



**NANYANG
TECHNOLOGICAL
UNIVERSITY**

CE1107/CZ1107: DATA STRUCTURES AND ALGORITHMS

Binary Trees

College of Engineering

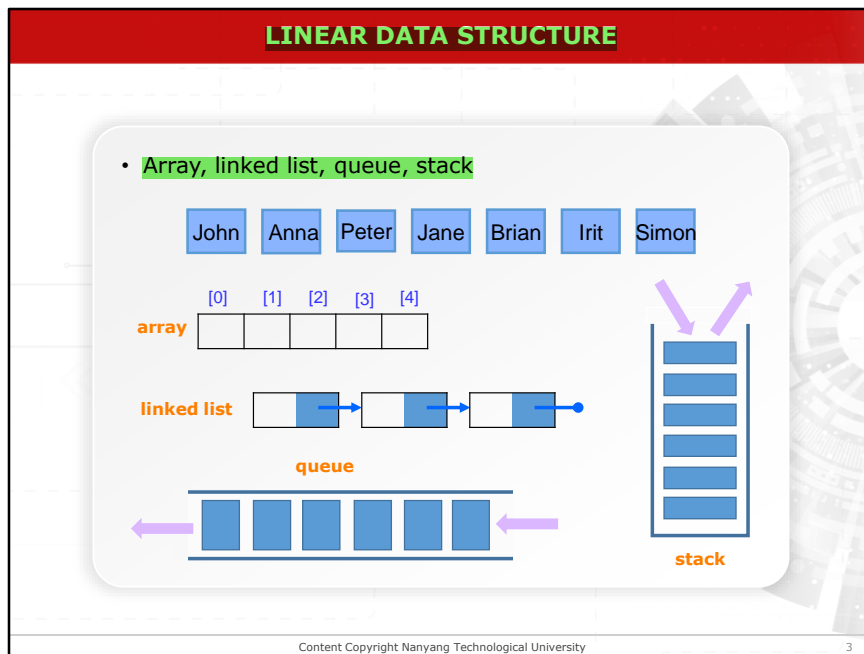
School of Computer Science and Engineering

This lecture is on Binary Trees

OUTLINE

• Non-linear data structures

- Tree data structure
 - Binary trees
- Implement binary tree nodes in C
- Binary Tree Traversal
- Tree traversal order
 - Pre-order
 - In-order
 - Post-order
- Application examples
 - Count nodes in a binary tree
 - Find grandchild nodes
 - Calculate height of every node
- Level-by-level traversal
- Preorder traversal with a stack

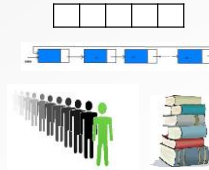


If it is a linear data structure, we can store data as;

- an array
- linked list
- queue
- stack

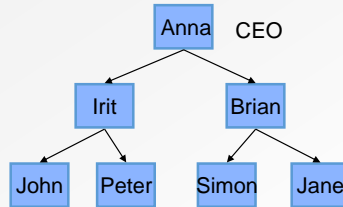
DATA STRUCTURES SO FAR...

- **Linear**
 - Items all arranged one after another
 - Random access
 - Arrays
 - Sequential access
 - Linked list
 - Limited-access sequential
 - Stacks
 - Queues
- Used them to store lists of numbers, lists of people, lists of moves, etc
 - Linear data



NON-LINEAR DATA STRUCTURE

- Suppose you have a set of names



Tree

- Company organization

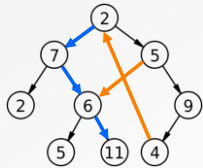
Not good to use linear data structure to store hierarchical relationships

Non-linear Data structures

Suppose you have some names of few people, who represent a company. Then, to arrange their names according to their position in the company, you have to use a 'tree'.

TREE DATA STRUCTURE

- Still using **nodes + links** representation
- New idea:
 - **Each node can have links to more than one other node**
 - **No loop**



Observe that:

- **If we follow one path of a tree, we get a linked list**

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Tree data structure

The key points of tree data structure can be listed as below.

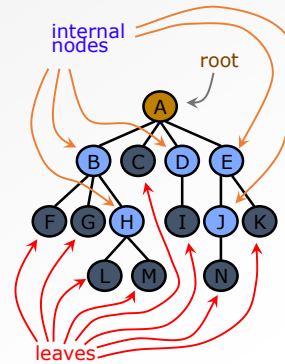
1. Each node can have links to more than one other node. - In linear data structure, for an example if we consider queue; a node can be linked with only one other node. But in non-linear data structure, this concept changes.
2. No loop - As per the example show in the slide, if you put a link from node. 5 to node 6, the result would be a loop between 2,7,6,5 nodes. Same result would be given if you form a link from node 4 to node 2.
3. A tree data structure contains more than one linked list.

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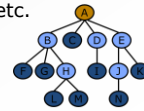
TREE DATA STRUCTURE

- A tree is composed of nodes
- Each node contains a value
- Types of nodes
 - **Root:** only one in a tree, has no parent.
 - **Internal** (non-leaf): Nodes with children are called internal nodes
 - **Leaf:** nodes without children are called leaves



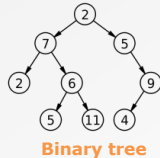
WHY TREES?

- Model layouts with hierarchical relationships between items
 - Chain of command in the army
 - Personnel structure in a company
 - (Binary tree structure is limited because each node can have **at most two children**)
- Tree structures also allow us to
 - Some problems require a tree structure: some games, most optimization problems, etc.
 - **Allow us to do the following very quickly:**
(we'll see that in the following lectures)
 - **Search for a node with a given value**
 - **Add a given value to a list**
 - **Delete a given value from a list**

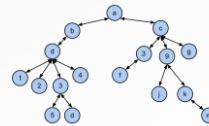


TREE DATA STRUCTURE

- Tree data structure looks like... a tree:
 - Only one root node (no nodes points to it)
 - Each node branches out to some number of nodes
 - **Each node has only one "parent" node** – the node pointing to it (except the root node)



Binary tree

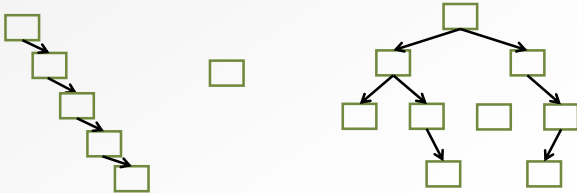


General tree

- General tree
 - Each node can have links to any number of other nodes
- **Binary tree (we'll work with this in our course)**
 - **Each node can have links to at most two other nodes**

POSSIBLE TREE CONFIGURATIONS

- We'll see later why not all trees configurations are desirable/useful
- Has to do with balance of a tree



The slide illustrates three different tree configurations. The first configuration on the left is a skewed tree with five nodes connected in a single chain, representing an unbalanced tree. The second configuration in the middle is a single isolated node, which is also considered a tree. The third configuration on the right is a balanced tree with a root node, two children, and four grandchildren, totaling seven nodes.

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From the given examples in the slide;

- The first one can be recognized as a tree, but it is an unbalanced tree.
- The second one has only one node, yet it can be defined as a tree
- The third one is also a tree.

