Control-Flow Analysis

Chapter 8, Section 8.4

Chapter 9, Section 9.6

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Control-Flow Graphs

- Control-flow graph (CFG) for a procedure/method
  - A node is a basic block: a single-entry-single-exit sequence of three-address instructions
  - An edge represents the potential flow of control from one basic block to another
- Uses of a control-flow graph
  - Inside a basic block: local code optimizations; done as part of the code generation phase
  - Across basic blocks: global code optimizations; done as part of the code optimization phase
  - Aspects of code generation: e.g., global register allocation

Control-Flow Analysis

- Part 1: Constructing a CFG
- Part 2: Finding dominators and post-dominators
- Part 3: Finding loops in a CFG
  - What exactly is a loop?We cannot simply say "whatever CFG subgraph is generated by *while*, *do-while*, and *for* statements" – need a general graph-theoretic definition
- Part 4: Static single assignment form (SSA)
- Part 5: Finding control dependences
  - Necessary as part of constructing the program dependence graph (PDG), a popular IR for software tools for slicing, refactoring, testing, and debugging

Part 1: Constructing a CFG

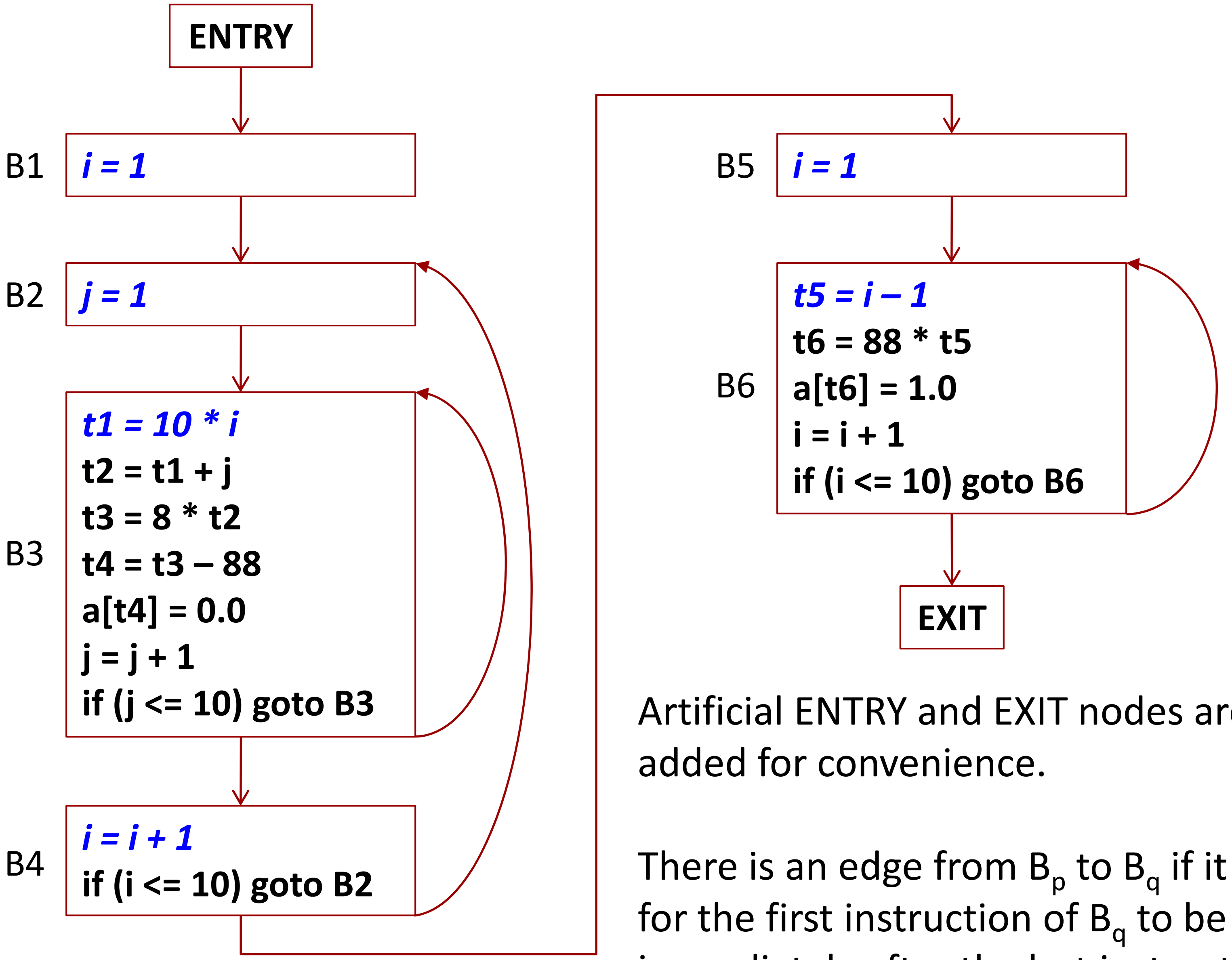
- Basic block: maximal sequence of consecutive three-address instructions such that
  - The flow of control can enter only through the first instruction (i.e., no jumps into the middle of the block)
  - The flow of control can exit only at the last instruction
- Given: the entire sequence of instructions
- First, find the leaders (starting instructions of all basic blocks)
  - The first instruction
  - The target of any conditional/unconditional jump
  - Any instruction that immediately follows a conditional or unconditional jump

Constructing a CFG

- Next, find the basic blocks: for each leader, its basic block contains itself and all instructions up to (but not including) the next leader

1. i = 1	First instruction
2. j = 1	Target of 11
3. t1 = 10 * i	Target of 9
4. t2 = t1 + j	
5. t3 = 8 * t2	
6. t4 = t3 – 88	
7. a[t4] = 0.0	
8. j = j + 1	
9. if (j <= 10) goto (3)	
10. i = i + 1	Follows 9
11. if (i <= 10) goto (2)	
12. i = 1	Follows 11
13. t5 = i – 1	Target of 17
14. t6 = 88 * t5	
15. a[t6] = 1.0	
16. i = i + 1	
17. if (i <= 10) goto (13)	

Note: this example sets array elements a[i][j] to 0.0, for 1 <= i,j <= 10 (instructions 1-11). It then sets a[i][i] to 1.0, for 1 <= i <= 10 (instructions 12-17). The array accesses in instructions 7 and 15 are done with offsets computed as described in Section 6.4.3, assuming row-major order, 8-byte array elements, and array indexing that starts from 1, not from 0.



