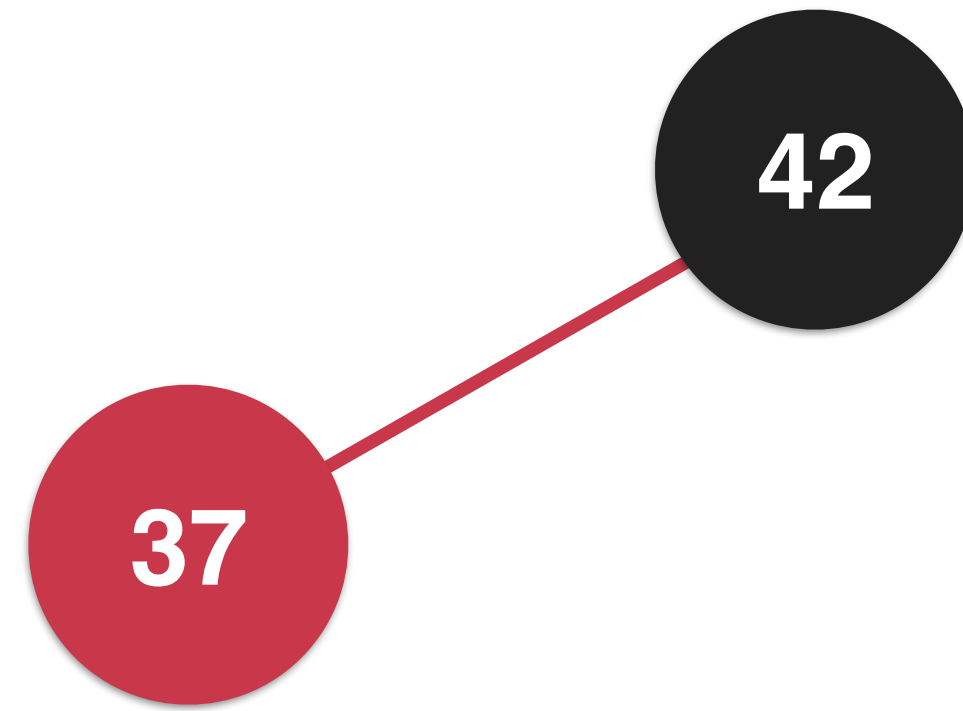
实践： 保持根节点为黑色节点

# 红黑树添加新元素

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# 红黑树添加新元素



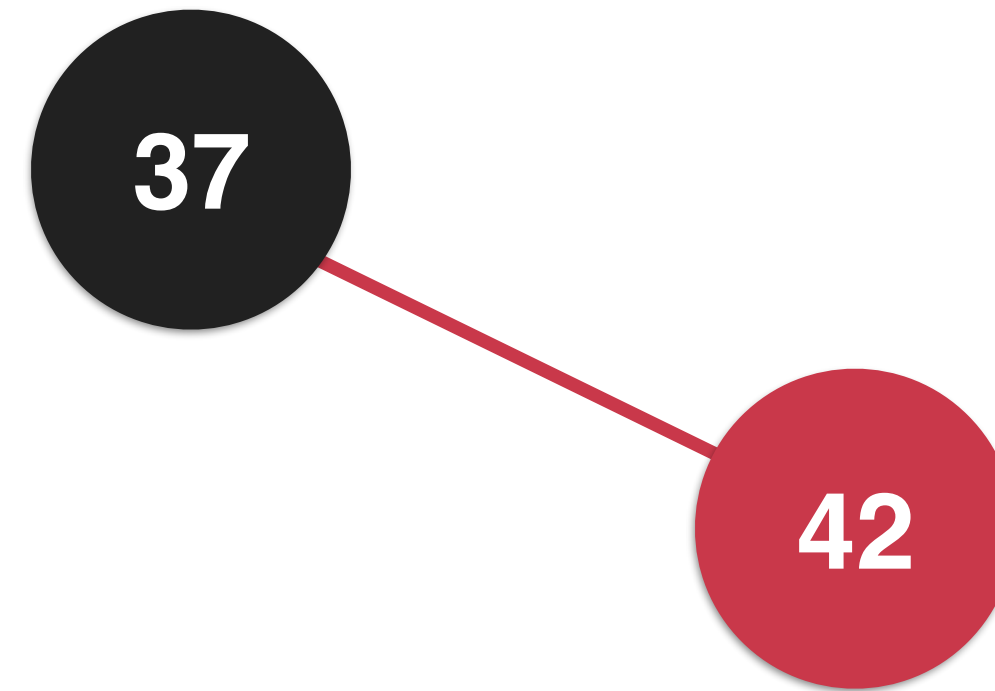
# 红黑树添加新元素

42

37

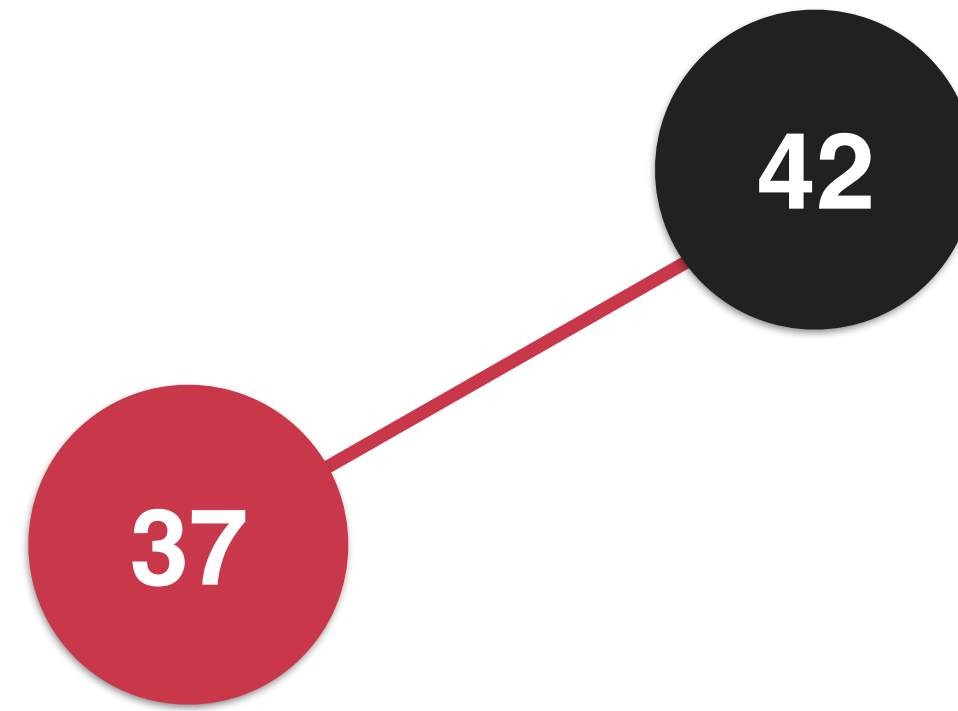
# 红黑树添加新元素

此时需要进行左旋转



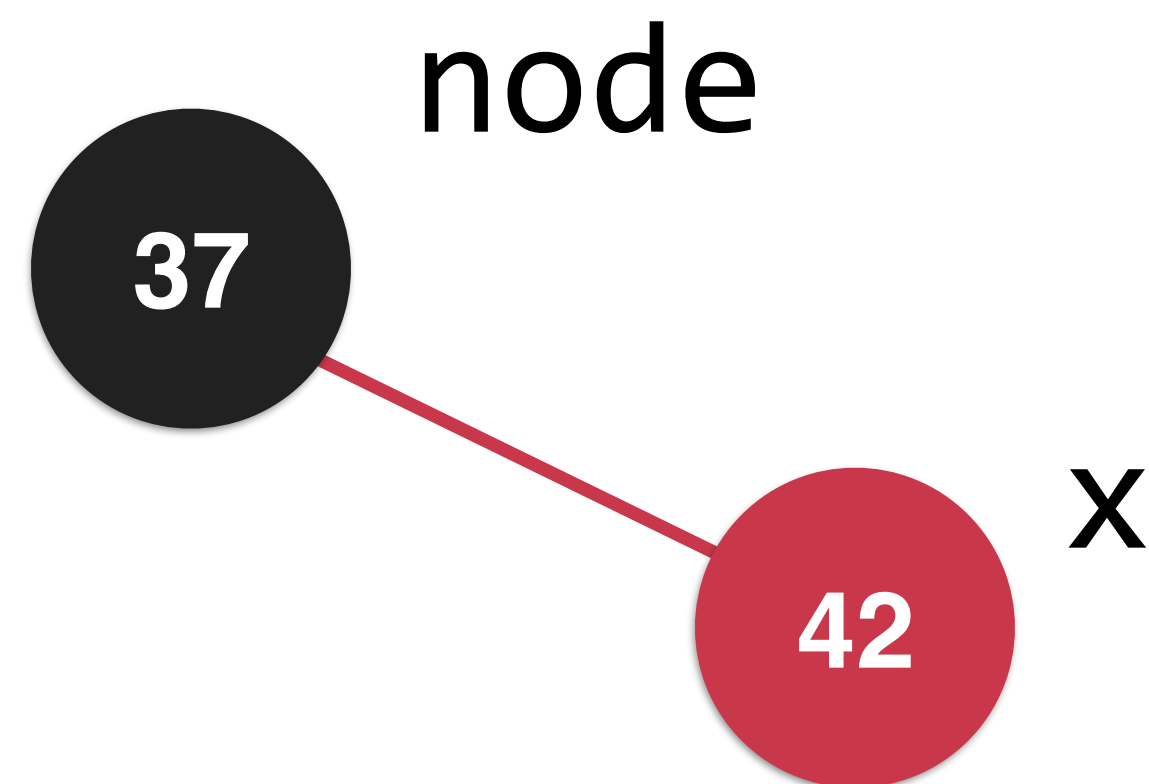
# 红黑树添加新元素

此时需要进行左旋转

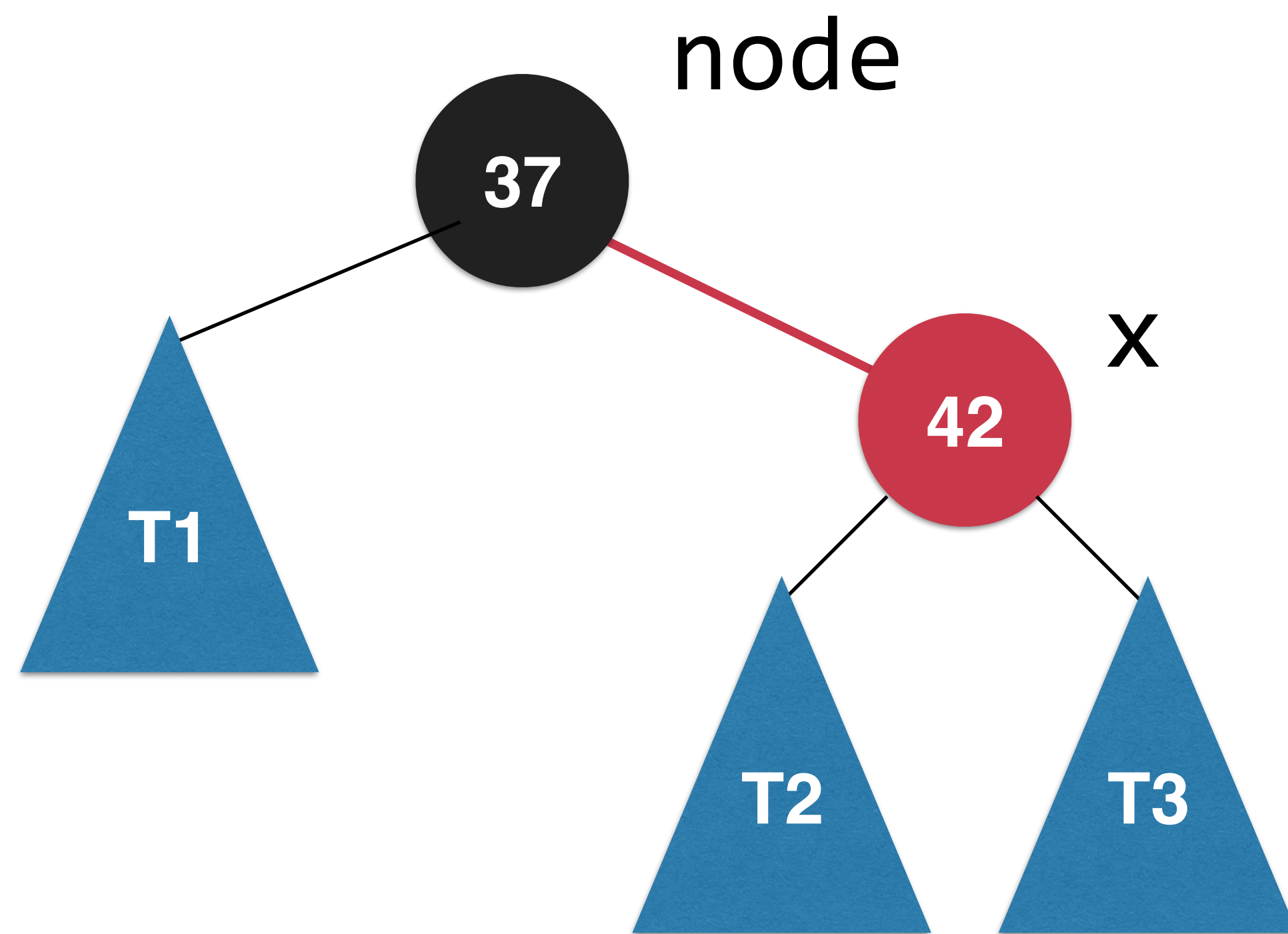


# 左旋转

此时需要进行左旋转



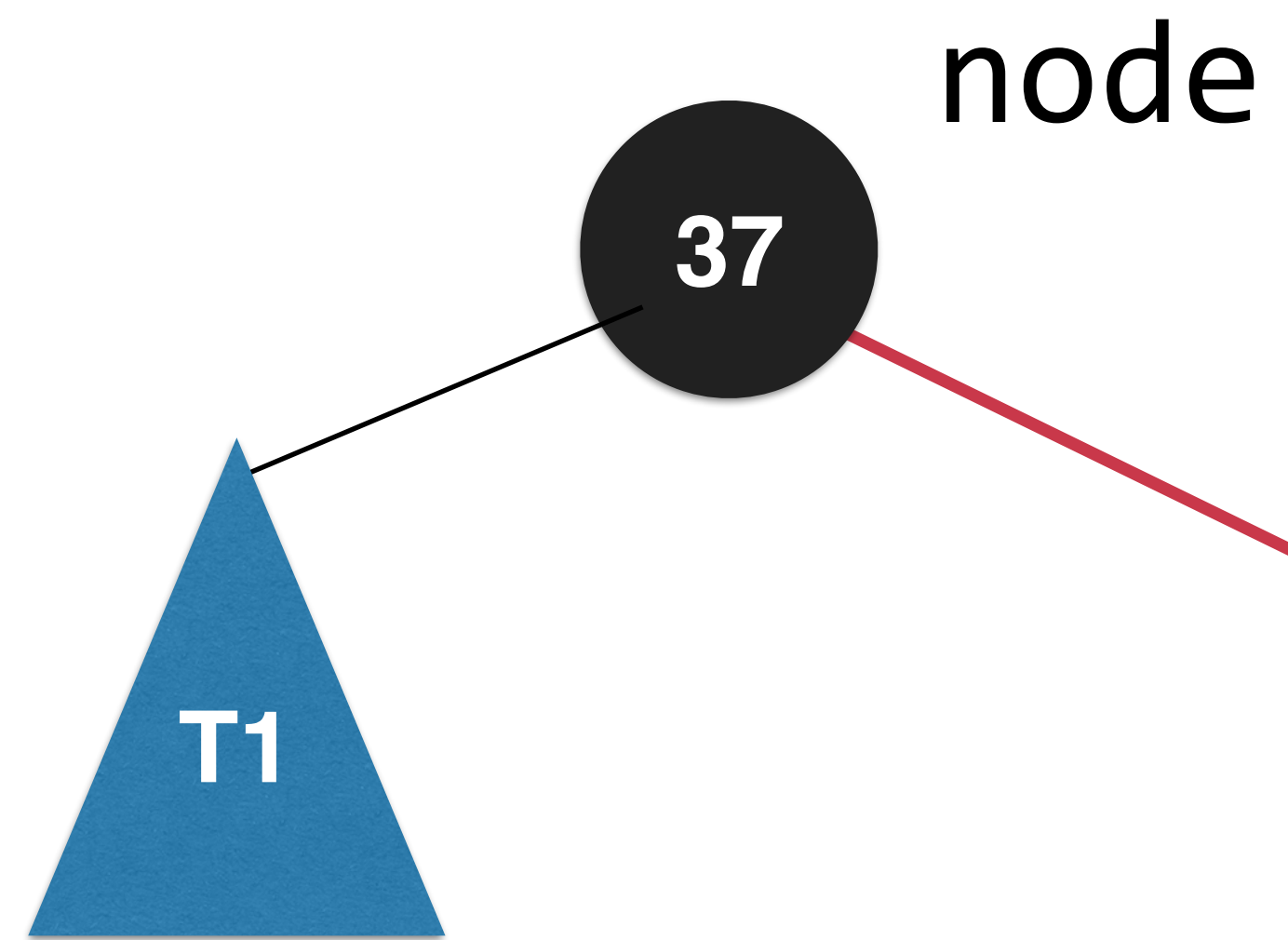
# 左旋转



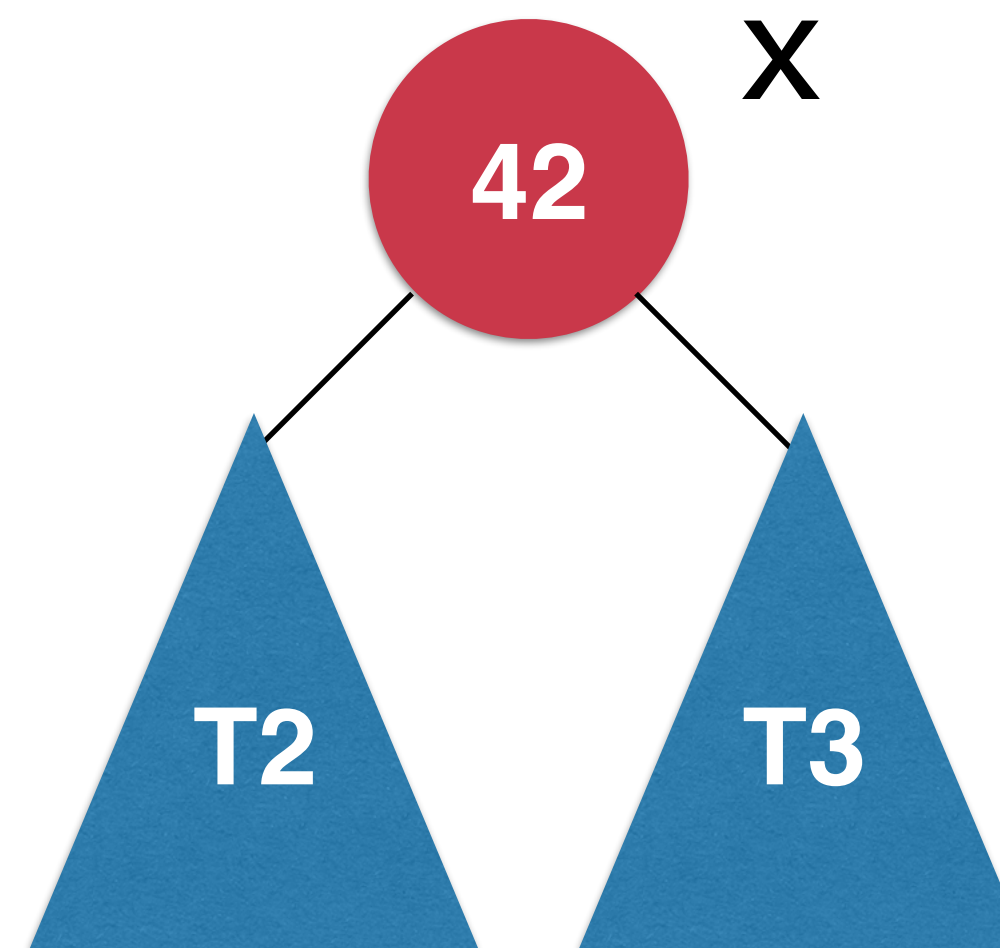
`node.right = x.left`



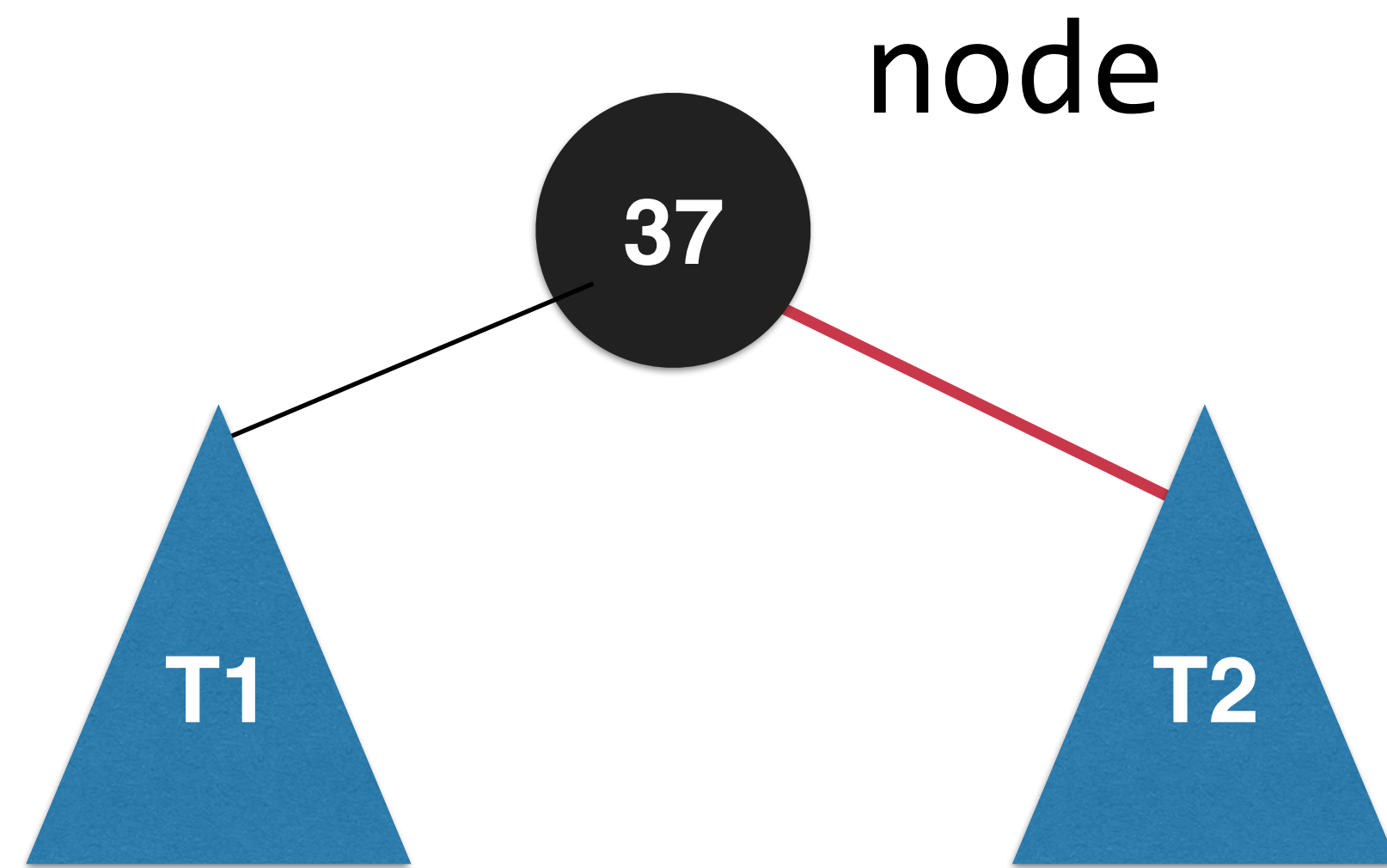
# 左旋转



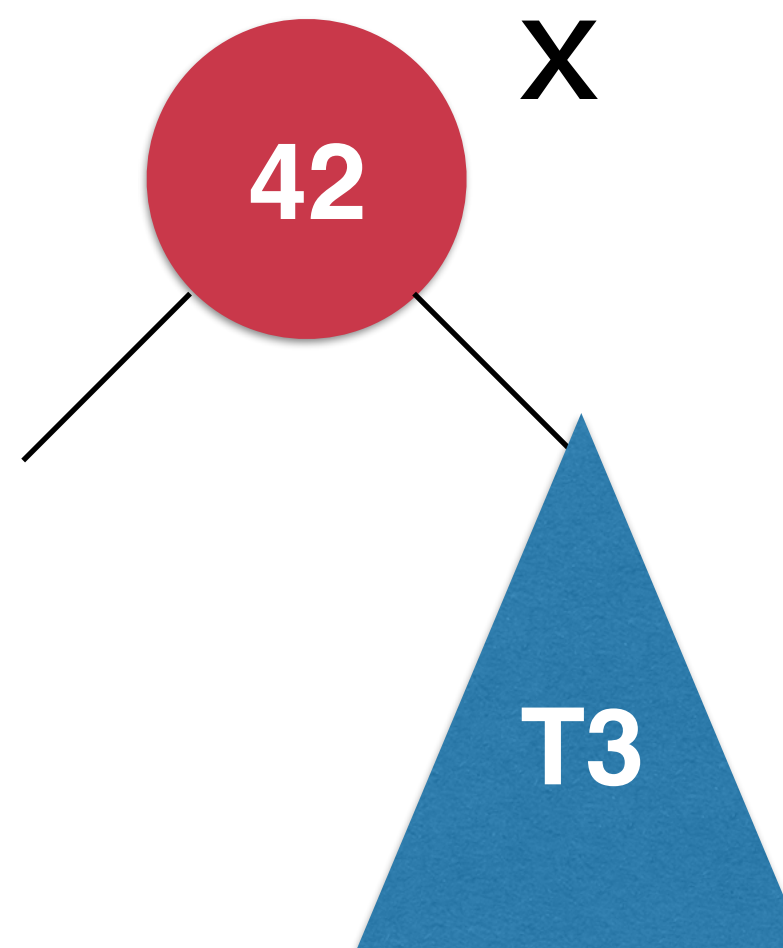
`node.right = x.left`



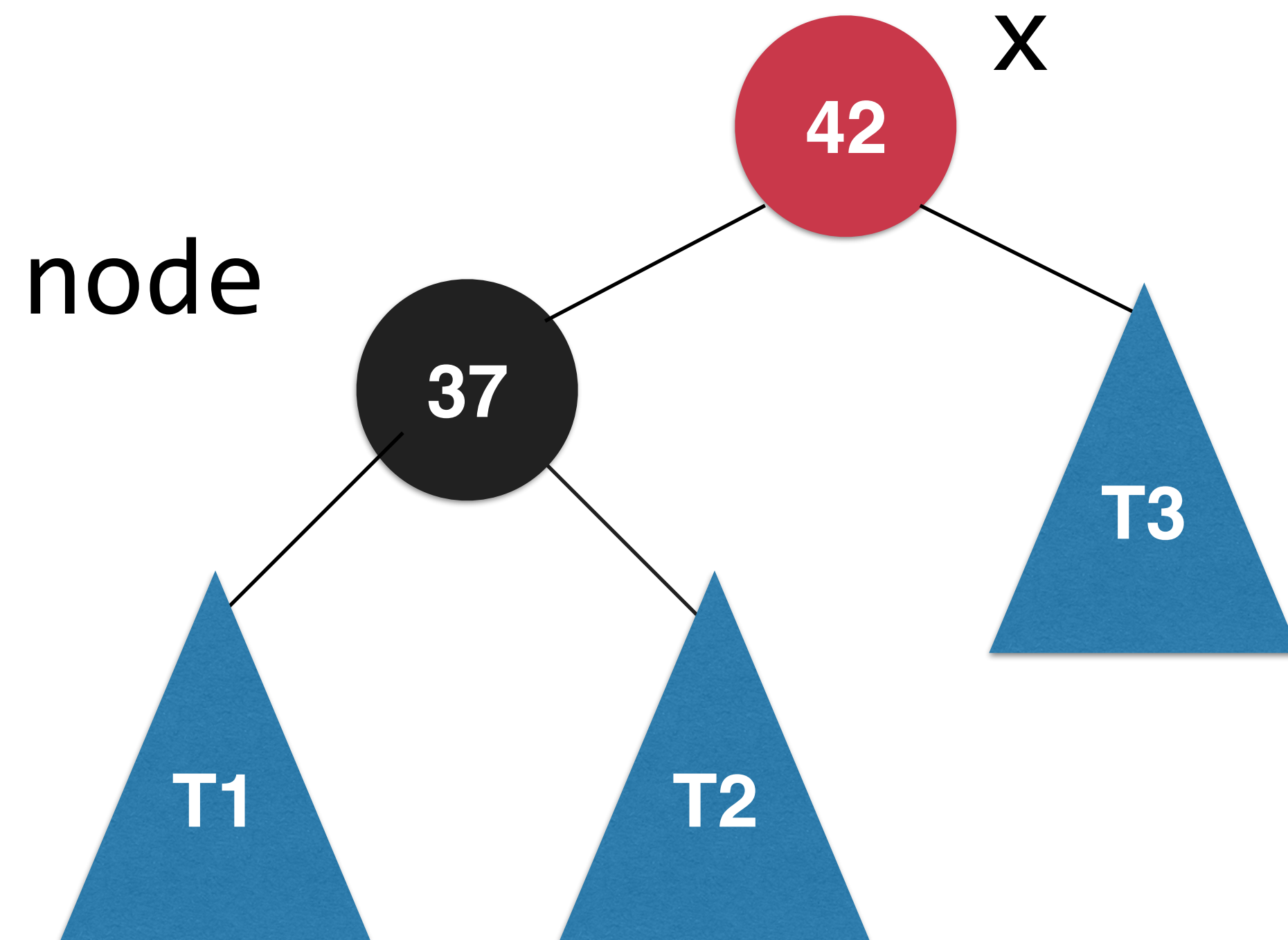
# 左旋转



`node.right = x.left`  
`x.left = node`

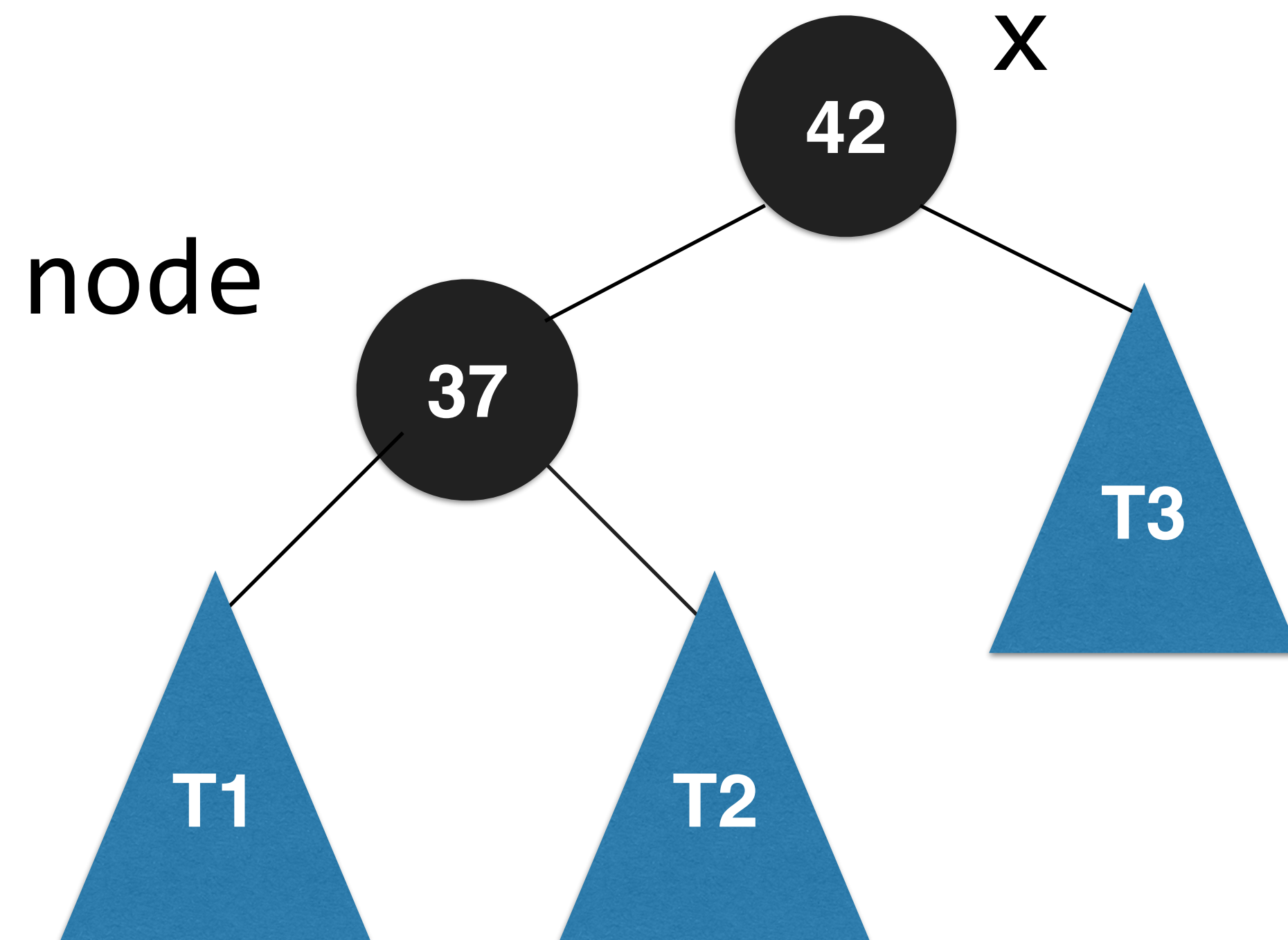


# 左旋转



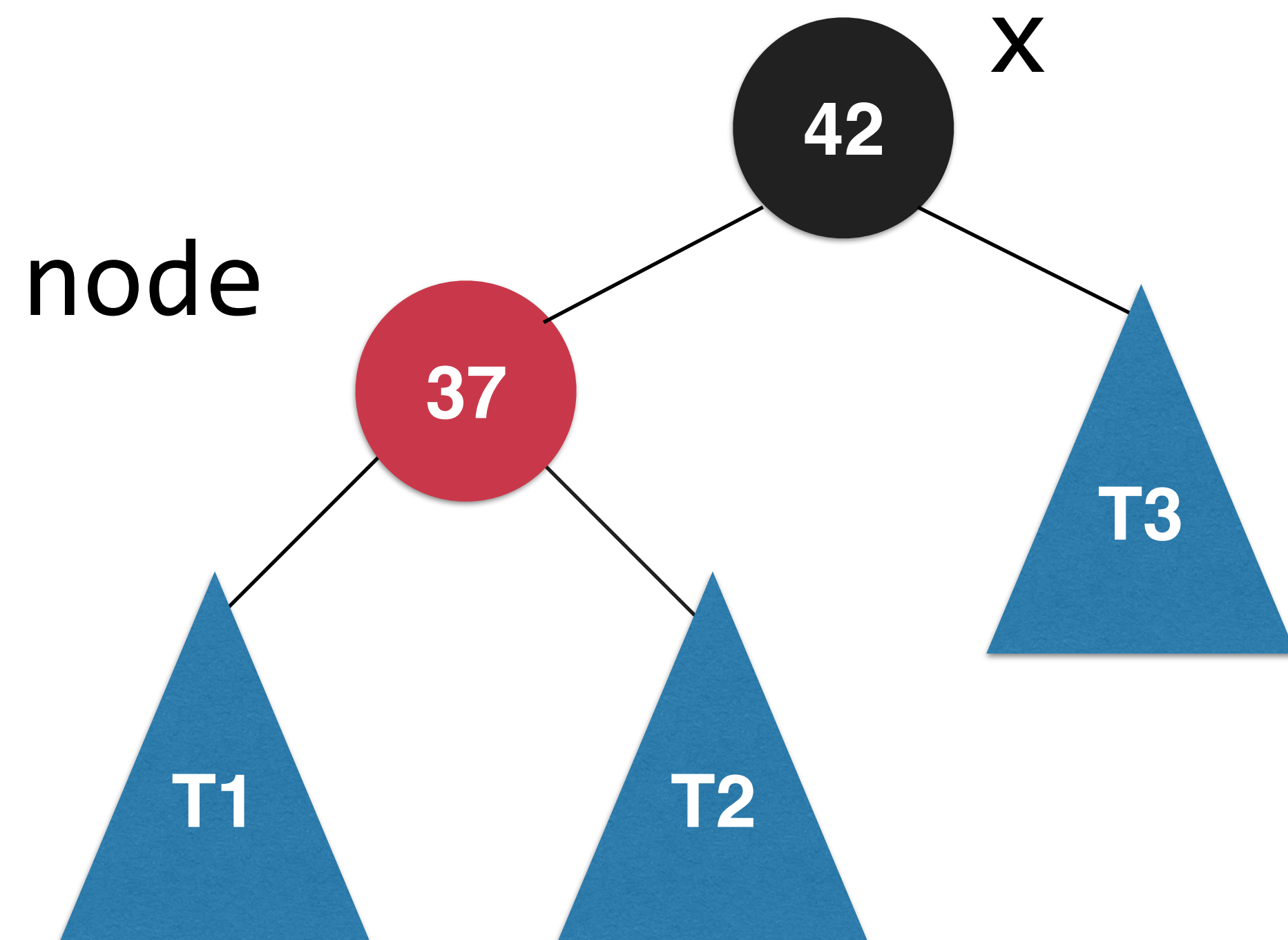