OUTLINE

- Non-linear data structures
- Tree data structure
 - Binary trees
- **Implement binary tree nodes in C**
- Binary Tree Traversal
- Tree traversal order
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 - In-order
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IMPLEMENTATION

- Recall implementation of LinkedList
 - Node has link to **at most one** other node
 - Defined a ListNode with one **next** pointer and a data **item**
- BinaryTree
 - Node has link to **at most TWO** other nodes
 - Define a BTreeNode with
 - Two pointers
 - A data item

```
typedef struct _listnode{
    int item;
    struct _listnode *next;
}ListNode;
```

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The C structure we have implemented for list node is;

```
typedef struct _listnode {
    int item;
    struct _listnode *next;
} ListNode;
```

The same method can be used for tree structure as well. But binary tree node has links to at most two other nodes. Therefore, there should be two pointers.

BTNode

- Start with a simple BTNode that stores an integer
 - The type of item can be character, string, or structure, etc.

```

typedef struct _bnode{
    int item;
    struct _bnode *left;
    struct _bnode *right;
} BTNode;
      
```

```

typedef struct _listnode{
    int item;
    struct _listnode *next;
} ListNode;
      
```

BTNode

Pointer to left child BTNode Pointer to right child BTNode

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The C structure for binary tree node

- The structure for binary tree can be defined as;

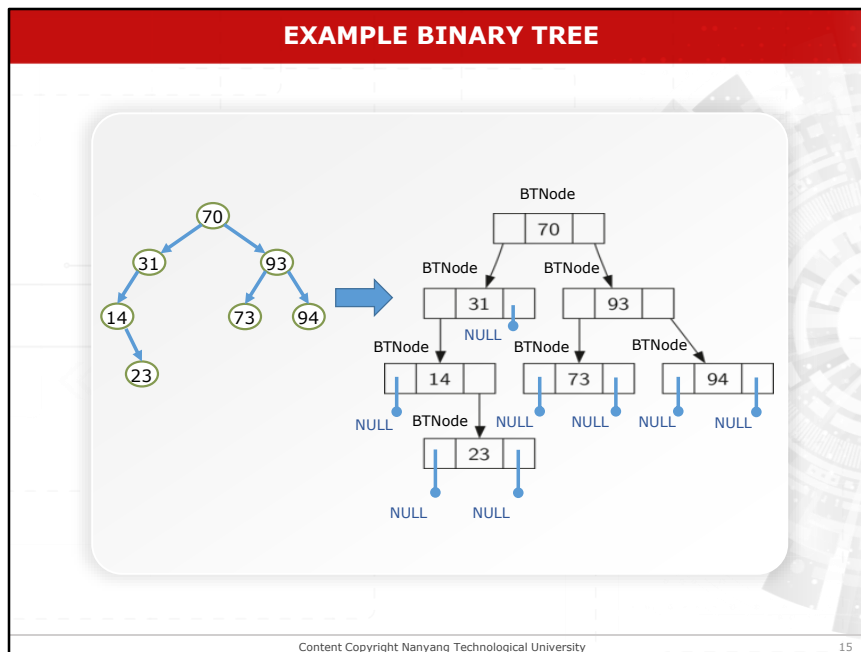
```
Typedef struct _bnode { ... }
```

- We have to define one integer variable to store values for the node.

```
int item;
```

- Then we have to declare two pointers which pointers to the left and right child nodes.

```
struct _bnode *left;
struct _bnode *right;
```



We have constructed the node structure based on the given tree.

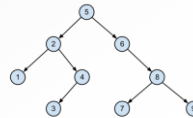
- Node 70 is the root node which has the right pointer points at node 93 and the left pointer points at node 31.
- Node 31 has only a left pointer which points at node 14, and node 14 has only a right pointer points at node 23.
- Node 93 has two pointers where left pointer points at node 73 and the right pointer points at node 94.

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TREE TRAVERSAL

- Given a linear data structure and a particular item, very obvious what the "next" item is
 - Each node has an obvious "previous" and "next" node
- Trees are non-linear structures
 - How to extract data from a binary tree?
 - What is the traversal sequence?
left/left/left, then left/left/right, then...?
- Need a systematic way to visit every node in the tree
 - Clearly defined steps
 - No repeated visits to nodes

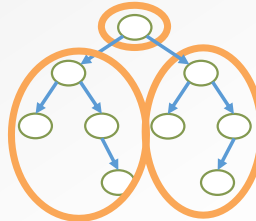


TREE TRAVERSAL

- Why is this important?
 - Tree traversal is foundation for many functions
- Very common function template:

Traverse tree

- At each node, perform some operation



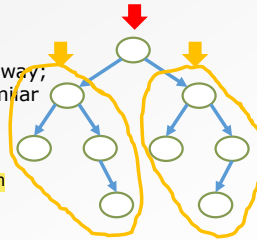
- Example task: count # of nodes in a tree

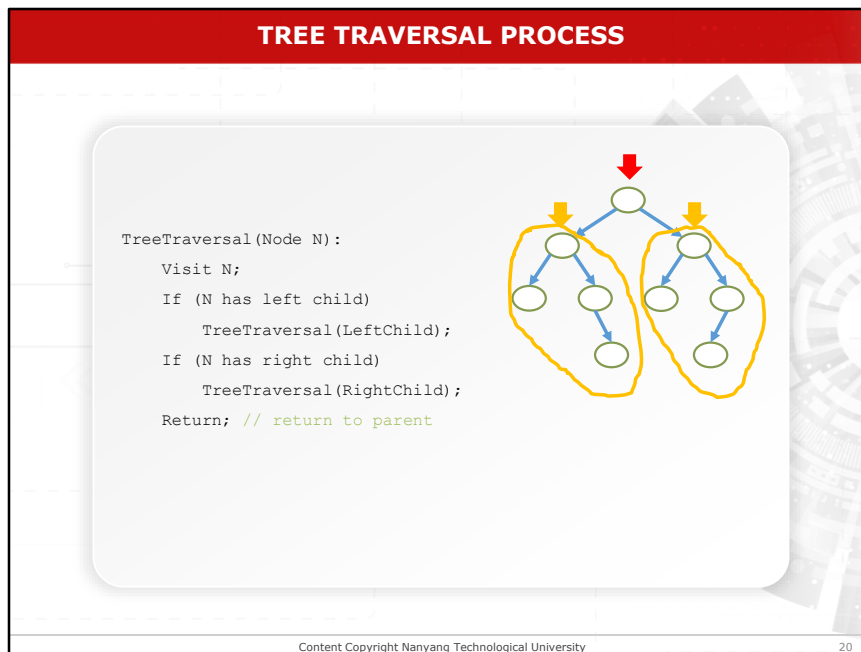
At every node N, size of that subtree

= size of N's left subtree
+ size of N's right subtree
+ N itself

TREE TRAVERSAL

- **Tree traversal is recursive**
 - Recursion: is the process of repeating items in a **self-similar** way; divide a problem into several similar sub-problems.
 - **At each node**
 - Visit the node and both children
- **Initial case + repeating case**
 - (Visit root) + (visit children)
- When combined, **guarantees that all nodes will be visited once and only once**



