worksheet_18

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1 Worksheet 18

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1.0.1 Topics

- Linear Regression

1.0.2 Optional Challenge

Every day my alarm goes off at seemingly random times… I've recorded the times at which it goes off for the past year of so (1 - 355 days). Today is day 356. Can you predict when my alarm will ring using `data.csv`?

Please fill out the piazza poll if you think you found the answer.

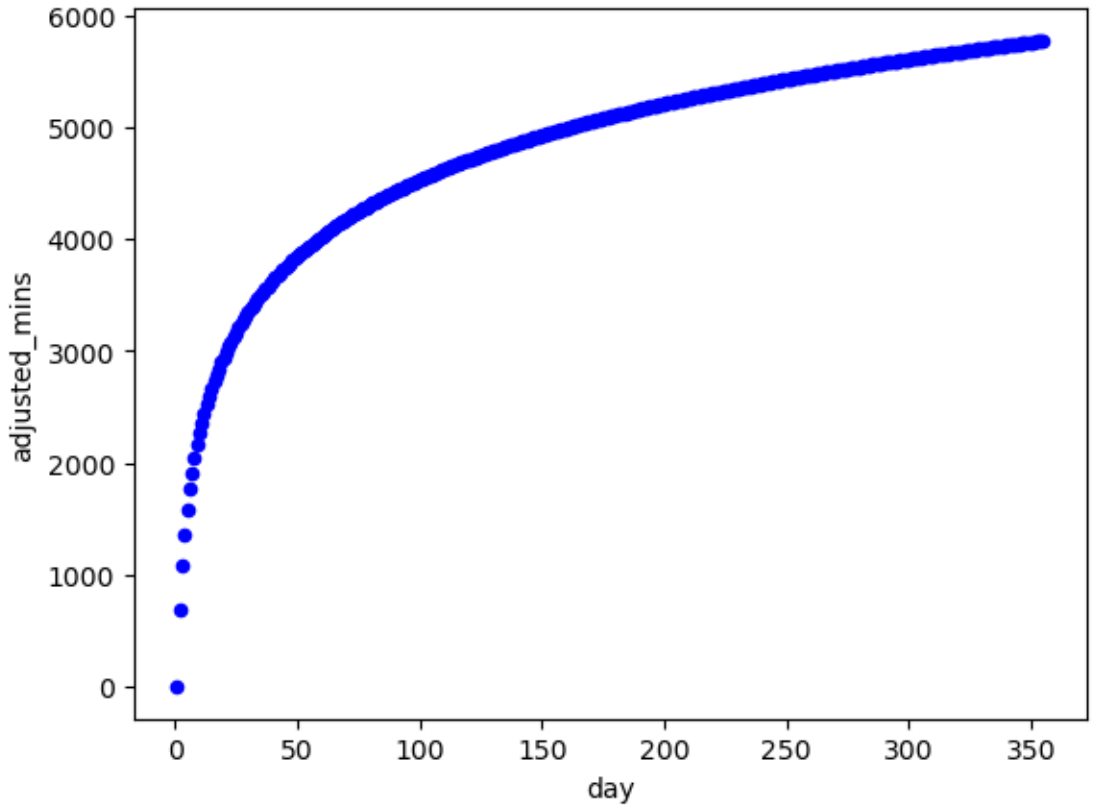
1.1 Linear Regression

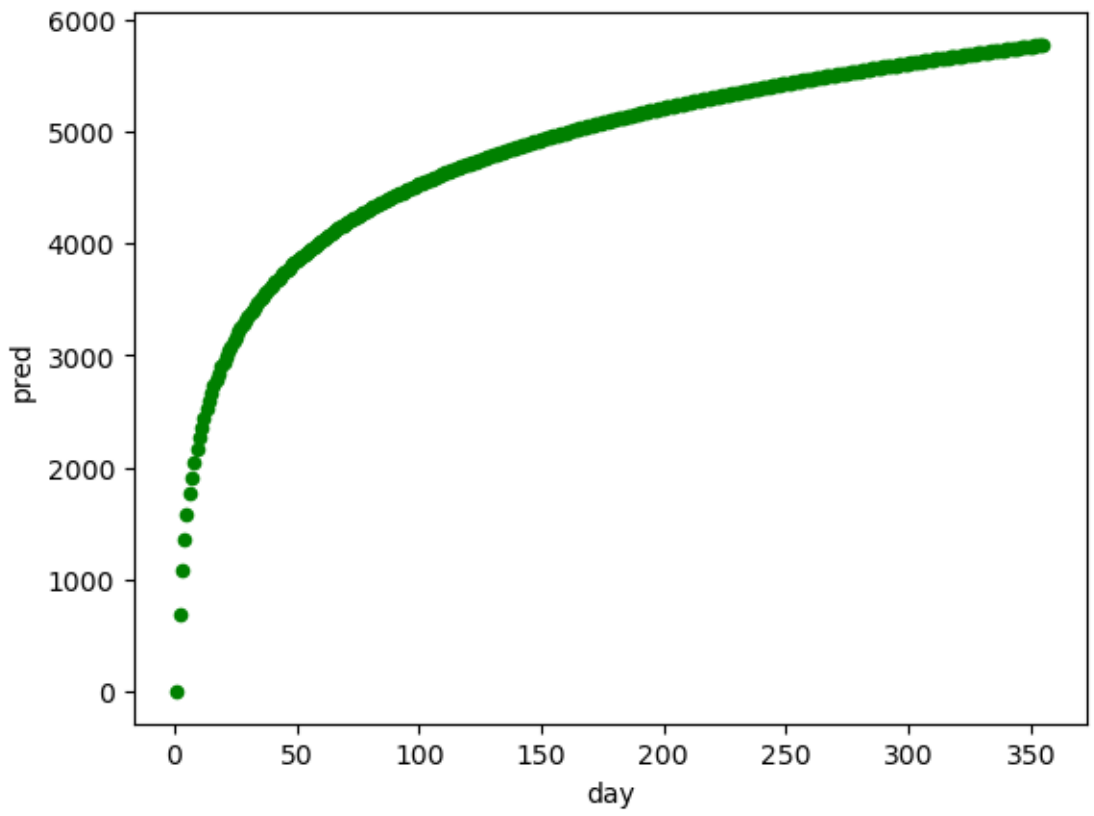
Where does randomness come from?

[1]:
```
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np

alarms = pd.read_csv('data.csv')
alarms['time'] = pd.to_datetime(alarms['alarm (%H:%M:%S)'], format='%H:%M:%S')
alarms['total_minutes'] = (alarms['time'] - alarms['time'][0]).dt.
 ↪total_seconds() / 60

cycles = 0
adjusted = np.zeros(355)
for i in range(1, 355):
    if alarms['total_minutes'][i] < alarms['total_minutes'][i-1]:
        cycles += 1
    adjusted[i] = alarms['total_minutes'][i] + 1440*cycles

alarms['adjusted_mins'] = adjusted
```

```
intercepts = np.ones(355)
adj_mins = alarms['adjusted_mins'].to_numpy()
X = np.array([intercepts, np.log(alarms['day'].to_numpy())]).T

beta_hat = np.linalg.solve(X.T @ X, X.T @ adj_mins)

print(beta_hat)

alarms['pred'] = beta_hat[0] + beta_hat[1] * np.log(alarms['day'].to_numpy())
alarms.plot.scatter(x='day', y='adjusted_mins', c = 'b')
alarms.plot.scatter(x='day', y='pred', c = 'g')

print((beta_hat[0] + beta_hat[1] * np.log(356)) % 1440)
```

