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A new method for removing mixed noises

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Abstract We first introduce a similarity assumption to describe the similarity phenomenon in natural images, and establish a similarity principle which supplies a simple mathematical justification for the non-local means filter in removing Gaussian noises. Using the similarity principe in an adapted way, we then propose a new algorithm, called mixed noise filter (MNF) to remove simultaneously a mixture of Gaussian and random impulse noises. Our experiments show that our new filter improves significantly the trilateral filter in removing mixed noises, and that it is as efficient as the non-local means filter in removing Gaussian noises, and as good as the trilateral filter in removing random impulse noises.

Keywords mixed noises, Gaussian noises, random impulse noises, image restoration, nonlinear filters, trilateral filter, non-local means filter, denoising, similarity principle

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1 Introduction

A numerical image is presented by an $M \times N$ matrix $u = \{u(i) : i \in I\}$, where $I = \{0, 1, \ldots, M-1\} \times \{0, 1, \ldots, N-1\}$, and $0 \leqslant u(i) \leqslant 255$. Noise can be systematically introduced into images during acquisition and transmission. Two noise models can adequately represent most noise added to images: additive Gaussian noise and random impulse noise [1]. The additive Gaussian noise model is: $v(i) = u(i) + n(i)$, where $u = \{u(i) : i \in I\}$ is the original image, $v = \{v(i) : i \in I\}$ the noisy one, and n is the Gaussian noise: $n(i)$ are independent and identically distributed Gaussian random variables with mean 0 and standard deviation σ. The random impulse noise model is: $v(i) = n(i)$ with probability p, and $v(i) = u(i)$ with probability $1-p$, where p is the impulse probability (the proportion of the occurrence of the impulse noise), $n(i)$ are independent random variables uniformly distributed on $\{0, 1, \ldots, 255\}$. In this article we always denote by u the original image, and v the noisy one. A fundamental problem of image processing is to effectively remove noises from an image while keeping its features intact.

Some important denoising methods have been found in recent years. To remove Gaussian noises, the most efficient methods include the non-local means filter introduced by Buades, Coll and Morel [2], its variants (see e.g. [3]), and some filters using nonlinear total variation or wavelet (see e.g. [4, 5]). To

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remove random impulse noises, the trilateral filter recently proposed by Garnett et al. [1], and its variants [6], have proven very powerful; see also [7–14] for some other methods. The trilateral filter has also been the most efficient method to remove a mixed noise composed of a Gaussian noise and a random impulse noise. Unfortunately, this filter does not work very well when the Gaussian noise is relatively high. The non-local means filter is very efficient for removing a Gaussian noise, but its mathematical justification has not been so satisfactory, and it is not adapted to remove mixed noises.

In this article, we first introduce the similarity assumption to give a mathematical description of the similarity of patches in natural images, using probability theory. We next present the similarity principle to give a mathematical explanation for the non-local means filter; our principle is much simpler than the consistence theorem of Buades et al. [2] established under several conditions such as strict stationarity and β-mixing conditions. We then give a new filter, which we call the mixed noise filter (MNF for short), to remove a mixture of Gaussian and random impulse noises. This filter uses the similarity principle in an adapted way, and combines the basic ideas of the trilateral filter and the non-local means filter. For the new filter, an important point is to construct a new weighted "norm", which we call impulse controlled weighted norm. Our experiments show that our new filter is significantly better than the trilateral filter in removing mixed noises.

The rest of the paper is organized as follows. In section 2, after a short recall of the non-local means filter, we introduce the similarity assumption and establish the similarity principle. In section 3, we give a brief presentation of the trilateral filter, whose main ideas will be used in the definition of our new filter. In section 4, we introduce the impulse controlled weighted norm, define our new filter called the mixed noise filter (MNF), and show some experimental results to compare our filter with the trilateral filter and the non-local means filter. Section 5 is a brief conclusion.

