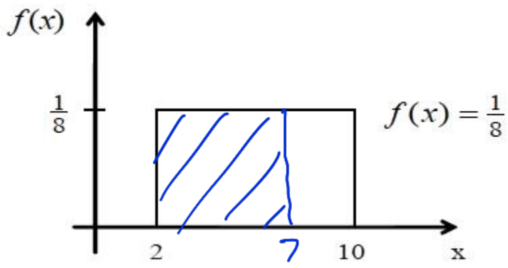
1. The figure below shows the graph of the uniform continuous distribution of a random variable that takes on values on the interval from 2 to 10. Find:



(a) $P(X < 7)$
(b) $E(X)$
(c) $Var(X)$

(a) $P(X<7) = \int_{-\infty}^{7} f(x)\,dx = \int_{2}^{7} \frac{1}{8}\,dx$

$= \frac{1}{8}x\Big]_2^7 = \frac{1}{8}(7-2)$

(b) $E[X] = \frac{a+b}{2} = \frac{2+10}{2} = 6$

OR $\int_2^{10} x f(x)\,dx$

(c) $Var[X] = \frac{(a-b)^2}{12}$

OR $\underbrace{E[X^2]}_{\int_2^{10} x^2 f(x)dx} - E[X]^2$

6^2

2. The waiting time for one to be served in a queueing system is a random variable having an exponential distribution with an average of 4 minutes.

(a) Determine the variance of the waiting time.

(b) What is the probability that one has to wait for at least 10 minutes before being served?

let X = waiting time, $X \sim Exp(\frac{1}{4})$ where $\lambda = \frac{1}{4}$

(a) $Var[X] = (\frac{1}{\lambda})^2 = 16$

(b) $P(X \geq 10) = \int_{10}^{\infty} \frac{1}{4} e^{-\frac{x}{4}}\,dx$

$= e^{-\frac{10}{4}}$

3. The cumulative distribution function of the r.v. X is given below:

$$F(x) = \begin{cases} 0, & x < 1 \\ 1 - x^{-3}, & x \geq 1 \end{cases}$$

(a) Determine the probability density function of X.

(b) Calculate E[X] and var[X].

(a) p.d.f $f(x) = \dfrac{dF(x)}{dx} = \begin{cases} \dfrac{d0}{dx} & x<1 \\ \dfrac{d(1-x^{-3})}{dx} & x \geq 1 \end{cases} = \begin{cases} 0 & x<1 \\ 3x^{-4} & x \geq 1 \end{cases}$

(b) $E[X] = \int_1^\infty \underline{x \cdot 3x^{-4}}\, dx$

$= \left[-\dfrac{3x^{-2}}{2} \right]_1^\infty$

$= \dfrac{3}{2}$

$Var[X] = E[X^2] - E[X]^2$

$= \int_1^\infty 3x^{-2} dx - \left(\dfrac{3}{2}\right)^2$

$= \left[-3x^{-1}\right]_1^\infty - \left(\dfrac{3}{2}\right)^2 = \dfrac{3}{4}$

4. Given a r.v. having the normal distribution with μ=16.2 and σ^2=1.5625, find the probabilities that it will take on a value (use the standard normal distribution table)

(a) greater than 16.8

(b) between 13.6 and 18.8

$X \sim N(\mu, \sigma^2)$

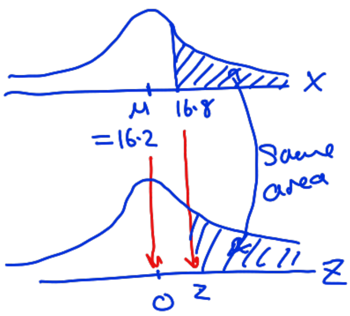
(a) $P(X > 16.8)$

$= P\left(Z > \dfrac{X-\mu}{\sigma}\right)$

$= P\left(Z > \underbrace{\dfrac{16.8-16.2}{\sqrt{1.5625}}}_{=0.48}\right)$

$= 1 - \underline{P(Z < 0.48)}$

from z-table, we get 0.6844



(b) $P(13.6 < X < 18.8)$

$= P\left(\dfrac{13.6-16.2}{1.25} < Z < \dfrac{18.8-16.2}{1.25}\right)$

$= P(-2.08 < Z < 2.08)$

$= P(Z < 2.08) - P(Z < -2.08)$

$= 0.9812 - 0.0188$

from z-table

5. Studies have shown that 22% of all patients taking a certain antibiotic will get a headache. Use the normal approximation to the binomial distribution to find the probability that among 50 patients taking this antibiotic

(a) at least 10 will get a headache

(b) at most 15 will get a headache

p

n

let X = # of patients got headache

$X \sim B(50, 0.22)$

$\therefore np = 50 \times 0.22 = 11$

$nq = 50 \times (1-0.22) = 39$

$\Rightarrow$ can be approximated by $Y \sim N(\mu, \sigma^2)$

$\mu = np = 11$

$\sigma^2 = npq = 8.58$

with continuity correction

(a) $P(X \geq 10) \approx P(Y \geq 9.5)$

$= P\left(Z \geq \frac{9.5-11}{\sqrt{8.58}}\right)$

$= P(Z > -0.51) = P(Z < 0.51)$ from Z table

$= 0.6950$

(b) $P(X \leq 15)$ with continuity correction

$\approx P(Y \leq 15.5)$

$= P\left(Z \leq \frac{15.5-11}{\sqrt{8.58}}\right)$

$= P(Z < 1.54)$ use Z-table

$= 0.9382$