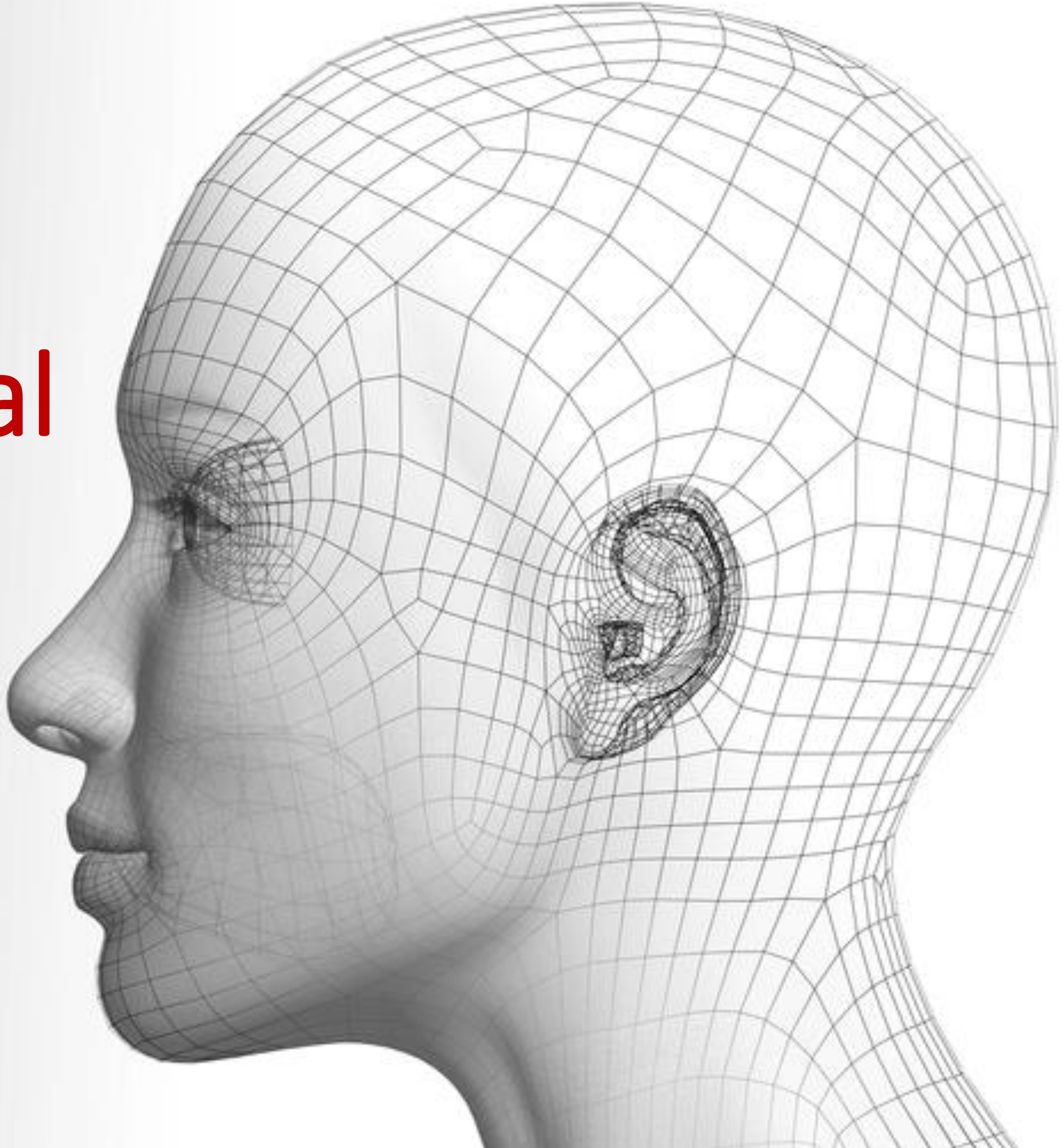
SC 4001 CE/CZ 4042 Neural Network and Deep Learning

Last update: 9 Oct 2024

Tutorial 6
Convolutional Neural Networks I



Question 1

1. The first hidden layer of a convolution neural network (CNN) has a convolution layer consisting of two feature maps with filters $\boldsymbol{w_1}$and $\boldsymbol{w_2}$ and biases = 0.1, and neurons having sigmoid activation functions, and a pooling layer with a pooling windows of size 2x2:

$$\boldsymbol{w}_1 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \text{ and } \boldsymbol{w}_2 = \begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

The input layer is of 6x6 size and receives an input image $\boldsymbol{I}$:

Filters=weights

$$\boldsymbol{I} = \begin{pmatrix} 0.7 & 0.1 & 0.2 & 0.3 & 0.3 & 05 \\ 0.8 & 0.1 & 0.3 & 0.5 & 0.1 & 0.0 \\ 1.0 & 0.2 & 0.0 & 0.3 & 0.2 & 0.7 \\ 0.8 & 0.1 & 0.5 & 0.6 & 0.3 & 0.4 \\ 0.1 & 0.0 & 0.9 & 0.3 & 0.3 & 0.2 \\ 1.0 & 0.1 & 0.4 & 0.5 & 0.2 & 0.8 \end{pmatrix}$$

a. Find the outputs at the first convolution layer if
   i. padding = 0 and strides = (1,1)
   ii. padding = 1 and strides = (2,2)

b. Find the outputs at the first pooling layer for Part (a), assuming strides of (2, 2) and pooling is
   i. max pooling
   ii. mean pooling

t6q1.ipynb

Question 1

$$\boldsymbol{w}_1 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \boldsymbol{w}_2 = \begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \text{ and } b = 0.1.$$

$$\boldsymbol{I} = \begin{pmatrix} 0.7 & 0.1 & 0.2 & 0.3 & 0.3 & 05 \\ 0.8 & 0.1 & 0.3 & 0.5 & 0.1 & 0.0 \\ 1.0 & 0.2 & 0.0 & 0.3 & 0.2 & 0.7 \\ 0.8 & 0.1 & 0.5 & 0.6 & 0.3 & 0.4 \\ 0.1 & 0.0 & 0.9 & 0.3 & 0.3 & 0.2 \\ 1.0 & 0.1 & 0.4 & 0.5 & 0.2 & 0.8 \end{pmatrix}$$

Synaptic inputs to the feature map with filter $\boldsymbol{w}_1$:

$\boldsymbol{u}_1 = Conv(\boldsymbol{I}, \boldsymbol{w}_1) + b_1$

Padding = 0 and strides = (1,1)

$\boldsymbol{u}_1(1,1) = 0.7\times0 + 0.1\times1 + 0.2\times1 + 0.8\times1 + 0.1\times0 + 0.3\times1 + 1.0\times1 + 0.2\times1 + 0.0\times0 + 0.1 = 2.7$

