Project

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1. Code Description

This code is used to get the corresponding stress history from a given strain history, while the input and output information are fulfilled by reading and writing corresponding files. The initial shear modulus $G_{\max}$, the reference strain γ_r the Poisson's ratio v, and the number of yield surfaces NYS in ask input for the model. The translation of yield surface and accumulation of stress and strain with be stored for further used.

2. Discretization the backbone curve

The discretization is realized by discrete the shear stress in to uniform pieces. And the last yield surface is treated as perfectly plastic. The maximum number of yield surfaces is fifty, as Fortran 77 asked for the dimension of a matrix when first claimed. The discretized backbone curve is shown in figure 1. There are seven yield surfaces in this match. As the slope of backbone curve is decreasing, this discretizing method gives a good capture of the low strain part but missed the relatively bigger strain part, which may be enough for soil. Also we can add more yield surfaces for a better match if necessary.

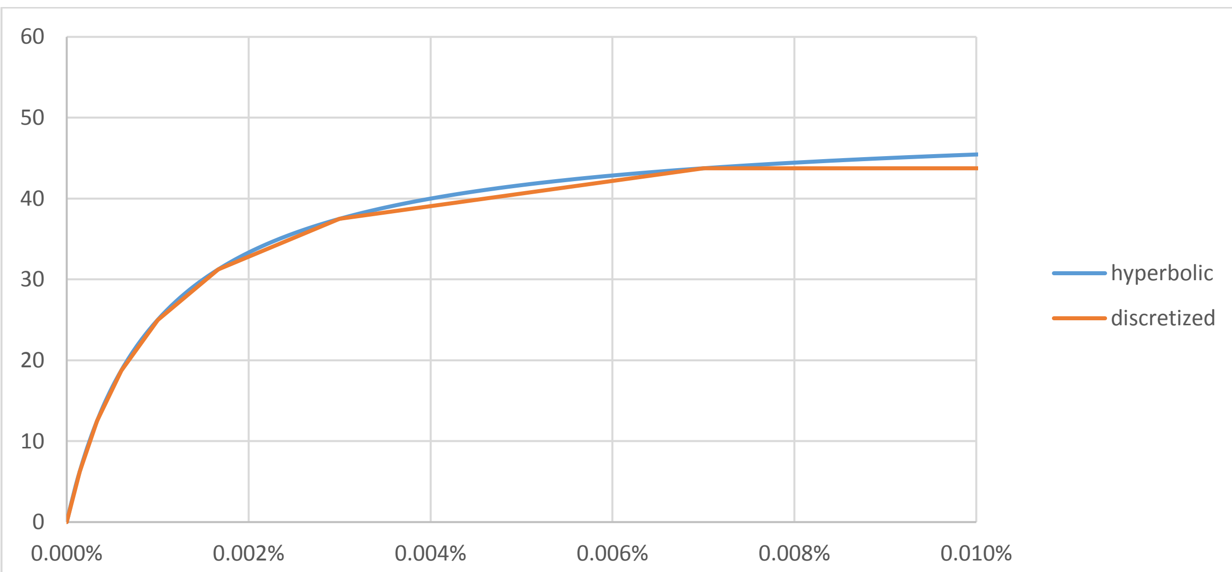

Figure 1. Discretization of backbone curve

3. Algorithm for multi-yield surfaces model

STEP 1

m=0

STEP 2 Get the elastic predictor

Decomposed the strain increment in to hydrostatic and deviatoric part. Hydrostatic part:

$$\Delta\varepsilon_v = tr\left(\Delta\boldsymbol{\varepsilon}\right) \tag{Eq.1}$$

Deviatoric part:

$$\Delta\mathbf{e} = \Delta\boldsymbol{\varepsilon} - \frac{1}{3}\Delta\varepsilon_v\mathbf{I} \tag{Eq.2}$$

The elastic predictor, hydrostatic part:

$$\Delta p_{n+1} = B\cdot\Delta\varepsilon_v \tag{Eq.3}$$

Deviatoric part

$$\mathbf{s}_{n+1}^{tr} = \mathbf{s}_n + H_0\cdot\Delta\mathbf{e} \tag{Eq.4}$$

STEP 3 Check for overshooting of next yield surface, $f^{(m+1)}$

$$K^{tr} = \left\|\mathbf{s}_{n+1}^{tr} - \boldsymbol{\alpha}_n^m\right\| \tag{Eq.5}$$

If $K^{tr}=0$, go to step 6

$$\mathbf{Q}_n^m = \frac{\mathbf{s}_{n+1}^{tr} - \boldsymbol{\alpha}_n^m}{\left\|\mathbf{s}_{n+1}^{tr} - \boldsymbol{\alpha}_n^m\right\|} \tag{Eq.6}$$

If $f^{(m+1)} = K^{tr} - K^{m+1} \le 0$, go to step 6,

If m=NYS, go to step 6,

Otherwise, m=m+1.