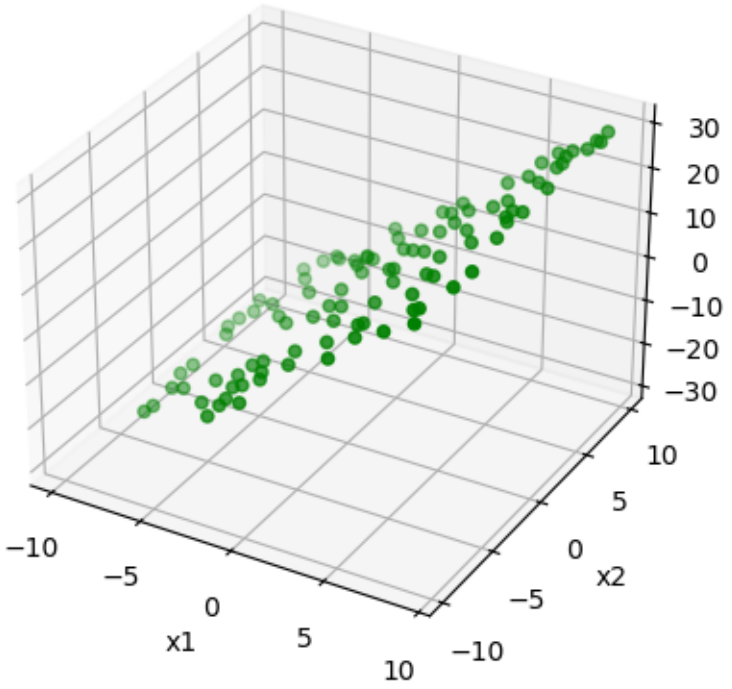
```
[-1.08077269e-02  9.83050156e+02]
15.340760939289794
```





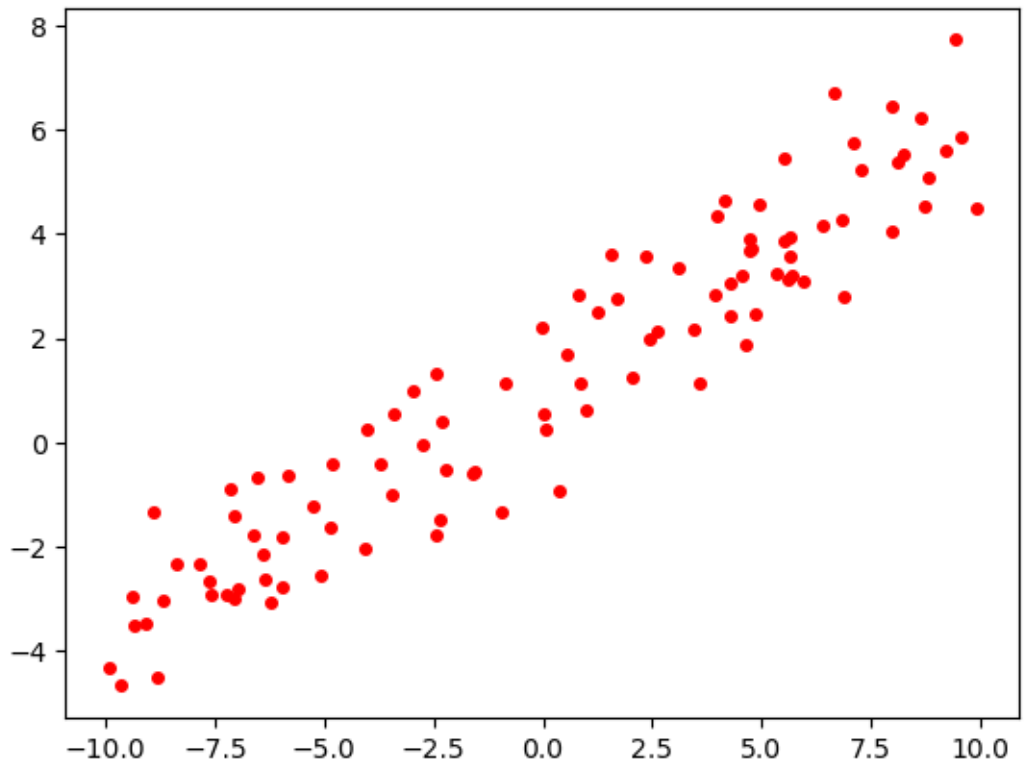
[2]:
```
%matplotlib inline
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

SAMPLE_SIZE = 100

# it's possible for y = 3 * x_1 + (1/4) * x_2
# but we don't know to look for x_2 and we only have x_1 as a feature
x1 = -10.0 + 20.0 * np.random.random(SAMPLE_SIZE)
x2 = -10.0 + 20.0 * np.random.random(SAMPLE_SIZE)
y = 3 * x1 + (1/4) * x2

# Create the figure
fig = plt.figure()

# Add an axes
ax = fig.add_subplot(111,projection='3d')

# and plot the point
ax.scatter(x1 , x2 , y,  color='green')
```

```
ax.set_xlabel("x1")
ax.set_ylabel("x2")
ax.set_zlabel("y")
plt.show()
```



