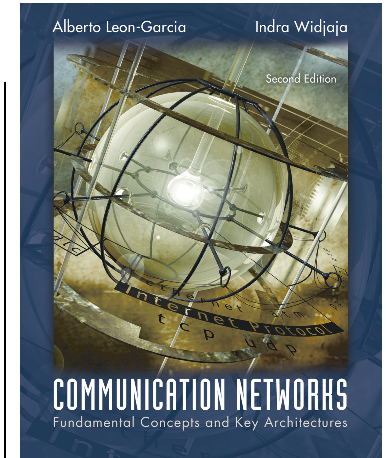
Digital Transmission

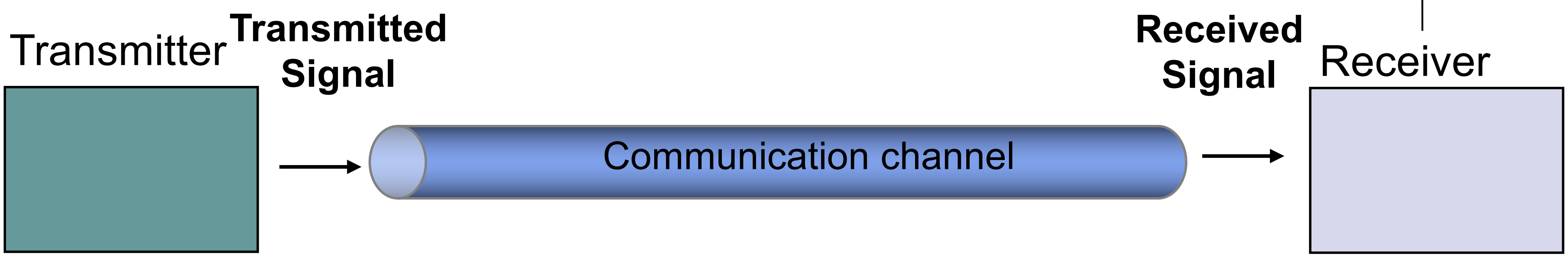


Digital Transmission

Media

Error Detection

Digital Transmission



Transmitter

- Accepts bits from data link layer
- Converts bits into digital modulated signal

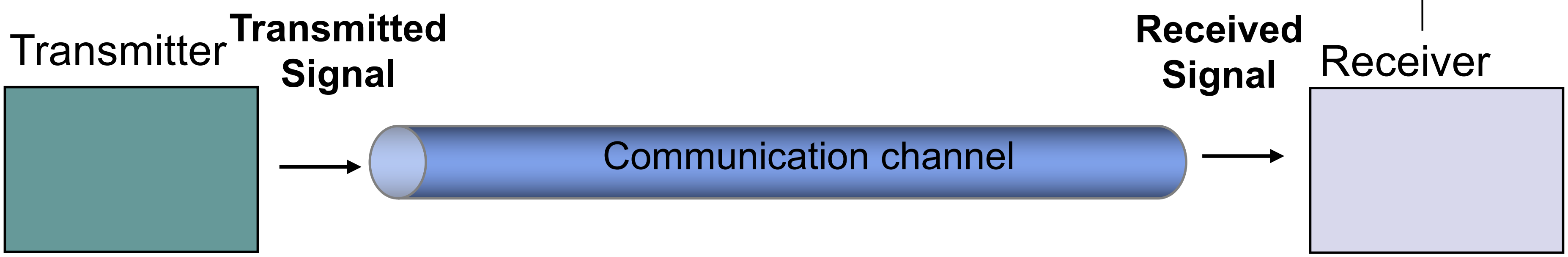
Receiver

- Recovers bits from received signal
- Transfers bits to its data link layer

Key questions

- How many ***bits per second*** can be transmitted reliably across the channel?
- How much ***power and bandwidth*** is consumed in the process?
- How do we control ***error rate*** in delivered bits?

Transmission Impairments



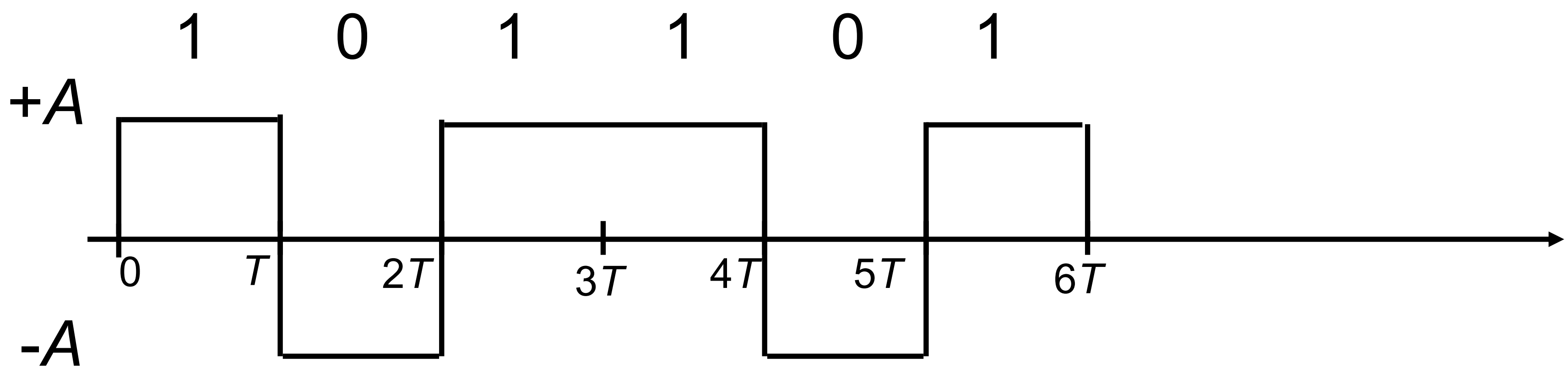
Communication Channels

- Pair of copper wires
- Coaxial cable
- Radio
- Light in optical fiber
- Light in air
- Infrared

Transmission Impairments

- Signal attenuation
- Signal distortion
- Spurious noise
- Interference from other signals

Digital Binary Signal



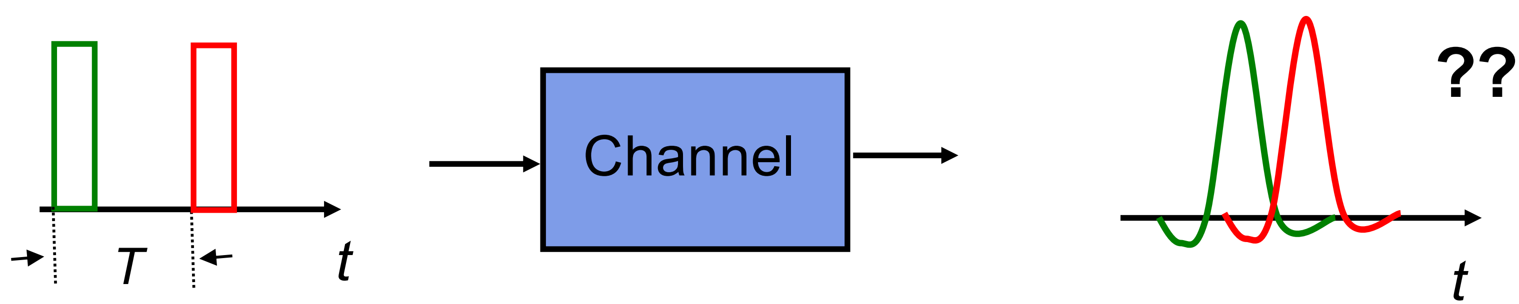
Bit rate = 1 bit / T seconds

For a given communications medium:

- How do we increase transmission speed?
- How do we achieve reliable communications?
- Are there limits to speed and reliability?

Pulse Transmission Rate

- Objective: Maximize pulse rate through a channel, that is, make T as small as possible



- If input is a narrow pulse, then typical output is a spread-out pulse with ringing
- Question: How frequently can these pulses be transmitted without interfering with each other?
- Answer: 2 x W_c pulses/second

where W_c is the bandwidth of the channel

Multilevel Pulse Transmission

- If channel bandwidth W_c, we can transmit $2W_c$ pulses/sec (without interference)
- If pulses amplitudes are either $-A$ or $+A$, then each pulse conveys 1 bit, so

  Bit Rate = 1 bit/pulse x $2W_c$ pulses/sec = $2W_c$ bps
- If amplitudes are from $\{-A, -A/3, +A/3, +A\}$, then bit rate is 2 x $2W_c$ bps
- By going to $M = 2^m$ amplitude levels, we achieve

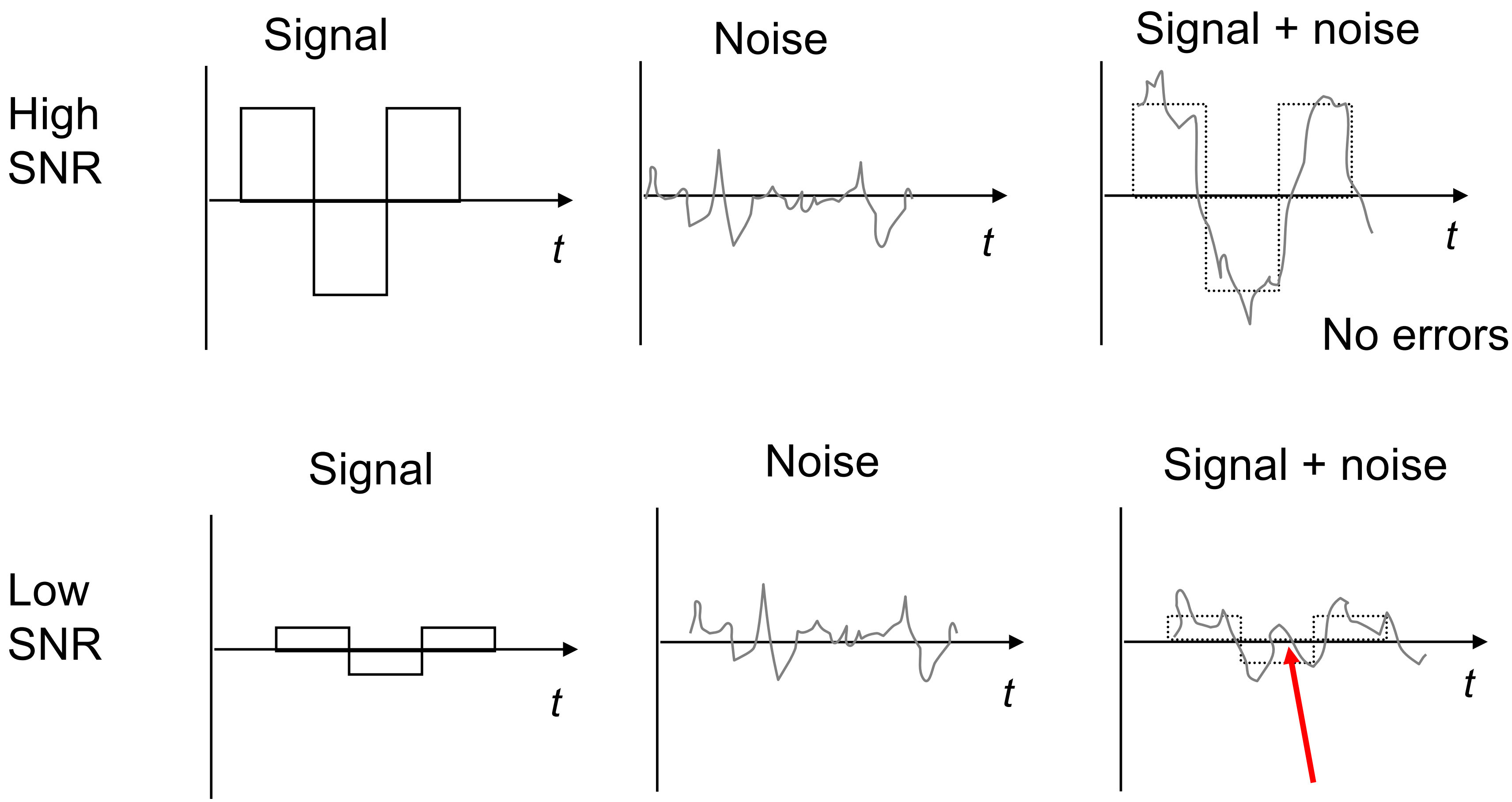
Bit Rate = m bits/pulse x $2W_c$ pulses/sec = $2mW_c$ bps

