MH1812 Tutorial
Chapter 3: Predicate Logic

Q1: Consider the predicates $M(x, y)$ = "x has sent an email to y", and $T(x, y)$ = "x has called y". The predicate variables x, y take values in the domain $D = \{\text{students in the class}\}$. Express these statements using symbolic logic.

1. There are at least two students in the class such that one student has sent the other an email, and the second student has called the first student.
2. There are some students in the class who have emailed everyone.

Q2: Consider the predicate $P(x, y)$="x is enrolled in the class y", where x takes values in the domain $S = \{\text{students}\}$, and y takes values in the domain D=$\{\text{courses}\}$. Express each statement by an English sentence.

1. $\exists x \in S, P(x, \text{MH1812})$.
2. $\exists y \in D, P(\text{Carol}, y)$.
3. $\exists x \in S, (P(x, \text{MH1812}) \wedge P(x, \text{CZ2002}))$.
4. $\exists x \in S, \exists x' \in S, \forall y \in D, ((x \neq x') \wedge (P(x, y) \leftrightarrow P(x', y)))$.

Q3: Consider the predicate $P(x, y, z)$ = "$xyz = 1$", for $x, y, z \in \mathbb{R}, x, y, z > 0$. What are the truth values of the following statements? Justify your answer.

1. $\forall x, \forall y, \forall z, P(x, y, z)$.
2. $\exists x, \exists y, \exists z, P(x, y, z)$.
3. $\forall x, \forall y, \exists z, P(x, y, z)$.
4. $\exists x, \forall y, \forall z, P(x, y, z)$.

Q4:

1. Express
$$\neg(\forall x, \forall y, P(x, y))$$
in terms of existential quantification.
2. Express
$$\neg(\exists x, \exists y, P(x, y))$$
in terms of universal quantification.

Q5: Consider the predicate $P(x, y) =$ "x is enrolled in the class y", where x takes values in the domain $S = \{\text{students}\}$, and y takes values in the domain $C = \{\text{courses}\}$. Form the negation of these statements:

1. $\exists x, (P(x, \text{MH1812}) \wedge P(x, \text{CZ2002}))$.
2. $\exists x, \exists y, \forall z, ((x \neq y) \wedge (P(x, z) \leftrightarrow P(y, z)))$.

Q6: Show that $\forall x \in D, P(x) \to Q(x)$ is equivalent to its contrapositive.

$\neg Q(x) \to \neg P(x)$

Q7: Show that

$$\neg(\forall x, P(x) \to Q(x)) \equiv \exists x, P(x) \wedge \neg Q(x).$$