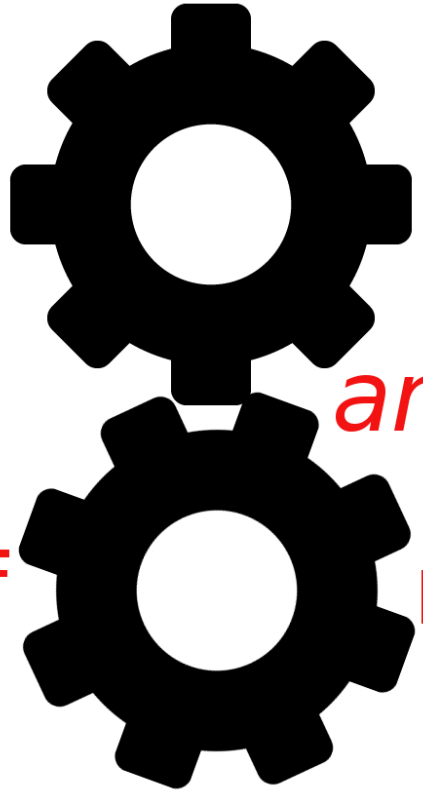


C de analysis *and* transf rmation

Loop transformations

Simone Campanoni
simonec@eecs.northwestern.edu

Outline

- Simple loop transformations
- Loop invariants based transformations
- Induction variables based transformations
- Complex loop transformations

Simple loop transformations

Simple loop transformations are used to

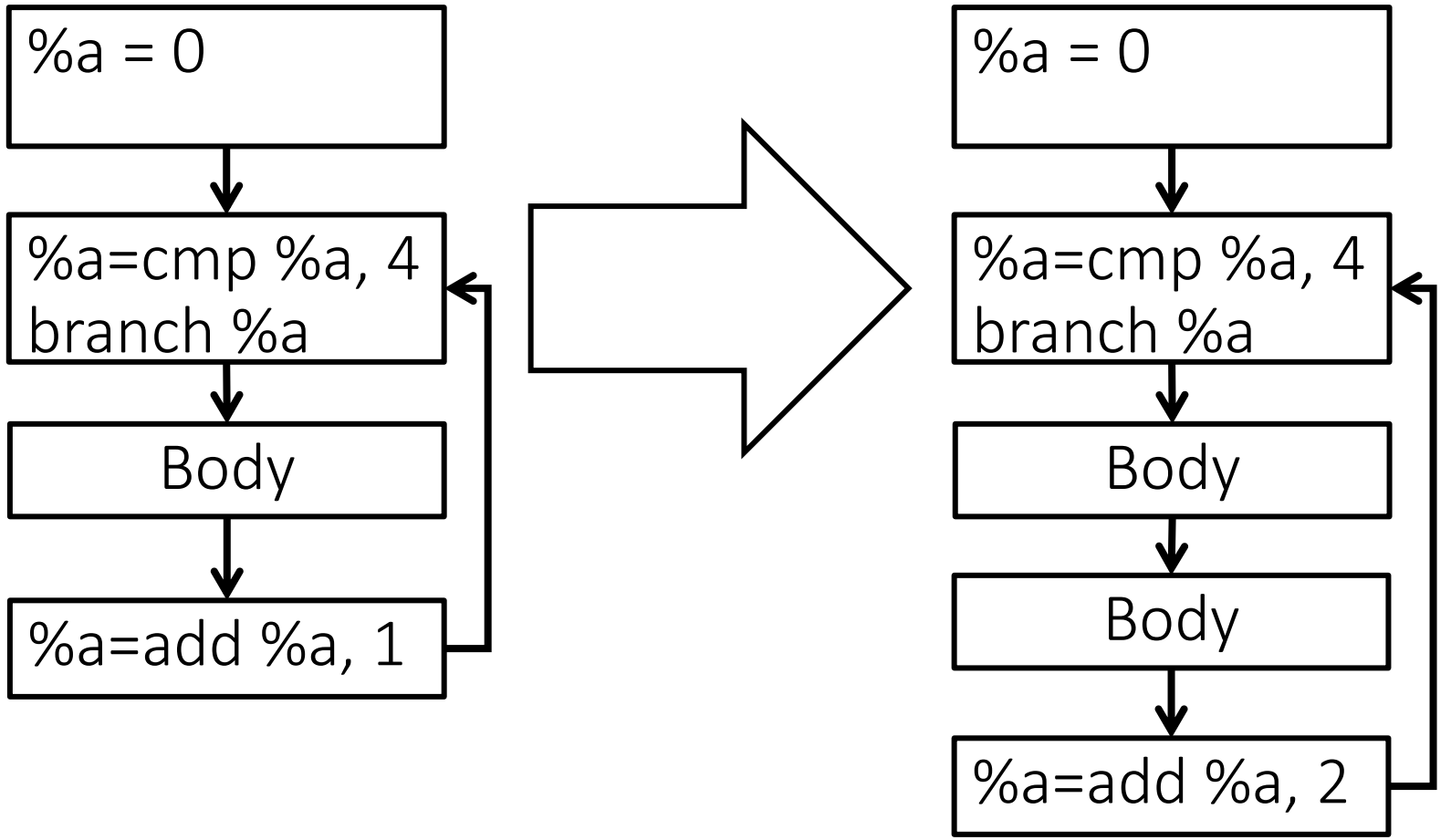
- Increase performance/energy savings

  and/or

- Unblock other transformations
  - E.g., increase the number of constant propagations
  - E.g., Extract thread-level parallelism from sequential code
  - E.g., Generate vector instructions

Loop unrolling

```
for (a=0; a < 4; a++){
   … // Body
}
```



Loop unrolling in LLVM: requirements

- The loop you want to unroll must be in LCSSA form

Loop unrolling in LLVM: dependences

```
void getAnalysisUsage(AnalysisUsage &AU) const override {
  AU.addRequired<AssumptionCacheTracker>();
  AU.addRequired<DominatorTreeWrapperPass>();
  AU.addRequired<LoopInfoWrapperPass>();
  AU.addRequired<ScalarEvolutionWrapperPass>();

  return ;
}
```

Loop unrolling in LLVM: headers

```
#include "llvm/Analysis/OptimizationRemarkEmitter.h"
#include "llvm/IR/Dominators.h"
#include "llvm/Transforms/Utils/LoopUtils.h"
#include "llvm/Transforms/Utils/UnrollLoop.h"
#include "llvm/Analysis/AssumptionCache.h"
#include "llvm/Analysis/ScalarEvolution.h"
#include "llvm/Analysis/ScalarEvolutionExpressions.h"
```

Loop unrolling in LLVM

Get the results of the required analyses

```
auto& LI = getAnalysis<LoopInfoWrapperPass>().getLoopInfo();
auto& DT = getAnalysis<DominatorTreeWrapperPass>().getDomTree();
auto& SE = getAnalysis<ScalarEvolutionWrapperPass>().getSE();
auto& AC = getAnalysis<AssumptionCacheTracker>().getAssumptionCache(F);
```

Fetch a loop

```
for (auto i : LI){
  auto loop = &*i;
  ...
}
```

```
void getAnalysisUsage(AnalysisUsage &AU) const override {
  AU.addRequired<AssumptionCacheTracker>();
  AU.addRequired<DominatorTreeWrapperPass>();
  AU.addRequired<LoopInfoWrapperPass>();
  AU.addRequired<ScalarEvolutionWrapperPass>();

  return ;
}
```

```
auto& LI = getAnalysis<LoopInfoWrapperPass>().getLoopInfo();
auto& DT = getAnalysis<DominatorTreeWrapperPass>().getDomTree();
auto& SE = getAnalysis<ScalarEvolutionWrapperPass>().getSE();
auto& AC = getAnalysis<AssumptionCacheTracker>().getAssumptionCache(F);
```

Loop unrolling in LLVM: API

Loop to unroll

```
auto forceUnroll = false;
auto allowRuntime = false;
auto allowExpensiveTripCount = true;
auto preserveCondBr = false;
auto preserveOnlyFirst = false;
auto unrolled = UnrollLoop(
  loop, 2,
  tripCount,
  forceUnroll,
  allowRuntime, allowExpensiveTripCount,
  preserveCondBr, preserveOnlyFirst,
  0, 0,
  false,
  &LI, &SE, &DT, &AC, &ORE,
  true);
```

Unroll factor

Loop unrolling in LLVM: API

```
auto tripCount = SE.getSmallConstantTripCount(loop);
```

It is 0, or the number of iterations known by SCE

```
auto forceUnroll = false;
auto allowRuntime = false;
auto allowExpensiveTripCount = true;
auto preserveCondBr = false;
auto preserveOnlyFirst = false;
auto unrolled = UnrollLoop(
  loop, 2,
  tripCount,
  forceUnroll,
  allowRuntime, allowExpensiveTripCount,
  preserveCondBr, preserveOnlyFirst,
  0, 0,
  false,
  &LI, &SE, &DT, &AC, &ORE,
  true);
```

Maximum number of iterations of this loop

```
auto& LI = getAnalysis<LoopInfoWrapperPass>().getLoopInfo();
auto& DT = getAnalysis<DominatorTreeWrapperPass>().getDomTree();
auto& SE = getAnalysis<ScalarEvolutionWrapperPass>().getSE();
auto& AC = getAnalysis<AssumptionCacheTracker>().getAssumptionCache(F);
```

