CZ3005 Tutorial 5

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Question 5.1

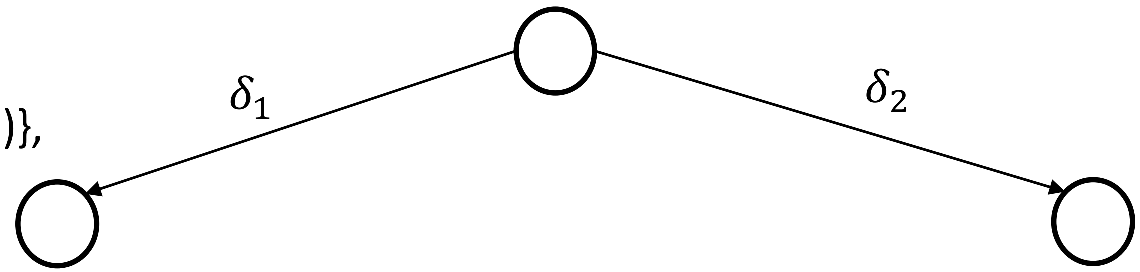
Find all extensions of the following default theories T = ⟨Δ, Φ⟩

1. $T = \langle \Delta = \left\{ \frac{R(x):\neg P(x)}{\neg P(x)}, \frac{Q(x):P(x)}{P(x)} \right\}, \Phi = \{R(N) \land Q(N)\} \rangle$
2. $T = \langle \Delta = \left\{ \frac{Summer:\neg Rain}{Sun\_Shining} \right\}, \Phi = \{\neg Sun\_Shining \land Summer\} \rangle$

Question 5.1(1)

$$T = \langle \Delta = \left\{ \delta_1 = \frac{R(N):\neg P(N)}{\neg P(N)}, \delta_2 = \frac{Q(N):P(N)}{P(N)} \right\}, \Phi = \{R(N) \wedge Q(N)\} \rangle$$

In-set={R(N), Q(N)}, Out-set={Ø}



In-set={R(N), Q(N), ¬ P(N)},
Out-set={P(N)}

Successful & Closed

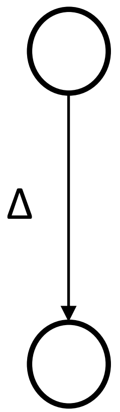
In-set={R(N), Q(N), P(N)},
Out-set={¬ P(N)}

Successful & Closed

Question 5.1(2)

$$T = \langle \Delta = \left\{ \frac{Summer : \neg Rain}{Sun\_Shining} \right\}, \Phi = \{\neg Sun\_Shining \wedge Summer\} \rangle$$

In-set={¬ Sun_Shining, Summer}, Out-set={∅}



In-set={¬ Sun_Shining, Summer, Sun_Shining}, Out-set={Rain}

Failed & Closed

Question 5.2

The fuzzy variable X is described by a set of fuzzy labels over the interval [0, 24] as shown in the Figure Q5.2.

- Describe the fuzzy labels using the trapezoidal membership function (TPMF) denoted by: TPMF[a, b, c, d] for each of the labels.
- State the type of fuzzy partitioning of the space provided by these four membership functions over the interval.

