Computational Continuum Modelling
Practical 5: SLIC for the Euler equations

The next step in developing Euler equations solvers is to move to a second order, slope limited numerical method. SLIC is the natural candidate since it can be built directly on your FORCE scheme, performing the limited reconstruction and the half-time step update before calling the FORCE update.

It is suggested that all tests are implemented initially with the Minbee limiter before moving onto the other, more more restrictive limiters. In all cases, it is important to make sure your code can deal with the case in which the ratio variable, r, is singular. All four limiters have the same behaviour in the case $r \to \infty$, so this should be clear.

When implementing second order methods, the opportunity for implementation bugs increases substantially. It can sometimes be challenging to work out when there is a subtle bug (wild oscillations and incorrect wave speeds are easy bugs to spot). A stable second order scheme should always provide sharper capturing of shock waves and contact discontinuities than a first order one, and should always converge quicker (globally) - there should be a much more noticeable difference between 100 and 200 cells for SLIC than for FORCE. Several of Toro's five tests can make this even harder - some of them are designed such that some combinations of scheme and limiter will fail; when in doubt, fall back to Minbee and test 1.

A few things to do when code is not working as expected:

- Force $\xi(r) = 0$ - this should revert to a FORCE scheme. If there is something wrong with the half time step update, this should become apparent.
- Construct simple initial data and check things are working as expected.
    1. Constant initial data - nothing should happen
    2. Stationary contact discontinuity (e.g. Toro's first test with $p_L = p_R = 1$) - again nothing should happen
    3. Moving contact discontinuity - as last test, but with a constant, non-zero velocity in the domain - feature should propagate at this velocity (and gently smear)
- Output data after a single time step - this can highlight if something is going wrong for a single variable only. For example, an error in the momentum update would be seen at time step 1, even if energy and density are correctly evolved. By time step 2, this would propagate into the other variables.

Exercises:

The five tests from Practical 4 should be repeated for the SLIC solver. For the Minbee limiter, demonstrate that the second order method results in more accurate solutions for these tests. Then consider other limiters, and look at the effect these have on the simulations.

When presenting results for these tests, it is up to you as to how to convey the information. For example, it is not probably not worth creating plots for all five tests for all four limiters, and certainly not at a variety of resolutions too.