In the absence of noise, the bit rate can be increased without limit by increasing m

Noise & Reliable Communications

- All physical systems have noise
  - Electrons always vibrate randomly at non-zero temperature
  - Motion of electrons induces noise
- Presence of noise limits accuracy of measurement of received signal amplitude
- Errors occur if signal separation is comparable to noise level
- Bit Error Rate (BER) increases with decreasing signal-to-noise ratio
- Noise places a limit on how many amplitude levels can be used in pulse transmission

Signal-to-Noise Ratio



$$\text{SNR} = \frac{\text{Average signal power}}{\text{Average noise power}}$$

$$\text{SNR (dB)} = 10 \log_{10} \text{SNR}$$

Shannon Channel Capacity

$$C = W_c \log_2 (1 + SNR) \text{ bps}$$

- Arbitrarily reliable communications is possible if the transmission rate $R < C$.
- If $R > C$, then arbitrarily reliable communications is not possible.
- "Arbitrarily reliable" means the BER can be made arbitrarily small through sufficiently complex coding.
- C can be used as a measure of how close a system design is to the best achievable performance.
- Bandwidth W_c & SNR determine C

Example

- Find the Shannon channel capacity for a telephone channel with W_c = 3400 Hz and SNR = 10000

$$C = 3400 \log_2 (1 + 10000)$$
$$= 3400 \log_{10} (10001)/\log_{10} 2 = 45200 \text{ bps}$$

Note that SNR = 10000 corresponds to

SNR (dB) = $10 \log_{10}(10000) = 40$ dB

Examples of Channels

Channel	Bandwidth	Bit Rates
Telephone voice channel	3-4 kHz	33 kbps
Copper pair	1 MHz	1-6 Mbps (~1 mile)
Coaxial cable	500 MHz (6 MHz channels)	38 Mbps/ channel
5 GHz radio (IEEE 802.11)	300 MHz (11 channels)	54 Mbps / channel
Optical fiber	Many TeraHertz	40-100 Gbps / wavelength

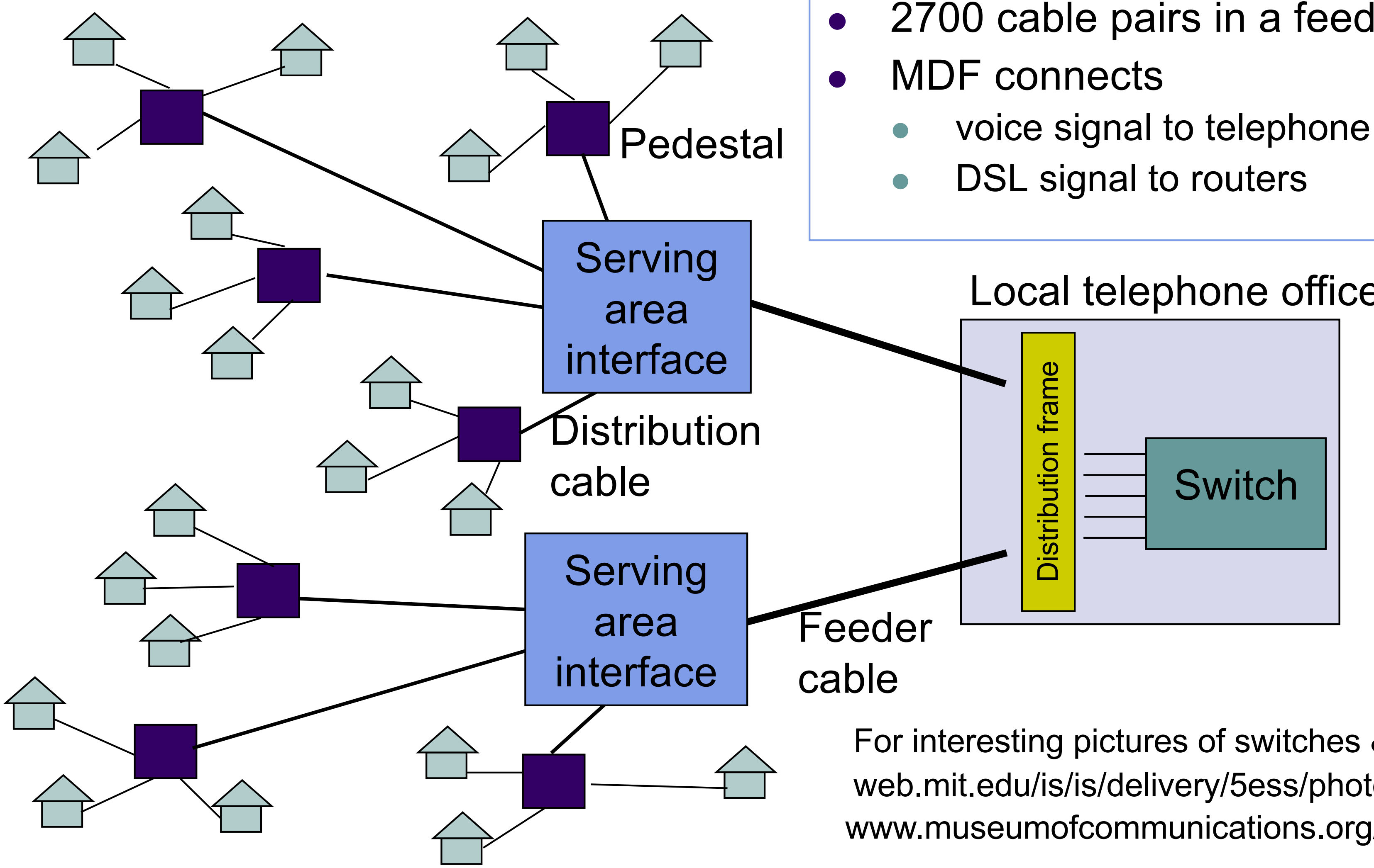
Bit Rates of Digital Transmission Systems

System	Bit Rate	Observations
Telephone copper pair	33.6-56 kbps	4 kHz telephone channel
Ethernet twisted pair	10 Mbps, 100 Mbps	100 meters of unshielded twisted copper wire pair
Cable modem	.5Mbps-38-343Mbps	Shared CATV return channel; 6Mz TV channel; 8 combined channels
ADSL twisted pair	64-640 kbps in, 1.536-6.144 Mbps out	Coexists with analog telephone signal
FTTC+copper	100Mbps, 1 Gbps	Fiber to the curb
2.4 GHz radio	2-11-54 Mbps	IEEE 802.11 wireless LAN
28 GHz radio	Multi-Gbps	mmwave radio in 5G
Optical fiber	10-40-100-200-400 Gbps	Ethernet
Optical fiber	>400Gbps, >petabit/s	1 wavelength; multiple wavelengths

Telephone Local Loop

Local Loop: "Last Mile"

