MH1812 Tutorial
Chapter 4: Proof Techniques

Q1: Let q be a positive real number. Prove or disprove the following statement: if q is irrational, then $\sqrt{q}$ is irrational.

Q2: Prove using mathematical induction that the sum of the first n odd positive integers is n^2.

Q3: Prove using mathematical induction that $n^3 - n$ is divisible by 3 whenever n is a positive integer. Can you modify your argument to show a stronger result that $n^3 - n$ is always divisible by 6?

Q4: Prove by mathematical induction that

$$1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1).$$

Q5: Prove using mathematical induction that for every integer $n \geq 1$ and real number $x \geq -1$,

$$(1+x)^n \geq 1 + nx.$$

$\forall n \geq 1, \forall x \geq -1$

Q6: Prove using mathematical induction that

$$2^n > n^2 + 6, \quad n \geq 5.$$