a) Create and plot a dataset of 100 (Y, X) points where `Y = 1 + 0.5 * X + eps`.

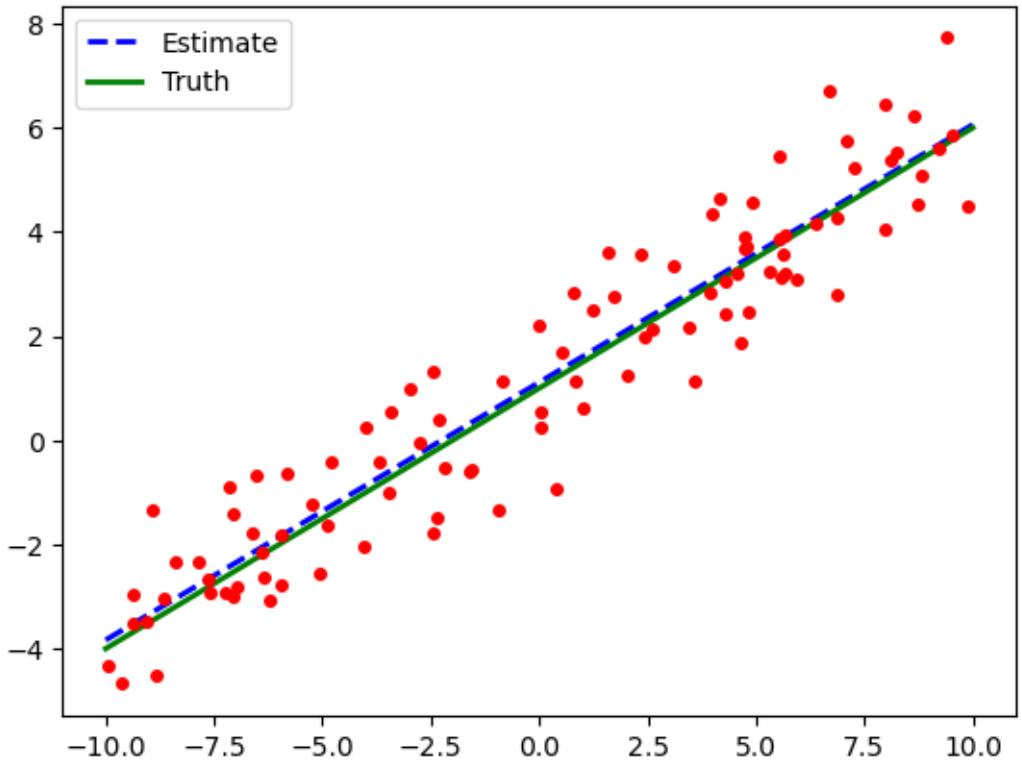
[3]:
```
import numpy as np
import matplotlib.pyplot as plt

SAMPLE_SIZE = 100

true_beta = np.array([ 1 , 0.5 ])
x = -10.0 + 20.0 * np.random.random(SAMPLE_SIZE)
y = true_beta[0] + true_beta[1] * x + np.random.randn(SAMPLE_SIZE)
plt.plot(x,y,'ro',markersize=4)
plt.show()
```



b) Plot the least squares estimate line through the scatter plot.

