Introduction to Computational Multiphysics
Practical 5: Magnetohydrodynamics.

This task this practical is to develop a one-dimensional MHD code. In 1D, divergence errors cannot build up (we have a constant B_x which will not evolve). The MHD code should be a modification of your SLIC code, so we also do not need to worry about the issues with intermediate variables in Riemann solvers.

The changes you will need to make are to go from a 3-component array to a 7-component array of variables (or 4-component if you start with part 1 of the first ACM practical), and change how the flux and the conversions between primitive and conserved variables happen. You will also need to ensure that your maximum wave speed is given by a max $= |\mathbf{v}| + c_f$, using the fast wave speed.

All tests should be in Cartesian coordinates with transmissive boundary conditions.

Exercises:

1. Check that your code still works for the Euler equations

   Use Toro's first test,

   $$\mathbf{w}_{\mathrm{L}} = (\rho, v_x, p)^T = (1, 0, 1)$$

   $$\mathbf{w}_{\mathrm{R}} = (\rho, v_x, p)^T = (0.125, 0, 0.1)$$

   where we also have $v_y = v_z = 0$ and $\mathbf{B} = 0$ everywhere, and $\gamma = 1.4$ and the initial discontinuity is at $x = 0.5$ on a domain $x \in [0, 1]$. This should not change the solution that you have seen before, i.e. it is a test you don't evolve the magnetic field when you shouldn't. Ensure that your results match those you've produced previously, and that magnetic field and y- and z-velocity are zero (don't worry about producing plots of these tests, just verify for yourself).

2. The Brio and Wu test, named for the authors of the paper "*An upwind differencing scheme for the equations of ideal magnetohydrodynamics*" with initial data:

   $$\mathbf{w}_{\mathrm{L}} = (\rho, v_x, v_y, v_z, p, B_x, B_y, B_z)^T = (1, 0, 0, 0, 1, 0.75, 1, 0)$$

   $$\mathbf{w}_{\mathrm{R}} = (\rho, v_x, v_y, v_z, p, B_x, B_y, B_z)^T = (0.125, 0, 0, 0, 0.1, 0.75, -1, 0)$$

   Whilst this is an obvious modification of Toro's test 1, to follow the results from Brio and Wu, the domain should be $x \in [0, 800]$ and the initial interface at $x = 400$, with a final time $t = 80$. For this test, $\gamma = 2$. If you read the original paper, they talk about using a fixed time step - you do not need to do this, use your maximum wave speed approach to get optimal time steps (this shows how much has changed since 1987...)

You should plot density, pressure, $x-$ and y-velocity and B_y for this case. All other variables should remain zero, or constant (you should check this, but don't need to plot it.) Results are in this paper, you can probably get better results than they could though with more suitable numerical methods and modern computational power.

3. Repeat the previous test, but switch the values of B_y and B_z. There should be no differences between the results, except all y-quantities become z-quantities. Plot v_z and B_z to demonstrate this.

4. Ryu and Jones test 2A, from Ryu and Jones, "*Numerical Magnetohydrodynamics in Astrophysics: Algorithm and Tests for One-dimensional Flow*" (1995). A test which has non-zero components for all variables, and thus has a full seven-wave structure.

$$\mathbf{w}_\mathrm{L} = (\rho, v_x, v_y, v_z, p, B_x, B_y, B_z)^T = \left(1.08, 1.2, 0.01, 0.5, 3.6/\sqrt{4\pi}, 2/\sqrt{4\pi}, 2/\sqrt{4\pi}, 0.95\right)$$

$$\mathbf{w}_\mathrm{R} = (\rho, v_x, v_y, v_z, p, B_x, B_y, B_z)^T = \left(1, 0, 0, 0, 4/\sqrt{4\pi}, 2/\sqrt{4\pi}, 2/\sqrt{4\pi}, 1\right)$$

   In this case, the domain is $x \in [0, 1]$ with the initial discontinuity at $x = 0.5$, and this is the first test which uses the standard $\gamma = 5/3$. The final time is $t = 0.2$. The same set of variables as the Brio and Wu test should be plotted, and the results are again in the paper.

If you wish to test your MHD code a little more, see Ryu and Jones. You can also use Dai and Woodward's paper, "*An Approximate Riemann Solver for Magnetohydrodynamics*", unfortunately, this second paper gives solutions for most tests as tables only, and does contain some typos and differences in scaling, quantifying the values of the intermediate states, but not plotting them. In these tables, the initial data is given by rows 'R1' and 'R8', the domain and initial discontinuity are the same as the Dai and Woodward test above, and B_x and γ are given as a note below each table.