Vibrastic 101

Artificial Intelligence Crash Course

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Overview

- Introduction
- All about Machine Learning
- Let's build our own

Course Organization

- Theory
- Challenge
- Project
- Question Answer

Refreshing

Python main characteristics:

- dynamic type system
- interpreted (actually: compiled to bytecode, `*.pyc` files)
- multi-paradigm: imperative, procedural, object-oriented, (functional), *literate*; do whatever you want
- **indentation is important!**
- Python is a high-level, dynamically typed multiparadigm programming language.
- Python code is often said to be almost like pseudocode, since it allows you to express very powerful ideas in very few lines of code while being very readable.

Refreshing (cntd.)

This course **assumes** that you have some programming experience at least:

- Java (static type system, compiled, object-oriented, verbose)
- C/C++ (static type system, compiled, multi-paradigm, low-level)
- Matlab? R?

In this *refreshing*, we're gonna review:

- Basic Python: Basic data types, containers, loops, functions and classes.
- **Pytorch** highlight

Basic Python : Data types

Numeric types

- Integers and floats work as you would expect from other languages:

```
x = 3; print(x, type(x))
y = 2.5; print(type(y))
```

```
print(x, x + 1, x - 1, x * 2, x ** 2)
print(y, y + 1, y * 2, y ** 2)
```

```
x += 1 # added to 4
x *= 2 # mutiplied to 8
```

Boolean

```python
t, f, aa, bb = True, False, True, False
print(t, f, type(t))
```

```python
print(t and f) # Logical AND;
print(t or f)  # Logical OR;
print(not t)   # Logical NOT;
print(t != f)  # Logical XOR;
```

```python
day = "Sunday"
if day == 'Sunday':
    print('Sleep!!!')
else:
    print('Go to work')
```

String

```python
hello = 'hello'
world = "world"
print(hello, len(hello))
```

```python
hw = hello + ' ' + world  # String concatenation
print(hw)
```

```python
hw12 = '%s %s! your number is: %d' % (hello, world, 12)  # sprintf style string formatting
print(hw12)
```

```python
s = "hello"
print(s.capitalize())
print(s.upper())
print(s.replace('l', '(ell)'))
print('  world '.strip())
```

List

```python
xs = [3, 1, 2]    # Create a list
print(xs, xs[2])
print(xs[-1])     # Count from the end of the list
```

```python
xs[2] = 'foo'    # Lists can contain elements of different types
print(xs)
```

```python
xs.append('bar') # Add a new element to the end of the list
print(xs)
```

```python
xs =  xs + ['thing1', 'thing2'] # Adding lists (the += op works too)
print(xs)
```

```python
x = xs.pop()     # Remove and return the last element of the list
print(x, xs)
```

Slicing

```
nums = list(range(5)) # range is a built-in function (more on this later)
print(nums)
```

```
print(nums[2:4])    # Get a slice from index 2 to 4 (exclusive); prints "[2, 3]"
```

```
print(nums[2:])     # Get a slice from index 2 to the end; prints "[2, 3, 4]"
```

```
print(nums[:2])     # Get a slice from the start to index 2 (exclusive); prints "[0, 1]"
```

```
print(nums[:])      # Get a slice of the whole list; prints ["0, 1, 2, 3, 4]"
```

```
print(nums[:-1])    # Slice indices can be negative; prints ["0, 1, 2, 3]"
```

```
nums[2:4] = [8, 9]  # Assign a new sublist to a slice
print(nums)         # Prints "[0, 1, 8, 9, 4]"
```

Loops

- Basic loop

```
for i in range(10):
    print(i)
```

- You can loop over the elements of a list like this:

```
animals = ['cat', 'dog', 'monkey']
for animal in animals:
    aa = animal + ' :)'
    print(aa)
```

- If you want access to the index of each element within the body of a loop, use the built-in `enumerate` function:

```
animals = ['cat', 'dog', 'monkey']
for idx, animal in enumerate(animals):
    print('Item number %d is a %s' % (idx + 1, animal))
```

Challenge

Write loops to draw triangle!

```
h = 3

*
**
***
```

```
h = 5

*
**
***
****
*****
```

List comprehension

- When programming, frequently we want to transform one type of data into another. For example, consider the following code that computes square numbers:

```
nums, squares = [0, 1, 2, 3, 4], []
for x in nums:
    squares.append(x ** 2)
```

- You can make this code simpler using a **list comprehension**:

```
nums = [0, 1, 2, 3, 4]
squares = [x ** 2 for x in nums]
```

- List comprehensions can also contain conditions:

```
nums = [0, 1, 2, 3, 4]
even_squares = [x ** 2 for x in nums if x % 2 == 0]
```

Dictionaries

A dictionary stores (key, value) pairs, similar to a `Map` in Java or an object in Javascript. You can use it like this:

```
d = {'cat': 'cute', 'dog': 'furry'}  # Create a new dictionary with some data
```

```
print(d['cat'])       # Get an entry from a dictionary; prints "cute"
print('cat' in d)
```

```
d['fish'] = 'wet'     # Set an entry in a dictionary
print(d['fish'])      # Prints "wet"
```

It is easy to iterate over the keys in a dictionary:

```
d = {'person': 2, 'cat': 4, 'spider': 8}
for animal in d:
    legs = d[animal]
    print('A %s has %d legs' % (animal, legs))
```

Basic Python : Functions

Python functions are defined using the `def` keyword. For example:

```python
def sign(x):
    if x > 0:
        return 'positive'
    elif x < 0:
        return 'negative'
    else:
        return 'zero'
```

```python
for x in [-1, 0, 1]:
    print(sign(x))
```

Function (cntd.)

We will often define functions to take optional keyword arguments, like this:

```
def hello(name, loud=False):
    if loud:
        print('HELLO, %s' % name.upper())
    else:
        print('Hello, %s!' % name)
```

```
hello('Bob')
hello('Fred', loud=True)
```

Challenge

Write this as a python function $\rightarrow \Sigma_i i^2$

```python
def sigma(i):
    bla bla bla...
```

```python
sigma(2) # will print 5
```

Basic Python : Classes and object oriented programming

- The syntax for defining classes in Python is straightforward.
- Remember to include `self` as the first parameter of the class methods.

```python
class Greeter():
    # Constructor
    def __init__(self, name):
        self.name = name  # Create an instance variable

    # Instance method
    def greet(self, loud=False):
        if loud:
            print('HELLO, %s!' % self.name.upper())
        else:
            print('Hello, %s' % self.name)
```

```python
g = Greeter('Fred')  # Construct an instance of the Greeter class
g.greet()            # Call an instance method; prints "Hello, Fred"
g.greet(loud=True)   # Call an instance method; prints "HELLO, FRED!"
```

- Inheritance

```
class Question(Greeter):
    def ___init__(self, name):
        super(Question, self).__init__(name)

    def ask(self):
        print('Are you %s?' % self.name)
```