Artificial ENTRY and EXIT nodes are often added for convenience.

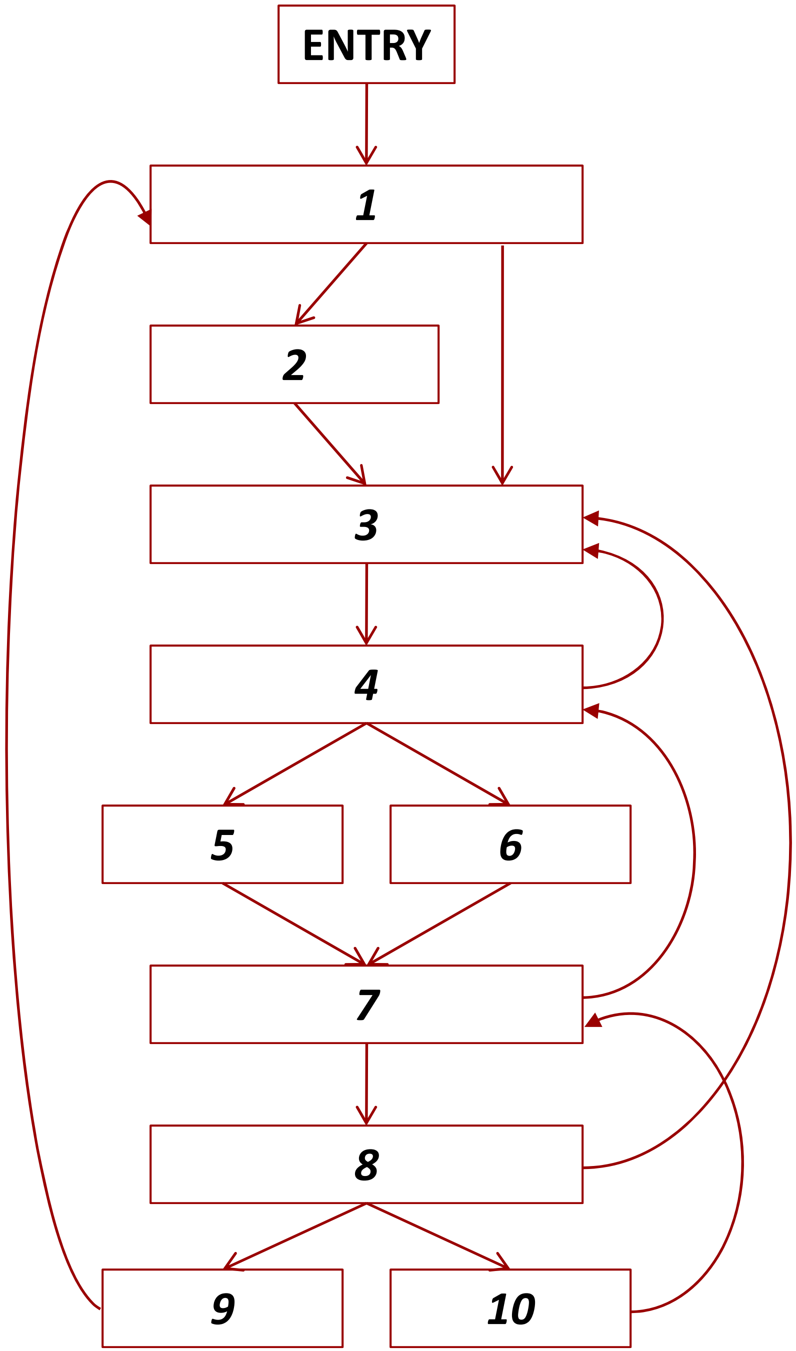
There is an edge from B_p to B_q if it is possible for the first instruction of B_q to be executed immediately after the last instruction of B_p . This is conservative: e.g., **if (3.14 > 2.78)** still generates two edges.

Practical Considerations

- The usual data structures for graphs can be used
  - The graphs are sparse (i.e., have relatively few edges), so an adjacency list representation is the usual choice
    - Number of edges is at most 2 * number of nodes
- Nodes are basic blocks; edges are between basic blocks, not between instructions
  - Inside each node, some additional data structures for the sequence of instructions in the block (e.g., a linked list of instructions)
  - Often convenient to maintain both a list of successors (i.e., outgoing edges) and a list of predecessors (i.e., incoming edges) for each basic block

Part 2: Dominance

- A CFG node *d* dominates another node *n* if every path from ENTRY to *n* goes through *d*
  - Implicit assumption: every node is reachable from ENTRY (i.e., there is no dead code)
  - A dominance relation $dom \subseteq$ Nodes × Nodes: *d dom n*
  - The relation is trivially reflexive: *d dom d*
- Node *m* is the immediate dominator of *n* if
  - $m \neq n$
  - *m dom n*
  - For any $d \neq n$ such *d dom n*, we have *d dom m*
- Every node has a unique immediate dominator
  - Except ENTRY, which is dominated only by itself



