



ML Bootcamp - Causality Class

Causality as an AI frontier

Tools & Methods for Causality

Coffee Break

Optimization of Causal Effects

Lunch Break

Causality as an AI frontier

Causality as an AI frontier

Examples of Causal Failures

A Famous Failure ...

... and some Easy Fixes

Tools & Methods for Causality

Coffee Break

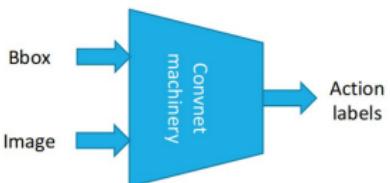
Optimization of Causal Effects

Lunch Break

Struggles of modern AI

Deep Learning is solving AI, right ?

Example: detection of the action “*giving a phone call*”



(Oquab et al., CVPR 2014)
~70% correct (SOTA in 2014)

Struggles of modern AI

Deep Learning is solving AI, right ?

Example: detection of the action “*giving a phone call*”



Not giving a phone call.

Giving a phone call ?????

Example from Léon Bottou '15

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Struggles of modern AI

Deep Learning is solving AI, right ?

Example: detection of the action "*giving a phone call*"



"Phone Call" action
causes
"Person" + "Phone" in
image
(but not vice versa !)

Example from Léon Bottou '15

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Struggles of modern AI (2)

With loads of data we can solve AI problems, right ?

The screenshot shows an Amazon product page for a 'Mobile Edge EXP' backpack. The main image is a black and grey backpack with white accents. To the right, product details are listed:

- Other products by [Mobile Edge](#)
- 4.5 stars (18,000 reviews)
- List Price: \$49.99
- Price: \$48.32
- You Save: \$1.67 (3%)
- Availability: In Stock
- Want it delivered Tue at checkout. [See details](#)
- 21 used & new avail

Below the main image, there's a link to 'see larger image and other views' with three smaller thumbnail images. Further down, there's a section titled 'Better Together' suggesting the backpack be paired with an HP Pavilion DV2610US laptop, with a total list price of \$1,123.99 and a special offer of \$898.31.

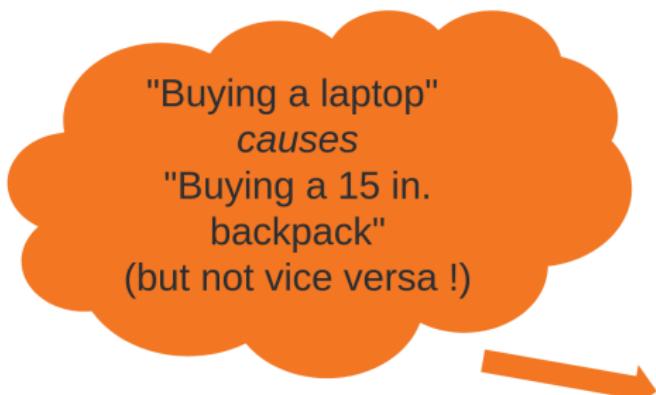


Example from Bernard Schoelkopf '17

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Struggles of modern AI (2)

With loads of data we can solve AI problems, right ?



amazon.com

Hello. Sign in to get personalized recommendations
Your Amazon.com Today's Deals

Shop All Departments Search Electronics

Electronics Browse Brands Top Sellers

Prime

Mobile Edge EXP
Other products by **Mobile**
4.5 stars (1,186 customer reviews)
List Price: \$49.99
Price: \$48.32
You Save: \$1.67 (3%)
Availability: In Stock!

Want it delivered **Tue** at checkout. [See details](#)

[21 used & new available](#)

[see larger image and other views](#)

[Share your own customer images](#)

Better Together
Buy the item with [HP Pavilion DV2610US 14.1" Entertainment](#) Hewlett-Packard today!

Total List Price: \$3,123.99
Buy Together Today: \$898.31
[Buy both now!](#)

criteo.

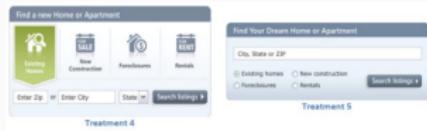
Example from Bernard Schoelkopf '17

12

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A/B test: Experiment where

- subjects are *randomly* assigned to groups
- groups are treated *differently*



The image shows two side-by-side versions of a real estate search interface. Both versions have a header "Find a new Home or Apartment". Below the header, there are four categories: "Existing homes" (highlighted in green), "New construction", "Foreclosures", and "Rentals". There are input fields for "Enter Zip", "Enter City", and "State", along with a "Search Listings" button.

Treatment 4: This version has a "Find Your Dream Home or Apartment" header. It includes a "Cty, State or ZIP" input field, checkboxes for "Existing homes", "New construction", "Foreclosures", and "Rentals", and a "Search Listings" button.

Treatment 5: This version has a "Find Your Dream Home or Apartment" header. It includes a "Cty, State or ZIP" input field, checkboxes for "Existing homes", "New construction", "Foreclosures", and "Rentals", and a "Search Listings" button.

Goals

- identify an effect: *is A influencing B ?*
- evaluate the impact of a change: *by how much ?*

Can't we rely on *expertise* ?

Can't we rely on *available data* ?



