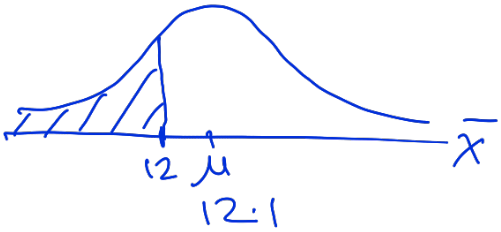
Problem 1

To determine whether a bottling machine is working satisfactorily, a production line manager randomly samples ten 12-ounce bottles every hour and measures the amount of beverage in each bottle. The mean $\bar{x}$ of the 10 fill measurements is used to decide whether to readjust the amount of beverage delivered per bottle by the filling machine.

If records show that the amount of fill per bottle is normally distributed, with a standard deviation of .2 ounce (σ), and if the bottling machine is set to produce a mean fill per bottle of 12.1 ounces (μ), what is the approximate probability that the sample mean $\bar{x}$ of the 10 test (n) bottles is less than 12 ounces?

$$P(\bar{X} < 12)$$

$$= P\Big(Z < \underbrace{\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}}_{Z_1 = ?}\Big)$$



$$\bar{X} \sim N\Big(\underset{=12.1}{\mu}, \frac{\sigma^2}{n}\Big)$$

use Z-table, we get

$\simeq 0.057$

Problem 2

A college C would like to have 1050 freshmen, and cannot accommodate more than 1060. Assume that each applicant accepts with probability $p = 0.6$ and that the acceptance can be modeled with Binomial distribution. If the college accepts 1700 (n) freshmen, what is the probability that it will have too many acceptances?

Let X = no. of acceptances $\Rightarrow X \sim B(n, p)$ 1700, 0.6

$$P(X > 1060)$$

$$\simeq P\Big(Z > \frac{1060.5 - 1020}{20.2}\Big)$$

continuity correction

$\therefore \mu = np = 1020$

$\sigma = \sqrt{npq} = 20.2$

use Z-table, we get

$\simeq 0.0214$

Problem 3

p

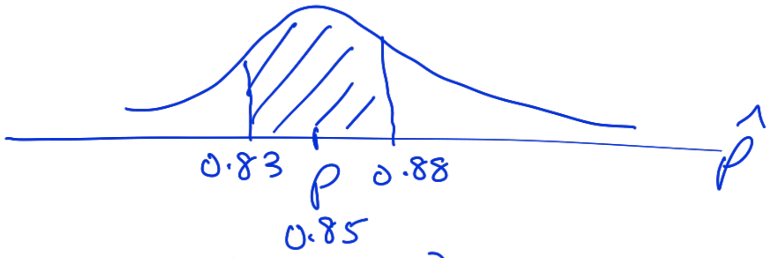
The proportion of individuals with an Rh-positive blood type is 85%. You have a random sample of $n = 500$ individuals. What is the probability that the sample proportion $\hat{p}$ lies between 83% and 88%?

$$\hat{p} = \frac{X}{n}$$

$$E[\hat{p}] = \frac{E[X]}{n} = \frac{\cancel{n}p}{\cancel{n}}$$

$$Var[\hat{p}] = \frac{1}{n^2} Var[X] = \frac{\cancel{n}pq}{n^{\cancel{2}}}$$

$$\hat{p} \sim N\left(p, \frac{pq}{n}\right)$$



$$P(0.83 < \hat{p} < 0.88)$$

$$= P\left(\frac{0.83 - 0.85}{\sqrt{pq/n}} < Z < \frac{0.88 - 0.85}{\sqrt{pq/n}}\right)$$

Problem 4

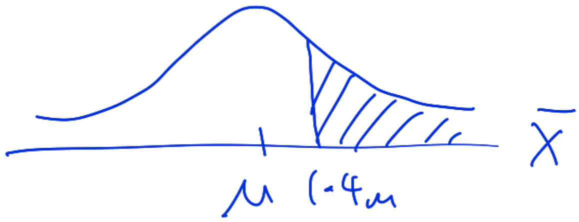
μ σ

A housing survey was conducted to determine the price of a typical home in Topanga, CA. The mean price of a house was roughly \$1.3 million with a standard deviation of \$300,000. There were no houses listed below \$600,000 but a few houses above \$3 million. What is the probability that the mean of 60 randomly chosen houses in Topanga is more than \$1.4 million?

n

let x = price of a house

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$



$$P(\bar{X} > 1.4m)$$

$$= P\left(Z > \frac{1.4m - 1.3m}{0.3m/\sqrt{60}}\right)$$

use z-table, we get

$$\simeq 0.0049$$

Problem 5

Suppose that an insurance company has 10,000 policy holders. The expected yearly claim per policyholder is \$240 with a standard deviation of \$800. What is the approximate probability that the total yearly claims $S_{10,000} > \$2.6$ million?

μ n σ

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

multiply N & D by n,

$$\Rightarrow \frac{n\bar{X} - n\mu}{\sigma\sqrt{n}} \sim N(0,1)$$

$$\cancel{n}\frac{\Sigma X}{\cancel{n}} \quad \frac{\Sigma X - n\mu}{\sigma\sqrt{n}} \sim N(0,1)$$

$$\Rightarrow \Sigma X \sim N(E[\bar{X}], \ var[\bar{X}])$$

$E[\bar{X}] = n\mu = 10000 \times 240$, $var[\bar{X}] = n\sigma^2$

$$\therefore P(\Sigma X > 2.6m)$$

$$= P\left(Z > \frac{2.6m - 2.4m}{\sqrt{10000}\ 800}\right)$$

use Z-table, we get

$$\simeq 0.0062$$