Advanced Continuum Modelling
Practical 3: Geometric source terms

This practical requires you to modify your existing Euler equation codes to work in cylindrical and spherical symmetry.

Your one-dimensional code can be modified to work in both coordinate systems (as well as Cartesian coordinates), and your two-dimensional code can be modified to work in cylindrical coordinates. Using first-order in time splitting, and a first order ODE solver are recommended - get these working before trying anything more adventurous!

Because cylindrical coordinates require $r > 0$, your domain will always start at zero, and transmissive boundary conditions do not make sense here. The technical term for the boundary conditions you need is **geometric boundaries** - vector quantities normal to the boundary are reflected. However, they are often referred to as **reflective boundaries**, because for the Euler equations, coding this behaviour is identical. For these conditions, for generality, we assume that cell 0 is the first cell of the computational domain (not the first of the vector):

- Density, pressure and z-velocity (if 2D) are treated exactly as:

$$\rho_{-1} = \rho_0, \quad \rho_{-2} = \rho_1$$

  and the same for other variables. Basically your variables are **mirrored** along $r = 0$

- For r-velocity, we need to mirror **and** reverse the velocity

$$v_{r,-1} = -v_{r,0}, \quad v_{r,-2} = -v_{r,1}$$

Also note that r and z can be treated as x and y in your code.

Exercises:

There are two key tests to consider, the cylindrical explosion from the previous practical, and a spherical explosion, both detailed in chapter 17 of Toro's book. In all cases, we have

$$\mathbf{w}_{\text{in}} = (\rho, v_x, v_y, p)^T = (1, 0, 0, 1)$$

$$\mathbf{w}_{\text{out}} = (\rho, v_x, v_y, p)^T = (0.125, 0, 0, 0.1)$$

with $\mathbf{w}_{\text{in}}$ is the region inside a circle, radius $R = 0.4$. In these tests, the circle should be centred on $(0, 0)$, and the r-domain should run from $r \in [0, 1]$. A final time of $t = 0.25$ should be used.

- **1D tests**: When setting up 1D initial data, you are effectively using Toro's test 1, just with the initial discontinuity at a slightly different position. The only change between the spherical, cylindrical and Cartesian tests is the application of your source term. Try to match Toro's results through a slice (as this is 1D, you can use a high resolution).

- **2D tests**: In a domain $z \in [-1, 1]$, you can set up initial data using a hemisphere $\sqrt{r^2 + z^2} = 0.4$. In otherwords, this is identical initial data to the cylindrical explosion test, but with a different domain centre (everything is shifted down and left by 1), and a reduced r-domain size. Cylindrical source terms should then allow this to reproduce the spherical explosion.

You can also consider following Toro's other example, the cylindrical (or spherical) implosion - reverse the 'in' and 'out' variables in the initial data above.