

MH1812

Extra Exercises of Combinatorics

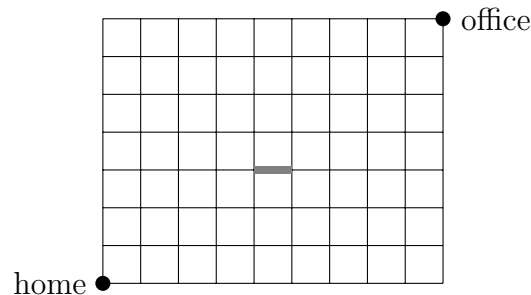
These are additional combinatorial questions and may not be discussed in the tutorials.

Q1: In how many ways can 5 indistinguishable rooks be placed on an 8-by-8 chessboard so that

- (a) no rook can attack each other?
- (b) no rook can attack each other and neither the first row nor the first column is empty?

Q2: A secretary works in a building located 9 blocks east and 7 blocks north of his home. Every day he walks 16 blocks to work. (See the map that follows.)

- (a) How many different routes are possible for him?
- (b) How many different routes if the block (coloured in grey) in the easterly direction, which begins 4 blocks east and 3 blocks north of his home, is under water (and he cannot swim)? (Hint: count the routes that use the block under water.)



Q3: Show that

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

for all integers $n \geq k \geq 1$ by

- (a) direct calculation;
- (b) a combinatorial argument that relates choosing k items from n items (the left-hand side) to choosing $k-1$ items from $n-1$ items and choosing k items from $n-1$ items (the right-hand side).

Q4: Show that

$$\binom{k}{k} + \binom{k+1}{k} + \cdots + \binom{n}{k} = \binom{n+1}{k+1}$$

for all integers $n \geq k \geq 1$ by

- (a) mathematical induction on n ;
- (b) a combinatorial argument.