SC2001 Unbounded Knapsack Problem

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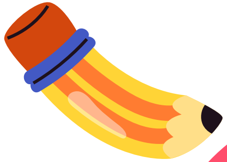


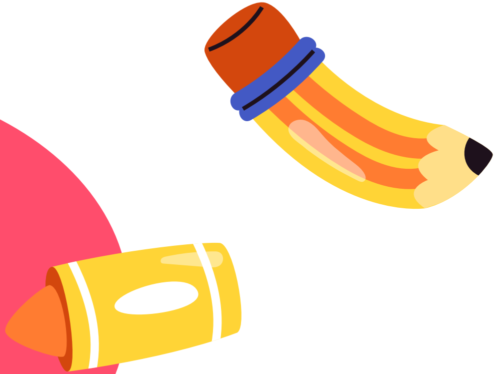
Presentation Overview

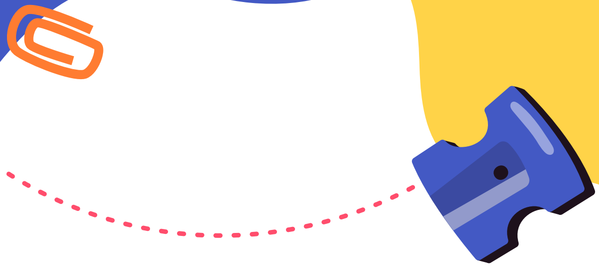
1. Recursive Definition

1. Subproblem graph

1. Bottom-up DP code

1. Running Results





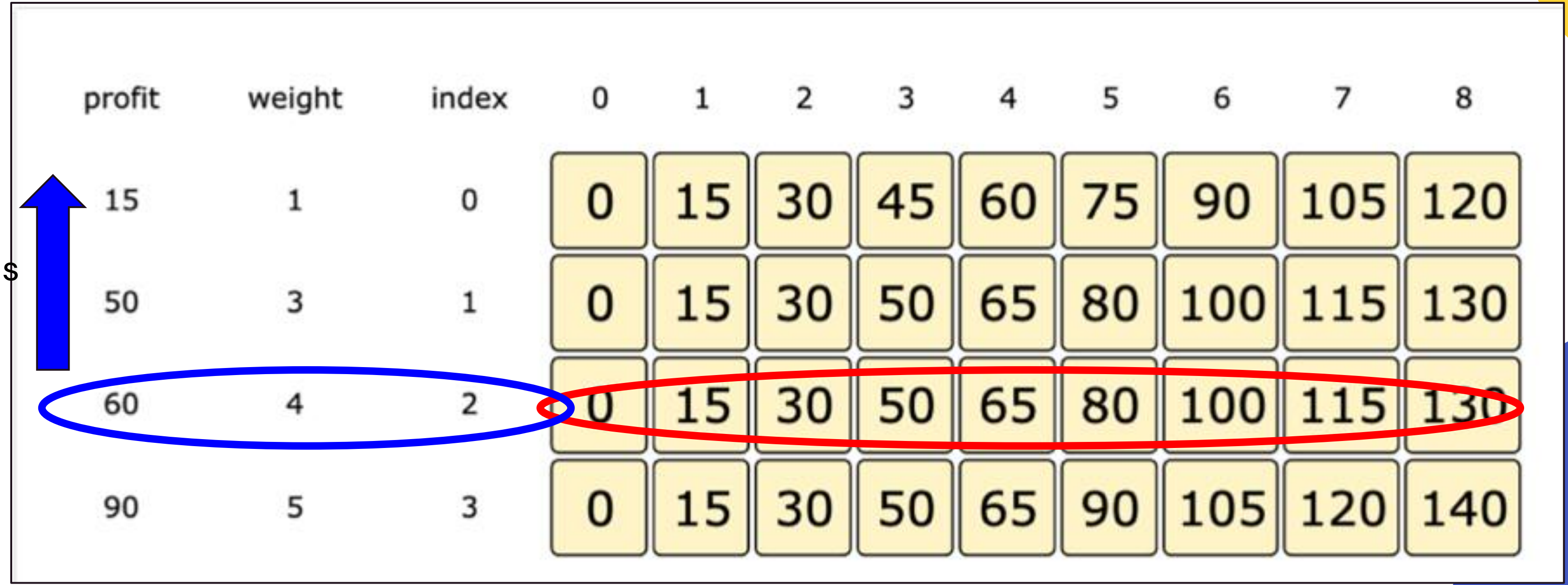
01

Recursive Definition

Understanding how to solve the Problem

GENERIC EXAMPLE

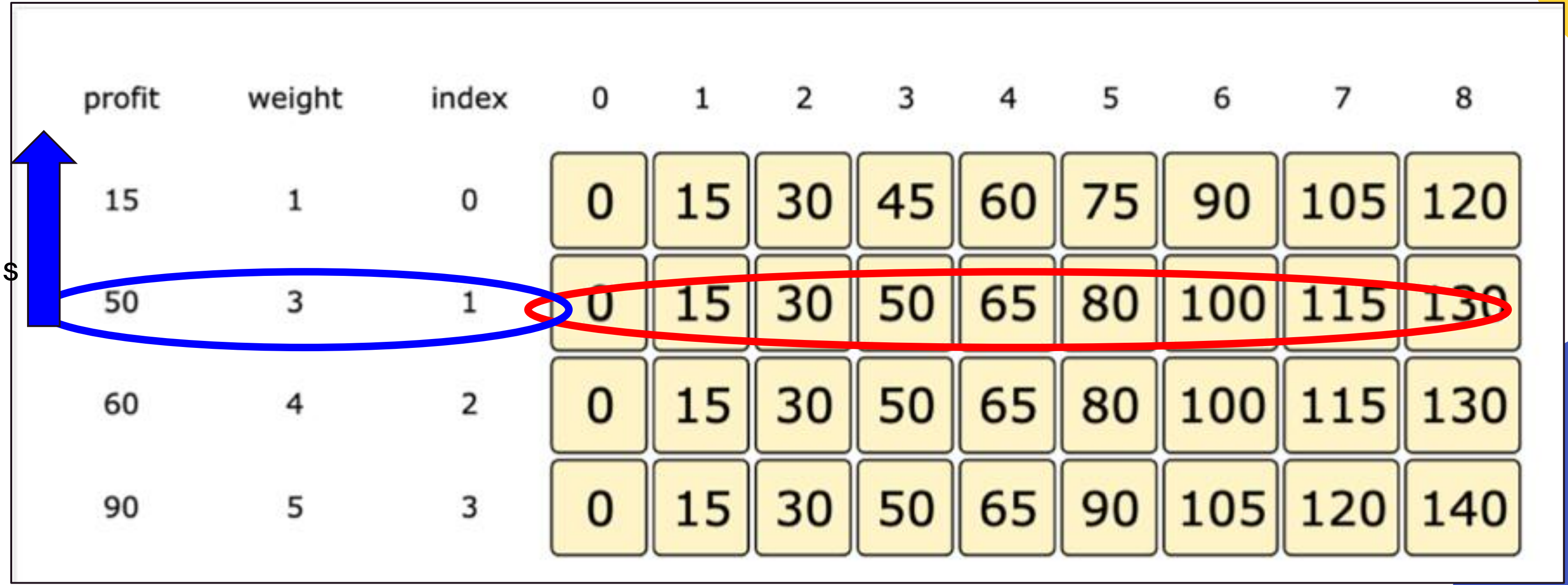
Only considers combinations for these items



profit	weight	index	0	1	2	3	4	5	6	7	8
15	1	0	0	15	30	45	60	75	90	105	120
50	3	1	0	15	30	50	65	80	100	115	130
60	4	2	0	15	30	50	65	80	100	115	130
90	5	3	0	15	30	50	65	90	105	120	140

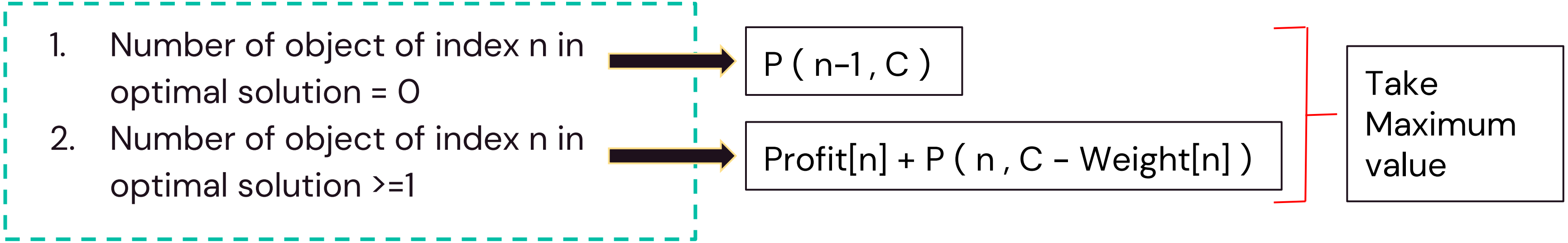
GENERIC EXAMPLE

Only considers combinations for these items



profit	weight	index	0	1	2	3	4	5	6	7	8
15	1	0	0	15	30	45	60	75	90	105	120
50	3	1	0	15	30	50	65	80	100	115	130
60	4	2	0	15	30	50	65	80	100	115	130
90	5	3	0	15	30	50	65	90	105	120	140

