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Data Analysis with Mixed-Integer Optimisation for Scheduling Royal Mail Deliveries

MENG INDIVIDUAL PROJECT

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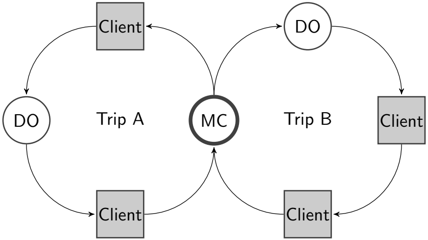
Problem Description

Context of the Problem

- Each MC is responsible for a broad geographic region.
- Trips are performed by each MC's fleet of HGV vehicles.



Atomic Block Definition

A **single round-trip** that commences at the Exeter MC, and after stopping at various external locations concludes again at the MC.

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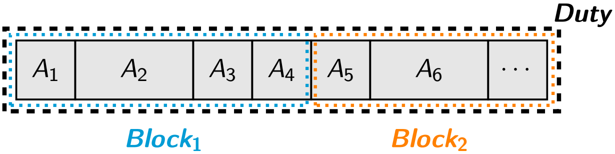
Breakdown of an Atomic Block



Structure of a driver's Duty: *Activities* (A_i), inside *Atomic Blocks* which themselves create a *Duty*.

What is an Activity?

Types of *Activities* featured in the dataset

The set of activities signifying the *task by task* actions performed by drivers, as observed in the data exploration.

Activity	Description
Start/End	The *beginning*, and *end* of a duty.
Travel	A *travel leg* between locations.
Load/Unload	*Loading* and *offloading* of mail units.
Meal Relief	A *meal-allowance* break.
Distribution/Processing	Non-essential administrative tasks.
Park Vehicle	*Parking* of HGV at end of duty.
Check	Scheduled *servicing* of HGV.
Clean	Scheduled *cleaning* of HGV.

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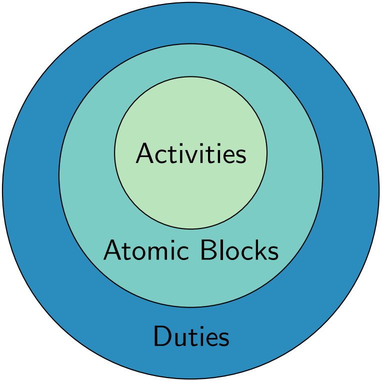
High-level Overview



The *duty* assigned to each driver is a collection of *blocks*, which themselves contain *activities*.

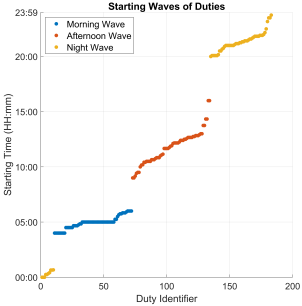
Building Blocks

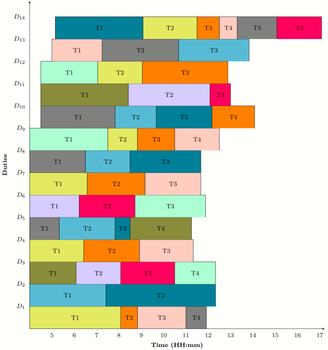
- Activity: Intervals of jobs performed inside an `Atomic Block`.
- Atomic Block: A round-trip starting and concluding at the MC.
- Duty: The total time that a driver works for, in a day.

Royal Mail Historical Schedules

What is a Schedule?

The set of all the **duties** assigned to each HGV driver for a day of the week.





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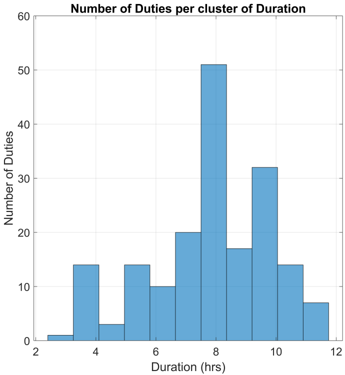
Evaluation of Historical Schedules

Motivation for Sensitivity Analysis

- How good are the historical schedules, and can they be improved?
- Our goal is to propose schedules with favourable characteristics.