STEP 4 Compute stress correction $\Delta\mathbf{s}^{pl}$

Pick out the corresponding $H^{'}$ to the asked yield surface form initial input.

$$d\lambda = \frac{K^{tr} - K^m}{H_n^{'m} + H_0} \tag{Eq.7}$$

If m=1

$$K^{pl} = H_0 \cdot d\lambda \tag{Eq.8}$$

If m>1

$$d\lambda = d\lambda \frac{H_n^{'m\text{-}1} \text{-} H_n^{'m}}{H_n^{'m-1}} \tag{Eq.9}$$

$$K^{pl} = d\lambda H_0 \left(1 - \frac{H^{'m} + H_0}{H^{'m\text{-}1} + H_0} \right) \tag{Eq.10}$$

STEP 5 Updates

$$\mathbf{s}_{n+1}^{tr} = (K^{tr} - K^{pl})\mathbf{Q}^m + \boldsymbol{\alpha}_n^m \tag{Eq.11}$$

Go to step 3, the first result get form this step is the result for M2

STEP 6 Final update

If m>0 compute the translation of $f^{(m)}$,

For m<NYS,

$$\boldsymbol{\mu} = \left(K_n^{m+1} - K_n^m \right) \mathbf{Q}^m + \boldsymbol{\alpha}_n^{m+1} - \boldsymbol{\alpha}_n^m \tag{Eq.12}$$

$$\boldsymbol{\alpha}_{n+1}^m = \boldsymbol{\alpha}_n^m + d\lambda \frac{H_n^{'m}}{\mathbf{Q}_n^m : \boldsymbol{\mu}} \boldsymbol{\mu} \tag{Eq.13}$$

For m=NYS

$$\boldsymbol{\alpha}_{n+1}^m = \boldsymbol{\alpha}_n^m + d\lambda \cdot H_n^{'m} \mathbf{Q}^m \tag{Eq.14}$$

If m>1, translate the inner surfaces $f^{(i)}$ to be tangent to $f^{(m)}$ at the stress point $\mathbf{s}_{n+1}$

$$\boldsymbol{\alpha}_{n+1}^i = \mathbf{s}_{n+1} - \frac{K^i}{K^m} \left(\mathbf{s}_{n+1} - \boldsymbol{\alpha}_{n+1}^m \right) \quad \left(i = 1, 2, 3.......m\text{-}1 \right) \tag{Eq.15}$$

STEP 7

$$\boldsymbol{\sigma}_{n+1} = \mathbf{s}_{n+1}^{tr} + \frac{1}{3} p_{n+1} \mathbf{I} \tag{Eq.16}$$

Output result.

For simplicity, the stresses and strains are handled as a vector rather than a matrix during the programming.

4. Application

The material information is read from a file named SYS.TXT

SYS.TXT

5000000 0.00001 0.33 7

The numbers is written in the order of $G_{\max}$, γ_r, v, NYS.

The strain history is read form a file named STRAIN.TXT

STRAIN.TXT

0 0 0 0 0 0

0 0 0.0001 0 0

…………….

Each line is a record of strain which have six components $\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{12}, \varepsilon_{23}, \varepsilon_{13}$.

The stress history is written to a file named STRESS.TXT, the format is similar to the strain history.

4.1 Match the backbone curve

A purely shear strain history is applied to this model with seven yield surfaces. The output result is plot in figure 2, which shows a perfectly overlapping on the input backbone curve. The stress have the same unit with the initial shear modulus in all the figures in this report.