2 The non-local means filter and the similarity principle

2.1 The non-local means filter (NL-means)

As shown in [2], the non-local means filter is very efficient in removing Gaussian noises. It is based on the similarity phenomenon existing very often in natural images. The algorithm can be described as follows. Let $v = \{v(i) : i \in I\}$ be a noisy image with an additive Gaussian noise, and $NL(v)$ be the restored image. $\forall i \in I$, $NL(v)(i)$ is defined as a weighted mean of the noisy image v:

$$NL(v)(i) = \frac{\sum_{j\in I} w(i,j)v(j)}{\sum_{j\in I} w(i,j)}, \tag{1}$$

where the weights $\{w(i,j)\}_{j\in I}$ depend on the similarity of the windows N_i and N_j:

$$w(i,j) = \mathrm{e}^{-\frac{||v(N_i)-v(N_j)||^2_{2,a}}{2h^2}}. \tag{2}$$

Here $h > 0$ is a control parameter, and $v(N_j) = (v(k) : k \in N_j)$ is the vector composed of the grey values $v(k)$ in the window N_j with center j, and $||v(N_i) - v(N_j)||^2_{2,a}$ is a kind of weighted norm:

$$||v(N_i) - v(N_j)||^2_{2,a} = \sum_{k\in N_i} a(i,k)|v(k) - v(Tk)|^2, \tag{3}$$

where $T = T_{ij}$ is the translation mapping i to j (so that $N_j = TN_i$), $a(i,k) > 0$ are some fixed weights, usually chosen to be a decreasing function of the Euclidean norm $\|i - k\|$ [2].

In practice, in (1) we usually use relatively large windows to replace the whole image I. They are called practical windows, while N_i are called small patches. Practical windows are usually chosen to be of size 21×21 or 15×15, and small patches of size 9×9 or 7×7 [2].

2.2 Similarity assumption and similarity principle

We first give a mathematical description of the similarity of patches in natural images, which we call similarity assumption.

For $i \in I$, let $N_i(d)$ be the window with center i and size $d \times d$ (usually $d = 3, 5, \ldots$), and set $N_i^0(d) = N_i(d) \backslash \{i\}$. We sometimes simply write N_i and N_i^0 for $N_i(d)$ and $N_i^0(d)$, respectively.

Asumption 1 (Similarity assumption)**.** For each fixed $i \in I$, there is a subset I_i of I such that $\{v(N_j) : j \in I_i\}$ is a family of random variables having the same distribution as $v(N_i)$.

We then say that the windows N_j $(j \in I_i)$ are similar to the window N_i. In other words, two small patches are similar if they are realizations of the same law.

In natural images, there are often many similar small patches. We describe this similarity phenomenon by saying that the similarity assumption holds with $|I_i|$ relatively large for each i, where $|I_i|$ is the cardinality of I_i.

We next present a limit theorem, we call similarity principle, based on the similarity assumption with $|I_i| \to \infty$.

Theorem 1 (Similarity principle)**.** Under the similarity assumption, for each $i \in I$,

$$\lim_{|I_i| \to \infty} v^0(i) = u(i), \quad \text{almost surely,}$$

where

$$v^0(i) = \frac{\sum_{j \in I_i} w^0(i,j) v(j)}{\sum_{j \in I_i} w^0(i,j)}, \tag{4}$$

and

$$w^0(i,j) = \mathrm{e}^{-\frac{\|v(N_i^0) - v(N_j^0)\|^2}{2h^2}}. \tag{5}$$

The similarity principle gives a simple mathematical explanation for the non-local means filter. It demonstrates that when $|I_i|$ is large enough, the estimated value $v^0(i)$ is very close to the original image $u(i)$.

When the windows N_j and N_i are similar, the event $\{\|v(N_j) - v(N_i)\| \leqslant \theta\}$ has probability close to 1 for $\theta > 0$ large enough. In practice, we take some threshold θ, and we say that the windows N_j and N_i are similar if $\|v(N_j) - v(N_i)\| \leqslant \theta$. If a window N_j is not similar to N_i, then $\|v(N_j) - v(N_i)\|$ is large, so that the corresponding weight $w^0(i,j)$ can be neglected. Also, it is not hard to see that we can replace $w^0(i,j)$ approximately by $w(i,j)$. Therefore we have

$$NL(v)(i) \approx v^0(i) \approx u(i). \tag{6}$$

This shows that $NL(v)(i)$ is a reasonable estimator of $u(i)$.

As the similarity principle is based on the similarity assumption with a sufficient large number of similar windows, we can presume that we could introduce a kind of "degree of similarity", and show that the larger the "degree of similarity", the better the quality of restoration. This will be done in a forthcoming paper.

