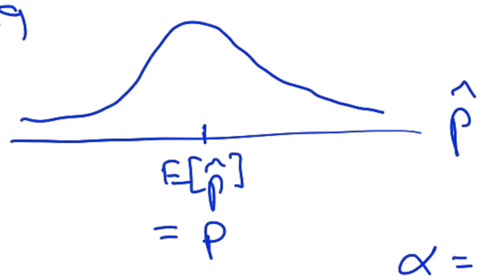
Problem 1

A four-sided (tetrahedral) die is tossed 1000 times, and 290 fours are observed. Is there evidence to conclude that the die is biased, that is, say, that more fours than expected are observed? (Use $\alpha = 0.05$)

$\hat{p} = \frac{290}{1000} = 0.29$



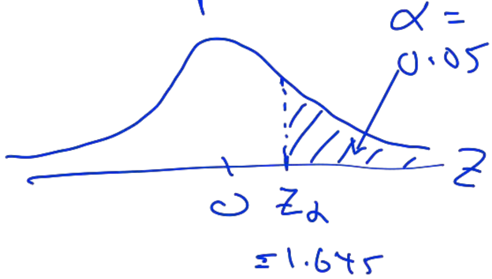
$H_0: p = 0.25$

$H_A: p > 0.25$

From Z-table, $Z_\alpha = 1.645$

Test statistic

$$Z_1 = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{0.29 - 0.25}{\sqrt{0.25(0.75)/1000}} = 2.92$$



Since $Z_1 > Z_\alpha$, we reject H_0

i.e. the die is biased towards "4"

Problem 2

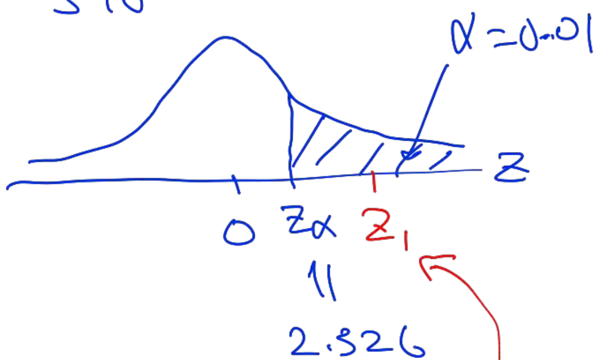
Let p equal the proportion of drivers who use a seat belt in a state that does not have a mandatory seat belt law. It was claimed that $p = 0.14$. An advertising campaign was conducted to increase this proportion. Two months after the campaign, $y = 104$ out of a random sample of $n = 590$ drivers were wearing seat belts. Was the campaign successful? (Use $\alpha = 0.01$)

$\hat{p} = \frac{104}{590} = 0.176$

$H_0: p = 0.14$

$H_A: p > 0.14$

From Z-table, $Z_\alpha = 2.326$



Test statistic Z_1

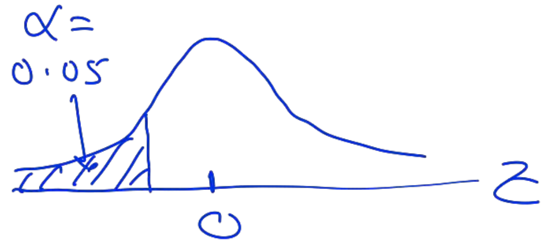
$$Z_1 = \frac{\hat{p} - p}{\sqrt{pq/n}} = 2.52$$

Since $Z_1 > Z_\alpha$, $\therefore$ we reject H_0

i.e. the campaign was successful

Problem 3

Boys of a certain age are known to have a mean weight of $\mu = 85$ pounds. A complaint is made that the boys living in a municipal children's home are underfed. As one bit of evidence, $n = 25$ boys (of the same age) are weighed and found to have a mean weight of $\bar{x} = 80.94$ pounds. It is known that the population standard deviation σ is 11.6 pounds. Based on the available data, what should be concluded concerning the complaint using p-value approach? (Use $\alpha = 0.05$)



$H_0: \mu = 85$

$H_A: \mu < 85$

Test statistic $z_1 = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} = -1.75$

p-value $= P(Z < z_1)$

$= P(Z < -1.75) = 0.04$

Since p-value $< \alpha$, $\therefore$ reject H_0

i.e. boys were underfed.

Problem 4

To compare customer satisfaction levels of two competing cable television companies, 174 customers of Company 1 and 355 customers of Company 2 were randomly selected and were asked to rate their cable companies on a five-point scale, with 1 being least satisfied and 5 most satisfied. The survey results are summarized in the following table

$Var[y] = Var[\bar{x}_1] + Var[\bar{x}_2]$

$= \dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}$

Company 1	Company 2
$n_1 = 174$	$n_2 = 355$
$\bar{x}_1 = 3.51$	$\bar{x}_2 = 3.24$
$s_1 = 0.51$	$s_2 = 0.52$

$y = \bar{x}_1 - \bar{x}_2$

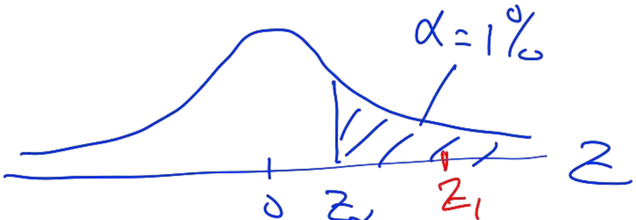
$E[y] = E[\bar{x}_1] - E[\bar{x}_2]$

$= \mu_1 - \mu_2$

Test at the 1% level of significance whether the data provide sufficient evidence to conclude that Company 1 has a higher mean satisfaction rating than does Company 2.

$H_0: \mu_1 - \mu_2 = 0$

$H_A: \mu_1 - \mu_2 > 0$



$= 2.326$

from z-table

Test statistic

$$z_1 = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}} = 5.684$$

Since $z_1 > z_\alpha$, $\therefore$ reject H_0