MH1812 Tutorial
Chapter 2: Propositional Logic

Q1: Decide whether the following statements are propositions. Justify your answer.

1. $2 + 2 = 5$.
2. $2 + 2 = 4$.
3. $x = 3$.
4. Every week has a Sunday.
5. Have you read "Catch 22"?

Q2: Show the second law of de Morgan:

$$\neg(p \vee q) \equiv \neg p \wedge \neg q.$$

Q3: Show that second absorption law $p \wedge (p \vee q) \equiv p$ holds.

Q4: These two laws are called distributivity laws. Show that they hold:

1. Show that $(p \wedge q) \vee r \equiv (p \vee r) \wedge (q \vee r)$.
2. Show that $(p \vee q) \wedge r \equiv (p \wedge r) \vee (q \wedge r)$.

Q5: Verify $\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) \equiv \neg p$ by

- constructing a truth table,
- developing a series of logical equivalences.

Q6: Using a truth table, show that:

$$\neg q \to \neg p \equiv p \to q.$$

Q7: Show that $p \vee q \to r \equiv (p \to r) \wedge (q \to r)$.

Q8: Are $(p \to q) \vee (q \to r)$ and $p \to r$ equivalent statements?

Q9: Show that this argument is valid:

$$\neg p \to F;$$
$$\therefore p.$$

Q10: Show that this argument is valid, where C denotes a contradiction.

$$\neg p \to C;$$
$$\therefore p.$$

Q11: Determine whether the following argument is valid:

$$\neg p \to r \wedge \neg s;$$
$$t \to s;$$
$$u \to \neg p;$$
$$\neg w;$$
$$u \vee w;$$
$$\therefore t \to w.$$

Q12: Determine whether the following argument is valid:

$$p;$$
$$p \vee q;$$
$$q \to (r \to s);$$
$$t \to r;$$
$$\therefore \neg s \to \neg t.$$