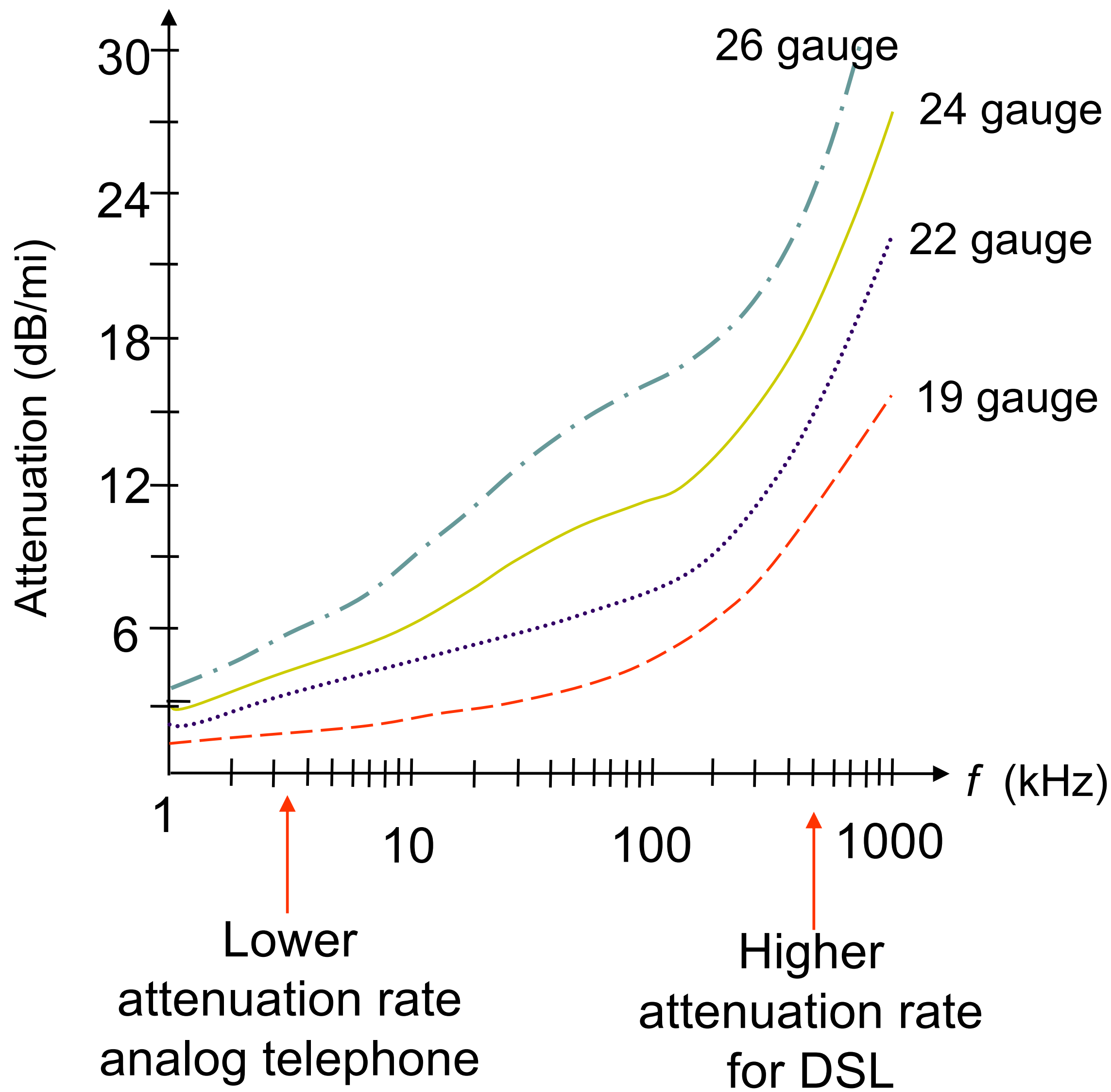
- Copper pair from telephone to CO
- Pedestal to SAI to Main Distribution Frame (MDF)
- 2700 cable pairs in a feeder cable
- MDF connects
  - voice signal to telephone switch
  - DSL signal to routers



For interesting pictures of switches & MDF, see
web.mit.edu/is/is/delivery/5ess/photos.html
www.museumofcommunications.org/coe.html

Twisted Pair

Twisted pair
- Two insulated copper wires in regular spiral pattern to minimize interference
- Various thicknesses, e.g. 0.016 inch (24 gauge)
- Low cost
- Attenuation increases with distance
- Telephone loop from customer to CO
- Intra-building wiring closet to desktop



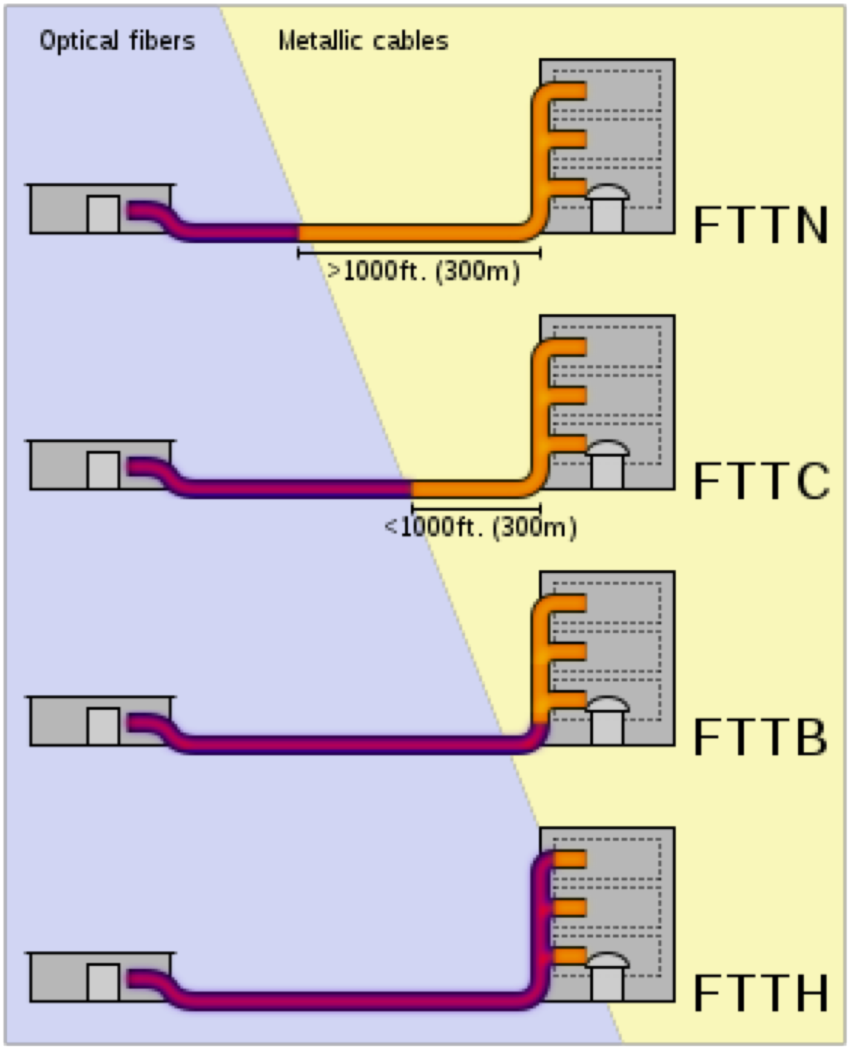
Twisted Pair Bit Rates

Table 3.5 Data rates of 24-gauge twisted pair

Standard	Data Rate	Distance
T-1	1.544 Mbps	18,000 feet, 5.5 km
DS2	6.312 Mbps	12,000 feet, 3.7 km
1/4 STS-1	12.960 Mbps	4500 feet, 1.4 km
1/2 STS-1	25.920 Mbps	3000 feet, 0.9 km
STS-1	51.840 Mbps	1000 feet, 300 m

- Twisted pairs can provide high bit rates at short distances
- Asymmetric Digital Subscriber Loop (ADSL)
  - Internet Access
  - Lower 3 kHz for voice
  - Upper band for data
  - 64 kbps inbound
  - 640 kbps outbound
- Much higher rates possible at shorter distances
  - Strategy for telephone companies is to bring fiber close to home & then twisted pair
  - Higher-speed access + video

Fiber to the x



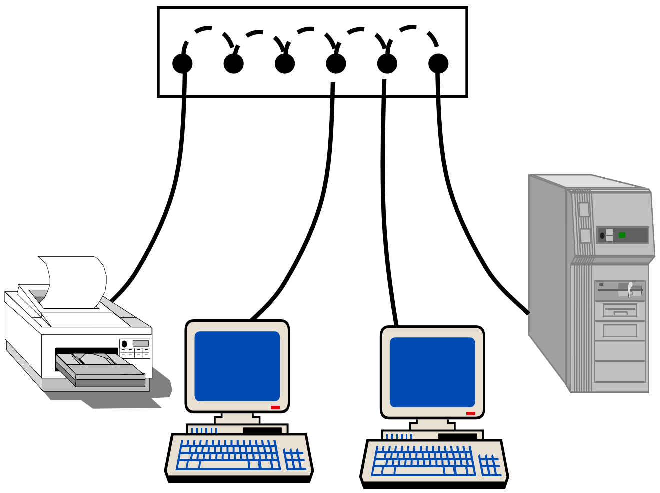
Node

Curb

Building

Home

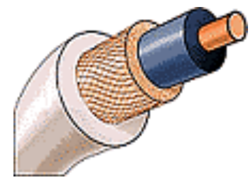
Ethernet LANs



In office or home networks

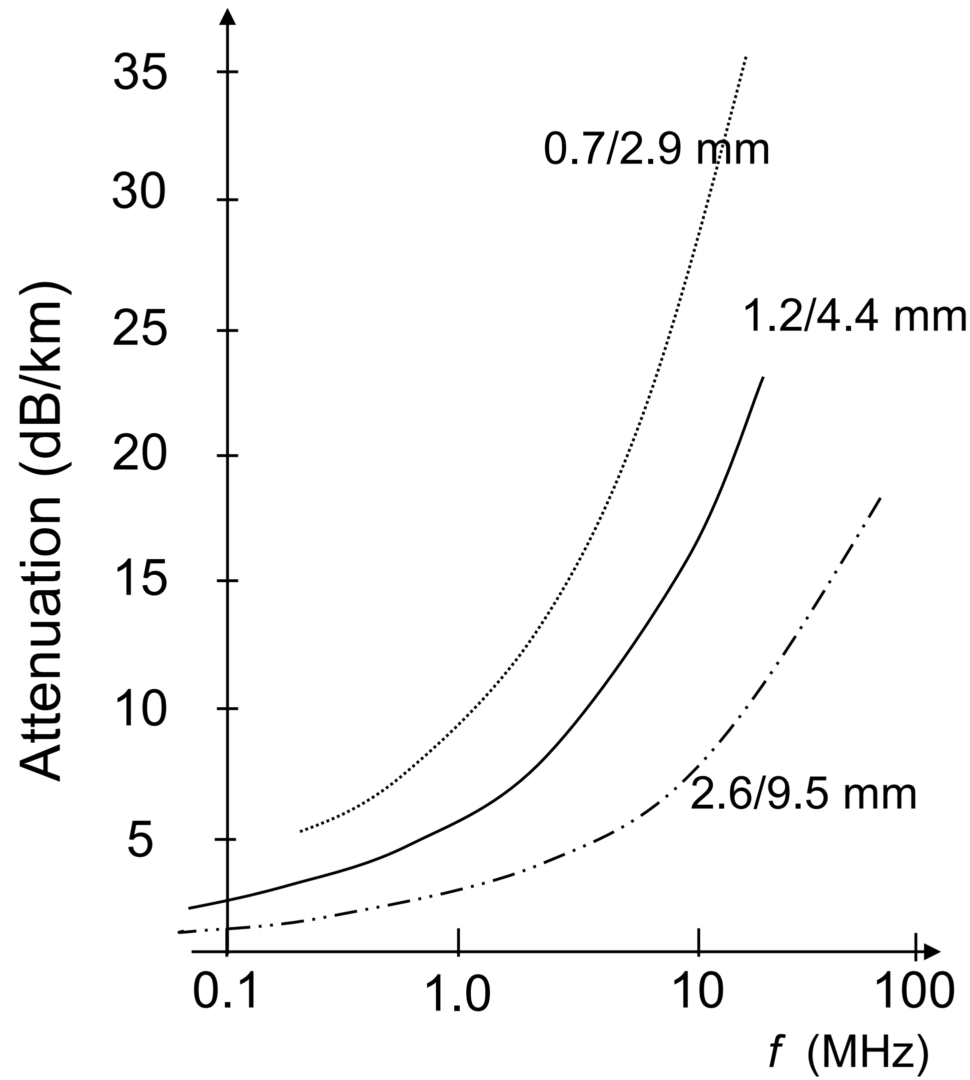
- Cat 3 Unshielded twisted pair (UTP): ordinary telephone wires
- Cat 5 UTP: tighter twisting to improve signal quality
- Shielded twisted pair (STP): costly
- 10BASE-T Ethernet
  - 10 Mbps, Twisted pair
  - Two Cat3 pairs,100 meters
- 100BASE-T4 *Fast* Ethernet
  - 100 Mbps, Twisted pair
  - Four Cat3 pairs, 100 meters
  - Cat5 & STP provide other options
- 1000BASE-T GigE
  - Four Cat5 pairs, 100 meters
- Many optical fiber options
  - 500m – many km