```
node.right = x.left  
x.left = node  
x.color = node.color
```

# 左旋转



```
node.right = x.left  
x.left = node  
x.color = node.color
```

# 左旋转



```
node.right = x.left  
x.left = node  
x.color = node.color  
node.color = RED
```

实践：左旋转

# 颜色翻转和右旋转

# 红黑树添加新元素

42



# 红黑树添加新元素



42

# 红黑树添加新元素

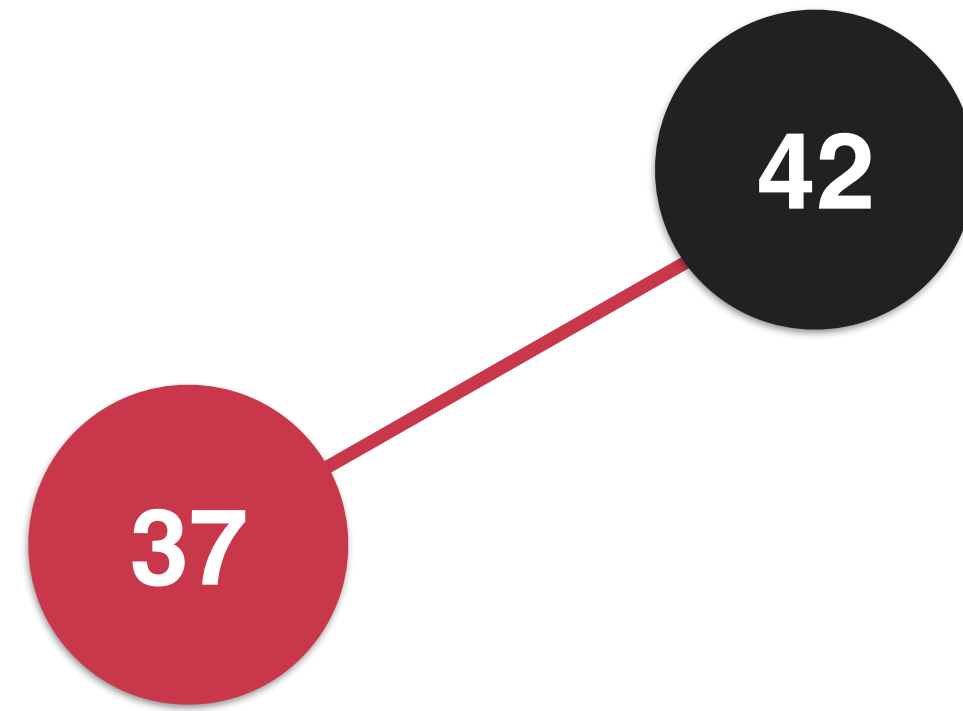
42

# 红黑树添加新元素

37

42

# 红黑树添加新元素



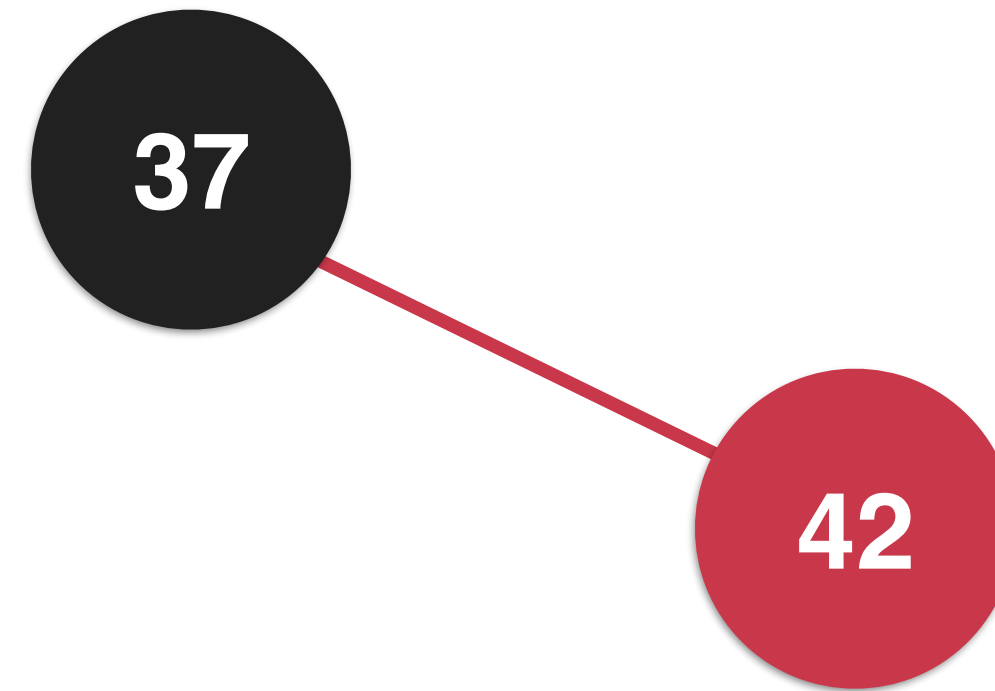
# 红黑树添加新元素

42

37

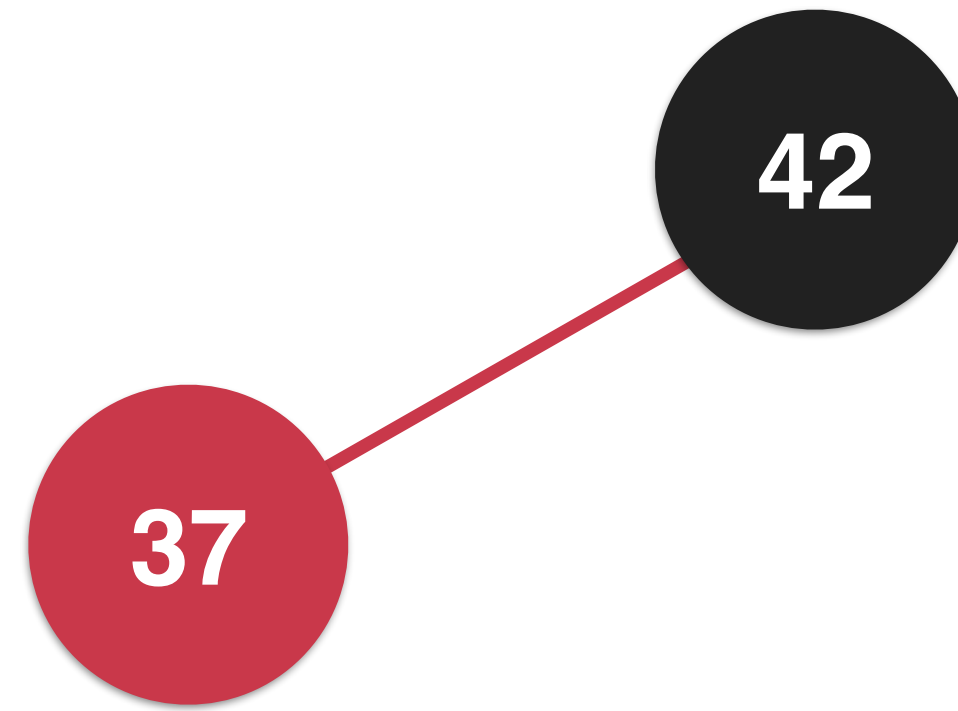
# 红黑树添加新元素

此时需要进行左旋转



# 红黑树添加新元素

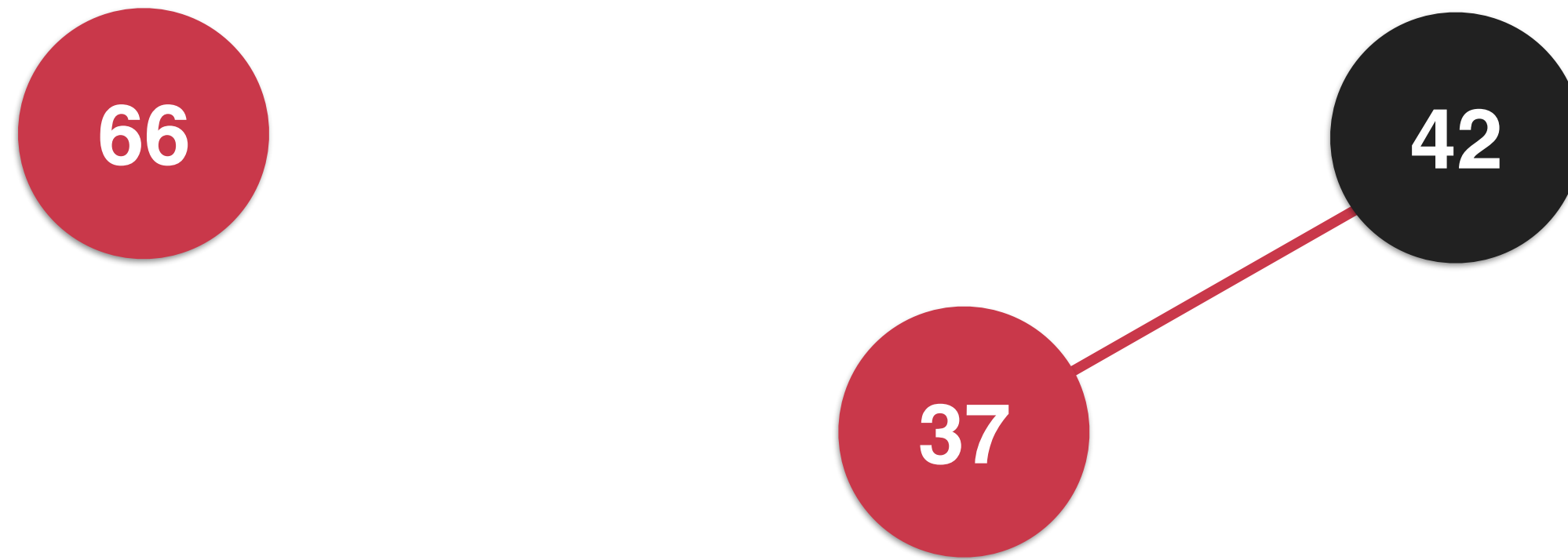
此时需要进行左旋转



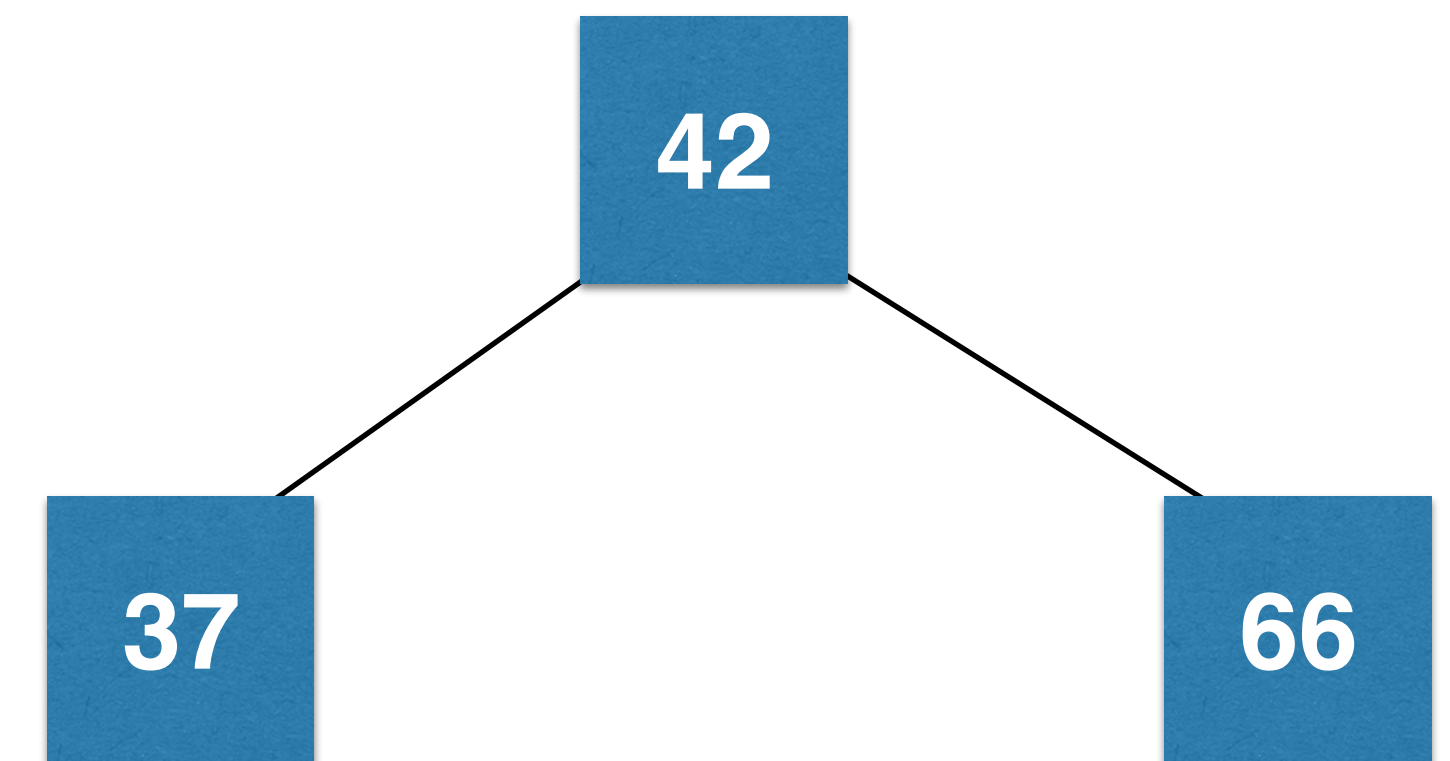
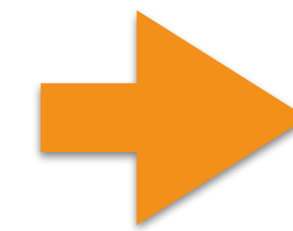
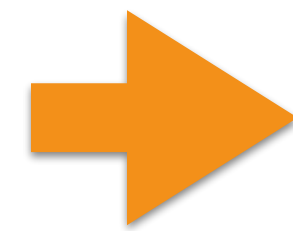
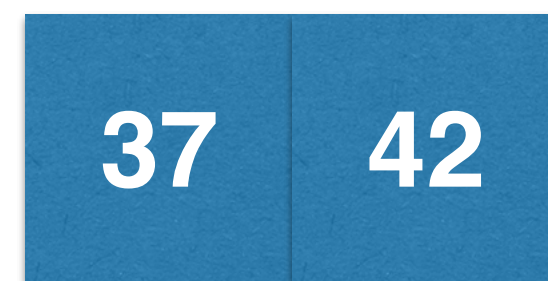
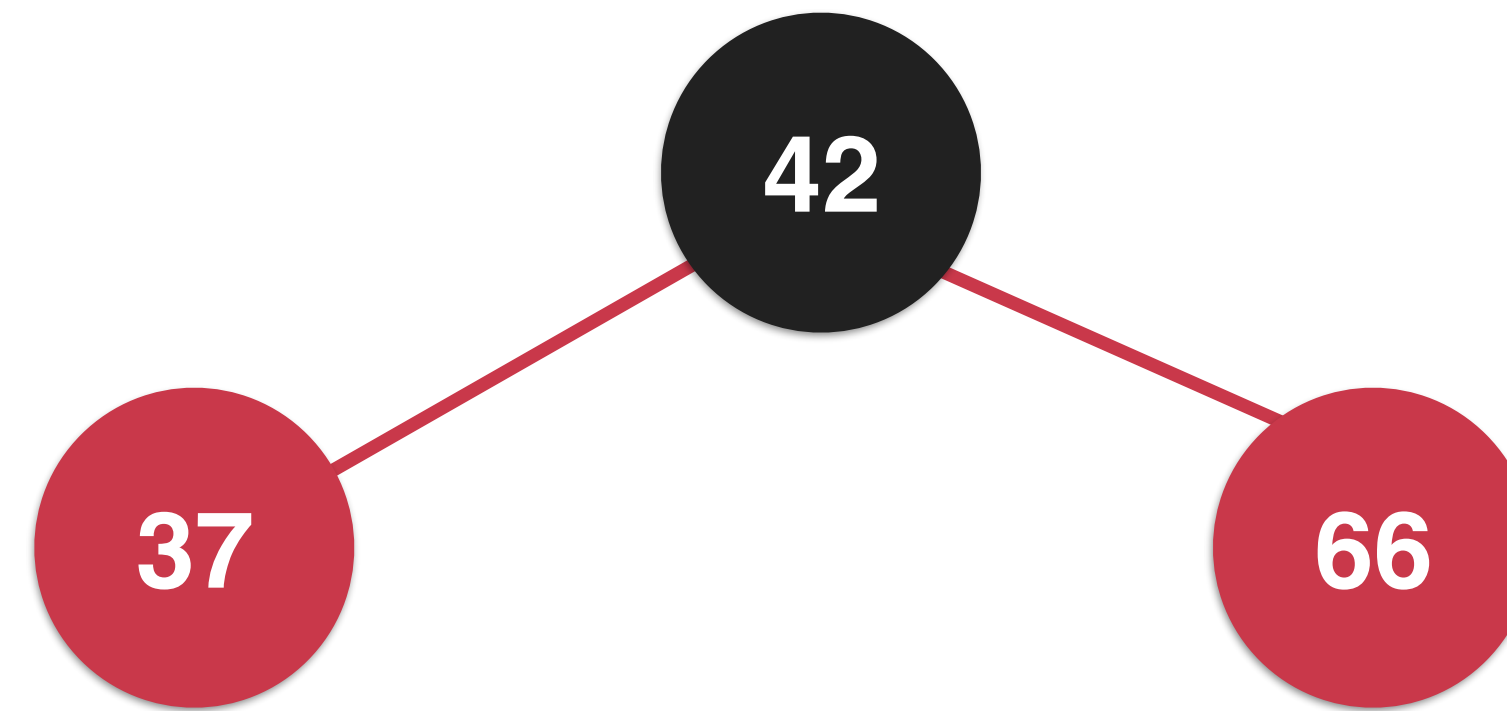
向红黑树中的“3-node”添加元素