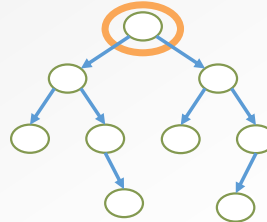


Tree Traversal Process

- The pseudo code of the tree traversal process:
- Since we try to count the number of nodes in the whole tree, we have to start with the root node. Therefore **Node N is the root node.**
- First we will **visit the root node**
- Then we **check whether the root node has a leftchild.** If it has, we then **traverse the left subtree of the root which start with leftchild.**
- Then we **check the right child of the root in the same manner.**

TREE TRAVERSAL TEMPLATE #1Pseudocode

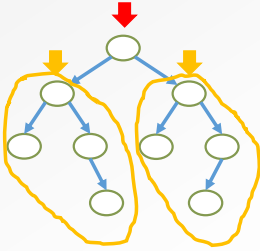
```
TreeTraversal(Node N):  
  Visit N;  
  If (N has left child)  
    TreeTraversal(LeftChild);  
  If (N has right child)  
    TreeTraversal(RightChild);  
  Return; // return to parent
```



In main(), call TreeTraversal(root)

TREE TRAVERSAL TEMPLATE #2

- **Current function:**
 - Need to check for existence of left and right children before following them
- **New version:**
 - Always follow links to children
 - Then check if the link is NULL
 - In other words, **not actually pointing at a BTreeNode**



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We can improve the discussed code by checking if a node is NULL or NOT NULL after visiting the node.

TREE TRAVERSAL TEMPLATE #2

Pseudocode

```

TreeTraversal2(Node N) :
    If N==NULL return;
    Visit N;
    TreeTraversal2 (LeftChild);
    TreeTraversal2 (RightChild);
    Return; // return to parent

In main(), call TreeTraversal2(root)

```

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Tree traversal template #2

Here, we points at a node and first check if the node is NULL or NOT NULL.

If the node is NULL, the code will return. Otherwise it visits N and pass the TreeTraversal (LeftChild) and TreeTraversal (RightChild) functions.

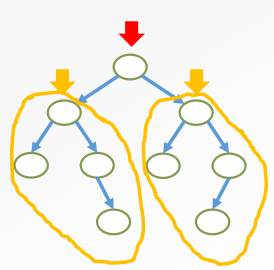
This code has only one 'if' statement, but the previous template we had two 'if' statements.

TreeTraversal2() IMPLEMENTATION

```

Void TreeTraversal2(BTNode *cur){
    If (cur == NULL) return;
    PrintNode(cur); // visit cur
    TreeTraversal2(cur->left);
    TreeTraversal2(cur->right);
}

```



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Tree Traversal2() implementation

The code for the 2nd template.

- First we define the `TreeTraversal2()` function where a pointer 'cur' will be passed with the `BinaryTreeNode` type.

```
Void TreeTraversal2(NTNode *cur){...}
```

- Then we check if the pointer points at a `NULL` node. If yes, we will return. Otherwise we print the Node.

```
if (cur==NULL) return;
printNode cur;
```

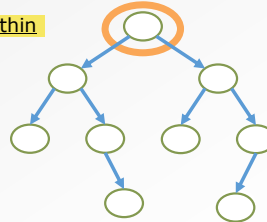
Next, we continue on accessing `cur->left` and `cur->right`.

```
TreeTraversal2 (cur->left);
TreeTraversal2 (cur->right);
```


TREETRAVERSAL() FEATURES

- **Recursive**

- TreeTraversal() is called from within its own body
- initial call TreeTraversal(root)



- **Depth-first**

- The traversal goes as deep as possible before backtracking and going sideways
- Not level-by-level! (that is called breadth-first)

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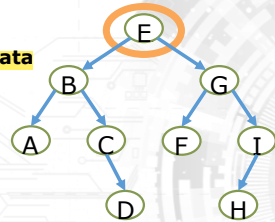
THREE "STANDARD" WAYS TO TRAVERSAL

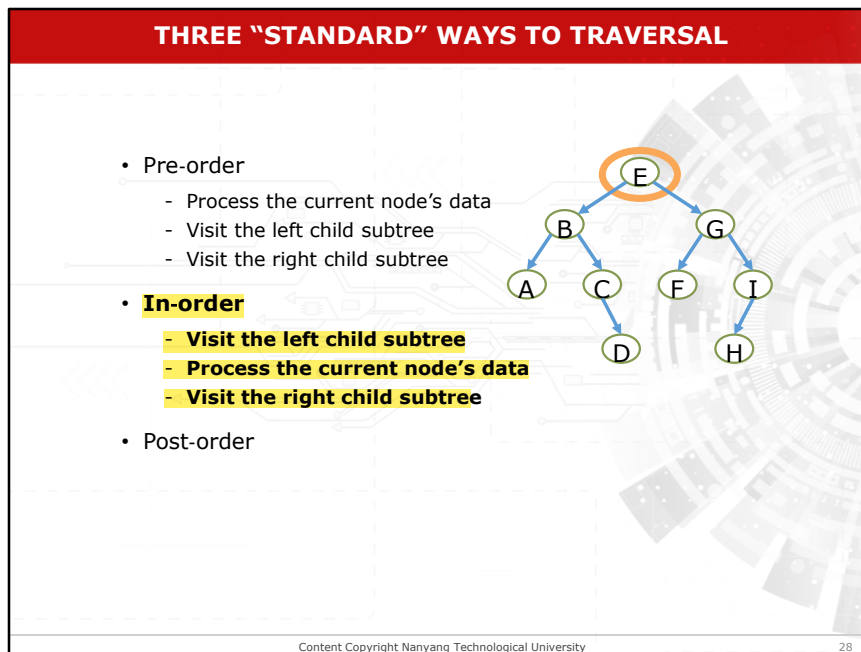
- Pre-order

- Process the current node's data
- Visit the left child subtree
- Visit the right child subtree

- In-order

- Post-order





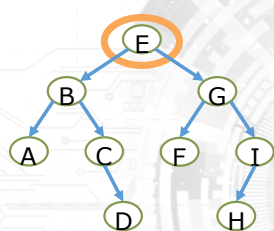
In-order traversal

- In in-order traversal we first visit the left child subtree.
 - For an example if we consider the given tree in the slide, the root node F is where we start traversing. Therefore at first we move the pointer from F to B. Now for B also, first we have to visit the left child subtree. Therefore we move the pointer from B to A.
 - Now for A there are no left child subtree therefore we proceed to process the current node's data.
 - Then we visit the right child subtree; but A does not have a right child. Therefore we move the pointer back to B.
 - Since B's left child subtree is already completed, we can proceed to B's data.
 - Then we will visit to right child subtree of B which is C. Now where have to visit C's left child subtree; but C does not have a left child subtree. Therefore we can proceed with C's data and move the pointer to it's right child subtree which is D.

- For D there is no left child or right child sub tree, therefore we proceed with D's data and return to it's parent C.
- C is already visited therefore we return to B and then to F.
- Now we have already visited the left child subtree of F. Therefore we can proceed with F's data.

