Maths/LA/Tut4
Vector Space

23 Sep 2020

Tutorial 4 Help links

Youtube link: playlist

https://www.youtube.com/playlist?list=PLki3aFwg-9ew2XuhVtgoeU9OmfPvWl_BS

PDF

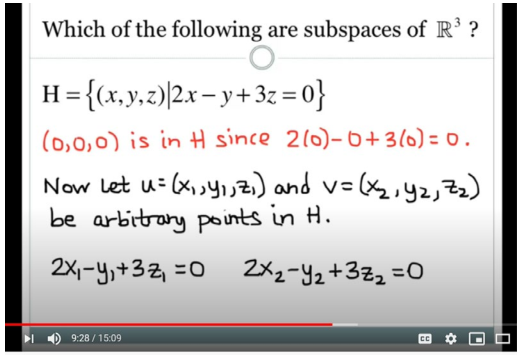
Q1-8: https://www.dropbox.com/s/x3nx736gn64ngkt/Tut4_Q1_8_ces.pdf?dl=0

Q9-16: https://www.dropbox.com/s/vfwt6has3ievpvl/Tut4_Q9_16_ces.pdf?dl=0

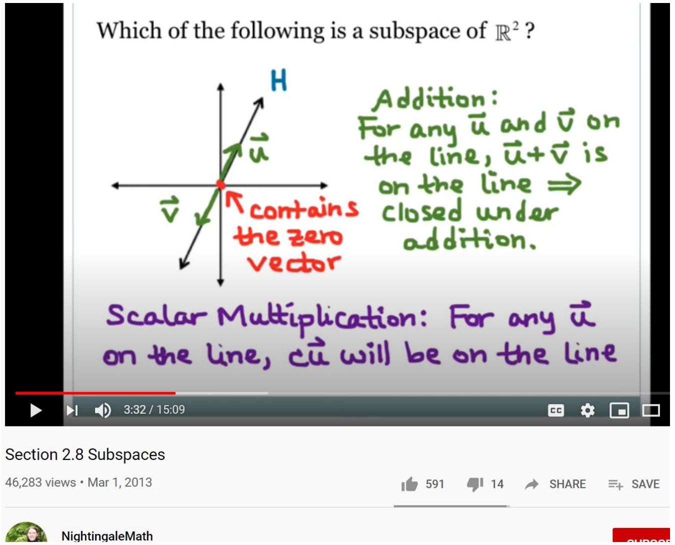
NightingGale: Good Reference for Sub-space, Basis, NullSpace, and ColSpace

1) NightingaleMath: Great explanation

    a) Subspace: https://www.youtube.com/watch?v=u4Og9zGParQ
    b) Basis of subspace: https://youtu.be/BeAko80xC_U
    c) NullSpace of A: https://youtu.be/-3BaYAHSINY
    d) Col Space of A: https://youtu.be/gtkY5Kjy_Jo



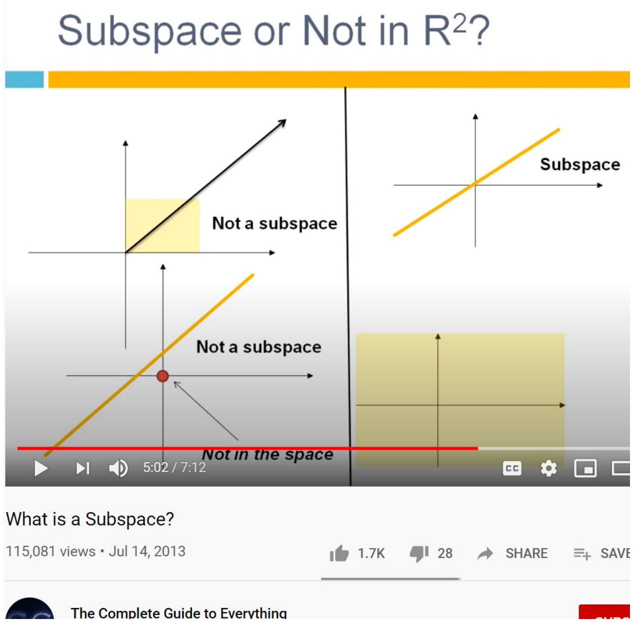
@time 9:28 for Q2b (similar)



What is a sub-space

Ref:

1) The complete guide to everything - “What is a Sub-space”: https://www.youtube.com/watch?v=0gHg5X6ng_4
2) Writeup by Clark University: https://mathcs.clarku.edu/~ma130/subspaces.pdf
3) Dr Peyam: “Null space is a subspace” https://www.youtube.com/watch?v=BwRhkXXqcRg



How to find Dimension, Basis, Span, Row/Col space?

