CLT_HW

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2021/3/5

Q1

MA 677

NO.______ DATE______ CLT HW

Q1.

$\mu = 4 \qquad \sigma = 5/12$

We want to find $P(X \geq \frac{250}{60})$

$$P(X \geq \frac{250}{60}) = 1 - P(Z < \frac{X-\mu}{\sigma/\sqrt{n}})$$

$$= 1 - P(Z < \frac{\frac{25}{6} - 4}{\frac{5}{12}/\sqrt{60}})$$

$$= 1 - P(Z < \frac{\sqrt{60}/6 \times 12}{5})$$

$$= 1 - P(Z < 3.01)$$

$$= 1 - 0.9990$$

$$= 0.001$$

The probability that the machine will produce at least 250 feet in an hour is 0.001.

Q2

$\lambda = 5. \qquad X = 5.5$

$$P(X < 5.5) = P(Z \leq \frac{5.5 - 5}{\sqrt{5/125}})$$

$$= P(Z \leq \frac{0.5}{0.2})$$

$$= P(Z \leq 2.5)$$

Q3

NO. DATE

Q3.

$$\sigma/\sqrt{n} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.1(0.9)}{n}}$$

We want to find n that makes:

$$P(X < 0.13) \geq 0.99$$

$$P(X < 0.13) = P\left(Z \leq \frac{(0.13 - 0.1)\sqrt{n}}{\sqrt{0.09}}\right)$$

$$= P\left(Z \leq \frac{0.03\sqrt{n}}{0.3}\right)$$

$$P\left(Z \leq \frac{\sqrt{n}}{10}\right) \geq 0.99$$

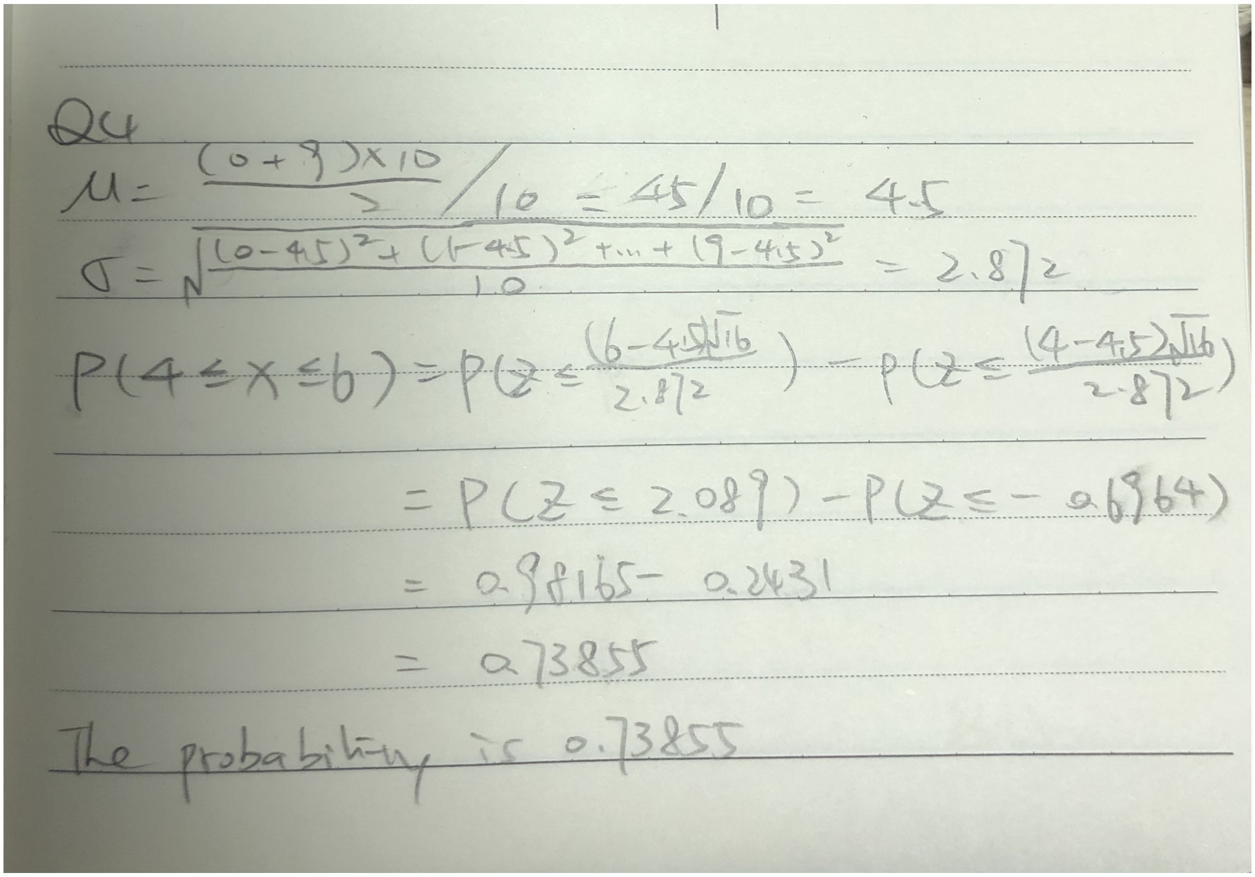
$$\frac{\sqrt{n}}{10} \geq 2.32635$$

$$\sqrt{n} \geq 23.2635$$

$$n \geq 541.19 \approx 542.$$

$\therefore$ The smallest random sample should be 542

Q4



Q4

$$\mu = \frac{(0+9)\times 10}{2} / 10 = 45/10 = 4.5$$

$$\sigma = \sqrt{\frac{(0-4.5)^2 + (1-4.5)^2 + \dots + (9-4.5)^2}{10}} = 2.872$$

$$P(4 \le x \le 6) = P\left(Z \le \frac{(6-4.5)\sqrt{16}}{2.872}\right) - P\left(Z \le \frac{(4-4.5)\sqrt{16}}{2.872}\right)$$

