EECS203002 Ordinary Differential Equations

2025 Bonus Project: Hot Bath with Heat Loss

Deadline: 2025/12/08 (Mon) 08:00

I. Introduction

Life is stressful. After a long week full of exams, deadlines, and projects, you decide to take a hot bath to relax. However, you only have a tub initially filled with cold water. To make a comfortable hot bath, you open the hot spring water faucet while draining water at the same time so that the water level in the tub remains constant.

The bathtub also loses heat to the surrounding air in the bathroom. In this project, you are asked to write a program to **sketch the temperature dynamics of the bath water, taking both mixing and heat loss into account, and to demonstrate your program to show how it works.**

II. Problem Description

Consider a bathtub that always contains 100 L of water.

Let $T(t)$ denote the temperature (°C) of the water in the tub at time t.

- At time $t = 0$, the tub contains water with initial temperature T_0.
- Hot spring water with temperature T_{in} flows into the tub at the **same rate** R (L/min).
- The mixture in the tub is kept perfectly stirred, so its temperature is uniform and equal to $T(t)$ at any time.
- At the same time, water flows out of the tub at the **same rate** R (L/min), so the total volume of water in the tub stays equal to 100 L.
- The bathroom has a constant room temperature T_{room}. The tub loses heat to the room according to Newton's law of cooling: the rate of heat loss is proportional to the difference $T(t) - T_{\text{room}}$. We denote the heat loss coefficient by $k > 0$.

III. Goal

- **A.** Fix T_0, T_{in}, T_{room}, and k.
  Investigate how different flow rates R (slow vs. fast inflow/outflow) influence the temperature dynamics of the bath water.
  Plot the curves $T(t)$ for several different values of R on the same figure, and mark each curve with the corresponding value of R on the Cartesian coordinate plane.
  (Adding 1 point to your final grade of ODE)

- **B.** Fix T_{in}, T_{room}, R, and k.
  Investigate the impact of different initial temperatures T_0 of the bath water (e.g., very cold vs. lukewarm water before adding hot spring water).
  Plot the curves $T(t)$ for different values of T_0 and mark the values of T_0 on the Cartesian coordinate plane.
  (Adding 1 point to your final grade of ODE)

- **C.** Fix T_0, T_{in}, T_{room}, and R.
  Investigate how different heat loss coefficients k affect the dynamics.
  For example, a larger k corresponds to a poorly insulated tub (fast heat loss), and a smaller k corresponds to a well-insulated tub.
  Plot the curves $T(t)$ for different values of k, and mark the parameter values on the Cartesian coordinate plane.
  (Adding 1 point to your final grade of ODE)

IV. Language

C/C++, Python, MATLAB, or other programming languages.

V. Requirements / Grading

- **A.** Upload the source code of your program to the eeclass platform.
- **B.** Demonstrate your project by downloading the source code from the eeclass platform and then compiling/running the file in your environment (using your own machine) and explain the results. The time slots for demonstration will be determined and announced on the eeclass platform later.
- **C.** The correctness of your results, including the correct implementation of the ODE solution (analytical or numerical) and correct plotting of the curves.

- **D.** Plagiarism is not allowed. If you are caught, you will **FAIL this ODE course**, not just get 0 points in the bonus project.

VI. Hint: Energy Balance Model for the Bath Temperature

To derive the differential equation for the bath temperature, you may start from the principle of energy conservation. Let

$$E(t) = \rho c V\, T(t)$$

be the total thermal energy of the water in the tub, where ρ is the density of water, c is the specific heat, and V is the water volume.

The rate of change of energy satisfies:

$$\frac{dE}{dt} = Q_{\text{mix}} + Q_{\text{cool}}$$

The two contributions are:

- Mixing term (hot water inflow and outflow):

  $$Q_{\text{mix}} = \rho c R\,(T_{\text{in}} - T(t))$$

  where R is the inflow/outflow rate and T_{in} is the temperature of the incoming hot water.

- Heat loss to the room (Newton's law of cooling):

  $$Q_{\text{cool}} = -\,k(T(t) - T_{\text{room}})$$

  where k is the heat loss coefficient in Newton's law of cooling. Using these expressions in the energy balance will lead you to the ODE for $T(t)$.

VII. Example

The following figure is an example result of this project, Goal A, when we fix the inflow temperature T_{in}, the room temperature T_{room}, the heat loss coefficient k, but vary R (flow rate).