- Two treatment variants, *prescribed by physician*:
 - $Treatment_1$: open surgical procedure
 - $Treatment_2$: ultrasonic probes and small puncture procedure



	$Treatment_1$	$Treatment_2$
Total	273/350= 78%	289/350= 83%

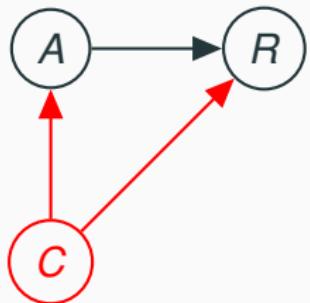
Table 1: Recovery rate



Is an A/B test¹ necessary to conclude ?

¹in medicine and social science the proper name is Randomized Control Trial

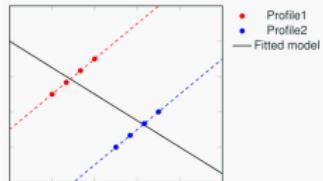
- Simpson's paradox
- *Confounder*: stone size



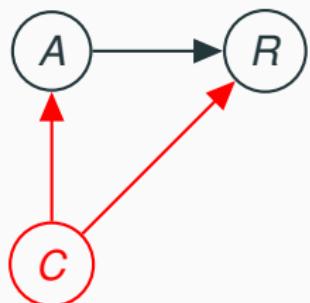
	<i>Treatment₁</i>	<i>Treatment₂</i>
Total	$273/350 = 78\%$	$289/350 = 83\%$
Small stone	$81/87 = 93\%$	$234/270 = 87\%$
Big stone	$192/263 = 73\%$	$55/80 = 69\%$

Table 2: Recovery rate

- Confounding bias pulls the observed causal relation away from the true association
- Here it inverted the true causal effect



- Simpson's paradox
- *Confounder*: stone size



	<i>Treatment₁</i>	<i>Treatment₂</i>
Total	$273/350 = 78\%$	$289/350 = \textbf{83\%}$
Small stone	$81/87 = \textbf{93\%}$	$234/270 = 87\%$
Big stone	$192/263 = \textbf{73\%}$	$55/80 = 69\%$

Table 3: Recovery rate

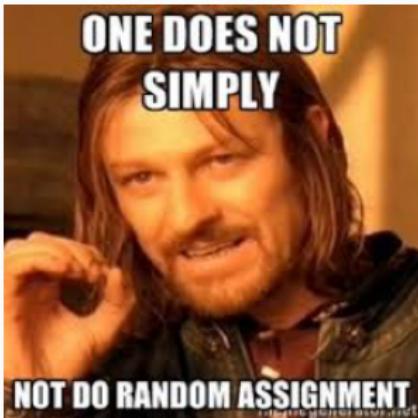
- Fix #1: put potential confounders in the model

- not possible to put *all* confounders in the model
- **Fix #2: break confounders to avoid the effects of confounders**



- assign treatment at random to avoid selection bias

- Beware of observables
- Randomize to evaluate the causal impact of a change



Tools & Methods for Causality

Causality as an AI frontier

Tools & Methods for Causality

Formalism & Fundamental Results

Causal Discovery

Application: Brain Research

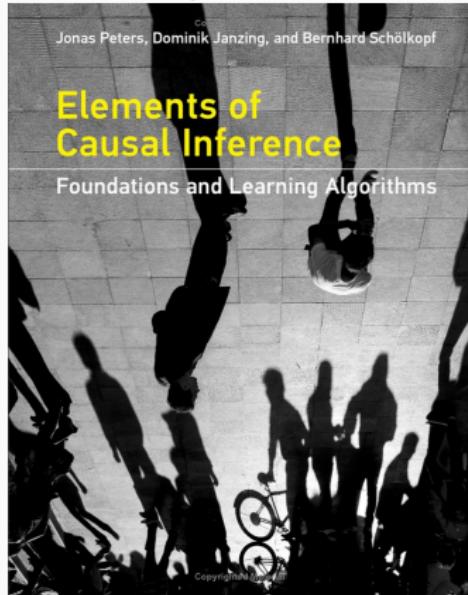
Counterfactual Reasoning

Coffee Break

Optimization of Causal Effects

Lunch Break

This section is heavily lifted from Peters, Janzic and Schoelkopf book and presentation



Notation

- A, B event
- X, Y, Z random variable
- x value of a random variable
- \Pr probability measure
- P_X probability distribution of X
- p density
- p_X or $p(X)$ density of P_X
- $p(x)$ density of P_X evaluated at the point x
- always assume the existence of a joint density, w.r.t. a product measure



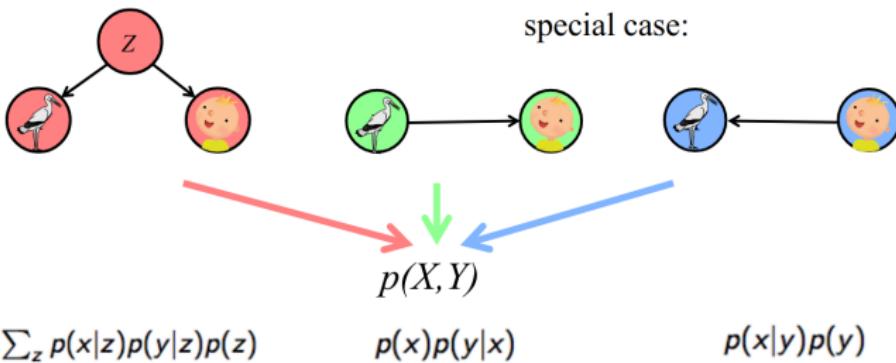
Common Cause Principle (Reichenbach)