This example is artificial: it does not have an EXIT node; nodes 4 and 8 have more than 2 outgoing edges

ENTRY *dom n* for any *n*
1 dom n for any *n* except ENTRY
2 does not dominate any other node
3 *dom* 3, 4, 5, 6, 7, 8, 9, 10
4 *dom* 4, 5, 6, 7, 8, 9, 10
5 does not dominate any other node
6 does not dominate any other node
7 *dom* 7, 8, 9, 10
8 *dom* 8, 9, 10
9 does not dominate any other node
10 does not dominate any other node

Immediate dominators:

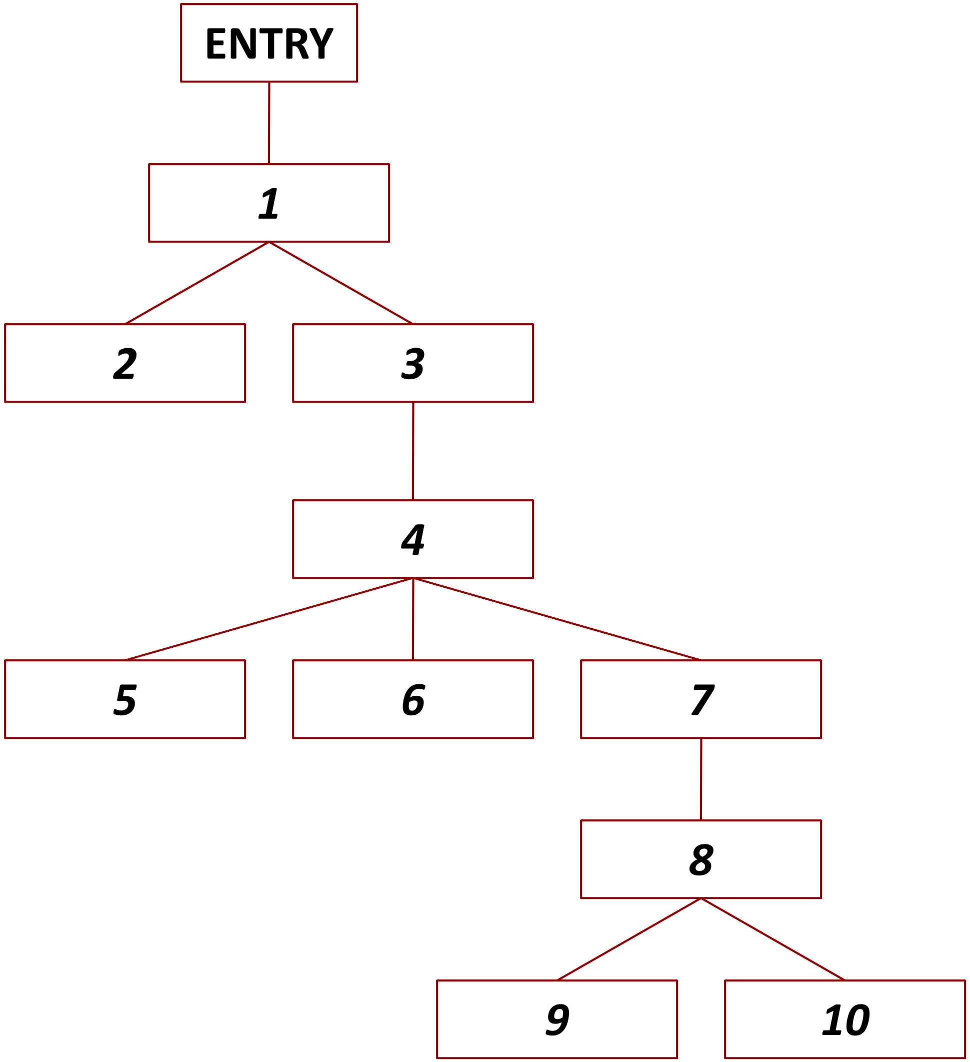
1 $\rightarrow$ ENTRY	2 $\rightarrow$ 1
3 $\rightarrow$ 1	4 $\rightarrow$ 3
5 $\rightarrow$ 4	6 $\rightarrow$ 4
7 $\rightarrow$ 4	8 $\rightarrow$ 7
9 $\rightarrow$ 8	10 $\rightarrow$ 8

A Few Observations

- For any acyclic path from ENTRY to *n*, all dominators of *n* appear along the path, always in the same order (for all such paths)
- Dominance is a transitive relation: *a dom b* and *b dom c* means *a dom c*
- Dominance is an anti-symmetric relation: *a dom b* and *b dom a* means that *a* and *b* must be the same
  - Reflexive, anti-symmetric, transitive: **partial order**
- If *a* and *b* are two dominators of some *n*, either *a dom b* or *b dom a*