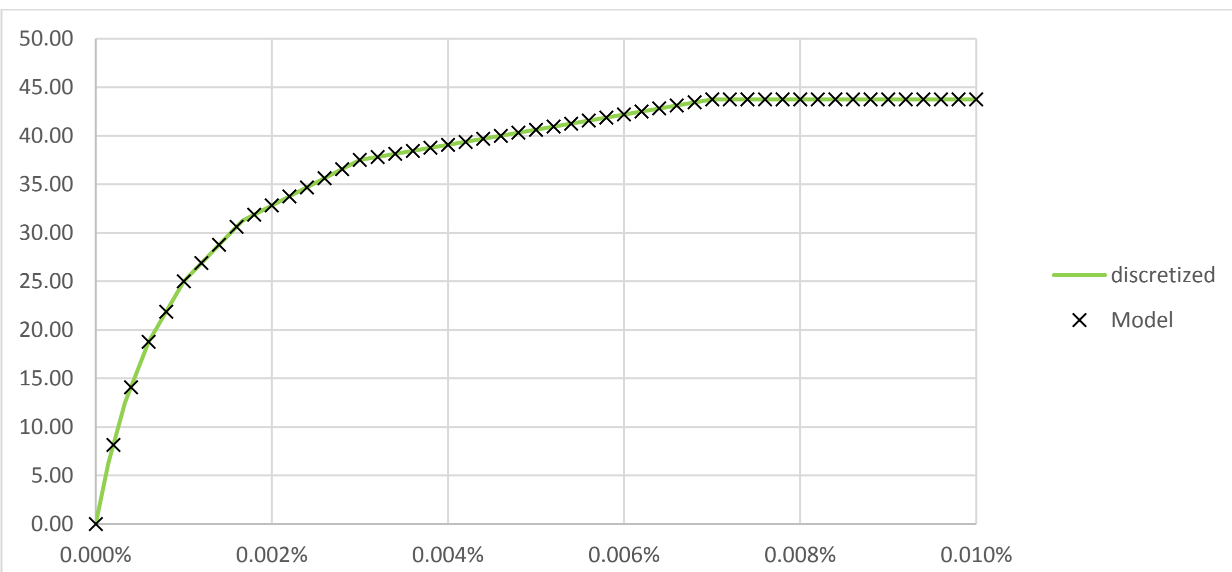

Figure 2. Capture the backbone curve

4.2. Purely hydrostatic monotonic loading.

A purely hydrostatic monotonic strain history is applied to this model, the result in figure 3 shows that the stress keeps increasing in a straight line even for large strain. This is due to the cylindrical yield surface assumption, and the suppositional stress point moves along the hydrostatic axis which never cross any yield surfaces. This gives a result that all the normal component of stress remains equal and increase forever.

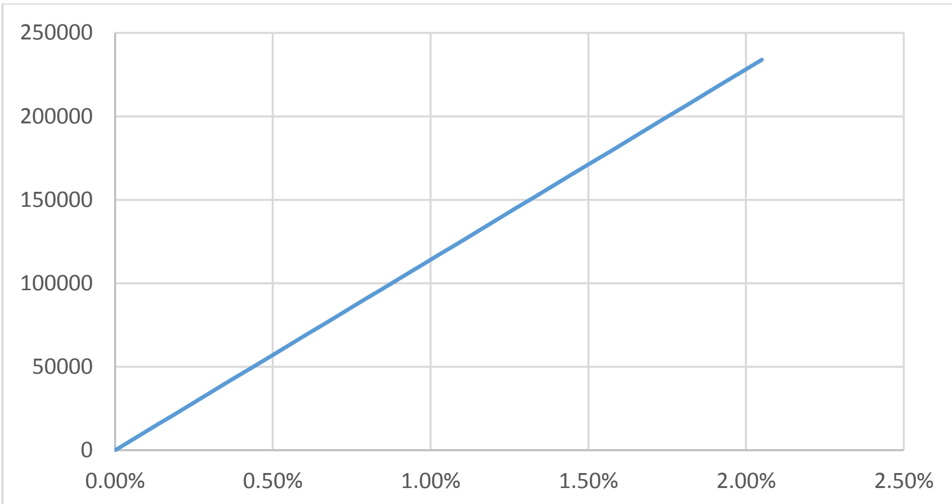

Figure 3. Normal stress vs normal strain

4.3. Sinusoidal variation of a purely shear loading

A purely shear loading is applied to this model, with a frequency of 10 Hz and the magnitude increased from 0.0001% to 0.01% and 34 yield surface. The result is in the figure4.