THREE "STANDARD" WAYS TO TRAVERSAL

- Pre-order
 - Process the current node's data
 - Visit the left child subtree
 - Visit the right child subtree
- In-order
 - Visit the left child subtree
 - Process the current node's data
 - Visit the right child subtree
- **Post-order**
 - Visit the left child subtree
 - Visit the right child subtree
 - Process the current node's data



```
graph TD; E((E)) --> B((B)); E --> G((G)); B --> A((A)); B --> C((C)); C --> D((D)); G --> F((F)); G --> I((I)); I --> H((H));
```

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Post-order traversal

In post-order traversal; first we visit the left child subtree, then we visit the right child subtree and finally process the current node's data.

TREE TRAVERSAL - PRINT

- Recall the TreeTraversal() template (TT) - **Pre-order** :

- Simple task at each node: print out data in that node

```
void TreeTraversal(BTNode *cur){  
    if (cur == NULL)  
        return;  
  
    // Do something with the current node's data  
  
    TreeTraversal(cur->left); //Visit the left child node  
    TreeTraversal(cur->right); //Visit the right child node  
}
```

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Tree traversal-Print

For the pre-order traversal, we first check whether the current node is NULL or NOTNULL.

If the current node is NOT NULL we visit the current node then visit the left child subtree and finally visit the right child subtree.

TREE TRAVERSAL - PRINT

- Recall the TreeTraversal() template (TT) – **Pre-order** :

- Simple task at each node: print out data in that node

```
void TreeTraversal(BTNode *cur){  
    if (cur == NULL)  
        return;  
  
    printf("%c",cur->item);  
  
    TreeTraversal(cur->left); //Visit the left child node  
    TreeTraversal(cur->right); //Visit the right child node  
}
```

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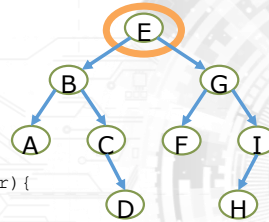
Once the pointer 'cur' points at the current node, we can print the data of the current node by issuing the given code.

```
print ('%c' , cur->item);
```


TREE TRAVERSAL PRE-ORDER: PRINT

Output:

E B A C D G F I H



```
void TreeTraversal_pre(BTNode *cur){  
    if (cur == NULL)  
        return;  
  
    printf("%c ", cur->item);  
  
    TreeTraversal_pre(cur->left); //Visit the left child node  
    TreeTraversal_pre(cur->right); //Visit the right child node  
}
```

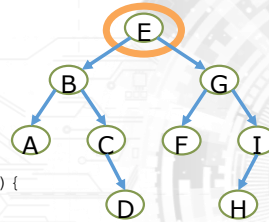
The output of the code would be E, B, A, C, D, G, F, I and H.

TREE TRAVERSAL IN-ORDER: PRINT

Output:

A B C D E F G H I

```
void TreeTraversal_in(BTNode *cur) {  
    if (cur == NULL)  
        return;  
  
    TreeTraversal_in(cur->left); //Visit the left child node  
    printf("%c ", cur->item);  
    TreeTraversal_in(cur->right); //Visit the right child node  
}
```



In in-order traversal, after checking whether the current node is NULL or NOT NULL, we check the left child subtree.

```
TreeTraversal_in(cur->left);
```

Then we print the current node value and check the right child subtree.

```
printf("%c", cur->item);  
TreeTraversal_in(cur->right);
```

The output of the code would be A, B, C, D, F, G, H and I

TREE TRAVERSAL POST-ORDER: PRINT

Output:

A D C B F H I G E

```
void TreeTraversal_post(BTNode *cur){  
    if (cur == NULL)  
        return;  
  
    TreeTraversal_post(cur->left); //Visit the left child node  
    TreeTraversal_post(cur->right); //Visit the right child node  
    printf("%c ", cur->item);  
}
```

```
graph TD  
    E((E)) --> B((B))  
    E --> G((G))  
    B --> A((A))  
    B --> C((C))  
    C --> D((D))  
    G --> F((F))  
    G --> I((I))  
    I --> H((H))
```

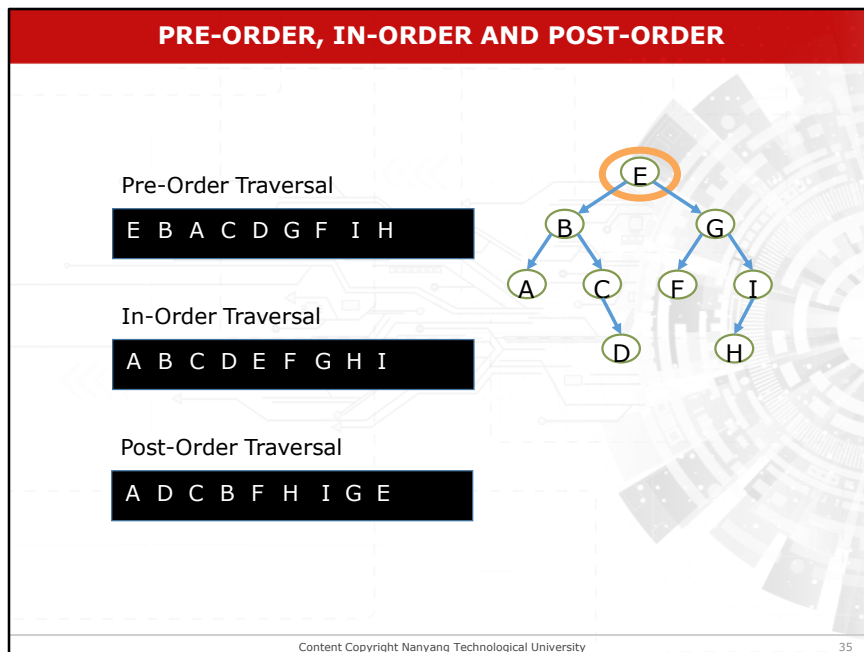
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In post-order traversal, we can print the value of the current node after checking it's left and right child subtree.

The output would be A, D, C, B, F, H, I, G and E.

Root node is the last node to be printed in post-order traversal.

Different traversal types outputs different results.



In post-order traversal, we can print the value of the current node after checking it's left and right child subtree.

The output would be A, D, C, B, F, H, I, G and E.

Root node is the last node to be printed in post-order traversal.

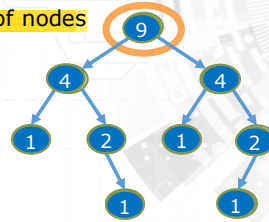
Different traversal types outputs different results.

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COUNT NODES IN A BINARY TREE

- Recursive definition:
 - Number of nodes in a tree
 $\equiv 1$
+ number of nodes in left subtree
+ number of nodes in right subtree
- Each node returns the number of nodes in its subtree



COUNT NODES IN A BINARY TREE

- Each node returns the number of nodes in its own subtree
- Leaf nodes return 1
Information **propagates upwards** as TreeTraversal returns from visiting leaf nodes
- Which is the first/last count to be returned?