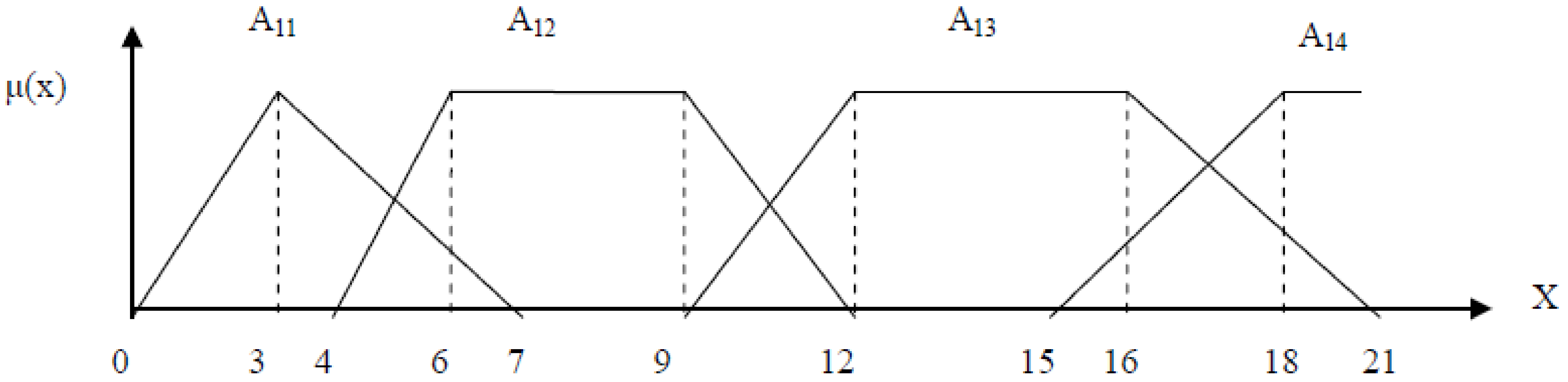
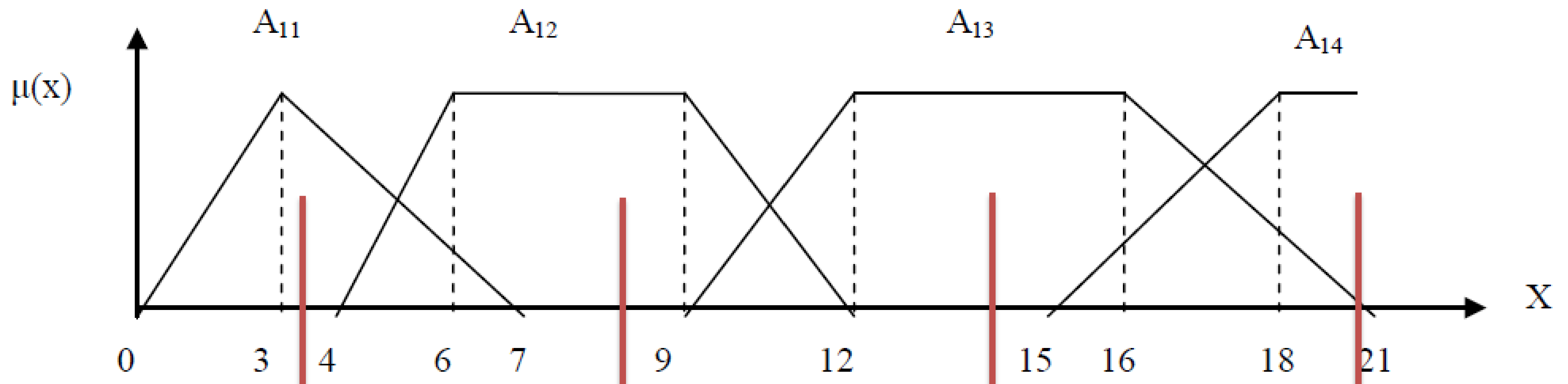