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Simplifying Assumptions

Key Simplifications	
(1)	We consider a **duty** as a *machine* and a **block** as a *job*, freely assigning blocks to duties.
(2)	The **duration** of a **block** stays **constant**, irrespective of the timing of its occurrence.
(3)	The **routes** are **fixed**, and we **maintain** the **order** with which we visit external locations.

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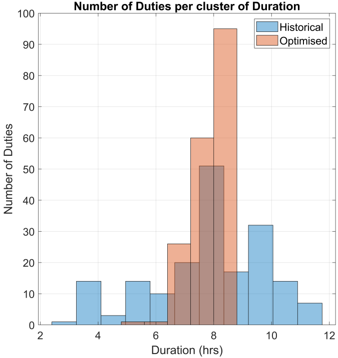
Load Balancing

Model Description

- Objective: find a more **balanced schedule** in terms of **duty length**.
- We minimise the **maximum length of a duty**.
- We execute all blocks featured in the historical schedule.
- Each driver executes at most one block per unit of time.

$$
\begin{aligned}
\text{minimize} \quad & y \\
\text{subject to} \quad & y \geq \sum_{j=1}^{n} x_{i,j} p_j && \forall\, i \in D \\
& \sum_{i=1}^{m} x_{i,j} = 1 && \forall\, j \in B \\
& y \geq 0 \\
& x_{i,j} \in \{0, 1\} && \forall\, j \in B,\ i \in D
\end{aligned}
$$

Numerical Results



(%) Reduction	
Makespan	Maximum Difference
28%	72%

Results Interpretation

A more **balanced** schedule. We redistributed the workload to reduce the amount of overtime that drivers need to perform.

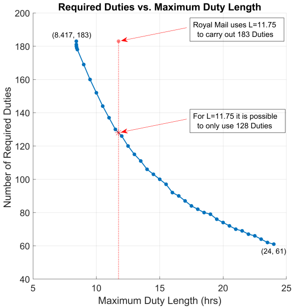
Number of Duties vs Maximum Duty Length

Model Description

- We fix an upper threshold of duty length L and execute all blocks while **minimising the amount of duties.**
- We vary L and conduct a Sensitivity Analysis to determine the minimum number of duties that give us a feasible schedule.

$$
\begin{array}{lll}
\text{minimize} & \sum_{i=1}^{m} y_i & \\
\text{subject to} & y_i \geq x_{i,j} & \forall\ i \in D,\ j \in B \\
& \sum_{i=1}^{m} x_{i,j} = 1 & \forall\ j \in B \\
& \sum_{j=1}^{n} x_{i,j} p_j \leq L & \forall\ i \in D \\
& y \geq 0 & \\
& x_{i,j} \in \{0,1\} & \forall\ j \in B,\ i \in D
\end{array}
$$

Numerical Results



Results Interpretation

There is a trade-off between the maximum duty length and the number of duties. It is possible to schedule **all blocks** with around **30% fewer duties.**

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Number of Blocks with Limited Driver Time

Model Description

- Minimise the maximum duty L and determine the **maximum number of blocks** that can **still be completed.**
- We preserve the duties at the historical level of 183 duties.
- We conduct a Sensitivity Analysis by varying L to determine the maximum number of blocks that can be processed for each L.

$$
\begin{aligned}
\text{maximize} \quad & \sum_{i=1}^{m} x_{i,j} & \\
\text{subject to} \quad & \sum_{i=1}^{m} x_{i,j} \leq 1 & \forall\, j \in B \\
& \sum_{j=1}^{n} x_{i,j} p_j \leq L & \forall\, i \in D \\
& y \geq 0 & \\
& x_{i,j} \in \{0,1\} \qquad \forall\, j \in B,\ i \in D &
\end{aligned}
$$

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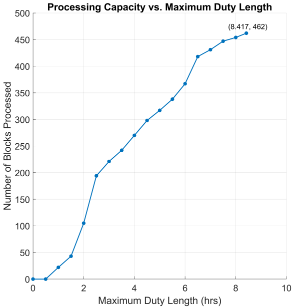
Numerical Results



Results Interpretation

Reducing the maximum time a driver works for by **25%** only reduces the number of blocks that can be processed by **10%**.

Eliminating Redundant Activities

What is **useful** time?

We classify the activities as **useful** or **not** to signify which activities must be maintained when minimising *non-useful* time.