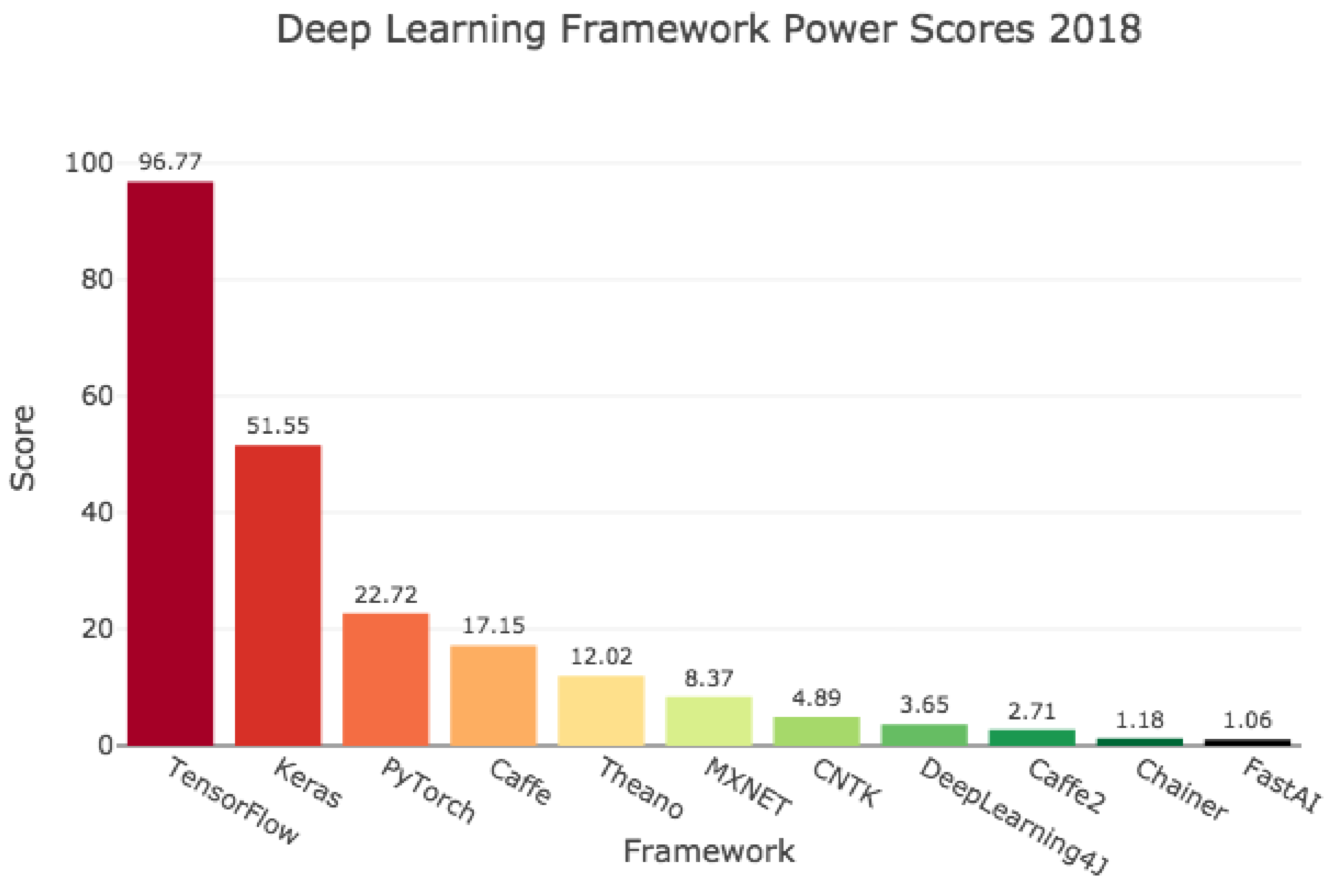
```
q = Question('Fred')
q.ask()
```

Basic Python : Import statement

- We have seen already the `import` statement in action.
- Python has a huge number of libraries included with the distribution.
- Most of these variables and functions are not accessible from a normal Python interactive session.
- Instead, you have to import them.
- You can also make your own module
- Browse here : https://pypi.org/

Machine Learning Framework

- **TensorFlow** by Google
- **Keras** by Francois Chollet
- **PyTorch** by Facebook



One of PyTorch feature ... that's loved by researchers

Autograd

- To help us to praise this feature, let's do some **basic math** beforehand
- Solve these !

$$f(x, y) = xy \qquad \frac{\partial}{\partial x} f(3) = ?$$

Solution

$$\frac{\partial}{\partial x} f(x, y) = y \qquad \rightarrow \qquad \frac{\partial}{\partial x} f(1, 2) = 2$$

Quite easy, right?

Challenge

How about...

$$g(x, y) = xy^2 - x^2y \qquad \frac{\partial}{\partial x} g(2, 5) = ?$$

Here is Pytorch come to the play!

- Import the library

```
import torch
```

- Declare our variable and function

```
x = torch.tensor(1.0, requires_grad = True)
y = torch.tensor(2.0)
f = x*y
```

- Get our number

```
f.backward()
print(x.grad.data)
```

Challenge

Solve this with Pytorch

$$g(x, y) = xy^2 - x^2y \qquad \frac{\partial}{\partial x} g(2, 5) = ?$$

Now we're set!

Preparation

- Open Google Colab
- Put this on the cell in case we need them during the course

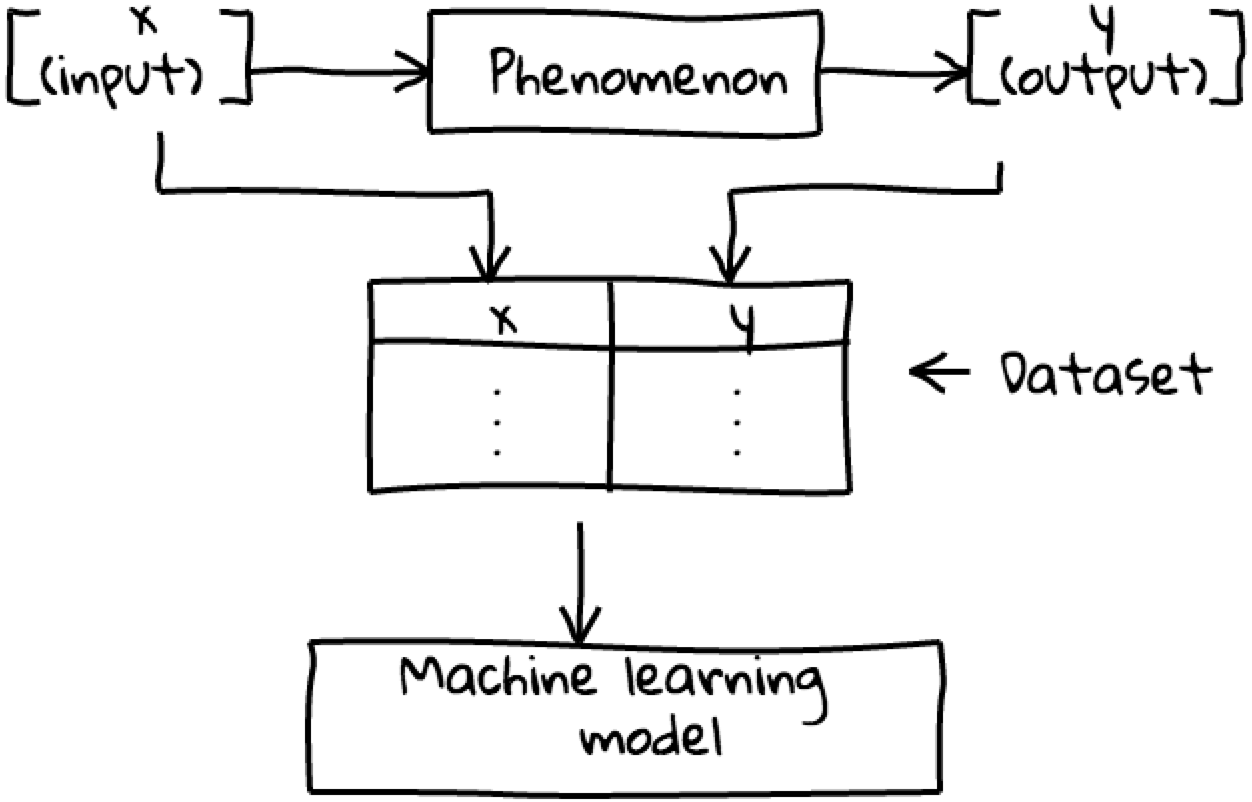
```
import random, math
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.cm as cm
import torch
import torch.nn as nn
import torch.nn.functional as F
import torch.optim as optim
from torch.autograd import Variable
import torchvision.transforms as transforms
import torchvision.datasets as dsets
```

Hold on!

Definition

- **Machine Learning (ML)** : A subset of artificial intelligence involved with the creation of algorithms which can modify itself without human intervention to produce desired output- by feeding itself through structured data.
- **Deep Learning** : Same, but has numerous layers

Definition (cntd.)