Figure Q5.2

Question 5.2



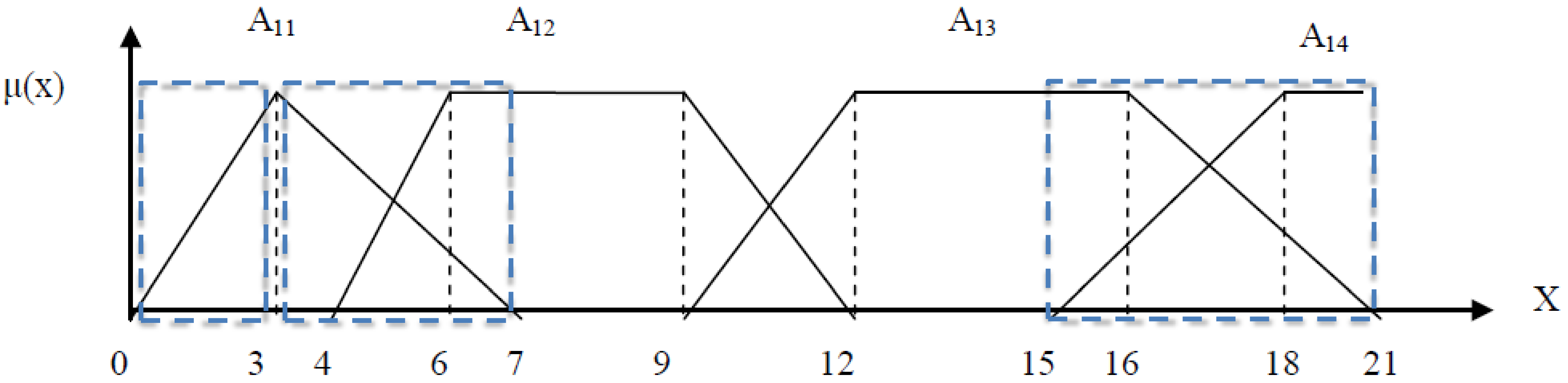
A_{11}=TPMF[0, 3, 3, 7] – triangular MF

A_{13}=TPMF[9, 12, 16, 21]

A_{12}=TPMF[4, 6, 9, 12]

A_{14}=TPMF[15, 18, 21, 21]

Question 5.2



This is a non-pseudo fuzzy partition. The interval overlaps between A_{11} and A_{12} and between A_{13} and A_{14} (the bounded regions) are non pseudo fuzzy partition, i.e. summation of MF in the overlap region is $\neq 1$

Question 5.3

A set of fuzzy variables s-quality, f-quality and t-payment are defined by the respective set of membership functions:

s-quality:	fuzzy term/label "poor" μ_{sq1} :	tpmf[0, 0, 4, 5]
	fuzzy term/label "good" μ_{sq2} :	tpmf[4, 5, 6, 7]
	fuzzy term/label "excellent" μ_{sq3} :	tpmf[6, 7, 10, 10]
f-quality:	fuzzy term/label "lousy" μ_{fq1} :	tpmf[0, 0, 2, 3]
	fuzzy term/label "delicious" μ_{fq2} :	tpmf[7, 8, 10, 10]
t-quality:	fuzzy term/label "cheap" μ_{tq1} :	tpmf[0, 2, 2, 3]
	fuzzy term/label "average" μ_{tq2} :	tpmf[3, 4, 4, 5]
	fuzzy term/label "generous" μ_{tq3}:	tpmf[4, 5, 5, 9]

Question 5.3(i)