Activity	Useful Time
Start/End	✓
Travel	✓
Load/Unload	✓
Meal Relief	✓
Distribution/Processing	✗
Park Vehicle	✗
Check	✗
Clean	✗

Table 1: List of *useful* or *not* activities.

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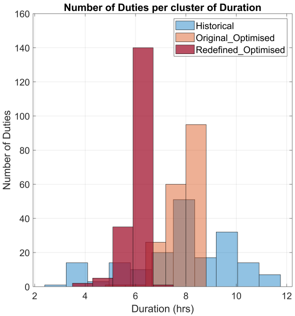
Redefining the Problem

Actions Taken

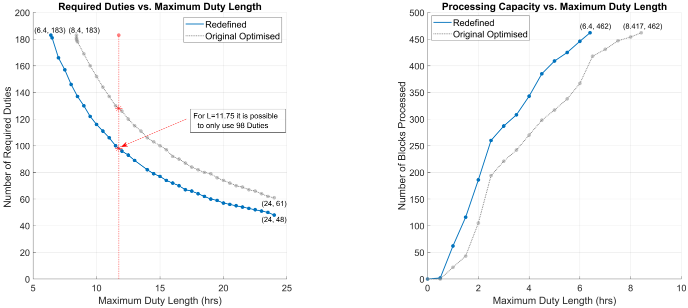
Artificially created more space for optimisation in the data by deleting activities that we classified as **non-useful working time**.



(%) Reduction

Schedule	**Makespan**	**Maximum Difference**
Historical	28%	72%
Redefined	36%	78%

Redefined Sensitivity Analysis



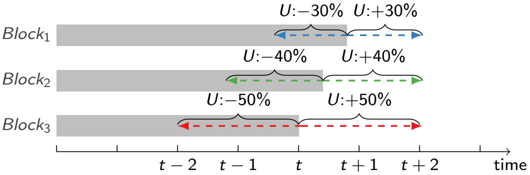
(a) Number of duties required as a function of the Maximum Duty Length (in 30min intervals), up to 24 hours per duty.

(b) Number of blocks processed as a function of the Maximum Duty Length (in 30min intervals), up to the historical average duty length.

Dealing with Uncertainty

Motivation for Uncertainty Study

- Initial study to determine the schedules' level of **robustness**.
- Robustness is evaluated by measuring **deadline deviations** and **delays to makespan**.
- Our goal is to determine which methodology provides the most robust schedule.



We simulate uncertainty by applying *box uncertainty sets* of different magnitudes on the blocks of an instance.

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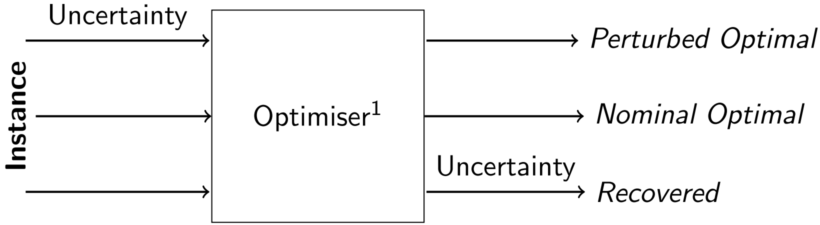
Incorporating Uncertainty into Schedules



[1] Applies Makespan Scheduling Formulation from slide 9

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Numerical Results

Schedule		Optimised	Recovered	Makespan	Overrun Duties
Undisturbed					
Nominal		✓		08:25	0
Disturbed					
Instance					
U: ±30%	`reduced`	✓		10:00	71
			✗	10:46	44
	`augmented`	✓		10:12	73
			✗	10:46	60
U: ±40%	`reduced`	✓		10:44	79
			✗	11:29	66
	`augmented`	✓		11:26	89
			✗	11:41	68
U: ±50%	`reduced`	✓		10:44	68
			✗	12:15	58
	`augmented`	✓		10:52	82
			✗	12:19	79

Results Interpretation

The **perturbed optimal** schedule has an improved makespan compared to the **recovered** schedule. However, it has substantially more delayed duties.