$\boldsymbol{u}_1(1,2) = 0.1\times0 + 0.2\times1 + 0.3\times1 + 0.1\times1 + 0.3\times0 + 0.5\times1 + 0.2\times1 + 0.0\times1 + 0.3\times0 + 0.1 = 1.4$

$\boldsymbol{u}_1(2,1) = 0.8\times0 + 0.1\times1 + 0.3\times1 + 1.0\times1 + 0.2\times0 + 0.0\times1 + 0.8\times1 + 0.1\times1 + 0.5\times0 + 0.1 = 2.4$

Question 1

$$\boldsymbol{u}_1 = \begin{pmatrix} 2.7 & 1.4 & 1.4 & 1.9 \\ 2.4 & 2.0 & 2.0 & 2.1 \\ 1.7 & 2.0 & 2.6 & 2.6 \\ 2.8 & 2.0 & 3.1 & 2.0 \end{pmatrix}$$

Similarly, $\boldsymbol{u}_2 = Conv(\boldsymbol{I}, \boldsymbol{w}_2) + b_2 = \begin{pmatrix} 0.3 & 0.0 & -0.2 & 2.0 \\ 0.3 & 0.4 & 0.6 & 0.8 \\ -0.1 & 0.8 & 1.1 & -0.3 \\ 0.2 & -0.1 & -0.2 & 0.3 \end{pmatrix}$

Feature maps at the convolutional layer

$$\boldsymbol{y}_1 = f(\boldsymbol{u}_1) = \frac{1}{1 + e^{-\boldsymbol{u}_1}} = \begin{pmatrix} 0.94 & 0.8 & 0.8 & 0.87 \\ 0.92 & 0.88 & 0.88 & 0.89 \\ 0.85 & 0.88 & 0.93 & 0.93 \\ 0.94 & 0.88 & 0.96 & 0.88 \end{pmatrix}$$

$$\boldsymbol{y}_2 = f(\boldsymbol{u}_2) = \frac{1}{1 + e^{-\boldsymbol{u}_2}} = \begin{pmatrix} 0.57 & 0.5 & 0.45 & 0.55 \\ 0.57 & 0.60 & 0.65 & 0.69 \\ 0.48 & 0.69 & 0.75 & 0.43 \\ 0.55 & 0.48 & 0.45 & 0.57 \end{pmatrix}$$

Output spatial size

= (Input size – Filter size +2(Padding))/Stride + 1

= (6 – 3)/1 + 1 = 4

Question 1

Feature maps at the convolutional layer:

$$\boldsymbol{y} = \begin{pmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \boxed{\begin{matrix} 0.94 & 0.8 \\ 0.92 & 0.88 \end{matrix}} & \begin{matrix} 0.8 & 0.87 \\ 0.88 & 0.89 \end{matrix} \\ \begin{matrix} 0.85 & 0.88 \\ 0.94 & 0.88 \end{matrix} & \begin{matrix} 0.93 & 0.93 \\ 0.96 & 0.88 \end{matrix} \end{pmatrix} \\ \begin{pmatrix} 0.57 & 0.5 & 0.45 & 0.55 \\ 0.57 & 0.60 & 0.65 & 0.69 \\ 0.48 & 0.69 & 0.75 & 0.43 \\ 0.55 & 0.48 & 0.45 & 0.57 \end{pmatrix} \end{pmatrix}$$

Pooling 2×2 and strides = 2

Max-pooling:

$$o = \begin{pmatrix} \begin{pmatrix} 0.94 & 0.89 \\ 0.94 & 0.96 \end{pmatrix} \\ \begin{pmatrix} 0.60 & 0.69 \\ 0.69 & 0.75 \end{pmatrix} \end{pmatrix}$$

Mean-pooling:

$$p_{ave} = \begin{pmatrix} \begin{pmatrix} 0.88 & 0.86 \\ 0.89 & 0.92 \end{pmatrix} \\ \begin{pmatrix} 0.56 & 0.58 \\ 0.55 & 0.55 \end{pmatrix} \end{pmatrix}$$

Question 1

$$\boldsymbol{w}_1 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \boldsymbol{w}_2 = \begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \text{ and } b = 0.1.$$

$$\begin{array}{cc} & \begin{array}{ccc} 0 & 0 & 0 \end{array} \\ \begin{array}{c} 0 \\ 0 \\ \\ \\ \\ \\ \end{array} & \boldsymbol{I} = \begin{pmatrix} 0.7 & 0.1 & 0.2 & 0.3 & 0.3 & 05 \\ 0.8 & 0.1 & 0.3 & 0.5 & 0.1 & 0.0 \\ 1.0 & 0.2 & 0.0 & 0.3 & 0.2 & 0.7 \\ 0.8 & 0.1 & 0.5 & 0.6 & 0.3 & 0.4 \\ 0.1 & 0.0 & 0.9 & 0.3 & 0.3 & 0.2 \\ 1.0 & 0.1 & 0.4 & 0.5 & 0.2 & 0.8 \end{pmatrix} \end{array}$$

Synaptic inputs to the feature map with filter $\boldsymbol{w}_1$:

$\boldsymbol{u}_1 = Conv(\boldsymbol{I}, \boldsymbol{w}_1) + b_1$

Padding = 1 and strides = (2,2)

$$\boldsymbol{u}_1(1,1) = 0.0 \times 0 + 0.0 \times 1 + 0.0 \times 1 + 0.0 \times 1 + 0.7 \times 0 + 0.1 \times 1 + 0.0 \times 1 + 0.8 \times 1 + 0.1 \times 0 + 0.1 = 1.0$$

Question 1

$$\boldsymbol{w}_1 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \boldsymbol{w}_2 = \begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \text{ and } b = 0.1.$$

$$\begin{array}{c} \begin{array}{cccccc} & 0 & 0 & 0 & & \end{array} \\ \boldsymbol{I} = \begin{pmatrix} 0.7 & 0.1 & 0.2 & 0.3 & 0.3 & 05 \\ 0.8 & 0.1 & 0.3 & 0.5 & 0.1 & 0.0 \\ 1.0 & 0.2 & 0.0 & 0.3 & 0.2 & 0.7 \\ 0.8 & 0.1 & 0.5 & 0.6 & 0.3 & 0.4 \\ 0.1 & 0.0 & 0.9 & 0.3 & 0.3 & 0.2 \\ 1.0 & 0.1 & 0.4 & 0.5 & 0.2 & 0.8 \end{pmatrix} \end{array}$$

Synaptic inputs to the feature map with filter $\boldsymbol{w}_1$:

$\boldsymbol{u}_1 = Conv(\boldsymbol{I}, \boldsymbol{w}_1) + b_1$

Padding = 1 and strides = (2,2)

$$\boldsymbol{u}_1(1,1) = 0.0 \times 0 + 0.0 \times 1 + 0.0 \times 1 + 0.0 \times 1 + 0.7 \times 0 + 0.1 \times 1 + 0.0 \times 1 + 0.8 \times 1 + 0.1 \times 0 + 0.1 = 1.0$$

$$\boldsymbol{u}_1(1,2) = 0.0 \times 0 + 0.0 \times 1 + 0.0 \times 1 + 0.1 \times 1 + 0.2 \times 0 + 0.3 \times 1 + 0.1 \times 1 + 0.3 \times 1 + 0.5 \times 0 + 0.1 = 0.9$$

Question 1

$$\boldsymbol{w}_1 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \boldsymbol{w}_2 = \begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \text{ and } b = 0.1.$$

$$\begin{array}{cc} & \begin{array}{cccccc} & & & 0 & 0 & 0 \end{array} \\ \boldsymbol{I} = & \begin{pmatrix} 0.7 & 0.1 & 0.2 & 0.3 & 0.3 & 05 \\ 0.8 & 0.1 & 0.3 & 0.5 & 0.1 & 0.0 \\ 1.0 & 0.2 & 0.0 & 0.3 & 0.2 & 0.7 \\ 0.8 & 0.1 & 0.5 & 0.6 & 0.3 & 0.4 \\ 0.1 & 0.0 & 0.9 & 0.3 & 0.3 & 0.2 \\ 1.0 & 0.1 & 0.4 & 0.5 & 0.2 & 0.8 \end{pmatrix} \end{array}$$

Synaptic inputs to the feature map with filter $\boldsymbol{w}_1$:
$\boldsymbol{u}_1 = Conv(\boldsymbol{I}, \boldsymbol{w}_1) + b_1$

Padding = 1 and strides = (2,2)

$$\boldsymbol{u}_1(1,1) = 0.0 \times 0 + 0.0 \times 1 + 0.0 \times 1 + 0.0 \times 1 + 0.7 \times 0 + 0.1 \times 1 + 0.0 \times 1 + 0.8 \times 1 + 0.1 \times 0 + 0.1 = 1.0$$

$$\boldsymbol{u}_1(1,2) = 0.0 \times 0 + 0.0 \times 1 + 0.0 \times 1 + 0.1 \times 1 + 0.2 \times 0 + 0.3 \times 1 + 0.1 \times 1 + 0.3 \times 1 + 0.5 \times 0 + 0.1 = 0.9$$

$$\boldsymbol{u}_1(1,3) = 0.0 \times 0 + 0.0 \times 1 + 0.0 \times 1 + 0.3 \times 1 + 0.3 \times 0 + 0.5 \times 1 + 0.5 \times 1 + 0.1 \times 1 + 0.0 \times 0 + 0.1 = 1.5$$

Question 1

$$\boldsymbol{w}_1 = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \boldsymbol{w}_2 = \begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} \text{ and } b = 0.1.$$

$$\boldsymbol{I} = \begin{array}{c} \\ 0 \\ 0 \\ 0 \\ \\ \\ \end{array} \begin{pmatrix} 0.7 & 0.1 & 0.2 & 0.3 & 0.3 & 05 \\ 0.8 & 0.1 & 0.3 & 0.5 & 0.1 & 0.0 \\ 1.0 & 0.2 & 0.0 & 0.3 & 0.2 & 0.7 \\ 0.8 & 0.1 & 0.5 & 0.6 & 0.3 & 0.4 \\ 0.1 & 0.0 & 0.9 & 0.3 & 0.3 & 0.2 \\ 1.0 & 0.1 & 0.4 & 0.5 & 0.2 & 0.8 \end{pmatrix}$$

Synaptic inputs to the feature map with filter $\boldsymbol{w}_1$:
$\boldsymbol{u}_1 = Conv(\boldsymbol{I}, \boldsymbol{w}_1) + b_1$

Padding = 1 and strides = (2,2)

$\boldsymbol{u}_1(1,1) = 0.0 \times 0 + 0.0 \times 1 + 0.0 \times 1 + 0.0 \times 1 + 0.7 \times 0 + 0.1 \times 1 + 0.0 \times 1 + 0.8 \times 1 + 0.1 \times 0 + 0.1 = 1.0$

$\boldsymbol{u}_1(1,2) = 0.0 \times 0 + 0.0 \times 1 + 0.0 \times 1 + 0.1 \times 1 + 0.2 \times 0 + 0.3 \times 1 + 0.1 \times 1 + 0.3 \times 1 + 0.5 \times 0 + 0.1 = 0.9$

$\boldsymbol{u}_1(1,3) = 0.0 \times 0 + 0.0 \times 1 + 0.0 \times 1 + 0.3 \times 1 + 0.3 \times 0 + 0.5 \times 1 + 0.5 \times 1 + 0.1 \times 1 + 0.0 \times 0 + 0.1 = 1.5$

$\boldsymbol{u}_1(2,1) = 0.0 \times 0 + 0.8 \times 1 + 0.1 \times 1 + 0.0 \times 1 + 1.0 \times 0 + 0.2 \times 1 + 0.0 \times 1 + 0.8 \times 1 + 0.1 \times 0 + 0.1 = 2.0$

Question 1

$$\boldsymbol{u}_1 = \begin{pmatrix} 1.0 & 0.9 & 1.5 \\ 2.0 & 2.0 & 2.1 \\ 2.0 & 2.0 & 2.0 \end{pmatrix}$$

Similarly, $\boldsymbol{u}_2 = Conv(\boldsymbol{I}, \boldsymbol{w}_2) + b_2 = \begin{pmatrix} 1.0 & 1.0 & 0.7 \\ 0.1 & 0.4 & 0.8 \\ 0.3 & -0.1 & 0.3 \end{pmatrix}$

Feature maps at the convolutional layer

$$\boldsymbol{y}_1 = f(\boldsymbol{u}_1) = \frac{1}{1 + e^{-\boldsymbol{u}_1}} = \begin{pmatrix} 0.73 & 0.71 & 0.82 \\ 0.88 & 0.88 & 0.89 \\ 0.88 & 0.88 & 0.88 \end{pmatrix}$$

$$\boldsymbol{y}_2 = f(\boldsymbol{u}_2) = \frac{1}{1 + e^{-\boldsymbol{u}_2}} = \begin{pmatrix} 0.73 & 0.73 & 0.67 \\ 0.52 & 0.60 & 0.69 \\ 0.57 & 0.48 & 0.57 \end{pmatrix}$$