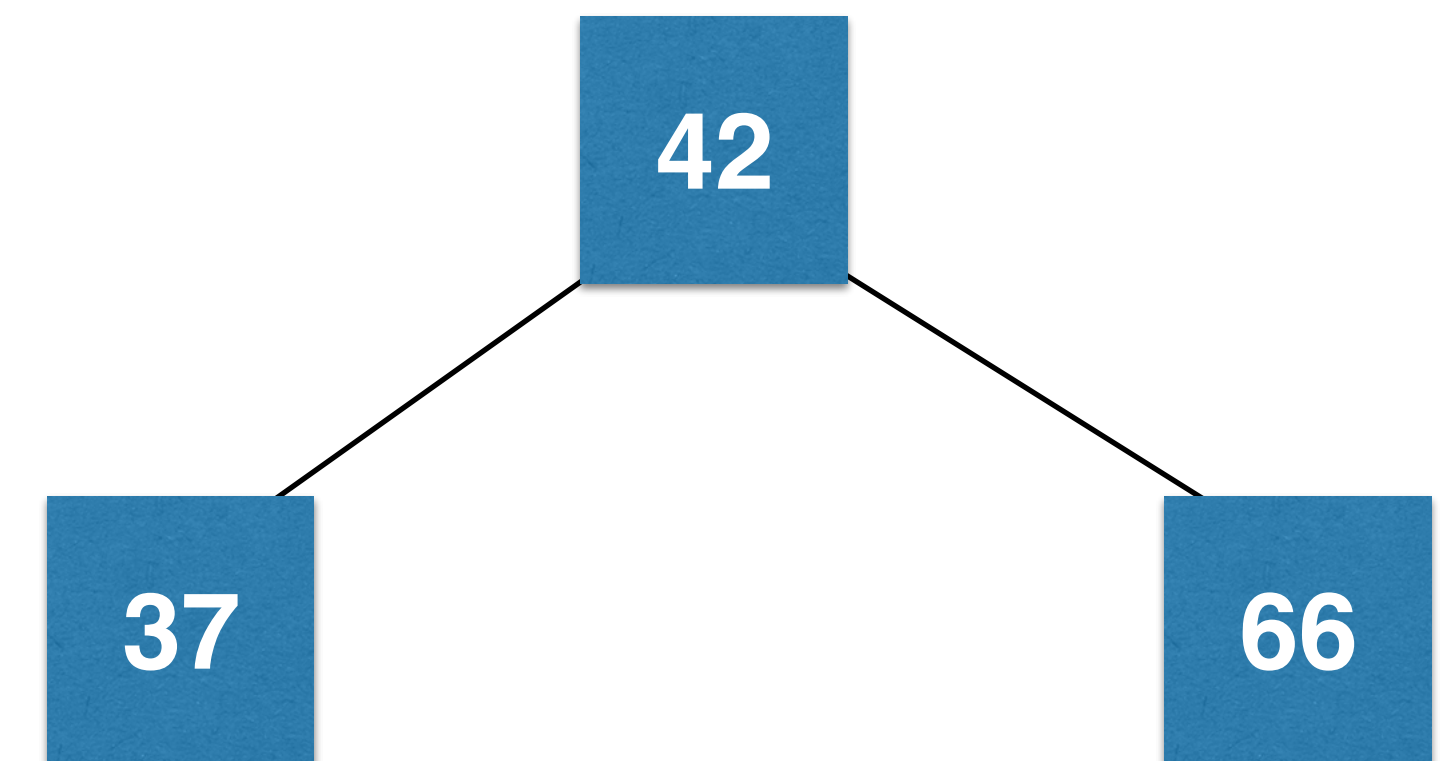
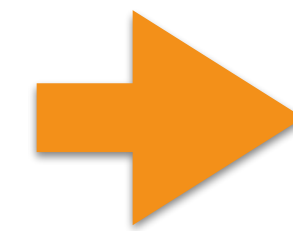
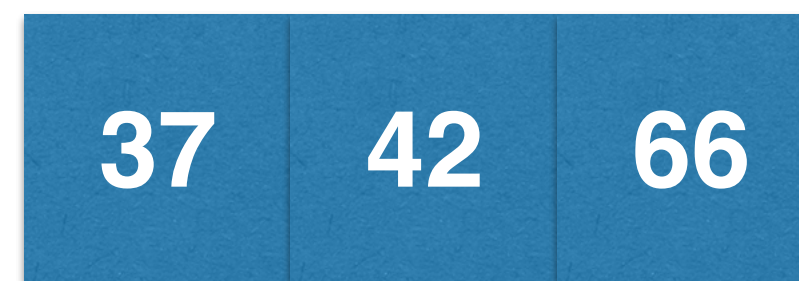
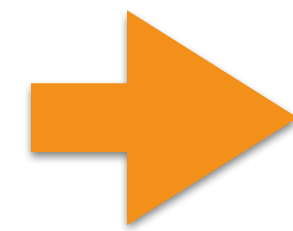
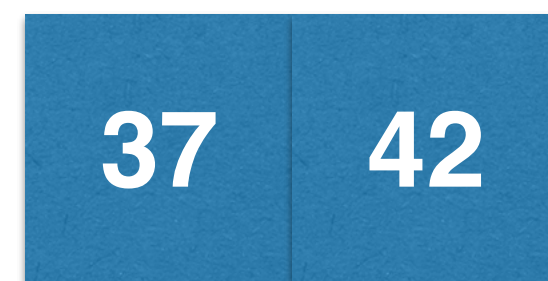
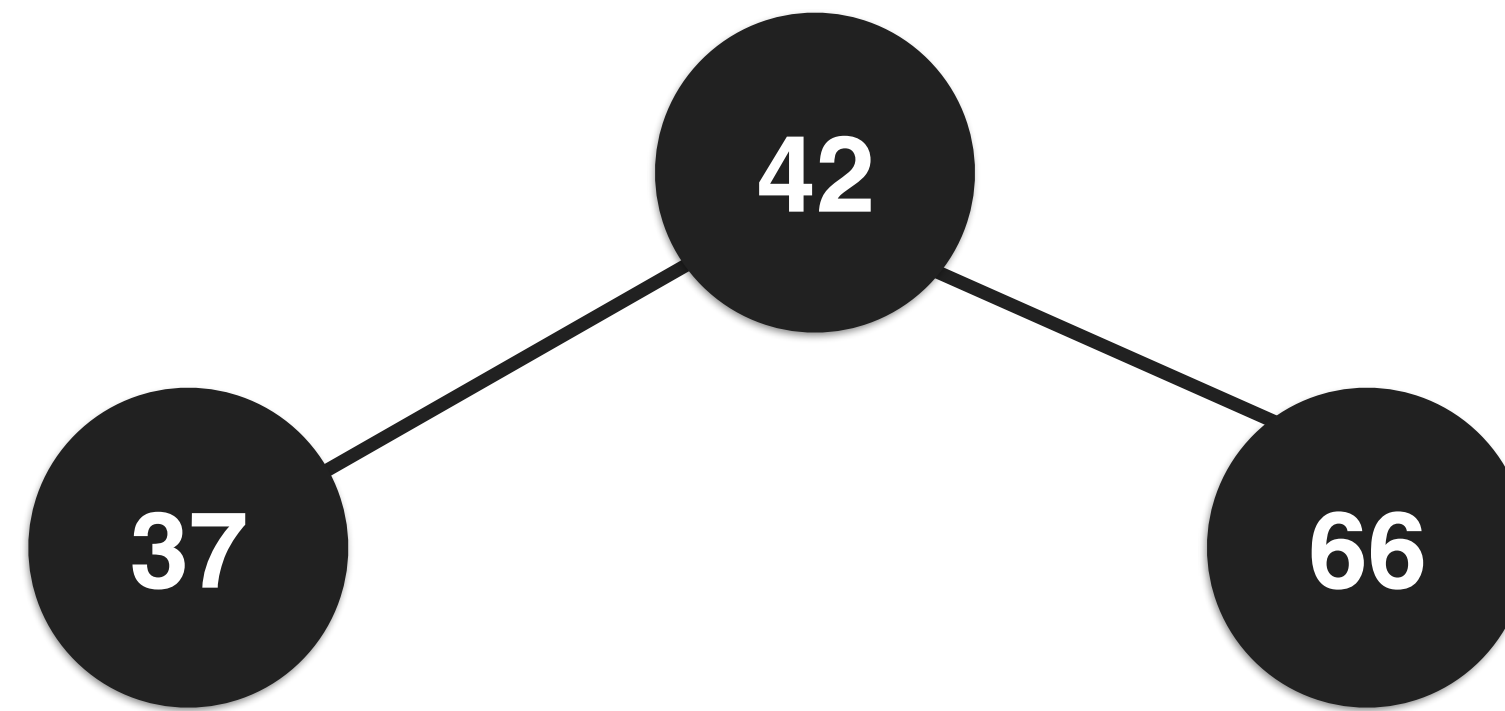
# 红黑树添加新元素



# 红黑树添加新元素



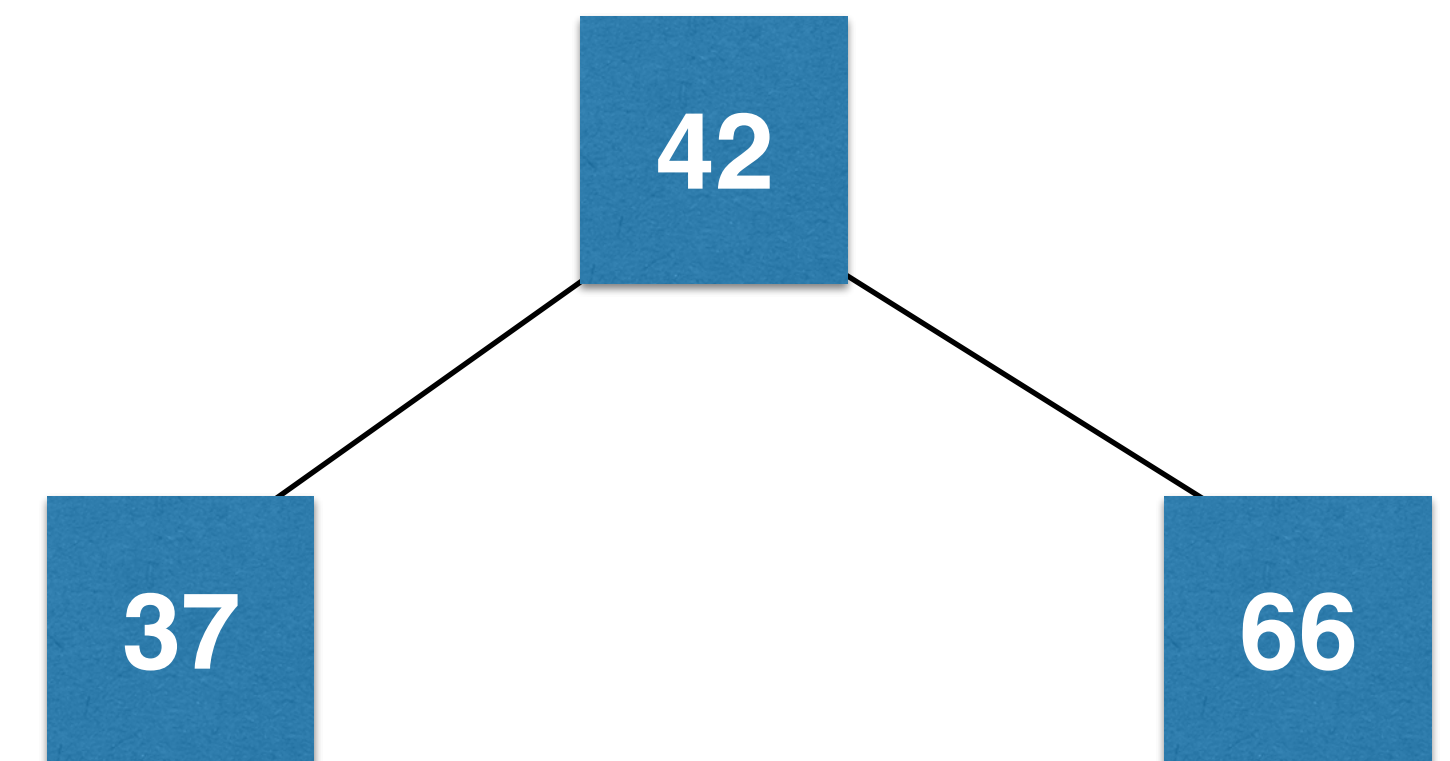
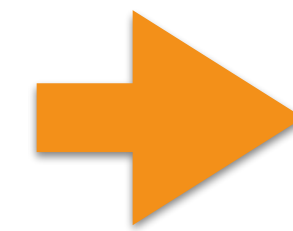
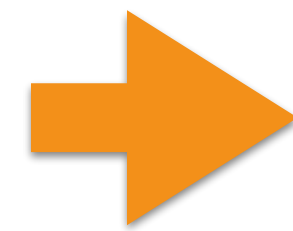
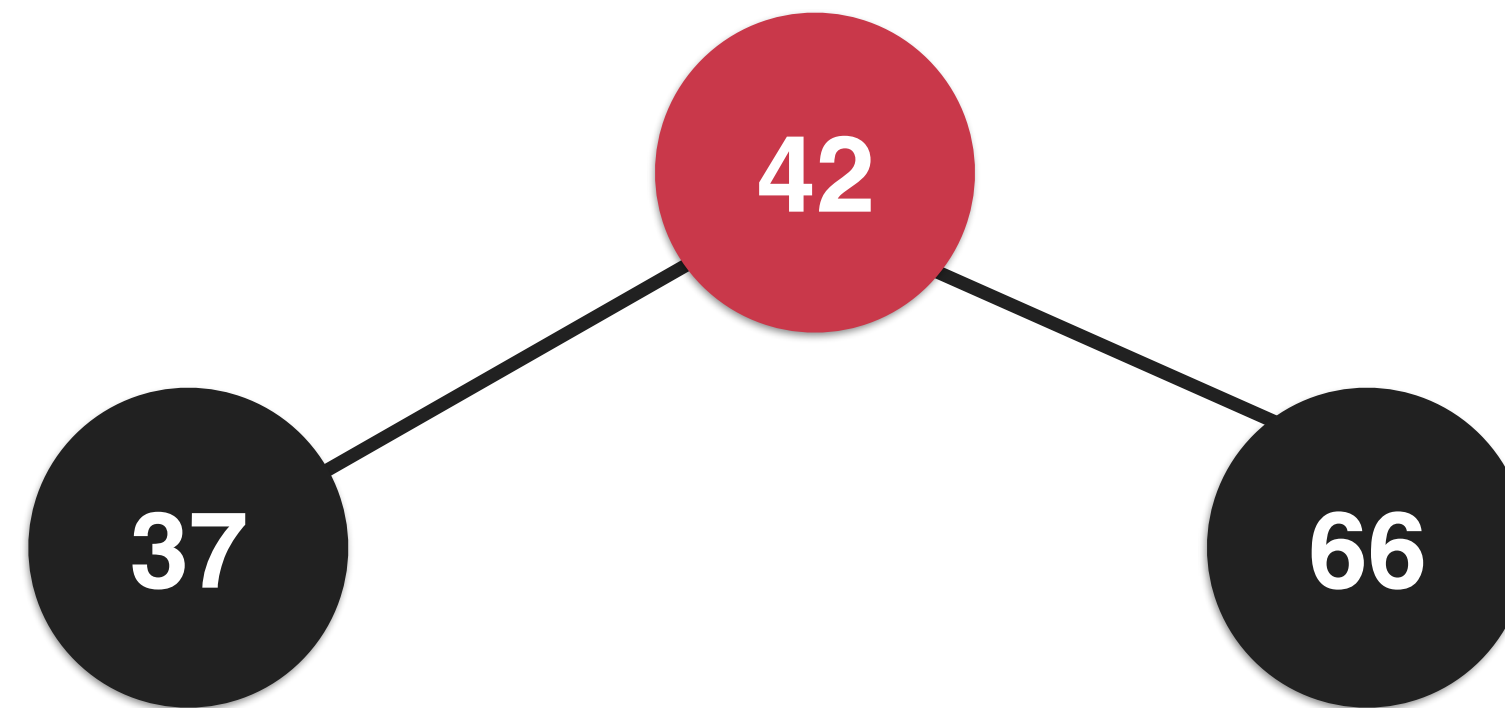
# 红黑树添加新元素