Possibilities to consider given capacity C and object of index n



1. Number of object of index n in optimal solution = 0
2. Number of object of index n in optimal solution >=1

P (n-1 , C)

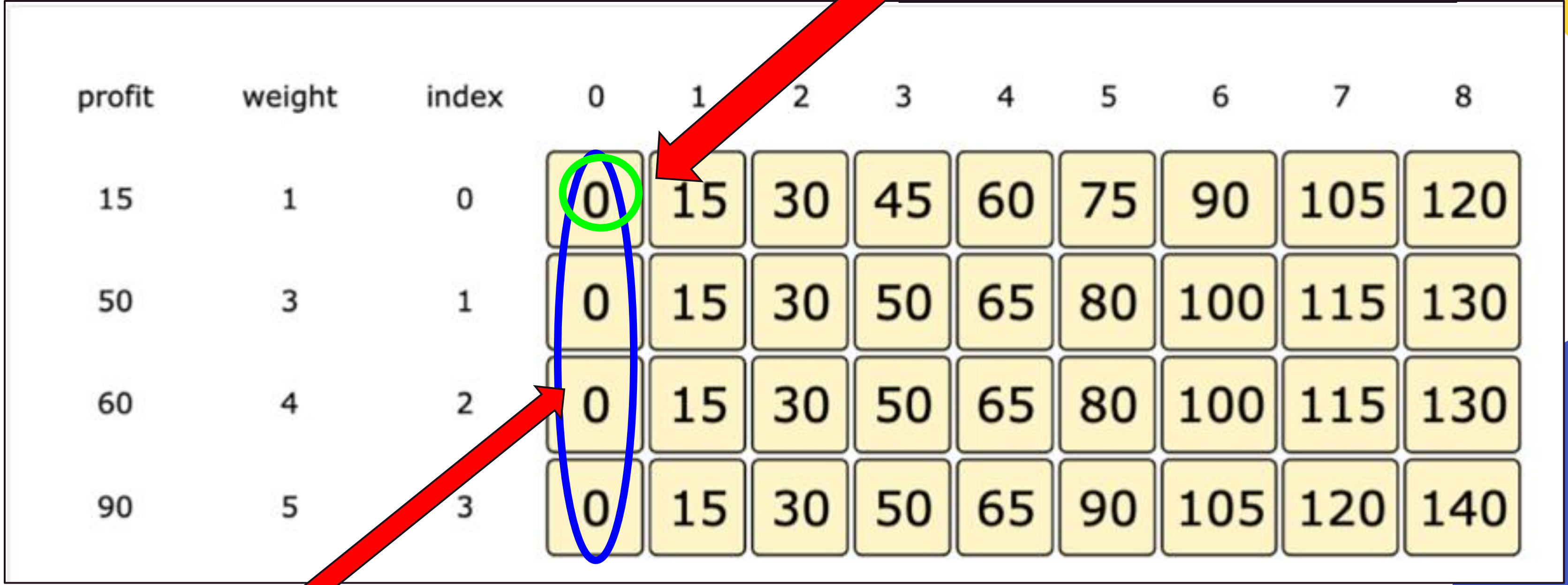
Profit[n] + P (n , C - Weight[n])

Take Maximum value

GENERIC EXAMPLE

Our Base Cases

Weight of First item > Bag capacity

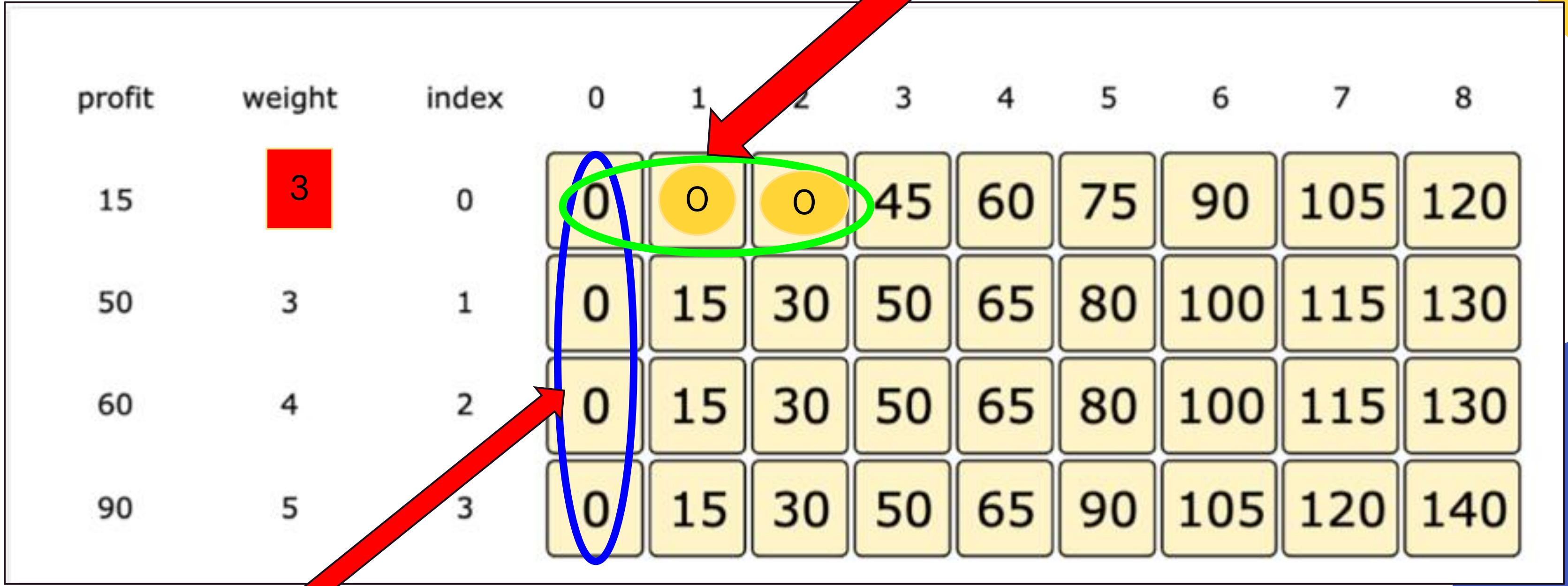


profit	weight	index	0	1	2	3	4	5	6	7	8
15	1	0	0	15	30	45	60	75	90	105	120
50	3	1	0	15	30	50	65	80	100	115	130
60	4	2	0	15	30	50	65	80	100	115	130
90	5	3	0	15	30	50	65	90	105	120	140

Bag Capacity=0

GENERIC EXAMPLE

Our Base Cases



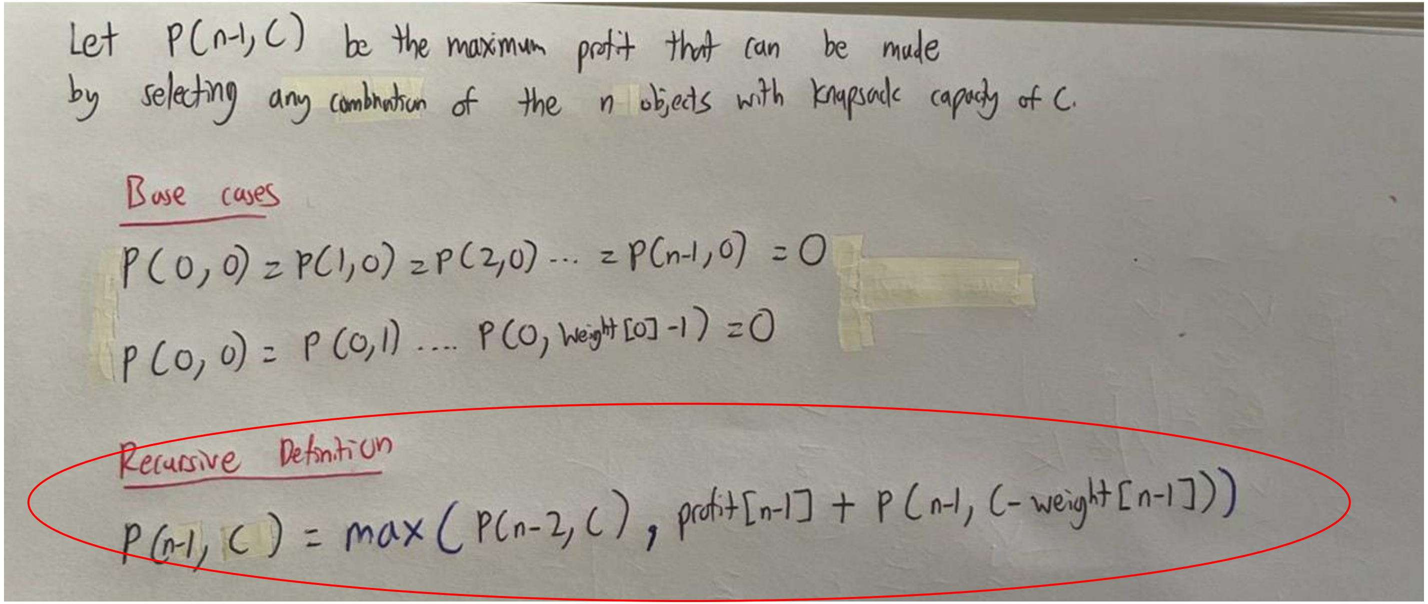
profit	weight	index	0	1	2	3	4	5	6	7	8
15	3	0	0	0	0	45	60	75	90	105	120
50	3	1	0	15	30	50	65	80	100	115	130
60	4	2	0	15	30	50	65	80	100	115	130
90	5	3	0	15	30	50	65	90	105	120	140