- (i) if X and Y are statistically dependent, then there exists Z causally influencing both;
- (ii) Z screens X and Y from each other (given Z , X and Y become independent)



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special case:



$$\sum_z p(x|z)p(y|z)p(z)$$

$$p(x)p(y|x)$$

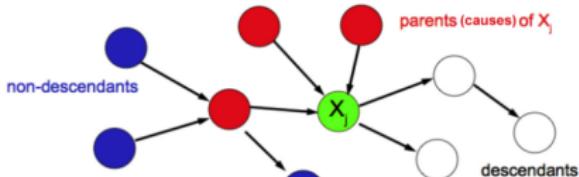
$$p(x|y)p(y)$$

Definition of a Structural Causal Model

(Pearl et al.)



- directed acyclic graph G with vertices X_1, \dots, X_n
(following arrows does not lead to loops)
- Semantics: vertices = observables, arrows = direct causation
- $X_i := f_i(\text{PA}_i, U_i)$, with independent RVs U_1, \dots, U_n that possess a joint density
- U_i stands for “unexplained” (alternatively “noise” or “exogenous variable”)
- this is also called a *(nonlinear) structural equation model*



Pearl's do-notation

- Motivation: goal of causality is to infer the effect of interventions
 - distribution of Y given that X is set to x :

$$p(Y|do X = x) \text{ or } p(Y|do x)$$

- don't confuse it with $P(Y|x)$
- can be computed from p and G



Difference between seeing and doing

$$p(y|x)$$

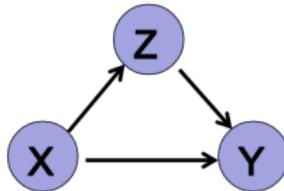
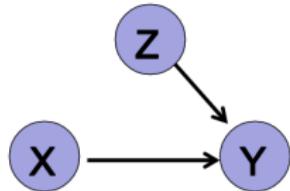
probability that someone gets 100 years old given that we know that he/she drinks 10 cups of coffee per day

$$p(y|do\,x)$$

probability that some randomly chosen person gets 100 years old after he/she has been forced to drink 10 cups of coffee per day

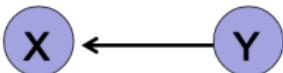


Examples for $p(.|do x) = p(.|x)$

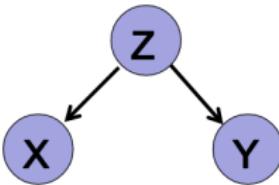


Examples for $p(.|do x) \neq p(.|x)$

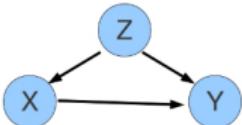
- $p(Y|do x) = P(Y) \neq P(Y|x)$



- $p(Y|do x) = P(Y) \neq P(Y|x)$



Example: controlling for confounding



$X \not\perp\!\!\!\perp Y$ partly due to the confounder Z and partly due to $X \rightarrow Y$

- causal factorization

$$p(X, Y, Z) = p(Z)p(X|Z)p(Y|X, Z)$$

- replace $P(X|Z)$ with δ_{Xx}

$$p(Y, Z|do x) = p(Z) \delta_{Xx} p(Y|X, Z)$$

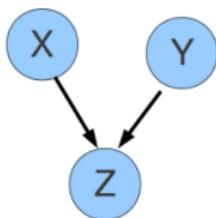
- marginalize

$$p(Y|do x) = \sum_z p(z)p(Y|x, z) \neq \sum_z p(z|x)p(Y|x, z) = p(Y|x).$$

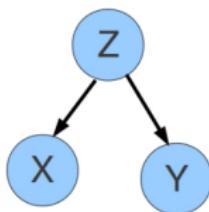


Asymmetry under inverting arrows

(Reichenbach 1956)



$$\begin{aligned} X \perp\!\!\!\perp Y \\ X \not\perp\!\!\!\perp Y | Z \end{aligned}$$



$$\begin{aligned} X \not\perp\!\!\!\perp Y \\ X \perp\!\!\!\perp Y | Z \end{aligned}$$



Algorithmic construction of causal hypotheses

IC algorithm by Verma & Pearl (1990) to reconstruct DAG from p

idea:

1. Construct skeleton
2. Find v-structures
3. direct further edges that follow from
 - graph is acyclic
 - all v-structures have been found in 2)



Causal Inference Method

Prefer the causal direction that can better be fit with an additive noise model.