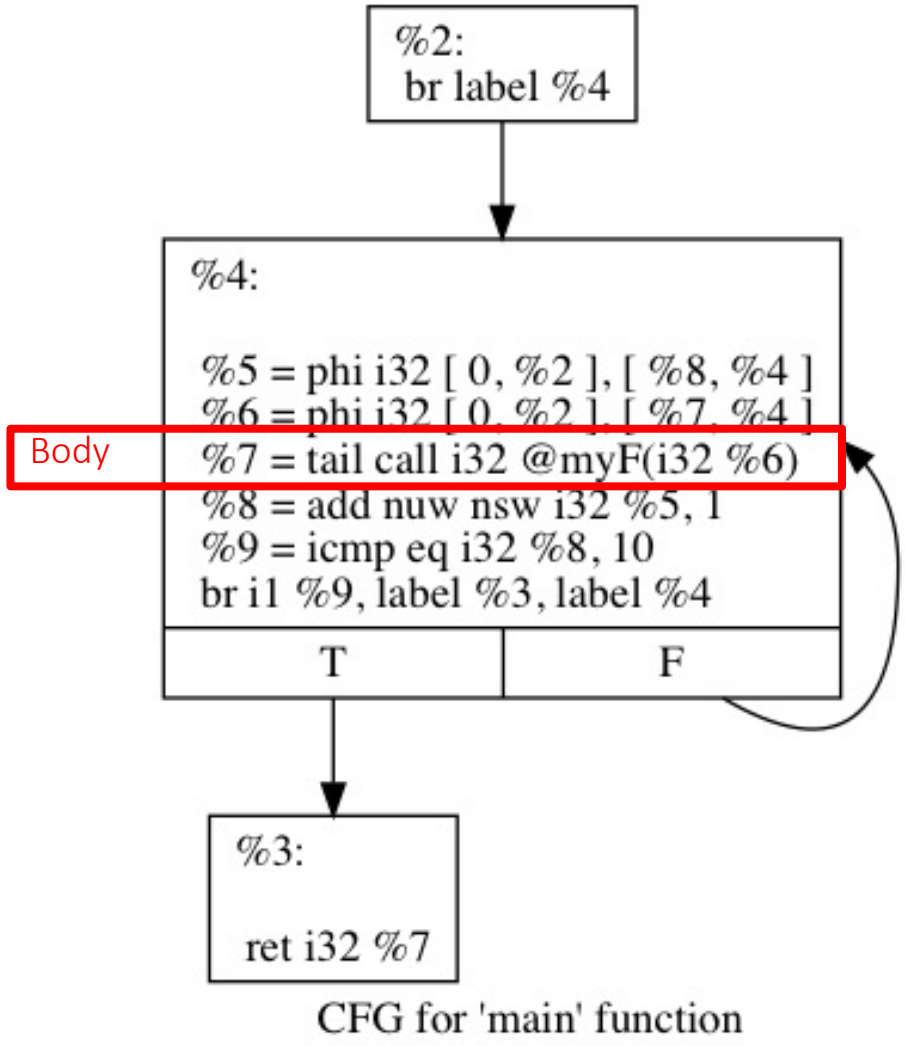
```
void getAnalysisUsage(AnalysisUsage &AU) const override {
  AU.addRequired<AssumptionCacheTracker>();
  AU.addRequired<DominatorTreeWrapperPass>();
  AU.addRequired<LoopInfoWrapperPass>();
  AU.addRequired<ScalarEvolutionWrapperPass>();

  return ;
}
```

Loop unrolling in LLVM: result

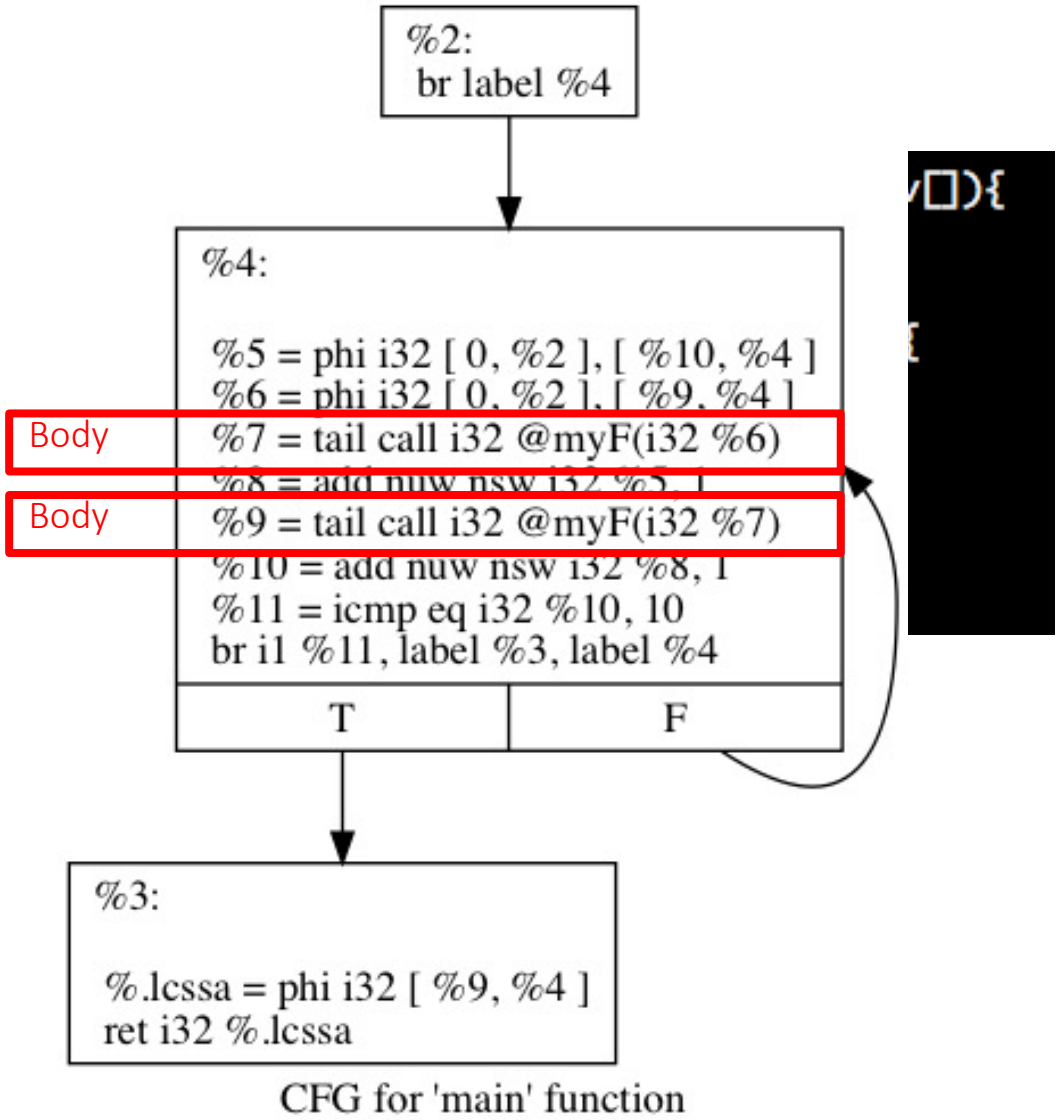
```
auto forceUnroll = false;
auto allowRuntime = false;
auto allowExpensiveTripCount = true;
auto preserveCondBr = false;
auto preserveOnlyFirst = false;
auto unrolled = UnrollLoop(
  loop, 2,
  tripCount,
  forceUnroll,
  allowRuntime, allowExpensiveTripCount,
  preserveCondBr, preserveOnlyFirst,
  0, 0,
  false,
  &LI, &SE, &DT, &AC, &ORE,
  true);
```

```
switch (unrolled){
  case LoopUnrollResult::FullyUnrolled :
    errs() << "   Fully unrolled\n";
    return true ;

  case LoopUnrollResult::PartiallyUnrolled :
    errs() << "   Partially unrolled\n";
    return true ;

  case LoopUnrollResult::Unmodified :
    errs() << "   Not unrolled\n";
    break ;

  default:
    abort();
}
```

Loop unrolling in LLVM: example



CFG for 'main' function



CFG for 'main' function

Loop unrolling in LLVM: Demo

- Detail: LLVM_loops/README
- Pass: LLVM_loops/llvm/7
- C program: LLVM_loops/code/12
- C program: LLVM_loops/code/0

Loop unrolling: the trip count

```
auto forceUnroll = false;
auto allowRuntime = false;
auto allowExpensiveTripCount = true;
auto preserveCondBr = false;
auto preserveOnlyFirst = false;
auto unrolled = UnrollLoop(
  loop, 2,
  tripCount,
  forceUnroll,
  allowRuntime, allowExpensiveTripCount,
  preserveCondBr, preserveOnlyFirst,
  0, 0,
  false,
  &LI, &SE, &DT, &AC, &ORE,
  true);
```

```
 8 int main (int argc, char *argv[]){
 9   auto r = 0;
10
11   for (auto i=0; i < 10; i++){
12     r = myF(r);
13   }
14
15   return r;
16 }
```

```
 7 int main (int argc, char *argv[]){
 8   auto r = 0;
 9
10   for (auto i=0; i < argc; i++){
11     r = myF(r);
12   }
13
14   return r;
15 }
```

```
auto tripCount = SE.getSmallConstantTripCount(loop);
```

Loop unrolling: the trip multiple

```
auto forceUnroll = false;
auto allowRuntime = false;
auto allowExpensiveTripCount = true;
auto preserveCondBr = false;
auto preserveOnlyFirst = false;
auto unrolled = UnrollLoop(
  loop, 2,
  tripCount,
  forceUnroll,
  allowRuntime, allowExpensiveTripCount,
  preserveCondBr, preserveOnlyFirst,
  0, 0,
  false,
  &LI, &SE, &DT, &AC, &ORE,
  true);
```

```
auto tripMultiple = SE.getSmallConstantTripMultiple(loop);
```