For the proof of the similarity principle, we need the law of large numbers for l-dependent random variables. For a fixed integer $l \geqslant 0$, a sequence of random variables $X_1, X_2, \ldots$ is called to be l-dependent if each subsequence $X_{i_1}, X_{i_2}, \ldots$ is independent whenever $|i_m - i_n| > l$ for all $m, n \geqslant 1$ (that is, all random variables of the sequence with distances $> l$ are independent of each other).

Lemma 1. If $X_1, X_2, \ldots$ is a sequence of l-dependent and identically distributed real random variables with $E|X_1| < \infty$, then

$$\lim_{n \to \infty} \frac{X_1 + \cdots + X_n}{n} = \mathbb{E}X_1 \quad \text{almost surely.}$$

This lemma is a direct consequence of the usual law of large numbers for independent random variables, since $\forall j \in \{1, \ldots, l+1\}$, $\{X_{i(l+1)+j} : i \geqslant 0\}$ is a sequence of independent and identically distributed real random variables, and for each $n \in \mathbb{N}$, we have

$$X_1 + \cdots + X_n = \sum_{j=1}^{l+1} \sum_{i=0}^{m-1} X_{i(l+1)+j} + \sum_{1 \leqslant j \leqslant k} X_{m(l+1)+j},$$

where $m, k \in \mathbb{N}$ are integers with $n = m(l+1) + k$, $0 \leqslant k \leqslant l$.

Proof of the similarity principle. For $x \in \mathbb{R}^{|N_i^0|}$ (here $|N_i^0|$ is the cardinality of the set N_i^0) and $j \in I_i$, let

$$\omega^0(x, j) = \mathrm{e}^{-\frac{||x - v(N_j^0)||_{2,a}^2}{2h^2}}.$$

Since $v(N_{j_1}^0)$ and $v(N_{j_2}^0)$ are independent of each other when $N_{j_1}^0$ and $N_{j_2}^0$ are disjoint, the sequence of real random variables $\{\omega^0(x, j) : j \in I_i\}$ is l-dependent for $l \in \mathbb{N}$ large enough. Therefore, by the preceding lemma,

$$\lim_{|I_i| \to \infty} \frac{1}{|I_i|} \cdot \sum_{j \in I_i} \omega^0(x, j) = \mathbb{E}\omega^0(x, j_0) \quad \text{almost surely},$$

where $j_0 \in I_i$ is fixed. For the same reason,

$$\lim_{|I_i| \to \infty} \frac{1}{|I_i|} \cdot \sum_{j \in I_i} w^0(x, j) v(j) = \mathbb{E}\omega^0(x, j_0)\mathbb{E}v(j_0) = \mathbb{E}\omega^0(x, j_0)u(i) \quad \text{almost surely;}$$

here we have used the independence of $\omega^0(x, j_0)$ and $v(j_0)$, and the fact that

$$\mathbb{E}v(j_0) = \mathbb{E}v(i) = u(i).$$

Therefore, for each $x \in \mathbb{R}^{|N_i^0|}$

$$\lim_{|I_i| \to \infty} \sum_{j \in I_i} \omega^0(x, j) v(j) / \sum_{j \in I_i} \omega^0(x, j) = u(i) \quad \text{almost surely.}$$

Since $v(N_j^0)$ and $v(N_i^0)$ are independent of each other except for a finite number of $j \in I_i$, we can replace x by $v(N_i^0)$ in the above equality. This completes the proof.

3 The trilateral filter

The trilateral filter is an improvement of the bilateral filter [15], adapted to remove a random impulse noise and its mixture with a Gaussian noise. It is based on the ROAD (rank ordered absolute difference) statistic defined by

$$\mathrm{ROAD}(i) = r_1(i) + \cdots + r_m(i),$$

where $r_k(i)$ is the kth smallest term in $\{|u(i) - u(j)| : j \in N_i^0(d)\}$, m is a constant usually taken in $\{2, 3, \ldots, 7\}$. We take $m = 4$ as in [1]. If i is an impulse noisy point, then the value of $\mathrm{ROAD}(i)$ is large; otherwise it is small. If either i or j is an impulse noisy point, then the value of $J(i, j)$ is close to 1; otherwise it is close to 0 (cf. [1]).