Implementation:

- Compute a function f as non-linear regression of X on Y
- Compute the residual

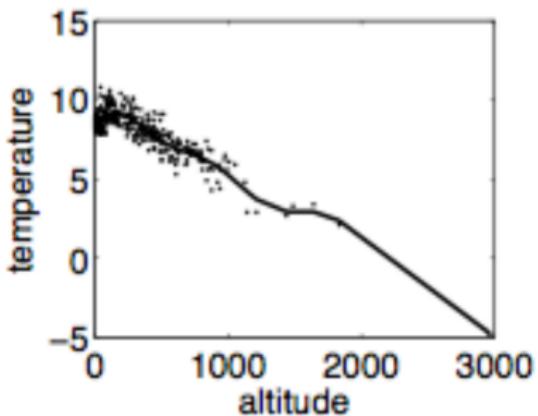
$$N_Y := Y - f(X)$$

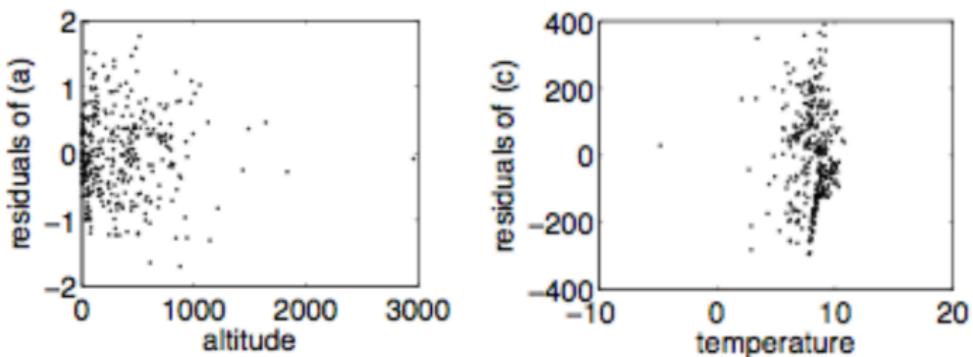
- check whether N_Y and X are statistically independent (uncorrelated is not enough)



Experiments

Relation between altitude (cause) and average temperature (effect)
of places in Germany





Our independence tests detect strong dependence.
Hence the method prefers the correct direction

$\text{altitude} \rightarrow \text{temperature}$



Success Story

Causal inference in brain research

Grosse-Wentrup, Janzing, Siegel, Schölkopf, NeuroImage 2016

Let X, Y be some brain state features and S some randomized experimental condition (i.e., a parentless node!) Assume

$$\begin{aligned} S &\not\perp X \\ S &\not\perp Y \\ S &\perp\!\!\!\perp Y \mid X \end{aligned}$$

then Markov condition and faithfulness imply

$$S \rightarrow X \rightarrow Y$$

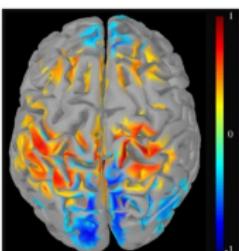
applied to:

X : γ -power in the parietal cortex

Y : γ -power in the medial prefrontal cortex

S instruction to up- or down-regulate X

(conditional independence verified via regression)



Question:
 X and Y are
correlated... but
why ?

Success Story

Causal inference in brain research

Question:
X and Y are
correlated... but
why ?

Grosse-Wentrup, Janzing, Siegel, Schölkopf, NeuroImage 2016

Let X, Y be some brain state features and S some randomized experimental condition (i.e., a parentless node!) Assume

$$\begin{aligned} S &\not\perp\!\!\!\perp X \\ S &\not\perp\!\!\!\perp Y \\ S &\perp\!\!\!\perp Y | X \end{aligned}$$

then S is a common cause and faithfulness imply

$$S \rightarrow X \rightarrow Y$$

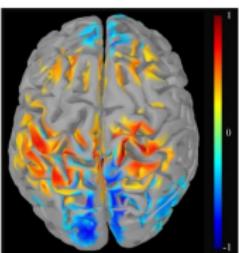
applied

X : γ -power in the parietal cortex

Y : γ -power in the medial prefrontal cortex

S instruction to up- or down-regulate X

(conditional independence verified via regression)



Success Story

Causal inference in brain research

(conditional)
Independence
tests

Brain state features and S some randomized experimental
conditions (i.e., a T -node!)

$$\begin{aligned} S &\perp\!\!\!\perp X \\ S &\perp\!\!\!\perp Y \\ S &\perp\!\!\!\perp Y | X \end{aligned}$$

then Markov condition and faithfulness imply

$$S \rightarrow X \rightarrow Y$$

applied

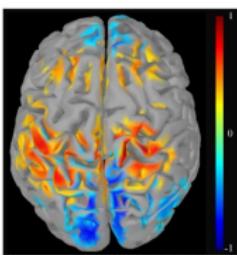
X : γ -power in the parietal cortex

Y : γ -power in the medial prefrontal cortex

S instruction to up- or down-regulate X

(conditional independence verified via regression)

Question:
 X and Y are
correlated... but
why ?



Success Story

Causal inference in brain research

(conditional)
Independence
tests

Brain state features and S some random
confounders (e.g., a participant's mood!)

Gilkopf, NeuroImage 2016

Robust conclusion
about the
underlying causal
mechanism

Question:
 X and Y are
correlated... but
why?

$$\begin{aligned} S &\not\perp\!\!\!\perp X \\ S &\not\perp\!\!\!\perp Y \\ S &\perp\!\!\!\perp Y | X \end{aligned}$$

then M is causal and faithfulness implies

$$S \rightarrow X \rightarrow Y$$

intervention

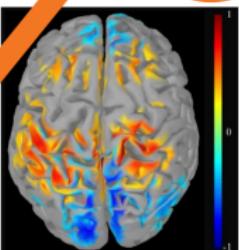
applied

X : γ -power in the parietal cortex

Y : γ -power in the medial prefrontal cortex

S instruction to up- or down-regulate X

(conditional independence verified via regression)



Success Story

Causal inference in brain research

(conditional)
Independence
tests

Brain state features and S some random
confounder (e.g., a participant's mood!) Assume