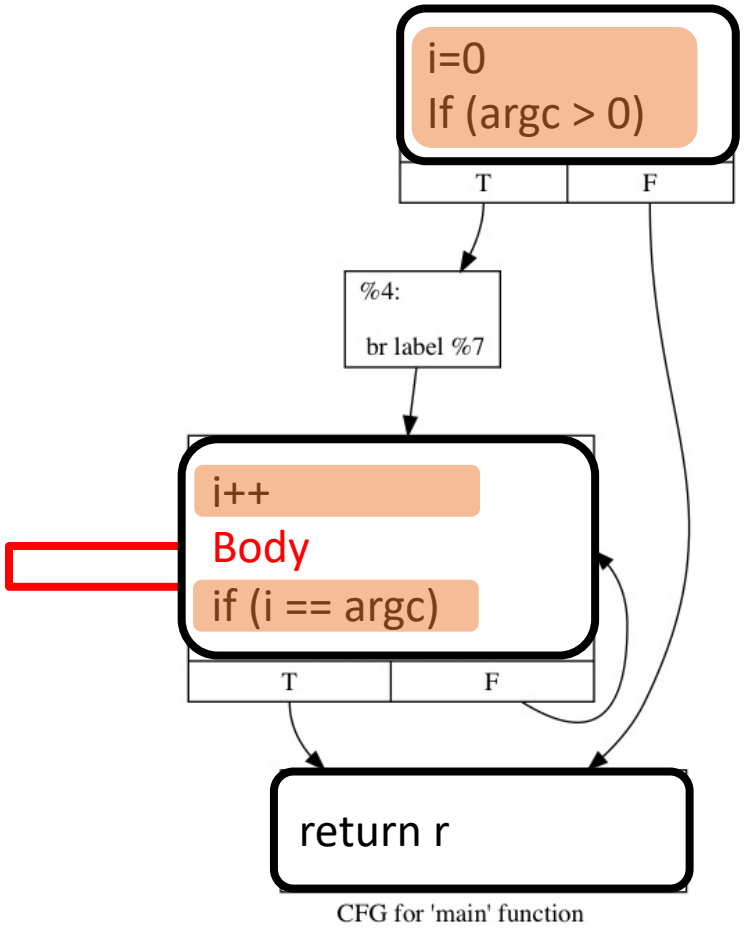
Largest constant divisor of the trip count

```
auto unrolled = UnrollLoop(
  loop, 2,
  tripCount,
  forceUnroll,
  allowRuntime, allowExpensiveTripCount,
  preserveCondBr, preserveOnlyFirst,
  tripMultiple, 0,
  false,
  &LI, &SE, &DT, &AC, &ORE,
  true);
```

Loop unrolling in LLVM: Demo 2

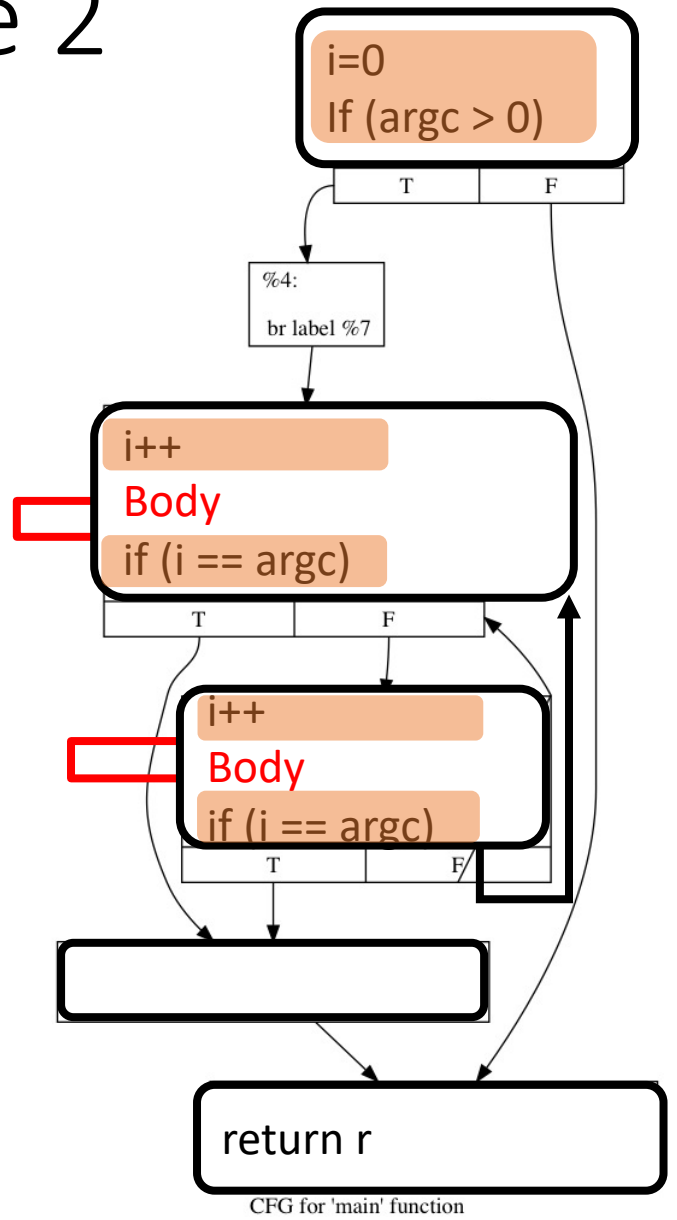
- Detail: LLVM_loops/README
- Pass: LLVM_loops/llvm/8
- C program: LLVM_loops/code/0

Loop unrolling in LLVM: example 2



CFG for 'main' function

```
 7 int main (int argc, char *argv[]){
 8   auto r = 0;
 9
10   for (auto i=0; i < argc; i++){
11     r = myF(r);
12   }
13
14   return r;
15 }
```

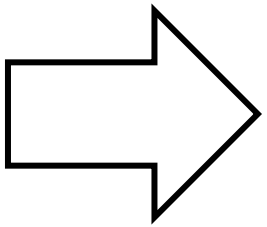


CFG for 'main' function

There is still the same amount of loop overhead!

Loop unrolling in LLVM: the runtime checks

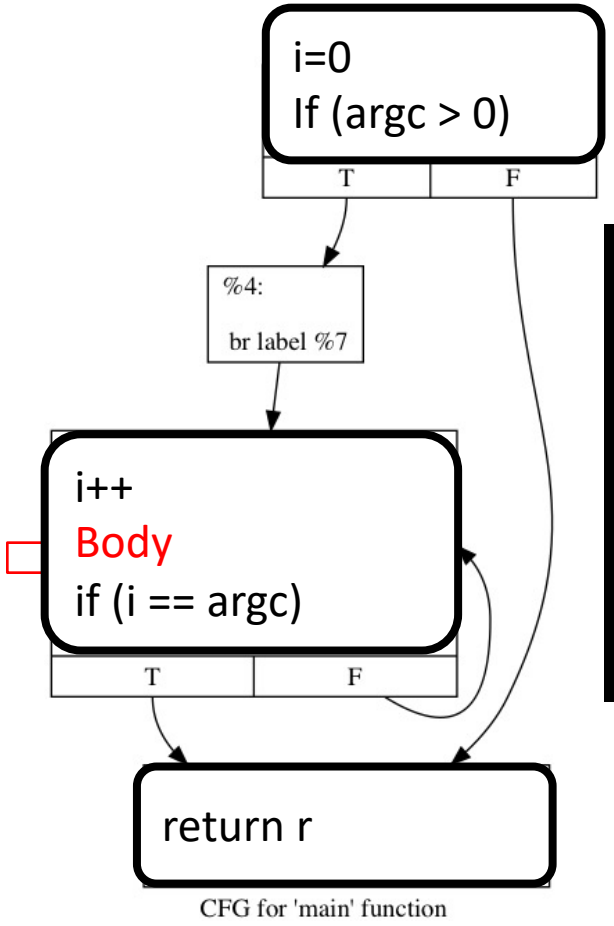
```
auto forceUnroll = false;
auto allowRuntime = false;
auto allowExpensiveTripCount = true;
auto preserveCondBr = false;
auto preserveOnlyFirst = false;
```



```
auto forceUnroll = false;
auto allowRuntime = true;
auto allowExpensiveTripCount = true;
auto preserveCondBr = false;
auto preserveOnlyFirst = false;
```

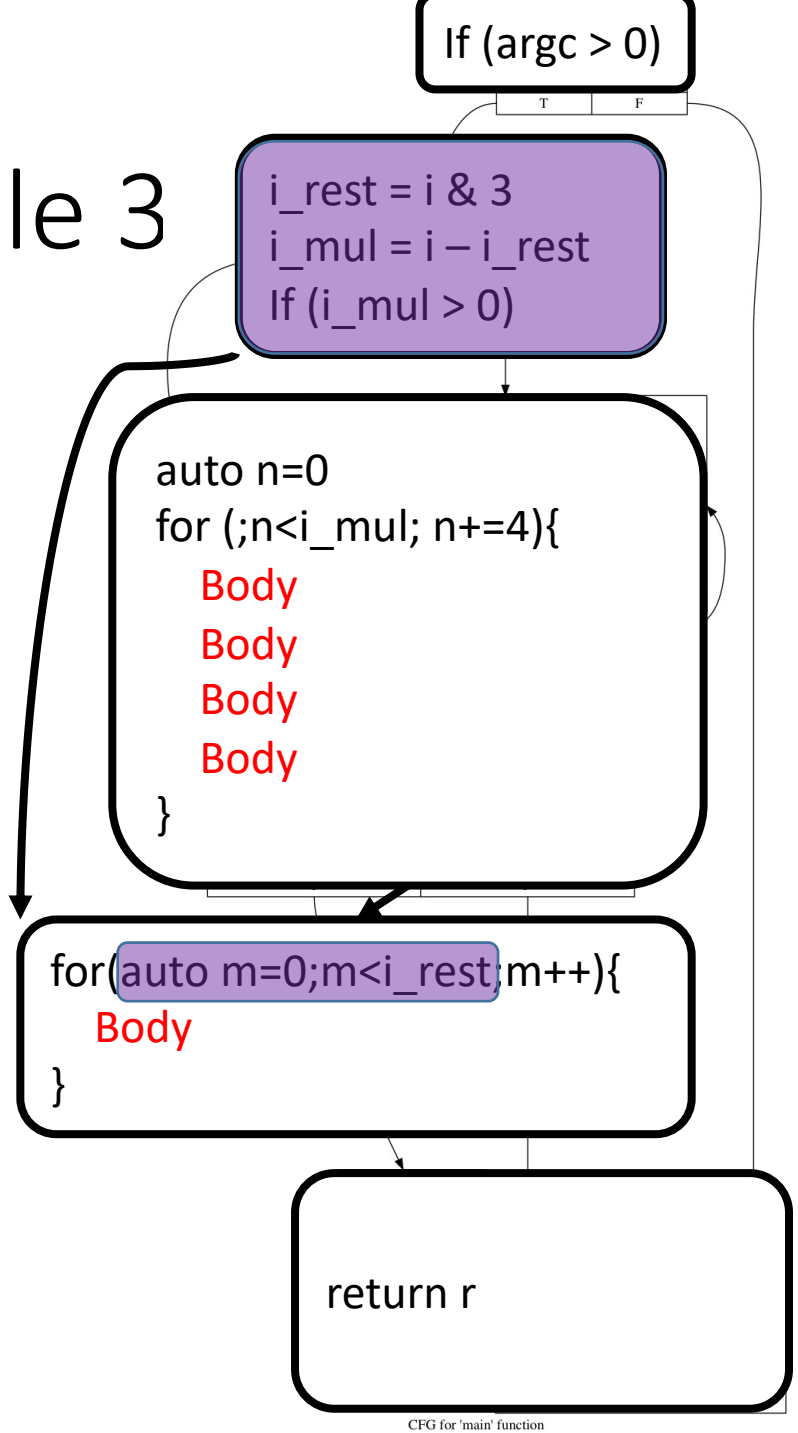
```
auto unrolled = UnrollLoop(
  loop, 2,
  tripCount,
  forceUnroll,
  allowRuntime, allowExpensiveTripCount,
  preserveCondBr, preserveOnlyFirst,
  tripMultiple, 0,
  false,
  &LI, &SE, &DT, &AC, &ORE,
  true);
```