Dominator Tree

- The parent of *n* is its immediate dominator



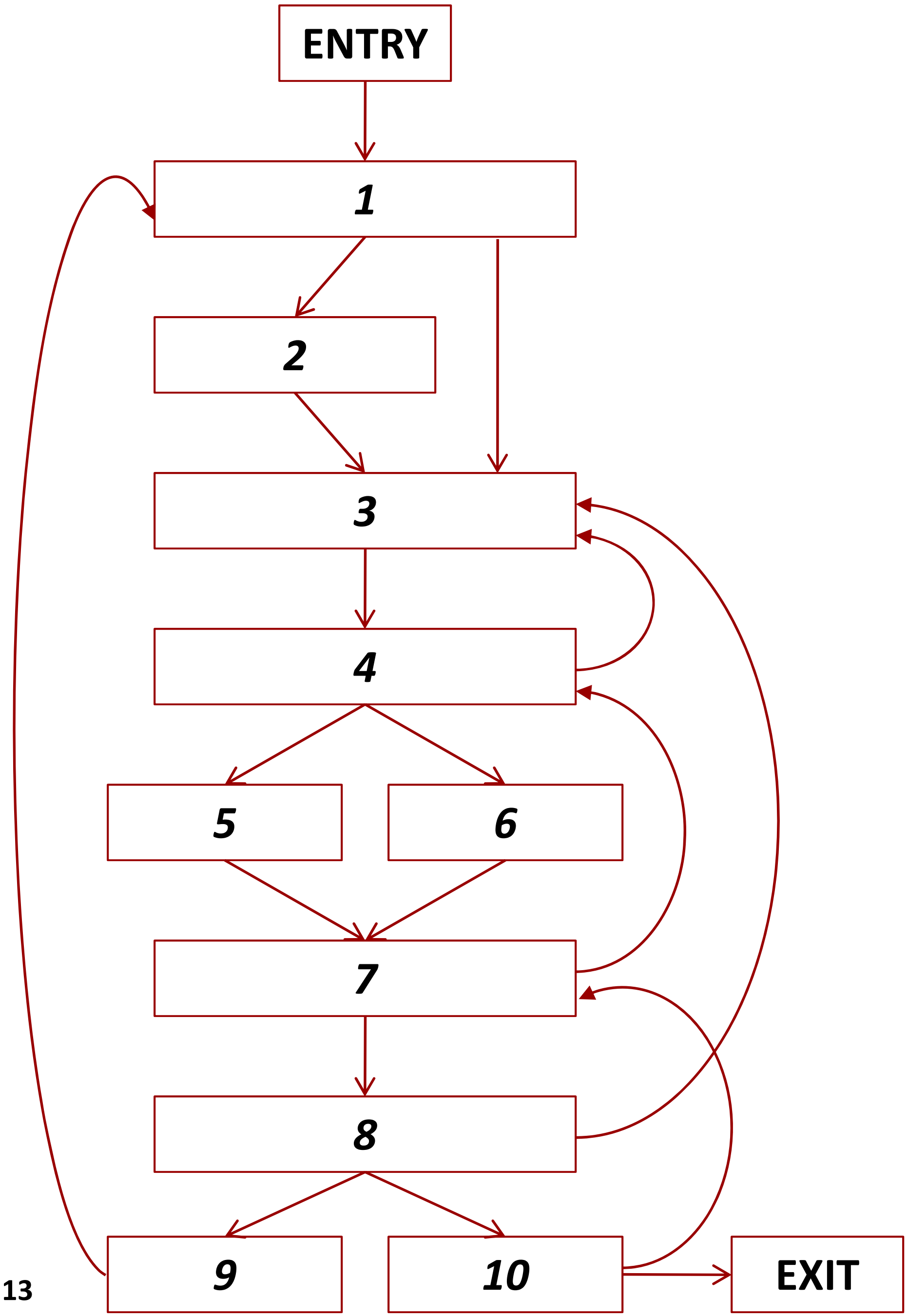
The path from *n* to the root contains all and only dominators of *n*

Constructing the dominator tree: the classic $O(N\alpha(N))$ approach is from *T. Lengauer and R. E. Tarjan. A fast algorithm for finding dominators in a flowgraph. ACM Transactions on Programming Languages and Systems, 1(1): 121–141, July 1979.*

Many other algorithms: e.g., see *K. D. Cooper, T. J. Harvey and K. Kennedy. A simple, fast dominance algorithm. Software – Practice and Experience, 4:1–10, 2001.*

Post-Dominance

- A CFG node *d* post-dominates another node *n* if every path from *n* to EXIT goes through *d*
  - Implicit assumption: EXIT is reachable from every node
  - A relation $pdom \subseteq$ Nodes × Nodes: *d pdom n*
  - The relation is trivially reflexive: *d pdom d*
- Node *m* is the immediate post-dominator of *n* if
  - $m \neq n$; m $pdom$ n; $\forall d \neq n.\ d$ $pdom$ $n \Rightarrow d$ $pdom$ m
  - Every *n* has a unique immediate post-dominator
- Post-dominance on a CFG is equivalent to dominance on the reverse CFG (all edges reversed)
- Post-dominance tree: the parent of *n* is its immediate post-dominator; root is EXIT



