N D Q H

1. A jar contains four coins: a nickel (5¢), a dime (10¢), a quarter (25¢), and a half-dollar (50¢). Three coins are randomly selected without replacement from the jar.

(a) List all the possible outcomes in sample space S.

(b) What is the probability that the total amount drawn will equal 60¢ or more?

Outcomes	Amt
{N, D, Q}	40¢
{N, D, H}	65¢
{N, Q, H}	80¢
{D, Q, H}	85¢

$$Prob(Amt \geq 60¢) = \frac{3}{4}$$

2. A mother prepares nine popsicles of different flavours: three of orange, three of cherry and three of grape, for a party of four children. If every child is allowed to choose a popsicle of his/her favourite flavour, what is the probability that all of them will get their choices?

Total # of ways for 4 children to choose the 3 flavours

$= 3 \times 3 \times 3 \times 3 = 81$

# of ways for anyone of them not getting his/her flavour

= # of ways for all of them to choose the same flavour

$= 3$

$P(\text{anyone of them not getting his/her flavour}) = \frac{3}{81} = \frac{1}{27}$

$\therefore P(\text{all of them getting their flavours}) = 1 - \frac{1}{27}$

3. When two events are mutually exclusive, they cannot both happen when the experiment is performed. Once event B has occurred, event A cannot occur, i.e. $P(A|B) = 0$ or $P(A \cap B) = 0$, and vice versa. The occurrence of event B certainly affects the probability of occurrence of event A. Therefore, mutually exclusive events must be dependent.

When two events are independent, the occurrence of event B does not affect the probability of occurrence of event A, i.e. $P(A|B) = P(A)$ or $P(A \cap B) = P(A)P(B)$, and vice versa. Event A may still occur even if event B has occurred. Therefore, independent events cannot be mutually exclusive.

Use the relationships above to fill in the table below:

$P(A) + P(B) - P(A \cap B)$
$\shortparallel$

P(A)	P(B)	Conditions	P(A|B)	P(A∩B)	P(AUB)
0.3	0.4	mutually exclusive	0	0	0.7
0.3	0.4	independent	0.3	0.12	0.58
0.1	0.5		0	0	0.6
0.2	0.5		0.2	0.1	0.6

4. A blood disease is found in 2% of the persons in a certain population. A new blood test will correctly identify 96% of the persons with the disease and 94% of the persons without the disease.

(a) What is the probability that a person who is called positive by the blood test actually has the disease?

(b) What is the probability that a person who is called negative by the blood test actually does not have the disease?

(c) Comment on the results obtained in part (a) & (b).

Let D = event that one has the disease

T = event that the test result is +ve

$P(D) = 0.02 \Rightarrow P(D') = 1 - 0.02 = 0.98$

$P(T|D) = 0.96 \Rightarrow P(T'|D) = 1 - 0.96 = 0.04$

$P(T'|D') = 0.94 \Rightarrow P(T|D') = 1 - 0.94 = 0.06$

(a) $P(D|T) = \dfrac{P(D \cap T)}{P(T)} = \dfrac{P(T|D)P(D)}{P(T \cap D) + P(T \cap D')}$

$= \dfrac{P(T|D)P(D)}{P(T|D)P(D) + P(T|D')P(D')} = 0.246$

(b) $P(D'|T') = \dfrac{P(T'|D')P(D')}{P(T'|D')P(D') + P(T'|D)P(D)} = 0.999$

(c) Getting a +ve result is a weak suggestion that one has the disease.
Whereas a -ve result almost guarantee that one does not have the disease.

5. (a) A magician has in his pocket a fair coin and a doctored coin where both sides are heads. If he randomly picks a coin to flip, and obtains a head, what is the probability that he picks the fair coin?

(b) If he flips the same coin the second time and obtains a head again, what is the probability that it is a fair coin?

F = fair coin , H = head

(a) $$P(F|H) = \frac{\overset{1/2}{P(H|F)}\,\overset{1/2}{P(F)}}{\underset{1/2}{P(H|F)}\,\underset{1/2}{P(F)} + \underset{1}{P(H|F')}\,\underset{1/2}{P(F')}} = \frac{1}{3}$$

$\frac{1}{2} \times \frac{1}{2}$

(b) $$P(F|HH) = \frac{P(HH|F)\,P(F)^{1/2}}{\underset{1/2 \times 1/2}{P(HH|F)}\,\underset{1/2}{P(F)} + \underset{1 \times 1}{P(HH|F')}\,\underset{1/2}{P(F')}} = \frac{1}{5}$$