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Count nodes in a binary tree

Q: Which is the **first count to be returned?**

A: The **most left leaf node** provides the first count

Q: Which is the **last count to be returned?**

A: The **root node** provides the last count

countNode()

- Return the size of your subtree to your parent node
- Leaf nodes must return 1 to parent node
- Root node returns size of entire tree

```
graph TD; Root(( )) --- L1L(( )); Root --- R1R(( )); L1L --- L2LL(( )); L1L --- L2LR(( )); R1R --- L2RR(( )); R1R --- L2RL(( )); L2LR --- L3LLL(( )); L2RL --- L3RLR(( ));
```

```
void TreeTraversal(BTNode *cur){  
    if (cur == NULL)  
        return;  
    //may do something with cur;  
    TreeTraversal(cur->left);  
    TreeTraversal(cur->right);  
    //may do something with cur;  
}
```

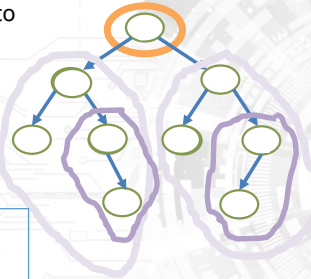
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We use TreeTraversal Template to count the no of nodes in Binary Tree

countNode()

- Return the size of your subtree to your parent node
- Leaf nodes must return 1 to parent node
- Root node returns size of entire tree

```
int countNode(BTNode *cur) {  
    if (cur == NULL)  
        return ???;  
  
    countNode(cur->left);  
    countNode(cur->right);  
    ??? //sum and get total;  
}
```



countNode()

- Leaf nodes must return 1
 - "Null" nodes should return 0
- Leaf node returns $1 + 0 + 0$

```

int countNode(BTNode *cur) {
    if (cur == NULL)
        return 0;

    l = countNode(cur->left);
    r = countNode(cur->right);
    return 1+r+l;
}

```

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- As learned, we first have to check if the current node is NULL or NOT NULL, if NULL we should return 0;

if (cur==NULL)
 return 0;
- Then we can define two variable as "l" and "r" where "l" gets the count of the nodes in left child subtrees and "r" gets the count of the nodes in the right child subtrees.

l =countNode(cur->left);
 r= countNode(cur->right);
- After get the values for "l" and "r", we can get the sum of of "l" and "r", add 1 to the sum and return the value;

return 1+r+l ;

countNode()

- Leaf nodes must return 1
 - "Null" nodes should return 0
- Leaf node returns $1 + 0 + 0$

```
int countNode(BTNode *cur) {  
    if (cur == NULL)  
        return 0;  
  
    return (countNode(cur->left)  
        + countNode(cur->right)  
        + 1);  
}
```

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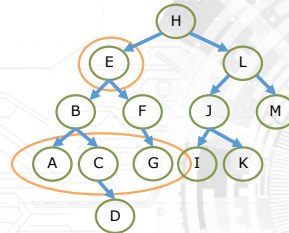
We can write the CountNode() function in even lesser complex way;
 $\text{return (countNode (cur->left) + countNode(cur->right)+1);}$

OUTLINE

- Non-linear data structures
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FIND GRANDCHILDREN

- Given a node X, find all the nodes that are X's grandchildren
- Given node E, we should return grandchild nodes A, C, and G
- What if we want to find k-level grandchildren?
 - Need a way to keep track of how many levels down we've gone



X->left->left
 X->left->right
 X->right->left
 X->right->right

2-level grandchildren

Find grandchildren

As per the given tree, E's grandchildren are A, C and G.

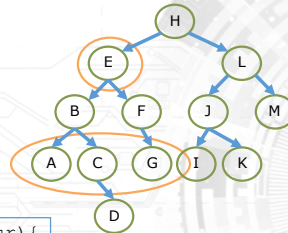
For the given X node, X's grandchildren can be listed as;

x->left-> left
 x->left->right
 x->right->left
 x->right->right

But the above list is only for the 2nd level grandchildren.

FIND GRANDCHILDREN

- We want to go down k "levels"
- Use a counter to track how far down we've gone
- At each `TreeTraversal(child)`, increment counter



```
void TreeTraversal(BTNode *cur) {  
    if (cur == NULL)  
        return;  
    // check counter  
    TreeTraversal(cur->left);  
    TreeTraversal(cur->right);  
}
```

Do something with the current node's data

Visit the left child node

Visit the right child node

To find the K -level grandchildren of a tree we use the `TreeTraversal` template.

FIND GRANDCHILDREN

```

void main( ){ ...
    if (X = null) return;
    findgrandchildren(X, 0);
}

1. void findgrandchildren(
    BTNode *cur, int c)
2.     if (cur == NULL) return;
3.     if (c == k) {
4.         printf("%d ", cur->item);
5.         return;
6.     }
7.     if (c < k) {
8.         findgrandchildren(cur->left, c+1);
9.         findgrandchildren(cur->right, c+1);
10.    }

```

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Find grandchildren

- First we declare the `findgrandchildren()` function. We should pass the pointer and a counter which we use to track how far we go down in the tree.
- Same as in the TreeTraversal template we check whether the current node is NULL or NOT NULL.

```
if (cur==NULL) return;
```
- If the counter is equal to K, we print the value of the current node.

```
if (C==K){
    printf("%d",cur->item);
    return;
}
```
- Otherwise ($C < K$) we go on traversing the left child subtree and the right child subtree. We have to **increase C by one because we are going down of the tree by 1 level.**

```
if (C<K){
    findgrandchildren(cur->left, C+1);
    findgrandchildren(cur->right, C+1);
}
```

```
findgrandchildren(cur->right, C+1);  
}
```


FIND GRANDCHILDREN

```

void main( ){ ...
    if (X = null) return;
    findgrandchildren(X, 0);
}

void findgrandchildren(
    BTreeNode *cur, int c){
    if (cur == NULL) return;

    if (c == k){
        printf("%d ", cur->item);
        return;
    }
    if (c < k){
        findgrandchildren(cur->left, c+1);
        findgrandchildren(cur->right, c+1); }
}

```

if k=2, we call
findgrandchildren(H,0),
what is the output?
How about k=3?
How about
findgrandchildren(H,1)?

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- If K=2, findgrandchildren (H,0) what is the output? 2 steps down
B, F, J and M
- If K=3;
A,C,G,I and K 3 steps down
- Findgrandchildren (H,1) when K=3? 2 steps down
B, F, J and M

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CALCULATE HEIGHT OF EVERY NODE

- Height of a node = number of links from that node to the deepest leaf node
- How does each node calculate its height?
 - What is the height of node D, C, H?
- We found:
 - leaf.height = 0
 - Non-leaf node X

$X.\text{height} = \max(X.\text{left.height}, X.\text{right.height}) + 1$

- Does information propagate upwards or downwards?

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Calculate height of every node

- For an example, if we want to find the height of H we have to check the height of its children E and L.
- For E we have to check B and F. For B we have to check A and C.
- A is a leaf node. Therefore A's height = 0
- C's child D also a leaf node. Therefore D's height is 0.
- C's height = D's height + 1 = 1
- For B we have to first find the max height of its children and then add 1 to it.
- B's height = $\max[B.\text{left.height}, B.\text{right.height}] + 1$

$$= \max[C, 1] + 1$$

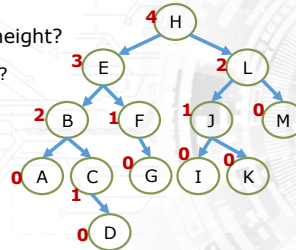
$$= 1 + 1$$

$$= 2$$

- Likewise we can fill the height of each node as in the slide.
- This time the information propagates upwards.