Highlight

- **Learning** : Construction and study of systems that can learn from data.
- **Adaptation** : The capacity to adapt implies to be able to modify what has been learn in order to cope with those modifications.
- **Flexibility and robustness** : Self-organization
- **Provide explanations** : Explanations are necessary to validate and find directions for improvement.
- **Discovery/creativity** : Capacity of discovering processes and/or relations previously unknown

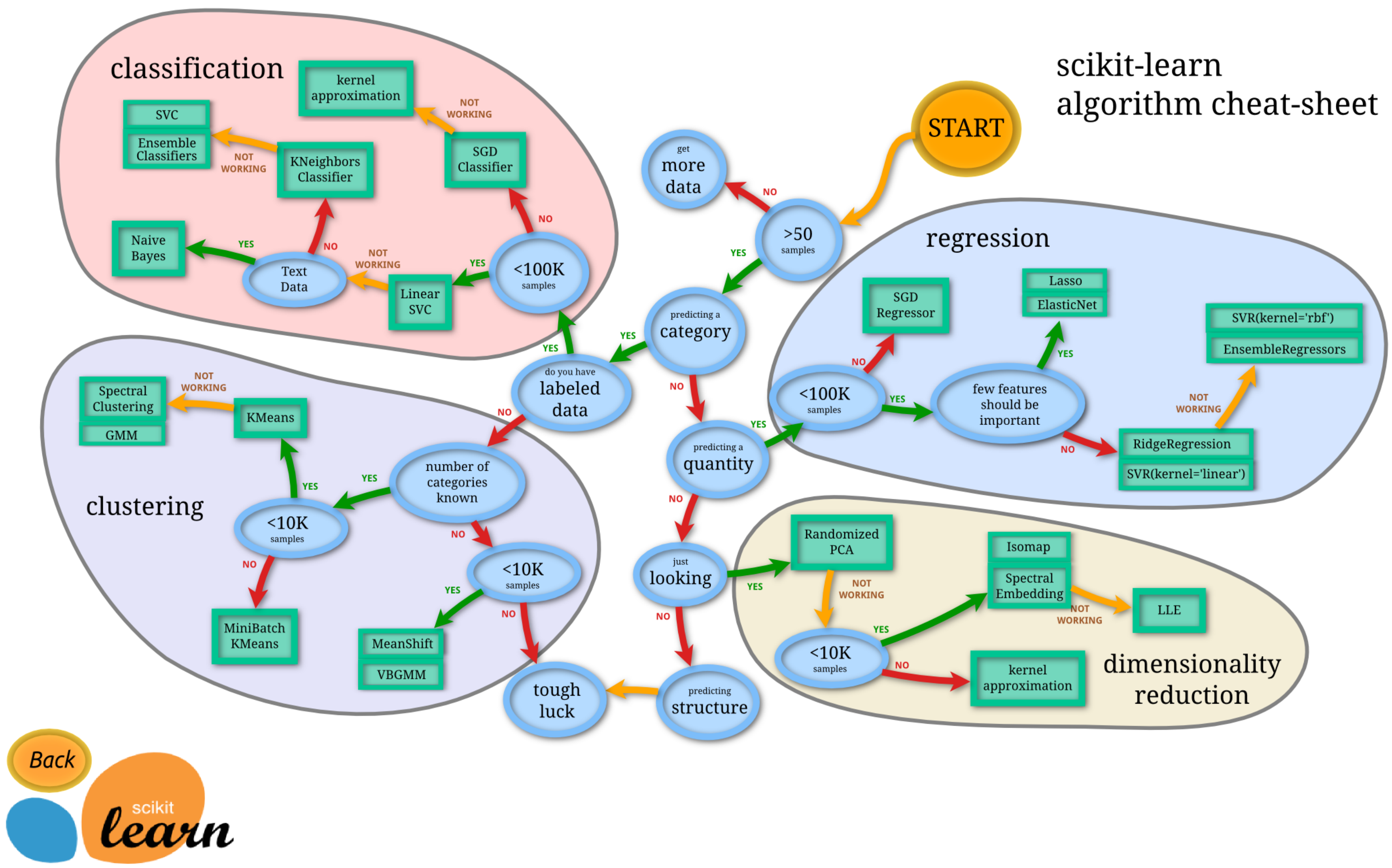
In other words

- Having a **process** $\vec{F} : \mathcal{D} \rightarrow \mathcal{I}$ that **transforms** a given $\vec{x} \in \mathcal{D}$ in a $\vec{y}$.
- Construct on a dataset $\Psi = \{\langle \vec{x}_i, \vec{y}_i \rangle\}$ with $i = 1, \ldots, N$.
- Each $\langle \vec{x}_i, \vec{y}_i \rangle$ represents an **input** and its corresponding **expected output**: $\vec{y}_i = \vec{F}(\vec{x}_i)$.
- **Optimize** a **model** $\mathcal{M}(\vec{x}; \vec{\theta})$ by adjusting its parameters $\vec{\theta}$.
  - Make $\mathcal{M}()$ to be as similar as possible to $\vec{F}()$ by optimizing one or more error (loss) functions.

Classification of ML

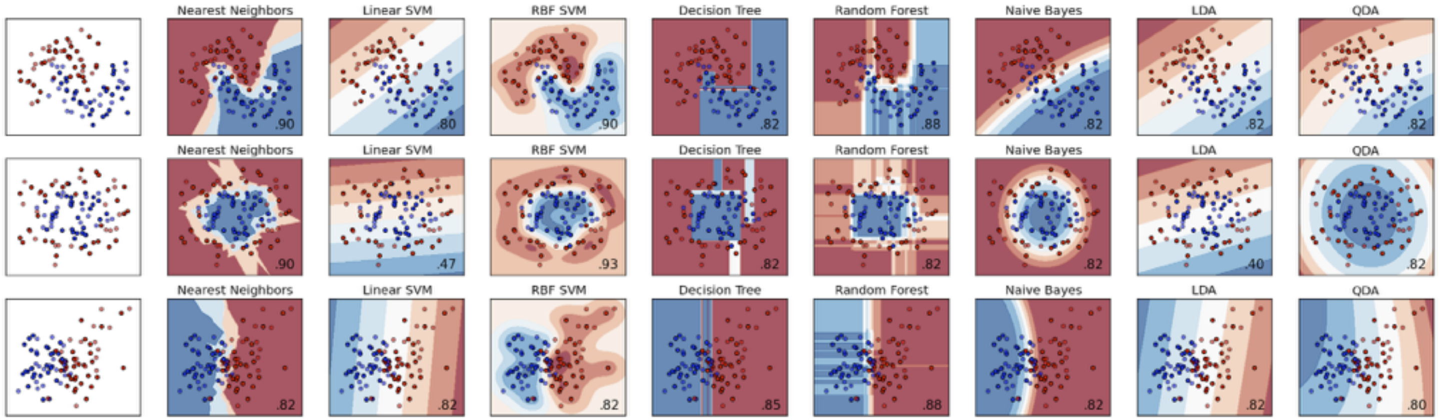
- **Classification**: $\vec{F} : \mathcal{D} \rightarrow \{1, \ldots, k\}$; $\vec{F}(\cdot)$ defines 'categories' or 'classes' labels.
- **Regression**: $\vec{F} : \mathbb{R}^n \rightarrow \mathbb{R}$; it is necessary to predict a real-valued output instead of categories.
- **Clustering**: group a set of objects in such a way that objects in the same group (*cluster*) are more *similar* to each other than to those in other groups (clusters).
- **Synthesis**: generate new examples that are similar to those in the training data

Classification of ML (cntd.)



Classification of ML (cntd.)

Many ML methods



- different assumptions on data
- different scalability profiles at training time
- different latencies at prediction time
- different model sizes (embedability in mobile devices)

Another Classification of ML

- **Supervised Learning** : Allows you to collect data or produce a data output from the previous experience.
- **Unsupervised Learning** : Finds all kind of unknown patterns in data.
- **Reinforced Learning** : It can be understood using the concepts of agents, environments, states, actions and rewards.

Supervised Learning $^{\text{hands on}}$

- Sometimes we can observe the pairs $\langle \vec{x}_i, \vec{y}_i \rangle$:
- We can use the $\vec{y}_i$'s to provide a *scalar feedback* on how good is the model $\mathcal{M}(\vec{x}; \vec{\theta})$.
- That feed back is known as the *loss function*.
- Modify parameters $\vec{\theta}$ as to improve $\mathcal{M}(\vec{x}; \vec{\theta}) \rightarrow$ *learning*.

