L7 practice problems

Answers:

1.

    a. $F(X,Y,Z) = X'Y + X'Y'Z$

            $= X'Y(Z' + Z) + X'Y'Z$

            $= X'YZ' + X'YZ + X'Y'Z$

            $= \sum m (1, 2, 3)$

    b. $F(A,B,C,D) = ([(A+B')' + C] ' + D)'$ - apply DeMorgans theorem

            $= [(A+B')' + C]\ \ D'$

            $= (A'B + C) D'$

            $= A'BD' + CD'$

            $= A'B (C+C') D' + (A'B' + A'B + AB' + AB) CD'$

            $= A'B'CD' + A'BCD' + A'BC'D' + AB'CD' + ABCD'$

            $= \sum m (2, 6, 4, 10, 14)$

            $= \pi M (0, 1, 3, 5, 7, 8, 9, 11, 12, 13, 15)$

(a)

X Y Z	F
0 0 0	0
0 0 1	1
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	0
1 1 0	0
1 1 1	0

(b)

A B C D	F
0 0 0 0	0
0 0 0 1	0
0 0 1 0	1
0 0 1 1	0
0 1 0 0	1
0 1 0 1	0
0 1 1 0	1
0 1 1 1	0
1 0 0 0	0
1 0 0 1	0
1 0 1 0	1
1 0 1 1	0
1 1 0 0	0
1 1 0 1	0
1 1 1 0	1
1 1 1 1	0

2. $F(X, Y, Z) = \sum m (1, 2, 3)$

3.   F(A, B, C, D) = π M (0, 1, 3, 5, 7, 8, 9, 11, 12, 13, 15)

4a.   Z = ABC + AB'(A'C')'

        = ABC + AB'(A'' + C'')   - DeMorgan's theorem

        = ABC + AB' + AB'C

        = ABC + AB' (1 + C)

        = ABC + AB'

        = A(BC + B')   - apply absorption law X+X'Y = X+Y

        = A(C + B')

        = AC + AB'   (SOP)

4b.   X = (A' + B)(A + B + D)D'

        = (AA' + A'B + A'D + AB + BB + BD)D'

        = (A'B + AB + B + A'D + BD)D'

        = (B(1 + A' + A) + A'D + BD)D'

        = (B + A'D + BD)D'

        = BD' + A'DD' + BDD'

        = BD' + 0 + 0

        = BD'   (SOP)

A minimum-cost SOP expression will have the minimum number of products, and each product will have the minimum number of variables.
Similarly, a minimum-cost POS expression will have the minimum number of sums, and each sum will have the minimum number of variables.