Coaxial Cable

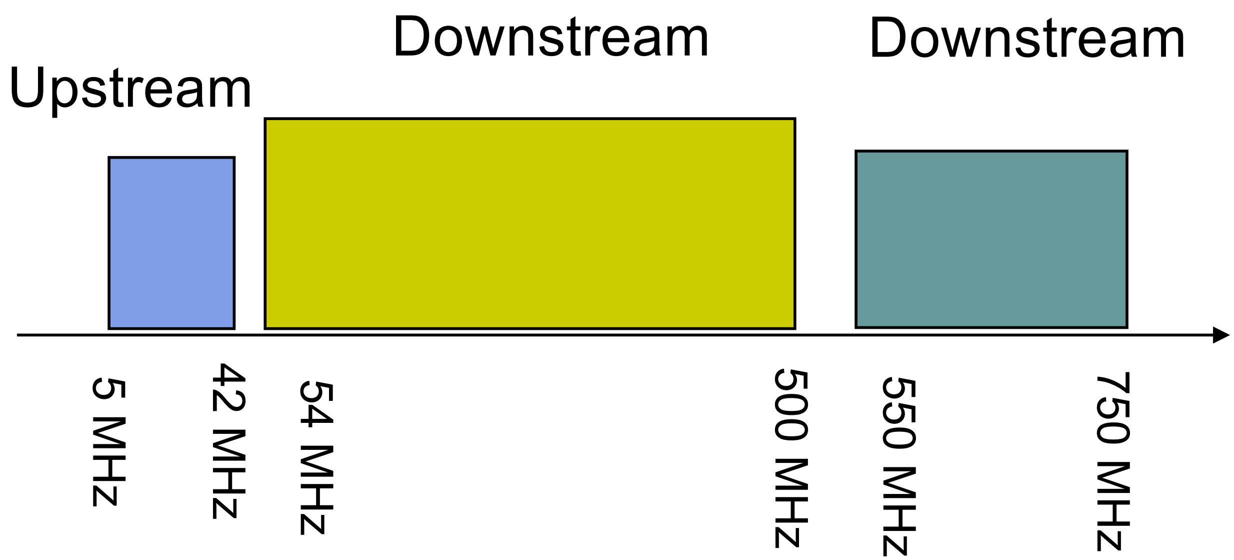


Twisted pair

- Cylindrical braided outer conductor surrounds insulated inner wire conductor
- High interference immunity
- Higher bandwidth than twisted pair
- Hundreds of MHz
- Cable TV distribution
- Original Ethernet LAN medium

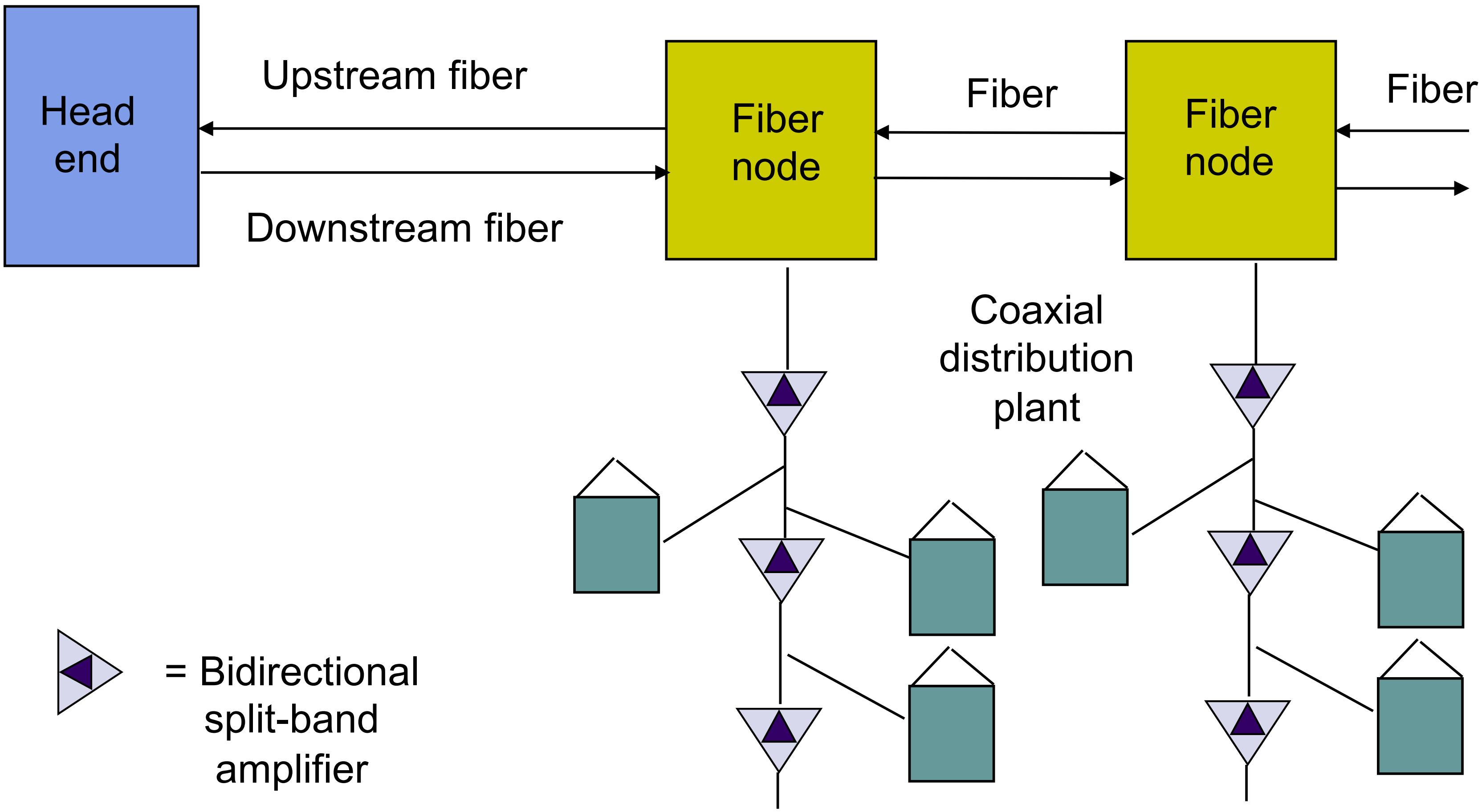


Cable Modem & TV Spectrum



- Cable TV network originally unidirectional
- Cable plant upgraded to bidirectional
- 1 analog TV channel is 6 MHz, can support very high data rates
- Cable Modem: *shared* upstream & downstream
  - 5-42 MHz upstream into network; 2 MHz channels; 500 kbps to 4 Mbps
  - >550 MHz downstream from network; 6 MHz channels; 36 Mbps

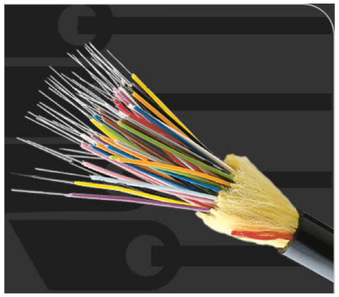
Cable Network Topology



Optical Fiber Properties

Advantages

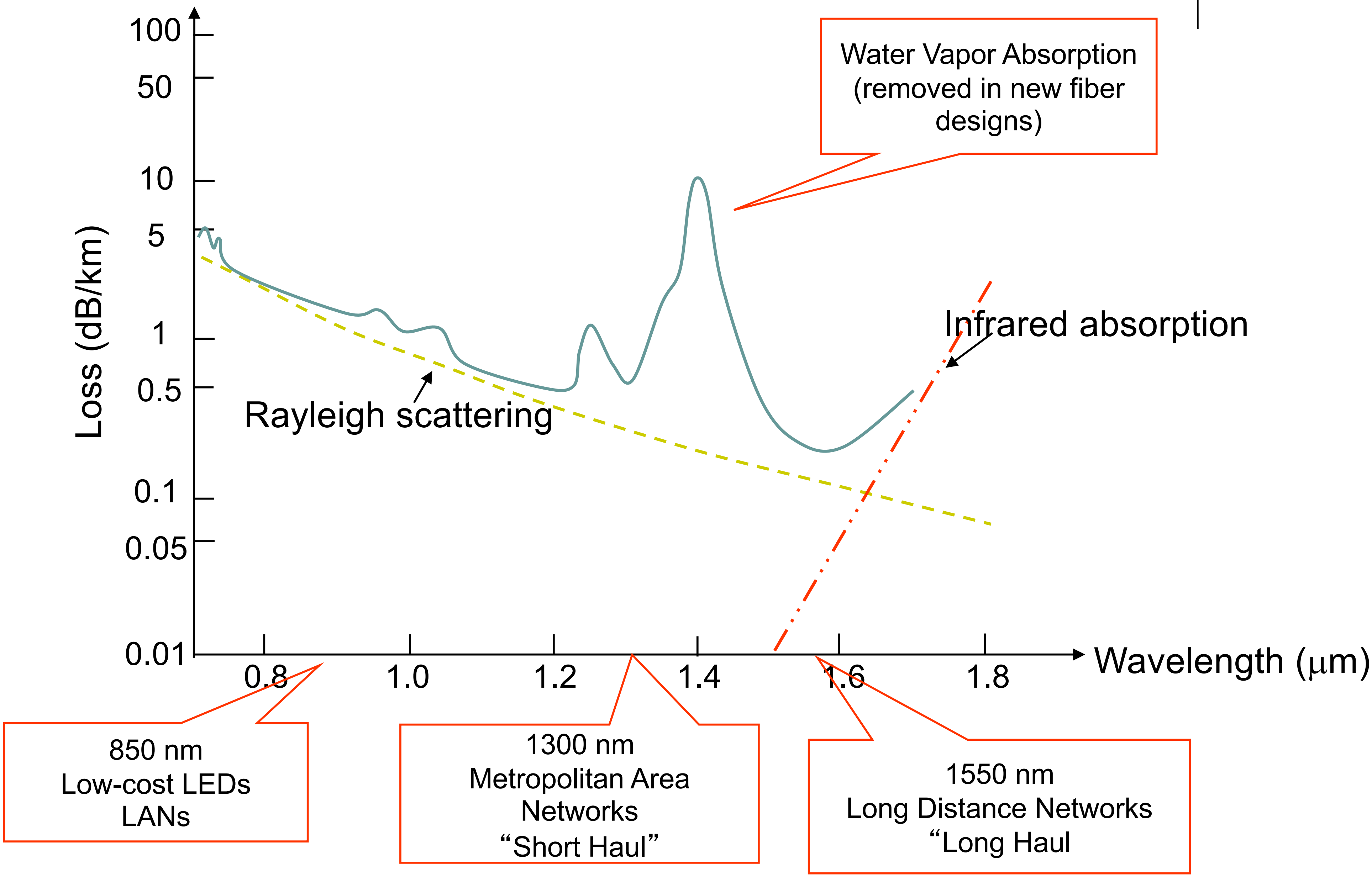
- ***Very low attenuation***
- ***Noise immunity***
- ***Extremely high bandwidth***
- Security: Very difficult to tap without breaking
- No corrosion
- More compact & lighter than copper wire



