Computational Continuum Modelling
Practical 2: The advection equation (again)

The aim of this practical is to produce code that can solve the advection equation in a stable and general manner. The code should be able to take in:

- Number of points
- Advection speed (a)
- Final time
- Choice of numerical method to use
- Initial data

From this, the code should be able to solve the advection equation on a periodic domain using a theoretically stable time step. The key goal is to reproduce the set of plots shown on slide 17 of the lecture - solutions for Lax-Friedrichs, Lax-Wendroff and Warming-Beam methods (and also to add the upwind method to these tests).

Note that the Warming-Beam method uses information from cells $i-1$ and $i-2$, is a single ghost cell enough for this?

Exercises:

1. Reproduce the set of plots shown on slide 17 of the lecture - the initial data is
$$u_0(x) = e^{-8x^2}, \qquad -1 \leq x \leq 1$$
for the Gaussian curve and
$$u_0(x) = \begin{cases} 1 & 0.3 \leq x \leq 0.7 \\ 0 & \text{otherwise} \end{cases}$$
for the square wave. A CFL number $C = 0.8$ should be used, and $a = 1$ in the advection equation. In order to reproduce the plots after $t = 1$, it may be necessary to 'shift' the data plotted. These tests are also described in Section 5.5 of Toro's book. Comment on the accuracy of the results.

2. Check that your code is stable for other values of a, in particular $a = -1$ (Note, Warming-Beam method will need to be adjusted).

3. Computing error - it is possible to define error in the case of these tests:

$$\Delta_e = u(x_i, t^n) - u_i^n.$$

An overall quantification of the error can then be given by

$$\text{Error}_{L1} = \Delta x \sum_i |\Delta_e| .$$

By running the simulation with 100, 200, 400, 800 and 1600 cells, how does this error value change for the different methods and cases?