[4]:
```
intercept = np.ones(np.shape(x)[0])
X = np.array([intercept, x]).T
beta_hat = np.linalg.solve(X.T @ X, X.T @ y)
print(beta_hat)
x_plot = np.linspace(-10,10,100)
y_est = beta_hat[0] + beta_hat[1] * x_plot
plt.plot(x_plot, y_est,'b--', lw=2, label='Estimate')
plt.plot(x_plot, true_beta[0] + true_beta[1] * x_plot, lw=2, c='g',␣
 ↪label='Truth')
plt.plot(x, y,'ro',markersize=4)
plt.legend()
plt.show()
```

```
[1.11395025 0.49481614]
```

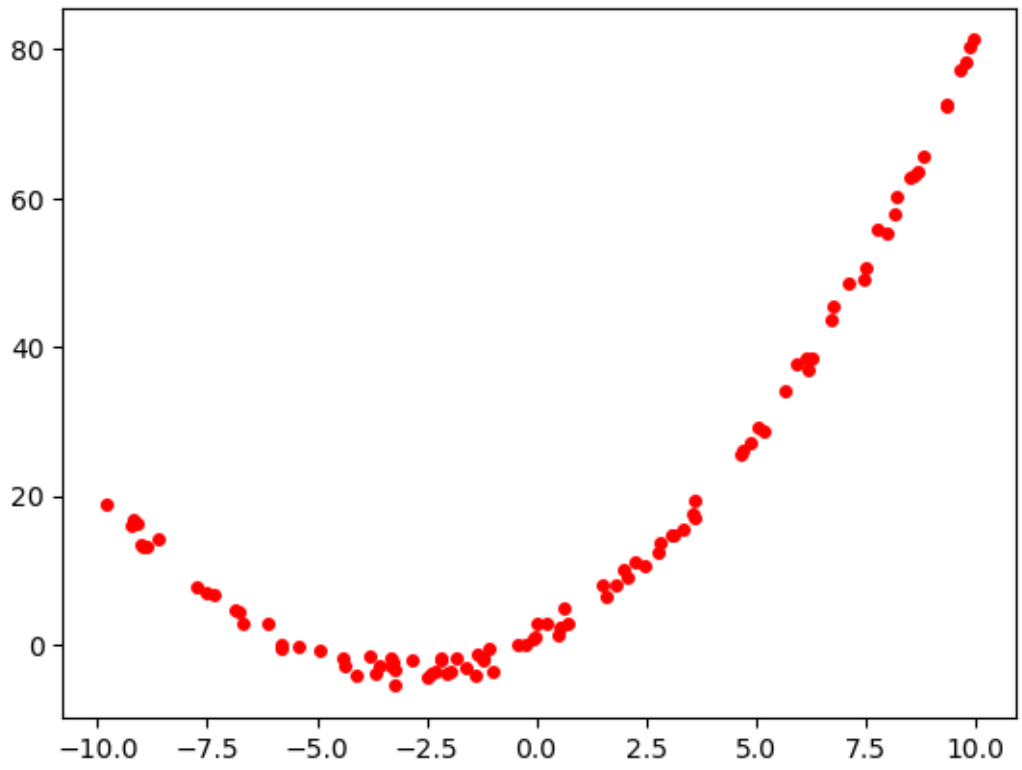


c) How does the estimate of `beta` compare to the parameters we used to generate the data?