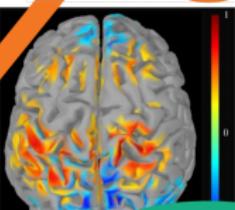
$$\begin{aligned} S &\perp\!\!\!\perp X \\ S &\perp\!\!\!\perp Y \\ S &\perp\!\!\!\perp Y | X \end{aligned}$$

then $M = S \rightarrow X \rightarrow Y$ and faithfulness implies

$$S \rightarrow X \rightarrow Y$$

intervention applied
 X : γ -power in the parietal cortex
 Y : γ -power in the medial prefrontal cortex
 S instruction to up- or down-regulate X
(conditional independence verified)

Robust conclusion
about the
underlying causal
mechanism



Question:
 X and Y are
correlated... but
why?

Prediction of the effect of future interventions:

- If we would change $P_X \Rightarrow P_Y$ would change
- If we would change $P_Y \Rightarrow P_X$ won't change

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Formalism & Fundamental Results

Causal Discovery

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Interventions (e.g. A/B tests) are powerful causal tools, yet can be costly

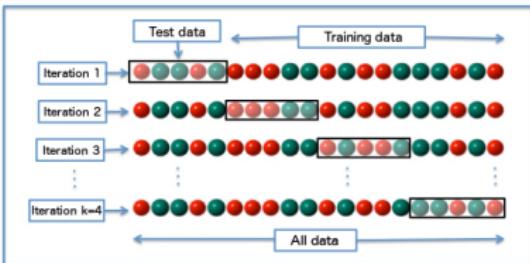
- candidate change must be production-ready (tests, interaction with other tests, certifications ...)
- a poor candidate can make you loose money real fast (or die ...)



Need to find a good candidate *before* A/B test (and roll-out !)

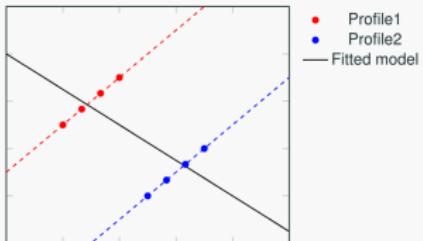
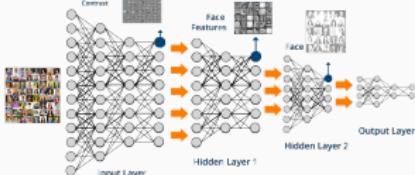
Validate model on historical data

- technical metrics: RMSE, LLH, F1, ...
- variance estimation: cross-validation



But technical metrics may differ from application/business metrics. E.g. better click prediction doesn't necessarily translates into additional revenue...

After all we got nowadays lots of powerful learning models ...



Why can't we predict performance from historical data ?

Policy $\pi : X \rightarrow A$

$$\mathbb{E}_{\pi'}[R(\pi'(X))] \approx \mathbb{E}_\pi[\hat{R}(\pi'(X))] ?$$

- dangerous when π influences the next X
- even fanciest model not immune to confounders

In digital advertising, auctions are run to win ad placements.
Idea is to replay past, won auctions.

Utility metric:

$$U(p') = \sum_{i \in \mathcal{D}} (v_i - c_i) \mathbb{I}[p'_i v_i > c_i]$$

- p / p' : reference/candidate value prediction model
- r : value of placement if auction won
- c : best opponent bid = cost of winning auction

Noise injection:

$$c \sim \Gamma(a, b); \mathbb{E}[\Gamma(a, b)] = c$$

Still, no access to lost auctions ...

- Goal: evaluate multiple possible policies and what would happen if we were to apply them.
- Idea: randomize the production policy to estimate *all* possible variants

When you can intervene (i.e. randomize), counterfactual reasoning is the most powerful tool to optimize a system.

- For each display draw a random variable for color selection
- Choose one of two colors:
 - Red with probability 0.80
 - Green with probability 0.20

	Proba(red)	Proba(green)
Production	80 %	20 %
Test	20 %	80 %
	Clicks(red)	Clicks(green)
Production	1000	300
Test	?	?

	Proba(red)	Proba(green)
Production	80 %	20 %
Test	20 %	80 %

	Clicks(red)	Clicks(green)
Production	1000	300
Test	250	1200

$$\text{Clicks}_{\text{Test}} = \frac{1}{4} \times \text{Clicks}_{\text{Prod}}(\text{Red}) + 4 \times \text{Clicks}_{\text{Prod}}(\text{Green})$$

with $\frac{1}{4} = \mathbb{P}_{\text{Test}}(\text{Red})/\mathbb{P}_{\text{Prod}}(\text{Red})$

$4 = \mathbb{P}_{\text{Test}}(\text{Green})/\mathbb{P}_{\text{Prod}}(\text{Green})$

$$\begin{aligned} Clicks_{Test} &= \frac{\mathbb{P}_{Test}(Red)}{\mathbb{P}_{Prod}(Red)} \times Clicks_{Prod}(Red) \\ &\quad + \frac{\mathbb{P}_{Test}(Green)}{\mathbb{P}_{Prod}(Green)} \times Clicks_{Prod}(Green) \end{aligned}$$

$$\mathbb{E}_{Test}[R] \simeq \sum_{i \in samples} w_i \times R_i$$

Where:

- $w_i := \frac{\mathbb{P}_{Test}(A=a_i)}{\mathbb{P}_{Prod}(A=a_i)}$
- a_i the action sampled (under Prod distribution) on line i
- R_i reward observed on line i

Coffee Break



Questions/Discussion welcome during the break :-)

Optimization of Causal Effects

Causality as an AI frontier

Tools & Methods for Causality

Coffee Break

Optimization of Causal Effects

Learning under changing distribution: Counterfactual Learning

Application: Recommender Systems

Lunch Break

Predicting

- annotating categories
- predicting a click/sale probability
- recognize items in images

Acting

- placing a bid
- recommending products
- changing our margin

What could go wrong ?

engine begins to buy some new traffic / FBL reaction /
Advertiser adjust CPC / Publisher adapts reserve prices...

