Advanced Continuum Modelling
Practical 4: Beyond the ideal equation of state

Whilst it is important to consider various equations of state, finding test problems is hard. Throughout the literature, when going beyond ideal gas, there is a tendency to go straight to multiphysics simulations. It's not that shock tube tests don't exist, it's just that papers get distracted by exciting simulations. I've found a few tests that are effectively single-material (though both are actually implemented in multiphysics codes). All tests in this practical are 1D tests.

Whilst object-orientated programming allows for nice ways to incorporate multiple equations of state, it is recommended that you wait until you've got working underlying code before trying to implement equation of state classes. Initially, it is probably best to use simple if statements inside any function that deals with equation of state-specific variable conversions, such as primitive to conservative calculations.

Exercises:

1. Stiffened gas:

   Finding tests for stiffened gas Riemann problems is generally not easy - as soon as people start using these, they often head straight to multiphysics and consider air/water interfaces. Two tests are available on the Clawpack website (developed, in part, by Leveque), see `http://www.clawpack.org/riemann_book/html/Euler_Tammann.html`.

   The first test is a modification of the Sod test (Toro's first test),

   $$\mathbf{w}_L = (\rho, v_x, p)^T = (1500, 0, 3000 \times 101325), \quad x < 0.5$$

   $$\mathbf{w}_R = (\rho, v_x, p)^T = (1000, 0, 101325), \quad x > 0.5$$

   In this test, $\gamma = 7.15$ and $p_\infty = 3 \times 10^8$ Pa. Unfortunately, a final time is not given, so you'll have to reach one by trial and error. Try something around 10^{-4} s to start with, and see what happens, as this is roughly the order of magnitude of length of domain/speed of sound in water, so the waves should stay in the domain.

   A second test has

   $$\mathbf{w}_L = (\rho, v_x, p)^T = (1000, -v, 2 \times 101325), \quad x < 0.5$$

$$\mathbf{w}_R = (\rho, v_x, p)^T = (1000, v, 2\times 101325)\,, \quad x > 0.5$$

with no value for v given. If a value of $v = 100$ is used, this test develops a strong rarefaction test (similar to Toro's test 2). However, in this case, it is designed to show one area where using the stiffened gas for water leads to unphysical behaviour - a sustained region of negative pressure. As we shall see later, this would normally result in cavitation. Again, the final time is undefined, and a similar value to the previous test should be used.

Consider also running these tests with the equation of state parameters for water given in the lecture - do you see a difference?

2. JWL equation of state:

   This is very much a preliminary test if you are going to look at detonations and explosives, and it is ideal for this practical since it uses only the Euler equations with the JWL equation of state, avoiding all chemistry effects. Here we have

$$\mathbf{w}_L = (\rho, v_x, p)^T = \left(1700, 0, 10^{12}\right)\,, \quad x < 0.5$$

$$\mathbf{w}_R = (\rho, v_x, p)^T = \left(1000, 0, 5\times 10^{10}\right)\,, \quad x < 0.5$$

   This test is run to $t = 12\times 10^{-6}\,\mathrm{s}$, and equation of state parameters are:

$$\rho_0 = 1840 \quad A = 854.5\times 10^9 \quad B = 20.5\times 10^9 \quad R_1 = 4.6 \quad R_2 = 1.35$$

$$\Gamma = 0.25, \Delta\varepsilon = 0, C = 0$$

   Results for this test are in Shyue "A Fluid-Mixture Type Algorithm for Compressible Multicomponent Flow with Mie-Grüneisen Equation of State". This paper also considers the speed of sound; this is something you can derive for yourself, but may not want to.

Feel free to create your own shock tube tests for these equations of state too. Specially for water, where you have a good idea of reasonable initial data.