Supervised Learning hands on

- import library

```
import random
import numpy as np
import matplotlib.pyplot as plt
```

- replicable random seed

```
random.seed(42)
```

- create input

```
x = np.arange(100)
```

Supervised Learning $^{\text{hands on}}$

- let's suppose that we have a phenomenon such that $y_{\text{real}} = \sin\left(\frac{\pi x}{50}\right)$

```
y_real = np.sin(x*np.pi/50)
```

- introducing some uniform random noise to simulate measurement noise

```
y_measured = y_real + (np.random.rand(100) - 0.5)
```

- plot the real vs measured

```
plt.scatter(x,y_measured, marker='.', color='b', label='measured')
plt.plot(x,y_real, color='r', label='real')
plt.xlabel('x'); plt.ylabel('y'); plt.legend(frameon=True);
```

Supervised Learning hands on

- let's use one of supervised method : Support Vector Machine

```
from sklearn.svm import SVR
clf = SVR() # using default parameters
```

- training

```
clf.fit(x.reshape(-1, 1), y_measured)
```

- predicting the output

```
y_pred = clf.predict(x.reshape(-1, 1))
```

Supervised Learning hands on

- plotting the result

```
plt.scatter(x, y_measured, marker='.', color='blue', label='measured')
plt.plot(x, y_pred, 'g--', label='predicted')
plt.xlabel('X'); plt.ylabel('y'); plt.legend(frameon=True);
```

- We observe for the first time an important negative phenomenon: overfitting.

Supervised Learning hands on

- We will be dedicating part of the course to the methods that we have for control overfitting.

```
clf = SVR(C=1e3, gamma=0.0001)
clf.fit(x.reshape(-1, 1), y_measured)
```

- predicting the output

```
y_pred_ok = clf.predict(x.reshape(-1, 1))
```

- plotting the result

```
plt.scatter(x, y_measured, marker='.', color='b', label='measured')
plt.plot(x, y_pred, 'g--', label='overfitted')
plt.plot(x, y_pred_ok, 'm-', label='not overfitted')
plt.xlabel('X'); plt.ylabel('y'); plt.legend(frameon=True);
```

Unsupervised Learning

In some cases we can just observe a series of items or values, e.g., $\Psi = \{\vec{x}_i\}$:

- It is necessary to find the *hidden structure* of *unlabeled data*.
- We need a measure of correctness of the model that does not requires an expected outcome.
- Although, at first glance, it may look a bit awkward, this type of problem is very common.
- Related to anomaly detection, clustering, etc.

Unsupervised Learning hands on

- Let's generate a dataset that is composed by three groups or clusters of elements, $\vec{x} \in \mathbb{R}^2$.

```
x_1 = np.random.randn(30,2) + (5,5)
x_2 = np.random.randn(30,2) + (10,0)
x_3 = np.random.randn(30,2) + (0,2)
```

- See the plot

```
plt.scatter(x_1[:,0], x_1[:,1], c='red', label='Cluster 1', alpha =0.74)
plt.scatter(x_2[:,0], x_2[:,1], c='blue', label='Cluster 2', alpha =0.74)
plt.scatter(x_3[:,0], x_3[:,1], c='green', label='Cluster 3', alpha =0.74)
plt.legend(frameon=True); plt.xlabel('$x_1$'); plt.ylabel('$x_2$');
plt.title('Three datasets');
```

Unsupervised Learning hands on

- Merge all data

```
x = np.concatenate(( x_1, x_2, x_3), axis=0)
```

- See the plot

```
plt.scatter(x[:,0], x[:,1], c='m', alpha =0.74)
plt.title('Training dataset');
```

Unsupervised Learning hands on

- let's use one of the unsupervised method : KMeans

```
from sklearn.cluster import KMeans
clus = KMeans(n_clusters=3)
```

- fit and predict the data

```
clus.fit(x)
labels_pred = clus.predict(x)
```

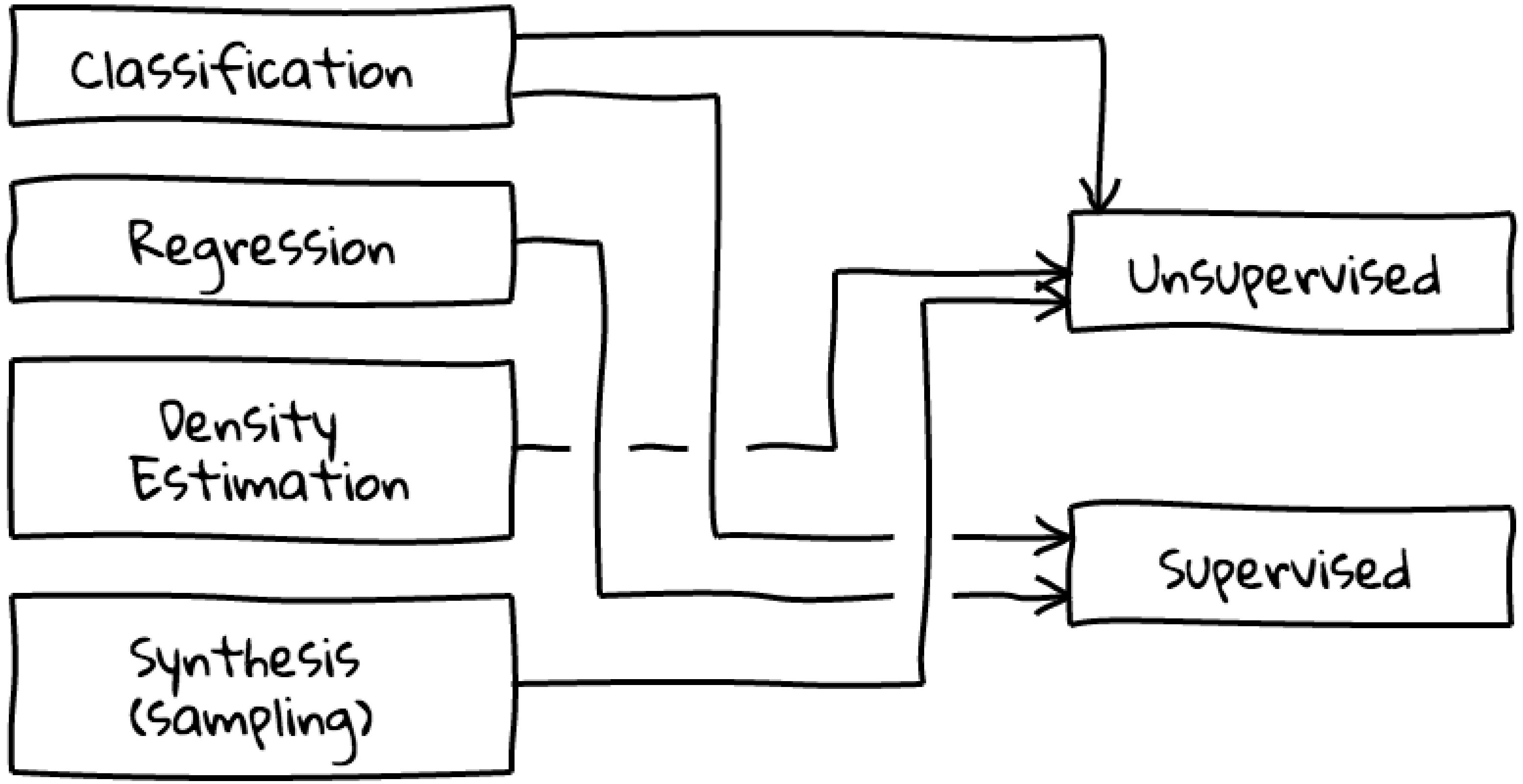
- See the plot

```
cm=iter(plt.cm.Set1(np.linspace(0,1,len(np.unique(labels_pred)))))
for label in np.unique(labels_pred):
    plt.scatter(x[labels_pred==label][:,0], x[labels_pred==label][:,1],
                c=next(cm), alpha =0.74, label='Pred. cluster ' +str(label+1))
plt.legend(loc='upper right', bbox_to_anchor=(1.45,1), frameon=True);
plt.xlabel('$x_1$'); plt.ylabel('$x_2$'); plt.title('Clusters predicted');
```

Reinforced Learning

- Inspired by behaviorist psychology;
- How to take actions in an environment so as to maximize some notion of cumulative reward?
- Differs from standard supervised learning in that correct input/output pairs are never presented,
- ...nor sub-optimal actions explicitly corrected.
- Involves finding a balance between exploration (of uncharted territory) and exploitation (of current knowledge)
- see : https://www.youtube.com/watch?v=yEOEqaEgu94 (4 minutes view)

