Advanced Continuum Modelling
Practical 2: Two-dimensional Euler equations part 2

This practical takes the work from the partially-two dimensional solver of the previous practical, and makes it fully two-dimensional.

To do this, we now need to add the y-dimensional flux update to the system. This should be done in a dimensionally split manner, but feel free to use the unsplit method as an intermediate step if it helps with debugging.

In two dimensions, you now have two flux functions, $\mathbf{f}$ and $\mathbf{g}$. It is up to you whether you want to deal with this through separate functions (easier to read) or a single flux function which also takes a direction argument to compute the correct flux (more generalisable, and avoids some code duplication).

Similar choices can be made for the numerical flux, where the direction variable could allow you to switch between using Δx or Δy, and/or whether you compare (i, j) and $(i+1, j)$ or (i, j) and $(i, j+1)$.

Exercises:

In all tests, boundary conditions are transmissive for all variables.

1. Repeat Toro's tests 1 and 3 from the previous practical, but with your left state existing where $y < 0.5$, and right state where $y > 0.5$. Check that results are identical (when rotated 90°).

2. Now set up one of these tests with the left state $x - y < 1$, or similar, such that the initial Riemann problem discontinuity is not aligned to the computational grid. How well do results match? Generate output often at early times, what happens at your boundaries?

3. Now for a real 2D test, Toro's 'Cylindrical explosion', where explosion means area of high pressure propagating outwards. The initial data (equation 17.3 in Toro's book) is:

$$\mathbf{w}_{\text{in}} = (\rho, v_x, v_y, p)^T = (1, 0, 0, 1)$$

$$\mathbf{w}_{\text{out}} = (\rho, v_x, v_y, p)^T = (0.125, 0, 0, 0.1)$$

where we have a square domain ($x \in [0, 2]$, $y \in [0, 2]$), and $\mathbf{w}_{\text{in}}$ is the region inside a circle, radius $R = 0.4$, centred on $(1, 1)$, and $\mathbf{w}_{\text{out}}$ is the rest of the domain. How does a 1D slice out of this simulation compare to Toro's test 1?

4. 2D Riemann problems:

$$\mathbf{w} = (\rho, v_x, v_y, p)^T = \begin{cases} (0.5313, 0, 0, 0.4)^T & x > 0.5, y > 0.5 \\ (1, 0.7276, 0, 1)^T & x < 0.5, y > 0.5 \\ (0.8, 0, 0, 1)^T & x < 0.5, y < 0.5 \\ (1, 0, 0.7276, 1)^T & x > 0.5, y < 0.5 \end{cases}$$

and

$$\mathbf{w} = (\rho, v_x, v_y, p)^T = \begin{cases} (1, 0.75, -0.5, 1)^T & x > 0.5, y > 0.5 \\ (2, 0.75, 0.5, 1)^T & x < 0.5, y > 0.5 \\ (1, -0.75, 0.5, 1)^T & x < 0.5, y < 0.5 \\ (3, -0.75, -0.5, 1)^T & x > 0.5, y < 0.5 \end{cases}$$

These tests, originally from Liu and Lax, "Solution of two-dimensional Riemann problems of gas dynamics by positive schemes" (1998), which the University doesn't have access to, can be found in Pan and Xu "A Compact Third-Order Gas-Kinetic Scheme for Compressible Euler and Navier-Stokes Equations" 10.4208/cicp.141214.140715s.