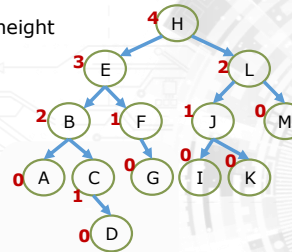
CALCULATE HEIGHT OF EVERY NODE

- Height of a node = number of links from that node to the deepest leaf node
- How does each node calculate its height?
 - What is the height of node D, C, H?
- Go through entire tree:
calculate and store height of
each node in the item field



CALCULATE HEIGHT OF EVERY NODE

- We want each node to report its height
 - Leaf node must report 0



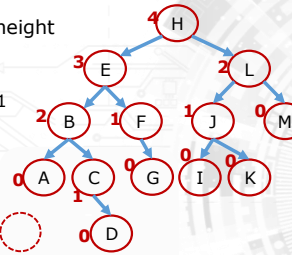
```
int TreeTraversal(BTNode *cur){  
    if(cur == NULL)  
        return ;  
  
    int l = TreeTraversal(cur->left);  
    int r = TreeTraversal(cur->right);  
  
    // do something here. Max( left, right)?  
  
    return ;  
}
```

We can use the TreeTraversal template for this example.

CALCULATE HEIGHT OF EVERY NODE

- We want each node to report its height
 - Leaf node must report 0
 - At "null" condition, must report -1

```
int TreeTraversal(BTNode *cur) {
    if (cur == NULL) {
        return -1;
    }
    int l = TreeTraversal(cur->left);
    int r = TreeTraversal(cur->right);
    int c = max(l, r) + 1;
    return c;
}
```



We define two integer variables as "l" and "r" where "l" gets the height of current node's left child subtree and "r" gets the height of current node's right child subtree. Then we can calculate C;

$\text{int } c = \max(l, r) + 1;$

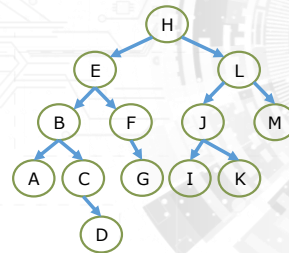
Q: At the beginning of the code, we check whether the current node is NULL or NOT NULL. If the current node is NULL should we return C?

A: No, If we return) to a NULL node; as in the example A is 1 level up to the null node therefore according to the defined code A's height would be 0+1 which is 1.

This is not correct because A is a leaf node. Therefore for the NULL node we have to pass -1.

QUESTIONS

- Does the tree traversal order matter?
- Depth of a node = number of links from that node to the root node. How does each node calculate its depth?



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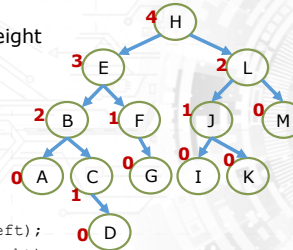
Post Order Traversal used Here:

- We have used the post-order traversal for this example.
- The tree traversal order should be selected
- Based on the problem
- Depth of a node can be calculated by considering root node's depth as 0. Here we propagate the information downwards

CALCULATE HEIGHT OF EVERY NODE

- **Height** of a node = number of links from that node to the deepest leaf node
- We want each node to report its height
 - Leaf node must report 0
 - At "null" condition, must report -1

```
int TreeTraversal(BTNode *cur) {  
    if (cur == NULL)  
        return -1;  
  
    int l = TreeTraversal(cur->left);  
    int r = TreeTraversal(cur->right);  
  
    int c = max (l, r) + 1;  
    return c;  
}
```



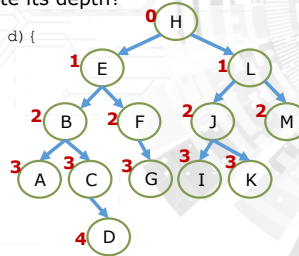
Post Order Traversal used Here:

- We have used the post-order traversal for this example.
- The tree traversal order should be selected based on the problem
- Depth of a node can be calculated by considering root node's depth as 0. Here we propagate the information downwards

QUESTIONS

- Does the tree traversal order matter?
- **Height** of a node = number of links from that node to the deepest leaf node
- **Depth** of a node = number of links from that node to the root node. How does each node calculate its depth?

```
void TreeTraversal(BTNode *cur, int d){
    if(cur == NULL)
        return;
    //print cur->item and d;
    TreeTraversal(cur->left, d+1);
    TreeTraversal(cur->right, d+1);
    return;
}
```



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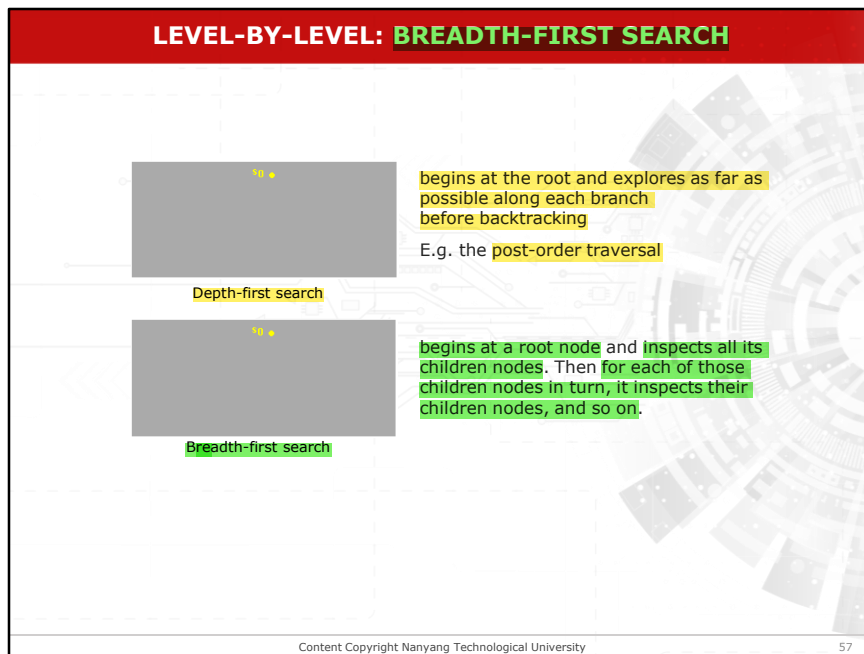
55

Here we propagate the information downwards

- Depth of a node can be calculated by considering root node's depth as 0.
- `printf("Depth of Node %c is : %d ", cur->item, d);`

OUTLINE

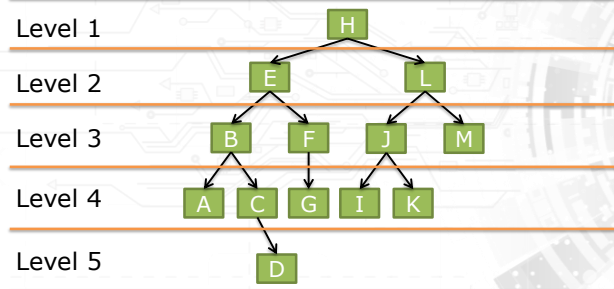
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- **Level-by-level traversal**
- Preorder traversal with a stack



There are two types of searching mechanisms.