$$= P(Z \le 2.089) - P(Z \le -0.6964)$$

$$= 0.98165 - 0.2431$$

$$= 0.73855$$

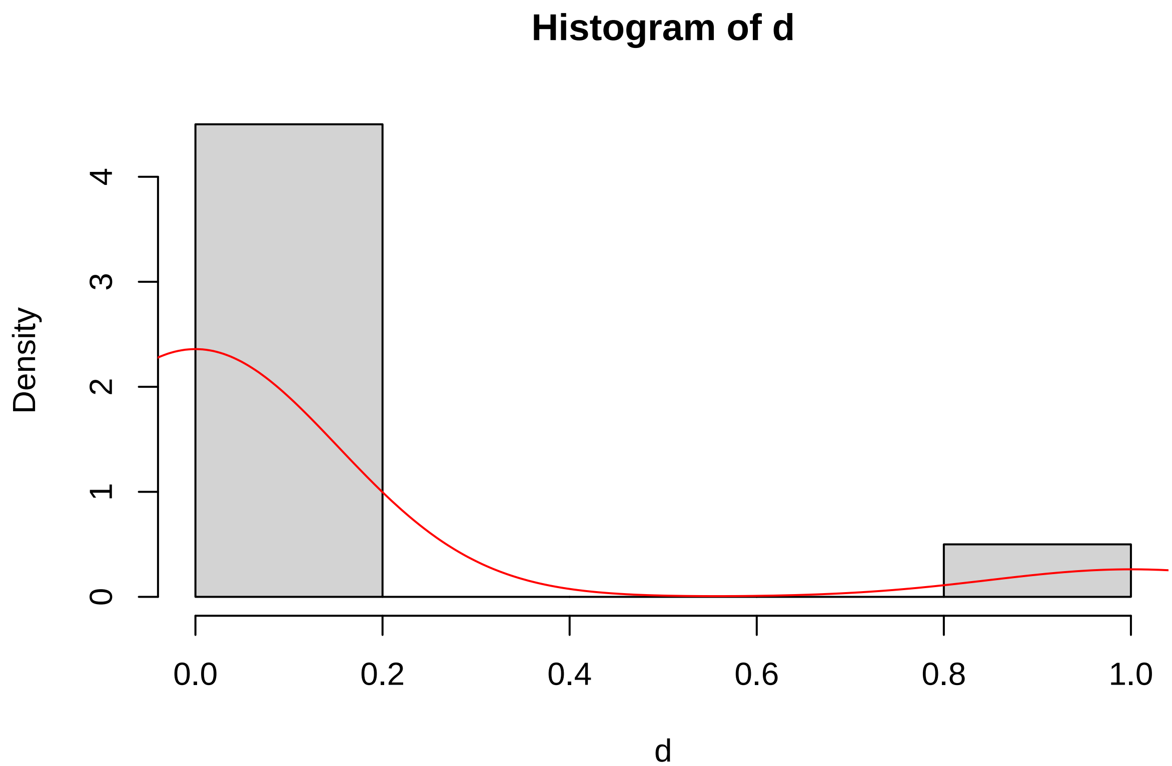
The probability is 0.73855

Q5

Choose a skewed binomial distribution