# 红黑树添加新元素

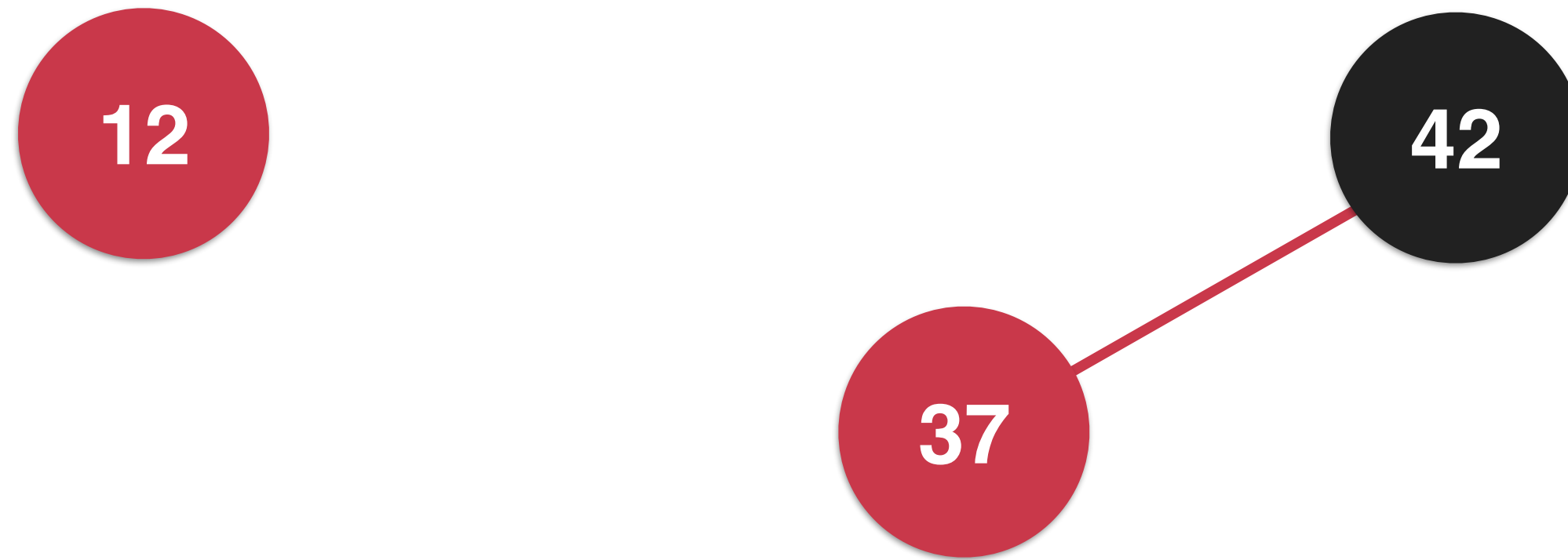
颜色翻转

flipColors

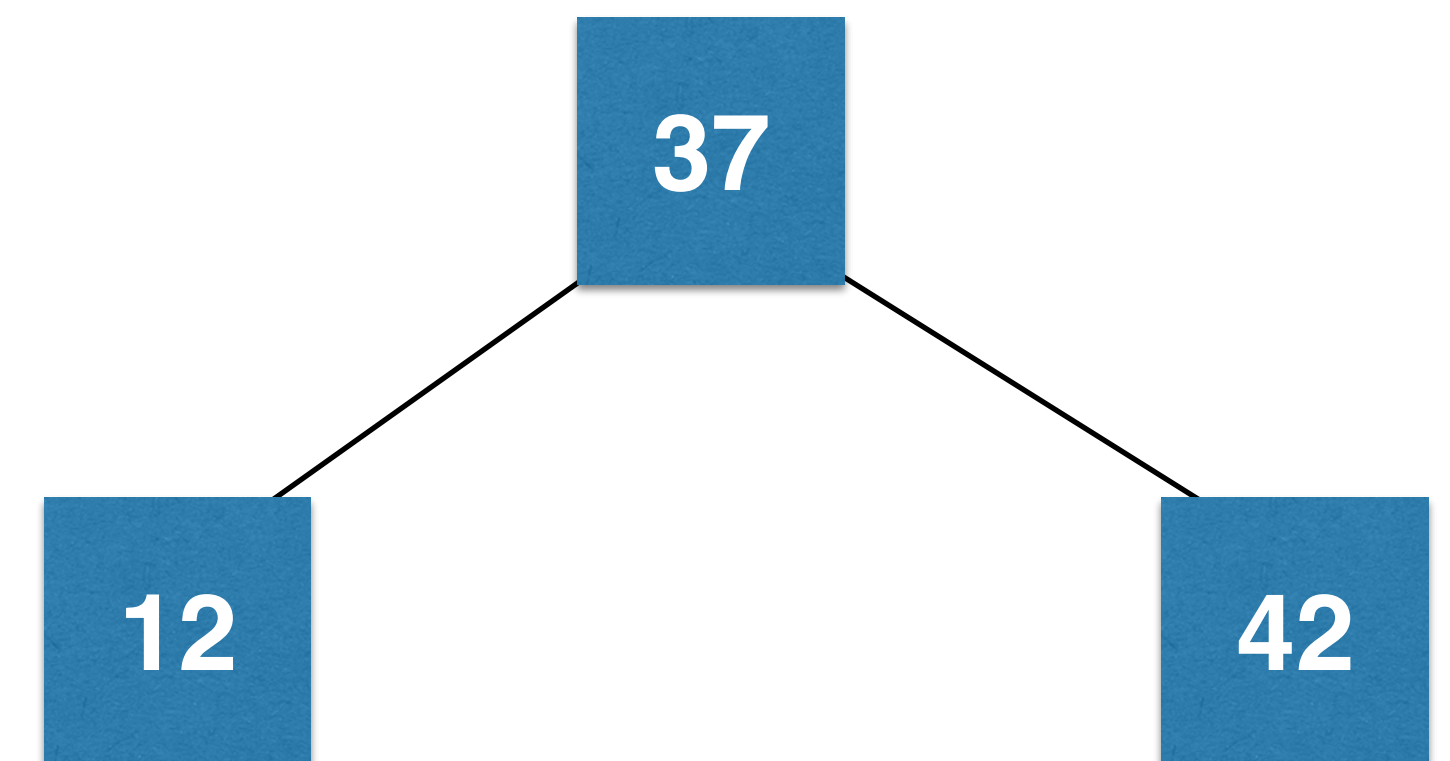
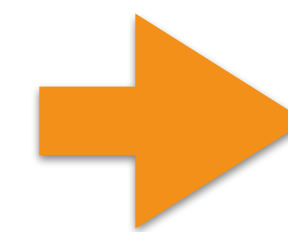
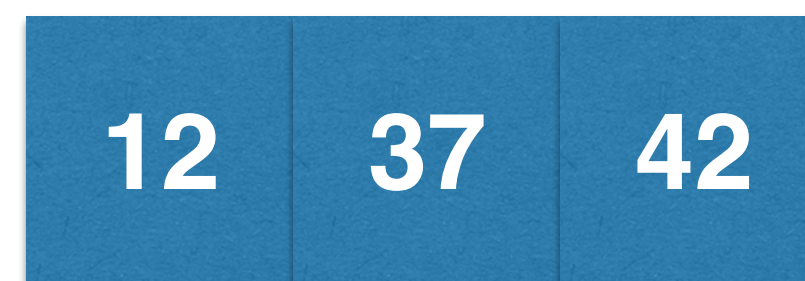
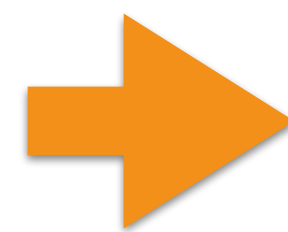
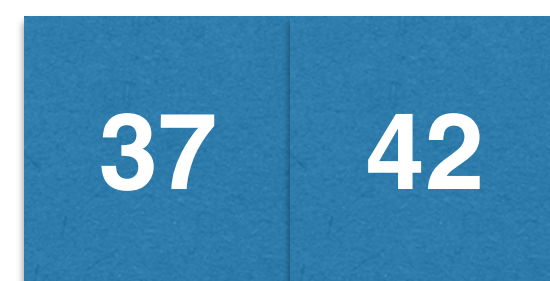
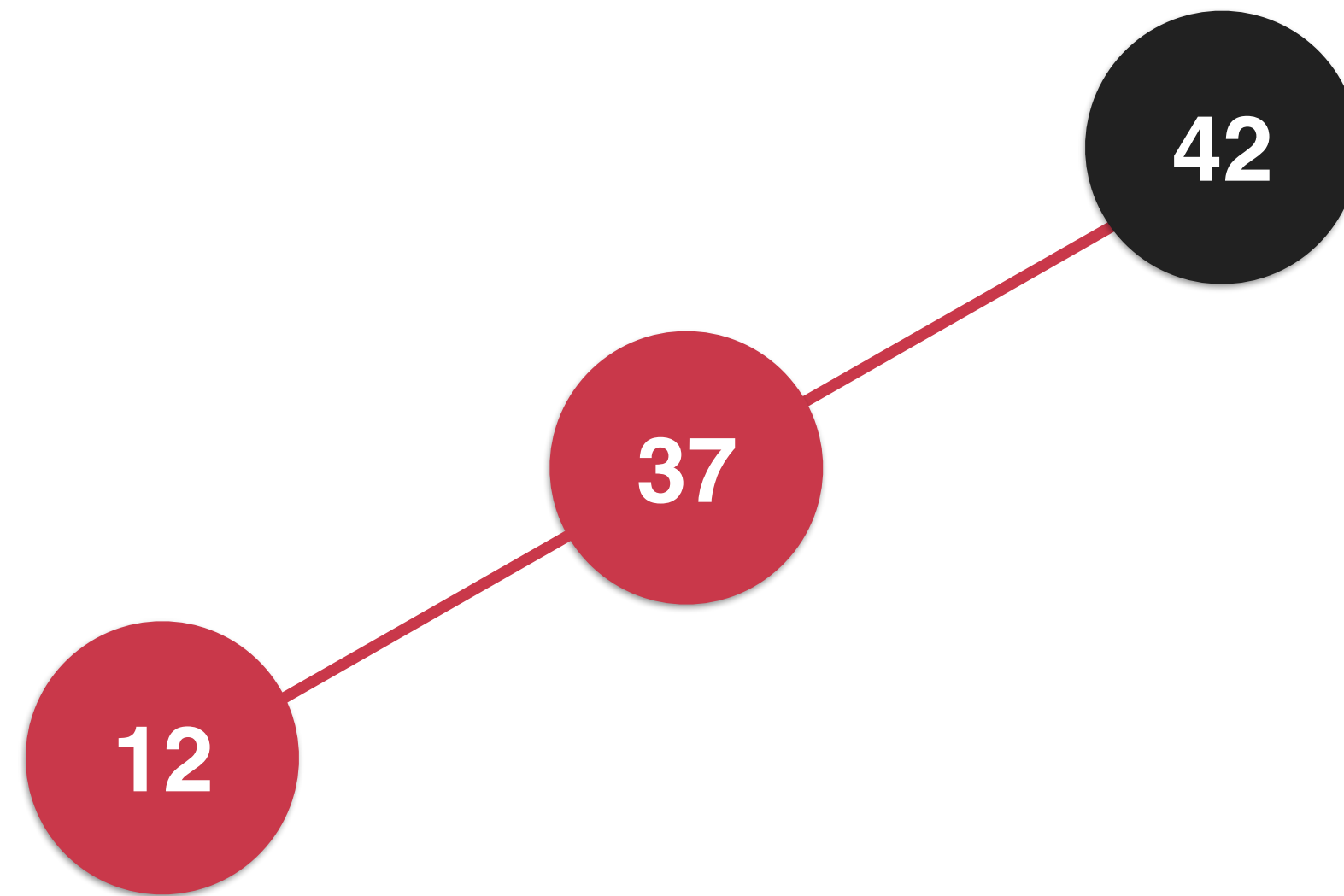


实践： 颜色翻转

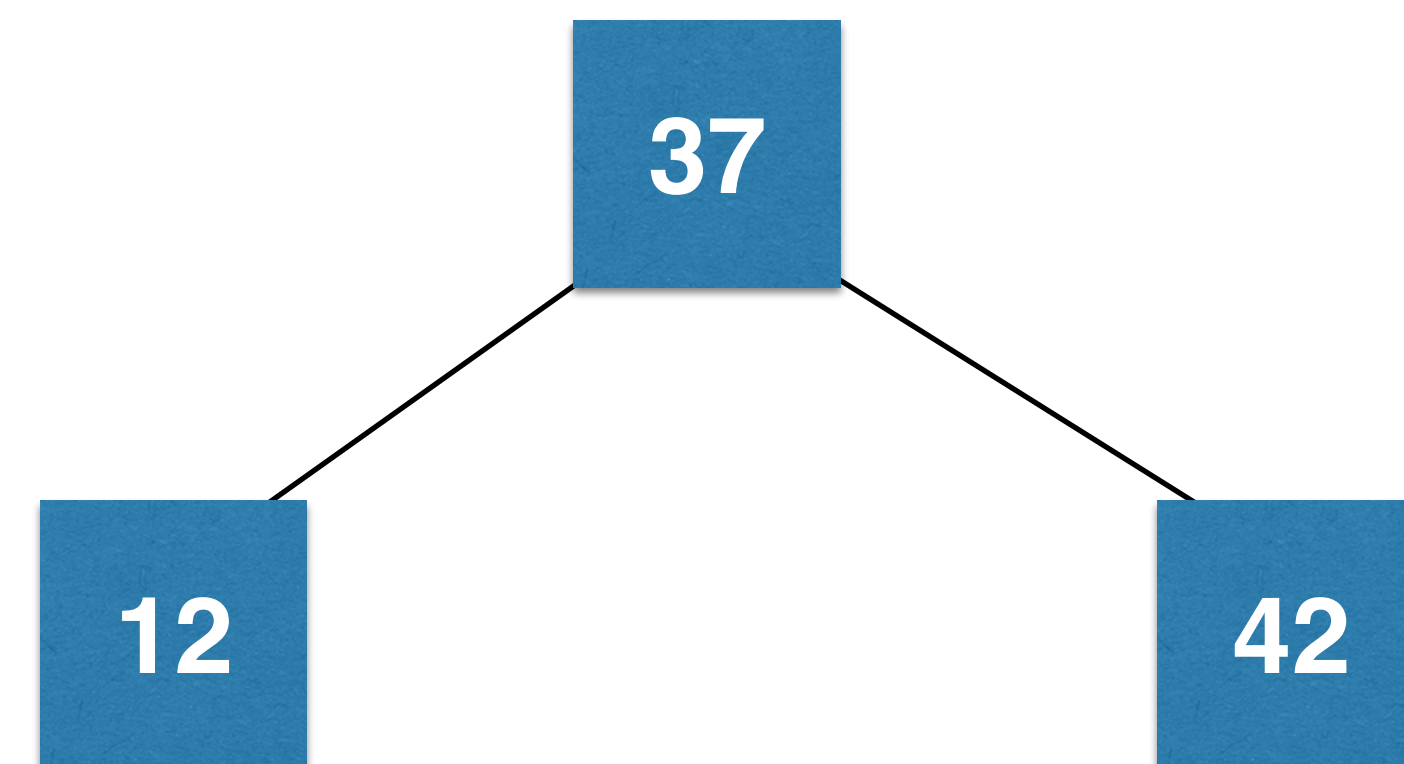
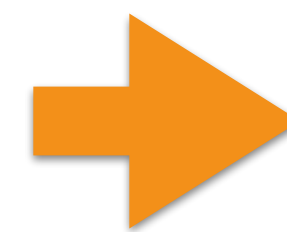
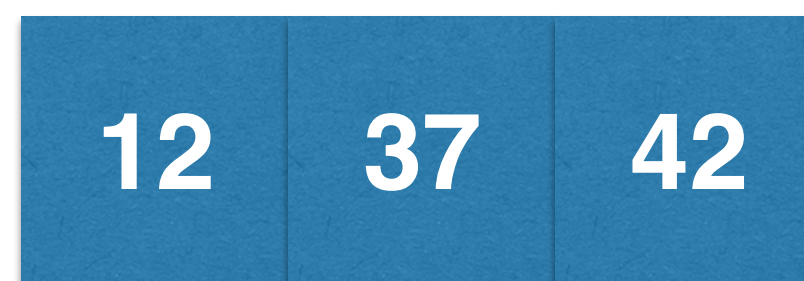
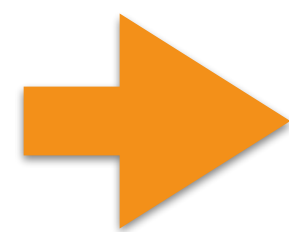
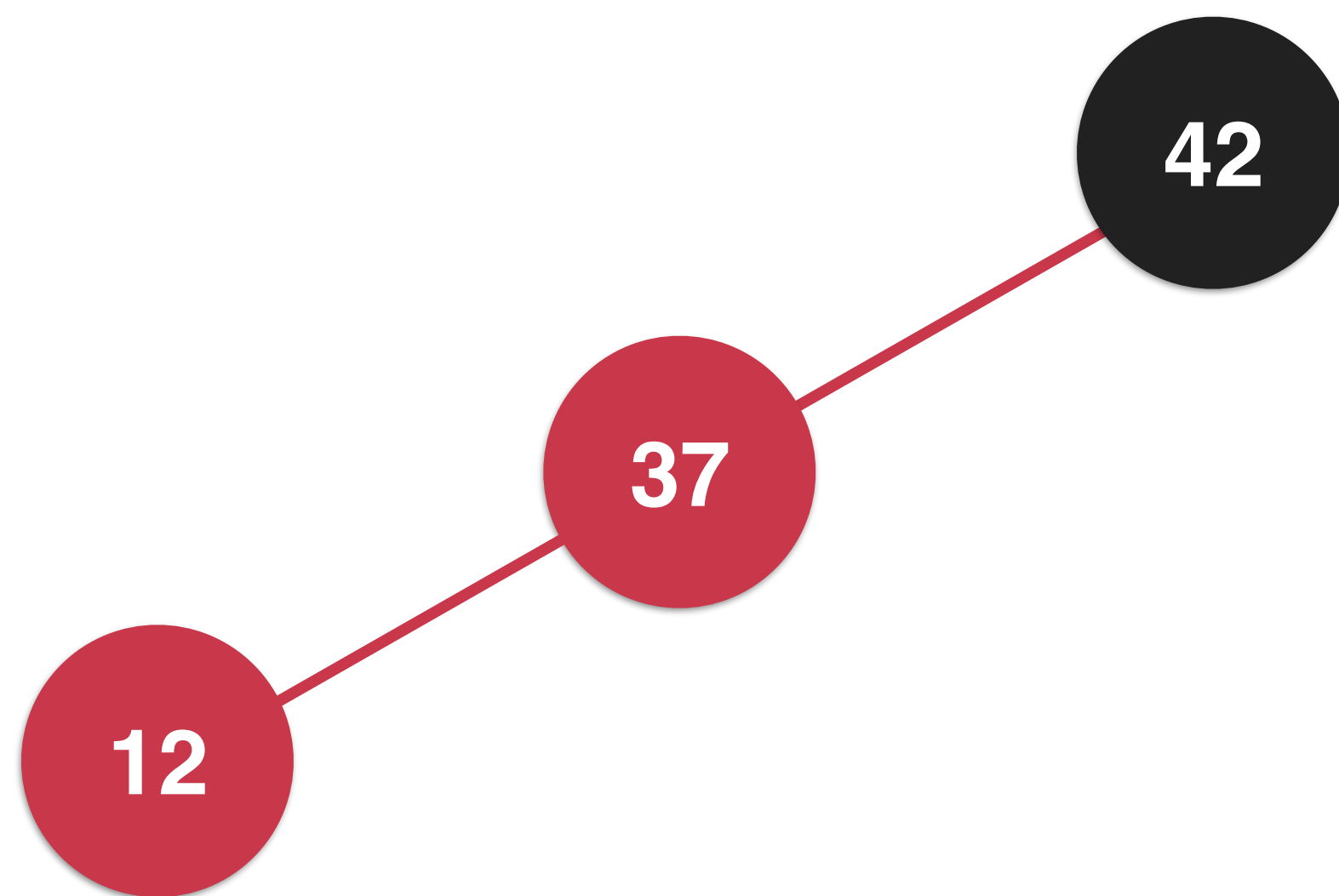
# 红黑树添加新元素



# 红黑树添加新元素

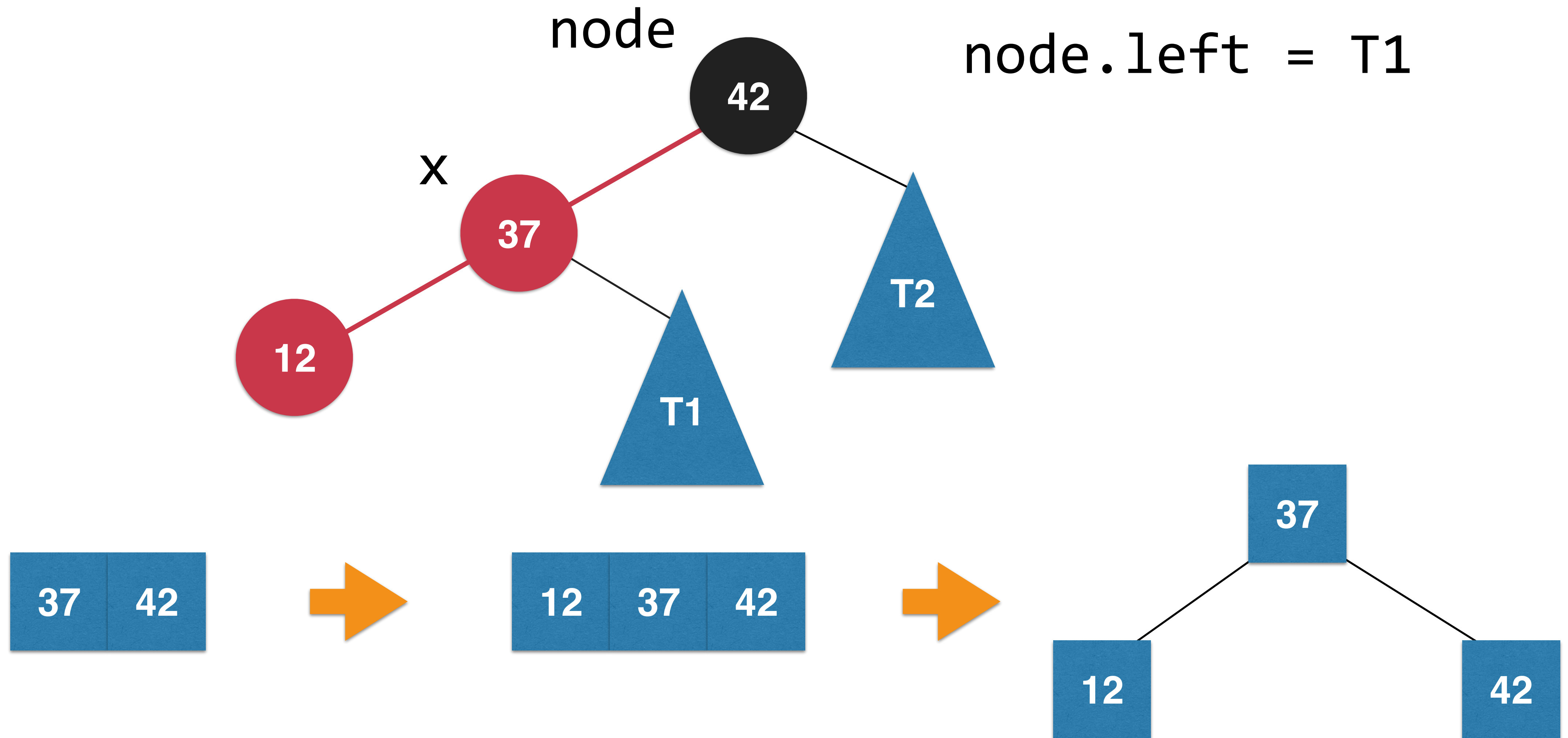


# 右旋转

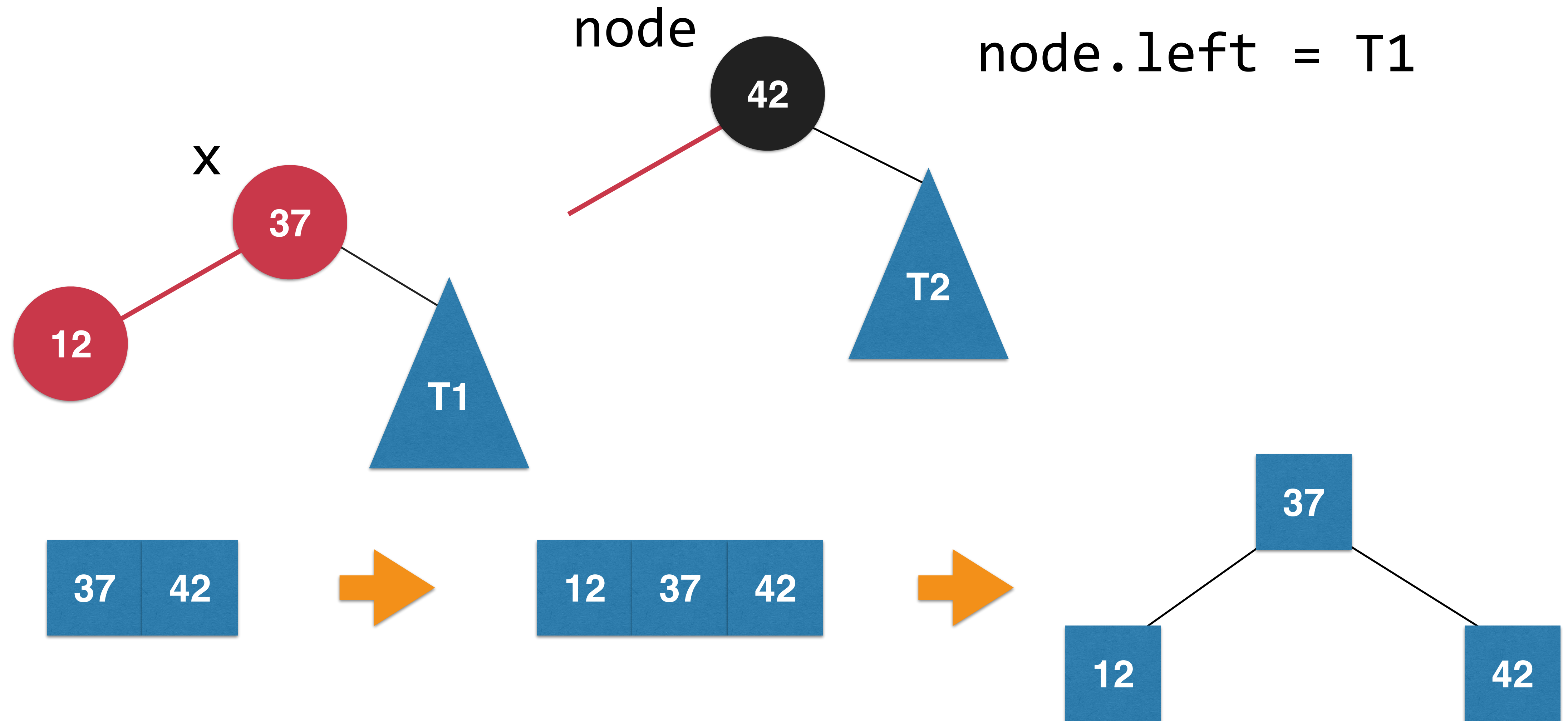




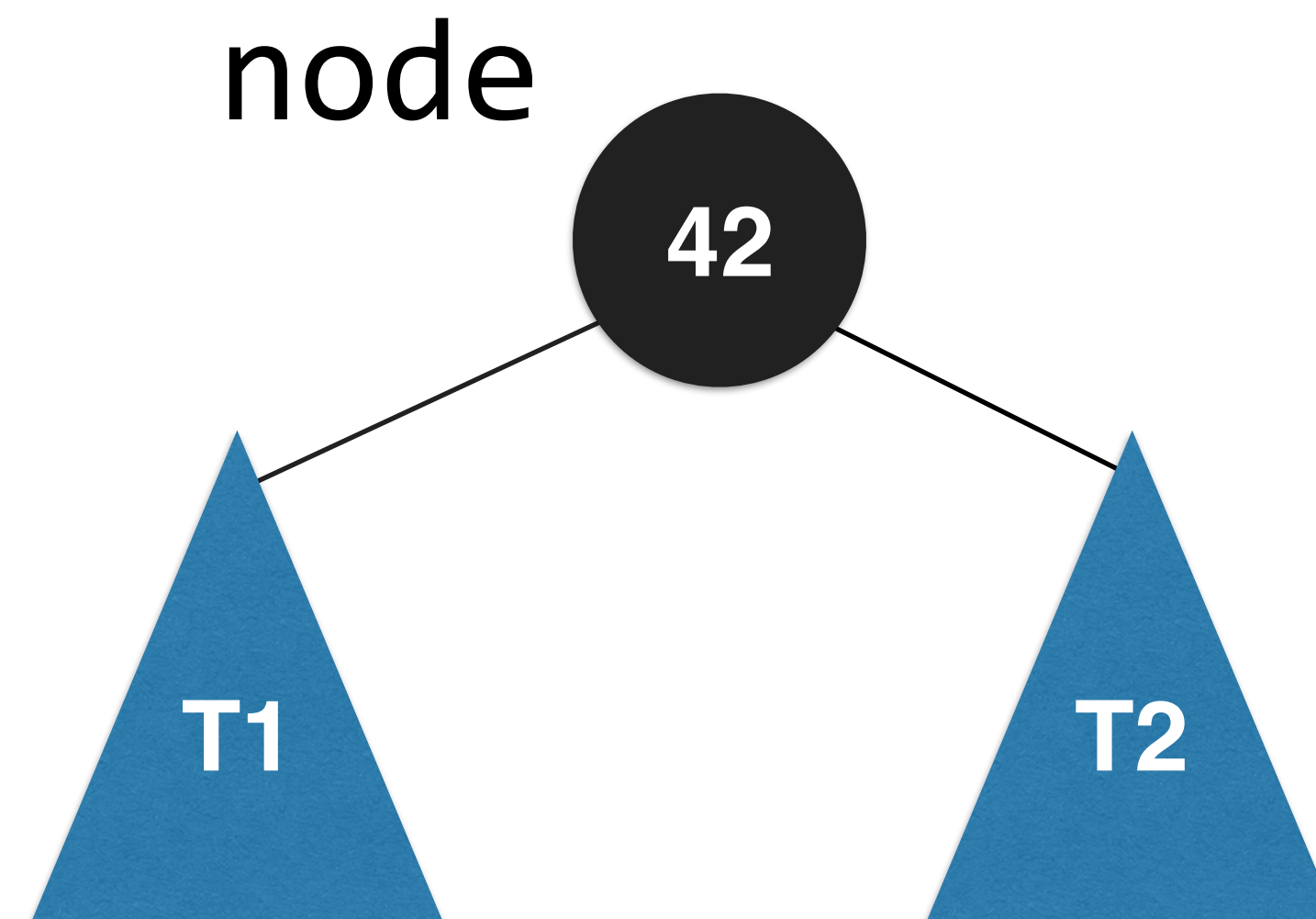
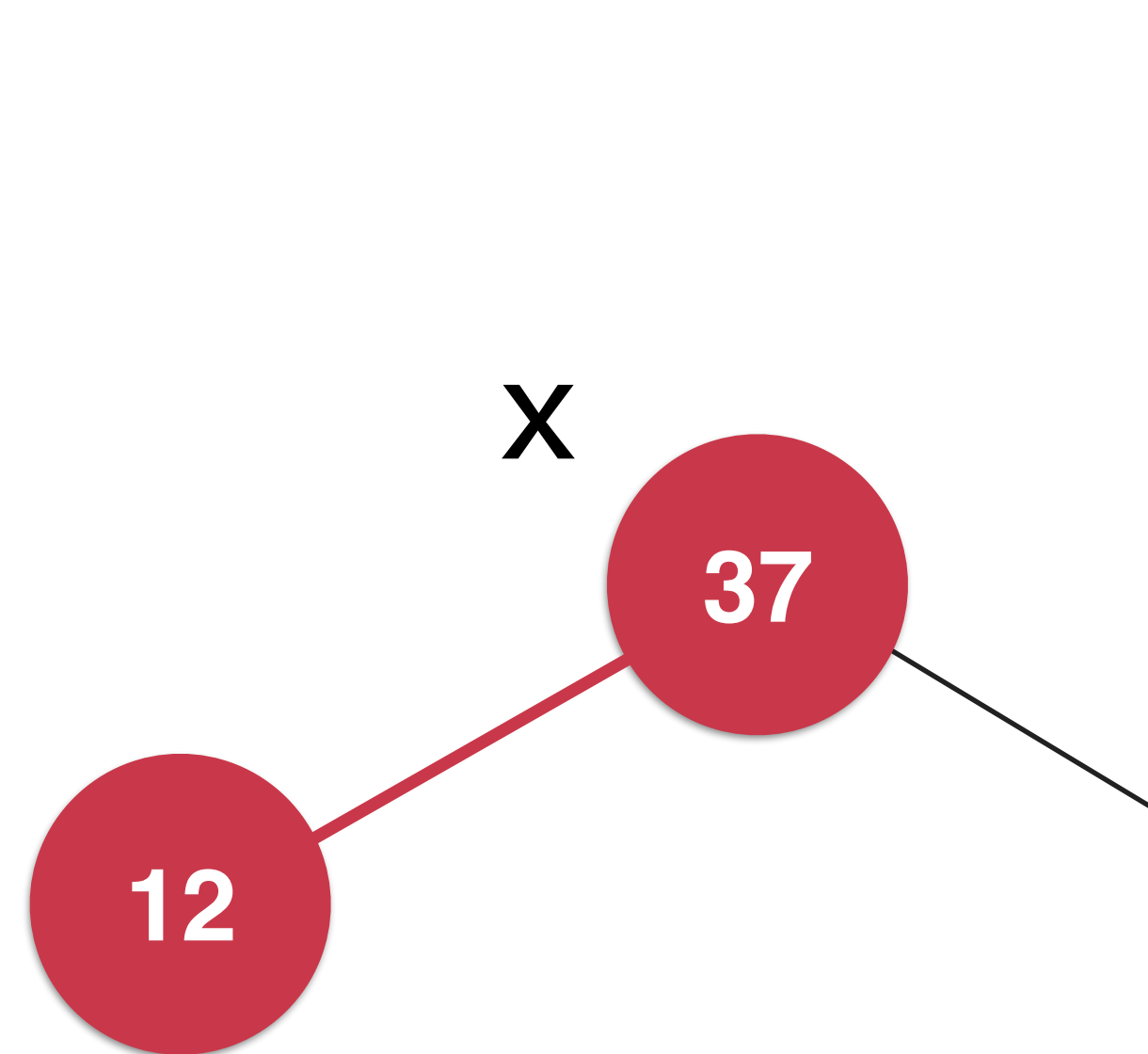
# 右旋转