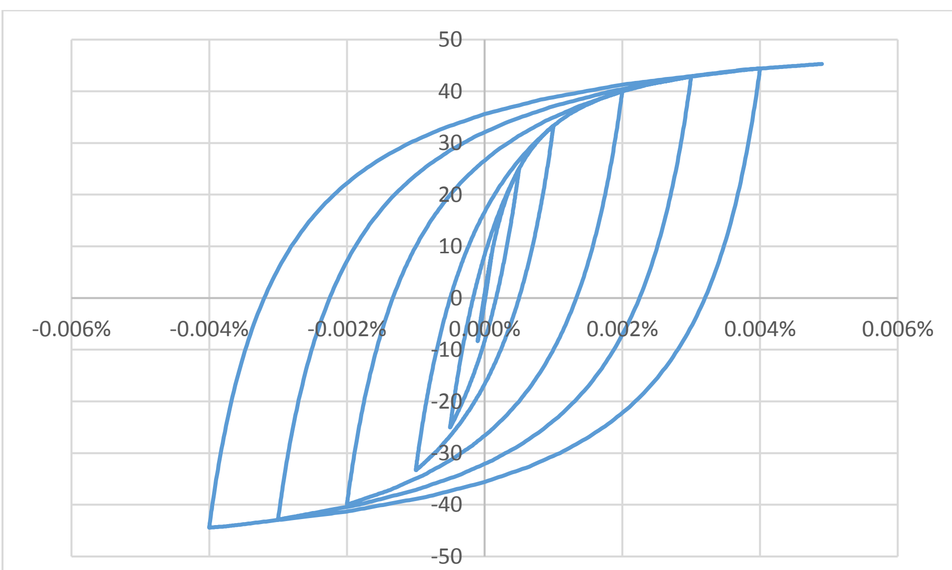

Figure 4. Historic loops of shear stress

The historical loop increased in size and rotated to horizontal axis as the magnitude of strain increased. This is because the shear modulus decrease for large strain follow the information given by backbone curve. In addition, the more yield surfaces used the smoother curve we get and also gives a better reproduction of the input backbone curve.

4.4. Biaxial shear loading

A given biaxial shear strain history is inputted to the model and get the corresponding stress history shown in figure 5 and 6. The changes in e two shear strain component both have contribution for the soil to reach yield condition, then the strain-stress loop behaviors as disordered because this two component are coupled.

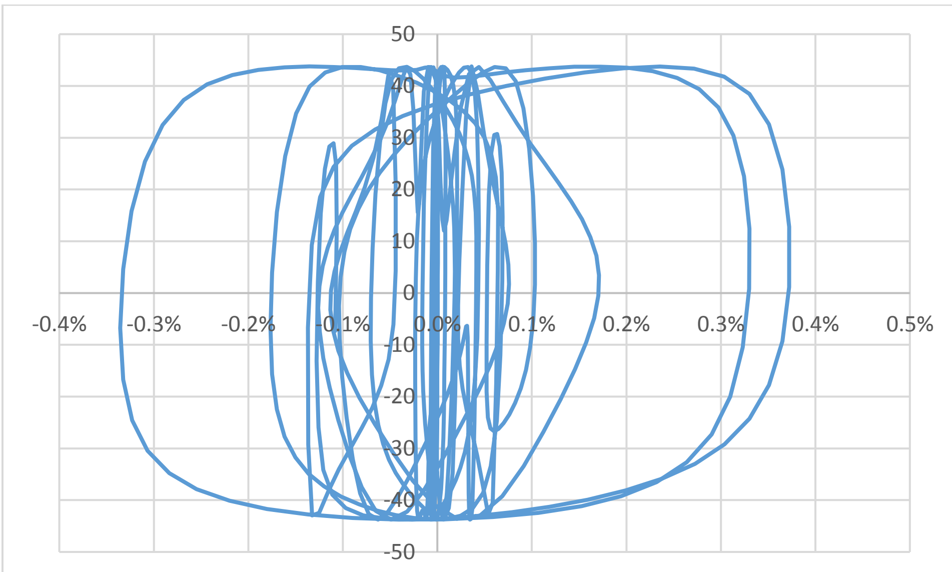

Figure 5. Shear stress history (a)