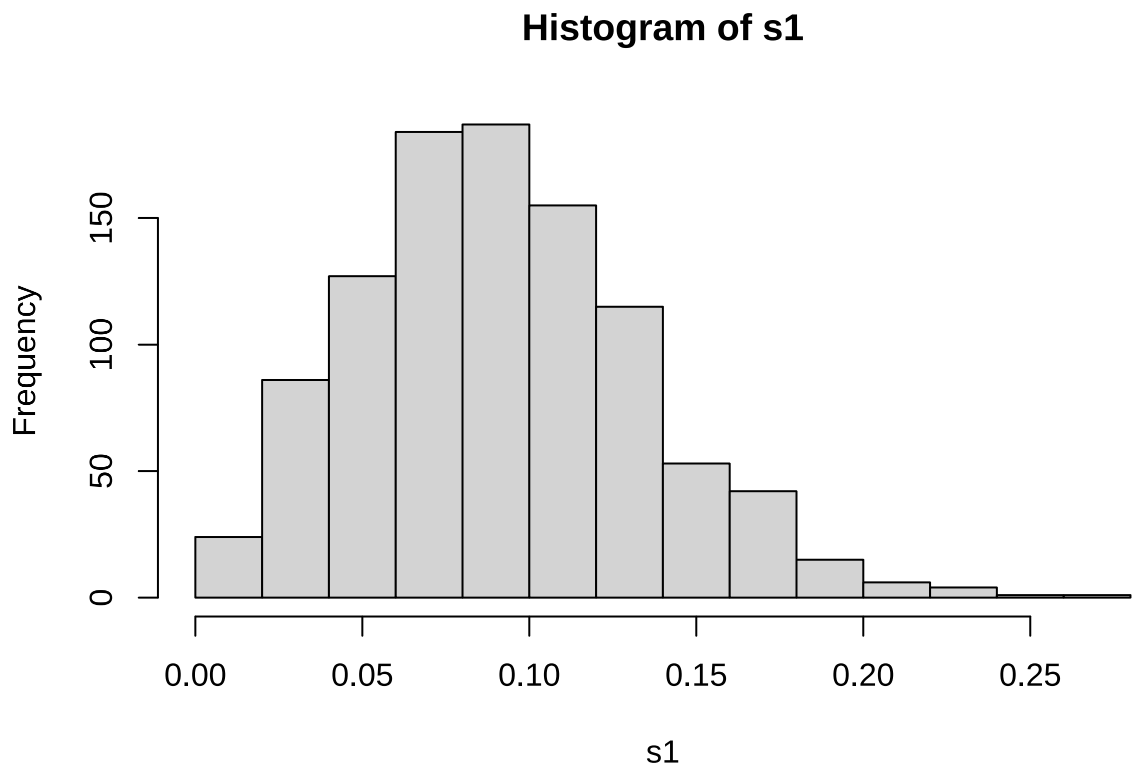
```r
#The skewed binomial distribution
set.seed(677)
n<-20
p<-0.1
d<-rbinom(n,1,p)

#histogram of this distribution
hist(x = d, freq = FALSE)
lines(x = density(x = d), col = "red")
```