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Lexicographic Load Balancing

Model Description

- We minimise the **LexOpt Weigthed Sum** to obtain a new schedule. We then compare it to the **recovered** *wrt.* robustness.

$$
\begin{array}{lll}
\text{minimise} & \sum_{i=1}^{m} w_i y_i & \\
\text{subject to} & y_i \geq y_{i+1} & \forall\ i \in D \\
& y_i \geq \frac{1}{m-i+1}(\sum_{j=1}^{n} p_j - \sum_{q=1}^{i} -1 y_q) & \forall\ i \in D \\
& y_i \geq \sum_{j=1}^{n} p_j * x_{i,j} & \forall\ i \in D \\
& \sum_{i=1}^{m} x_{i,j} = 1 & \forall\ j \in B \\
& y_i \geq 0 & \\
& x_{i,j} \in \{0,1\} & \forall\ j \in B,\ i \in D \\
& \text{where } w_i = 2^{m-i} &
\end{array}
$$

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Numerical Results

Schedule		Lexicographic	Recovered	Makespan	Overrun Duties
Undisturbed					
Nominal		✓		08:25	0
Disturbed					
Instance					
U: ±30%	reduced	✓		10:49	43
			✗	10:46	44
	augmented	✓		10:53	58
			✗	10:46	60
U: ±40%	reduced	✓		12:16	64
			✗	11:29	66
	augmented	✓		11:45	68
			✗	11:41	68
U: ±50%	reduced	✓		12:15	57
			✗	12:15	58
	augmented	✓		12:22	79
			✗	12:19	79

Results Interpretation

The **LexOpt** generated schedule consistently outperforms or matches the **recovered** *wrt.* deadline deviations. It is only marginally beaten in terms of makespan delays.

Concluding Remarks

Future Directions	
(1)	Schedules that satisfy the **Meal-Relief** constraint enforced by EU regulations.
(2)	Incorporating Deadline Constraints.
(3)	Investigating the *VRP* aspect of the problem.
(4)	Study the effect of Ellipsoidal Uncertainty Sets.

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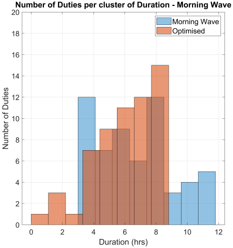
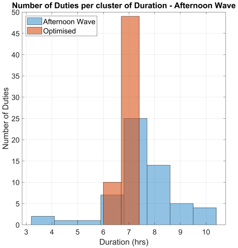
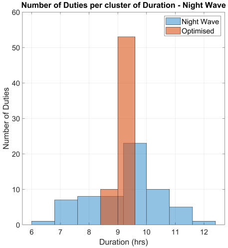
Appendix - Departure Waves Scheduling



(a) Morning



(b) Afternoon



(c) Night

Results Interpretation

Optimising the problem in *wave sub-instances* generates schedules with an **08:25** makespan similar to the results seen in slide 10.

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Appendix - Pre-emptive Load Balancing

Model Description

- Objective: find the **theoretical optimal schedule** wrt. **duty length**.
- We minimise the **maximum length of a duty**.
- We execute all blocks featured in the historical schedule.
- Each driver executes at most one activity per unit of time.

$$
\begin{aligned}
\text{minimise} \quad & y \\
\text{subject to} \quad & y \geq \sum_{j=1}^{n} x_{i,j} && \forall\ i \in D \\
& \sum_{i=1}^{m} x_{i,j} = p_j && \forall\ j \in B \\
& y \geq \sum_{i=1}^{m} x_{i,j} && \forall\ j \in B \\
& y, x_{i,j} \geq 0
\end{aligned}
$$

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Appendix - Numerical Results

Historical	
Makespan	**Optimality Gap**
08:05	3.1%

Redefined	
Makespan	**Optimality Gap**
06:32	6.6%

Results Interpretation

We achieve a lower-bound with respect to the maximum duty length observed in the **theoretical optimal schedules**. They are merely theoretical limits that are practically unrealisable.

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Appendix - Glossary

These terms are used interchangeably throughout the dissertation, to not tire the reader by using the same term repeatedly.

- Timetable = Schedule = Itinerary
- Duty = Shift
- Break = Meal-Relief
- HGV = Heavy Goods Vehicles = 7.5 tonne lorries
- HGV Driver = Employee
- Model = Formulation
- Maximum Difference[2] = Measure of the difference in the distribution of load between the heaviest and least loaded duties.

[2] Stein C. et al. Scheduling When You Do Not Know the Number of Machines.