By definition, the restored image by the trilateral filter is

$$TriF(v)(i) = \frac{\sum_{j \in N_i(L)} w(i, j) v(j)}{\sum_{j \in N_i(L)} w(i, j)}, \tag{7}$$

where

$$w(i, j) = w_S(i, j) w_R(i, j)^{J_I(i,j)} w_I(j)^{1 - J_I(i,j)} \tag{8}$$

is composed of the spatial factor $w_S(i, j)$, the radiometric $w_R(i, j)$, the impulsive weight $w_I(j)$, and the joint impulsive weight $J_I(i, j)$, defined by $w_S(i, j) = \mathrm{e}^{\frac{-\|i-j\|^2}{2\sigma_S^2}}$, $w_R(i, j) = \mathrm{e}^{\frac{-|v(i) - v(j)|^2}{2\sigma_R^2}}$,

$$w_I(j) = \mathrm{e}^{-\frac{ROAD(j)^2}{2\sigma_I^2}}, \tag{9}$$

$$J_I(i, j) = \mathrm{e}^{-\frac{((ROAD(i) + ROAD(j))/2)^2}{2\sigma_J^2}}, \tag{10}$$

σ_S, σ_R, σ_I and σ_J being controlling parameters. We mention that instead of defining the joint impulsive weight $J_I(i, j)$, Garnett et al. [1] initially introduced $J(i, j) = 1 - J_I(i, j)$ as the joint impulsivity. We find more convenient to use $J_I(i,j)$ instead of $J(i,j)$, especially when we will define our new mixed noise filter (cf. (11)).

It is known [1] that $J_I(i,j)$ approaches 0 if either i or j is corrupted with impulse noise, and approaches 1 otherwise.

4 A new method for remove mixed noises: mixed noise filter

The trilateral filter [1] has been shown to be very efficient in removing a mixed noise composed of a Gaussian noise and a random impulse noise. Unfortunately, it does not work very well when the Gaussian noise is relatively high. We shall improve it by a new filter, using our similarity principle and the basic idea of trilateral filter.

4.1 The impulse controlled weighted norm

A key idea in our new filter is the introduction of a new weighted norm, that we call impulse controlled weighted norm. For windows N_i and $N_j = TN_i$, the impulse controlled weighted norm of $v(N_i) - v(N_j)$, denoted $||v(N_i) - v(N_j)||_M$, is defined by

$$||v(N_i) - v(N_j)||_M^2 = \frac{\sum_{k\in N_i} J_I(k,T(k))\,|v(k) - v(T(k))|^2}{\sum_{k\in N_i} J_I(k,T(k))}, \tag{11}$$

(recall that $T = T_{ij}$ is the translation in the plan mapping i to j, and $J_I(k,T(k))$ is the joint impulsive weight defined in (10)). If k or $T(k)$ is an impulse noisy point, then $J_I(k,T(k))$ is close to 0, so that impulse noisy points contribute little in the weighted norm.

The word “norm” is justified by the fact that, for a fixed window N_i and given weights $a_k = a_{i,k} > 0$, $k \in N_i$, the following equality defines a norm $||.||_M$ on $\mathbb{R}^{|N_i|}$:

$$||x||_M^2 = \sum_{k\in N_i} a_k x_k^2;$$

it is the norm induced by the inner product

$$\langle x, y\rangle = \sum_{k\in N_i} a_k x_k y_k,$$

$$x = (x_k : k \in N_i), \quad y = (y_k : k \in N_i).$$

4.2 Mixed noise filter (MNF)

We now use the similarity principle and the basic idea of the trilateral filter to give a new filter for removing mixed noises. Let D be a positive integer. We usually take $D > d$. We use $N_i^0(D)$ as practical windows, and $N_j(d)$ as small patches.

Our new filter, called mixed noise filter (MNF for short), is by definition:

$$MNF(v)(i) = \frac{\sum_{j\in N_i^0(D)} w(i,j)v(j)}{\sum_{j\in N_i^0(D)} w(i,j)}, \tag{12}$$

where $MNF(v)$ represents the restored image of the observed image v,

$$w(i,j) = w_I(j)w_M(i,j), \tag{13}$$

with

$$w_M(i,j) = \mathrm{e}^{-\frac{||v(N_i)-v(N_j)||_M^2}{2\sigma_M^2}} \tag{14}$$

(recall that $w_I(j)$ is defined in (9)).

