1. Suppose that the probabilities are 0.2, 0.4, 0.3 and 0.1 that the number of wills filed on any day at Kusu Island will be 0, 1, 2, or 3. X $P(x)$

(a) What is the probability of having at least 2 wills filed per day?
(b) Find the expected number of wills filed per day.
(c) Find the variance of the number of wills filed per day.

let X = # of wills filed / day

(a) $P(X \geq 2) = P(X=2) + P(X=3)$

(b) $E[X] = \sum X\,P(X) = 0 \times 0.2 + 1 \times 0.4 + \cdots + 3 \times 0.1$

$\mu = 1.3$

(c) $Var[X] = \sum (X-\mu)^2 P(X) = 0.81$

OR $= E[X^2] - E[X]^2$

$= \sum X^2 P(X) - \mu^2$

2. Given that $f(x) = k/2^x$, is a discrete probability function for a r.v. that can take on the values $x = 0, 1, 2, 3$ and 4. Find k and tabulate the cumulative probability $P(X \leq x)$.

x	0	1	2	3	4
p.m.f $f(x) = \frac{k}{2^x}$	$\frac{k}{2^0}$	$\frac{k}{2^1}$	$\frac{k}{2^2}$	$\frac{k}{2^3}$	$\frac{k}{2^4}$
$P(X \leq x)$	$\downarrow$	$\rightarrow +$	$\rightarrow +$	$\rightarrow +$	$\rightarrow 1$

$\Rightarrow \sum \frac{k}{2^x} = 1$, we get $k = \frac{16}{31}$

3. A biased die is rolled 50 times and the number of twos appeared is 10. If the die is rolled for another 10 times, determine the following:

$\Downarrow$ $p = \frac{10}{50} = 0.2$

(a) the probability that we get a two exactly 3 times.
(b) the expected number of twos.
(c) the variance of the number of twos.

let X = # of twos seen in $n=10$ trials

$X \sim B(10, 0.2)$

(a) $P(X=3) = \binom{n}{x} p^x (1-p)^{n-x} = 0.201$

(b) $E[X] = \sum x P(X=x) = np$

(c) $Var[X] = np(1-p)$

4. The number of calls coming per minute into a hotel reservation center is Poisson random variable with mean 3.

(a) Find the probability that no calls come in a given 1minute period.
(b) Assume that the number of calls arriving in two different minutes are independent. Find the probability that at least two calls will arrive in a given two minutes period.

let X = # of calls coming in 1 min

$X \sim Pois(\mu=3)$ where $P(X=x) = \frac{e^{-\mu}\mu^x}{x!}$

(a) $P(X=0) = \frac{e^{-3}3^0}{0!}$

(b) let X_1 & X_2 = # of calls in the 1st & 2nd min

$P(X_1+X_2 \geq 2) = 1 - P(X_1+X_2 < 2)$

$= 1 - [\underbrace{P(X_1=0 \text{ and } X_2=0)}_{P(X_1=0)\,P(X_2=0)} + P(X_1=1 \text{ \& } X_2=0) + \underbrace{P(X_1=0 \text{ \& } X_2=1)}_{P(X_1=0)\,P(X_2=1)}]$

...

$= \frac{e^{-3}3^0}{0!} \cdot \frac{e^{-3}3^1}{1!}$

5. The probability that a student fails Subject A exam is 0.05. If the student failed the subject, he will have to re-take it the following semester. Let X be the number of times he attempted to pass the subject.

(a) Determine and name the probability distribution of X.

(b) Find the probability that a student will pass the subject with no more than 2 attempts.

(c) Find the average number of attempts to pass the subject.

(a) Geometric Dist. $P(X=x) = (0.05)^{x-1}(0.95)^{1}$

(b) $P(X \leq 2) = P(X=1) + P(X=2)$

$= 0.95 + 0.05 \times 0.95$

(c) $E[X] = \frac{1}{p} = \frac{1}{0.95}$