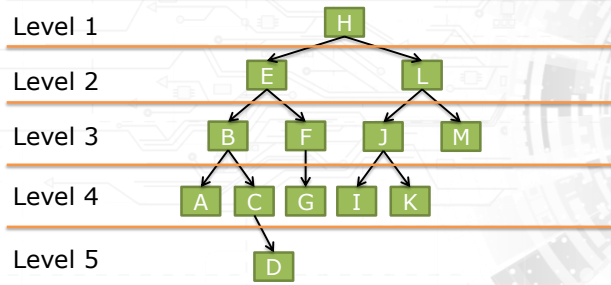
- Depth - first search
- Breadth – first search

LEVEL-BY-LEVEL TREE TRAVERSAL

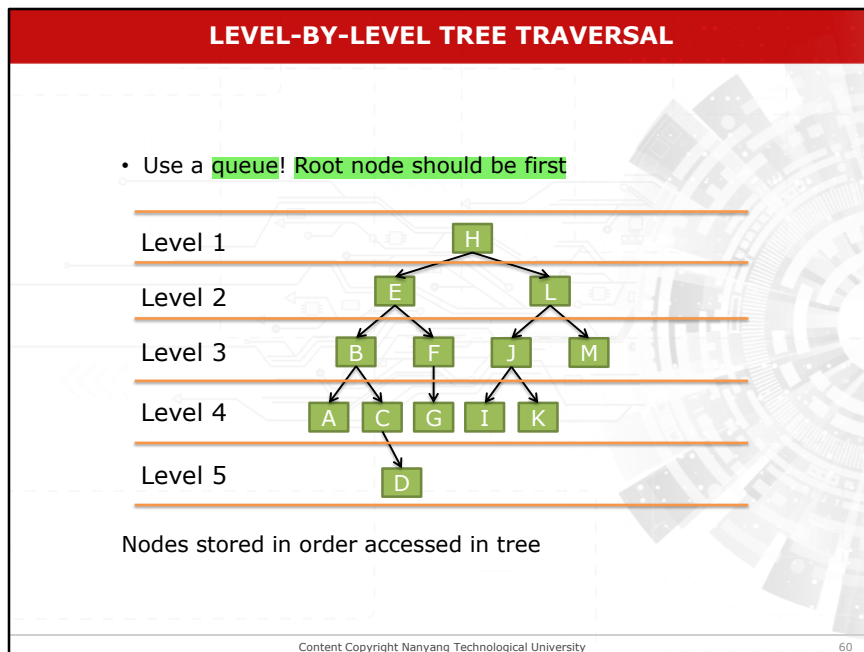


LEVEL-BY-LEVEL TREE TRAVERSAL

- Hint: Make use of another data structure



Nodes stored in order accessed in tree...

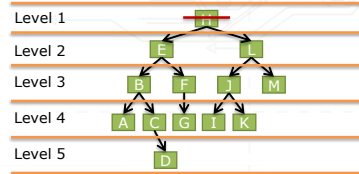


I have a queue which is capable of holding binary tree nodes

You need to change the internal node structure then your queue can hold binary tree nodes.

LEVEL-BY-LEVEL TREE TRAVERSAL

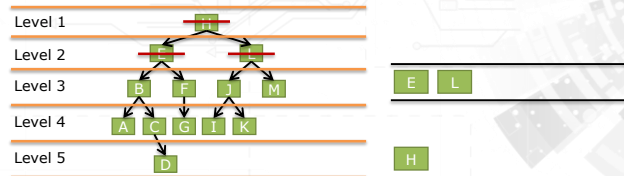
- Enqueue the root, H



H

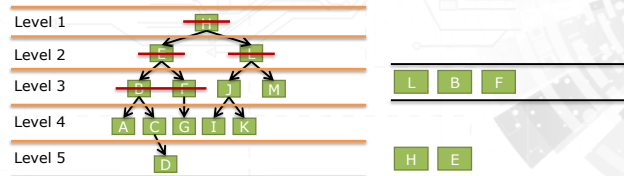
LEVEL-BY-LEVEL TREE TRAVERSAL

- Enqueue the root, H
- Dequeue H, and enqueue H's children



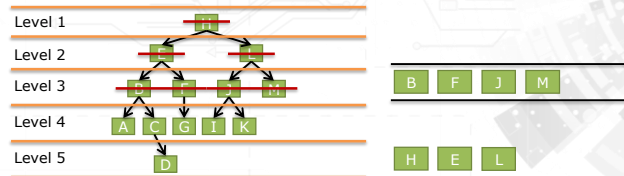
LEVEL-BY-LEVEL TREE TRAVERSAL

- Enqueue the root, H
- Dequeue H, and enqueue H's children
- Dequeue E, and enqueue E's children



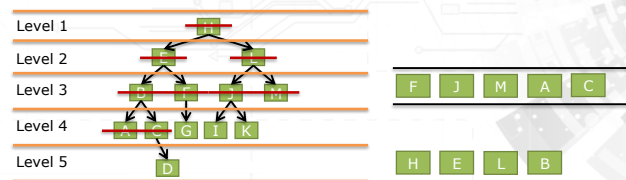
LEVEL-BY-LEVEL TREE TRAVERSAL

- Enqueue the root, H
- Dequeue H, and enqueue H's children
- Dequeue E, and enqueue E's children
- **Dequeue L, and enqueue L's children**



LEVEL-BY-LEVEL TREE TRAVERSAL

- Enqueue the root, H
- Dequeue H, and enqueue H's children
- Dequeue E, and enqueue E's children
- Dequeue L, and enqueue L's children
- **Dequeue B, and enqueue B's children**



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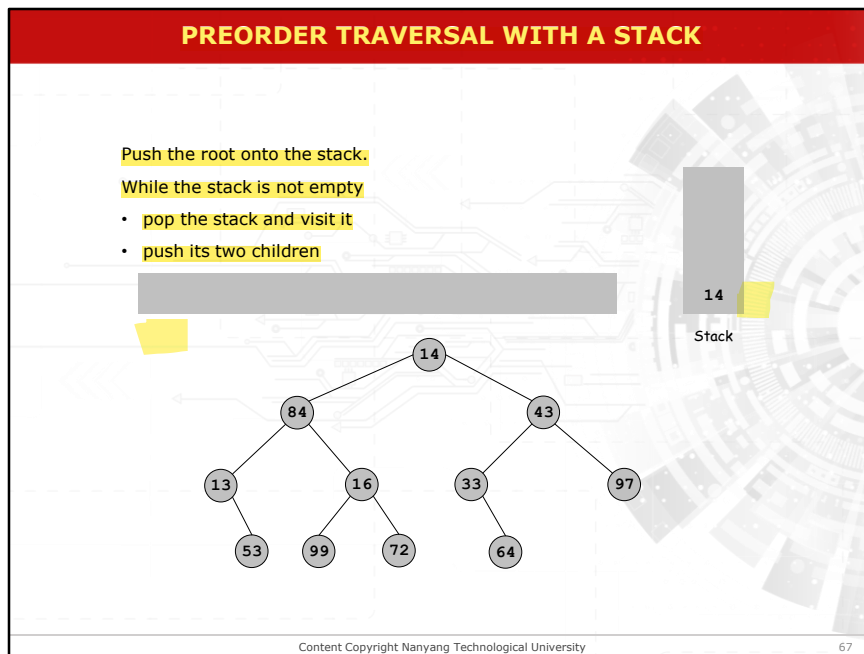
65

I don't touch level 3 nodes until I process level 2 nodes.