Figure 1 Original Lena image (512×512).

We notice that, for each impulse noisy point j in $N_i^0(D)$, $w(i,j)$ is close to 0. Hence our new filter can be regarded as an application of the similarity principle to the remained image (which contains only Gaussian noises) obtained after filtering the impulse noisy points by the impulse controlled weighted norm.

Here for simplicity we have not considered the spatial weights $w_S(i,j)$. In a forthcoming paper, we shall show how the introduction of spatial weights can sometimes improve the quality of restoration.

4.3 Experiments and comparisons

We now present some experimental results to compare our new filter, the mixed noise filter (MNF), with the trilateral filter and the non-local means filter (NL-means).

As usual we use the value of PSNR (peak signal-to-noise ratio) to measure the quality of a restored image: if u is the original image, $\tilde{u}$ the restored one, then

$$PSNR(\tilde{u}) = 10\log_{10}(255^2/MSE), \quad MSE = \frac{1}{|I|}\sum_{i\in I}[\tilde{u}(i) - u(i)]^2$$

(see e.g. [1]). The larger the value of PSNR, the better the quality of restoration.

All images used in our experiments are of size 512×512. One of the original images is shown in Figure 1.

In Figures 2 and 3, we compare our mixed noise filter with the trilateral filter in removing a mixed noise. To the image Lena, we add a Gaussian noise and a random impulse noise with different values of σ and p, and we compare the restored images and their PSNR values. For convenience, the PSNR values appeared in Figures 2 and 3 are also presented in Table 1. In Table 2, we do further comparisons in removing mixed noises, using the images Boats and Peppers.

The experimental results show that our new filter MNF clearly outperforms the trilateral filter in removing a mixed noise, both visually and quantitatively, especially when the Gaussian noise is relatively high.

In Table 3, we compare our new filter MNF with the non-local means filter in removing Gaussian noises with different values of standard deviation σ; in Table 4, we compare our MNF with the trilateral filter in removing impulse noises, with different values of impulse probability p.

In both tables, we use the images Lena, Peppers and Boats. See also Figure 4 for a comparison between trilateral filter and MNF in removing simple Gaussian noises.

The results show that the new filter MNF is equally efficient as the trilateral filter in removing impulse noises, and that it works like the non-local means filter in removing Gaussian noises.

4.4 Selection of control parameters

To remove mixed noises, the trilateral filter [1] needs four parameters: σ_S (for the spatial weight) σ_I (for the impulse weight), σ_J (for the joint impulsive weight), and σ_R (for the radiometric). In our new filter MNF, we have neglected the spatial factor, so that we need naturally three control parameters: σ_I, σ_J

Figure 2 Comparison between trilateral filter and MNF (1). (a) The noisy image a ($\sigma = 10, p = 0.2$); (b) Image a restored by Trilateral, PSNR=31.79; (c) Image a restored by MNF, PSNR=32.84; (d) the noisy image b ($\sigma = 20, p = 0.2$); (e) Image b restored by Trilateral, PSNR=27.73; (f) Image b restored by MNF, PSNR=30.23.

Figure 3 Comparison between trilateral filter and MNF (2). (a) The noisy image c ($\sigma = 30, p = 0.2$); (b) Image c restored by Trilateral, PSNR=24.92; (c) Image c restored by MNF, PSNR=27.33**; (d) the noisy image d ($\sigma = 10, p = 0.4$); (e) Image d restored by Trilateral, PSNR=28.75; (f) Image d restored by MNF, PSNR=29.96.

and σ_M. We believe that these parameters should depend on the values of σ and p. Our experimental results show that the following two empirical formulas are good for the choice of parameters in the MNF, in removing mixed noises:

$$\sigma_I = \sigma_J = 100 + \sigma - 160p, \tag{15}$$

Table 1 PSNR values in removing mixed noises with Trilateral Filter and MNF (1)