Ref

1) MIT Strang – Vector Space:
   a) https://www.youtube.com/watch?v=n98ilenWoak
   b) https://www.mathworks.com/videos/differential-equations-and-linear-algebra-54-independence-basis-and-dimension-117459.html
   c) https://www.mathworks.com/videos/differential-equations-and-linear-algebra-55-the-big-picture-of-linear-algebra-117460.html

2) Linear basis and span: 3Blue1Brown:
   a) https://www.youtube.com/watch?v=k7RM-ot2NWY

3) Great Worked Example and Writeup:
   a) https://yutsumura.com/how-to-find-a-basis-for-the-nullspace-row-space-and-range-of-a-matrix/
   b) Writup by Upenn (Moore):
      https://www.math.upenn.edu/~moose/240S2013/slides7-22.pdf

   c) JHU Row and Col Space using Anton-Rorres book:
   http://www.math.jhu.edu/~jmb/note/rowcol.pdf

   d) TAMU:
   https://www.math.tamu.edu/~fnarc/psfiles/find_bases.pdf

Yutsumura worked examples

Problem 708

Let $A = \begin{bmatrix} 2 & 4 & 6 & 8 \\ 1 & 3 & 0 & 5 \\ 1 & 1 & 6 & 3 \end{bmatrix}$.

(a) Find a basis for the nullspace of A.

(b) Find a basis for the row space of A.

(c) Find a basis for the range of A that consists of column vectors of A.

(d) For each column vector which is not a basis vector that you obtained in part (c), express it as a linear combination of the basis vectors for the range of A.

How to extend a basis

how to extend a basis

Asked 7 years, 1 month ago Active 1 year ago Viewed 26k times

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This is a very elementary question but I can't find the answer in my book at the moment.

If I have, for example, two vectors v_1 and v_2 in $\mathbb{R}^5$ and knowing that they are linear independent , how can I extend this two vectors to a basis of $\mathbb{R}^5$. I know that I just have to add v_i for $3 \leq i \leq 5$ such that they are linear independent but how can I do that? Is there an easy algorithm?

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An easy solution, if you are familiar with this, is the following:

Put the two vectors as rows in a 2×5 matrix A. Find a basis for the null space $\text{Null}(A)$. Then, the three vectors in the basis complete your basis.

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edited Aug 12 '13 at 15:56
Ataraxia
5,693 3 22 49

answered Aug 12 '13 at 15:02
N. S.
121k 9 124 228

Ref:
1) https://math.stackexchange.com/questions/465870/how-to-extend-a-basis

Linear Transformation

A linear transformation T from vector space V to vector space W
(Pg 204/Lay/4th Edition)

1) Kernel = Null Space = a subspace of V (input)

i) set of **u (subspace of V)**
s.t the transformation of T on u, i.e T(u) = 0 (vector).

ii) Null space of a matrix A (mxn) is the subspace of R^n, i.e, the set of
x \in R^n s.t Ax = 0. Theorem 2 (pg 199) the null space is a subspace of R^n

2) Range of T == set of **all vectors** in W of the form T(x) for **x in V**,

Ref:

1) Video James Hamlin: "Matrix Transformation": https://www.youtube.com/watch?v=a7KC9Jx5qlM

2) Great Writeup by ClarkU "Matrix and Linear transformation" : https://mathcs.clarku.edu/~ma130/lintrans.pdf

3) Interactive Linear Algebra: "Matrixes as functions" https://textbooks.math.gatech.edu/ila/matrix-transformations.html

Interactive Linear Algebra: "Matrixes as functions"