Remark



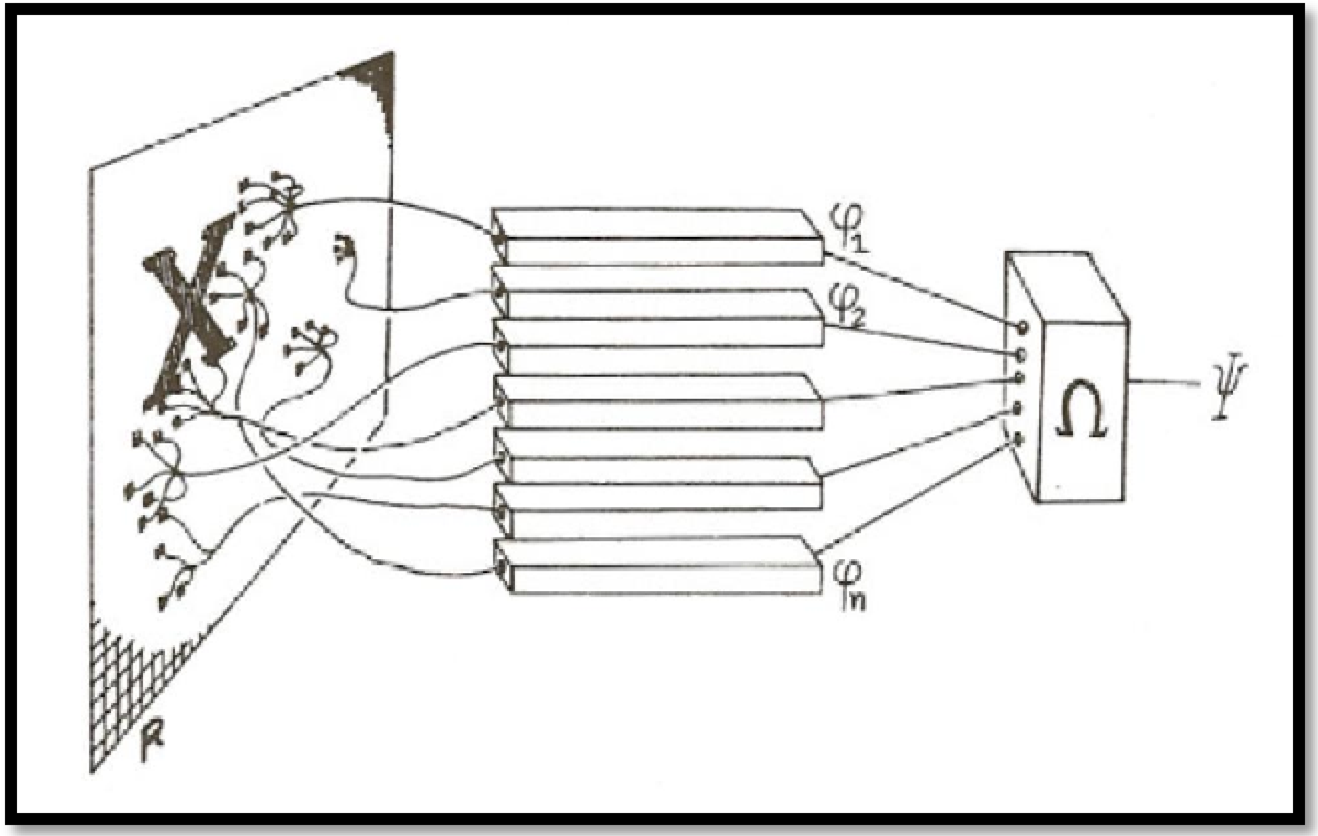
Classification
Regression
Density Estimation
Synthesis (sampling)
Unsupervised
Supervised

Let's focus on Artificial Neural Network

- Not all methods are applicable in real life
- Most featured method
- Nature inspired
- Efficient computation
- Evolutionary optimization

Artificial Neural Network

Artificial neuron



Artificial Neural Network

Artificial neuron as a neuron abstraction

In general terms, an input $\vec{x} \in \mathbb{R}^n$ is multiplied by a weight vector $\vec{w}$ and added a bias b producing the net activation, net. net is passed to the *activation function* $f()$ that computed the neuron's output $\hat{y}$.

$$\hat{y} = f\left(\mathrm{net}\right) = f\left(\vec{w} \cdot \vec{x} + b\right) = f\left(\sum_{i=1}^{n} w_i x_i + b\right).$$

Artificial Neural Network

The perceptron

The Perceptron and its learning algorithm pioneered the research in neurocomputing.

- The perceptron is an algorithm for learning a linear binary classifier.
- That is a function that maps its input $\vec{x} \in \mathbb{R}^n$ (a real-valued vector) to an output value $f(\vec{x})$ (a single binary value) as,

$$f(\vec{x}) = \begin{cases} 1 & \text{if } \vec{w} \cdot \vec{x} + b > 0\,, \\ 0 & \text{otherwise;} \end{cases}$$

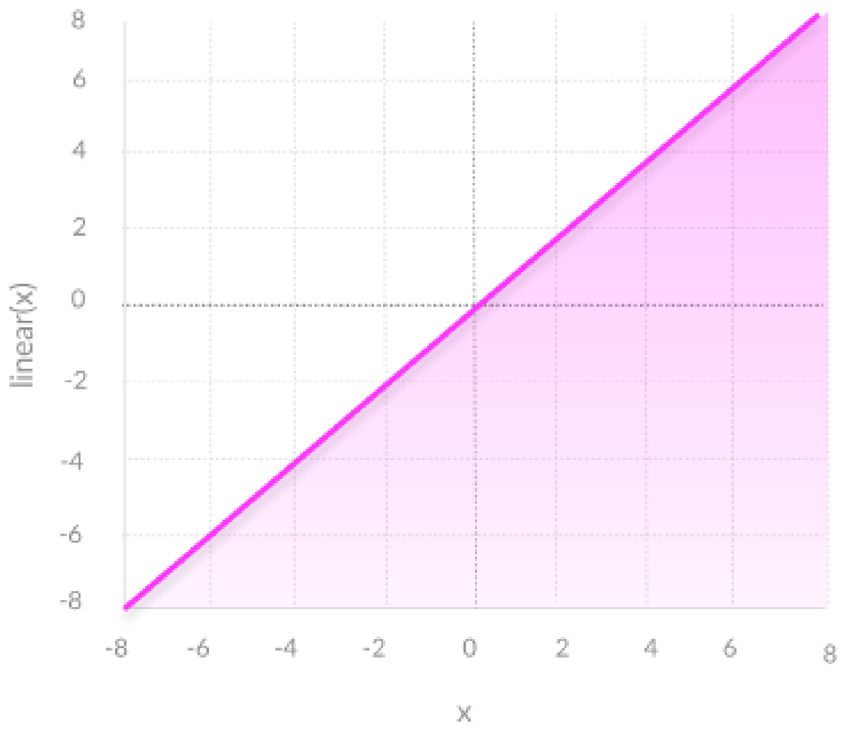
where $\vec{w}$ is a vector of real-valued *weights*, $\vec{w} \cdot \vec{x}$ is the *dot product* $\sum_{i=1}^{n} w_i x_i$, and b is known as the *bias*.

Activation functions

Linear Function `nn.Linear`

$$f(x) = x + b$$

- Disadvantages
  - Not possible to use backpropagation
  - All layers of the neural network collapse into one
  - Limited power to handle complexity