Interpolation vs extrapolation

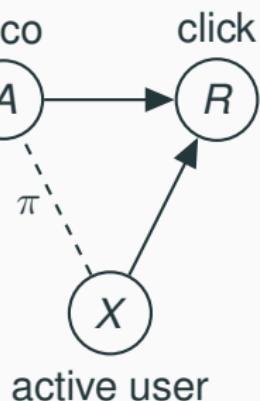
Recommender System $A \sim \pi(X)$: best-of or personalized

Data Collection: dormant (best-of) – active (personalized)

Objective: learn to personalize on dormant



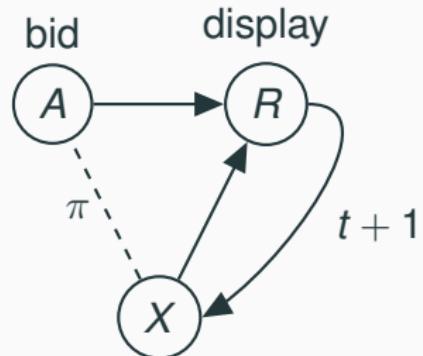
- Personalized products only known in the context of active users
- Need to explore reco on dormants



Bidder: $\text{bid} \sim \pi(\text{nb_displays_last_day})$

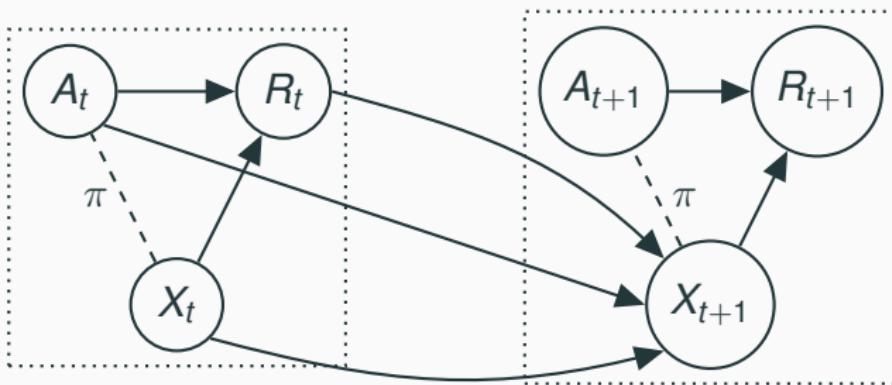
Objective: Change bidder to bid more on low fatigue and less on high fatigue.

- Increasing bids on low fatigue will increase the fatigue
- Several solutions:
 - consider an action to be a sequence of bids
 - move to reinforcement learning setting



`nb_displays_last_day`

Reinforcement Learning

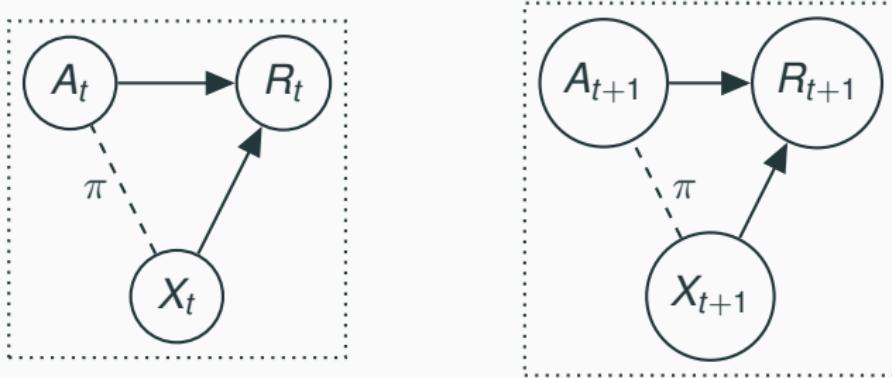


Reward is a bit complex:

$$\mathbb{E}_\pi[R] = \sum_x d^\pi(x) \sum_a \pi(a|x) \mathbb{E}[R|x, a]$$

where $d_\pi(x)$ is the stationary distribution of states under π .