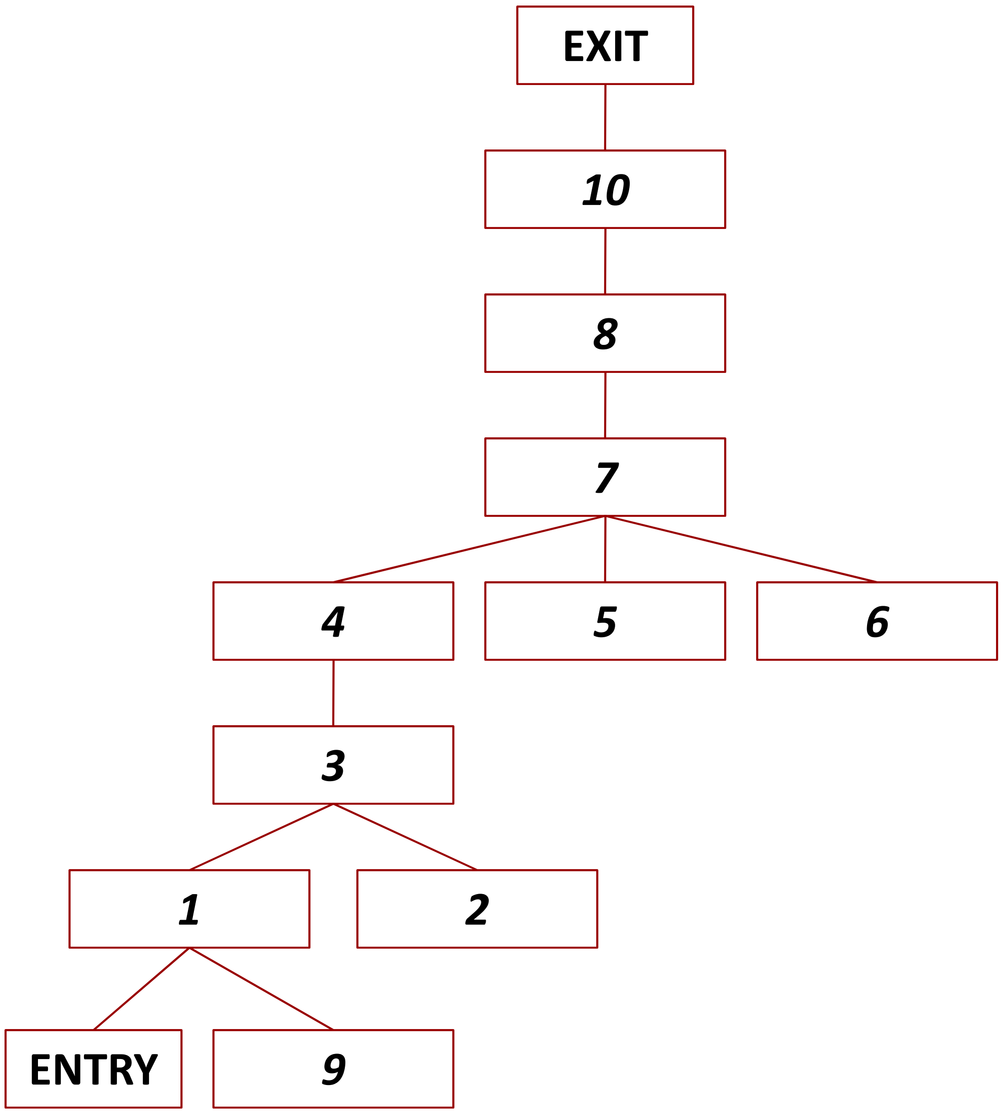
Extend the previous example with EXIT

ENTRY does not post-dominate any other *n*
1 *pdom* ENTRY, 1, 9
2 does not post-dominate any other *n*
3 *pdom* ENTRY, 1, 2, 3, 9
4 *pdom* ENTRY, 1, 2, 3, 4, 9
5 does not post-dominate any other *n*
6 does not post-dominate any other *n*
7 *pdom* ENTRY, 1, 2, 3, 4, 5, 6, 7, 9
8 *pdom* ENTRY, 1, 2, 3, 4, 5, 6, 7, 8, 9
9 does not post-dominate any other *n*
10 *pdom* ENTRY, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
EXIT *pdom n* for any *n*

Immediate post-dominators:

ENTRY $\rightarrow$ 1	1 $\rightarrow$ 3
2 $\rightarrow$ 3	3 $\rightarrow$ 4
4 $\rightarrow$ 7	5 $\rightarrow$ 7
6 $\rightarrow$ 7	7 $\rightarrow$ 8
8 $\rightarrow$ 10	9 $\rightarrow$ 1
10 $\rightarrow$ EXIT	

Post-Dominator Tree



The path from *n* to the root contains all and only post-dominators of *n*

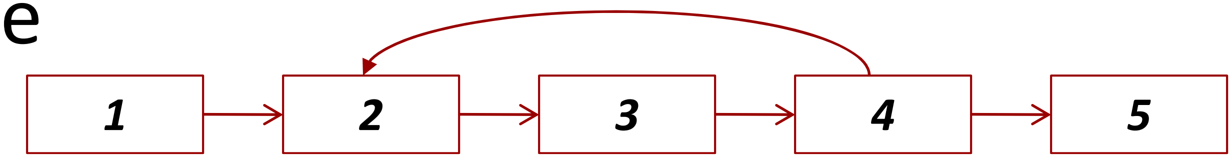
Constructing the post-dominator tree: use any algorithm for constructing the dominator tree; just "pretend" that the edges are reversed

Computing the Dominator Tree

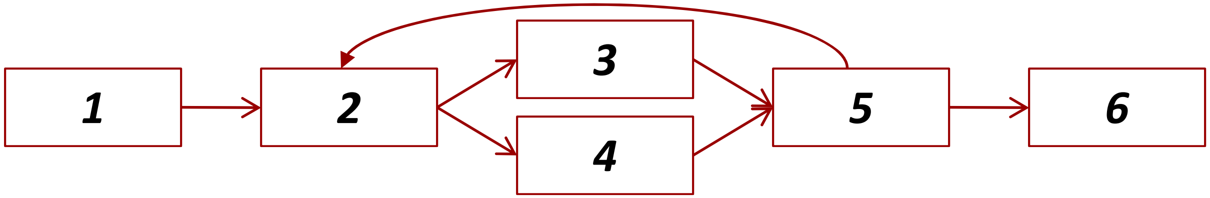
- Theoretically superior algorithms are not necessarily the most desirable in practice
- Our choice (for the default project): Cooper et al.,
- Formulation and algorithm based on insights from dataflow analysis
  - Essentially, solving a system of mutually-recursive equations – more later …
- I expect you to read the paper carefully and to implement the algorithm for computing the dominator tree