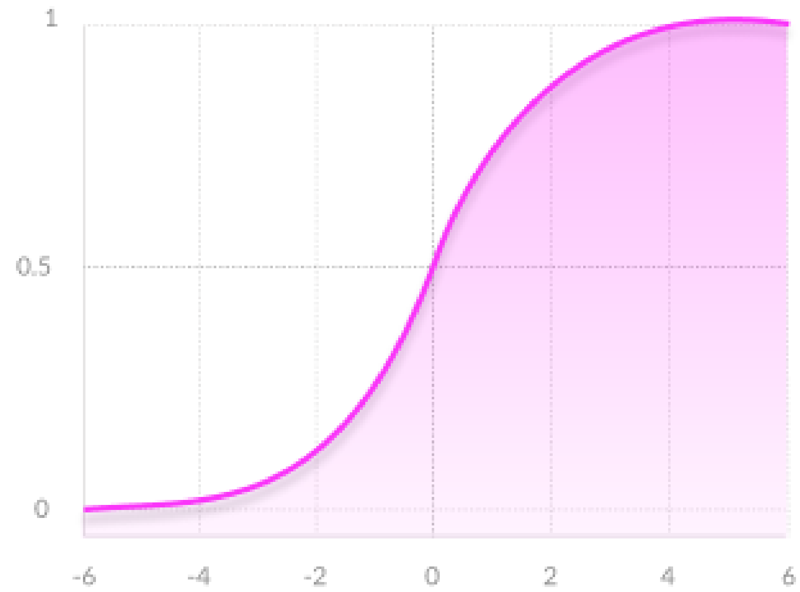


Activation functions

Sigmoid Function `nn.Sigmoid`

$$f(x) = \frac{1}{1+e^{-x}}$$

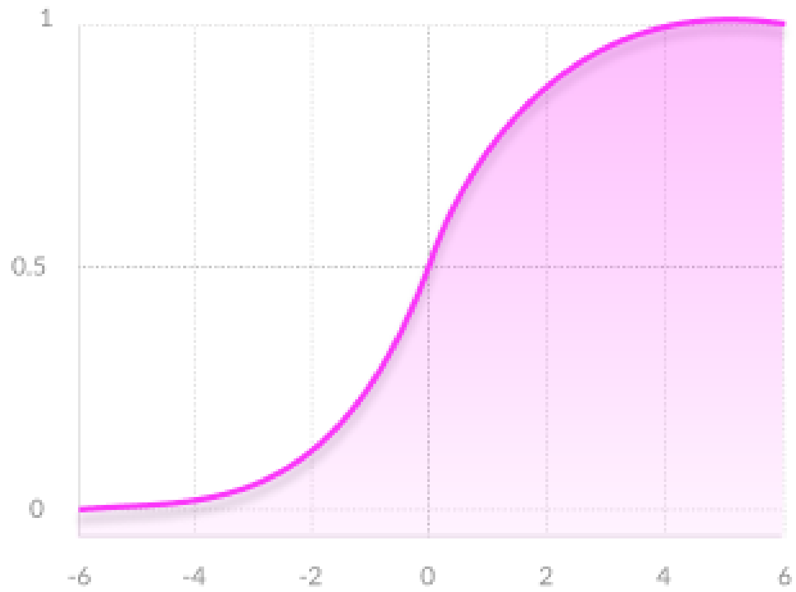
- Advantages
  - Smooth gradient
  - Clear predictions
- Disadvantages
  - Vanishing gradient
  - Outputs not zero centered
  - Computationally expensive



Activation functions

Hyperbolic Tangent Function `nn.Tanh`

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

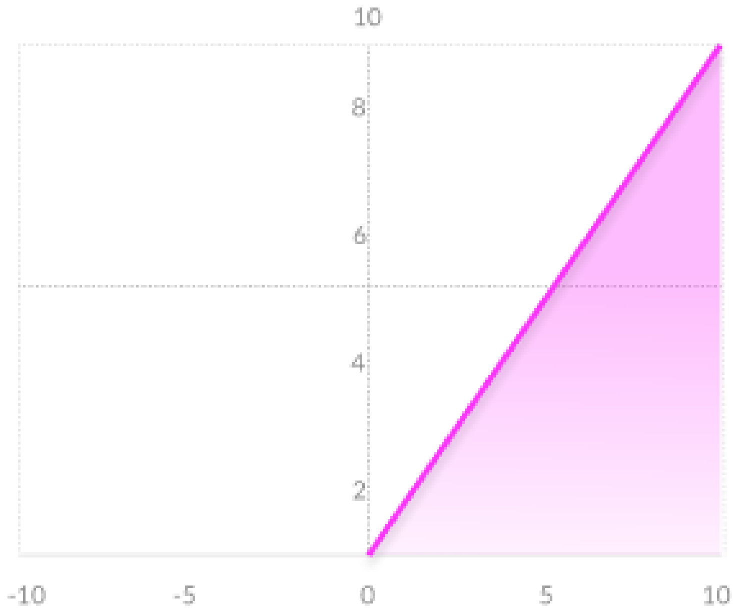


- Advantages
  - Zero centered : strong negative, neutral, and positive values.
  - Otherwise like the Sigmoid function.
- Disadvantages
  - Like the Sigmoid function

Activation functions

ReLU Function `nn.ReLU`

$$f(x) = max(0, x)$$



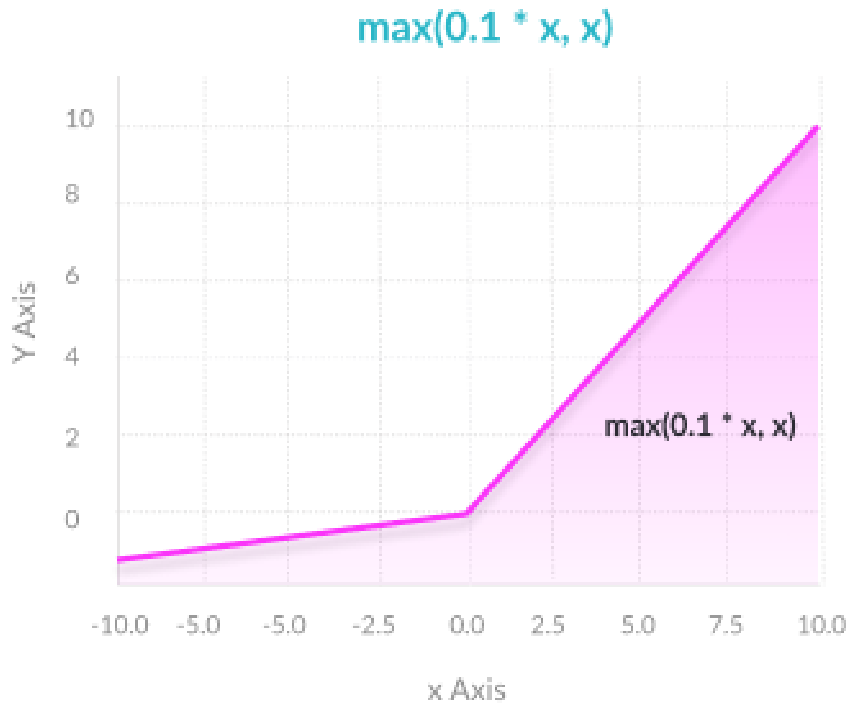
- Advantages
  - Computationally efficient : network converge very quickly
  - Non-linear : allows for backpropagation
- Disadvantages
  - The Dying ReLU problem

Activation functions

Leaky ReLU Function `nn.LeakyReLU`

$$f(x) = max(0, x)$$

- Advantages
  - Prevents dying ReLU problem
  - Otherwise like ReLU
- Disadvantages
  - Results not consistent

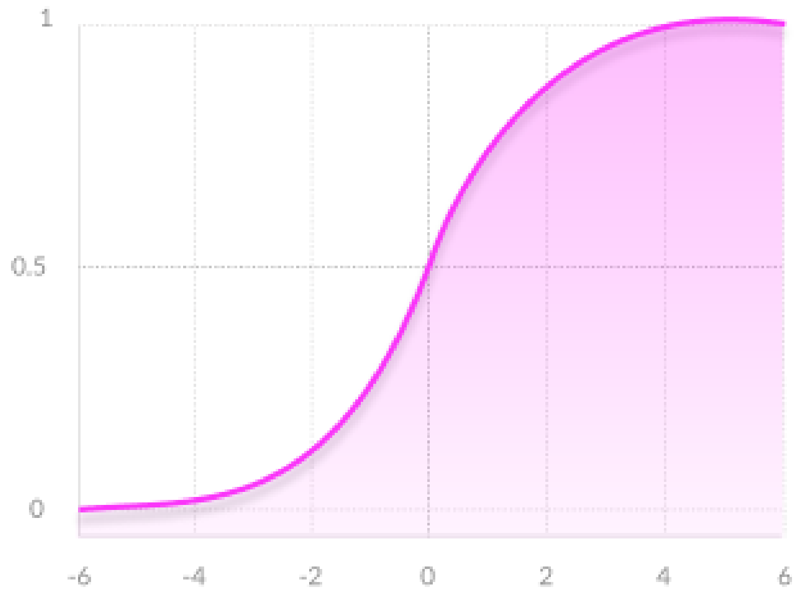


Activation functions

Sotmax Function `nn.Softmax`

$$f(x_i) = \frac{e^{x_i}}{\sum_j e^{x_j}}$$

- Advantages
  - Able to handle multiple classes only one class in other activation functions
  - normalizes the outputs for each class between 0 and 1
  - Useful for output neurons



Perceptron learning

Learning goes by calculating the prediction of the perceptron, $\hat{y}$, as

$$\hat{y} = f\left(\vec{w} \cdot \vec{x} + b\right) = f(w_1 x_1 + w_2 x_2 + \cdots + w_n x_n + b)\ .$$

After that, we update the weights and the bias using the perceptron rule:

$$w_i = w_i + \alpha(y - \hat{y})x_i\ ,\ i = 1, \ldots, n\,;$$
$$b = b + \alpha(y - \hat{y})\,.$$

Here $\alpha \in \left(0, 1\right]$ is known as the *learning rate*. Or can be further enhanced using momentum :

$$\vec{w}(t+1) = \vec{w}(t) + \alpha\Delta\vec{w}(t) + \beta\Delta\vec{w}(t-1),$$

where $\beta \in \mathbb{R}^+$ is known as the momentum rate.