Noisy images	Lena, $\sigma = 10, p = 0.2$	Lena, $\sigma = 20, p = 0.2$	Lena, $\sigma = 30, p = 0.2$	Lena, $\sigma = 10, p = 0.4$
PSNR by Trilateral	31.79(31.64)	27.73	24.92	28.75
PSNR by MNF	32.84	30.23	27.33**	29.96

Table 2 PSNR values in removing mixed noises with Trilateral Filter and MNF (2)

Noisy images	Boats, $\sigma = 10, p = 0.2$	Boats, $\sigma = 20, p = 0.2$	Peppers, $\sigma = 10, p = 0.2$	Peppers, $\sigma = 20, p = 0.2$
PSNR by Trilateral	30.21	27.11	29.00	26.75
PSNR by MNF	30.88	28.56	29.68	28.4

Notes:

(1) The non-local means filter does not work well for mixed noises or impulse noises. We therefore have not shown the corresponding results for this filter in removing such noises.

(2) The values of PSNR about the Trilateral Filter: the values in parentheses are taken from [1]; the values marked with * are obtained by searching for the best control parameters; other values are obtained by using the parameters suggested in [1]: $\sigma_S = 5$, $\sigma_I = 40$, $\sigma_J = 50$.

(3) The sizes of windows in MNF: we usually take 7×7 for practical windows, and 3×3 for small patches. For the results marked with **, we use 9×9 or 15×15 for practical windows, and 5×5 or 7×7 for small patches.

Figure 4 Comparison of Trilateral Filter and MNF (3). (a) The noisy image e ($\sigma = 10$); (b) Image e restored by Trilateral, PSNR=33.23*(33.23); (c) Image e restored by MNF, PSNR= 34.86.

Table 3 PSNR values in removing Gaussian noises with trilateral filter, MNF & NL-means

Noisy images	Lena, $\sigma = 10$	Peppers, $\sigma = 30$	Boats, $\sigma = 20$
PSNR by Trilateral	33.23*(33.23)	25.18	24.92
PSNR by MNF	34.86	28.55**	30.30
PSNR by NL-Means	35.01	28.67	30.94

Table 4 PSNR values in removing impulse noises with trilateral filter and MNF

Noisy images	Lena, $p = 0.2; 0.3; 0.4$	Peppers, $p = 0.2; 0.3; 0.4$	Boats, $p = 0.2; 0.3; 0.4$
PSNR by Trilateral	(35.03); (33.16); (31.36)	30.08; 29.43; 27.75	31.99; 30.29; 28.16
PSNR by MNF	34.62; 33.12; 31.43**	30.07; 29.42; 28.12**	32.58; 30.51; 28.36**

$$2\sigma_M^2 = 18\sigma + 400(p + p^2) + 0.4\sigma^2 p. \tag{16}$$

We point out that the values of parameters, or the coefficients in the above formula, can also vary in some intervals. The dependance of the filter on the values of σ_I and σ_J in a neighborhood of the suggested value given above is not very noticeable. As both parameters are used to control the influence of impulse points, it seems feasible to take them equal: $\sigma_I=\sigma_J$. This is shown by our experimental results: using the above simple formula already gives good result. But sometimes we can get better results if we choose different values for σ_I and σ_J. For example, to the noisy image peppers with $\sigma = 30$, if we use $\sigma_I = 300$, $\sigma_J = 25$ in our MNF, then the PSNR value of restored image may be more than 29.9, which is larger than that obtained by the non-local means filter (see Table 3). The dependence of the filter on the value of σ_M is more noticeable. Sometimes, taking a different value in the neighborhood of the suggested value given above may improve the restoration result, especially in the case of a simple gaussian noise or a simple impulse noise.

5 Conclusions

In this paper, we first present a similarity principle, which gives a simple mathematical justification for the non-local means filter adapted to remove Gaussian noises. Based on this principle and using an impulse controlled weighted norm, we then propose a new filter, called mixed noise filter (MNF), to remove the mixture of a Gaussian noise and a random impulse noise. Our experimental results show that our new filter clearly outperforms the trilateral filter to remove mixed noise, especially when the Gaussian noise is relatively large. It works equally well as the trilateral filter or the non-local means filter to remove a random impulse noise or a Gaussian noise, respectively. Another advantage of our new filter is that, compared with the trilateral filter, we have less factors in the weights (therefore also less control parameters), and a simple formula for the control parameters.

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