Logged Bandit Feedback



Reward is a bit complex:

$$\mathbb{E}_\pi[R] = \sum_x \mathbb{P}(x) \sum_a \pi(a|x) \mathbb{E}[R|x, a]$$

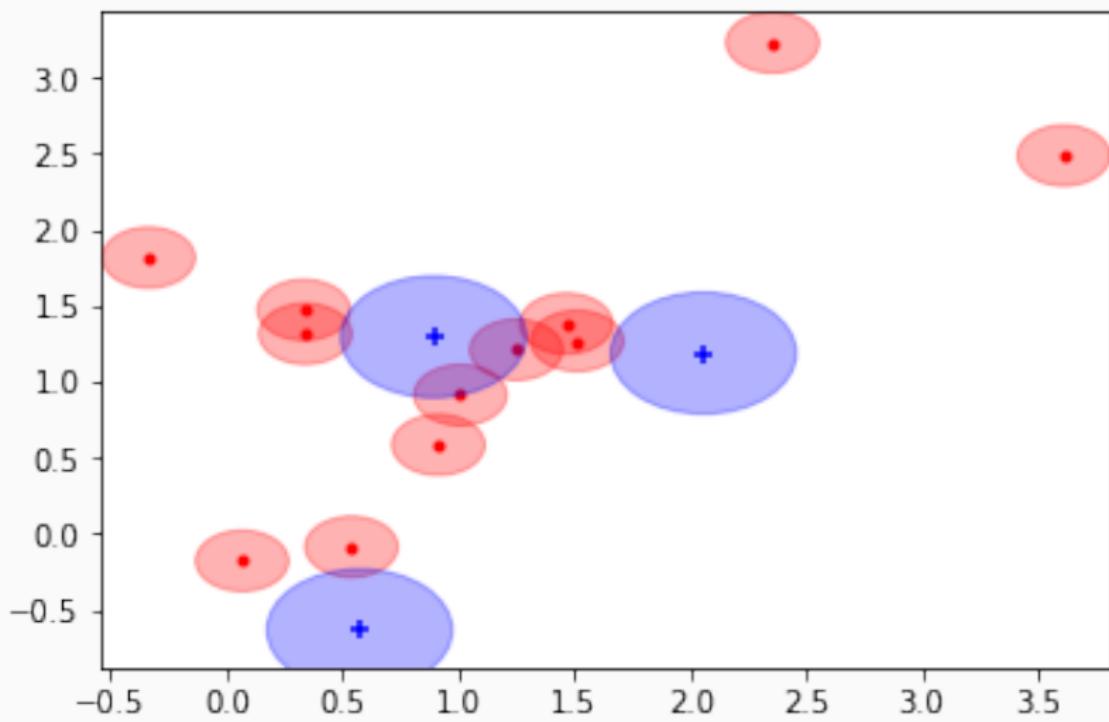
where $\mathbb{P}(x)$ does not depends on π .

Counterfactual Risk Minimization a.k.a. Policy Gradient++

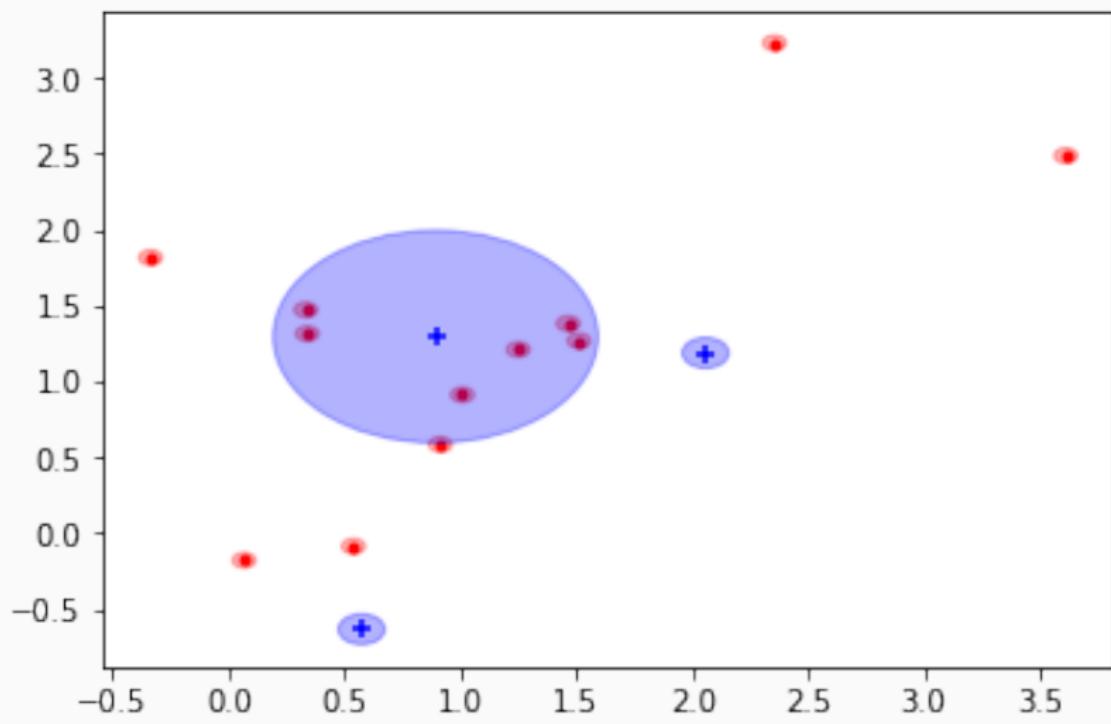
$$R(\pi_\theta, \pi_0) = \frac{1}{N} \sum_i r_i w_i(\theta) + \lambda \sqrt{\frac{V_N(r_i w_i(\theta))}{N}}$$

where $w_i(\theta) = \frac{\pi_\theta(a_i|x_i)}{\pi_0(a_i|x_i)}$.