Study of Learning Rate α hands on

- error function $\rightarrow E(\boldsymbol{X}, \boldsymbol{y}; \boldsymbol{w}) = \frac{1}{2} \left\| \boldsymbol{X} \cdot \boldsymbol{w} - \boldsymbol{y} \right\|_2^2$.

```
def error(X, y, w):
    return 0.5*np.linalg.norm(X.dot(w) - y)**2
```

- the gradient $\rightarrow \nabla \boldsymbol{w} = \nabla_{\boldsymbol{w}} E(\boldsymbol{X}, \boldsymbol{y}; \boldsymbol{w}) = \boldsymbol{X}^T \cdot (\boldsymbol{X} \cdot \boldsymbol{w} - \boldsymbol{y})$.

```
def linear_regression_gradient(X, y, w):
    return X.T.dot(X.dot(w)-y)
```

- gradient descent loop

```
def gradient_descent(X, y, w_0, alpha, max_iters):
    'Returns the values of the weights as learning took place.'
    w = np.array(w_0, dtype=np.float64)
    w_hist = np.zeros(shape=(max_iters+1, w.shape[0]))
    w_hist[0] = w
    for i in range(0,max_iters):
        delta_weights = -alpha*linear_regression_gradient(X_bias, y, w)
        w += delta_weights
        w_hist[i+1] = w
    return w_hist
```

- plot contour

```
def plot_contour(X_data, y_data, bounds, resolution=50, cmap=cm.viridis,
                 alpha=0.3, linewidth=5, rstride=1, cstride=5, ax=None):
    (minx,miny),(maxx,maxy) = bounds

    x_range = np.linspace(minx, maxx, num=resolution)
    y_range = np.linspace(miny, maxy, num=resolution)
    X, Y = np.meshgrid(x_range, y_range)

    Z = np.zeros((len(x_range), len(y_range)))

    for i, w_i in enumerate(x_range):
        for j, w_j in enumerate(y_range):
            Z[j,i] = error(X_data, y_data, [w_i, w_j])
```

(continuing from previous page)

```
    if not ax:
        fig = plt.figure(figsize=(6,6))
        ax = fig.gca()
        ax.set_aspect('equal')
        ax.autoscale(tight=True)
    cset = ax.contourf(X, Y, Z, 30, cmap=cmap, rstride=rstride,
                       cstride=cstride, linewidth=linewidth, alpha=alpha)
    cset = ax.contour(X, Y, Z, 10, cmap=cmap, rstride=rstride,
                       cstride=cstride, linewidth=linewidth)
    plt.clabel(cset, inline=1, fontsize=7)
    return Z
```

- try initialize variables

```
X = np.array([[0.0], [1.0], [2.0], [3.0], [4.0]])
X_bias = np.hstack((X, np.ones((N, 1))))
y = np.array([10.5, 5.0, 3.0, 2.5, 1.0])

w_0 = [-3,2]
alpha = 0.05
max_iters = 25
```

- run learning

```
w_hist = gradient_descent(X_bias, y, w_0, alpha, max_iters)

plot_hist_contour(X_bias, y, w_hist, w_norm, title='end='+str(w_hist[-1]), show_legend=True)
```

- function to run learning on several alpha

```
def alphas_study(alphas):
    fig = plt.figure(figsize=(11,7))
    for i,alpha in enumerate(alphas):
        ax = fig.add_subplot(2,3,i+1)
        w_hist = gradient_descent(X_bias, y , w_0, alpha, max_iters)
        plot_hist_contour(X_bias, y, w_hist, w_norm, ax=ax, title='$\\alpha='+str(alpha)+'$')
    plt.legend(scatterpoints=1, ncol=3, bbox_to_anchor=(-0.2,-0.2), frameon=True);
    plt.tight_layout()
```

- the alpha

```
alphas = np.linspace(0.02,0.07,6)
```

- study the alpha

```
alphas_study(alphas)
```

Study of Momentum Rate β hands on

- gradient descent with momentum

```
def gradient_descent_with_momentum(X, y, w_0, alpha, beta, max_iters):
    w = np.array(w_0, dtype=np.float64)
    w_hist = np.zeros(shape=(max_iters+1, w.shape[0]))
    w_hist[0] = w
    omega = np.zeros_like(w)
    for i in range(max_iters):
        delta_weights = -alpha*linear_regression_gradient(X, y, w) + beta*omega
        omega = delta_weights
        w += delta_weights
        w_hist[i+1] = w
    return w_hist
```

- set the variables

```
alpha = 0.05
beta = 0.5
max_iters = 25
```

- run the momentum learning

```
w_hist = gradient_descent(X_bias, y, (-3,2), alpha, max_iters)
w_hist_mom = gradient_descent_with_momentum(X_bias,y, (-3,2), alpha, beta, max_iters)
```

- compare plot

```
def comparison_plot():
    fig = plt.figure(figsize=(9,4.5))
    ax = fig.add_subplot(121)
    plot_hist_contour(X_bias, y, w_hist, \
        w_norm, ax=ax, title='Gradient descent')
    ax = fig.add_subplot(122)
    plot_hist_contour(X_bias, y, w_hist_mom, \
        w_norm, ax=ax, title='Gradient descent with momentum', show_legend=True)
    plt.tight_layout()
```

- the plot

```
comparison_plot()
```

- study alpha and momentum

```
def alphas_study_with_momentum(alphas, beta):
    fig = plt.figure(figsize=(11,7))
    for i,alpha in enumerate(alphas):
        ax = fig.add_subplot(2,3,i+1)
        w_hist = gradient_descent_with_momentum(X_bias, y , w_0, alpha, beta, max_iters)
        plot_hist_contour(X_bias, y, w_hist, w_norm, ax=ax, title='$\\alpha='+str(alpha)+'$')
    plt.legend(scatterpoints=1, ncol=3, bbox_to_anchor=(-0.2,-0.2), frameon=True);
    plt.tight_layout()
```

- run it

```
alphas_study_with_momentum(alphas, 0.5)
```

Multilayer Perceptron

The composition of layers of perceptrons can capture complex relations between inputs and outputs in a hierarchical way. In order to proceed we need to improve the notation we have been using. That for, for each layer $1 \geq l \geq L$, the activations and outputs are calculated as:

$$\mathrm{net}_j^l = \sum_i w_{ji}^l x_i^l \qquad | \qquad y_j^l = f^l(\mathrm{net}_j^l)\,,$$

where:

- y_j^l is the jth output of layer l,
- x_i^l is the ith input to layer l,
- w_{ji}^l is the weight of the j-th neuron connected to input i,
- net_j^l is called net activation, and
- $f^l(\cdot)$ is the activation function of layer l, e.g. $\tanh()$, in the hidden layers and the identity in the last layer (for regression)

Training MLPs with Backpropagation

- Backpropagation of errors is a procedure to compute the **gradient of the error function with respect to the weights** of a neural network.
- We can use the gradient from backpropagation to apply **gradient descent**!

A math flashback

The **chain rule** can be applied in composite functions as,

$$(f \circ g)'(x) = (f(g(x)))' = f'(g(x))\, g'(x).$$

or, in Leibniz notation,

$$\frac{\partial f(g(x))}{\partial x} = \frac{\partial f(g(x))}{\partial g(x)} \cdot \frac{\partial g(x)}{\partial x}$$

The **total derivative** of $f(x_1, x_2, ... x_n)$ on x_i is

$$\frac{\partial f}{\partial x_i} = \sum_{j=1}^{n} \frac{\partial f}{\partial x_j} \cdot \frac{\partial x_j}{\partial x_i}$$

To apply gradient descent we need... to calculate the gradients

Applying the chain rule,

$$\frac{\partial \ell}{\partial w_{ji}^l} = \overbrace{\frac{\partial \ell}{\partial \mathrm{net}_j^l}}^{\delta_j^l} \underbrace{\frac{\partial \mathrm{net}_j^l}{\partial w_{ji}^l}}_{\frac{\partial\left(\sum_i w_{ji}^l x_i^l\right)}{\partial w_{ji}^l} = x_i^l}$$

hence we can write

$$\frac{\partial \ell}{\partial w_{ji}^l} = \delta_j^l x_i^l$$

What about the hidden layers ($1 \leq l < L$)?