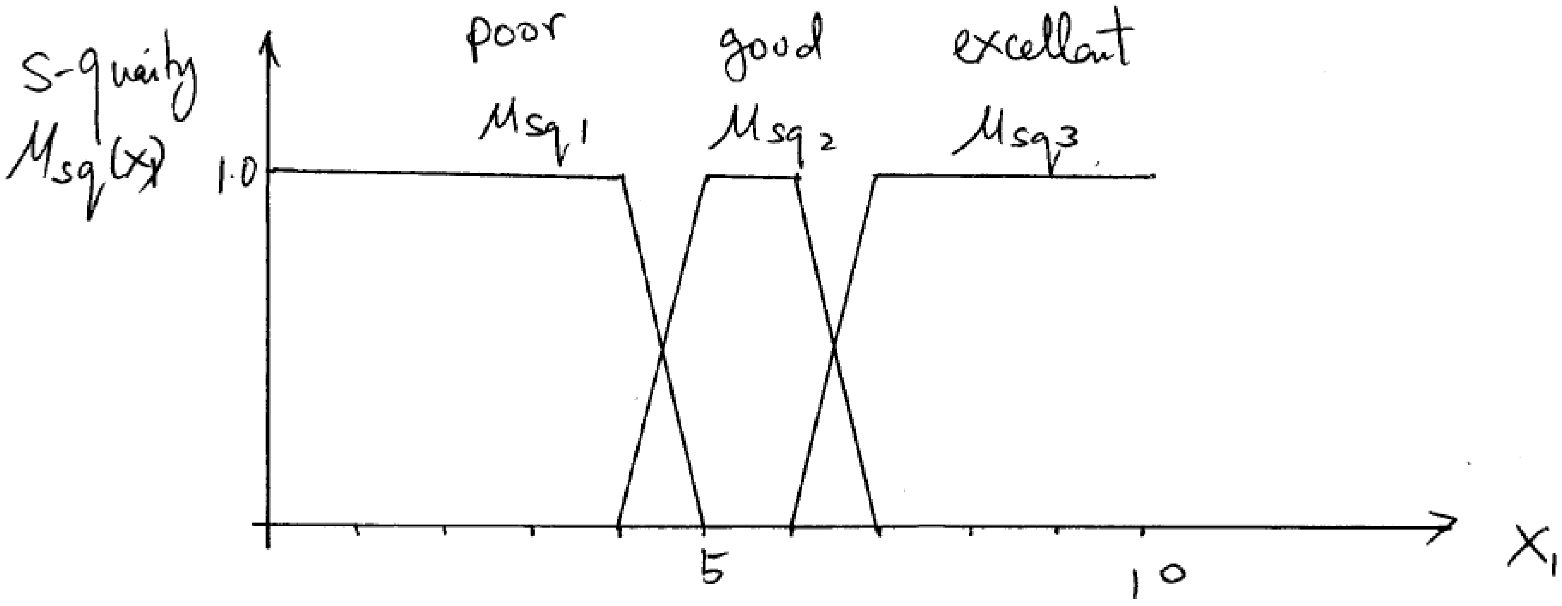
Draw the fuzzy partitions for each of the fuzzy variable over the domain [0, 10]. State the type of fuzzy partitioning for each of the dimensions.

s-quality:

tpmf[0, 0, 4, 5]

tpmf[4, 5, 6, 7]

tpmf[6, 7, 10, 10]



pseudo fuzzy partition

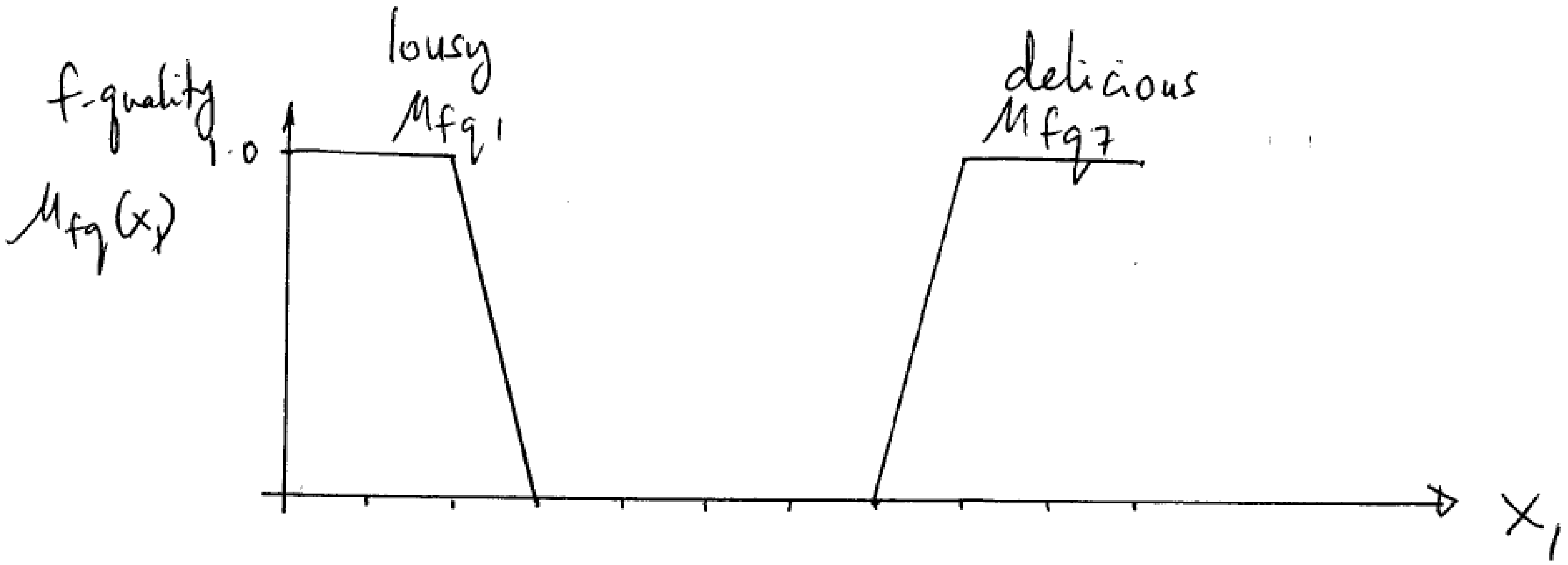
Question 5.3(i)