Transformations

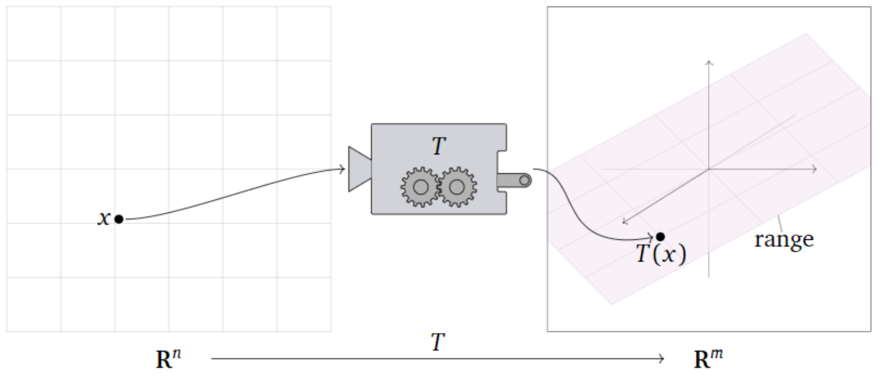
At this point it is convenient to fix our ideas and terminology regarding functions, which we will call *transformations* in this book. This allows us to systematize our discussion of matrices as functions.

Definition. A ***transformation*** from $\mathbf{R}^n$ to $\mathbf{R}^m$ is a rule T that assigns to each vector x in $\mathbf{R}^n$ a vector $T(x)$ in $\mathbf{R}^m$.

- $\mathbf{R}^n$ is called the ***domain*** of T.
- $\mathbf{R}^m$ is called the ***codomain*** of T.
- For x in $\mathbf{R}^n$, the vector $T(x)$ in $\mathbf{R}^m$ is the ***image*** of x under T.
- The set of all images $\{T(x) \mid x \text{ in } \mathbf{R}^n\}$ is the ***range*** of T.

The notation $T: \mathbf{R}^n \longrightarrow \mathbf{R}^m$ means "T is a transformation from $\mathbf{R}^n$ to $\mathbf{R}^m$."

It may help to think of T as a "machine" that takes x as an input, and gives you $T(x)$ as the output.



Rank Nullity Theorem

1) Proof by Peyam:
https://www.youtube.com/watch?v=eHZLkruAIfM

2) Proof by Technion:
https://www.youtube.com/watch?v=407N5AXs1aM

The Column Space and the Null Space of a Matrix

- Suppose that A is a $m \times n$ matrix. Then

$$\dim \mathrm{Null}(A) + \dim \mathrm{Col}(A) = n.$$

  Why:

  - $\dim \mathrm{Null}(A) =$ number of free variables in row reduced form of A.
  - a basis for $\mathrm{Col}(A)$ is given by the columns corresponding to the leading 1's in the row reduced form of A.

- The dimension of the Null Space of a matrix is called the "nullity" of the matrix.

- The dimension of the Column Space of a matrix is called the "rank" of the matrix.

Kernel and Nullity of Matrix

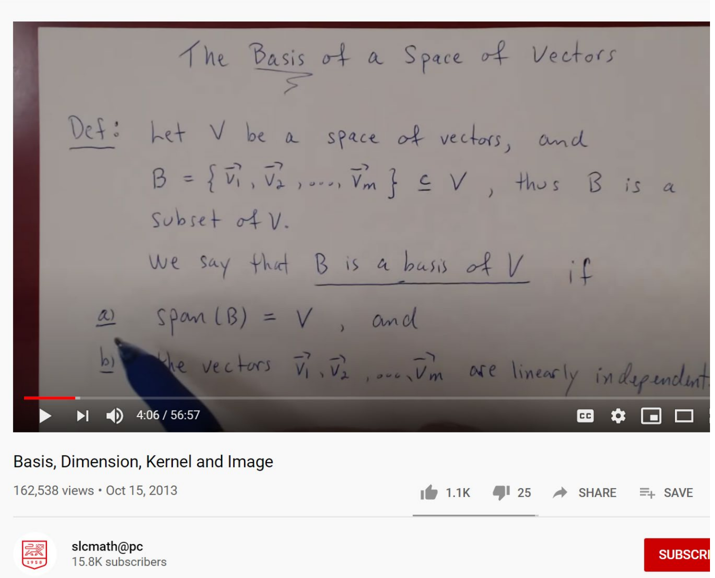
UNSW:

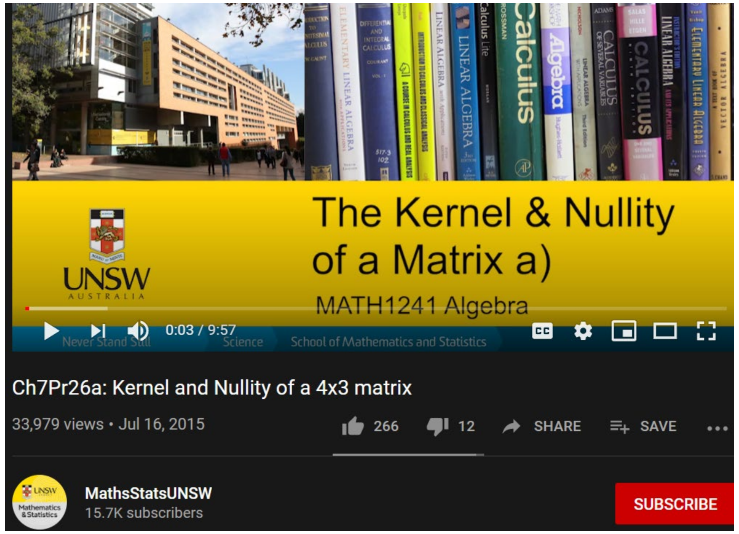
https://www.youtube.com/watch?v=7_wxkyVglxo&feature=youtu.be

https://math.unm.edu/~loring/links/linear_s06/nullity.pdf

http://www.maths.usyd.edu.au/u/geoffp/lm-ss/amlect12.pdf

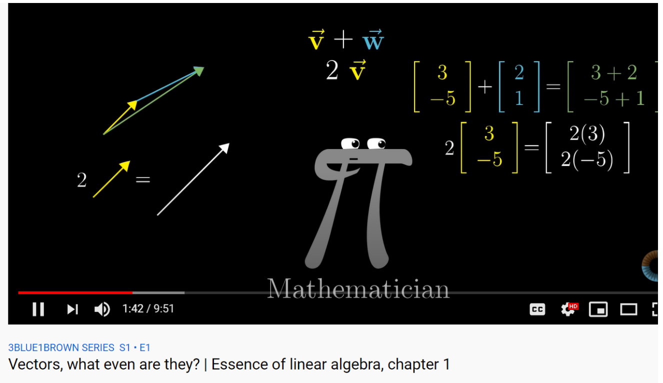
slcmath@pc: https://www.youtube.com/watch?v=2laflhB7ajk

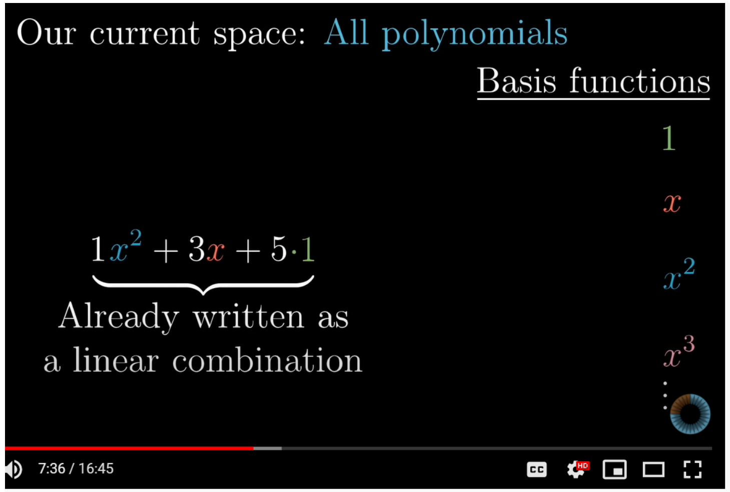


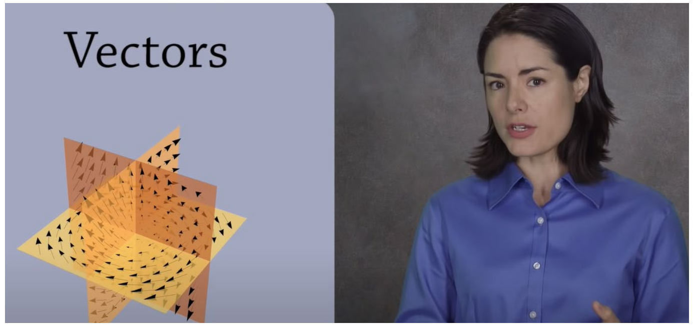


Abstract Vector Space Background

- What is (Abstract) Vector Space
  - 3Blue1Brown: https://youtu.be/TgKwz5Ikpc8
  - Socratica: https://www.youtube.com/watch?v=ozwodzD5bJM







Random notes for Future:

https://mathcs.clarku.edu/~ma130/isomorphism.pdf

Clark: https://mathcs.clarku.edu/~ma130/

Writing proof: https://mathcs.clarku.edu/~ma130/writingProofs.PDF