The estimate is close to the parameters we used to generate the data.

d) Create and plot a dataset of 100 (Y, X) points where `Y = 1 + 3 * X + .5 * X^2 + eps`.

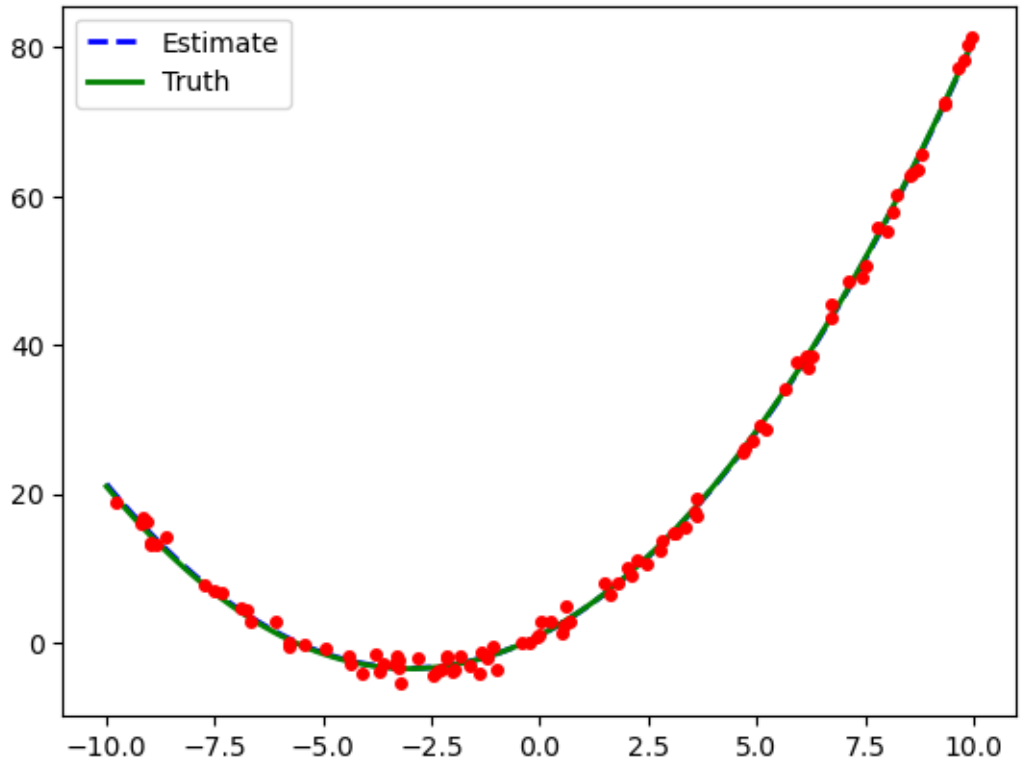
[5]:
```
SAMPLE_SIZE = 100

true_beta = np.array([ 1 , 3, 0.5 ])
x = -10.0 + 20.0 * np.random.random(SAMPLE_SIZE)
y = true_beta[0] + true_beta[1] * x + true_beta[2] * x * x + np.random.
 ↪randn(SAMPLE_SIZE)
plt.plot(x,y,'ro',markersize=4)
plt.show()
```



e) Plot the least squares estimate line through the scatter plot.

[6]:
```
intercept = np.ones(np.shape(x)[0])
X = np.array([intercept, x, x**2]).T
beta_hat = np.linalg.solve(X.T @ X, X.T @ y)
print(beta_hat)
x_plot = np.linspace(-10,10,100)
y_est = beta_hat[0] + beta_hat[1] * x_plot + beta_hat[2] * x_plot **2
plt.plot(x_plot, y_est,'b--', lw=2, label='Estimate')
plt.plot(x_plot, true_beta[0] + true_beta[1] * x_plot + true_beta[2] * x_plot *␣
 ↪x_plot, lw=2, c='g', label='Truth')
plt.plot(x, y,'ro',markersize=4)
plt.legend()
plt.show()
```

```
[0.98198296 2.97468036 0.50096126]
```