Similarly, I don't process level 2 nodes until I finished processing level 1 nodes.

OUTLINE

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- Level-by-level traversal
- **Preorder traversal with a stack**



I have a stack which is capable of holding binary tree nodes

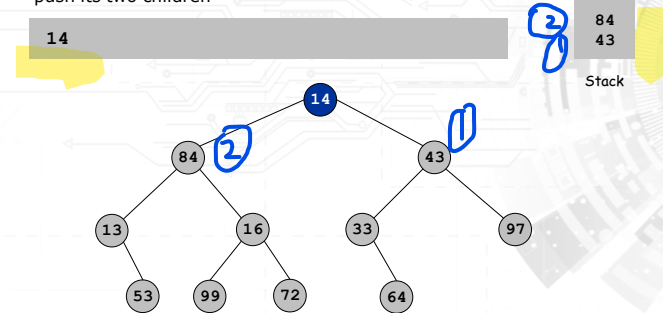
You need to change the internal node structure then your stack can hold binary tree nodes.

PREORDER TRAVERSAL WITH A STACK

Push the root onto the stack.

While the stack is not empty

- pop the stack and visit it
- push its two children



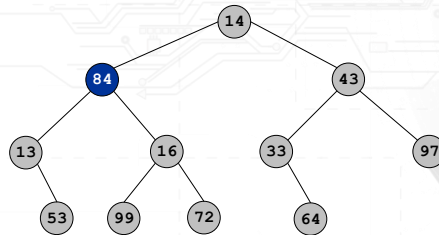
PREORDER TRAVERSAL WITH A STACK

Push the root onto the stack.

While the stack is not empty

- pop the stack and visit it
- push its two children

14 84



13
16
43
Stack

PREORDER TRAVERSAL WITH A STACK

Push the root onto the stack.

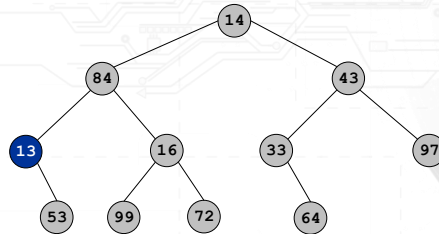
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13

53
16
43

Stack



PREORDER TRAVERSAL WITH A STACK

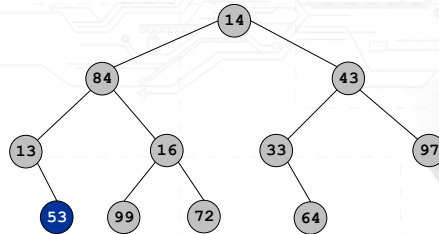
Push the root onto the stack.

While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53

16
43
Stack



PREORDER TRAVERSAL WITH A STACK

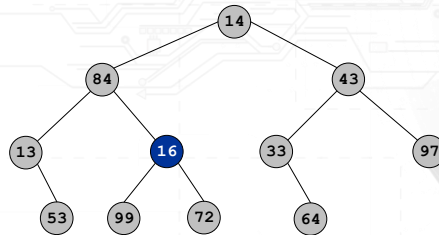
Push the root onto the stack.

While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16

99
72
43
Stack



PREORDER TRAVERSAL WITH A STACK

Push the root onto the stack.

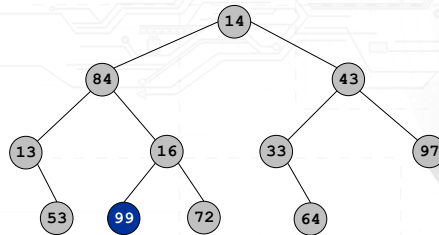
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16 99

72
43

Stack



PREORDER TRAVERSAL WITH A STACK

Push the root onto the stack.

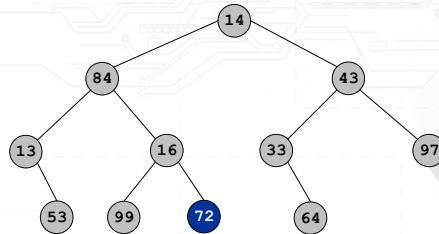
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16 99 72

43

Stack



PREORDER TRAVERSAL WITH A STACK

Push the root onto the stack.

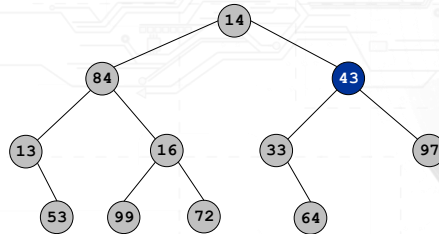
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16 99 72 43

33
97

Stack



PREORDER TRAVERSAL WITH A STACK

Push the root onto the stack.

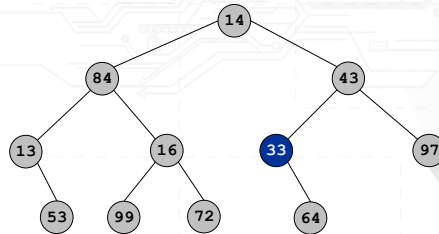
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16 99 72 43 33

64
97

Stack



PREORDER TRAVERSAL WITH A STACK

Push the root onto the stack.

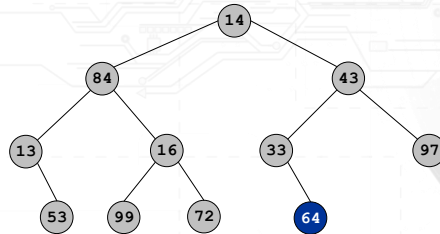
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16 99 72 43 33 64

97

Stack



PREORDER TRAVERSAL WITH A STACK

Push the root onto the stack.

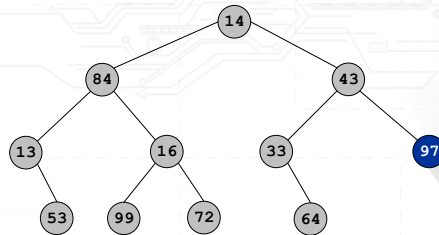
While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16 99 72 43 33 64 97



Stack



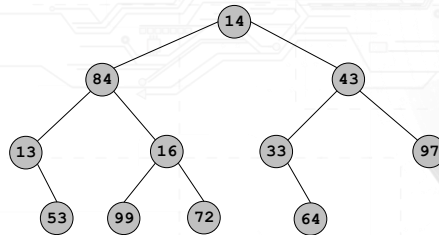
PREORDER TRAVERSAL WITH A STACK

Push the root onto the stack.

While the stack is not empty

- pop the stack and visit it
- push its two children

14 84 13 53 16 99 72 43 33 64 97



Stack

YOU SHOULD BE ABLE TO

- Binary tree Traverse:
 - Pre-order
 - In-order
 - Post-order
- Write recursive binary tree functions using the TreeTraversal template as a starting point
- Based on the traversal of the binary tree, do a lot of things: print, count numbers, count height/depth, find grandchildren,..., etc.

<http://nova.umuc.edu/~jark/idsv/lesson1.html>