Simulation of Matter under Extreme Conditions
Introduction

Stephen Millmore

Laboratory for Scientific Computing, University of Cambridge

Simulation of Matter under Extreme Conditions

- This course follows on from Numerical Methods for Compressible Fluid Dynamics
- Various extensions to the work from this course are considered, whilst maintaining a regular Cartesian mesh and a single system of evolution equations
- Six lectures and six practicals
- Again, lectures will be recorded
- Practicals will initially build on Compressible Fluid Dynamics work, and will then extend to incorporate the topics from this course
- Again, progress in the practicals may be used when assigning research projects
- And also again, additional practical sessions are scheduled for after the exams

- The lectures

1. Mesh generation for continuum modelling (Part 1) (Dr N. Gokhale)
2. Mesh generation for continuum modelling (Part 2) (Dr N. Gokhale)
3. Source terms and equations of state (Part 1)
4. Equations of state (Part 2) and magnetohydrodynamics (Part 1)
5. Magnetohydrodynamics (Part 2)
6. Elastoplastic solids

- Some topics are longer than others, hence the lecture structure

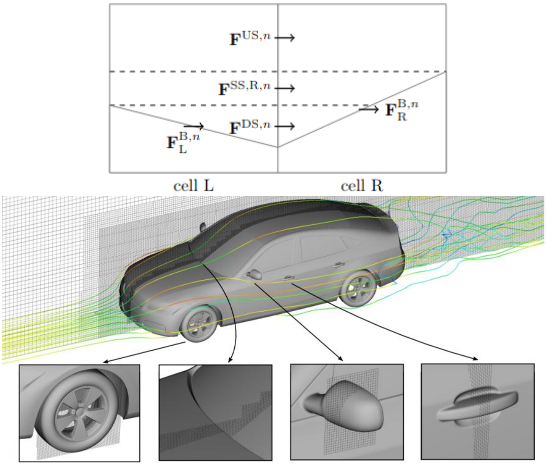
Advancing our modelling

- In Numerical Methods for Compressible Fluid Dynamics, we developed a range of numerical methods for solving non-linear hyperbolic systems of equations
- With this focus, we made several assumptions about what we were going to solve:
    1. Everything was inviscid and had no body forces acting upon it
    2. $p = (\gamma - 1)\,\rho\varepsilon$ - everything is an ideal gas (whatever that means)
    3. Every (compressible inviscid continuum) material can be described by the Euler equations
    4. Everything within a simulation is the same material
- To the best of my knowledge, none of this is true
- In this course, we shall expand our modelling to deal with points 1-3 here
- Dealing with point 4 is probably the most challenging, and requires its own course (Multiphysics Modelling for Four States of Matter)

Do we need to start again?

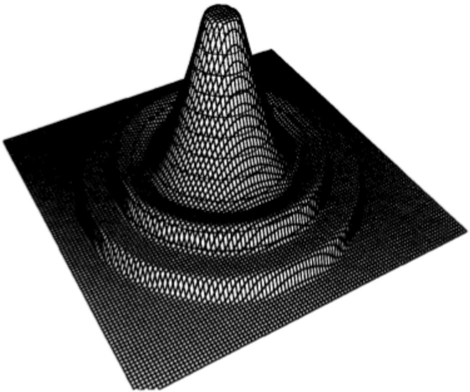
- Despite all the simplifications we made during Numerical Methods for Compressible Fluid Dynamics, this course builds naturally upon that material
- The numerical methods we developed are valid for **all** systems of conservation laws, we will continue to be working with these
- This is why we spent so much time considering vector forms and flux vectors
- We have seen that the one-dimensional methods are can still be used when we increase the dimensionality of the system
- The key idea is that any code written for solving the compressible Euler equations **does not need to be re-written from scratch** as we include more physics in the system
- In most cases, function calls need to change, or new functions need to be implemented, within an existing framework

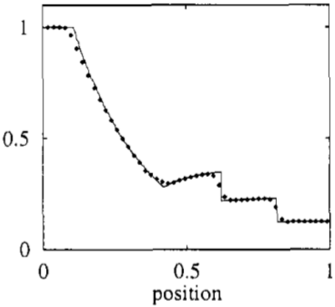
Cut cell methods



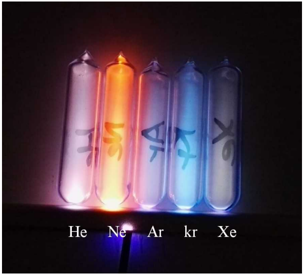
Source terms

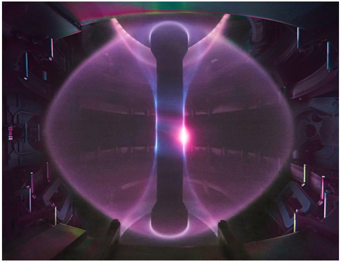
$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathbf{f}(\mathbf{u}) = \mathbf{s}(\mathbf{u}, \mathbf{x})$$





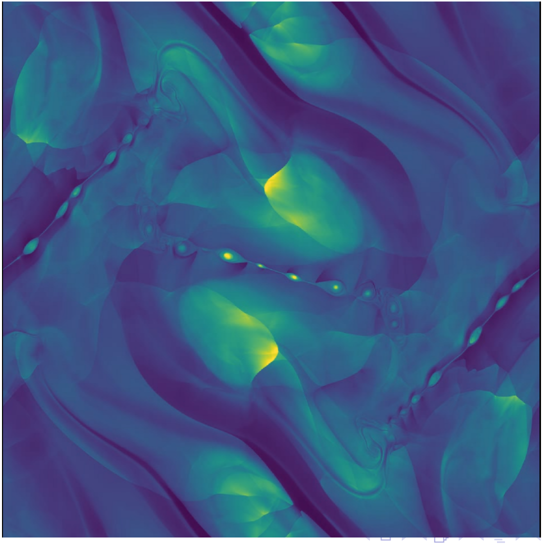
Equations of state





Magnetohydrodynamics

$$\mathbf{u} = \begin{pmatrix} \rho \\ \rho v_x \\ \rho v_y \\ \rho v_z \\ E \\ B_x \\ B_y \\ B_z \end{pmatrix}$$



Elastoplastic solids