Weight of First item > Bag capacity

Bag Capacity=0

Q1 Recursive Definition:

For N objects, N-1 is our Maximum object index, hence we have the following:



Let $P(n-1, C)$ be the maximum profit that can be made by selecting any combination of the n objects with knapsack capacity of C.

Base cases

$P(0,0) = P(1,0) = P(2,0) \dots = P(n-1,0) = 0$

$P(0,0) = P(0,1) \dots P(0, Weight[0]-1) = 0$

Recursive Definition

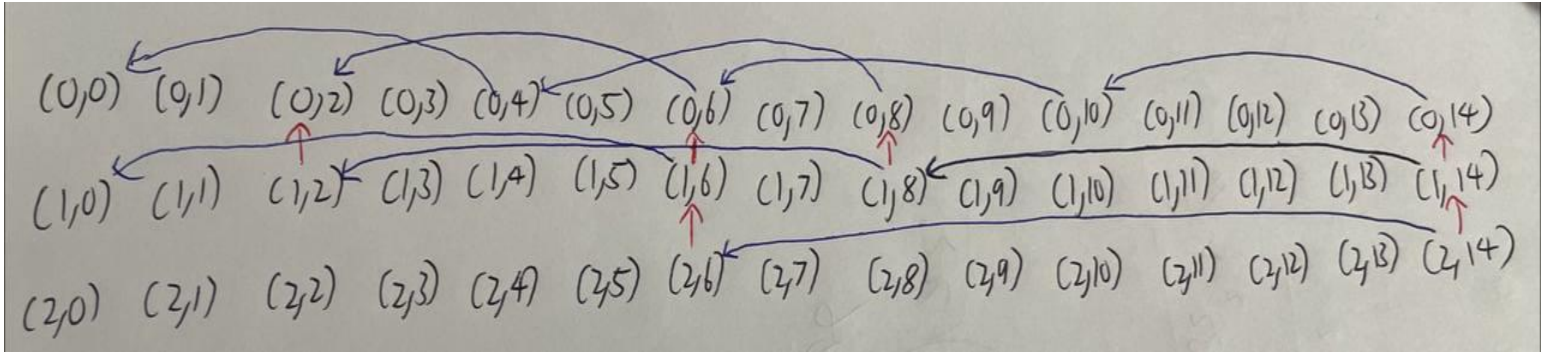
$P(n-1, C) = max(P(n-2, C), profit[n-1] + P(n-1, C - weight[n-1]))$

02

Subproblem Graph

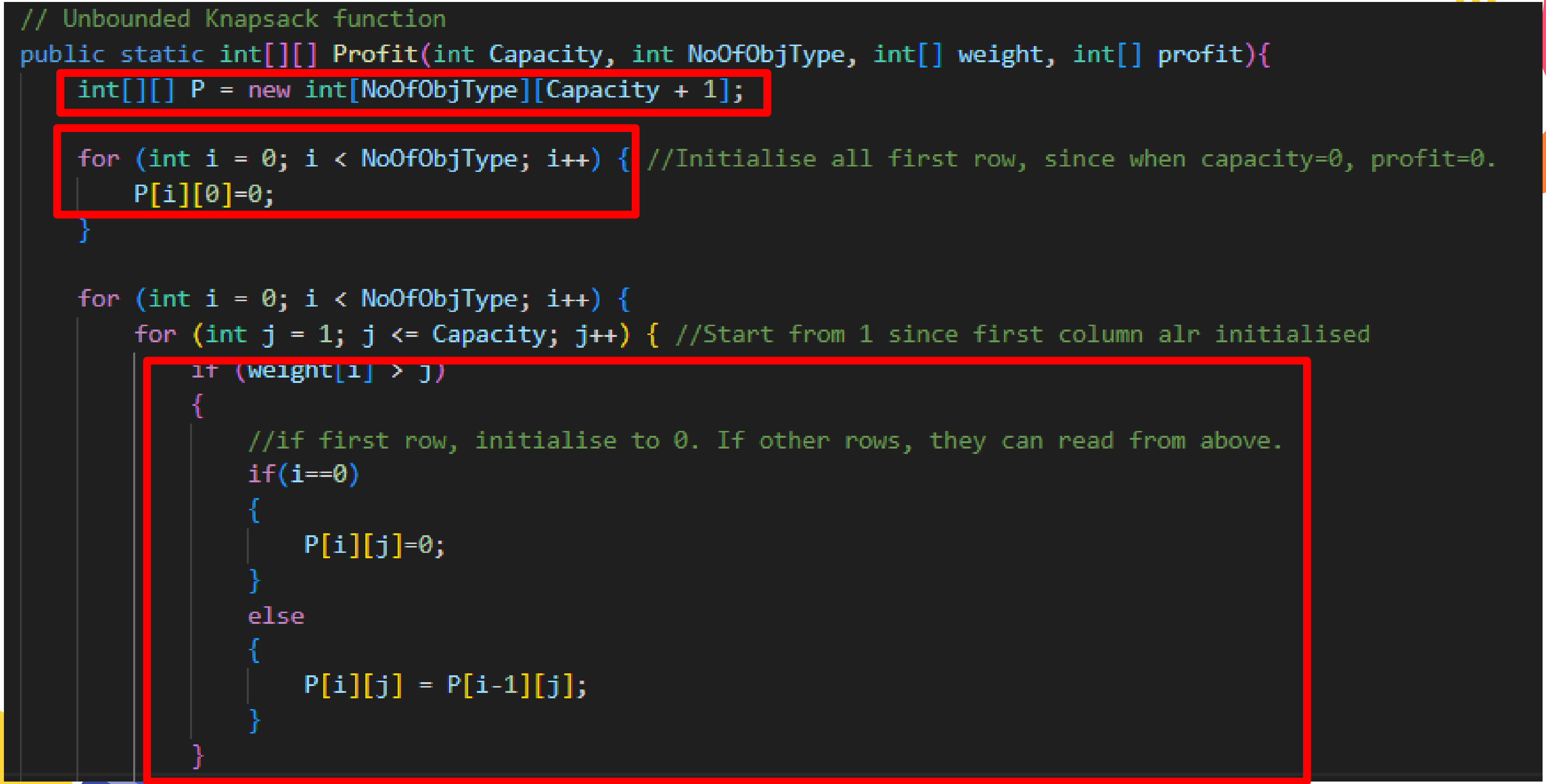
Q2 Subproblem graph for P(14) given the following items:

	0	1	2
w_i	4	6	8
p_i	7	6	9



03

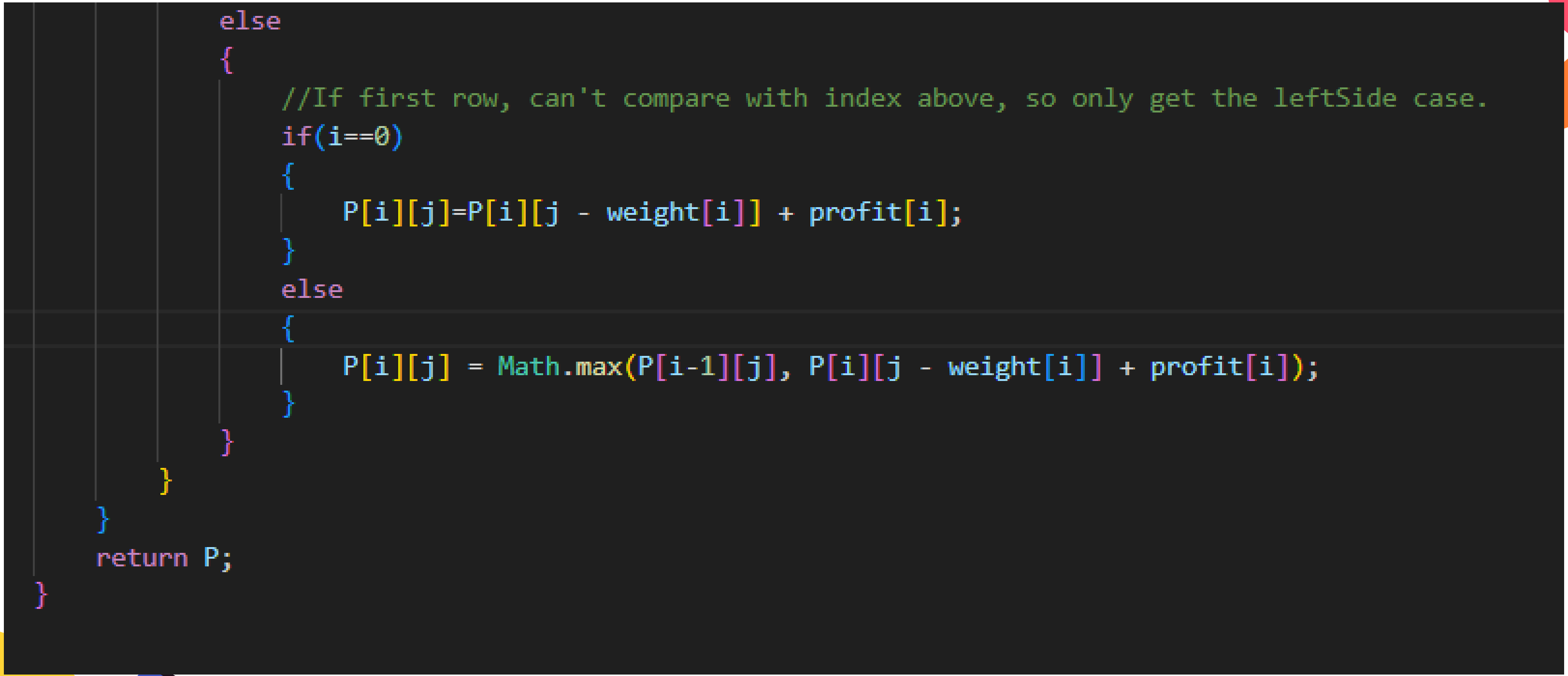
Algorithm



```
// Unbounded Knapsack function
public static int[][] Profit(int Capacity, int NoOfObjType, int[] weight, int[] profit){
    int[][] P = new int[NoOfObjType][Capacity + 1];

    for (int i = 0; i < NoOfObjType; i++) { //Initialise all first row, since when capacity=0, profit=0.
        P[i][0]=0;
    }

    for (int i = 0; i < NoOfObjType; i++) {
        for (int j = 1; j <= Capacity; j++) { //Start from 1 since first column alr initialised
            if (weight[i] > j)
            {
                //if first row, initialise to 0. If other rows, they can read from above.
                if(i==0)
                {
                    P[i][j]=0;
                }
                else
                {
                    P[i][j] = P[i-1][j];
                }
            }
```

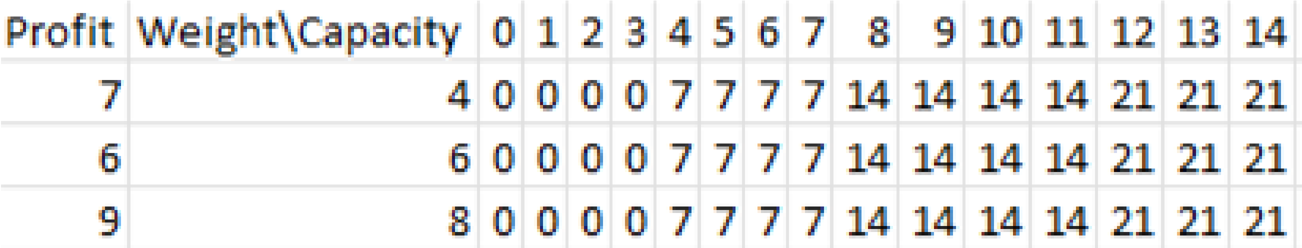


```
            else
            {
                //If first row, can't compare with index above, so only get the leftSide case.
                if(i==0)
                {
                    P[i][j]=P[i][j - weight[i]] + profit[i];
                }
                else
                {
                    P[i][j] = Math.max(P[i-1][j], P[i][j - weight[i]] + profit[i]);
                }
            }
        }
    }
    return P;
}
```

04 Results

a)Result of P(14): object table 1

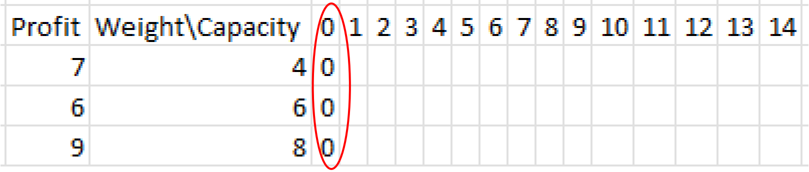
i	0	1	2
Wi	4	6	8
pi	7	6	9



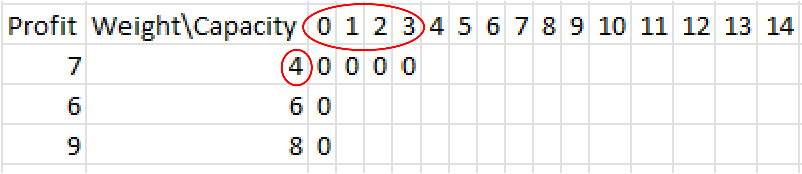
Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
6	6	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
9	8	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21

a)Result of P(14): object table 1,steps 1-2

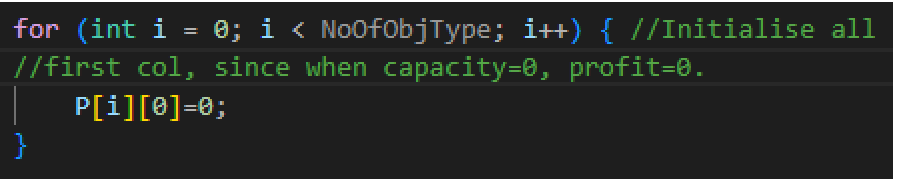
i	0	1	2
Wi	4	6	8
pi	7	6	9



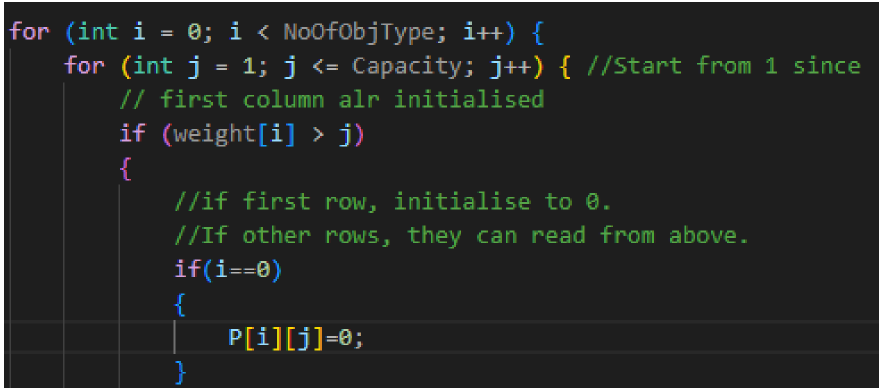
Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0														
6	6	0														
9	8	0														



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0	0	0	0											
6	6	0														
9	8	0														



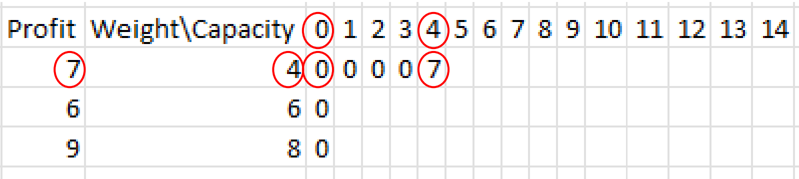
```
for (int i = 0; i < NoOfObjType; i++) { //Initialise all
//first col, since when capacity=0, profit=0.
    P[i][0]=0;
}
```