Question 1

Feature maps at the convolutional layer:

$$\boldsymbol{y} = \begin{pmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 0.73 & 0.71 & 0.82 \\ 0.88 & 0.88 & 0.89 \\ 0.88 & 0.88 & 0.88 \end{pmatrix} \\ \begin{pmatrix} 0.73 & 0.73 & 0.67 \\ 0.52 & 0.60 & 0.69 \\ 0.57 & 0.48 & 0.57 \end{pmatrix} \end{pmatrix}$$

Pooling VALID 2x2 strides = 2:

Max-pooling:

$$o = \begin{pmatrix} (0.88) \\ (0.73) \end{pmatrix}$$

Mean-pooling:

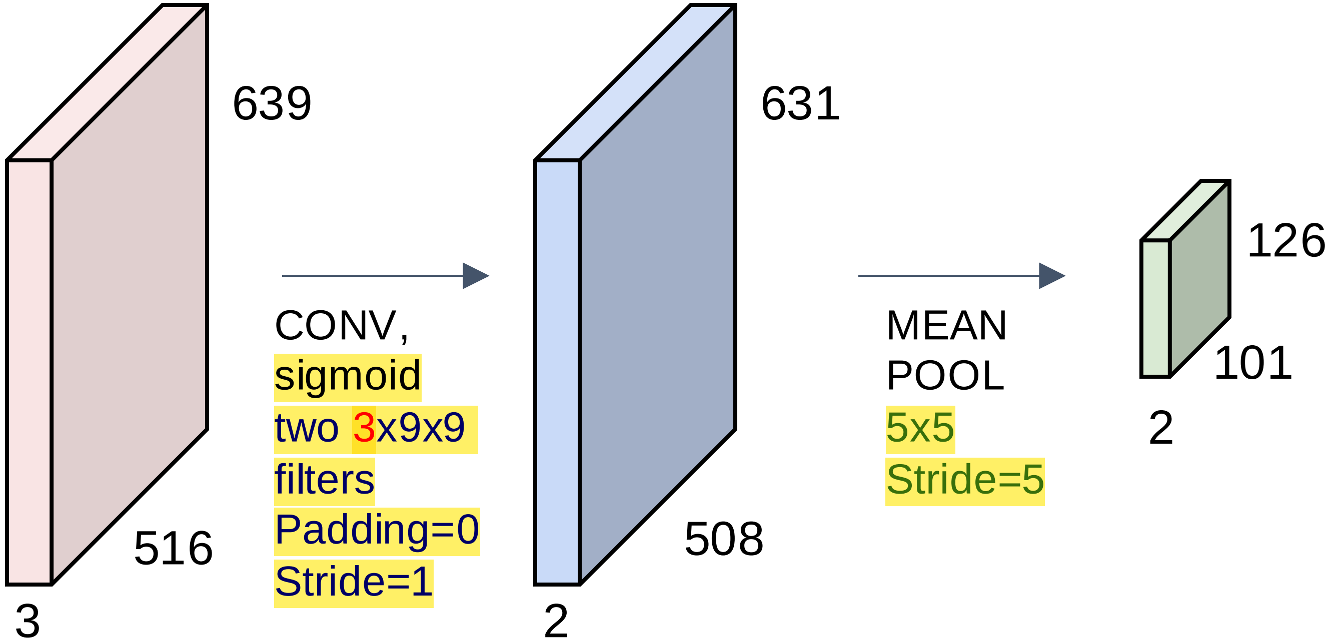
$$p_{ave} = \begin{pmatrix} (0.80) \\ (0.65) \end{pmatrix}$$

Question 2

2. Given ‘3wolfmoon.jpg’ color image of size 639×516.
    a. Initialize weights and biases of a convolutional layer with two kernels of size 9×9. Note that the input image is in color and has three channels.
    b. Display the feature maps at the convolution layer, assuming sigmoid activation functions. Use VALID padding and strides = 1.
    c. Display the outputs of a mean pooling layer with a pooling window size 5×5 and strides = 5.

t6q2.ipynb

Question 2



639
516
3
CONV,
sigmoid
two 3x9x9
filters
Padding=0
Stride=1
631
508
2
MEAN
POOL
5x5
Stride=5
126
101
2

Question 2

```
# open image and normalize
img = Image.open('3wolfmoon.jpg')
img = np.asarray(img, dtype='float32') / 256.
print(img.shape)
```

```
---------------------------------------------------------------------------
FileNotFoundError                         Traceback (most recent call last)
<ipython-input-2-becdc6409bf5> in <module>()
      1 # open image and normalize
----> 2 img = Image.open('3wolfmoon.jpg')
      3 img = np.asarray(img, dtype='float32') / 256.
      4 print(img.shape)

/usr/local/lib/python3.7/dist-packages/PIL/Image.py in open(fp, mode)
   2841
   2842     if filename:
-> 2843         fp = builtins.open(filename, "rb")
   2844         exclusive_fp = True
   2845

FileNotFoundError: [Errno 2] No such file or directory: '3wolfmoon.jpg'
```

You need to mount Google Drive locally and use the correct path

https://colab.research.google.com/notebooks/io.ipynb#scrollTo=u22w3BFiOveA

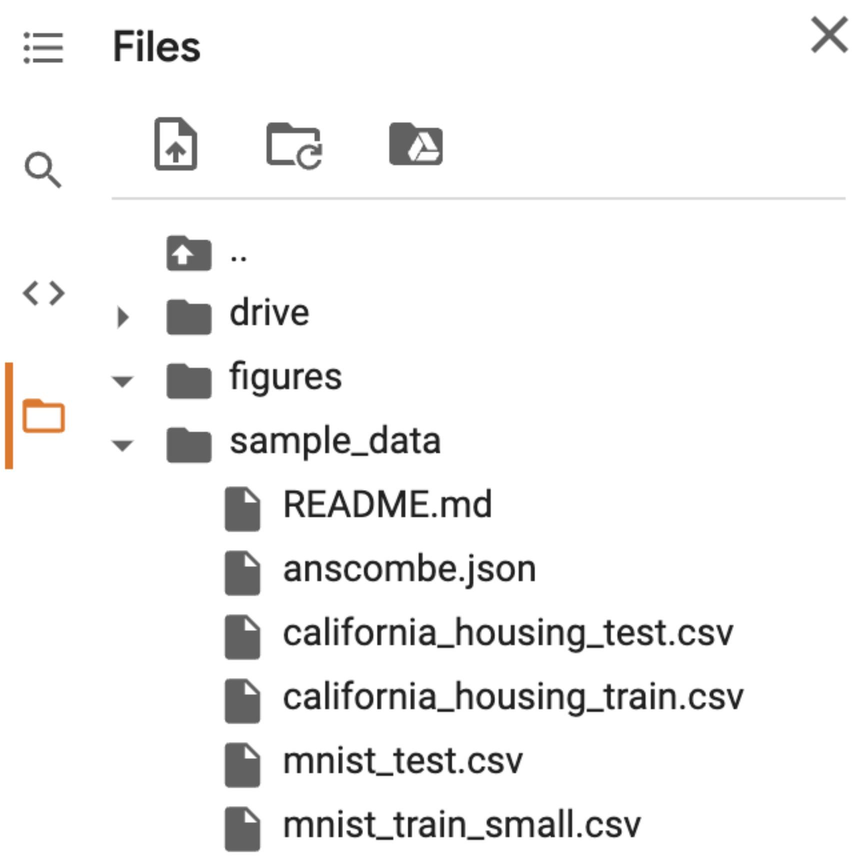
Question 2

```
# mount google drive
from google.colab import drive
drive.mount('/content/drive')
```

```
Mounted at /content/drive
```

```
# open image and normalize
img = Image.open('/content/drive/MyDrive/3wolfmoon.jpg')
img = np.asarray(img, dtype='float32') / 256.
print(img.shape)
```