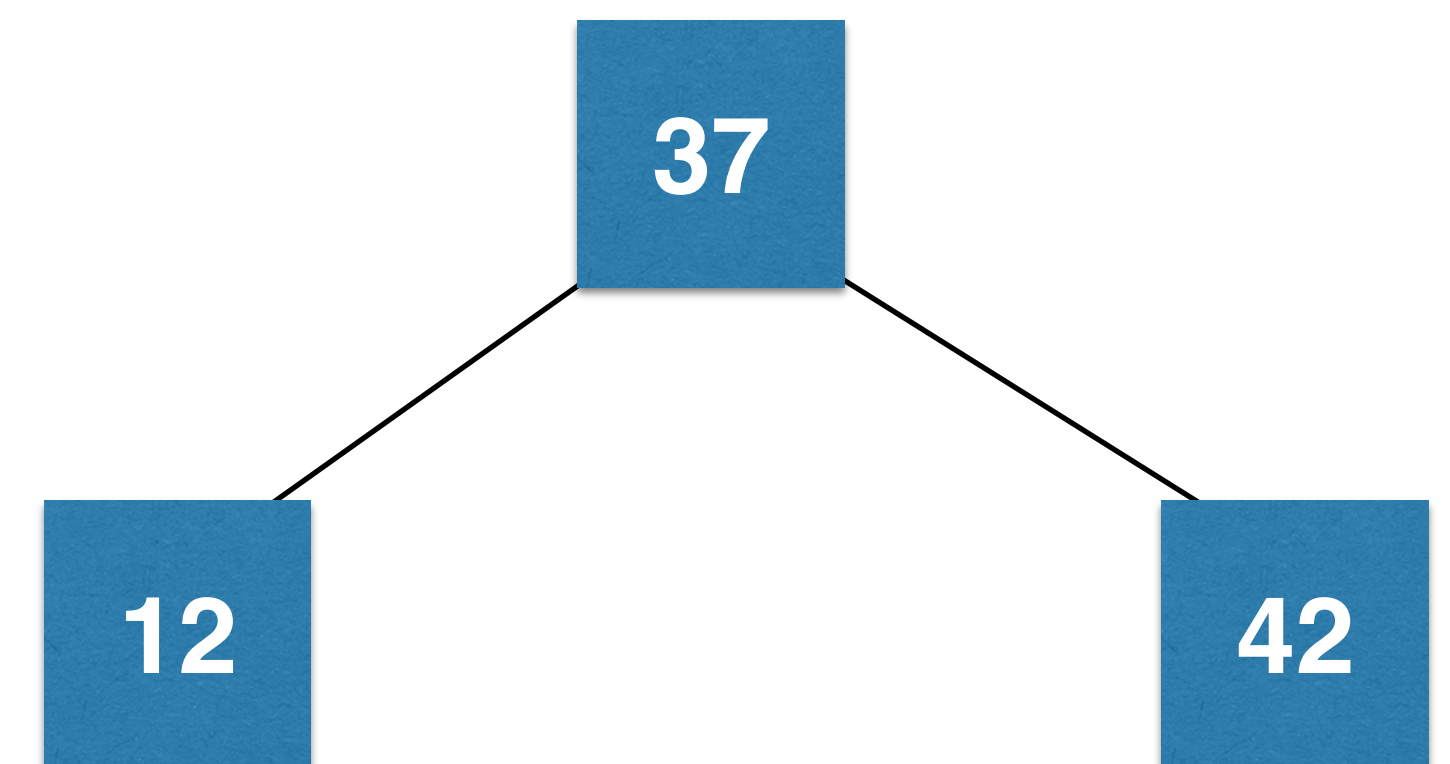
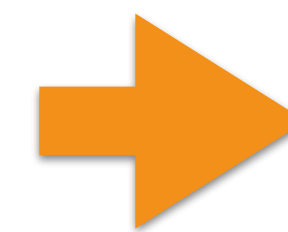
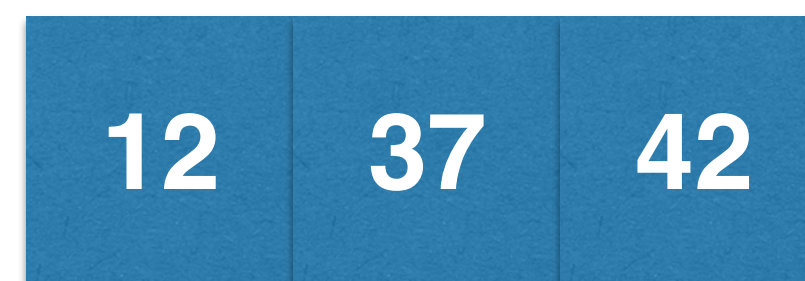
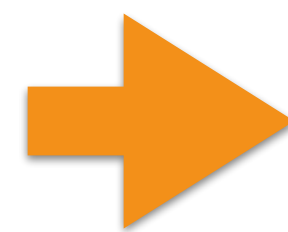
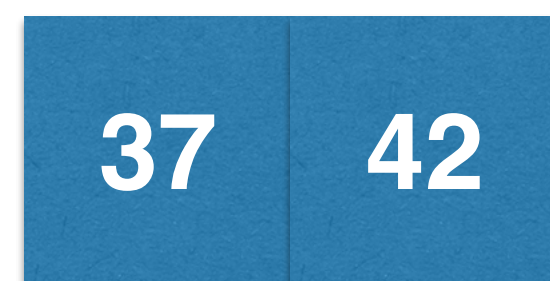
# 右旋转



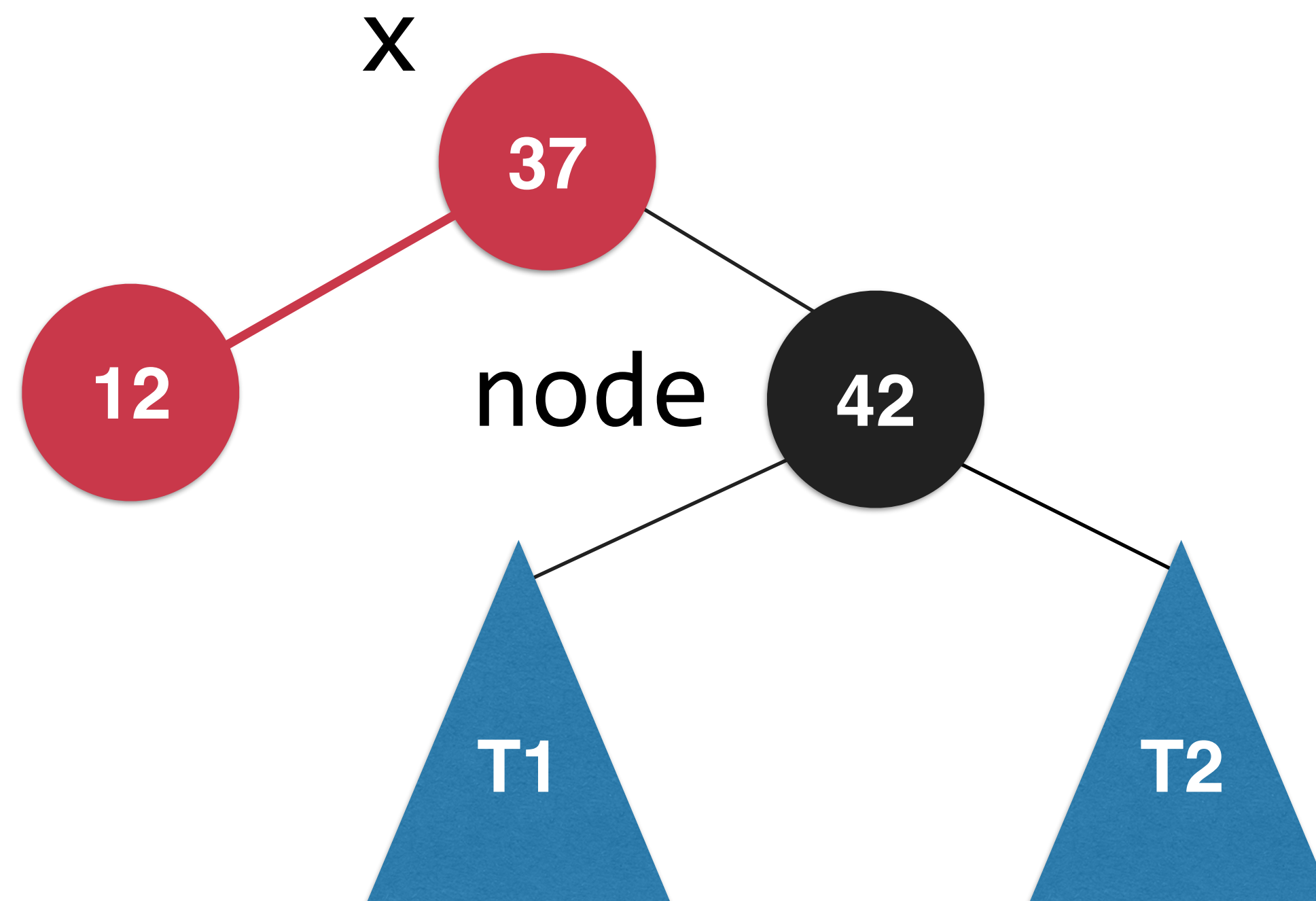
# 右旋转



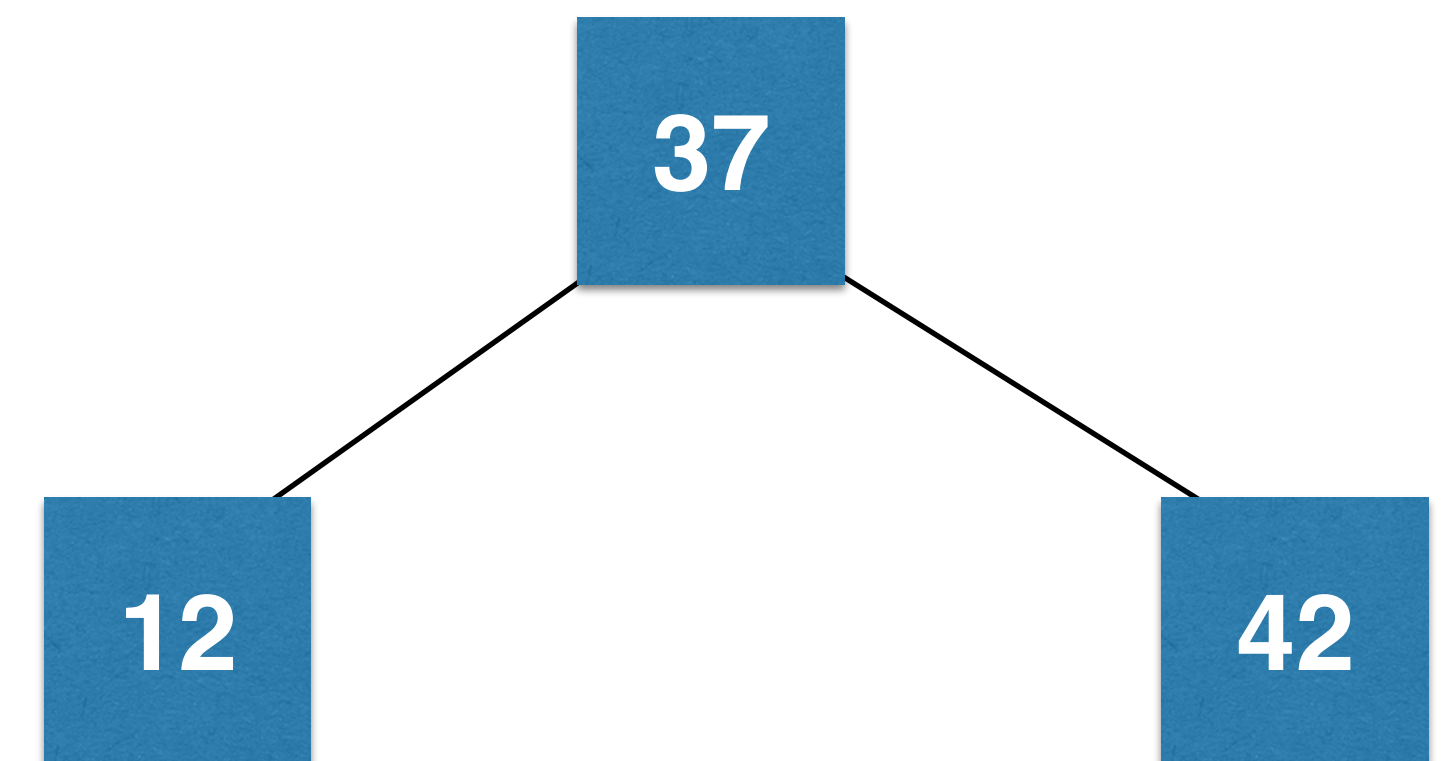
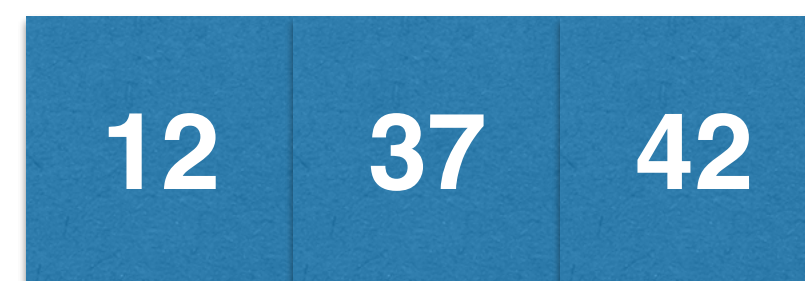
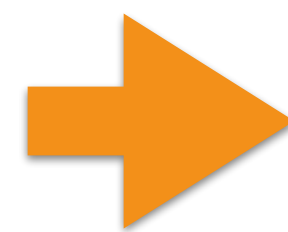
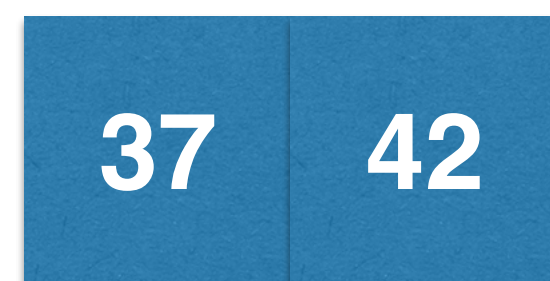
`node.left = T1`  
`x.right = node`



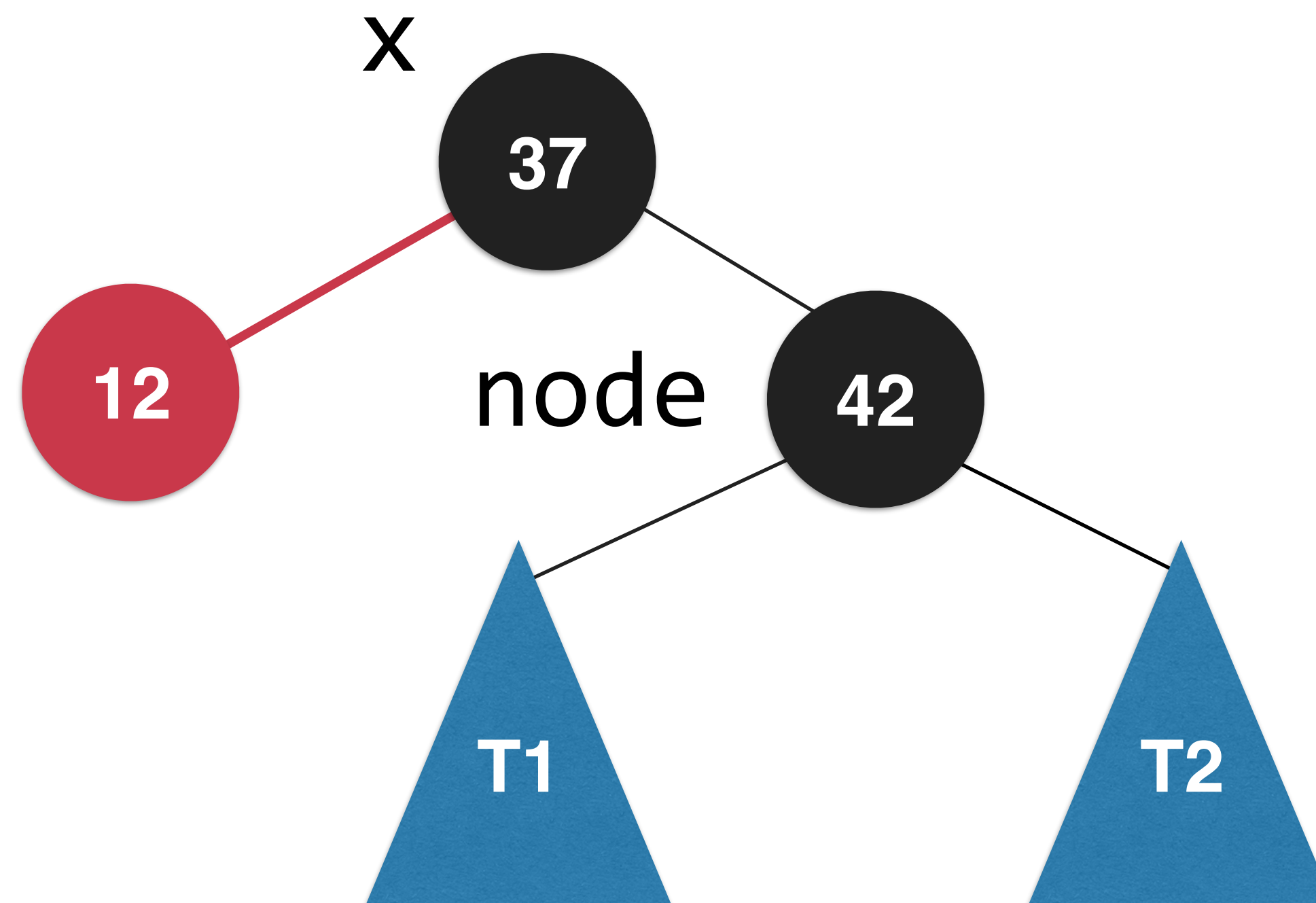
# 右旋转



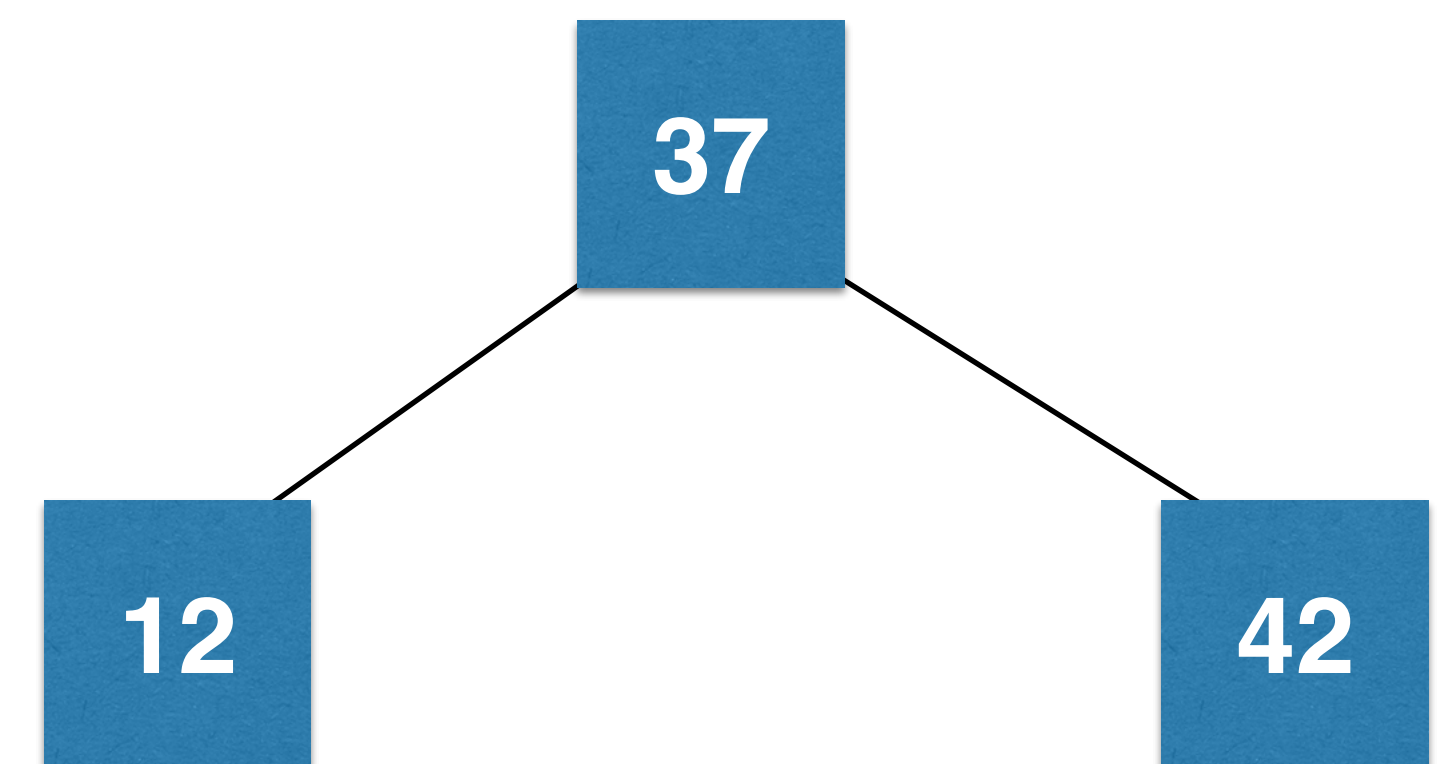
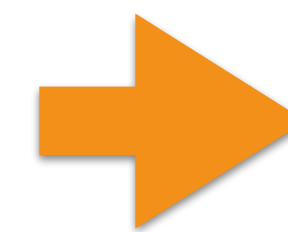
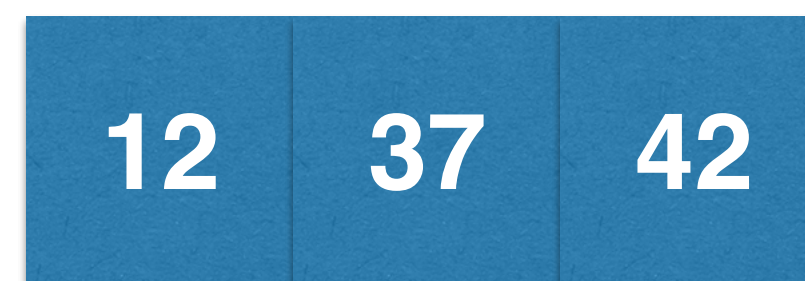
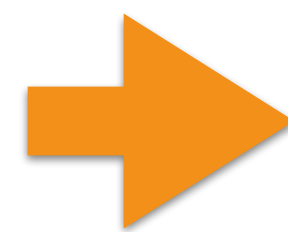
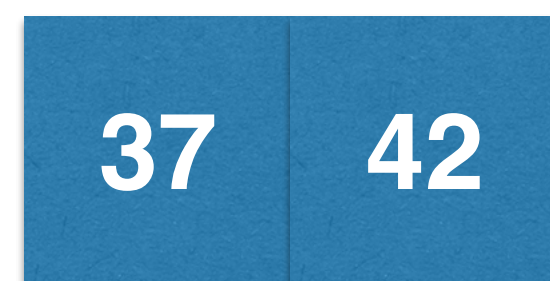
```
node.left = T1  
x.right = node  
x.color = node.color
```