Draw the fuzzy partitions for each of the fuzzy variable over the domain [0, 10]. State the type of fuzzy partitioning for each of the dimensions.

f-quality:

tpmf[0, 0, 2, 3]

tpmf[7, 8, 10, 10]



non-pseudo fuzzy partition

Question 5.3(i)

Draw the fuzzy partitions for each of the fuzzy variable over the domain [0, 10]. State the type of fuzzy partitioning for each of the dimensions.

t-quality:

tpmf[0, 2, 2, 3]

tpmf[3, 4, 4, 5]

tpmf[4, 5, 5, 9]

non-pseudo fuzzy partition

Question 5.3(ii)

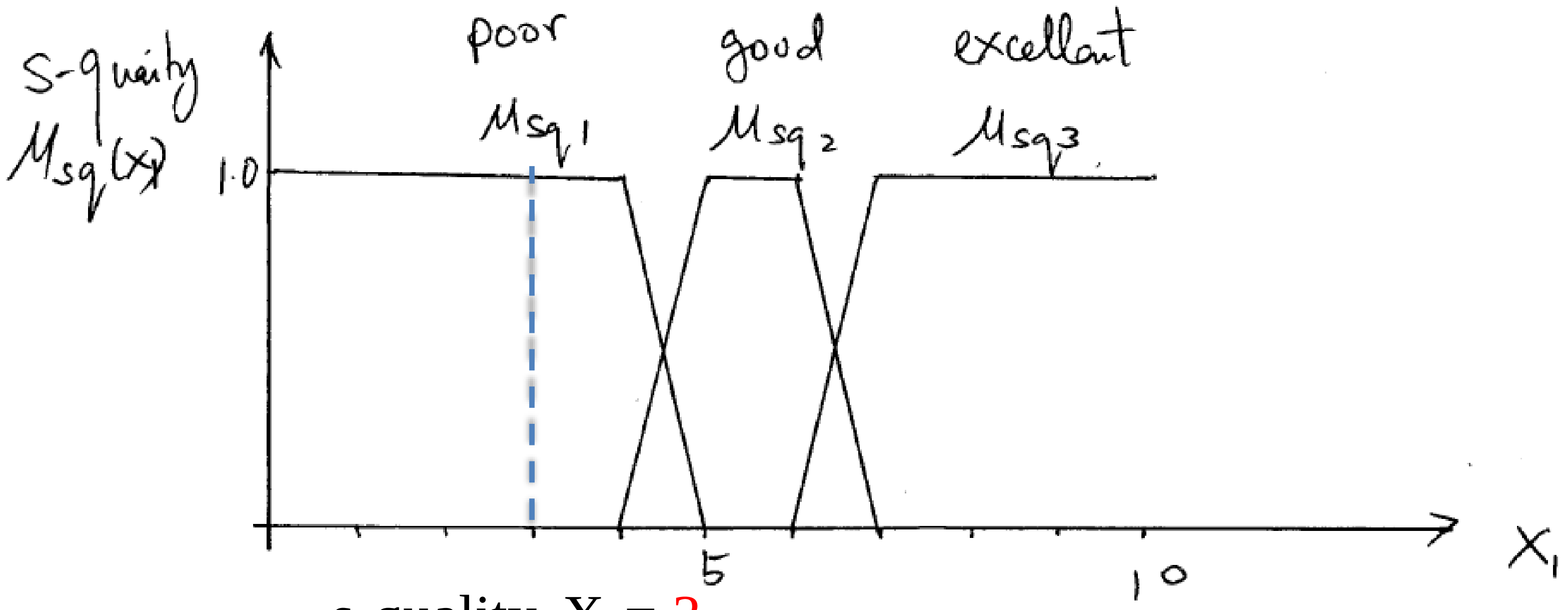
The above fuzzy labels are used in the formulation of a fuzzy expert rule system for tipping. The amount of tips (t-quality) derived from the fuzzy rules are based on the service quality (s-quality) and the food quality (f-quality).