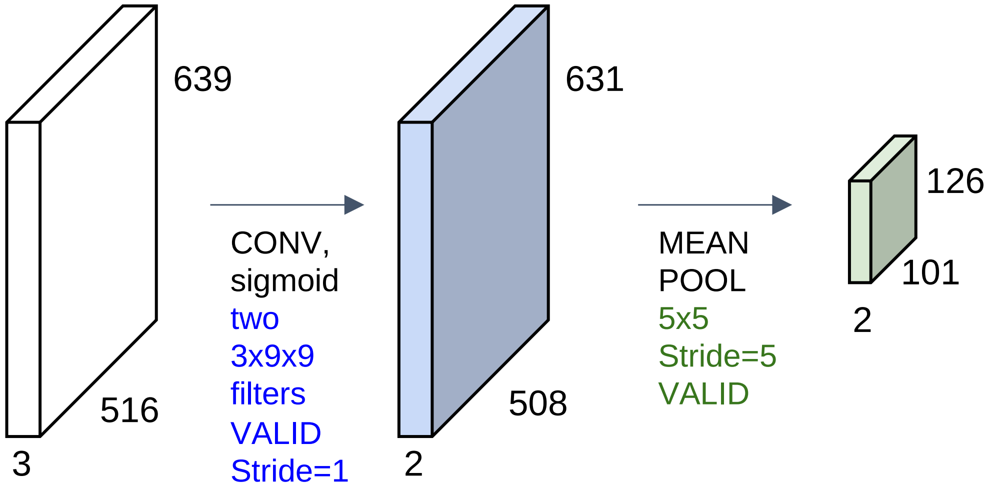
```
(639, 516, 3)
```



You can copy the path from the browser

Question 2

Plot the three channels of the input image



Original image

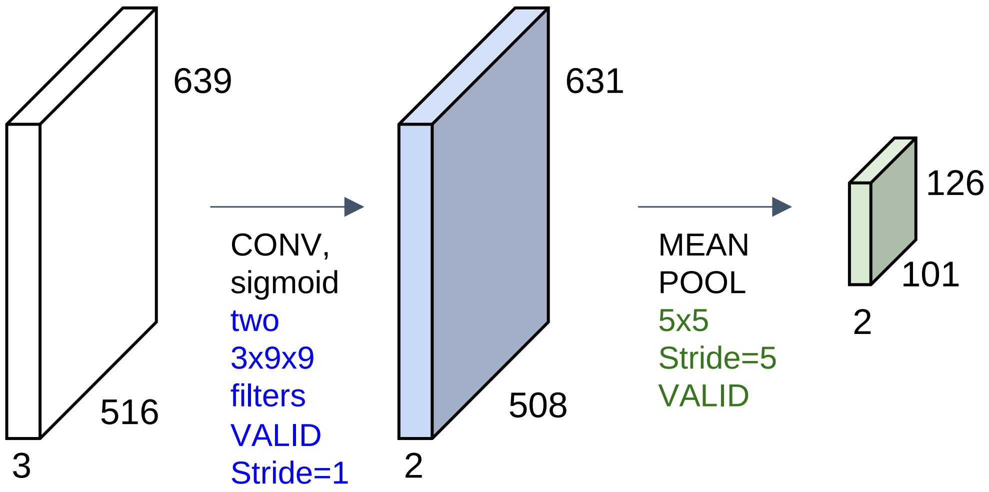
Red

Green

Blue

Question 2

Plot the original image and the first and second components of conv output



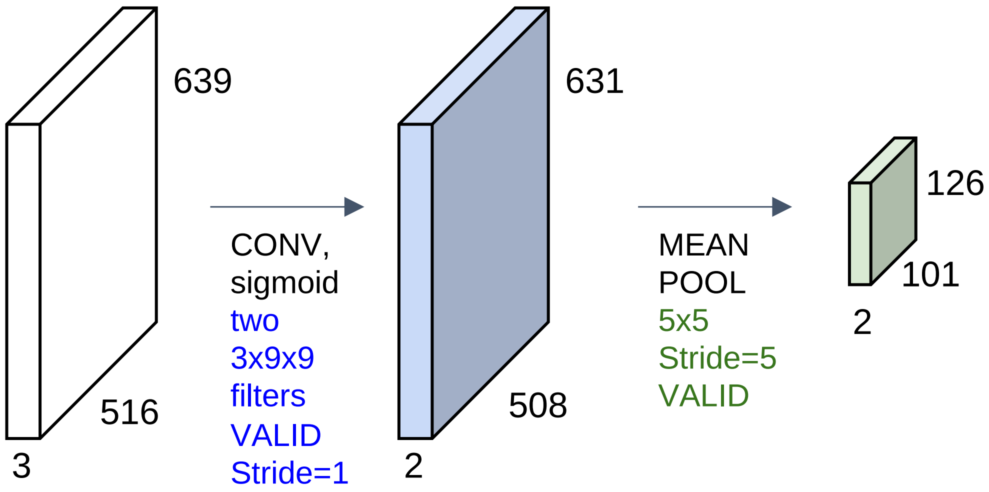
Original image

Feature map 1

Feature map 2

Question 2

Plot the original image and the first and second components of pooling output



Original image

Pooled feature map 1

Pooled feature map 2

Question 3

3. Design a CNN with one hidden layer to recognize digit images in MNIST database:

   http://yann.lecun.com/exdb/mnist/

   The convolution layer consists of 25 filters of dimensions 9x9 and the pooling layer has a pooling window size 4x4. Assume VALID padding and default strides for both convolution and pooling layer. Train the network with mini batch gradient decent learning with learning factor $\alpha = 10^{-3}$ and batch size = 128.