# 右旋转

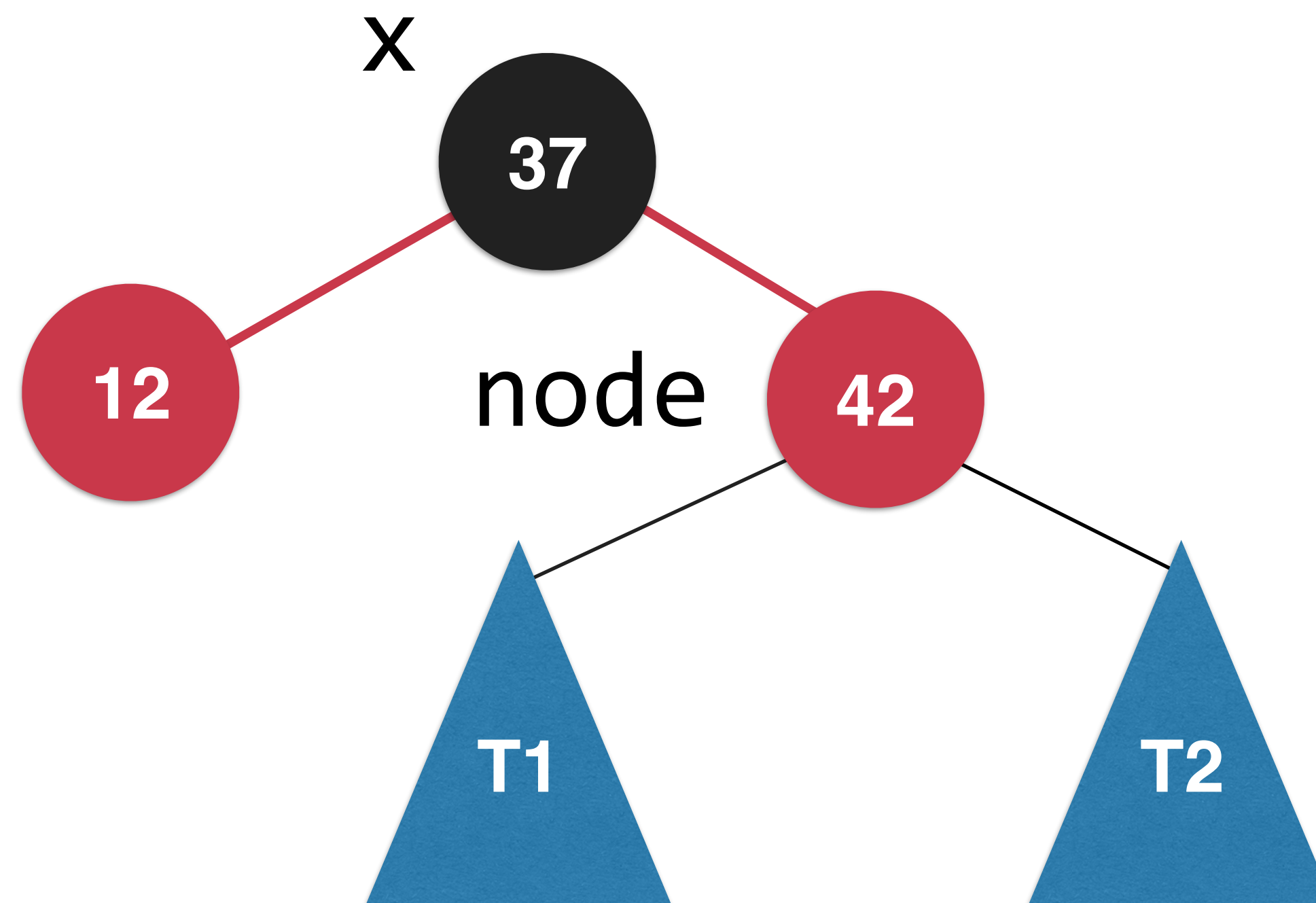


```
node.left = T1  
x.right = node  
x.color = node.color  
node.color = RED
```





# 右旋转

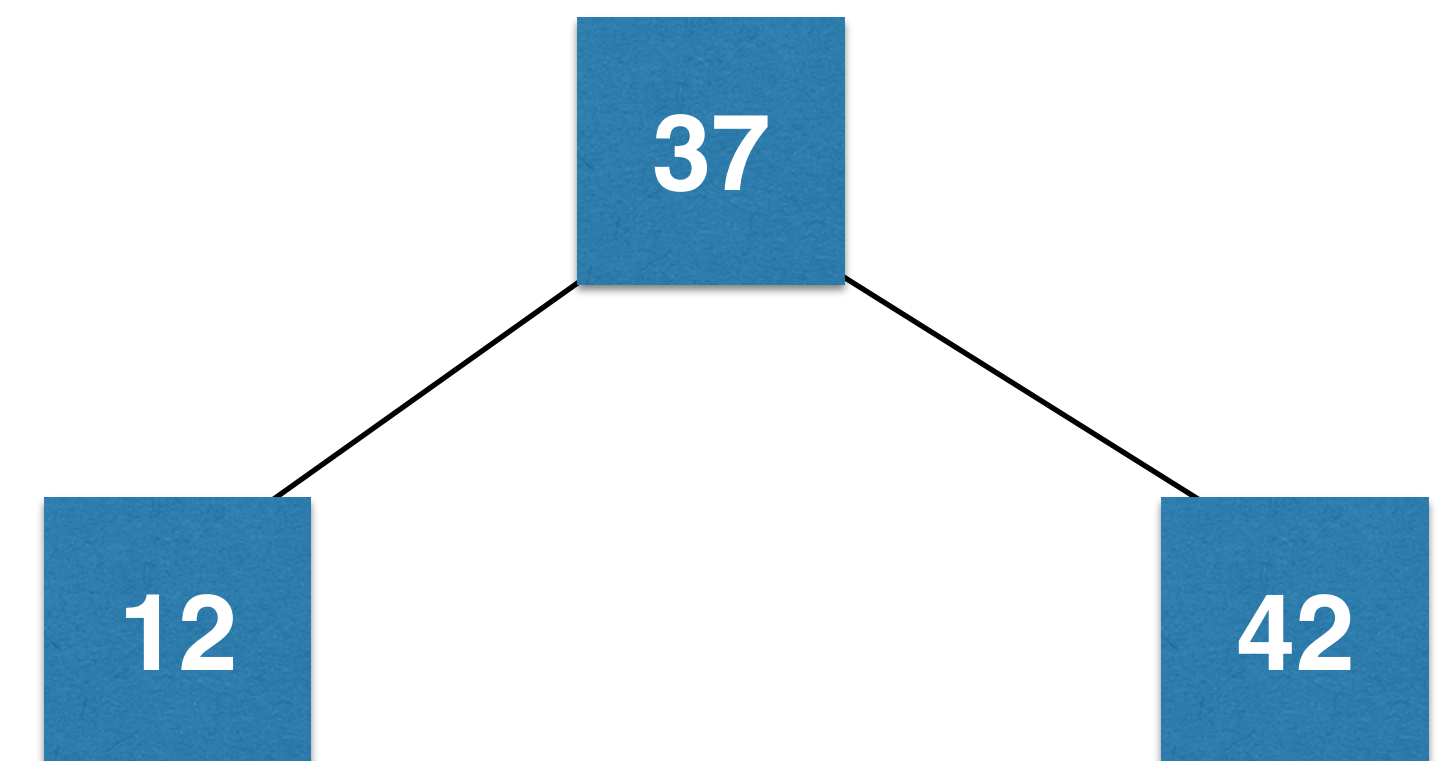
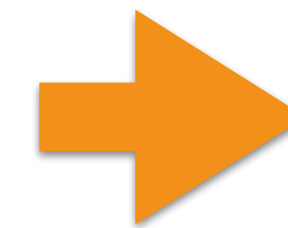
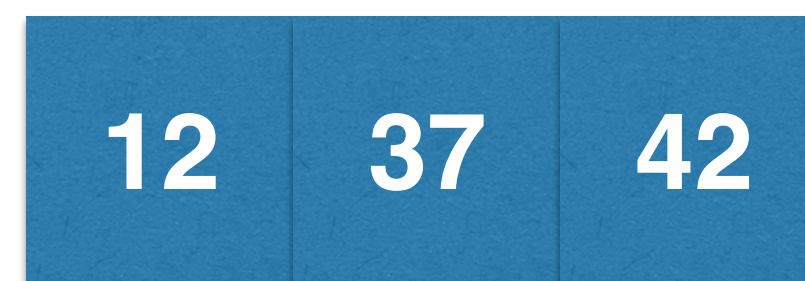
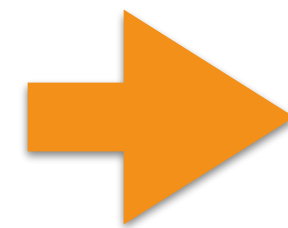
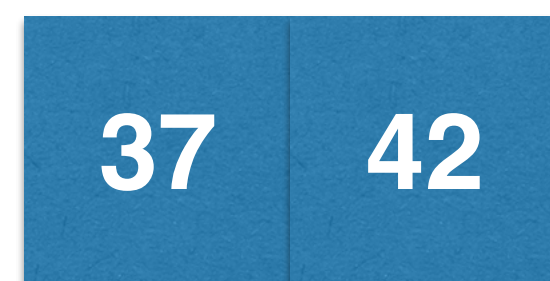


`node.left = T1`

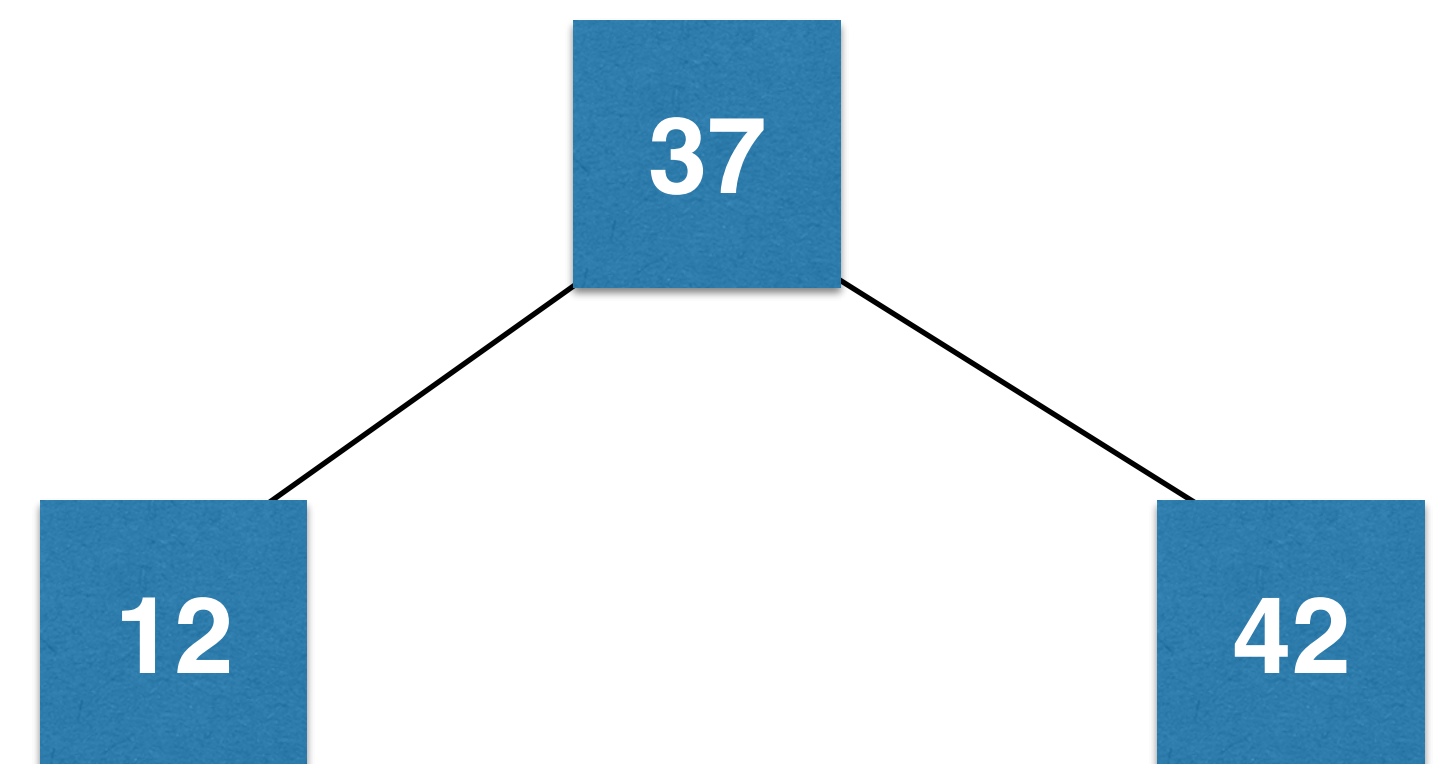
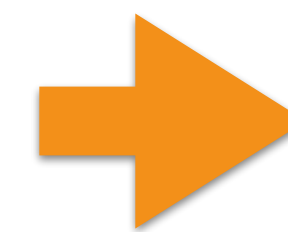
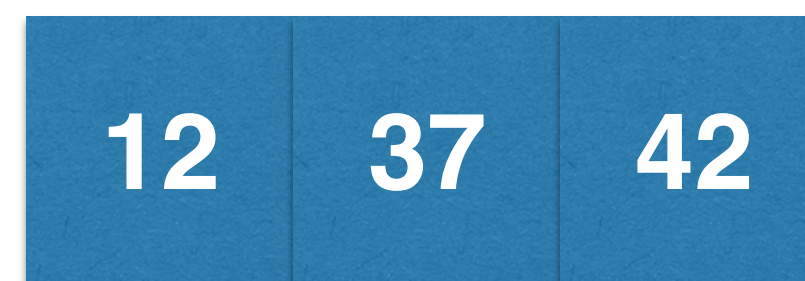
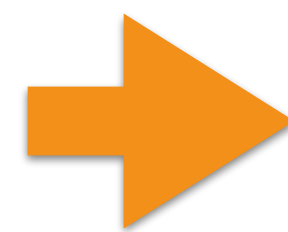
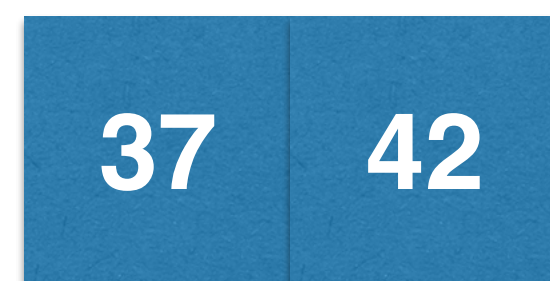
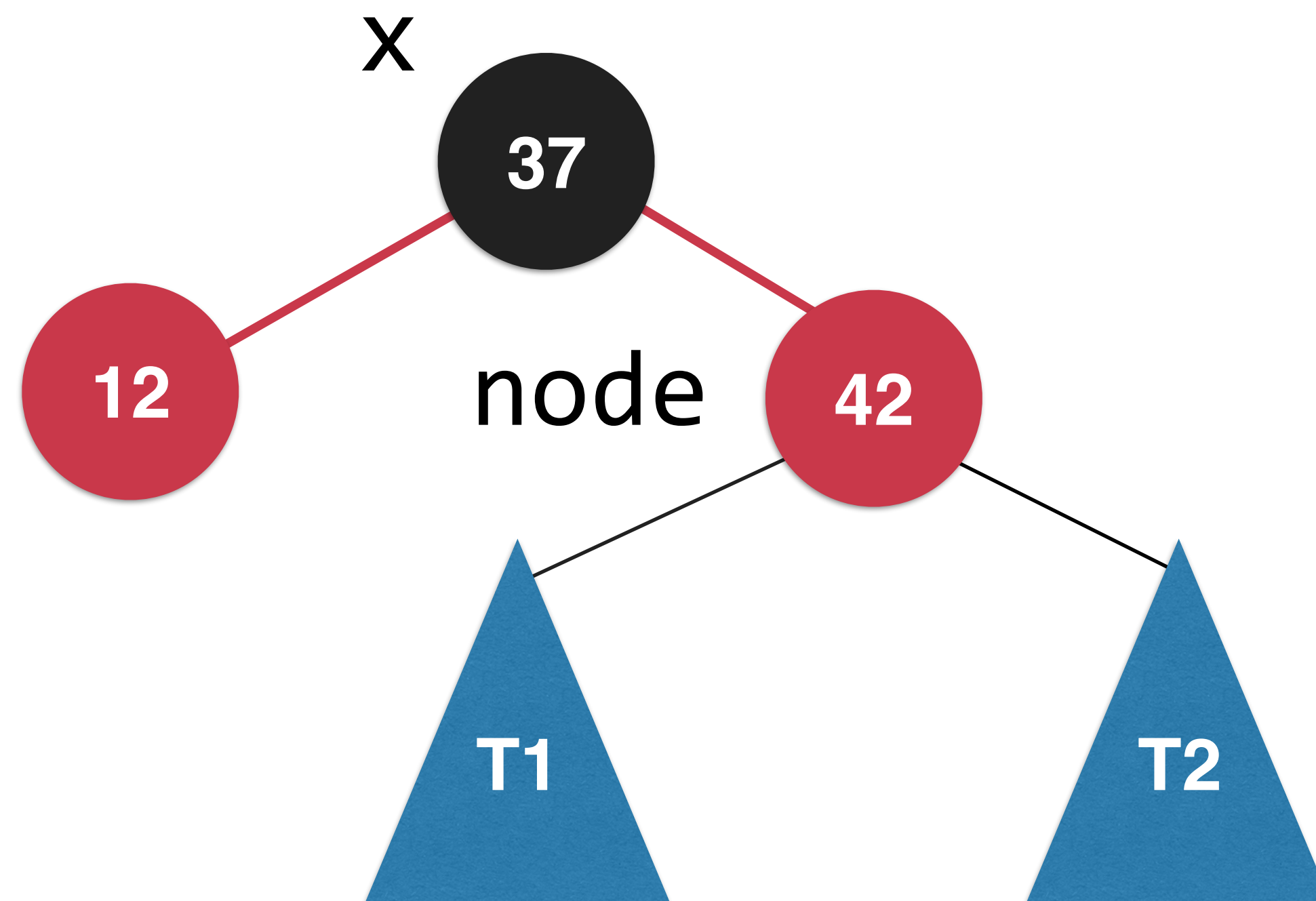
`x.right = node`

`x.color = node.color`

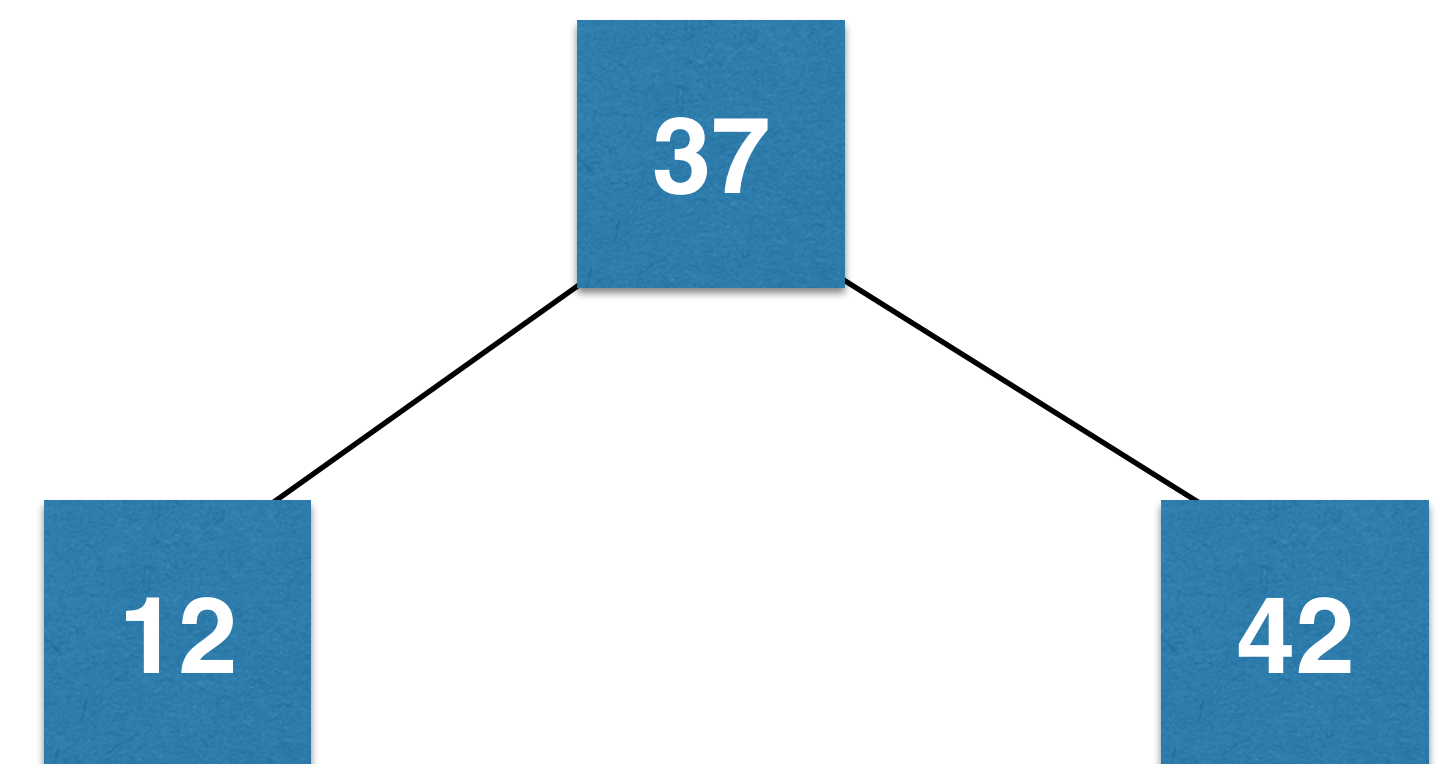
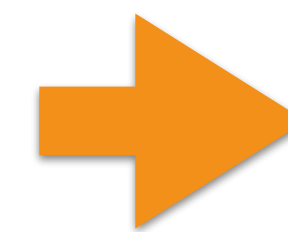
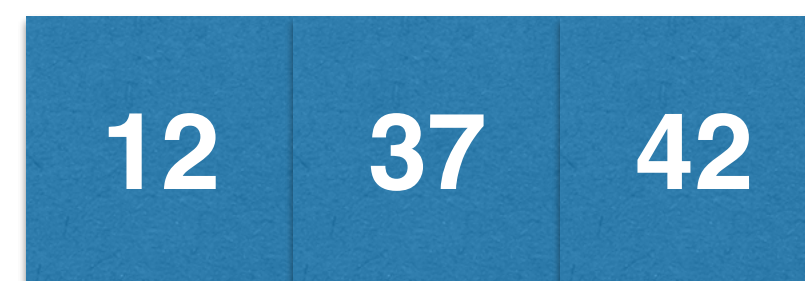
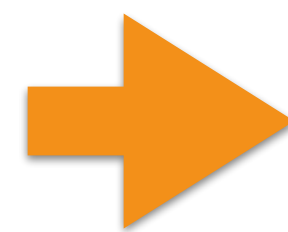
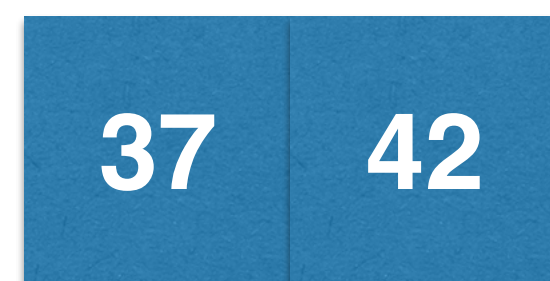
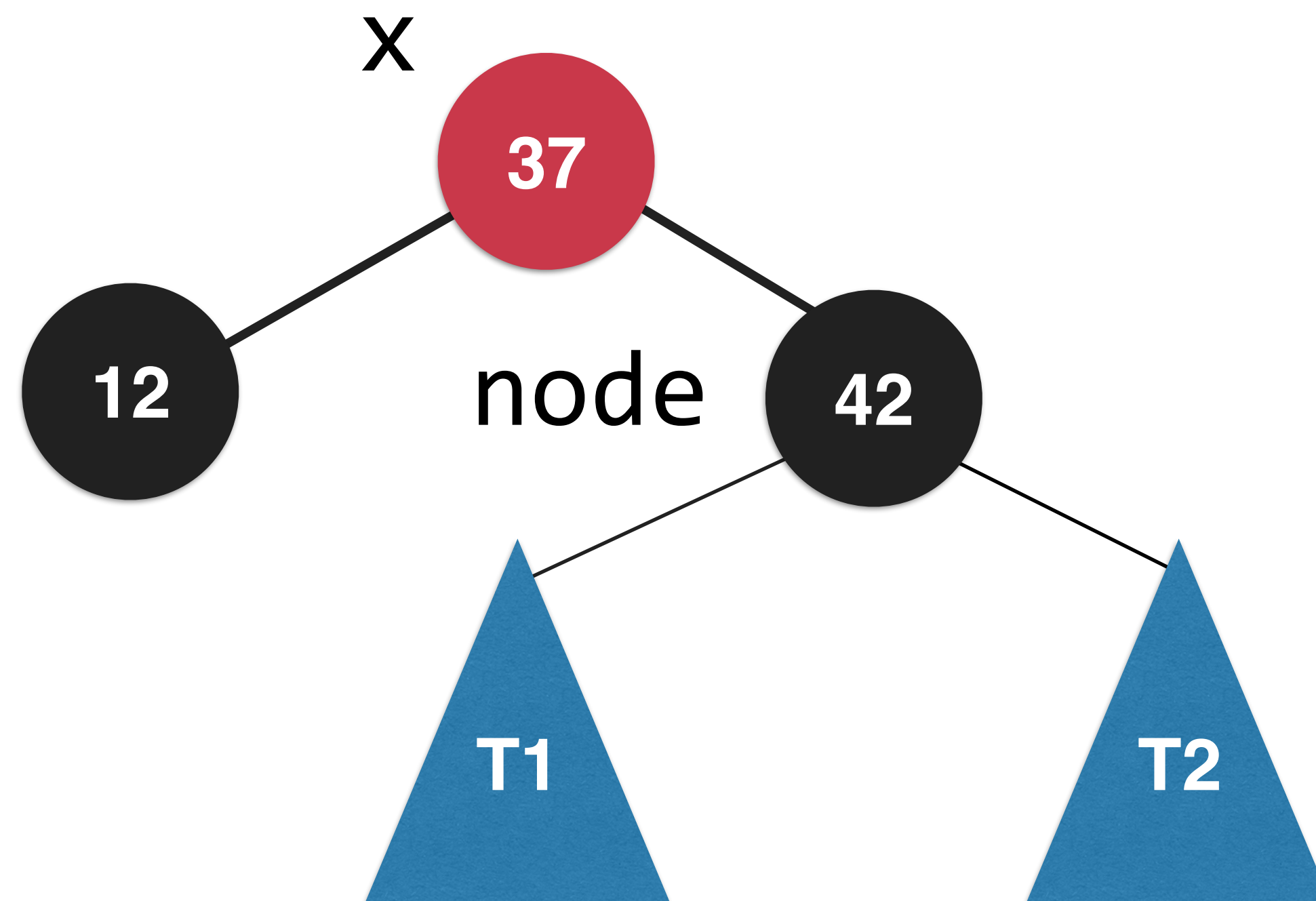
`node.color = RED`