Part 3: Loops in CFGs

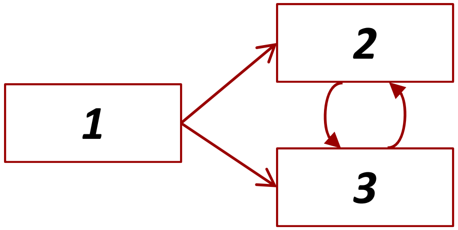
- Cycle: sequence of edges that starts and ends at the same node
  - Example:



- Strongly-connected component (SCC): a maximal set of nodes such as each node in the set is reachable from every other node in the set
  - Example:

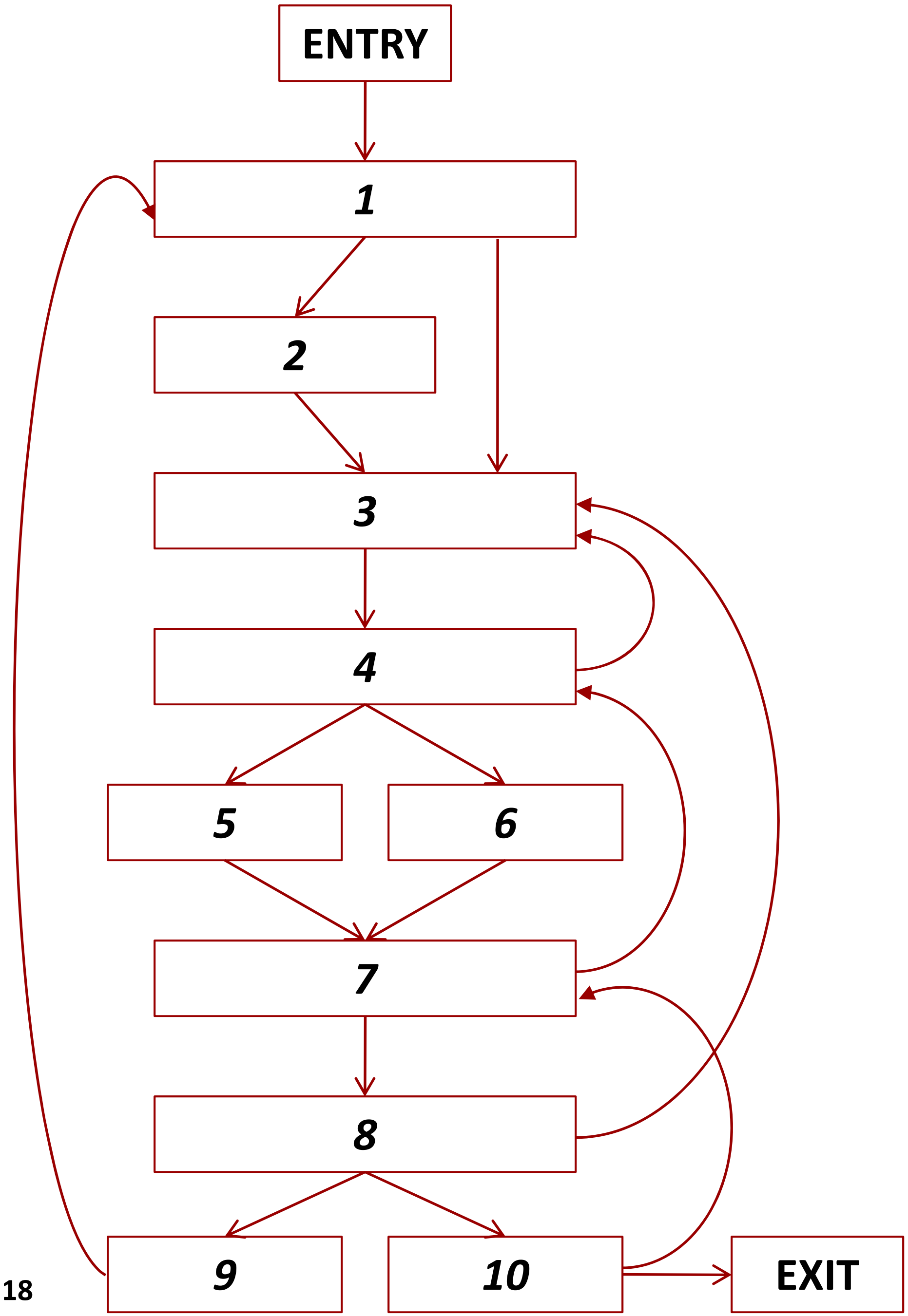


- Loop: informally, a strongly-connected component with a single entry point
  - An SCC that is not a loop:



Back Edges and Natural Loops

- Back edge: a CFG edge (n,h) where h dominates n
  - Easy to see that n and h belong to the same SCC
- Natural loop for a back edge (n,h)
  - The set of all nodes m that can reach node n without going through node h (trivially, this set includes h)
  - Easy to see that h dominates all such nodes m
  - Node h is the header of the natural loop
- Trivial algorithm to find the natural loop of (n,h)
  - Mark h as visited
  - Perform depth-first search (or breadth-first) starting from n, but follow the CFG edges in reverse direction
  - All and only visited nodes are in the natural loop



Immediate dominators:

$1 \to$ ENTRY　　$2 \to 1$　　$3 \to 1$

$4 \to 3$　　$5 \to 4$　　$6 \to 4$

$7 \to 4$　　$8 \to 7$　　$9 \to 8$

$10 \to 8$　　EXIT $\to 10$

Back edges: **$4 \to 3$, $7 \to 4$, $8 \to 3$, $9 \to 1$, $10 \to 7$**

Loop(**$10 \to 7$**) = { 7, 8, 10 }

Loop(**$7 \to 4$**) = { 4, 5, 6, 7, 8, 10 }

Note: Loop(**$10 \to 7$**) $\subseteq$ Loop(**$7 \to 4$**)