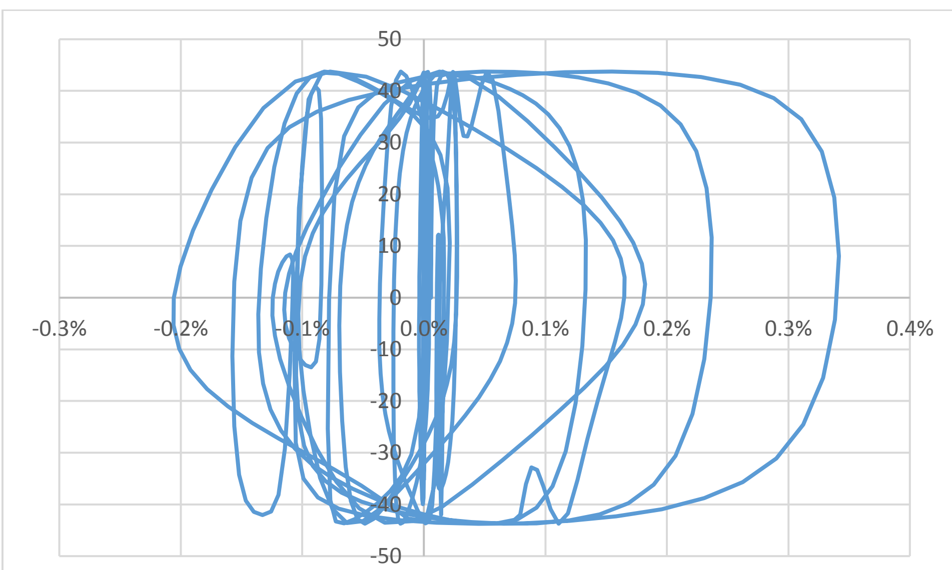

Figure 6. Shear stress history (b)

4.5. Sinusoid tri-axial loading

A sinusoid tri-axial strain history applied to this model via the following process: First increase ε_{11} ε_{22} ε_{33} at same time till to 0.001%; then increase ε_{11} to 0.002% only; at last, change ε_{11} as a function: $\varepsilon_{11}=0.002\%+0.001\%\sin(20\pi t)$ The result is plotted in figure 7. During the hydrostatic loading part, the line go along the q axis, this is similar to section 4.2. Then the line moves up as a result of monotonic loading, at certain point which the sinusoid strain applied, a loop is obtained because of the similar reason of shear loading. If we change the magnitude of sinusoid load, we will end up with loops with different shape, like section 4.3.

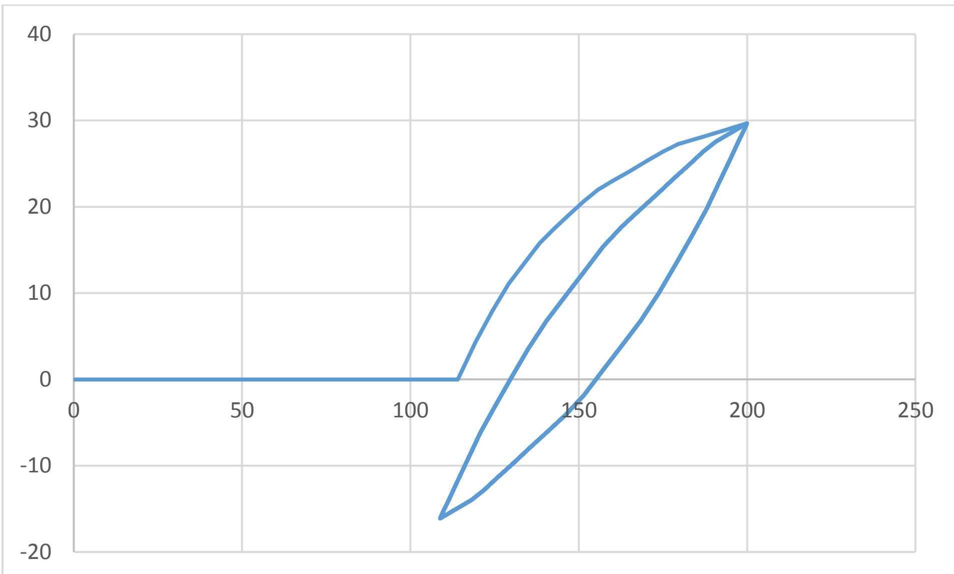

Figure 7. p-q digram

5. References

[1]. Mourad Zeghal. Lecture notes. Constitutive laws in geomechanic (RPI CIVI 6961). Spring 2014.