# 颜色翻转



# 颜色翻转

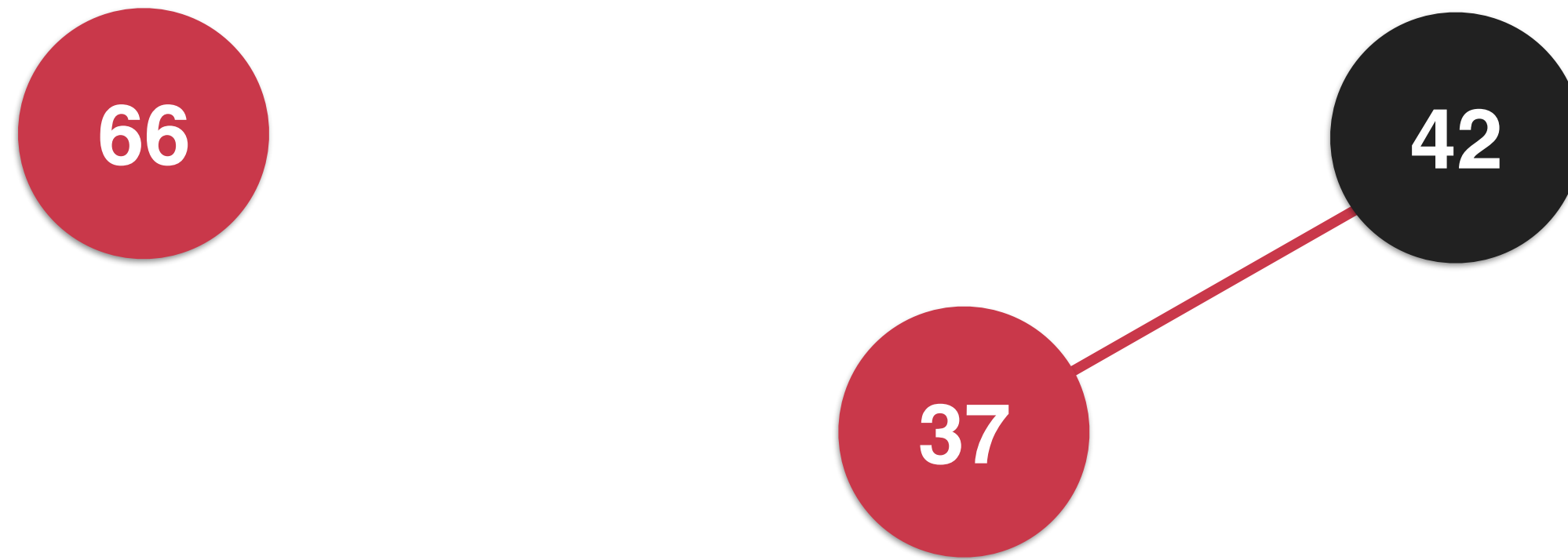




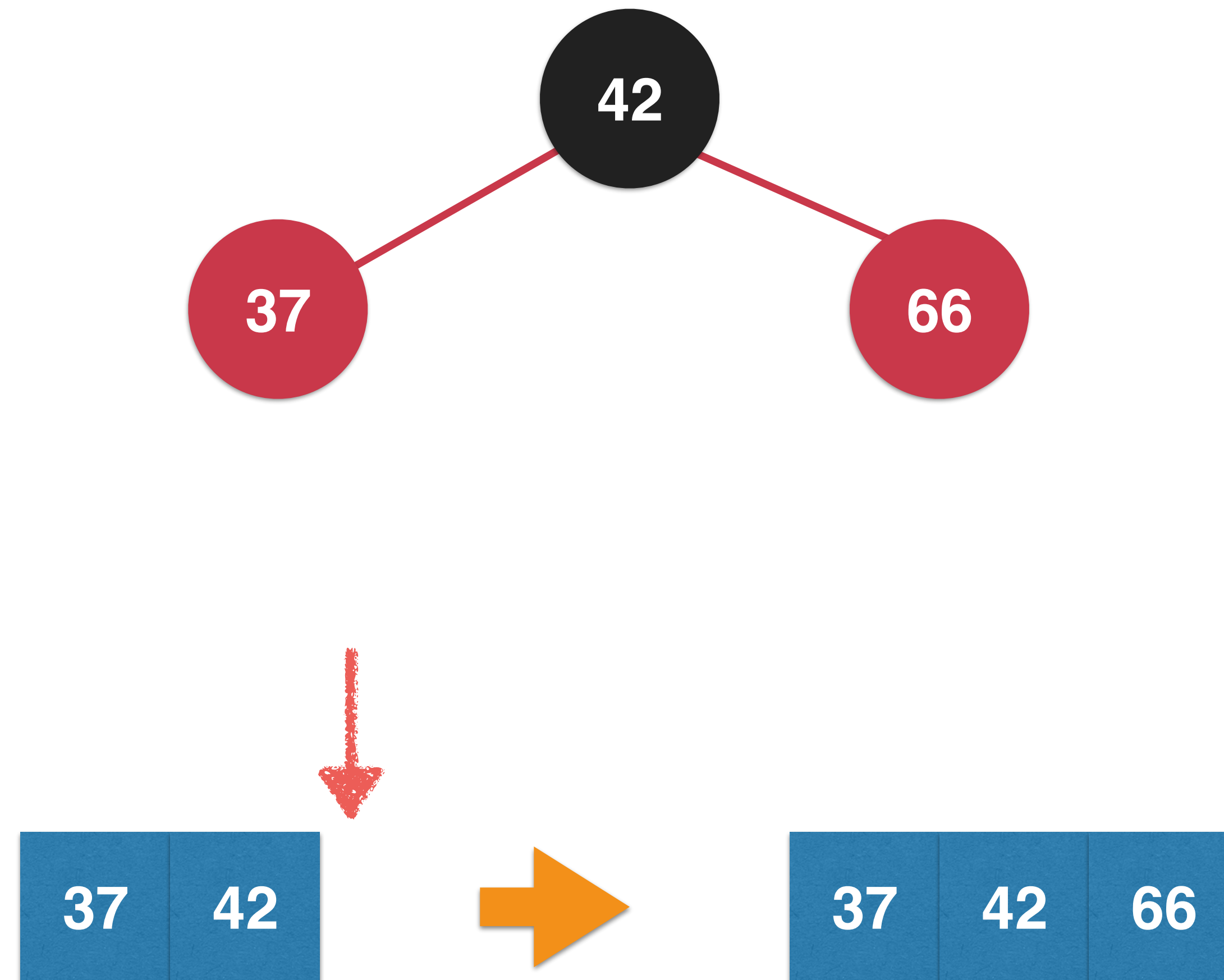
实践：右旋转

向红黑树中添加新节点

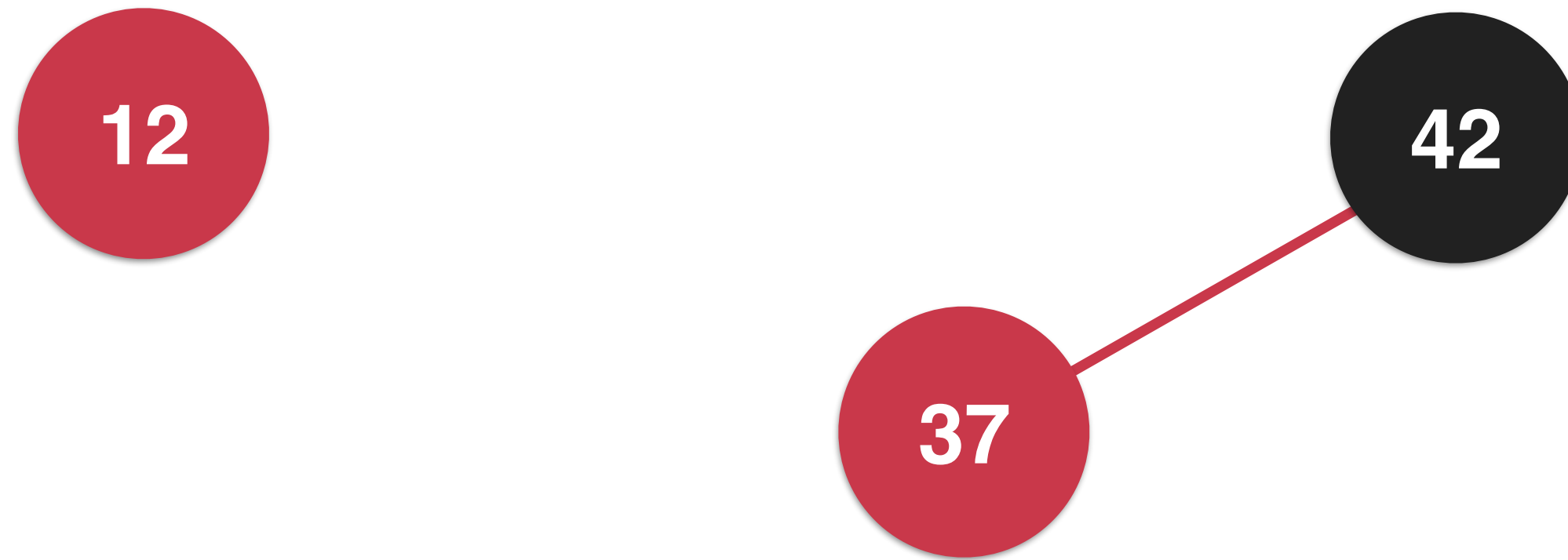
# 红黑树添加新元素



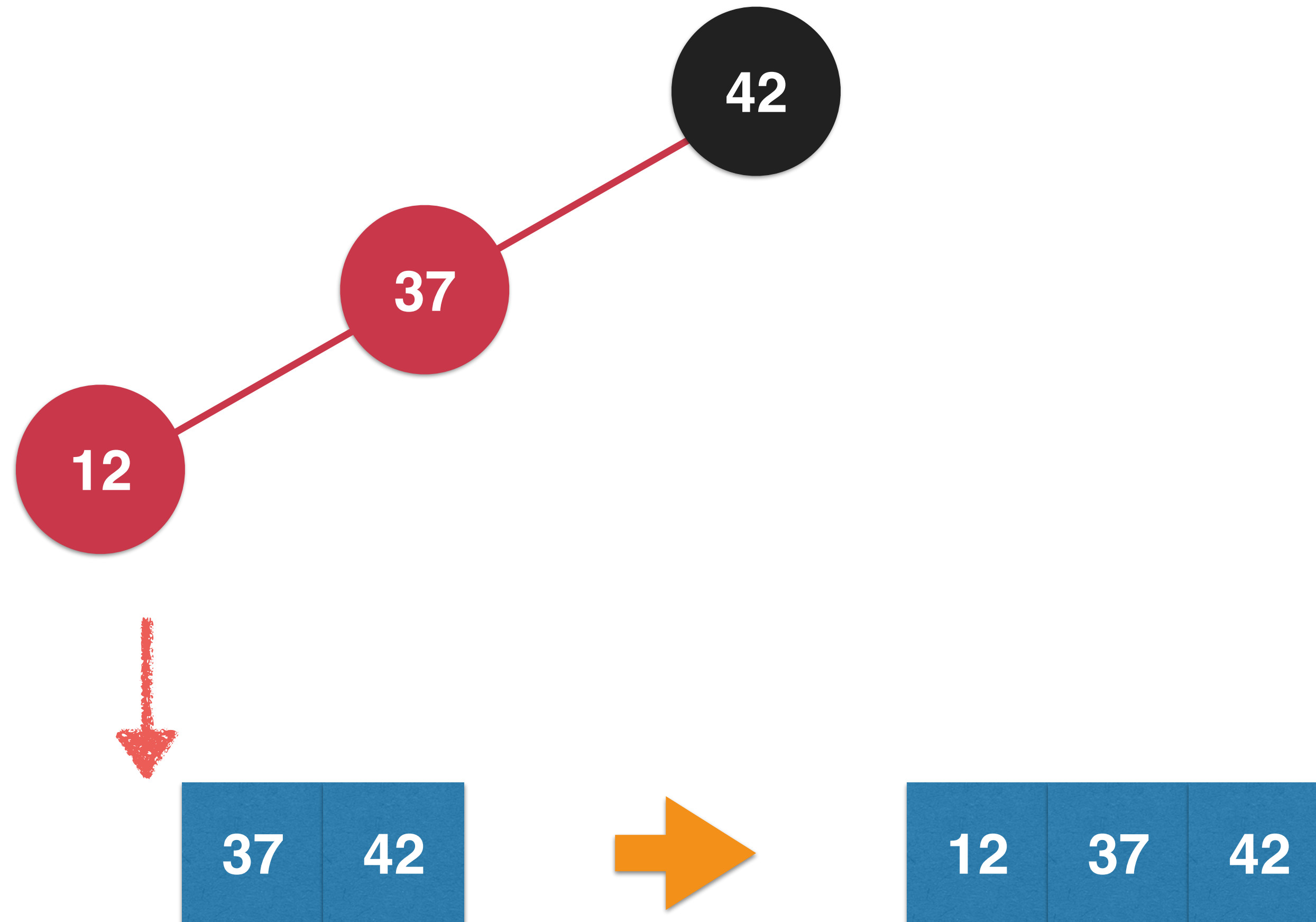
# 红黑树添加新元素



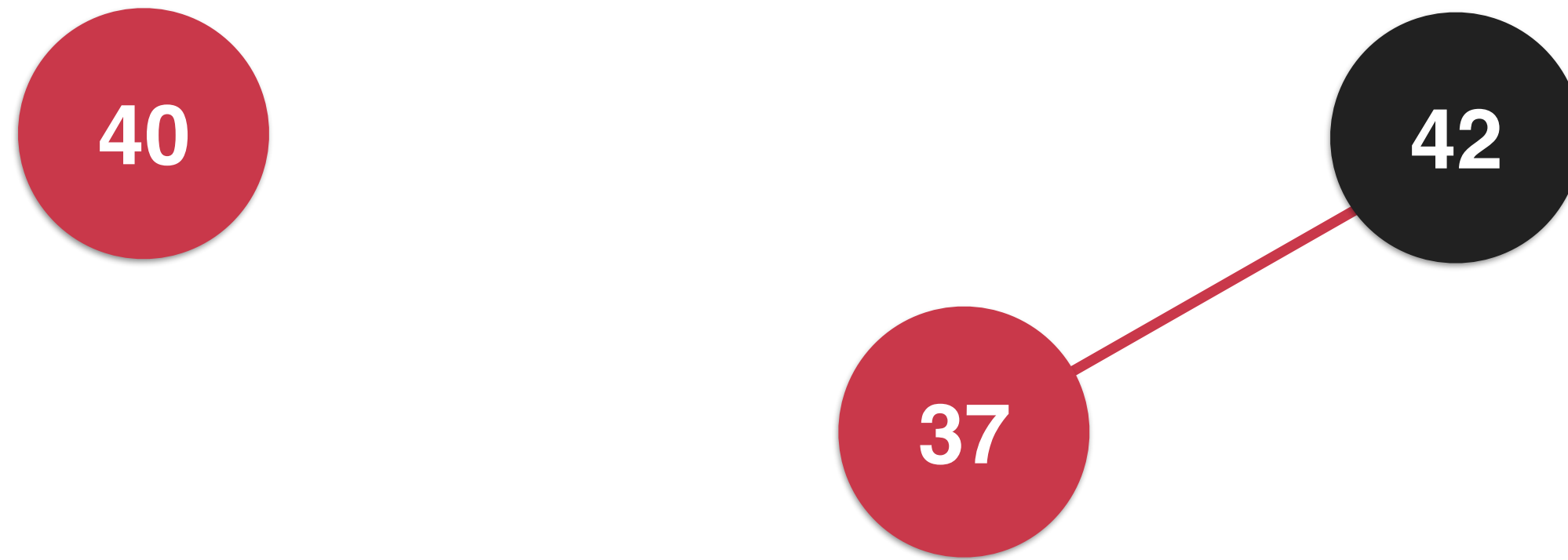
# 红黑树添加新元素



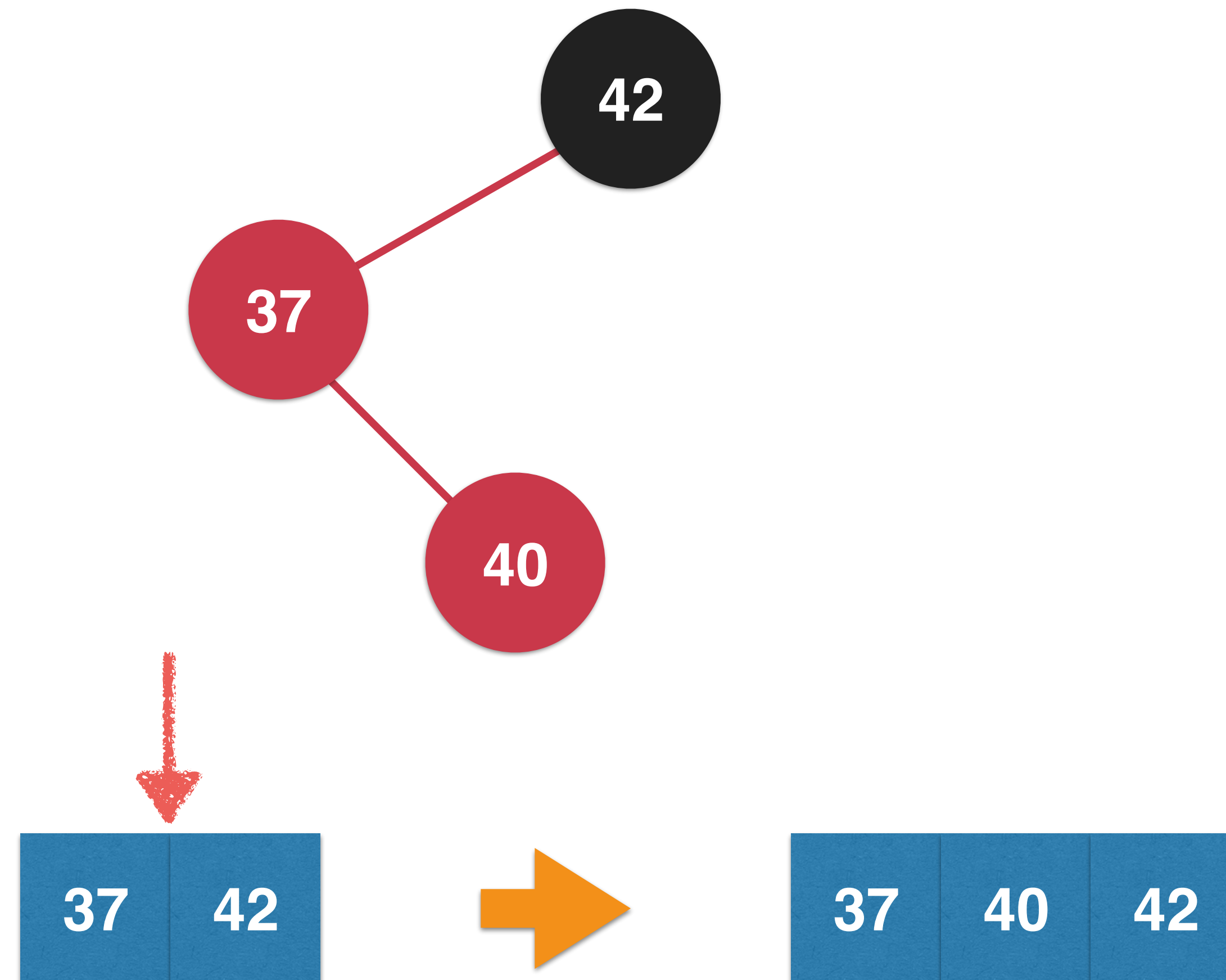
# 红黑树添加新元素



# 红黑树添加新元素

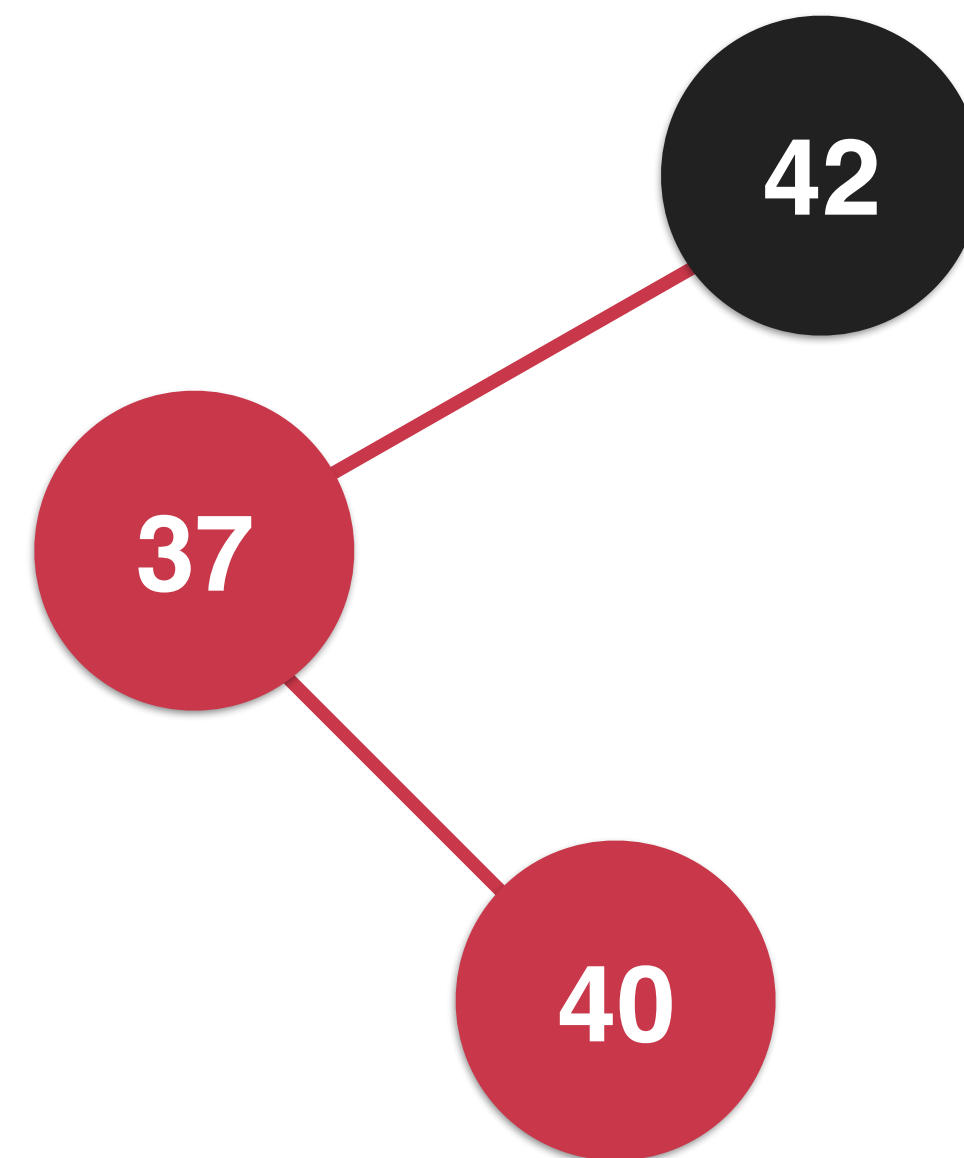


# 红黑树添加新元素



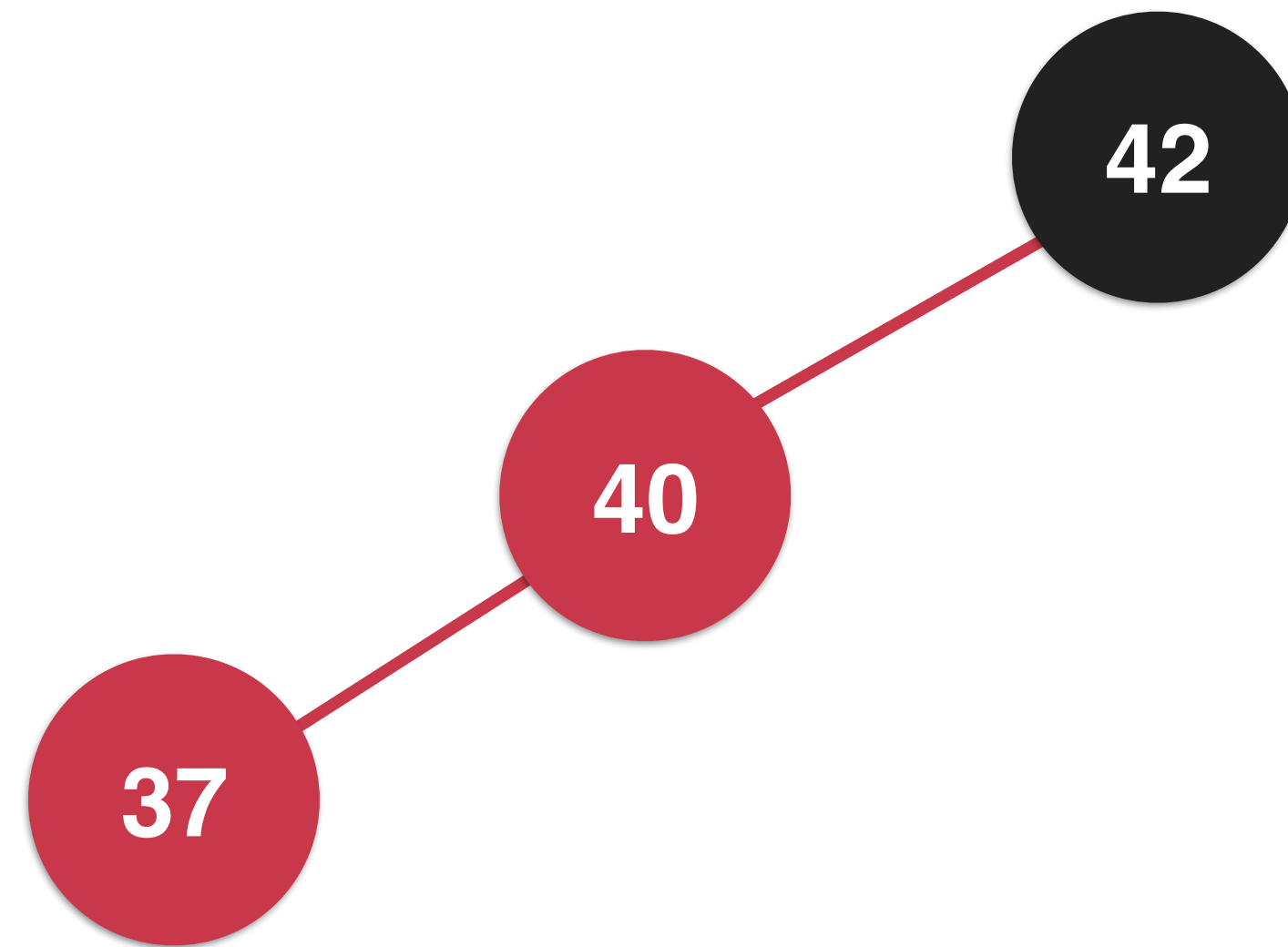


# 红黑树添加新元素



左旋转

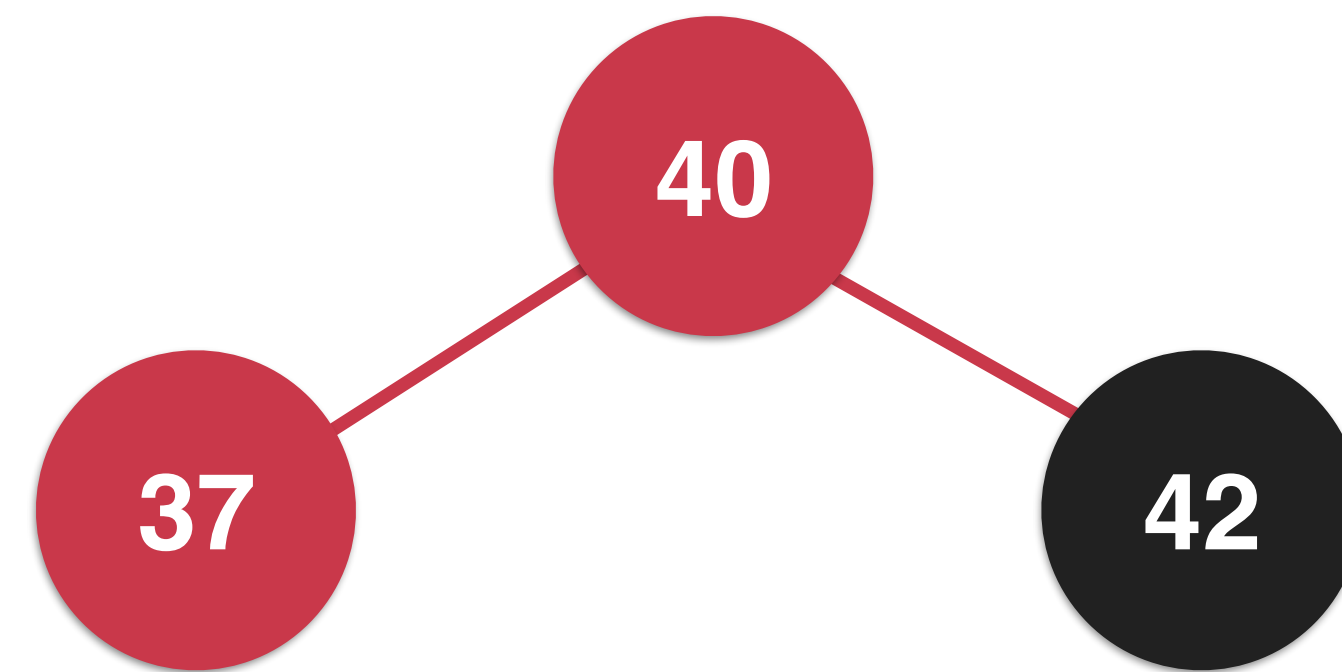
# 红黑树添加新元素



左旋转

右旋转

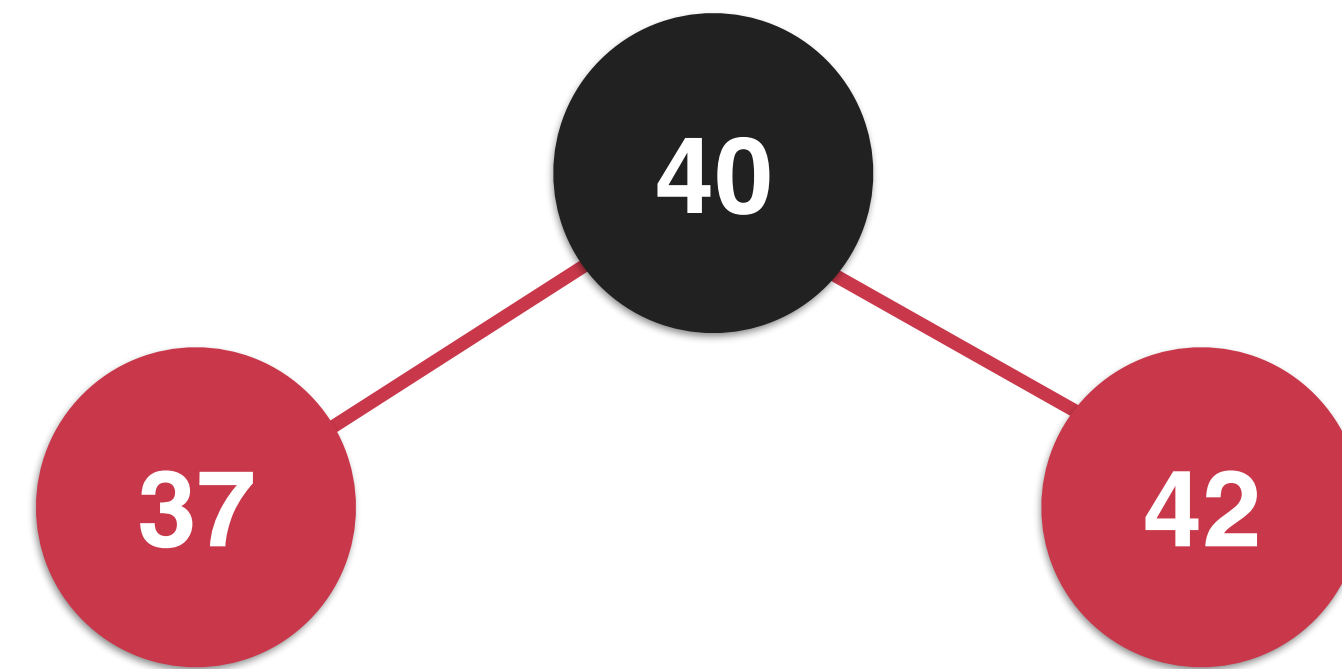
# 红黑树添加新元素



左旋转

右旋转

# 红黑树添加新元素

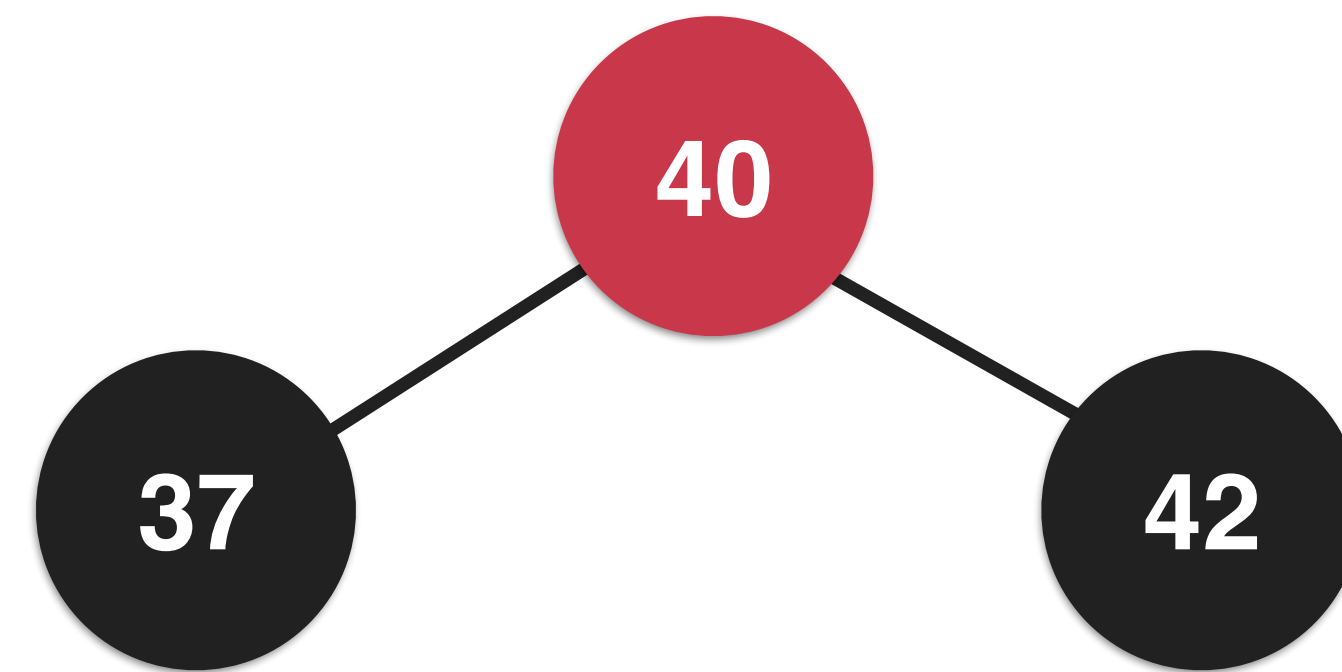


左旋转

右旋转

颜色翻转

# 红黑树添加新元素



左旋转

右旋转

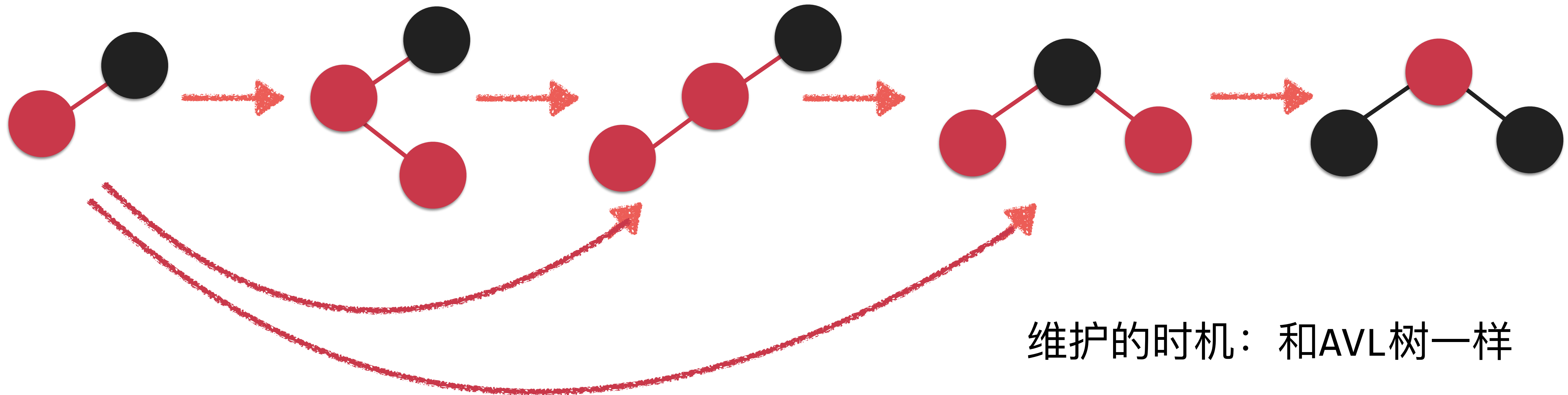
颜色翻转

# 红黑树添加新元素

左旋转

右旋转

颜色翻转



维护的时机：和AVL树一样

