# MH1812 Tutorial
# Chapter 1: Elementary Number Theory

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Q1: Show that 2 is the only prime number which is even.

Q2: Show that if $n^2$ is even, than $n$ is even, for $n$ an integer.

Q3: The goal of this exercise is to show that $\sqrt{2}$ is irrational. We provide a step by step way of doing so.

1. Suppose by contradiction that $\sqrt{2}$ is rational, that is $\sqrt{2} = \frac{m}{n}$, for $m$ and $n$ integers with no common factor. Show that $m$ has to be even.
2. Compute $m^2$, and deduce that $n$ has to be even too, a contradiction.

Q4: Show the following two properties of the integers modulo $n$:

1. $(a \bmod n) + (b \bmod n) \equiv a + b \pmod{n}$.
2. $(a \bmod n) \cdot (b \bmod n) \equiv a \cdot b \pmod{n}$.

Q5: Compute the addition table and the multiplication tables for integers modulo 4.

Q6: Show that $\frac{p(p+1)}{2} \equiv 0 \pmod{p}$ for $p$ an odd prime.

Q7: Find the last digit of $7^{9999}$.

Q8: Find the last digit of $8^{9999}$.

Q9: Consider the following sets $S$, with respective operator $\Delta$.

1. Let $S$ be the set of odd integers and $\Delta$ be the multiplication. Is $S$ closed under $\Delta$? Justify your answer.
2. Let $S$ be the set of nonzero rational numbers $\mathbb{Q} \setminus \{0\}$ and $\Delta$ be the division. Is $S$ closed under $\Delta$? Justify your answer.
3. Let $S$ be the set of positive integers $\mathbb{Z}^+$ and $\Delta$ be the subtraction. Is $S$ closed under $\Delta$? Justify your answer.
4. Let $S$ be the set of irrational numbers and $\Delta$ be the addition. Is $S$ closed under $\Delta$? Justify your answer.