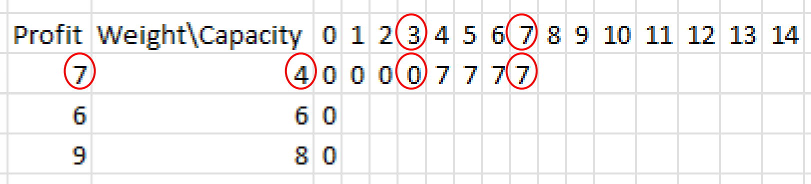
```
for (int i = 0; i < NoOfObjType; i++) {
    for (int j = 1; j <= Capacity; j++) { //Start from 1 since
        // first column alr initialised
        if (weight[i] > j)
        {
            //if first row, initialise to 0.
            //If other rows, they can read from above.
            if(i==0)
            {
                P[i][j]=0;
            }
```

a)Result of P(14): object table 1,step 3

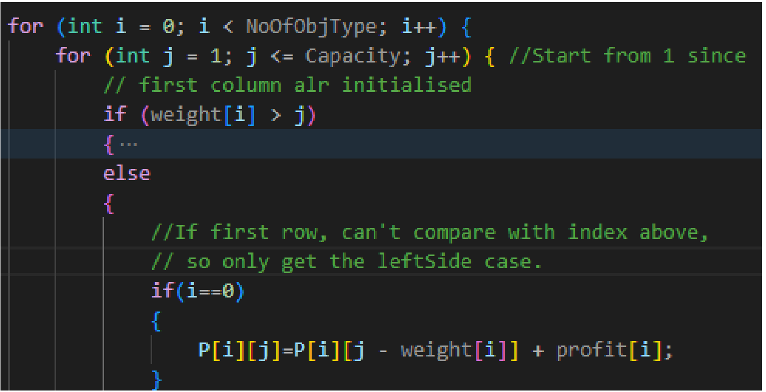
i	0	1	2
Wi	4	6	8
pi	7	6	9



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0	0	0	0	7										
6	6	0														
9	8	0														



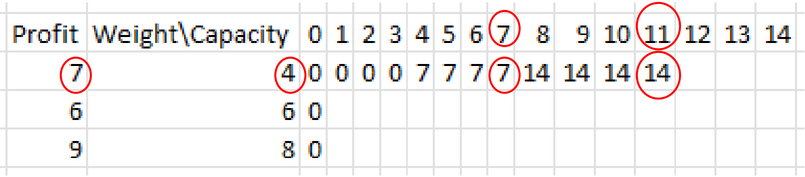
Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0	0	0	0	7	7	7	7							
6	6	0														
9	8	0														



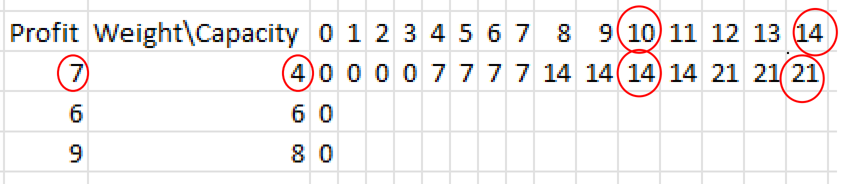
```
for (int i = 0; i < NoOfObjType; i++) {
    for (int j = 1; j <= Capacity; j++) { //Start from 1 since
        // first column alr initialised
        if (weight[i] > j)
        {⋯
        else
        {
            //If first row, can't compare with index above,
            // so only get the leftSide case.
            if(i==0)
            {
                P[i][j]=P[i][j - weight[i]] + profit[i];
            }
```

a)Result of P(14): object table 1,step 3(cont)

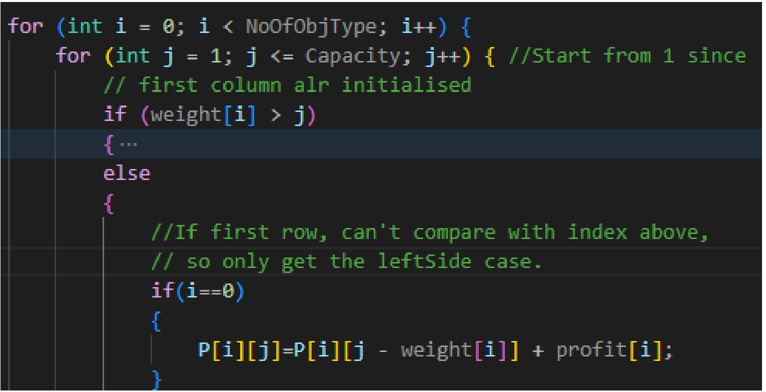
i	0	1	2
Wi	4	6	8
pi	7	6	9



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0	0	0	0	7	7	7	7	14	14	14	14			
6	6	0														
9	8	0														



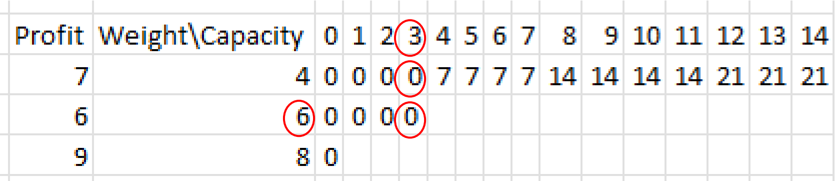
Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
6	6	0														
9	8	0														



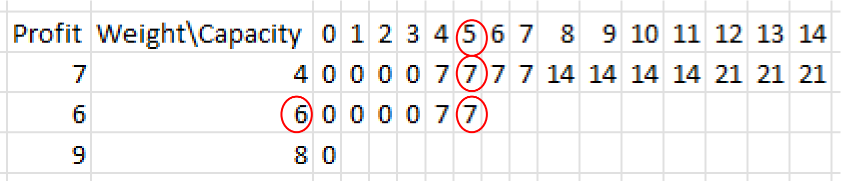
```
for (int i = 0; i < NoOfObjType; i++) {
    for (int j = 1; j <= Capacity; j++) { //Start from 1 since
        // first column alr initialised
        if (weight[i] > j)
        {…
        else
        {
            //If first row, can't compare with index above,
            // so only get the leftSide case.
            if(i==0)
            {
                P[i][j]=P[i][j - weight[i]] + profit[i];
            }
```

a)Result of P(14): object table 1,step 4

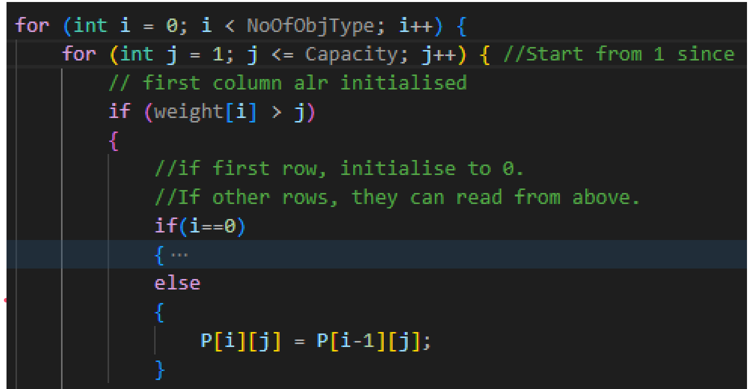
i	0	1	2
Wi	4	6	8
pi	7	6	9



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
6	6	0	0	0	0											
9	8	0														



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
6	6	0	0	0	0	7	7									
9	8	0														



```
for (int i = 0; i < NoOfObjType; i++) {
    for (int j = 1; j <= Capacity; j++) { //Start from 1 since
        // first column alr initialised
        if (weight[i] > j)
        {
            //if first row, initialise to 0.
            //If other rows, they can read from above.
            if(i==0)
            {···
            else
            {
                P[i][j] = P[i-1][j];
            }
```

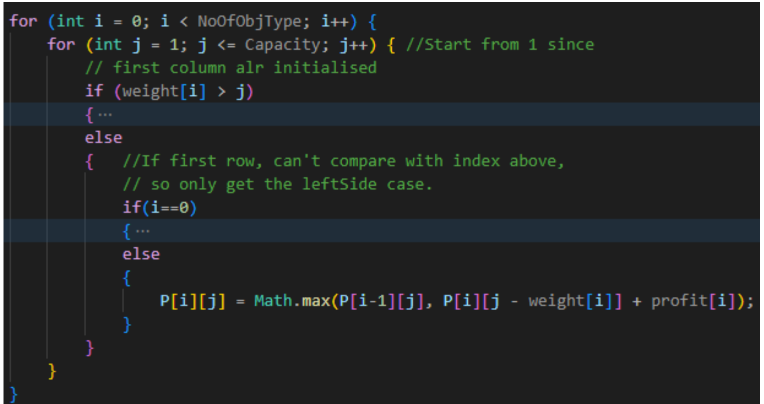
a)Result of P(14): object table 1,step 5

i	0	1	2
Wi	4	6	8
pi	7	6	9

max(7,6) = 7

Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
6	6	0	0	0	0	7	7	7								
9	8	0														

Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
6	6	0	0	0	0	7	7	7								
9	8	0														

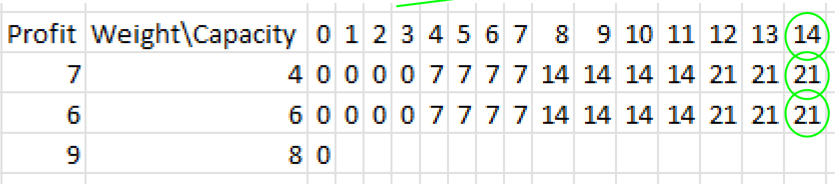


```
for (int i = 0; i < NoOfObjType; i++) {
    for (int j = 1; j <= Capacity; j++) { //Start from 1 since
        // first column alr initialised
        if (weight[i] > j)
        {…
        else
        {   //If first row, can't compare with index above,
            // so only get the leftSide case.
            if(i==0)
            {…
            else
            {
                P[i][j] = Math.max(P[i-1][j], P[i][j - weight[i]] + profit[i]);
            }
        }
    }
}
```