Here are 4 fuzzy rules:

R1. If service is poor then tip is cheap.
R2. If service is excellent and food is delicious then tip is generous.
R3. If food is lousy then tip cheap.
R4. If service is good and food is delicious then tip is average.

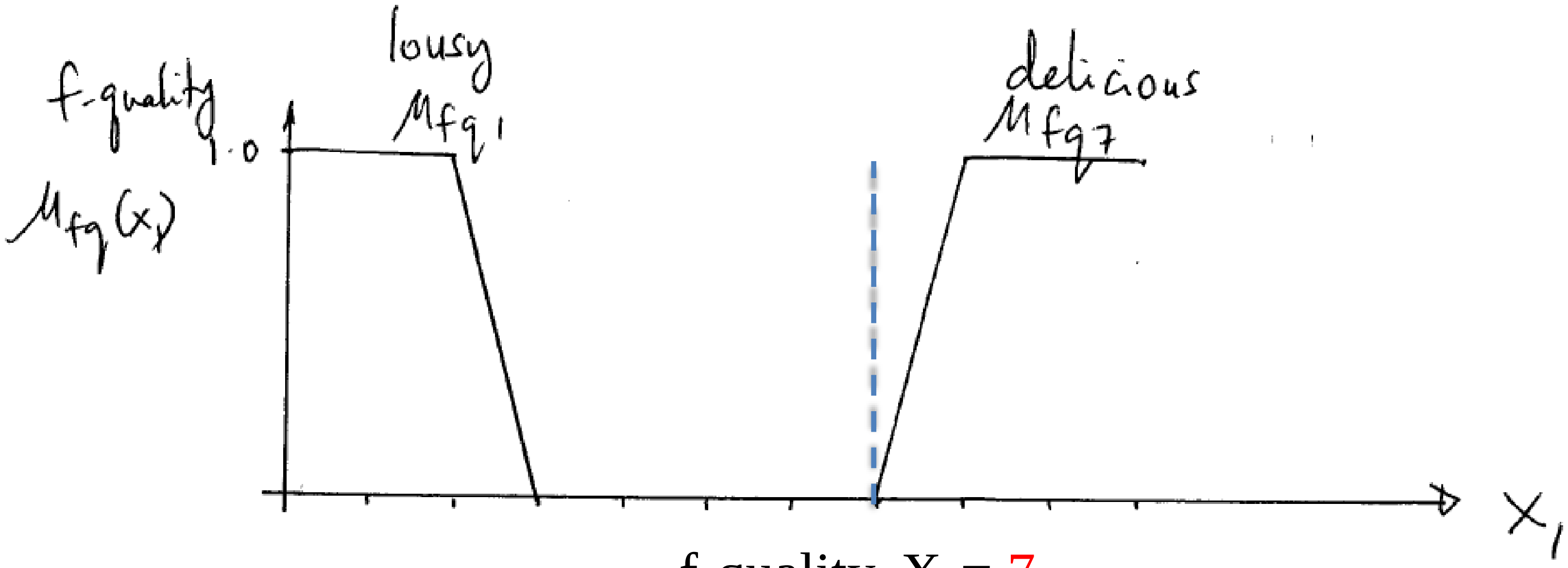
Determine the membership for the resultant tip if the scores for s-quality is 3 and f-quality is 7.