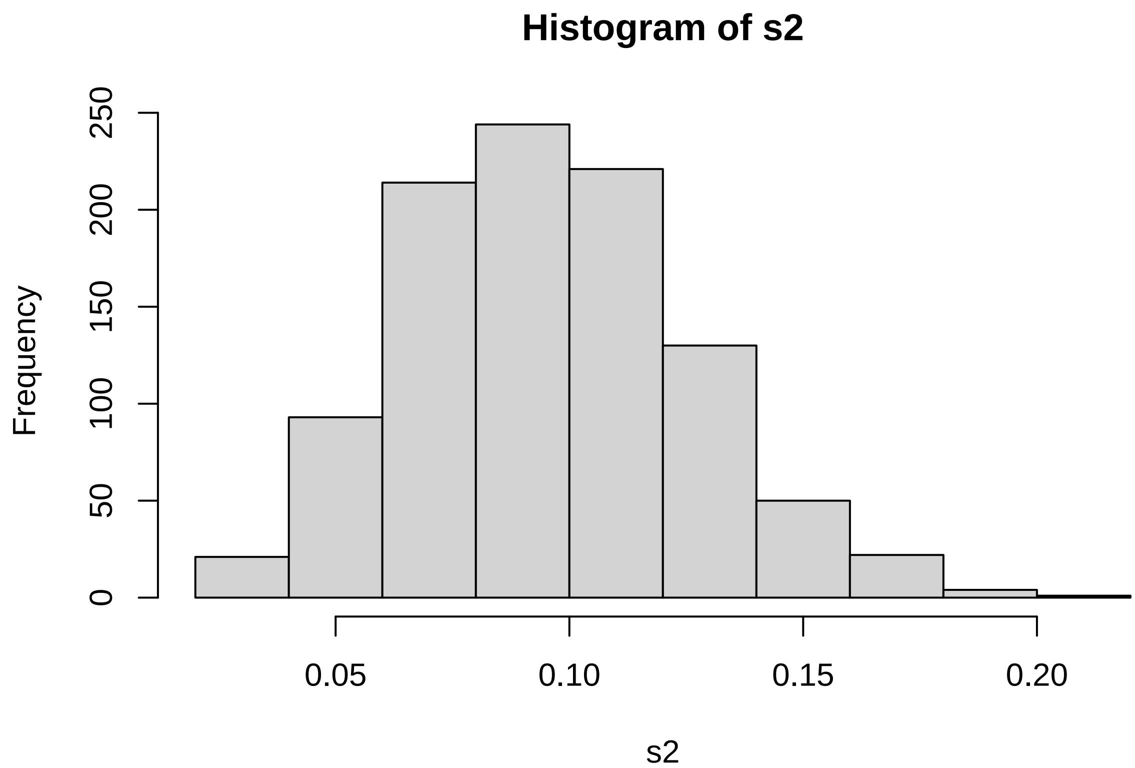


Normal approximation to the binomial distribution

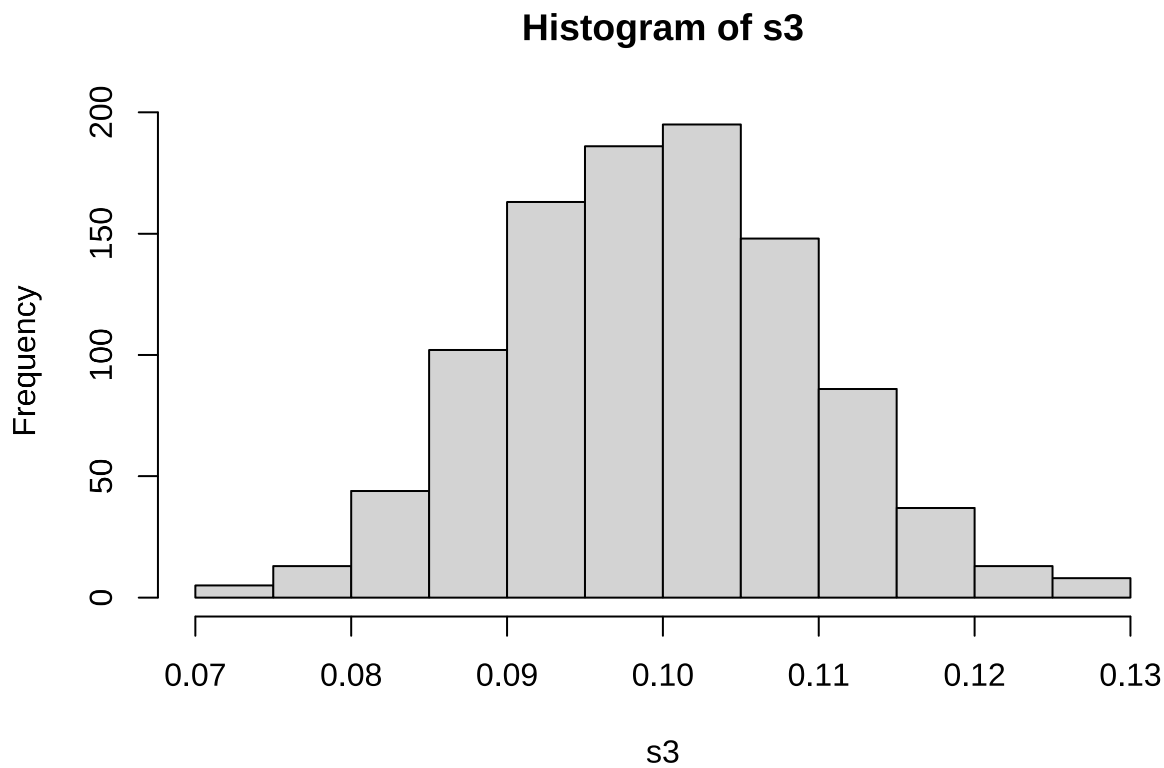
```r
#Visually check that when n groups up, the binomial distribution approximate to normal.