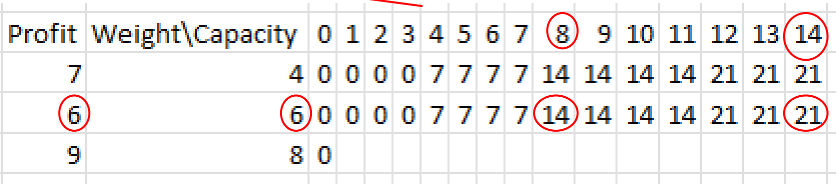
a)Result of P(14): object table 1,step 5(cont)

i	0	1	2
W_i	4	6	8
p_i	7	6	9

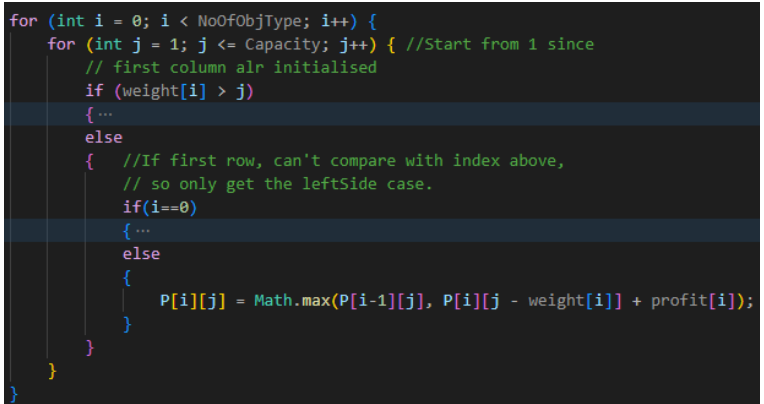
max(21,20) = 21



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
6	6	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
9	8	0														



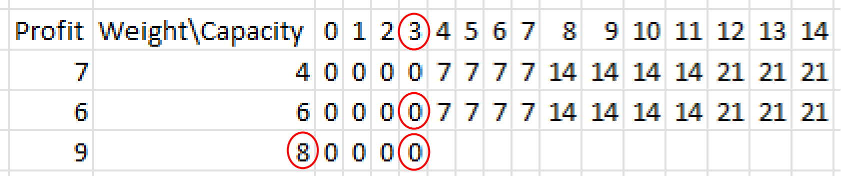
Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
6	6	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
9	8	0														



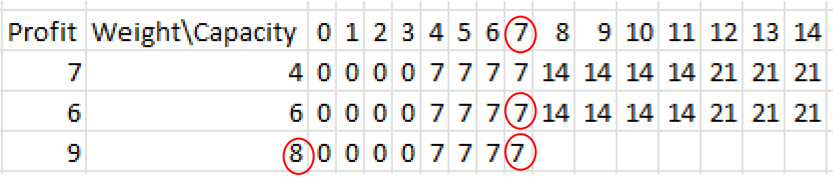
```
for (int i = 0; i < NoOfObjType; i++) {
    for (int j = 1; j <= Capacity; j++) { //Start from 1 since
        // first column alr initialised
        if (weight[i] > j)
        {…
        else
        {   //If first row, can't compare with index above,
            // so only get the leftSide case.
            if(i==0)
            {…
            else
            {
                P[i][j] = Math.max(P[i-1][j], P[i][j - weight[i]] + profit[i]);
            }
        }
    }
}
```

a)Result of P(14): object table 1,step 4

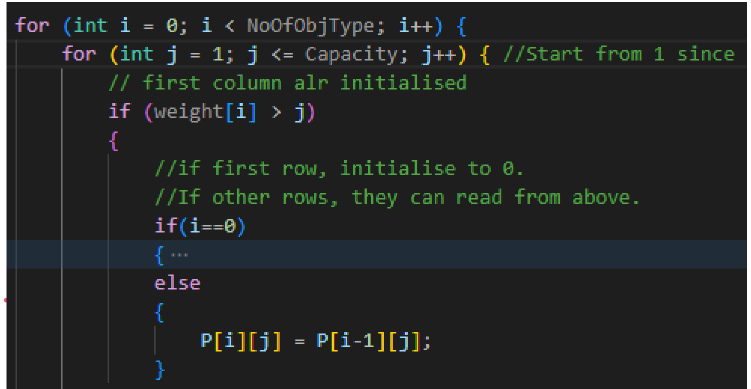
i	0	1	2
Wi	4	6	8
pi	7	6	9



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
6	6	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
9	8	0	0	0	0											



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
6	6	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
9	8	0	0	0	0	7	7	7	7							

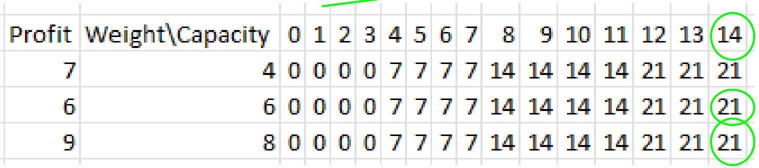


```
for (int i = 0; i < NoOfObjType; i++) {
    for (int j = 1; j <= Capacity; j++) { //Start from 1 since
        // first column alr initialised
        if (weight[i] > j)
        {
            //if first row, initialise to 0.
            //If other rows, they can read from above.
            if(i==0)
            {…
            else
            {
                P[i][j] = P[i-1][j];
            }
```

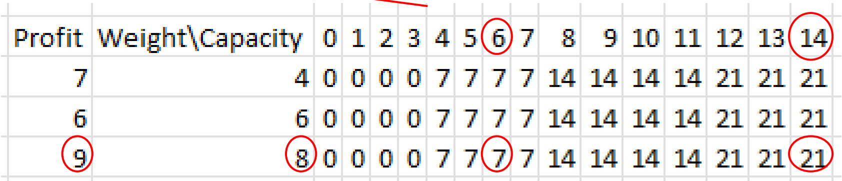
a)Result of P(14): object table 1,step 5

i	0	1	2
Wi	4	6	8
pi	7	6	9

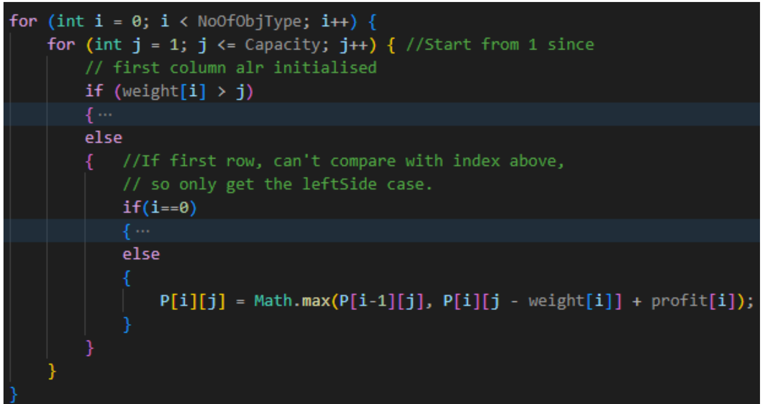
max(21,16) = 21



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
6	6	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
9	8	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21



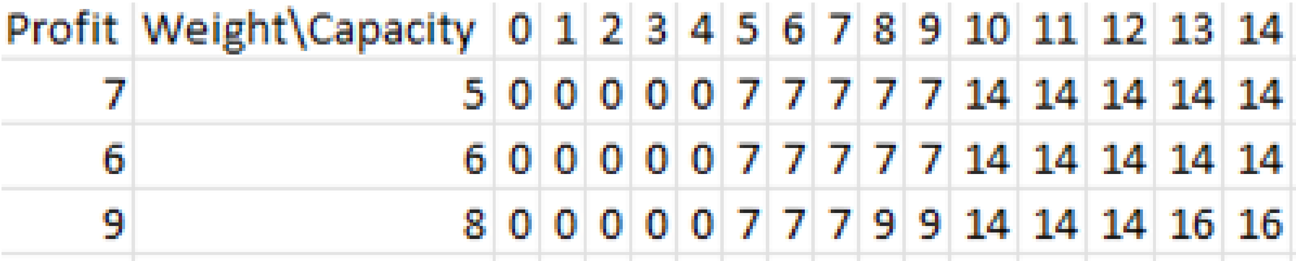
Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	4	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
6	6	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21
9	8	0	0	0	0	7	7	7	7	14	14	14	14	21	21	21



```
for (int i = 0; i < NoOfObjType; i++) {
    for (int j = 1; j <= Capacity; j++) { //Start from 1 since
        // first column alr initialised
        if (weight[i] > j)
        {⋯
        else
        {   //If first row, can't compare with index above,
            // so only get the leftSide case.
            if(i==0)
            {⋯
            else
            {
                P[i][j] = Math.max(P[i-1][j], P[i][j - weight[i]] + profit[i]);
            }
        }
    }
}
```