Disadvantages

- New types of optical signal impairments & dispersion
  - Polarization dependence
  - Wavelength dependence
- Limited bend radius
  - If physical arc of cable too high, light lost or won’t reflect
  - Will break
- Difficult to splice
- Mechanical vibration becomes signal noise

Very Low Attenuation



Huge Available Bandwidth

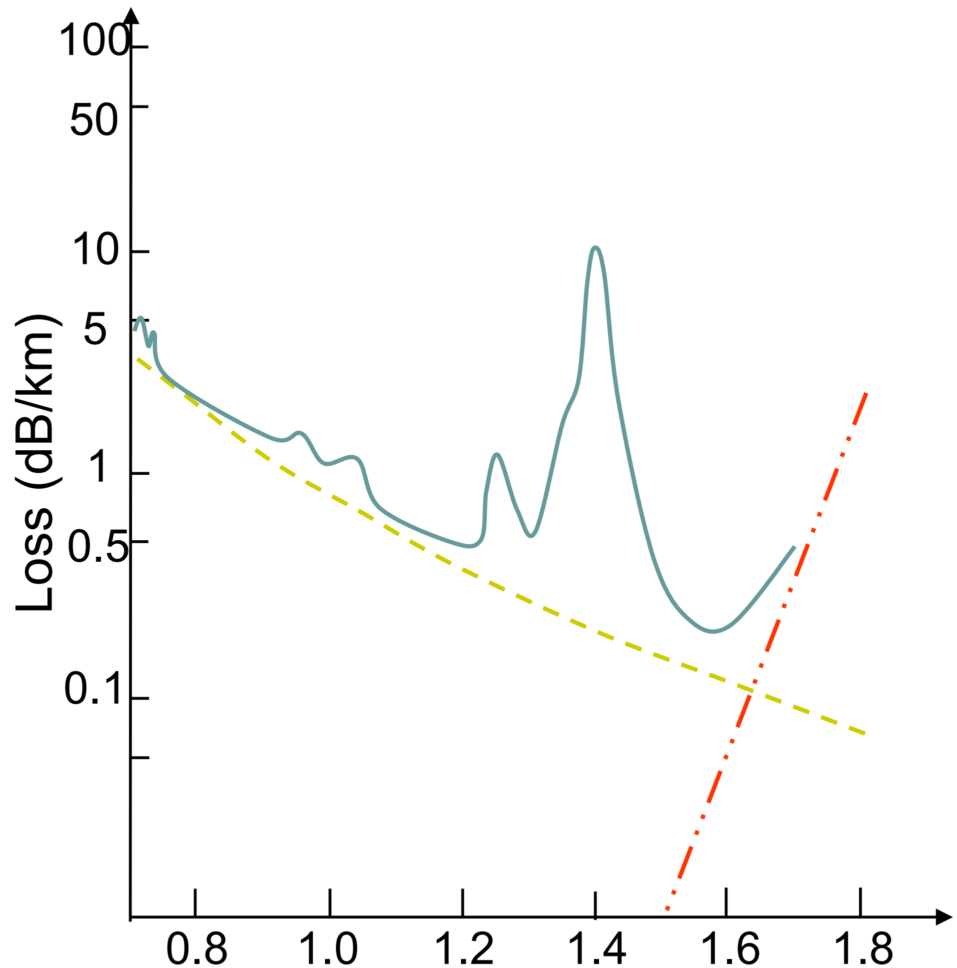
- Optical range from λ_1 to $\lambda_1 + \Delta\lambda$ contains bandwidth

$$B = f_1 - f_2 = \frac{v}{\lambda_1} - \frac{v}{\lambda_1 + \Delta\lambda}$$

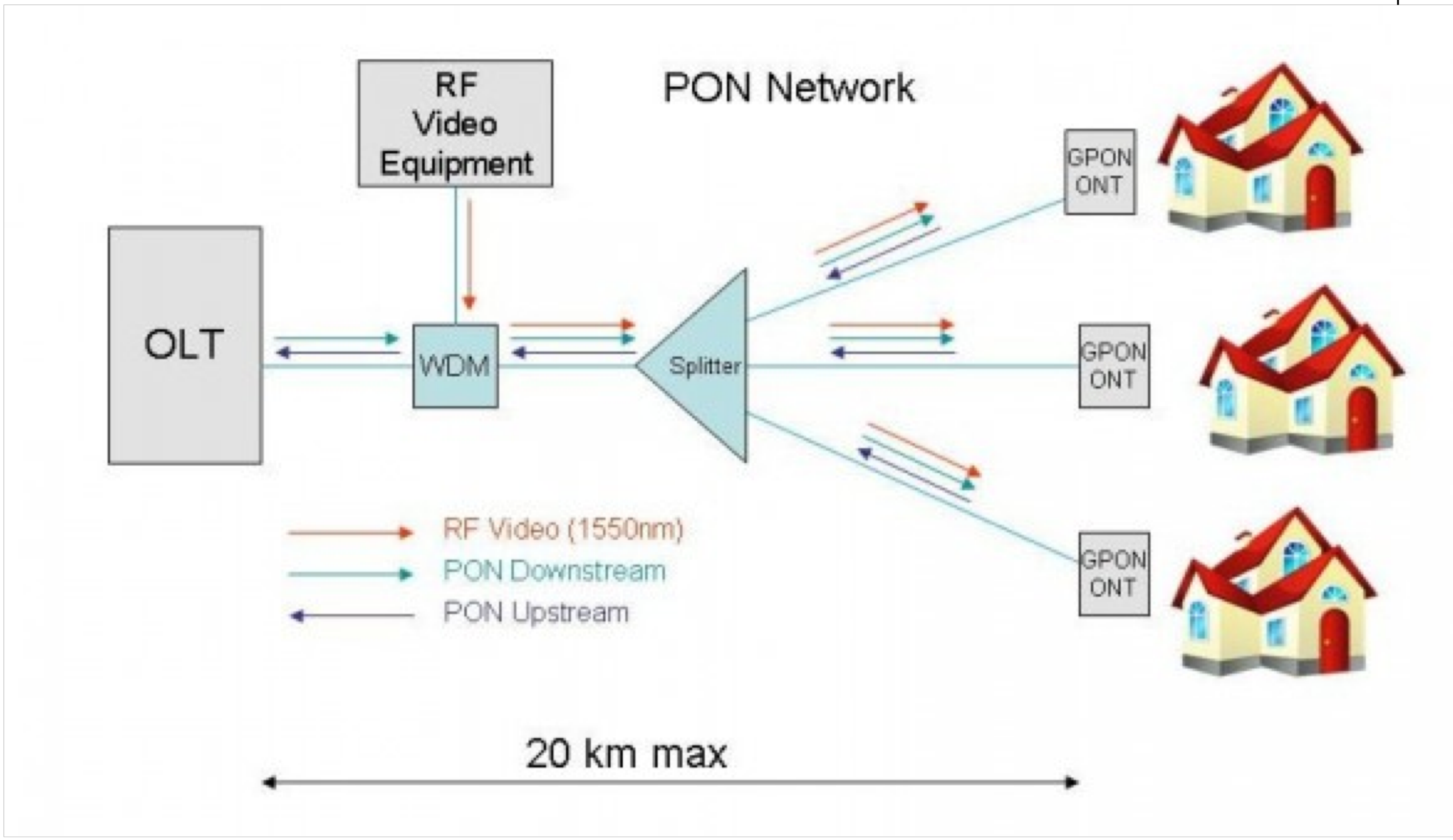
$$= \frac{v}{\lambda_1}\left\{\frac{\Delta\lambda / \lambda_1}{1 + \Delta\lambda / \lambda 1}\right\} \approx \frac{v\,\Delta\lambda}{\lambda_1^2}$$

- Example: $\lambda_1 = 1450$ nm
  $\lambda_1 + \Delta\lambda = 1650$ nm:

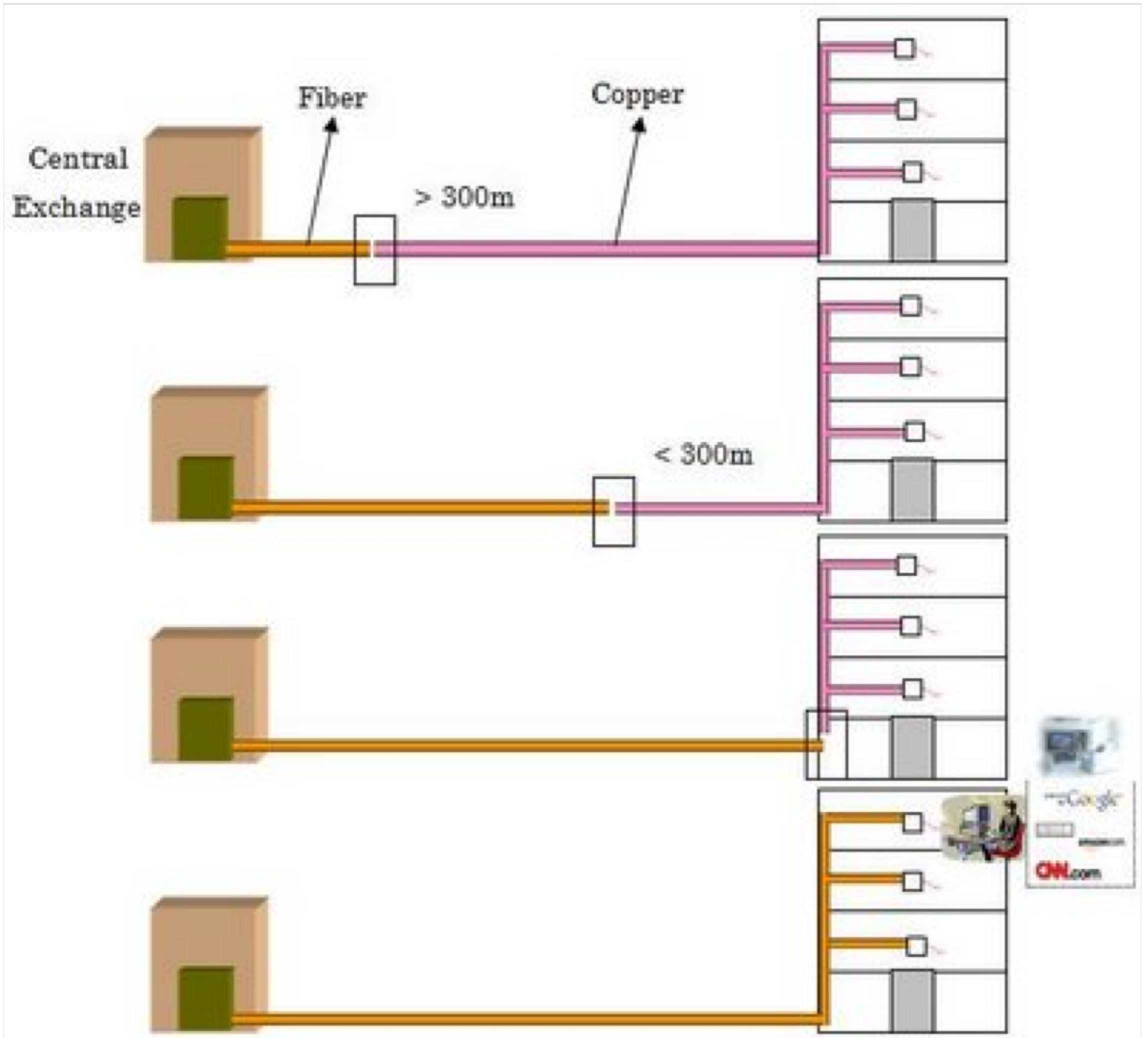
$$B = \frac{2(10^8)\text{m/s } 200\text{nm}}{(1450 \text{ nm})^2} \approx 19 \text{ THz}$$



Fiber to the Home



Fiber to the Home Evolution



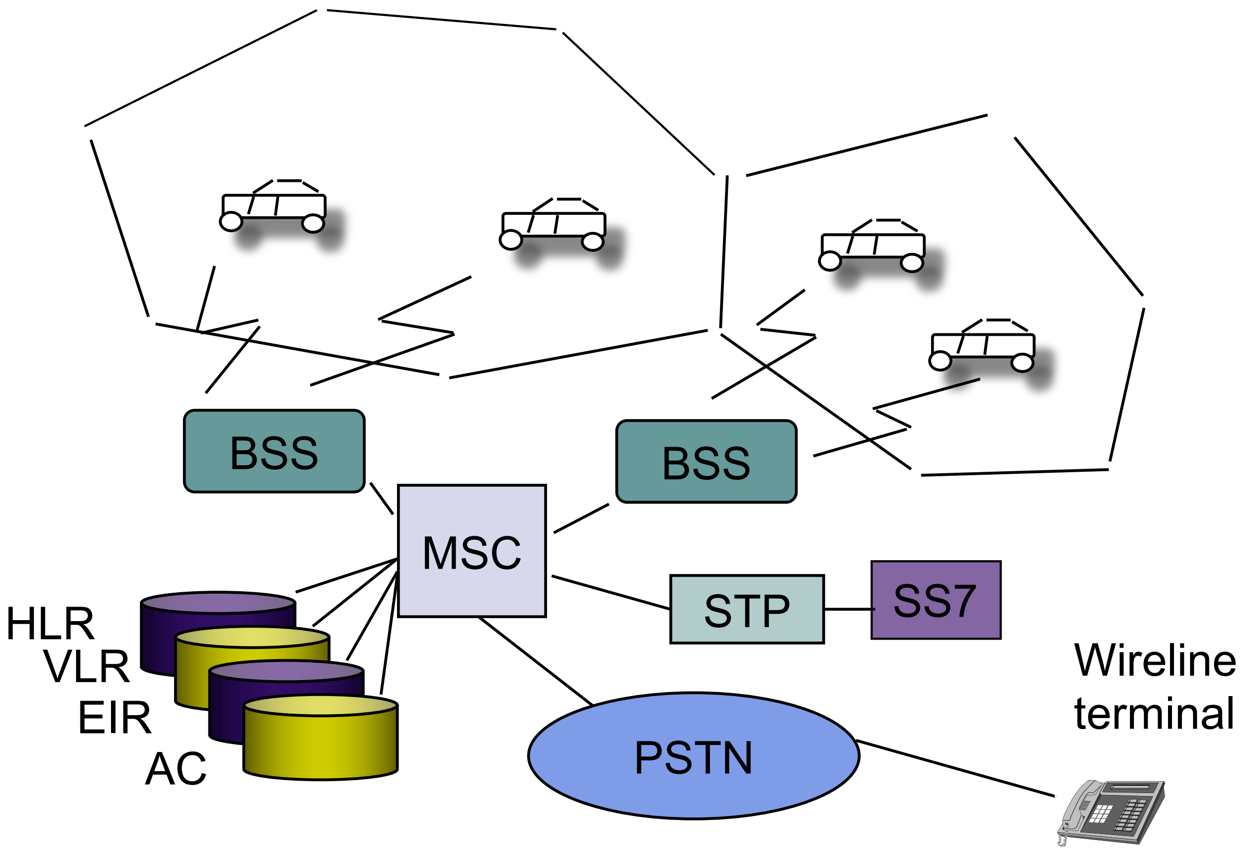
From CO to
Multiprovider Point of Presence

Cellular Communications

Two basic concepts:

- Frequency Reuse
  - A region is partitioned into *cells*
  - Each cell is covered by *base station*
  - Power transmission levels controlled to minimize inter-cell interference
  - Spectrum can be reused in other cells
- Handoff
  - Procedures to ensure continuity of call as user moves from cell to another
  - Involves setting up call in new cell and tearing down old one

Cellular Network



Base station

- Transmits to users on *forward channels*
- Receives from users on *reverse channels*

Mobile Switching Center

- Controls connection setup within cells & to telephone network

AC = authentication center
BSS = base station subsystem
EIR = equipment identity register
HLR = home location register

MSC = mobile switching center
PSTN = public switched telephone network
STP = signal transfer point
VLR = visitor location register

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RADIO SPECTRUM ALLOCATIONS IN CANADA

The Spectrum

Radio waves use the electromagnetic spectrum. The lowest frequencies have the longest radio waves and the highest frequencies have the shortest radio waves.

Radio waves are characterized according to their frequency, the unit for which is the hertz (Hz). The frequency is determined by the number of complete waves propagated through a medium past a fixed point in one second. Thus, the frequency of a signal where one wave passes a fixed point in one second is one hertz. A kilohertz (kHz) represents 1000 waves passing a point in one second, or 1000 hertz. One megahertz (MHz) is 1000 kilohertz and a gigahertz (GHz) is 1000 megahertz.

The spectrum is divided into a number of frequency bands, each possessing characteristics peculiar to it which determine the usage appropriate to that band. Each band has been allocated by international agreement at a World Radiocommunication Conference (WRC) to one or more radio services or for specific usages. Sponsored by the International Telecommunication Union (a United Nations agency), WRCs are held to extend, review and revise frequency allocations among the various uses.

After WRC conferences — and when Canada's needs change — Industry Canada allocates specific frequency bands to services to satisfy domestic communications requirements as shown on this chart. The official regulatory provisions that pertain to frequency allocations in Canada are contained in the Canadian Table of Frequency Allocations and the related spectrum policies.

Among radio spectrum users are broadcasters, taxis, building and other construction trades, air transportation, radio amateurs, marine transportation, telecommunications carriers, electrical power utilities, trucking companies, police, and federal, provincial, territorial and municipal departments and agencies.

This chart is based on the 2007 Canadian Table of Frequency Allocations, which was developed from decisions of World Radio Conferences, including WRC-03. The chart provides a graphic representation of Canadian electromagnetic spectrum allocations between 9 Hz and 275 GHz.

For further information on spectrum or radio matters, contact the Spectrum and Radio Policy Directorate, Industry Canada, Ottawa (e-mail: dgtp-dsrs@ic.gc.ca) or one of its regional offices in Moncton, Montréal, Toronto, Winnipeg or Vancouver.

ATTRIBUTION DES FRÉQUENCES RADIOÉLECTRIQUES AU CANADA

Le spectre

Les ondes radioélectriques utilisent le spectre électromagnétique. Aux fréquences les plus basses correspondent les ondes radio les plus longues et aux fréquences les plus élevées, les ondes radio les plus courtes.

Les ondes radio se caractérisent par leur fréquence, qui se mesure en hertz (Hz). La fréquence est déterminée par le nombre d'ondes complètes franchissant un point fixe d'un support en une seconde. On dira donc d'un signal pour lequel une onde franchit un point fixe en une seconde qu'il a une fréquence de 1 hertz. Le kilohertz (kHz) équivaut à 1 000 ondes par seconde, soit 1 000 hertz, le mégahertz, à 1 000 kilohertz et le gigahertz (GHz), à 1 000 mégahertz.

Le spectre se compose de bandes de fréquences possédant chacune des particularités qui en déterminent l'utilisation. Chaque bande est attribuée à un ou plusieurs services radio ou à des usages déterminés par voie d'accords internationaux signés à une Conférence mondiale des radiocommunications (CMR). Organisées sous l'égide d'un organisme des Nations Unies, l'Union internationale des télécommunications, les CMR ont pour but d'étendre, d'étudier et de réviser l'attribution des bandes de fréquences.