s1<-c()
for(i in 1:1000){
  n<-rbinom(50,1,p)
  s1[i]=mean(n)
}
hist(s1)
```



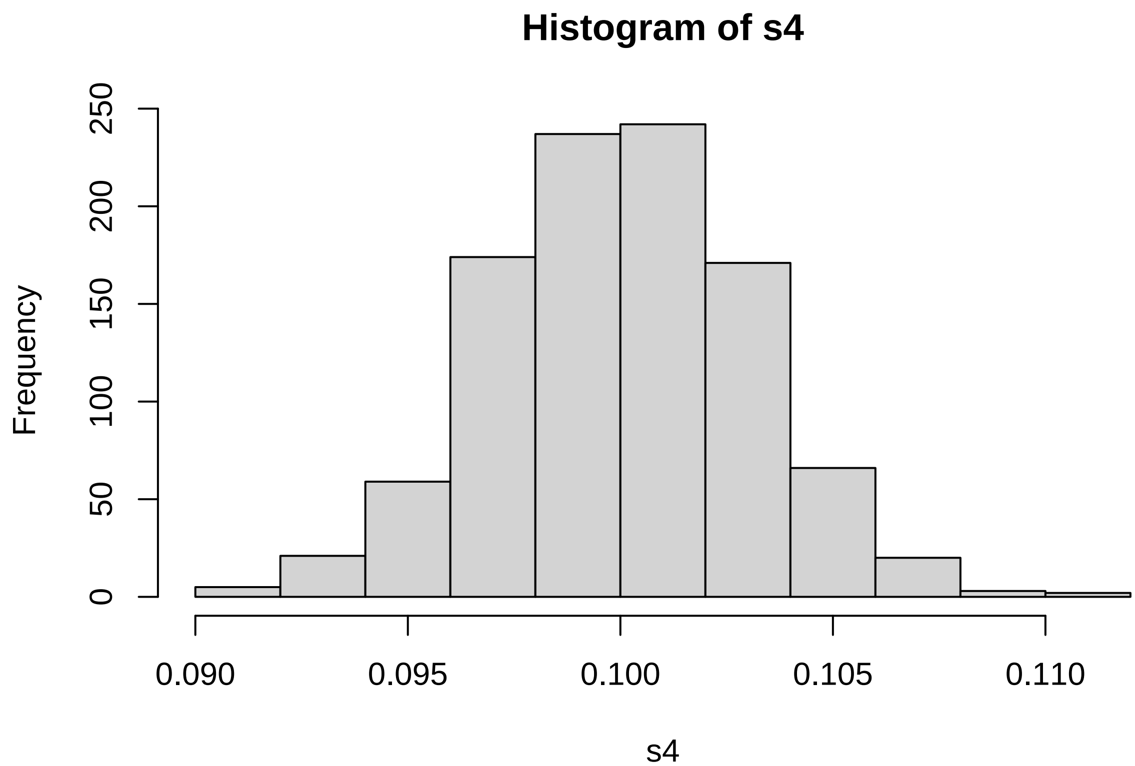

```
s2<-c()
for(i in 1:1000){
  n1<-rbinom(100,1,p)
  s2[i]=mean(n1)
}
hist(s2)
```