Loop unrolling in LLVM: example 3




```
 7 int main (int argc, char *argv[]){
 8   auto r = 0;
 9
10   for (auto i=0; i < argc; i++){
11     r = myF(r);
12   }
13
14   return r;
15 }
```

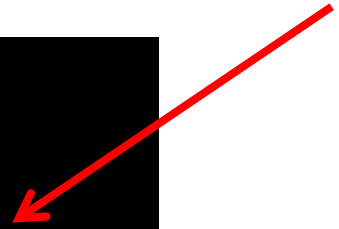
Runtime checks



Loop unrolling in LLVM: the runtime checks

```
auto forceUnroll = false;
auto allowRuntime = true;
auto allowExpensiveTripCount = true;
auto preserveCondBr = false;
auto preserveOnlyFirst = false;
```

```
auto unrolled = UnrollLoop(
  loop, 2,
  tripCount,
  forceUnroll,
  allowRuntime, allowExpensiveTripCount,
  preserveCondBr, preserveOnlyFirst,
  tripMultiple, 0,
  false,
  &LI, &SE, &DT, &AC, &ORE,
  true);
```



Loop unrolling in LLVM: API

```
auto forceUnroll = false;
auto allowRuntime = true;
auto allowExpensiveTripCount = true;
auto preserveCondBr = false;
auto preserveOnlyFirst = false;
```

```
auto unrolled = UnrollLoop(
  loop, 2,
  tripCount,
  forceUnroll,
  allowRuntime, allowExpensiveTripCount,
  preserveCondBr, preserveOnlyFirst,
  tripMultiple, 0,
  false,
  &LI, &SE, &DT, &AC, &ORE,
  true);
```

```
auto& LI = getAnalysis<LoopInfoWrapperPass>().getLoopInfo();
auto& DT = getAnalysis<DominatorTreeWrapperPass>().getDomTree();
auto& SE = getAnalysis<ScalarEvolutionWrapperPass>().getSE();
auto& AC = getAnalysis<AssumptionCacheTracker>().getAssumptionCache(F);
```

```
OptimizationRemarkEmitter ORE(&F);
```

Loop unrolling in LLVM: API

```
auto forceUnroll = false;
auto allowRuntime = true;
auto allowExpensiveTripCount = true;
auto preserveCondBr = false;
auto preserveOnlyFirst = false;
```

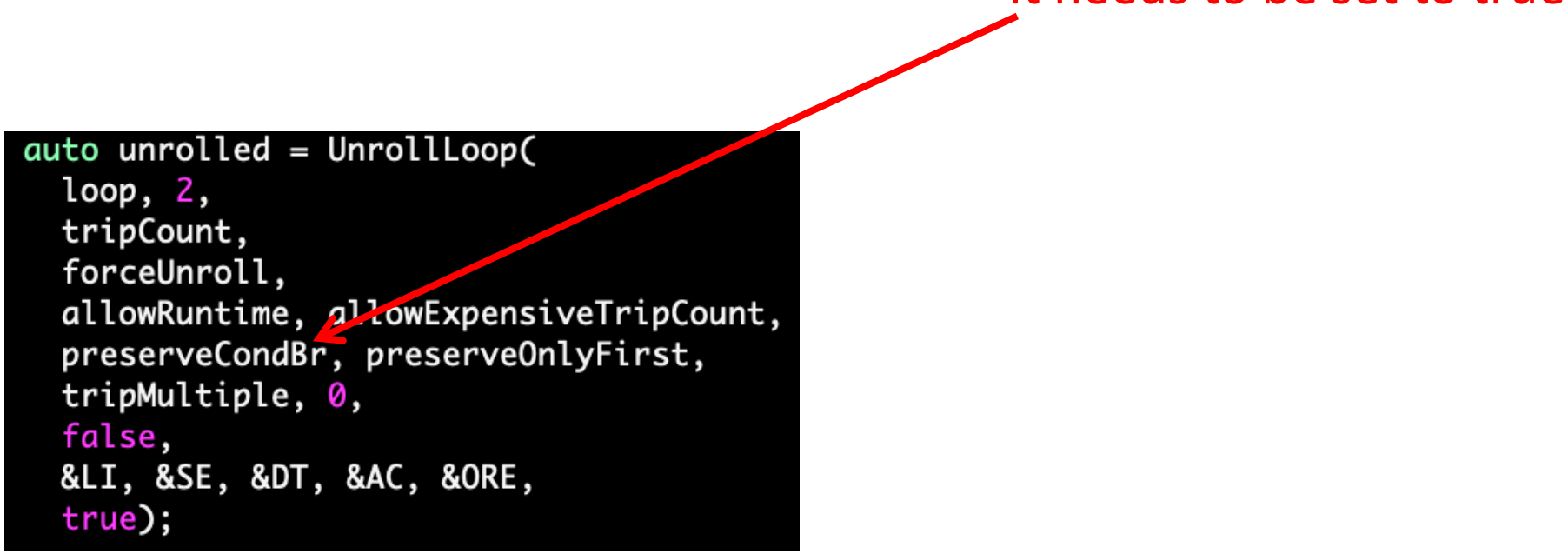
```
auto unrolled = UnrollLoop(
  loop, 2,
  tripCount,
  forceUnroll,
  allowRuntime, allowExpensiveTripCount,
  preserveCondBr, preserveOnlyFirst,
  tripMultiple, 0,
  false,
  &LI, &SE, &DT, &AC, &ORE,
  true);
```

Normalize the generated loop to LCSSA

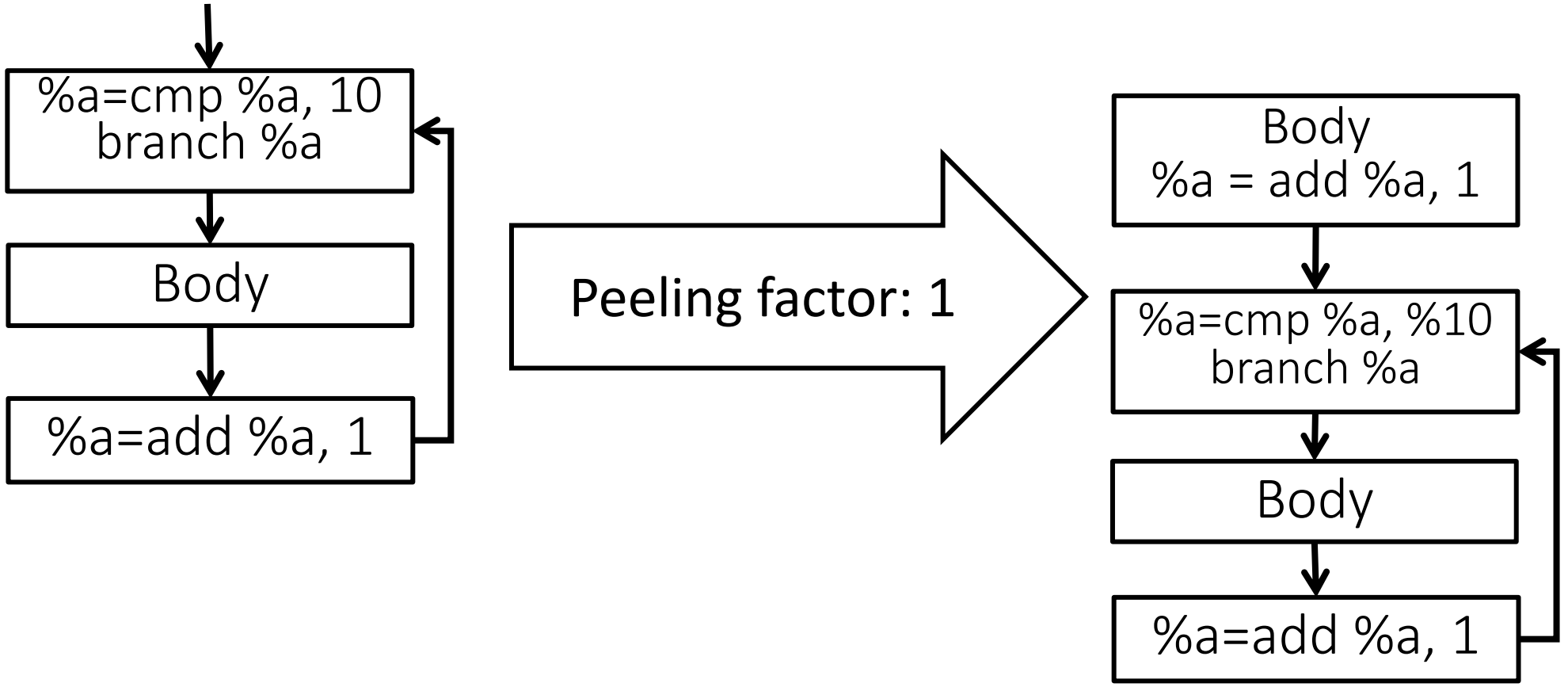
Code example

```
for (auto i=0; i < argc; i++){
  r = myF(r);
  if (r == 50) break ;
}
```

```
auto unrolled = UnrollLoop(
  loop, 2,
  tripCount,
  forceUnroll,
  allowRuntime, allowExpensiveTripCount,
  preserveCondBr, preserveOnlyFirst,
  tripMultiple, 0,
  false,
  &LI, &SE, &DT, &AC, &ORE,
  true);
```