Loop(**$4 \to 3$**) = { 3, 4, 5, 6, 7, 8, 10 }

Note: Loop(**$7 \to 4$**) $\subseteq$ Loop(**$4 \to 3$**)

Loop(**$8 \to 3$**) = { 3, 4, 5, 6, 7, 8, 10 }

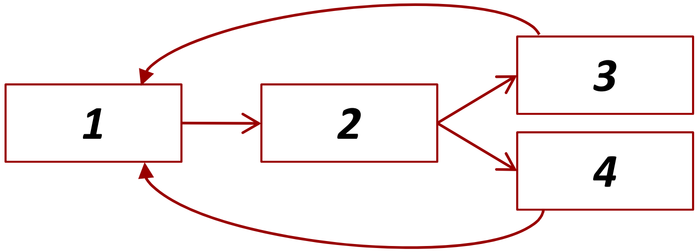
Note: Loop(**$8 \to 3$**) = Loop(**$4 \to 3$**)

Loop(**$9 \to 1$**) = { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }

Note: Loop(**$4 \to 3$**) $\subseteq$ Loop(**$9 \to 1$**)

Loops in the CFG

- Find all back edges; each target *h* of at least one back edge defines a loop *L* with *header(L) = h*
- *body(L)* is the union of the natural loops of all back edges whose target is *header(L)*
  - Note that *header(L)* $\in$ *body(L)*
- Example: this is a single loop with header node 1



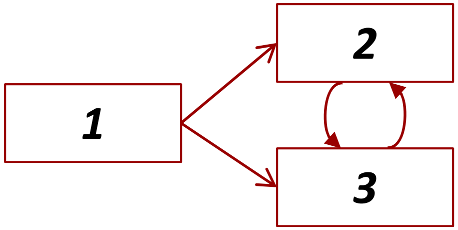
- For two CFG loops L_1 and L_2
  - *header*(L_1) is different from *header*(L_2)
  - *body*(L_1) and *body*(L_2) are either disjoint, or one is a proper subset of the other (nesting – inner/outer)

Flashback to Graph Algorithms

- Depth-first search in the CFG [Cormen et al. book]
  - Set each node's color as *white*
  - Call DFS(ENTRY)
  - DFS(*n*)
    - Set the color of *n* to *grey*
    - For each successor *m*: if color is *white*, call DFS(*m*)
    - Set the color of *n* to *black*
- Inside DFS(*n*), seeing a grey successor *m* means that (*n*,*m*) is a retreating edge
  - Note: *m* could be *n* itself, if there is an edge (*n*,*n*)
- The order in which we consider the successors matters: the set of retreating edges depends on it

Reducible Control-Flow Graphs

- For reducible CFGs, the retreating edges discovered during DFS are all and only back edges
  - The order during DFS traversal is irrelevant: all DFS traversals produce the same set of retreating edges
- For irreducible CFGs: a DFS traversal may produce retreating edges that are not back edges
  - Each traversal may produce different retreating edges
  - Example:



    - No back edges
    - One traversal produces the retreating edge 3 → 2
    - The other one produces the retreating edge 2 → 3

Reducibility (1/2)

- A number of equivalent definitions
  - One of them we already saw
- The graph can be reduced to a single node with the application of the following two rules
  - Given a node *n* with a single predecessor *m*, merge *n* into *m*; all successors of *n* become successors of *m*
  - Remove an edge n → n
- Try this on the graphs from slides 18, 17, and 20

Reducibility (2/2)

- The essence of irreducibility: a SCC with multiple possible entry points
  - If the original program was written using **if-then**, **if-then-else**, **while-do**, **do-while**, **break**, and **continue**, the resulting CFG is always reducible
  - If **goto** was used by the programmer, the CFG could be irreducible (but, in practice, it typically is reducible)
- Optimizations of the intermediate code, done by the compiler, could introduce irreducibility
- Code obfuscation: e.g., Java bytecode can be transformed to be irreducible, making it impossible to reverse-engineer a valid Java source program