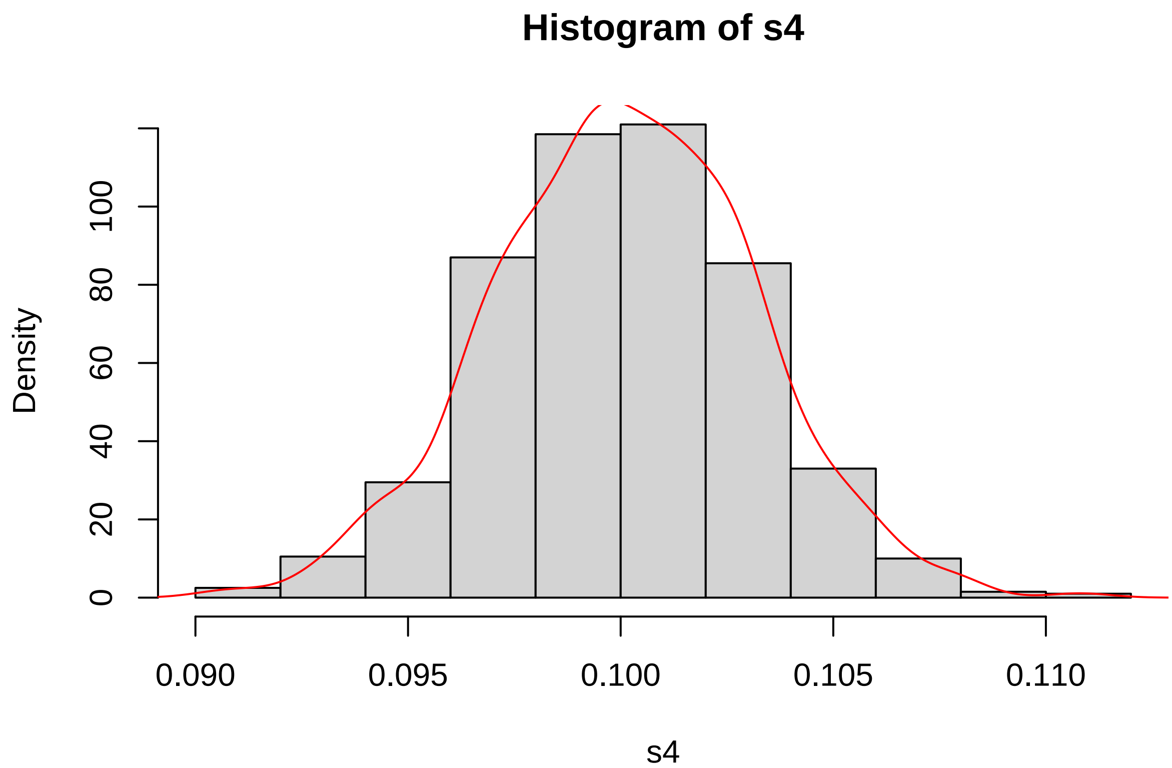

```r
s3<-c()
for(i in 1:1000){
  n3<-rbinom(1000,1,p)
  s3[i]=mean(n3)
}
hist(s3)
```




```r
s4<-c()
for(i in 1:1000){
  n4<-rbinom(10000,1,p)
  s4[i]=mean(n4)
}
hist(s4)
```