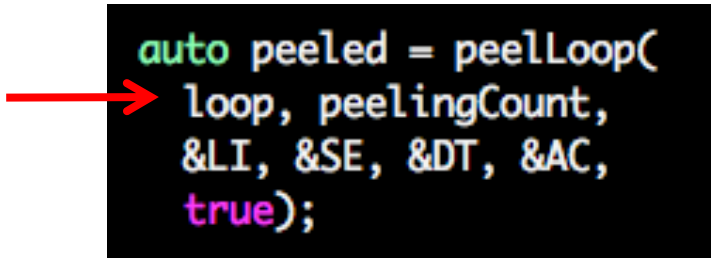


Loop peeling



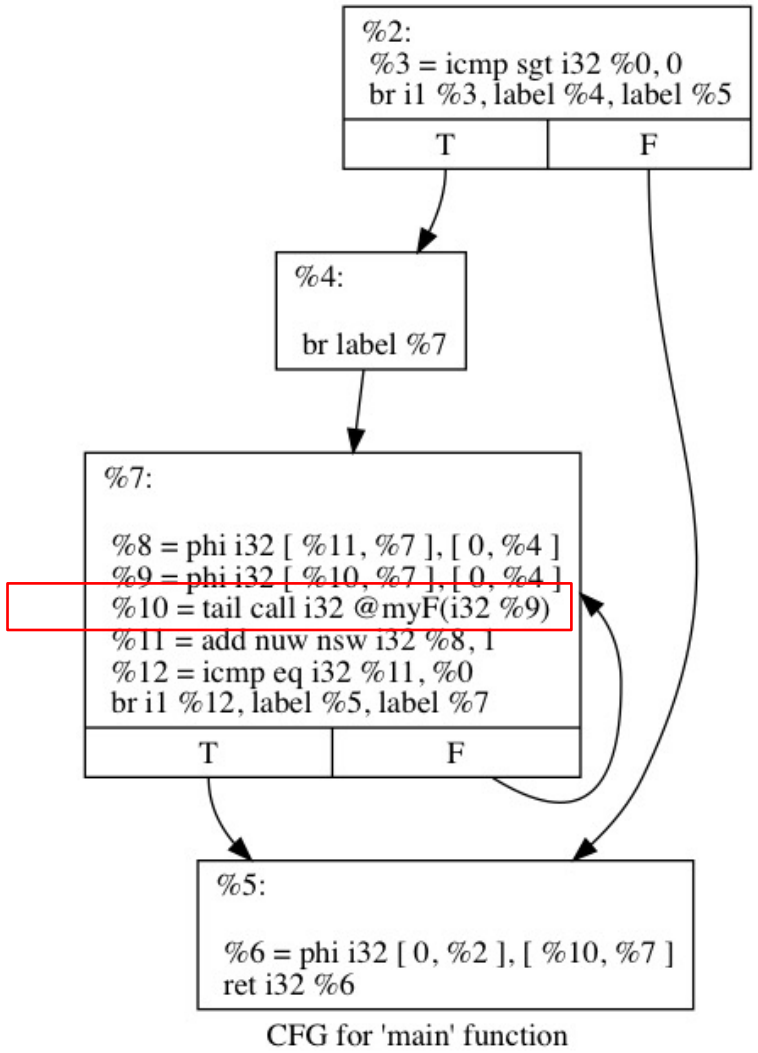
Loop peeling in LLVM

- API

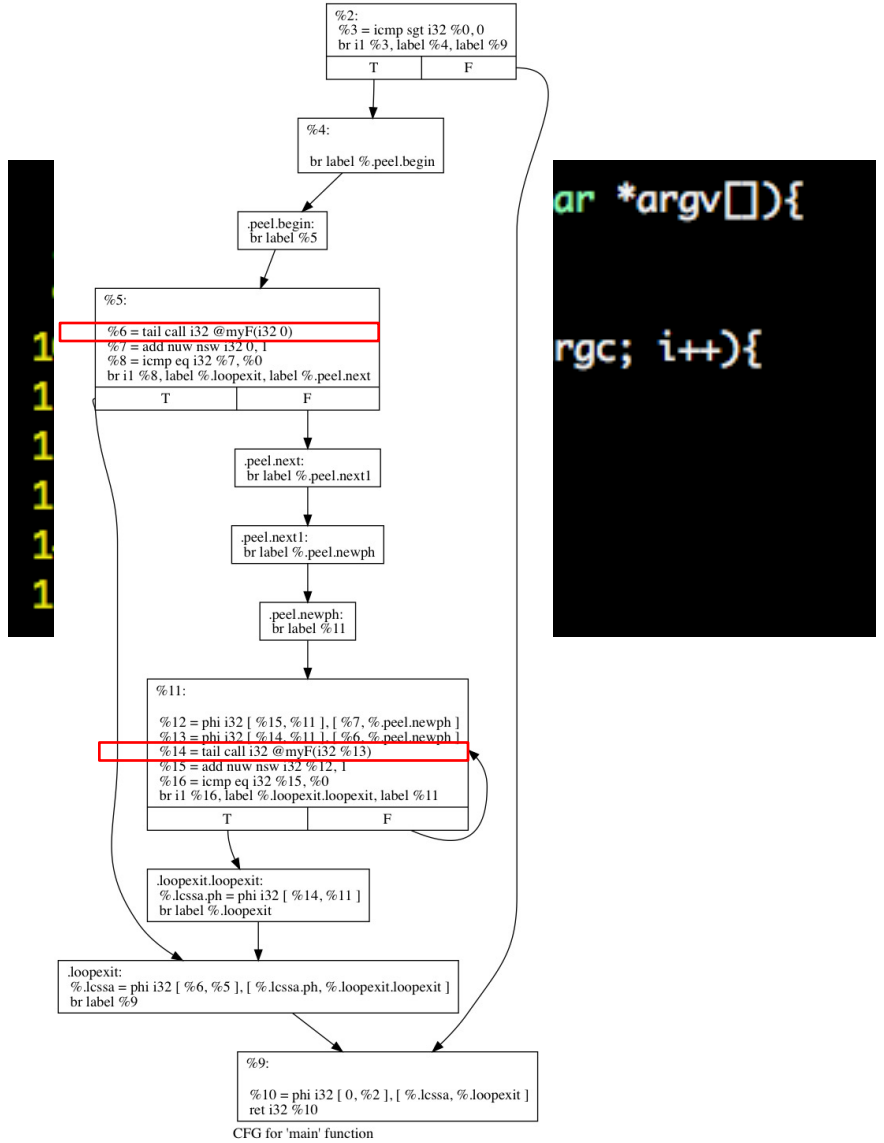


  - No trip count
  - No flags
  - (almost) always possible
  - To check if you can peel, invoke the following API: bool canPeel(Loop *loop)

Loop peeling in LLVM: example



CFG for 'main' function



CFG for 'main' function

Loop unrolling and peeling together

```
auto unrolled = UnrollLoop(
  loop, 2,
  tripCount,
  forceUnroll,
  allowRuntime, allowExpensiveTripCount,
  preserveCondBr, preserveOnlyFirst,
  tripMultiple, 0,
  false,
  &LI, &SE, &DT, &AC, &ORE,
  true);
```

Fetching analyses outputs from a module pass

- From a function pass

```
auto& LI = getAnalysis<LoopInfoWrapperPass>().getLoopInfo();
auto& DT = getAnalysis<DominatorTreeWrapperPass>().getDomTree();
auto& SE = getAnalysis<ScalarEvolutionWrapperPass>().getSE();
auto& AC = getAnalysis<AssumptionCacheTracker>().getAssumptionCache(F);
```

- From a module pass

```
auto& LI = getAnalysis<LoopInfoWrapperPass>(F).getLoopInfo();
auto& DT = getAnalysis<DominatorTreeWrapperPass>(F).getDomTree();
auto& SE = getAnalysis<ScalarEvolutionWrapperPass>(F).getSE();
auto& AC = getAnalysis<AssumptionCacheTracker>().getAssumptionCache(F);
```

Outline

- Simple loop transformations

- Loop invariants based transformations

- Induction variables based transformations

- Complex loop transformations

Optimizations in small, hot loops

- Most programs: 90% of time is spent in few, small, hot loops

```
while (){
  statement 1
  statement 2
  statement 3
}
```

- Deleting a single statement from a small, hot loop might have a big impact
  (100 seconds -> 70 seconds)

Loop example

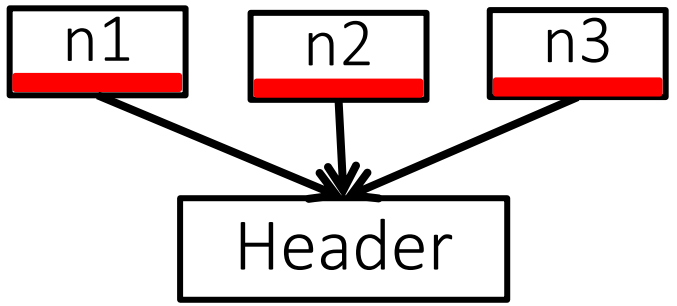
```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
   do {
3:   a = 1;
4:   y = x + N;
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
     }
10:  x++;
11:} while (x < N);
```