Part 4: Static Single Assignment (SSA) Form

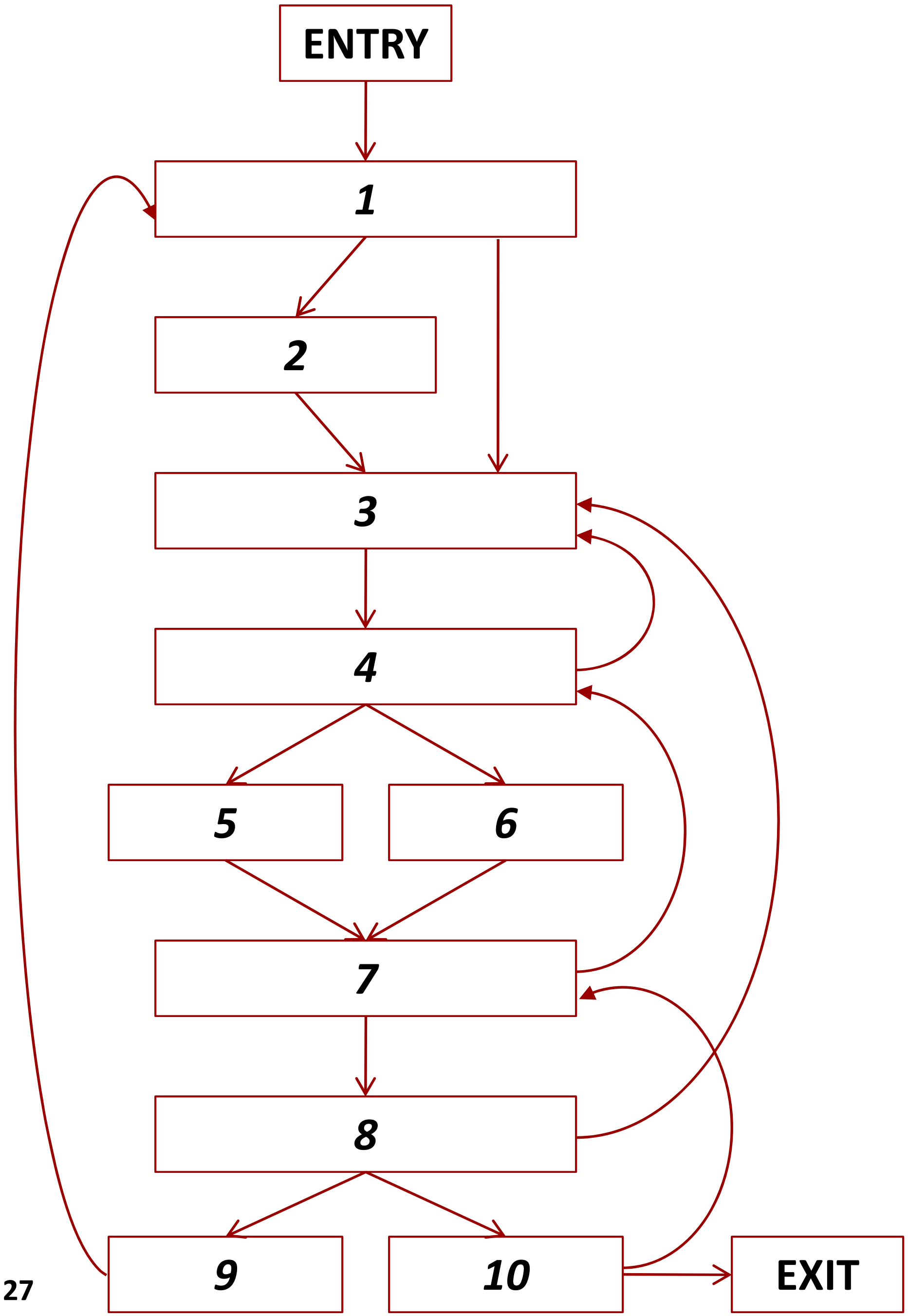
- Source: Cytron et al., ACM TOPLAS, Oct. 1991
  - Section 1 (ignore Section 1.1)
  - Section 2
  - Section 3 (ignore Section 3.1)
  - Section 4 (ignore the detailed proofs in Section 4.3)
- The key issues
  - Insert ϕ-functions at join points (Sections 3 and 4)
  - Rename the variables so that **each use (read)** of a variable is reached by **exactly one definition (write)** of that variable – i.e., by a **single assignment**
    - Section 5.2 discusses this issue, but we will not
- Alternative for dom. frontiers: Cooper et al. 2001

Part 5: Control Dependence: Informally

- A node *n* is control dependent on a node *c* if
  - There exists an edge e_1 coming out of *c* that definitely causes *n* to execute
  - There exists some edge e_2 coming out of *c* that is the start of some path that avoids the execution of *n*
- The decision made at *c* affects whether *n* gets executed: if e_1 is followed, n definitely is executed; if e_2 is followed, there is the possibility that n is not executed at all
  - Thus, *n* is control dependent on *c* – the control-flow leading to *n* depends on what *c* does

Control Dependence: Formally

- (part 1) *n* is control dependent on *c* if
  - $n \neq c$
  - *n* does not post-dominate *c*
  - there exists a path from *c* to *n* such that *n* post-dominates every node on the path except *c*
- (part 2) *n* is control dependent on *n* if
  - there exists a path from *n* to *n* (with at least one edge) such that *n* post-dominates every node on the path
    - this implies that *n* has two outgoing edges
    - this case applies to the header of a loop
- See Cytron et al., 1991, Section 6 for more details
  - *c* belongs to DF(*n*) but computed on the *reverse* CFG



Consider all branch nodes *c*: 1, 4, 7, 8, 10

ENTRY does not post-dominate any other *n*
1 *pdom* ENTRY, 1, 9
2 does not post-dominate any other *n*
3 *pdom* ENTRY, 1, 2, 3, 9
4 *pdom* ENTRY, 1, 2, 3, 4, 9
5 does not post-dominate any other *n*
6 does not post-dominate any other *n*
7 *pdom* ENTRY, 1, 2, 3, 4, 5, 6, 7, 9
8 *pdom* ENTRY, 1, 2, 3, 4, 5, 6, 7, 8, 9
9 does not post-dominate any other *n*
10 *pdom* ENTRY, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
EXIT *pdom* *n* for any *n*

2 is control dependent on 1
3, 4, 5, 6 are control dependent on 4
4, 7 are control dependent on 7
9, 1, 3, 4, 7, 8 are control dependent on 8
7, 8, 10 are control dependent on 10

Finding All Control Dependences

- Consider all CFG edges (*c*,*x*) such that *x* does not post-dominate *c* (therefore, *c* is a branch node)
- Traverse the post-dominator tree bottom-up
  - *n* = *x*
  - while (*n* != parent of *c* in the post-dominator tree)
    - report that *n* is control dependent on *c*
    - *n* = parent of *n* in the post-dominator tree
  - Example: for CFG edge (8,9) from the previous slide, traverse and report 9, 1, 3, 4, 7, 8 (stop before 10)
- Other algorithms exist, but this one is simple and works quite well [Cooper et al., 2001]