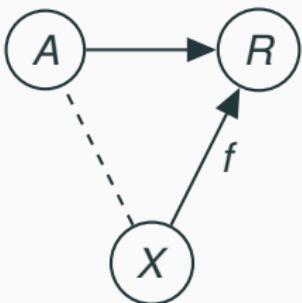
What we would like



What happens



List(k products) click



Display and List(candidate products)

Action: choosing a list of products, from a list of candidates.

Sampling the list of products:

- Computing one prediction per candidate product
 - $\text{Proba(a Sale matched to this product } | \text{ it is displayed})$
- Product Score := $\text{Prediction}^{\alpha}$ (Usually $\alpha = 4$)
- Repeat k times
 - Pick one product with probabilities proportional to scores
 - Add it to the list of displayed products
 - Remove it from the candidates

Probability of one banner – Example

$\mathbb{P}(A = (p_a, p_b)) ?$

Product	p_a	p_b	p_c	p_d	Total
Prediction	2	3	1	2	

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Computing banner probability with chain rule:

$$\mathbb{P}(A = (p_a, p_b)) = \mathbb{P}(\text{First} = p_a) \times \mathbb{P}(\text{Second} = p_b | \text{First} = p_a)$$

Here:

$$\mathbb{P}(A = (p_a, p_b)) = 4/18 \times 9/14$$

$$\mathbb{E}_{Test}(R) \approx \sum_{i \in displays} r_i \times \frac{\mathbb{P}_{Test}(A = a_i)}{\mathbb{P}_{Prod}(A = a_i)}$$

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We log:

- a description of all the candidates
- their predictions from prod model
- the chosen action (which products were displayed)
- the reward r_i

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We can recompute:

- products scores for prod model and $\mathbb{P}_{Prod}(A = a_i)$
- the predictions of a test model on each candidate
- products scores for test model and $\mathbb{P}_{Test}(A = a_i)$

Three difficulties:

- Non recommendable items
- Large importance weights
- Independance beween displays ?

What reco really does:

- Score each candidate
- Repeat
 - Sample one item
 - Check if this item is recommendable
- Until it found k recommendable items

We do not know if non selected items are recommendable.

Banner of size 1. $\mathbb{P}(Action = (p_b))?$

Product	p_a , Not recommendable	p_b	Total
Score	1.0	1.0	2.0
$\mathbb{P}(\text{Selected First})$	1/2	1/2	1

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$$\mathbb{P}(Action = display(p_b)) = 1$$

But when we select p_b first, we estimate:

$$\mathbb{P}(Action = display(p_b)) = 0.5$$

The problem with large weights

Product	P_a	P_b	P_c	P_d	Total
Score Prod	1	1	1	100	103
Score Test	1000	100	10	1	1111

$$\mathbb{P}_{Prod}(A = (P_a, P_b, P_c)) = \frac{1}{103 \times 102 \times 101} \approx 10^{-6}$$

$$\mathbb{P}_{Test}(A = (P_a, P_b, P_c)) = \frac{1000 \times 100 \times 10}{1111 \times 111 \times 11} \approx 0.7$$

$$IW(A = (P_a, P_b, P_c)) = \mathbb{P}_{Test}/\mathbb{P}_{Prod} \approx 10^{+6}$$

With a probability of 10^{-6} , we get a weight of 10^{+6}

=> Huge variance

Capped estimator:

$$\sum_{i \in \text{Displays}} r_i \times \text{Min}(w_i, C)$$

Lower variance, but biased. This estimator tells that policies very different from prod would perform very badly.

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In practice, no value with both:

- acceptable variance
- acceptable bias

Idea:

- $\mathbb{E}(\text{Min}(w_i, C)) < 1$
- Trying to compensate this by a global factor, estimating how the weights were reduced on average.

Normalized estimator: =

$$\sum_{i \in \text{Displays}} r_i \times \text{Min}(w_i, C) \times \frac{N}{\sum_{i \in \text{Displays}} \text{Min}(w_i, C)}$$

- Two kind of displays :
 - Low context ($\mathbb{E}(R)$ low whatever the action)
 - High context ($\mathbb{E}(R)$ high whatever the action)
- On High context Test algo = Prod algo
 - $w = 1$ on high context
- On low context: Test algo choose actions (almost) never chosen by Prod
 - $w = 0$ or » C on low context
 - $\sum \text{Min}(w_i, C) \approx 0$

$$\begin{aligned}
 \text{Normalized} &= \sum_{i \in \text{Displays}} R_i \times \text{Min}(w_i, C) \times \frac{N}{\sum_{i \in \text{Displays}} \text{Min}(w_i, C)} \\
 &\approx \left(\sum_{\text{highcontext}} R_i \times \text{Min}(w_i, C) \right) \times \frac{N}{\sum_{\text{highcontext}} \text{Min}(w_i, C)} \\
 &\approx \mathbb{E}(R | \text{highcontext}) \times N
 \end{aligned}$$

Lunch Break



LUNCH BREAK

Questions/Discussion welcome during lunch break :-)

This course contains material from

- Elements of Causal Inference (Peters, Janzing, Schoelkopf - MIT Press - 2017)
- Causality Tutorial (Calauzènes, Gilotte, Diemert, Aslan - Criteo ML Big Days - 2018)