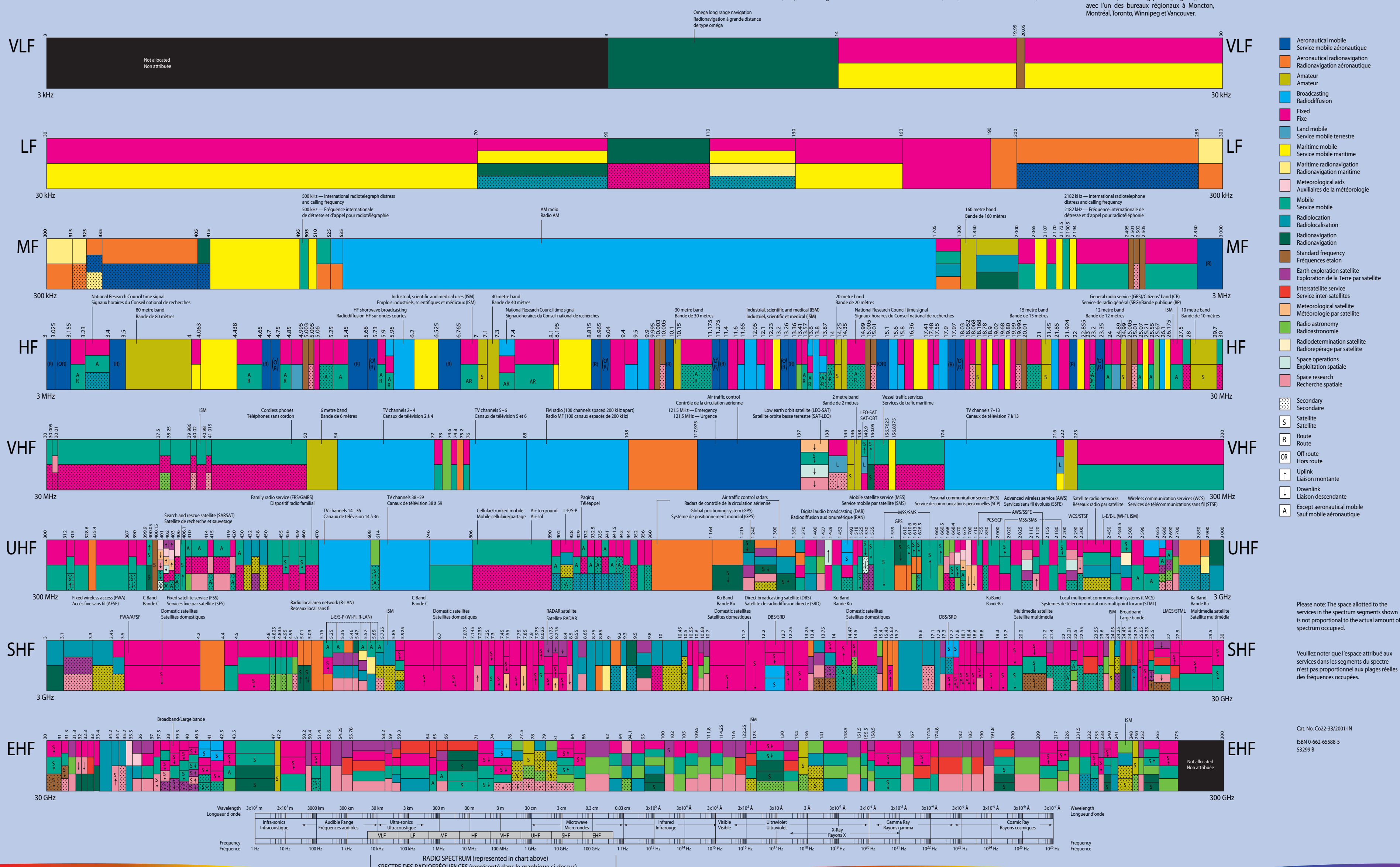
À l'issue de chacune de ces conférences et quand des changements s'imposent au Canada, Industrie Canada attribue des bandes de fréquences particulières à certains services, de manière à s'adapter aux besoins du pays en matière de communications, comme l'illustre le graphique ci-dessous. Les dispositions officielles de la réglementation touchant l'attribution des fréquences au Canada figurent dans le Tableau canadien d'attribution des bandes de fréquences et dans les politiques connexes d'utilisation du spectre.

Parmi les utilisateurs du spectre radioélectrique, on compte les radiodiffuseurs, les compagnies de taxi, l'industrie du bâtiment et d'autres secteurs de la construction, les transporteurs aériens, les radioamateurs, les transporteurs maritimes, les entreprises de télécommunications, les services publics d'électricité, les entreprises de camionnage, la police, ainsi que les ministères ou organismes fédéraux, provinciaux, territoriaux et municipaux.

Ce graphique est fondé sur la version 2007 du Tableau canadien d'attribution des bandes de fréquences, résultant des diverses Conférences mondiales des radiocommunications, notamment la CMR-03. Il fournit la représentation graphique des attributions de fréquences radioélectriques au Canada, entre 9 et 275 GHz.

Pour obtenir plus de renseignements sur le spectre ou les radiocommunications, veuillez communiquer avec la Direction des politiques du spectre et de la radiocommunication d'Industrie Canada à Ottawa (courriel: dgtp-dsrs@ic.gc.ca), ou avec l'un des bureaux régionaux à Moncton, Montréal, Toronto, Winnipeg et Vancouver.

Canada



Please note: The space allotted to the services in the spectrum segments shown is not proportional to the actual amount of spectrum occupied.

Veuillez noter que l'espace attribué aux services dans les segments du spectre n'est pas proportionnel aux plages réelles des fréquences occupées.

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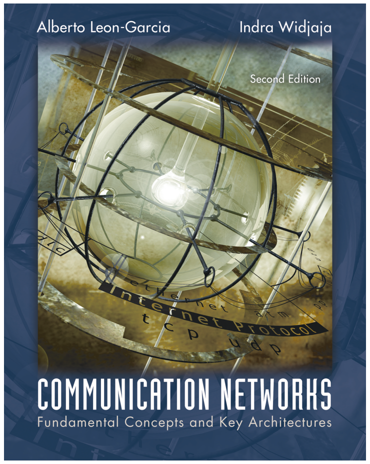
Chapter 3
Digital Transmission Fundamentals



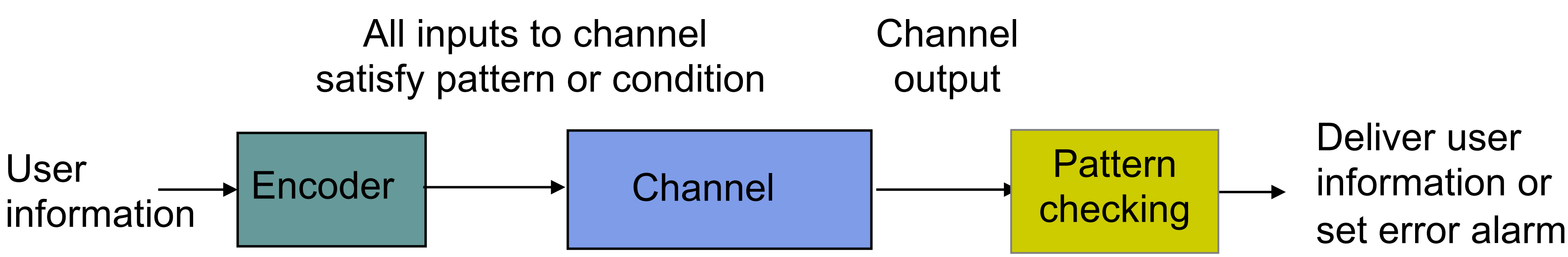
Error Detection and Correction

Error Control

- Data applications require error-free transfer
- Voice & video applications tolerate some errors
- Error control provides an acceptable error rate
- Two basic approaches:
  - Error ***detection*** & retransmission (ARQ)
  - Forward error ***correction*** (FEC)

Key Idea

- All transmitted data blocks ("codewords") satisfy a pattern
- If received block doesn't satisfy pattern, it is in error
- Redundancy: Only a subset of all possible blocks can be codewords
- Blindspot: when channel transforms a codeword into another codeword



Single Parity Check

- Append an overall parity check to *k* information bits

Info Bits: $b_1, b_2, b_3, \ldots, b_k$

Check Bit: $b_{k+1} = b_1 + b_2 + b_3 + \ldots + b_k$ modulo 2

Codeword: $(b_1, b_2, b_3, \ldots, b_{k,}, b_{k+!})$

- All codewords have even # of 1s
- Receiver checks to see if # of 1s is even
  - All error patterns that change an odd # of bits are detectable
  - All even-numbered patterns are undetectable
- Parity bit used in ASCII code

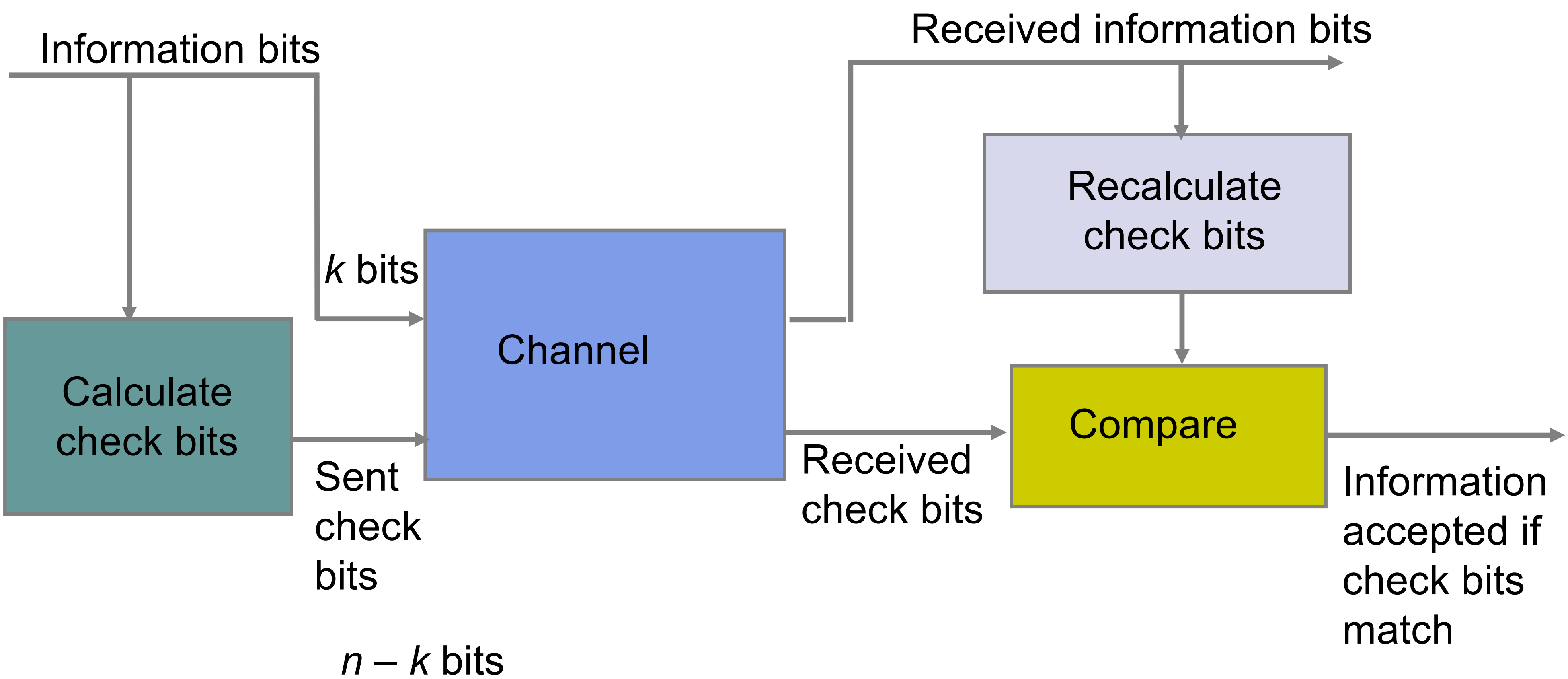
Example of Single Parity Code

- Information (7 bits): (0, 1, 0, 1, 1, 0, 0)
- Parity Bit: $b_8 = 0 + 1 + 0 + 1 + 1 + 0 = 1$
- Codeword (8 bits): (0, 1, 0, 1, 1, 0, 0, 1)