b)Result of P(14): object table 2

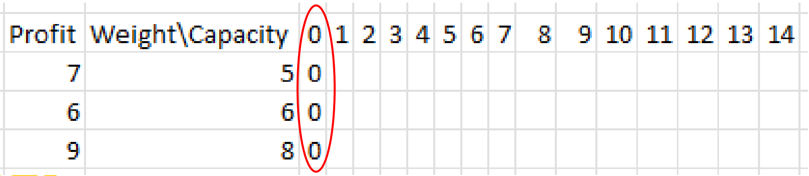
i	0	1	2
Wi	5	6	8
pi	7	6	9



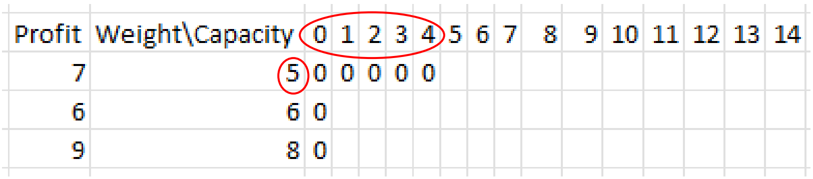
Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	5	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
6	6	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
9	8	0	0	0	0	0	7	7	7	9	9	14	14	14	16	16

a)Result of P(14): object table 2,steps 1-2

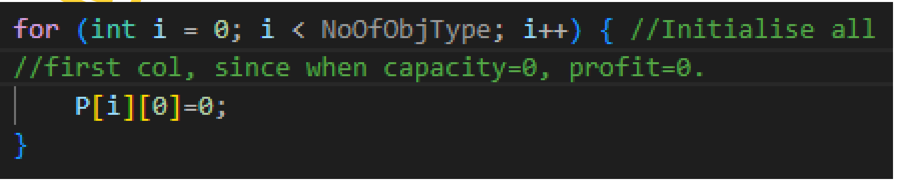
i	0	1	2
Wi	5	6	8
pi	7	6	9



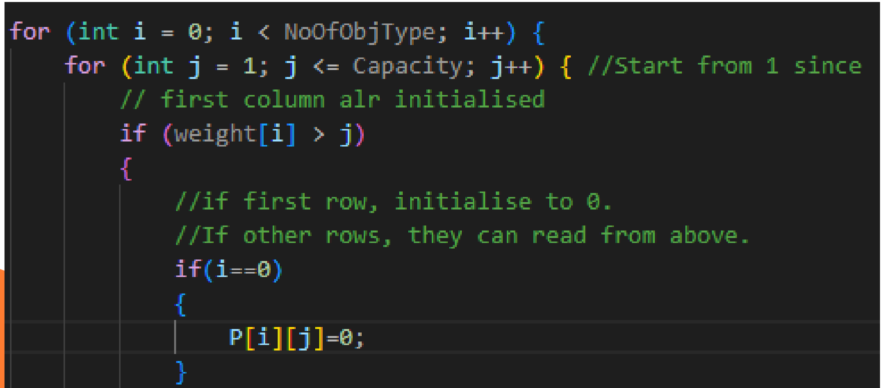
Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	5	0														
6	6	0														
9	8	0														



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	5	0	0	0	0	0										
6	6	0														
9	8	0														



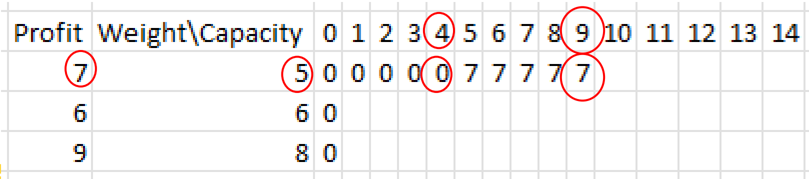
```
for (int i = 0; i < NoOfObjType; i++) { //Initialise all
//first col, since when capacity=0, profit=0.
    P[i][0]=0;
}
```



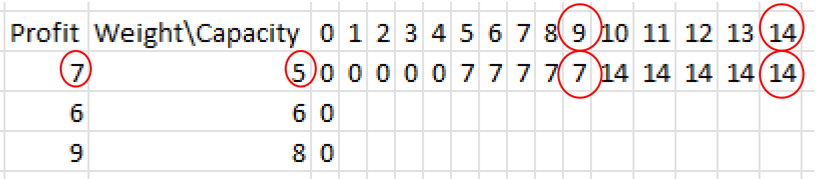
```
for (int i = 0; i < NoOfObjType; i++) {
    for (int j = 1; j <= Capacity; j++) { //Start from 1 since
        // first column alr initialised
        if (weight[i] > j)
        {
            //if first row, initialise to 0.
            //If other rows, they can read from above.
            if(i==0)
            {
                P[i][j]=0;
            }
```

a)Result of P(14): object table 1,step 3

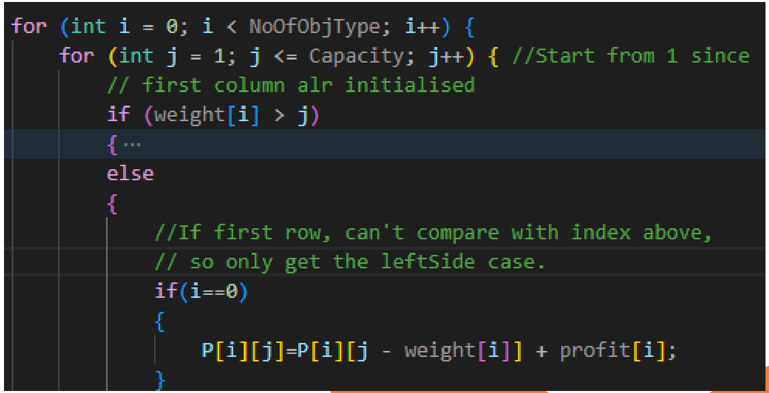
i	0	1	2
Wi	5	6	8
pi	7	6	9



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	5	0	0	0	0	0	7	7	7	7	7					
6	6	0														
9	8	0														



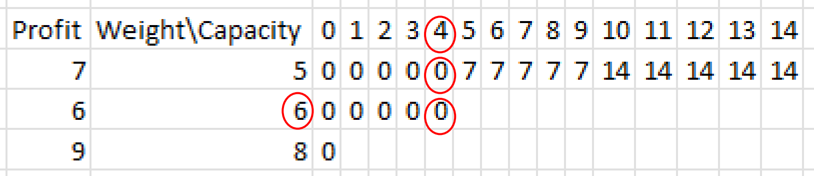
Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	5	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
6	6	0														
9	8	0														



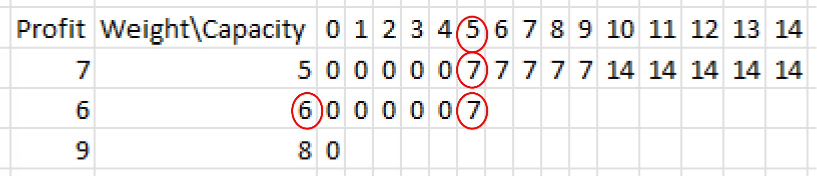
```
for (int i = 0; i < NoOfObjType; i++) {
    for (int j = 1; j <= Capacity; j++) { //Start from 1 since
        // first column alr initialised
        if (weight[i] > j)
        {…
        else
        {
            //If first row, can't compare with index above,
            // so only get the leftSide case.
            if(i==0)
            {
                P[i][j]=P[i][j - weight[i]] + profit[i];
            }
```

a)Result of P(14): object table 1,step 4

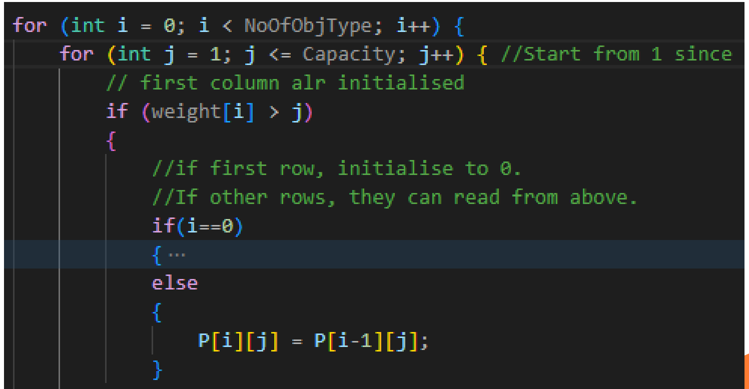
i	0	1	2
Wi	5	6	8
pi	7	6	9



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	5	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
6	6	0	0	0	0	0										
9	8	0														