   Plot

   a. The training and test errors against learning epochs.
   b. Final filter weights
   c. Feature maps at the convolution and pooling layers for a representative test pattern
   d. Repeat training by introducing decay parameter $\beta = 10^{-6}$ and momentum term with $\gamma = 0.5$, and compare the learning curves

t6q3a.ipynb

t6q3b.ipynb

Question 3

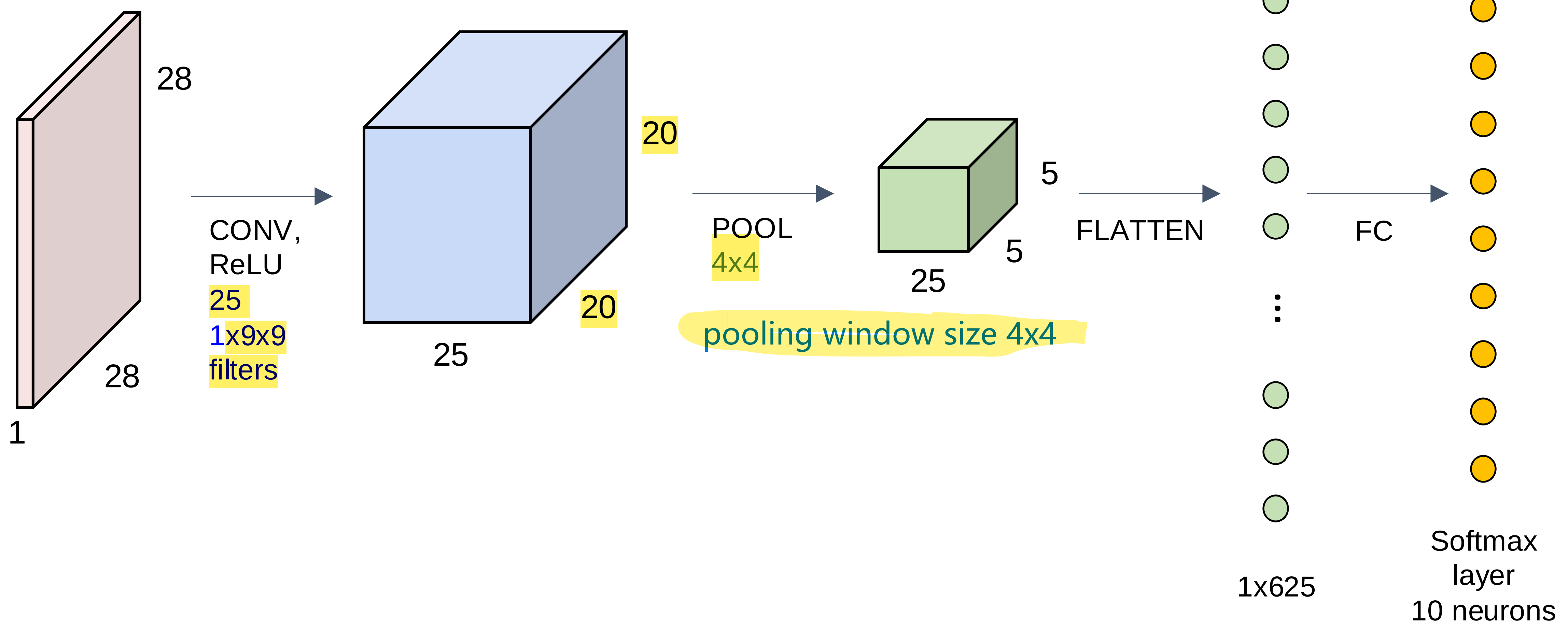
MNIST

- Size-normalized and centred 1x28x28 =784 inputs
- Training set = 60,000 images
- Testing set = 10,000 images

http://yann.lecun.com/exdb/mnist/

Question 3

Output height = Output width = (input size - filter size)/stride + 1
= (28-9)/1 + 1 = 20



Question 3

Change runtime type

Runtime type

Python 3

Hardware accelerator

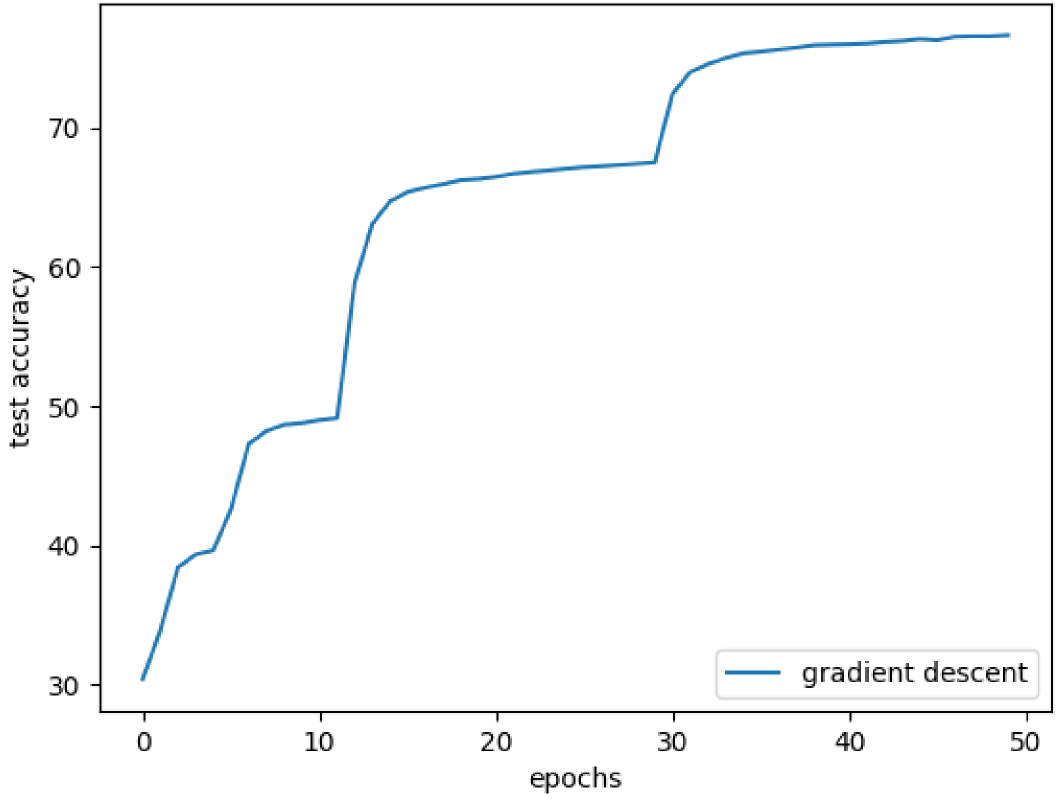
- CPU
- T4 GPU (selected)
- A100 GPU
- V100 GPU
- TPU