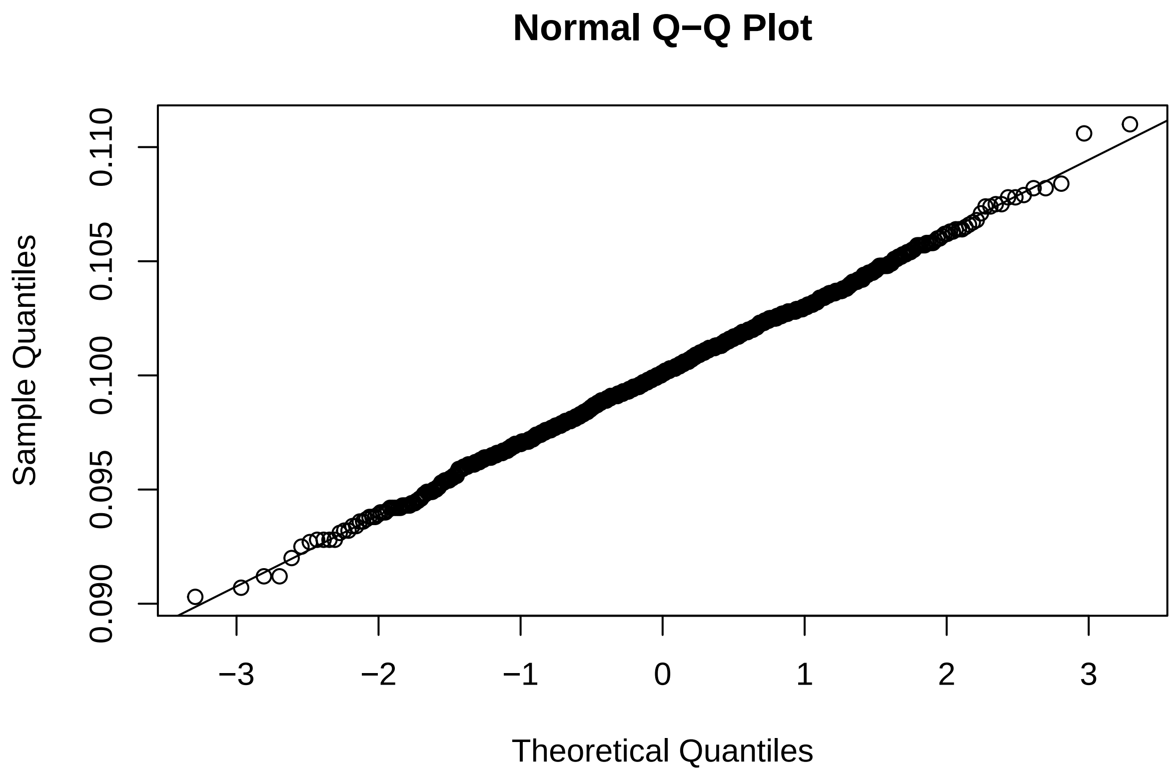

```r
#normality check
hist(x = s4, freq = FALSE)
lines(x = density(x = s4), col = "red")
```




```r
shapiro.test(s4)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  s4
## W = 0.99887, p-value = 0.7968
```

```r
qqnorm(s4)
qqline(s4)
```



The histogram for the generated data shows a belled curve. The shapiro test for the distribution with n=10000 shows p-value greater than 0.05, which indicates the sample is normally distributed. The Q-Q Plot also shows a straight line showing the normality.