- **Observation**: each statement in that loop will contribute to the program execution time
- **Idea**: what about moving statements from inside a loop to outside it?
- Which statements can be moved outside our loop?
- How to identify them automatically? (code analysis)
- How to move them? (code transformation)

Hoisting code

- In order to “hoist” a loop-invariant computation out of a loop, we need a place to put it
- We could copy it to all immediate predecessors of the loop header...

```
for (auto pBB : predecessors(H)){
  p = pBB->getTerminator();
  inv->moveBefore(p);
}
```



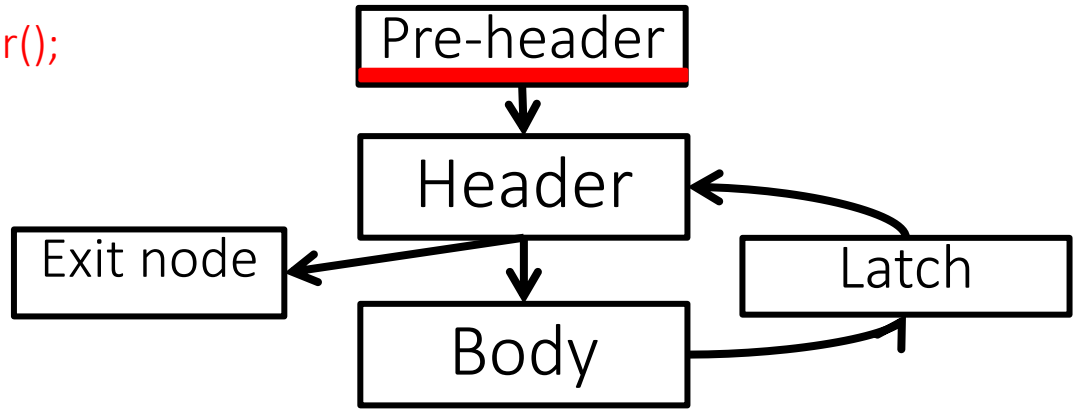
Is it correct?

- ...But we can avoid code duplication (and bugs) by taking advantage of loop normalization that guarantees the existence of the pre-header

Hoisting code

- In order to “hoist” a loop-invariant computation out of a loop, we need a place to put it
- We could copy it to all immediate predecessors of the loop header...

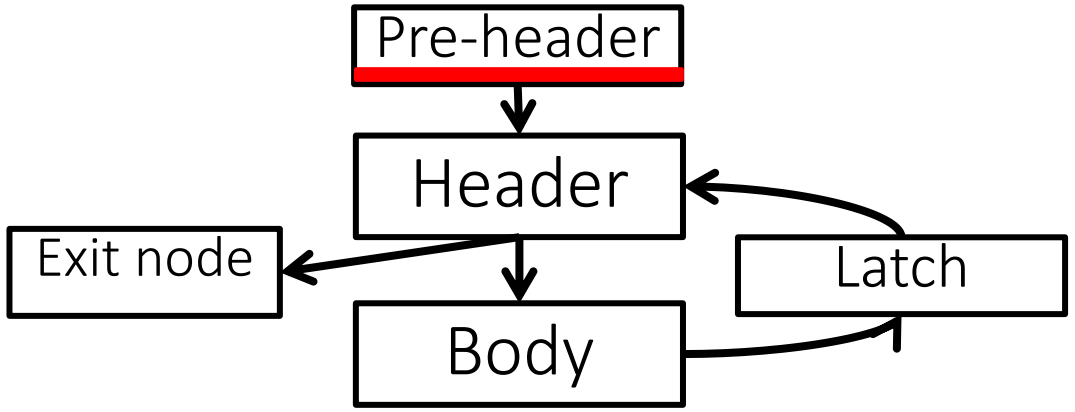
```
pBB = loop->getLoopPreheader();
p = pBB->getTerminator();
inv->moveBefore(p);
```



- ...but we can avoid code duplication (and bugs) by taking advantage of loop normalization that guarantees the existence of the pre-header

Can we hoist
all invariant instructions of a loop L
in the pre-header of L?

```
for (inv : invariants(loop)){
  pBB = loop->getLoopPreheader();
  p = pBB->getTerminator();
  inv->moveBefore(p);
}
```



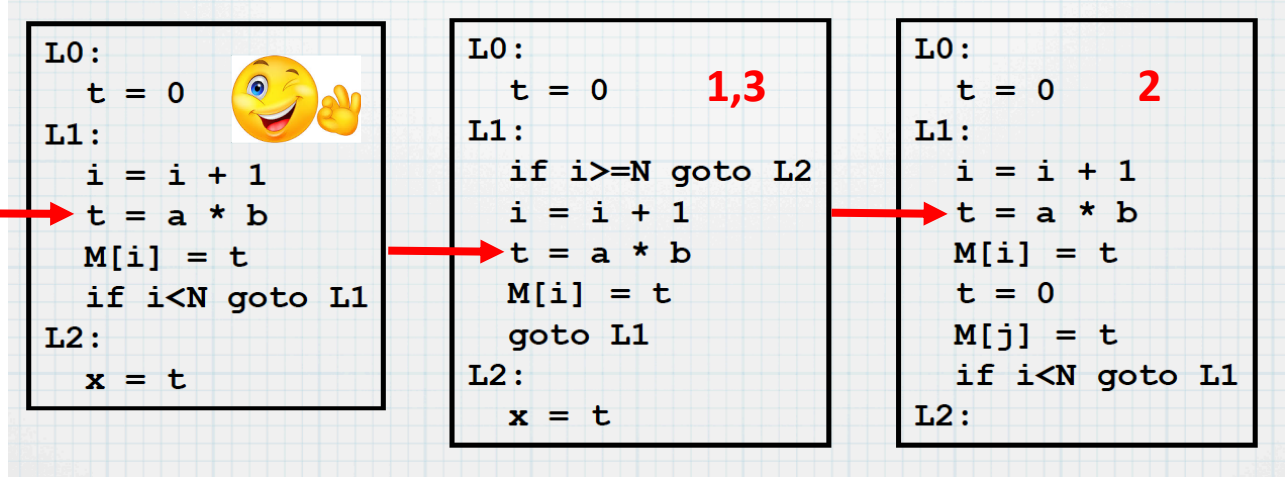
Hoisting conditions

Loop invariant code motion

- For a loop-invariant definition

(d) t = x op y

- We can hoist d into the loop's pre-header if ??
  1. d dominates all loop exits at which t is live-out, and
  2. there is only one definition of t in the loop, and
  3. t is not live-out of the pre-header

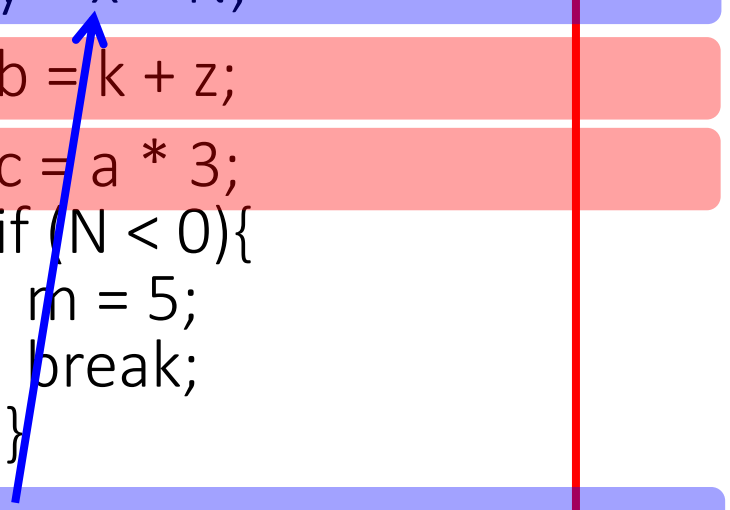


Outline

- Simple loop transformations
- Loop invariants based transformations
- Induction variables based transformations
- Complex loop transformations

Loop example

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
   do {
3:   a = 1;
4:   y = x + N;
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
     }
10:  x++;
11:} while (x < N);
```



Assuming a,b,c,m are used after our code

Do we have to execute 4 for every iteration?

Do we have to execute 10 for every iteration?

Loop example

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
   do {
3:   a = 1;
4:   y = x + N;
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
     }
10:  x++;
11:} while (x < N);
```

y=N

Do we have to execute 4 for every iteration?

Compute manually values of x and y
for every iteration
What do you see?

Do we have to execute 10 for every iteration?

Loop example

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
   do {
3:  a = 1;
4:
5:  b = k + z;
6:  c = a * 3;
7:  if (N < 0){
8:    m = 5;
9:    break;
    }
10: x++;y++;
11:} while (x < N);
```

y=N

Do we have to execute 4 for every iteration?

Do we have to execute 10 for every iteration?

Loop example

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
   do {
3:  a = 1;
4:
5:  b = k + z;
6:  c = a * 3;
7:  if (N < 0){
8:    m = 5;
9:    break;
    }
10: x++;y++;
11:} while (y < (2*N));
```

y=N

Do we have to execute 4 for every iteration?

Do we have to execute 10 for every iteration?

Loop example

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
   do {
3:   a = 1;
4:
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
     }
10:  y++;
11:} while (y < (2*N));
```

y=N

Do we have to execute 4 for every iteration?

Do we have to execute 10 for every iteration?

Loop example

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
   do {
3:   a = 1;
4:
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
     }
10:  y++;
11:} while (y < tmp);
```

y=N;tmp=2*N;

Do we have to execute 4 for every iteration?

x, y are induction variables

Do we have to execute 10 for every iteration?

Is the code transformation worth it?

