Computational Continuum Modelling
Practical 1: The advection equation

Following the structure of the code shown in lectures (or otherwise), implement a numerical solver for the advection equation

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$

Unless otherwise stated, throughout this practical, chosen values for this equation and its numerical solution are:

- $x \in [0:1]$
- $t \in [0:1]$
- $a = 1$
- 100 points in the domain, spacing $\Delta x = 1/100$

Two different functions should be considered for initial data:

$$u_0(x) = \sin(x)$$

or

$$u_0(x) = \begin{cases} 1 & 0.25 \leq x \leq 0.75 \\ 0 & \text{otherwise} \end{cases}$$

Your code should output both data at both the initial and final time of the simulation. This data should be plotted and stored in a document. Any plotting software is fine, all my plots are produced in gnuplot, which is freely available and runs well on linux machines. If you are new to plotting software, the command:

```
plot 'advectionResults.dat' with linespoints
```

will plot data (as output in the lecture slides) from the appropriately named file.

Exercises:

1. Run the code with the spatial derivative approximated through forward, backward and centred differences, and plot the results.
2. Now set $a = -1$ and repeat the tests for the three derivatives - describe the results (it may not be necessary to plot them)

3. Now try $a = 0.5$ and $a = 2$ - plot any interesting results (i.e. where behaviour has changed from previous tests)

4. Can you adjust t and Δt such that you get the same results for these tests as you did for the first? Recalling the physical quantities in the origin of the advection equation, and using dimensional analysis, may help.

5. Now return to $a = 1$, choose an appropriate differencing scheme, and set $\Delta t = 0.9\Delta x$ - what happens to the solution as you vary the final time (compare times 1,2,5 or 10) - what do you think is happening here?

6. Repeat the previous test, but keep the final time fixed, and vary the number of points used instead - what do you think is happening here?