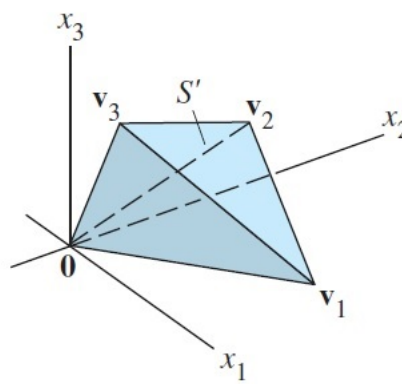
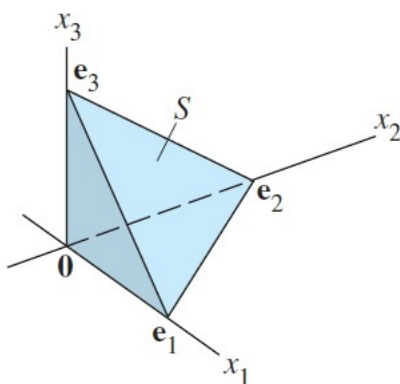


Tutorial 3

Determinants

1. If a 3×3 matrix A has $|A| = -1$, find $|\frac{1}{2}A|$, $|-A|$, $|A^2|$ and $|A^{-1}|$.
2. Reduce $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 2 \end{bmatrix}$ to U to find $|A|$ as the product of pivots.
3. Using variables a, b, c , construct a 3×3 skew-symmetric matrix ($A = -A^T$). Show that the determinant of such a matrix is equal to 0.
4. Find the volume of the parallelepiped with one vertex at the origin and adjacent vertices at $(1, 3, 0)$, $(-2, 0, 2)$ and $(-1, 3, -1)$.
5. In the figure below, let S be the tetrahedron in \mathbb{R}^3 with vertices at the vectors $\mathbf{0}, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ (\mathbf{e}_i 's are standard unit vectors) and let S' be the tetrahedron with vertices at vectors $\mathbf{0}, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.
 - a. Find the standard matrix for the linear transformation that maps S to S' .
 - b. Find a formula for the volume of the tetrahedron S' . (Volume of a tetrahedron = $(1/3) \times (\text{area of base}) \times \text{height}$).



Answers

1. $-1/8, 1, 1, -1$
2. -1

3.

4. 18

5. a. $A = [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \mathbf{v}_3]$ b. $(1/6) \times \text{abs}(|A|)$

End