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
A :y=N;tmp=2*N;
    do {
3:  a = 1;

5:  b = k + z;
6:  c = a * 3;
7:  if (N < 0){
8:    m = 5;
9:    break;
     }
10: y++;
11:} while (y < tmp);
```

Induction variable elimination

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
    do {
3:  a = 1;
4:  y = x + N;
5:  b = k + z;
6:  c = a * 3;
7:  if (N < 0){
8:    m = 5;
9:    break;
     }
10: x++;
11:} while (x < N);
```

… and after Loop Invariant Code Motion …

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
A :y=N;tmp=2*N;
3 :a=1;
5 :b=k+z;
6: c=a*3;
   do{
7:   if (N < 0){
8:     m = 5;
9:     break;
      }
10:  y++;
11:} while (y < tmp);
```

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
   do {
3:   a = 1;
4:   y = x + N;
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
      }
10:  x++;
11:} while (x < N);
```

… and with a better Loop Invariant Code Motion …

```
 1: if (N>5){ k = 1; z = 4;}
 2: else {k = 2; z = 3;}
A :y=N;tmp=2*N;
3 :a=1;
5 :b=k+z;
6: c=a*3;
7: if (N < 0){
8:    m=5;
   }

   do{
10:   y++;
11:} while (y < tmp);
```

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}

   do {
3:   a = 1;
4:   y = x + N;
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
     }
10:  x++;
11:} while (x < N);
```

… and after dead code elimination …

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
```

```
3 :a=1;
5 :b=k+z;
6: c=a*3;
7: if (N < 0){
8:    m=5;
   }
```

Assuming a,b,c,m are used after our code

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
```

```
   do {
3:   a = 1;
4:   y = x + N;
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
      }
10:  x++;
11:} while (x < N);
```

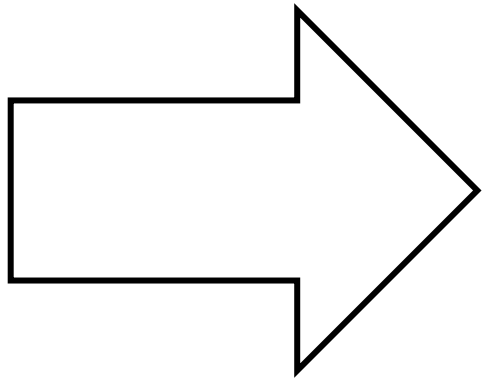
Induction variable elimination

- Suppose we have a loop variable
  - i initially set to i_0; each iteration i = i + 1
- and a variable that linearly depends on it
  - $x = i * c_1 + c_2$

- We can
  - Initialize $x = i_o * c_1 + c_2$
  - Increment x by c_1 each iteration

Is it faster?

```
1: i = i_o
2: do {
3:   i = i + 1;
     ...
A:   x = i* c1 + c2
B:} while (i < maxI);
```



```
  1: i = i_o
N1:x = i_o * c1 + c2
  2: do {
3:   i = i + 1;
     ...
A:   x = x + c1
B:} while (i < maxI);
```

On some hardware, adds are much faster than multiplies

- Strength reduction

Many optimizations rely on IVs

- Like induction variable elimination we have seen before
- or like loop unrolling to compute the trip count

```
auto tripMultiple = SE.getSmallConstantTripMultiple(loop);
```

Induction variable elimination: step 1

①Iterate over IVs

k = j * c1 + c2

- where IV j =(i, a, b), and
- this is the only def of k in the loop, and
- there is no def of i between the def of j and the def of k

```
i = …
…
j = i …
…
k = j …
```

②Record as k = (i, a*c1, b*c1+c2)

Induction variable elimination: step 2

For an induction variable k = (i, c1, c2)

① Initialize k = i * c1 + c2 in the pre-header

② Replace k's def in the loop by k = k + c1

- Make sure to do this after i's definition

Outline

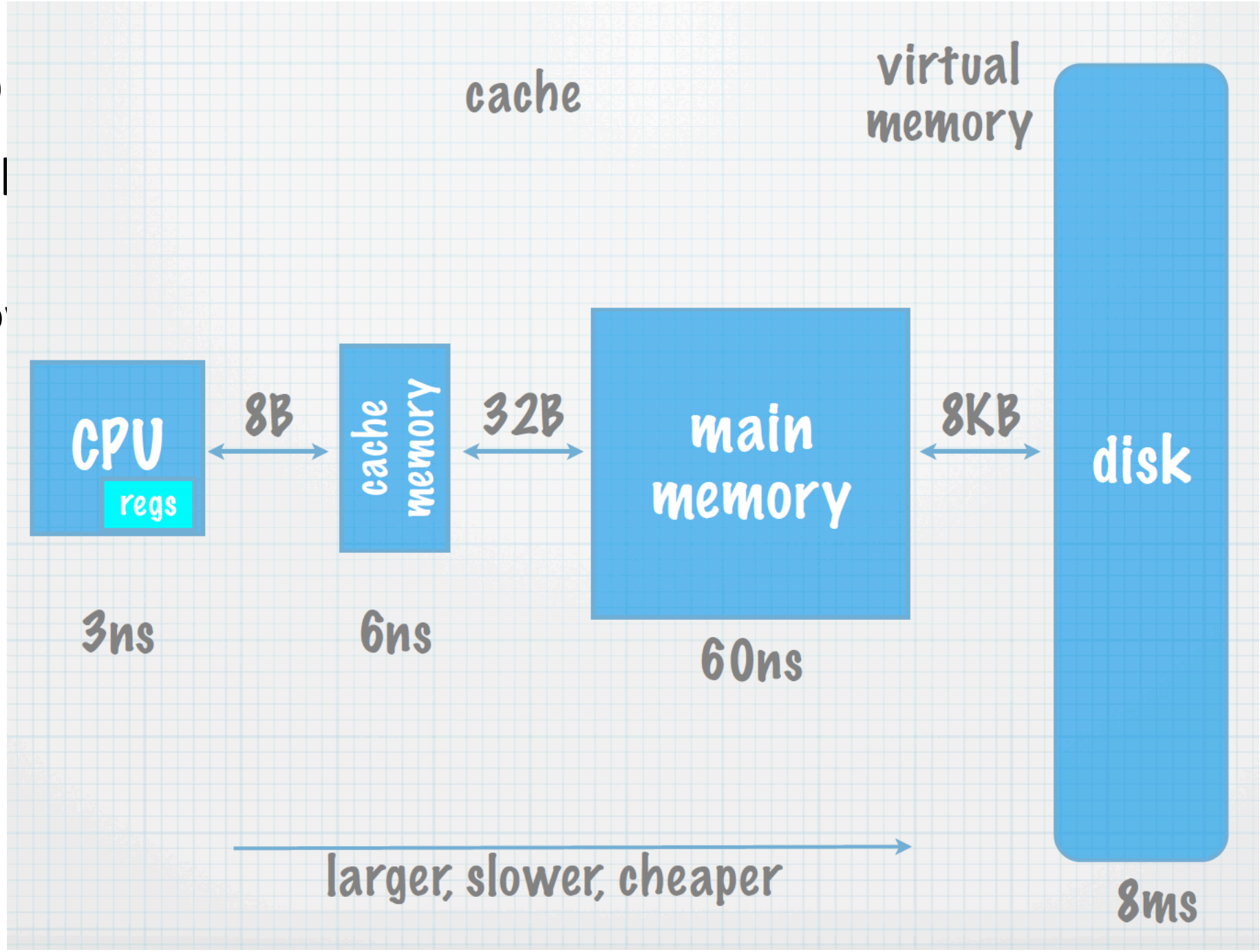
- Simple loop transformations
- Loop invariants based transformations
- Induction variables based transformations
- Complex loop transformations

Loop transformations

- Restructure a loop to expose more optimization opportunities and/or transform the “loop overhead”
  - Loop unrolling, loop peeling, …
- Reorganize a loop to improve memory utilization
  - Cache blocking, skewing, loop reversal
- Distribute a loop over cores/processors
  - DOACROSS, DOALL, DSWP, HELIX

Loo
for

- Ho


cache
virtual memory
CPU
regs
8B
cache memory
32B
main memory
8KB
disk
3ns
6ns
60ns
larger, slower, cheaper
8ms

Goal: improve cache performance

- **Temporal locality**

  A resource that has just been referenced
  will more likely be referenced again in the near future

- **Spatial locality**

  The likelihood of referencing a resource is higher
  if a resource near it was just referenced

- Ideally, a compiler generates code
  with high temporal and spatial locality
  for the target architecture
  - What to minimize: bad replacement decisions

What a compiler can do

- Time:
  - When is an object accessed?
- Space:
  - Where does an object exist in the address space?
- These are the two “knobs” a compiler can manipulate

Manipulating time and space

- Time: reordering computation
  - Determine when an object will be accessed, and predict a better time to access it

- Space: changing data layout
  - Determine an object's shape and location, and determine a better layout

First understand cache behavior …

- When do cache misses occur?
  - Use locality analysis
- Can we change the visitation order to produce better behavior?
  - Evaluate costs
- Does the new visitation order still produce correct results?
  - Use dependence analysis

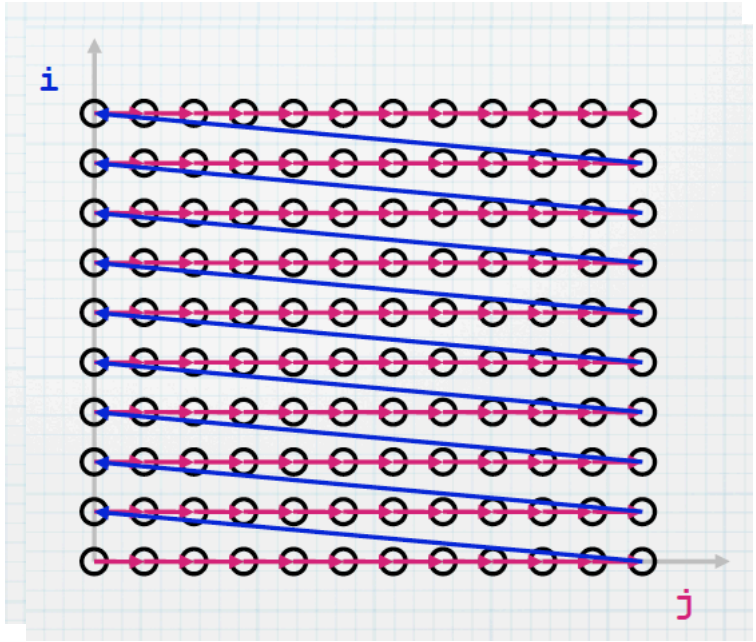
… and then rely on loop transformations

- loop interchange
- cache blocking
- loop fusion
- loop reversal
- …

Code example

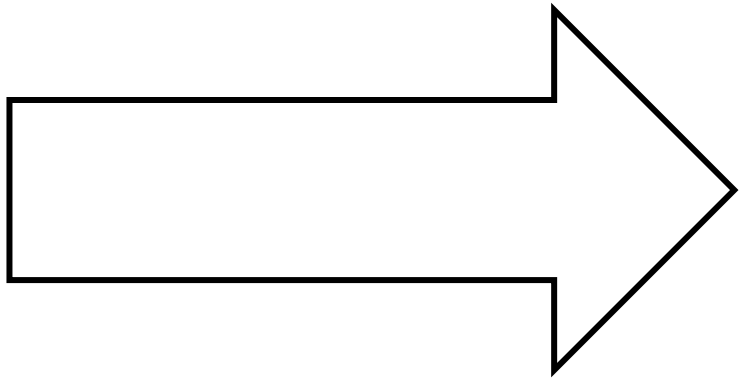
```
double A[N][N], B[N][N];
…
for i = 0 to N-1{
  for j = 0 to N-1{
     … = A[i][j] …
  }
}
```

Iteration space for A



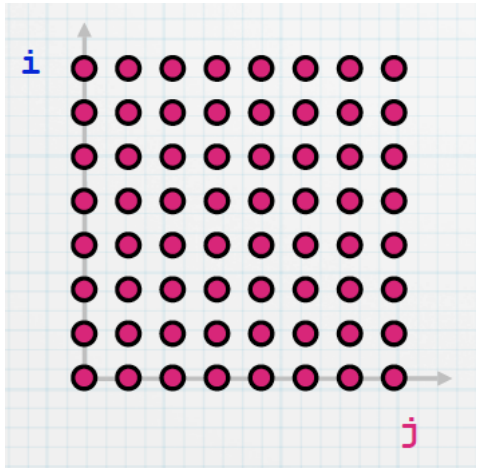
Loop interchange

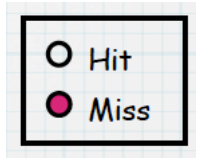
```
for i = 0 to N-1
  for j = 0 to N-1
    … = A[j][i] …
```



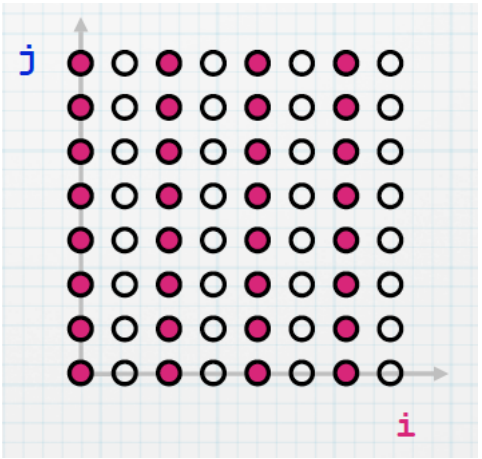
```
For j = 0 to N-1
  for i = 0 to N-1
    … = A[j][i] …
```

Assumptions: N is large; A is row-major; 2 elements per cache line





A[][] in C? Java?



Java (similar in C)

To create a matrix:

double [][] A = new double[3][3];

A is an array of arrays

A is not a 2 dimensional array!

Java (similar in C)

To create a matrix:

