MH1812 Tutorial
Chapter 7: Set Theory

Q1: Deduce that the cardinality of the power set $P(S)$ of a finite set S with n element is 2^n.

Q2: Consider the set $A = \{1, 2, 3\}$ and let $P(A)$ denote the power set of A.

- Compute the cardinality of $P(A)$ using the binomial theorem approach.
- Compute the cardinality of $P(A)$ using the counting approach.

Q3: Let $P(C)$ denote the power set of C. Given $A = \{1, 2\}$ and $B = \{2, 3\}$, determine:

$$P(A \cap B), P(A), P(A \cup B), P(A \times B).$$

Q4: Prove by contradiction that for two sets A and B

$$(A - B) \cap (B - A) = \emptyset.$$

Q5: Let $P(C)$ denote the power set of C. Prove that for two sets A and B

$$P(A) = P(B) \iff A = B$$

dk

Q6: Let $P(C)$ denote the power set of C. Prove that for two sets A and B

$$P(A) \subseteq P(B) \iff A \subseteq B.$$

dk

Q7: Show that the empty set is a subset of any set[1]. dk

Q8: Show that for two sets A and B

$$A \neq B \equiv \exists x[(x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A)].$$

Q9: Prove that for the sets A, B, C, D

$$(A \times B) \cup (C \times D) \subseteq (A \cup C) \times (B \cup D).$$

Does the equality hold?

[1]Yes, the empty set is a subset of itself.

Q10: Does the equality

$$(A_1 \cup A_2) \times (B_1 \cup B_2) = (A_1 \times B_1) \cup (A_2 \times B_2)$$

hold?

Q11: How many subsets of $\{1, \ldots, n\}$ are there with an even number of elements? Justify your answer.

Q12: Prove the following set equality:

$$\{7a + 9b \mid a, b \in \mathbb{Z}\} = \mathbb{Z}.$$

Q13: Let A, B, C be sets. Prove or disprove the following set equality:

$$A - (B \cup C) = (A - B) \cap (A - C).$$

Q14: For all sets A, B, C, prove that

$$\overline{(A - B) - (B - C)} = \overline{A} \cup B.$$

using set identities.

Q15: This exercise is more difficult. For all sets A and B, prove $(A \cup B) \cap \overline{A \cap B} = (A - B) \cup (B - A)$ by showing that each side of the equation is a subset of the other.

Q16: The symmetric difference of A and B, denoted by $A \triangle B$, is the set containing those elements in either A or B, but not in both A and B.

1. Prove that $(A \triangle B) \triangle B = A$ by showing that each side of the equation is a subset of the other.
2. Prove that $(A \triangle B) \triangle B = A$ using a membership table.

Q17: In a fruit feast among 200 students, 88 chose to eat durians, 73 ate mangoes, and 46 ate lychees. 34 of them had eaten both durians and mangoes, 16 had eaten durians and lychees, and 12 had eaten mangoes and lychees, while 5 had eaten all 3 fruits. Determine, how many of the 200 students ate none of the 3 fruits, and how many ate only mangoes?

Q18: Let A, B, C be sets. Prove or disprove the following set equality:

$$A \times (B - C) = (A \times B) - (A \times C).$$