Introduction to Computational Multiphysics
Practical 1: Mixed-material exact Riemann solvers

Exact Riemann solvers solve the equation

$$f_R\left(p^*, \mathbf{u}_R\right) + f_L\left(p^*, \mathbf{u}_L\right) + \Delta v = 0.$$

One important feature of this formulation is that f_R and f_L are computed completely independently, there is no requirement for these to be computed from the same material. As a result, Riemann problems can be extended to multi-material cases, with each constant state belonging to a different material.

Exercises:

The first task is to modify your exact Riemann problem solver to allow for an interface between two different ideal gases. This is fairly straightforward, effectively you can now consider $\mathbf{u}_L$ to have an ideal gas constant γ_L, and $\mathbf{u}_R$ to have an ideal gas constant γ_R. To achieve this, a material's adiabatic index should now be passed as a function to $f_{L/R}(p^*, \mathbf{u}_{L/R}, \gamma_{L/R})$.

Note that this will only be used to get an exact solution. In a few more practicals, we will get to numerical simulations.

To test your exact solver, first use the moving contact discontinuity from the first lecture. Further tests are available from Fedkiw et al., "A Non-oscillatory Eulerian Approach to Interfaces in Multimaterial Flows (the Ghost Fluid Method)" in Section 5.2. Test A from the paper can be run immediately. Tests B-D require some extra calculation. These consider a shock interacting with an interface, so are not set up as a Riemann problem. However, there is enough information to work out when the solution evolves to form a Riemann problem, and from this, compute the exact solution.

The second task is to modify your exact Riemann solver to allow one (or both) of the materials to be a stiffened gas. This requires a slight modification of the functions for the solution:

$$f\left(p^*, \mathbf{u}_K\right) = \begin{cases} \left(p^* - p_K\right)\left(\frac{A_K}{p^*+B_K}\right)^{1/2} & p^* > p_K \\ \frac{2c_{s,K}}{\gamma_K-1}\left[\left(\frac{p^*+p_{\infty,K}}{p_K+p_{\infty,K}}\right)^{\frac{\gamma_K-1}{2\gamma_K}} - 1\right] & p^* \leq p_K \end{cases}$$

$$A_K = \frac{2}{\left(\gamma_K+1\right)\rho_K}, \qquad B_K = \frac{\gamma_K - 1}{\gamma_K + 1}p_K + \frac{2\gamma_K p_{\infty,K}}{\gamma_K+1}.$$

The key test for this modification comes from a water-gas shock tube test from Chinnayya, Daniel and Saurel, "*Modelling detonation waves in heterogeneous energetic materials*", (2003), described in section 7.1 and plotted in figure 12. The initial data for this test is

$$(\rho, v, p)_1^T = \left(1000, 0, 10^9\right)^T, \quad \gamma_L = 4.4, p_{\infty,L} = 6 \times 10^8$$

$$(\rho, v, p)_2^T = \left(50, 0, 10^5\right)^T, \quad \gamma_R = 1.4, p_{\infty,R} = 0$$

In the paper there is a typo in the initial pressure for the water region (not saying there isn't also a typo in mine...). For this test, the initial interface is at $x = 0.7$, with material 1 present in $x < 0.7$, and the final time stated is $t = 237.44\,\mu$s, though as with previous tests, this will need to be modified for the exact solution from the time a Riemann problem exists within the solution.