We can express the loss ℓ as a function of the activations of the subsequent layer,

$$\ell = \ell\left(\mathrm{net}_1^{l+1}, \ldots, \mathrm{net}_K^{l+1}\right) \ ,$$

therefore, applying total derivatives,

$$\frac{\partial \ell}{\partial \hat{y}_j^l} = \frac{\partial \ell\left(\mathrm{net}_1^{l+1}, \ldots, \mathrm{net}_K^{l+1}\right)}{\partial \hat{y}_j^l} \ .$$

For the output layer ($l = L$)

$$\delta_j^L = \frac{\partial \ell}{\partial \mathrm{net}_j^L} = \overbrace{\frac{\partial \ell}{\partial \hat{y}_j^L}}^{\frac{\partial\left(\frac{1}{2}\sum_j\left(y_j - \hat{y}_j^L\right)^2\right)}{\partial \hat{y}_j^L} = \left(y_j - \hat{y}_j^L\right)} \cdot \underbrace{\frac{\partial \hat{y}_j^L}{\mathrm{net}_j^l}}_{f'(\mathrm{net}_j^L)} = \left(y_j - \hat{y}_j^L\right) f'(\mathrm{net}_j^L).$$

therefore

$$\frac{\partial \ell}{\partial w_{ji}^L} = \left(y_j - \hat{y}_j^L\right) f'(\mathrm{net}_j^L) x_i^L$$

Back-propagating the errors to the hidden layer

The δs of the subsequent layers are used to calculate the δs of the more internal ones.

$$\delta_j^l = \frac{\partial \ell}{\partial \mathrm{net}_j^l} = \frac{\overbrace{\partial \ell}^{\sum_k \delta_k^{l+1} w_{kj}^{l+1}}}{\partial \hat{y}_j^l} \underbrace{\frac{\partial \hat{y}_j^l}{\partial \mathrm{net}_j^l}}_{f'(\mathrm{net}_j^l)} = \sum_k \left(\delta_k^{l+1} w_{kj}^{l+1}\right) f'(\mathrm{net}_j^l)$$

Briefly, in each layer (we will omit the sample index k and layer l)

$$\delta_j = \begin{cases} \hat{y}_j - y_j & \text{in the output layer,} \\ \\ f'(\text{net}_j) \sum_k \frac{\partial \ell}{\partial \hat{y}_k} & \text{otherwise.} \end{cases}$$

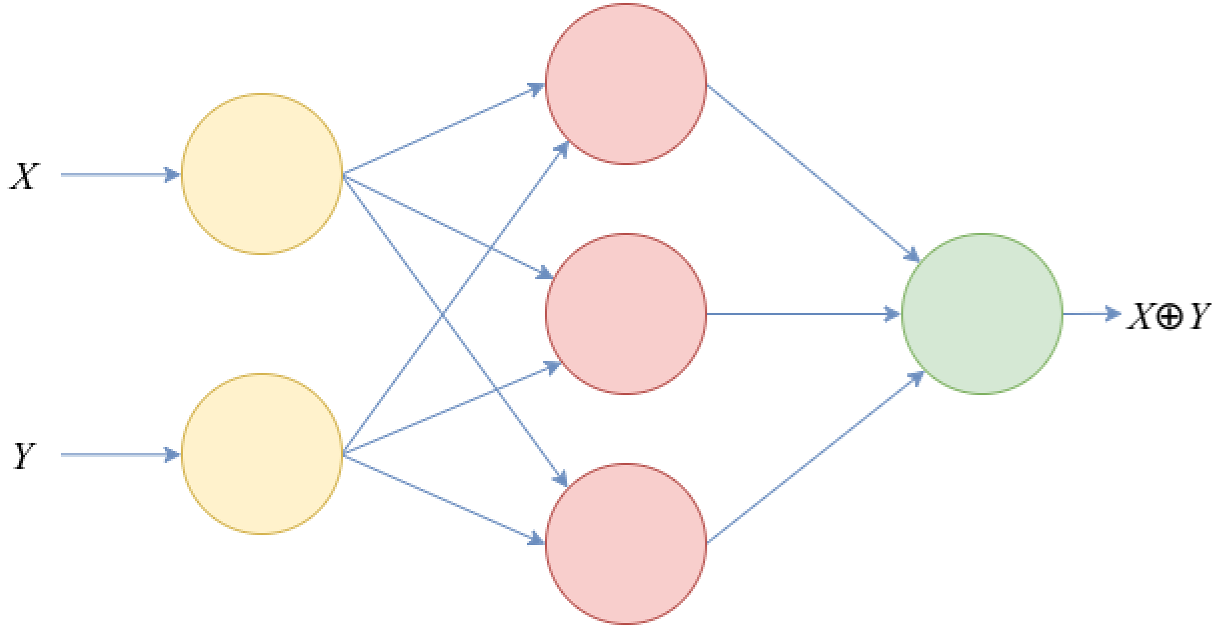
$$\frac{\partial \ell}{\partial w_{ji}} = \delta_j x_i \, ; \quad \frac{\partial \ell}{\partial x_i} = \delta_j w_{ji} \, .$$

where

- all nodes k are in the layer after j;
- net_j is known from propagation: $\sum_i w_{ji} x_i$;
- actually you do not have to save a_j because $g'(a_j)$ usually can be computed from y_j, e.g.
  - identity function: $f'(\text{net}_j) = 1$,
  - $\tanh$: $f'(\text{net}_j) = 1 - y_j^2$;
- $\frac{\partial \ell}{\partial w_{ji}}$ will be used to update the weight w_{ji} in gradient descent;
- $\frac{\partial \ell}{\partial x_i}$ will be passed to the previous layer to compute the deltas;

Build MLP in PyTorch

- Let's create network to model XOR gate



- The XOR truth table

x	y	XOR
0	0	0
0	1	1
1	0	1
1	1	0

- Input pair

```
inputs = list(map(lambda s: Variable(torch.Tensor([s])), [
    [0, 0],
    [0, 1],
    [1, 0],
    [1, 1]
]))
```

- The target

```
targets = list(map(lambda s: Variable(torch.Tensor([s])), [
    [0],
    [1],
    [1],
    [0]
]))
```

- The network

```
class XOR(nn.Module):
    def __init__(self):
        super(XOR, self).__init__()
        self.fc1 = nn.Linear(2, 3, True)
        self.fc2 = nn.Linear(3, 1, True)

    def forward(self, x):
        x = F.sigmoid(self.fc1(x))
        x = self.fc2(x)
        return x
```

- Initialize the network

```
net = XOR()
```

- Epoch, criterion and optimizer

```
EPOCHS = 50000
criterion = nn.MSELoss()
optimizer = optim.SGD(net.parameters(), lr=0.01)
```

- Training loop

```
print("Training loop:")
for idx in range(0, EPOCHS):
    for input, target in zip(inputs, targets):
        optimizer.zero_grad()   # zero the gradient buffers
        output = net(input)
        loss = criterion(output, target)
        loss.backward()
        optimizer.step()    # Does the update
    if idx % 5000 == 0:
        print("Epoch {: >8} Loss: {}".format(idx, loss.data.numpy()))
```

- The results:

```
print("Final results:")
for input, target in zip(inputs, targets):
    output = net(input)
    print("Input:[{},{}] Target:[{}] Predicted:[{}] Error:[{}]".format(
        int(input.data.numpy()[0][0]),
        int(input.data.numpy()[0][1]),
        int(target.data.numpy()[0]),
        round(float(output.data.numpy()[0]), 4),
        round(float(abs(target.data.numpy()[0] - output.data.numpy()[0])), 4)
    ))
```

- Inference

```
output = net(Variable(torch.Tensor([1, 0])))
print(output)
```

Repository

All material in this course can be cloned from https://github.com/linerocks/vibrastic101

References

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End of slide

Thank You !