Question 5.3(ii)



s-quality, $X_1 = 3$

Question 5.3(ii)



f-quality, $X_1 = 7$

Question 5.3(ii)

R1. If service is poor then tip is cheap. // Fuzzy implication is the AND operator (i.e. min()).

s-quality X_1=3: $\mu_{sq1}(X_1=3) = 1$

$\min(1, \mu_{tq1}) = \mu_{tq1}$

R2. If service is excellent and food is delicious then tip is generous.

$\min(\mu_{sq3}(X_1=3) = 0, \mu_{fq2}(X_2=7) = 0) = 0$

$\min(0, \mu_{tq3}) = 0$

R3. If food is lousy then tip cheap.

f-quality X_2=7: $\mu_{fq1}(X2=7) = 0$

$\min(0, \mu_{tq1}) = 0$

R4. If service is good and food is delicious then tip is average.

$\min(\mu_{sq2}(X_1=3) = 0, \mu_{fq2}(X_2=7) = 0) = 0$

$\min(0, \mu_{tq2}) = 0$

Question 5.3(ii)

Fuzzy output is the aggregate of the outputs of each fuzzy rule.

Union operation - each of the rules is an alternative match.

$U(\mu_{tq1}, 0, 0, 0) = Max(\mu_{tq1}, 0, 0, 0) = \mu_{tq1}$

Either take maximum membership or Centroid defuzzification for the output μ_{tq1} (i.e. tip is cheap).

Question 5.3(iii)

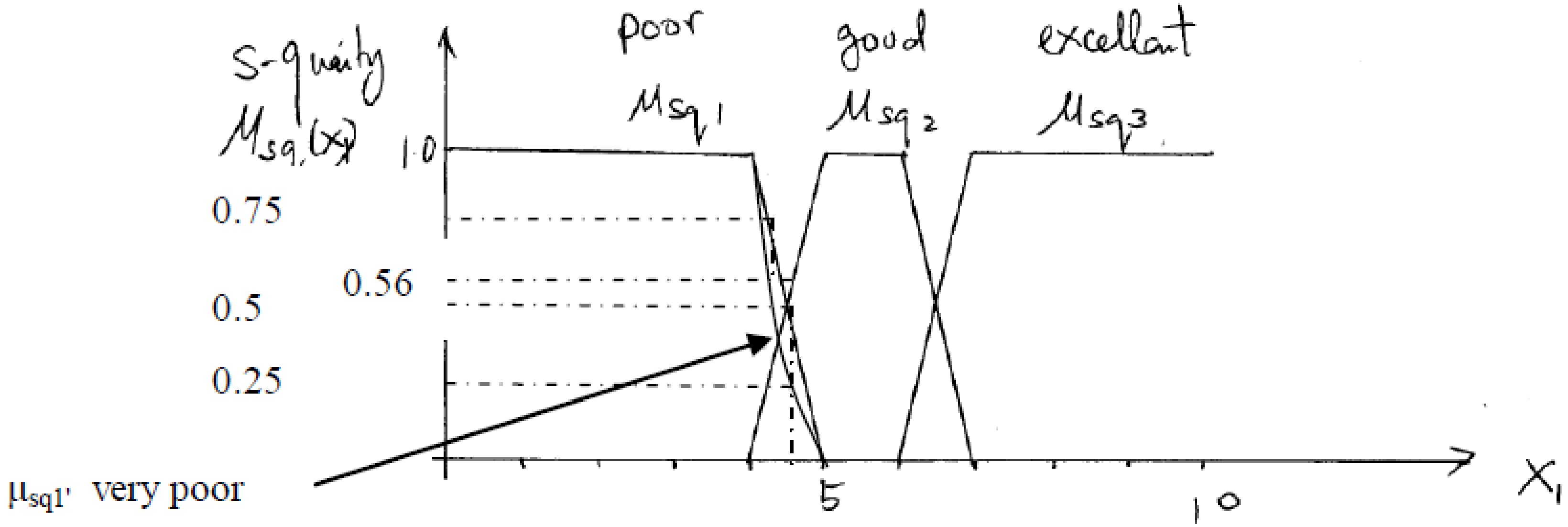
Linguistic modifiers or hedges are used to change the semantics of the linguistic labels. What will the fuzzy memberships for s-quality and t-quality be like if a rule is given as:

R1'. If service is very poor then tip is very cheap.

