

Game Theory Problems

1. Use the following one-shot, normal form game to answer the questions below.

Strategy	D	E	F
A	100, 125	300, 250	200, 100
B	250, 0	500, 500	750, 400
C	0, -100	400, 300	-100, 350

- (a) Find each player's dominant strategy, if it exists.
- (b) Find the Nash equilibrium.

2. In a two-player, one-shot simultaneous-move game each player can choose strategy A or strategy B. If both players choose strategy A, each earns a payoff of \$500. If both players choose strategy B, each earns a payoff of \$100. If player 1 chooses strategy A and player 2 chooses strategy B, then player 1 earns \$0 and player 2 earns \$650. If player 1 chooses strategy B and player 2 chooses strategy A, then player 1 earns \$650 and player 2 earns \$0.

- (a) Write the above game in normal form.
- (b) Find each player's dominant strategy, if it exists.
- (c) Find the Nash equilibrium (or equilibria) of this game.
- (d) Rank strategy pairs by aggregate payoff.
- (e) Can the outcome with the highest aggregate payoff be sustained in equilibrium? Why or why not?

3. Consider a two-player, sequential-move game where each player can choose to play *right* or *left*. Player 1 moves first. Player 2 observes player 1's actual move and then decides to move *right* or *left*. If player 1 moves *right*, player 1 receives \$0 and player 2 receives \$15. If both players move *left*, player 1 receives -\$10 and player 2 receives \$8. If player 1 moves *left* and player 2 moves *right*, player 1 receives \$10 and player 2 receives \$10.

- (a) Write the above game in extensive form.
- (b) Find the Nash equilibrium outcomes to this game.
- (c) Which of the equilibrium outcomes is most reasonable? Explain.

4. Use the following normal-form game to answer the questions below.

Strategy	C	D
A	10, 10	60, -5
B	-5, 60	50, 50

- (a) Identify the one-shot Nash equilibrium.
- (b) Suppose the game is infinitely repeated and the interest rate is 5%. Can the players achieve payoffs that are better than the one-shot Nash equilibrium? Explain.

5. While there is a degree of differentiation among general merchandise retailers like Target and Kmart, weekly newspaper circulars announcing sales provides evidence that these firms engage in price competition. This suggests that Target and Kmart simultaneously choose to announce one of two prices for a given product: a regular price or a sale price. Suppose that when one firm announces the sale price and the other announces the regular price for a particular product, the firm announcing the sale price attracts 50 million extra customers to earn a profit of \$5 billion, compared to the \$3 billion earned by the firm announcing the regular price. When both firms announce the sale price, the two firms split the market equally (each getting an extra 25 million customers) to earn profits of \$1 billion each. When both firms announce the regular price, each company attracts only its 50 million loyal customers and the firms each earns \$3 billion in profits. If you were in charge of pricing at one of these firms, would you have a clear-cut pricing strategy? If so, explain why. If not, explain why not and propose a mechanism that might solve your dilemma.

6. Coca-Cola and PepsiCo are the leading competitors in the market for cola products. In 1960 Coca-Cola introduced Sprite, which today is the worldwide leader in the lemon-lime soft drink market and ranks fourth among all soft drinks worldwide. Prior to 1999, PepsiCo did not have a product that competed directly against Sprite and had to decide whether to introduce such a soft drink. By not introducing a lemon-lime soft drink, PepsiCo would continue to earn \$200 million profit, and Coca-Cola would continue to earn a \$300 million profit. Suppose that by introducing a new lemon-lime soft drink, one of two possible strategies could be pursued: (1) PepsiCo could trigger a price war with Coca-Cola in both the lemon-lime and cola markets, or (2) Coca-Cola could acquiesce and each firm maintain its current 50/50 split of the cola market and split the lemon-lime market 30/70 (PepsiCo/Coca-Cola). If PepsiCo introduced a lemon-lime soft drink and a price war resulted, both companies would earn profits of \$100 million. Alternatively, Coca-Cola and PepsiCo would earn \$275 million and \$227 million respectively, if PepsiCo introduced a lemon-lime soft drink and Coca-Cola acquiesced and split the markets as listed above. If you were a manager at PepsiCo, would you try to convince your colleagues that introducing the new soft drink is the most profitable strategy? Why or why not?