```
double [][] A = new double[3][];
A[0] = new double[3];
A[1] = new double[3];
A[2] = new double[3];
```

Java (similar in C)

To create a matrix:

```
double [][] A = new double[3][];
A[0] = new double[10];
A[1] = new double[5];
A[2] = new double[42];
```

A is a jagged array

C#: [][] vs. [,]

```
double [][] A = new double[3][];
A[0] = new double[3];
A[1] = new double[3];
A[2] = new double[3];
```

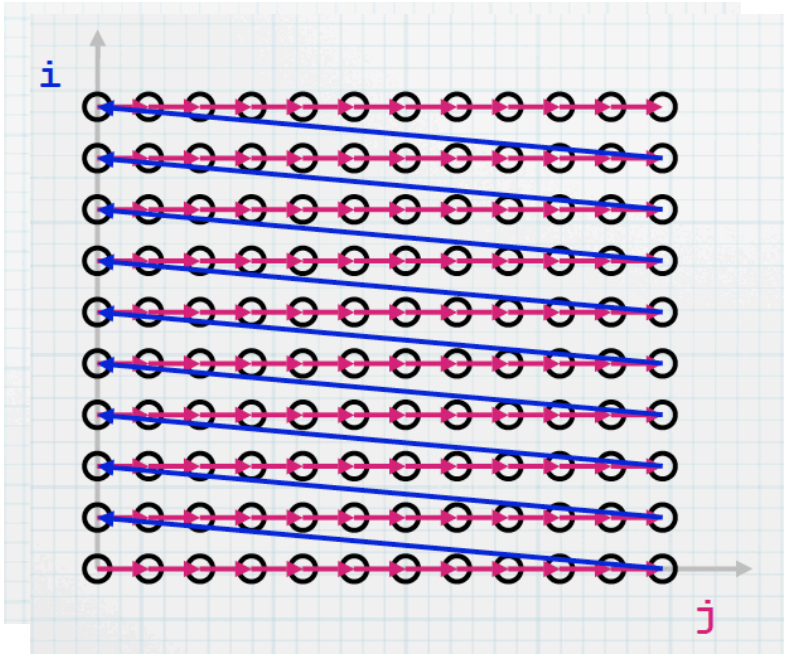
```
double [,] A = new double[3,3];
```

The compiler can easily choose between raw-major vs. column-major

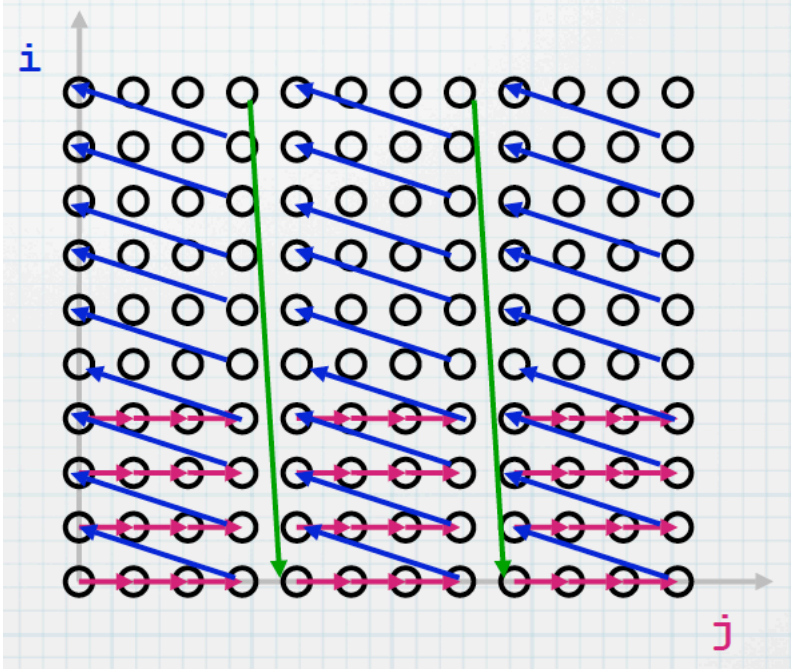
```c
#include <stdio.h>

int main (){
  int a[2][4];

  printf("0x%p\n", &a[0][0]);
  printf("0x%p\n", &a[0][1]);
  printf("  Distance: %d bytes\n", ((unsigned int)(&a[0][1])) - ((unsigned int)(&a[0][0])));

  printf("0x%p\n", &a[0][0]);
  printf("0x%p\n", &a[1][0]);
  printf("  Distance: %d bytes\n", ((unsigned int)(&a[1][0])) - ((unsigned int)(&a[0][0])));

  return 0;
}
```

Cache blocking (a.k.a. tiling)

```
for i = 0 to N-1
  for j = 0 to N-1
    f(A[i], A[j])
```

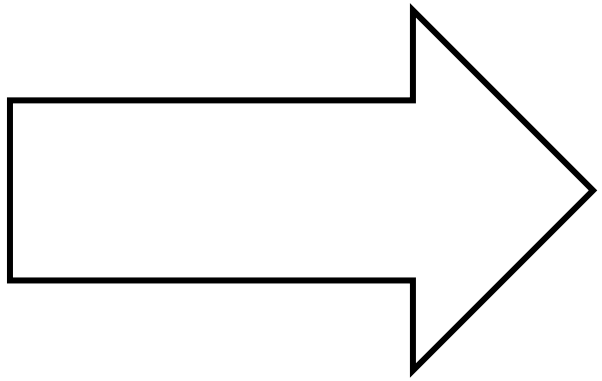



```
for JJ = 0 to N-1 by B
  for i = 0 to N-1
    for j = JJ to min(N-1,JJ+B-1)
      f(A[i], A[j])
```



Loop fusion

```
for i = 0 to N-1
  C[i] = A[i]*2 + B[i]

for i = 0 to N-1
  D[i] = A[i] * 2
```



```
for i = 0 to N-1
  C[i] = A[i] * 2 + B[i]
  D[i] = A[i] * 2
```

- Reduce loop overhead
- Improve locality by combining loops that reference the same array
- Increase the granularity of work done in a loop

Locality analysis

- Reuse:
  Accessing a location that has been accessed previously
- Locality:
  Accessing a location that is in the cache

- Observe:
  - Locality only occurs when there is reuse!
  - … but reuse does not imply locality

Steps in locality analysis

- Find data reuse
- Determine “localized iteration space”
  - Set of inner loops where the data accessed by an iteration is expected to fit within the cache
- Find data locality
  - Reuse ∩ localized iteration space ⇒ locality