Responses to reviewer 6

Corrections to your comments are highlighted in color ■. In the revised version, we have made the following changes to the symbols and terminologies used in the manuscript.

Original version	Revised version
Controller,disturbance	Player I, Player II
$a \in \mathcal{A}$, $d \in \mathcal{D}$	$a \in \mathcal{A}$, $b \in \mathcal{B}$
$\mathscr{A}$, $\mathscr{D}$	$\mathscr{A}_0$, $\mathscr{B}_0$
Given time horizon $\bar{T}$	T or $T_1, ..., T_M$
Given admissible cost $\bar{J}$	J or $J_1, ..., J_M$
$\phi_{t_0}^{t_1}(., s_0, a, d)$	$\phi_{s,t}^{a,b}(.)$
$\min\limits_{a(.)} \max\limits_{d(.)} \ t_f$	$\sup\limits_{\beta(.)} \inf\limits_{a(.)} \ t_f$ (Emphasize that Player II plays nonanticipative strategies)
$J_{t_0}^{t_1}(s_0, a, d)$	$\mathcal{J}_{t_0}^{t_1}(s, a, b)$
$\min\limits_{a(.)} \max\limits_{d(.)} \ J_{t_0}^{t_1}(s_0, a, d)$	$\sup\limits_{\beta(.)} \inf\limits_{a(.)} \ \mathcal{J}_{t_0}^{t_1}(s, a, \beta[a])$
Given final time t_f	$\bar{T}$ (The symbol of the free final time t_f remains unchanged)
$\widehat{W}_{t_f}^c(.)$	$W_{\bar{T}}^c(., 0)$, $\left(W_{\bar{T}}^c(s, \tau) = \sup\limits_{\beta(.)} \inf\limits_{a(.)} \ \mathcal{J}_{\tau}^{\bar{T}}(s, a, \beta[a]) \right)$

1. The paper formulates a cost-limited reachability problem such that a set of initial states from which a given set is reachable under the constraints for a given cost function. The problem is an extension of a standard reachability problem. The paper defines a modified system and proposes a method for obtaining the problem using the system. This approach is straightforward. Moreover, the paper insists that the approach allows for significant storage space saving. But, its evidence is not shown in the paper. The formulated problem is novel. So, I suggest the paper is revised as Technical Communiques.

Response: The advantage of the proposed method in terms of storage space are reflected in the following application scenario: In the level set method, to compute the BRT of time horizon T, one needs to solve the following HJ PDE:

$$\begin{cases} \dfrac{\partial V}{\partial t}(s,t) + \min\left[0, \min\limits_{a\in\mathcal{A}} \max\limits_{b\in\mathcal{B}} \dfrac{\partial V}{\partial s}(s,t) f(s,a,b)\right] = 0 \\ \text{s.t.} V(s,T) = l(s) \end{cases} \tag{1}$$

Then the BRT is:

$$R_K(T) = \{s_0 \in \mathbb{R}^n | V(s_0, 0) \leq 0\} \tag{2}$$

This means that to save $R_K(T)$, one needs to save value function $V(., 0)$. In this case, the storage space consumed is proportional to $\prod_{i=1}^n N_i$, where N_i is the number of grid points in the ith dimension of the Cartesian grid. For $T_1, ..., T_M \in [0, \infty)$, the representations of the BRTs of these time horizons are:

$$\begin{aligned} &R_K(T_1) = \{s_0 \in \mathbb{R}^n | V(s_0, T - T_1) \leq 0\} \\ &... \\ &R_K(T_M) = \{s_0 \in \mathbb{R}^n | V(s_0, T - T_M) \leq 0\} \end{aligned} \tag{3}$$

Since the value functions $V(., T - T_1), ..., V(., T - T_M)$ are each different, the storage space consumption required to save these BRTs is proportional to $M \prod_{i=1}^n N_i$, which is M times as much as the original. In the proposed method, only $W_{\bar{T}}^c(., 0)$ ($\bar{T} > \max(T_1, ..., T_M)$) needs to be saved to save $R_K(T_1), ..., R_K(T_M)$. These BRTs are represented as:

$$\begin{aligned} &R_K(T_1) = \left\{s_0 \in \mathbb{R}^n | W_{\bar{T}}^c(s_0, 0) < T_1\right\} \\ &... \\ &R_K(T_M) = \left\{s_0 \in \mathbb{R}^n | W_{\bar{T}}^c(s_0, 0) < T_M\right\} \end{aligned} \tag{4}$$

The storage space consumed does not vary with M and is always proportional to $\prod_{i=1}^{n} N_i$, which is one part in M of that of the level set method. We visually display the superiority of this storage approach in the first example of the revised manuscript.

In addition, the revised version includes a description of the information pattern and also compares the proposed method with the level set method in more detail, resulting in a longer manuscript that does not meet the requirements of a Technical Communique.

2. In Eq. (6), "maxt_f" should be "max t_f".

Response: This problem has been fixed in the revised version.

3. In Eq. (10), it is unclear whether the symbol "t_f" is a given constant or a decision variable.

Response: t_f is not a given constant, but a free final time. This point is emphasized in the revised version. We have replaced a large number of symbols in the revised version to make the paper more readable.