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	5	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
6	6	0	0	0	0	0	7									
9	8	0														

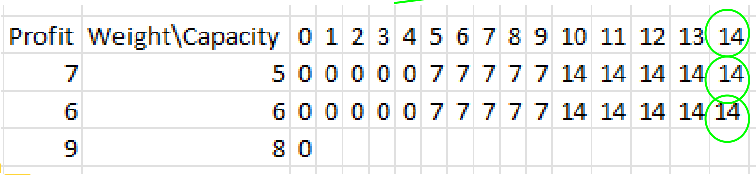


```
for (int i = 0; i < NoOfObjType; i++) {
    for (int j = 1; j <= Capacity; j++) { //Start from 1 since
        // first column alr initialised
        if (weight[i] > j)
        {
            //if first row, initialise to 0.
            //If other rows, they can read from above.
            if(i==0)
            {…
            else
            {
                P[i][j] = P[i-1][j];
            }
```

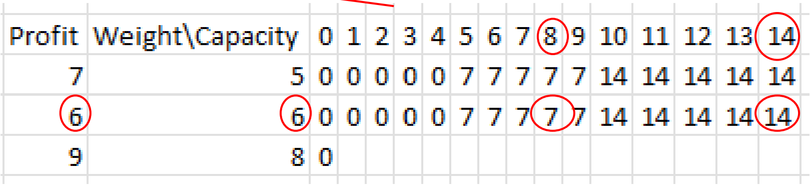
a)Result of P(14): object table 1,step 5

i	0	1	2
Wi	5	6	8
pi	7	6	9

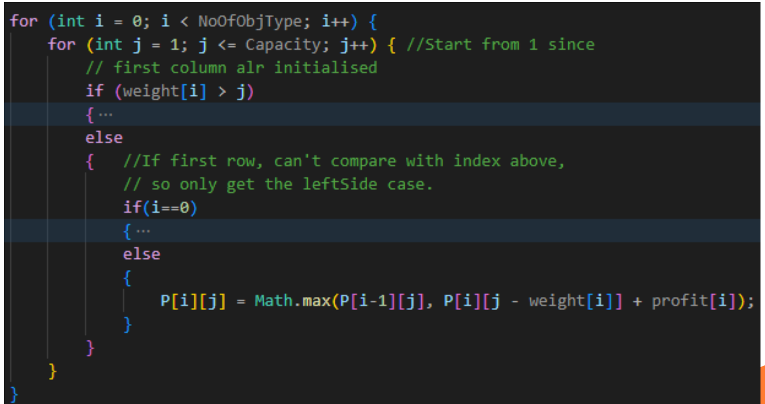
max(14,13) = 14



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	5	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
6	6	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
9	8	0														



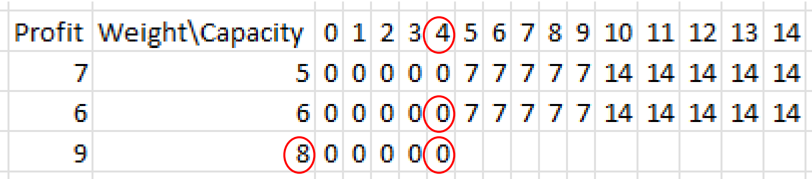
Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	5	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
6	6	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
9	8	0														



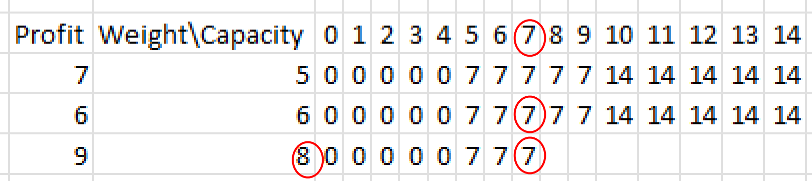
```
for (int i = 0; i < NoOfObjType; i++) {
    for (int j = 1; j <= Capacity; j++) { //Start from 1 since
        // first column alr initialised
        if (weight[i] > j)
        {…
        else
        {   //If first row, can't compare with index above,
            // so only get the leftSide case.
            if(i==0)
            {…
            else
            {
                P[i][j] = Math.max(P[i-1][j], P[i][j - weight[i]] + profit[i]);
            }
        }
    }
}
```

a)Result of P(14): object table 1,step 4

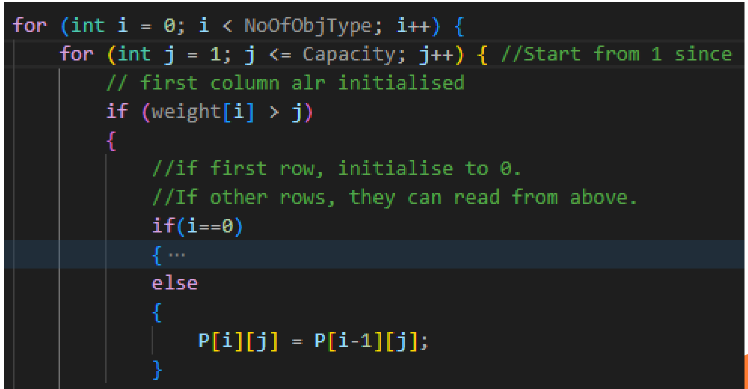
i	0	1	2
Wi	5	6	8
pi	7	6	9



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	5	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
6	6	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
9	8	0	0	0	0	0										



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	5	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
6	6	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
9	8	0	0	0	0	0	7	7	7							

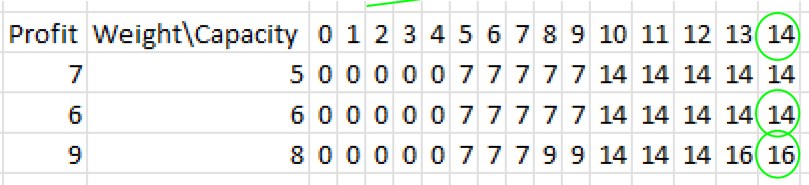


```
for (int i = 0; i < NoOfObjType; i++) {
    for (int j = 1; j <= Capacity; j++) { //Start from 1 since
        // first column alr initialised
        if (weight[i] > j)
        {
            //if first row, initialise to 0.
            //If other rows, they can read from above.
            if(i==0)
            {…
            else
            {
                P[i][j] = P[i-1][j];
            }
```

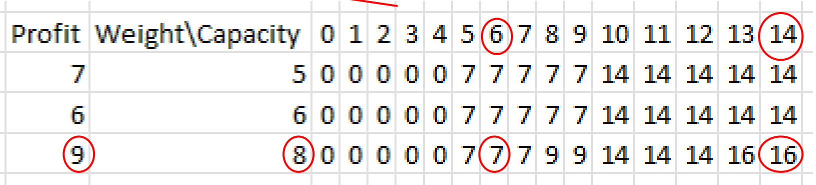
a)Result of P(14): object table 1,step 5

i	0	1	2
W_i	5	6	8
p_i	7	6	9

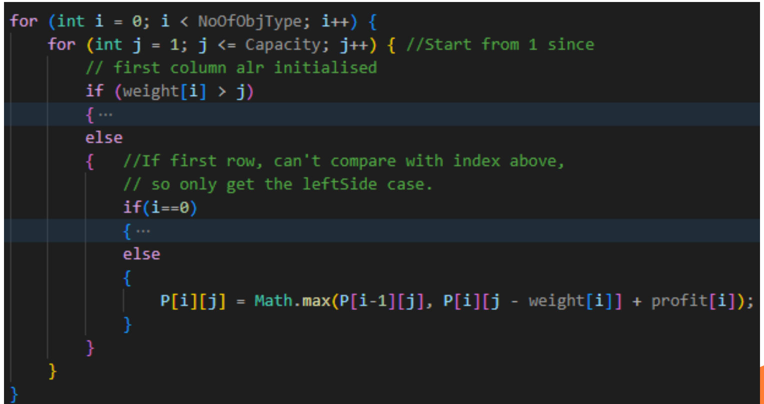
max(14,16) = 16



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	5	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
6	6	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
9	8	0	0	0	0	0	7	7	7	9	9	14	14	14	16	16



Profit	Weight\Capacity	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
7	5	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
6	6	0	0	0	0	0	7	7	7	7	7	14	14	14	14	14
9	8	0	0	0	0	0	7	7	7	9	9	14	14	14	16	16



```
for (int i = 0; i < NoOfObjType; i++) {
    for (int j = 1; j <= Capacity; j++) { //Start from 1 since
        // first column alr initialised
        if (weight[i] > j)
        {···
        else
        {   //If first row, can't compare with index above,
            // so only get the leftSide case.
            if(i==0)
            {···
            else
            {
                P[i][j] = Math.max(P[i-1][j], P[i][j - weight[i]] + profit[i]);
            }
        }
    }
}
```

Thank You!