Want access to premium GPUs? **Purchase additional compute units**

Cancel   Save

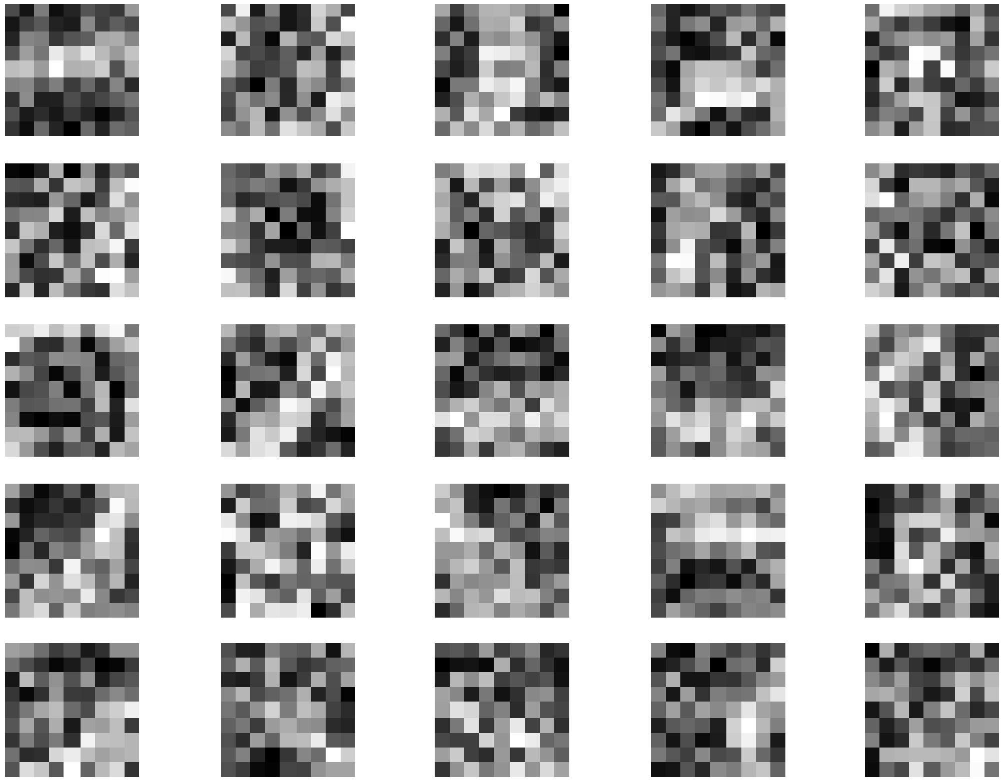
Question 3a

Plot the learning curve (I only show the test curve) - you should be able get better learning curve if you run for more epochs