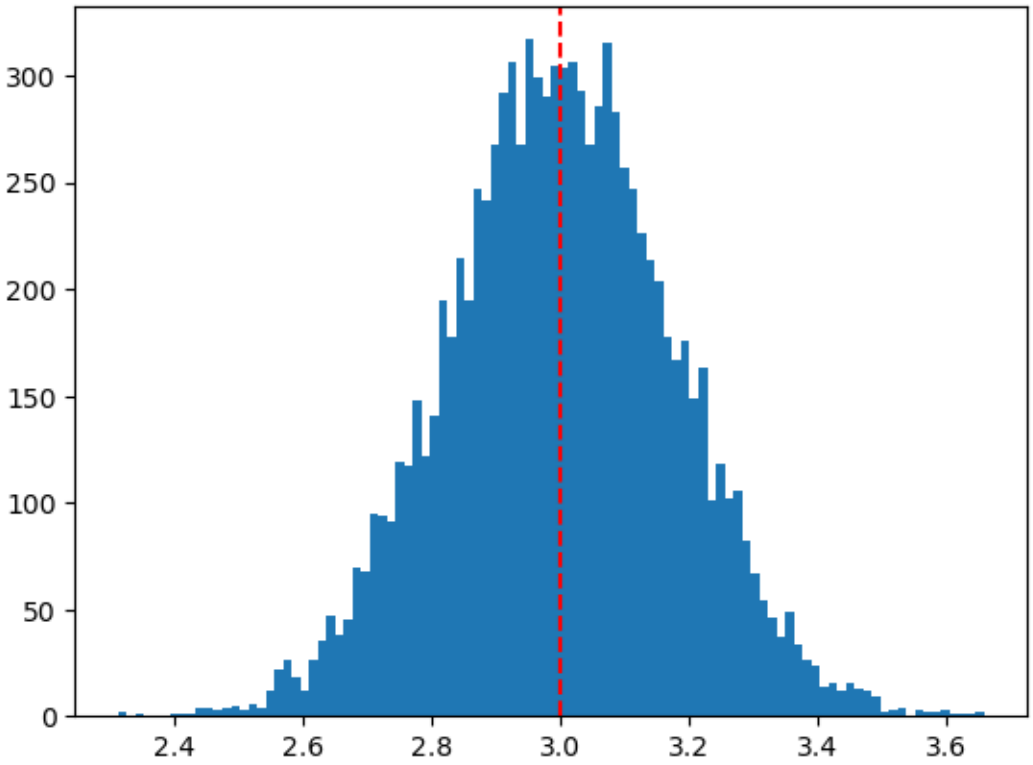


f) How does the estimate of `beta` compare to the parameters we used to generate the data?

The estimate is close to the parameters we used to generate the data.

g) Let's repeat d) and f) a large number of times to see how close our estimates are on average and what that distribution looks like.

[7]:
```
import numpy as np
import matplotlib.pyplot as plt

betas = []
true_beta = np.array([ 1 , 3, 0.5 ])
for _ in range(10000):
    x = -1 + 2 * np.random.random(SAMPLE_SIZE)
    y = true_beta[0] + true_beta[1] * x + true_beta[2] * x * x + np.random.
 ↪randn(SAMPLE_SIZE)
    intercept = np.ones(np.shape(x)[0])
    X = np.array([intercept, x, x**2]).T
    betas.append(np.linalg.solve(X.T @ X, X.T @ y))

plt.hist(np.array(betas)[:, 1], bins=100)
plt.axvline(x=true_beta[1], c='r', linestyle='dashed')
plt.show()
```


300
250
200
150
100
50
0
2.4
2.6
2.8
3.0
3.2
3.4
3.6