Advanced computational algorithms for PDEs: Practical #1

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1 The FORCE scheme

- Within the finite volume scheme *EulerFV1d.cpp* implement the First–Order Centred (FORCE) flux.
- Assess the performance of the FORCE scheme using tests 1, 2 and 3 of Toro's book (the initial conditions of which are listed for convenience in Tab.1). For each test, compare the numerical solution with the exact solution. This can be computed using the ComputeExactSolution routine already available within *EulerFV1d.cpp*; see Fig.1 as a reference.
- (Optional) Consider also tests 4, 5 and 6 of Toro's book and display the obtained numerical solution against the exact one.

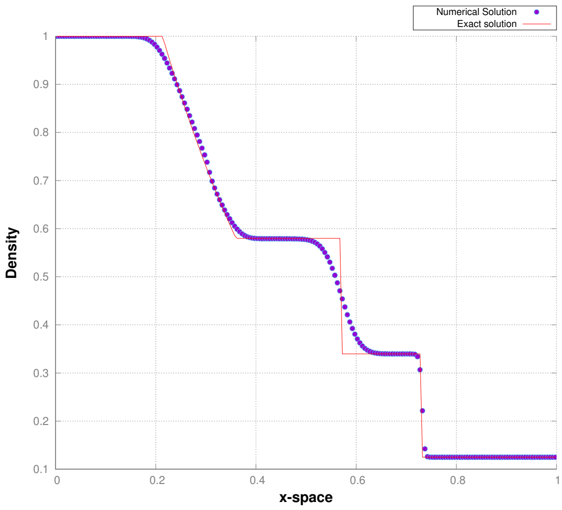

Figure 1: Test 1; HLL scheme; CFL $= 0.9$; 200 cells are used.

2 The HLLC Approximate Riemann Solver

- Modify the basic HLL Riemann solver (already available within the finite volume scheme *EulerFV1d.cpp*) to obtain the three-wave HLLC approximate Riemann solver.
- Assess the performance of the resulting numerical scheme using tests 1, 2 and 3 of Toro's book.
- Compare the results of HLL and HLLC for an isolated contact discontinuity (tests 6 and 7 in Tab.1).
- Consider the advection of a chemical species of concentrations q by the flow speed u, i.e:

$$\partial_t q + u \partial_x q = 0 \tag{1}$$

  Modify the HLLC Riemann solver accordingly to include the advection of the quantity q.
- (Optional) Implement and assess the behaviour of different wave speed estimates (e.g. Davis and Einfeldt).

	ρ_L	u_L	p_L	ρ_R	u_R	p_R	x_0	T_{out}
Test 1	1.0	0.75	1.0	0.125	0.0	0.1	0.3	0.2
Test 2	1.0	-2.0	0.4	1.0	2.0	0.4	0.5	0.15
Test 3	1.0	0.0	1000.0	1.0	0.0	0.01	0.5	0.012
Test 4	5.99924	19.5975	460.894	5.99242	-6.19633	46.0950	0.4	0.035
Test 5	1.0	-19.59745	1000.0	1.0	-19.59745	0.01	0.8	0.012
Test 6	1.4	0.0	1.0	1.0	0.0	1.0	0.5	2.0
Test 7	1.4	0.1	1.0	1.0	0.1	1.0	0.4	2.0

Table 1: Data for seven test problems with exact solution.

3 The TV Flux-Vector Splitting (further work)

- Within the finite volume scheme *EulerFV1d.cpp* implement the flux-vector splitting technique proposed by Toro and Vázquez-Cendón. As usual, consider the tests listed in Tab.1 to assess the performance of the resulting numerical scheme.

- (Optional) Test the behaviour of the TV scheme for the Woodward and Colella blast wave problem. The initial condition in pressure consists of three constant states separated by two discontinuities, namely

$$p(x,0) = \begin{cases} p_L = 1000, & \text{if x} < 0.1 \\ p_M = 0.01, & \text{if } 0.1 < \text{x} < 0.9 \\ p_R = 100, & \text{if x} > 0.9 \end{cases} . \tag{2}$$

  Density and particle velocity are instead constant with $\rho(x,0) = 1$ and $u(x,0) = 0$. The compuatational domain is the interval $[0,1]$ with reflective boundary conditions (*note: these have to be implemented in the code as part of the exercise*). Display the solution for density at the output time $T_{out} = 0.038$ units.