MH1812 Tutorial
Chapter 9: Functions

Q1: Consider the set $A = \{a, b, c\}$ with power set $P(A)$ and intersection $\cap$ function: $P(A) \times P(A) \to P(A)$, i.e., for any $x, y \in P(A)$, $f(x, y) = x \cap y$. What is its domain? its co-domain? its range? What is the cardinality of the pre-image of $\{a\}$?

Q2: Show that $\sin : \mathbb{R} \to \mathbb{R}$ is not one-to-one.

Q3: Show that $\sin : \mathbb{R} \to \mathbb{R}$ is not onto, but $\sin : \mathbb{R} \to [-1, 1]$ is.

Q4: Is $h : \mathbb{Z} \to \mathbb{Z}$, $h(n) = 4n - 1$, onto (surjective)?

Q5: Is $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^3$, a bijection (one-to-one correspondence)?

Q6: Consider $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ and $g : \mathbb{R} \to \mathbb{R}$, $g(x) = x + 5$. What is $g \circ f$? What is $f \circ g$?

Q7: Consider $f : \mathbb{Z} \to \mathbb{Z}$, $f(n) = n + 1$ and $g : \mathbb{Z} \to \mathbb{Z}$, $g(n) = n^2$. What is $g \circ f$? What is $f \circ g$?

Q8: Given two functions $f : X \to Y$, $g : Y \to Z$. If $g \circ f : X \to Z$ is one-to-one, must both f and g be one-to-one? Prove or give a counter-example.

Q9: Show that if $f : X \to Y$ is invertible with inverse function $f^{-1} : Y \to X$, then $f^{-1} \circ f = i_X$ and $f \circ f^{-1} = i_Y$.

Q10: Prove or disprove $\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$, for x, y two real numbers.

Q11: If you pick five cards from a deck of 52 cards, prove that at least two will be of the same suit.

Q12: If you have 10 black socks and 10 white socks, and you are picking socks randomly, you will only need to pick three to find a matching pair.