- If single error in bit 3 : (0, 1, 1, 1, 1, 0, 0, 1)
  - # of 1's =5, odd
  - Error detected

- If errors in bits 3 and 5: (0, 1, 1, 1, 0, 0, 0, 1)
  - # of 1's =4, even
  - Error not detected

Checkbits & Error Detection



How good is the single parity check code?

- *Redundancy*:
  - 1 redundant bit per k information bits:
  - overhead = $1/(k + 1)$
- *Coverage*:
  - An error pattern is a binary $(k + 1)$-tuple with 1s where errors occur and 0's elsewhere
  - All error patterns with odd # of errors can be detected
  - Of 2^{k+1} binary $(k + 1)$-tuples, ½ are odd, so 50% of error patterns can be detected
- Is it possible to detect more errors if we add more check bits?
- Yes, with the right codes & more check bits

What if bit errors are random?

- Many transmission channels introduce bit errors at random, independently of each other, and with probability p
- Some error patterns are more probable than others:

$$P[10000000] = p(1 - p)^7 = (1 - p)^8(p/(1-p)) \text{ and}$$

$$P[11000000] = p^2(1 - p)^6 = (1 - p)^8(p/(1-p))^2$$

- In any worthwhile channel $p < 0.5$, and so $(p/(1 - p) < 1$
- It follows that patterns with 1 error are more likely than patterns with 2 errors and so forth
- What is the probability that an undetectable error pattern occurs?

Single parity check code with random bit errors

- Undetectable error pattern if even # of bit errors:

$$P[\text{error detection failure}] = P[\text{undetectable error pattern}]$$
$$= P[\text{error patterns with even number of 1s}]$$

$$= \binom{n}{2} p^2(1-p)^{n-2} + \binom{n}{4} p^4(1-p)^{n-4} + \ldots$$

- Example: Evaluate above for $n = 32$, $p = 10^{-3}$

$$P[\text{undetectable error}] = \binom{32}{2} (10^{-3})^2 (1 - 10^{-3})^{30} + \binom{32}{4} (10^{-3})^4 (1 - 10^{-3})^{28}$$

$$\approx 496\ (10^{-6}) + 35960\ (10^{-12}) \approx 4.96\ (10^{-4})$$

- For this example, roughly 1 in 2000 error patterns is undetectable

Internet Checksum

- Several Internet protocols (e.g. IP, TCP, UDP) use check bits to detect errors in the *IP header* (or in the header and data for TCP/UDP)
- A checksum is calculated for header contents and included in a special field.
- Checksum recalculated at every router, so algorithm selected for ease of implementation in software
- Let header consist of L, 16-bit words,

  $\mathbf{b}_0, \mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_{L-1}$
- The algorithm appends a 16-bit checksum $\mathbf{b}_L$

Checksum Calculation

The checksum $\mathbf{b}_L$ is calculated as follows:

- Treating each 16-bit word as an integer, find

$$\mathbf{x} = \mathbf{b}_0 + \mathbf{b}_1 + \mathbf{b}_2 + \ldots + \mathbf{b}_{L-1} \text{ modulo } 2^{16}-1$$

- The checksum is then given by:

$$\mathbf{b}_L = -\,\mathbf{x} \quad \text{modulo } 2^{16}-1$$

Thus, the headers must satisfy the following ***pattern***:

$$\mathbf{0} = \mathbf{b}_0 + \mathbf{b}_1 + \mathbf{b}_2 + \ldots + \mathbf{b}_{L-1} + \mathbf{b}_L \text{ modulo } 2^{16}-1$$

- The checksum calculation is carried out in software using one's complement arithmetic

Internet Checksum Example

Use Modulo Arithmetic

- Assume 4-bit words
- Use mod 2^4-1 arithmetic
- $\underline{b}_0$=1100 = 12
- $\underline{b}_1$=1010 = 10
- $\underline{b}_0$+$\underline{b}_1$=12+10=7 mod15
- $\underline{b}_2$ = -7 = 8 mod15
- Therefore
- $\underline{b}_2$=1000

Use Binary Arithmetic

- Note 16 =1 mod15
- So: 10000 = 0001 mod15
- leading bit wraps around

$b_0 + b_1$ = 1100+1010
=10110
=10000+0110
=0001+0110
=0111
=7

Take 1s complement

b_2 = -0111 =1000

Polynomial Codes

- Polynomials instead of vectors for codewords
- Polynomial arithmetic instead of check sums
- Implemented using shift-register circuits
- Also called *cyclic redundancy check (CRC)* codes
- Most data communications standards use polynomial codes for error detection
- Polynomial codes also basis for powerful error-correction methods

Binary Polynomial Division

- Division with Decimal Numbers

$$
\begin{array}{r l}
34 & \leftarrow \text{quotient} \\
\hline
35\ )\ 1222 & \leftarrow \text{dividend} \\
105 & \\
\hline
172 & \\
140 & \\
\hline
32 & \leftarrow \text{remainder}
\end{array}
$$

divisor

dividend = quotient x divisor +remainder

$$1222 = 34 \times 35 + 32$$

- Polynomial Division

$$
\begin{array}{r l}
x^3 + x^2 + x & = q(x) \text{ quotient} \\
\hline
x^3 + x + 1\ )\ x^6 + x^5 & \leftarrow \text{dividend} \\
x^6 + \quad x^4 + x^3 & \\
\hline
x^5 + x^4 + x^3 & \\
x^5 + \quad x^3 + x^2 & \\
\hline
x^4 + \quad x^2 & \\
x^4 + \quad x^2 + x & \\
\hline
x & = r(x) \text{ remainder}
\end{array}
$$

divisor

Note: Degree of r(x) is less than degree of divisor

Polynomial Coding

- Code has binary *generating polynomial* of degree $n\text{–}k$

$$g(x) = x^{n-k} + g_{n-k-1}x^{n-k-1} + \ldots + g_2x^2 + g_1x + 1$$

- *k information bits* define polynomial of degree $k - 1$

$$i(x) = i_{k-1}x^{k-1} + i_{k-2}x^{k-2} + \ldots + i_2x^2 + i_1x + i_0$$

- Find *remainder polynomial* of at most degree $n - k - 1$

$$\begin{array}{r} q(x) \\ g(x)\ \overline{)\ x^{n-k}\, i(x)} \\ r(x) \end{array}$$

$$x^{n-k}i(x) = q(x)g(x) + r(x)$$

- Define the *codeword polynomial* of degree $n - 1$

$$\boxed{\underbrace{b(x)}_{n \text{ bits}} = \underbrace{x^{n-k}i(x)}_{k \text{ bits}} + \underbrace{r(x)}_{n-k \text{ bits}}}$$

There are 2^k codewords

Polynomial example: $k = 4$, $n–k = 3$

Generator polynomial: $g(x)= x^3 + x + 1$

Information: (1,1,0,0) $i(x) = x^3 + x^2$

Encoding: $x^3i(x) = x^6 + x^5$

$$
\begin{array}{r l}
 & x^3 + x^2 + x \\
\cline{2-2}
x^3 + x + 1 \,) & x^6 + x^5 \\
 & x^6 + \phantom{x^5 +{}} x^4 + x^3 \\
\cline{2-2}
 & \phantom{x^6 +{}} x^5 + x^4 + x^3 \\
 & \phantom{x^6 +{}} x^5 + \phantom{x^4 +{}} x^3 + x^2 \\
\cline{2-2}
 & \phantom{x^6 + x^5 +{}} x^4 + \phantom{x^3 +{}} x^2 \\
 & \phantom{x^6 + x^5 +{}} x^4 + \phantom{x^3 +{}} x^2 + x \\
\cline{2-2}
 & \phantom{x^6 + x^5 + x^4 + x^3 + x^2 +{}} x
\end{array}
$$

$$
\begin{array}{r l}
 & 1110 \\
\cline{2-2}
1011 \,) & 1100000 \\
 & 1011 \\
\cline{2-2}
 & 1110 \\
 & 1011 \\
\cline{2-2}
 & 1010 \\
 & 1011 \\
\cline{2-2}
 & 010
\end{array}
$$

Transmitted codeword:

$b(x) = x^6 + x^5 + x$

$\Longrightarrow$ $\underline{b} = (1,1,0,0,0,1,0)$

The *Pattern* in Polynomial Coding

- All codewords satisfy the following **pattern**:

$$b(x) = x^{n-k}i(x) + r(x) = q(x)g(x) + r(x) + r(x) = q(x)g(x)$$

- All codewords are a multiple of *g(x)!*
- To check if an arbitrary polynomial *p(x)* is a codeword divide it by *g(x)* and check if remainder is zero

Standard Generator Polynomials

CRC = cyclic redundancy check

- CRC-8:

  $= x^8 + x^2 + x + 1$ ATM

- CRC-16:

  $= x^{16} + x^{15} + x^2 + 1$ Bisync

  $= (x + 1)(x^{15} + x + 1)$

- CCITT-16:

  $= x^{16} + x^{12} + x^5 + 1$ HDLC, XMODEM, V.41

- CCITT-32: IEEE 802, DoD, V.42

  $= x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x + 1$

See Wikipedia on “cyclic redundancy check” for many more generator polynomials