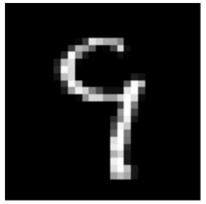


Question 3b

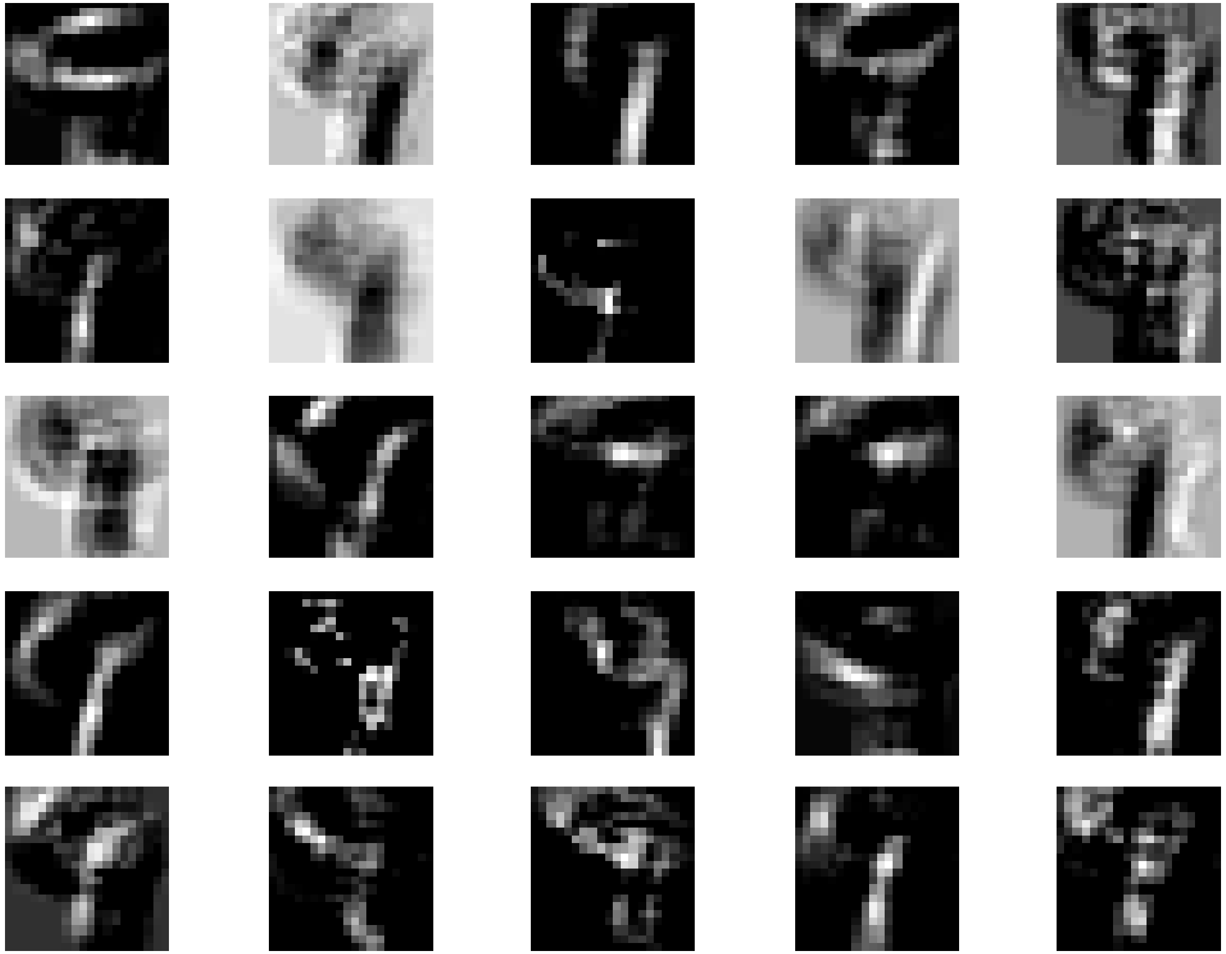
Plot filters learned in the conv layer



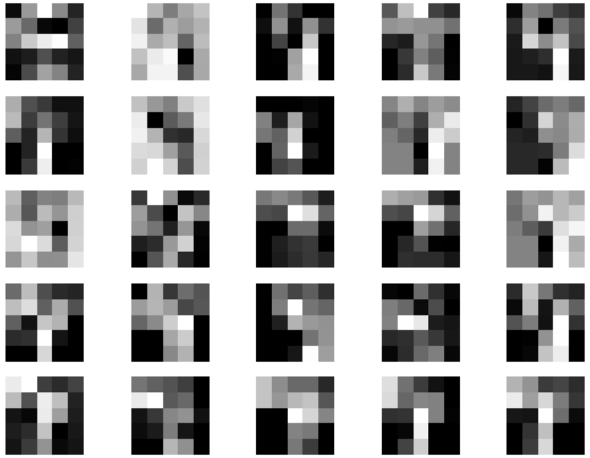
Question 3c



Input 28x28

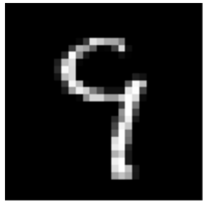


Feature maps of the convolution layer 25x20x20



Feature maps of pooling layer 25x5x5

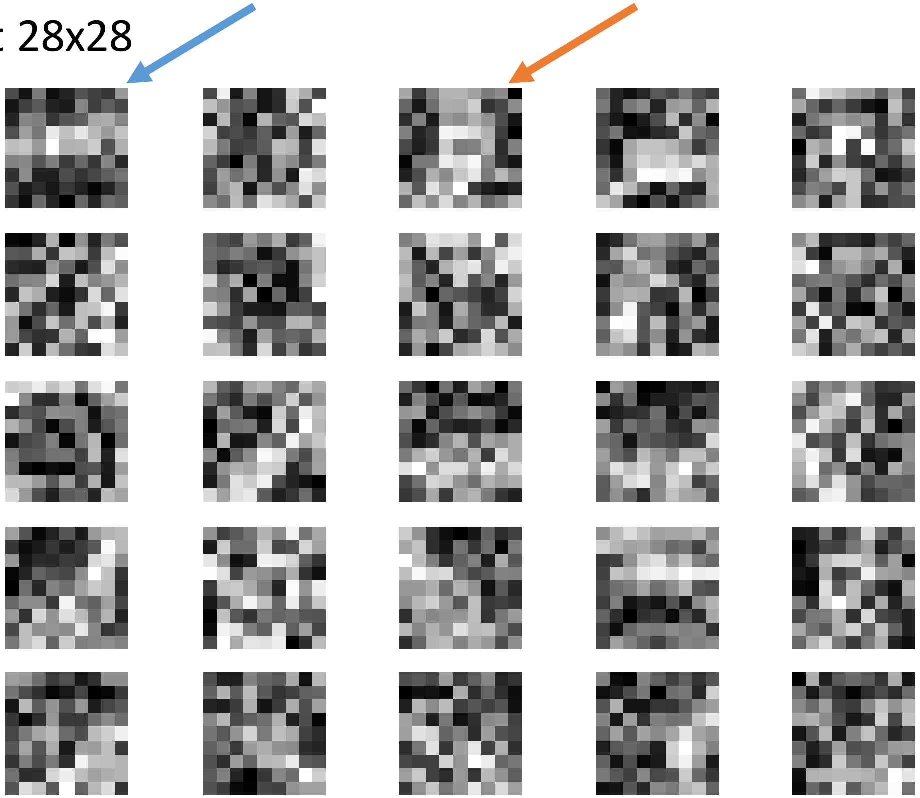
Question 3c



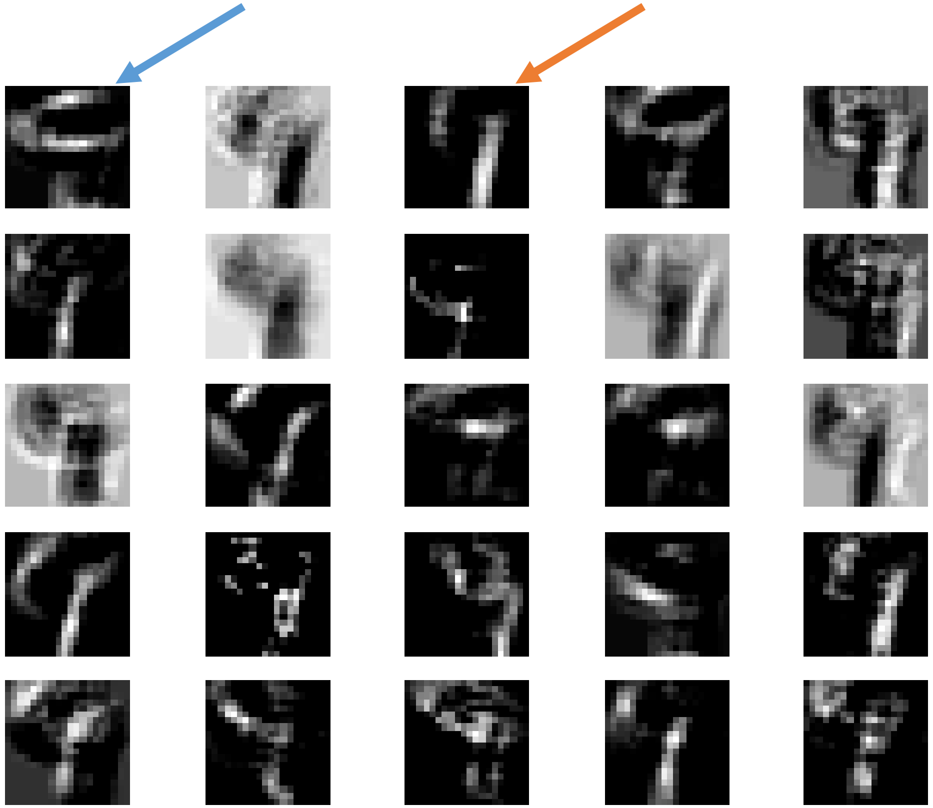
Input 28x28

Compare the filters learned and the corresponding feature maps.

What do you see?



Filters learned in the conv layer



Feature maps of the convolution layer 25x20x20

Question 3d

Gradient descent:

$$W = W - \alpha \nabla_W J$$

Weight decay (regularization):

$$J_1(W, b) = J(W, b) + \beta_2 \sum_{ij} (w_{ij})^2$$

Weight decay:

$$W = W - \alpha(\nabla_W J + \beta W)$$

Sgd with momentum:

$$V = \gamma V - \alpha \nabla_W J$$
$$W = W + V$$

Sgd with decay and momentum:

$$V = \gamma V \boxed{- \alpha(\nabla_W J + \beta W)}$$
$$W = W + V$$