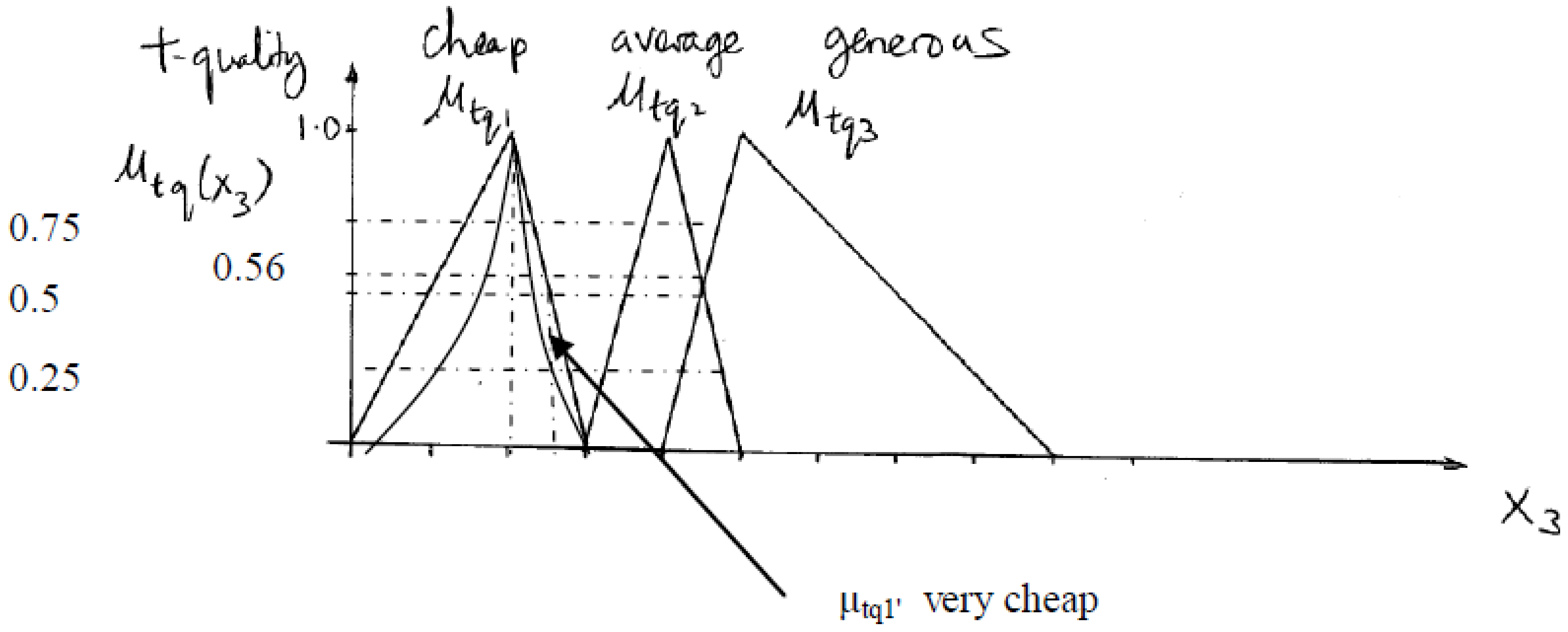
The very fuzzy linguistic hedge is the **square** operator.

Therefore the membership functions for very poor and very cheap will be modified using the square operator on μ_{tq1} and μ_{tq3}.

Question 5.3(iii)



Question 5.3(iii)



Thank you!