添加节点后回溯向上维护

实践： 向红黑树中添加新节点

# 红黑树的性能



# 实践：红黑树的性能

# 红黑树的性能总结

对于完全随机的数据，普通的二分搜索树很好用！

缺点：极端情况退化成链表（或者高度不平衡）

对于查询较多的使用情况，AVL树很好用！

红黑树牺牲了平衡性（ $2\log n$ 的高度）

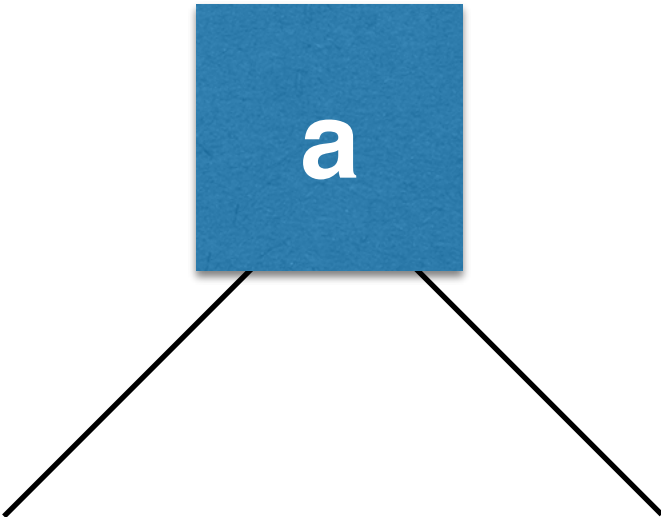
统计性能更优（综合增删改查所有的操作）

更多和红黑树相关的问题

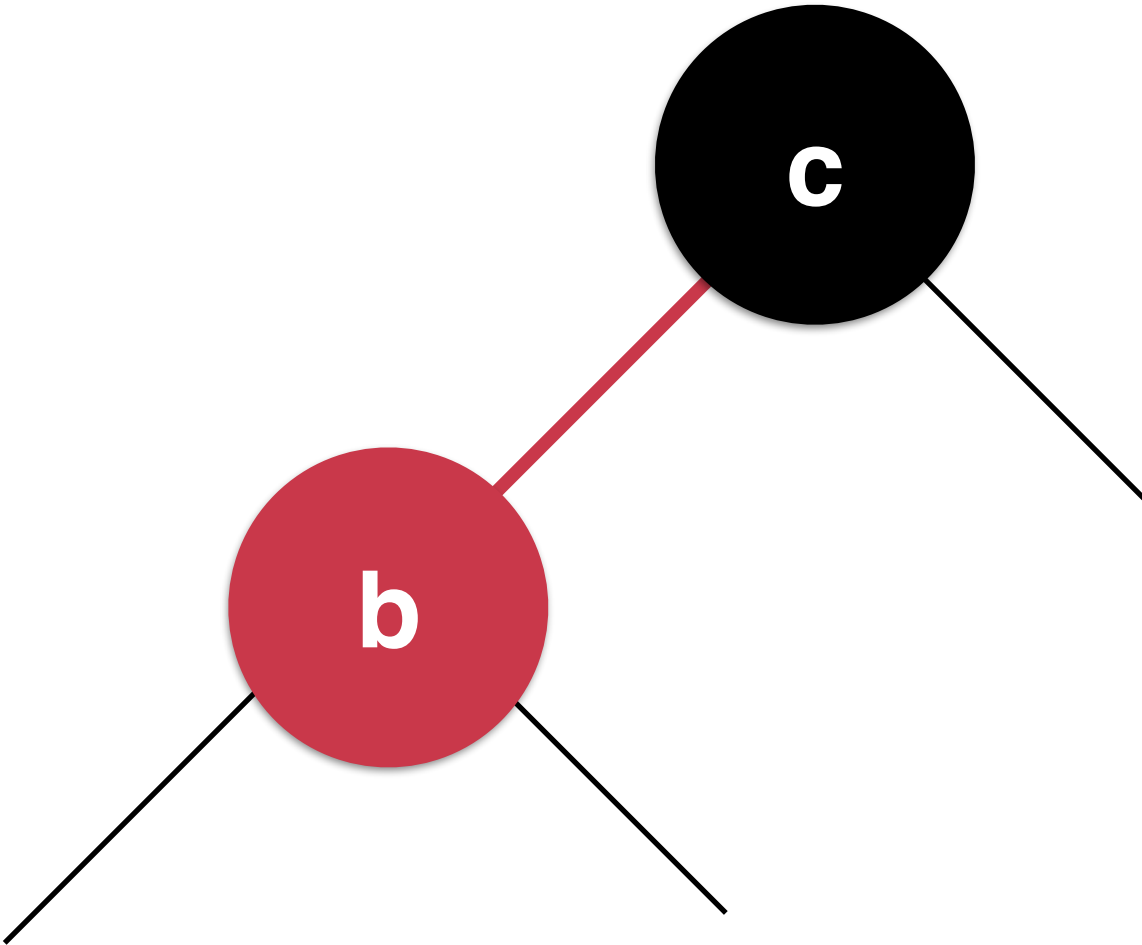
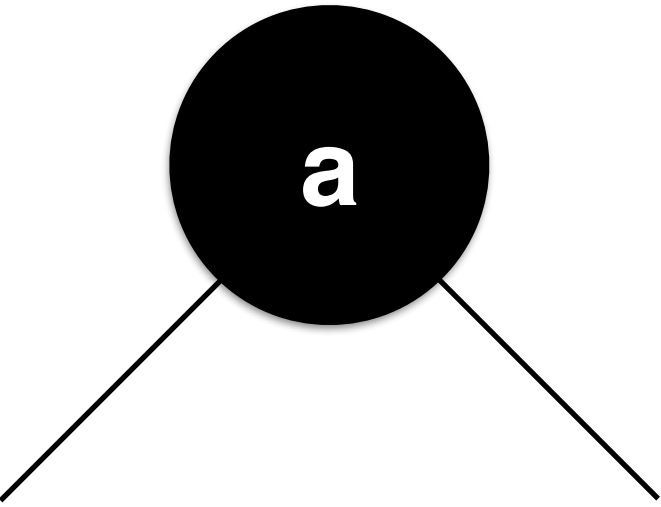
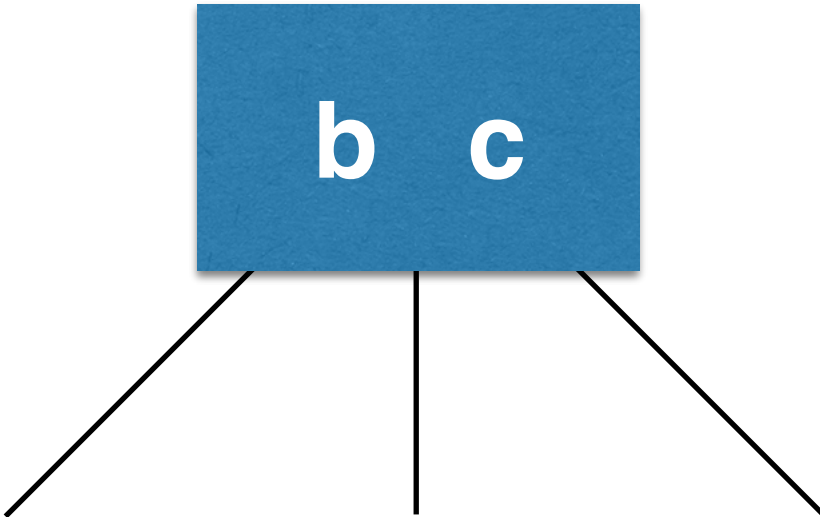
# 红黑树中删除节点

# 左倾红黑树

2节点

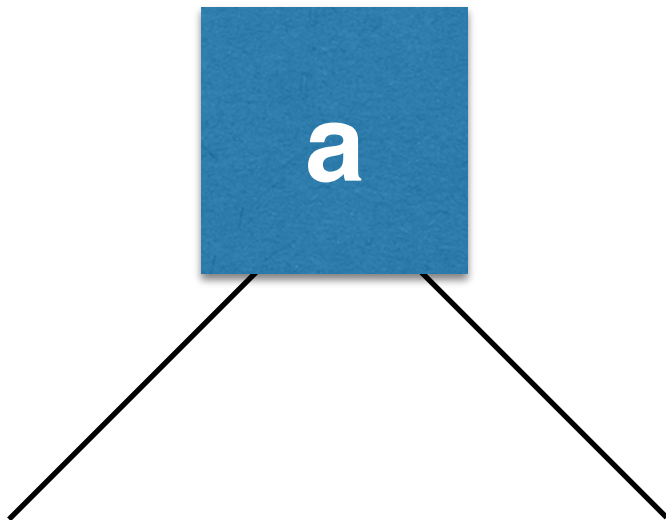


3节点

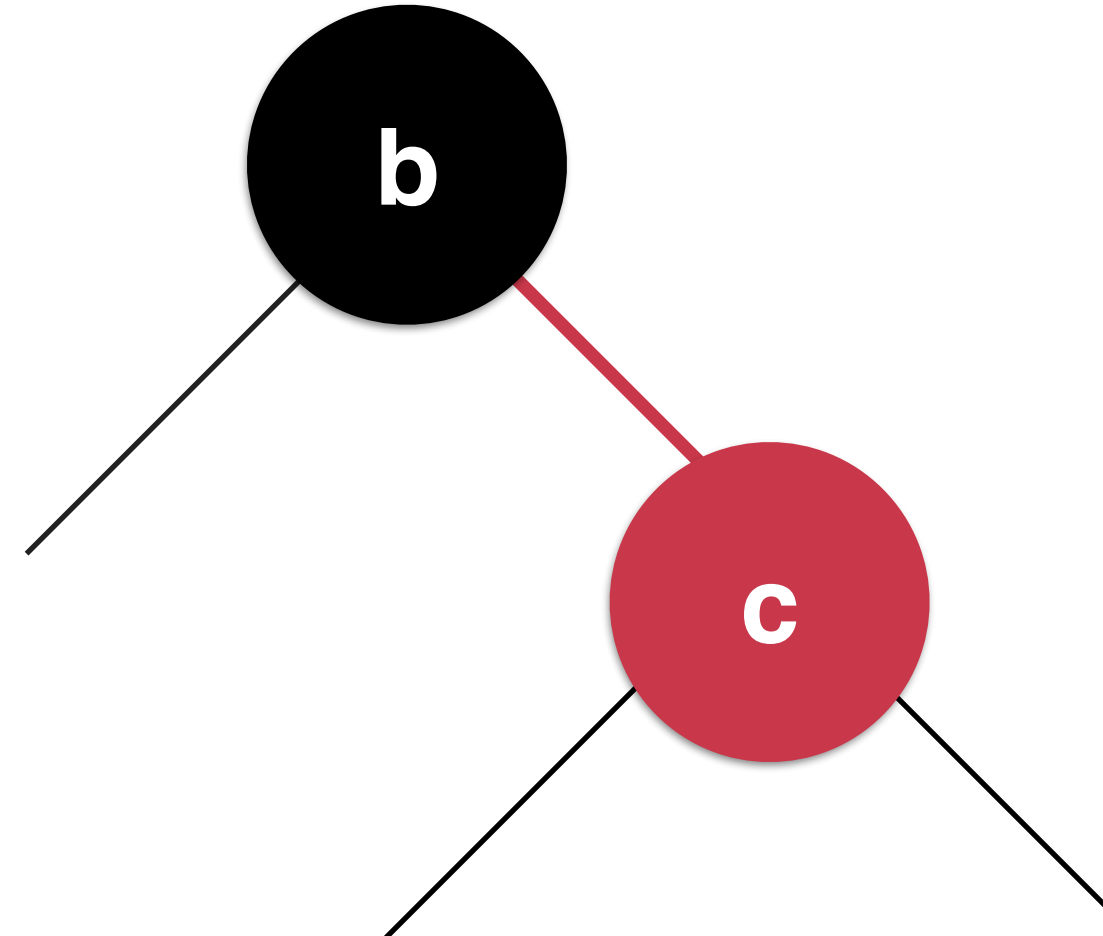
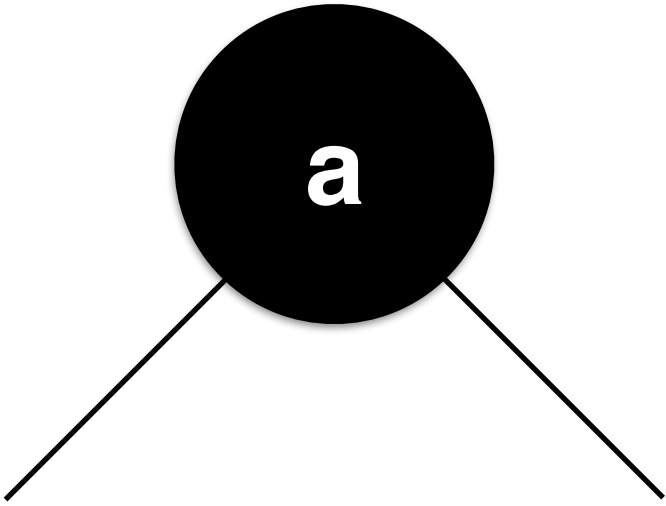
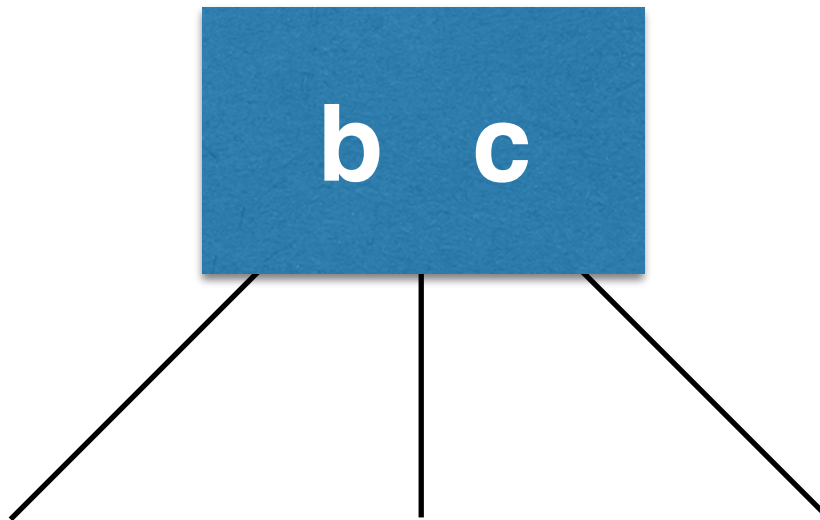


# 右倾红黑树

2节点



3节点



红黑树统计性能更优

# 另一种统计性能优秀的树结构： Splay Tree（伸展树）

局部性原理：刚被访问的内容下次高概率被再次访问



# 基于红黑树的Map和Set

java.util中的TreeMap和TreeSet基于红黑树：)

# 红黑树的其他实现

算法导论中红黑树的实现

# 红黑树

# 其他

欢迎大家关注我的个人公众号：是不是很酷



# 玩儿转数据结构

liuyubobobo